Evaluation of a Gradient Free and a Gradient Based Optimization Algorithm for Industrial Beverage Pasteurisation Described by Different Modeling Variants

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Abstract

Inspired by Krones Group in Holte/Copenhagen, the intention of this thesis is to simulate and optimize pasteurisation processes. Based on different modes of heat transfer, a mathematical model to describe the thermal processes in canned beverages is developed. The main goal of the thesis is to optimize the thermal treatment of the product. Both the derivative-free COBYLA and the gradient-based MMA optimization algorithm are described and evaluated. The optimal results obtained with the NLopt library in Python are compared and different modeling variants of the optimization problem are developed with regard to their meaningfulness and numerical behaviour.

1. Introduction

Pasteurisation is a method used in the food production industry to extend the shelf life of products, mostly beverages. This is achieved by heating the product to a product-specific lethal temperature to inactivate particular micro-organisms. A *Pasteurisation Unit* (PU) is defined as one minute of thermal treatment at 60 °C. Canned beverages are pasteurised by transporting them on a conveyor belt through a tunnel pasteur, where spray water in different temperatures gently heates, pasteurises and cooles the product.

2. Modelation of the Problem Description

Both conductive and convective heat transfer take place during the thermal treatment of canned beverages. In order to reduce the computational time, the convective heat transfer within the liquid is omitted and a simplified model to describe the dependence of the PUs on the spray water temperature is obtained. For a tunnel pasteur with four pasteurisation zones the following optimization problem is considered:

$$\begin{aligned} &\underset{u}{\text{minimize}}\ (\mathrm{PU}_{\text{end}}(u) - \mathrm{PU}_{\text{des}})^2 \\ &\text{subject to } (u_i - u_p(t_i))^2 \leq 25^2,\ i = 4:6, \\ &\qquad\qquad (u_i - u_{i+1})^2 \leq 0,\ i = 4:6, \\ &\qquad\qquad 58 \leq u_i \leq 65,\ i = 4:6. \end{aligned} \tag{1}$$

The vector $u \in \mathbb{R}^{10}$ describes the spray water temperature in each zone, PU_{end} the final amount of PUs and PU_{des} the desired amount of PUs in the product. The constraints regulate the temperature difference between the product and the spray water, the temperature difference between the four pasterurisation zones and the bounded temperatures.

Figure 1 visualizes the optimization task: Find the optimal spray water temperature in the four pasteurisation zones in order to guarantee a certain amount of PUs in the product once it leaves the tunnel pasteur.

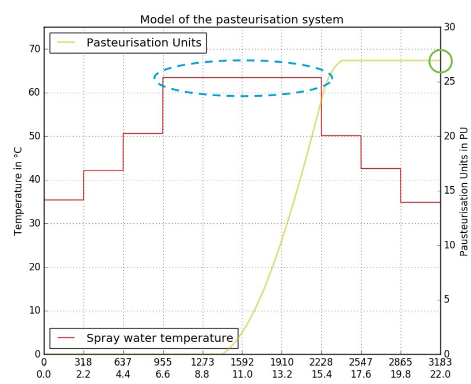

Figure 1: Scheme of the optimization problem

3. MMA

The gradient-based *method of moving asymptotes* (MMA) is introduced in [1] to solve optimization problems of

*Supervised by Professor Claus Führer, claus@maths.lth.se the following form:

$$\begin{aligned} &\underset{x\in\mathbb{R}^n}{\text{minimize}}\ c_0(x) \\ &\text{subject to } c_i(x) \leq 0,\ i = 1:m \\ &\qquad\qquad c_j^{\min} \leq x_j \leq c_j^{\max},\ j = 1:n \end{aligned} \tag{2}$$

with $c_j^{\max} \in \mathbb{R}$ given and $c_i \in C^2(\mathbb{R})$ for $i = 0 : m$.
Approximating the optimal solution is based on the following scheme: A distinction is made between inner and outer iterations. In any outer iteration step, the objective and constraint functions c_i are approximated by an element of a parameter depending family of convex functions g_i. In the inner iteration step, the parameters of this approximation are changed iteratively to fulfill certain conservativity conditions. The approximation method is visualized in Figure 2.

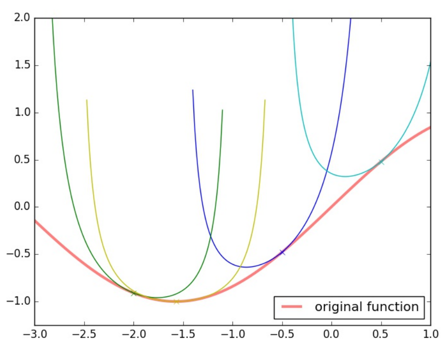

Figure 2: Convex approximations in different points with MMA

4. COBYLA

The derivative-free *constraint optimization by linear approximation* described in [2] is based on the idea of the Nelder and Mead method to solve the following optimization problem:

$$\begin{aligned} &\underset{x\in\mathbb{R}^n}{\text{minimize}}\ \ c_0(x) \\ &\text{subject to } c_i(x) \geq 0,\ i = 1:m \end{aligned} \tag{3}$$

with c_i real-valued functions for $i = 0 : m$.
A simplex in $\mathbb{R}^n$ is constructed and used to approximate the cost and constraint functions by linear functions in the $n + 1$ vertices. Thus, the original constrained problem (3) is approximated by a LP problem which is solved. A new data point in the optimization space is found by using the optimal solution of the LP problem and the original objective and constraint functions. This information is used to improve the approximating LP problem. When the solution cannot be improved anymore, the step size is reduced to refine the search. When the step size becomes sufficiently small, the algorithm finishes.

5. Results for the Pasteurisation Process

Both algorithms are available via the NLopt library in Python and are used to optimize the pasteurisation process. Coupling two neighboured pasteurisation zones enables to visualize the level curves of the cost function in a contour plot.

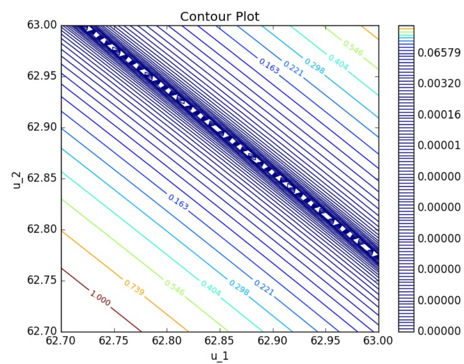

Figure 3: Contour plot of the cost function in (1)

The zero level is a line, in consequence the minimum is not unique. Taking some of the constraints into the cost function allows to modify the cost function. The level plot of the modified cost function shows that a unique minimum exists.

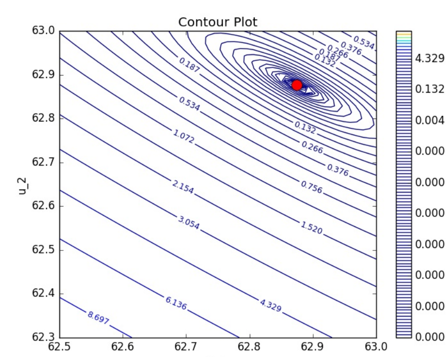

Figure 4: Contour plot of the modified cost function

The MMA algorithm fails for very small feasible regions, which can be seen in Table 1.

	COBYLA		MMA	
Type of the constraint handling	Calls of the cost function	Level of the cost function	Calls of the cost function	Level of the cost function
as equality constraint	48	10^{-9}	5	10^{-1}
by penalty terms	89	10^{-9}	21	10^{-6}
feasible region 10^{-1}	48	10^{-9}	8	10^{-8}
feasible region 10^{-2}	49	10^{-9}	7	10^{-9}
feasible region 10^{-3}	49	10^{-9}	7	10^{-7}
feasible region 10^{-4}	52	10^{-8}	38	10^{-16}
feasible region 10^{-5}	50	10^{-8}	40	10^{-6}
feasible region 10^{-6}	52	10^{-8}	64	10^{-4}
feasible region 10^{-7}	52	10^{-8}	78	10^{-4}

Table 1: Comparison of two different optimization algorithms. Note the influence of the width of the feasible region for the performance of MMA

In [3] a method to modify the MMA algorithm is presented in order to make it applicable to equality constraints.

6. Conclusion and Future Work

The MMA algorithm is a serious alternative to derivative free routines, even if the gradient has to be approximated numerically. The future work detected in this thesis includes aspects from algorithm design and development as well as the study of different model variants and optimization setups:

- Extending MMA
- Optimization with dynamic scenarios
- Optimization of the overall processing time
- Optimization of the energy consumption
- Optimization of a more detailed model

References

[1] K. Svanberg, "A class of globally convergent optimization methods based on conservative convex separable approximations," *SIAM Journal on Optimization*, vol. 12 (2), pp. 555–573, 2002.

[2] M. J. D. Powell, "A Direct Search Optimization Method that Models the Objective and Constraint Functions by Linear Interpolation," in *Advances in Optimization and Numerical Analysis* (S. Gomez and J.-P. Hennart, eds.), ch. 4, pp. 51–67, Dordrecht: Kluwer Academic Publishers, 1994.

[3] K.-U. Bletzinger, "Explicit approximation of equality constraints," *Transactions on the Built Environment*, vol. 2, 1993.