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The Uncertainty of Risk

Volatility of Volatility in the Swedish Equity Market

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Abstract

In addition to market volatility, a stylized theoretical model and recent empirical findings suggests the existence of a premium for market volatility of volatility. By developing seven different measures of volatility of volatility and through the method of principal component analysis, we investigate if this aggregate uncertainty is priced in the Swedish equity market. We find no strong evidence for a volatility of volatility effect in the cross-section of stock returns. Moreover, we find no persistence in stock exposure to volatility of volatility over time. This suggests that investors are unable to distinguish high volatility of volatility from low volatility of volatility stocks and hence, they cannot adequately price the aggregate market uncertainty in the Swedish equity market.

Keywords: *Uncertainty, Volatility of Volatility, Aggregate risk factors, ICAPM*

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1 Introduction

Economic risk defined as market volatility is a fundamental aspect of investment behavior which governs the financial markets and the economy as a whole. All investors face a tradeoff dilemma between risk and return. Based on their expectations of future uncertainty, investors seek to make the risk-return tradeoff which best matches their risk preferences. In some sense, the aggregate market volatility is a known unknown, i.e. market participants are aware of its existence and based on market fundamentals and historical information; formulate their expectation of the future market risk. In 1993, the Chicago Board Options Exchange (CBOE) launched the volatility index “VIX” which is considered to be the primary indicator of expected market volatility and the best proxy of implied market volatility. Nowadays, investors can also trade options and futures on the VIX index which is essentially instruments directly based on the market’s expectations of future volatility (CBOE, 1993).

The measurement and modeling of risk in terms of historical volatility have received increasing attention in the last decades, especially after the financial crisis of 2008. Under the Basel plan, banks and other financial institutions are today obliged to calculate and report value at risk, which is based on historical volatility, on a day to day basis. A deep understanding of the fundamental nature of risk is undoubtedly vital, both for practitioners and scholars alike. Interestingly, the common perception about risk which has been based on historical volatility neglects the possibility of uncertainty in the risk itself. Economic uncertainty, which could pave the way for a financial crisis, may not only depend on the volatility but also on the volatility of volatility, i.e. economic uncertainty do not only entail a higher level of volatility but may also result in more fluctuations in the volatility itself. This might imply that investors do not make investment decisions based on historical volatility alone but also based on the probability that the volatility itself will change. In that case, the existence of volatility of volatility is not only interesting from a risk management perspective but would also have important theoretical implications for asset pricing models (Nolte et al. (2005)).

Expanding the VIX index, in 2006 the CBOE launched the VVIX index which represents the expected volatility of the 30-day forward price of the VIX index. Derived from call and put options with VIX as the underlying, the VVIX index is a direct measure of the risk neutral expectation of volatility of volatility (cboe.com). Although VVIX is an important indicator frequently used by the VIX trading community which represents a multibillion dollar market, the concept of volatility of volatility has received little attention in the literature. Most of the earlier work addressing market uncertainty has largely focused on volatility risk as an aggregate risk factor (e.g. Ang et al. (2006b), Adrian & Rosenberg (2008) Delisle et al. (2011), Cremers et al (2015)).

A small but growing strand of literature has however examined the volatility of volatility risk (e.g. Wang, Kirby & Clark (2013), Baltussen et al. (2014), Huang & Shaliastovich (2014), Chen et al. (2016)). Specifically, this later strand expands the asset pricing literature by incorporating the market volatility of volatility as a priced risk factor. Inspired by this later literature development, our study aims to investigate the aggregate effect of volatility of volatility in the Swedish equity market. Unlike previous studies which mainly focus on the U.S. equity market, we contribute to earlier literature by using the SIX index, which is the Swedish equivalent of VIX, as our measure of implied volatility of the Swedish equity market. Further, we extend the earlier literature by developing several measures of volatility of volatility and through the method of principal component analysis obtain its underlying feature in order to test if it is a priced risk factor in the cross-section of stock returns. We find no strong evidence for a volatility of volatility effect. Moreover, we find no persistence in stock exposure to volatility of volatility over time. Suggesting that, investors might be unable to distinguish between stocks with high exposure to volatility of volatility from stocks with low exposure.

As previously mentioned, market volatility risk is by now a thoroughly investigated phenomenon. Based on the seminal contribution of Merton's (1973) Intertemporal Capital Asset Pricing model (ICAPM), several studies have found that systematic market volatility is a priced risk factor in the cross-section of stock returns. Using the innovations of VIX, Ang et al. (2006b) found that high sensitivity to the changes in implied market volatility is associated with a lower average stock return. The negative risk premium associated with the innovations in market volatility has been widely confirmed by several other studies (Delisle et al. (2011), Cremers et al (2015), Adrian & Rosenberg (2008)).

Despite the comprehensive focus on volatility, not much has been done when it comes to uncertainty about volatility. One could argue that since risk averse investors are prepared to pay for insuring against high volatility, uncertainty aversion might depend on both volatility and its higher moments. Ellsberg (1961) showed that uncertainty is distinct from risk and has a strong effect on investor behavior, implying that uncertainty should have a systematic effect on asset prices. One technical problem has been the difficulty of adequately measuring uncertainty. Although risk can be adequately captured by return volatility, measuring uncertainty is however non-trivial (Knight (2012)). Due to these difficulties, some studies have focused on the variance risk premium as a way of capturing market uncertainty (e.g. Wang, Kirby & Clark (2013), Bollerslev et al. (2009), Drechsler & Yaron (2011)).

The difference between implied and realized volatility constitutes the variance risk premium. A positive variance risk premium implies that the realized volatility has been anticipated by the investors and that they have opportunity to hedge against certain risks. However, when the implied volatility is lower than the realized, i.e. when the variance risk premium is negative, the market participants are taken by surprise which is also one of the characteristics of a market crash (Wang, Kirby & Clark (2013)).

Although the concept of implied volatility and variance risk premium is central, there have been some inconsistencies in the literature regarding the forecasting ability and measurement efficiency of implied volatility (Christensen & Prabhala (1998)). Much of the historical criticism has been due to the use of the traditional models like Black-Scholes for the calculation of option implied volatility. More recent literature has however developed the use of a "model free" approach which is independent of the Gaussian assumptions of Black-Scholes and is also used in the calculation of both VIX and SIX (e.g. Carr & Madan (1998), Britten-Jones & Neuberger (2000)).

However, despite the recent progress in the literature, it is hard to adequately quantify the variance risk premium. Wang, Kirby & Clark (2013) argues that because of these measurement errors, the concept of variance risk premium remains only theoretical. From an asset pricing perspective however, the adequate quantification of the variance risk premium is crucial. Based on the equilibrium model of Bollerslev et al. (2009) and Drechsler & Yaron (2011), one can show that the variance risk premium is fundamentally driven by market volatility of volatility. This implies that market volatility of volatility can more adequately capture the underlying effect of variance risk premium which is fundamentally related to how market participants perceive uncertainty about risk.

Based on this, a few studies have focused on the asset pricing implications of volatility of volatility. Using high frequency intraday data, Chen et al. (2016) calculate the realized volatility of the VIX index as a measure of volatility of volatility. By developing a market based three-factor model, they found that market volatility of volatility risk is priced in the cross-section of stock returns. Focusing on the volatility of volatility of individual stocks, Baltussen et al. (2014) found that stocks with high uncertainty underperform stocks with low uncertainty by 10 percent annually. Huang & Shaliastovich (2014) showed that volatility of volatility is a separate risk factor distinct from volatility.

All these studies use however different measures for volatility of volatility. Huang & Shaliastovich (2014) use the VVIX index as a direct measure of volatility of volatility. Baltussen et al. (2014) employ a rolling window procedure on implied volatility in order to obtain the realized volatility of implied volatility. More importantly, each study use only one measure of volatility of volatility. Interestingly, Wang, Kirby & Clark (2013) develop several measures including both parametric models like GARCH and non-parametric measures. Although they found that volatility of volatility have some explanatory power for the equity premium and predictive power for stock returns, they fail to find a significant relation between variance risk premium and volatility of volatility. One given reason for this is their short time series and their use of daily data instead of high frequency intraday data.

Inspired by this emerging strand of literature, our study aims to follow the footsteps of Wang, Kirby & Clark (2013) and test if volatility of volatility is a priced risk factor in the Swedish stock market. By using the SIX volatility index and high frequency intraday data on OMXS30 futures, we develop seven different measures of volatility of volatility in order to adequately capture its underlying features. Further, through the method of principal component analysis, we are able to capture the underlying component which constitutes the volatility of volatility. In order to test if volatility of volatility is a systematic risk factor, we follow the large strand of literature investigating the contemporary relation between factor loadings and realized returns (Fama & French (1993), Fama-MacBeth (1973); among others).

Although we control for market volatility and account for other known risk factors, like Fama & French (1993) three factors and the momentum portfolio, the focus of our study is however solely on aggregate volatility of volatility. Our study distinguishes itself from previous work in several aspects. First, unlike Chen et al. (2016), we use several measures of volatility of volatility. Also, in contrast to Wang, Kirby & Clark (2013), we use high frequency intraday data with a time horizon of 10 years. More importantly, we combine the information content of seven different measures into one singe risk factor. When it comes to estimating factors, most studies rely on either a model driven method like GARCH estimation or data driven approach like principal component analysis. By combining data driven and model driven methods, we are able to extract the relevant factor without imposing too much structure on the data.

Unlike all previous studies, we intend to focus on stocks listed on OMX Stockholm PI. Using different measures, including VVIX, the major focus of previous research has been on the volatility of volatility of the S&P 500 index. This is however mostly due to the fact that VIX and VVIX which are the main implied measures of uncertainty are both based on S&P 500 option prices. By using SIX as a local measure of implied volatility in the Swedish stock market; it is interesting to investigate if the risk-return and uncertainty-return relations also hold in the Swedish stock market. To the best of our knowledge, the price of volatility of volatility risk in the Swedish equity market has not yet been investigated. Besides contributing to a better understanding of the asset pricing implications of volatility of volatility in the Swedish market, this study might also inspire to the construction of a VSIX index in the future. The rest of this paper is organized as follows. Next we present the literature review followed by the methodology and empirical approach. Thereafter follows a presentation and analysis of the obtained results. The final chapter concludes.

2 Literature Review

In order to get an in depth understanding of volatility of volatility we initially start with volatility. The concept of time varying volatility was first introduced by arbitrage pricing theory (APT) and the multifactor models of Ross (1976) and Merton (1973) and it is a thoroughly investigated phenomenon (Ang et al. (2006b)). Merton's (1973) Intertemporal capital asset pricing model (ICAPM), implies that changing investment opportunities should have a systematic effect on asset prices and consumption. By further calibrating Merton's (1973) ICAPM, Campbell (1993, 1996) and Chen (2002) showed that aggregate market volatility is a state variable that systematically affects the cross-section of asset returns and should be negatively priced. Ang et al. (2006b) empirically confirmed this theory and found that innovations in aggregate market volatility carry a significantly negative risk premium.

The underlying mechanism in which the innovations in market volatility is priced, works through the deterioration in the investment opportunity set which arises from increasing market volatility. Specifically, higher market volatility induces changes in the expectation of future market returns and tends to be followed by downward market movements. Due to this, risk averse investors will demand assets that are positively correlated with aggregate volatility innovations in order to hedge against this increased uncertainty about market returns. This will in turn result in a higher price and thus a lower return for these assets (Adrian & Rosenberg (2008), Ang et al. (2006b)). This effect is however due to decreased consumption and increased precautionary savings in the presence of uncertainty about market returns (Chen (2002)).

The question here is if this uncertainty not only comprises volatility but also volatility of volatility. In that case the deterioration of investment opportunities will also depend on volatility of volatility. The existence of a volatility of volatility factor might also be motivated in a multifactor factor model as in Ross (1976)'s arbitrage pricing theory or Merton's (1973) ICAPM (Baltussen et al. (2014)). The classical ICAPM is however often used in the motivation of a traditional risk-return tradeoff relation. For a theoretical argument concerning uncertainty in terms of volatility of volatility a more dynamic model is needed.

Although this study extends the works of Ang et al. (2006b) and is inspired by their methodological approach for testing aggregate risk factors, our study is also related to a different strand of literature. Based on utility theory, Knight (1921) showed that there is a difference between risk which can be assessed by numerical probabilities and uncertainty (also called ambiguity) where the investors cannot objectively assess the probabilities. Knightian uncertainty was later confirmed by Ellsberg (1961) and became a famous phenomenon in the economic literature.

Based on this, Segal (1987) developed a new framework in utility theory based on second-order beliefs. This framework entails not only the concept of risk aversion but also second-order risk aversion or uncertainty aversion. In this framework, investors do not only care about the fixed distribution and volatility of asset returns but they are also affected by the possibility that this distribution itself might change. Intuitively, one could argue that each possible probability distribution has a certain volatility parameter, so an entire range of probability distributions would in itself constitute a distribution of volatilities (Agarwal et al. (2014)). This would imply that market uncertainty in terms of volatility of volatility might comprise an uncertainty premium which is due to the inability to correctly assess the expected market returns.

A study which shows that uncertainty can essentially be captured by volatility of volatility is Baltussen et al. (2014). Based on second-order beliefs, they develop a two stage utility framework in which agents do not only care about uncertainty about returns but also uncertainty about risk. Their study focuses however on idiosyncratic uncertainty of individual stocks and finds that volatility of volatility as a stock characteristic is negatively priced. Based on ambiguity theory, they offer two alternative explanations for this finding. Firstly, investors may have a preference for betting on high uncertainty events which would result in a higher price and thus lower returns for stocks with high uncertainty. Their second explanation is based on the "limited participation phenomenon" which implies that uncertainty averse investors will avoid participation in high uncertainty stocks. As a result, these stocks will only be held by investors with more optimistic view.

Although we have a similar theoretical approach and one of the volatility of volatility measures in this study is inspired by Baltussen et al. (2014), we formulate however aggregate uncertainty as a systematic undiversifiable risk factor as opposite to a idiosyncratic diversifiable uncertainty. By extending their framework to a representative agent setting, financial theory would predict that stock returns would depend solely on factor exposure to uncertainty (Baltussen et al. (2014)). The theoretical motivation for an aggregate risk factor will however depend on the initial assumptions in the model. As noted by Wang, Kirby & Clark (2013), one reason for the absence of a volatility of volatility factor in the literature might be due to the initial assumptions in equilibrium models. For instance, if one assumes the consumption growth or the volatility of consumption growth to be constant, there will be no volatility of volatility term.

A motivation for an aggregate uncertainty risk factor can be related to the work of Bollerslev et al. (2009) and Drechsler & Yaron (2011) which looks at the theoretical and pricing implications of the variance risk premium. Defined as the difference between the risk neutral expected and realized return volatility, the variance risk premium can directly capture attitudes towards uncertainty. This premium can also be understood in terms of a trading strategy where the variance risk premium essentially constitutes a hedge against chocks to fundamental state variables such as the volatility in consumption growth. By calibrating Bansal & Yaron's (2004) long run risk model, Bollerslev et al. (2009) develops a general equilibrium model where agents are assumed to have Epstein & Zin (1989) recursive preferences and prefer early resolution of uncertainty. Their model allows for a dynamic volatility structure, where the consumption growth does not only have a stochastic volatility but also a stochastic volatility of volatility. Based on this model, they show that aggregate economic uncertainty can essentially be captured by the time-varying variance risk premium.

Interestingly, their equilibrium model implies that the variance risk premium is itself driven by the volatility of volatility which constitutes the true source of uncertainty in the economy. Wang, Kirby & Clark (2013) empirically investigate this result by developing different volatility of volatility measures which also has been the motivation for most of the volatility of volatility measures used in our study. Building on Bollerslev et al. (2009) and Drechsler & Yaron's (2011) equilibrium model, Wang, Kirby & Clark (2013) show that volatility of volatility can be estimated directly from realized and implied volatility. Although they fail to find a significant relation between volatility of volatility and variance risk premium, this could largely be due to their use of daily data instead of high frequency intraday data (Corsi et al. (2008)).

Our study intends to further develop Wang, Kirby & Clark's (2013) methodological approach in measuring volatility of volatility by using high frequency data and most importantly, formulating uncertainty as an aggregate risk factor. A study which also tests for an aggregate uncertainty factor is Huang & Shaliastovich (2014). Based on option theory, they develop a model in which the stochastic volatility captures the aggregate volatility of volatility as a separate source of risk. In opposite to our study however, they look at the cross-section of equity index options and VIX options. They found strong support for a negative market price of volatility of volatility.

Another study which is both theoretically and methodologically related to our work is Chen et al (2016). They explicitly solve the equilibrium model of Bansal & Yaron (2004) and Bollerslev et al (2009) in order to extract the aggregate asset prices. Based on this, they develop a market-based three-factor model for cross-sectional asset prices. By using the Fama-MacBeth (1973) cross-sectional regression approach they found that market volatility of volatility is indeed a priced risk factor in the cross-sectional of stock returns. Specifically, they found that stocks with negative return sensitivities to volatility of volatility have higher future stock returns, even after controlling for Fama and French (1993) risk factors. They also find a significant price effect of volatility of volatility in the cross-section of variance risk premium which supports the initial hypothesis stated by Wang, Kirby & Clark's (2013).

Our distinction between volatility risk and uncertainty in terms of volatility of volatility risk can also be related to the literature regarding aggregate jump risk. Cremers et al (2015) develops an equilibrium model in which the representative agents are both risk and crash averse which results in both volatility risk and jump risk being separately priced. In this sense, investors seek protection both against changing investment opportunities and tail events which is characterized by extreme economic events like a market crash. By developing option strategies that result in the loadings of these two risk factors being orthogonal, Cremers et al (2015) found both volatility risk and jump risk to be separately and negatively priced. An analogy can also be drawn to volatility of volatility. Wang, Kirby & Clark (2013) argues that volatility of volatility is a crucial variable for capturing extreme events and tail risk. Similar to Cremers et al (2015), we also impose a clear distinction between aggregate volatility and aggregate volatility of volatility.

The above studies suggest several channels through which volatility of volatility could be priced. Building on Ang et al. (2006b), aggregate uncertainty may change the investments opportunity set. Due to this, higher uncertainty will worsen the risk-return tradeoff and decrease expected future market returns or increase expected future volatility (Chen et al (2014)). As argued by Baltussen et al. (2014), investors might also have preference for uncertainty about risk or simply be uncertainty averse, resulting in the "limited participation phenomenon". The theoretical implications of the variance risk premium might suggest that, in addition to the traditional risk-return tradeoff, there should also exist an uncertainty-return tradeoff where uncertainty averse investors demand a premium for exposure to volatility of volatility risk. In that case, investors will acquire stocks with high sensitivity to innovations in volatility of volatility in order to hedge against aggregate uncertainty. This implies that volatility of volatility risk should be negatively priced in the cross-section of asset returns.

3 Methodology

This section starts with a review of the dataset used in the study followed by the calculation of the volatility of volatility measures. We then present the methodological approach of principal component analysis and the regression models which will be used when testing the risk factors. In order to test if volatility of volatility is a priced risk factor in the cross-section of equity returns, we initially start with the estimation of the seven volatility of volatility series. It is also important to note that since we use implied and realized volatility as the input in some of the volatility of volatility measures, this will result in both realized volatility of implied volatility and realized volatility of realized volatility. Although we acknowledge that none of these are implied volatility of implied volatility like the VVIX index, we will however combine all these measures into a single volatility of volatility factor through the method of PCA. By using PCA, we will extract the joint underlying feature of all volatility of volatility series into one single risk factor. The extracted risk factor will then be tested with two different methods in order to investigate if it is priced in the Swedish equity market. The first method is based on the Fama & French (1993) methodology of testing the contemporary relation between factor loadings and realized returns. Initially, we estimate factor loadings of volatility of volatility for individual stocks using a rolling annual period with daily data. Second, we sort stocks into 5 quintile portfolios based on factor loadings over the one-year period. Finally, we run time series regressions of these portfolios controlling for Fama & French 3- factors, innovations in market volatility and a momentum factor. The second methodology we use is based on Fama-Macbeth (1973) and is used to estimate the specific risk premiums of the risk factors. In this approach we start by estimating factor loadings using monthly data over the entire sample period. Second, we run a cross sectional regression each month on the estimated factor loadings from the first step. This gives us a time series of risk premiums of volatility of volatility and in order to test if the factor is priced in the market, we conduct a t-test on the time series averages. The Fama-Macbeth regression approach enables us to more consistently estimate the market price of the volatility of volatility factor while controlling for other known determinants of stock returns.

3.1 Data

We use stocks listed on the Swedish stock exchange during the period between 2004-08 and 2013-03, which covers a total of 2185 trading days. We use firms within the OMXSPI index which is a weighted index containing firms from the major stock lists in Sweden (large cap, mid cap and small cap). The sample is limited to firms that have been listed during the entire sample period; hence firms that have been newly listed or delisted during the sample period have been excluded. This approach can cause something that is called survival ship bias, a concept that we will discuss more in detail in a latter section of this paper. We limit the sample to ordinary common shares and when a company has two shares listed we use the one that is most actively traded. To limit the problem with illiquid stocks we exclude stocks with more than 350 unchanged daily price observations during the sample period, which makes our final sample containing 112 stocks.

The stock price data, OMXS30 price index and the SIX Index which is all daily data are obtained from Thomson Reuters Data stream. The SIX index also serves as our measure of aggregate volatility in the Swedish equity market. To compute realized aggregate volatility, which will be discussed in detail in the next section, we use five-minute intraday high-frequency data of OMXS30 futures with a rolling 30 day maturity obtained from the data provider TradeNode AB. We obtain monthly returns for the Fama & French three factor portfolios and the risk free rate, which is a 3-month Swedish treasury bill, from the data library by Professor Stefano Marmi (Stefano Marmi, 2015). All of the data sets range from May 2004 until March 2013. The restriction of May 2004 is due to the unavailability of the SIX index before this period. The upper limit is determined by the unavailability of monthly returns for the Fama & French three factor portfolios after March 2013. Next, follows a review of implied volatility, realized volatility and OMXS30 returns which constitutes the main inputs used in the calculation of volatility of volatility.

3.2 Implied volatility (SIX)

As a measure of investor's future expectations of market risk, implied volatility serves as a predictor of future or expected volatility. Also, derived from option prices with the market index as the underlying, implied volatility is forward-looking by nature as opposed to past realized volatility (Baltussen et al. (2014)). The option implied volatility index VIX, is launched by the Chicago Board of Exchange (CBOE) and is the main volatility indicator of the S&P 500 index. Similarly, SIX volatility index, launched by SIX Financial Information in 2004, is the leading volatility indicator in the Swedish equity market. Based on implied volatility of OMXS30 index call and put options, the SIX index serves as a good proxy for the expected volatility in the Swedish market one month ahead (Six, 2016). The yearly SIX index data is retrieved from Thomson Reuters Datastream and transformed into daily volatility by dividing with the square-root of 252 (the number of trading days in a year) in order to make it comparable with our realized volatility measure. This index will serve as our measure of implied market volatility (IV) and is used as input in the calculation of VVOL.

3.3 Realized volatility

Unlike IV, the past realized volatility reflects the actual return variation and can be calculated in a model free fashion using high frequency intraday returns. Following Corsi et al. (2008) and Bollerslev et al. (2009), the logarithmic price is assumed to follow a geometric brownian motion where the realized variance process at day t with sampling frequency n is defined as

$$RV_t^2 = \sum_{i=1}^{n} r_{t,i}^2 = \sum_{i=1}^{n} \left[p_{t+\frac{i}{n}\Delta} - p_{t+\frac{i-1}{n}\Delta}\right]^2 \tag{1}$$

where $i = 1, \dots, n$ and $r_{t,i}$ denotes the continuously compounded within day return. In the limit where $n \to \infty$, this realized variance will converge to the true integrated variance process. Based on high frequency intraday data, this nonparametric approach provides a much more reliable and accurate estimation of the true underlying return variation compared to the use of traditional sample variances based on daily data (Barndorff-Nielsen & Shephard (2002)). In order to calculate the realized volatility, hereafter RV, we use intraday (5-minute frequency) price data on OMXS30 index futures with 30 days maturity, obtained from TradeNode AB database (Tradenode, 2016). One advantage with the use of OMXS30 index futures is the high liquidity compared to using the OMXS30 index directly. Comparing the OMXS30 future data with the OMXS30 index results in a cross correlation of almost 1. Tables with test results can be found in appendix 1.

3.4 OMXS30 Index

The OMXS30 is a market weighted price index which consists of the 30 most actively traded shares on NASDAQ Stockholm. The index is reweighted twice a year based on the market capitalization of each security. Besides giving a good representation of the Swedish equity market, the low number of shares included in the index guarantees a high liquidity (Nasdaqomx, 2014). We use the OMXS30 index daily returns both as a proxy of the market portfolio and as input for one of the VVOL measures which will be discussed in detail in the next section. The choice of OMXS30 is also a matter of consistency, since both implied and realized volatility is based on this index and ought to represent the volatility of the Swedish equity market. The use of OMXS30 as a proxy for the market portfolio instead of OMXSPI which is a somewhat broader index is however not a major concern since the two indexes are highly correlated. Appendix 2 provides a graphical representation of the two indexes and which have a correlation of 0,99.

3.5 Volatility of volatility measures

The VVOL measures developed in this paper is based on the previous work of Wang, Kirby & Clark (2013) and Baltussen et al. (2014). Most of the VVOL measures are estimated from the underlying realized and implied volatility series which serve as our main inputs. In some other VVOL measures, we use the OMXS30 returns directly.

We estimate seven different VVOL series based on two different ideas. The first class of VVOL measures is based on a non-parametric approach, one of which is a direct method and the other is a rolling window approach. The direct method is applied directly on the OMXS30 future intraday returns while the rolling window is applied to both the realized and implied volatility series. The second category is based on GARCH modeling and consists of three different GARCH models. The first GARCH model which we call GARCH-2 is only based on the OMXS30 future intraday returns while the second GARCH model called GARCH-3 is applied to both the realized and implied volatility. The last GARCH model is a so called Nested GARCH-4 model which is based on a three step estimation procedure where we use daily returns of the OMXS30 index. In the table 1 we illustrate the different categories and VVOL measures. The following section describes the different estimation models and the methods applied in generating the VVOL measures.

Table 1. *Overview of VVOL measures categorized by category, estimation model and underlying data series.*

Category	**Model**	**Data series**	**VVOL measures**
Nonparametric	Direct method	Intraday returns	D-VVOL
	Rolling window	RV	RW-RV VVOL
		SIX	RW-IV VVOL
GARCH	GARCH-2	Intraday returns	GA2-VVOL
	GARCH-3	RV	GA3-RV VVOL
		SIX	GA3-IV VVOL
	Nested GARCH-4	Daily returns	GA4-VVOL

3.5.1 Nonparametric - Direct estimation of VVOL

Following Wang, Kirby & Clark (2013), the scaled returns are approximately Gaussian and follows the simple process

$$r = \sigma\varepsilon$$

Where $\sigma \geq 0$, $\varepsilon \sim N(0,1)$ and $Corr(\sigma, \varepsilon) = 0$.

Taking expectations and assuming that $Corr(\sigma^2, \varepsilon^2) = 0$, we have

$$E(r^2) = E(\sigma^2)E(\varepsilon^2) = E(\sigma^2)$$

Since $\varepsilon \sim N(0,1)$, we have that $E(\varepsilon^2) = 1$ and $E(|\varepsilon|) = \sqrt{\frac{2}{\pi}}$ which yields

$$E(|r|) = E(\sigma)E(|\varepsilon|) = \sqrt{\frac{2}{\pi}}E(\sigma)$$

Based on this, the sample estimates of $E(\sigma^2)$ and $E(\sigma)$ are defined as

$$E(\sigma^2) = \frac{1}{N}\sum_{i=1}^{N} r_i^2$$

and

$$E(\sigma) = \sqrt{\frac{\pi}{2}}\frac{1}{N}\sum_{i=1}^{N}|r_i|$$

Using the above result, we get a daily measure of the realized variance of volatility defined as

$$Var_t(\sigma) = E_t(\sigma^2) - E_t(\sigma)^2 = \frac{1}{N}\sum_{i=1}^{N} r_{t,i}^2 - \frac{\pi}{2}\left[\frac{1}{N}\sum_{i=1}^{N}|r_{t,i}|\right]^2 \tag{2}$$

where $r_{t,i}$ is the intraday return of day t at time i and $N = 96$ for each day.

By taking the square root of the absolute value of this variance estimate, we get the volatility of volatility measure denoted by

$$\sqrt{|Var(\sigma)|} = \text{D-VVOL}$$

3.5.2 Nonparametric - Rolling window estimation of VVOL

Based on Baltussen et al. (2014), we use a similar approach for modeling the volatility of volatility. Although Baltussen et al. (2014) use a rolling window approach in order to model the implied volatility of volatility for individual assets; we intend to apply the same method in order to model the market volatility of volatility. This estimation is applied to both RV and IV in order get a measure for both the volatility of realized volatility and the volatility of implied volatility. Similar to Baltussen et al. (2014) which uses a rolling window of one month, we use a rolling window of 22 trading days where we calculate the daily standard deviation of the volatility series based on the past 22 days for each day. Thus, we calculate volatility of volatility on day t as follows:

$$VVOL_t = \sqrt{\frac{1}{22}\sum_{j=t-21}^{t}\left(\sigma_j - \bar{\sigma}_t\right)^2} \tag{3}$$

Where

$$\bar{\sigma}_t = \frac{1}{22}\sum_{j=t-21}^{t}\sigma_j$$

and $\sigma_j = RV_j$ or IV_j.

Based on this procedure, we obtain the two measures RW-RV VVOL and RW-IV VVOL.

3.5.3 Realized volatility of GARCH volatility (GARCH-2)

Wang, Kirby & Clark (2013) develop a realized volatility of volatility measure where they first apply GARCH model estimation to the daily returns series and then calculate the monthly realized volatility of GARCH volatility. Although Wang, Kirby & Clark (2013) use daily returns in order to obtain a monthly measure of VVOL, we apply GARCH model estimation on the intraday returns in order to obtain a daily measure of VVOL.

Initially, we assume that the returns follows a simple GARCH (1,1) process defined as:

$$r_t = \mu_r + h_t \varepsilon_t \tag{4}$$

$$h_t^2 = \alpha_0 + \alpha_1 h_{t-1}^2 + \beta_1 u_{t-1}^2 \tag{5}$$

Where ε_t is a white noise process and r_t is thus assumed to follow a standard geometric Brownian motion. From this GARCH process, we then get an estimate of the intraday GARCH volatility $\hat{h}_{t,i}$for each time i.

Based on this volatility estimate, we then calculate the realized volatility of volatility for day t defined as

$$RV_t(\hat{h}) \equiv \sqrt{\sum_{i=1}^{n} \left[\hat{h}_{t+\frac{i}{n}\Delta} - \hat{h}_{t+\frac{i-1}{n}\Delta}\right]^2} \tag{6}$$

where $n = 96$ for each day t and $RV(\hat{h})$ = GA2-VVOL.

Following this theoretical framework, we start by estimating the mean equation and work on the squared residuals from this estimation. After unit-root testing the squared residuals for stationary, we apply several tests on this series and based on ACF, PACF, heteroskedastisity tests for ARCH effects, AIC and BIC information criteria, we estimate the optimal lag length to be an GARCH (1,1) model. Test statistics and estimation output are included in appendix 4.1.

3.5.4 Volatility of realized/implied volatility (GARCH-3)

According to Corsi et al. (2008), the realized variance can be defined as

$$RV_t^2 \equiv \sum_{i=1}^{n} \left[p_{t+\frac{i}{n}\Delta} - p_{t+\frac{i-1}{n}\Delta}\right]^2$$

where $p_{t,i}$ is the logarithmic price of day t at time i. Based on this, it can be shown that the logarithm of realized volatility is normally distributed with time varying variance defined as

$$\frac{RV_t - \sqrt{\int_{t-1}^{t} \sigma^2(s)\, ds}}{\sqrt{\frac{Q_t^*}{2MRV_t}}} \xrightarrow{d} N(0,1)$$

Where the convergence depends on $n \to \infty$, i.e. when the number of within day price observations approaches infinity. The standard deviation of the realized volatility is approximated by:

$$\sqrt{\frac{Q_t^*}{2MRV_t}}$$

Following Corsi et al. (2008), the above result implies that the logarithm of realized volatility follows a GARCH (1,1) process denoted by

$$y_t = \mu_y + \sqrt{h_t}\varepsilon_t \tag{7}$$

$$h_t = \omega + \alpha_1 h_{t-1} + \beta_1 u_{t-1}^2 \tag{8}$$

where $y_t = RV_t$ or IV_t and ε_t is a white noise sequence. Also here, the volatility process is assumed to follow a standard geometric Brownian motion. By applying this GARCH estimation on the daily volatility series, we obtain a daily measure of VVOL. The above model is applied to both the realized and implied volatility in order to obtain the two measures GA3-RV VVOL and GA3-IV VVOL.

For the estimation, we start by unit-root testing the initial volatility series and estimate the simple mean equation of the volatility process. After demeaning the series, we apply several tests on the squared residuals, i.e. ACF and PACF, testing for ARCH effects, AIC and BIC information criteria. Based on the test results we estimate the optimal lag length to be GARCH (1,1). Test results and tables can be found in appendix 4.2.

3.5.5 The nested GARCH model (Nested GARCH-4)

Wang, Kirby & Clark (2013) develop a three-step GARCH estimation procedure in order to estimate the volatility of volatility based on the daily return series. Since the underlying volatility estimation is based on returns, this method cannot be applied to the SIX volatility series. Assuming that the return follows the process:

$$r_t = \mu_r + \sigma_t \varepsilon_{r,t} \tag{9}$$

where $\varepsilon_{r,t}$ is a i.i.d. white noise process and μ_r could be a constant or a ARMA process by itself. The stochastic volatility evolution is then defined by a GARCH process

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \beta_1 u_{t-1}^2 + q_t \varepsilon_{\sigma,t} \tag{10}$$

where $u_{t-1} = \sigma_{t-1}\varepsilon_{r,t-1}$ and $\varepsilon_{\sigma,t}$ is another i.i.d. white noise process independent of $\varepsilon_{r,t}$. We also have the usual restrictions $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$. As illustrated above, part of the volatility process is assumed to be a deterministic GARCH while another part is assumed to be stochastic, where q_t is the volatility of volatility which also follows a GARCH process

$$q_t^2 = \alpha_q + \rho_q q_{t-1}^2 + \varphi_q \eta_{t-1}^2 \tag{11}$$

where $\eta_{t-1} = q_{t-1}\varepsilon_{\sigma,t-1}$, $\rho_q \geq 0$, $\varphi_q \geq 0$ and $\rho_q + \varphi_q < 1$.

Based on the above framework, we start by estimating the mean equation for the daily OMXS30 return series. Since we are interested in the second order volatility in (11), we simply assume that the return series follows the mean equation defined in (9). After demeaning the return series we apply several tests for stationary and autocorrelation on the squared residuals from the first-step estimation. In the second-step, we use the squared residual series in order to estimate the parameters α_0, α_1 and β_1.

Based on the obtained parameters, we calculate the first GARCH volatility process σ_t^2, but more importantly, we save the residuals from this process in order to continue to the third final step. In the final step, we treat the squared residuals from the GARCH model as an observed time series and conduct the same stationary tests as in the previous case. Following the same procedure as above, based on the squared residual series, we estimate the parameters α_q, ρ_q and φ_q from which we calculate the volatility of volatility series q_t^2. By taking the square-root of this estimate, we obtain the VVOL measure defined as

$$\sqrt{q_t^2} = \text{GA4-VVOL}$$

Test results are included in appendix 4.3. A detailed account of the estimation method can also be found in Wang, Kirby & Clark (2013). It should however be noted that although this method will result in an unbiased estimation of the coefficients, the three-step estimation procedure will inevitably lead to some loss in efficiency.

3.6 Principal Component

As mentioned earlier, we will use the method of principal component analysis, which is essentially a form of factor analysis, in order to capture the underlying common component in our seven VVOL measures. This component will then be used as a proxy for the true underlying VVOL factor. This section starts with a review of the general method of principal component analysis followed by a description of the computational approach and testing method used in the estimation of the risk factors.

3.6.1 Technical disposition

Initially, assume we have a $T \times K$ matrix denoted Y, containing T observations on K variables. The question then is if these K variables can be explained by linear functions of a small set of other variables denoted principal components (Campbell & MacKinlay (1997)).

The first principal component could be defined as

$$PC_{1t} = x_1' y_t$$

where y_t is the $t;th$ row vector in the matrix Y and x_1 is a $K \times 1$ vector yet unknown, but we continue as if it were known. Here, x_1 is the solution to the maximization problem

$$\max_{x_1} \; x_1' V x_1$$

Subject to

$$x_1' x_1 = 1$$

where V is a $K \times K$ covariance matrix defined as

$$V = E[(y_t - E[y_t])(y_t - E[y_t])'] = Y'Y \tag{12}$$

Here, the eigenvector associated with the largest eigenvalue of V is the solution to x_1. Consequently, the first principal component denoted $PC_{1t} = x_1' y_t$, will explain the largest proportion of the covariance matrix V.

With the same procedure, we obtain the second principal component denoted $PC_{2t} = x_2' y_t$, by solving the same maximization problem for x_2.

$$\max_{x_2} \; x_2' V x_2$$

But now with the additional restriction

$$x_1' x_2 = 0$$

This additional restriction will ensure orthogonality between x_2 and x_1. Due to this procedure, the eigenvectors, containing the loadings on each principal component, are mutually orthogonal and thus, all of the obtained principal components are uncorrelated. The above process could be repeated until all K principal components are computed.

Following Theil's (1971) linear regression analogy, one can also more intuitively, view principal component analysis, PCA for short, as a linear regression where we essentially try to find the explanatory variables that best explains the dependent variable. So each subsequent principal component minimizes the sum of squares of the residuals that are left over from earlier principal components. Consequently, no other variable will explain the data as good as the first principal component. Based on this, the goal of PCA is to find the set of linear combinations in the data that describes as much of the observed variation as possible. The advantage with PCA is that one can obtain the efficient dimensionality of the data without losing too much information (Frye (1997)).

PCA has traditionally been employed as a data driven method on interest rates and assets or portfolio returns in order to reduce the data dimensionality to a more manageable threshold and extract the market risk factors which are assumed to be the underlying source of covariation in the data. Unlike model driven techniques, PCA imposes less structure on the data without strong assumptions about the data generating process (Loretan (1997)). In our case, the Y matrix will contain the seven VVOL measures and the aim with PCA is to isolate the common factor which in itself is assumed to be a VVOL series. Due to this, the question of dimensionality i.e. the number of principal components to be retained, becomes important and will be further discussed in the next section.

Usually the dimensionality question will depend on the correlation structure in the data. If the data is highly correlated, only one or two PC's will be sufficient. However, if the mutual correlation in the data is weak, more PC's might be needed (Loretan (1997)). Due to the high correlation in the VVOL data set, the first principal component explains over 70 percent of the total variation in the data, as presented in appendix 5. As PCA detects factors that are linear combinations of the data, the first principal component is also the most suitable proxy of the actual VVOL factor since it is essentially a one-to-one transformation of the seven VVOL series. We will however also look at more formal indicators in order to determine the sufficient number of PC's.

Although lose in structure and easy to employ, PCA requires some distributional qualities of the data. Specifically, stationarity is an important requirement in order to avoid spurious relations in the data. Stationarity also ensures the confidence of an obtained risk factor to be stable over time (Kritzman et al. (2010)). In order to ensure stationarity, we conduct unit-root tests to all of the VVOL series. Test results and tables can be found in appendix 5. Another issue is unit sensitivity of the obtained principal components. Specifically, if the underlying elements in the Y matrix are measured in different unites; the obtained principal components will be effected. A common practice is therefore to standardize the underlying variables (Dotsis (2015)). This is however not a major concern in our case since all the elements in the Y matrix are measured in volatility units.

3.6.2 Factor computation

The principal component analysis is based on the Eviews add-in "StatFact" which is a user written program released by IHS. Due to the increasing use of factor models in the literature and as a response to the "big data" paradigm, the StatFact application is developed as a flexible and convenient interface form which one can extract the relevant factors through the method of PCA (Rahal (2015)). Besides the actual factors, the program also provides several options for standardization or normalization of the initial data. The underlying modeling and estimation approach in StatFact is based on Stock & Watson (2002).

Focusing on forecasting a single time series by using several predictors, Stock & Watson (2002) use PCA in order to reduce the number of predictors to a few numbers of factors. Specifically, it is assumed that the K-dimensional predictor series denoted by Y_t, have a factor model representation

$$Y_t = \Lambda F_t + e_t \tag{13}$$

Where e_t is a $K \times 1$ vector of error terms. Following the classical PCA methodology described above, the principal component estimator of the factor matrix $\hat{F}$ is calculated as

$$\hat{F} = \frac{Y'\hat{\Lambda}}{K} \tag{14}$$

In our case, where $T > K$, i.e. when the number of time series observations is larger than the number of variables, the factor loading matrix $\hat{\Lambda}$ is calculated as $\sqrt{K}$ times the eigenvectors associated with the largest eigenvalues of $Y'Y$.

The program also delivers several information criteria developed by Bai & Ng (2002) which could work as guidance for the sufficient number of factors extracted. This information criteria is however based on the assumption that both T and K converge to infinity i.e. for very small T or K, this information criteria is insufficient. Since in our case the cross-sectional dimension is only $K = 7$, we instead rely on other conventional indicators like fraction of variance explained and the size of the eigenvalues in order to determine the efficient dimensionality of the data (Loretan (1997)).

Initially we start by unit-root testing all of the VVOL series for stationarity. Without transforming the data in any way, we then employ the StatFact application in order to obtain the stationary factors from the VVOL data. Based on the conventional indicators, we only use the first factor as our measure of VVOL which will then be used in further analysis. Test results and tables will be presented in the result section. A more detailed description of the StatFact application can also be found in Rahal (2015).

3.7 Fama & French regressions

In order to test the hypothesis that stocks with higher sensitivity to volatility of volatility have lower returns, we follow the methodology of Ang et al (2006a). Using this methodology, we estimate factor loadings on a rolling 12-month period by running the following regression using daily data:

$$r_t^i - r_f = \alpha^i + \beta_{mkt}^i(r_t^{mkt} - r_f) + \beta_{\Delta SF}^i \Delta VVOL + \varepsilon_t^i \quad (15)$$

Where, $r_t^i - r_f$ and $r_t^{mkt} - r_f$ is the excess return on stock i and the market portfolio, $\Delta VVOL$ is the daily change in the stationary factor constructed from the volatility of volatility series. Finally, β_{mkt}^i and $\beta_{\Delta SF}^i$ are factor loadings for the market portfolio and volatility of volatility respectively. Empirically, other risk factors have explained the cross-section of stock returns, but we do not incorporate these factors in the estimation of volatility of volatility loadings in equation (15). Ang et al (2006a) argue that adding additional risk factor only adds a lot of noise to the regression. Instead, we control for Fama and French (2003) 3-factors together with a momentum factor and innovations in volatility after we sorted stocks into portfolios based on volatility of volatility loadings. This approach is similar to previous literature examining the relationship between factor loadings and stock returns.

At the beginning of each calendar month we then sort stocks into 5 quintile portfolios based on their sensitivity to volatility of volatility over the following year. The quintile 1 portfolio contains stocks with lowest sensitivity to volatility of volatility and the quintile 5 portfolio contains stocks with highest sensitivity to volatility of volatility. We also construct a 5 minus 1 portfolio, which is long quintile portfolio 5 and short quintile portfolio 1. The portfolio return is calculated using annual returns over a rolling 12-month period. By using realized factor loadings and average returns over the same time period we can control for the potential problem with time varying risk factors. The returns are calculated on both equally weighted and value weighted portfolios. All portfolios are weighted based on market capitalization at the beginning of the 12-month period. By using an overlapping estimation window we induce moving average affect into the regression. To adjust for this issue, we compute the standard errors using 12-lags according to Newy & West (1987).

In order to control for other risk factors, we estimate a time series regression on the portfolios excess returns while controlling for the market portfolio, F&F- 3 factors, the momentum portfolio and Δinnovations in market volatility. The F&F 3-factors include the market portfolio, a HML-portfolio capturing the market-to-book effect and a SMB portfolio capturing the size effect. The fourth factor is the WML portfolio which captures the momentum effect.

3.8 Fama & MacBeth regressions

In order to estimate the risk premium of volatility of volatility, we use the two stage methodology from Fama-Macbeth (1973). The Fama-Macbeth approach is a widely used method when testing if a given risk factor is priced in the market. First, we start by estimating the factor loadings for each individual stock with the same specification used in equation (15), but now with monthly data. Furthermore, here we use the entire sample period which will result in one beta estimation for each asset. In the second step, we conduct cross-sectional regressions every month with the stocks excess returns as dependent variables and the estimated factor loadings from the previous step as explanatory variables. The coefficient estimates from this second-step regressions will gives us a times series of risk premiums for each risk factor. The second-step cross-sectional regression model is specified below in equation (16).

$$r_t^i - r_f = \alpha^i + \gamma_{mkt}^i \beta_{mkt}^i + \gamma_{\Delta VVOL}^i \beta_{\Delta VVOL}^i + \gamma_{\Delta VOL}^i \beta_{\Delta VOL}^i + \gamma_{SMB}^i \beta_{SMB}^i + \gamma_{HML}^i \beta_{HML}^i + \gamma_{WML}^i \beta_{WML}^i + \varepsilon_t^i \qquad (16)$$

Where, $r_t^i - r_f$ is the excess return on stock i and γ_j is the estimated risk premium for the aggregate risk factor j. Here, $\Delta VVOL$ is the monthly innovation in the volatility of volatility, ΔVOL is the monthly innovation in volatility. Finally, $r_{mrk} - r_f$, SMB, HML and WML are the F&F- 3 factors and the momentum portfolio respectively. In order test if a given risk premium is statistically significant, we conduct a simple t-test on the time series average of the estimated risk premiums.

3.9 Method reflection

By using the Swedish market in our study, we put a number of limitations on our measurement of volatility of volatility and on the number of stocks in our sample. The implied volatility of the Swedish stock market, measured by SIX, was first launched in 2004 which puts a limit to our sample period. Although, a longer time period is preferable and could result in more adequate estimations, we conclude that the chosen time frame of 103 months is sufficient to test our hypothesis. Furthermore, we use stocks in the OMXSPI index from august 2004 to march 2013 excluding non-liquid stocks. This procedure limits our sample to 112 stocks, which is small compared to other studies. Moreover, our sample is tilted towards large stocks and if smaller, non liquid stocks capture important volatility of volatility characteristics related to uncertainty of risk, this may bias our result. Our study also lacks the measure of implied volatility of implied volatility. Furthermore, Huang & Shaliastovich (2014) argues that the power of VVIX is that it is model free, forward looking and captures the risk neutral expectations of volatility of volatility. By looking at the correlation between realized variance of VIX and implied variance of VIX (VVIX), they found a strong relation between the variables, however as the estimated correlation between the variables was 0.53 it is obvious that the VVIX captures other characteristics that we might not succeed to seize in our 7 volatility of volatility measures.

For simplicity, we limit our sample to stocks that have been listed during the entire sample period, which means that we do not include stocks that have been listed or delisted during the sample period in our study. This method can cause survivorship bias, which may affect our result. If these delisted stocks have a different relation with respect to market uncertainty which is captured by volatility of volatility, this might introduce bias in our result. The problem with companies that fail to survive during our sample period may be that they are riskier and hence share common components related to volatility of volatility. The problem with newly listed companies is that younger firms tend to be more risky, which again can be captured by volatility of volatility. For example, Baltussen et al (2014) found that stocks with high idiosyncratic volatility of volatility in their sample tend to be younger. Although we acknowledge this problem, survivorship bias is something we cannot test nor adjust for. We thereby keep in mind that it can affect our result and proceed with caution.

4 Results

4.1 Realized Volatility and Implied Volatility

As illustrated in figure 1, we can see that the evolution of RV and IV are indeed very similar. The two volatility series have a correlation of 0,81. We can also see that implied volatility seems to have a slightly higher mean compared to realize volatility which also supports the theory of a positive variance risk premium. This higher long term mean is also presented in appendix 2, where IV and RV have a mean of 0,0129 and 0,0114 respectively. Interestingly, the realized volatility seems to be higher during some periods, especially during the financial crises of 2008. This is also in line with a negative variance risk premium during a market meltdown where the market participants are taken by surprise. In accordance with the results of Bollerslev et al. (2009) and Drechsler & Yaron (2011) concerning the variance risk premium, this finding might imply that the market price of volatility is negative. RV also has a substantially higher standard deviation compared to IV which is also illustrated in figure 1. The smoothness of IV compared to RV might also be due to the forward looking nature of IV which reflects the investors' expectations and hence their ability to discount. Descriptive statistics for RV and IV are included in appendix 3.

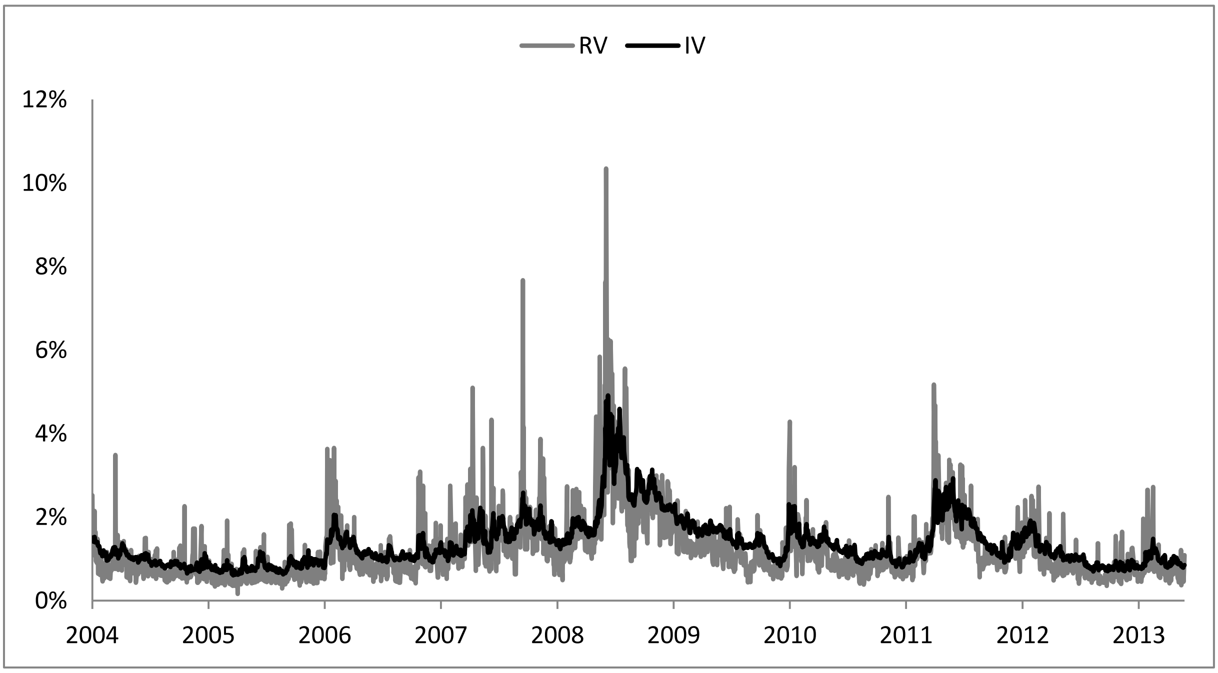

Figure 1. *Time series plot of Implied (IV) and Realized volatility (RV) over the period of May 2004 – Mar 2013.*

4.2 Volatility of Volatility

Figure 2 and 3 illustrates all the non-parametric and parametric volatility of volatility series respectively. Starting with the non-parametric measures, we can see that they follow a similar pattern as the volatility series in figure 1 with spikes during the same time periods, especially in the mid 2008. The volatility of volatility measures also seem to be lower than the underlying volatility series. This is however in contrast to previous findings which imply that volatility of volatility, measured by the VVIX index, is higher and more volatile than the volatility series measured by VIX (Huang & Shaliastovich (2014)). Also the level and long term mean of our estimated VVOL series are highly affected by the measurement and estimation method as illustrated in table 2. Among the nonparametric volatility of volatility measures, the RW-RV seems to be most preannounced with higher spikes and more volatile features. This can also be seen in table 2 where RW-RV has the highest mean and standard deviation among the non-parametric measures. This is however probably due to the underlying realized volatility series which has a more volatile characteristic then the implied volatility. Similarly, RW-IV VVOL shares the more subtle feature of the underlying implied volatility. Interestingly, the direct volatility of volatility measure (D-VVOL) which is directly estimated from the intraday data seems to have the lowest mean and standard deviation among all the measures. This estimate is however strongly dependent on the assumption that the scaled returns follow the simple process described in section 3.5.1.

Turning to the model driven parametric measures, we can see that almost all of them are substantially higher than the non-parametric measures. They also seem to have a more volatile feature except for the GARCH-2 measure (GA2) which is more similar to the non-parametric measure D-VVOL. This can also be seen in table 3 where GA2 and D-VVOL have a correlation of 0,974. This is might be due to the fact that GARCH-2 is essentially a semi-parametric measure which is a combination of GARCH estimation and the rolling window procedure. Again, GA3-RV and GA3-IV are highly affected by their underlying volatility series with GA3-RV having a higher mean and standard deviation compared to GA3-IV. After confirming the assumption of a long term mean and volatility clustering in the underlying volatility series, as presented in appendix 4.2, we can also see that both GA3-RV and GA3-IV share a very similar pattern during periods of high volatility. Similar to the findings of Wang, Kirby & Clark (2013), the GA4 (Nested-GARCH) is the most pronounced with higher standard deviation and substantially more persistent spikes then the rest of the volatility of volatility measures. This is most likely due to the two step estimation procedure which probably entails some loss in efficiency as mentioned before. Theoretically, the GA4 is based on the residuals from the first stage GARCH model which in turn is based on the first stage residual series which might distort the second stage GARCH estimation. The GA4 share however the same patterns and spikes as the rest of the VVOL series. Looking at the correlation matrix in table 3, we can see that most of the volatility of volatility measures are highly correlated with correlations around 70%. Except for D-VVOL and GA2 which, as mentioned before, seem to be highly correlated with each other but are less correlated with the rest of the VVOL measures. The high correlation among the VVOL measures also supports our methodological approach of PCA and speaks for a common underlying component which might drive the estimated VVOL measures.

Table 2. ***Descriptive statistics over all volatility of volatility measures.*** *The table presents sample mean, median standard deviation, skewness, kurtosis, minimum and maximum values based on daily observations over the entire sample period.*

	D-VVOL	RW-RV	RW-IV	GA2	GA3-RV	GA3-IV	GA4
Mean	0.00062	0.00392	0.00155	0.00284	0.00604	0.00425	0.01502
Median	0.00047	0.00302	0.00123	0.00210	0.00389	0.00235	0.01289
Std. Dev.	0.00055	0.00294	0.00121	0.00273	0.00552	0.00489	0.00729
Maximum	0.00864	0.02321	0.00978	0.04315	0.05717	0.03738	0.04945
Minimum	2.8E-05	0.00093	0.00036	0.00015	0.00212	0.00057	0.00669
Skewness	4.07368	2.93767	3.06460	4.28766	3.53704	2.67891	1.86703
Kurtosis	33.0138	14.7914	16.4546	35.7441	20.9342	12.2118	7.12327

Table 3. ***Pairwise correlation over the volatility of volatility measures.*** *The table presents sample correlation between the VVOL measures based on daily observations over the entire sample period.*

	D-VVOL	RW-RV	RW-IV	GA2	GA4	GA3-RV	GA3-IV
D-VVOL	1.000						
RW-RV	0.538	1.000					
RW-IV	0.492	0.896	1.000				
GA2	0.974	0.559	0.520	1.000			
GA4	0.487	0.758	0.810	0.523	1.000		
GA3-RV	0.556	0.847	0.806	0.581	0.818	1.000	
GA3-IV	0.541	0.730	0.757	0.572	0.885	0.880	1.000

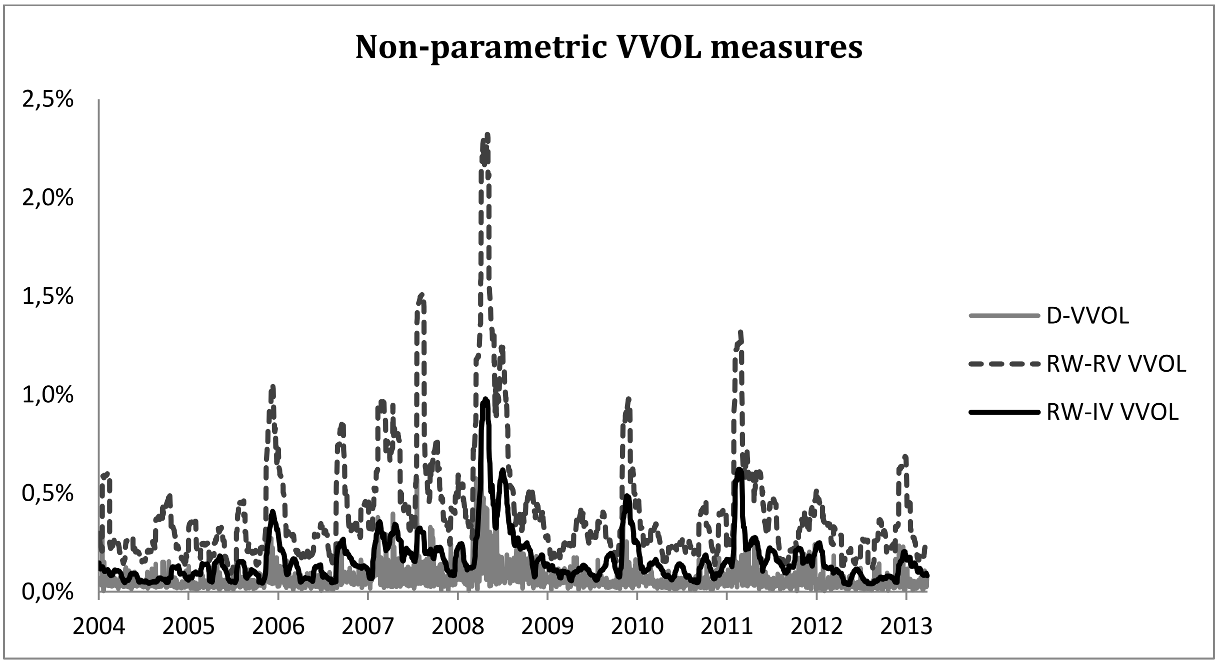

Figure 2. *Time series plot of the non-parametric VVOL measures over the period of May 2004 – Mar 2013.*

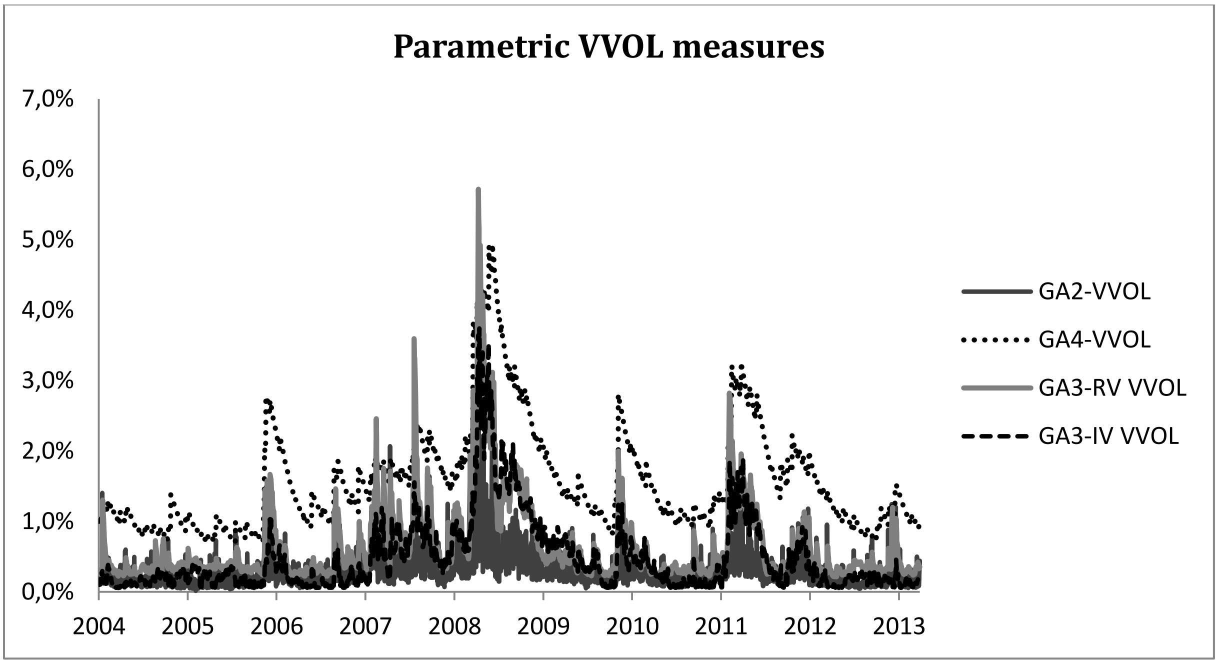

Figure 3. *Time series plot of the parametric VVOL measures over the period of May 2004 – Mar 2013.*

4.3 Aggregate Risk Factors

In table 4, we report summery statistics of all the risk factors, and the volatility of volatility factor on level (VVOL) which is obtained through PCA. Starting with the first row of table 4, we can see that the VVOL factor has a first order autocorrelation of 0,995. In line with Ang et al. (2006b)'s approach of measuring the innovations in volatility, this further support our approach of taking the first difference instead of ARMA modeling in order to obtain the innovations of volatility of volatility. In table 4 we can also see that the means of the innovations in volatility of volatility (ΔVVOL) and volatility (ΔVOL) are almost zero, with (ΔVOL) having a slightly higher standard deviation than (ΔVVOL). The mean excess return on the market portfolio is positive as expected. However, Fama & French (1993) factors SMB and HML seems to have negative mean returns, implying that the size and value premiums in the Swedish equity market might be negative. This might be the result of important differences between the U.S. and Swedish equity market concerning the size and liquidity of the two markets. However, with the high standard deviation of SMB and HML, none of the risk factors are significantly different from zero and no definite conclusions can be drawn. The momentum portfolio (WML) seems to have a positive mean return with the highest standard deviation and first order autocorrelation among the risk factors. All the other risk factors seem to have a fairly low first order autocorrelation.

Table 4. ***Summary statistics on the risk factors.*** *The table presents the sample mean, median, standard deviation, skewness and kurtosis of the risk factors used in both the time series and cross-sectional regression approach. The summary statistics is based on monthly observations over the entire sample period, resulting in 104 observations. AR(1) refers to the first order autocorrelation. P(10) and P(90) indicates the 10th and 90th percentiles, respectively.*

	Mean	Median	Std. Dev	Skewness	Kurtosis	AR(1)	P(10)	P(90)
VVOL	0.006599	0.005279	0.003817	2.312746	9.690044	0.995	0.0036	0.011
ΔVVOL	-1.8E-05	-0.00035	0.00236	2.20310	9.59294	0.043	-0.0021	0.001
ΔVOL	-3.4E-05	-0.00040	0.00318	0.18947	7.85399	-0.14	-0.0027	0.003
Rm-Rf	0.00528	0.00733	0.05166	-0.64909	4.21789	0.148	-0.0489	0.060
SMB	-0.00367	-0.00820	0.04038	0.65087	8.45190	-0.16	-0.0429	0.039
HML	-0.00187	-0.00120	0.02402	0.25456	3.05336	-0.13	-0.0281	0.030
WML	0.01133	0.01350	0.04298	-1.44212	7.80408	0.204	-0.0277	0.062

Table 5 presents the correlations between the aggregate risk factors. Initially, we can see that the correlation between ΔVOL and market excess return is negative and relatively high. This is consistent with theory and might suggest that periods of high volatility is followed by market downturns. Interestingly, the correlation between ΔVVOL and market excess return is also negative but slightly lower. This might suggest that both ΔVOL and ΔVVOL have some common underlying component. The correlation between ΔVOL and ΔVVOL (0,513) implies that the two variables are modestly correlated which entails an acceptable level of multicollinearity in the regression approach. The linear relation between ΔVOL and ΔVVOL might also been created by constriction. Both ΔVOL and ΔVVOL seem to have the highest correlation with the market portfolio which might distinguish them from the other risk factors.

Table 5. *Pairwise correlation between the risk factors.* *The table presents sample correlation between the risk factors based on monthly observations over the entire sample period.*

	ΔVOL	ΔVVOL	SMB	HML	WML	Rm-Rf
ΔVOL	1.000					
ΔVVOL	0.513	1.000				
SMB	0.275	0.120	1.000			
HML	-0.030	-0.012	-0.264	1.000		
WML	0.092	0.005	-0.039	-0.189	1.000	
Rm-Rf	-0.717	-0.557	-0.404	0.161	-0.240	1.000

Table 6. *Yearly summary statistics on ΔVVOL.* *The table presents yearly summary statistics on the change in the volatility of volatility factor based on daily frequency. P(10) and P(90) indicates the 10^{th} and 90^{th} percentiles, respectively.*

Year	Mean	Median	Std. Dev.	P(10)	P(90)
2004	0.0043	0.0042	0.000665	0.00364	0.00541
2005	0.0039	0.0039	0.000454	0.00345	0.00455
2006	0.0056	0.0045	0.002741	0.00336	0.01022
2007	0.0068	0.0071	0.001833	0.00432	0.00911
2008	0.0127	0.0092	0.006349	0.00682	0.02299
2009	0.0098	0.0085	0.003682	0.00593	0.01494
2010	0.0061	0.0052	0.002330	0.00399	0.00941
2011	0.0080	0.0056	0.004183	0.00401	0.01413
2012	0.0061	0.0062	0.001573	0.00408	0.00843
2013	0.0041	0.0039	0.000796	0.00345	0.00519
Total	0.0065	0.0052	0.003816	0.00367	0.01157

The above table (table 6) presents yearly summery statistics on the innovations in volatility of volatility (ΔVVOL). We can see that the total average mean is 0,0065 with the lowest level in year 2005 (0,0039) and the highest during the financial crises in 2008 (0,0127) followed by a sharp decline in 2010 (0,0061). The highest level is also given by the 90th percentile in 2008 which is some orders of magnitude larger than the previous years. Interestingly, the year 2008 also have the highest standard deviation (0,0063).

4.4 Fama & French regressions with Portfolios Sorts

In the following section, we present the Fama & French regression results of the value weighted and equally weighted portfolio sorts in section 4.4.1 and 4.4.2 respectively. The obtained findings are then analyzed in section 4.4.3.

4.4.1 Value Weighted Portfolios Sorted by $\Delta VVOL$ Exposure

Table 7 reports value weighted excess return on the five quintile portfolios and the 5 minus 1 hedge portfolio. Although our main focus is on the value weighted portfolio sorts in table 7, we also present the results for equally weighted portfolio sorts in table 8 as a robustness test. Initially, in the first row of table 7, we present the mean excess return without controlling for other risk factors. In rows 2 to 5, we present the mean excess return while controlling for additional aggregate risk factors. We find no clear pattern in the excess returns of the 5 quintile portfolios and the average annual excess return of the portfolios varies between 5,3% to 11,4%. The mean excess return of the quintile 4 portfolio is negatively significant at the 10% level controlling for F&F 3-factors and at the 5% level controlling for F&F 4-factors.

Turning to the 5 minus 1 hedge portfolio in the last column, we can see that the spread in return between the quintile 5 and the quintile 1 portfolio is 4,1% per year on average, meaning that stocks with the highest sensitivity to volatility of volatility tend to have higher returns than stocks with the lowest sensitivities in our sample. Controlling for the market portfolio and F&F 4-factors yields a positive and significant alpha value at the 5% level and at the 10% level when controlling for F&F 3-factors. An interesting result is that firms with higher loadings of volatility of volatility tend to be larger. The average firm size is around 12,5 billion for the quintile 1 portfolio, 22,6 for the quintile 2 portfolio, 32,5 billion for the quintile 3- portfolio and around 45 billion for both the quintile 4 and quintile 5 portfolio.

Table 7. ***Value weighted portfolios sorted by ∆VVOL loadings estimated based on a yearly rolling window.*** *The table presents value weighted quintile portfolios formed at each month based on their exposure to ∆VVOL over the following year. Quintile 1 (Q1) refers to the stocks with lowest loadings and quintile 5 (Q5) to the highest respectively. The last column (5 min 1) refers to the hedge portfolio which is long the stocks in quintile 5 and shorts the stocks in quintile 1. Each portfolio is updated every month and value weighted based on the relative market capitalization for each firm included in the portfolio. Mean return refers to the sample average of the quintile portfolios. CAPM alpha refers to the average portfolio return, controlling for excess market return. FF-3 alpha controls for Fama & French (1993) three factors (excess market return, SMB and HML). FF-4 alphas additionally controls for the momentum strategy portfolio (WML). The fifth row is the average portfolio returns, controlling for innovations in market volatility measured by the change in SIX. The row labeled ∆VVOL loadings refers to the average loadings for each portfolio based on the entire sample period. The last row shows the average firm size for stocks included in each portfolio in millions SEK. Newey & West (1987) heteroskedastic-robust p-values using 12 lags are reported in parentheses. ***, ** and * indicates 1%, 5% and 10 % significance levels respectively.*

	Q1	**Q2**	**Q3**	**Q4**	**Q5**	**5 min 1**
Mean return	0.050352 (0.5504)	0.113928** (0.0297)	0.089654** (0.0267)	0.053279 (0.2382)	0.091746 (0.1207)	0.041393 (0.1157)
CAPM alpha	-0.086404 (0.2660)	0.022351 (0.7341)	-0.003738 (0.9182)	-0.046445 (0.1945)	-0.020054 (0.6494)	0.066350** (0.0108)
FF-3 alpha	-0.085456 (0.2569)	0.023506 (0.6829)	-0.003764 (0.9410)	-0.045626* (0.0896)	-0.018524 (0.5942)	0.066932* (0.0770)
FF-4 alpha	-0.084096 (0.1925)	0.027008 (0.6313)	-0.006856 (0.9136)	-0.051816** (0.0267)	-0.017695 (0.6118)	0.066400** (0.0137)
∆VOL alpha	0.049581 (0.6105)	0.113346** (0.0203)	0.089153* (0.0714)	0.052751 (0.2049)	0.091232 (0.2151)	0.041651 (0.1487)
∆VVOL loadings	-7.7319*** (0.0000)	-3.230781*** (0.0000)	-0.892426** (0.0499)	1.257963*** (0.0058)	5.279848*** (0.0000)	
Average firm size (Mkr)	12539	22663	32464	45070	44695	

4.4.2 Equally Weighted Portfolios Sorted by $\Delta VVOL$ Exposure

Table 8 presents the result for equally weighted portfolio sorts. The average excess return varies between 11,4% and 21,8% per year and is, in general, higher than for the value weighted portfolios, which indicates that smaller companies have higher returns. Similar to the value weighted portfolio sorts, table 8 shows no clear monotonically increasing or decreasing patterns in excess returns. Interestingly, the quintile portfolios are now more significant when controlling for innovations in market volatility as shown in the last row. This further supports the fact that the pattern is not monotonic. In contrast to the previous findings of Chen et al (2016), we can see that the hedge portfolio is positively significant at 1% level when controlling for the market portfolio and the volatility factor.

Table 8. ***Equally weighted portfolios sorted by $\Delta VVOL$ loadings estimated based on a yearly rolling window.*** *The table presents equally weighted quintile portfolios formed at each month based on their exposure to $\Delta VVOL$ over the following year. Quintile 1 (Q1) refers to the stocks with lowest loadings and quintile 5 (Q5) to the highest respectively. The last column (5 min 1) refers to the hedge portfolio. Newey & West (1987) heteroskedastic-robust p-values using 12 lags are reported in parentheses. ***, ** and * indicates 1%, 5% and 10 % significance levels respectively.*

	Q1	**Q2**	**Q3**	**Q4**	**Q5**	**5 min 1**
Mean return	0.145600* (0.0779)	0.171138** (0.0266)	0.153137** (0.0106)	0.113717* (0.0781)	0.218175** (0.0160)	0.072575*** (0.0084)
CAPM alpha	-0.011482 (0.8652)	0.042941 (0.4665)	0.028531 (0.3971)	-0.009827 (0.8066)	0.047254 (0.4204)	0.058735*** (0.0071)
FF-3 alpha	-0.009726 (0.9018)	0.044327 (0.2366)	0.029054 (0.4480)	-0.008677 (0.7688)	0.048911 (0.3098)	0.058638** (0.0319)
FF-4 alpha	0.008726 (0.8982)	0.041474 (0.2619)	0.028509 (0.3695)	-0.007969 (0.17829)	0.051225 (0.2645)	0.042499 (0.8054)
ΔVOL alpha	0.144695* (0.0881)	0.170365*** (0.0065)	0.152374*** (0.0058)	0.113098* (0.0657)	0.217997** (0.0443)	0.072602*** (0.0033)

4.4.3 The Contemporaneous Relation between $\Delta VVOL$ and Stock Returns

Motivated by theoretical arguments and empirical findings, we expect market volatility of volatility to be a negatively priced risk factor in the cross-section of stock returns. The result from the portfolio sorts indicates that the outcome is ambiguous. If volatility of volatility is negatively priced, we expect a clear pattern of monotonically descending returns from portfolio 1 to portfolio 5 and a negative return on the hedge portfolio, which clearly is not the case. Instead, the hedge portfolio seems to have a positive and significant excess return in most of the specifications, especially in the equally weighted portfolio sorts of table 8. This is in contrast with theory and implies that volatility of volatility might have a positive market price which results in stocks with high sensitivity to innovations in volatility of volatility earning higher returns. An interesting fact in the value-weighted regressions is that the quintile 4 portfolio which contains stocks with high loadings on volatility of volatility has a negative and significant alpha value. This result is in line with previous findings that volatility of volatility is a negatively priced risk factor. The overall result from the portfolio sorts is however inconclusive and we cannot conclude that stocks with high exposure to volatility of volatility have lower nor higher returns than other stocks in the sample. These results might suggest that investors in the Swedish equity market are not uncertainty averse and hence, does not require insurance against shocks in volatility of volatility. Another explanation might be that investors are not even aware of the existence of volatility of volatility and seek only insurance against shocks to aggregate market volatility. Although the results from the portfolio sorts shows no clear contemporaneous relation between sensitivity to $\Delta VVOL$ and average returns, one also need to take into account other known determinants of excess returns in order to make a robust conclusion of the underlying relation. We address this issue through multivariate Fama-MacBeth regressions presented in the following section.

4.5 Fama–MacBeth regressions and the Price of Risk

Table 9 presents the result of the Fama-Macbeth regressions. Specifically, the table includes the time series average of the coefficient estimates from monthly Fama-Macbeth regressions which is essentially the estimated risk premiums of the aggregate risk factors. Initially, in the first specification (model 1) we include only the volatility risk factor. We can see that column 1 shows a negative risk premium for market volatility (ΔVOL). This is in line with the findings of previous literature and in accordance with theory which implies that investors are prepared to pay for insuring against market volatility risk (e.g. Ang et al. (2006b), Cremers et al (2015)). However, in contrast to the earlier studies, we do not find this effect to be significant. Also the effect is rather small relative to the previous findings with a very low R-squared of only 0,68%. In the second specification (model 2) we only include the market volatility of volatility risk factor. In the second column, we can first see that the R-squared rises to 5,6%, but more importantly, the volatility of volatility risk premium (ΔVVOL) seems to be more negative (-0.000494) than the volatility risk premium (-0.000165), suggesting that investors might pay more in order to hedge against volatility of volatility risk. Nevertheless, the ΔVVOL risk premium is not significant. Although, the p-value of ΔVVOL seems to be lower than that of ΔVOL.

Turning to the third specification (model 3), we include ΔVVOL while controlling for ΔVOL. Interestingly, we now see that the ΔVVOL risk premium becomes significant at the 10% level and has not changed dramatically compared to the previous specification in model 2. This change in significance might also be due to the positive correlation between ΔVOL and ΔVVOL (0.513) as shown in table 5. The ΔVOL risk premium seems to have decreased (-0.000328) but is still not significant and remains insignificant in all of the other specifications. This is similar to the results of Chen et al. (2016) who found the market volatility factor to be significant only in their first specification but insignificant in their other models. Also including the market risk factor in model 4, the price of ΔVVOL seems to remain significant at the 10% level. Now, while controlling for both of the volatility factors and the market portfolio at the same time, we are able to also estimate the economic significance ΔVVOL. We do this by using the standard deviation of the estimated Fama-Macbeth betas presented in appendix 6. Using the ΔVVOL risk premium in model 4 (-0.000583), a two standard deviation increase in the stocks sensitivity to ΔVVOL results in a 0.61% drop in expected rate of return per month ($2 \times -0.000583 \times 5.259637 = -0.0061327$). The same calculation for the price of ΔVOL in model 4 (-0.000371) results in a 0,37% decrease in the expected rate of return. Although none of the volatility premiums are strongly significant, the magnitude of this economic effect is rather high. Interestingly, the ΔVVOL effect is almost twice in magnitude compared to ΔVOL. Again, this might imply that investors are more averse towards market uncertainty measured by volatility of volatility than they are of market volatility which would make ΔVVOL an important aggregate risk factor in the cross-section of stock returns. In the rest of the specifications (model 5, 6 and 7), we also control for Fama & French (1993) factors SMB, HML and the momentum factor WML. Similar to Ang et al. (2006b), Chen et al. (2016) and Baltussen et al (2014), we do not find any of these other risk factors to be significant. Ang et al. (2006b) argues that the market portfolio, SMB and HML factors are estimated imprecisely due to the time period and their set of base assets used. Baltussen et al (2014) refers to the fact that the market beta does not pay off anymore and that previously documented return anomalies have disappeared in recent years. Although our result could have a similar explanation, the most probable cause in our case could be due to survivorship bias as explained in section 3.9. Also Chen et al. (2016) found only the volatility of volatility factor to be statistically significant in their specifications.

Nevertheless, we can see that the ΔVVOL factor is somewhat robust when controlling for other determinants of stock returns and remains at a 10% significant level in all of the models. Although it is tempting to make a case for an aggregate market uncertainty factor, it is hard to find this effect to be distinct from ΔVOL. The specifications in model 1, 2 and 3 were also an attempt to disentangle the ΔVOL and ΔVVOL effects. We can however see that ΔVVOL becomes significant only when including ΔVOL which suggest that the two factors might be intertwined. Besides from the fact that 10% significance level is generally too low for making a case for a risk factor explanation, the Fama–MacBeth regression result is also in contrast with the results from the portfolio sorts in section 4.4.

The weak significance of ΔVVOL might also depend on the liquidity and ownership structure of the Swedish equity market. Although U.S. investors make up a significant share of the equity owners in the Swedish market (30,8 % in 2015), the majority investors might have different preferences and risk appetite (SCB, 2015). The Swedish equity market is also smaller compared to the U.S. market. These arguments would however not explain the contrasting results of Fama–MacBeth regression and portfolio sorts which makes the empirical support for an aggregate volatility of volatility factor inconclusive.

Table 9. ***Multivariate Fama-MacBeth regressions.*** *The table presents average coefficient estimates from monthly Fama-MacBeth regressions. Each month we perform cross-sectional regressions where we regress monthly excess returns against the estimated loadings for each stock in order to obtain a time series of risk premiums. ΔVOL and $\Delta VVOL$ represents the innovations in volatility and volatility of volatility respectively. Rm-Rf, SMB and HML represent Fama & French (1993) three factors. WML is the momentum strategy portfolio. The adjusted $\boldsymbol{R^2}$ are presented in the last row. One-sided p-values are reported in parentheses. ***, ** and * indicates 1%, 5% and 10 % significance levels respectively.*

Model	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**
Constant	0.009358**	0.004246	0.005201	0.005419*	0.005814*	0.005742*	0.004408
	(0.030817)	(0.156144)	(0.101225)	(0.084618)	(0.05844)	(0.060151)	(0.124556)
ΔVOL	-0.000165		-0.000328	-0.000371	-0.000369	-0.000379	-0.000222
	(0.360952)		(0.241912)	(0.211083)	(0.211986)	(0.216766)	(0.31707)
ΔVVOL		-0.000494	-0.000556*	-0.000583*	-0.000622*	-0.000629*	-0.000527*
		(0.11829)	(0.098082)	(0.083793)	(0.058949)	(0.06264)	(0.091481)
Rm-Rf				0.004284	0.004210	0.004352	0.006501
				(0.258217)	(0.262248)	(0.256962)	(0.16341)
SMB					-0.000892	-0.000950	-0.000520
					(0.425679)	(0.423114)	(0.457622)
HML						-0.000263	0.001325
						(0.474497)	(0.37179)
WML							0.007095
							(0.132319)
Adj-$\boldsymbol{R^2}$	0.006480	0.056295	0.068771	0.061800	0.055780	0.047077	0.109811

4.6 Is Volatility of Volatility a Persistent Risk Factor?

In table 10 we present factor loadings for the 5 quintile portfolios during the time period before, after and at the time when the portfolios are constructed. Given our rolling estimation period of 12 months, the portfolio formation at time 0 is correlated up to 12 months before and after its formation, hence the time period 25 to 13 months before and 13 to 25 after its formation is the closest 12 month period we can analyze without inducing a spurious correlation between two time periods.

The result of this table indicates that volatility of volatility is a persistent risk factor, at least for stocks with a low exposure to volatility of volatility. Stocks in quintile portfolio 1 and 2 tend to have highly negative exposures to volatility of volatility both during the time period before the portfolio formation is made, but also during the time period after its formation. For quintile portfolio 3 to 5 the volatility of volatility persistence is weaker and all three portfolios have slightly negative exposure to volatility of volatility both before and after the portfolio formation is made. Baltussen et al (2014) found that idiosyncratic volatility of volatility for individual stocks is persistent for all five quintiles. Our result, concerning the exposure to aggregate volatility of volatility, is similar to theirs for highly negative exposures, but we do not find a volatility of volatility persistence stocks that have high exposures to volatility of volatility.

Table 10. ***Persistence in volatility of volatility loadings over time.*** *The table presents volatility of volatility loadings for a given quintile portfolio 25 to 13 months before and 13 to 25 months after the portfolio formation is made. Within each portfolio, all stocks have been given equal weights. Given our rolling estimation period of 12 months, a 2 year period around the portfolio formation has been excluded in order to not induce a spurious relationship.*

	-13 to -25	**0**	**+13 to +25**
Q1	-1.378	-7.732	-2.117
Q2	-0.680	-3.231	-1.340
Q3	-0.595	-0.892	-0.854
Q4	-0.402	1.258	-0.872
Q5	-0.483	5.280	-0.900

4.7 Is the Volatility of Volatility a Negatively Priced Risk Factor in the Swedish Stock Market?

As indicated by our results in the previous sections, we do not find the market price of volatility to be significant in the Swedish equity market which is in contrast to the findings of Ang et al. (2006b). Nevertheless, we find the price of market volatility to be negative. This is in accordance with the theory of deteriorating investment opportunities that arise from increasing market volatility. More importantly however, our result indicates that there is no or a weak relationship between sensitivity to volatility of volatility and the cross-section of stock returns. These results differs from earlier work by Chen et al (2016) and Huang & Shaliastovich (2014) who found that volatility of volatility has a significantly negative market price.

Besides from the low significance level of 10 %, the overall results from the Fama-Macbeth regressions in table 9 is however encouraging. Not only do we find the volatility of volatility risk premium to be negative but it also seems to be robust in all of the specifications. This finding might imply that volatility of volatility is an important risk factor that is relevant for equity returns. However, due to the contrasting outcomes between portfolio sorts and Fama-Macbeth regressions, our results should be interpreted with caution.

Interestingly, our findings are also similar to Baltussen et al (2014) who found that only idiosyncratic volatility of volatility is priced in the market, as opposite to our aggregate volatility of volatility factor. Based on the preference based explanation of Baltussen et al (2014), one could argue that uncertainty about risk is imbedded in the investor's utility functions and that investors are averse to uncertainty about risk, which would result in the limited participation phenomenon as explained in section 2.

However, based on the results in table 9, we cannot rule out the possibility of an aggregate risk factor explanation. Our contrasting results could also depend on the underlying characteristics of the Swedish equity market. One could argue that the volatility of volatility effect might be U.S. specific and the effect could differ between the U.S. and the smaller Swedish stock market. A small open economy like Sweden is however largely affected by the global economy as a hole and U.S. financial markets specifically. As previously mentioned, a large part of the ownership interest in the Swedish equity market also consists of U.S. investors which might result in similar investor behavior in the two markets.

Although our inconclusive findings concerning volatility of volatility could be given several explanations, the result in section 4.7 indicates that stock exposure to market volatility of volatility is not persistent for stocks with high exposure to the risk factor. Persistence in exposure to market volatility of volatility is a necessary condition for the explanations of a negative market price of risk as argued by Chen et al (2016) and Huang & Shaliastovich (2014) and for a negative relationship between idiosyncratic volatility of volatility and stock returns as proposed by Baltussen et al (2014). Even if investors are risk averse and dislikes idiosyncratic volatility of volatility, they may not be able to distinguish high volatility of volatility from low volatility of volatility stocks hence, the preferences over these stocks should not matter. The explanations given by Huang & Shaliastovich (2014) and Chen et al (2016) are also highly dependent on how investors can separate stocks into high and low exposure to volatility of volatility. Without a strong evidence for this persistency, we must reject the hypothesis that volatility of volatility is a state variable that affects the cross-section of equity returns in the Swedish equity market.

5 Conclusion

Recent studies have found a negative relationship between volatility of volatility and the cross section of stock returns. Building on the Intertemporal capital asset pricing model (ICAPM), arbitrage pricing theory (APT) and ambiguity aversion, the existence of a negatively priced volatility of volatility factor has been motivated. In our study, we use seven different measures of volatility of volatility and by the use of principal component analysis; we extract the underlying risk component of market volatility of volatility from these measures. Focusing on the Swedish equity market, we find that uncertainty about risk, i.e. volatility of volatility has no or weak explanatory power in the cross section of stock returns. By forming portfolios on volatility of volatility loadings and controlling for previously documented risk factors, we find no relationship between portfolios sorted on volatility of volatility and realized returns. Based on Fama-Machbeth regressions, we find a negative but weakly priced volatility of volatility risk factor. Interestingly, our results suggests that firms with high exposure to market volatility of volatility tend to be larger and have higher market beta, which indicates that our measure of market volatility of volatility captures important aspects of risk that is distinct from the market beta. Moreover, we do not find persistence in stock exposure to volatility of volatility over the studied time period. Persistence in exposure to market volatility of volatility is a necessary condition for investors to be able to hedge variations in the risk factor and consequently affect stock returns. The weak relationship between volatility of volatility and the cross-section of stock returns can thus be explained by discontinuity in volatility of volatility over time, especially for stocks with high exposure to the risk factor. One could argue that even if investors are risk averse and dislike volatility of volatility risk, they may fail to distinguish high volatility of volatility from low volatility of volatility stocks and hence, be unable to use these stocks in order to hedge variations in the risk factor. As initially mentioned, aggregate market volatility could be seen as a known unknown. Our findings might suggest that aggregate market volatility of volatility could be an unknown unknown in the Swedish equity market. Although, our estimated volatility of volatility factor is based on the theoretical result of the variance risk premium, it would be interesting to investigate the empirical implications of the variance risk premium in the Swedish equity market. Another move forward in the investigation of market uncertainty would be to develop the equivalent of VVIX for the Swedish market which would result in a purely implied volatility of implied volatility measure.

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Appendix

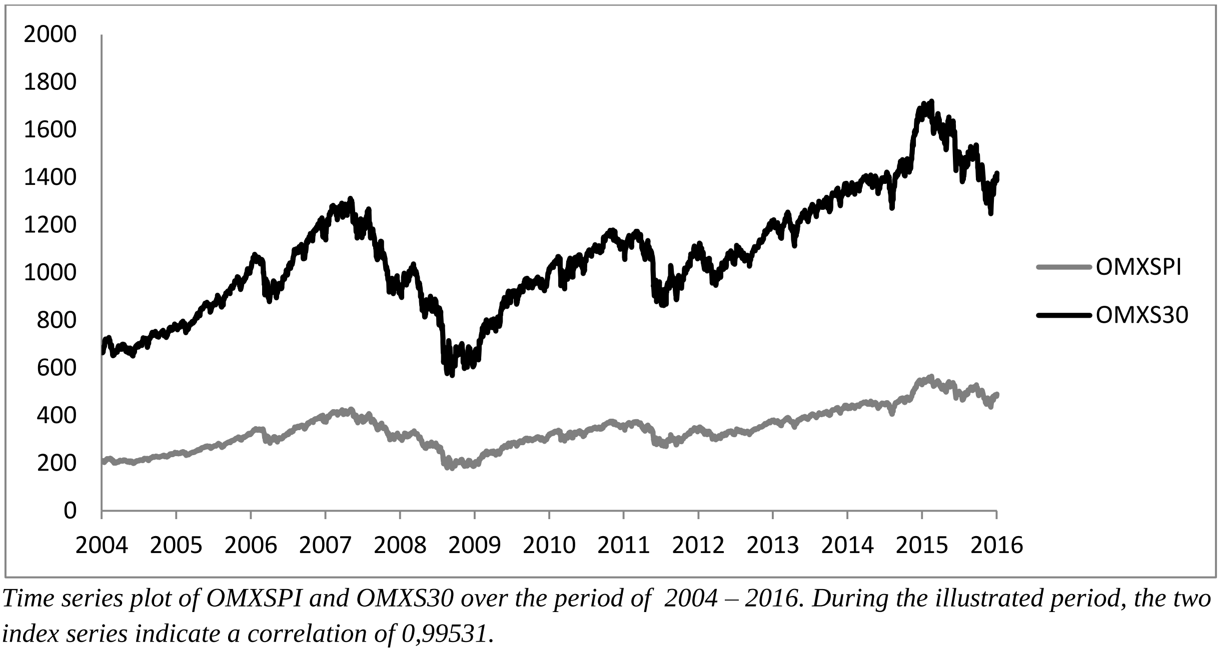
Appendix .1

- Descriptive statistics of OMXS30 price index and OMXS30 futures.

	OMXS30 PRICE INDEX	OMXS30 FUTURES
Mean	1069.168	1064.440
Median	1054.490	1047.260
Maximum	1719.930	1710.595
Minimum	567.6100	571.5396
Std. Dev.	246.9530	249.1376
Skewness	0.377248	0.362078
Kurtosis	2.737882	2.692200
Jarque-Bera	77.69953	75.40651
Probability	0.000000	0.000000
Sum	3125179.	3111357.
Sum Sq. Dev.	1.78E+08	1.81E+08
Correlation	0.9992	0.9992
Observations	2923	2923

Appendix .2

- Graphical illustration of OMXSPI and OMXS30 price index.



Time series plot of OMXSPI and OMXS30 over the period of 2004 – 2016. During the illustrated period, the two index series indicate a correlation of 0,99531.

Appendix .3

- Descriptive statistics of implied volatility (IV) and realized volatility (RV).

	IV	RV
Mean	0.012921	0.011434
Median	0.011229	0.009504
Maximum	0.049085	0.103441
Minimum	0.005858	0.001616
Std. Dev.	0.005653	0.007373
Skewness	2.008743	3.158961
Kurtosis	8.764468	21.94810
Jarque-Bera	6253.443	50533.23
Probability	0.000000	0.000000
Sum	39.28114	34.75795
Sum Sq. Dev.	0.097131	0.165225
Correlation	0.806786	0.806786
Observations	3040	3040

Appendix .4 - GARCH estimation

The section below presents the statistical tests conducted in the GARCH estimations.

Appendix 4.1 GARCH-2

- Unit root test of log returns

Null Hypothesis: LOGRETURN has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=88)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-402.1182	0.0001
Test critical values:	1% level	-3.430204	
	5% level	-2.861359	
	10% level	-2.566714	

*MacKinnon (1996) one-sided p-values.

- Unit root test of squired residuals

Null Hypothesis: SQUIRED RESIDUALS has a unit root
Exogenous: Constant
Lag Length: 21 (Automatic - based on SIC, maxlag=88)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-104.2136	0.0001
Test critical values:	1% level	-3.430204	
	5% level	-2.861359	
	10% level	-2.566714	

*MacKinnon (1996) one-sided p-values.

- GARCH (1,1) estimation output

Dependent Variable: LOGRETURN
Method: ML - ARCH (Marquardt) - Normal distribution
Included observations: 299853 after adjustments
Convergence achieved after 19 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1.25E-06	5.28E-07	2.375008	0.0175
		Variance Equation		
C	2.93E-08	8.00E-11	365.6610	0.0000
RESID(-1)^2	0.273840	0.000405	675.4043	0.0000
GARCH(-1)	0.806166	0.000224	3592.341	0.0000

R-squared	-0.000001	Mean dependent var	2.59E-06
Adjusted R-squared	-0.000001	S.D. dependent var	0.001382
S.E. of regression	0.001382	Akaike info criterion	-10.82452
Sum squared resid	0.572941	Schwarz criterion	-10.82437
Log likelihood	1622886.	Hannan-Quinn criter.	-10.82447
Durbin-Watson stat	2.051657		

- ARCH Effect Heteroskedasticity test of the squared residuals

Heteroskedasticity Test: ARCH - Pre-estimation
Null Hypothesis: No autocorrelation
Lag Length: 4

F-statistic	217.8549	Prob. F(1,299850)	0.0000
Obs*R-squared	217.6982	Prob. Chi-Square(1)	0.0000

Heteroskedasticity Test: ARCH - After estimation
Null Hypothesis: No autocorrelation
Lag Length: 4

F-statistic	2.827124	Prob. F(1,299850)	0.0927
Obs*R-squared	2.827116	Prob. Chi-Square(1)	0.0927

Appendix 4.2 GARCH-3

- Unit root test of realized volatility (RV)

Null Hypothesis: RV has a unit root
Exogenous: Constant
Lag Length: 8 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.196932	0.0000
Test critical values:	1% level	-3.432318	
	5% level	-2.862295	
	10% level	-2.567216	

*MacKinnon (1996) one-sided p-values.

- Unit root test of squired residuals (RV)

Null Hypothesis: Squired residuals_RV has a unit root
Exogenous: Constant
Lag Length: 18 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.718823	0.0000
Test critical values:	1% level	-3.432325	
	5% level	-2.862298	
	10% level	-2.567218	

*MacKinnon (1996) one-sided p-values.

- GARCH (1,1) estimation output (RV)

Dependent Variable: RV
Method: ML - ARCH
Included observations: 3040
Convergence achieved after 28 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.008350	5.76E-05	144.8724	0.0000
Variance Equation				
C	8.85E-07	4.24E-08	20.89633	0.0000
RESID(-1)^2	0.232084	0.009005	25.77283	0.0000
GARCH(-1)	0.764457	0.004031	189.6653	0.0000

R-squared	-0.174886	Mean dependent var	0.011434
Adjusted R-squared	-0.174886	S.D. dependent var	0.007373
S.E. of regression	0.007992	Akaike info criterion	-7.835017
Sum squared resid	0.194121	Schwarz criterion	-7.827097
Log likelihood	11913.23	Hannan-Quinn criter.	-7.832170
Durbin-Watson stat	0.478570		

- ARCH Effect Heteroskedasticity test of the squared residuals (RV)

Heteroskedasticity Test: ARCH - Pre-estimation
Null Hypothesis: No autocorrelation
Lag Length: 4

F-statistic	161.0711	Prob. F(10,3019)	0.0000
Obs*R-squared	1054.160	Prob. Chi-Square(10)	0.0000

Heteroskedasticity Test: ARCH - After estimation
Null Hypothesis: No autocorrelation
Lag Length: 4

F-statistic	0.395484	Prob. F(10,3019)	0.9492
Obs*R-squared	3.964056	Prob. Chi-Square(10)	0.9490

- Unit root test of implied volatility (IV)

Null Hypothesis: IV has a unit root
Exogenous: Constant
Lag Length: 5 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.541289	0.0071
Test critical values:	1% level	-3.432316	
	5% level	-2.862294	
	10% level	-2.567216	

*MacKinnon (1996) one-sided p-values.

- Unit root test of squired residuals (IV)

Null Hypothesis: Squired residuals_IV has a unit root
Exogenous: Constant
Lag Length: 26 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.421637	0.0003
Test critical values:	1% level	-3.432330	
	5% level	-2.862301	
	10% level	-2.567219	

*MacKinnon (1996) one-sided p-values.

- GARCH (1,1) estimation output (IV)

Dependent Variable: IV
Method: ML - ARCH
Included observations: 3040
Convergence achieved after 29 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.009898	2.65E-05	373.9315	0.0000
Variance Equation				
C	2.68E-07	2.94E-08	9.113482	0.0000
RESID(-1)^2	0.830441	0.075604	10.98404	0.0000
GARCH(-1)	0.173960	0.047109	3.692683	0.0002

R-squared	-0.286162	Mean dependent var	0.012921
Adjusted R-squared	-0.286162	S.D. dependent var	0.005653
S.E. of regression	0.006412	Akaike info criterion	-9.035843
Sum squared resid	0.124927	Schwarz criterion	-9.027922
Log likelihood	13738.48	Hannan-Quinn criter.	-9.032995
Durbin-Watson stat	0.029838		

- ARCH Effect Heteroskedasticity test of the squared residuals (IV)

Heteroskedasticity Test: ARCH - Pre-estimation
Null Hypothesis: No autocorrelation
Lag Length: 4

F-statistic	2394.134	Prob. F(10,3019)	0.0000
Obs*R-squared	2690.703	Prob. Chi-Square(10)	0.0000

Heteroskedasticity Test: ARCH - After estimation
Null Hypothesis: No autocorrelation
Lag Length: 4

F-statistic	0.908770	Prob. F(10,3019)	0.5240
Obs*R-squared	9.093440	Prob. Chi-Square(10)	0.5233

Appendix 4.3 GARCH-4 (Nested GARCH)

- Unit root test of log returns

Null Hypothesis: LOGRETURN has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-42.13791	0.0000
Test critical values:	1% level	-3.432313	
	5% level	-2.862293	
	10% level	-2.567215	

*MacKinnon (1996) one-sided p-values.

- Unit root test of squired residuals

Null Hypothesis: SQUIRED RESIDUALS has a unit root
Exogenous: Constant
Lag Length: 12 (Automatic - based on SIC, maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.403511	0.0000
Test critical values:	1% level	-3.432321	
	5% level	-2.862296	
	10% level	-2.567217	

*MacKinnon (1996) one-sided p-values.

- ARCH Effect Heteroskedasticity test of the squared residuals

Heteroskedasticity Test: ARCH - Pre-estimation
Null Hypothesis: No autocorrelation
Lag Length: 4

F-statistic	69.96365	Prob. F(10,3018)	0.0000
Obs*R-squared	570.0393	Prob. Chi-Square(10)	0.0000

- First step ARMA estimation

Dependent Variable: SQUIRED RESIDUALS
Method: Least Squares
Included observations: 3038 after adjustments
Convergence achieved after 7 iterations
MA Backcast: 5/11/2004

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.993651	0.002395	414.8991	0.0000
MA(1)	0.923818	0.008151	113.3364	0.0000

R-squared	0.175906	Mean dependent var	0.000191
Adjusted R-squared	0.175635	S.D. dependent var	0.000505
S.E. of regression	0.000459	Akaike info criterion	-12.53588
Sum squared resid	0.000639	Schwarz criterion	-12.53192
Log likelihood	19044.00	Hannan-Quinn criter.	-12.53445
Durbin-Watson stat	2.107140		

Inverted AR Roots	.99
Inverted MA Roots	.92

- Second step ARMA estimation

Dependent Variable: SQUIRED RESIDUALS
Method: Least Squares
Included observations: 3038 after adjustments
Convergence achieved after 23 iterations
MA Backcast: 5/11/2004

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.989221	0.004098	241.3646	0.0000
MA(1)	0.950565	0.008692	109.3573	0.0000

R-squared	0.055951	Mean dependent var	2.36E-07
Adjusted R-squared	0.055640	S.D. dependent var	2.38E-06
S.E. of regression	2.31E-06	Akaike info criterion	-23.11745
Sum squared resid	1.62E-08	Schwarz criterion	-23.11349
Log likelihood	35117.41	Hannan-Quinn criter.	-23.11603
Durbin-Watson stat	2.118820		

Inverted AR Roots	.99
Inverted MA Roots	.95

- Second step-ARCH Effect Heteroskedasticity test of the squared residuals

Heteroskedasticity Test: ARCH - After estimation
Null Hypothesis: No autocorrelation
Lag Length: 4

F-statistic	0.000668	Prob. F(1,3036)	0.9794
Obs*R-squared	0.000669	Prob. Chi-Square(1)	0.9794

Appendix .5 - Principal component analysis

- Unit root tests of volatility of volatility measures

Null Hypothesis: D-VVOL has a unit root
Exogenous: Constant
Lag Length: 8 (Automatic - based on SIC. maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.277487	0.0000
Test critical values:	1% level	-3.432348	
	5% level	-2.862308	
	10% level	-2.567223	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: RW-RV VVOL has a unit root
Exogenous: Constant
Lag Length: 24 (Automatic - based on SIC. maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.604533	0.0001
Test critical values:	1% level	-3.432359	
	5% level	-2.862313	
	10% level	-2.567226	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: RW-IV VVOL has a unit root
Exogenous: Constant
Lag Length: 23 (Automatic - based on SIC. maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.132094	0.0009
Test critical values:	1% level	-3.432359	
	5% level	-2.862313	
	10% level	-2.567226	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: GA2-VVOL has a unit root
Exogenous: Constant
Lag Length: 9 (Automatic - based on SIC. maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.345099	0.0000
Test critical values:	1% level	-3.432348	
	5% level	-2.862309	
	10% level	-2.567223	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: GA3-RV VVOL has a unit root
Exogenous: Constant
Lag Length: 5 (Automatic - based on SIC. maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.287370	0.0000
Test critical values:	1% level	-3.432346	
	5% level	-2.862307	
	10% level	-2.567223	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: GA3-IV VVOL has a unit root
Exogenous: Constant
Lag Length: 5 (Automatic - based on SIC. maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.739168	0.0037
Test critical values:	1% level	-3.432346	
	5% level	-2.862307	
	10% level	-2.567223	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: GA4-VVOL has a unit root
Exogenous: Constant
Lag Length: 11 (Automatic - based on SIC. maxlag=28)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.493050	0.0083
Test critical values:	1% level	-3.432350	
	5% level	-2.862309	
	10% level	-2.567224	

*MacKinnon (1996) one-sided p-values.

- Principal component output

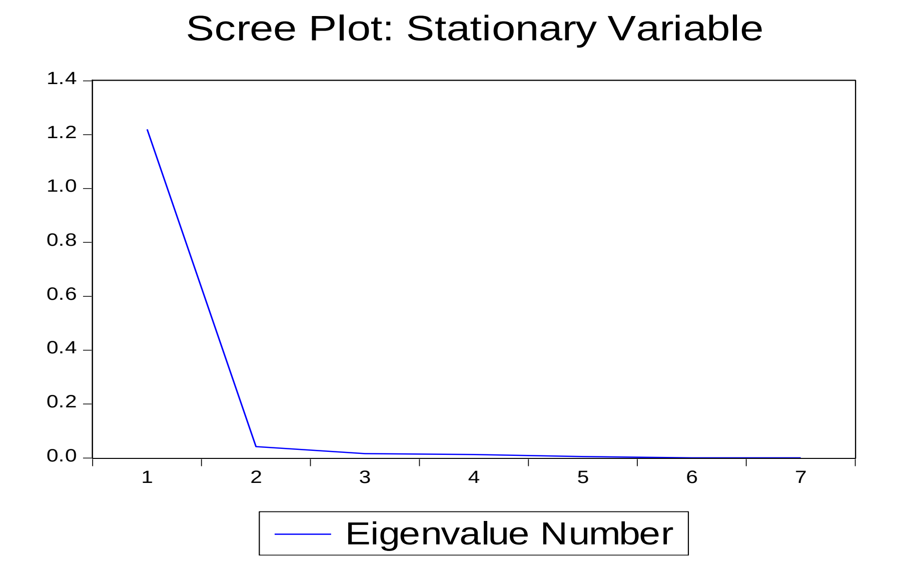
Principal Components Analysis
Included observations: 2998
Computed using: Ordinary correlations
Extracting 7 of 7 possible components

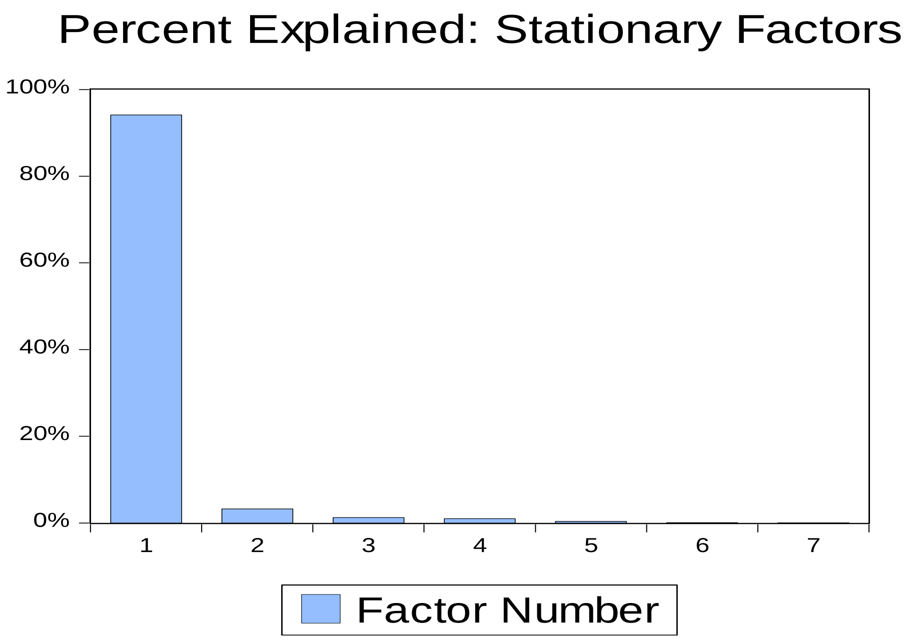
Eigenvalues: (Sum = 7, Average = 1)

Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
1	5.176873	4.097688	0.7396	5.176873	0.7396
2	1.079185	0.707236	0.1542	6.256059	0.8937
3	0.371950	0.180938	0.0531	6.628008	0.9469
4	0.191012	0.100811	0.0273	6.819020	0.9741
5	0.090200	0.024300	0.0129	6.909220	0.9870
6	0.065900	0.041020	0.0094	6.975120	0.9964
7	0.024880	---	0.0036	7.000000	1.0000

Eigenvectors (loadings):

Variable	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7
D_VVOL	0.325980	0.636663	0.020504	0.029244	-0.003363	-0.016971	0.697726
RW_RV_VVOL	0.393110	-0.185654	0.573297	-0.185599	0.428022	0.514393	-0.008750
RW_IV_VVOL	0.390336	-0.251225	0.456154	0.390578	-0.597080	-0.259479	0.007907
GA3_RV_VVOL	0.405250	-0.179629	-0.098081	-0.686770	0.061456	-0.564274	-0.007188
GA3_IV_VVOL	0.395887	-0.177311	-0.556201	-0.134131	-0.420218	0.554792	0.010270
GA2_VVOL	0.336780	0.608226	-0.006205	0.065715	0.002279	-0.021294	-0.715420
GA4_VVOL	0.390467	-0.251861	-0.379229	0.564078	0.529071	-0.202805	0.032511





- Stationary factors and volatility of volatility correlation matrix

	D-VVOL	RW-RV VVOL	RW-IV VVOL	GA3RV VVOL	GA3-IV VVOL	GA2-VVOL	GA4-VVOL
STATIONARYFACTORS_1	0.570201	0.835496	0.854038	0.914945	0.937671	0.604367	0.976544
STATIONARYFACTORS_2	0.551417	0.685305	0.587929	0.857063	0.730036	0.562097	0.454628
STATIONARYFACTORS_3	-0.529741	-0.081785	0.050344	0.122507	0.350052	-0.520217	0.312817
STATIONARYFACTORS_4	-0.482368	0.221240	0.107536	0.069592	-0.293961	-0.501024	-0.164946
STATIONARYFACTORS_5	-0.049072	-0.425113	-0.389762	0.031050	-0.165401	-0.053071	-0.156349
STATIONARYFACTORS_6	0.047373	0.021564	0.431378	0.060341	0.041486	0.058570	0.107354
STATIONARYFACTORS_7	-0.140364	0.096092	0.090248	0.078685	0.062203	0.083786	0.148156

Appendix .6 Cross-sectional Fama-Macbeth Betas

- Descriptive statistics of cross-sectional Fama-Macbeth betas estimated based on the entire time period of August 2004 to March 2013 using monthly data.

	HML-BETA	Rm-Rf-BETA	SMB-BETA	ΔVOL-BETA	ΔVVOL-BETA	WML-BETA
Mean	0.411984	0.994308	-0.123828	-11.97126	-14.33159	-0.504527
Median	0.396266	0.980747	-0.161581	-11.86391	-13.82056	-0.408921
Maximum	2.130592	2.311940	1.967541	-2.348951	-2.157940	0.208477
Minimum	-1.710101	0.142778	-1.589700	-26.66926	-34.47223	-3.361062
Std. Dev.	0.651656	0.419671	0.587385	4.991741	5.259637	0.499811
Skewness	-0.268836	0.454786	0.752760	-0.349435	-0.693746	-2.352397
Kurtosis	4.201432	3.133622	4.207339	2.898912	4.681334	12.11627