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Models of Bankruptcy Prediction Since the Recent Financial Crisis: KMV, Naïve, and Altman's Z- score

NEKN02

by

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Abstract

Corporate bankruptcy prediction has become a popular research topic since 1960s, and default risk management plays a more significant role among investors, debtors and the lenders. The Altman's z-score model, the KMV model, and the Naïve model are well-known and widely used bankruptcy prediction models. This paper focuses on examining the bankruptcy prediction accuracy of these three models in the U.S. market since the Financial Crisis from 2007 to 2012. After comparing each model's performance, we find that the Naïve model has the best default prediction power in the whole industries. Since the financial industry includes high leveraged firms, pervious researches usually do not specifically analyze financial firms. After analyzing different industries, we find that KMV model has the best bankruptcy prediction power for all non-financial firms, while the Naïve is the most effective prediction model for financial firms. For all industries, we recommend to use the Naïve model to predict the default risks.

Keywords: Default risk, Altman's Z-score, KMV model, Naïve model, Receiver Operating Curve.

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1. INTRODUCTION

Qualitatively and quantitatively predicting a firm's bankruptcy probability is valuable for both creditors and investors, especially during financial crisis period, because the market is more volatile and risky. Most commonly used models are Altman's z-score, KMV model and the Naïve model, but they provide relatively different prediction accuracy depending on different research situations. Thus, we focus on studying prediction accuracy of these three models since the current financial crisis. The Altman's z-score model is an accounting-based measure, while the KMV model and the Naïve model are the market-based measures. Meanwhile, these three models are fairly useful to investors and creditors.

Altman's Z-score (1968) model combines several different accounting variables to predict a firm's probability of default, and they can be observed from the firms' financial statements. On the other hand, the market-based model predicts a firm's bankruptcy probability mainly through option view of a firm's equity. The market equity variables can be observed from the stock market, which include both the financial statements information and the market information. Thus, we want to study which model's variables include more available information regard the probability of default.

This paper attempts to understand which model can provide a better bankruptcy prediction since the financial crisis in 2007. Additionally, since financial firms are different due to high leverage, this paper assesses whether the prediction models are different between the financial industry and the non-financial industry.

According to our research results, we find that the Naïve model provides the best prediction performance for all firms. For all non-financial firms, the KMV model has the best bankruptcy prediction power, while the Naïve model is the most effective prediction model for financial firms. Meanwhile, Altman's Z-Score model is considered as a relatively weak prediction model.

In this paper, we choose one accounting-based model and two market-based methods respectively, and test their bankruptcy prediction power between 2007 and 2012. The contribution of this paper is to help the financial institution to use the better model to evaluate firm's credit default risk in the American market when the economic is close to collapse. We also test if these models have predictive power for financial firms, and test the cut-off points for each model in different industries.

The paper continues as follows: Chapter 2 includes the previous research on comparing accounting-based models and market-based models, and the research about KMV model and Naïve model. In Chapter 3, we report the description of our models, such as Altman's Z-score, KMV model, and Naïve model, the methodology of testing model, the Receiver Operating Characteristic curve and Area Under the Curve. Then, our data selection procedure is discussed in Chapter 4. In Chapter 5 and 6, we describe summary statistics, and present our results. Finally, we make our conclusion of the paper in Chapter 7.

2. Previous Research

Corporate bankruptcy prediction is a popular study area, and several researchers have relied on accounting measures or market measure as variables to study the bankruptcy prediction, and these researches provide different comparison results.

Some literatures show that the market-based models have better predictive power for default. For instance, Hillegeist et al. (2004) compare Altman's z-score (1968), an accounting-based model, with Black-Scholes-Merton Probability of Bankruptcy (BSM-PB) which is a market-based model. They select data from 1979 to 1997, and use discrete hazard model to test and make comparison for each model. Their results show that the BSM-PB model outperforms the Altman's z-score, but they also illustrate that the z-score is still statistically significant when pooling in regressions with BSM-PB. Moreover, Wu et al. (2010) apply Receiver Operating Characteristics (ROC) curve to test the predictive power between accounting-based models such as Altman's z-score (1968), and Hillegeist's BSM-PB model (2004). They find that the probability of default will increase if the profitability decreases, and the results also show that Altman's z-score has poor predictability of default relative to Hillegeist's BSM-PB performance. Korablev and Dwyer (2007) compare Moody's KMV EDF with Altman's z-score (1968) and simplified Merton model. They collect data from 1996 to 2006 and test the models respectively to different geographical areas, including North America, Europe, and Asia. Their results demonstrate that the Moody's KMV EDF model is superior to other models no matter which area and firm size. Also, Campbell et al. (2008) use a dynamic logit model with accounting and market variables and compare these models. They find that market-based ratios outperform the book-based ratios and propose that market prices incorporate current information more rapidly. According to these researches, the market-based models provide better bankruptcy prediction performance.

On the other hand, some researches have suggested that both accounting-based and market-based methods have effective power to predict default. Reisz and Perlich (2004) analyze the data from 1988 to 2002 in United States, and find that Altman's (1968) Z-score has a slightly better prediction power in bankruptcy over one year period, but they find the KMV model and other market-based models are more effective over longer horizons. Agarwal and Taffler (2008) study the public non-financial firms in the United Kingdom from 1985 to 2001, their results show that the z-score is slightly more accurate, but both accounting-based method and market-based method reflect important evidence about default. Warren Miller (2009) compares Altman's Z-score, and market-based models under Morningstar and tested their ability to predict default. He uses the cumulative accuracy profile and accuracy ratio to estimate their predictive power. The result shows the market-based model outperforms accounting-based model, but z-score has the same predictive power when the companies are relatively safe. Furthermore, according to Bharath and Shumway's research (2008), they conclude that the Merton DD model is a suitable way of forecasting default, but not a statistical sufficiently for default. From these researches, it is hard to say which model can provide better prediction performance.

There are no previous research about comparing default prediction models for financial firms, however, some paper discuss if the distance to default can be used in financial industry. Crosbie and Bohn (2003) state that the KMV model can be used for financial institutions. Yet, Alistair Milne (2014) collects 41 largest banks in the world and prove that the distance to default has poor predictive power during financial crisis.

To sum up, according to previous researches, we find there is no consistent result of the best bankruptcy prediction model. Also, there is no identical result if the model can be applied to financial institutions. In addition, most researches do not compare default prediction models in financial industry. Therefore, this is our purpose and motivation to do this research. From Korablev and Dwyer's (2007) research, we know that the KMV model is better the Merton model. Also, according to Bharath and Shumway's (2008) research, it shows the Naïve model is an effective bankruptcy prediction model. Therefore, we choose the KMV model and the Naïve model to be the representatives of the market-based bankruptcy prediction models. Through the empirical results in our research, we want to test the best prediction model. Moreover, we want to study the best model in financial industries and non-financial industries.

3. MODEL & METHODOLOGY

This chapter introduces the main bankruptcy prediction models and the methodology. First, we will describe the Altman's Z-score model, the KMV model, and the Naïve model. Then, we will introduce our methodology, which can be used to compare each model's prediction accuracy.

3.1 Model Description: Altman's Z-score

The Altman's z-score, published by Edward I. Altman (1968), is commonly referred to as the z-score model. Because the model is easy to apply, it becomes one of the most famous and widely used bankruptcy prediction models for the public manufacturing firms. Altman, Danovi, and Falini (2010) state that after the original Altman's z-score model is published, the model is developed by Edward I. Altman during the following year, and the model's predicting accuracy and applicability have been improved. The developed models also can be applied for private firms and non-manufacturing companies. However, z-score (1968) model is still one of most wildly used corporate bankruptcy prediction model. This paper will focus on the original Altman's z-score model since our research samples are all public firms.

The Altman's z-score model is estimated by a linear combination of five ratios with five weighted coefficients. They are liquidity ratios, cumulative profitability ratios, asset productivity ratios, market based financial leverage ratios, and capital turnover ratios. The liquidity ratio measures the firm's distress risk, and it means if the firm has enough working capital to cover its short-term debt. The cumulative probability ratio represents the amount of reinvested earnings or losses. The asset productivity ratio, which is similar to return of assets, shows the firm's ability of generating profits based on its assets before tax and interest. The market based financial leverage ratio is valued how far the firm's assets can decline before firms go bankruptcy. The capital turnover ratio shows the evaluation of the firm's ability to generate sales by using its assets.

To reasonably weight each ratio, Altman (1968) selects a group of firms with bankruptcy announcements and another matched firms which have survived. When analyzing the selected data, Altman proves that half of the 66 original data sample corporations have filed bankruptcy, and all the business database are manufactures and they are matched by approximate size of assets. Based on the Altman's research results, Altman creates the z-score linear equation

$$\boldsymbol{Z - Score = 1.2A + 1.4B + 3.3C + 0.6D + 0.999E} \qquad \textbf{(1)}$$

Where:

A = Working Capital/Total Asset ***(WC/TA)***

B = Retained Earnings/Total Assets ***(RE/TA)***

C = Earnings before Interest & Tax/Total Assets ***(EBIT/TA)***

D = Market Value of Equity/Total Liabilities ***(ME/TL)***

E = Sales/Total Assets ***(S/TA)***

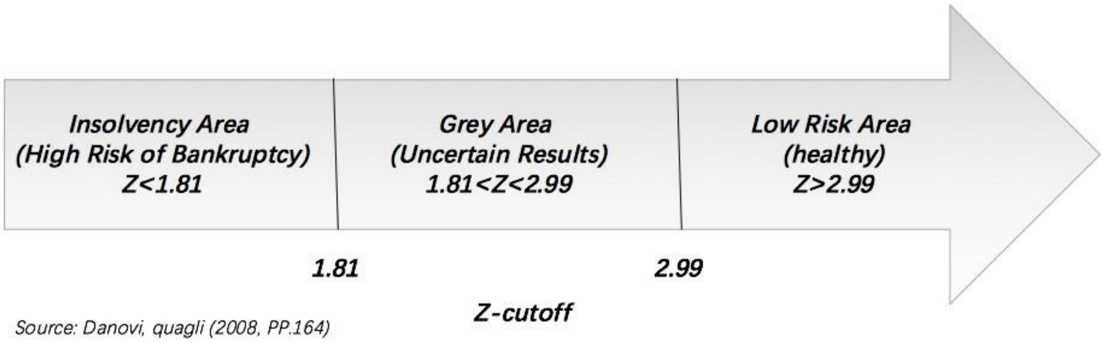

Figure 3.1: The Classification Areas of Altman's Z-score

(Note: if z-score is higher than 2.99, firms have low default risk; if z-score is lower than 1.81, firms have higher risk to default; if z-score is between 1.81 and 2.99, the default risk is median.)

This paper accesses the application of the Altman's z-score model (1968) to examine the model's prediction accuracy. According to the Altman's z-score model, the z-scores include three classification intervals, which show on **Figure 3.1**. When the Z-score is below 1.81, it means that the firm probably faces high risk to go bankruptcy, while the score above 2.99

means the firm is unlikely to go bankruptcy. That is, lower score indicates higher likelihood of bankruptcy, and vice versa.

3.2 Model Description: KMV Model and Naïve Model

KMV model and Naïve model are based on Merton model. Merton (1974) proposes a method to produce probability of default through Black-Scholes (1973) option pricing model.

3.2.1 Background: Merton Model

The Merton model incorporates more market information, which is the largest distinct between Altman's z-score and Merton. The basic idea of the Merton model is that the firm will default if its market asset value is less than its liabilities. On the other hand, if the market asset is greater than its liabilities, the firm paybacks the debt and distributes the remaining earnings to its equity holders and remains solvent. Therefore, the equity $E_T = \max(A_T\text{-}D, 0)$, where we call the amount of debt (D) to be paid at the maturity date as the default point. The equity of the firm is regarded as a call option on the underlying assets with a strike price, which equals to the firm's book value of its debt (D). In this model, the most important variables are market asset value and asset volatilities. However, these two variables are unobservable. Therefore, we use market equity value and equity volatility, and solve the simultaneous equations with the two variables to estimate market value of asset and volatility of asset.

There are some assumptions on the validity of the Merton model, which include: (1) the capital market is frictionless, which means that there are no transaction costs or taxes; (2) there are no arbitrage opportunities; (3) the risk-free interest rate is constant; (4) there are no bankruptcy costs; (5) the firm issues only zero-coupon bonds mature at time T; (6) Assets are log-normal distributed; (7) Assets follow the geometric Brownian motion (GBM).

We can use the Merton model for the firm's equity

$$\mathbf{E} = \mathbf{A} * \mathbf{N}(\boldsymbol{d_1}) - \mathbf{D}\boldsymbol{e^{-rT}} * \boldsymbol{N}(\boldsymbol{d_2}) \qquad \textbf{(2)}$$

$$\boldsymbol{d_1} = \frac{\ln\left(\frac{A}{D}\right) + [\mu_A + 0.5\sigma_A{}^2]T}{\sigma_A\sqrt{T}} \qquad \textbf{(3)}$$

$$\boldsymbol{d_2} = \boldsymbol{d_1} - \boldsymbol{\sigma_A}\sqrt{\boldsymbol{T}} \qquad \textbf{(4)}$$

where E is the equity value of the firm, μ_A is an estimated expected annual return of the firm's assets. We apply $\mu_A = R_f + 0.06$, and N(·) is the cumulative standard normal distribution function. Since asset value and asset volatilities are unknown variables and we only have one equation, we include the Ito's lemma which shows the equity value is a function of the firm value and volatility

$$\boldsymbol{\sigma_E} = \left(\frac{A}{E}\right)\frac{\partial E}{\partial A}\boldsymbol{\sigma_A} = \left(\frac{A}{E}\right)\boldsymbol{N}(\boldsymbol{d_1})\boldsymbol{\sigma_A} \qquad \textbf{(5)}$$

where d_1 is from equation (3). Therefore, we can solve equation (2) and (5) simultaneously and get the values of A and σ_A; then, we can use them to calculate Distance to Default (DD) of the firm, defined by

$$\mathbf{DD} = \frac{\ln\left(\frac{A}{D}\right) + [\mu_A - 0.5\sigma_A{}^2]T}{\sigma_A\sqrt{T}} \qquad \textbf{(6)}$$

DD can be seen as the normalized distance between the asset value of the firm (A) and the face value of its debt (D).

The probability of default (PD), or called expected default frequency (EDF) is

$$\mathbf{PD} = \mathbf{N}(-\mathbf{DD}) = \boldsymbol{EDF} \qquad \textbf{(7)}$$

3.2.2 KMV Model

Moody's KMV improves some disadvantages of Merton model. One of them is the setting of default point. In practice, the firm's liability would not mature at the same date, which means that the firm does not have to repay the total liabilities. Therefore, in KMV, the default point is suggested as the **short-term debt + 0.5* long-term liabilities**. Jarrow and Turnbull (2000) suggest another disadvantage in Merton model: since the financial data have the property of fat-tails, the risk will be easily underestimated under normal distribution. Thus, the KMV model has improved the drawback; that is, the model becomes less dependent on the underlying distributional assumptions. Crosbie and Bohn (2003) propose a process with three steps to calculate Moody's KMV probabilities of default. (1) Estimate the market value of assets and asset volatility; (2) calculate the distance-to-default; (3) transform the distance-to-default into probability of default.

The method to estimate the distance to default is slightly different from the Merton's model. In the KMV world, distance to default (DD) is calculated

$$\mathbf{DD} = \frac{\boldsymbol{Market\ Value\ of\ Assets - Default\ Point}}{\boldsymbol{Market\ Value\ of\ Assets * Asset\ volatility}} \qquad \textbf{(8)}$$

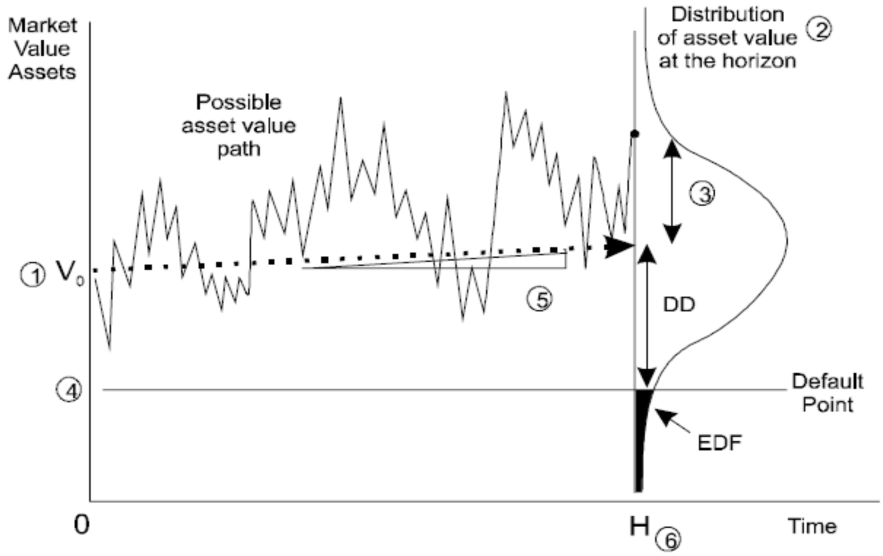

Figure 3.2: Calculate Distance to Default (Crosbie and Bohn, 2003)

*(Note: ①asset value at T=0； ②the distribution of asset value at the horizon; ③ volatility of future asset value; ④ the level of default point = short-term debt+0.5*long-term debt; ⑤the expected growth rate of asset value over the horizon; ⑥the length of horizon.)*

To sum up, KMV model needs six variables, which include market equity value, equity volatility, long-term debt, short-term debt, risk-free rate, and time horizon. In our research, we set the time horizon as one year. Then, we have to solve the equation (2) and (5) simultaneously to get the market value and volatility of asset. We also use the equation (8) to calculate distance to default for KMV model. The summary is shown in **Figure 3.2**.

3.2.3 Naïve Model

Bharath and Shumway (2008) propose an alternative model, the Naïve model, and they prove that the Naïve model has better predictive power than other models. They simplify the Merton model which can be simply calculated for the distance to default instead of solving equation (2) and (5). The advantage of Naïve model is that the variables can be easily observed or estimated, and it is a single equation model.

Bharath and Shumway's Naïve model approximates the default point as the face value of the firm's debt

$$\textbf{Naïve } \mathbf{D = F} \tag{9}$$

They suggest the volatility of each firm's debt is related to the equity volatility, they add five percentage points and quarter time equity volatility to let volatility related to default risk. Therefore, they approximate the debt volatility as

$$\textbf{Naïve } \boldsymbol{\sigma_D = 0.05 + 0.25 * \sigma_E} \tag{10}$$

The total volatility of the firm can be approximated by equity volatility and debt volatility, and the weights are corresponding to the proportion of the firm's value respectively.

$$\textbf{Naïve } \boldsymbol{\sigma_V = \frac{E}{E+F}\sigma_E + \frac{F}{E+F}(\textbf{naive } \sigma_D)} \tag{11}$$

Next, they set the firm's stock return, r, which is a year before maturity as the expected asset return on the firm. Then we can use the market equity value, total debt value from equation (9), risk-free rate, volatility of firm value from equation (11), and time horizon to estimate the Naïve distance to default

$$\textbf{Naïve DD} = \frac{\ln[(E+F)/F]+(\mu_A-0.5 \text{ Naïve } \sigma_V)T}{\text{Naïve } \sigma_V\sqrt{T}} \tag{12}$$

We can map the distance to default into probabilities

$$\mathbf{PD_{naive}} = \mathbf{N(-Naïve\ DD)} \tag{13}$$

3.2.4 Estimation of Equity Volatility

To estimate the volatility of equity for KMV model and Naïve model, we firstly calculate the log return of the firm's common stock prices on each trading day. We then calculate the standard deviation of the returns and obtain the estimated daily volatility of equity. We use daily data in a whole year preceding the annual observations. However, this method has some disadvantages. First, it is the average of the volatility in the whole year; hence it ignores volatility change thorough the time. Second, outliers may cause severe influence on the standard deviation. Therefore, we apply exponentially weighted moving average (EWMA) presented by RiskMetrics (1996) to estimate equity volatility

$$\boldsymbol{\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)e_{t-1}^2} \tag{14}$$

where σ_t^2 is the estimated daily volatility at time t, σ_{t-1}^2 is the estimated daily volatility at time t-1, e_{t-1}^2 is the daily log return of common stock price, and λ is a constant which is usually set at 0.94. At time T=0, we use the $\sigma_{t-1}^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2$ and $e_{t-1}^2 = 0$ as the start value where the u_{n-i}^2 is log return of daily stock price follow Hull (2015). To obtain yearly volatility, we multiply the daily volatility with squared root of the number of trading days in a year, which is assumed to be 252 days, thus

$$\boldsymbol{\sigma_{E,yearly} = \sqrt{252} * \sigma_{E,daily}} \tag{15}$$

3.3 Methodology Description for Model's Accuracy Comparison

We apply Receiver Operating Characteristic (ROC), Area under the Curve (AUC), and Accuracy Ratio (AR) to test Altman's z-score, KMV model and Naïve model. They are the most widely used methods to test the validation of the models' default predication accuracy and measure their discriminative power. Reisz and Perlich (2004), Korablev and Dwyer (2007), Agarwal and Taffler (2008), Warren Miller (2009), and Wu et al. (2010) also apply the ROC curve to make comparison between accounting based models and market-based models. Afik, Arad, and Galil (2016) also use this method to find the best default point in the Merton model. Therefore, ROC curve and AUC model have become a popular method for testing the accuracy.

Table 3.1: Four Types of Predicted Situations for Default Events

	Actual Default (T=1)	**Actual Non-default (T=1)**
Predicted Default (T=0)	True Default (TD)	False Default (FD) *(Type Ⅱ error)*
Predicted Non-default (T=0)	False Non-default (FN) *(Type Ⅰ error)*	True Non-default (TN)
Sum	N_d = TD+FN	N_n=FD+TN

(Note: True default (TD) is predicted to default and truly defaulted; False Non-default (FN)is predicted not to default but actually default; False default (FD) is estimated to default but actually not default; True non-default (TN) is estimated not to default and truly remain solvent.)

To construct ROC curve, firstly we need to sort our companies into four categories. (1) True default (TD) is for companies which are predicted to default and truly defaulted. (2) False non-default (FN) is for firms which are predicted not to default but actually default. (3) False default (FD) is for companies which are estimated to default but actually not default. (4) True non-default (TN) is for firms are estimated not to default and truly remained solvent. The four categories are shown in **Table 3.1**.

After calculating numbers of companies in each category, we have the total number of default as N_d, and the total number of non-default as N_n. Then, we can use these numbers to calculate the false positive rate (FPR) and true positive rate (TPR). The FPR, which is also called false alarm rate, relates to the rate at which the solvent firms are classified to default category, therefore FPR = FD/ N_n. In contrary, TPR, also named as hit rate, is the rate at which firms are precisely predicted to default, that is TPR = TD/ N_d. Thus, FPR and TPR are the two rates to construct the x-axis and y-axis of the ROC curve respectively.

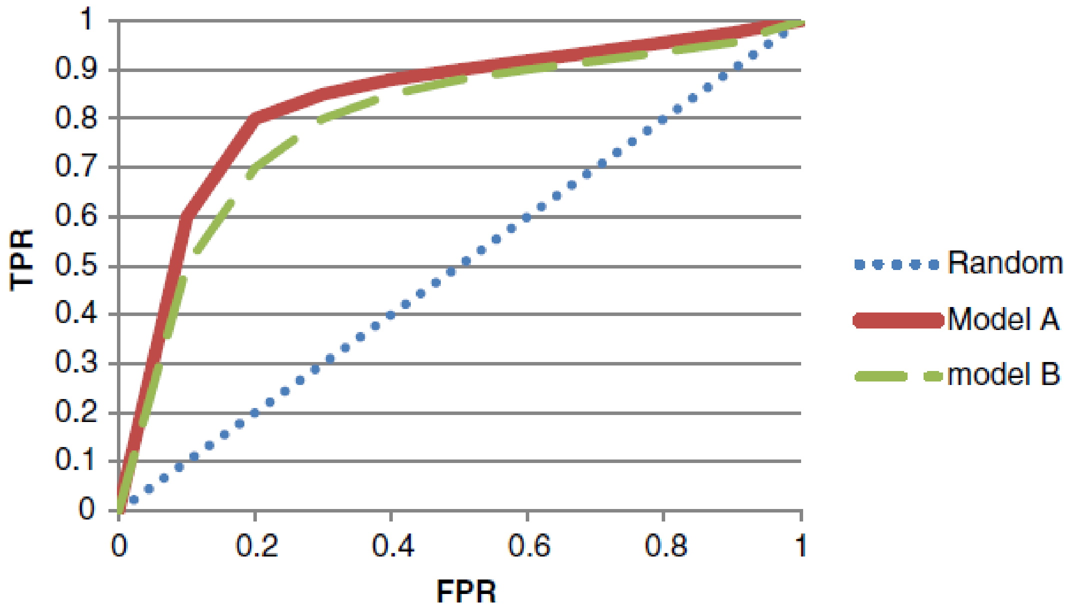

Figure 3.3: Illustration of ROC Curves (Afik, Arad, and Galil, 2016)

(Note: true positive rate (TPR) is the rate precisely predicted as default; false positive rate (FPR) is the rate of solvent firms classifies to default category. The random model is a 45 degree line which does not have predictive power. The higher the curve, the better the prediction power of the model. Therefore, Model A is better than Model B because Model A lies above Model B.)

As can be seen, **Figure 3.3** shows an example of ROC curve; the random line is a 45 degree straight line, which shows there is no predictive power. Therefore, the closer the estimated curve to the random line, the less the power to predict default. Then, model A is always superior to model B, that is, model A has a better power to predict. If the curves cross, model A does not dominate model B, then compare the Area Under the Curve (AUC) of each model. The value of AUC is in the range of [0.5, 1] and the random model has the AUC equals to 0.5. Therefore, the higher the AUC, the higher the predictive power of the model. Another curve is Cumulative Accuracy Profile (CAP, or power curve), which is similar to ROC curve but with different x-axis. The x-axis of CAP is the fraction of the total obligors instead of the fraction of total non-default firms. The fraction of area between scoring models and the random models, and area between random model and perfect model can be defined as Accuracy Ratio (AR). The value of AR lies between 0 and 1. AR for the perfect model is equal to 1, and the random model is equal to 0. We can calculate the AR directly from its relationship with AUC proved by Engelmann et al (2003), which is AR = 2*AUC-1. Therefore, through this method, we can easily compare which alternative model has a better predictive power to default probability.

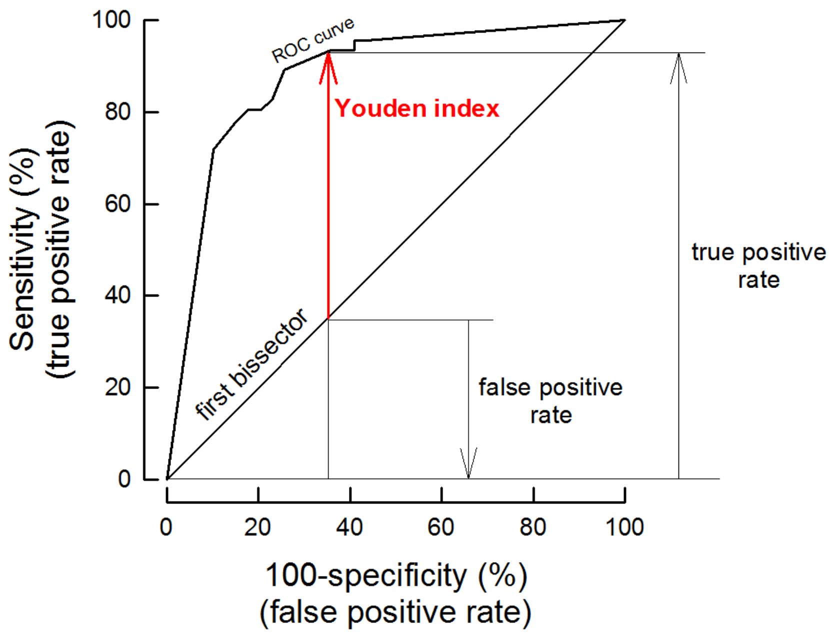

Figure 3.4: Youden's Index (Michils, Louis, Peché, and Muylem, n.d.)

(Note: Youden's index is the first derivative of ROC curve, where a point with a tangent line parallel to the random line is the cut-off point. The value below the cut-off point have higher probabilities that the event will happen. The figure is available at: http://ppt.cc/gUZ6N)

We can use cut-off point of the ROC curve to decide below which level of the score (here is the Altman's Z-score, KMV and Naïve models' DD) can have higher probabilities of the events happen. Youden (1950, cited in Michils, Louis, Peché, and Muylem, n.d.) suggested Youden's J-statistic (also called Youden's index) to test the difference between true positive rates and false positive rates. As showed in **Figure 3.4,** the maximized J-statistic is at the point which the first derivative function of J equals to zero in the ROC curve.

$$\mathbf{J(x) = F(x) - x} \qquad \textbf{(15)}$$

Where x = FPR, and the first derivative of J(x) is

$$\mathbf{J'(x) = F'(x) - 1} \qquad \textbf{(16)}$$

When F'(x) equal to one, which means that the point has a tangent line which is parallel to the random line, a gain in FPR results in a loss of the same amount in TPR. Therefore, we can find the point as the cut-off point of the ROC curve, and this point represents if the number calculated is less than the cut-off point, we have high probability of the events occur.

4. DATA COLLECTION AND DATA STATISTIC

This chapter will introduce how we select our data sample and where we get our data. The second half will present the summary statistic of the selected data.

4.1 Data Collection

Our data is from S&P Capital IQ. The data period is from 2007 to 2012, which covers the recent financial crisis. Therefore, we collect public firms in the United States. Their accounting and market data are collected from 2006 to 2011, and the bankruptcy announcement companies are from 2007 to 2012. There are totally 17,647 companies, and 392 of which announced bankruptcy during the period. However, not all firms' data are complete, and some financial data is missing. Therefore, we have to select companies with comprehensive data in KMV model, Naïve model, and Altman's Z-score model. This results shows in the **Table 4.1**.

Table 4.1: Individual Year Bankruptcy and Non-bankruptcy Observations

All Industries				
Year	**Sample Size**	**Non-bankruptcy**	**Bankruptcy**	**Bankruptcy Ratio**
2007	3327	3293	34	1,022%
2008	2418	2364	54	2,233%
2009	2354	2317	37	1,572%
2010	2305	2290	15	0,651%
2011	2421	2401	20	0,826%
2012	2510	2495	15	0,598%
Total	15335	15160	175	1,141%

Table 4.1: Individual Year Bankruptcy and Non-bankruptcy Observations (cont.)

Non-financial Industries				
Year	**Sample Size**	**Non-bankruptcy**	**Bankruptcy**	**Bankruptcy Ratio**
2007	2680	2648	32	1,194%
2008	2053	2011	42	2,046%
2009	2002	1974	28	1,399%
2010	1948	1938	10	0,513%
2011	2053	2036	17	0,828%
2012	2143	2130	13	0,607%
Total	12879	12737	142	1,103%

Financial Industry				
Year	**Sample Size**	**Non-bankruptcy**	**Bankruptcy**	**Bankruptcy Ratio**
2007	647	645	2	0,309%
2008	365	353	12	3,288%
2009	352	343	9	2,557%
2010	357	352	5	1,401%
2011	368	365	3	0,815%
2012	367	365	2	0,545%
Total	2456	2423	33	1,344%

The variables for Altman's z-score includes total current assets, total assets, total current liabilities, total liabilities, retained earnings, EBIT, market value of equity, and sales. For financial firms, we use EBT excluding unusual expenses as a proxy for EBIT, since most financial firms do not have interest expense items in financial statements.

We collect the financial data for KMV model and Naïve model, which includes market value of equity, total current liabilities, total non-current liabilities, and daily stock price. We choose the risk-free rate suggested by Campbell et al. (2008), who measure the risk-free rate using the Treasury bill (T-bill) rate. We use coupon equivalent of T-bill rate since it is the bill's yield based on the price and it is used as discount in one year. Then, we follow their method using a constant market premium and estimating μ_A as $R_f + 0.06$.

By using Matlab, we compute the equity volatility, solve the equation (2) and (5) in KMV model to estimate the market value of assets and asset volatility, and achieve the distance to default finally. Then, we use IBM SPSS Statistics to build the ROC curve and AUC.

4.2 Summary Statistic

In this section, we compare the summary statistic of each variable in Altman's Z-score, KMV model, and Naïve model. Meanwhile, we show the difference of each variable's mean, standard deviation, median, maximum, and minimum numbers between bankruptcy firms and non-bankruptcy firms in different industries.

4.2.1 Summary Statistic: Z-Score Model

As can be seen from **Table 4.2**, the means of z-score for bankruptcy firms in both financial industry and non-financial industry are less than the means of non-bankruptcy firms. Looking into each variable, we find the means of **Working Capital/Total Asset, Retained Earnings/Total Asset,** and **Earnings Before Interest and Tax/Total Asset** are all negative for financial firms. Due to the current financial crisis, companies' performance becomes worse, and financial crisis induces negative working capital, negative retained earnings and negative earnings before interest and tax. For these three variables, the means for bankruptcy firms are smaller than non-bankruptcy firms' in financial industry. However, for non-financial companies, there are no certain tendency between default firms and non-default firms. Therefore, in general, we can say these three variables in financial industry have influence when calculating z-score, whereas in non-financial industry, they are not.

We can also infer that in financial firms, high leverage induces higher value in liabilities, and working capital becomes negative. Hence, if the difference between current asset and current liability becomes smaller, financial firms will be less likely to default. This is probably because of financial firms' capital requirement regulation. Since non-default financial firms have higher reserve, they can endure more risk and remain solvency.

The variable, *Market Equity Value/Total Liabilities*, shows large difference in means between default firms and non-default firms, and non-default firms have larger mean than default firms. It could be speculated that the market can differentiate unlikely default firms from high default risk firms. Therefore, this variable influences the final score for both industries. The last variable, Sales/Total Asset, shows close results but with higher mean for non-bankruptcy firms in both industries. We can infer that this variable has slight influence if the firm will go bankruptcy next year or not. Thus, a firm's ability to utilize its asset to generate revenue determines default risk level.

Table 4.2: Altman's Z-score Summary Statistic

a. Bankruptcy Financial Industry

	WC/TA	*RE/TA*	*EBIT/TA*	*ME/TL*	*Sales/TA*	*Z-Score*
Mean	-1,61	-3,80	-0,79	0,12	0,10	-9,70
STD	6,02	20,54	4,21	0,27	0,28	49,73
Mid	-0,65	0,00	-0,01	0,03	0,01	-0,80
Max	0,57	0,23	0,02	1,47	1,14	1,28
Min	-33,40	-112,55	-23,09	0,00	-0,03	-272,96

c. Bankruptcy Non-financial Industry

	WC/TA	*RE/TA*	*EBIT/TA*	*ME/TL*	*Sales/TA*	*Z-Score*
Mean	-0,91	-10,92	-1,20	2,60	0,98	-17,79
STD	4,53	25,01	3,49	8,77	0,93	48,19
Mid	0,05	-2,23	-0,25	0,38	0,75	-2,39
Max	0,87	0,72	0,49	91,84	3,87	57,28
Min	-40,09	-148,98	-30,61	0,00	0,00	-284,54

b. Non-Bankruptcy Financial Industry

	WC/TA	*RE/TA*	*EBIT/TA*	*ME/TL*	*Sales/TA*	*Z-Score*
Mean	-0,39	-1,17	-0,05	1,91	0,23	-0,90
STD	1,25	22,46	1,32	16,71	1,38	35,91
Mid	-0,63	0,05	0,01	0,17	0,04	-0,51
Max	0,99	1,70	1,03	640,00	41,86	386,56
Min	-39,17	-711,67	-44,56	0,00	-0,52	-1144,27

d. Non-Bankruptcy Non-financial Industry

	WC/TA	*RE/TA*	*EBIT/TA*	*ME/TL*	*Sales/TA*	*Z-Score*
Mean	-0,53	-10,05	-1,68	12,54	1,02	-11,69
STD	14,61	222,90	120,31	238,80	1,15	688,66
Mid	0,22	0,10	0,06	2,41	0,82	2,95
Max	1,00	2,39	7,72	25500,00	51,63	15308,34
Min	-823,00	-19120,00	-13540,00	0,00	-0,17	-71176,30

(Note: a. Bankruptcy firms in financial industry; b. Non-bankruptcy firms in financial industry; c. Bankruptcy firms in non-financial industry; d. Non-bankruptcy firms in non-financial industry.)

4.2.2 Summary Statistic: KMV and Naïve Model

In both financial industry and non-financial industry **(Table 4.3)**, bankruptcy firms have lower means in distance to default for both KMV and Naïve model. Looking further into each variable, bankruptcy firms have higher means in equity volatility, asset volatility, debt volatility and naïve firm volatility than non-bankruptcy firms. The volatilities are lower in financial industries because their common stock prices are relatively stable than non-financial firms'. The reason for low volatility in financial industry is the special property and government regulations. However, due to the high leverage in financial industries, financial firms have higher short-term debt, long-term debt and total debt, especially in non-bankruptcy financial firms. It can be inferred that due to depression period, people would intend to save money in lower default-risk financial firms, and those savings are liabilities. Furthermore, for non-financial industries, firms with lower bankruptcy probabilities have higher liabilities since they can take loans with lower costs and easier to raise money from the market. Thus, non-financial firms have higher means of liabilities for non-default firms than default firms.

For estimated market value of asset, it is obvious that non-bankruptcy firms have higher means than bankruptcy firms no matter in which industry. However, in financial industry, market asset value is less than short-term liabilities. This is also due to the high leverage property of financial firms. Comparing market asset value with short-term liabilities, it can be seen that financial firms have less mean in market asset value than in short-term debts. Moreover, for bankruptcy financial firms, there is a large difference between market asset value and short-term debt; thus, this large difference can distinguish default firms from non-default firms. Stock return also shows difference between bankruptcy firms and non-bankruptcy firms. For companies announced bankruptcy, they tend to have lower stock returns than non-bankruptcy companies.

To sum up, we find that all models result in lower means of distance to default (DD) for default firms, and higher means of distance to default (DD) for non-default firms. In Altman's z-score, *Market Equity Value/Total Liability* has the largest influence when determining the score. In KMV model and Naïve model, default firms have higher means of volatilities and lower means of asset values. During this period, the firm with lowest z-score and DD remains solvent.

Table 4.3: KMV and Naïve Summary Statistic

e. **Bankruptcy** Financial Industry

	σ_E	*ME*	*STD*	*LTD*	*VA*	σ_A	*KMV DD*	*Return*	*F*	σ_D	σ_V	*Naïve DD*
Mean	1,218	666,265	10385,250	3090,003	2759,783	0,388	-30,954	-1,014	13475,260	0,354	0,427	-2,619
Median	0,978	46,400	1358,300	191,700	797,008	0,295	-10,880	-1,008	1588,900	0,294	0,311	-2,766
STD	0,855	3033,771	43568,470	12932,430	6700,740	0,609	103,496	0,715	56310,480	0,214	0,410	2,028
Max	3,518	17513,400	250831,000	74128,900	28040,170	3,511	0,814	0,168	324959,900	0,929	2,469	1,526
Min	0,160	2,360	1,840	0,000	2,360	0,006	-601,733	-2,470	1,840	0,090	0,096	-6,773

f. **Non-Bankruptcy** Financial Industry

	σ_E	*ME*	*STD*	*LTD*	*VA*	σ_A	*KMV DD*	*Return*	*F*	σ_D	σ_V	*Naïve DD*
Mean	0,487	2498,311	14500,930	9176,499	14478,590	0,244	-12,567	-0,146	23677,430	0,172	0,247	2,221
Median	0,349	282,900	995,200	203,800	1077,843	0,150	-0,264	-0,033	1504,500	0,137	0,177	0,978
STD	0,544	9569,123	83229,710	48246,090	82328,410	0,462	416,783	0,581	115765,000	0,136	0,317	37,637
Max	13,218	139705,700	1130370,000	1311234,000	1220107,000	13,218	6677,535	1,622	1448814,000	3,355	8,992	1798,732
Min	0,000	0,001	0,001	0,000	0,001	0,000	-19313,900	-8,517	0,001	0,050	0,003	-9,891

g. **Bankruptcy** Non-financial Industry

	σ_E	*ME*	*STD*	*LTD*	*VA*	σ_A	*KMV DD*	*Return*	*F*	σ_D	σ_V	*Naïve DD*
Mean	1,608	34,994	434,120	574,184	518,344	1,261	-9,300	-1,252	1008,304	0,452	0,793	-1,356
Median	1,286	9,780	10,700	3,640	13,628	0,969	-0,188	-1,039	19,900	0,372	0,682	-1,367
STD	1,116	120,114	4346,099	5850,168	5213,356	1,100	86,973	1,265	10191,930	0,279	0,530	2,286
Max	7,077	1340,800	51562,700	69500,500	61900,120	7,076	1,915	1,588	121063,200	1,819	4,779	5,281
Min	0,237	0,003	0,000	0,000	0,003	0,052	-1031,030	-6,551	0,147	0,109	0,128	-7,181

h. **Non-Bankruptcy** Non-financial Industry

	σ_E	*ME*	*STD*	*LTD*	*VA*	σ_A	*KMV DD*	*Return*	*F*	σ_D	σ_V	*Naïve DD*
Mean	0,615	2891,498	663,293	1188,572	3708,398	0,509	1,256	-0,128	1851,865	0,204	0,460	4,133
Median	0,439	322,750	60,400	53,200	416,756	0,354	1,865	-0,006	133,200	0,160	0,343	3,646
STD	0,589	11944,620	3625,849	7112,527	15374,630	0,547	44,370	0,728	10523,070	0,147	0,400	4,332
Max	14,730	278704,200	142247,400	327323,800	380486,900	14,730	45,003	4,413	469571,200	3,733	12,072	44,272
Min	0,008	0,002	0,001	0,000	0,002	0,008	-3615,670	-8,923	0,001	0,052	0,025	-13,284

(Note: e. Bankruptcy firms in financial industry; f. Non-bankruptcy firms in financial industry; g. Bankruptcy firms in non-financial industry; h. Non-bankruptcy firms in non-financial industry.)

5. RESULTS ANALYSIS

This chapter will introduce our analysis results. According to our empirical results, we conclude the best bankruptcy prediction in all industries since the current financial crisis. Then, we conclude our recommendations relating to the best prediction models in non-financial industries and financial industries.

5.1 All Industries

For different bankruptcy prediction models, Area under the Curve (AUC) can be effectively used to compare each model's performance. The value of AUC is between 0.5 and 1, and the random AUC value equals to 0.5. In other word, the model with higher AUC value means that this model has better prediction ability. Moreover, accuracy ratio is another measure can be applied to estimate the prediction power comparison. The perfect model's accuracy ratio equals 1, and the random model's equals to 0. Thus we prefer the prediction model with higher accuracy value.

Table 5.1: Area Under the Curve for All Industries

All Industries: Area Under the Curve							
Area Value (Asymptotic Sig.)	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**Whole Period**
KMV	0.767****	0.811***	0.873***	0.937***	0.898***	0.936***	0.859***
Naïve	0.888***	0.850***	0.862***	0.931***	0.917***	0.946***	0.875***
Z-Score	0.776***	0.802***	0.776***	0.880***	0.842***	0.893***	0.806***

*(Note: *** significant at 1% or lower; ** significant at 5% or lower; * significant at 10% or lower)*

We begin by examining all the firms, and we tested the prediction performance based on the Altman's z-score model, the KMV model, and the Naïve model. All the testing results are statistic significant, and more details can be seen the **Table 5.1**. We use previous one year's accounting and market information, and compare with the next year bankruptcy events. The individual year's AUC (Area under the Curve) shown in the **Figure 5.1** tells us that the Naïve model has the best bankruptcy prediction except year 2009 and year 2010. However, in 2009 and 2010, the KMV model can provide best bankruptcy prediction. Except the year 2007, the Altman's z-score model always has the poorest prediction performance among the three models. In general, the Naïve model has the best individual year's prediction power.

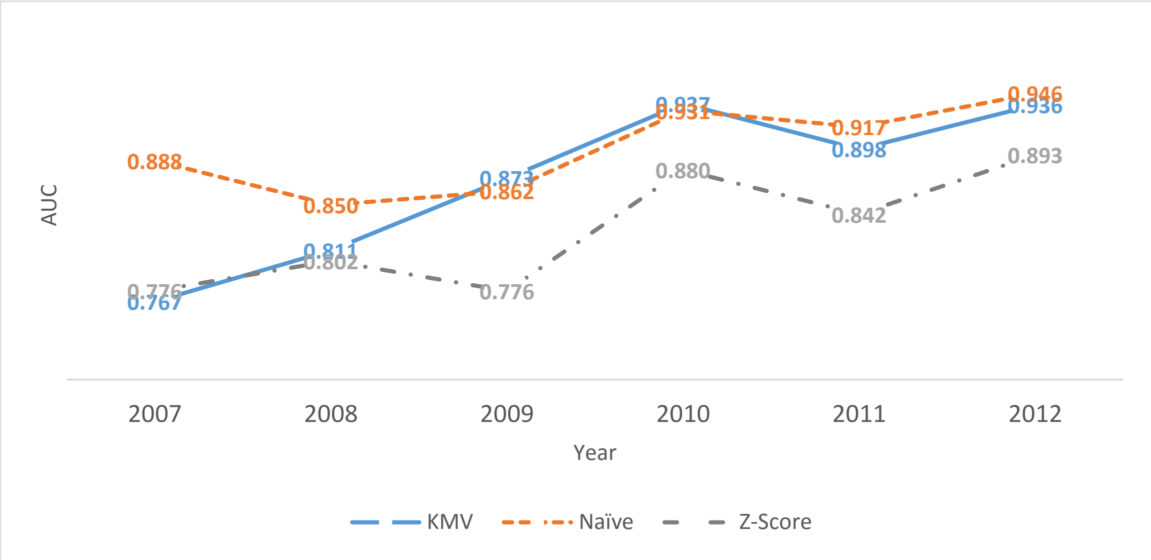

Figure 5.1: All industries: Area Under the Curve for Individual Years

(Note: the larger the AUC value, the greater the predictive power of the model)

Then, we turn to test the whole period's observations together. There are totally 15,335 events during this period, and 175 of the events are bankruptcy during the whole period. **Figure 5.2** illustrates that all the models have the power to predict bankruptcy events, because the curves are all above the reference line significantly, and the curves show concave shapes. Since there are cross points among the curves, we compare the Area Under the Curve (AUC), which demonstrates that the larger the area, the better the predictive ability. Our testing results in **Table 5.1** shows that the AUC for the KMV, the Naïve, and the Z-score, which are 0.859, 0.875, 0.806 respectively, are great satisfied. The best model is the Naïve, followed by KMV and Z-Score. It is not surprised that Z-Score has the lowest predictive power, but the model is still statistical significantly. Consequently, we summarize that these three models have different bankruptcy prediction power, but their abilities are all effective.

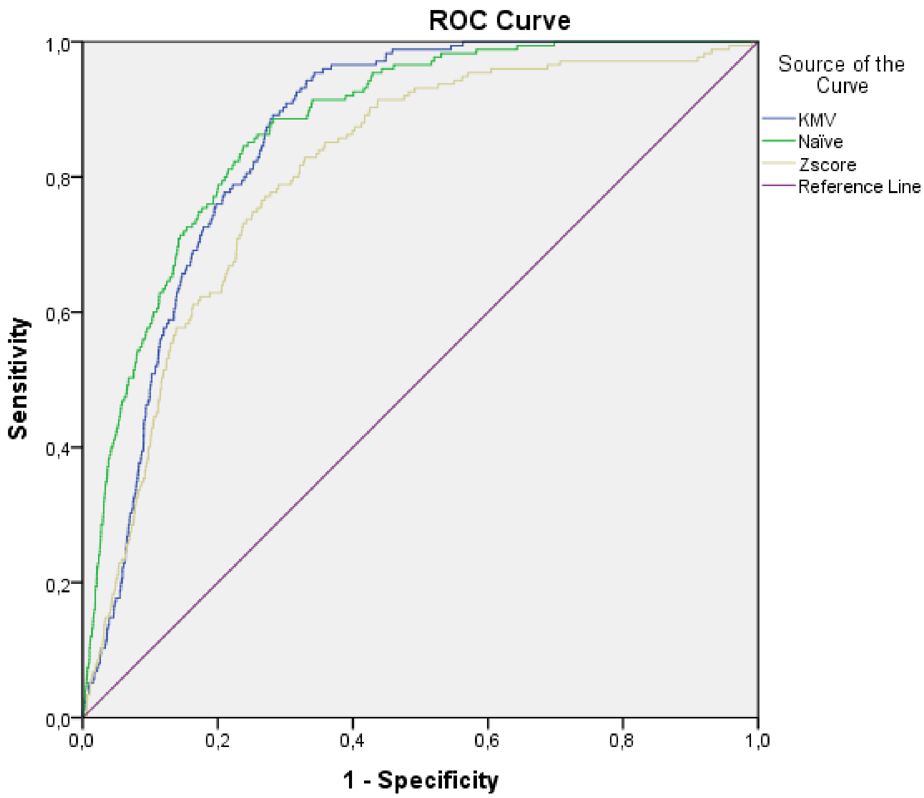

Figure 5.2: All Industries: Area Under the Curve for the Whole Period

(Note: The higher the curve, the better predictive power. Since there are cross points, we have to compare the Area Under the Curve.)

After calculation, we find the accuracy ratio range of the KMV model is from 0.534 to 0.874 between 2007 and 2012. The lowest accuracy ratio of the Naïve model is 0.692, and the highest ratio is 0.892. The Altman's z-score's range is from 0.514 to 0.786. Thus, the Naïve model has the best accuracy ratio range since it is more stable. Additionally, we find that all the models have the lowest accuracy ratio in the beginning of the financial crisis period; on the other hand, when the economics becomes better, our model accuracy ratios are higher.

When we analyze whole period, we still can conclude that the Naïve model has the best prediction performance, and the KMV model has the similar performance. On the other hand, there is more than 10% of accuracy ratio difference between the Altman's z-score and the others. It means the Altman's z-score has the poorest prediction power during whole period. It can be inferred that since KMV and Naïve model incorporate more market information, they can reflect market influence more instantly than Altman's z-score. We can also infer that since stock returns can reflect market information directly and immediately, Naïve model can explain the phenomenon slightly better and so has marginal precision than KMV model.

To sum up, for years, the Naïve model has the most stable and accurate bankruptcy prediction. For the whole period, the Naïve model still provides the best prediction performance. Meanwhile, the Altman's z-score model's bankruptcy prediction power is the worst comparing to the other two models.

Table 5.2: All Industries: Cut-off Points for Individual Years and Whole Period

Cut-off Points Table							
Year	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**Whole Period**
KMV	0.1392	0.4505	0.1017	0.8565	0.9830	1.0019	0.5885
Naïve	0.6089	0.4371	-1.3958	0.5247	1.5240	-0.5622	0.2437
Z-Score	-0.1618	0.9439	-0.1755	-0.3297	0.1825	-0.1717	0.3155

(Note: a firm's calculating value base on each model is lower than the critical value shown in the table, and the firm faces high probability to go bankruptcy)

For each model, we can use the cut-off point to identify a firm's critical value of defaults. The cut-off point of the ROC curve represents that if a company's calculated value is less than the critical value, a company has higher bankruptcy probabilities. **Table 5.2** shows that each model's critical value of bankruptcy prediction in each year. For the whole period, the cut-off point of z-score is 0.3155, which is lower than the original Altman's z-score bankruptcy standard, 1.81. In a word, when the firm's z-score is below 0.3155, the firm probably faces higher risk to go bankruptcy. For years, all individual year's cut-off points are still lower than 1.81, the critical bankruptcy z-score value.

For all companies, we find that all these three models are effective, but the Naïve model can provide the best prediction performance. Next, we discuss the other question in our research, and compare the models' predictive power between the financial industry and the non-financial industry. The following, we will show the results.

5.2 Non-Financial Industry and Financial Industry

We focus on the non-financial firm samples, and study if the bankruptcy predictive power for these three models has the same ranking as in the whole industry samples.

From **Figure 5.3 and 5.4**, the KMV model and the Naïve model provide the similar bankruptcy prediction performance during each individual year; on the other hand, the KMV has slightly better prediction power than the Naïve model. Altman z-score's performance is still provides poorest. Then, after integrating the whole period samples, we find that all the three models have effective default predictive abilities, and our test results are statistic significant. The AUC **(Table 5.3)** for each model is 0.904, 0.883, and 0.823 respectively; thus KMV model has the superior predictive power for non-financial firms, and Naïve and z-score are followed by.

In a word, for all non-financial firms, the Naïve model's prediction ability and the KMV model prediction ability are similar, and all models can provide satisfied bankruptcy prediction performance. According to our empirical results, we recommend to use the KMV model to predict bankruptcy risk for non-financial firms.

Table 5.3: Area Under the Curve for Non-financial Industry

Non-financial Industry: Area Under the Curve							
Area Value (Asymptotic Sig.)	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**Whole Period**
KMV	0.890***	0.862***	0.867***	0.971***	0.950***	0.950***	0.904***
Naïve	0.895***	0.871***	0.845***	0.962***	0.934***	0.954***	0.883***
Z-Score	0.802***	0.817***	0.782***	0.889***	0.853***	0.912***	0.823***

*(Note: *** significant at 1% or lower; ** significant at 5% or lower; * significant at 10% or lower)*

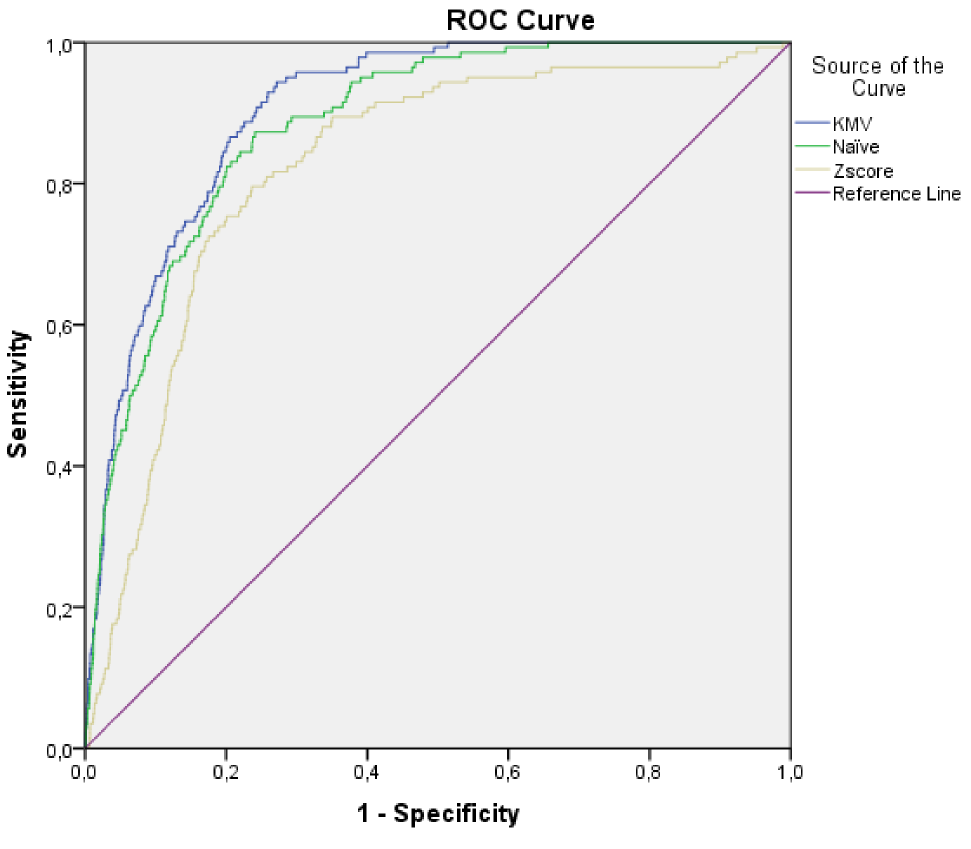

Figure 5.3: Non-financial Industries: Area Under the Curve for the Whole Period

(Note: The higher the curve, the better predictive power. If the curves cross, compare Area Under the Curve.)

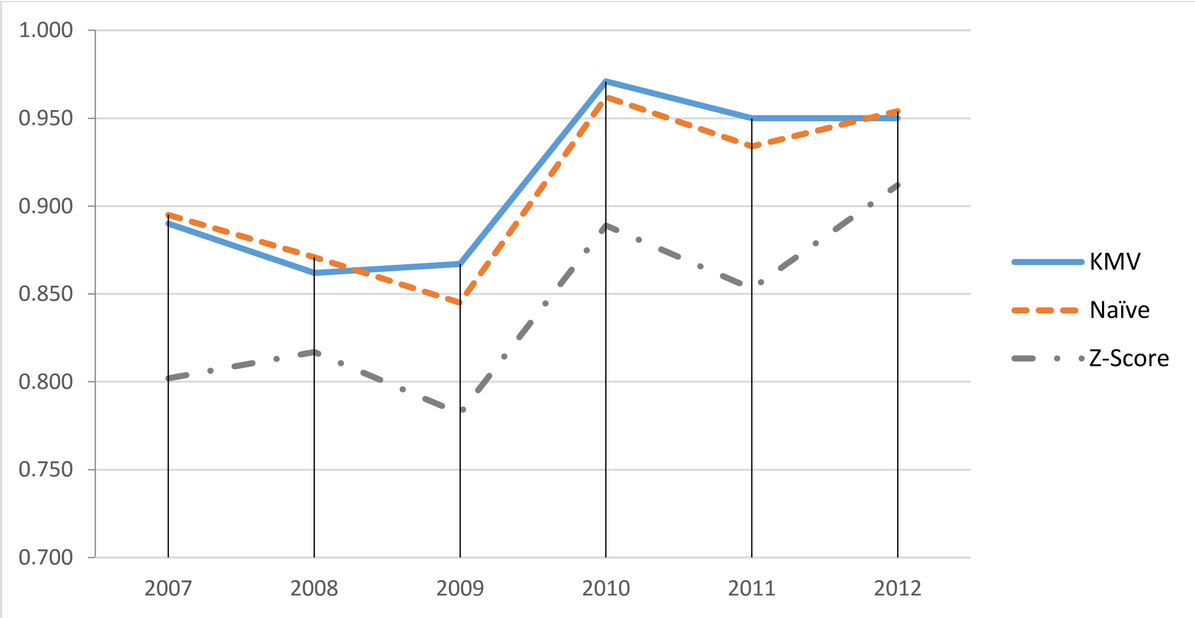

Figure 5.4: Non-financial Industry: Area Under the Curve for Individual Years

(Note: the larger the AUC value, the greater the predictive power of the model)

Finally, we focus on the all financial firms, and study the best bankruptcy prediction model during since 2007 to 2012.

Table 5.4: Financial Industry: Area Under the Curve for Individual Years and the Whole Period

Area Under the Curve (FIN Firms)							
Year	***2007***	***2008***	***2009***	***2010***	***2011***	***2012***	***Whole Period***
KMV	0.555	0.759***	0.914***	0.844***	0.969***	0.979**	0.747***
Naïve	0.805	0.811***	0.906***	0.810**	0.820*	0.941**	0.862***
Z-Score	0.685	0.648*	0.821***	0.965***	0.754	0.700	0.742***

*(Note: *** significant at 1% or lower; ** significant at 5% or lower; * significant at 10% or lower)*

Since the testing results **in Table 5.4** are not statistic significant in 2007, 2011, and 2012, we cannot compare the individual year prediction power for these three years. On the other hand, we find that each year's best prediction models are all different between 2008 and 2010. Then, we integrate the whole period samples together; it shows that the AUC of each model is 0.747, 0.862, and 0.742 for KMV, Naïve, and z-score. Thus, during the whole period, we find that Naïve model has the most effective power to predict bankruptcy for financial firms, while KMV model and Altman's z-score have similar but just fair predictive power for bankruptcy prediction.

In a word, according to our empirical testing results in **Figure 5.5**, we summarize that the Naïve model can relatively provide the best bankruptcy prediction performance for all the financial firms during whole period.

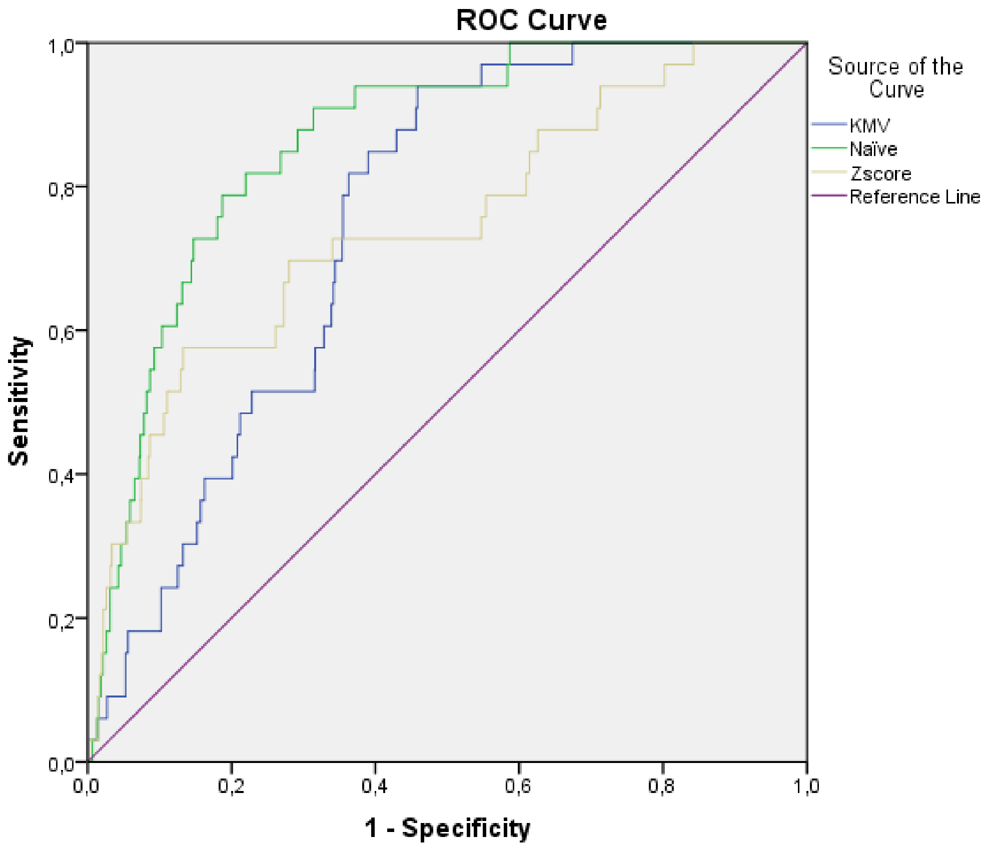

Figure 5.5: Financial Industries: Area Under the Curve for the Whole Period

(Note: The higher the curve, the better predictive power. If the curves cross, compare Area Under the Curve.)

According to the **Table 5.5**, it shows that the cut-off points for non-financial firms will be higher than all companies' cut-off points, because they do not include financial companies' data, which are almost negative. As a result, the cut-off points for the three models are 0.7948, 0.4887, and 1.0058. In addition, Altman suggest that if the score below 1.81 for non-financial firms, they will have high probabilities to default. In comparison, in our research, the cut-off point for non-financial firms is at 1.0058, which means that when the z-score is below this level, firms tend to default. However, in financial industry, the cut-off points for the three models are negative. It is mainly because of the large liabilities which induce many of the ratios to be negative. However, ROC curve still shows the models can be used, and the cut-off points of these three models are -3.091, -0.8641, and -0.664 respectively.

Table 5.5: Cut-off Points for Individual Year and Whole Period

Cut-off Points Table							
Non-financial Industry							
Year	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**FIN Crisis**
KMV	0,1392	0,4505	0,1017	0,8565	0,983	1,0019	0,5888
Naïve	0,6089	0,4371	-1,3958	0,5247	1,524	-0,5622	0,1895
Z-Score	-0,1618	0,9439	-0,1755	-0,3297	0,1825	-0,1717	0,048
Financial Industry							
KMV	-14.817	-11,915	-0,7996	-1,7822	-12,463	-1,0163	-3,091
Naïve	1,4841	-1,6654	-2,596	-2,0184	1,4984	-2,7964	0,8641
Z-Score	-0,5605	-0,5561	-0,6886	-0,8889	-0,6541	-0,4932	-0,664

(Note: a firm's calculating value base on each model is lower than the critical value shown in the table, and the firm faces high probability to go bankruptcy)

5.3 Results Summary

To sum up, the KMV model has the best bankruptcy prediction power for all non-financial firms, while the Naïve model is the most effective prediction model for financial firms. Meanwhile, it should be noted that for Altman's z-score (1968), which is considered as a relatively weak prediction model in the financial industry by some previous researches, is regarded as a useful model in our research even though the model is not effective as KMV and Naïve model.

6. CONCLUSION

Since financial crisis, default risk prediction becomes more and more important for financial institutions and investors. We apply three of the most prevalent models including Altman's Z-Score, KMV model and Naïve model, and answer the question: which default predict models have the best power in forecasting bankruptcy after the financial crisis in 2007. In addition, many of the previous researches exclude financial firms from the sample since financial firms are high leverage, they think massive amount of liabilities in financial firms cause the results to be biased. However, we think these models still have predictive power in financial industry, we compare the prediction power between financial firms and non-financial firms.

We choose three models to predict default risk: accounting-based method, Altman's Z-Score, and market-based method, KMV model and Naïve model. To compare the power of these models, we conduct Receiver Operating Characteristics (ROC) curve and calculate the Area Under the Curve (AUC). The larger the Area Under the Curve, the better the prediction power of the model. Then we estimate the cut-off points for each model and illustrate firms with distance to default or Z-Score below the cut-off point have higher probability of default.

The sample covers U.S public firms with total 15,335 events, including solvent and insolvent events, and 175 of which are default after the financial crisis. There are 2,456 events in financial industry and 12,879 events in non-financial industry.

In conclusion, during financial crisis, Altman's Z-Score, KMV and Naïve model are effective in predicting bankruptcy even for financial firms. For all firms, the Naïve model has the best prediction power, and the Z-score is the poorest. Yet, when we compare non-financial firms during the six years from 2007 to 2012, the KMV model provides the best bankruptcy prediction performance. According to our empirical test results, we recommend to use the Naïve model to predict bankruptcy risk of the financial firms. Although there is no model can outperform consistently other models, the market-based model are generally better than the accounting- based model. Both KMV model and Naïve model are market-based measures, and their better performance probably is because the models incorporate more market information.

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