The smile of currency derivatives – PCA modelling of the FX effect

Master Thesis

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Abstract

This paper investigates the non-flat volatility surface of foreign exchange options, a so-called volatility smile. Foreign exchange options are especially interesting due their liquidity and frequency in risk management. To propose a non-complex model of the determinants of variation in the smile, a principal component approach is suggested. As this approach allows for explanation of the development of the smile with a streamlined three-factor model, this suggests an easy interpreted model with application in risk management. To evade problems with noise in data, this paper proposes to apply the principal component analysis on fixed-delta volatility deviations from at-the-money volatility. Furthermore the paper seeks to capture potential links between current market condition and movement in the smile. The results show that foreign exchange volatility dynamics are dependent upon current market conditions.

Keywords: *foreign exchange, option, risk management, implied volatility, volatility smile, principal components*

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Abbreviations

ARCH	Auto-regressive conditional heteroscedasticity
ATM	At-the-money
BF	Butterfly
BS	Black and Scholes
CME	Chicago Mercantile Exchange
EUR	Euros
EWMA	Exponentially weighted moving average
FTSE	The Financial Times Stock Exchange
FX	Foreign Exchange
GBM	Geometric Brownian Motion
ITM	In-the-money
IV	Implied volatility
OTC	Over-the-counter
OTM	Out-of-the-money
PCA	Principal Component Analysis
PC	Principal Component
PHLX	Philadelphia Stock Exchange
RR	Risk reversal
S&P	Standard and Poor's
USD	US dollars

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1. Introduction

> With the rapid innovation and growth of the derivate securities, the management of volatility risk in its many forms has become an important topic for researchers and practitioners. This is due to the sensitivity of derivatives to market volatilities and the need to manage this risk accurately and at low cost (Zhu & Avellaneda, 1997, p.81).

The prelusive quote pretty much says it all. Building capacity to model and capture volatility dynamics is of interest for any agent that engage in risk management or speculation.

In spite of being the most widely used and accepted option pricing model in the world, the Black and Scholes (1973) (henceforth *BS*) option pricing model does not stand unchallenged. Commonly the constancy assumption of the volatility parameter in the model is challenged with empirical findings that show that a non-dynamic volatility structure is unrealistic. Furthermore, it is suggested that the expectations market participants have regarding volatility provide a better estimate of the volatility parameter in the option pricing formula. If the BS model would be consistent with reality, option contracts with different strikes and maturities on the same underlying asset would entail the same expected - *implied* - volatilities. However, since the financial crisis of 1987 asymmetry in volatility structures has been observed which manifests itself in a non-flat volatility surface that for fixed maturities commonly resembles the shape of a skew or smile.

This paper investigates such a volatility smile of foreign exchange (hereinafter *FX*) options. The reason for this is threefold. First, it is observed that volatility dynamics are asset specific (Skiadopoulus, Hodges & Clewlow, 1999) and that *"*the smile construction procedure and the volatility quoting mechanisms are FX specific and differ significantly from other markets*"* (Reiswich & Wustyp, 2012, p.58). Whereas a great abundance of research work has been carried out on equity options, this has been done to a lesser extent on FX options. Second, if it is theorized that implied volatility has an impact as a predictor of volatility (see for example Latané and Rendleman (1976); Jorion (1995); Christensen and Prabhala (1998)) then there is evidence that points in the direction of FX implied volatilities being even more relevant (see Jorion (1995)). Third, the FX market is one of the most liquid markets in the world and derivatives on currencies are frequently being used by agents who engage in hedging and speculation. To be able to not only enhance understanding of FX volatility dynamics but also to provide an efficient and easily interpreted modelling tool would therefore be most valuable for risk managers and speculators. For example, financial institutions commonly take long positions in exchange traded instruments to neutralize a short position in an over-the-counter option (Hull, 2009). As volatility is one of the determinants of an options delta and commonly used for risk management purposes, the dynamics of volatility becomes especially interesting.

Scholars frequently use different approaches on the topic of volatility dynamics, such as stochastic volatility modelling (see Hull and White (1987)), generalized least squares (see Day and Lewis (1988)) and vector autoregression (see Chalamandaris and Tsekrekos (2010)). This paper takes on a *principal component analysis* approach, a method that is appealing in that it extracts a set of uncorrelated factors out of a dataset comprised of linearly dependent explanatory variables. Such *collinearity* is frequently observed in financial data (Badshah, 2008) and makes distinction of important risk factors challenging or impossible[1]. Principal component analysis requires *stationary* data, which implies time-invariant mean and autocovariance, and finite variance. As this is not necessarily the case with implied volatility data, this paper utilizes a technique suggested to overcome issues such as noisy and non-stationary data. The methodology, as developed by Alexander (2001), models fixed-delta implied volatility deviations from at-the-money counterpart and has as of yet not been used on FX derivatives, to the best knowledge of the author of this paper.

Moreover, the approach implies a minimized set of variables that explain as much as possible of the variation in data. It is commonly suggested that three risk factors in form of level, slope oscillation and convexity together account for up to 90-95% of the variability in option volatility dynamics (see Derman and Kamal (1997); Zhu & Avellaneda (1997); Skiadopoulus *et al*. (1999); Alexander (2001); Cont and da Fonseca (2002)). Especially risk managers ought to be appreciative of the straightforward interpretations that follow thereof. Besides, as allegedly the volatility dynamics are dependent upon market conditions (see Derman (1999)) presumably

[1] See Farrar and Glauber (1967) for a full discussion on consequences of collinearity in data
[2] PCA is scale-dependent so in cases with variables with different units, these need to be standardized. Standardization of covariance implies transformation to correlation matrix and standardization of variables.

also sensitivity of the three risk factors to changes in the price of the underlying varies with time. This would then indicate that reformation of the smile is conditioned on the prevailing market regime.

The objective of this paper is therefore twofold. First it aims to confirm capability of the proposed model on the FX implied volatility smile. Secondly, it seeks to study and potentially quantify the sensitivity in risk factors to changes in the price of the underlying.

The findings provided thereof speak in favour of a streamlined three-factor model with capacity to adequately expose the FX characteristic dynamics of the smile. Not to mention, results produce evidence for the three risk factors being time-varyingly sensitive to innovations in the underlying. This suggests that movements in the smile are contingent upon market conditions. Most importantly, we have at hand a non-complex and easy-to-construe model, which as such is thought to have application to both option pricing and delta hedging.

This paper is structured as follows. The following chapter outlines the theoretical framework that constitutes the foundation of this paper, succeeded by an overview of previous research. Then a chapter follows in which derivation of the methodology and the rationale behind it is provided, after which results are reported in conjunction with a discussion of the implications thereof. The paper is concluded with a summary of the implications of the findings.

2. Theoretical framework

It is clear that an enhanced comprehension of the mechanics and dynamics of the derivative instruments becomes pivotal when studying FX induced volatility surfaces, an ambition that is sought to be met within the next section, 2.1. In the same way Section 2.2 shed light on the technicalities involved in the statistical method PCA; by doing so the author of this paper believes to give insight to why and how this approach is applicable for the investigation of the FX smile.

2.1 Foreign exchange options

2.1.1 Characteristics

The foreign exchange (FX) option provides a hedge for foreign exchange exposure with the underlying being the exchange rate (Hull, 2009). As a derivative instrument it is thus similar to other types of options in functionality and purpose, but differs significantly in valuation and quoting. FX options are priced with a model that allows for interest parity such as the one proposed by Garman and Kohlhagen (1983), unlike for example traditional BS that uses only one interest rate (Grabbe, 1983).

FX options are either over-the-counter (OTC) or exchange traded. The former market mainly relates to financial institutions such as banks offering derivative instruments on the interbank market or to their corporate clients for hedging purposes. Transactions being bilateral, data on these are more limited (Levich, 2012). Nevertheless is this one of the most liquid derivative markets in the world (Reiswich & Wustyp, 2010). Exchange traded options on the other hand are publicly quoted on a number of exchanges such as Philadelphia Stock Exchange (PHLX) and Chicago Mercantile Exchange (CME) (Grabbe, 1983). These derivative markets are less liquid than their OTC counterparts (Hull, 2009).

Quoting of FX options contrasts those of other options (Reiswich & Wustyp, 2010). Rather than price the markets expected – implied - volatility of the option is used by the FX market to denote option value.

Exchanges rates are quoted based on the currency pair in question. Spot and forward rates display what number of units of domestic currency is required to buy one unit of foreign currency. “Foreign” and “domestic” in this context do not refer to any specific geographical location, but rather to the *numeraire* currency that buys some other currency. A quote to trade Euros for USD is therefore “EURUSD”.

Furthermore, FX options are traded both as European and American style options (Hull, 2009). American style FX options give the holder the right – at any time – to exercise before expiry date, which then always has a value. Hence, the premium of an American FX option is higher than that of a European style option.

A special type of derivative is the option on FX future. This is the right to enter a FX futures contract upon a certain maturity date. In this case the underlying is in itself a derivative instrument, in the form of a standardized contract on a beforehand-agreed exchange rate. Unlike an option the future contract is an obligation. By the organizational structure of exchange trading practically all credit risk has been eliminated, making futures a popular choice. Options on futures are normally American (Hull, 2009).

2.1.2 The Black and Scholes option pricing framework

In 1973 Fischer Black and Myron Scholes developed a model for pricing of options. Even after more than 40 years the model still maintains acceptance and although modifications have been proposed to accommodate for alternatives to the standard stock option there are as of yet no other models to compete successfully.

The model is derived assuming an ideal market for the underlying asset and the option (Black & Scholes, 1973). That implies

a) a constant short-term interest rate that is known

b) a constant volatility of the underlying asset

c) log-returns of the underlying that are normally distributed at option expiry:

$$\ln\left(\frac{s_T}{s_t}\right) \sim \mathcal{N}\left(\left(r - \frac{\sigma^2}{2}\right)\tau, \sigma\sqrt{\tau}\right) \tag{2.1}$$

where

r: drift term in form of the short term interest rate,

T: time of expiry,

t: time of today,

$\tau = T - t$: time to expiry,

s_t: the price of the underlying at time t,

σ: the standard deviation of the underlying asset

d) that no dividends are being paid out

e) that the option cannot be exercised before maturity date *f) a frictionless market*

g) that short-selling of the underlying is possible

As the representation above assumes constant parameters and in addition the strike price is constant, then in the standard BS model the value of the option varies with price of the underlying asset and time-to-maturity only. The value of the option, w, is then

$$w(s_t, c, \tau, \sigma) = s_t \mathcal{N}(\theta d_+) - c e^{-r\tau} \mathcal{N}(\theta d_-)\theta \qquad (2.2)$$

where

$$d_\pm = \frac{\ln\left(\frac{s_t}{c}\right) + \left(r \pm \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}},$$

$\mathcal{N}(.)$: the standard normal cumulative density function,

c: strike price,

$\theta = +1$ for a call, $\theta = -1$ for a put,

That the BS does not allow for stochastic volatility is normal criticism of the model as it implies mispricing if volatility is not constant (Jorion, 1995). If volatility were deterministic in nature, the volatility parameter could be interpreted as a time-to-expiry average volatility. For stochastic volatility, however, the arbitrage arguments behind the BS model would fall short.

With the advent of the option on foreign exchange rate came also the need for an option pricing model that allows for interest rate parity. Garman and Kohlhagen (1983) therefore adjusts the standard BS model by the inclusion of a foreign interest rate so that

$$w(s_t, c, \tau, \sigma) = s_t e^{-r_f\tau} \mathcal{N}(\theta d_+) - c e^{-r\tau} \mathcal{N}(\theta d_-)\theta \qquad (2.3)$$

where

$$d_\pm = \frac{\ln\left(\frac{s_t}{c}\right) + \left(r - r_f \pm \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}},$$

r_f: the foreign short term interest rate

Black (1976) extends the BS model to allow for writing options on future contracts. Initially it was set up for commodity futures but it does the job for other types of futures as well, such as FX futures. The relationship

$$f_t = s_t e^{(r-r_f)\tau} \tag{2.4}$$

where

f_t : the future price at time t

gives a value of the option, a function of future price and time-to-maturity, that is

$$w(f_t, c, \tau, \sigma) = \theta e^{-r\tau}[f_t \mathcal{N}(\theta d_+) - c\mathcal{N}(\theta d_-)] \tag{2.5}$$

where

$$d_\pm = \frac{\ln(f_t) - \ln(c)}{\sigma\sqrt{\tau}} \pm \frac{\sigma\sqrt{\tau}}{2}$$

Operating within the BS framework, the model prices European options. Jorion (1995) points out that deriving implied volatility using Equation 2.5 would give a minor upward bias in the estimate. He emphasizes, though, that the shorter the maturity the smaller the error and that in some contexts approximation works well enough.

2.1.3 Moneyness

Moneyness is the umbrella term for the relative position of the price of the underlying asset in relation to the exercise price of the option (Whaley, 2006). As the intrinsic value of the option is a function of strike price and price of the underlying, the moneyness level is dependent upon the difference between the two prices. For example, upon maturity a call option has the intrinsic value of

$$w(s, T) = \begin{cases} 0, & s_T < c \\ s_T - c, & s_T \geq c \end{cases} \tag{2.6}$$

which obviously increases with an increase of s. Thus the higher the intrinsic value, the deeper *in-the-money*, ITM, the option is. Although the same call option would have zero intrinsic value in any level *out-of-the-money,* OTM - in any state where the price of the underlying asset is below exercise price, that is - the further below the price goes, the farther OTM the option goes. The option is only said to be *at-the-money,* ATM, when the exercise price is equal to that of the underlying.

For a put option the intrinsic value is

$$w(s,T) = \begin{cases} c - s_T, & s_T < c \\ 0, & s_T \geq c \end{cases} \tag{2.7}$$

which then increases with a decrease in s. The argumentation on moneyness for put option is analogue to that of the call option.

2.1.4 Delta hedging

The *delta* of an option is defined as the sensitivity in option value with respect to the price of the underlying asset. Considering the option value being a function of price of the underlying asset and time-to-maturity, the delta is the slope of this function and quantifies the impact of the change in the underlying asset's price to the price of the option (Hull, 2009).

Algebraically, the option's delta, Δ, is the partial derivative of the option value with respect to the price of the underlying, so that in the BS framework

$$\Delta_{BS} = \frac{\partial w(s_t, c, \tau, \sigma)}{\partial s} \tag{2.8}$$

Using Equations 2.2, 2.3 and 2.5 would then render deltas of $\theta\mathcal{N}(d_+)$, $\theta e^{-r_f\tau}\mathcal{N}(d_+)$ and $\theta\mathcal{N}(d_+)$ for the standard BS representation, the FX option and the option on an FX future respectively.

Alexander (2001) points out that for non-constant volatility it becomes more complicated. The delta of the option then becomes a function of the BS delta, Δ_{BS}, and the partial derivatives of the option value with respect to the volatility, $\partial w/\partial\sigma$ -

the *vega* of the option - and the volatility with respect to the price of the underlying, $\partial\sigma/\partial s$, so that

$$\Delta = \Delta_{BS} + (\partial w/\partial\sigma)(\partial\sigma/\partial s) \qquad (2.9)$$

Obliquely, the delta of the option tells the number of units that need to be bought of the underlying to cover the short position of the contract. Let us assume we have an FX option on the EURUSD currency pair. Furthermore we consider an exercise FX rate of 1.40 and delta of 0.5. Then the long position gives the right to purchase 100,000 EUR for 140,000 USD upon maturity. On the other side of the transaction, buying $0.5 * 100,000 = 50{,}000$ EUR would be sufficient to hedge the position. Alternatively, to hedge a short position in the future market, a delta of 0.5 would require 50,000 EUR to cover a nominal amount of 100,000 EUR (Reiswich & Wustyp, 2010 & 2012).

2.1.5 Implied volatility

Implied volatility (hereinafter *IV)* is the *ex ante* market forecast of volatility of the option's underlying asset and it mirrors the expectation of the investors (Day & Lewis, 1988).

Latané and Rendleman (1976), Jorian (1995) and Christensen and Prabhala (1998) provide favourable evidence for IVs being superior historical volatilities as estimates of future volatility. They point out that in the BS framework out of the 5 required inputs in the model, only the volatility parameter is unobservable. Black and Scholes (1972), albeit being able to show empirically that historical volatility works well as input within the framework, themselves point out that a standard deviation metric that exposes volatility over the maturity over the option would improve preciseness in option value if only had it been observable.

Mathematically the IV is obtained by using the option pricing model (i.e. the relevant version of BS) inversely. As the option value is a function of the investors expectation of the volatility during the life of option, the market price of the option at hand would allow for solving for implied volatility in the options pricing model. As IV estimates are noisy, different maturities and strike prices give different estimates, hence the price of the option is a function of the IV (Day & Lewis, 1988). Differently put, plugging in the IV estimate would render a theoretical option value equal to the market price (Badshah, 2008), so that

$$w_{theoretical}\left(s_t, c, \tau, \sigma_t^{imp}(c,T)\right) = w_{market}(c,T) \tag{2.10}$$

2.1.6 The FX implied volatility smile

If market option prices were to be consistent with the option pricing model based upon constant volatility, *stricto sensu* the volatility structure of the option would be totally non-dynamic. However, empirically it is shown that IVs alter both in the maturity as well as in the moneyness dimension (see Zhu and Avellaneda (1997); Skiadopoulus *et al*. (1999); Badshah (2008); Chalamandaris and Tsekrekos (2010)). The three-dimensional volatility function that depends on moneyness, *m,* and time-to-maturity, τ, is the *implied volatility surface:*

$$I_t(m,\tau) = \sigma_t(m(s_t),\tau) \tag{2.11}$$

It is suggested that IVs for ATM options adjust upwards with the increase of expiration days of the options, a phenomenon commonly referred to as the *term structure of volatility*. In a similar way asymmetry in the magnitude of IV due to changes in moneyness is generally referred to as the *volatility skew* (Derman, 1999).

The term *volatility smile* is coined due to the fact that for some type of options, such as FX options, the IV surface for a certain τ has a non-flat profile that is similar to the shape of a smile. This implies a volatility surface function whose second derivative is positive as it increases the further the option goes OTM, decreases when it is getting nearer ATM, and increases the deeper ITM it goes (Nowak & Sibetz, 2012). Figure 2.1 is a graphical representation of the FX smile.

For FX options the convention is to use delta as a measure of moneyness and IV for these options are therefore quoted somewhat differently than other options (Reiswich

& Wustyp, 2012). Typical delta call levels are 10 (ITM), 25 (ITM), 50 (ATM), 75 (OTM) and 90 (OTM). By put-call-parity OTM for a call option implies ITM for a put and vice versa (Nowak & Sibetz, 2012). Hence we would have that a 75 delta call equals 25 delta put and that 90 delta call similarly becomes 10 delta put, as depicted in Figure 2.1. In the market only the 50 delta call is quoted as implied volatility σ_{ATM} whereas the other quotes are their positions in relation to σ_{ATM}.

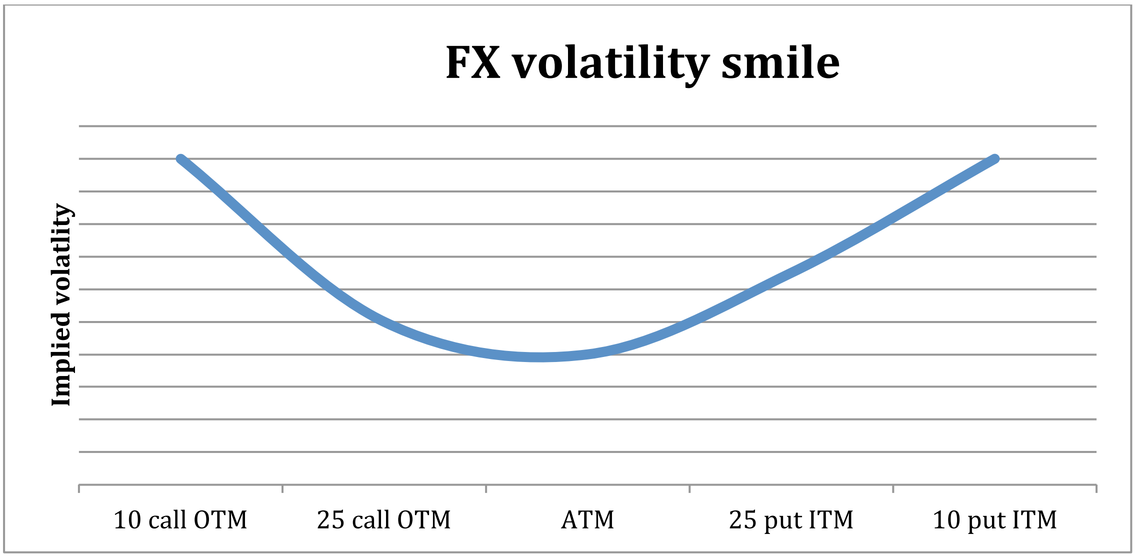

Figure 2.1 The FX volatility smile. Source: Author's own calculations.

The difference between call and put price volatilities is referred to as *risk reversal.* For example, the volatility of a 25 delta call, σ_{25C}, minus that of 25 delta put, σ_{25P}, gives the 25 delta risk reversal, RR_{25}:

$$RR_{25} = \sigma_{25C} - \sigma_{25P} \tag{2.12}$$

Thus, for any delta level, D, the risk reversal is:

$$RR_D = \sigma_{DC} - \sigma_{DP} \tag{2.13}$$

The measure of the distance of the averaged volatility of a delta call and put and the ATM IV at a certain delta level is called *butterfly.* Hence:

$$BF_D = \frac{1}{2}(\sigma_{DC} + \sigma_{DP}) - \sigma_{ATM} \tag{2.14}$$

Market quotes risk reversals and butterflies rather than volatilities for the different levels of delta (Reiswich & Wustyp, 2012); to get σ_{DP} and σ_{DC} use the relationships

$$\sigma_{DP} = \sigma_{ATM} + \frac{1}{2} RR_D + BF_D \tag{2.15}$$

$$\sigma_{DC} = \sigma_{ATM} - \frac{1}{2} RR_D + BF_D \tag{2.16}$$

2.2 Principal Component Analysis (PCA)

Collinearity in data is problematic as it introduces challenges in model specification. Increasing interdependence within the matrix of explanatory variables causes its corresponding correlation matrix to approach singularity. As a consequence it propels growing indeterminacy of structural relationships (Farrar & Glauber, 1967). One technique that enables conversion of potentially correlated variables to a set of linearly independent factors is principal component analysis (henceforth PCA). It is a statistical method originally proposed by Karl Pearson (1901) and developed and named by Harold Hotelling (1933). By fitting a multidimensional ellipsoid to the dataset in question and imposing orthogonality on its axes, a new set of uncorrelated variables referred to as *principal components* (hereinafter *PCs*) is provided. PCA strives to reduce dimensionality in the dataset by maximizing the cumulative contribution to overall variation in a minimized set of variables.

2.2.1 Orthonormal linear transformation

The focal point being the variation of the dataset, the analysis is based on the covariance matrix[2] (Joliffe, 2002). Let $\boldsymbol{Y}$ be a matrix comprised by a multivariate dataset

$$\boldsymbol{Y}_{T\times k} = [\boldsymbol{y}_1, \boldsymbol{y}_2, \dots, \boldsymbol{y}_k] \tag{2.17}$$

where

$\boldsymbol{y}_i$: a T $*$ 1 vector for $i = 1,2, \dots, k$

T: number of days in examined period,

k: the number of variables

[2] PCA is scale-dependent so in cases with variables with different units, these need to be standardized. Standardization of covariance implies transformation to correlation matrix and standardization of variables.

The symmetric corresponding covariance matrix is henceforth referred to as $\widehat{\mho}$. The *eigenvector* matrix $\boldsymbol{W}$ is obtained by spectral decomposition of $\widehat{\mho}$**:**

$$\widehat{\mho}_{k\times k} = \boldsymbol{W}\Lambda\boldsymbol{W}' \qquad (2.18)$$

where $\Lambda = diag[\lambda_1, \lambda_{2,}\lambda_3 \ldots.. \lambda_k]$ and comprises the diagonal *eigenvalues* that are related to the eigenvector matrix. The latter consists of k numbers of eigenvectors. Hence $\boldsymbol{W_{k\times k}} = [\boldsymbol{w_1}, \boldsymbol{w_2}, \ldots.., \boldsymbol{w_k}]$ is the orthogonal matrix in which the eigenvector corresponding to the eigenvalue λ_m is the *m*th column so that $\boldsymbol{w}_m = [w_{1m}, w_{2m}, \ldots. w_{km}]'$. The expansion of Equation 2.18 gives

$$\widehat{\mho} = \sum_{m=1}^{k} \lambda_m \boldsymbol{w}_m \boldsymbol{w}'_m \qquad (2.19)$$

By the optimization of Equation 2.19 the orthogonal eigenvector matrix is obtained. This is *de facto* an iterative process that includes solving of a sequence of optimization problems. The eigenvector $\boldsymbol{w}_m$ with the mth highest eigenvalue λ_m is obtained by optimizing $\lambda_m \boldsymbol{w}_m \boldsymbol{w}'_m$ with the restriction of $\boldsymbol{w}_m \boldsymbol{w}'_m = 1$ to ensure vector unity. The entire eigenvector matrix is therefore solved for by the optimization of $\lambda_m \boldsymbol{w}_m \boldsymbol{w}'_m$ for $m = 1,2,3, \ldots, k.$ Furthermore, to impose orthogonality *j* numbers of constraints are added for any $m > 1$; thus the optimization is additionally subject to $\boldsymbol{w}_j \boldsymbol{w}'_m = 0$ for all $1 \leq j < m$.

Hotelling (1933) explains how the first PC has the greatest mean square correlation in the PC-vector. By ordering the components according to size of their respective eigenvalues, PC1, PC2, PC3,…, PC*k* are extracted.

The principal component matrix $\boldsymbol{P}$ is defined by orthonormal linear transformation (Joliffe, 2002) of $\boldsymbol{Y}$ so that

$$\boldsymbol{P}_{T\times k} = \boldsymbol{YW} \tag{2.20}$$

where

$$\boldsymbol{P}_{T\times k} = [\boldsymbol{p}_1, \boldsymbol{p}_2, \dots, \boldsymbol{p}_k],$$

$$\boldsymbol{p}_i: the\ i\text{th principal component, a vector with the dimension } T * 1$$

That relationship at hand, it is implied that for Y we have

$$\boldsymbol{Y} = \boldsymbol{P}\,\boldsymbol{W}^{-1} \tag{2.21}$$

2.2.2 Dimensionality reduction

If some of the PC:s are found to contribute little enough to question the explanatory significance of them it can be supposed that the number of independent components are less than the variables that are measured (Hotelling, 1933). In fact, the "objective in many applications of PCA is to replace the p elements of $\mathbf{x}$ by a much smaller number of m of PCs, which nevertheless discard very little information" (Joliffe, 2002, p.111). That implies that a minimized number of PCs explains as much of the variation in the dataset as possible. Due to reduction in dimensionality the result should be more efficient and simpler computation (Badshah, 2008).

There are various rules one might adhere to when choosing the number of PCs, many of which are more or less *ad hoc* approaches or rule-of-thumbs. A standard approach according to Joliffe (2002) is to consider the cumulative contribution to the total variance. The approach involves choosing an m-sized subset of the PC matrix whose combined weights account for more than a threshold somewhere in the 70-90% range. Another is referred to as *Kaiser's rule* (Kaiser, 1960) which suggests the exclusion of PCs with individual variances smaller than 1. A third and a fourth method are a *scree graph*[3] and *log-eigenvalue diagram*[4].

[3] A scree graph plots eigenvalues against k that enables qualified studying of steepness of the slope. The m number of components is determined at the mth point where the steepness of the slope decreases. See Joliffe (2002).
[4] The logarithms of eigenvalues are plotted against k. See Joliffe (2002).

2.2.3 Applications in finance

As PCA by construction overcomes issues such as collinearity, it becomes a feasible alternative for the examination of financial data as correlation in that context regularly is observed. Not the least the inherited straightforward interpretation makes the approach popular and frequently used typically on term structures. For example, upon analysing the US treasury curve Litterman and Scheinkman (1991 cited in Forzani and Tolmasky (2003)) suggest that three factors translated to level, slope oscillation and convexity explains the majority of variability. As volatility surfaces share some structural characteristics with interest rate term structure it is plausible that the former fruitfully could be modelled in a similar fashion. As a result, studies of volatility dynamics using PCA favourably suggest that main variation in IV data could be accounted for by the first three principal components. Skiadopoulus *et al*. (1999); Alexander (2001); Cont and da Fonseca (2002); Bonney *et al.* (Bonney *et al*., 2008); all propose that approximately 70-90% is explained by three components.

A limitation with a PCA approach is the requirement of stationarity; a violation would give spurious results. First- and second-order moments define the stationarity condition by requiring that variance be finite, and mean and autocovariance time-independent (see Joliffe (2002); Brooks (2014)). Empirically this is a property that is not necessarily observed in volatility data (see for example Zhu and Avellaneda (1997)). To apply PCA on the volatility smile one interesting technique is therefore rather to model daily smile change in terms of fixed-delta deviations from ATM IV as proposed by Alexander (2001). The approach is interesting in the way it reduces noise in data and should resolve issues related to non-stationarity one risks encountering, by applying PCA on the volatility surface.

3. Literature review

Traditionally work that is carried out on IV dynamics tends to be equity option based. For example, Latané and Rendleman (1976) focus their investigation on stock options. In so doing they produce important pieces of evidence for their IVs being better forecasts of volatility in the Black and Scholes option pricing model. It is worth to note that Black and Scholes (1972) themselves suggest that implied volatility - had it been observable - is a better estimate. Also Christensen and Prabhala (1998) when undertaking a study on S&P100 index options, produce results in support of IV being a better predictor of future volatility than the historical counterpart. Their findings are not the least interesting in that they contrast earlier research, which commonly noted biasedness and inefficiency in estimates of IV. Moreover, the authors do hypothesize that volatility structures of FX options do not differ from those of the studied index options. However, they do not themselves carry out any tests on this category of options in support of this. Jorion (1995) on the other hand does produce results supportive of FX IV being superior in performance that of its historical counterpart. His research involves options on FX futures traded on CME and strives to quantify the improvement in prediction when substituting sample standard deviation with IV.

Obviously, because of the findings favouring the theory that IVs better reflect market expectations and thus impact option value, extensive effort is therefor made to enhance understanding and modelling of these market dynamics.

Derman (1999) points out that asymmetric volatility surfaces have been into effect no longer than since the stock market crash of 1987. He observes periodical trends in stock index volatility dynamics and look into the variation in these in relation to that of the index. Thereinafter he proposes a set of deterministic “sticky” models corresponding to these market regimes, emphasizing the relevance of volatility as a predictor for stock market profit. It is worth to note, however, that the models are not tested on other areas of hedging.

For exchange traded FX options, Taylor and Xu (1994) research four FX options traded in PHLX. Their work focuses mainly on term structure of IV and uses regression and *Kalman*[5] *filtering* methods. Their results provide evidence in support of volatility term structure dynamics for exchange traded FX options. It is worth to note however that the moneyness dimension is ignored and in addition research is done before the advent of the very liquid euro. Chalamandaris and Tsekrekos (2010) study the IV dynamics of OTC options on a multitude of currencies against the euro.

[5] A process that finds an optimal estimator by ”filtering” noisy data, adaptive to real time data processing. Is separates the ”noise” and the ”signal” (Kalman (1960)).

Using vector autoregression they suggest a non-flat profile of the OTC FX volatility surface.

PCA as a statistical technique is not unusual in finance. As financial markets frequently present a high degree of correlation, PCA has become a useful tool in that it efficiently extracts independent risk factors in a correlated system of data (Badshah, 2008). The similarities in structure of interest rate term and IV maturities make the PCA approach commonly used technique also for modelling of the IV surface. For example, Skiadopoulus *et al*. (1999) use PCA on IV of S&P500 future options. They group options into "buckets" based on their maturities, and average the IVs within buckets. By performing PCA on each bucket separately they suggest that two risk factors account for 78% of the variation in the volatility skew, albeit only 60% when studying variation in the entire volatility surface. This contrasts work by Derman and Kamal (1997) who, by rather studying OTC options, propose that three risk factors explain 95% of surface variability. Skiadopoulus *et al*. (1999) then assume a noisier structure for a future option than that of an OTC option. Interestingly, the authors speculate that the volatility surface be asset specific, but refer to future research for support. Rather than studying maturity buckets individually Fengler, Härdle and Villa (2003) claim different maturity groups might follow a common *eigenstructure*. Hence they take on a *common principal components analysis* approach, which allows for dimensionality reduction to only a very small amount of components shared by maturity groups.

Cont and da Fonseca (2002) put emphasis on the time dimension, pointing out not only how most studies focus on ATM volatility but also how they tend to overlook the time aspect. They use *Karhunen-Loéve decomposition*[6] to describe the random fluctuation of the volatility surface with a small number of risk factors. When investigating FTSE and S&500 options respectively not only do they find that IV data have mean-reverting property and show highly positive autocorrelation, but also do they suggest that variance of daily log-variations in IV ought to be explainable by two or possibly three PCs. Furthermore they propose that IV surface has a non-flat shape and the adjustments of the price of the underlying are not correlated with changes in

[6] A dimension reduction technique for stochastic processes

IV. It cannot be ruled out, however, that the estimations of IVs suffer from measurement errors. Such issue is typical for any index option study[7].

On the other hand, Alexander (2001) proposes that FTSE100 index and IV levels do move *in tandem* and that level, slope oscillation and convexity are the focal determinants of the volatility skew. She develops a technique to model the volatility skew by performing PCA on daily changes in deviations of fixed-strike IV from the ATM counterparts. This way she proposes a way to expose volatility sensitivity to changes in price of the underlying. The author extends her model by quantifying a time-varying sensitivity parameter. Bonney, Shannon and Uys (2008) follow Alexander when applying this methodology on index options, but instead choose to model the South African Top40 skew. The researchers aim to decompose volatility risk in the less liquid but emerging market setting of the Top40 option market. However, to the knowledge of the author of this thesis, the methodology for which we are indebted to Alexander (2001) has as of yet not been used on FX options.

Overall, not very many attempts seem to have been made to model the FX smile using PCA. Zhu and Avellaneda (1997) ignore the smile but use the technique to model the volatility term structure of FX options. They study a historical dataset that comprises IVs from options on no less than 13 currency pairs. In so doing they observe non-stationary IV data. Furthermore, they observe strong heteroscedastic property in the implied volatility process, an issue they choose to resolve by the utilization of an E-ARCH[8] model. Interestingly they show that PCA efficiently can model level, slope oscillation and convexity, and that their cumulative contribution total about 95% of the FX volatility term structure variation.

[7] Estimation of index volatility might be subject to distortion (Jorion, 1995). The higher the number of assets included in the underlying portfolio, the lower the likelihood that all are traded at the same time point in time as the option. Also, bid-ask spread adds on errors-in-variables issue.

[8] Exponential ARCH

4. Methodology

As previously mentioned this paper suggests the FX smile to be modelled by PCA approach to overcome issues with collinearity in data. Moreover, to evade problems with violations of the stationary condition - typically observed in volatility data - this paper takes on a specific PCA approach developed by Alexander (2001). She models the volatility skew as fixed-strike IV deviations from ATM IV. The following sections outline the steps involved in the application of the model to the FX smile, with the inclusion of motivation and derivation of the technique as detailed in Section 4.1. The succeeding Section 4.2 outlines the PCA approach undertaken on the model proposed in Section 4.1. The concluding section describes the suggested quantification process of time-varying sensitivity of volatility to changes in future price.

4.1 Modelling of volatility

Alexander (2001) constructs her model based on the assumptions of Derman (1999) who proposes parameterization of the volatility skew[9] according to different market regimes, commonly referred to as "sticky models"[10]. Let σ_0 and s_0 be the initial values at the first observation of the skew. Then denote current IV of an option with strike c and time-to-maturity τ with $\sigma_{c,t}(\tau)$ and current ATM IV with $\sigma_{ATM,t}(\tau)$, then in the Derman framework we observe three market regimes:

a) According to the *sticky-strike rule* the volatility skew should be parameterized as

$$\sigma_{c,t}(\tau) = \sigma_0 - b(\tau)(c - s_0) \tag{4.1}$$

where $b(\tau)$ is the slope of the volatility skew for maturity τ. Then for an ATM-option, as strike price equates current price, s_t, we would have

[9] Derman's work is carried out on S&P500. As previously mentioned, for FX options it is normal rather to refer to the "volatility smile" due the dynamics of the volatility of the FX options. For argumentation purposes however, to keep consistency with Derman's work, the term "skew" will be used.

[10] "Sticky" in the context of option trading is a reference to something that does not change. Derman points out that it is sometimes more straightforward to construct models around what does not change, which thus is the basic principle for his "sticky models".

$$\sigma_{ATM,t}(\tau) \ = \sigma_0 - b(\tau)(s_t - s_0) \tag{4.2}$$

implying that only $\sigma_{ATM}(\tau)$ changes with a change in s.

b) The *sticky-delta rule* says that the market skew is parameterized as

$$\sigma_{c,t}(\tau) \ = \sigma_0 - b(\tau)(c - s_t) \tag{4.3}$$

and for ATM volatility

$$\sigma_{ATM,t}(\tau) \ = \sigma_0 \tag{4.4}$$

indicating a regime in which $\sigma_c(\tau)$ changes with change in s, whereas $\sigma_{ATM}(\tau)$ stays unaffected upon such alteration.

c) The *sticky-tree* model implies that

$$\sigma_{c,t}(\tau) \ = \sigma_0 - b(\tau)(c + s_t) + 2b(\tau)s_0 \tag{4.5}$$

and then

$$\sigma_{ATM,t}(\tau) \ = \sigma_0 - 2b(\tau)(s_t - s_0) \tag{4.6}$$

In this regime, both fixed-strike and ATM volatilities are affected by a jump in the price of the underlying asset.

Alexander (2001) points out that in this framework - regardless of market regime - the fixed-strike deviation from the ATM IV always is the same for any τ:

$$\sigma_{c,t}(\tau) \ - \sigma_{ATM,t}(\tau) \ = \ -b(\tau)(c - s_t) \tag{4.7}$$

Alexander (2001) argues that the linear relationship in Equation 4.7 entails that the deviations of fixed-strike volatilities and the ATM volatility will change with the same amount - namely $b(\tau)$ - as the price of the underlying moves. This would imply one *single* significant risk factor of a change in fixed-strike deviations. Hence she challenges Derman's sticky models by applying PCA on daily changes in volatility skew $\Delta(\sigma_c - \sigma_{ATM})$ and in so doing she finds it to be an efficient method when used on index options. More importantly, she emphasizes the utility on FX options as well.

4.2 PCA

4.2.1 Principal components analysis of daily changes in the FX volatility smile

By Equation 2.17, Chapter 2 matrix $\boldsymbol{Y}$ of daily changes in the FX volatility smile is constructed so that

$$\boldsymbol{Y}_{(T-1)\times k} = [\Delta(\sigma_{\mathrm{D1}} - \sigma_{ATM}), \Delta(\sigma_{\mathrm{D2}} - \sigma_{ATM}), \ldots, \Delta(\sigma_{\mathrm{Dk}} - \sigma_{ATM})] \quad (4.8)$$

where

k: the number of deltas

$\Delta(\sigma_{\mathrm{D}i} - \sigma_{ATM})$: a $(T-1) * 1$ vector of daily changes in volatility skew

By applying PCA framework on the corresponding covariance matrix eigenvalues and eigenvectors are obtained by solving the optimization problem as per Equation 2.19, Chapter 2. Finally the PC matrix is given by Equation 2.20, Chapter 2.

4.2.2 Dimensionality reduction

In Alexander's work (2001) the first three PCs account for up to 90% of the variation in the dataset. Considering that the remaining PCs altogether only make up for 10% of the variability, in line with PCA theory and in alignment with the Alexander's methodology, the focus in this paper will also be on the first three components.

This leaves us with a relationship where a change in fixed-delta deviation from ATM IV is

$$\Delta_t(\sigma_D - \sigma_{ATM}) \approx w_{D1}p_{1,t} + w_{D2}p_{2,t} + w_{D3}p_{3,t} \quad (4.9)$$

where

$p_{i,t}$: the ith principal component at time t

4.3 Quantification of sensitivity of PCs to changes in future price

The time-varying sensitivity, $\gamma_{i,t}$, of the PCs to changes in the price of the underlying asset (future price), Δf_t, is parameterized by the implied relationship between change in price (future price) and the *i*th principal component (Alexander, 2001):

$$p_{i,t} = \gamma_{i,t}\Delta f_t + \varepsilon_{i,t} \quad (4.10)$$
$$\varepsilon_{i,t} \sim IID\ \mathcal{N}(0,1)$$

The gamma matrix $\Gamma_{(T-2)*3} = [\gamma_{1,t}, \gamma_{2,t}, \gamma_{3,t}]$ is estimated by using the correlation

$$\gamma_{i,t} = \frac{cov_{t-1}(p_{i,t}, \Delta f_t)}{var_{t-1}(\Delta f_t)} \qquad \text{for } i = 1,2,3 \quad (4.11)$$

To evade extensive parameterization Alexander (2001) suggests that an exponentially weighted moving average (EWMA) process can be utilized to estimate the sensitivity coefficient so that

$$cov_{t-1}(p_{i,t}, \Delta f_t) = \hat{\sigma}_{p_i,\Delta f;t} = (1-\lambda)p_{i,t-1}\Delta f_{t-1} + \lambda\hat{\sigma}_{p_i,\Delta f;t-1} \quad (4.12)$$

As $var_{t-1}(\Delta f_t) = cov_{t-1}(\Delta f_t, \Delta f_t)$ that implies that

$$var_{t-1}(\Delta f_t) = \hat{\sigma}_t^2 = (1-\lambda)\Delta f_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2 \quad (4.13)$$

Note that initial values $\hat{\sigma}_0^2$ and $\hat{\sigma}_{p_i,\Delta f_t;0}$ are the unconditional variance and covariance respectively. Ultimately a smoothing parameter λ of 0.94 as provided by JP Morgan/Reuter's RiskMetrics is used as per proposal.

4.4 Data

For the dataset we are indebted to OptionWorks®. It comprises quotes of American style EURO FX Future Options (EURUSD) traded on Chicago Mercantile Exchange (CME). This choice is similar to Wang (2007) and Jorion's (1995). They both choose to investigate options on FX futures traded on CME. The choice of the currency pair EURUSD comes rather natural, as this is the most traded currency pair in the world. As such it is a currency pair more likely than any other to be involved in hedging activities. The choice of CME FX options on futures is supported by Jorion's (1995) argumentation. Firstly, the contracts in question are actively traded. Secondly, the fact that both the option and the underlying asset are being traded side-by-side in the exchange negotiates issues such as non-synchronous trading. A third reason is of a more practical nature. Specified market quotes on options on futures are publicly available, in that they are exchange traded (Hull, 2009).

The data range from 15th June 2009 – 11th December 2009 and are 126 daily quotes for the 3-month maturity on ATM, risk reversals and butterflies, with deltas of 10 and 25 respectively. As both Alexander (2001) (1-, 2- and 3-month) and Bonney *et al.* (2008) (3-month) include 3-month maturities for their studies, the same maturity is used for the work of this paper as well. Furthermore the dataset includes daily future prices.

Jorion (1995) points out that the fact that options of futures that are traded on CME are American would render a biased estimate of the IV if used inversely in Black (1976). However, for shorter maturities this estimation error ought to be very small. Also, in this particular case data is already on quoted IV.

Daily IVs for 10 delta put, 25 delta put, 25 delta call and 10 delta call are obtained using the relationships as per Equations 2.15 & 2.16, Chapter 2.

4.5 Delimitations

As with any methodology some compromises are made. Firstly, this work does not investigate volatility dynamics of several maturities. This is in line with the research work performed by Bonney *et al*. (2008) but not by Alexander (2001) who investigates in total three maturities. Obviously it would add depth to any examination to include more dimensions.

Secondly - as has been pointed out - with investigation of exchange traded options comes some advantages. However, there is also a downside. In fact, OTC FX options are the most traded FX derivatives. As market prices are functions of IV, obviously how these derivatives are being priced on OTC market would provide very interesting and valuable insight on market volatility dynamics. Without further research, at this point it cannot be ruled out that some valuable insights about FX markets volatility dynamics are being lost as focus is being held on the less liquid exchange traded market.

The third delimitation concerns measurement errors as - as is being pointed out by Reiswich and Wustyp (2012) - Equations 2.15 & 2.16, Chapter 2 represent a somewhat *ad hoc* way to compute the IVs for the different delta levels. However, as the method is being used and with acceptable results (see for example Chalamandaris and Tsekrekos (2010)) these errors - if any - are expected to be rather small.

5. Results and analysis

This section outlines the empirical results of the two-step methodology detailed in the previous chapter. It displays the results of the PCA carried out on the dataset as specified. That is being followed up by a report of the sensitivity analysis of the PCs to changes in price of the underlying future. The results reported herein are aimed to fulfil the twofold objective of this paper, which is the validation of the proposed model on the FX smile in combination with the study and quantification of the sensitivity in risk factors to changes in future price.

5.1 Principal Component Analysis

5.1.1 Covariance matrix

We know from the theoretical framework that PCA is based upon the covariance matrix, hence covariances of the daily changes in fixed-delta deviations are computed and the results thereof are displayed in Table 5.1. It can be noted that autocovariances between the delta calls and delta puts are negative.

Δ	$\sigma_{10c} - \sigma_{ATM}$	$\sigma_{25c} - \sigma_{ATM}$	$\sigma_{25P} - \sigma_{ATM}$	$\sigma_{10P} - \sigma_{ATM}$
$\sigma_{10c} - \sigma_{ATM}$	1.27E-06	5.15E-07	-2.33E-07	-4.84E-07
$\sigma_{25c} - \sigma_{ATM}$	5.15E-07	2.98E-07	-2.03E-07	-3.16E-07
$\sigma_{25P} - \sigma_{ATM}$	-2.33E-07	-2.03E-07	3.84E-07	5.94E-07
$\sigma_{10P} - \sigma_{ATM}$	-4.84E-07	-3.16E-07	5.94E-07	1.37E-06

Table 5.1 The covariance matrix, corresponding to daily changes in fixed-delta deviations.

5.1.2 Eigenvalues

When carrying out PCA on the covariance matrix it is shown that the first three components account for the majority of the variability in the dataset, which is shown in Table 5.2. The table reports the PCA decomposition with corresponding eigenvalues for all the four PCs. We recall that PC1 typically translates to level and here it alone accounts for 66.85% of the total variance. Representing the slope oscillation, the contribution of PC2 is 28.10%, which is still a very relevant portion of changes in the smile. In addition the convexity component contributes with 4.00% to total variability. All in all, this results in the cumulative contribution of 98.96% for PC1, PC2 and PC3.

Component	Eigenvalue	Contribution	Cumulative
PC1	2.20E-06	66.85%	66.85%
PC2	9.26E-07	28.10%	94.95%
PC3	1.32E-07	4.00%	98.96%
PC4	3.43E-08	1.04%	100.00%

Table 5.2 Eigenvalues of the covariance matrix

5.1.3 Eigenvectors

Δ	$\boldsymbol{w_1}$	$\boldsymbol{w_2}$	$\boldsymbol{w_3}$
$\boldsymbol{\sigma_{10c} - \sigma_{ATM}}$	-0.5868	0.7280	0.2036
$\boldsymbol{\sigma_{25c} - \sigma_{ATM}}$	-0.3029	0.2080	-0.4906
$\boldsymbol{\sigma_{25P} - \sigma_{ATM}}$	0.3267	0.2619	0.7452
$\boldsymbol{\sigma_{10P} - \sigma_{ATM}}$	0.6761	0.5985	-0.4032

Table 5.3 Eigenvectors of the covariance matrix

Table 5.3 shows a first eigenvector $\boldsymbol{w_1}$ containing weights reasonably similar in proportion but different in signs. Furthermore and aligned with expectation it can be noted that weights increase with lower deltas, as risk is assumed to increase with distance to ATM. These results signify that a shift in PC1 renders asymmetric changes in the fixed-delta deviations. A positive adjustment would give negative changes for the first two fixed-delta deviations, namely 10C and 25C, but positive for 25P and 10P. Table 5.3 also displays figures for 10C and 10P that in absolute value are reasonably similar in magnitude. The same relationship is noted for changes in 25C and 25P. Moving in different directions, this impels a non-parallel adjustment of the volatility surface and a slightly "tilting" effect in the evolution of the smile. A graphical example of this is depicted in Figure 5.1. This result is counterintuitive, as it would be expected that fixed-delta calls and puts IV levels would move *in tandem.* Contrary it is shown that an interpretation of PC1 as a *stricto sensu* parallel shift of the smile is an oversimplification.

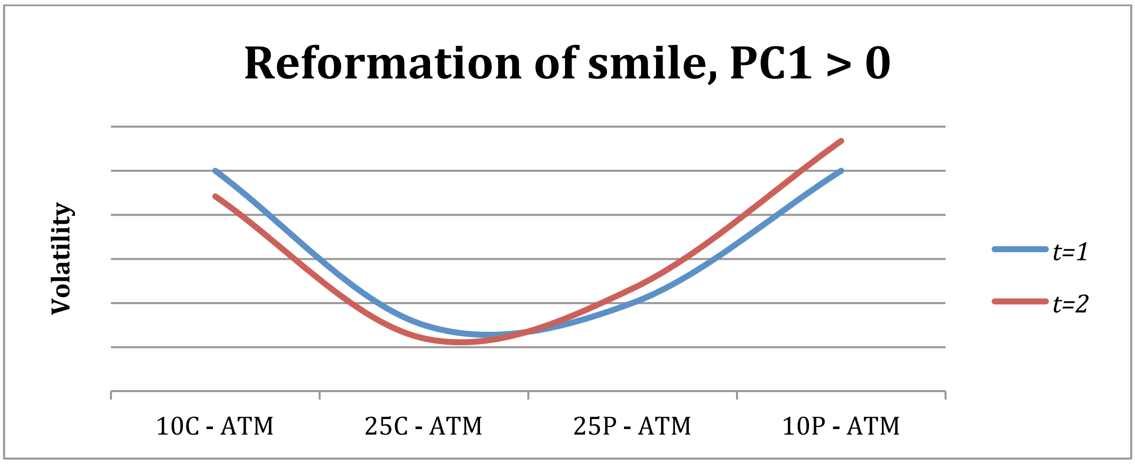

Figure 5.1 Example of how a positive PC1 reforms the smile. The negative weights of the fixed-delta call deviations propels a negative shift in the left tail of the smile, whereas the positive weights for the fixed-delta put deviations move the right tail upwards. Source: Author's own calculations.

Table 5.3 reports that the second eigenvector $\boldsymbol{w}_2$ comprises only positive weights, determining the impact of innovation in PC2, which translates to slope oscillation component. This suggests that directional changes in slopes of fixed-delta deviations are equally determined by PC2. This therefore involves a modification of the range of the smile upon a given alteration of PC2. A positive slope oscillation component therefore implies a narrower range of the smile, of which an example is given in Figure 5.2, whereas a negative component suggests widening of the smile like shown in Figure 5.3. Furthermore, just like for $\boldsymbol{w}_1$ and for the same reason it can be noted that values of the weights increase with higher/lower deltas.

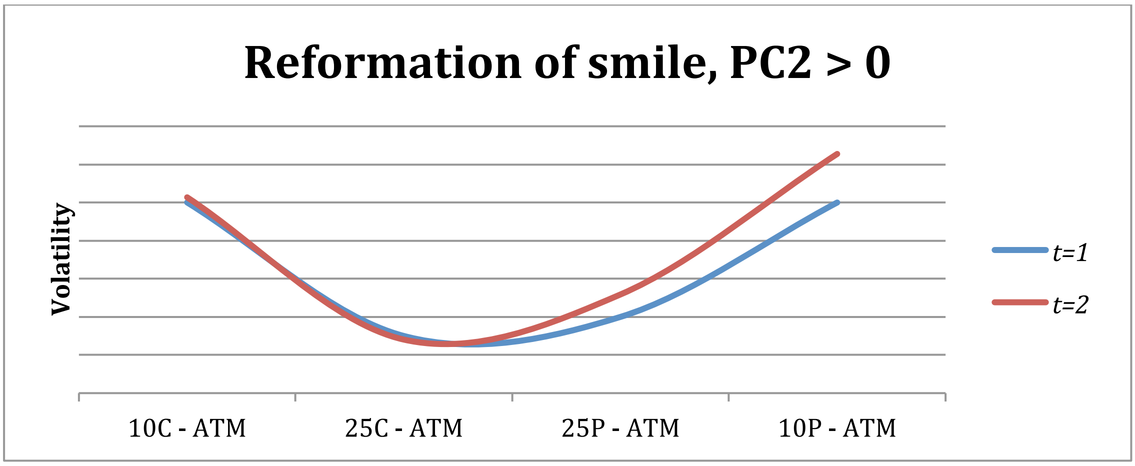

Figure 5.2 Example of how a positive PC2 reforms the smile, given the same positive adjustment in PC1 as in Figure 5.1. As weights in the eigenvector are positive all slopes are adjusted positively, causing a narrowed smile. Also note how the small but noticeable disproportion between call and put levels manifests itself. This is most obvious at 25P and 25C as the former is seemingly more affected than 25C. Source: Author's own calculations.

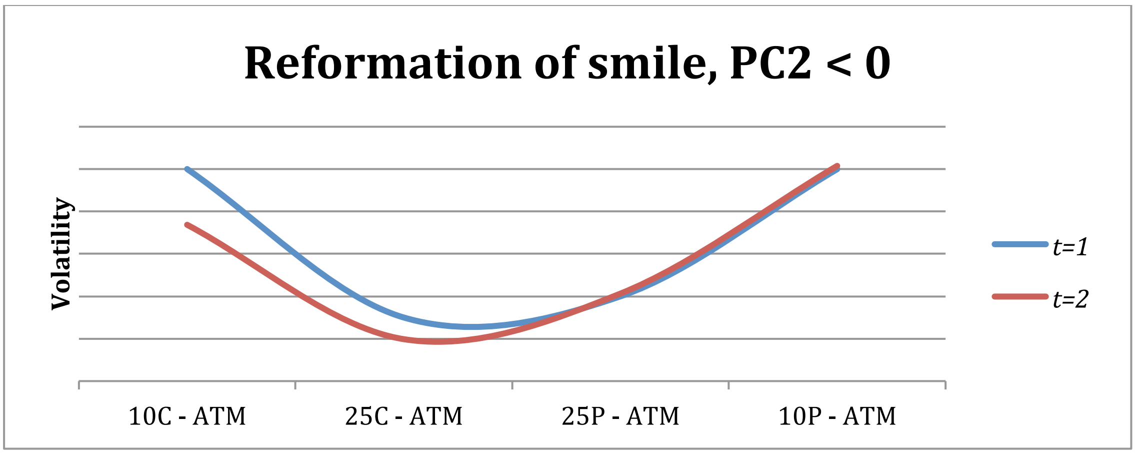

Figure 5.3 Example of how a negative PC2 reforms the smile, given the same positive adjustment in PC1 as in Figure 5.1. Again, as all weights in the eigenvector are positive, the negative PC2 impels a widened smile. Disproportion between weights in this case causes the more obvious reformation to occur in the left tail. Source: Author's own calculations.

For the third eigenvector $\boldsymbol{w}_3$ that corresponds to PC3 - which translates to convexity - it should be noted how signs vary within vector. That weights and signs differ between fixed-delta deviations implies that convexity of the smile to some extent will be adjusted for a given change in PC3. A given innovation in PC3 would then adjust the curvature of the smile negatively in two fixed-delta levels and positively in the remaining two. Moreover, it is interesting to see that 10C and 10P counteract, so that a negative change in 10C is correlated with a positive change in 10P. Examples of both positive and negative PC3 are given in Figure 5.4 & 5.5.

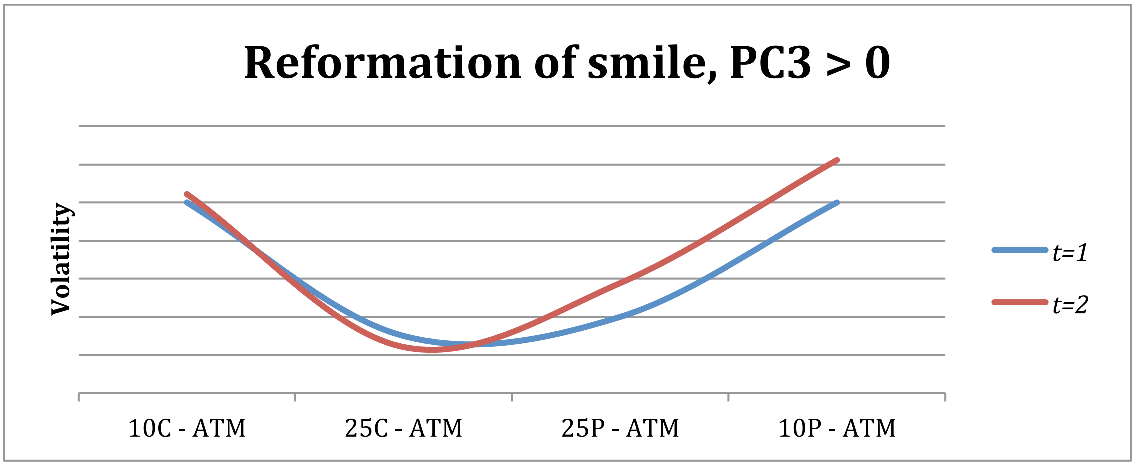

Figure 5.4 Example of how a positive PC3 reforms the smile, given positive PC1 & PC2. A smoothening of the curvature can be observed in 10C and 25P fixed-delta deviations, whereas convexity is sharpened at 25C and 10P, most noticeably at 25C. Especially 25C and 25P are affected, which stands in relation to their proportional weights in the eigenvector. Source: Author's own calculations.

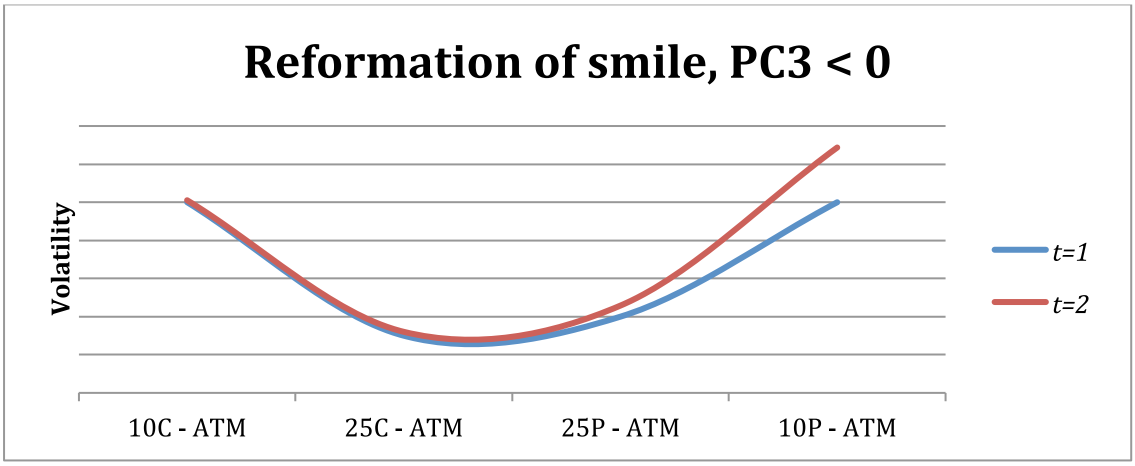

Figure 5.5 Example of how a negative PC3 reforms the smile, given positive PC1 & PC2. Most noticeably again 25C and 25P fixed-delta deviations are most impacted and obviously in the opposite way as in Figure 5.4. In other words, that means smoothening of the curvature at 25C and sharpening at 25P. Source: Author's own calculations.

5.2 Sensitivity of PCs to changes in the future price

Equation 4.10 in the methodology chapter algebraically explains the relationship between change in future price and each PC. The relationship implies certain sensitivity in the component to changes in price, which is quantified by its corresponding gamma coefficient. Figure 5.6 graphs the gamma coefficients, $\gamma_1, \gamma_2, \gamma_3$, for the three PCs and already by a first qualitative assessment a few discrepancies in behaviour can be noted. First, γ_1, typically is higher in absolute value and overall positive. Second, γ_1 seems to show low stability in terms of variation, but high in terms of shifts of signs. Third, γ_3 seems to be more stable than the other two and typically closer to zero. A subsample of the gamma matrix is reported in Table 5.4; refer to Appendix A.4 for the full sample.

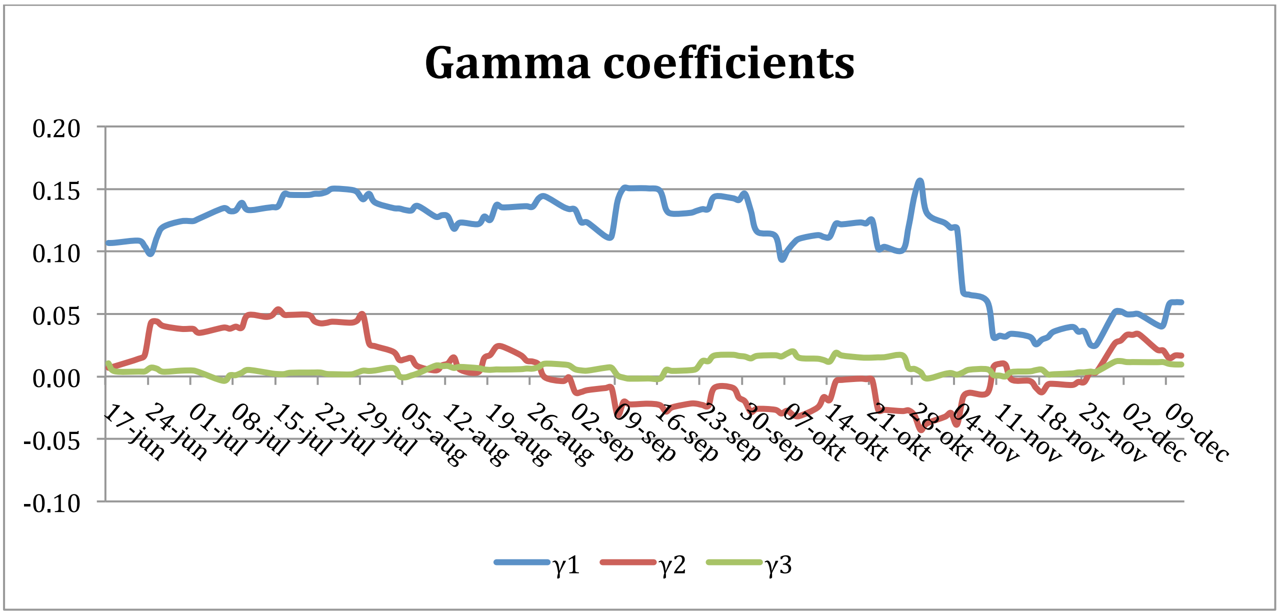

Figure 5.6 The time-varying gamma coefficients of the three PCs for the entire sample period.

Date	γ_1	γ_2	γ_3
02/11/09	0.1228	-0.0323	0.0021
03/11/09	0.1188	-0.0292	0.0027
04/11/09	0.1183	-0.0384	0.0015
05/11/09	0.0676	-0.0166	0.0033
06/11/09	0.0654	-0.0131	0.0054
09/11/09	0.0598	-0.0132	0.0057
10/11/09	0.0315	0.0075	0.0012

Table 5.4 Subsample of the gamma matrix ranging from 2nd November 2009 to 10th November 2009.

5.2.1 The gamma coefficient of the level component

That γ_1 - capturing sensitivity in PC1 – overall has a higher level in absolute value is aligned with expectancy due to the high contribution to the covariance matrix as has been noted in Table 5.2. Also the constant positive sign comes as no surprise, as an augmentation of the future price clearly should induce a positive shift in volatility.

Moreover, Figure 5.6 gives the impression that γ_1 is a highly varying parameter, which obviously is related especially to the big drop in level upon a certain point in time, namely 5th November 2009. However, before and after the drop the coefficient still has a fairly narrow range, between 0.0936 and 0.1563 before the drop and 0.0254 and 0.0676 after.

5.2.2 The gamma coefficient of the slope oscillation component

The second gamma coefficient tracks sensitivity of the slope oscillation component to change in the future price. In other words it quantifies the tendency to narrow or widen the range of the smile. Therefore it is an interesting result to see how this propensity differs over the sample period. In two periods, from 17th June 2009 to 27th August 2009 and 10th November 2009 to 12th November 2009, change in future price shows positive correlation with PC2. In these time periods a positive change in future price implies a narrowed smile, a scenario in fashion similar to that illustrated in Figure 5.2. This contrasts the mid-period that ranges from 28th August 2009 to 9th November 2009 that shows typically negative co-movement between the two variables, which would lead to a wider smile for positive shifts in future price. Figure 5.3 shows a simplified example of such reformation of the smile. γ_2 ranges between -0.0429 and 0.0537.

5.2.3 The gamma coefficient of the convexity component

γ_3 - exposing sensitivity to shifts in the convexity component - is typically but not always positive and shows overall the greatest stability. Moreover it is already noted that this is the parameter that congregates closest to zero, with a range between -0.0035 and 0.0200. Obviously this tells us that PC3 is the risk factor less sensitive to changes in the future price, which is in line with expectation, as we recall the contribution of only 4% for the convexity component from Table 5.2.

5.3 Implications

In comparison with research on index options (see Alexander (2001); Bonney *et al.* (2008)) dimensionality reduction in this paper involves an even smaller compromise with an explanatory model that explains more than 98% of the change in the volatility smile. Setting this in perspective, results close to 80% (Bonney *et al.*, 2008) and up to 90% (Alexander, 2001) are reported. The high cumulative contribution of PC1, PC2, PC3 is in line with expectancy. At the one hand, Bonney *et al.* (2008) point out that it is the liquidity level of the dataset that might produce the difference between in explanatory level of PCA, which is aligned with the fact that the FX market is considered one of the most liquid markets. On the other hand, reduction of dimensionality in this case is a smaller compromise *per se* due to the smaller range of initial dimensions.

Attention should be paid to that some of the results in this paper - derived from FX derivatives - contrast previous work on index option. When Alexander (2001) and Bonney *et al*. (2008) report typically positive weights in the first eigenvector, the results of this paper rather point in the direction of asymmetrical changes in the volatility smile. It cannot be ruled out that this is due to estimation errors potentially induced by the somewhat *ad hoc* way IVs are computed as discussed in the data section. To do so, further research is called for.

The quantification of sensitivity in PCs to changes in the underlying future price further shows that the model captures time-varying effects in the volatility smile. A comparison with other options – namely index options - is relevant as the proposed model is specified to capture volatility dynamics of different types of options but is tested sparsely on FX derivatives. When the volatility surface empirically differs in shape between options, it is interesting how the properties of the gamma matrix differ from that of the index option counterpart. The empirical results of this paper are in alignment with Alexander (2001; Alexander, 2001; Alexander, 2001; Alexander, 2001; Alexander, 2001; Alexander, 2001; Alexander, 2001) and Bonney *et al*. (2008) in terms of a γ_1 that does not change signs and typically is higher in absolute value. They do however contradict in respect to stability of the gamma coefficients as index option induced γ_1:s are reported to be more stable, and γ_2:s and γ_3:s less so.

Another implication of the reported results is that the gamma coefficients show susceptibility to shifts in market regime. The gamma coefficients that clearest show affection to these are γ_1 and γ_2. We recall from Figure 5.6 the two periods from 17th June 2009 to 27th August 2009 and 10th November 2009 to 12th November 2009 with a narrowing smile and a period in-between with a widening smile. On the one hand γ_1 is clearly affected by a change in market regime upon the obvious drop on 5th November 2009. As Table 5.4 shows, on this date the entire gamma matrix is affected by a shift in market regime. On the other hand, γ_1 does not show any major change in sensitivity *level* upon the shift at 28th August 2009 although some higher instability in the period that ranges thereafter could be noted. As mentioned γ_3 typically congregates close to zero, yet a somewhat higher sensitivity level where convexity changes appear more is observed in the mid-period. Furthermore, on 5th November 2009 a *de facto* jump of 123.86% is reported.

Albeit it therefore could be suggested that the proposed model captures the FX specific dynamics of the volatility surface, yet it reveals little about the reasons behind. These are rather inherited in the mechanics of the derivative *per se* in addition to the technicalities of the underlying future. It has neither been the intention nor the expectation of this paper to try and explain the exact causations of the FX specific volatility effects. To gain insight, presumably an adjusted or alternative approach would be required.

6. Conclusion

This paper proposes a PCA approach to model the dynamics of implied volatility smile of FX future option. The latter are especially interesting due their liquidity and the application for risk managers and other agents engaging in hedging activities. The methodology suggested uses fixed-delta implied volatility deviations from the at-the-money implied volatility to capture these dynamics. PCA is appealing in that it allows for the extraction of uncorrelated explanatory factors in a set of correlated variables, frequently observed in financial data. The proposed adaptation of the PCA approach overcomes issues such as violation of the stationary condition, which is rather the norm in volatility data.

The findings show that that we have at hand a model that well captures the volatility dynamics of FX options and provide further evidence that these can be explained by three risk factors alone. It is shown that the contribution of these risk factors total more than 98% to the variability in the smile. These findings are favourable for this paper's objective to validate conformity of the model on FX options. Furthermore the paper seeks to expose and quantify time-varying sensitivity in the three relevant risk factors. The results provided show that sensitivity indeed is time-dependent and hence the shape of the volatility surface is affected by a shift in market regimes.

Whereas most of the results in this paper are in alignment with expectation, others are less anticipated such as the non-parallel movement of the smile upon a given shift in future price. Whereas the methodology of this paper does accomplish in displaying the volatility dynamics of the dataset in question, it is less successful in concretizing the instigating sources of the effects. What exactly are the roots of these FX specifics have not been under the radar of this paper, but the results herein prompt there are incentives for future research of this topic. Moreover, as research up to this point favours that liquidity level plays a role in model specification, investigation of the even more liquid over-the-counter market could provide some interesting insights with application to risk management.

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Appendix

A.1 Daily changes in the FX smile

Date	10C - ATM	25C - ATM	25P - ATM	10P - ATM
16/06/09	-0.000875	-0.000554	0.001008	0.000094
17/06/09	-0.000779	0.000077	0.000179	0.001229
18/06/09	-0.000339	-0.000024	-0.000678	-0.001483
22/06/09	-0.001857	-0.000844	0.000052	0.000313
23/06/09	0.000640	-0.000085	0.001498	0.002418
24/06/09	0.001244	0.000601	-0.001317	-0.002505
25/06/09	-0.002275	-0.000608	0.000408	0.001767
26/06/09	-0.001206	-0.000589	0.000835	0.001162
29/06/09	-0.000338	-0.000088	0.000396	0.000094
01/07/09	-0.001208	-0.000253	0.000151	0.000696
02/07/09	0.001046	0.000037	-0.000566	-0.002674
06/07/09	-0.000045	0.000595	-0.000861	0.000177
07/07/09	-0.000185	-0.000052	-0.000588	-0.000934
08/07/09	0.001076	0.000759	-0.001035	-0.001387
09/07/09	0.000043	-0.000414	0.001284	0.002253
10/07/09	0.000813	0.000241	-0.000247	-0.001214
13/07/09	0.000190	0.000203	0.000059	0.001310
14/07/09	-0.001005	-0.000159	-0.000613	-0.001858
15/07/09	-0.001658	-0.000796	0.001460	0.002740
16/07/09	0.000470	-0.000446	0.000550	-0.000636
17/07/09	-0.000039	0.000186	-0.000216	0.000170
20/07/09	-0.001264	-0.000354	0.000428	0.000501
21/07/09	0.000997	0.000167	0.000016	0.000279
22/07/09	-0.000425	-0.000217	-0.000037	0.001143
23/07/09	0.000560	0.000510	-0.000572	-0.001602
24/07/09	-0.000137	0.000081	-0.000626	-0.000863
27/07/09	0.000849	0.000016	0.000566	-0.000004
28/07/09	-0.000865	-0.000073	-0.000497	-0.000543
29/07/09	0.002379	0.000910	-0.000781	-0.001210
30/07/09	0.000016	0.000018	-0.000057	-0.000401
31/07/09	-0.001134	-0.000766	0.000864	0.001325
03/08/09	-0.001536	-0.000636	-0.000054	0.001435
04/08/09	-0.000800	-0.000259	0.000779	-0.000188
05/08/09	0.000024	-0.000147	0.000753	0.000219
06/08/09	0.001727	0.000831	-0.000564	-0.000496
07/08/09	0.000957	0.000560	-0.001092	-0.000559
10/08/09	-0.000532	-0.000092	-0.000646	-0.001680
11/08/09	0.001672	0.000281	0.000137	0.000411
12/08/09	0.001485	0.001029	-0.000213	-0.000157
13/08/09	-0.002988	-0.001424	0.000058	-0.000335
14/08/09	0.000759	0.000388	-0.000148	-0.000543
17/08/09	-0.000033	-0.000045	-0.001219	-0.002489
18/08/09	0.000373	0.000271	0.000275	0.000480
19/08/09	-0.000837	-0.000875	0.001181	0.002579
20/08/09	0.000767	0.000770	-0.000224	-0.000833

Table A.1 Panel A. Daily changes in the FX smile. Subset ranging from 16th June 2009 to 20th August 2009.

Date	10C - ATM	25C - ATM	25P - ATM	10P - ATM
21/08/09	-0.001346	-0.000602	0.000218	0.000240
24/08/09	0.001139	0.000561	-0.000033	0.000221
25/08/09	-0.000292	-0.000103	-0.000123	-0.000160
26/08/09	0.001351	0.000964	-0.000688	-0.001099
27/08/09	-0.001654	-0.001117	0.000589	0.000752
28/08/09	0.000384	-0.000047	0.000019	0.000285
31/08/09	0.000458	0.000326	0.000180	0.000565
01/09/09	0.001925	0.000401	-0.000111	-0.000299
02/09/09	0.000700	0.000495	-0.000546	-0.000870
03/09/09	-0.000350	-0.000175	-0.000234	-0.000621
04/09/09	0.000993	0.000432	0.000096	-0.000976
07/09/09	0.000338	0.000092	0.000080	0.000216
08/09/09	-0.003634	-0.001633	0.000509	0.001879
09/09/09	-0.000249	-0.000210	0.001354	0.003624
10/09/09	-0.000876	-0.000267	-0.000241	0.000091
11/09/09	0.000466	0.000034	0.000404	0.000312
14/09/09	-0.000053	0.000061	-0.000459	-0.000835
16/09/09	-0.000524	-0.000406	0.000422	-0.000702
17/09/09	0.001516	0.000904	-0.000148	0.000213
18/09/09	-0.000387	-0.000046	-0.000709	-0.000916
21/09/09	0.000487	0.000614	-0.000355	-0.000470
22/09/09	-0.001025	-0.001007	0.000822	0.000667
23/09/09	-0.000798	-0.000185	-0.000039	-0.000062
24/09/09	0.000908	0.000912	-0.001472	-0.002068
25/09/09	0.000712	-0.000099	-0.000065	-0.001012
28/09/09	0.001361	0.000445	-0.000013	0.000133
29/09/09	0.002592	0.000907	-0.000353	-0.000964
30/09/09	-0.000569	0.000106	-0.000508	-0.001190
01/10/09	-0.000086	0.000278	-0.000150	0.000212
02/10/09	-0.000131	-0.000159	0.000030	-0.000259
05/10/09	0.000571	0.000514	-0.000686	-0.001580
06/10/09	-0.000771	-0.000722	0.001122	0.001176
07/10/09	0.001545	0.001024	-0.000590	-0.000483
08/10/09	-0.001126	-0.000631	0.000035	0.000912
09/10/09	0.000144	0.000135	-0.000617	-0.001226
12/10/09	-0.000002	0.000409	0.000737	0.000903
13/10/09	-0.000657	-0.000473	-0.000513	0.000047
14/10/09	0.000079	-0.000860	0.001690	0.001993
15/10/09	0.000153	0.000475	-0.000449	0.000675
16/10/09	0.000350	-0.000032	-0.000046	-0.000875
19/10/09	-0.000302	-0.000122	0.000154	0.000204
20/10/09	0.000996	0.000553	-0.000407	-0.000781
21/10/09	-0.001100	-0.000350	-0.000488	-0.001482
22/10/09	0.000742	0.000459	-0.000505	-0.000924
23/10/09	-0.000216	0.000218	-0.000039	0.000840
26/10/09	0.001916	0.000578	-0.000520	-0.001708
27/10/09	0.003672	0.001449	-0.001473	-0.002862
28/10/09	0.002545	0.000926	-0.000255	-0.000864
29/10/09	-0.000039	-0.000018	-0.000456	0.000053
30/10/09	0.000322	0.000505	-0.000439	-0.000462

Table A.1 Panel B. Daily changes in the FX smile. Subset ranging from 21st August 2009 to 30th October 2009.

Date	10C - ATM	25C - ATM	25P - ATM	10P - ATM
02/11/09	0.001689	0.000414	0.000026	-0.000269
03/11/09	0.001814	0.000331	0.000449	0.000768
04/11/09	0.001247	0.000349	-0.000112	-0.000375
05/11/09	-0.001724	-0.000011	-0.000773	-0.000020
06/11/09	-0.001100	-0.000402	0.000574	0.001380
09/11/09	0.001853	0.001026	-0.000200	0.000047
10/11/09	-0.000843	-0.000377	-0.000539	-0.001867
11/11/09	0.000180	-0.000225	0.000649	0.000411
12/11/09	0.001304	0.000590	-0.000220	0.000619
13/11/09	0.000231	0.000080	-0.000227	-0.000916
16/11/09	-0.000218	-0.000044	-0.000208	-0.000864
17/11/09	0.000730	0.000404	-0.000152	-0.000154
18/11/09	0.000155	0.000329	0.000047	0.001085
19/11/09	0.001248	0.000810	-0.000918	-0.001814
20/11/09	0.000706	0.000233	-0.000443	-0.000411
23/11/09	0.000104	-0.000005	0.000175	0.000095
24/11/09	-0.000332	-0.000082	0.000228	0.000024
25/11/09	0.000657	0.000085	0.000082	0.000132
26/11/09	0.000330	0.000136	-0.000016	0.000062
27/11/09	-0.000191	0.001136	-0.001708	-0.002791
30/11/09	0.000769	-0.000446	0.000465	0.001033
01/12/09	0.000561	0.000111	0.000323	0.000701
02/12/09	0.000322	0.000130	-0.000021	0.000088
03/12/09	-0.001171	-0.000184	-0.000260	-0.001197
04/12/09	0.000468	0.000033	-0.000658	-0.000286
07/12/09	0.000334	0.000183	0.000551	0.001718
08/12/09	0.001990	0.001214	-0.000595	-0.001670
09/12/09	0.000336	-0.000174	0.000304	0.001472
10/12/09	-0.000417	-0.000385	-0.000372	-0.000521
11/12/09	0.001055	0.000494	-0.000015	-0.000229

Table A.1 Panel C. Daily changes in the FX smile. Subset ranging from 2nd November 2009 to 11th December 2009.

A.2 Daily changes in future price

Date	Future	Date	Future	Date	Future
16/06/09	0.005893	**18/08/09**	0.005770	**19/10/09**	0.004507
17/06/09	0.012099	**19/08/09**	0.010027	**20/10/09**	-0.001710
18/06/09	-0.006288	**20/08/09**	0.001599	**21/10/09**	0.010890
22/06/09	-0.003004	**21/08/09**	0.007999	**22/10/09**	-0.000967
23/06/09	0.021389	**24/08/09**	-0.004803	**23/10/09**	-0.002409
24/06/09	-0.015691	**25/08/09**	0.001999	**26/10/09**	-0.014279
25/06/09	0.006599	**26/08/09**	-0.006701	**27/10/09**	-0.005108
26/06/09	0.008973	**27/08/09**	0.013099	**28/10/09**	-0.008758
29/06/09	0.000590	**28/08/09**	-0.008501	**29/10/09**	0.012543
01/07/09	0.006593	**01/09/09**	-0.011514	**30/10/09**	-0.011808
02/07/09	-0.012223	**02/09/09**	0.005800	**02/11/09**	0.002532
06/07/09	-0.006493	**03/09/09**	-0.002200	**03/11/09**	-0.005007
07/07/09	-0.003278	**04/09/09**	0.005911	**04/11/09**	0.019393
08/07/09	-0.007702	**07/09/09**	-0.000003	**05/11/09**	-0.002865
09/07/09	0.018470	**08/09/09**	0.017992	**06/11/09**	-0.003148
10/07/09	-0.008503	**09/09/09**	0.005400	**09/11/09**	0.016080
13/07/09	0.002390	**10/09/09**	0.004004	**10/11/09**	-0.001707
14/07/09	-0.003804	**11/09/09**	0.000899	**11/11/09**	-0.001707
15/07/09	0.019500	**14/09/09**	0.002097	**12/11/09**	-0.009907
16/07/09	0.001800	**16/09/09**	0.010898	**13/11/09**	0.002993
17/07/09	-0.000836	**17/09/09**	0.002499	**16/11/09**	0.008910
20/07/09	0.007936	**18/09/09**	-0.002701	**17/11/09**	-0.013079
21/07/09	-0.002400	**21/09/09**	-0.004303	**18/11/09**	0.008419
22/07/09	0.003258	**22/09/09**	0.011299	**19/11/09**	-0.002184
23/07/09	-0.002358	**23/09/09**	0.000699	**20/11/09**	-0.006360
24/07/09	0.001200	**24/09/09**	-0.014401	**23/11/09**	0.011738
27/07/09	0.002753	**25/09/09**	0.001099	**24/11/09**	0.000367
28/07/09	-0.006701	**28/09/09**	-0.007403	**25/11/09**	0.016269
29/07/09	-0.016901	**29/09/09**	-0.002218	**26/11/09**	-0.000010
30/07/09	0.006949	**30/09/09**	0.007680	**27/11/09**	-0.017463
31/07/09	0.017700	**01/10/09**	-0.011126	**30/11/09**	0.002995
03/08/09	0.015500	**02/10/09**	0.005296	**01/12/09**	0.010392
04/08/09	-0.002056	**05/10/09**	0.007287	**02/12/09**	-0.000007
05/08/09	0.004099	**06/10/09**	0.004896	**03/12/09**	-0.000407
06/08/09	-0.008643	**07/10/09**	-0.004157	**04/12/09**	-0.026182
07/08/09	-0.017100	**08/10/09**	0.010638	**07/12/09**	-0.000753
10/08/09	-0.004063	**09/10/09**	-0.006809	**08/12/09**	-0.013790
11/08/09	0.001599	**12/10/09**	0.007974	**09/12/09**	0.003399
12/08/09	0.006599	**13/10/09**	0.003258	**10/12/09**	0.000291
13/08/09	0.004999	**14/10/09**	0.008290	**11/12/09**	-0.010022
14/08/09	-0.009401	**15/10/09**	0.002255		
17/08/09	-0.008803	**16/10/09**	-0.002911		

Table A.2 Daily changes in future price. Full sample.

A.3 PC matrix

Date	PC1	PC2	PC3
16/06/09	0.001074	-0.000433	0.000807
17/06/09	0.001324	0.000231	-0.000558
18/06/09	-0.001019	-0.001317	0.000036
22/06/09	0.001574	-0.001326	-0.000051
23/06/09	0.001774	0.002287	0.000313
24/06/09	-0.003036	-0.000814	-0.000013
25/06/09	0.002847	-0.000618	-0.000573
26/06/09	0.001945	-0.000086	0.000198
29/06/09	0.000418	-0.000104	0.000232
01/07/09	0.001306	-0.000476	-0.000289
02/07/09	-0.002618	-0.000979	0.000851
06/07/09	-0.000315	-0.000029	-0.001014
07/07/09	-0.000699	-0.000858	-0.000074
08/07/09	-0.002137	-0.000161	-0.000365
09/07/09	0.002043	0.001630	0.000261
10/07/09	-0.001452	-0.000149	0.000353
13/07/09	0.000733	0.000980	-0.000545
14/07/09	-0.000818	-0.002037	0.000166
15/07/09	0.003543	0.000650	0.000037
16/07/09	-0.000391	0.000013	0.000980
17/07/09	0.000011	0.000055	-0.000329
20/07/09	0.001328	-0.000581	0.000033
21/07/09	-0.000441	0.000932	0.000020
22/07/09	0.001076	0.000320	-0.000469
23/07/09	-0.001753	-0.000595	0.000084
24/07/09	-0.000732	-0.000763	-0.000186
27/07/09	-0.000321	0.000767	0.000589
28/07/09	0.000001	-0.001100	-0.000293
29/07/09	-0.002745	0.000993	-0.000056
30/07/09	-0.000305	-0.000240	0.000114
31/07/09	0.002076	0.000035	0.000254
03/08/09	0.002047	-0.000406	-0.000619
04/08/09	0.000676	-0.000545	0.000621
05/08/09	0.000424	0.000315	0.000550
06/08/09	-0.001785	0.000986	-0.000277
07/08/09	-0.001466	0.000193	-0.000668
10/08/09	-0.001007	-0.001581	0.000132
11/08/09	-0.000744	0.001558	0.000139
12/08/09	-0.001359	0.001145	-0.000298
13/08/09	0.001977	-0.002656	0.000269
14/08/09	-0.000979	0.000269	0.000072
17/08/09	-0.002047	-0.001842	0.000111
18/08/09	0.000113	0.000687	-0.000046
19/08/09	0.002886	0.001062	0.000099
20/08/09	-0.001320	0.000161	-0.000053

Table A.3 Panel A. PC matrix. Subset ranging from 16th June 2009 to 20th August 2009.

Date	PC1	PC2	PC3
21/08/09	0.001206	-0.000904	0.000087
24/08/09	-0.000699	0.001070	-0.000156
25/08/09	0.000054	-0.000362	-0.000037
26/08/09	-0.002053	0.000346	-0.000268
27/08/09	0.002010	-0.000832	0.000347
28/08/09	-0.000013	0.000445	0.000000
31/08/09	0.000073	0.000787	-0.000160
01/09/09	-0.001489	0.001277	0.000233
02/09/09	-0.001327	-0.000051	-0.000156
03/09/09	-0.000238	-0.000724	0.000090
04/09/09	-0.001342	0.000253	0.000455
07/09/09	-0.000054	0.000415	-0.000005
08/09/09	0.004064	-0.001727	-0.000317
09/09/09	0.003102	0.002299	-0.000400
10/09/09	0.000578	-0.000703	-0.000264
11/09/09	0.000059	0.000639	0.000253
14/09/09	-0.000702	-0.000646	-0.000046
16/09/09	0.000094	-0.000775	0.000691
17/09/09	-0.001068	0.001380	-0.000331
18/09/09	-0.000610	-0.001026	-0.000215
21/09/09	-0.000905	0.000109	-0.000277
22/09/09	0.001625	-0.000341	0.000629
23/09/09	0.000470	-0.000667	-0.000076
24/09/09	-0.002688	-0.000773	-0.000526
25/09/09	-0.001093	-0.000125	0.000552
28/09/09	-0.000849	0.001160	-0.000005
29/09/09	-0.002562	0.001406	0.000208
30/09/09	-0.000669	-0.001237	-0.000067
01/10/09	0.000061	0.000083	-0.000351
02/10/09	-0.000041	-0.000276	0.000178
05/10/09	-0.001783	-0.000602	-0.000010
06/10/09	0.001833	0.000286	0.000559
07/10/09	-0.001737	0.000894	-0.000433
08/10/09	0.001480	-0.000396	-0.000261
09/10/09	-0.001157	-0.000762	-0.000002
12/10/09	0.000729	0.000816	-0.000016
13/10/09	0.000393	-0.000683	-0.000303
14/10/09	0.002114	0.001514	0.000894
15/10/09	0.000076	0.000497	-0.000809
16/10/09	-0.000802	-0.000287	0.000405
19/10/09	0.000402	-0.000083	0.000031
20/10/09	-0.001413	0.000265	-0.000057
21/10/09	-0.000409	-0.001888	0.000181
22/10/09	-0.001365	-0.000050	-0.000078
23/10/09	0.000617	0.000381	-0.000519
26/10/09	-0.002624	0.000357	0.000407
27/10/09	-0.005010	0.000876	0.000093
28/10/09	-0.002441	0.001462	0.000222
29/10/09	-0.000085	-0.000120	-0.000360
30/10/09	-0.000798	-0.000052	-0.000323

Table A.3 Panel B. PC matrix. Subset ranging from 21st August 2009 to 30th October 2009.

Date	PC1	PC2	PC3
02/11/09	-0.001289	0.001161	0.000268
03/11/09	-0.000499	0.001967	0.000232
04/11/09	-0.001127	0.000726	0.000150
05/11/09	0.000749	-0.001472	-0.000914
06/11/09	0.001887	0.000092	-0.000155
09/11/09	-0.001431	0.001538	-0.000294
10/11/09	-0.000830	-0.001951	0.000364
11/11/09	0.000453	0.000500	0.000466
12/11/09	-0.000598	0.001385	-0.000438
13/11/09	-0.000853	-0.000423	0.000208
16/11/09	-0.000510	-0.000739	0.000171
17/11/09	-0.000705	0.000483	-0.000101
18/11/09	0.000559	0.000843	-0.000532
19/11/09	-0.002504	-0.000249	-0.000096
20/11/09	-0.000908	0.000201	-0.000136
23/11/09	0.000062	0.000178	0.000116
24/11/09	0.000310	-0.000185	0.000133
25/11/09	-0.000295	0.000596	0.000100
26/11/09	-0.000198	0.000301	-0.000036
27/11/09	-0.002677	-0.002021	-0.000745
30/11/09	0.000534	0.001207	0.000305
01/12/09	0.000217	0.000936	0.000017
02/12/09	-0.000175	0.000308	-0.000050
03/12/09	-0.000152	-0.001675	0.000140
04/12/09	-0.000694	0.000004	-0.000296
07/12/09	0.001090	0.001454	-0.000303
08/12/09	-0.002859	0.000546	0.000039
09/12/09	0.000950	0.001169	-0.000213
10/12/09	-0.000112	-0.000793	0.000037
11/12/09	-0.000929	0.000730	0.000054

Table A.3 Panel C. PC matrix. Subset ranging from 2nd November 2009 to 11th December 2009.

A.4 Gamma matrix

Date	γ1	γ2	γ3
17/06/09	0.1066	0.0067	0.0103
18/06/09	0.1069	0.0081	0.0042
22/06/09	0.1086	0.0142	0.0039
23/06/09	0.1040	0.0173	0.0040
24/06/09	0.0980	0.0427	0.0070
25/06/09	0.1113	0.0440	0.0061
26/06/09	0.1195	0.0404	0.0038
29/06/09	0.1242	0.0380	0.0046
01/07/09	0.1243	0.0380	0.0047
02/07/09	0.1264	0.0349	0.0033
06/07/09	0.1347	0.0391	-0.0035
07/07/09	0.1323	0.0382	0.0009
08/07/09	0.1329	0.0398	0.0010
09/07/09	0.1389	0.0390	0.0030
10/07/09	0.1332	0.0490	0.0052
13/07/09	0.1348	0.0476	0.0032
14/07/09	0.1354	0.0490	0.0023
15/07/09	0.1362	0.0537	0.0019
16/07/09	0.1460	0.0493	0.0019
17/07/09	0.1453	0.0493	0.0029
20/07/09	0.1452	0.0492	0.0031
21/07/09	0.1461	0.0442	0.0032
22/07/09	0.1463	0.0425	0.0031
23/07/09	0.1477	0.0429	0.0020
24/07/09	0.1502	0.0438	0.0018
27/07/09	0.1493	0.0430	0.0016
28/07/09	0.1476	0.0445	0.0030
29/07/09	0.1417	0.0493	0.0046
30/07/09	0.1461	0.0264	0.0044
31/07/09	0.1391	0.0242	0.0048
03/08/09	0.1347	0.0197	0.0067
04/08/09	0.1343	0.0132	0.0001
05/08/09	0.1331	0.0139	-0.0007
06/08/09	0.1328	0.0146	0.0008
07/08/09	0.1365	0.0082	0.0024
10/08/09	0.1278	0.0048	0.0087
11/08/09	0.1290	0.0088	0.0082
12/08/09	0.1280	0.0104	0.0084
13/08/09	0.1182	0.0152	0.0068
14/08/09	0.1231	0.0055	0.0076
17/08/09	0.1219	0.0034	0.0067
18/08/09	0.1280	0.0147	0.0056
19/08/09	0.1253	0.0173	0.0053
20/08/09	0.1372	0.0237	0.0056
21/08/09	0.1353	0.0239	0.0055
24/08/09	0.1360	0.0171	0.0058
25/08/09	0.1362	0.0126	0.0063
26/08/09	0.1358	0.0119	0.0062
27/08/09	0.1426	0.0094	0.0076

Table A.4 Panel A. Gamma matrix. Subset ranging from 17th June 2009 to 27th August 2009

Date	γ1	γ2	γ3
28/08/09	0.1441	-0.0007	0.0102
31/08/09	0.1357	-0.0037	0.0096
01/09/09	0.1339	-0.0008	0.0088
02/09/09	0.1334	-0.0126	0.0057
03/09/09	0.1233	-0.0124	0.0048
04/09/09	0.1232	-0.0110	0.0046
07/09/09	0.1121	-0.0093	0.0069
08/09/09	0.1121	-0.0093	0.0069
09/09/09	0.1405	-0.0309	0.0008
10/09/09	0.1506	-0.0202	-0.0009
11/09/09	0.1505	-0.0223	-0.0018
14/09/09	0.1505	-0.0218	-0.0016
16/09/09	0.1485	-0.0230	-0.0017
17/09/09	0.1335	-0.0281	0.0053
18/09/09	0.1302	-0.0247	0.0044
21/09/09	0.1309	-0.0217	0.0050
22/09/09	0.1324	-0.0218	0.0061
23/09/09	0.1339	-0.0229	0.0123
24/09/09	0.1341	-0.0233	0.0123
25/09/09	0.1439	-0.0090	0.0168
28/09/09	0.1426	-0.0091	0.0173
29/09/09	0.1411	-0.0169	0.0165
30/09/09	0.1462	-0.0200	0.0159
01/10/09	0.1322	-0.0284	0.0144
02/10/09	0.1159	-0.0260	0.0165
05/10/09	0.1125	-0.0267	0.0169
06/10/09	0.0936	-0.0296	0.0160
07/10/09	0.1006	-0.0275	0.0184
08/10/09	0.1065	-0.0310	0.0200
09/10/09	0.1102	-0.0317	0.0149
12/10/09	0.1131	-0.0248	0.0142
13/10/09	0.1117	-0.0166	0.0131
14/10/09	0.1118	-0.0188	0.0119
15/10/09	0.1222	-0.0041	0.0189
16/10/09	0.1217	-0.0028	0.0168
19/10/09	0.1232	-0.0018	0.0152
20/10/09	0.1224	-0.0022	0.0150
21/10/09	0.1251	-0.0028	0.0151
22/10/09	0.1022	-0.0268	0.0153
23/10/09	0.1037	-0.0267	0.0154
26/10/09	0.1009	-0.0277	0.0169
27/10/09	0.1195	-0.0271	0.0067
28/10/09	0.1450	-0.0314	0.0060
29/10/09	0.1563	-0.0429	0.0033
30/10/09	0.1309	-0.0377	-0.0017
02/11/09	0.1228	-0.0323	0.0021
03/11/09	0.1188	-0.0292	0.0027
04/11/09	0.1183	-0.0384	0.0015
05/11/09	0.0676	-0.0166	0.0033
06/11/09	0.0654	-0.0131	0.0054
09/11/09	0.0598	-0.0132	0.0057

Table A.4 Panel B. Gamma matrix. Subset ranging from 28th August 2009 to 9th November 2009

Date	γ1	γ2	γ3
10/11/09	0.0315	0.0075	0.0012
11/11/09	0.0326	0.0100	0.0007
12/11/09	0.0319	0.0093	0.0000
13/11/09	0.0341	-0.0026	0.0035
16/11/09	0.0317	-0.0036	0.0040
17/11/09	0.0257	-0.0090	0.0051
18/11/09	0.0295	-0.0127	0.0054
19/11/09	0.0315	-0.0064	0.0016
20/11/09	0.0359	-0.0059	0.0018
23/11/09	0.0396	-0.0068	0.0024
24/11/09	0.0358	-0.0044	0.0033
25/11/09	0.0359	-0.0044	0.0033
26/11/09	0.0254	0.0036	0.0039
27/11/09	0.0254	0.0036	0.0039
30/11/09	0.0513	0.0263	0.0117
01/12/09	0.0521	0.0287	0.0123
02/12/09	0.0498	0.0332	0.0115
03/12/09	0.0498	0.0332	0.0115
04/12/09	0.0498	0.0338	0.0115
07/12/09	0.0413	0.0214	0.0114
08/12/09	0.0409	0.0208	0.0115
09/12/09	0.0579	0.0146	0.0100
10/12/09	0.0594	0.0168	0.0096
11/12/09	0.0593	0.0167	0.0096

Table A.4 Panel C. Gamma matrix. Subset ranging from 10th November 2009 to 11th December 2009