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Strategic Asset Allocation

The Impact of Foreign Exposure on Portfolio Choice

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Abstract

Plenty of research has been made on strategic asset allocation, but the focus on foreign market exposure is sparse. This paper analyzes how the investor's portfolio choice is affected when exposed to four foreign assets, one domestic asset, and the risk free interest rate. Two models are developed using dynamic programming for an investor with iso-elastic utility and lognormal asset returns. The problem is derived under the assumption of constant investment opportunities, making the analysis more mathematically convenient. In addition, the investor's certainty equivalent of wealth is derived and assessed throughout the investment horizon and as diversification possibilities increases with the models. The optimal portfolio choice problem is solved using empirical data between 1996 and 2016. Attained values of the portfolio weights show quite extreme numbers, as many other mean-variance models do. The result provides no clear conclusion regarding the investor´s portfolio choice of risky assets. However, the investor's certainty equivalent of wealth turned out to be exponentially increasing over time but did not change with diversification possibilities.

Keywords: Strategic asset allocation, Dynamic programming, Foreign exposure, Portfolio choice, Certainty equivalent of wealth

Contents

1. Introduction

In the world of economics, the consensus says that individuals and institutions are interested in how to allocate their money in the most efficient manner. The individual might want to save money for his or her retirement, or maybe as a bequest for possible heirs, while the institutions invest on behalf of its owner's and investor's interests.

An investor is assumed to have some kind of preferences, that is, a measurement of utility on his lifetime investments. This utility could be split into two parts; the terminal wealth that is being held at the time of the investor's death, and the consumption throughout the lifetime. During the investor's lifetime, different decisions can be made on how to invest, known as *strategic asset allocation*[1]. This is the choice of how to invest in a broad variety of asset classes, such as bonds, stocks, real estate, commodities, exchange rates, hedge funds, the risk free interest rate, etc., to meet the investor's goals. However, the allocations will vary due to changes in investment opportunities, investment horizon, and macro-economic risk factors.

In the 1990s and early 2000s theories regarding strategic asset allocation increased as a consequence of the finding that stock returns might be predictable by measuring of parameters such as interest rates and divided-to-price ratio. Another reason was the general technical advancement with increasing computing power, hence making the way of solving complex problems based on large statistical data possible (Diris, 2011).

There are several ways to implement the strategic asset allocation problem. Already in the 1920's, Frank Ramsey (1928), derived a multi-period consumption savings decision, but under the assumption that the individual was restricted to invest in a single asset paying a certain return. Almost 25 years later, Harry Markowitz (1952), published a paper where he had developed a

[1] Strategic asset allocation, portfolio choice, and portfolio optimization are used synonymously.

framework for what is known as *static portfolio optimization,* which later lead to the invention of *CAPM*.[2] A static portfolio does not take into account changing conditions within the investment period, making it less valuable for long-term investors. However, his work is considered a starting point for what is known as Modern Portfolio Theory (MPT).

Later on, Jan Mossin (1968) published an article, *"Optimal Multiperiod Portfolio Policies"*, that took Ramsey's work from 1928 one step further by solving for the individual's optimal multi-period portfolio decisions under *uncertainty*. Yet, his work was based upon the assumption that there were no interim consumption decisions, only a utility of terminal wealth. But one year later, Paul Samuelson (1969) and Robert Merton (1969) derived a more complete solution regarding the same problem. This time, there was both a consumption–savings choice as well as an asset allocation decision involving risky securities included in each period. To reach this outcome, their models involved the technique of *stochastic dynamic programming*.

Moreover, Merton (1971) published another pioneering article regarding the optimal consumption and portfolio choice problem in a continuous-time model. It included treatment of the Itô process and Geometric Brownian Motions[3], just like his previous article from 1969. These theories and methods in specific will act as the foundation for the modelling approaches of this study.

In 1997 an additional article on strategic asset allocation was published by Brennan, Schwartz & Lagnado (1997). They implemented the theories of Merton, using numerical dynamic programming for an investor with assumed power utility over wealth. In addition, they investigated the certainty equivalent level of wealth on the investor's portfolio choice. Their research will also be of guidance throughout this study. However, this study will not have any short-sales constraints in contrary to their work. The reason is because short selling the type

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[2] Capital Asset Pricing Model.
[3] Also knows as Wiener process

of assets within this study is in reality possible from the investor perspective and hence more realistic.

Already during the literature phase of this study, it became clear that little research regarding portfolio optimization within foreign markets existed. As a consequence, this study is based upon earlier research of similar problems but will try to deviate from the regular outline in two ways. First, the study is performed from the perspective of a Swedish investor point of view. This kind of research applied on a Swedish perspective is to this day not known to the author. Since every country has its own individual economic features, the study might mainly address to Swedish investors and institutions, but can hopefully appeal to additional actives within financial economics and portfolio choice theory. Secondly, based on the question formulation, the impact of *direct foreign risk exposure* on strategic asset allocation is investigated. The investment opportunity set contains five different assets. Namely the Swedish, American, and British stock markets, as well as the USD/SEK and the GBP/SEK exchange rates. In addition, the investor's certainty equivalent of wealth will be evaluated.

Given the above, the thesis aims to answer the following question formulation:

How does a foreign risk exposure opportunity set of assets, from a Swedish perspective, impact an investor's strategic asset allocation and certainty equivalent of wealth?

The applied framework for the thesis is in line with what was used by Merton (1971). It does not take into account features such as human capital nor labor income for the representative agent. It does not seek to extend the optimal portfolio choice problem into a larger part, but rather make use of those structures already implied by earlier research. Hence, the used model is guided by parsimony with a limited set of assets within the opportunity set rather than an extensive question formulation.

Further. To make the optimization problem more mathematically convenient, the problem is solved through the special case of log normally distributed asset prices, which will be clarified in chapter 2. Changing investment opportunities, on the other hand, would most probably provide a different and more realistic outcome.

Readers should also note that there was no particular reason to why the American and the British markets were chosen to represent the foreign portfolios. It is only the author's belief that the data is considerably reliable and that the countries are easy and relevant to invest in from a Swedish perspective.

The purpose of this thesis is to increase the understanding of how investors behave when exposed to foreign investment opportunities. Hopefully, the study could also act supportive for both private and institutional investors and contribute to further research within portfolio choice theory.

To summarize, the objective of this thesis is to investigate how the strategic asset allocation of an investor could take place when exposed to foreign currencies and investment possibilities. This is done by a stochastic programming approach given a range of commonly used properties and restrictions from earlier research. The optimal portfolio problem is solved under the assumption of constant investment opportunities. By intuition, the numerical results will be similar to those that would have been attained from an optimization problem using CAPM. To further extend the level of advancement, the certainty equivalent of wealth is taken into consideration, in analogy with the research by Brennan, Schwartz & Lagnado (1997). Even though portfolios of constant investment opportunities might be less realistic than those of changing conditions, the author believes that the results could be interesting as a foreign diversification possibility analysis.

The second chapter of this thesis will explain the theoretical framework of the dynamic programming approach on a more detailed level. Chapter 3 describes the methodology and the modelling approaches are specified. In addition, information about the gathering of data and the software used is given. Chapter 4 provides the results when the models are applied on empirical data. It is presented both numerically and graphically and followed by a discussion and analysis of the outcome. Lastly, chapter 5 offers a conclusive and summary of the entire study. Weaknesses and suggestions to continued research are discussed.

2. Theoretical Framework

This chapter aims to explain the fundamental theories that the thesis is based upon. A suitable elaboration around the theoretical framework is in some way problematic to provide. To derive the concepts of continuous time would be very extensive and time-inefficient, and that is not the intention of this study. On the other hand, some kind of introduction is necessary for readers that are not familiar with these types of theories and models from before.

2.1 Expected Utility and Risk Aversion

The price of an asset is determined partly by the investor's preferences for taking on risk inherent in the asset, and partly by the distribution of the asset's risky future payments. For a theory of asset valuation to be satisfied, it has to consider how investors allocate their money among assets with different future payments. The standard approach for modeling investor choices over risky assets is explored looking at the development of expected utility theory.

The main characteristics of utility functions are best described by looking at their derivatives. The first order derivative describes the investor's change in satisfaction with change in wealth, and the second order derivative describes how the investor looks upon risk tolerance with changes in wealth. It is nevertheless essential which type of utility function that is chosen for a study, as taking a certain utility function for granted could jeopardize the validity of the conclusions from an entire study (Campbell & Viceira, 2002).

As mentioned in the introduction section, this study uses earlier research as a foundation for the models applied. One of these studies is the *"Strategic asset allocation"* by Brennan, Schwartz & Lagnado (1997) where a power utility function is used in their research of changing investment opportunities.

$$U(W) = \frac{W^{1-\gamma} - 1}{1 - \gamma}$$

It satisfies the criteria for risk aversion if the symbol for relative risk aversion, $\gamma > 0$, which should be treated realistic as people are observed to be risk averse in general. Moreover, in contrary to exponential utility functions, it relies on the distributional assumption of log normal returns. Applying power utility on a multi-period investment horizon is therefore preferred as the product of two lognormal returns is still lognormal (Campbell & Viceira, 2002). In this thesis, a power utility function is used as well, but specified to the special case of an iso-elastic utility function,

$$J(W,t) = \frac{W^{1-\gamma}}{1-\gamma} f(t)$$

This is the same type of utility function as Lundtofte (n.d) uses in the derivation of dynamic programming with *constant* investment opportunities.

2.1.1 The Special Case of Log Utility

In the modelling approaches of this study, the asset prices are considered to be lognormally distributed. The special case concludes that all of the expected rate of returns, μ_i, (including the risk-free interest rate), and its variances σ_i, are constants. This means that the investment opportunities are constant over time, hence making portfolio choice decisions independent of the state variables over the considered time period. As a consequence, the investor's utility function, J, only depends on the investor's wealth, W, and time, t. This is a rather common method to simplify the analysis of continuous time portfolio choice (Pennacchi, 2008).

2.2 The Continuous Time Framework

Modeling variables after a continuous random process, rather than discrete, allows for different behavioral assumptions as well as sharper model results. By intuition, the time interval $[0, T]$ can be divided into N time periods. Continuous time deviates from discrete time by allowing for N to go to infinity ($N \to \infty$), creating an infinite number of "decision points". This could be applied on the movement of the price of an asset. Thus, the continuous time framework can be seen as a better approximation of reality in finance than the discrete framework (Lundtofte, n.d).

In order to model the process of portfolio optimization in continuous time properly, the theory known as dynamic programming is used. This process includes properties known as Brownian motions and Itô's lemma.[4]

2.2.1 Bellman's Principle of Optimality

Now, one might ask - How is it possible to trace back the optimal portfolio choice in a multi-period optimization problem? The problem can be solved using backward dynamic programming and is better described out of a discrete time setting. Pennacchi (2008) explains it rather intuitive. Consider that the individual optimizes his wealth at the final time $T-1$. Then, the multi-period problem has become a single-period one. It is possible to characterize the problem for this single period and once that is done, it can be solved for the decisions made at time $T-2$. This procedure can continue until the current date so that the utility is maximized at time $t=0$. The theory is called Bellman's Principle of Optimality, and was put into words by Bellman (1957).

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman, 1957, Chap. III. 3.)

2.3 The Certainty Equivalent of Wealth

During the modeling approaches of this study, the certainty equivalent of wealth is investigated. As the name hints, it can be described as the guaranteed return that an agent would accept rather than taking a chance on a higher, but uncertain, return (Investopedia, n.d.). Or as Brennan, Schwartz & Lagnado

[4] To keep this thesis rather intuitive, the more mathematically rigorous analysis of Brownian motions and the Itô process are referred to the literature. See for example Pennacchi (2008).

(1997) describes it – "that amount of wealth such that the investor is indifferent between receiving it for sure at the horizon, and having his current wealth today *and* the opportunity to invest it optimally up to the horizon" (Brennan, Schwartz, & Lagnado, 1997, p22). Thus, the variance of the certainty equivalent is a better metric to measure relative risk of investment strategies. In this particular study the certainty equivalent of wealth is compared both between models of different diversification possibilities as well as between investors of different level of risk aversion.

3. Method

3.1 Modeling Approaches

With the fundamentals of the dynamic programming approach from chapter 2, the model of this thesis can be defined. As it turns out, there will be two tests performed out of two models. The first model, Model 1, describes an investor that can choose between investing in three risky assets; a domestic stock portfolio, S, a foreign stock portfolio, S_F and the foreign currency, Q. In addition, the investor can invest in cash with a guaranteed rate of return, r. As the study aims to investigate foreign investment exposure, an extension to the first model is created, Model 2. In the second model, an additional foreign market is introduced to further examine the portfolio choice problem as diversification possibilities increases. For exact derivation of the models, see Pennacchi (2008) or Lundtofte (n.d).

3.2 Model 1

The joint stochastic process of the opportunity set for Model 1 is assumed to be of the following form:

$$\frac{dS}{S} = \mu_S dt + \sigma_S dz_S$$

$$\frac{dQ}{Q} = \mu_Q dt + \sigma_Q dz_Q$$

$$\frac{dS_F}{S_F} = \mu_{S_F} dt + \sigma_{S_F} dz_{S_F}$$

**Here, μ_i represents the mean return and σ_i is the variance. dt is the drift term and dz_i represents the increments to the Wiener processes.*

In the opportunity set, dS/S represents the rate of return for the domestic stock portfolio OMXS30, and dQ/Q is the evolution of the exchange rate USD/SEK. dS_F/S_F is the rate of return of the foreign stock portfolio, the S&P500. However, in order to get the stochastic process for the foreign stock portfolio in terms of domestic value, the price of the foreign stock portfolio has to be multiplied with the exchange rate for every time period. This is done by applying Itô's product rule. The new process is called dL/L for convenience.

$$\frac{dL}{L} = \frac{d(QS_F)}{QS_F} = \mu_{S_F}dt + \sigma_{S_F}dz_{S_F} + \mu_Q dt + \sigma_Q dz_Q + \sigma_{S_F}\sigma_Q\rho_{S_F Q}dt$$
$$= (\mu_{S_F} + \mu_Q + \sigma_{S_F}\sigma_Q\rho_{S_F Q})dt + \sigma_{S_F}dz_{S_F} + \sigma_Q dz_Q$$

$^*\rho_{ij}$ *is interpreted as the correlation coefficient between the increments to the two Brownian motions z_i and z_j.*

The proportions invested in the different assets are defined as α for the domestic stock portfolio, β for the currency, and δ for the foreign stock portfolio. Thus, the stochastic process for wealth, W, is:

$$\frac{dW}{W} = \alpha\frac{dS}{S} + \beta\frac{dQ}{Q} + \delta\frac{dL}{L} + (1-\alpha-\beta-\delta)rdt$$

$$= \left[\alpha(\mu_S - r) + \beta\left(\mu_Q - r\right) + \delta(\mu_L - r) + r\right]dt + \alpha\sigma_S dz_S + \beta\sigma_Q dz_Q + \delta\sigma_L dz_L$$

r is the risk free rate of return, α is the portfolio weight in the OMXS30, β is the portfolio weight in the USD/SEK, and δ is the portfolio weight in the S&P500.

In general, the HJB-equation without intermediate consumption is given by

$$0 = \max_{\alpha,\beta,\delta}\{L[J]\}$$

Here, $L[J]$ is the Dynkin operator, which is attained from the drift term of the value function, $J(W_t, t)$, after applying Itô's lemma. The derivation becomes

$$dJ(W_t,t) = \frac{\partial J}{\partial t}dt + \frac{\partial J}{\partial W}dW_t + \frac{1}{2}\frac{\partial^2 J}{\partial W^2}(dW_t)^2$$

$$= \left(\frac{\partial J}{\partial t} + \frac{\partial J}{\partial W} W\big(\alpha(\mu_S - r) + \beta(\mu_Q - r) + \delta(\mu_L - r) + r\big) + \frac{1}{2}\frac{\partial^2 J}{\partial W^2} W^2\big(\alpha^2\sigma_S^2 + \beta^2\sigma_Q^2 + \delta^2\sigma_L^2 + 2\alpha\beta\rho_{SQ} + 2\alpha\delta\rho_{SL} + 2\beta\delta\rho_{QL}\big)\right) dt + (\dots)dz_S + (\dots)dz_Q + (\dots)dz_L$$

The dz_i:s are not interesting as only the drift term, dt, is relevant for the HJB-equation. Hence, the HJB-equation is given by

$$0 = \max_{\alpha,\beta,\delta} \left\{ \frac{\partial J}{\partial t} + \frac{\partial J}{\partial W} W\big(\alpha(\mu_S - r) + \beta(\mu_Q - r) + \delta(\mu_L - r) + r\big) + \frac{1}{2}\frac{\partial^2 J}{\partial W^2} W^2\big(\alpha^2\sigma_S^2 + \beta^2\sigma_Q^2 + \delta^2\sigma_L^2 + 2\alpha\beta\rho_{SQ} + 2\alpha\delta\rho_{SL} + 2\beta\delta\rho_{QL}\big)\right\}$$

Note that the correlation matrix $\mathbf{\Phi}$ has to be positive definite.[5]

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \rho_{SQ} & \rho_{SL} \\ \rho_{SQ} & 1 & \rho_{QL} \\ \rho_{SL} & \rho_{QL} & 1 \end{bmatrix}$$

The same method that (Pennacchi, 2008, p.334) describes is used to solve for optimal portfolio weights for constant investment opportunities in matrix formation. Denote $\mathbf{\Omega} \equiv [\sigma_{ij}]$ to be the $n \times n$ covariance matrix whose i, j^{th} element is σ_{ij}, and denote the i, j^{th} element of the inverse of $\mathbf{\Omega}$ to be v_{ij}, that is, $\mathbf{\Omega^{-1}} \equiv [v_{ij}]$. The optimal portfolio weights, symbolized by $\boldsymbol{\omega}$, will look like the following when written in matrix formation.

$$\boldsymbol{\omega} = \frac{\frac{\partial J}{\partial W}}{-W\frac{\partial^2 J}{\partial W^2}} \mathbf{\Omega^{-1}}(\boldsymbol{\mu} - r\mathbf{1})$$

[5] See Apendix for proof.

Where $\boldsymbol{\mu}$ is the vector of mean returns belonging to the different investment opportunities and **1** is a vector of ones. For readers familiar with dynamic programming, notice that although the investor has a long-term horizon, the absence of state variables imply that he only has a myopic demand and no hedging demand. Therefore, he allocates a constant fraction of his wealth into the risky assets.

To examine the optimal portfolio weights separately, the first order condition w.r.t. α, β and δ can be computed.

$$\alpha^* = \frac{\frac{\partial J}{\partial W}}{-W\frac{\partial^2 J}{\partial W^2}}\left(v_{\alpha S}(\mu_S - r) + v_{\alpha Q}\left(\mu_Q - r\right) + v_{\alpha L}(\mu_L - r)\right)$$

$$\beta^* = \frac{\frac{\partial J}{\partial W}}{-W\frac{\partial^2 J}{\partial W^2}}\left(v_{\beta S}(\mu_S - r) + v_{\beta Q}\left(\mu_Q - r\right) + v_{\beta L}(\mu_L - r)\right)$$

$$\delta^* = \frac{\frac{\partial J}{\partial W}}{-W\frac{\partial^2 J}{\partial W^2}}\left(v_{\delta S}(\mu_S - r) + v_{\delta Q}\left(\mu_Q - r\right) + v_{\delta L}(\mu_L - r)\right)$$

An iso-elastic value function is assumed of the form $J(W,t) = \frac{W^{1-\gamma}}{1-\gamma}f(t)$, where $f(t)$ is a function of time and γ is the coefficient for risk aversion. Thus,

$$\frac{\partial J}{\partial W} = W^{-\gamma}f(t)$$

$$\frac{\partial^2 J}{\partial W^2} = -\gamma W^{-\gamma-1}f(t)$$

Inserting these into the expression for the optimal portfolio weights matrix formation yields

$$\boldsymbol{\omega} = \frac{1}{\gamma}\boldsymbol{\Omega}^{-1}(\boldsymbol{\mu} - r\mathbf{1})$$

For the separate portfolio weights the expression will look like

$$\alpha^* = \frac{1}{\gamma}\Big(v_{\alpha S}(\mu_S - r) + v_{\alpha Q}\big(\mu_Q - r\big) + v_{\alpha L}(\mu_L - r)\Big)$$

$$\beta^* = \frac{1}{\gamma}\Big(v_{\beta S}(\mu_S - r) + v_{\beta Q}\big(\mu_Q - r\big) + v_{\beta L}(\mu_L - r)\Big)$$

$$\delta^* = \frac{1}{\gamma}\Big(v_{\delta S}(\mu_S - r) + v_{\delta Q}\big(\mu_Q - r\big) + v_{\delta L}(\mu_L - r)\Big)$$

It is possible to solve for the value function being correctly predicted. A function $g(t)$ is defined such that $f(t) = e^{-\rho t}g(t)^{-\gamma}$. In this case, the value function is given by

$$J(W,t) = e^{-\rho t}g(t)^{-\gamma}\frac{W^{1-\gamma}}{1-\gamma}$$

Where ρ now represents the impatience parameter of the economy and t is the time passed from initial time.

The derivatives can be computed once again

$$\frac{\partial J}{\partial t} = -\rho J - \gamma e^{-\rho t}g(t)^{-\gamma-1}g'(t)\frac{W^{1-\gamma}}{1-\gamma} = \left(-\rho - \gamma\frac{g'(t)}{g(t)}\right)J$$

$$\frac{\partial J}{\partial W} = e^{-\rho t}g(t)^{-\gamma}W^{-\gamma} = (1-\gamma)\frac{J}{W}$$

$$\frac{\partial^2 J}{\partial W^2} = -\gamma e^{-\rho t}g(t)^{-\gamma}W^{-\gamma-1} = -\gamma(1-\gamma)\frac{J}{W^2}$$

Inserting these derivatives together with the optimal portfolio weights into the HJB equation gives the following

$$0 = \left(-\rho - \gamma \frac{g'(t)}{g(t)}\right) J + (1-\gamma)\frac{J}{W} W \boldsymbol{\omega}'(\boldsymbol{\mu} - r\mathbf{1}) + \frac{1}{2}\left(-\gamma(1-\gamma)\frac{J}{W^2}\right) W^2 \boldsymbol{\omega}'\boldsymbol{\Omega}\boldsymbol{\omega}$$

Simplifying and cancelling out the J:s

$$0 = -\rho - \gamma \frac{g'(t)}{g(t)} + (1-\gamma)\boldsymbol{\omega}'(\boldsymbol{\mu} - r\mathbf{1}) - \frac{1}{2}\gamma(1-\gamma)\boldsymbol{\omega}'\boldsymbol{\Omega}\boldsymbol{\omega}$$

Simplifying once again

$$0 = -\gamma \frac{g'(t)}{g(t)} + \left[(1-\gamma)r + (1-\gamma)\boldsymbol{\omega}'(\boldsymbol{\mu} - r\mathbf{1}) - \frac{1}{2}\gamma(1-\gamma)\boldsymbol{\omega}'\boldsymbol{\Omega}\boldsymbol{\omega} - \rho\right]$$

Defining $h(t) = \frac{1}{g(t)}$ and inserting into the above equation, the following is obtained

$$0 = \gamma \frac{h'(t)}{h(t)} + \left[(1-\gamma)r + (1-\gamma)\boldsymbol{\omega}'(\boldsymbol{\mu} - r\mathbf{1}) - \frac{1}{2}\gamma(1-\gamma)\boldsymbol{\omega}'\boldsymbol{\Omega}\boldsymbol{\omega} - \rho\right]$$

Multiplying both sides by $h(t)$ and dividing by γ

$$0 = h'(t) + k_0 h(t)$$

Where k_0 is a scalar

$$k_0 = \frac{1}{\gamma}\left[(1-\gamma)r + (1-\gamma)\boldsymbol{\omega}'(\boldsymbol{\mu} - r\mathbf{1}) - \frac{1}{2}\gamma(1-\gamma)\boldsymbol{\omega}'\boldsymbol{\Omega}\boldsymbol{\omega} - \rho\right]$$

$$k_0 = \frac{1}{\gamma}\left[(1-\gamma)r + (1-\gamma)\left(\frac{1}{\gamma}\boldsymbol{\Omega}^{-1}(\boldsymbol{\mu} - r\mathbf{1})\right)'(\boldsymbol{\mu} - r\mathbf{1}) - \frac{1}{2}\gamma(1-\gamma)\left(\frac{1}{\gamma}\boldsymbol{\Omega}^{-1}(\boldsymbol{\mu} - r\mathbf{1})\right)'\boldsymbol{\Omega}\left(\frac{1}{\gamma}\boldsymbol{\Omega}^{-1}(\boldsymbol{\mu} - r\mathbf{1})\right) - \rho\right]$$

$0 = h'(t) + k_0 h(t)$ is an inhomogeneous differential equation, which has the solution

$$h(t) = \left(A - \frac{1}{k_0}e^{k_0 t}\right)e^{-k_0 t}$$

where A is a constant.

Given the boundary condition $J(W,T) = e^{-\rho T}\frac{W^{1-\gamma}}{1-\gamma}$, $g(T) = h(T) = 1$ and therefore, after solving for A

$$h(t) = \left(1 + \frac{1}{k_0}\right)e^{k_0(T-t)} - \frac{1}{k_0}$$

The function g is obtained as the inverse of h

$$g(t) = \frac{1}{h(t)} = \frac{1}{\left(1 + \frac{1}{k_0}\right)e^{k_0(T-t)} - \frac{1}{k_0}} = \frac{k_0}{(1 + k_0)e^{k_0(T-t)} - 1}$$

Hence, the value function is given by

$$J(W,t) = e^{-\rho t}\left(\frac{(1 + k_0)e^{k_0(T-t)} - 1}{k_0}\right)^{\gamma}\frac{W^{1-\gamma}}{1-\gamma}$$

3.2.1 Certainty Equivalent of Wealth, Model 1

Although the above calculations might be interesting to evaluate from a financially theoretical perspective when applied on empirical data, the results will not differ very much from what could have been attained with the regular CAPM. As mentioned in chapter 1, the certainty equivalent of wealth is taken into consideration to further extend the investigation.

Given the assumption of an iso-elastic utility function, the value function can be defined as

$$e^{-\rho t}\frac{\overline{W}^{1-\gamma}}{1-\gamma} = J(W,t)$$

To get the certainty equivalent, CE, solve for $\overline{W}$

$$\overline{W} = \left(\frac{(1-\gamma)}{e^{-\rho t}} e^{-\rho t} \left(\frac{(1+k_0)e^{k_0(T-t)}-1}{k_0}\right)^{\gamma} \frac{W^{1-\gamma}}{1-\gamma}\right)^{\frac{1}{1-\gamma}} = CE$$

When simplified it becomes

$$\overline{W} = W\left(\frac{(1+k_0)e^{k_0(T-t)}-1}{k_0}\right)^{\frac{\gamma}{1-\gamma}} = CE$$

3.3 Model 2

Model 2 contains two additional assets in the opportunity set. This enables further analysis on how the investor behaves as the diversification possibilities increases. The new model extends Model 1 to include the possibility to invest in the British pound represented by Q_{UK}, and the British stock portfolio, S_{UK}.

The new joint stochastic process looks like the following.

$$\frac{dS}{S} = \mu_S dt + \sigma_S dz_S$$

$$\frac{dQ}{Q} = \mu_Q dt + \sigma_Q dz_Q$$

$$\frac{dS_F}{S_F} = \mu_{S_F} dt + \sigma_{S_F} dz_{S_F}$$

$$\frac{dQ_{UK}}{Q_{UK}} = \mu_{Q_{UK}} dt + \sigma_{Q_{UK}} dz_{Q_{UK}}$$

$$\frac{dS_{UK}}{S_{UK}} = \mu_{S_{UK}} dt + \sigma_{S_{UK}} dz_{S_{UK}}$$

Here, dS/S is the rate of return for the domestic stock portfolio OMXS30. dQ/Q is the evolution of the USD/SEK exchange rate. dS_F/S_F is the American stock portfolio S&P500. dQ_{UK}/Q_{UK} is the GBP/SEK exchange rate. Finally, dS_{UK}/S_{UK} is the British stock portfolio represented by the FTSE100.

Just like in the previous model, the foreign assets have to be normalized with the exchange rate to make it comparable to the domestic asset. Hence, the price of the foreign stock portfolio is multiplied with the exchange rate for every time period. It is defined dL/L, for the American stock portfolio, and dK/K for the British stock portfolio for convenience. Applying Itô's lemma and using Itô's product rule gives the following.

$$\frac{dL}{L} = \frac{d(QS_F)}{QS_F} = \mu_{S_F} dt + \sigma_{S_F} dz_{S_F} + \mu_Q dt + \sigma_Q dz_Q + \sigma_{S_F}\sigma_Q\rho_{S_F Q} dt$$

$$= (\mu_{S_F} + \mu_Q + \sigma_{S_F}\sigma_Q\rho_{S_F Q})dt + \sigma_{S_F} dz_{S_F} + \sigma_Q dz_Q$$

$$\frac{dK}{K} = \frac{d(Q_{UK}S_{UK})}{Q_{UK}S_{UK}}$$
$$= \mu_{S_{UK}}dt + \sigma_{S_{UK}}dz_{S_{UK}} + \mu_{Q_{UK}}dt + \sigma_{Q_{UK}}dz_{Q_{UK}}$$
$$+ \sigma_{S_{UK}}\sigma_{Q_{UK}}\rho_{S_{UK}Q_{UK}}dt$$
$$= (\mu_{S_{UK}} + \mu_{Q_{UK}} + \sigma_{S_{UK}}\sigma_{Q_{UK}}\rho_{S_{UK}Q_{UK}})dt + \sigma_{S_{UK}}dz_{S_{UK}} + \sigma_{Q_{UK}}dz_{Q_{UK}}$$

The proportions invested in the different assets are defined as α for the domestic stock portfolio, β for the USD/SEK currency, δ for the American stock portfolio, η for the GBP/SEK currency, and θ for the British stock portfolio. Thus, the stochastic process for wealth, W, becomes:

$$\frac{dW}{W} = \alpha\frac{dS}{S} + \beta\frac{dQ}{Q} + \delta\frac{dL}{L} + \eta\frac{dQ_{UK}}{Q_{UK}} + \theta\frac{dK}{K} + (1 - \alpha - \beta - \delta - \eta - \theta)rdt$$

$$= \left[\alpha(\mu_S - r) + \beta\left(\mu_Q - r\right) + \delta(\mu_L - r) + \eta\left(\mu_{Q_{UK}} - r\right) + \theta(\mu_K - r) + r\right]dt$$
$$+ \alpha\sigma_S dz_S + \beta\sigma_Q dz_Q + \delta\sigma_L dz_L + \eta\sigma_{Q_{UK}}dz_{Q_{UK}} + \theta\sigma_K dz_K$$

Assuming everything is equal to Model 1 except for the additional portfolio weights, the same structure can be followed and the new HJB-equation without intermediate consumption, given by $0 = \max_{\alpha,\beta,\delta,\eta,\theta}\{L[J]\}$, is maximized. The optimal portfolio weights are given by

$$\boldsymbol{\omega} = \frac{1}{\gamma}\boldsymbol{\Omega}^{-\mathbf{1}}(\boldsymbol{\mu} - r\mathbf{1}) \qquad (*)$$

Here, $\boldsymbol{\omega}$ are the optimal portfolio weights written in matrix formation, $\boldsymbol{\mu}$ is a vector of excess returns from the different assets, and $\mathbf{1}$ is a vector of ones.

For the separate portfolio weights the expression will look like

$$\alpha^* = \frac{1}{\gamma}\Big(\upsilon_{\alpha S}(\mu_S - r) + \upsilon_{\alpha Q}\left(\mu_Q - r\right) + \upsilon_{\alpha L}(\mu_L - r) + \upsilon_{\alpha Q_{UK}}\left(\mu_{Q_{UK}} - r\right)$$
$$+ \upsilon_{\alpha K}(\mu_K - r)\Big)$$

$$\beta^* = \frac{1}{\gamma}\Big(\upsilon_{\beta S}(\mu_S - r) + \upsilon_{\beta Q}\big(\mu_Q - r\big) + \upsilon_{\beta L}(\mu_L - r) + \upsilon_{\beta Q_{UK}}\big(\mu_{Q_{UK}} - r\big) + \upsilon_{\beta K}(\mu_K - r)\Big)$$

$$\delta^* = \frac{1}{\gamma}\Big(\upsilon_{\delta S}(\mu_S - r) + \upsilon_{\delta Q}\big(\mu_Q - r\big) + \upsilon_{\delta L}(\mu_L - r) + \upsilon_{\delta Q_{UK}}\big(\mu_{Q_{UK}} - r\big) + \upsilon_{\delta K}(\mu_K - r)\Big)$$

$$\eta^* = \frac{1}{\gamma}\Big(\upsilon_{\eta S}(\mu_S - r) + \upsilon_{\eta Q}\big(\mu_Q - r\big) + \upsilon_{\eta L}(\mu_L - r) + \upsilon_{\eta Q_{UK}}\big(\mu_{Q_{UK}} - r\big) + \upsilon_{\eta K}(\mu_K - r)\Big)$$

$$\theta^* = \frac{1}{\gamma}\Big(\upsilon_{\theta S}(\mu_S - r) + \upsilon_{\theta Q}\big(\mu_Q - r\big) + \upsilon_{\theta L}(\mu_L - r) + \upsilon_{\theta Q_{UK}}\big(\mu_{Q_{UK}} - r\big) + \upsilon_{\theta K}(\mu_K - r)\Big)$$

Note that the correlation matrix $\mathbf{\Phi_M}$ for this model also has to be positive definite.[6]

$$\mathbf{\Phi_M} = \begin{bmatrix} 1 & \cdots & \rho_{SK} \\ \vdots & \ddots & \vdots \\ \rho_{SK} & \cdots & 1 \end{bmatrix}$$

3.3.1 Certainty Equivalent of Wealth, Model 2

Just like in Model 1, the certainty equivalent of wealth is calculated in Model 2. This enables comparison between the investors as the diversification possibilities increases. The certainty equivalent is computed and assessed once again.

[6] See Apendix for proof.

$$\bar{W} = W\left(\frac{(1+k_0)e^{k_0(T-t)}-1}{k_0}\right)^{\frac{\gamma}{1-\gamma}} = CE$$

3.4 Data

The collection of data for this study was made using the software program *Datastream 5.1* with an investigation period from January 1st 1996 to January 1st 2016. Since the thesis is written from a Swedish market perspective, the domestic stock portfolio is measured by the Swedish stock market index Nasdaq OMXS30. To clarify once again, the foreign stock market portfolios used are the American S&P 500 market index and the currency that the investor can invest in is the SEK/USD exchange rate in Model 1. In Model 2, the additional stock market portfolio used is the British FTSE100 and the exchange rate is the SEK/GBP. The risk free rate, r, is proxied by the average interest rate given by Handelsbanken, on a 20-year period. The test is performed using *Microsoft Excel*.

4. Result and analysis

In this part of the study, the parameters are estimated and put into the equations in order to get a numerical value of the portfolio weights and the certainty equivalent of wealth.

4.1 Parameters

Following the reasoning from part 2.1 regarding the relative risk aversion coefficient, the value of γ has to be larger than zero for a power utility function. However, to stay in line with earlier research by Brennan, Schwartz & Lagnado (1997), the control problem in this study is solved with an initial numerical value of γ set to 5. The reason for this rather extreme value of the risk aversion coefficient is, as the authors states, "to offset the treatment of the parameters of the stochastic process as known when they are in fact estimated and therefore subject to estimation error" (Brennan, Schwartz & Lagnado, 1997, p17).

In order to keep the equation simple, the value of the risk free interest rate is set to the average of the 20-year period given collected data. The test is also performed on a daily basis. Hence, the value of r is 0,0139 %.

Earlier research points out that the factor $e^{-\rho} = 0{,}95$, implying that the impatience parameter ρ, would equal a number of 0,05.[7]

The value of W is the amount of wealth that the investor starts his investing period with. This amount will have great impact on the size of the value function and thus the certainty equivalent. But as long as the value of W is consistent between the models, it does not matter for relative comparison, which is why W is set to an arbitrary number of 100.

[7] E-mail correspondence with Frederik Lundtofte, Lund University, May 11th 2016

Since the gathered data for the study range over a period of 20 years, the value of T is set to 20 and t is set to zero. However, in the analysis, CE is examined as t approaches 20.

4.2 Empirical Results for Model 1

From the definitions in chapter 3, the portfolio weights and the certainty equivalent of wealth could be estimated. To extend the research, and for the reason of comparison, the estimations were performed twice again when varying the risk aversion coefficient of the investor. This was done with both an increased and decreased preference for risk.

4.2.1 Varying the Risk Aversion

In analogy with earlier research, the initial value was set to $\gamma = 5$. The lower boundary for the risk aversion coefficient was set to $\gamma = 2$. The motivation is, as mentioned in chapter 2, that the value of γ for a power utility function has to be positive. However, a value of 1, which is equal to log utility, would also imply dividing with 0 in the equation for the CE. Altough a possibly lower number could be achieved, it would not add any significance to the outcome of the study. Following the same motivation for increasing risk aversion, the value was set to $\gamma = 10$.

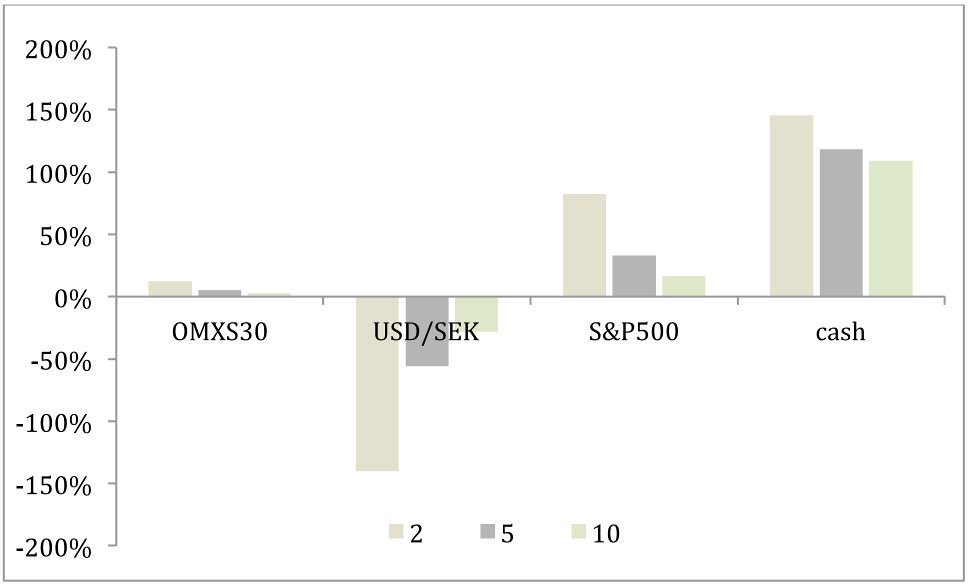
The results are presented in Table 1 below and the optimal portfolio weights are offered in graphical form in Figure 1. As can be seen, the numbers for the optimal portfolios are quite extreme.

Table 1: Result Model 1

Index	γ −value		
	2	5	10
OMXS30	13%	5%	3%
USD/SEK	-140%	-56%	-28%
S&P500	82%	33%	16%

cash	145%	118%	109%
CE	0,38	2,54	3,60

Figure 1: Graphical result of portfolio weights, Model 1. Different values of the risk aversion coefficient 2, 5, and 10.



4.3 Empirical results for Model 2

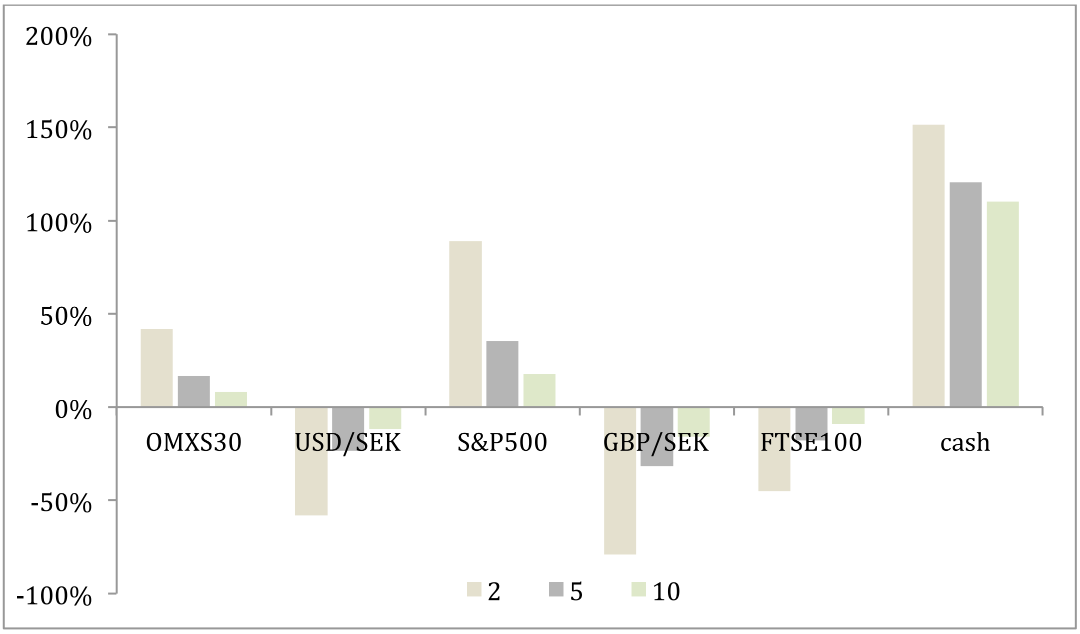
Known from the modeling approaches of the thesis, the question formulation is tested in two models. The second model is performed with the same values for the risk aversion coefficient as the first model to make them comparable. The results are presented in Table 2 and Figure 2 below.

Table 2: Result Model 2

Index	γ −value		
	2	5	10
OMXS30	42%	17%	8%
USD/SEK	-58%	-23%	-12%
S&P500	89%	36%	18%

GBP/SEK	-79%	-32%	-16%
FTSE100	-45%	-18%	-9%
cash	151%	121%	110%
CE	0,38	2,54	3,60

Figure 2: Graphical result of portfolio weights, Model 2. Different values of the risk aversion coefficient 2, 5, and 10.



4.4 Analysis

4.4.1 The Optimal Portfolio Choice

As can be seen, the individual's tendency to invest in other currencies increases as his risk aversion decreases and vice versa. Nevertheless, as long as the investor is risk averse, the general arrangement is that the investor wants to short sell the foreign currencies. Why? This might be as a consequence of several things. Even though most risks associated with exchange rates might be looked upon as common knowledge it is still relevant to state some of them. And since the type of investor in this particular thesis is undecided (it could be an individual as well as a financial institution), it is easier to analyze the exchange risk from the perspective of a corporation. In a working paper from the IMF by Papaioannou (2006), the most general risks are stated as follows.

1. Transaction risk – this is basically the risk from cash flows. It is caused by the effect that exchange rate fluctuations have on the obligations of a company to make or receive payments denominated in a foreign currency.
2. Translation risk – which is the risk of variations of the value of assets and liabilities denominated in foreign currency
3. Broader economic risk – reflects the impact of exchange rate variations and competitiveness

At first, neither of these risks relates rather relevant to this study as the investment in currencies is supposedly done in a speculative manner.

Now, there is a difference between exchange rate risk and exchange rate exposure. Maurice D. Levi (2005) elaborates around the definition of the latter. He defines it as “the sensitivity of changes in the real domestic-currency value of assets or liabilities to changes in exchange rates” (Levi, 2005, p192). This could lead to several interpretations. First, a measure of *sensitivity* becomes a description of how the home-currency value (in this case the SEK) of something changes when the exchange rate changes. As an example, the exposure could be calculated like the following.

$$\text{Exposure} = \frac{\Delta \text{V}(\text{SEK})}{\Delta \text{S}\left(\frac{\text{SEK}}{\text{USD}}\right)}$$

**In the case of multiple exchange rates that are changing at the same time, the equation could be extended to estimate the level of exposure.*

In this case the investor cares about the Swedish krona values to the exchange rate against the dollar. The bigger is the ratio, the larger is the exposure to the dollar. Secondly, by real domestic-currency values, the interpretation is that the exposure is the sensitivity of changes in real (i.e. inflation-adjusted) SEK values of assets or liabilities to changes in exchange rates. This implies that the exposure for an American and a Swedish agent on the same asset or liability is different.

All of the above could be treated as rather intuitive, but what is interesting is the connection between the exposure and the risk. Keep in mind that assets and liabilities are balance sheet items which could affect the value of a firm significantly. If the exposure exists on assets and liabilities, there are many ways that the values of them could be affected by exchange rates. The simplest way is through the translation of foreign-currency values into domestic-currency values, i.e. translation risk (Levi, 2005). And here is where the connection between risk and exposure occurs.

As an example, if a Swedish company depends on imports that become more expensive in SEK when the USD value of the SEK decreases, the company may become less profitable and its share price could decline. But it gets even more complex than this.

Muller and Verschoor (2006) performed a study of 817 multinational firms that are exposed to exchange rate variations. They estimate the impact of exchange rate variations on the firms' stock market returns. During the period of 1988-2002, 22 % of the firms had significant exposure to the GBP exchange rate and 14 % to the USD. Even though all of the 817 companies are not part of the OMXS30, the S&P500, and the FTSE100, the author believes that the outcome could be applied on the indices as well.

Conclusively, the reason for the investor to take a short position in the USD/SEK exchange rate, while investing in the S&P500, could have been seen as a hedge towards exchange exposure (or risk). However, this statement does not hold for Model 2, as the investor in this case is short selling both the GBP/SEK as well as the FTSE100. According to the Efficient Market Hypothesis (EMH) it is impossible for the investor to outperform the market. So for an investor to process all the information regarding exchange risk and exposure within the investment opportunity set, must seem merely impossible.

Given the reasoning above, there is no clear conclusion that can be derived regarding the investor´s portfolio choice of risky assets.

Anyhow, the ratio invested in cash from the first model varies between 109 % and 145 % depending on the level of risk aversion. A similar result, 110-151 %, is observed from Model 2. This larger proportion of allocation in cash is easier to relate to earlier research. Brennan, Schwartz & Lagnado (1997) found that a short term investor allocates a majority of his wealth into cash. The reason for this, as the authors' states, is that cash is riskless over a short period but is not riskless for someone with a longer horizon. Although the investing period for the investor is 20 years, which could be seen as a rather lengthy horizon, the situation of constant investment opportunities causes the investor to act as if he only had a myopic demand. This makes the study analogous to the case of a short term investment period.

Regarding the type of extreme portfolio weights that the investor chooses, Anson (2012) argues that this appears to be a general problem for mean-variance portfolio optimizers. As a cause of estimation error maximization, very large portfolio weights are generally allocated to the assets with the highest mean return and the lowest volatility and vice versa. Furthermore, neither of the models includes any additional cost for short selling, which might cause an unrealistic tendency towards extreme negative portfolio weights.

4.4.2 The Certainty Equivalent of Wealth

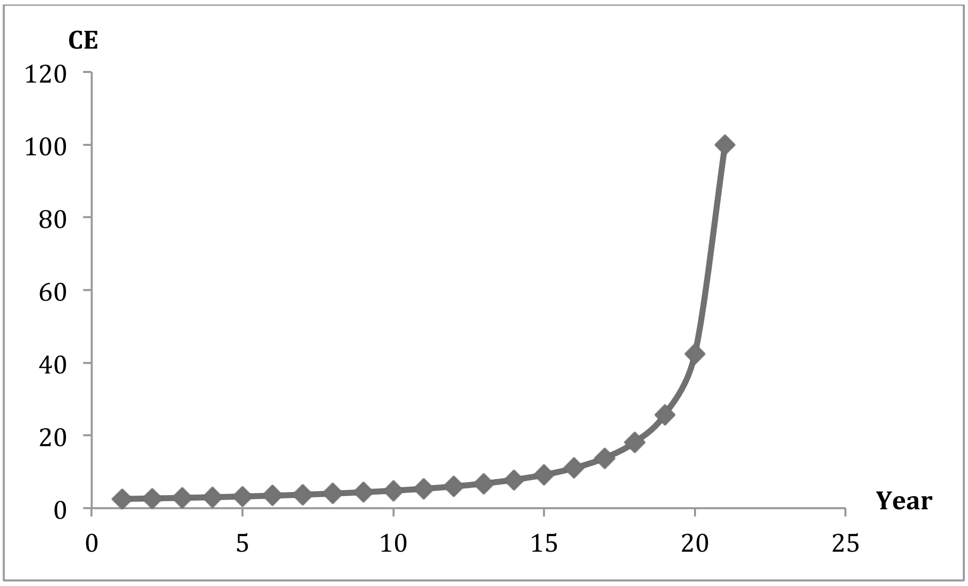
Given the definition in chapter 2, the certainty equivalent of wealth can be analyzed. Similar research of the CE on dynamic programming is to the author's knowledge sparse. So in similarity with the portfolio weights, the work of Brennan, Schwartz & Lagnado (1997) can be compared to the outcome.

To repeat, the certainty equivalent of wealth is such that the investor is indifferent between receiving it for sure at the horizon and having current wealth *and* the opportunity to invest in the available opportunities up to the horizon. Brennan, Schwartz & Lagnado (1997) found evidence that the volatility of the CE is much greater than the volatility of wealth. Their myopic certainty equivalent, which does not attempt to hedge against shifts in the investment opportunity set and thus is comparable to the certainty equivalent of this study, had a very high volatility. In addition, their unconstrained and normalized CE was much greater than an unconstrained CE, which reflects the ability to take more advantage of investment possibilities by taking short positions. They also found encouraging evidence that the CE is decreasing in r, which relates to the fact that the expected return on a stock is negatively related to the short-term interest rate.

The obtained values for the CE in this study was 0,38 for $\gamma = 2$, 2,54 for $\gamma = 5$, and 3,60 for $\gamma = 10$. Naturally, the CE increases with risk aversion. This could be interpreted as when the investor gets more risk averse, he wants a larger amount for sure at the end of the horizon, to give up investment opportunities today. However, the results shows that the values of the CE does not change between the models. This is kind of counter-intuitive, as one could expect that the CE would change as diversification possibilities increases. Similar studies are to the author not known. Therefore, the result encourages further studies of comparable tests and consequently, only one CE is presented in the graph below.

To compare the result with the research by Brennan, Schwartz & Lagnado (1997), the CE is investigated over the time horizon when keeping the risk aversion constant at $\gamma = 5$. The result is presented in Figure 3 below.

Figure 3: Change of the Certainty equivalent of wealth over 20 years, $\gamma = 5$.



As can be seen, the CE increases exponentially until it finally reaches 100 (initial wealth) at the end of the investment horizon. The result could be interpreted as if the investor's preference for receiving a certain amount of money at the horizon but keeping his current wealth *and* the opportunity to invest in the available assets up to the horizon stays relatively unaffected through the beginning of the time period. At the end of the investment horizon, the preference increases more drastically. From an intuitive aspect, the result seems reasonable. For example, the outcome could be linked to the regular problem of an individual's increase in risk aversion with age, which suggests that a young person should invest in riskier assets and an old person should invest in less riskier ones.

The only similarity to the study by Brennan, Schwartz & Lagnado (1997) is that the CE is increasing as the investor approaches the time horizon. However, their research showed high volatility in the CE through the entire time period, which is not the case for this study. They also showed that a majority of the changes in the CE came from changes in state variables, which highlights the biased outcome from constant investment opportunities. As a conclusion, the result shows no evidence of similarity to the earlier research except that the CE was increasing over the investment horizon. Again, the discrepancy from earlier research is possibly as a consequence of constant investment opportunities, but also different type of data.

5. Conclusion

The amount of research on strategic asset allocation using dynamic programming is extensive. A majority of the studies elaborates around the case where the investment opportunity set contains only one risky asset, a riskless bond and the risk-free interest rate. In this thesis, the purpose was to deviate from earlier research in the sense that the problem would be of multiple-choice and with an emphasis on foreign exchange rate exposure. In addition, the certainty equivalent of wealth of the investor has been investigated. To reach a clear solution to the problem was never the intention, nor within capability, of the study. Hopefully, the thesis could contribute to further research within portfolio choice theory.

The methodology of the study began by a selection of a suitable population of investigation. Earlier research was of modest contribution in the decision of investigated assets and time period. Those choices were rather guided by intuition. After the data had been collected, the optimal portfolio choice and the certainty equivalent of wealth could be calculated using the modelling approaches in *Microsoft Excel*.

Considering the question formulation of the thesis "*How does a foreign risk exposure opportunity set of assets, from a Swedish perspective, impact an investor's strategic asset allocation and certainty equivalent of wealth?*" the answer is that no pattern regarding the portfolio choice could be observed. The numerical results indicated quite extreme portfolio weights, probably as a consequence of mean-variance modelling and non-additional costs for short-selling. Part of the portfolio choice could be vaguely connected to earlier research, but only from an intuitive standpoint. However, the investor's certainty equivalent of wealth turned out to be exponentially increasing over time but did not change with diversification possibilities. The analysis of the CE showed no evidence of similarity to the earlier research except that the CE was increasing over the investment horizon. The dissimilarity is probably a cause of the constant investment opportunities in this study.

5.1 Suggestions to continued research

In this thesis, the limitations are stated in the first chapter. However, as the outcome did not provide any certain explanation on how investor's behave when exposed to foreign exchange rates, continued research is encouraged. As a first step, the inclusion of state variables within the opportunity set would be interesting, hence allowing for a hedging demand of the investor. In addition, the multiple-choice problem could also, with the help of better computing power, be extended to include a much larger sample of markets and assets.

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7. Appendix

7.1 Correlation Matrices

For the correlation matrix to be positive definite, the determinant of the sub-matrices has to be greater than or equal to zero. As can be seen, the requirement holds for both models. This could also be solved with the help of a software program, e.g. *Wolfram Alpha*.

7.1.1 Model 1

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \rho_{SL} & \rho_{SQ} \\ \rho_{SL} & 1 & \rho_{LQ} \\ \rho_{SQ} & \rho_{LQ} & 1 \end{bmatrix}$$

	S	*Q*	*L*
S	1	0,542627	0,664731
Q	0,542627	1	0,775902
L	0,664731	0,775902	1

Sub-matrix 1
1

Determinant 1
1

Sub-matrix 2	
1	0,542627
0,542627	1

Determinant 2
0,705556

Sub-matrix 3		
1	0,542627	0,664731
0,542627	1	0,775902
0,664731	0,775902	1

Determinant 3
0,221402

7.1.2 Model 2

$$\Phi_M = \begin{bmatrix} 1 & \cdots & \rho_{SK} \\ \vdots & \ddots & \vdots \\ \rho_{SK} & \cdots & 1 \end{bmatrix}$$

	S	Q	L	Q_{UK}	K
S	1	0,542627	0,664731	0,578249	0,833026
Q	0,542627	1	0,775902	0,937043	0,709238
L	0,664731	0,775902	1	0,741895	0,766726
Q_{UK}	0,578249	0,937043	0,741895	1	0,77162
K	0,833026	0,709238	0,766726	0,77162	1

Sub-matrix 1
1

Determinant 1
1

Sub-matrix 2	
1	0,542627
0,542627	1

Determinant 2
0,705556

Sub-matrix 3		
1	0,542627	0,664731
0,542627	1	0,775902
0,664731	0,775902	1

Determinant 3
0,221402

Sub-matrix 4			
1	0,542627	0,664731	0,578249
0,542627	1	0,775902	0,937043
0,664731	0,775902	1	0,741895
0,578249	0,937043	0,741895	1

Determinant 4
0,025411

Sub-matrix 5				
1	0,542627	0,664731	0,578249	0,833026
0,542627	1	0,775902	0,937043	0,709238
0,664731	0,775902	1	0,741895	0,937124
0,578249	0,937043	0,741895	1	0,742129
0,833026	0,709238	0,766726	0,77162	1

Determinant 5
0,003665