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A SIMPLE MODEL FOR WAVE GENERATION UNDER VARYING WIND CONDITIONS

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ABSTRACT

The Hanson Larson evolution model assumes that wave height grows or decays from an initial condition towards an equilibrium wave height through a simple exponential growth. The model uses wind input data, fetch length and depth measurements. Using this input parameters, it both determines the wave height and period values, thus predicts wave climate. The model is developed using deep water conditions where group velocity is assumed be constant.

The model is validated using measured wave data from the Baltic Sea. The wave and input wind data are retrieved from the Swedish meteorological and hydraulic institute's (SMHI) data base. The wind stations used in the analysis have a correlation coefficient of 0.72. The wind inputs were also checked for their reliability to represent the wind condition at the wave buoy. This was done by comparing the wind speed and wind direction measured at the wind stations. Whenever the difference between speeds was less than 5 m/s and wind direction difference less than 45 degrees, the wind input was taken as reliable.

The wave heights calculated were in a fair agreement with observed values for most of the trial runs, except when wind direction was abruptly changing. The error observed was more for wave periods as compared to wave height, but the growth and decay pattern calculated by the model was similar as the observed pattern.

Other two more models based on wave energy analysis are also tested as to predict wave height and wave period, from wind input data. In comparison to the Hanson Larson evolution model, these models resulted in more disagreement between calculated and observed values, rendering them less useful.

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1. INTRODUCTION

1.1 General Background

To determine wave characteristics (e.g., height, period, direction) along a coast is an important part of solving coastal engineering problems. Such parameters are usually predicted using analytical or numerical models. Such models use empirical as well as some basic physical mathematical relationship among several other parameters (wind direction, speed, depth) to make forecasting. Most of the numerical models use spectral representations to describe wave climate. However, analytic models employ empirical formulae that use wind input data to calculate wave height and wave period.
One commonly used technique is a method developed by the US army Corps of engineers (SPM, 1984). In the method, a wind speed data will be mainly used in determination of the wave height and period once it is identified whether a fetch limited or duration limited conditions prevails. However the wind direction data will not be considered. In addition, the method does not handle wind speed variability very well.

Wave evolution models, that assumes the wave height grows or decays exponentially to an equilibrium condition from an initial wave height, are becoming important and useful. These models consider wave directions and also take temporal variation of wind speeds into considerations. More over models of such type are easy to use as their input is limited and easily accessible. The input data includes wind speed, wind directions, fetch lengths and depths. They also come handy to consultants in cases of time constrains as the models can be setup and run in short time. The source code can be changed easily by the user.

1.2 Problem Statement

Estimating wave properties from a time series of wind data (e.g., speed, direction) is an integral part of describing the design conditions for structures and activities in the coastal areas. Determining the wave height induced by a wind with uniform speed is relatively easy; however, complications arise when the wind is changing direction and speed.

Temporal changes in the wind conditions result in wave decay and growth that have direct implications for wave height calculation. If the wind condition changes, the wave height that develops is influenced by the existing conditions before the change occurred. There have been some simple mathematical models that attempt to take this into account without resorting to solving the full energy-balance equation for the wave evolution (WAM, 1994). For example, Dahlerus and Egermayer (2005); [see also Hanson and Larson, 2008] proposed one such expression assuming that the wave height evolution from existing wave conditions follows a simple exponential response function. The equations assume that a wave height tends to grow or decay to an equilibrium wave height depending on the wind speed conditions.

The mathematical relationship that define such evolution of wave heights have been used in different research papers as a basis formula for wave height calculations, although the formula has not been properly validated with measured wind and wave data. Also, other formulations using the same principle, but employing other quantities than the wave height (e.g., wave energy flux) may be employed to yield simple predictive formulas for time-varying wind conditions.

The main aim of the thesis is to validate the wave evolution models with wind and wave data from the Baltic Sea. Some alternative formulations to this equation are also investigated as well and compared to the data in order to find the most suitable expression.

1.3 Research Questions

The main question this thesis answered is the applicability of the evolution model and wind conditions where it can be used. Through the process of answering the main question, solutions to other specific questions were also given. Some of these research questions raised in the thesis are:

1. What affects the wave height evolution over a sea, especially for time-varying wind conditions in deep waters?
2. What kind of methods are employed in wave height forecasting procedures, especially simplified ones that do not include extensive numerical modelling?
3. How does wind speed and its variability affect wave evolution in deep waters?
4. What is the effect of wind direction and its variability on wave evolution?
5. Is the existing model and associated mathematical relationships suggested valid for wave height forecasting over the Baltic Sea?

6. Is it possible to develop alternative formulations to the existing model that yields improved predictions for time-varying wind conditions?

1.4 Objectives

The main objectives of the thesis are to validate the evolution model, to check the time response coefficient and also to see possibilities of developing other simpler models that predict the wave climate. Some of the general and specific objectives are listed below.

1. To analyse if the existing model is applicable to describe wave height growth and decay properly for time-varying wind conditions (speed and direction).
2. To formulate alternative wave evolution models to the existing one.
3. To validate the existing model, as well as any new models, with wind and wave data measurements collected from stations over the Baltic Sea. If necessary, coefficient values in the models are calibrated with the available data.
4. To propose a robust, reliable, and easy-to-use model for predicting the wave evolution under time-varying wind conditions.

1.5 Methodology

First of all, wind and wave data measurements from stations operated by the Swedish meteorological and Hydrological institute (SMHI) are collected and properly compiled. The data collected from different wind stations are correlated to each other to check how much the measurements agree in order to find representative wind input. Thus, for a selected wave station a number of wind stations, close to the wave buoy were investigated. The wave data employed were of statistical quantities, for example, significant wave height, peak spectral wave period, and peak spectral wave direction.

Then calculation of wave heights are carried out based on the suggested wave evolution equations. For this a FORTRAN code was written to handle all the data. The use of FORTRAN is motivated by the fact that calculation times in FORTRAN would be considerably shorter compared to other programs such as MATLAB.

Validation of the results then followed. The results obtained using the existing model and new models developed are validated with the measured wave heights from SMHI.

In case the model predictions are not in a satisfactory agreement with the measurements, improvement of the models needs to be made, including finding more suitable coefficient values.

1.6 Structure of the Thesis

The flow of the thesis with the main topics included in each section are given in figure 1.

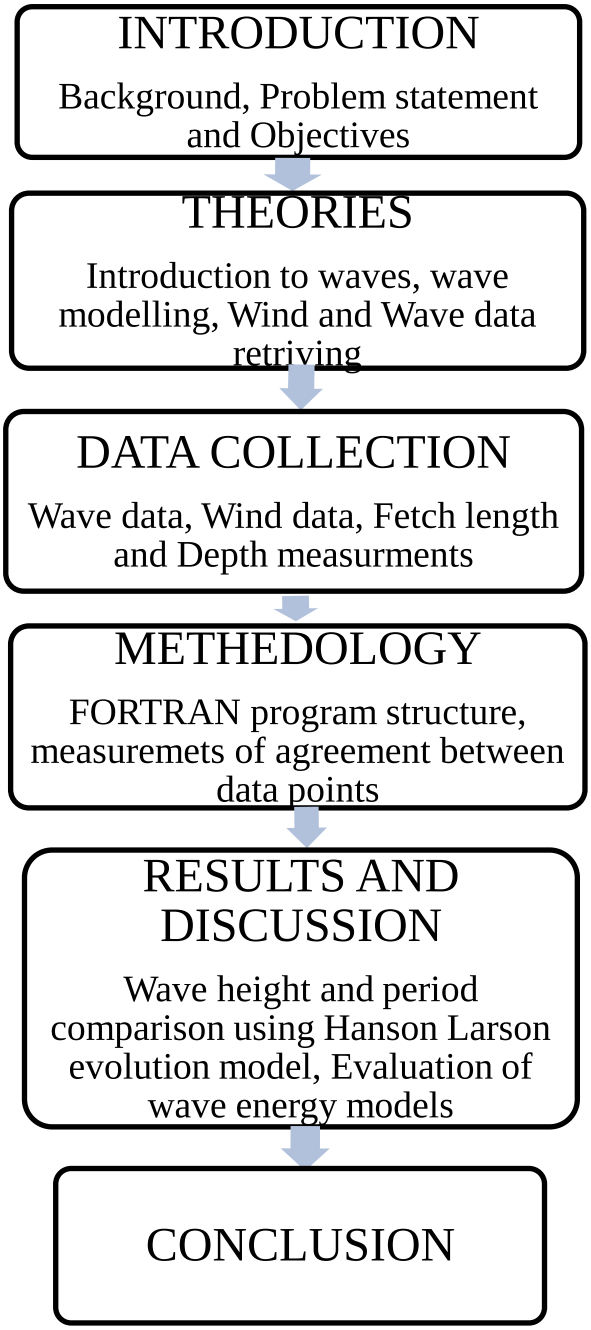


Figure 1: Thesis structure

2. INTRODUCTION TO WAVE THEORY

2.1 General

The creation of waves over a water surface can be attributed to different factors. Some are caused by natural forces from the subsurface, like earthquakes while others are caused by wind. No matter what the causes are, waves are usually described by wave height (H), length (L), period (T) and direction (θ). Wave height is the maximum vertical distance between a trough and a crest. While a wave period denotes the time it takes for two wave crests to pass a single point. And wave length is the maximum distance between two consecutive crests. (SPM, 1984). This parameters are visually depicted in figure 2.

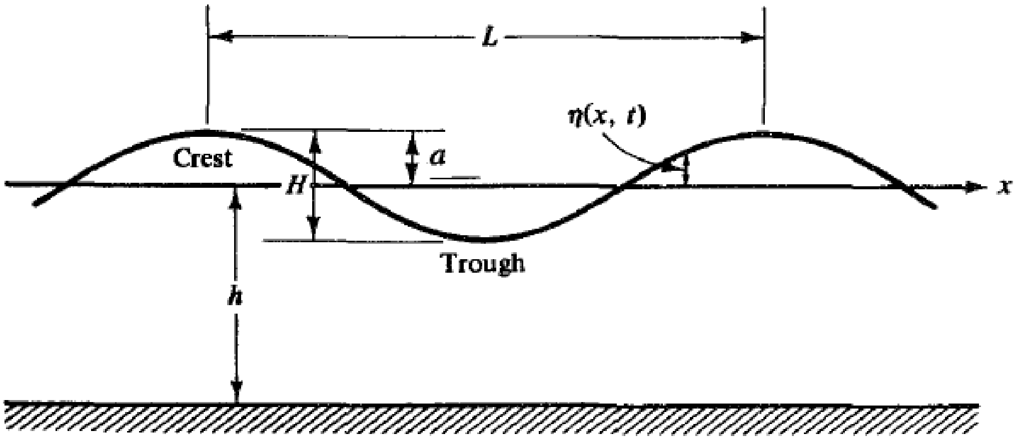


Figure 2: Wave characteristics (h is the wave height, a is the wave amplitude, L is the wave length, h is the depth η (x, t) is the measure of wave surface departure from the mean water level) [source: Dean and Dalrymple, 1984]

A wave over a sea or an ocean is a very complex phenomenon hard to describe using simple mathematical expressions. The difficulty of describing waves arises from the fact that waves lack linearity and their three dimensional nature.

However, there have been a lot of attempts to describe waves since the study of waves began. The commonly and widely accepted way of describing waves is by considering them as a summation of several sinusoidal functions of different phase. Each function applies a correction to the previous one. In this regard, the Airy wave theory is the first approximation that uses one sinusoidal function, thus it is also known as first order wave theory (SPM, 1984).

2.2 Airy Wave Theory

The Airy wave theory is the simplest of all the wave theories and could well describe the major characteristics of any given wave. It is based on several assumptions, the important ones of which, are listed below.

1. the fluid, water in this case, is homogenous and incompressible
2. the surface tension can be neglected
3. Coriolis effect, which results from the earth's rotation is neglected
4. pressure at the free surface is uniform and constant
5. the fluid is ideal or inviscid.
6. the bed is assumed horizontal, fixed an impermeable
7. waves are of two dimensions

Even if, the density of sea water over a stretch of several miles cannot be constant, over a short length, it is safe to assume density remains the same. (Holthuijsen, 2007). The application of Airy theory is also over a short length of the sea. The density profile over a vertical section in an ocean is also variable at different depths, especially at the estuaries where a river water (fresh water) joins a sea water. This would urge the application of caution when describing such phenomenon using the Airy wave theory.

Water is a relatively inviscid fluid meaning its viscosity is very low. So the viscous forces that result internally are minimal and could be ignored without it having a significant effect for practical applications (Holthuijsen, 2007).

The assumption that the bed is horizontal, usually tends to put restrictions on calculation and analysis in shallow waters, however such problems could be curbed by using local depth values wherever necessary and applying other wave theories that would better treat waves in shallow water conditions (SPM, 1984).

In deriving the equations, care must be taken, as to make the equations linear, by ignoring some kinematic and dynamic force elements. Nevertheless, the assumptions are valid as long as the wave height does not get large compared to the wavelength.

Based on the assumptions listed above, two major equations can be developed. The first of the equations is based on the concept of conservation of mass. This is the principle of continuity. The second one is based on the principle of conservation of momentum. Equation 1, shown below, describes the principle of continuity while equations 2a, 2b and 2c describe momentum conservation.

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial x} = 0 \qquad (1)$$

where u_x, u_y *and* u_z are the respective velocity components in the X, Y and Z principal directions.

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (2a)$$

$$\frac{\partial u_y}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad (2b)$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \qquad (2c)$$

where u_x, u_y *and* u_z are the respective velocity components in the X, Y and Z principal directions, P is pressure (Pa), g is acceleration due to gravity(m/s^2).

2.3 Higher Order Wave Theories

High order wave theories are formulated in order to refine the results obtained from a simple linear theory of Airy's and make the formulations more representative of the real conditions in the sea. These theories are better equipped to explain phenomenon like mass transport by waves.

The most famous of the higher wave theories, the second order wave theory, developed by Stokes is primarily based on addition of the two sinusoidal functions. This will result in the increase in wave peak estimation and a fattening of wave troughs. The phase velocity, the group velocity and wave length are all the same as in the linear wave theory (Mehaute, 1976).

The application of second order wave theories finds more importance where waves are asymmetrical about the normal sea level and water particle orbits are open unlike Airy's linear wave theory which most applies for symmetrical wave and closed orbits (SPM, 1984).

The third order Stoke's wave theory is based on the summation of three consecutive wave periods of T, T/2 and T/3. However since the results obtained for wave heights tend towards infinity as the depth gets smaller and smaller, the application of third wave linear theories is limited in shallow waters (Mehaute, 1976).

2.4 Wave Height Distribution

When one goes out to the sea, one will see several waves each with different wave heights. However, the need to represent the wave climate with certain definite wave parameters is of paramount importance for a researcher or an engineer designing structures on the sea. With regard to this, two very important terms have been proposed; the significant wave height and the maximum wave height.

Assume there are N number of waves measured in a given period of time. Arranging those from the largest to the smallest in descending order and assigning 1 to the largest and N to the smallest, the significant wave height denoted by H_s, would be defined as the average of the highest N/3 waves. The maximum wave height would be the highest of all the wave heights measured.

Another important statistical parameter which is used in wave height descriptions is the root mean square wave height (H_{rms}) it is defined by the expression given in equation 3.

$$H_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} H_i^2} \qquad (3)$$

where H_{rms} is the root mean square wave height, N is the total number of waves measured, Hi is individual wave heights.

Through the Rayleigh's probability density function, the root mean square wave height is also related to the maximum and significant wave heights through the relations given by equations 4a and 4b (Dean and Dalrymple, 1984).

$$H_{\max} = \sqrt{2}H_{rms} \qquad (4a)$$

$$H_s = 1.416H_{rms} \qquad (4b)$$

The significant wave period definition is dependent on the technique used to measure wave data. If visual observations are used, the significant wave period will be the average period of 10 to 15 of successive prominent waves. On the other hand, if gauge measurements are used, the significant period would be the average period of all waves whose troughs are below and whose crests are above the mean water level (SPM, 1984).

2.5 Wave and Energy Spectrum

The depiction of wave climate of the sea is better represented by wave spectrum (SPM, 1984). A spectral approach assumes each wave as a sum of several sinusoidal terms. This emanates from direct approach first used by Joseph Fourier (1768-1830), to write every continuous function over a given interval as a sum of sines and cosine functions. Waves are better represented by mathematical expression of the type given in equation 5.

$$\eta(t) = \sum_{i=1}^{N} a_i \cos(\omega_i t - \phi_i) \qquad (5)$$

where $\eta(t)$ time dependant wave departure from the mean water level, a_i is the amplitude, ω_i is the frequency and ϕ_i is the phase of the ith wave at time t.

The airy wave theory described above is the first approximation of the type shown in equation 5 with an N value of one. The plot of the square amplitude versus the frequency results in the energy spectrum which is a discrete spectra, since the frequency component is discrete. But the most common plot is the energy density plot which is a continuous plot. The area under this plot is the measure of the total energy contained in the wave field.

The use of Fourier series to accommodate for the variability and randomness of the wave surface can be better illustrated in figure 3. As the number of added functions increases, the sum of these functions will form shapes that better represent the wave form seen out in nature.

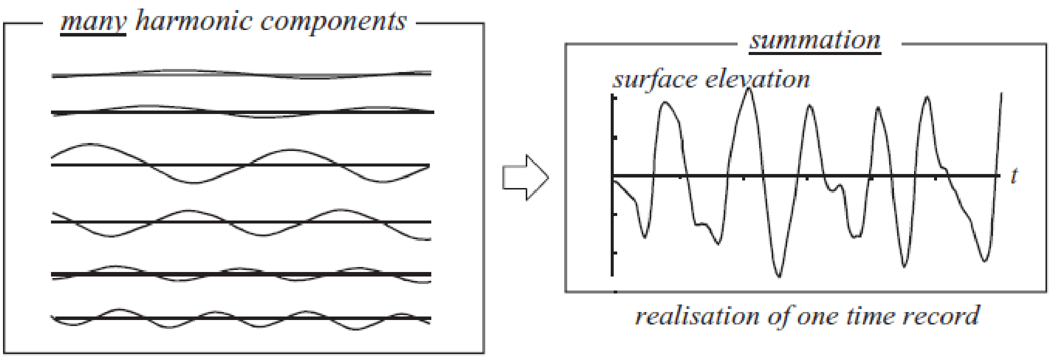


Figure 3: Fourier series sum of harmonic waves to simulate the "real" shape of the wave (Source: Holthuijsen, 2007)

2.6 Wind Waves

The mechanisms by which wind effects the formations of waves over a sea surface is through pushes, frictional drag and pressure differences.When wind blows over the sea, the first types of waves that grow on the surface are of small wave heights. These are called capillary waves. If wind continues to blow for longer times, the capillary waves will be pushed even further to be turned in to gravity waves characterized by a longer wave length and relatively large wave heights. Gravity waves, unlike capillary waves, do not die out immediately when the wind stops bowing. Rather they continue to traverse until they find something to fetch up against (Kinsman, 1984).

The transfer of stress from the wind to the water surface is a major mechanism through which wind waves are generated (WMO, 1998). The turbulence created in the air would create a perturbation over the ocean surface thus leading to the development of a wave. The stress generated by the wind is also a prominent way by which energy is transferred from the wind to the wave. However even if the energy transferred acts at all points throughout the wave profile, its effect are different at different points. For instance, at points which are moving in the direction of the blow, like the crest, the drag will have a speeding effect. Conversely for points which are moving in the opposite direction as the wind, the effect is more of dragging and slowing down (Kinsman, 1984).

Another mechanism by which energy is transferred from the wind to the waves is through pressure difference between the air and surface of the sea. Because of this difference all the energy coming from the wind is lost to the sea in terms of sea surface turbulence.

The growth of a wave and how high it could get depends on several factors as wind speed, duration of blow and fetch length. The wind speed is the measure of the wind force. The more the wind is moving fast, the more push it exerts on the waves and facilitate their growth. The duration of blow is also equally important. A longer duration of wind blow in one direction would obviously allow for more wave build up; however the growth is also dependant on the available fetch.

As described by different researchers, the rate of growth of the wave is exponential, and depends on the existing state of the sea. Figure 4 shown below indicates the different stages a wave goes through when a constant wind field blows for some time.

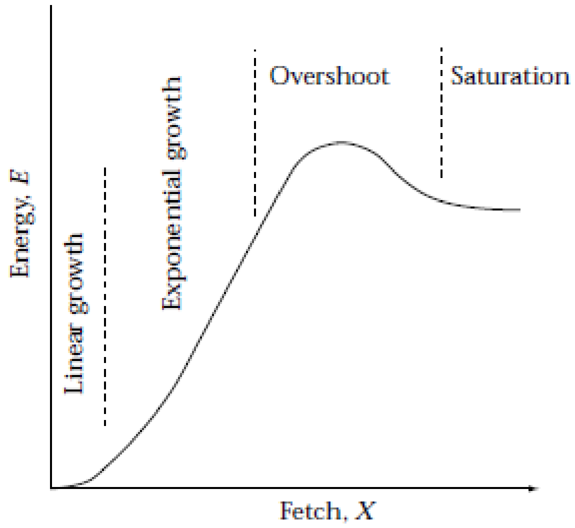


Figure 4: Various growth stages and development of wave energy with increase in fetch length

3. WIND WAVE MODELLING

3.1 General

Modelling of wave generation from wind stress field was a great interest to many researchers in the fields of oceanography and climatology (Holthuijsen, 2007). The outputs are also important for off shore activities like gas and oil exploration (WMO, 1998). Understanding wind and wave interaction is crucial to describe global weather patterns and also long term climatic conditions. Such endeavors can only be undertaken through models that could describe the interactions well.

The modelling of waves over a sea surface most of the time requires a case where there is a spatial homogeneity and steadiness in time. But over a large scale this is practically impossible. So the best way to account for such chaotic condition of the sea is to form a grid where it is assumed that the wave conditions are almost similar. Wave models could be generally classified into analytical models and numerical models.

3.2 Analytic Models

The analytical models described here are the shore protection manual (SPM) method and Hanson Larson evolution model.

3.2.1 SPM method of wave prediction.

The method heavily relies on the fetch and duration limited conditions advanced after the end of World War II by Sverdrup and Munk (Weigel, 1964). Duration limited conditions only occur when the wave heights reached by the wind wave are mainly controlled by the duration of the wind blow. The fetch distance over which the wind is blowing is assumed to be sufficient but not limiting.

On the other hand, fetch limited condition occurs in cases where the wave reaches equilibrium by the time it reaches the end of the fetch distance. Unlike the duration limited case, the major controlling factor is not the duration of blow, actually wind is assumed to blow over the whole fetch length for the entire time (SPM, 1984).

In deep water conditions (which this study also focuses on), the main parameters used in the analysis of fetch limited situation are the fetch length and the effective wind speed (SPM, 1984).

The predicted parameters H_m and T_m are calculated using equations 6a, 6b and 6c shown below.

$$\frac{gH_m}{u_A^2} = 1.6*10^{-3}(\frac{gF}{u_A^2})^{\frac{1}{2}} \qquad (6a)$$

$$\frac{gT_m}{u_A} = 2.857*10^{-1}(\frac{gF}{u_A^2})^{\frac{1}{3}} \qquad (6b)$$

$$\frac{gt}{u_A} = 68.8*(\frac{gF}{u_A^2})^{\frac{2}{3}} \qquad (6c)$$

where H_m wave height (m), T_m is wave period (sec), U_A is wind stress factor (m/s), F is fetch length (m), T is duration of wind (sec), g is acceleration due to gravity (m/s^2)

However, the maximum wave height attained will not indefinitely increase when both the duration and the fetch length increase. It would eventually reach a maximum and cannot grow more. This case is called a fully developed wave condition. For a fully developed sea condition SPM (1984) sets the following governing relations shown in equation 7a, 7b and 7c.

$$\frac{gH_m}{u_A^2} = 2.433*10^{-1} \qquad (7a)$$

$$\frac{gT_m}{u_A} = 8.134 \qquad (7b)$$

$$\frac{gt}{u_A} = 7.15*10^4 \qquad (7c)$$

where H_m= spectrally based significant wave height (m), T_m= the period of the peak of the wave spectrum (sec), t = duration (sec), u_A= the wind stress factor (m/s)

The equations have been plotted on special nomographs, which makes it visually possible to get the peak periods and the significant wave heights (see appendix 4). The procedures for calculating wave heights and spectral wave periods form SPM nomograph involves;

i) correcting wind speed for factors like wind measurement height, location, temperature and duration of blow.

ii) calculating wind stress factor using equation 8 shown below.

$$u_A = 0.71*u^{1.23} \qquad (8)$$

where u_A is wind stress factor (m/s), u is correction adjusted wind speed (m/s)

iii) determining whether fetch limited, duration limited or fully developed wave condition exists.

iv) finally the significant wave height and spectral wave period can be read from nomographs. Note that in cases of shallow water, there are numerous nomographs drawn for various water depth.

3.2.2 Hanson Larson method

The development of the Hanson Larson method was to curb the wave decay and growth problem, which was not represented in the SPM method. (Hanson and Larson, 2008). In the model, wave height growth and decay is governed by a first order differential equation of type shown in equation 9.

$$\frac{dH}{dt} = \lambda(H_{eq} - H) \qquad (9)$$

where H is the wave height (m), H_{eq} is the equilibrium wave height (determined from SPM in meters), t is the time (sec), and λ a coefficient describing the time response of the system (sec^{-1})

Solving equation 9, assuming H=0 at t=0 results in equation 10, which indicates that wave height grows (or decays) from a specific wave height (H_o) at the beginning of the calculation towards an equilibrium wave height (H_e).

$$H = H_e + \left(H_o - H_e\right)\exp\left(-\lambda t\right) \qquad (10)$$

The question arises as to how to estimate the coefficient λ; this parameter determines at which speed equilibrium conditions are attained. One possibility is to make use of the semi-empirical equation from SPM (1984) that specifies the time (t_M) it takes for an entire area with a specific fetch length (F) to contribute to the build-up of waves (t_M corresponds to equilibrium conditions). This equation is given by,

$$t_M = 32.15\left(\frac{F^2}{u_A}\right)^{1/3} \qquad (11)$$

In the SPM method, if $0 \le t \le t_M$, where t is the duration of the wind (sec), an equivalent fetch (F_e) is calculated from Equation 12:

$$F_e = \left(\frac{t}{32.15} \right)^{3/2} u_A^{1/2} \qquad (12)$$

This fetch length is used in the equation for wave height (*H*) (SPM, 1984):

$$H = 5.112 * 10^{-4} u_A F_e^{1/2} \qquad (13)$$

On the other hand, if the entire fetch of the study area contributes to the wave build-up, the maximum wave height ($H_{max} = H_e$) is:

$$H_{\max} = 5.112 * 10^{-4} u_A F^{1/2} \qquad (14)$$

Thus, the ratio between the wave build-up during t and t_M is obtained from Equations 13 and 14:

$$\frac{H}{H_{\max}} = \left(\frac{F_e}{F} \right)^{1/2} \qquad (15)$$

However, since $F_e / F = \left(t / t_M \right)^{3/2}$ from Equations 12, Equations 15 can be expressed as:

$$\frac{H}{H_{\max}} = \left(\frac{t}{t_M} \right)^{3/4} \qquad (16)$$

This equation is valid for $0 \le t \le t_M$; if $t > t_M$, $H = H_{max}$.

Equation 16 may be employed to describe the wave growth from an initially calm sea to equilibrium conditions, when the fetch determines H_{max}. In order to estimate the rate coefficient in Equation 10, it is fitted against Equation 16 which is assumed to represent the "truth" and an optimal value on the rate parameter is determined that minimizes the difference between the equations. However, before this procedure, Equation 10 is rewritten to yield a representation shown in equation 17.

$$H = H_e + \left(H_o - H_e\right)\exp\left(-\mu\frac{t}{t_M}\right) \qquad (17)$$

where $\mu = \lambda t_M$ is taken to be a constant. In the fitting procedure and setting $H_o = 0$ yields:

$$\frac{H}{H_e} = 1 - \exp\left(-\mu\frac{t}{t_M}\right) \qquad (18)$$

Figure 5 shows the result of fitting Equation 18 towards Equation 16; the best-fit value obtained on the rate coefficient was μ=2.17.

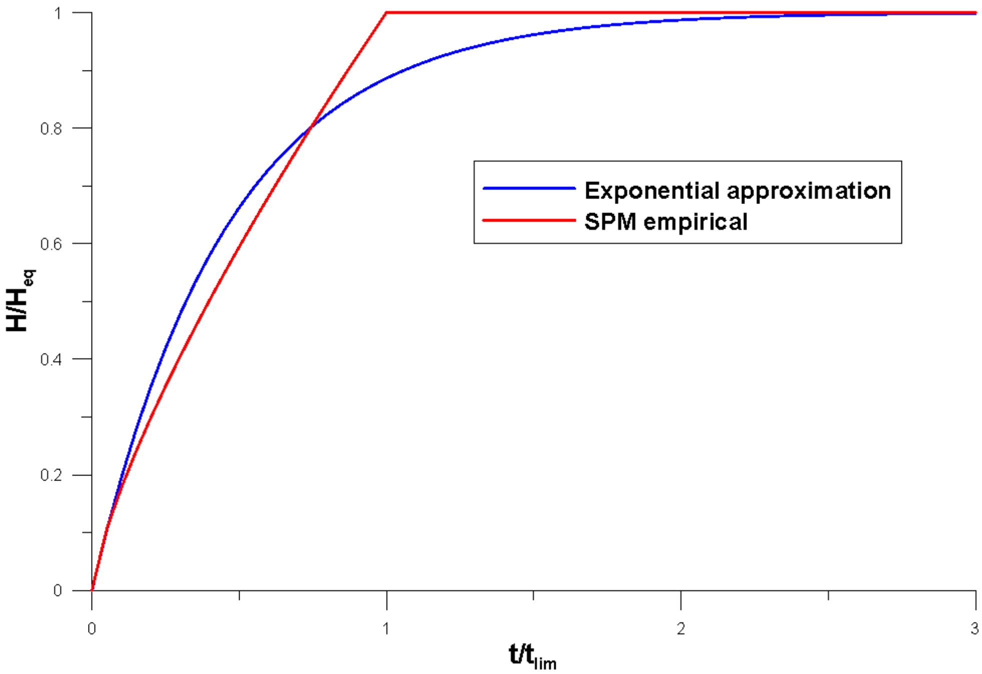


Figure 5: Least-square fit of an exponential function to the SPM formula for wave growth with time.

The same kind of approach can be employed for the wave period (T); for example Equation 18 may be used with T/T_{max} instead of H/H_{max}. The maximum period is given by (SPM, 1984):

$$T_{\max} = 6.238 * 10^{-2} \left(U_A F\right)^{1/3} \qquad (19)$$

An alternative, however, would be to use the same approach that resulted in Equation 16, but for T and T_{max}. Carrying out such a derivation gives:

$$\frac{T}{T_{\max}} = \left(\frac{t}{t_M}\right)^{1/2} \qquad (20)$$

Combining Equations 16 and 20 yields the following relationship:

$$T = T_{\max}\left(\frac{H}{H_{\max}}\right)^{2/3} \qquad (21)$$

In the above equations from SPM (1984) deep-water conditions were assumed. For shallow water and another wave evolution equation than Equation 10, the above analysis probably should be modified.

3.2.3 Growth model based on wave energy flux

Assume that the wave growth (decay) occurs towards an equilibrium wave energy flux (at a particular location and represented by the subscript e). The governing equation may be expressed as,

$$\frac{d}{dt}\left(EC_g\right) = \kappa\left(\left(EC_g\right)_e - EC_g\right) \qquad (22)$$

where E is the wave energy (J), C_g the group speed (m/s), κ a coefficient,
If the water depth is constant, C_g does not vary and may be eliminated from Equation 22. Using linear wave theory for E, Equation 22 reduces to:

$$\frac{d}{dt}\left(H^2\right) = \kappa\left(H_e^2 - H^2\right) \qquad (23)$$

This differential equation may be solved to yield (setting $H = H_o$, for $t = 0$):

$$H(t) = \left(H_e^2 - \left(H_e^2 - H_o^2\right)\exp\left(-\kappa t\right)\right)^{1/2} \qquad (24)$$

where *H (t)* is a wave height as a function of time. In this case it is not straightforward to determine the corresponding wave period; however, the simplest approach is to use a growth and decay formula similar to the one derived for wave height in the Hanson Larson method, and it follows the equation presented in equation 25.

$$T = T_e + (T_o - T_e)\exp(-\kappa t) \qquad (25)$$

where T is the wave period (sec), T_e is the equilibrium wave period (sec), T_o is initial wave period (sec).

Alternative to equation 25, another assumption is the relation between wave height and period as depicted in equation 26, where α is a coefficient usually taken as 5.3.

$$T = \alpha\sqrt{H} \qquad (26)$$

If Equation 26 is generalized to $T = \alpha H^n$, where n is an arbitrary power, the solution will be:

$$H = \left(H_e^{2+n} - \left(H_e^{2+n} - H_o^{2+n} \right) \exp\left(-\kappa t \right) \right)^{1/(2+n)} \quad (27)$$

Assuming deep water, $C_{go} = gT / 4\pi$, and Equation 23 can be developed using Equation 26 to give:

$$\frac{d}{dt}\left(H^{5/2} \right) = \kappa \left(H_e^{5/2} - H^{5/2} \right) \quad (28)$$

where C_{go} is the group velocity in deep water (m/s)

The solution to Equation 28 is:

$$H(t) = \left(H_e^{5/2} - \left(H_e^{5/2} - H_o^{5/2} \right) \exp\left(-\kappa t \right) \right)^{2/5} \quad (29)$$

In this case κ will have a different value than in Equation 24. After *H* has been calculated, *T* is given by Equation 26.

But the coefficient (κ) in equations 24 and 29 could both be represented as μ/t_M. Using similar fitting technique used in the Hanson Larson method, the value of μ can be determined as 1.31 and 1.25 in equations 24 and 29 respectively.

If deep water condition is not assumed, the relationship between C_g and *T* (*H*) is more complex and analytical solution is probably not possible.

3.3 Numerical Models

Numerical models, are becoming very important these days, due to better understanding of the wind-sea interactions and advancements in computing technology. Several researches also have shown greater interests in this field of study since wave climates control global weather patterns and it is applied in a range of various sectors (WAM, 1994).

Most numerical wave models follow more or less similar procedural flow charts of the sample shown in figure 6. They try to solve conditions of the sea after some time elapse of δt after manipulating existing wave state through processes like wave growth, refraction, dissipation, nonlinear interaction and atmospheric forcing (WMO, 1998).

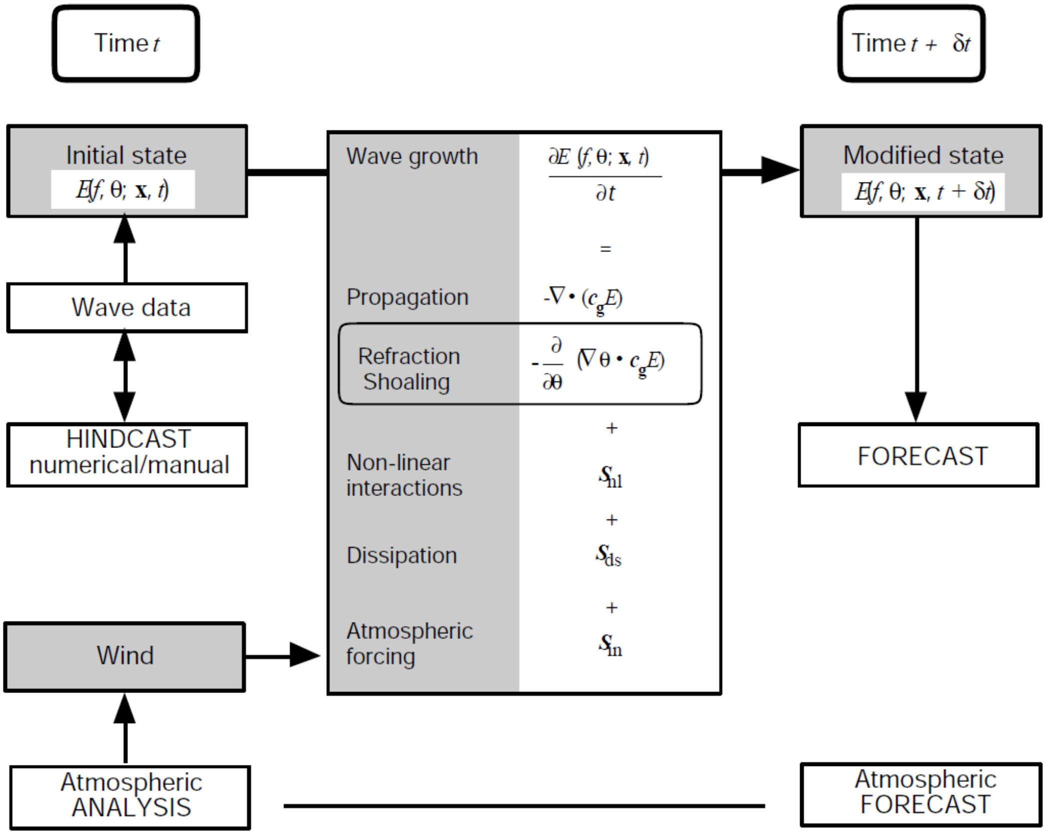


Figure 6: Schematic operational flowchart of most models (Source WMO, 1998)

The common equation solved by most wave models is the one shown below in equation 30.

$$\frac{\partial E}{\partial t} + \nabla.\left(C_g E\right) = S = S_{in} + S_{nl} + S_{dl} \qquad (30)$$

where $E=E\ (f,\theta,X,t)$ is the two dimensional wave spectrum depending on frequency and direction, $C_g = C_g\ (f,\theta)$ is group water velocity, S is total source term, S_{in} is energy input by the wind, S_{nl} is non-linear energy transfer by wave-wave interaction, S_{dl} is dissipation, t is time

The term in the right hand side is the sum of the energy density ($\frac{\partial E}{\partial t}$) and the group velocities in the principal two directions while the left hand side show the storage term composed of the generation, wave-wave interaction and dissipation components.

The classification of wave models in to first, second and third generation is generally attributed to each model's handling of the spectral shape and particularly to the model's handling of the non-linear energy transfer term (S_{nl}) term in equation 30. First generation models do not have this term. The transfer is handled by the two other terms namely the S_{in} and S_{ds}. On the other hand, second generation wave models handle the non-linear energy term implicitly having a parametric reference spectrum, while the third generation models explicitly handle the non-linear energy transfer parameter (WMO, 1998).

The findings of the SWAMP group published in 1984 mainly led to the development of third generation wave models. The report found out that there is a variability and some level of disagreement between model results of the time and a new model that handles the spectral shape better is a necessity. However, advancements in computer technology have also immensely contributed for rapid changes in wave modelling (WAM, 1994).

The most commonly used models these days are third generation types, and the known types that fall in this category are the WAM, SWAM and WaveWatch III which are briefly described below.

3.3.1 WAM

The main difficulties identified by the SWAMP group were problems in developing spectral shape and non-linear transfer of energy. According to the WAMDI group (1988), the problem with spectral shape was solved in two ways. One way was through developing a source function that has the same degree of freedom as the spectrum itself. The second way was to specify the source function to make the energy balance closed. The model is based on a prescribed source functions that are based on series of numerical integration of transport equations (WAMDI group, 1988, Synder et.al., 1981). Both modifications appear in the WAM model.

The real strength of the WAM model is its ability to be used universally. Previous models have been successful at regional basis because they can only take a specified spectral shape, but WAM and other third generation models made it possible for the spectral shape to change and take any form. (Street, 2011). Figure 7, shows plot of measured and calculated significant wave heights using WAM model. As depicted, the model's better representation of existing condition is clearly discernible.

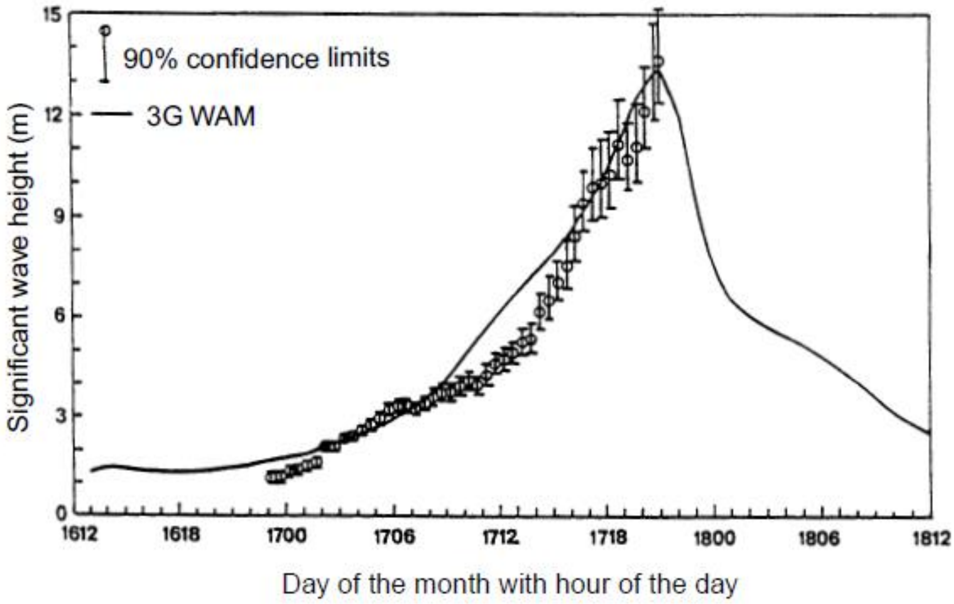


Figure 7: Comparison between measured wave heights and calculated wave heights using third generation wave models during Hurricane Camille in 1969 (source: WMO, 1998)

3.3.2 SWAN

SWAN is a third generation wave model developed by Delft University of Technology. The model can analyse conditions involving a temporary wave growth and decay in coastal and inland water conditions (TU delft, 2016).

The wave model usually requires wind input data, sea current information and sea bottom conditions. The model has flexibility in accommodating non-structural grids. Such types of grids are not the usual rectangular or quadrilateral types (structural grids) rather they take different forms, of which triangular pattern is the most common. Sometimes the triangular grids are also combined with the commonly used rectangular grids to form the so called hybrid grids. Such approaches prove to be helpful especially around coastal areas and islands, whose geometry cannot simply be handled by structural grid types (SWAN, 2015).

However, SWAN is a bit inferior as compared to the WAM and WaveWatch III with regards to efficiency. The latter two models are more efficient in describing the ocean climate well while SWAN is usually employed in cases of a transition analysis, where studies made over the large ocean are transitioned in the study of coastal conditions (SWAN, 2015).

3.3.3 WAVE WATCH III

The Wave Watch III is one of the third generation wave models developed by the Marine Modelling and Analysis Branch (MMAB) of the Environmental Modelling Centre (EMC) of the National Centres for Environmental Prediction (NCEP) and it is an improvement over the wave watch originally developed by the TU delft and WaveWatch II developed at NASA (Wave Watch III, 2015).

In this model the basic input spectrum is the wave number and direction spectrum, while the output consists of the frequency wave direction spectrum. The frequency wave direction spectrum is the more commonly known spectrum output from most third generation wave models. The source term used in WaveWatch III includes more terms than shown in equation 30, like the linear input term (s_{ln}), which takes in to account the linear growth at the initial phase of modelling. Moreover, in shallow water conditions, terms that account for bottom interaction (s_{bot}), depth induced breaking (s_{db}) and wave scattering (S_{sc}) are also considered. The other useful feature is that it provides a room for user defined terms represented by S_{xx}.

The model includes sub grid representation in cases of islands and coastal areas, which cannot easily be described by rectangular grids. The grids are generated by a special MATLAB program integrated to the system. (WaveWatch III, 2015).

4. DATA EMPLOYED

4.1 Study Area

The wind and wave data measurements used in this thesis project are from the Baltic Sea. The Baltic Sea is part of the Atlantic Ocean enclosed by Sweden, Finland, Germany, Denmark and the Baltic countries. It has a total catchment area of 1,641,650 km^2. It has an average depth of 55 meters and a maximum depth of 459 meters (Lepparanta and Myrberg, 2009).

There are several reasons for choosing this site. One main reason is that the Swedish Meteorological and Hydrological Institute (SMHI) has reliable wind and wave measurement stations over this sea. These data have been consistently measured for several years with good accuracy (SMHI, 2014a). Furthermore, the Baltic Sea is a closed basin of limited size, which makes it relatively simple to estimate the fetch length. This will make it easier to study wind-induced waves over the sea. Figure 8 depicts the Baltic Sea and some of the neighboring countries.

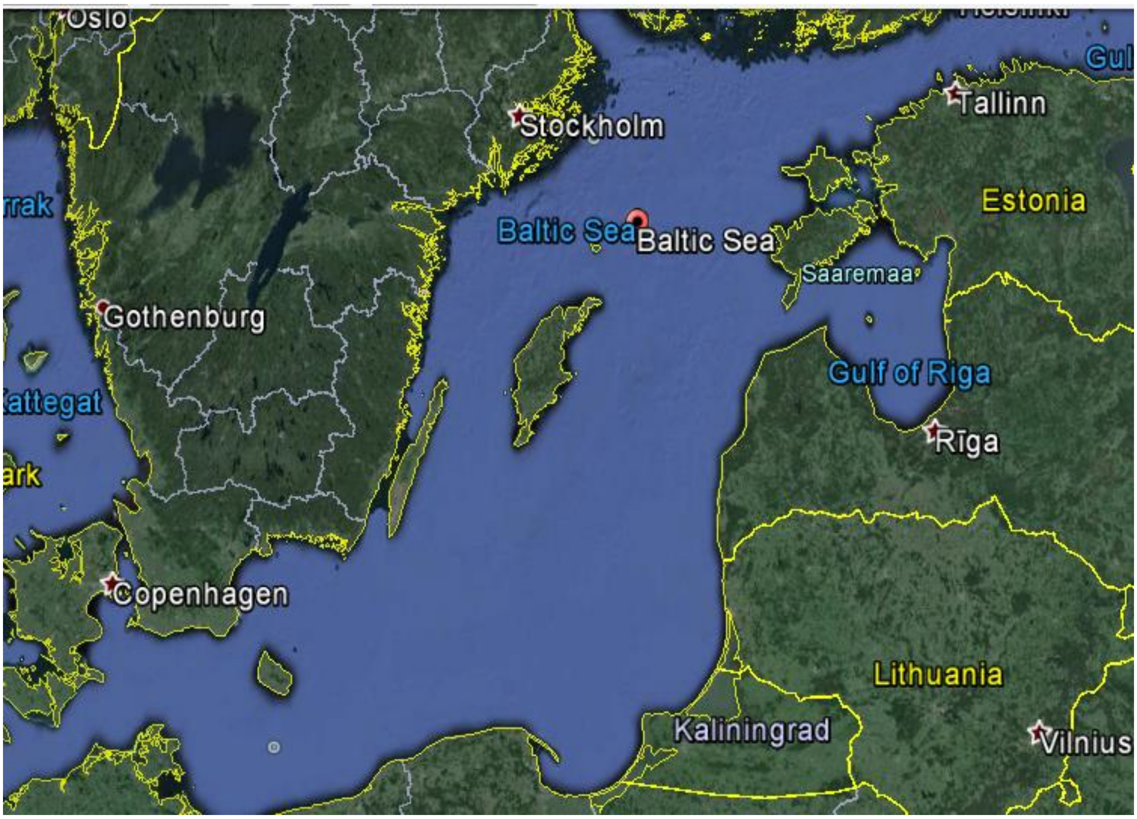


Figure 8: Baltic Sea (source: google earth)

Wind data from SMHI is aggregated from several automatic gauge stations that report wind speed and direction every hour. There are also manual stations that report wind data every three hour. The measurements are reported as mean wind speeds averaged over a 10 minutes interval (SMHI, 2014b).

Wave height and period measurement data have been recorded by SMHI starting in 1978. In the early days, wave height and period were the only parameters measured. The measurements were made by simple buoys placed in the ocean floor close to a light house tower around the coast for the buoys to get electricity (SMHI, 2014a).

However, recent advancements made it possible to manufacture better measuring devices that would also measure wave direction. Lower resolution data are collected from the buoy by satellites each hour while high resolution data are stored in the buoy until they are retrieved manually (SMHI, 2014a).

The measured wave heights are reported as significant wave heights. The measurement time span is usually 20 minutes. (See Appendix 1 and 2 for locations of wind stations and wave buoys administered by SMHI). The main data used in the thesis are wind series data (wind speed and wind directions), wave series data (wave height, period and directions), fetch length and depths in different directions. In this study, no field measurement was done. Rather available data sets were retrieved from its website.

4.2 Wave Data

The wave buoy used to test the model is at a station called Södra östersjön shown in figure 9 below located the Baltic Sea. This buoy was selected for it being far from the coast and islands, allows the study of waves representative of deep water conditions.

Figure 9: Wave and wind stations

The wave data available at the station is an hourly measured data of significant wave height, wave period and wave direction from June 16, 2005 to April 16, 2011. But the available wave data at the on station is not complete. Some wave data measurements have been missed.

As shown in table 1, the data can be generally broken down into 8 time periods where consecutive hourly data have been measured. The maximum wave height measured during the entire operation of the buoy was 7.4 meters high while the maximum wave period recorded was 9.62 seconds.

Table 1: Wave height and wave period as measured at Södra östersjön wave station

Date	Wave height (m)		Wave period (sec)	
	mean	maximum	mean	maximum
16/06/2005 to 18/06/2005	0.35	0.94	2.89	3.72
02/03/2006 to 21/07/2006	0.78	2.71	3.5	5.95
10/02/2007 to 21/11/2007	1.14	5.31	3.9	8.05
18/12/2007 to 21/02/2008	1.94	6.24	4.73	9.62
01/05/2008 to 05/05/2008	0.28	0.59	2.91	3.48
31/08/2008 to 30/09/2009	1.23	6.24	3.94	8.05
09/10/2009 to 17/08/2010	1.12	7.4	3.84	9.04
16/09/2010 to 16/04/2011	1.7	7.16	4.46	8.52

4.3 Wind Data

The measurement of wind speed and direction are very important for the determination of wave characteristics. There are different techniques of measuring wind data over the sea. Some of the most common are ship weather reports, fixed buoy reports, land station reports and satellite data (WMO, 1998). These days satellite data are more reliable and commonly used.

Thus ölands södra udde A and Hoburg stations shown above in figure 9 are used in the analysis. ölands södra udde A wind station has wind measurement data from June 16, 2005 to March 4, 2011 measured hourly while Hoburg wind station has a wind measurement values between January 1, 1939 and August 31, 2012 measured every three hours.

The maximum wind speed recorded at Hoburg wind station between the periods of June 16, 2005 and June 8, 2009 was 24 m/s while for the same period, the maximum at ölands södra udde A was 19.5 m/s. The mean wind speed at the two stations, however seems to be almost similar with values of 5.77 m/s and 6.18 m/s for Hoburg and ölands södra udde A wind stations respectively (See table 2). As regards to the wind directions, in both stations south westerly wind direction dominantely prevail, while easterly winds are the least prevalent (See figure 10). To make the comparisons between the two stations measurements done every three hours were employed.

Table 2: Wind speed statistics at Hoburg and ölands södra udde A wind stations

Wind stations	Mean wind speed(m/s)	Maximum wind speed (m/s)
ölands södra udde A	6.18	19.5
Hoburg	5.77	24

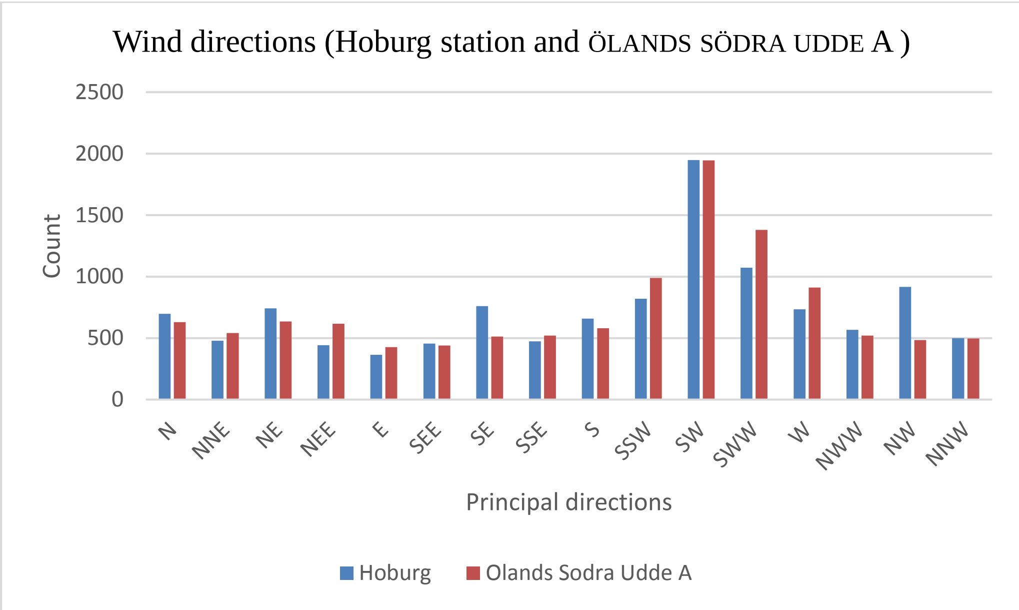


Figure 10: Wind directions at ölands södra udde A and Hoburg wind stations

A correlation coefficient calculated using wind speed recorded by the wind stations in the periods between 16/06/2005 and 08/06/2009 was found to be 0.73. In order to verify if the wind data inputs is representative of the condition at the buoy's location, a comparison was made between the two wind stations. Data points for which wind directions have a difference of less than 45 degrees and a wind speed difference of less than 5 m/s between the stations were considered valid and used in calculations.

4.4 Fetch Length and Depth Measurment

The fetch length, the maximum distance over which a wind blows, should be measured in all directions. Nevertheless, this is a tedious task. So a better way of accomplishing the task is to use representative directions and interpolate between these directions for the rest. In this study 16 principal directions, each separated with 22.5 degrees were used. (See appendix 3) The fetch length and accompanying depth in each of the 16 directions is given in table 3. As can be seen in the table the minimum depth is 58 meters below the normal sea surface which corresponds to a deep water condition for most of the wave lengths.

Table 3: Fetch length an depth measured in 16 different directions

Direction (measured from north in degrees)	**Fetch length(Km)**	**Average depth(m)**
0	160	121
22.5	471	172
45	226	140
67.5	153	100
90	141	100
112.5	153	120
135	146	110
157.5	168	125
180	140	116
202.5	130	100
225	255	75
247.5	356	80
270	283	70
292.5	144	58
315	160	63
337.5	273	55
360	160	121

4.5 Uncertainties in Wind and Wave Series Measurements

Most of the wave height and wave period data available at Södra östersjön are measured on hourly basis, with some skipped measurements. When the missed measurements are not for more than two or three consecutive hours, a simple linear interpolation was used to fill the gaps. But otherwise, only measurements for which consecutive wind direction and speed were recorded are used as inputs.

The wind data measurements from both ölands södra udde A and Hoburg stations are almost complete and entries were missed very rarely. So only simple interpolation would suffice to fill the missing gaps.

5. METHODOLOGY

The calculation of wave heights and wave periods, using the Hanson Larson evolution model and the alternate models described in 3.2.3 were carried out on a Visual Studios FORTRAN program. The layout of the programming follows a simple structure from defining wind inputs to executing the formula, as shown in figure 11.

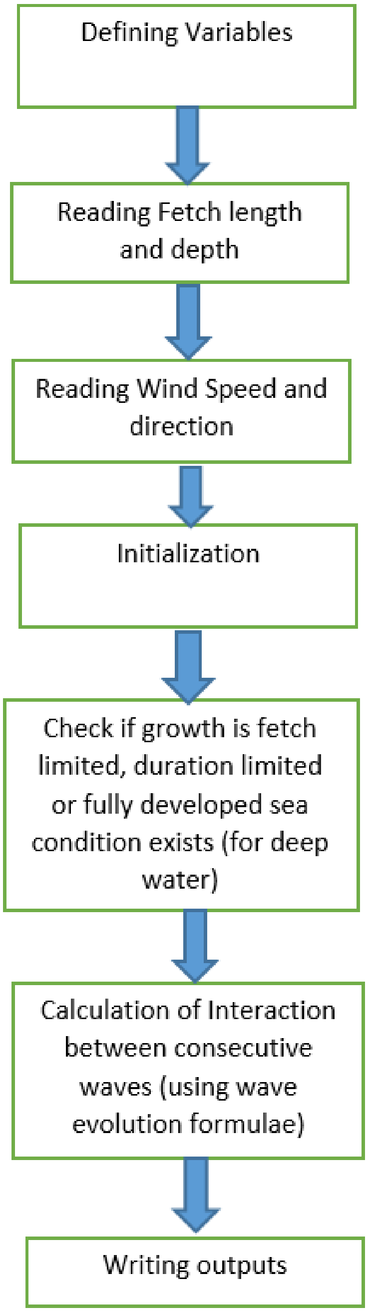


Figure 11: FORTRAN program structure for wave evolution model

5.1 Agreement Between Measured and Calculated Values

One method of studying interdependence between two measurements is correlation. Correlation between two data sets, measures how much one measurement is dependent on the other. However, it does not measure perfect agreement between them (Bland and Altman, 2003). A value of 1, in equation 31 tells that there is a full dependence between two quantities while a value of 0 indicates, no dependence. On the other hand a value of -1 shows when one quantity increases the other decreases.

$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2}\sqrt{n(\sum y^2) - (\sum y)^2}} \qquad (31)$$

where *r*= correlation coefficient, *n* is the number of data points.

Another statistical parameter used in this study is the Root Mean Square Error (RMSE). The root mean square error is usually employed in model evaluations (Chai and Draxler, 2014). The root mean square measures the average of the difference between two measurements and is calculated using equation 32.

$$RMSE = \sqrt{\frac{1}{n} * \sum_{1}^{n}(x - y)^2} \qquad (32)$$

In both equations 31 and 32, the variables represented by X and Y designate the measured (observed) and the model-calculated values respectivey. A Normalized Root Mean Square value (NRMSE) which considers the maximum (X_{max}) and minimum (X_{min}) measured values is calculated using equation 33.

$$NRMSE = \frac{RMSE}{X_{\max} - X_{\min}} \qquad (33)$$

6. RESULTS AND DISCUSSION

6.1. Comparison of Wave Heights

Wave height trial runs were made only for selected events for which wave data were available so that comparisons could be made. As previously mentioned, wind measurement data were assumed to represent the condition at the buoy when the difference in direction of the wind as measured by the two wind stations is 45 degrees or less and also the wind speed difference is less or equal to 5 m/s. Such is the criteria used in the analysis. Therefore, especial emphasis in the discussion is given to these periods of time which conform to the criteria.

Figures 12, 13 and 14 show the comparison between measured and calculated wave heights for some selected events for a relatively longer periods of time.

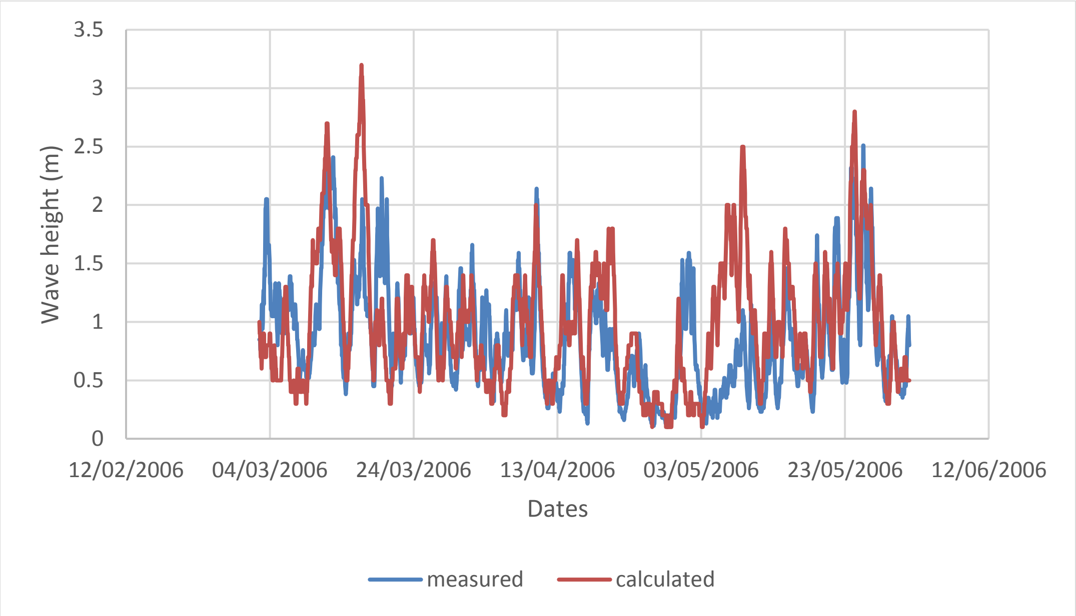


Figure 12: Wave height comparison between measured and calculated wave heights

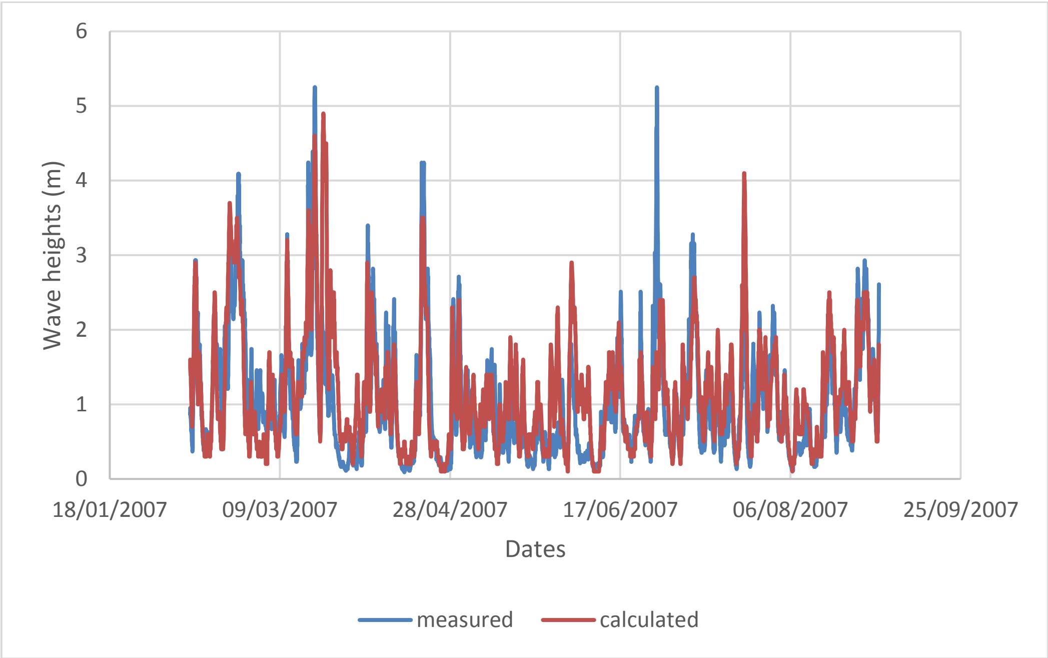


Figure 13: Wave height comparison between measured and calculated wave heights (10/02/2007 to 31/08/2007)

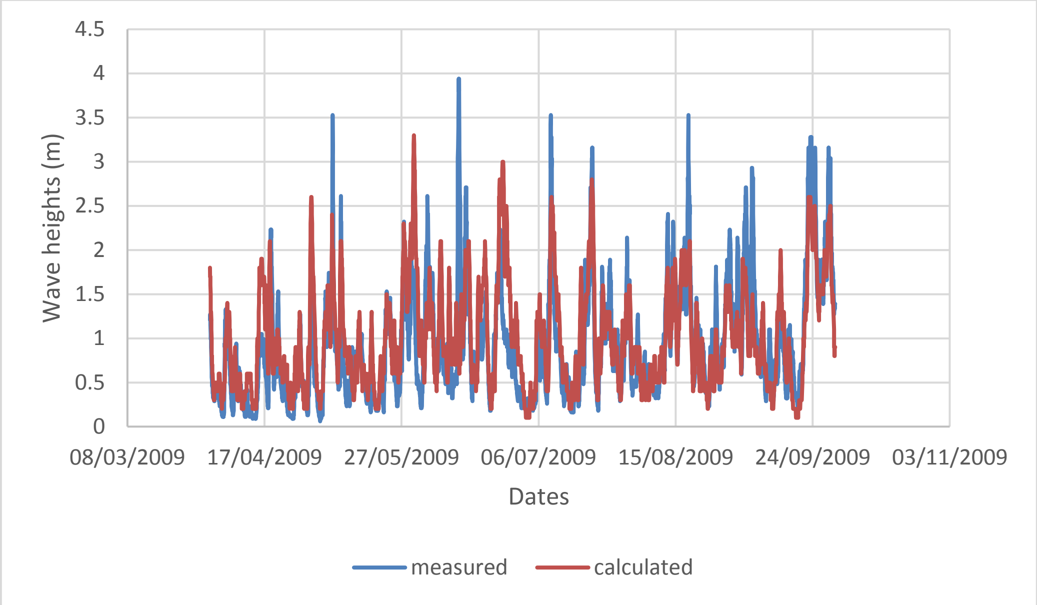


Figure 14: Wave height comparison between measured and calculated wave heights (01/04/2009 to 30/09/2007)

Not all the data points shown in figures 12, 13 and 14 conform to the already set out criteria that there should be a difference in wave direction between stations of less than 45 degree and wind speed difference of 5 m/s. So to have a better clarity and understanding of the relation between the measured and the calculated values, only some data points which conform to this rule have been plotted in figures 15, 17, 19, 21 and 23. More over the wind speeds and directions that correspond to the wave heights are shown below each figure. Note that the wind directions shown are measured in a range from 0 to 360 degrees and have a cyclic nature of sinusoidal functions. That means, for instance, the degrees 0 and 360 represent the same direction and the difference in direction between 358 degrees and 2 degrees is only 4 degrees.

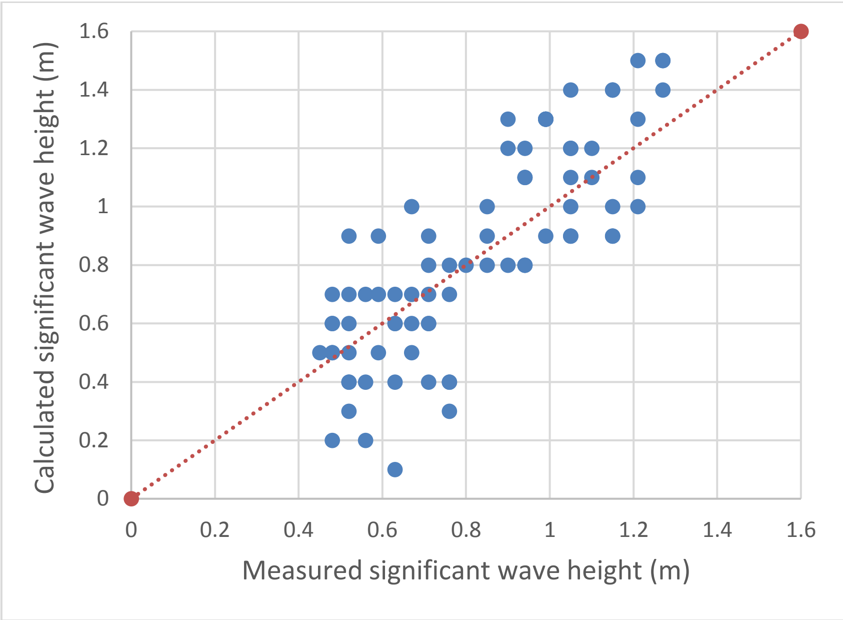


Figure 15: Significant wave height measured and calculated (22/03/2006 to 25/03/2006)

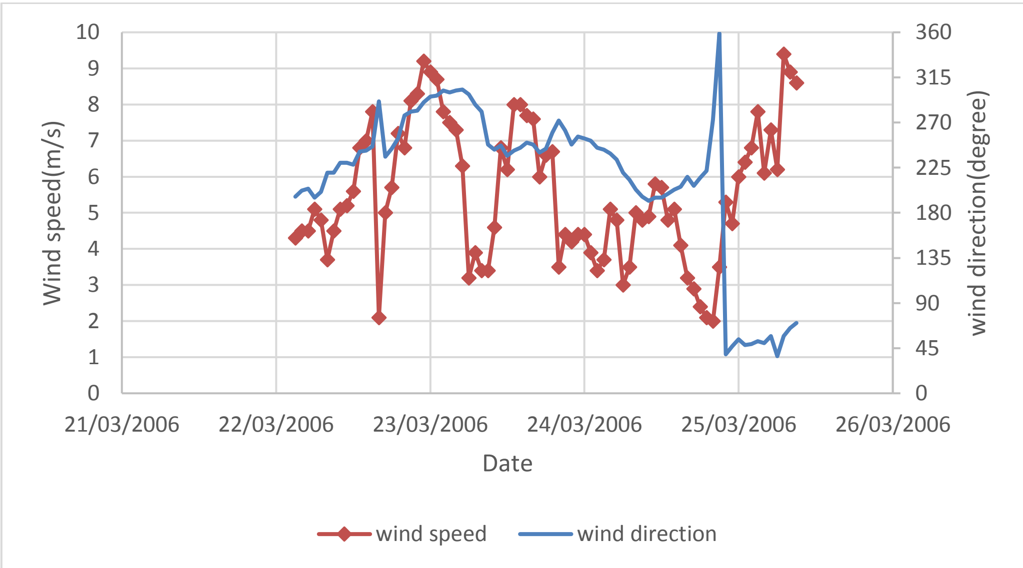


Figure 16: Wind speed and direction (22/03/2006 to 25/03/2006)

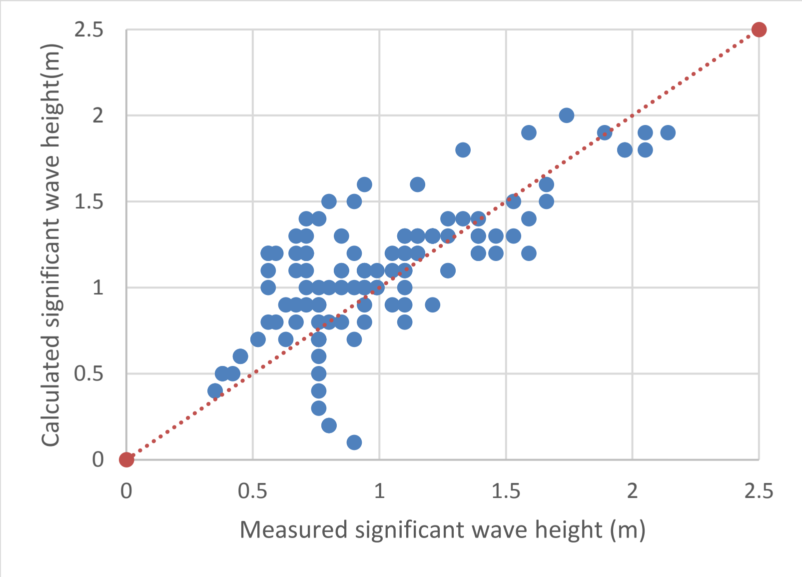


Figure 17: Significant wave height measured and calculated (06/04/2006 to 12/04/2006)

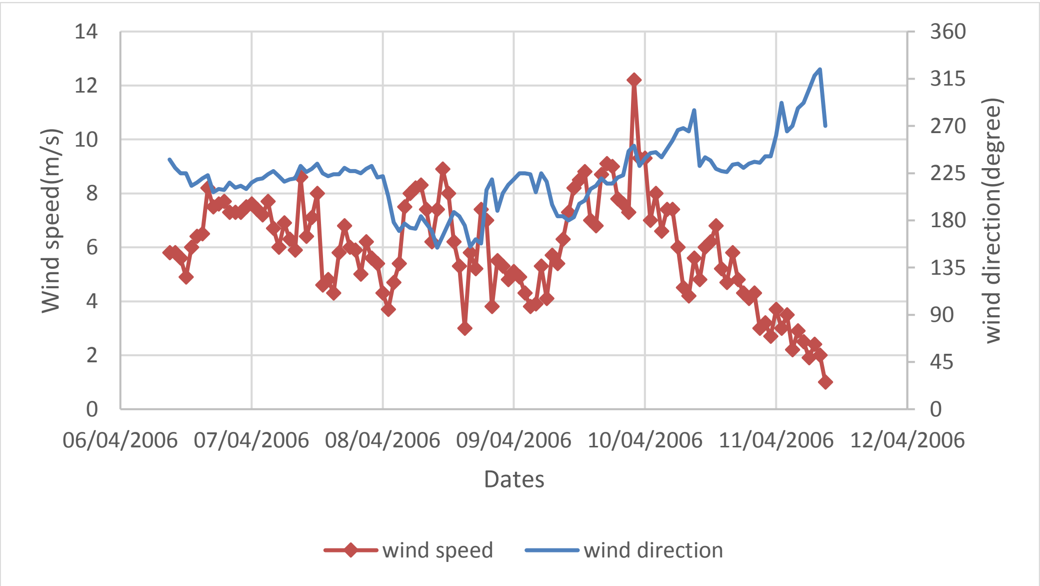


Figure 18: Wind speed and wind direction (06/04/2006 to 12/04/2006)

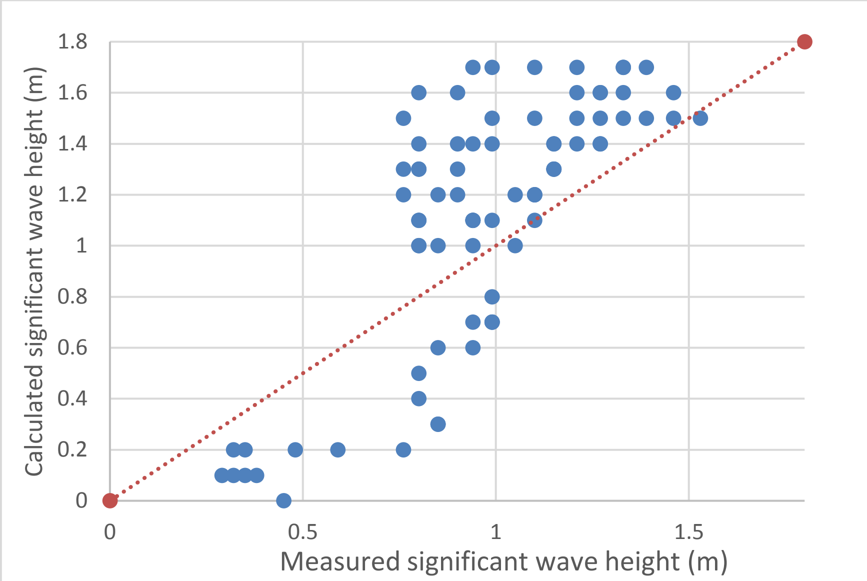


Figure 19: Significant wave height measured and calculated (11/06/2007 to 15/06/2007)

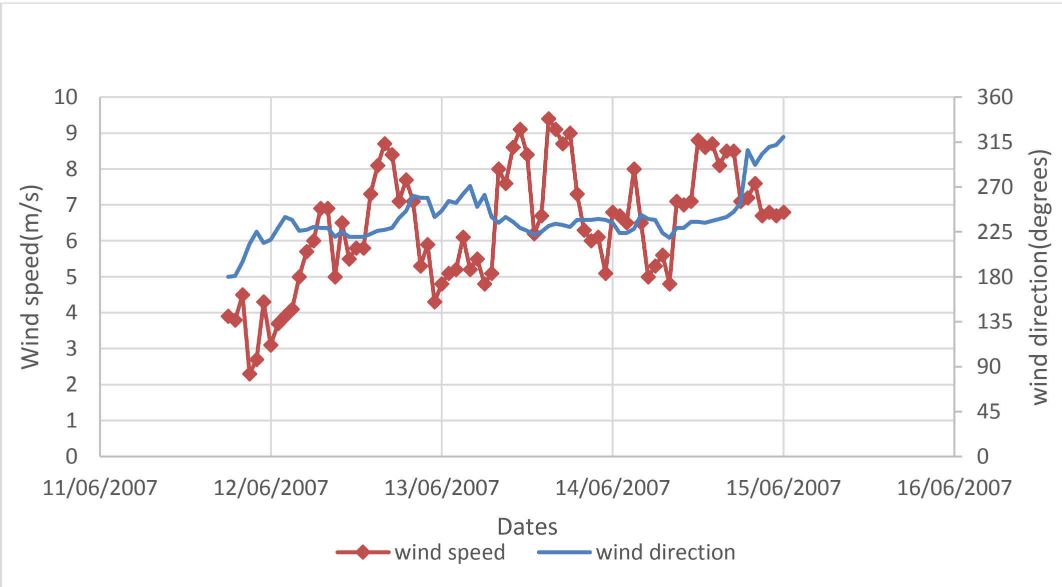


Figure 20: Wind speed and direction (11/06/2007 to 15/06/2007)

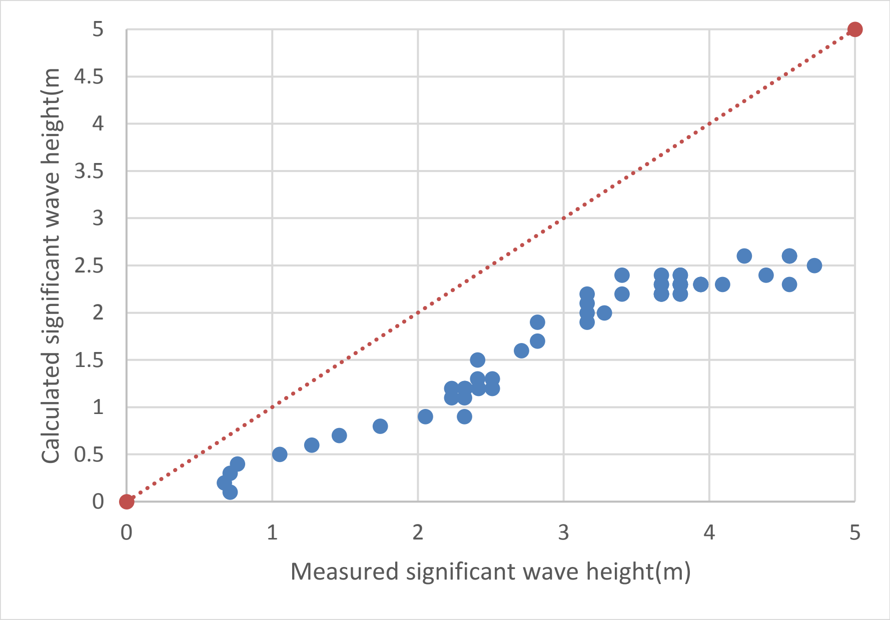


Figure 21: Significant wave height measured and calculated (13/02/2008 to 16/02/2008)

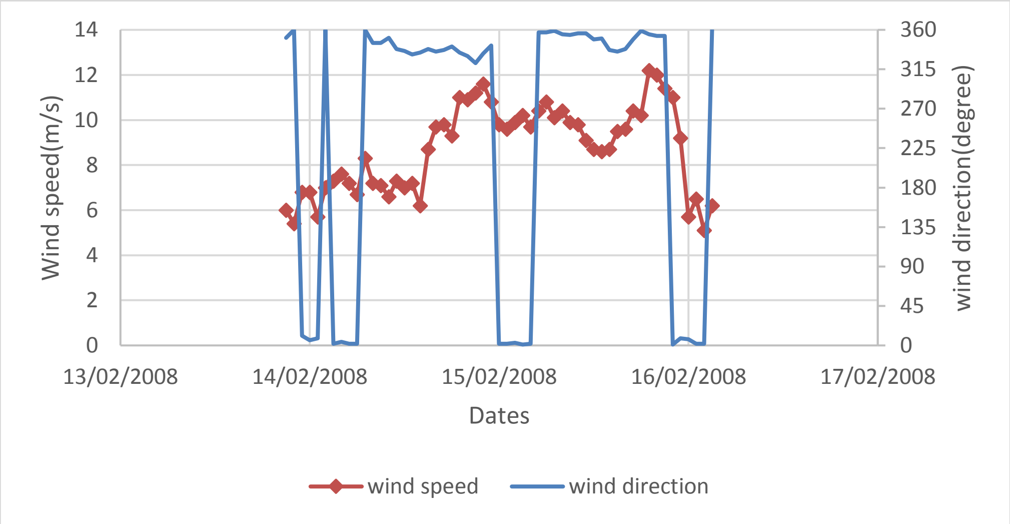


Figure 22: Wind speed and direction (13/02/2008 to 16/02/2008)

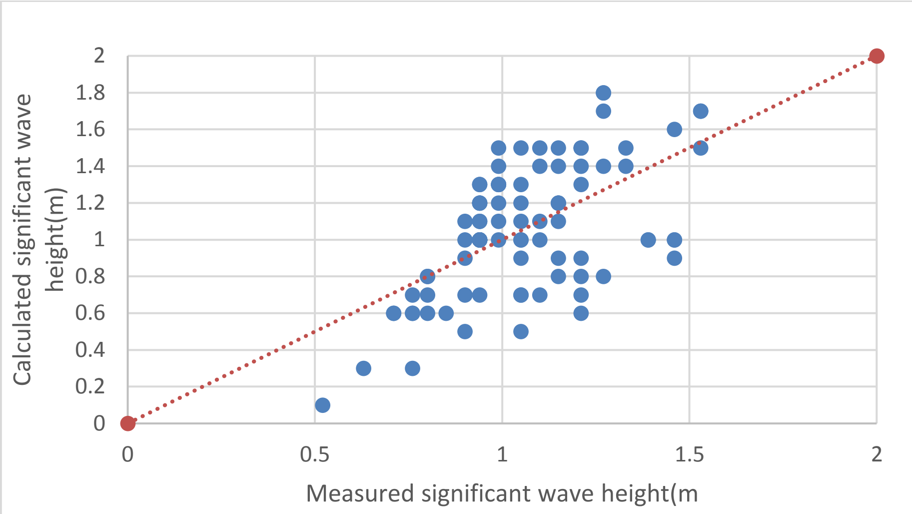


Figure 23: Significant wave height measured and calculated (02/09/2008 to 05/09/2008)

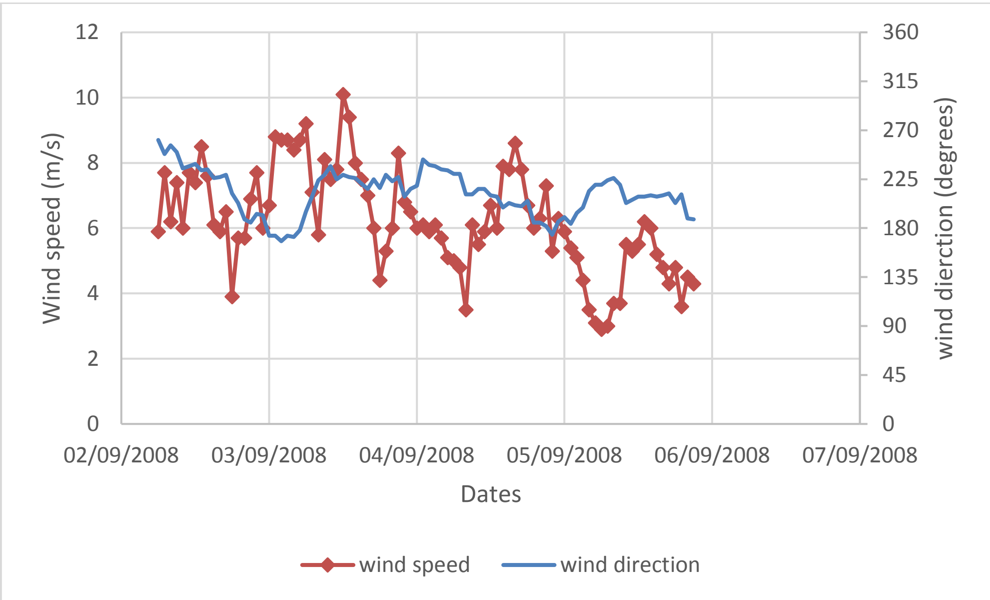


Figure 24: Wind speed and direction (02/09/2008 to 05/09/2008)

Most of the results obtained using Hanson Larson evolution model, more or less follow the measured pattern fairly. However, there are clear cases where the difference between the two values becomes irreconcilable. To have a clear picture of the overall difference between the calculated and the measured components, a normalized root mean square error calculation (NRMSE) has been made. Table 4 shows these values along with the mean, the maximum and the minimum deviation between measured and calculated wave heights values.

As can be clearly seen from figure 21 and also as confirmed by table 4, a maximum NRMSE value of 0.33 was registered for a period between 13/02/2008 and 16/02/2008, where as the lowest NRMSE recorded is 0.16.

Table 4: Comparison between measured and calculated wave heights

Dates	NRMSE	Difference between measured and Calculated		
		mean (m)	max (m)	min (m)
22/03/2006 to 25/03/2006	0.26	0.17	0.53	0
06/04/2006 to 11/04/2006	0.16	0.23	0.8	0
11/06/2007 to 15/06/2007	0.29	0.31	0.8	0
13/02/2008 to 16/02/2008	0.33	1.28	2.25	0.36
02/09/2008 to 05/09/2008	0.28	0.23	0.61	0

One can assume the NRMSE values tend to get bigger when the wind input becomes less representative. To investigate that, some analysis was done. The agreement between wind data measurements (thus the wind inputs validity to represent the wind condition at the buoy) is quantified by calculating the difference in wind speed and direction between wind stations. The more the difference in wind speed and direction between the stations, the less representative the wind input is, and vice versa. Table 5 tabulates the mean of these values. The maximum mean wind speed difference between the stations was 1.72 m/s for the period between 13/02/2008 and 16/02/2008 while the corresponding difference in wind direction was 28.61 degrees for the same period, which was also the highest among the recorded values. And this in turn corresponds to the highest NRMSE value of 0.33 from table 4.

However, the pattern will not follow for the rest of the entries. For instance, the second highest NRMSE value of 0.29 corresponds to the the lowest wind speed and direction difference values of 1.26 m/s and 9.26 degrees respectively. (See table 5). The assumption that NRMSE values tend to be higher when there is more disagreement between measured wind input values does not follow through. So it may be probably safe to conclude that the quality of wind inputs does not have a major impact on the observed disagreements between measured and calculated wave heights.

Table 5: Means of Wind speed difference and Wind direction difference between the two wind stations

Dates	Mean of wind direction difference (degrees)	Mean of wind speed difference (m/s)
22/03/2006 to 25/03/2006	20.81	1.61
06/04/2006 to 11/04/2006	19.07	1.51
11/06/2007 to 15/06/2007	9.26	1.26
13/02/2008 to 16/02/2008	28.61	1.72
02/09/2008 to 05/09/2008	12.4	1.48

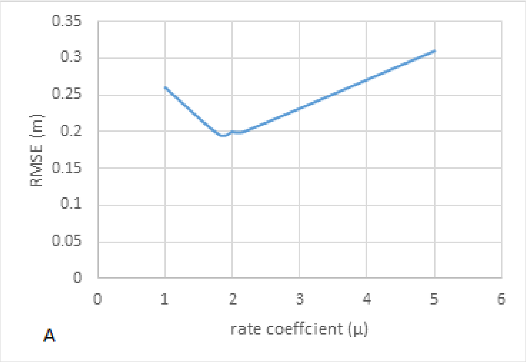
Rather the main reason behind the disagreement between the calculated and measured values is attributed to the prevailing wind speeds. Cases where there are calm wind conditions; that is less wind speed values resulted in more agreement. The NRMSE value for the dates between 13/02/2008 and 16/02/2008 was 0.33, the highest NMRSE value calculated. And table 6 also shows that the wind speed measured during this same period has the highest mean wind speed value of 8.74 m/s. The rest of the NMRSE values also follow similar patterns.

Table 6: The mean, the maximum and minimum wind speeds recorded during trial time periods

Dates	Mean (m/s)	Max (m/s)	Min(m/s)
22/03/2006 to 25/03/2006	5.54	9.4	2
06/04/2006 to 11/04/2006	5.98	12.2	1
11/06/2007 to 15/06/2007	6.39	9.4	2.3
13/02/2008 to 16/02/2008	8.74	12.2	5.1
02/09/2008 to 05/09/2008	6.23	10.1	2.9

Hanson Larson evolution model has rate coefficient (μ) that was calculated based on the best fit between the SPM curve and wave growth and decay function. But this value can be altered and agreement between the calculated and measured values can be checked.

In figure 25, the effect of changing the value of the rate coefficient on the RMSE values is shown. Wind input values from time periods of 22/03/2006 to 25/03/2006 (case A) and 06/04/2006 to 11/04/2006 (Case B) were used in the analysis. In both cases, least RMSE values were obtained when the rate coefficient is in the range between 1.8 and 2.2. Most of the rate coefficient values in this range gave an RMSE value of 0.2 for case A and 0.3 in case B. The rate coefficient of 2.17 determined from the fitting shown in figure 5 above, also falls in the range.



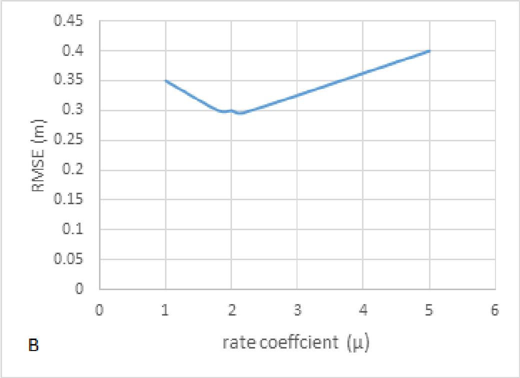


Figure 25: RMSE VS Rate coefficient values (A corresponds to time period 22/03/2006 to 25/03/2006 while B corresponds to 06/04/2006 to 11/04/2006)

6.2. Analysis of Effect of Wind Direction

According to Savile (1954) wind transfers energy to the water surface in the wind direction and all the directions 45 degree or less either side of the wind direction. So assuming that wind direction that does not change with more than 45 degrees between consecutive measurements can be considered to contribute to subsquent growth or decay, some conditions where the wind direction changes abruptly (with more than 45 degrees) are studied as to see the effect of the wind direction change on wave height calculations. The results are shown in table 7. The table shows the difference in wind direction between consecutive hourly measurements, the prevailing wind speed and also the percentage change from the measured wave height.

Table 7: Effect of wind direction on wave height modelling

Date	Time	Wind direction difference (Between consecutive hourly measurements in degrees)	Wind speed (m/s)	Difference between measured and calculated Wave heights (m)	Percentage change from the measured (%)
18/03/2006	7:00:00	92	2.7	0.01	1.69
22/03/2006	17:00:00	55	5	0.01	1.41
24/03/2006	21:00:00	86	3.5	0.12	23.08
08/04/2006	19:00:00	51	7	0.25	29.41
10/04/2006	10:00:00	53	4.8	0.19	11.95
11/04/2006	9:00:00	54	1	0.05	14.29
14/06/2007	19:00:00	53	7.2	0.39	32.23
16/07/2007	2:00:00	168	1.7	0.44	57.89
16/07/2007	3:00:00	175	3.2	0.39	54.93
16/07/2007	5:00:00	81	1.6	0.23	34.33
16/07/2007	10:00:00	75	2.5	0.22	57.89
17/07/2007	8:00:00	111	3.6	0.11	18.64
17/07/2007	9:00:00	106	5.7	0.07	11.11
17/07/2007	10:00:00	103	5.6	0.11	18.64
21/09/2008	4:00:00	55	5.9	0.64	68.09

As seen in table 7, the deviation from measured wave height increases when the wind direction between consecutive measurements changes more. In adition, the deviations exacerbate when there is a continual change in direction of wind as indicated on dates 16/07/2007 and 17/07/2007.

The model wave height calculations are sensitive to the variability of wind directions and speeds. To some extent, the model fails to predict the wave climate in cases where there is high wind speed and significant variability in wind direction,. Thus use of the model during a storm event must be done with precautions.

6.3. Comparison of Wave Periods

Similar formulae of wave evolution used in wave height forecasts have been used to predict the wave period. A time series comparison between the measured and calculated wave periods is depicted in figures 26 to 28.

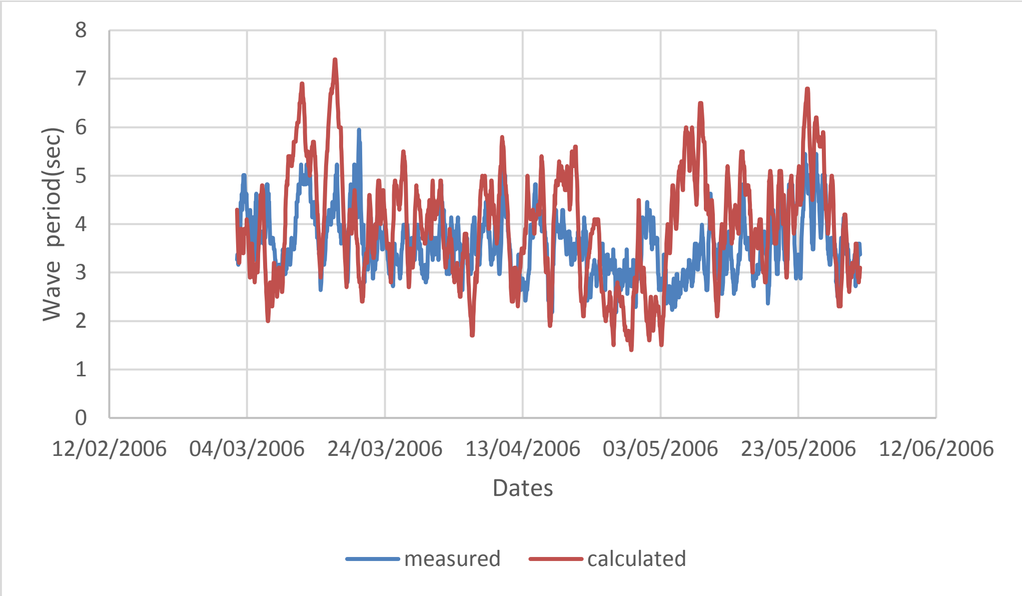


Figure 26: Wave period comparison between measured and calculated wave periods (02/03/2006 to 31/05/2006)

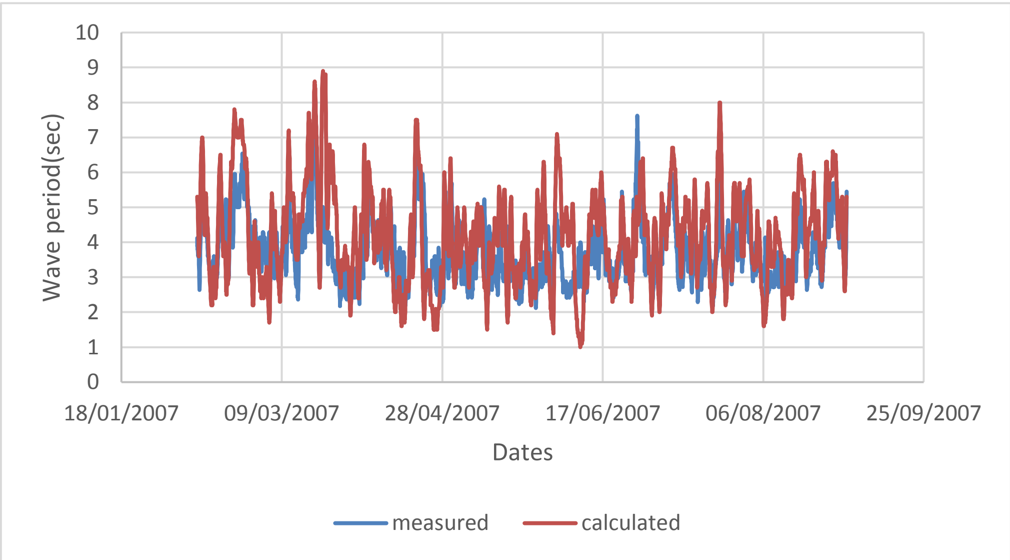


Figure 27: Wave period comparison between measured and calculated wave periods(10/02/2007 to 31/08/2007)

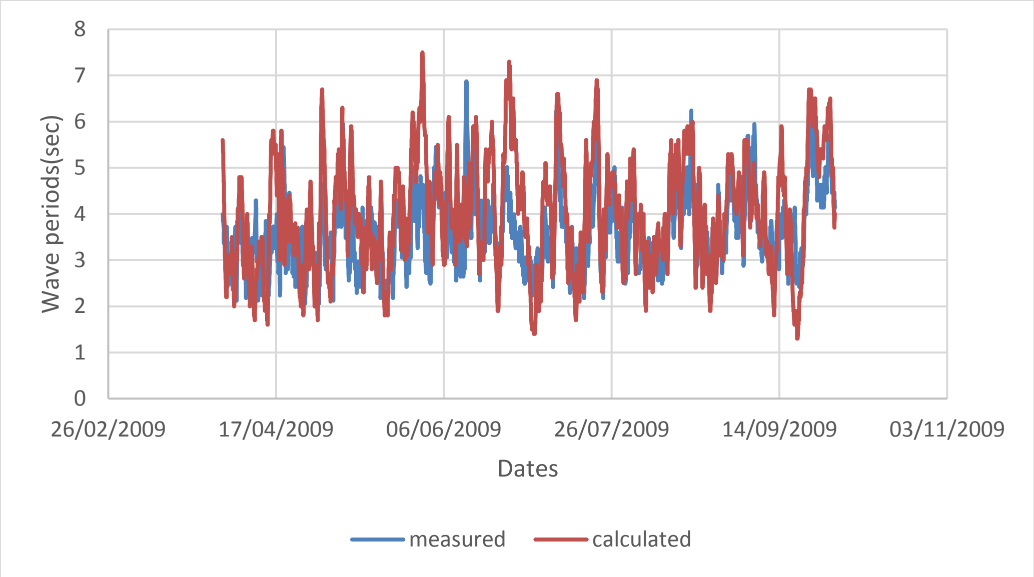


Figure 28: Wave period comparison between measured and calculated wave periods (01/04/2009 to 30/09/2009)

The period comparison shown in the figures 26 to 28 use all the data points within the stated time periods. As in the case of the wave heights, some specific periods of time are selected and presented in figures 29 to 33.

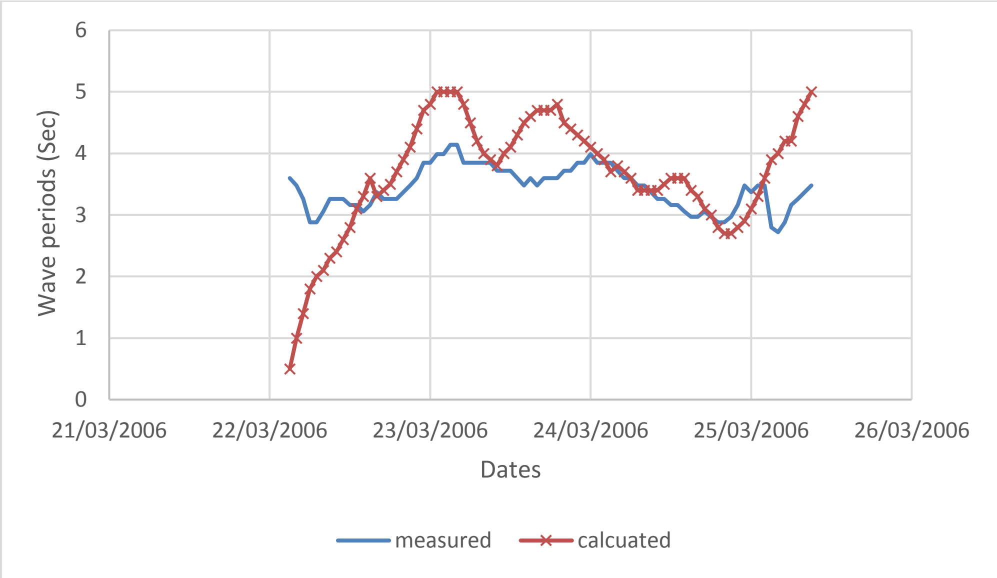


Figure 29: Wave period measured vs calculated (22/03/2006 to 25/03/2006)

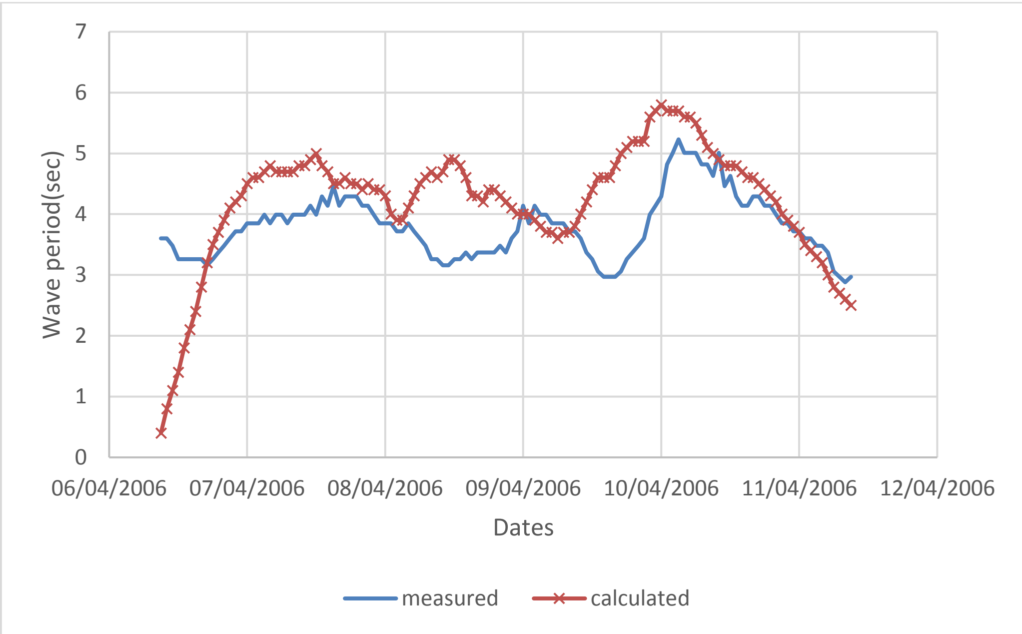


Figure 30: Wave period measured vs calculated (06/04/2006 to 11/04/2006)

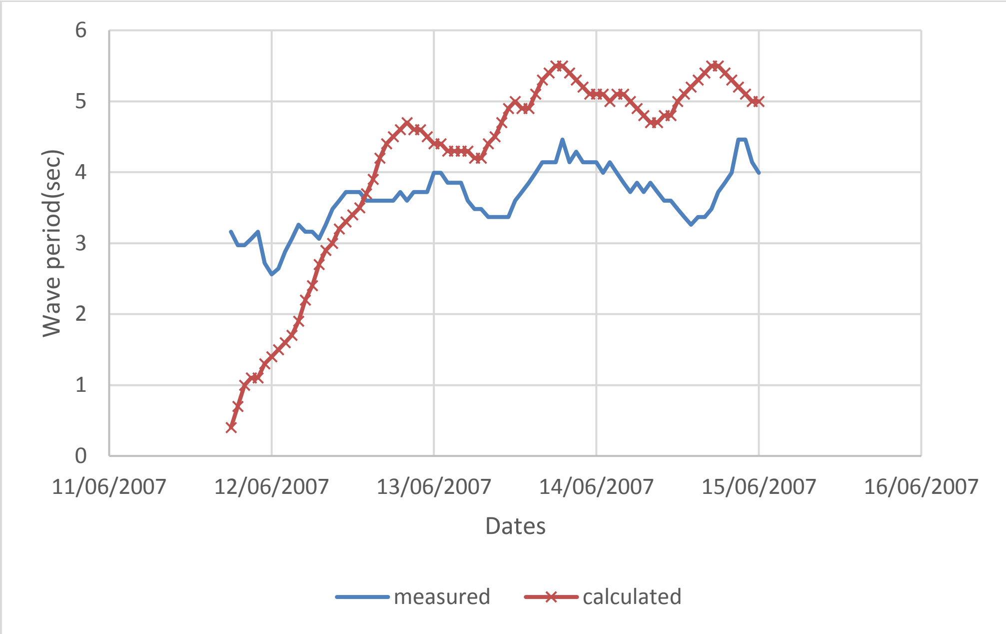


Figure 31: Wave period measure vs calculated (11/06/2007 to 15/06/2007)

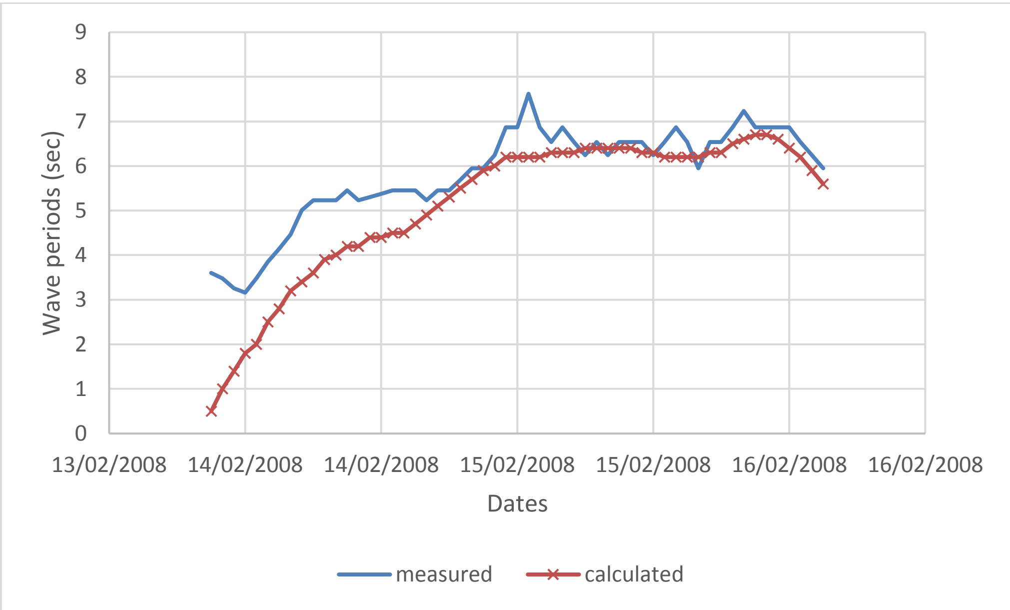


Figure 32: Wave period measured vs calculated (13/02/2008 to 16/02/2008)

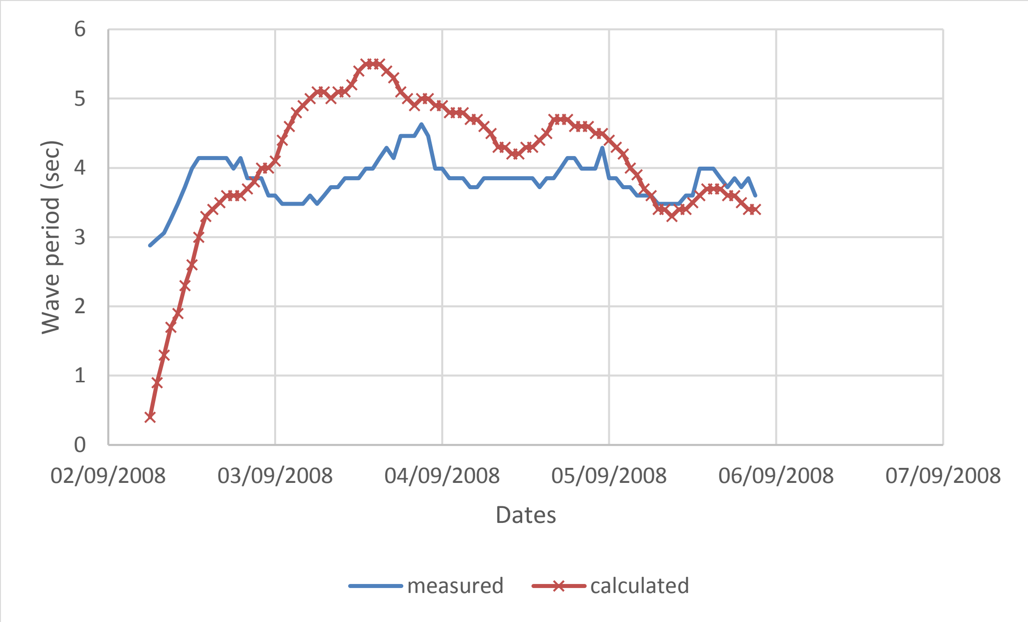


Figure 33: Wave period measured vs calculated (02/09/2008 to 05/09/2008)

The corresponding NRMSE values with the mean, the maximum and the minimum differences between measured and calculated wave periods have also been calculated (see table 8). As seen from figures 29 to 33, the difference between measured and calculated values of wave periods are higer at the beginning of each trial runs. This is due to the model's initalization properties of starting from a value of zero for every run, while the observed wave period at the begining of the trial runs is usually more than zero. Thus, the NRMSE values calculated in table 8 ignored these moments.

Table 8: Comparison between measured and calculated wave periods

Date	NRMSE	Difference between measured and calculated		
		mean (sec)	Maximum (sec)	minimum (sec)
22/03/2006 to 25/03/2006	0.48	0.62	3.1	0
06/04/2006 to 11/04/2006	0.35	0.72	3.2	0.02
11/06/2007 to 15/06/2007	0.57	1.08	2.76	0.1
13/02/2008 to 16/02/2008	0.16	0.71	3.1	0.05
02/09/2008 to 05/09/2008	0.45	0.74	2.48	0

The NRMSE values tabulated in table 8 show that the deviatitions between observed and calculated wave periods have escaleted as they are compared to their wave height counterparts . However, visually it can be seen that the calculated wave period follows similar patterns of growth and decay as the measured wave periods.

6.4 Analysis of Other Models.

The alternative models mentioned in section 3.2.3 were run with the wind data from ölands södra udde A. Results obtained in terms of normalized root mean square errors and also the mean of differences between the measured and the calculated wave heights are tabulated in tables 9 and 10 below. Table 9 represents respective values obtained for the evolution model represented by equation 24 while table 10 corresponds to equation 29.

The wave model represented by equation 24 has a maximum normalized root mean square value of 0.34. As in the Hanson Larson evolution model, this time period also corresponds to the period for which there is maximum mean wind speed. (See table 6 for mean wind speeds during each time period). This model can be seen to follow similar correspondence between wind speed and NRMSE values as in the Hanson Larson wave evolution model.

For the wave evolution model represented by equation 29 though, the highest NRMSE value of 0.36 was obtained for the minimum mean wind speed of 5.54 m/s while the second highest value of 0.34 corresponds to the highest mean wind speed value of 8.74 m/s.

Table 9: Comparison between measured and calculated wave heights (Using Equation 24)

Dates	NRMSE	Differences between measured and calculated		
		Mean (m)	Max (m)	Min (m)
22/03/2006 to 25/03/2006	0.3	0.21	0.53	0.01
06/04/2006 to 11/04/2006	0.2	0.29	0.7	0
11/06/2007 to 15/06/2006	0.31	0.33	0.8	0.05
13/02/2008 to 16/02/2008	0.34	1.29	2.35	0.16
02/09/2008 to 05/09/2008	0.26	0.23	0.46	0.01

TABLE 10: Comparison between measured and calculated wave heights (Using equation 29)

Dates	NRMSE	Mean(m)	Max(m)	Min(m)
22/03/2006 to 25/03/2006	0.36	0.26	0.5	0.01
06/04/2006 to 11/04/2006	0.23	0.34	0.85	0
11/06/2007 to 15/06/2006	0.32	0.33	0.84	0.05
13/02/2008 to 16/02/2008	0.34	1.29	2.35	0.16
02/09/2008 to 05/09/2008	0.28	0.25	0.56	0

Furthermore, when comparing the NRMSE values between these models on one hand and the Hanson Larson evolution model on the other, more deviations from observed values (larger NRMSE values) is discerned in these models, rendering them less useful.

The models were also tested to predict wave periods using equations 25 and 26. The NRMSE values calculated for each model along with the mean of differences are given in tables 11 and 12. Higher deviations at the beginning of the runs, which are related to the model's initializations have been removed from the NRMSE calculations. The maximum NRMSE values obtained using equation 24 was 0.53 while the minimum was 0.28. Using equation 25, maximum NRMSE values of 1.53 and a minimum of 0.31 were obtained. The NRMSE values obtained for equation 25 are more or less similar to the ones obtained using Hanson Larson model. This is expected because similar equations were used both times, with slight variation on the time coefficient value. However, the discrepancies between observed and measured wave periods for equation 26 are higher, thus making it less applicable.

TABLE 11: Comparison of wave periods (Using equation 25)

Dates	NRMSE	Differences between measured and calculated		
		mean (sec)	max (sec)	min (sec)
22/03/2006 to 25/03/2006	0.44	0.63	3.3	0.01
06/04/2006 to 11/04/2006	0.31	0.65	3.4	0.01
11/06/2007 to 15/06/2006	0.53	0.99	2.96	0
13/02/2008 to 16/02/2008	0.28	1.2	3.3	0.05
02/09/2008 to 05/09/2008	0.42	0.78	2.58	0.02

TABLE 12: Comparison of wave periods (Using equation 26)

Dates	NRMSE	Differences between measured and calculated		
		mean (sec)	max (sec)	min (sec)
22/03/2006 to 25/03/2006	1.53	2.03	2.92	0.04
06/04/2006 to 11/04/2006	0.94	2.34	3.44	0.12
11/06/2007 to 15/06/2006	1.29	2.21	3.74	0.02
13/02/2008 to 16/02/2008	0.31	1.28	2.35	0.02
02/09/2008 to 05/09/2008	1.28	2.17	3.15	0.12

7. CONCLUSION

The Hanson Larson evolution model was initially developed to circumvent problems encountered in using SPM method of wave height and period hindcasting. More over the model also addresses the need to come up with simple and effective models that can be used in wave climate predictions.

The model, as has been discussed in previous sections predicts wave heights in a very good way in most cases. The calculated wave heights obtained using model trial runs were in good agreement with the measured values observed in the sea. However, high wind speeds have caused the discrepancy between calculated and measured values to escalate in some cases. The NRMSE values have shown a positive correlation with the wind speed. The higher the wind speed the higher the NRMSE value and vise versa.

The variability in wind directions are also not well handled by the model. In cases where there are abrupt and continuous changes in wind directions, the model values and observed wave heights do not conform well. Some cases resulted in deviations of more than 55% from measured values, especially in cases where there were sudden changes in wind directions each hour over a three hours' period.

As for the wave period, predictions made by the model are in fair agreement with the observed wave periods in the sea, although there are some rooms for improvement. The Hanson Larson evolution model was found out to result in less errors than other models suggested in equation 26, signifying both the wave height and period follow similar exponential growth and decay patterns.

In the study, three models have been tested for their predictability of the wave climate over the sea. Of the three models the Hanson Larson evolution model, the one based on simple exponential growth of the wave height towards the equilibrium wave heights (equations 9 and 24) resulted in more agreement between observed and calculated values, for both wave height and wave period. The model is a modification of equation 26 with an n-value of -1. The time coefficient rate of 2.17 was also found to result the least RMSE between observed and model values.

There are some improvements and additional investigations that could be done over the same evolution models though. One major area of further investigation could be in the line of using more buoys and checking if similar results could be obtained over the Baltic sea and other water bodies. Furthermore some more additional improvements can also be made in terms of handing shallow water conditions, while still keeping the model simple and easy to use.

8. REFERENCES

Publications

1. Bernard Le Mehaute, 1976, *An Introduction to Hydrodynamics and Water Waves*, Springer-Verlag publisher, New York, United states of America.
2. Blair Kinsman, 1984, *Wind Waves: their generation and propagation on the ocean surface,* Dover publications, New York, United states of America.
3. Dahlerus, C.J., Egermayer, D. (2005). *Run-Up and Sand Dune Erosion along the Coast of Ystad—Present Situation and Future Scenarios. M.Sc. Thesis*, LU: 2005:11, Department of Water Resources Engineering, Lund University, Lund, Sweden.
4. Irminger Street, S., 2011 *Wave Atlas of the South-Eastern Coast of Sweden – A 6-year simulation of waves using local wind measurements from meteorological stations around the Baltic Sea*. Department of Water Resources Engineering, Lund University, Lund, Sweden. MSc Report TVVR 11/5007
5. J.M. Bland and D.G. Altman, 2003, A*pplying the right statistics: Analysis of measurement studies*, ultrasound Obstet Gynecol volume 22, pp 85-93.
6. Larson M. and Hanson H. , 2008 *Extreme waves and water levels in the southern Baltic Sea: implications for flooding at present and future conditions*, Journal of Hydraulic Research 46(2), PP 292-302

7. Leo h. Holthuijsen, 2007 *Waves in oceanic and coastal waters*, Delft University of Technology and UNESCO-IHE, Cambridge University press, New York, United states of America.
8. Matti Lepparanta and Kai Myrberg, 2009, *Physical Oceanography of the Baltic Sea*, Springer publishing with Praxis Publishing, Chichester, United Kingdom.
9. Robert G. Dean and Robert A. Dalrymple, 1984, *Water Wave mechanics for Engineers and Scientists*, Prentice hall Inc. Englewood cliffs, new Jersey, United states of America.
10. Robert L. Weigel, 1964, *prentice hall International series in Theoretical and applied Mechanics: Oceanographic Engineering*, Prentice hall International, Englewood, New Jersey, United States of America.
11. Saville, T. 1954. *The Effect of Fetch Width on Wave Generation*, Technical memorandum 70, Beach Erosion Board, Washington, DC.
12. Sea wave Modelling Project (SWAMP), 1985 *Ocean wave modelling*, springer science and media, New York and Plenum press New York, United states of America.
13. Synder, R.L., F.W. Dobson, J.A. Elliott and R.B. long, 1981, *Array measurements of atmospheric pressure fluctuations above surface gravity waves*, Journal of fluid mechanics 102, PP 1-59.
14. T. Chai and R.R. Draxler, 2014, *Root mean Square Error (RMSE) or Mean Absolute Error (MAE)? Arguments against avoiding RMSE in the literature*, Geoscientific Model development 1247-1250

15. USACE, US army of corps of Engineers 1984, *Shore protection Manual (SPM)*, Coastal Engineering Research Centre Washington USA.
16. WAMDI, 1998, *The WAM Model- Third generation Ocean Wave Prediction Model*, American meteorological Society, Volume 18 PP 1775-1810
17. Wave modelling (WAM), 1994, *Dynamics and Modelling of Ocean waves*, Cambridge university press, Cambridge, United Kingdom.
18. World Meteorological organization (WMO), 1998, Guide to wave analysis and forecasting, second edition, Geneva Switzerland.

Electronic References

1. SWAN, 2015 Introduction to the SWAN Model, accessed on May 16, 2016 at http://swanmodel.sourceforge.net/online_doc/swanuse/node3.html
2. Swedish meteorological and Hydraulics Institute (SMHI), 2014a, *Measuring and Calculating waves*, accessed on February 03, 2016 at http://www.smhi.se/en/theme/measuring-and-calculating-waves-1.12267.
3. Swedish meteorological and Hydraulics Institute (SMHI), 2014b, observations in Sweden, accessed on February 03, 2016 at http://www.smhi.se/en/weather/sweden-weather/observations#.
4. Swedish meteorological and Hydraulics Institute (SMHI), 2016a, *Meteorologiska observationer*, accessed on February 15, 2016 at http://opendata-download-metobs.smhi.se/explore/?parameter=2.

5. Swedish meteorological and Hydraulics Institute (SMHI), 2016b, *Oceanografiska observationer*, accessed on February 15, 2016 at http://opendata-download-ocobs.smhi.se/explore/?parameter=0.
6. TU delft, 2016, SWAN model, accessed on May 16, 2016 at http://www.swan.tudelft.nl
7. Wave watch III, 2015, wave Watch III Model , accessed on May 16, 2016 at http://polar.ncep.noaa.gov/waves/wavewatch/

APPENDIX 1: WIND OBSERVATION

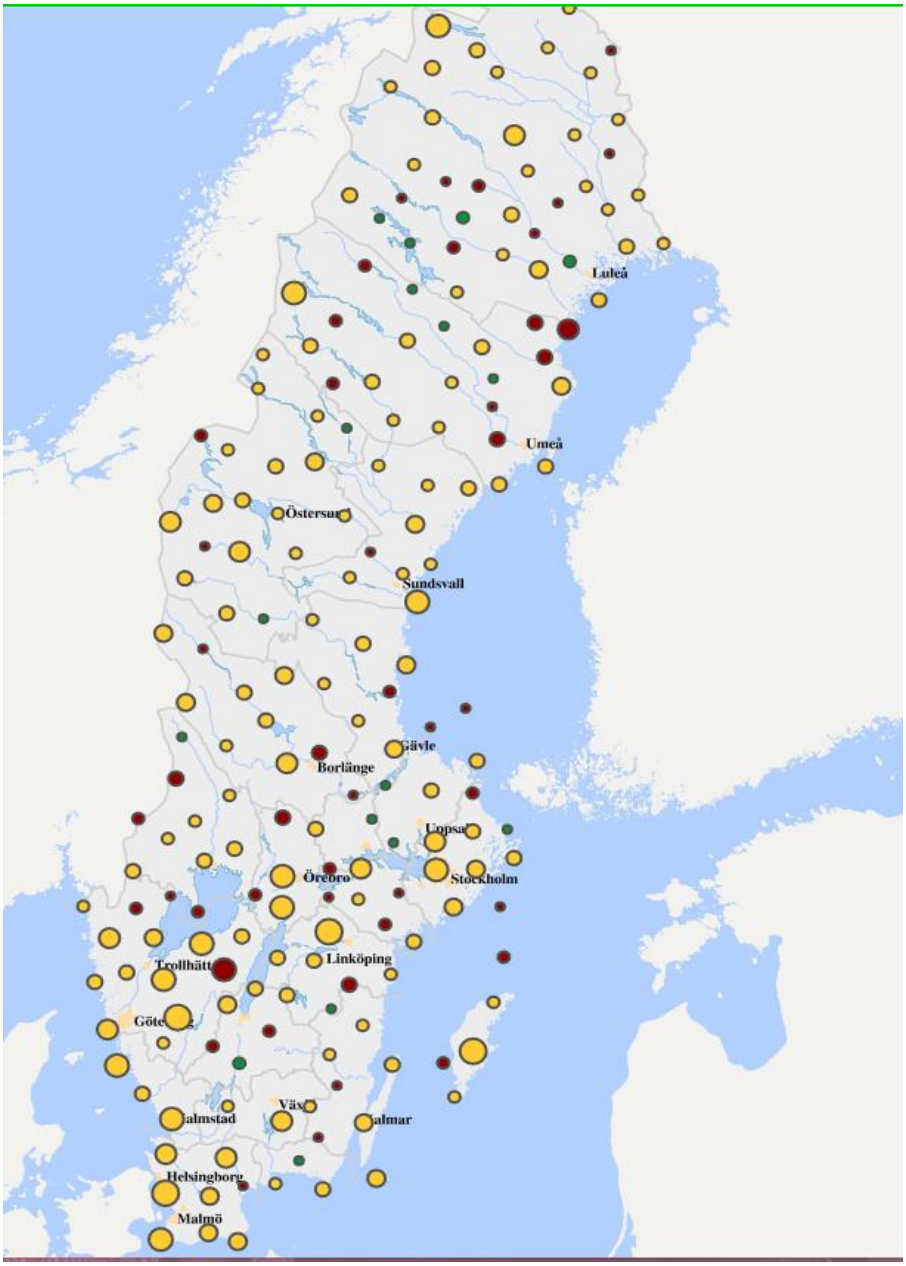


Figure 34: Wind stations administered by SMHI (Red dot: currently inactive; yellow and green: active wind stations) (source SMHI 2016a)

APPENDIX 2: OCEAN OBSERVATION

Figure 35: Wave buoys administered by SMHI over the Baltic (Red dot: currently inactive; yellow and green: active wave buoys) (source SMHI 2016b)

APPENDIX 3: FETCH MEASUREMENT DIRECTIONS

Figure 36: Principal directions in which fetch length were measured

APPENDIX 4: NOMOGRAPH FOR HINDCASTING WAVE HEIGHT AND PERIOD (DEEP WATER CONDITIONS)

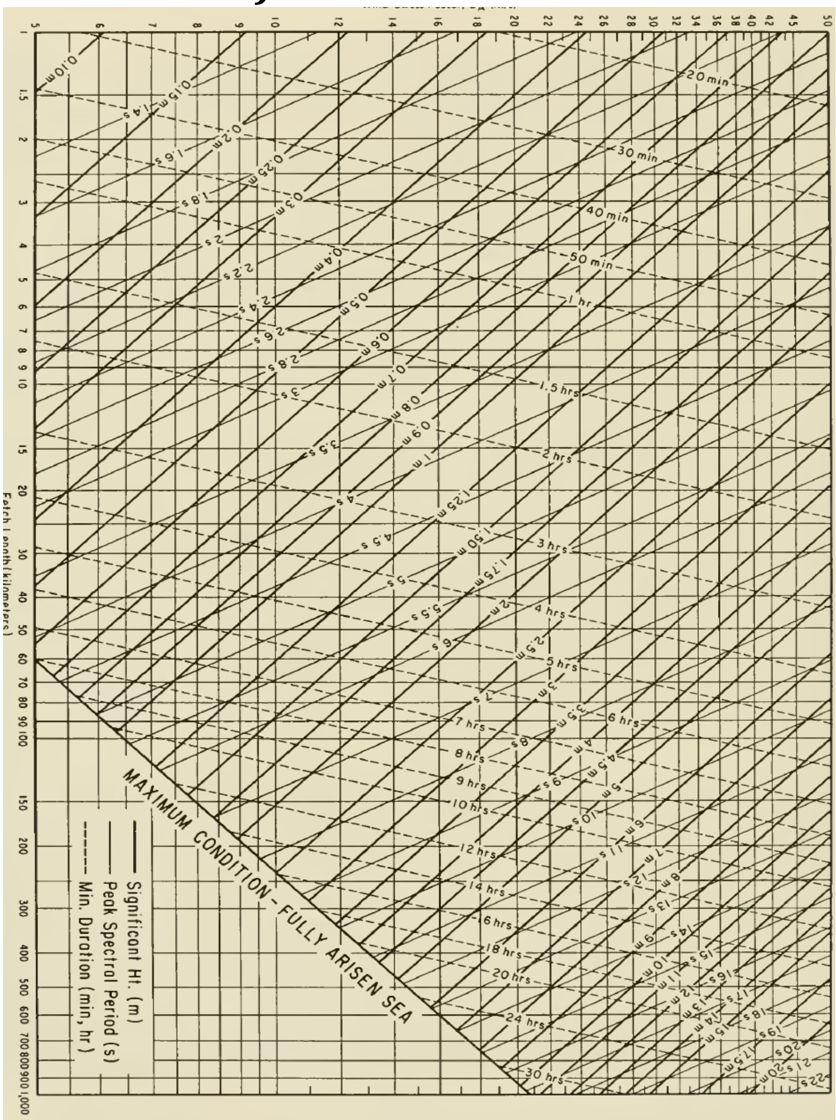


Figure 37: Nomographs of deep water significant wave prediction (metric units) (source: SPM, 1984)