

LUND UNIVERSITY SCHOOL OF ECONOMICS AND MANAGEMENT

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# Identifying asset pricing bubbles

Testing for explosive behavior in the NASDAQ  
and STOXX 600 Europe Technology indices

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By

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## **Abstract**

A forward recursive estimation method is used to examine stock market data on unit root against explosive behavior as an indication of financial exuberance. Through specific dividend-stock pricing modeling, the recursive implementation of a right-tailed ADF test allows for directly testing the price index series on explosive behavior and its corresponding dividend series on non-explosive behavior. In addition, the forward recursive estimation method enables us to date stamp periodically collapsing bubbles. Empirically applied, we find the dotcom bubble of the late 90's in the EU technology index (STOXX 600 Europe Technology) which is in line with financial exuberance on the NASDAQ. Moreover, both indices demonstrate explosive behavior around the financial crisis in 2008. Lastly, it is found that the model in smaller subsamples is highly sensitive to the initial starting point.

Keywords: Asset pricing bubble, explosive behavior, right-tailed ADF, forward recursive regression.

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## List of Abbreviations

ADF	Augmented Dickey-Fuller
AIC	Akaike information criterion
AR	Autoregressive
CPI	Consumer price index
DCF	Discounted Cash Flow
DF	Dickey-Fuller
DFC	Chow-type Dickey-Fuller
DIV	Dividend series
EU	European Union
FCFF	Free cash flow to firm
GDP	Gross domestic product
GSDAF	Generalized supremum augmented Dickey-Fuller
IC	Information criterion
IPO	Initial public offering
PI	Price index
S&P	Standard & Poor
supADF	Supremum augmented Dickey-Fuller

## 1. Introduction

*‘Clearly, sustained low inflation implies less uncertainty about the future, and lower risk premiums imply higher prices of stocks and other earning assets. We can see that in the inverse relationship exhibited by price/earnings ratios and the rate of inflation in the past. But how do we know when irrational exuberance has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade?’ (Greenspan, 1996)*

Asset price bubbles are in every investors’ mind when participating in the financial market. The question in mind is if current asset prices reflect the fundamental asset price or if they drift away from fair asset value for various reasons. Phrased differently in an investor’s perspective: is it time to buy, or time to sell? Failing to act in time when a bubble is in place can lead to major loss of value. Furthermore, tumbling financial markets can force whole economies into recessions or at least have economies struggle with lower growth. A well-known example of a crisis on the stock exchange with a severe economic impact is Black Tuesday in 1929. Due to the enormous impact, governments, central banks, and investors are eager to understand in which condition and stage financial markets are.

A bubble is defined by an asset, or asset group, being overvalued with respect to its fundamental value. In the stock market, this results in share prices rising far above the fundamental value. Alan Greenspan, former chairman of the Federal Reserve, phrased this phenomenon as ‘irrational exuberance’. These exuberant valuations of assets can continue for several years, and then suddenly enormous selloffs cause the market to collapse. This moment is considered as the bursting of the bubble, as it causes a steep drop in prices (Shiller, 2005).

There are multiple well-known examples on bubbles. The oldest known is the tulip mania, which occurred in 1636 in the Netherlands (Shiller, 2005). Right before the collapse of the bubble, the prices of single tulip bulbs at one point exceeded a tenfold of the annual salary of a skilled craftsman, (Shiller, 2005). More recent examples of bubbles are the dotcom bubbles in the end of the 90’s and the housing market bubble in 2008, causing the financial crisis. A more mathematical explanation of an asset pricing bubble, and how to test for those, is given in the methodology section on page 12.

Spectacular rises and dramatic bursts of bubbles are widely discussed in the academic world. A well-known and highly researched bubble in recent years is the dotcom bubble in the US,

which first created and then wiped out enormous amounts of wealth at the time of collapse in 2000. Intrigued by the prominent remark by Alan Greenspan in December 1996, where he questioned whether irrational exuberance has driven asset values to unjustifiable levels, the researchers Phillips, Wu and Yu (2011) presented in their paper a new econometric methodology built on forward recursive regressions in the time series context. Their econometric approach enables detecting financial exuberance and its origination and termination. In their research, financial exuberance is defined as non-stationary and explosive autoregressive processes in the time series. Phillips et al. (2011) applied the augmented Dickey-Fuller (ADF) test for a unit root in the null hypothesis against the alternative of an explosive root. In a forward recursive test procedure, they estimated an autoregressive model on the log real NASDAQ price index and log real NASDAQ dividend series for the period from 1973 to 2005 and several sub-periods. There was no evidence of explosive behavior found in the NASDAQ dividend series. The price index series however did demonstrate explosive behavior. They identified the origination of this explosive behavior, and so the financial exuberance, in 1995. At the same time, they empirically underlined the collapse of the NASDAQ price index between September 2000 and March 2001. Therefore, the empirically evidenced origination of financial exuberance found in 1995 by Phillips et al. (2011) provides certain empirical substance to Greenspan's remark from 1996.

In this research, we use the European technology stock market in order to find out if there was a similar asset pricing bubble process during the new-economy hype as in the NASDAQ in the late 1990's resp. early 2000's. In a further step, we employ the testing procedure using the approach in Phillips et al. (2011) to more current data in order to identify a current potential asset pricing bubble in the NASDAQ and the European technology stock market.

The examination of the more current data sample is motivated by the tremendous acquisitions Facebook and other technology driven multinationals made in the recent years. Examples of such acquisitions are the WhatsApp-deal and Instagram-takeover by Facebook, the Skype-acquisition by Microsoft and many more. In addition to these acquisitions, the emergence of no less than 174 unicorn start-ups, privately owned technology companies valued over \$1 billion, has contributed to the interest in the current data sample (Fortune, 2016). Some of these so-called Unicorns have received extremely high valuations, such as Uber with \$62 billion and Airbnb with \$25 billion (Fortune, 2016). Numerous unicorns are currently preparing IPO's, so these valuations by the private market suddenly matter to the public market. However, even though 2015 was branded 'the year of the unicorn', since the start of

2016 the exuberance regarding these tech companies has cooled down and investors are now beginning to fear the valuations and the sustainability of the business models of these unicorns (Mooney, 2016). This fear and the potential devaluations will possibly lead to the next phenomena: Unicorpses (Mooney, 2016). These are unicorns that go under due to the bursting of this potential bubble (Mooney, 2016). An important distinction to be made here is that Unicorns are privately owned, and this research focusses on the public market. Nonetheless, the suspicion is that the potential exuberance in the private market regarding technology companies can be seen in the public market as well.

Despite a great number of influential articles on asset price bubbles and how to test for those, there is not yet a unanimous decision on the best model/method. Many researchers contradict each other in their models and discussions. The majority of researchers focus on two USA indices for their data samples; the NASDAQ and the S&P 500. This research will identify whether a bubble has occurred in a European tech index; STOXX Europe 600 Technology. As such, we move away from the USA as the primary research region and focus on the influence of the dotcom bubble on the EU market. In addition to this historical research, an attempt to identify a potential asset bubble in the same STOXX index as well as in the NASDAQ will be made using more recent data. The suspicion is that the European index will show similar behavior to the NASDAQ index as we apply the following saying: If the US sneezes, Europe catches a cold.

There are numerous relevant articles on unit root testing, asset pricing bubbles, and irrational exuberance, which will be used in this research as a guideline and a fundament to elaborate on. See for example: Hausman (1978), Shiller (1980), West (1987), Diba and Grossman (1988), Evans (1991), Cunando, et al. (2005), Gurkaynak (2005), Phillips et al. (2011), Frömmel and Kruse (2011), and Homm and Breitung (2012). Several of these will be discussed in the literature review in the next section.

The remainder of this thesis is constructed as follows; section 2 contains the literature review. Section 3 elaborates on the definition of asset bubbles and market exuberance. In addition, the model specifications are discussed. In section 4 of this thesis, a description of our data is given. Section 5 regards the test results of this research, which in turn will be analyzed and discussed in section 6. In section 7 the conclusions are drawn based on previous discussions and our test results. This section also contains a discussion on the drawbacks of this thesis and suggestions for further research.

## 2. Literature Review

### 2.1 Phillips, Wu and Yu (2011)

Phillips et al. (2011) propose a new method to test for financial exuberance in their 2011 article. They define financial exuberance in the time series context as an explosive autoregressive (AR) process. Phillips et al. (2011) suggest testing for such behavior based on a recursive implementation of a right-sided unit root test and a sup test. This will allow them to date stamp the origination and the end (collapse) of explosive behavior in the NASDAQ index, between February 1973 and June 2005. As this research closely follows the Phillips et al. (2011) paper, the data and the results are described rather extensively below.

As mentioned, the sample period of Phillips et al. (2011) covers the period in between February 1973 and June 2005. The data consists out of 389 monthly observations on the composite price index (dividends not included) and the composite dividend yields of the NASDAQ. The NASDAQ composite dividend series are computed from these two series. The composite dividend series and the composite price index of the NASDAQ are the series used for testing in terms of natural logarithm. In addition to these series, the Consumer Price index (CPI) is obtained from the St. Louis Federal Reserve and used to convert these series of nominal values into real values. Figure one of the Phillips et al. (2011) article shows the plotted normalized series, and it can be clearly seen that the price and dividend progressed jointly and realized a steady increase in price / pay-out until the 1990's. The price series starts to distance it from the dividend series in the middle of the 1990's, as it realizes a steep increase in price. This rapid upward movement of the price series continues until the late 1990's. The mounting movement of the price series can be contributed to the popularity of the dotcom stocks. The value of the price index promptly decreased in the beginning of the year 2000, and it sustained this fall to the mid-1990's level. However, the dividend series remained constant throughout the entire rise and freefall of the price series. This development contributes to the notion of an asset pricing bubble.

In the first examination part of Phillips et al. (2011), they use the data for the log real NASDAQ price and log real NASDAQ dividend series over the full period from February 1973 to June 2005. They apply an ADF test on the full sample with  $H_0: \delta = 1$  and the alternative (right-tailed) hypothesis is  $H_1: \delta > 1$ . The inference results in the non-rejection of the null hypothesis for both price and dividend series implying no asset price bubble in the data. This is in line with the findings of Diba and Grossman (1988), but subject to the criticism from Evans (1991). Using forward recursive regression estimation technique with an

initial start-up sample of 39 observations (i.e. 10% of the full sample), Phillips et al. (2011) reject the null hypothesis in favor of the alternative hypothesis on the 1% level implying explosive behavior in the price series, but not in the dividend series. Based on the same estimation technique they date stamp the origination of the financial exuberance to July 1995 and the collapse to March 2001.

The second step of Phillips et al. (2011) is a rolling regression over the full sample size with an initial start-up size of 77 observations, rather than 39. This step was proposed by referees of the 2011 article in order to test the robustness of their results. The plotted graph of their results shows an origination date in the summer of 1995 (June) and the collapse in the fall of 2000 (September), similar to the results of the first test.

Due to the fact that numerous researches cover the 1990's decade, Phillips et al. (2011) also divided the full sample into a first sample period from January 1990 to December 1999 and a second sample period from January 1990 to June 2000. Applying an ADF test on the two subsamples, they find strong evidence of explosiveness in both subsamples in the price series, but not in the dividend series. The second subsample from January 1990 to June 2000 even shows stronger evidence of explosiveness in the price series than the first one. Using the forward recursive regression estimation technique like in the first part, Phillips et al. (2011) detect explosive behavior in both subsample periods as well.

The first two subsamples did not cover the full bubble period. Therefore, Phillips et al. (2011) constructed a third subsample, using the forward recursive testing method on data from January 1990 until June 2005. This third subsample was constructed in order to date stamp the collapse of the asset pricing bubble accurately. The test on this subsample is another forward recursive  $\text{supADF}_r$  with  $r \in [0.1, 1]$ , meaning 18 starting observations. Based on this test, the start of exuberance is date stamped at July 1995 and the end of exuberance at October 2000.

To confirm the understanding of the underlying model, the research of Phillips et al. (2011) has been replicated. Similar results were found on which the same conclusions were drawn, although with slightly different estimates. This might be due to the use of different lag lengths order in the regression equations (Eq. 6 and 7) which are not explicitly stated in the paper of Phillips et al. (2011). Similarly, Hogg and Breitung (2012) arrive at the same inferences with slightly different test statistics as well when replicating the testing from Phillips et al. (2011). The date stampings of the beginning of the bubble retrieved from the different testing procedures in the replications slightly deviate from the findings in Phillips et al. (2011).

On the other side, the date stampings of the termination of the bubble match fairly accurate with the results from Phillips et al. (2011). Overall, the same inferences could be made and so we move forward with our testing, based on this method proposed by Phillips et al. (2011).

## **2.2 Other relevant papers**

As was mentioned in the introduction section, there are multiple relevant papers on the topic of testing for an asset bubble. Gurkaynak (2005) provides a paper summarizing econometric methods for testing of asset price bubbles from research in the 1980's and 1990's. As in various models, the assumption is that the current stock price reflects the market's expectation of all future dividends discounted to present value. It is often referred to the present value model in literature. Shiller (1980) argues in variance bounds tests that the ex-post rational price can be defined as the present value of actual (not future) dividends. He suggests that the variance of the ex-post rational price should be at least of the same magnitude as the observed price which is based on expected dividends. If this is violated, he implies that there is a divergence between the fundamental asset price and the actual asset price. Based on this approach, Shiller (1980) showed the validity of the present value model, but several follow-up researches criticize that the test has problems whether the model violation is due to the presence of bubbles.

Other early research on asset bubbles by Diba and Grossman (1988), using standard unit root test and cointegration test applied to the S&P Composite Stock Price index and dividend series over the period 1871-1986, did not reveal any support for rational exuberant behavior. The idea behind their testing is that stock prices are as stationary as the dividend series. If so, then they argue that there is no rational bubble inherent which generates a nonstationary (bubble) component to the stock prices. These approaches are examined by Evans (1991) where he concluded that standard unit root and cointegration tests are inappropriate for detecting financial exuberance. Evans (1991) shows in simulations that the unit root test does not work well when bubbles collapse temporarily.

West (1987) elaborated the idea of a Hausman (1978) specification test on two sets of parameter estimates. The two sets represent two different ways of calculating the impact of dividends on the stock price which is defined as usual in terms of the present value of current and future dividends. The first approach makes use of the no-arbitrage asset pricing which means that the current stock price represents the fundamental value. One parameter estimate, which represents the discount rate under the no-arbitrage condition, is directly calculated in a regression of the stock price on a sample of lagged dividends. The parameter estimate is used

to define the autoregressive dividend process which yields in the fundamental stock price. The other set is constructed by considering a bubble term in the stock price equation. If the two parameter sets do not differ by applying a Hausman specification test, then it does not imply the presence of a bubble which is the null hypothesis in this test. Applying the tests to annual S&P and Dow Jones stock and dividend data, West (1987) finds the presence of bubbles in the data. Critiques from Dezbakshsh and Demirguc-Kunt (1990) state that the applied testing in small samples results in too many rejections of the null hypothesis. Further critique is given by Flood, Hodrick and Kaplan (1994) where they raise the issue that there might be other factors than dividends causing a bubble in asset prices.

Further research from Froot and Obstfeld (1991) models a rational bubble process entirely dependent on the level of dividends. This is similar to the approach of West's testing. Tying the level of dividends in a linear model to the fundamental price and in a nonlinear regression to price/dividend ratio enables Froot and Obstfeld (1991) to argue that if the coefficient estimate of the level of dividends is not significant, there is no sign of a bubble in the asset prices. Applying the tests to S&P data from 1900 to 1988 they find significant estimates resulting in the rejection of the null-hypothesis that there is no bubble. Froot and Obstfeld (1991) themselves criticize their results that the true model between the level of dividends and the fundamental price could also be nonlinear. Therefore, the whole process of how stock prices are built on can be different.

Cuñado, Gil-Alana and de Garcia (2005) examine in their study the order of integration of the NASDAQ price index and NASDAQ dividend series and its corresponding price-dividend ratio. In previous studies, model specifications were used to locate the change from an order of integration in the time series of  $I(0)$  to  $I(1)$  as an indication of a bubble. Cuñado, Gil-Alana and de Garcia (2005) apply a fractionally integrated modeling, i.e. fractional integration from  $I(0)$  to  $I(0,25)$ , ...,  $I(2,00)$ . They use sample data with daily, weekly and monthly data of the price index and dividend series. The findings are that the presence of bubbles depends on the sampling frequency applied in the testing. Evidence of the presence of bubbles is found when using monthly data, but this is not supported by the results based on daily and weekly data.

Frömmel and Kruse (2011) test the structural change in the long memory parameter of an autoregressive fractionally integrated moving average (ARFIMA) data generating process. The test includes the null hypothesis of constant memory against a change from stationary to non-stationary whereas non-stationary reflects the long memory. Applying the test procedure to the S&P dividend-price ratio data from 1871 to 2009, they find a structural break in S&P

data in July 1991. Applying a standard unit root test on the pre-break and post-break sample reveals the presence of a rational bubble in terms of a unit root in the post-break sample, but not in the pre-break sample. They also find that the bubble does not burst during their sample period due to the lack of evidence to change from a non-stationary process to a stationary process. Furthermore, Frömmel and Kruse (2011) apply the same testing procedure to S&P earnings-price ratio data for the same time period. They find the structural break in this particular time series in June 1991, and pre- and post-break analysis result in similar findings as in the S&P dividend-price ratio.

Homm and Breitung (2012) compare different testing methods for speculative bubbles in stock markets. This means the testing for a change from a random walk to an explosive process at an unknown point in the time series. Among the different testing methods, they conclude that the Phillips et al. (2011) test shows robust results at times of multiple breaks. This is especially useful to detect the origination and termination of financial exuberance. Additionally, Homm and Breitung (2012) suggest in a simulation framework that a sequential Chow-type Dickey-Fuller testing results in a strong predictor of the bubble starting date with the highest power among the different testing methods. Their testing procedure as such is interesting that it has a feature of forward recursive estimation method generating supremum Dickey-Fuller-Chow (DFC) test-statistics. This method demonstrates a similar approach as suggested by Phillips et al. (2011). Therefore, the methodology of Homm and Breitung (2012) is briefly described in the following.

The methodology of their sequential Chow-type Dickey-Fuller is based on an AR(1) process;  $y_t = \rho_t y_{t-1} + \epsilon_t$ , and goes as follows. The AR(1) model can be rewritten as

$$\Delta y_t = \delta (y_{t-1} \mathbf{1}_{\{t > [\tau T]\}}) + \epsilon_t \quad \text{Eq. 1}$$

under the assumption that  $\rho_t = 1$  for  $t = 1, \dots, [\tau T]$  and  $\rho_t - 1 = \delta > 0$  for  $t = [\tau T] + 1, \dots, T$ . The  $[\tau T]$  stands for the unknown time in the sample interval from  $\tau \in [0, 1]$  where the process changes from random walk to an explosive process.

In this rewritten model,  $\mathbf{1}_{\{\}} equals 1 when the  $\{t > [\tau T]\}$  statement is accurate and 0 otherwise. As such, the null of this test is  $H_0: \delta = 0$  and the alternative is  $H_1: \delta > 0$ .$

The Chow-Type DF statistic to test for a change from I(1) to explosive in the interval  $\tau \in [0, 1 - \tau_0]$  can be written as

$$\sup_{\tau \in [0, 1 - \tau_0]} DFC_{\tau} \quad \text{Eq. 2}$$

A sequence of DFC test-statistics is constructed, and the supremum is taken over that. Large values of the  $\sup_{\tau \in [0, 1 - \tau_0]} DFC_{\tau}$  lead to the rejection of the null hypothesis in favor of explosive behavior as the alternative hypothesis.

When replicating Phillips et al. (2011) and their own sequential Chow-type DF, they confirm not only financial exuberance in the NASDAQ price index, but also in American, British, and Spanish house price indices.

### 3. Econometric methodology

We previously stated in the literature review that many researches have been made on modeling time series in autoregressive processes and testing on unit root. As generally known, desirable properties in time series analysis would be a constant mean, constant variance, and constant autocovariance in the error terms. This means stationarity in the data series. In case of regressing non-stationary variables the following problems emerge (Brooks, 2014): First, non-stationary data implement a shock which persists as an infinite effect. Second, if two variables show the same non-stationary trends over time, the regression of one variable on the other variable would result in a high fitted model (high R-squared) even if the two variables are completely independent of each other (e.g. childbirth rate in Afghanistan regressed on crispbread consumption in Sweden). In statistical terms, this is named spurious regressions. Third, non-stationary variables computed in a regression model generate large test-statistics which lead to wrong inferences.

Considering a simple AR(1) process with a drift ( $y_t = \mu + \delta y_{t-1} + \varepsilon_t$ ) there are three possible cases how the time series behave (Brooks, 2014):

- (1) In the stationary case:  $\delta < 1 \Rightarrow \delta^T \rightarrow 0$  as  $T \rightarrow \infty$
- (2) In the unit root case:  $\delta = 1 \Rightarrow \delta^T = 1$  for all  $T$ . This will lead to the process of  $y_t = \mu + y_{t-1} + \sum_{t=0}^{\infty} \varepsilon_t$  as  $T \rightarrow \infty$ .
- (3) The explosive case:  $\delta > 1 \Rightarrow \delta^T \rightarrow \infty$  as  $T \rightarrow \infty$

The standard unit root testing procedure is to determine whether the process follows a unit root case or a stationary case. The appropriate method to test for a unit root in a time series is not through examining the (partial) autocorrelation function, as these series often show slowly

decaying autocorrelation functions, even in the case of a unit root (Brooks, 2014). The appropriate testing should be done through a Dickey-Fuller (DF) test. The DF test is a formal hypothesis testing procedure first created by Dickey and Fuller (Fuller, 1976; Dickey and Fuller, 1979). The null hypothesis of this test is that  $\delta = 1$  in

$$y_t = \delta y_{t-1} + \varepsilon_t. \quad \text{Eq. 3}$$

The alternative hypothesis is one-sided and is  $\delta < 1$ . As a result, we can state that in the DF test the  $H_0$  = the series has a unit root and the  $H_1$  = series is stationary (Brooks, 2014). An intercept, a time trend, both these variables, or neither can be added to Eq. 3 and as a result the equation can be written as follows:

$$y_t = \delta y_{t-1} + \mu + \lambda t + \varepsilon_t \quad \text{Eq. 4}$$

If we subtract  $y_{t-1}$  from both sides, we can rewrite the test as

$$\Delta y_t = \psi y_{t-1} + \mu + \lambda t + \varepsilon_t \quad \text{Eq. 5}$$

Where the original test of  $\delta = 1$  is now rewritten as a test of  $\psi = 0$ , due to  $\delta - 1 = \psi$ . The primary constriction with this proposed test is that it loses validity as soon as  $\varepsilon_t$  is anything else than white noise. The test would in such case become oversized, meaning that the volume of null hypotheses wrongfully rejected is higher than the nominal set size. The Augmented Dickey-Fuller (ADF) test offers the solution to this problem (Brooks, 2014). As the name suggests, this test is augmented by taking  $J$  lags of the dependent variable  $y$  (Brooks, 2014).

$$\Delta y_t = \psi y_{t-1} + \sum_{j=1}^J \phi_j \Delta y_{t-j} + \varepsilon_t \quad \text{Eq. 6}$$

Again an intercept and a time trend can be added:

$$\Delta y_t = \psi y_{t-1} + \mu + \lambda t + \sum_{j=1}^J \phi_j \Delta y_{t-j} + \varepsilon_t \quad \text{Eq. 7}$$

The lags in the dependent variable now assure a non-autocorrelation in  $\varepsilon_t$  by taking the dynamic structure of  $y_t$ , if present, into account. The null hypothesis remains the same ( $H_0: \psi = 0$ ) and the same critical values as in the DF test can be used. An important note is the decision on the number of lags  $J$  to be included. The frequency of data and an Information Criterion (IC) can be used to guide this decision (Brooks, 2014). The specification of the lag length in the test regression has a sensitive impact on the power of the test inferences (Brooks, 2014). Choosing a regression with an insufficient number of lags does not take care of all autocorrelation in the error term and therefore results in incorrect rejection or non-rejection of

the null hypothesis (Brooks, 2014). In contrast, the standard errors of the coefficients increase when choosing too many lags which result in biased hypotheses inferences (Brooks, 2014).

As introduced above, the standard testing for unit root reveals the behavior of the properties of the time series in terms of stationarity and unit root. The discussion in the research on rational bubble evolves around the motivation that stock price properties show explosive autoregressive behavior in certain sub-periods. This means that the stock price follows an autoregressive process of  $y_t = \mu + \delta y_{t-1} + \varepsilon_t$  with  $\delta > 1$  for those sub-periods. Several AR(1) simulated processes with  $\mu = 0$  and  $\varepsilon_t \sim i.i.d N(0,1)$  are shown in figure 1 where the time series follow stationarity ( $\delta = 0,9$ ), random walk ( $\delta = 1$ ) and non-stationarity (explosiveness with  $\delta > 1$ ).

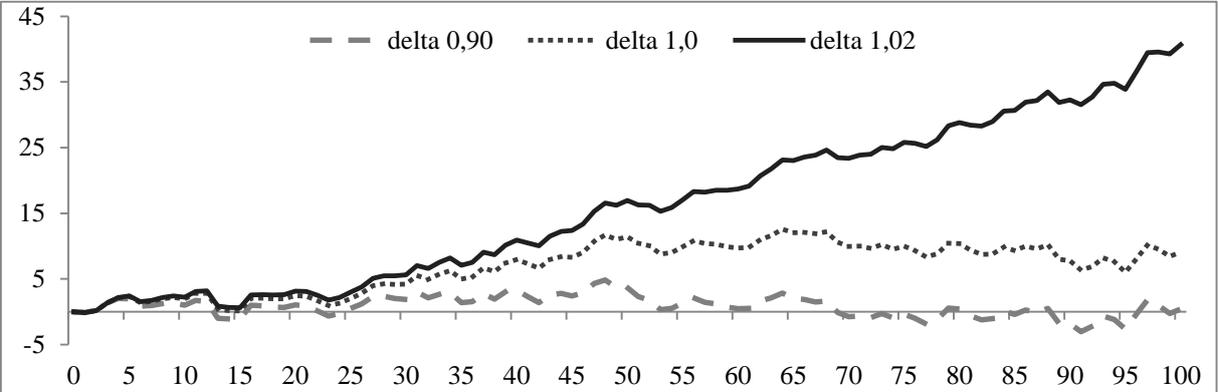


Figure 1: Simulated autoregressive processes with different behavior

In Phillips et al. (2011) the testing of rational bubbles is built on the methodology where the fundamental asset prices are determined by the market’s expectation of all future dividends discounted to present value. The fundamental asset prices are then tested on explosive behavior. The same methodology as proposed by Phillips et al. (2011) is used to test for rational bubbles in the European and American technology stock market in later sections.

**3.1 Asset pricing method**

In comprehensive formulations, Campbell and Shiller (1989) develop a dividend-stock price model on which Phillips et al. (2011) base their methodology. Campbell and Shiller (1989) propose that if nonstationary dividends cause nonstationary stock prices, it follows that dividends and stock prices are cointegrated. This is applied in various researches so that unit root testing and the testing of cointegration can be performed on the dividend-price relationship.

The first definition includes:

$$P_t = \frac{1}{1+R} E_t[P_{t+1} + D_{t+1}] = \frac{E_t[P_{t+1}+D_{t+1}]}{1+R} \quad \text{Eq. 8}$$

The current stock price  $P_t$  is determined by the current expectations of the sum of the future stock price and dividend  $D_{t+1}$  at the discount rate  $R$  to reach present value. Phillips et al. (2011) leave the discount rate constant over time which does not distort the implication on the bubble component explained later in this section.

In their research, Phillips et al. (2011) revert to Campbell and Shiller (1989) where the current stock price is defined in a logarithmic approximation by a fundamental price component and a bubble component:

$$p_t = p_t^f + b_t \quad \text{Eq. 9}$$

Where  $p_t^f$  is the fundamental price component and  $b_t$  is the bubble component. The fundamental price component is based on the assumption that the expected discounted value of the stock approaches zero in the infinity. This allows for the fundamental price component to be formulated in the way that it is solely determined by the expected present value of future dividends and its corresponding average log dividend-price ratio. In formula notation:

$$p_t^f = \frac{\kappa-\gamma}{1-\rho} + (1-\rho) \sum_{i=0}^{\infty} \rho^i E_t[d_{t+1+i}] \quad \text{Eq. 10}$$

Where  $d_t = \log(D_t)$ ,  $\gamma = \log(1+R)$ ,  $\rho = \frac{1}{1+e^{(\bar{d}-p)}}$  with  $(\bar{d}-p)$  as the average log dividend price-ratio and  $\kappa = -\log(\rho) - (1-\rho) \log(\frac{1}{\rho} - 1)$ .

The bubble component includes a growth factor based on the average log dividend-price ratio and follows the process:

$$b_t = (1+g)b_{t-1} + \varepsilon_{b,t} \text{ with } E_{t-1}[\varepsilon_{b,t}] = 0 \quad \text{Eq. 11}$$

Where  $g = e^{(\bar{d}-p)} > 0$  is the growth determined by the average log dividend-price ratio. This process implies that if the dividend-price ratio approaches zero, the bubble grows with a speed of  $(1+e^0)$ . If the dividend-price ratio goes to 1, the bubble grows with an extreme speed of  $(1+e^1)$ .

As can be seen from Eq. 9, if there is no bubble, the current stock price  $p_t$  is exclusively determined by the fundamental price component which in turn is based on the current dividend and average dividend-price ratio. In this case, it can be obtained that the current stock price  $p_t$  and the current dividend can be integrated of order 1 and consequently are cointegrated. Diba and Grossman (1988) applied a cointegration test on this relation which

implies that there is no evidence of a bubble if the current stock price and current dividend are cointegrated.

In the second case when there is a bubble component greater than zero, since the bubble process from Eq. 11 itself is constructed with explosive behavior, it results in explosive behavior of the current stock price  $p_t$  in Eq. 9. This way of modeling also leads to explosive behavior of  $\Delta p_t$ . Therefore, Diba and Grossman (1988) introduced a standard unit root test to  $\Delta p_t$ . In case of rejecting a unit root in  $\Delta p_t$ , they conclude that there is no evidence of a bubble in the current stock price  $p_t$  either. As mentioned in the literature review, Evans (1991) criticized that the unit root test does not work well when bubbles collapse occasionally. However, the way of modeling Eq. 9 and the bubble component process in Eq. 11 puts forward that testing  $p_t$  directly on explosiveness in combination with testing  $d_t$  directly on non-explosiveness reveals the sign of an asset pricing bubble. This sets forth that the discount rate is time invariant. Phillips et al. (2011) point out though that the current stock price could be induced explosive solely by the dividend and so the fundamental price component and the bubble component would be together explosively cointegrated. Therefore, the pattern of explosive dividend and price behavior would make conclusions about the asset pricing bubbles in the time series non-valid.

### **3.2 Practical implementation to test for asset pricing bubbles**

The standard right-tailed ADF has a weakness to detect multiple changes of non-explosive to explosive behavior and vice versa. We therefore follow the suggestions of Phillips et al. (2011) to take care of a potential existence of multiple bubbles by using forward recursive regressions and applying the ADF test for explosive behavior in the current stock price  $p_t$  and nonexplosive behavior in the dividend in our later testing implementation. The approach is to compute repeatedly right-sided ADF test statistics to test for explosive behavior in the sample data of the stock price series and separately the corresponding dividend series.

The time series are estimated in an autoregressive process by OLS specified as follows:

$$y_t = \mu + \delta y_{t-1} + \sum_{j=1}^J \phi_j \Delta y_{t-j} + \varepsilon_t \quad \text{Eq. 12}$$

Where  $y_t$  is the time series of either the log stock price or log dividend. We choose lag order  $J$  in accordance to Campbell and Perron (1991), which means starting with 12 for full sample resp. 6 lags for the subsample, where coefficients are sequentially tested for significance at 5% level. This leads to the model for which the coefficient of the last included lag is significant at the 5% level. The null hypothesis is  $H_0: \delta = 1$  and the alternative explosive

hypothesis is  $H_1: \delta > 1$ . The regression equation (Eq. 12.) is repeatedly estimated increasing the subset of the full sample data by one observation at each time. This generates supremum ADF test-statistics ( $supADF$ ) for each increment of the sample. In their testing, Phillips et al. (2011) define an initial subsample from the full sample and increase the initial subsample by one observation in each ADF-test until all observations of the full sample are included.

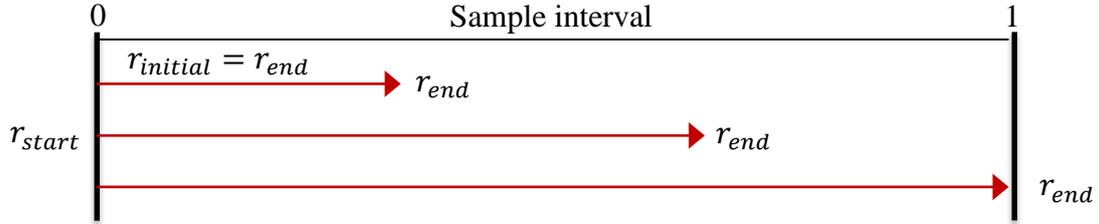


Figure 2: Process of the forward recursive estimation method

Figure 2 shows that the whole sample goes from 0 to 1. The initial subsample ( $r_{initial}$ ) is defined with a certain length at the starting point ( $r_{start}$ ) of the whole sample to the endpoint ( $r_{end}$ ) in between the whole sample, e.g. as a fraction of 10% of the whole interval. In the following, the initial subsample is repeatedly increased by one observation keeping the same starting point of the whole sample. In contrast to the right-tailed ADF testing of the full sample, the forward recursive ADF testing investigates each newly created sample which is incremented by one observation on a unit root against explosive behavior. The largest ADF test-statistic among all the created samples indicates whether there is explosive behavior in the full sample. If this ADF test-statistic exceeds the critical value, the null-hypothesis is rejected.

The ADF test-statistics and right-tailed critical values to test for a unit root against non-stationarity are based on the coefficient estimates as follows:

Full sample:  $ADF_1 = \frac{\hat{\delta}_{t-1}}{\hat{\sigma}_{\delta,t}}$  with critical values following the distribution of a Wiener process

$$\rightarrow \frac{\int_0^1 \tilde{W} dW}{(\int_0^1 \tilde{W}^2)^{1/2}}$$

Subsample:  $sup_{r \in [r_{initial}, 1]} ADF_r = sup_{r \in [r_{initial}, 1]} \frac{\hat{\delta}_{t-1}}{\hat{\sigma}_{\delta,t}}$  with critical values following the same distribution

$$\rightarrow sup_{r \in [r_{initial}, 1]} \frac{\int_0^1 \tilde{W} dW}{(\int_0^1 \tilde{W}^2)^{1/2}}$$

Where fraction  $r_{initial} = [0,1]$  represents the integer number of observations of the first subsample (e.g. the first subsample in Phillips et al. (2011) has 39 observations out of 389 observations). As in Phillips et al. (2011), we obtain the critical values for the full sample and

subsample by applying Monte Carlo simulation to the distribution process with 10'000 simulations.

### 3.3 Date stamping procedure

To recognize the origination and collapse of the bubble and accurately date stamp those, the recursive ADF test- statistics are compared against the right-tailed critical values of the asymptotic distribution of the standard Dickey-Fuller test-statistics. For practical implementation, Phillips et al. (2011) recommend to specify the critical values as follows:

$$cv_{\beta_n}^{adf}(s) = \frac{\log[\log(ns)]}{100} \quad \text{Eq. 13}$$

Where  $n = \text{sample size}$ ,  $s \in [0.1, 1]$  and  $\beta_n = \text{significance level}$ .

Assuming a sample size  $n = 500$  and  $s \in [0.1, 1]$ , it results that  $ns$  ranges between 50 and 500. Consequently, the critical values range between 0,0136 and 0,0183 in this particular example. The beginning of financial exuberance is marked when the ADF test-statistic from the repeatedly estimated regressions exceeds the critical value. The collapse is marked when the ADF test-statistic again returns to a smaller value than the critical value which indicates a change from non-stationarity to stationarity.

We apply a further test procedure to test the date stamping approach on robustness to the starting point of the estimation. Instead of using forward recursive regressions, we run regressions with rolling windows. The first window is set at the starting point of the sample including a fixed amount of observations of the full sample. The window is continuously rolled over to the next starting point keeping the fixed amount of observations of the full sample. Similar to the forward recursive technique, the ADF test-statistics are computed for each newly created starting point and ending point. Again, the ADF test-statistics are compared to the critical values in order to determine the origination and collapse of financial exuberance. The following figure 3 illustrates the procedure based on rolling windows.

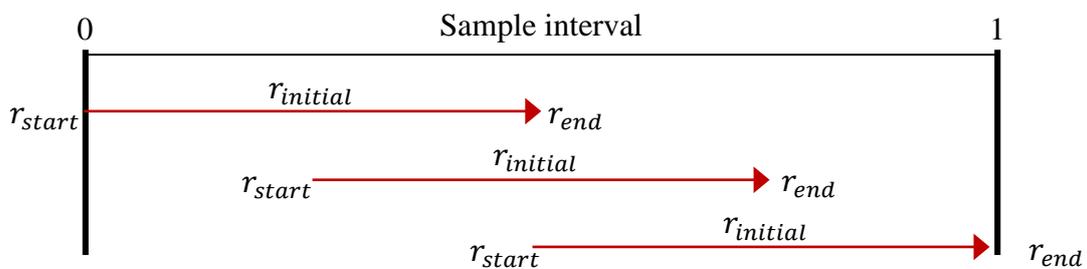


Figure 3: Process of the forward rolling estimation method

Retrieving the results from rolling windows regressions should reveal whether the ADF test-statistics from each newly created sample date stamp the origination and collapse of the bubble in an identical manor as with the use of forward recursive regressions. Finding similar date stamps would confirm the non-sensitivity of the estimation methods to the chosen starting point of the estimation period.

We follow the practical implementations from Phillips et al. (2011) based on the procedure described. We aim to investigate for similarities between the NASDAQ and STOXX data in the 1990's and early 2000's where most researches focused on detecting financial exuberance. As a follow-up, we aim to test for asset price bubbles in the NASDAQ and STOXX data in the aftermath of the dotcom bubble up until 2015 based on the practical implementations from Phillips et al. (2011).

Empirical results of this practical implementation will be displayed in section 5. In the next section, we will discuss the data used for the empirical application of the model.

#### **4. Description of Data**

We introduce our data sample consisting of the NASDAQ composite index and STOXX Europe 600 Technology index. For each index, two time series are extracted, which are the price index and dividend yield. For NASDAQ, the data sample ranges from the start of the index in 1973 to 2015 for both series. For STOXX, the price index series ranges from the start of the index in 1987 to 2015. The dividend yield sample starts in 1999, which is the first year the index records the dividend yield, and continues until 2015. We recognize this limitation for the research done on the period prior to 2000. However no European technology indices are available with longstanding historical track records and as such we move on with this limitation.

The data samples are extracted from Datastream International (Thomson Reuters), and monthly data is used in all times series. The data is transformed and expressed in natural logarithm used for the later testing in a similar fashion as described before in the section regarding Phillips et al. (2011)'s data. The Consumer Price Index (CPI) of the United States, extracted from the St Louis Federal Reserve database, is used to transform NASDAQ data samples from nominal to real values. A CPI of the European Union (EU) as a whole is not available for the full sample period of the STOXX index since the index started in 1987 where the EU has not yet been formed politically. Therefore, the STOXX data samples are transformed by taking the average CPI of the 10 European countries, which represent the most

significant economic importance in Europe. These countries are Germany, France, the UK, Italy, Sweden, Finland, Denmark, The Netherlands, Belgium and Switzerland. In addition of the economic importance of the chosen 10 European countries, the companies represented in the STOXX index are located in these countries and so they are most significant to this research. Of each country the individual CPI is taken in order to compute the average CPI.

In figure 4 and 5, the plotted time series of all four elements are shown. Both the price index series of the two indices and NASDAQ dividend series are normalized to 100 at the beginning of index launch. The dividend series of the STOXX index is normalized to 100 on July 1999; the first month in which data on the dividend yield of STOXX is available. It can be seen that the NASDAQ dividend series remained rather constant in the years leading up to 2002 / 2003, even during the dotcom bubble the dividend was a steady factor. In contrast, both the NASDAQ price series and the STOXX price series are constant from 1987 till 1995 and 1996 respectively, and then start rising. The price series of both indices move closely together until 1999 and experience the same enormous drop in the price series in the fall of 2000. Starting from 1999 and for the first six years, the dividend series of the two indices move relative closely together as well. In 2004, it can be seen that the NASDAQ dividend series realize an increase in yield. In 2009, for a short period of time, the dividend series of NASDAQ reached the same level as the price index series. This has not happened since 1991.

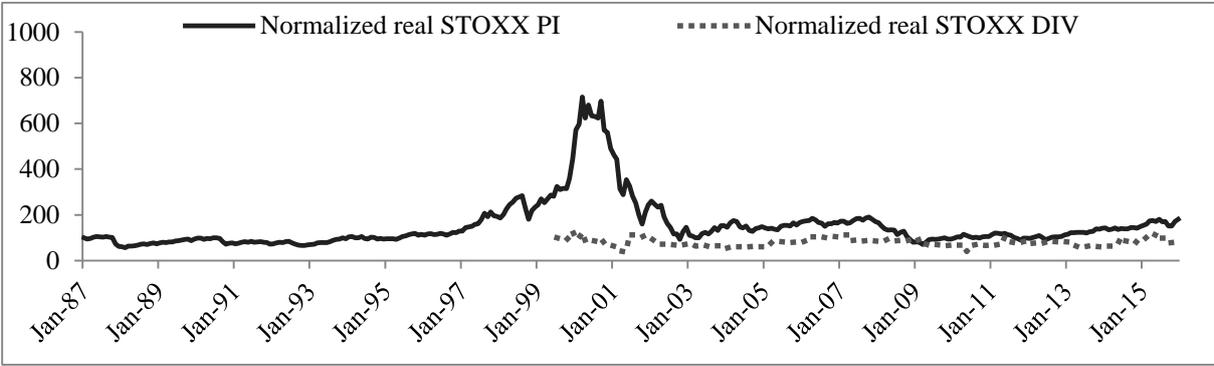


Figure 4: Plotted series of the STOXX PI and DIV, normalized in 1987 respect. 1999 at 100

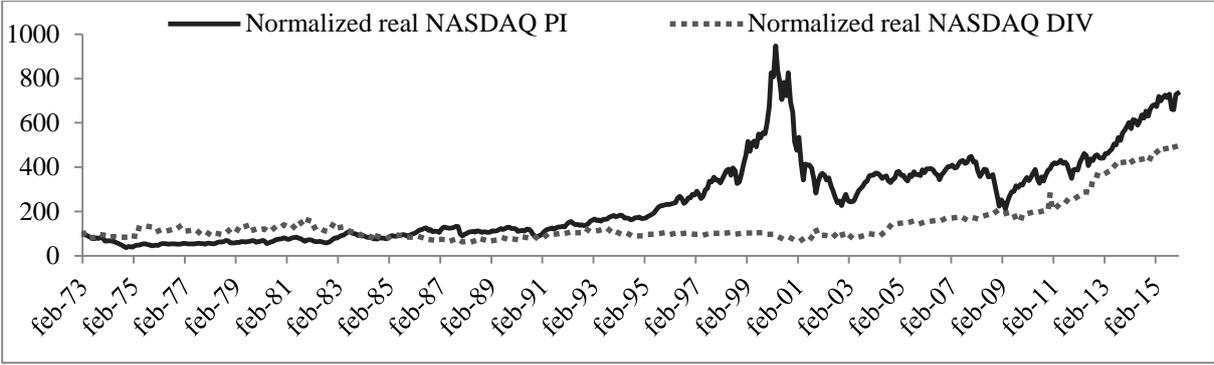


Figure 5: Plotted series of the NASDAQ PI and DIV, normalized in 1973 at 100

The price series of NASDAQ show relatively stable increasing progress from the dotcom bubble's recovery at the end of 2002 until January 2008. The subprime mortgage crisis is seen there as the series falls below the level of the dotcom bubble collapse within one year. From January 2009 and on however the price index of NASDAQ starts to recover and up to the end of the sample period in December 2015 it constantly increases, moving from a normalized value of 200 up to 730 in almost seven years.

Overall, it can be seen that the NASDAQ has rising values for both its series since the collapse of the dotcom, whereas the STOXX series remain rather constant. Also, the 2008 financial crisis, caused by the subprime mortgage crisis, is reflected more intensively in the NASDAQ price index, relative to its STOXX counterpart.

## **5. Empirical results**

### **5.1 Full sample period (1987 – 2015)**

When applying a standard right-tailed ADF test of the full sample of the log real STOXX price index from January 1987 to December 2015, the statistical value results in the non-rejection of the null-hypothesis<sup>1</sup>. This implies that the log real STOXX price index does not incorporate financial exuberance. In addition, the right-tailed ADF test of the full sample of the log real STOXX dividend series from July 1999 to June 2015 does not reveal explosive behavior in the data either<sup>2</sup>. As pointed out earlier, unit root testing has difficulties to detect stationarity resp. explosiveness when the bubbles collapse temporarily during the applied data period. As such, our non-rejection of the null-hypothesis based on log real STOXX price index and dividend series is consistent with the findings in Phillips et al. (2011) on the log real NASDAQ price index and dividend series. The lag length in the ADF test is specified in the approach of determining the last lag included at the 5% significance level. The lag length for the log real STOXX price index resp. log real STOXX dividend series is set at  $J=8$  resp.  $J=0$ .

With the purpose to overcome drawbacks of the standard unit root testing, we run forward recursive regressions in order to generate supADF test-statistics as suggested by Phillips et al. (2011). The initial start-up sample consists of 35 observations for the log real STOXX price index with starting point January 1987. The initial start-up sample represents 10% of the full sample, which consists out of 348 monthly observations, as similarly implemented by Phillips et al. (2011). The supADF test-statistic of 2,264 exceeds the critical value at the 1%

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<sup>1</sup> test statistics available upon request

<sup>2</sup> test statistics available upon request

significance level implying explosive behavior in the log real STOXX price index. In contrast, the log real STOXX dividend series does not reveal explosive behavior during July 1999 and December 2015. The summary of the test-statistics and critical values can be seen in table 1.

To briefly recapture the methodology, we want to find explosiveness in the price series and non-explosiveness in the dividend series to reveal financial exuberance. Due to the fact that there is no dividend data available for the exact same time period as for the price index, there is a lack in the inferences for the time period from 1987-1999.

Series	J	supADF	Significance	Critical values
STOXX PI	8	***2,264	***1%	1,994
STOXX DIV	0	0,247	**5%	1,429
			*10%	1,154

Table 1: test statistics of the full sample period (1987 – 2015)

In order to date stamp the beginning and collapse of the asset price bubble the forward recursive ADF test-statistics are compared to the critical values generated by applying Monte Carlo simulation to the distribution process with 10'000 simulations. In all the testing performed throughout this thesis, critical values are applied at a 95% confidence interval. As can be seen in figure 6, the first evidence of explosive behavior in the log real STOXX price index emerges in July 1997 and lasts until February 2001. In January 1998 and October 1998, the ADF test-statistics drop quickly below the critical values. In general, the log real STOXX dividend series do not behave explosively from July 1999 until December 2015. Interesting is however that following the collapse in the price index in February 2001, the dividend series contrarily exceeds the critical values for the consecutive two months, March and April 2001.

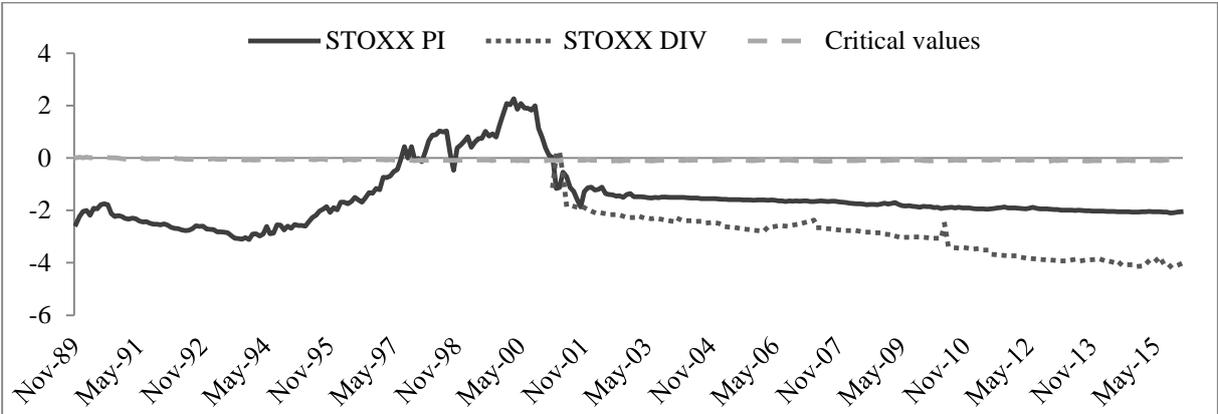


Figure 6: 1987-2015 ADF values, generated by the forward recursive estimation method

As applied in Phillips et al. (2011), we also construct a supplementary testing for the time period from 1987 to 2015 estimating regressions based on rolling windows. The initial window of the log real STOXX price index consists of 70 observations which is 20% of all observations from the data set. The starting point of the initial window is January 1987 and rolled over monthly until December 2015 keeping the same number of observations. The lag length is the same as for the forward recursive estimation method with  $J = 8$  for the price index resp.  $J = 0$  for the dividend series. The rolling ADF test-statistic of 1,945 exceeds the critical value at the 1% significance level. This again implies explosive behavior of the price index, as found in the forward recursive supADF testing.

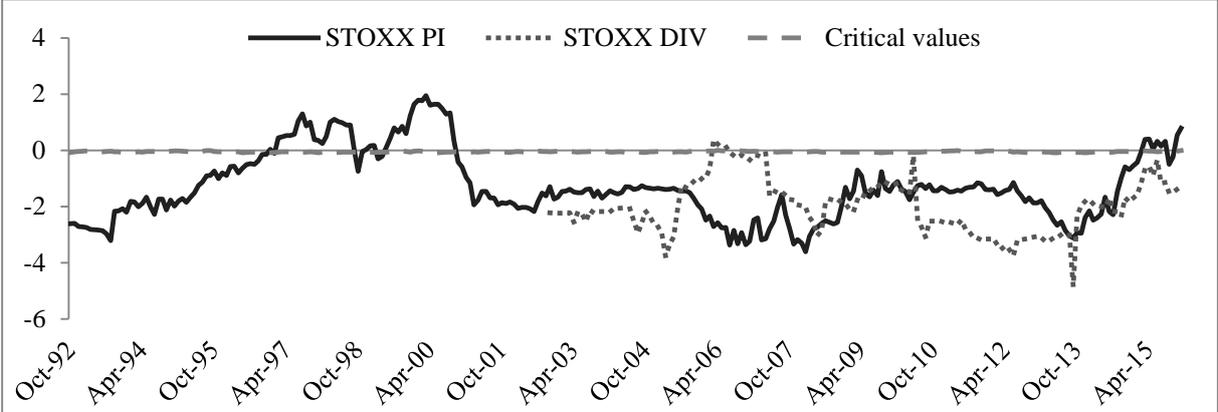


Figure 7: 1987-2015 ADF values, generated by the forward rolling estimation method

In figure 7, we see the origination of financial exuberance in the price index is dated to December 1996 and continues until October 2000. During this period, the test-statistics fall below critical values in the January 1997, October 1998 and March-April 1999. This pattern is similar to the first hints of financial exuberance found when applying forward recursive regressions. However, the date stamping in using rolling regressions reveals the bubble process in its origination and collapse earlier than in the forward recursive estimation method. The emergence of financial exuberance starts in December 1996 whereas its termination is date stamped in October 2000. Towards the end of the sample period, the test-statistics for the price index exceed the critical values from March 2015 until August 2015 and November 2015 until December 2015. This is however not revealed by the previous testing with forward recursive regressions.

The same test procedure of rolling window is applied to the log STOXX dividend series from July 1999 to December 2015 where the initial window contains 40 observations which is 20% of the entire data sample. In the date stamping, the dividend series exhibits explosiveness from March 2006 until June 2006 whereas the price index remains non-explosive.

In the following paragraph, we compare the STOXX data result retrieved by forward recursive resp. rolling window estimation to the findings on the NASDAQ price index and dividend series of Phillips et al. (2011). Applying the forward recursive estimation technique, we find explosive behavior in the STOXX price index, but not in the dividend series. This is also identical to the results found in the NASDAQ data by Phillips et al. (2011). Now, when it comes to date stamping the origination and collapse of the asset price bubble, the STOXX price index moves few times between explosive and non-explosive behavior for some short time periods from 1997 to 2001 whereas the NASDAQ price index constantly reveals explosiveness. It seems that the STOXX price index undergoes a slight double bubble induced by factors which do not impact the NASDAQ price index to break down twice in the same time period. Remarkably, the second collapse of the STOXX price index is pinpointed almost exactly in the same month as in the NASDAQ price index, i.e. STOXX price index in February 2001 compared to March 2001 for the NASDAQ price index. Looking at the origination, Phillips et al. (2011) date stamp the origination of the bubble for the NASDAQ price index in July 1995. Our date stamping for the STOXX price index reveals July 1997.

Comparing the results from estimations based on rolling windows between STOXX and NASDAQ, the collapse of the STOXX price index is dated in October 2000 which is somewhat earlier than found in using forward recursive regressions. However, this is in line with the findings by Phillips et al. (2011). They find the date stamp of the collapse of the NASDAQ price index in September 2000 using rolling windows which is earlier compared to March 2001 using forward recursive regressions. Furthermore, Phillips et al. (2011) date stamp the origination of the dotcom bubble in July 1995 consistently for both using forward recursive resp. rolling window regressions. This is different to our findings where the date stamp of the origination of financial exuberance happens with half a year difference between the two estimation methods.

## **5.2 Subsample period (2005 – 2015)**

The aftermath of the dotcom bubble until the end of 2015<sup>3</sup> is a period of interest for this research and so we originally constructed a subsample from 2002 to 2015. However, the test-statistics based on the forward recursive resp. rolling window estimation method led to highly inconsistent inferences in terms of bubble detection and date stamping. Thus, we decided to replace the subsample period from 2002 to 2015 with a subsample from 2005 to 2015.

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<sup>3</sup> The most recent available data when we started this research

This subsample represents the continuation of the data which Phillips et al. (2011) used in their research to test the NASDAQ on explosive behavior between 1973 and 2005. In addition, moving further away from the dotcom bubble would imply less contaminated data which in turn would suggest less sensitivity of the forward recursive estimation towards the initial value of the regression (Shi, 2010).

Standard right-tailed ADF tests on the 2005-2015 subsamples are applied to the log real price index and the log real dividend of both the STOXX as well as the NASDAQ index. The optimal lag lengths are computed on a 5% significance level with the maximum lag length set to six. The decreased number of maximum lags is due to the limited amount of observations in this subsample, which consists out of 126 monthly observations. As suggested by Phillips et al. (2011), a 10% initial startup window is used, resulting in a window of 12 observations. All series demonstrate zero significant lags on a 5% significance level, with the exception of the dividend series of NASDAQ. This particular series demonstrate one significant lag.

<b>Series</b>	<b><i>J</i></b>	<b>supADF</b>	<b>Significance</b>	<b>Critical values</b>
STOXX PI	0	**1,980	***1%	2,115
STOXX DIV	0	0,208	**5%	1,470
NASDAQ PI	0	***2,139	*10%	1,157
NASDAQ DIV	1	*1,258		

*Table 2: test statistics of the 2005 - 2015 subsample period*

Applying the forward recursive estimation method, the price index series of STOXX and NASDAQ demonstrate supADF test-statistics of 1,980 respectively 2,139. Consequently, the null hypothesis of a unit root can be rejected on a 95% confidence level for the price index series of STOXX and on a 99% confidence level for the NASDAQ price index series. The forward recursive ADF results in non-rejection of the null for the log real dividend series of both indices. However, the supADF test-statistic of the NASDAQ dividend series is relatively high and results in the rejection of the null-hypothesis on the 90% confidence interval. The summary of these results can be found in table 2.

Figure 8 and 9 display the log real series of STOXX price index and NASDAQ price index. Both series demonstrate exceeding test statistics relative to the critical values at the end of 2008 and the first half year of 2009, indicating explosive behavior. In addition to this relatively small bubble and after having formed a small peak in December 2010 and January 2011, the log real dividend series of NASDAQ demonstrates explosive behavior from February 2012 and onwards. This implied bubble forming in the log real dividend series of NASDAQ continues until the end of the sample period.

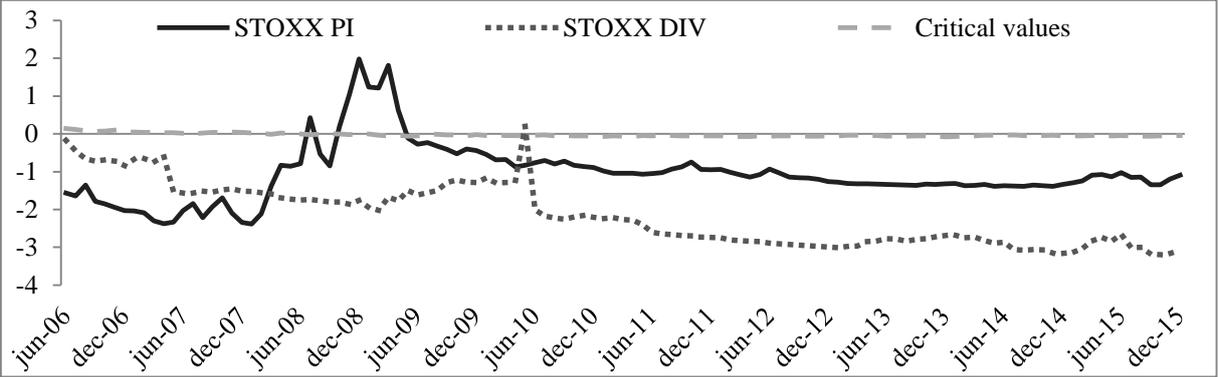


Figure 8: 2005-2015 STOXX ADF values, generated by the forward recursive estimation method

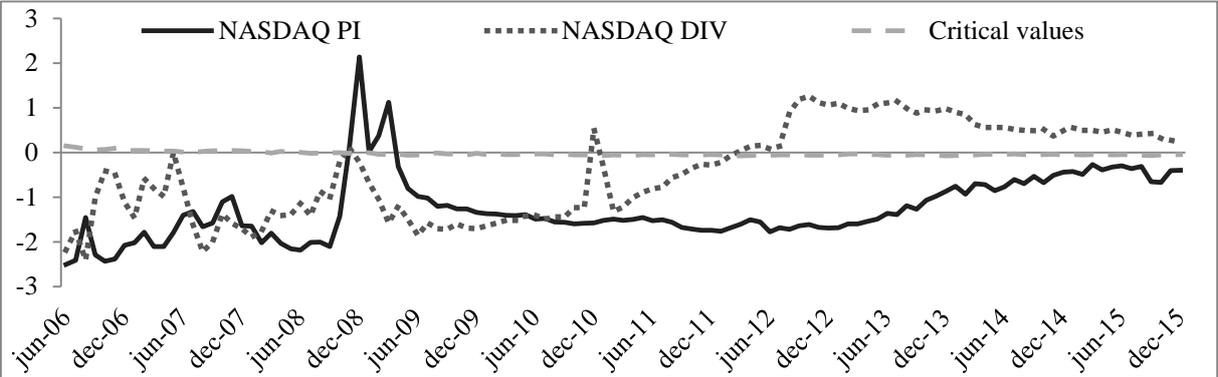


Figure 9: 2005-2015 NASDAQ ADF values, generated by the forward recursive estimation method

In this subsample, a supplementary forward rolling estimation method is implemented as well. The results of the rolling regressions imply explosive behavior in three out of four series. The log real series of STOXX price index, NASDAQ price index, and NASDAQ dividend series display test statistics exceeding the critical values on a 1% significance level, whereas the log real STOXX dividends series demonstrate explosive behavior on a 10% significance level when tested with a rolling ADF. However, the maximum significance level required to reject our null hypothesis has been set to 5% and as such this result does not lead to the rejection of the null<sup>4</sup>.

<sup>4</sup> Data available upon request

The rolling regression is based on a window size of 20% and as such it consists out of 25 observations. The same amount of lags per series is used as in the supADF. Turning to the date stamping procedure, the three series with 1% significance levels all exceed the critical values at the end of 2008 and the first half year of 2009, similar as in the forward recursive supADF testing. This is shown in figures 10 and 11.

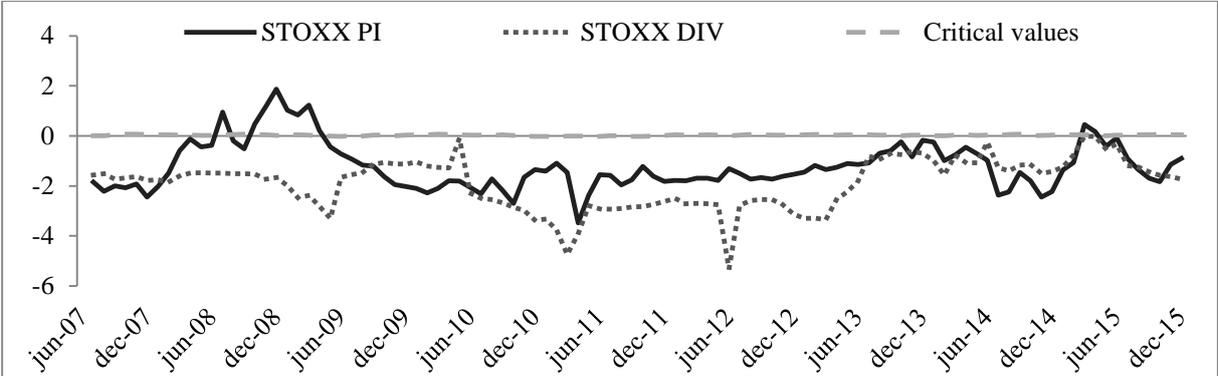


Figure 10: 2005-2015 STOXX ADF values, generated by the forward rolling estimation method

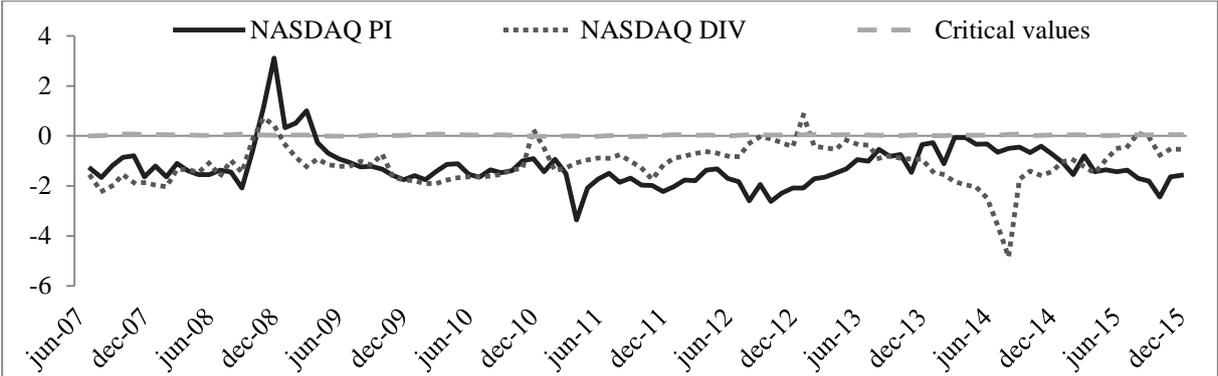


Figure 11: 2005-2015 NASDAQ ADF values, generated by the forward rolling estimation method

It can be seen from the plotted series in the figures 10 and 11 that the rolling ADF does not reveal the same explosive behavior in the NASDAQ dividend series as indicated by the forward recursive supADF results towards the end of the sample period between 2012 and 2015.

In the next section, the reported results will be analyzed more extensively, and their implications will be discussed.

**6. Analysis and discussion**

**6.1 NASDAQ, STOXX and the dotcom bubble**

The first objective of this thesis is to investigate the time series data of the STOXX price index and dividend series compared to the NASDAQ in the asset pricing bubble process between 1987 and 2005. We investigate whether there is similar explosive behavior in the

STOXX as in the NASDAQ. This would, to a certain extent, underline the statement: If the US sneezes, the EU catches a cold.

A comparison of the different test results retrieved from forward recursive and rolling window regressions on the STOXX price index and dividend series reveals that the STOXX closely follows the findings in the NASDAQ data displayed by Phillips et al. (2011) and others such as Homm and Breitung (2012) and Cuñado, Gil-Alana and de Garcia (2005). There are however two findings which stand out. First, the STOXX price index reaches explosive data behavior marginally later than found in the NASDAQ price index. This might be based on the fact that the United States of America led the development in internet technology as the base of the new economy hype during that time. However, the collapse of both indices is date stamped around the same time. Second, the STOXX price index exhibits characteristics of a double bubble with a short collapse in 1998 and an immediate re-bounce. A possible reason to explain this phenomenon is that the European financial markets might have been impacted more severely by the Russian financial crisis in 1998 than the US financial markets.

Overall, the STOXX price index and dividend series reveal similar financial exuberance compared to the findings on the NASDAQ data. This brings the thesis further to the second objective to apply the testing procedure proposed by Phillips et al. (2011) to more current data of the STOXX price index and dividend series as well as the NASDAQ price index and dividend series. The subsample is set from 2005 until 2015 in order to avoid any late distortions in the data caused by the dotcom bubble and to continue from the end of the sample used by Phillips et al. (2011).

## **6.2 Subsample 2005 – 2015**

Discussing the immediate continuation of the sample data used by Phillips et al. (2011) from June 2005 until December 2015 it can be found that the rejection resp. non-rejection of the null-hypotheses happen consistent regardless of the forward recursive or rolling window estimation method applied. In the analysis of this subsample, it can be stated that no typical rational bubble is found in this subsample and so we continue with discussing three other interesting findings which stand out. First, a ‘reversed bubble’ is found in 2008. Second, we find explosive behavior in the dividend series of NASDAQ yet not in the corresponding price index series, and third we found that the forward recursive estimation method is sensitive to the chosen initial starting point.

### **6.2.1 'Reversed bubble'**

Both the forward recursive estimation method and the rolling window method date stamp explosive STOXX and NASDAQ price index time series from late 2008 until second quarter in 2009. In the same time frame as September 2008, Lehman Brothers filed for bankruptcy and accelerated a global downswing in the different stock markets followed by an almost instantaneous partial recovery during 2009. So in contrary to an emergence followed by a collapse of the bubble as intuitively expected, the testing procedure recognizes explosive behavior in times of a 'reversed' bubble process as well. When examining the NASDAQ from 1973 until 2013, Shi and Song (2016) date stamp similar bubble behavior in the NASDAQ from May 2009 to August 2009. They claim it as "*the recovery phase of the subprime mortgage crisis in 2009*" (Shi & Song, 2016). In our research, and due to the macroeconomic environment and the irrationality of the investors in this certain period, this explosive behavior is not recognized as a typical rational bubble.

Furthermore, the NASDAQ dividend series demonstrate explosive behavior one month prior to its corresponding price index in 2008 and also terminates earlier than the price index. This is revealed by both the forward recursive regression estimates and the rolling window regression estimates. Hence, the NASDAQ price index and its corresponding dividend series behave explosively almost simultaneously. This is backed by a deduction of Phillips et al. (2011) that "*explosive characteristics in  $p_t$  could in principle arise from  $d_t$  and the two processes would then be explosively cointegrated*". So in this particular cointegrated case, and in the context of the Lehman collapse, this specific pattern of a reversed bubble cannot be judged uniquely as financial exuberance.

### **6.2.2 Sensitivity to the initial starting point**

The significant discrepancy in the results of the initially chosen 2002-2015 subsample indicates that the inferences based on the two different estimation methods evidently depend on the choice of the starting point ( $r_{start}$ ), as confirmed by Shi (2010). This non-detection by the forward recursive supADF might be induced by data close to the collapse of the dotcom bubble being included, whereas the regressions in the rolling window method lose the contaminated data as they start moving away from the starting point.

To shed some light on differences between the two regression methods, the residuals of the forward recursive regressions and the rolling window regressions were examined on the third (kurtosis) and fourth moment (skewness) in the subsample 2002-2015. If these two moments were significantly different in the different estimation methods, it would explain possible

outliers which would induce sensitivity to the distribution of the critical values. The data closer to the dotcom bubble may still incorporate a high degree of explosive behavior. This, in turn, might cause a different distribution of the ADF test-statistics with fatter tails which does not allow for detection of the bubble behavior in 2008/2009. On the other side, the rolling window estimation enables to move away from potentially contaminated data, and therefore would be less sensitive. However, the skewness and especially the kurtosis of the residuals of both regression methods do not differ significantly in the two regression methods<sup>5</sup>. Having no significant differences in the moments, we turn to a well-known issue that the power of unit root testing is sensitive towards the initial value ( $y_0$ , see eq. 12, equal to  $r_{start}$ ) in the data sample (Elliott & Müller, 2006). Even though ADF mitigates the sensitivity issue to a certain extent, it is hard to find conclusive explanations about the significant discrepancies between the inferences of the forward recursive supADF and the rolling window supADF in the subsample 2002-2015.

Both the forward recursive and the rolling window estimation method reveal bubble behavior in 2008/2009 when analyzing the subsample 2005-2015. This confirms that the forward recursive estimation method is more sensitive to the setting of the starting point of the sample period than the rolling window method. This finding is also supported in more recent literature; see for example Phillips, Shi and Yu (2013). As they elaborate in their paper (2013) data including multiple breaks might involve switches from non-stationary to stationary properties induced by possible linear or non-linear autoregressive data processes, which do not allow the forward recursive testing procedure to detect all inherent bubbles. This is where they introduce a further approach which also estimates right-tailed ADF test-statistics recursively but based on more flexible window sizes and the window's starting point. This approach can be found as generalized supADF (GSADF) test in their paper.

### ***6.2.3 Explosive behavior in the NASDAQ dividend series***

Another pattern in the NASDAQ price index and dividend series is detected from 2012 until 2015 when using forward recursive estimation method. Remarkably, the NASDAQ price index does not pick up explosiveness at the same time as the beginning of explosive behavior in its corresponding dividend series. The test results of the rolling window estimates do not perfectly confirm this pattern. However, both the forward recursive and the rolling window

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<sup>5</sup> Kurtosis results: 2,498 for rolling window and 2,889 for forward recursive in subsample 2002-2015

estimation methods applied to the full sample period 1973-2015 reveal the same patterns of explosive dividend series versus non-explosive price index series<sup>6</sup>.

It is hard to explain this behavior as rational investors would invest in stocks which pay out (higher) dividends and as such, due to the increase in demand, the price of the stock would increase as well. However, the price index series remains stationary and does not follow the explosive behavior of the dividend series. Therefore, further research on the calculation of the dividend yield of NASDAQ by Thomson Reuters was executed in order to find a potential explanation for this market irrationality. Thomson Reuters computes dividend yield as follows:

$$DY_t = \frac{\sum_1^n (D_t \cdot N_t)}{\sum_1^n (P_t \cdot N_t)} \cdot 100$$

Where,  $DY_t$  = aggregate dividend yield on day t,  $D_t$  = dividend per share on day t,  $N_t$  = number of shares in issue on day t,  $P_t$  = unadjusted share price on day t, and  $n$  = number of constituents in the index. Consequently, the yield is decreased by the presence of non-dividend paying constituents, which is important to know since the majority of the NASDAQ constituents do not engage in dividend payments. However, a growing number of constituents have started to pay out dividends to its shareholders in recent years. Between 2006 and 2014 the number of constituents actively paying out dividends almost doubled (+96%) as it went from 221 to 434. The year 2012, the origination year of the explosive behavior, demonstrates an increase of 52% in dividends per share and a 19% growth in dividend paying constituents. These results can be seen in Appendix II – recent NASDAQ dividend history. We believe this growing engagement in dividend pay-out policies by NASDAQ's constituents could well be the main reason for the development of explosive behavior in the dividend series. However, we also believe further research is required to confirm this assumption and investigate the 'dividend bubble' phenomena in more depth. In addition, this controversial dividend pattern does not unambiguously support our reasons of interest in this thesis which is the detection of current potential financial exuberance around recent large acquisitions in the technology sector and hype about unicorns. Nonetheless, we believe that the potential impact of unicorns on the stock market will be an interesting topic to further research.

In contrary to remarkable NASDAQ behavior, testing on the STOXX time series using the forward recursive estimation method reveals no signs of explosive behavior in the period of interest; neither in the price index nor the dividend series.

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<sup>6</sup> Data available upon request. Plotted ADF values can be found in Appendix I

### **6.3 Testing for robustness**

In order to test our results and confirm their robustness, two steps have been taken. STOXX data of the 2005-2015 subsample has been tested while being transformed by different Consumer Price Index rates. The second test for robustness has been performed by testing the full sample periods of both indices with lags based on Akaike Information Criterion (AIC).

As previously described, the data are transformed from nominal into real values. Since the current political European Union was formed later than the launch of the STOXX index, a uniform consumer price index was not collected methodically for the exact same time period. Therefore, we computed an average CPI for the most important economical countries in which the STOXX constituents are located. To check the results based on the average CPI on robustness, we tested the STOXX price index and dividend series in the subsample 2005-2015 by transforming the data using the methodically and uniformly collected CPI for 28 European countries since the constitution of the political EU. A third CPI is applied in which only countries take part having the euro as currency. Overall, all results are exactly consistent with the previously applied average CPI (see Appendix III). This gives support to the findings in two ways. First, the transformation from nominal into real values does not impact the results. Second, the approximation using the average CPI of the most important economic countries can be assumed to be appropriate for the time period prior to the methodically and uniformly collected CPI.

In the second robustness test we look at the lag lengths used in our tests. So far, the regression models and their corresponding lag length have been determined by selecting the last included lag at the 5% significance level according to the approach suggested by Campbell and Perron (1991). In order to check the results on robustness, we test the NASDAQ price index and dividend series from 1973 to 2005 and the STOXX price index and dividend series from 1987 to 2015 using an automatic selection of the lag length based on the AIC. The AIC determines the lag lengths more consistent among the regression models, based on different subsamples (Brooks, 2014). Nevertheless, the AIC will often include a larger amount of lags than other ICs such as Schwarz's Bayesian information criterion (Brooks, 2014). No individual IC is yet found superior to other ICs (Emerson, 2007) (Brooks, 2014). Since the lag lengths chosen based on Campbell and Perron (1991) resulted in regression models with mostly a high number of lags, the AIC would lean to this in that sense that the AIC would rather include a high number of lags as well. Therefore, the AIC seems to be the most appropriate IC.

It can be seen in appendix IV that choosing between different lag lengths results in different test-statistics. This is supported by Emerson (2007) who found that different information criterion often find different optimal lag lengths. The estimated test statistics show relatively large sensitivity to the chosen lag order (Emerson, 2007). However, in this research, it did not change the inferences made about the null hypotheses except that the null hypothesis of the NASDAQ price index is rejected on a 95% confidence interval using AIC instead of rejecting on a 99% confidence interval using the approach based on Campbell and Perron (1991).

## **7. Conclusion**

This research sets out to determine whether the European tech stock markets experienced a similar dotcom bubble as the NASDAQ in the 1990's and early 2000's. In a next step, both the markets were tested on asset pricing bubbles using more recent data (2005 – 2015). The main findings will be highlighted in this section.

Returning to the question posed at the beginning of this study, the suspicion was that the STOXX European Technology index closely follows the NASDAQ in the dotcom hype. This suspicion of similar behavior was confirmed as the index demonstrated the same bubble process. Turning to the second part of the research question posed in the introduction, no typical rational bubble was found in the NASDAQ and STOXX when testing the 2005-2015 subsample. However, we did find two interesting alternative outcomes. First, both indices were subject to explosive behavior in 2008/2009 in the aftermath of the Lehman collapse. Based on the tremendous market uncertainty during these times, the reversed order of the price movement (down and then up), and the short blaze of explosiveness in the data series, we entitle this as a reversed bubble rather than long-lasting financial exuberance. Second, explosive behavior in the dividend series of the NASDAQ data was found, starting in 2012, whereas the price index remains non-explosive. This is against intuition that explosively increasing dividends would rationally also induce explosively share prices in accordance to the dividend-stock price model.

In line with these investigations, we checked if bubbles are detected consistently when the data are divided into different subsamples. The study has confirmed the findings of Shi (2010) who found that the forward recursive estimation method is sensitive to the initial starting point. Differences of bubble detection respective non-bubble detection in the 2005-2015 subsample and the 2002-2015 subsample revealed the sensitivity of the model to the initial starting point for the practical testing implementation of the forward recursive estimation

method. The discrepancy between the inferences based on the forward recursive - and forward rolling estimation methods in the 2002-2015 subsample confirm this issue.

After an extensive period of research on asset pricing bubble criticism to our research and suggestions for further research have surfaced. Unfortunately, due to time constraints, these matters were outside of the scope of this research and as such only treated shortly in the following paragraphs.

When working on the methodology for this research, the following question kept going through our minds: why do we always use the present value of current and future expected dividends as the pricing mechanism of the fundamental stock price? Valuations through discounted dividend model or dividend-stock price model are sensitive to various factors (growth assumptions, accounting principles, etc.). In addition, dividend smoothing may create an inaccurate image of the intrinsic value. So instead, might a different valuation method, e.g. the DCF valuation with FCFF as input to the practical testing implementation suit better as an approximation of the fundamental stock price? Further research could answer to this.

Another suggestion for further research is related to the explosive behavior of the NASDAQ dividend series. Recent dividend paying history of the NASDAQ constituents reveals highly fluctuating figures. As such, we question whether dividend smoothing still is a current phenomenon. Does the NASDAQ composite demonstrate otherwise?

The most visible critique is that this study was limited by the absence of dividend data on the STOXX index. A rational bubble is defined in the dividend-stock price model by explosive behavior in the price index and non-explosive behavior in the dividend series. As we cannot exclude the possibility of explosive behavior of the dividend series due to this lack of data, no irrefutable conclusions can be drawn based on the explosive behavior of the price index.

We refer to another recent research for further criticism to our research. Shi (2010) argues that when multiple breaks occur in the same sample period, the proposed model by Phillips et al. (2011) typically diminishes in discriminatory power.

Final critique we have is that the practical approach by Phillips et al. (2011) does not extract the causes of the bubble (e.g. GDP downturn in China, etc.). The approach solely defines asset pricing bubble in term of explosive behavior in the time series. Therefore, the model does not have any explanatory power; it can only identify a certain phenomenon. Hence, it can be questioned if the testing procedures could be extended to reveal the cause of a bubble, which is subject to further research.

## 8. Appendices

### 8.1 Appendix I – Date stamping NASDAQ dividend bubble

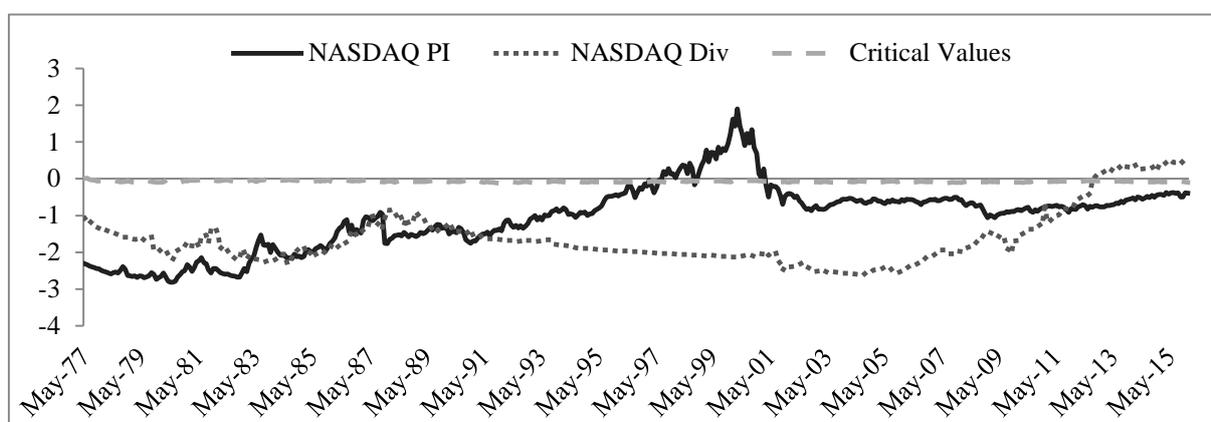


Figure 12: 1973-2015 NASDAQ ADF values, generated by the forward recursive estimation method

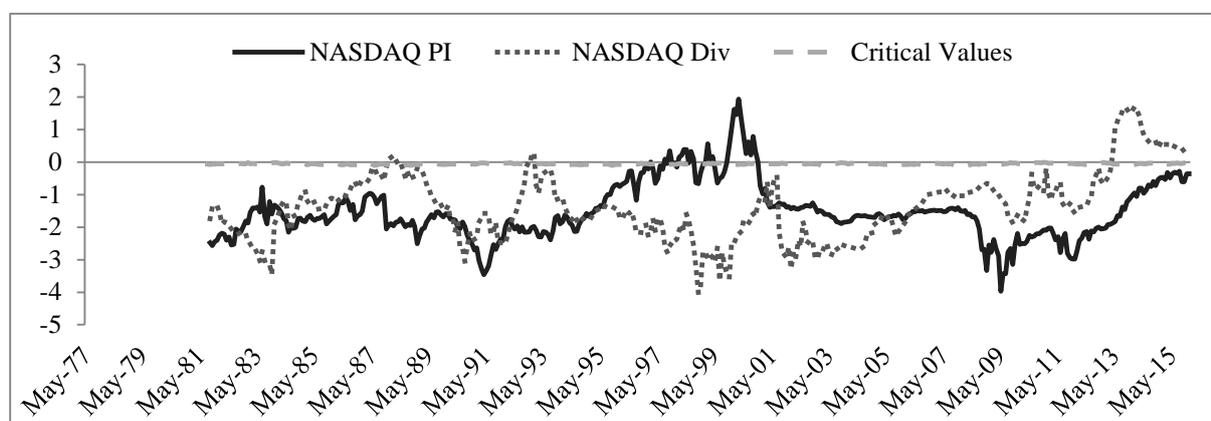


Figure 13: 1973-2015 NASDAQ ADF values, generated by the forward rolling estimation method

### 8.2 Appendix II – Recent NASDAQ dividend history

NASDAQ DIVIDEND HISTORY	2015	2014	2013	2012	2011	2010	2009	2008	2007	2006
Percentage Of Total Constituents Paying Dividend	15,8%	17,6%	16,8%	16,0%	13,5%	11,9%	10,7%	11,4%	11,3%	9,0%
Amount Of Dividend Paying Constituents	389	434	413	394	331	292	263	281	278	221
Annual Growth Rate Dividend Paying Constituents	-10%	5%	5%	19%	13%	11%	-6%	1%	26%	-
Total Dividends Paid Per Share (\$)	0,94	0,85	0,78	1,10	0,72	0,85	0,75	0,98	1,36	0,69
Annual Growth Rate Dividends Paid Per Share	11%	9%	-29%	52%	-15%	13%	-23%	-28%	98%	-
Total Dividends Paid (million\$)	57116	65949	71540	49886	35090	37264	43495	75193	32433	20852
Annual Growth Rate Dividends Paid	-13%	-8%	43%	42%	-6%	-14%	-42%	132%	56%	-
Average Dividend Paid Per Company (million\$)	147	152	173	127	106	128	166	268	117	94
Annual Growth Rate Average Dividends Paid	-3%	-12%	37%	19%	-17%	-23%	-38%	129%	24%	-

Total amount of constituents: 2463

Table 3: NASDAQ dividend history 2006-2015 (source: Orbis)

### 8.3 Appendix III – Testing on STOXX data with different CPI's

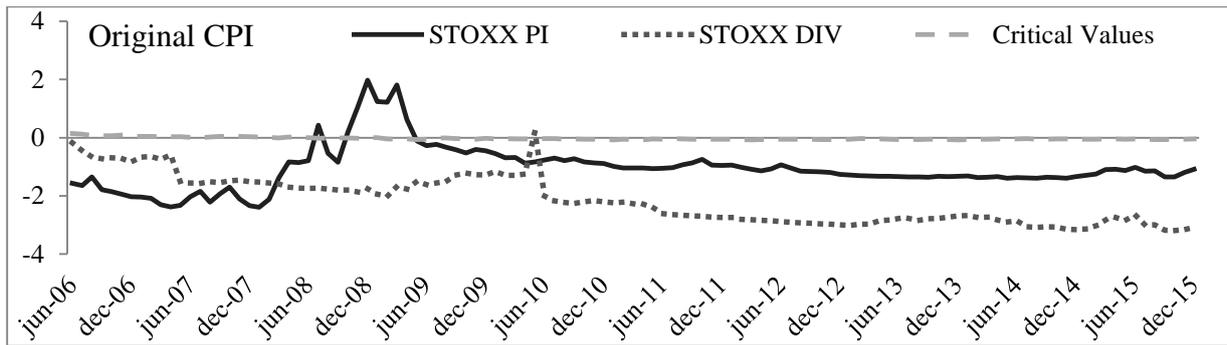


Figure 14: 2005-2015 STOXX ADF values, generated by the forward recursive estimation method

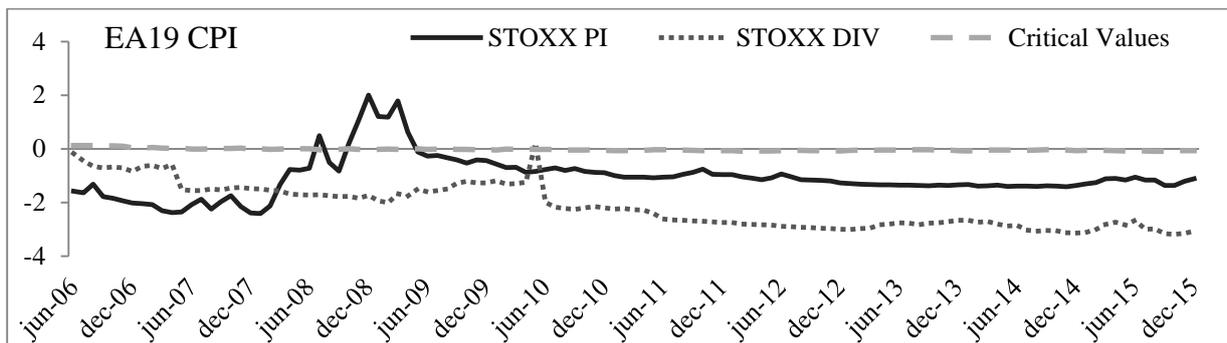


Figure 15: 2005-2015 forward recursive ADF values. Data transformed by CPI of Euro Area

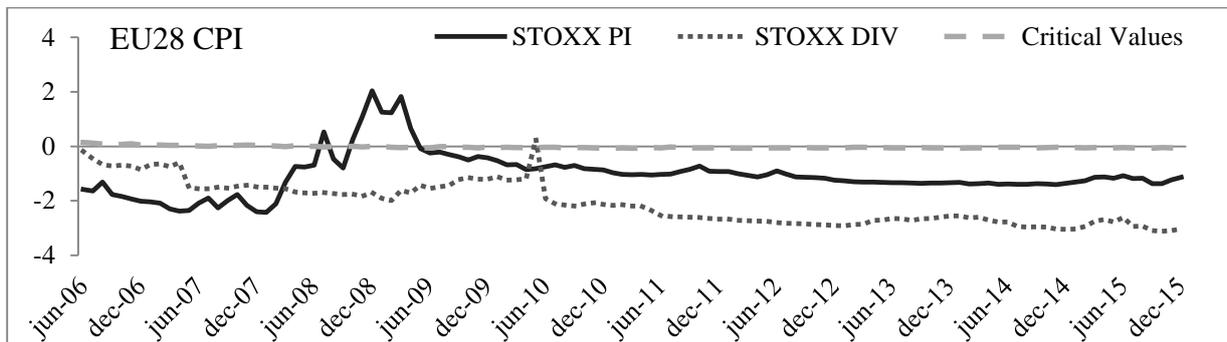


Figure 16: 2005-2015 forward recursive ADF values. Data transformed by CPI of European Union

### 8.4 Appendix IV – Different lag lengths

Index Series	NASDAQ 1973-2005		STOXX 1987-2015	
	price index	dividend	price index	dividend
AIC lag length	1	0	0	0
sup ADF	**1,896	-1,066	***2,809	0,247
Campbell & Perron lag length	10	12	8	0
sup ADF	***2,145	0,594	***2,264	0,247
critical values 1% ***	2,019	2,019	1,994	2,067
critical values 5% **	1,443	1,443	1,429	1,447
critical values 10% *	1,143	1,143	1,154	1,144

Table 4: Test results based on different lag lengths

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