

# Should Debt and Housing Market Dynamics Affect Monetary Policy?

Lessons from Sweden 1992-2015

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## Abstract

In this study, it will be investigated whether housing market dynamics and private indebtedness should be included in the Swedish Riksbank's formulation of monetary policy according to its mission of flexible inflation targeting. A basic model of the supply and demand sides will be chosen. This model includes a representation of the goods market equilibrium, short-run aggregate supply (SRAS) curve and expectations-augmented Phillips curve tied together through a representation of Okun's Law. Furthermore, the goals of the Riksbank will be given an analytical and mathematical interpretation in terms of a social loss function. Given the model and the social loss function an optimal rate-setting rule will be calculated. This set of models will then be used in regressions to make empirical inferences with respect to the Riksbank's view of social loss. In the next step, an alternative representation of the goods market equilibrium will be derived from a basic model regarding debt and housing market dynamics. This other goods market equilibrium will therefore include measures of private indebtedness and housing market dynamics. Using this new version of the equilibrium, an alternative rate-setting regime will be derived. Then by employing the empirical inferences regarding the Riksbank's view of social loss in the previous step and the alternative rate-setting rule in this step, counterfactual simulations of economic measures (i.e. inflation, unemployment rate, GDP, and the repo rate) will be produced. Based on these simulations, some conclusions will be drawn regarding the prudence of including housing market dynamics and private indebtedness into the model. The operational paradigm is Keynesian using forward-looking and rational expectations and the data concerns Sweden 1992Q1-2015Q4. Results largely indicate that housing market and debt dynamics should not be included in rate-setting deliberations by the Riksbank.

**Key words:** Monetary Policy, Private Indebtedness, Housing Prices, Flexible Inflation Targeting, Sweden.

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# 1 Introduction

Price stability has been widely regarded as the ultimate goal of monetary policy, which usually is interpreted as a low and steady inflation rate (Taylor, 1992). Low inflation is oftentimes interpreted as some reasonable target rate, e.g. two percent in the case of the Swedish Riksbank, while the steadiness usually indicates a certain tolerance interval of variation, for instance plus and minus one percentage point (see e.g. Heikensten & Vredin, 1998). Nevertheless, central banks usually find themselves at crossroads where, in the short term, adhering firmly to the inflation target would indicate significant losses in terms of other measures economic activity such as the GDP-output and the unemployment rate (Barro & Gordon, 1983; Taylor, 1993). In such situations, the central banks may decide to allow deviations from the inflation target - i.e. allow for discretion - in the short run while aiming for the target in the medium term. Such practice is dubbed *flexible inflation targeting* (Gali, 2008, pp. 95-102). Indeed, this is how the Riksbank operates which corresponds aptly with how its goals are formulated by the Swedish Parliament (Riksdagen).

Recently Goodfriend and King (2016) published a report, on behalf of the Swedish Parliament, evaluating the performance of the Riksbank in the period 2010-2015. The release of this report sparked new life into an already heated debate on whether the Riksbank has been conducting a far too contractionary monetary policy leading to an inflation rate lower than target (see e.g. Andersson & Jonung, 2014; Svensson, 2014a,b). As the National Institute of Economic Research - NIER - (2013) pointed out, the Riksbank had been systematically overestimating the inflation rate. The Riksbank had namely been referring to increasing levels of housing prices and private indebtedness for its conduct (Riksbank, 2010a, p. 18). Goodfriend and King (2016, pp. 90-93) conclude that Riksbank's decision was the result of several issues, two of which are worth mentioning. First, the government had failed to clarify which agency - the Riksbank or Finansinspektionen (Sweden's financial supervisory authority) - had jurisdiction over the consequences related to the dynamics of housing market and private indebtedness, i.e. financial stability. This put the Riksbank in the awkward situation of seeking to fulfill very contradictory goals. Second, the Riksbank relied too much on results from simulations in their rate-setting efforts. The authors conclude that - following the clarification of the government in 2013 that Finansinspektionen has jurisdiction over the issue of financial stability and received macro-prudential powers (Ibid., p. 100) - the Riksbank has been operating more smoothly in accordance with its mission. Nevertheless, this recent development begs a more comprehensive answer to the question:

*Should the Riksbank take into account the dynamics of the housing market and private indebtedness when setting the interest rate?*

This is the question that this thesis attempts at answering. The study focuses on drawing conclusions based on the Riksbanks' interpretation of its goals.

The Riksbank has set the inflation target at two percent with respect to the consumer price index (CPI) with an a tolerance interval of  $\pm 1$  percentage points until the mid-2010s (Goodfriend and King, 2016 p. 132). However, as Goodfriend and King (2016, p. 20) point out that "CPI inflation includes the impact of changes in interest rates on the imputed cost of owner-occupied housing. A measure of inflation excluding this impact of changes in interest rates is described as CPIX inflation. When interest rates rise to dampen demand and ultimately inflation, the initial impact is to push up CPI inflation. To counter this perverse effect, in its policy deliberations, the Executive Board consistently employs CPIX rather than CPI as the measure of inflation guiding policy." This line of reasoning is based on Heikensten and Vredin (1998, pp. 18-19) and - in more detail - Heikensten (1999). Moreover, these arguments parallel the independent findings of Jeske and Zheng (2013) who analyze US data. They (Ibid., p. 47) argue that the production of housing services has lower labor intensity than goods production, which in turn leads to nominal rigidities and associated price dispersion causing less volatility in the reallocation of labor in the housing sector in comparison to the goods sector. Hence, they conclude that lower weight should be put on the share of housing expenditures.

On the other hand, one could argue, as the Riksbank did in the early 2010s, that financial instability and volatility in the expectations regarding economic activities - e.g. inflation, output and unemployment - could have repercussions for the goals that a central bank has, prompting it to act on the shifts in housing prices

and private indebtedness - two of the main root causes of financial instability and tone-setters for the so-called animal spirits. This is in line with the findings of Akram and Eitrheim (2008). Their results emphasize the trade-off that central banks face between inflation and output stability (Ibid., p. 1252). Moreover, the findings of Bailliu, Kartashova, and Meh (2011, p. 27) illustrate that "a significant share of the funds borrowed against home equity was used for consumption and home renovation", driving up output. Too high levels of output can have detrimental effects on the volatility levels of the interest rates. Indeed, as Akram and Eithrem (2008) point out, responding to excess growth in housing prices could raise stability in inflation and output, which lie well within the scope of the Riksbank's mission. Moreover, one should also consider the fact that household indebtedness constitutes an important source of risk to household spending (Bailliu, Kartashova, and Meh, 2011), making output very sensitive to substantial negative consequences of any sharp correction in house prices. This sensitivity is potentially magnified if, as Disney, Bridges and Gathergood (2010, p. 494) suspects, any increases in housing prices over time prompts households to spend away their other financial assets.

The discussion above is also partly a debate regarding the trade-off of, on one hand, following an explicit rule for setting the interest rate and, on the other hand, allowing for various degrees of discretion (see e.g. Barro & Gordon, 1983; Taylor, 1993 and 2007), and at times has been interpreted as a trade-off between transparency and credibility (Heikensten & Vredin, 1998). In other words, while a monetary policy rule is transparent, predictable and easy to communicate, due to its simplicity (Ibid.), it may lack the nuance to react promptly and correctly to the market dynamics and hence miss its targets, affecting the credibility of the central banks (Barro & Gordon, 1983; Taylor, 1992). On the other hand, discretion is more difficult to communicate as situations to which the central banks has to react might be novel and complex (Ibid., Taylor, 1993). One partial result of studies similar to the present one, hence can be seen as attempting to evolve said monetary policy rules, so as to make them both easy to communicate while retaining the discretionary nuances. In other words, how would the monetary policy rule look like *if* housing market dynamics and private indebtedness were proper and relevant discretionary variables. However, such policy rules, regardless of their performance, should not be mechanically followed (Taylor, 1993), rather serve as a pedagogical framework for conceptualization and communication.

The outline of this study is as follows. First, a basic model of the supply and demand sides will be chosen. This model includes a representation of the goods market equilibrium, short-run aggregate supply (SRAS) curve and expectations-augmented Phillips curve tied together through a representation of Okun's Law. Furthermore, the goals of the Riksbank will be given an analytical and mathematical interpretation in terms of a social loss function. Given the model and the social loss function an optimal rate-setting rule will be calculated. This set of models will then be used in regressions to make empirical inferences with respect to the Riksbank's view of social loss.

In the next step, an alternative representation of the goods market equilibrium will be derived from a basic model regarding debt and housing market dynamics. This other goods market equilibrium will therefore include measures of private indebtedness and housing prices. Using this new version of the equilibrium, an alternative rate-setting regime will be derived. The theory regarding these models will be discussed in Section (2).

After introducing and motivating the theoretical models, regressions will be done to empirically assess the parameters of the different models. This is carried out in Section (3). These regressions partly establish certain economic equilibria in Sweden and partly help assess Riksbank's judgment of social loss with respect to its goals, namely low rate of inflation and sustainable levels of output and unemployment.

By employing the empirical inferences regarding the Riksbank's view of social loss in the previous step and the alternative rate-setting rule in this step, counterfactual simulations of economic measures (i.e. inflation, unemployment rate, GDP, and the repo rate) will be produced in Section (4). Based on these simulations, some conclusions will be drawn regarding the prudence of including housing market dynamics and private indebtedness into the model. Finally, some concluding remarks will be made in Section (5) together with some suggestions for future studies. The paradigm in which I will operate is Keynesian using forward-looking and rational expectations.

The majority results of this study suggests that the Riksbank should *not* incorporate the dynamics of housing prices and household indebtedness into its rate-setting deliberations. This is partially due to the fact that household debt is shown to have little effect on the factors formulated in the bank's goals. Moreover, these factors are shown to relay too much volatility into the path of the inflation rate. Finally, their inclusion seem to induce uncontrolled and undesired contraction in the economy as a whole. However, the results are not completely conclusive as the latter effect mentioned is based on average developments in economic activity. The true path is indeed well within the confidence interval of the counterfactual simulations, indicating non-significance of including housing market and debt dynamics into the Riksbank's decision-making process.<sup>1</sup>

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<sup>1</sup>Generally this last point regarding the lack of significance is echoing the problem of forecasting procedures that the Riksbank face and needs to be improved upon (see e.g. Andersson & Jonung, 2016; Goodfriend and King, 2016).

## 2 Theory

The theoretical framework of this study is based on the triad of equations corresponding to the *goods market equilibrium*, *Okun's Law* and *short-run aggregate supply* (SRAS). The goods market equilibrium is part of the demand side of the economy. It states the negative relationship between the deviation of real interest rate  $r$  from its Wicksellian (1907) *natural* level  $\bar{r}^*$  on one hand, and the output gap on the other (Sørensen & Whitta-Jacobsen, 2010, pp. 453-457) and is part of the Keynesian framework (ibid., pp. 451-453). Output gap here refers to the difference between the output of the economy  $y$  (usually measured in terms of GDP) and its *potential* level ( $\bar{y}$ ) - also called *long-term* or *trend* output. A simple representation of the equilibrium is found below:

$$y - \bar{y} = -\alpha_2(r - \bar{r}^*)$$

where  $\alpha_2 > 0$  is a coefficient. The real interest rate is of course defined as the difference between the nominal interest rate and inflation expectations i.e.  $i - \pi^e$ . Since, this study is concerned with the perspective of the central bank, the nominal interest rate is set equal to the monetary policy instrument  $i^p$ , which is called the *repo rate* in the case of the Swedish Riksbank. Hence, the equilibrium becomes:

$$y - \bar{y} = -\alpha_2(i^p - \pi^e - \bar{r}^*)$$

Moreover, as the credibility of the central bank in relation to the broad public is of concern in this study, the inflation expectations is set to that of the households. The choice of the inflation expectations of the household is further motivated by the fact that the repo rate is relevant for the private indebtedness of the households.

The equation above is very simplistic. In reality, for instance, there exists consistent demand shocks affecting the equilibrium. They can be represented in the following manner:

$$y - \bar{y} = -\alpha_2(i^p - \pi^e - \bar{r}^*) + z$$

where  $z$  is a stochastic variable with zero expectation. The effects of these shocks may be persistent over time. To represent that the equation must undergo two changes. First, it will be expressed as a time-series:

$$y_t - \bar{y}_t = -\alpha_2(i_t^p - \pi_t^e - \bar{r}_t^*) + z_t$$

where  $t$  is the time period. Now  $\{z_t\}$  is assumed to be an i.i.d. and stationary time-series process with zero expectation and finite standard deviation  $\sigma_z$ . Second, some lagged parameters should be incorporated to encapsulate the persistence. This can be carried out in different ways. One could for instance allow for lagged variables of the output gap or interest rate to appear in the equation. Here, however, a more general approach is ensued as we allow for lagged shocks to appear in the equilibrium directly. This is done by allowing some polynomial function  $\Phi$  of the lag operator  $B^2$  to affect the shocks  $z_t$ :

$$y_t - \bar{y}_t = \Phi_z(B)z_t - \alpha_2(i_t^p - \pi_t^e - \bar{r}_t^*)$$

where  $\Phi_z(B) = 1 + \sum_{k=1}^{n_z} \phi_k^z B^k$ . Here,  $n_z$  is the maximum number of lags of shock affecting the equilibrium. Moreover,  $\phi_k^z \in \mathbb{R}, k = 1, \dots, n_z$  are the parameters of the polynomial indicating in which way the lags affect the equilibrium.

Another issue is the fact that there usually seems to be a lag between the changes in the real interest rate and the effects it has on the output gap (see e.g. Svensson, 1997). For instance such lag might be one time-period, resulting in the following equation:

$$y_{t+1} - \bar{y}_{t+1} = -\alpha_2(i_t^p - \pi_t^e - \bar{r}_t^*) + \Phi_z(B)z_{t+1}$$

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<sup>2</sup>The lag operator  $B$  affects the process in this manner:  $B^k \cdot z_t = z_{t-k}$ .

In a general sense, such lagged effects could of course be any  $n_p$  number of periods:

$$y_{t+n_p} - \bar{y}_{t+n_p} = \Phi_z(B)z_{t+n_p} - \alpha_2(i_t^p - \pi_t^e - \bar{r}_t^*) \quad (1)$$

There is a stylized fact stating that this lag is indeed one year (Sørensen & Whitta-Jacobsen, 2010, p. 604). In a quarterly framework (i.e.  $t$  representing quarters) this means that  $n_p = 4$ . We have now arrived at the final representation of the goods market equilibrium used in this study. Hence we move on to the short-run aggregate supply (SRAS).

The short-run aggregate supply states the positive relationship between the *expected inflation gap* and the output gap. The expected inflation gap is the difference between the inflation  $\pi$  and the expected inflation  $\pi^e$ . A simple representation of SRAS curve is stated below:

$$\pi - \pi^e = \gamma(y - \bar{y})$$

or equivalently:

$$\pi = \pi^e + \gamma(y - \bar{y})$$

where  $\gamma > 0$  is a parameter. The positive relationship is due to the fact that higher output creates higher marginal production costs that is passed on to prices (Sørensen & Whitta-Jacobsen, 2010, pp. 503-505). As in the case of the goods market equilibrium a similar process could be carried out incorporating the temporal aspect of the curve, supply shocks  $s_t$ , persistence through a polynomial of the lag operator and lagged effect  $n_p$ :

$$\pi_{t+n_p} = \pi_t^e + \gamma(y_t - \bar{y}_t) + \Phi_s(B)s_{t+n_p} \quad (2)$$

where  $\{s_t\}$  is an i.i.d. and stationary stochastic process with zero mean and finite standard deviation  $\sigma_s$ ,  $\Phi_s(B) = 1 + \sum_{k=1}^{n_s} \phi_k^s B^k$ ,  $\phi_k^s \in \mathbb{R}$  is the polynomial of the lag operator with  $n_s$  as the maximum number of effective lagged shocks. Moreover,  $n_p$  similarly is the lagged period which is required for the output gap and expectations to influence the rate of inflation. There is a stylized fact stating that this lag is indeed one year, rendering the full time for monetary policy (i.e. the repo rate through the real interest rate) to fully influence the rate of inflation into two years (Sørensen & Whitta-Jacobsen, 2010, pp. 604-5). In a quarterly framework (i.e.  $t$  representing quarters) this fact corresponds to  $n_p = 4$ .

We move on now to Okun's Law. Okun (1962) found a negative relationship between unemployment and output. Later on some nuance was added to this relation, stating that the negative correlation is indeed between the output and *unemployment gaps* (Sørensen & Whitta-Jacobsen, 2010). Unemployment gap is defined as the deviation of the unemployment rate  $u$  and the *Non-Accelerating Rate of Unemployment* (NAIRU)  $\bar{u}$  - also dubbed long-term or trend unemployment.

$$u_t - \bar{u}_t = -\alpha_y(y_t - \bar{y}_t)$$

NAIRU, potential output and the natural rate of interest are closely related (Friedman, 1968; Okun, 1962; Phelps, 1967). NAIRU - and subsequently trend output and the natural interest rate - in each country depend on different long-term macroeconomic factors such as the degree of global competition, migration policies, labor unions' bargaining position, and indeed long-term factors affecting the cost of frequently used goods and services such as computers and electronics (Gordon, 1997, p. 29-30).

Once again, the temporal aspect of the curve, shocks  $\vartheta_t$  and persistence through a polynomial of the lag operator is incorporated:

$$u_t = \bar{u}_t - \alpha_y(y_t - \bar{y}_t) + \Phi_\vartheta(B)\vartheta_t \quad (3)$$

where  $\alpha_y > 0$  is a parameter, the lag polynomial is defined correspondingly ( $\Phi_\vartheta(B) = 1 + \sum_{k=1}^{n_\vartheta} \phi_k^\vartheta B^k$ ,  $\phi_k^\vartheta \in \mathbb{R}$ ), and  $n_\vartheta$  is the maximum number of effective lagged shocks.

Okun's law and the SRAS curve together connect the two economic factors of unemployment gap and in-

flation rate through the *Phillips curve*. Phillips (1958) found a stable negative relation between the rate of unemployment and the rate of change of money-wage rates in the UK (1861-1957). As wages are a key factor of the cost of output and consequently that of prices, the equation was improved upon to state the relationship between the rates of unemployment and inflation (Friedman, 1976). Furthermore, expectations were also introduced to the model to arrive at the *expectations-augmented* Phillips curve (Ibid.).

In an economy, the central bank observes these relationships and operates accordingly through a regime for setting the monetary instrument  $i^p$ , the repo rate in case of the Swedish Riksbank. This instrument then affects inflation, output and unemployment through these equations. The nature of this monetary policy regime depends on the central bank's goals and subsequently its view of social loss over time ( $L_t$ ). The Riksbank, for instance, has a target inflation ( $\pi^*$ ) of two percent, and moreover, at the same time aims at achieving sustainable growth and high employment. Such view of social loss could be represented as follows:

$$L_t = \frac{1}{2}[(\pi_t - \pi^*)^2 + \kappa_y(y_t - \bar{y}_t)^2 + \kappa_u(u_t - \bar{u}_t)^2] \quad (4)$$

where  $L_t$  is social loss at time  $t$  and the coefficients  $\kappa_\nu, \nu = y, u$  are the weight of social loss in output gap and lower-than-trend unemployment relative to that of inflation gap. The optimization problem for the Riksbank then becomes minimizing the present-value of current and all future social losses when setting its nominal interest rate today:

$$\min_{i_t^p} \mathbb{E}_t \left[ \sum_{j=t}^{\infty} \phi^{j-t} L_j(i_t^p) \right] \quad (5)$$

where  $\phi$  is the central bank's rate of time preference such that  $0 \leq \phi < 1$ . The box titled Model 1 summarizes the equations mentioned so far. They are discussed a bit more generally there and the concepts are more rigorously tied to each other.

This study intends to investigate the whether central banks, specifically the Swedish Riksbank, should incorporate developments in housing prices and private debt when formulating optimal monetary policy according to its goals, i.e. setting the repo rate. According to Goodfriend and King (2016, pp. 92-93), the Riksbank has already taken private debt and changing housing prices as a factor in discretionary monetary policy in the early 2010s. The model hence should be complemented with equation tying together private debt and housing costs to at least one of the other variables, most notably the interest rates ( $i, r$ ) and the income  $y$ . This is done in section (2.2). Observe further that the effect lag  $n_p$  is set to four capturing the stylized fact that it takes about two years ( $2n_p = 8$  quarters) for monetary policy have its full impact (Sørensen & Whitta-Jacobsen, 2010, pp. 604-605).

The goal is to use the models above to solve the minimization problem in (5) to arrive at different monetary policy rules for setting the repo-rate. These rules are then tested against the available macroeconomic time-series data in order to investigate which one is better tuned with the goals the Swedish Riksbank. Based on this last result, then one can draw conclusions with respect to the different social loss functions, and the importance of the different criteria relative to that of price stability, i.e. aiming for the inflation target. Before including the dynamics of the debt and housing market we first derive the optimal interest rule in the standard case excluding these factors.

## 2.1 The Optimal Interest Rate Rule

We are interested in estimating the value parameters which consist of  $\kappa_y$  and  $\kappa_u$  and the central bank's rate of time preference  $\phi$  in (5). Subsequently, they are to be used to derive a counterfactual model, using the same value parameters, while inserting into the model a link to the housing prices and private debt dynamics (see sections 2.2 and 4).

Now that we have motivated the need for the estimation, we proceed to establish the rule for setting the optimal interest rate. However, before doing so, it is crucial to reduce the general problem in (5) - i.e. the task of the central bank to minimize the social cost given its definition in (6). For the sake of simplicity we set the

lag of shocks taking effect  $n_p = 1$  in Model 1 while considering the the time periods annual, i.e.  $t$  being a year.

**Model 1: Flexible inflation targeting - Outside lag, no inside lag, persistence in shocks**

The model consists of the following equations:

$$L_t = \frac{1}{2}[(\pi_t - \pi^*)^2 + \kappa_y(y_t - \bar{y})^2 + \kappa_u(u_t - \bar{u})^2] \quad (6)$$

Social loss at quarter  $t$ ,

$$y_{t+n_p} - \bar{y} = \Phi_z(B)z_{t+n_p} - \alpha_2(i_t^p - \pi_t^e - \bar{r}^*) \quad \text{Goods market equil.} \quad (7)$$

$$\pi_{t+2n_p} = \pi_{t+n_p}^e - \alpha(u_{t+n_p} - \bar{u}) + \Phi_s(B)s_{t+2n_p} + \vartheta_{t+n_p} \quad \text{Phillips Curve} \quad (8)$$

$$\pi_{t+2n_p} = \pi_{t+n_p}^e + \gamma(y_{t+n_p} - \bar{y}) + \Phi_s(B)s_{t+2n_p} + \frac{\vartheta_{t+n_p}}{1-\alpha} \quad \text{SRAS curve} \quad (9)$$

$$\Phi_s(B) = 1 + \sum_{k=1}^{n_s} \phi_k^s B^k, \text{ where } \phi_1^s \neq 0 \text{ if } n_p > 1 \quad \text{Persistence (supply)} \quad (10)$$

$$\Phi_z(B) = 1 + \sum_{k=1}^{n_z} \phi_k^z B^k, \text{ where } \phi_1^z \neq 0 \text{ if } n_p > 1 \quad \text{Persistence (demand)} \quad (11)$$

**Formal representation**

$$\pi_t^e = \pi_{t+n_p|t}$$

**Type of Expectations**

$$\text{Forward-looking (New Keynesian)} \quad (12)$$

where index  $t$  is the time period,  $n_p$  is the lag at which shocks take effect,  $\pi_t^e$  is the expected inflation,  $\pi_{t+4|t}$  is the one-year forward-looking inflation expectations of households,  $u_t$  is unemployment and  $\bar{u}$  is the trend unemployment (i.e. NAIRU),  $\pi_t$  is the inflation rate,  $\pi^*$  the target rate of inflation,  $y_t$  the logarithmic transform of the output (GDP),  $\bar{y}$  is the logarithmic transformation of the output trend,  $i_t^p$  is the monetary instrument (repo-rate),  $\bar{r}^*$  is the natural risk-free interest rate,  $s_t$  and  $z_t$  are the supply and demand shocks respectively which together with  $\vartheta_t$  are i.i.d. random processes with zero expectation and finite variance, the coefficients  $\kappa_\nu, \nu = y, u$  are the weight of social loss in output gap and lower-than-trend unemployment relative to that of inflation gap,  $\alpha_2 > 0$  and  $0 < \alpha < 1$ , so that  $0 < \gamma \equiv \frac{\alpha}{1-\alpha}$ . Note that the expectations-augmented Phillips curve and the SRAS curve are deemed equivalent as it holds that,

$$u_{t+n_p} = \bar{u} - \frac{1}{1-\alpha}(y_{t+n_p} - \bar{y} + \vartheta_{t+n_p}) \quad (13)$$

which is a formal representation of Okun's Law. Observe further that the social loss function above is an interpretation of the task of Riksbank i.e. attaining the inflation target, and at the same time achieving sustainable growth and high employment. Note further that  $\Phi_s(B), \Phi_z(B)$  are polynomial functions of the lag operator  $B$ , with coefficients  $\phi_k^s, \phi_k^z, k = 1, 2, \dots$  respectively. In whichever equation they appear, stationarity is assumed. Moreover, the first coefficients of the polynomials are assumed to be non-zero ( $\phi_1^s, \phi_1^z \neq 0$ ), to insure continuous connection between the time-series models at every period. Furthermore,  $n_s$  and  $n_z$  are the highest lags effective in the time series models of supply and demand respectively.

Viewing the goods market equilibrium in (7) and the Phillips curve in (8), one can see that setting the nominal interest rate  $i_t = i_t^p - \bar{r}^*$ , will affect the two immediate following periods, affecting production one year ahead ( $y_{t+1} - \bar{y}$ ) through the goods market equilibrium and inflation two years ahead ( $\pi_{t+2}$ ) through the Phillips curve. Of course the nominal interest rate will also influence future production and inflation rates indirectly as an extension. However, these rates can also be adjusted directly, by future nominal rates of interest,  $i_{t+k}, k \geq 1$ . Hence, when setting the nominal interest rate, the central bank will only need to concern itself with immediate social loss for two periods ahead, reducing the optimization problem in (5) to the following:

$$\min_{i_t} \mathbb{E}_t [\phi L_{t+1}(i_t) + \phi^2 L_{t+2}(i_t)]$$

Since  $0 < \phi < 1$ , the problem is then equivalent to the expression below.

$$\min_{i_t} \mathbb{E}_t [L_{t+1}(i_t) + \phi L_{t+2}(i_t)] \quad (14)$$

Viewing the loss function in (6), we know that there are three elements in the loss function  $L_t$ , namely the inflation gap  $(\pi_t - \pi^*)$ , the output gap  $(y_t - \bar{y})$  and the unemployment gap  $(u_t - \bar{u})$ . In the next period social loss  $L_{t+1}$ , the next period inflation gap  $(\pi_{t+1} - \pi^*)$  cannot be influenced by setting the nominal interest rate at time  $t$ , as the effect of the interest rate takes two periods to impact inflation, as stated in the goods market equilibrium (7) and the Phillips curve (8). However, the output gap  $(y_{t+1} - \bar{y})$  and unemployment gap  $(u_{t+1} - \bar{u})$  are affected by the current nominal interest rate  $i_t$  directly through the Phillips curve and SRAS curve in (8) and (9) respectively. Hence, the reduced minimization problem above (14) is further reduced to the following:

$$\min_{i_t} \mathbb{E}_t \left[ \frac{1}{2} [\kappa_y (y_{t+1} - \bar{y})^2 + \kappa_u (u_{t+1} - \bar{u})^2] + \phi L_{t+2}(i_t) \right] \quad (15)$$

Similarly, we can identify the relevant elements in the two-period-ahead social loss  $L_{t+2}$ . The corresponding inflation gap  $(\pi_{t+1} - \pi^*)$  is affected by the current nominal interest rate ( $i_t$ ) through the goods market equilibrium (7) and the Phillips curve (8). However, the corresponding output gap  $(y_{t+2} - \bar{y})$  and unemployment gap  $(u_{t+2} - \bar{u})$ , though influenced *indirectly* by the current nominal interest rate  $i_t$ , can *directly* be adjusted by the future rate of nominal interest  $i_{t+1}$ . Hence, the central bank can afford not to concern itself with these entities at time  $t$ . Thereby, the minimization problem in 15 becomes even further reduced to the following form:

$$\min_{i_t} \mathbb{E}_t [Q], \text{ where, } Q = \frac{1}{2} [\kappa_y (y_{t+1} - \bar{y})^2 + \kappa_u (u_{t+1} - \bar{u})^2 + \phi (\pi_{t+2} - \pi^*)^2] \quad (16)$$

Using the final form above together with the equations in Model 1 the rule for setting the optimal nominal interest rate is found to be the following:

$$\begin{cases} i_t^p = \bar{r}_t^* + \pi_t^e + R (\pi_t - \pi^* + \gamma (y_t - \bar{y}_t)) \\ R \equiv \frac{\gamma \phi}{\alpha_2 (\kappa_y + \kappa_u / (1 - \alpha)^2 + \gamma^2 \phi)} \end{cases} \quad (17)$$

For detailed calculations see Section (7.1.1) in the Appendix. The parameters and variables are defined just as before. The reader should observe that this policy regime is a version of the standard Taylor (1993) rule for setting the monetary instrument. Next, we'll derive the optimal interest rule in the case where private debt and housing prices is inserted into Model 1. However, before doing so a model of debt and housing market dynamics is needed.

## 2.2 Debt and Housing Market Dynamics

There are many different ways to model the housing market dynamics. In this study, a relatively simplified one is utilized which is a modified version used by Sørensen & Whitta-Jacobsen (2010, pp. 406-421). Let's assume that a proportional property tax on the current value  $p_t^H H_t$  of the consumer's housing stock  $H_t$  is given by  $\tau$ , where  $p_t^H$  is the price of housing. Furthermore, a proportional income tax at the rate  $m$  is imposed, where a fraction  $d$  of interest expenses to be deducted from taxable income, where  $0 \leq d \leq 1$ . Finally, a fraction  $\delta p_t^H H_t$  is spent on repair and maintenance during each period to maintain the value of the housing stock. The parameter  $\delta$  may thus be interpreted as the depreciation rate for housing capital. Expenses on repair and maintenance are assumed not to be deductible from taxable income. Hence the total amount spent on housing at period  $t$  is given by:

$$\tilde{H}_t = M_t H_t, \quad M_t \equiv [r_t (1 - dm) + \delta + \tau] p_t^H \quad (18)$$

where  $r_t$  is the real interest rate. The sum borrowed to finance the housing consumption above and the consumption of non-durable goods  $C_t$  depends not only on the income at period  $t$  ( $Y_t$ ), but also on the present-

value of all expected future income,  $SY_t$ , defined below:

$$SY_t = \mathbb{E}_t \left( \sum_{k=1}^{\infty} \frac{Y_{t+k}}{\prod_{j=t+1}^{t+k} (1+r_j)} \right) \quad (19)$$

Furthermore, past debt (or savings),  $V_{t-1}$  must be taken into account. Hence, the households' expected debt at each period,  $V_t^e$ , is given by:

$$V_t^e = (1+r_t) \left[ V_{t-1} + C_t + \tilde{H}_t - (1-m)(Y_t + SY_t) \right] \quad (20)$$

$$V_t = V_t^e + v_t, \quad (21)$$

where  $v_t$  is i.i.d. noise with zero expectation and finite variance. In order to avoid the fallacy of perpetual debt (i.e. no-Ponzi-game rule) we assume that:

$$\lim_{t \rightarrow \infty} \frac{V_t^e}{1+r_t} = 0 \quad (22)$$

which together with (20) results in the all-familiar inter-temporal budget constraint (solvency restriction):

$$\sum_{t=1}^{\infty} \frac{C_t + \tilde{H}_t}{\prod_{j=0}^{t-1} (1+r_j)} = V_0 + (1-m) \sum_{t=1}^{\infty} \frac{Y_t}{\prod_{j=0}^{t-1} (1+r_j)} \quad (23)$$

where  $V_0$  is initial debt (or savings), and  $r_0 \equiv 0$ . Let the households have Cobb-Douglas utility of the form (constant returns to scale):

$$u_t = H_t^\eta C_t^{1-\eta}, 0 < \eta < 1. \quad (24)$$

At each period, households maximize the utility above.<sup>3</sup> Rewriting (20) and using (18), we attain that,

$$\begin{cases} C_t = R_t - M_t H_t \\ R_t \equiv \frac{V_t^e}{1+r_t} + (1-m)(Y_t + SY_t) - V_{t-1} \end{cases} \quad (25)$$

The maximization problem then becomes  $\max_{H_t} u_t$ . Using (24) and (25), the first-order condition gives the following:

$$\frac{du_t}{dH_t} = 0 \Rightarrow \eta H_t^{\eta-1} C_t^{1-\eta} = (1-\eta) M_t C_t^{-\eta} H_t^\eta$$

which results in,

$$\frac{C_t}{H_t} = \frac{1-\eta}{\eta} M_t \quad (26)$$

From (25) we have that:

$$\frac{C_t}{H_t} = \frac{R_t}{H_t} + M_t \quad (27)$$

Equations (26) and (27), hence yield the following:

$$\begin{cases} H_t^d = \frac{\eta R_t}{M_t} \\ R_t \equiv \frac{V_t^e}{1+r_t} + (1-m)(Y_t + SY_t) - V_{t-1} \\ M_t \equiv [r_t(1-dm) + \delta + \tau] p_t^H \end{cases} \quad (28)$$

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<sup>3</sup>Observe that, for simplicity, it has been assumed that households maximize their current utility ( $u_t$ ) at each period, rather than maximizing the present value of all their expected future utilities., i.e.  $U_t = u_t + \sum_{i=t+1}^{\infty} \frac{\mathbb{E}[u_i]}{(1+\phi)^{i-t}}$ , where  $\phi$  is the rate of time-preference.

which is the optimal demand for housing stock.

Now, consider the supply side. Let the housing investment be given by the following:

$$I_t^H = AX_t^\beta, \quad 0 < \beta < 1 \quad (29)$$

where  $A$  is the state of technology of construction, and  $X_t$  is input for construction in a broad sense. The fact that  $\beta$  is less than one, indicates diminishing returns to scale. The profit function,  $\Pi_t$ , is then given by,

$$\Pi_t = p_t^H I_t^H - P_t X_t \quad (30)$$

where  $P_t$  is the price of input. The supply side (firms) aim to maximize profit, i.e.  $\max_{I_t^H} \Pi_t$ . Using (29) and (30), the first order condition yields the following:

$$\frac{d\Pi_t}{dI_t^H} = 0 \Leftrightarrow p_t^H - \frac{P_t}{\beta A} \left( \frac{I_t^H}{A} \right)^{(1-\beta)/\beta} = 0$$

which yields the optimal supply

$$I_t^H = k \cdot \left( \frac{p_t^H}{P_t} \right)^{\beta/(1-\beta)}, \quad k = \beta^{\beta/(1-\beta)} A^{1/(1-\beta)}. \quad (31)$$

Moreover, let us now state the dynamics of the housing stock:

$$H_{t+1} = (1 - \delta)H_t + I_t^H \quad (32)$$

Equations (28), (31) and (32) provide the complete dynamic model for debt and housing. So far the calculations are fairly faithful to Sørensen & Whitta-Jacobsen (2010, pp. 406-421). They are summarized in the box below. However, the equations are non-linear. Let us now utilize this model to arrive at a simple linear dynamic.

#### Non-linear Housing Market Dynamics

$$\tilde{H}_t = M_t H_t, M_t \equiv [r_t(1 - dm) + \delta + \tau] p_t^H \quad \text{Amount spent on housing} \quad (33)$$

$$V_t^e = (1 + r_t) [V_{t-1} + C_t + \tilde{H}_t - (1 - m)(Y_t + SY_t)], V_t = V_t^e + v_t \quad \text{Households' expected debt} \quad (34)$$

$$H_t^d = \frac{\eta R_t}{M_t}, R_t \equiv \frac{V_t^e}{1 + r_t} + (1 - m)(Y_t + SY_t) - V_{t-1} \quad \text{Demand for housing stock} \quad (35)$$

$$I_t^H = k \cdot \left( \frac{p_t^H}{P_t} \right)^{\beta/(1-\beta)}, k = \beta^{\beta/(1-\beta)} A^{1/(1-\beta)} \quad \text{Housing investment} \quad (36)$$

$$H_{t+1} = (1 - \delta)H_t + I_t^H \quad \text{Housing stock dynamics} \quad (37)$$

Observe that from (35), we have that:

$$\frac{H_{t+1}}{H_t} = \frac{R_{t+1}}{R_t} \cdot \frac{M_t}{M_{t+1}} \cdot \frac{p_t^H}{p_{t+1}^H}$$

which yields,

$$\Delta \log(H_{t+1}) = \Delta \log(R_{t+1}) - \Delta \log(M_{t+1}) - \Delta \log(p_{t+1}^H) \quad (38)$$

where  $\Delta \log W_{t+1} = \log W_{t+1} - \log W_t$ . On the other hand, from (37) we have that,

$$\frac{H_{t+1}}{H_t} = (1 - \delta) \left( 1 + \frac{I_t^H}{(1 - \delta)H_t} \right) \quad (39)$$

It is reasonable that the amount invested in the housing at each period is less than the housing stock (even

adjusted for depreciation), i.e.  $|\frac{I_t^H}{(1-\delta)H_t}| < 1$ . Hence, by a Taylor expansion of order one we have that,

$$\log\left(1 + \frac{I_t^H}{(1-\delta)H_t}\right) \approx \frac{I_t^H}{(1-\delta)H_t}$$

Therefore, by (39) we have that,

$$\Delta \log(H_{t+1}) \approx \log(1-\delta) + \frac{I_t^H}{(1-\delta)H_t} \quad (40)$$

Using (38) and (40) above, we get,

$$\Delta \log(p_{t+1}^H) \approx \Delta \log(R_{t+1}) - \Delta \log(M_{t+1}) - \frac{I_t^H}{(1-\delta)H_t} - \log(1-\delta) \quad (41)$$

Let us now take look at the first three terms on the right-hand side of (41) separately. First Consider  $\Delta \log(M_{t+1})$ . By (35), we have that,

$$\log(M_t) = \log\left[(\delta + \tau)\left(1 + \left(\frac{r_t(1-dm)}{\delta + \tau}\right)\right)\right]$$

which results in,

$$\log(M_t) = \log(\delta + \tau) + \log\left(1 + \frac{r_t(1-dm)}{\delta + \tau}\right) \quad (42)$$

We know that  $0 < |r_t|, d, m, \delta, \tau < 1$ . Furthermore, it is safe to assume that the property tax and the depreciation rate  $(\delta + \tau)$  is usually much larger than the real interest rate  $(r_t)$  in order, especially when it is decreased by a factor  $1 - dm$ . Hence, it is reasonable to assume that,

$$\left|\frac{r_t(1-dm)}{\delta + \tau}\right| < 1$$

and consequently that

$$\log\left(1 + \frac{r_t(1-dm)}{\delta + \tau}\right) \approx \frac{r_t(1-dm)}{\delta + \tau}$$

by a Taylor expansion of order one. Then, by (42) we have that:

$$\Delta \log(M_{t+1}) \approx \alpha_{\Delta r} \Delta r_{t+1}, \quad \alpha_{\Delta r} = \frac{1-dm}{\delta + \tau} \quad (43)$$

Now consider  $\Delta \log(R_{t+1})$ . From (35) we have that  $R_t$  is a sum of several variables. Hence,  $\Delta \log(R_{t+1})$  a logarithmic transform of said sum cannot be directly rewritten as the sum of logarithmic transforms of its arguments. However, the following proportionality statement still holds:

$$\Delta \log(R_{t+1}) \propto [\log V_t - \Delta \log V_t + \log Y_{t+1} + \Delta \log Y_{t+1}]$$

In order to avoid collinearity between the pairs  $(\log V_t, \Delta \log V_t)$  and  $(\log Y_{t+1}, \Delta \log Y_{t+1})$  respectively, however, we only use the relative-difference variables in our approximation of  $\Delta \log(R_{t+1})$ :

$$\Delta \log(R_{t+1}) \approx \alpha_y \Delta y_{t+1} + \alpha_D \Delta D_t \quad (44)$$

where  $\Delta D_t \equiv \Delta \log V_t$ ,  $y_t \equiv \log Y_t$ , plus  $\alpha_y > 0$  and  $\alpha_D \in \mathbb{R}$  are coefficients. Observe that  $\alpha_y$  unambiguously being positive is due to the fact that  $\Delta \log(R_{t+1})$  is positively proportional to both  $\log Y_{t+1}$  and  $\Delta \log Y_{t+1}$ . On the other hand,  $\alpha_D$  could take any real value since  $\Delta \log(R_{t+1})$  is positively proportional to  $\log V_t$ , while negatively proportional to  $\log \Delta V_t$ .

Finally, consider  $I_t^H/H_t$ . From (35) and (36) we have that,

$$\frac{I_t^H}{H_t} = \frac{k(p_t^H)^{1/(1-\beta)}M_t}{P_t^{\beta/(1-\beta)}R_t}$$

Hence, by also utilizing the definitions of  $R_t$  and  $M_t$  in (35), it yields

$$\frac{I_t^H}{H_t} = \frac{k[r_t(1-dm) + \delta + \tau]}{(P_t p_t^H)^{\beta/(1-\beta)}[\frac{V_t^e}{1+r_t} - V_{t-1} + (1-m)(Y_t + SY_t)]} \quad (45)$$

Finally, we arrive at the following proportionality statement,

$$\frac{I_t^H}{H_t} \propto [r_t - \Delta P_t - Y_t - \Delta Y_t - p_t^H],$$

which again, by avoiding the the collinearity between  $Y_t$  and  $\Delta Y_t$  can be rewritten as,

$$\frac{I_t^H}{H_t} \propto [r_t - \Delta P_t - \Delta Y_t - p_t^H], \quad (46)$$

Notice that it is ambiguous how the quota  $I_t^H/H_t$  depends on the debt. However, there is a dependence.

Let us summarize our results so far,

$$\left\{ \begin{array}{l} \Delta \log(p_{t+1}^H) \approx \Delta \log(R_{t+1}) - \Delta \log(M_{t+1}) - \frac{I_t^H}{(1-\delta)H_t} - \log(1-\delta) \\ \Delta \log(M_{t+1}) \approx \alpha_{\Delta r} \Delta r_{t+1} \\ \left( \begin{array}{l} \Delta \log(R_{t+1}) \approx \alpha_y \Delta y_{t+1} + \alpha_D \Delta D_t, \\ \Delta D_t \equiv \Delta \log V_t, y_t \equiv \log Y_t \end{array} \right) \\ \frac{I_t^H}{H_t} \propto [p_t^H + r_t - \Delta P_t - \Delta Y_t], \end{array} \right.$$

The only variable in the proportionality statement of  $I_t^H/p_t^H$  which is not collinear with any parameters in  $\Delta \log(p_{t+1}^H)$ ,  $\Delta \log(M_{t+1})$  and  $\Delta \log(R_{t+1})$  is the variable  $\Delta P_t$ . Hence, a *linear and least-collinear approximation* of the housing market inflation ( $\pi_{t+1}^H \equiv \Delta \log(p_{t+1}^H)$ ) is given by:

$$\pi_{t+1}^H = \alpha_y \Delta y_{t+1} + \tilde{\alpha}_D \Delta D_t - \alpha_{\Delta r} \Delta r_{t+1} - \tilde{\alpha}_p \Delta P_t + \tilde{\theta}_{t+1} \quad (47)$$

where  $\{\tilde{\theta}_t\}$  is an AR(1) noise process,  $\Delta D_t \equiv \Delta \log V_t$ ,  $\Delta y_{t+1} \equiv \Delta \log Y_{t+1}$ , and  $\alpha_y, \alpha_{\Delta r}, \tilde{\alpha}_p > 0, \tilde{\alpha}_D \in \mathbb{R}$ . Hence, the model in (47) yields a dynamic relation for housing prices and debt at the same time. Recall also that  $r_t = i_t - \pi_t$ . Hence, the equation above relates debt and housing prices to inflation as well. Rearranging, the expression above we arrive at,

$$\Delta y_{t+1} = \frac{1}{\alpha_y} \left( \pi_{t+1}^H - \tilde{\alpha}_D \Delta D_t + \alpha_{\Delta r} r_{t+1} - \alpha_{\Delta r} r_t + \tilde{\alpha}_p \Delta P_t - \tilde{\theta}_{t+1} \right) \quad (48)$$

which is an instantaneous version of the goods market equilibrium in difference form. For this study's intents and purposes, a lagged version expressed in (output) gap format is needed however. Moreover, recall that  $r_t = i_t^p - \pi_t^e$ , in order to relate the real interest rate to expectations. Hence, in this study this equation will be used:

$$y_{t+n_p} - \bar{y}_{t+n_p} = \Phi_{\theta}(B)\theta_{t+n_p} - \alpha_r(i_t^p - \pi_t^e - \bar{r}_t^*) + \alpha_H \pi_t^H + \alpha_D \Delta D_t + \alpha_p \Delta P_t \quad (49)$$

$$\Phi_{\theta}(B) = 1 + \sum_{k=1}^{n_{\theta}} \phi_k^{\theta} B^k, \text{ where } \phi_1^{\theta} \neq 0 \text{ if } n_p > 1 \quad (50)$$

where  $y_t - \bar{y}$  is the output gap,  $\pi_t^H$  the housing market inflation,  $\Delta D_t \equiv \Delta \log V_t$  is the change in private debt, and  $\{\theta_t\}$  is an i.i.d. noise process with zero expectation and finite variance,  $\alpha_H, \alpha_r > 0, \alpha_D, \alpha_p \in \mathbb{R}$  are parameters,  $n_p$  is, as previously, the lag at which shocks take effect and  $n_{\theta}$  is the highest lag effective in the

new time series models of demand. In the alternative model, equations (49) and (50) replace the goods market equilibrium (7) and demand shocks (11) in Model 1 respectively.

The reader should observe that according to the model, the effects of housing price inflation and real interest rate have respectively a positive and negative impact on the output gap while the impacts of private indebtedness and construction costs are ambiguous. Indeed, as is shown in the regression results, construction costs seems to have a faded negative impact on the output gap, while any effect of private indebtedness is hardly observable. This is reasonable as increases in housing prices over time could prompt households to spend away their other financial assets (see e.g. Disney, Bridges & Gathergood, 2010, p. 494) driving up output and the negative effect of the real interest rate was already established for the standard goods market equilibrium in the beginning of this section. Moreover, any effect of increasing costs would entail higher prices, subsequently lower demand and finally diminishing output.

The inclusion of these parameters into the goods market equilibrium - while contested - is neither unreasonable nor controversial. Kearl and Mishkin (1977, p. 1583) argue that the demand for residential housing is one of the most volatile components of aggregate demand, subsequently affecting inflation and business cycles. Indeed, as Punzi (2013, pp. 609-610) points out, housing - being a durable good - yields a flow of housing services affecting aggregate demand and additionally represents a valuable asset that serves as a collateral for loans that could be used for further consumption. Other authors have included both housing prices, private indebtedness and even credit market dynamics into the demand side of the economy (e.g. Gelain, Lansing & Mendicino, 2012; Lambertini, Mendicino & Punzi, 2013). The minor novelty in the model of this study - to the best of this author's knowledge - is the inclusion of construction costs. Nevertheless, it is intuitive that if housing prices are relevant for driving the aggregate demand, so must be the cost of their construction.

The reader should observe that linear model in (49) assumes that private indebtedness, housing price inflation and construction costs operate *exclusively* in an exogenous manner. In other words, no effect of any other economic factor (e.g. inflation, output, unemployment, etc.) is assumed to influence the dynamics of either of the three factors. This is of course a simplifying aspect of the model.<sup>4</sup>

## 2.3 Optimal Interest Rule with Housing Market and Debt Dynamics

The derivation of the optimal interest rule where housing market and debt dynamics is included, is fairly similar to the previous one carried out for Model 1 in section (2.1). Setting the effect  $n_p = 1$  for simplicity, the general problem in (5) is similarly reduced:

$$\min_{i_t} \mathbb{E}_t[Q], \text{ where, } Q = \frac{1}{2} [\kappa_y(y_{t+1} - \bar{y})^2 + \kappa_u(u_{t+1} - \bar{u})^2 + (\pi_{t+2} - \pi^*)^2] \quad (51)$$

The arguments are equivalent to the ones in Section (2.2), as the central bank - in attempting to minimize the social cost given its definition in (6) - realizes that the nominal interest rate  $i_t = i_t^p - \bar{r}_t^*$ , will affect the two immediate following periods, affecting production one period ahead ( $y_{t+1} - \bar{y}_{t+1}$ ) through the alternative goods market equilibrium (49) and inflation two periods ahead ( $\pi_{t+2}$ ) through the Phillips curve (8). Its indirect effect on future production and inflation rates, however, can subsequently be adjusted directly, by tweaking future nominal rates of interest,  $i_{t+k}, k \geq 1$ .

Minimizing social loss, the alternative central bank rule for setting the optimal interest rate is given by the following expression:

$$\begin{cases} i_t^p = & \bar{r}_t^* + \pi_t^e + \tilde{R} [\pi_t - \pi^* + \gamma(y_t - \bar{y}_t)] + \frac{1}{\alpha_r} K_t \\ K_t \equiv & \alpha_H \pi_t^H + \alpha_D \Delta D_t + \alpha_P \Delta P_t \\ \tilde{R} \equiv & \frac{\gamma \phi}{\alpha_r (\kappa_y + \kappa_u) / (1 - \alpha)^2 + \gamma^2 \phi} \end{cases} \quad (52)$$

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<sup>4</sup>For studies seeking the impact of economic factors on housing prices and private indebtedness see Taylor (2007) or Eickmeier and Hofmann (2013). The latter authors find that monetary policy has a persistent on real estate wealth and private debt. Even Taylor (2007) found monetary policy to be a driver behind the housing and credit boom of the late 2000s.

For detailed calculations see Section (7.1.2) in the Appendix. As evident, this rule is very similar to the original optimal interest rate rule in (17). The major difference is the inclusion of the  $K_t$ -term which includes the housing market inflation  $\pi_t^H$ , difference in private debt  $\Delta D_t$  and the difference of the construction cost of new houses  $\Delta P_t$ .

Let us discuss for a moment the implications of the new rule in (52). As noted, this alternative rate-setting regime, just as the original rule (17), has positive dependency on the inflation gap  $(\pi_t - \pi^*)$ , i.e. the deviation of the inflation rate from the target. An increase in the inflation gap will induce the central bank to raise the the nominal rate of interest through their monetary instrument  $i_t^p$ , e.g. the repo rate in the case of the Swedish Riksbank. This means that the central bank contracts its policy to reduce inflation. A similar action will take effect with respect to the output gap. Indeed, higher demand is passed onto prices, rising the rate of inflation, hence a contraction is necessary. Some long-term and trend effects of available technology, competition levels, trade regulation, financial development, etc. are taken into account partly through the natural rate of real interest  $\bar{r}^*$  and partly through the long-term output  $\bar{y}$ . Moreover, higher inflation expectations  $(\pi^e)$  prompts the central bank to carry out a more contractive policy to rail in inflation. So far the components are the same as the original rule (17), which is a version of the Taylor (1993) rule.

The alternative rule (52), however, deviates from the original (17) in including the housing market inflation  $\pi_t^H$ , difference in private debt  $\Delta D_t$  and the difference of the construction cost of new houses  $\Delta P_t$ . According to the alternative rule, higher house price inflation prompts the central bank to conduct a more contractionary monetary policy by raising the monetary instrument  $i^p$ . This is reasonable since increases in housing prices over time could prompt households to spend away their other financial assets (see e.g. Disney, Bridges & Gathergood, 2010, p. 494) driving up output and subsequently prices and inflation. This feature was indeed incorporated into the alternative goods market equilibrium (49). The impacts of private indebtedness and construction costs are ambiguous for monetary policy, again a consequence of their ambiguous relation to output gap in the alternative goods market equilibrium (49). The ambiguous effect of the construction costs is reasonable as any effect of increasing costs would entail higher prices and rate of inflation while lowering demand and finally diminishing output. As is shown in the regression results however, construction costs seem to have a faded negative impact on the output gap, and hence prompting a more expansionary monetary policy by the central bank according to the rate setting regime (52). However, the results point to the conclusion that any effect of private indebtedness on output gap is hardly observable, making it irrelevant for the policy regime.

## 2.4 Methodological Framework

The general method consists of three main steps. First, the parameters in Model 1 will be estimated. These estimated parameters will subsequently be used to estimate the value parameters consisting of  $\kappa_y$  and  $\kappa_u$  in (6) and  $\phi$  in (5). As mentioned earlier, these parameters are not only unknown, but also estimating them cannot be carried out directly via regression. The reason is that we do not have access to the data regarding the social loss of the central bank. Hence, they are instead estimated indirectly through estimating  $R$  in the central bank's optimal rule for setting the interest rate (17). The value of  $R$  is related to the value parameters  $\kappa_y$ ,  $\kappa_u$  and  $\phi$ . In fact the expression:

$$M \equiv \frac{\gamma\phi}{(\kappa_y + \kappa_u)/(1 - \alpha)^2 + \gamma^2\phi} \quad (53)$$

is shared between both  $R$  in the original optimal rule for setting the interest rate (17) and  $\tilde{R}$  in the alternative rule (52). Hence, by estimating  $\alpha_2$  in (7) and  $R$  in (17), we can obtain  $M = \alpha_2 R$ . Subsequently, by first estimating  $\alpha_r$  in (49),  $\tilde{R}$  can be estimated as  $\tilde{R} = M/\alpha_r$ .

Hence, in the second step, we estimate the parameters in the model including housing market and debt dynamics, i.e. the Model 1 where the original goods market equilibrium (7) is substituted with the alternative version (49).<sup>5</sup> These new parameters together with  $M$  above from the first step are then employed to calibrate the parameters in the alternative optimal rule for setting the interest rate in (52).

The final step consists of simulating a counterfactual situation where the economy is allowed to develop ac-

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<sup>5</sup>Of course, the original equation insinuating demand persistence (11) is also replaced by the its alternative counterpart (50).

according to the alternative model and the alternative interest-setting regime. In practice, this is equivalent to simulations from the alternative model and the alternative interest-setting regime given the initial values of the different variables. Based on these simulations, an analysis will be done regarding the development of the social loss, inflation, output and unemployment. In particular, the success of the different interest rate rules will be compared in reducing the social loss and the gaps of inflation, output and unemployment.

### 3 Model Estimation

#### 3.1 Data Description

The quarterly time-series data is gathered from several different institutions. Households' one-year-ahead inflation expectations (excluding extreme values) is gathered from the National Institute of Economic Research (NIER)<sup>6</sup>. The repo rate<sup>7</sup> as well as the data for trend unemployment<sup>8</sup> is gathered from the Swedish Riksbank. The rest of the time-series data is collected from Statistics Sweden<sup>9</sup>. These include GDP (expenditure approach, current prices), inflation (CPI), construction cost index (CCI) for collectively built one- or two-dwelling buildings, real estate price index (REPI) for one- or two dwelling buildings, households' private debt, and the unemployment rate.

An issue that should be mentioned is the use of the *Hodrick-Prescott (HP) filter* to calculate certain trend levels. The HP-filter is defined as

$$HP = \sum_{t=1}^T (x_t - g_t) + \lambda \sum_{t=2}^T [(g_{t+1} - g_t) - (g_t - g_{t-1})]$$

where  $x_t$  is the data series of the variable in question,  $g_t$  is its trend levels and  $\lambda$  is an arbitrary smoothness parameter, which is set to 1600 as customary for quarterly data. The trend levels  $g_t$  are calculated by minimizing the value of the HP-filter above (Sørensen & Whitta-Jacobsen 2010, pp. 361-362). Specifically this filter was used to estimate the natural rate of real interest  $\bar{r}^*$  and the GDP-trend  $\bar{y}$ .

Another issue that should be pointed out is the presence of annual seasonality in the output gap ( $y_t - \bar{y}_t$ ) and unemployment gap ( $u_t - \bar{u}_t$ ). To avoid spurious regressions when regressing on these variables, an annual differentiation was performed. This is explicated in the models by the term  $\nabla_4 = 1 - B^4$ , where  $B$  is the one-period (i.e quarterly) lag operator. When subsequent simulation is performed this feature is also incorporated.

Variables	KPSS	ADF
$r - \bar{r}^*$	Y > [0.1000]	Y < [0.0010]
$\nabla_4(y - \bar{y})$	Y > [0.1000]	Y < [0.0010]
$\nabla_4(u - \bar{u})$	Y > [0.1000]	Y [0.0065]
$\pi^H$	N [0.0109]	Y < [0.0010]
$\Delta P$	Y > [0.1000]	Y < [0.0010]
$\Delta D$	N < [0.0100]	Y [0.0353]

Table 1: Stationarity tests of the economic variables: KPSS tests and Augmented Dickey-Fuller (ADF) test. Values in brackets are the p-values corresponding to the null hypothesis. The letters **Y** and **N** - indicating *Yes* and *No* respectively - summarize whether stationarity can be concluded at 95% significance. In the KPSS test, stationarity is the null hypothesis while the corresponding one for the ADF test is non-stationarity. Hence, in the former, high p-values indicate stationarity while for the latter the opposite holds.

The stationarity of the different variables is mentioned in Table 1. The tests used are the augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. In the ADF test, existence of unit roots (non-stationarity) is the null-hypothesis, while in the KPSS test the relation is reversed. The latter test is included to compensate for the general low power of the former. Indeed, not all series for which one cannot reject the unit hypothesis using the ADF test have necessarily orders of integration that exceed zero (Verbeek, 2012, pp. 261-267). For all the variables at least one of the tests suggest that the assumption of stationarity is plausible. Indeed, in most cases both of tests signal such plausibility.

<sup>6</sup> *Konjunkturinstitutet (KI)* in Swedish.

<sup>7</sup> The use of the repo rate as the monetary rate started at 1994Q2. Before that the *marginal rate* was used. Hence, in the data for the period 1992Q1-1994Q1, the marginal rate is used instead of the repo rate.

<sup>8</sup> From Monetary Policy Report October 2010 (Riksbank, 2010b).

<sup>9</sup> *Statistiska Centralbyrån (SCB)* in Swedish.

## 3.2 Estimation Methods & Diagnostics

Estimation, in this study, was done through OLS and the prediction error method (PEM).<sup>10</sup> In PEM, instead of minimizing the sum of the squared residuals between observed and fitted values, the sum of the difference between the squared residuals between observed values and first step predictions (i.e. the first step prediction errors) given the model are minimized (Jakobsson, pp. 155-8). The PEM and the usual OLS estimates (i.e. the ones using the fitted values) will coincide if the regressor (the independent variable) is uncorrelated with the measured noise values (the residuals) (Ali Akbari, 2014, p. 19). However, the PEM estimates generally have lower variance than the ones calculated through OLS. Additionally, the OLS regressions are accompanied by corresponding Newey-West hetereskedasticity-and-autocorrelation-consistent (NW-HAC) estimates (Verbeek, 2012, pp. 125-126.). In case moving average components of the noise process was introduced, their estimation was done *ex post* in OLS estimations, while done simultaneously in PEM.

Furthermore, in order to assure a comprehensive residual analysis several steps were conducted. To assess the independence criterion between noise and and the regressors, the crosscorrelation between the two was plotted. Moreover a 95% confidence interval (under the assumption of normality) was illustrated to judge the significance of the crosscorrelation values. The range of the lags for which crosscorrelation was investigated is plus and minus forty, indicating that up to four significant exceeding instances are allowed.<sup>11</sup> Moreover, as measures of fit the following were used:  $R^2$  (coefficient of determination), adjusted  $R^2$  and root mean square error (RMSE).

Additionally, in order to assess the whiteness of the residuals in the regressions the following five tests were invoked: the *Ljung-Box-Pierce test*, the *Monti test*, the *McLeod-Li test*, the *sign change test* and the *cumulative periodogram test*. All the tests were conducted at 1% rejection significance and hence their corresponding p-value is mentioned in the tables of regression results (except for the cumulative periodogram test which is a visual test where the cumulative periodogram should be within some confidence interval).<sup>12</sup> To assure normality of the residuals two tests were performed: the Jarque-Bera test and D'Agostino-Pearson's  $K^2$  test (D'AP- $K^2$ ). The latter performs better when sample size is small or medium (Jakobsson, 2013, p. 181). Oftentimes the normal probability plot is also illustrated.

Moreover, influence and outlier analysis was conducted. In order to identify potential influential points the measure of *leverage* was employed with the upper significance limit of 1 which implies that the leverage for such surpassing data point is so high as to force the regression line to pass exactly through that point (Rawlings, Pantula & Dickey, 1998, p. 361). In order to identify outlier points, the measure of *Cook's distance* - or simply *Cook's D* - was used which measures the effect of the data points on the regression parameter vector. Two cut-off thresholds are considered for the significance of Cook's D, namely, 1 and  $4/n$ , where  $n$  is the number of observations (Ibid., pp. 361-365). Exceeding these thresholds indicate high shifting influence of the data point upon the regression parameter. When performing OLS regressions, if any outlier was observed, a second estimation was performed while excluding the outlier(s). This is not possible, however, for the regressions employing PEM, as in said method the one-step-ahead predictions - rather than fitted values - are used to calculate the regression parameters. The eventual outliers are nonetheless mentioned even in such cases.

To produce the 95% confidence intervals of the simulation paths corresponding to the monetary variables the statistical tool of *bootstrap* was employed (Efron and Gong, 1983). Simply put, bootstraps are tools that aid the research process by providing numerical measures to establish the error or bias of the estimator in question (Ali Akbari, 2014, p. 16). They are invoked when it is reasonable to doubt whether the number of observations is large enough to guarantee a low enough variability, and hence, a high enough stability of the estimator. In our case, since the simulations are counterfactual, there is no path at all with which one could use to begin the estimation. Hence, this tool is crucial for the estimation.

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<sup>10</sup>The program used in order to conduct the calculations was the technical and numerical environment MATLAB. The functions used for the PEM estimations were mainly `pem` and also one instance of `bj` (for the estimation of the alternative goods market equilibrium in (58)). For the OLS regressions the function `fitlm` was employed.

<sup>11</sup>Four out of eighty is equal to five percent.

<sup>12</sup>See Jakobsson (2013, pp. 176-180) for details of these tests.

### 3.3 Results and Analysis

#### 3.3.1 Goods Market Equilibrium

For the goods market equilibrium the following final representation was chosen:

$$\nabla_4(y_{t+4} - \bar{y}_{t+4}) = \Phi_z(B)z_{t+4} - \alpha_2(i_t^p - \pi_t^e - \bar{r}_t^*) \quad (54)$$

with  $\pi_t^e = \pi_{t+4|t}^e$  as the one-year-ahead inflation expectations and demand shock according to  $\Phi_z(B) = 1 + \sum_{k=1}^2 \phi_k^z B^k$ , where it at least holds that  $\phi_1^z \neq 0$ . The variables in the equation are respectively the output gap ( $y_t - \bar{y}_t$ ), the real interest rate ( $r_t = i_t^p - \pi_t^e$ ), the natural rate of interest ( $\bar{r}_t^*$ ) and an i.i.d. normal noise process ( $\{z_t\}$ ) with zero expectation ( $\mathbb{E}[z_t] = 0$ ) and finite standard deviation ( $\sigma_z$ ). The equation above (54) is a version of the goods market equilibrium in (7) with effect lag being set to one year ( $n_p = 4$ ). As mentioned before, the output gap is differentiated with  $\nabla_4 = 1 - B^4$  in order to remove the annual seasonality in the data.

Table 2 shows the regression for the goods market equilibrium in (54) above. All the regressions indicate the parameter  $\alpha_2$  (corresponding to the effect of the real interest rate) being approximately equal to  $-1$ . These results are of the same order found by Laumas and Mehra (1977).<sup>13</sup> The significance of the parameters generally is high (except for  $\phi_2^z$  in PEM (II)), at least three out of five tests indicate whiteness of the residuals, and both normality tests are passed. Furthermore, a fairly satisfactory fit is observed with the coefficient of determination for e.g. the regression in the PEM (I) model being above 50%.

In all of the regressions, few leverage points of the residuals surpass the significance threshold indicating potential influence.<sup>14</sup> However, none of the corresponding measures of Cook's distance surpass any of the significance thresholds, which removes the possibility of the influential points being outliers. With respect to the cross-correlation functions, in the PEM (I) and (II) regressions up to four significant lags are observed. This is deemed acceptable as the confidence interval in the figures has 95% significance allowing for at most that number of exceedances. Moreover, these exceeding instances are also small in size. Hence, one can argue for the independence of the input and noise in the PEM (I) and (II) regressions. The same is not true for the OLS regression though as six exceedances are observed with at least one being significant in size.

#### 3.3.2 Short-Run Aggregate Supply

Table 3 shows the regression for the SRAS curve mentioned below.

$$\pi_{t+4} - \pi_t^e = \gamma \nabla_4(y_t - \bar{y}_t) + \Phi_s(B)s_{t+4} \quad (55)$$

with  $\pi_t^e = \pi_{t+4|t}^e$  as the one-year-ahead inflation expectations and supply shock according to  $\Phi_s(B) = 1 + \sum_{k=1}^2 \phi_k^s B^k$ , where it at least holds that  $\phi_1^s \neq 0$ . The variables in the equation are respectively the expected inflation gap ( $\pi_{t+8} - \pi_{t+4}^e$ ), the output gap ( $y_t - \bar{y}_t$ ) and an i.i.d. normal noise process ( $\{s_t\}$ ) with zero expectation ( $\mathbb{E}[s_t] = 0$ ) and finite standard deviation ( $\sigma_s$ ). The equation above (55) is a version of the SRAS curve in (9) with effect lag being set to one year ( $n_p = 4$ ). As mentioned before, the output gap is differentiated with  $\nabla_4 = 1 - B^4$  in order to remove the annual seasonality in the data.

Table 3 shows the regression for the goods market equilibrium in (55) above. All the regressions indicate the parameter  $\gamma$  (corresponding to the effect of the output gap) being approximately equal to 0.1. This result is of the same order as the stylized facts mentioned by Sørensen and Whitta-Jacobsen (2010, pp. 534 & 540).<sup>15</sup> The significance of the parameters generally is high, at least three out of five tests indicate whiteness of the residuals, and both normality tests are passed. Furthermore, a fairly satisfactory fit is observed with the

<sup>13</sup>These authors find the parameters be slightly lower and in the interval of  $(-0.2, -0.5)$ . However, their data concerns US in the period 1900-1974. Moreover, they include several different variables - amongst all the output - which reduces the effect of real interest rate, due to output and real interest rate being correlated.

<sup>14</sup>See section (7.2) in the Appendix for details. Figures 3, 4 and 5 show the diagnostics of the OLS, PEM (I) and PEM (II) regressions respectively.

<sup>15</sup>The values there are mentioned for US data from the period 1947-2007 and either static or adaptive expectations are used in the model estimations. The values are in the range of 0.08 - 0.36 with an outlier of  $-1$ .

coefficient of determination for e.g. the regression in the PEM (I) model being above 30%.

Method Submethod	OLS		PEM	
	Standard	NW-HAC	(I)	(II)
$\alpha_2$	-1.3154***	-1.3154**	-1.0370***	-0.8596**
S.E.	(0.2875)	(0.5300)	(0.3689)	(0.4023)
p-val.	[0.0000]	[0.0150]	[0.0061]	[0.0354]
$\phi_1^z$	0.6668***	0.6668***	0.6674***	0.6593***
S.E.	(0.0807)	(0.0898)	(0.0836)	(0.1259)
p-val.	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$\phi_2^z$				0.1574
S.E.				(0.1251)
p-val.				[0.2113]
<b>Fit</b>				
$R^2$ (%)	19.87		54.12	56.65
	44.19			
Adj. $R^2$ (%)	19.87		53.09	55.18
	44.19			
RMSE	0.0214		0.0192	0.0187
	0.0158			
<b>Whiteness test</b>				
Ljung-Box-Pierce	Y [0.0141]		N [0.0000]	N [0.0002]
McLeod-Li	Y [0.3746]		Y [0.6521]	Y [0.2660]
Monti	N [0.0004]		N [0.0001]	N [0.0002]
Sign change	Y [0.5176]		Y [0.1730]	Y [0.6002]
Cumul. periodogram	Y		Y	Y
<b>Normality test</b>				
Jarque-Bera	Y > [0.5]		Y [0.2127]	Y [0.2468]
D'AP- $K^2$	Y [0.4585]		Y [0.3076]	Y [0.3436]

Table 2: Regression results of the goods market equilibrium in (54). The estimations of  $\phi_1^z$  using OLS is done *ex post*, while all the estimations using PEM is performed simultaneously. In the fit section of the OLS method, the value at the top and bottom of each measure of fit corresponds to the *ex ante* and *ex post* regressions respectively. The whiteness and normality tests of the OLS regression is only reported for the *ex post* regressions as the *ex ante* regressions rarely passed any of the tests. Values in parentheses indicate standard errors and those in the brackets indicate p-values. The p-values of the whiteness and normality tests should be - contrary to the p-values of the parameters in the regression - a high value in order to pass the tests. The letters **Y** and **N** - indicating *Yes* and *No* respectively - summarize whether whiteness or normality can be concluded at 99% significance. (\*: 90% significance; \*\*: 95% significance; \*\*\*: 99% significance.)

In all of the regressions, few leverage points of the residuals surpass the significance threshold indicating potential influence.<sup>16</sup> Even fewer of the corresponding measures of Cook's distance surpass the significance thresholds, identifying some outliers. Excluding the outliers, however, does not affect the results significantly in the OLS regression in Table 3. With respect to the cross-correlation functions, in the OLS and PEM (II) regressions fewer than four are observed to have significant values, with slight exceedances. Hence, one can argue for the independence of the input and noise in the OLS and PEM (II) regressions. The same is not true for the PEM (I) regression though as seven exceeding instances are observed. However, none of them are very extreme. Hence independence is still arguably intact.

<sup>16</sup>For details of the diagnostics see section (7.2) in the Appendix. Figures 6, 7 and 8 show the diagnostics of the OLS, PEM (I) and PEM (II) regressions respectively.

Method Submethod	OLS			PEM	
	Standard	NW-HAC	Excl. Out.	(I)	(II)
$\gamma$	0.1335**	0.1335**	0.1186**	0.1138**	0.1041**
S.E.	(0.0537)	(0.0660)	(0.0556)	(0.0541)	(0.0449)
p-val.	[0.0149]	[0.0463]	[0.0361]	[0.0385]	[0.0227]
$\phi_1^s$	0.8442***	0.8442***	0.8846***	0.7349***	0.9712***
S.E.	(0.0598)	(0.0593)	(0.0613)	(0.0847)	(0.1018)
p-val.	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$\phi_2^s$					0.4595***
S.E.					(0.1066)
p-val.					[0.0000]
<b>Fit</b>					
$R^2$ (%)	28.87		27.70	34.84	53.89
	61.45		63.25		
Adj. $R^2$ (%)	28.87		27.70	33.30	52.25
	61.45		63.25		
RMSE	0.0128		0.0128	0.0088	0.0076
	0.0071		0.0069		
<b>Whiteness test</b>					
Ljung-Box-Pierce	Y [0.9423]		Y [0.6496]	N [0.0000]	N [0.0000]
McLeod-Li	Y [0.8144]		Y [0.5258]	Y [0.1620]	Y [0.8560]
Monti	Y [0.5172]		Y [0.3702]	Y [0.0282]	Y [0.4796]
Sign change	Y [0.6662]		Y [0.8273]	Y [0.0137]	Y [0.1634]
Cumul. periodogram	Y		Y	N	N
<b>Normality test</b>					
Jarque-Bera	Y [0.0579]		Y [0.0483]	Y > [0.5]	Y > [0.5]
D'AP- $K^2$	Y [0.0854]		Y [0.0605]	Y [0.6431]	Y [0.6134]

Table 3: Regression results of the SRAS in (55). The estimations of  $\phi_1^s$  using OLS is done *ex post*, while all the estimations using PEM is performed simultaneously. In the fit section of the OLS method, the value at the top and bottom of each measure of fit corresponds to the *ex ante* and *ex post* regressions respectively. The whiteness and normality tests of the OLS regression is only reported for the *ex post* regressions as the *ex ante* regressions rarely passed any of the tests. Values in parentheses indicate standard errors and those in the brackets indicate p-values. The p-values of the whiteness and normality tests should be - contrary to the p-values of the parameters in the regression - a high value in order to pass the tests. The letters **Y** and **N** - indicating *Yes* and *No* respectively - summarize whether whiteness or normality can be concluded at 99% significance. (\*: 90% significance; \*\*: 95% significance; \*\*\*: 99% significance.)

### 3.3.3 Okun's Law

Table 4 shows the regression for the goods market equilibrium mentioned below.

$$u_t - \bar{u}_t = \alpha_y(y_t - \bar{y}_t) + \Phi_{\vartheta}(B)\vartheta_t \quad (56)$$

with  $\Phi_{\vartheta}(B) = \nabla_4^{-1}(1 + \phi_1^{\vartheta}B)$ . The variables in the equation are respectively the output gap ( $y_t - \bar{y}_t$ ), the unemployment gap ( $u_t - \bar{u}_t$ ) and an i.i.d. normal noise process ( $\{\vartheta_t\}$ ) with zero expectation ( $\mathbb{E}[\vartheta_t] = 0$ ) and finite standard deviation ( $\sigma_{\vartheta}$ ) above (56) is a version of Okun's Law in (13). As mentioned before, both of the output and unemployment gaps are differentiated with  $\nabla_4 = 1 - B^4$  in order to remove the annual seasonality in the data.

Table 4 shows the regression for Okun's Law in (56) above. Except for the PEM (II) regression, all the other ones indicate the parameter  $\alpha_y$  (corresponding to the effect of the output gap) being approximately equal to  $-0.26$ . Apel and Jansson (1999) find the net effect of the output gap to be roughly of the same order,<sup>17</sup> and the findings here are even closer to the results of Prachowny (1993).<sup>18</sup> The significance of the parameters generally is high and at least one of the normality tests is passed in the OLS and PEM (I) regressions.

<sup>17</sup>They find the net effect to be around  $-2$  rather than  $-0.2$ . However, they are concerned with the period 1970-96, use a slightly different representation of the Okun's Law and their results are part of a larger state-space estimation of both the Phillips curve and Okun's Law through a Kalman filter. Hence, such small digression is reasonable.

<sup>18</sup>Prachowny (1993) finds the value to be about  $-0.6$ . The model, however, employs several different variables such as hours worked, used capital, etc. and is concerned with a more dated data sample from the USA.

However, the PEM (I) regression does not produce white noise, while introducing the persistence parameter  $\phi_1^\vartheta$  simultaneously in the PEM (II) regression seems to remove too much of the link between the output and unemployments gap, reducing  $\alpha_y$  in absolute value from 0.26 to about 0.08 in absolute values. Österholm (2016) - who uses a similar model to PEM (II) - also finds a value closer to the latter one, namely  $-0.06$ . The reduction above is also a steep break from the *ex ante* OLS regressions with values of  $\alpha_y$  close to  $-0.29$ . Moreover, the PEM (II) regression does not improve whiteness significantly as only one test (McLeod-Li) passes. Furthermore, normality of the residuals is worsened as none of the tests passes even the generous 99%-significance tests. Hence, the PEM (I) model is to be preferred over PEM (II) for the simulations later on, despite the lack of whiteness. Nevertheless, a fairly satisfactory fit is observed with the coefficient of determination for e.g. the regression in the PEM (I) model being above 36%.

In all of the regressions, few leverage points of the residuals surpass the significance threshold indicating potential influence.<sup>19</sup> Even fewer of the corresponding measures of Cook's distance surpass the significance thresholds, identifying some outliers. Excluding the outliers, however, does not affect the results significantly in the OLS regression in Table 4. With respect to the cross-correlation functions, in the OLS and PEM (I) regressions at most one lag is observed to have significant values. Hence, one can argue for the independence of the input and noise in the OLS and PEM (I) regressions. The same is not true for the PEM (II) regression though as about six or seven exceeding instances are observed. However, none of them are very extreme.

Method Submethod	OLS			PEM	
	Standard	NW-HAC	Excl. Out.	(I)	(II)
$\alpha_y$	-0.2913***	-0.2913***	-0.2921***	-0.2684***	-0.0820***
S.E.	(0.0369)	(0.0454)	(0.0350)	(0.0433)	(0.0277)
p-val.	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0039]
$\phi_1^\vartheta$	0.7941***	0.7941***	0.8110***		0.8573***
S.E.	(0.0638)	(0.0778)	(0.0711)		(0.0652)
p-val.	[0.0000]	[0.0000]	[0.0000]		[0.0000]
<b>Fit</b>					
$R^2$ (%)	40.67		43.93	37.40	72.61
	63.26		60.03		
Adj. $R^2$ (%)	40.67		43.93	36.73	72.02
	63.26		60.03		
RMSE	0.0090		0.0085	0.0090	0.0061
	0.0055		0.0052		
<b>Whiteness test</b>					
Ljung-Box-Pierce	Y [0.0946]		Y [0.0560]	N [0.0000]	N [0.0000]
McLeod-Li	Y [0.4242]		Y [0.7463]	N [0.0000]	Y [0.2882]
Monti	N [0.0045]		Y [0.0412]	N [0.0000]	N [0.0000]
Sign change	Y [0.8330]		Y [0.8292]	N [0.0000]	N [0.0054]
Cumul. periodogram	Y		Y	N	N
<b>Normality test</b>					
Jarque-Bera	Y [0.3293]		Y [0.1627]	Y [0.0171]	N [0.0095]
D'AP- $K^2$	Y [0.3083]		Y [0.2050]	N [0.0084]	N [0.0023]

Table 4: Regression results of Okun's Law in (56). The estimations of  $\phi_1^\vartheta$  using OLS is done *ex post*, while all the estimations using PEM is performed simultaneously. In the fit section of the OLS method, the value at the top and bottom of each measure of fit corresponds to the *ex ante* and *ex post* regressions respectively. The whiteness and normality tests of the OLS regression is only reported for the *ex post* regressions as the *ex ante* regressions rarely passed any of the tests. Values in parentheses indicate standard errors and those in the brackets indicate p-values. The p-values of the whiteness and normality tests should be - contrary to the p-values of the parameters in the regression - a high value in order to pass the tests. The letters **Y** and **N** - indicating *Yes* and *No* respectively - summarize whether whiteness or normality can be concluded at 99% significance. (\*: 90% significance; \*\*: 95% significance; \*\*\*: 99% significance.)

<sup>19</sup>For details see section (7.2) in the Appendix. Figures 6, 7 and 8 show the diagnostics of the OLS, PEM (I) and PEM (II) regressions respectively.

### 3.3.4 Optimal Rate-Setting Rule

Table 5 shows the regression for the optimal rate-setting rule mentioned below.

$$i_t^p - \pi_t^e - \bar{r}_t^* = R(\pi_t - \pi^* + \gamma \nabla_4(y_t - \bar{y}_t)) + \Phi_\eta(B)\eta_t \quad (57)$$

with  $\pi_t^e = \pi_{t+4|t}^e$  as the one-year-ahead inflation expectations,  $\pi^* = 2\%$  as the inflation target and  $\Phi_\eta(B) = (1 + \sum_{k=1}^2 \phi_k^\eta B^k)$ . The parameter  $\gamma$  is chosen from PEM (I) in Table 3. The variables in the equation are respectively the real interest rate  $r = i_t^p - \pi_t^e$ , the output gap  $(y_t - \bar{y}_t)$  and an i.i.d. normal noise process  $(\{\eta_t\})$  with zero expectation ( $\mathbb{E}[\eta_t] = 0$ ) and finite standard deviation ( $\sigma_\eta$ ) above (57) is a version of the original rate-setting rule in (17). As mentioned before, the output gap is differentiated with  $\nabla_4 = 1 - B^4$  in order to remove the annual seasonality in the data.

Method Submethod	OLS			PEM		
	Standard	NW-HAC	Excl. Out.	(I)	(II)	(III)
$R$	0.3394***	0.3394***	0.3432***	0.3394***	0.2468***	0.2324***
S.E.	(0.0524)	(0.0801)	(0.0534)	(0.0769)	(0.0666)	(0.0699)
p-val.	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0004]	[0.0013]
$\phi_1^\eta$	0.6710***	0.6710***	0.7786***		0.7745***	0.8442***
S.E.	(0.0797)	(0.0883)	(0.0784)		(0.0813)	(0.1212)
p-val.	[0.0000]	[0.0000]	[0.0000]		[0.0000]	[0.0000]
$\phi_2^\eta$						0.0898
S.E.						(0.1198)
p-val.						[0.4558]
<b>Fit</b>						
$R^2$ (%)	32.28		32.54	32.40	62.40	64.10
	45.13		54.63			
Adj. $R^2$ (%)	32.28		32.54	31.62	61.51	62.82
	45.13		54.63			
RMSE	0.0091		0.0092	0.0090	0.0070	0.0070
	0.0066		0.0060			
<b>Whiteness test</b>						
Ljung-Box-Pierce	Y [0.0745]		Y [0.6846]	N [0.0000]	N [0.0064]	Y [0.0325]
McLeod-Li	Y [0.8632]		Y [0.9808]	N [0.0042]	Y [0.6563]	Y [0.7110]
Monti	N [0.0026]		Y [0.5915]	N [0.0000]	N [0.0000]	N [0.0000]
Sign change	Y [0.1957]		Y [0.3770]	N [0.0009]	Y [0.7477]	Y [0.5919]
Cumul. periodogram	Y		Y	N	Y	Y
<b>Normality test</b>						
Jarque-Bera	Y [0.0347]		Y [0.0196]	Y > [0.5]	Y [0.0481]	Y [0.0283]
D'AP- $K^2$	Y [0.0299]		Y [0.0103]	Y [0.7315]	Y [0.0456]	Y [0.0214]

Table 5: Regression results of the optimal rate-setting rule in (57). The estimations of  $\phi_1^\eta$  using OLS is done *ex post*, while all the estimations using PEM is performed simultaneously. In the fit section of the OLS method, the value at the top and bottom of each measure of fit corresponds to the *ex ante* and *ex post* regressions respectively. The whiteness and normality tests of the OLS regression is only reported for the *ex post* regressions as the *ex ante* regressions rarely passed any of the tests. Values in parentheses indicate standard errors and those in the brackets indicate p-values. The p-values of the whiteness and normality tests should be - contrary to the p-values of the parameters in the regression - a high value in order to pass the tests. The letters **Y** and **N** - indicating *Yes* and *No* respectively - summarize whether whiteness or normality can be concluded at 99% significance. (\*: 90% significance; \*\*: 95% significance; \*\*\*: 99% significance.)

All regressions in Table 5 indicate that the parameter  $R$  is approximately between 0.23 and 0.34. These results are of the same order of the ones found by Clarida, Gali and Getler (1998).<sup>20</sup> According to Sørensen and Whitta-Jacobsen (2010, pp. 460-462) the value is between 0.25 and 0.5 for most central banks.<sup>21</sup> Taylor (1993) - after whom the monetary policy rule is widely named - recommended the value be set at 0.5. The

<sup>20</sup>The values found there are in the range 0.48 – 2.2. The lower range of values found in this study may be due to the other authors' use of slightly different models and the fact that their regression values concern monetary policy rules of other countries (namely Italy, France, UK, USA, Japan and Germany). Moreover, their results concern an older period, namely 1979-1993, which is characterized by higher inflation rates and a subsequently more intense attempts to deflate said rate.

<sup>21</sup>The central banks mentioned are that of Japan, Germany, United States and United Kingdom.

significance of the parameters generally is high (except for  $\phi_2^\eta$  in PEM (III)) and both of the normality tests are passed. However, the PEM (I) regression does not produce white noise. Nonetheless, the PEM (I) model will be preferred over PEM (II) for the simulations later on, due to the PEM (II) counterpart being problematic from a theoretical perspective. Moreover, the value of  $R$  in the PEM (I) regression is closer to its counterpart in the *ex ante* OLS regression. Nevertheless, a fairly satisfactory fit is observed with the coefficient of determination for e.g. the regression in the PEM (I) model being above 30%.

In all of the regressions, few leverage points of the residuals surpass the significance threshold indicating potential influence.<sup>22</sup> Even fewer of the corresponding measures of Cook's distance surpass the significance thresholds, identifying very few or no outliers. Excluding the outliers, however, does not affect the results significantly in the OLS regression in Table 5. With respect to the cross-correlation function, in the OLS regressions no lag is observed to have significant values. Hence, one can argue for the independence of the input and noise. However, this does not hold true for the PEM regressions as several exceeding instances are observed. However, none of them are very extreme and the lags showing significance far away from the scope of the model. Hence, independence is still tenable.

### 3.3.5 Alternative Goods Market Equilibrium

For the alternative goods market equilibrium the following final representation was chosen:

$$\nabla_4(y_{t+4} - \bar{y}_{t+4}) = \Phi_\theta(B)\theta_{t+4} - \alpha_r(i_t^p - \pi_t^e - \bar{r}_t^*) + \alpha_H\pi_t^H + \alpha_D\Delta D_t + \alpha_P\Delta P_t \quad (58)$$

with  $\pi_t^e = \pi_{t+4|t}^e$  as the one-year-ahead inflation expectations and demand shock according to  $\Phi_\theta(B) = 1 + \phi_1^\theta B$ , where  $\phi_1^\theta \neq 0$ . The variables in the equation are respectively the output gap ( $y_t - \bar{y}_t$ ), the real interest rate ( $r_t = i_t^p - \pi_t^e$ ), the natural rate of interest ( $\bar{r}_t^*$ ),  $\pi_t^H = \Delta \ln P_t^H$  as the quarterly inflation rate of housing prices,  $\Delta D$  the logarithmic and quarterly difference of the households' private debt,  $\Delta P_t$  as the quarterly difference of the construction costs and an i.i.d. normal noise process ( $\{\theta_t\}$ ) with zero expectation ( $\mathbb{E}[\theta_t] = 0$ ) and finite standard deviation ( $\sigma_\theta$ ). The equation above (58) is a version of the goods market equilibrium in (49) with effect lag being set to one year ( $n_p = 4$ ). As mentioned before, the output gap is differentiated with  $\nabla_4 = 1 - B^4$  in order to remove the annual seasonality in the data.

Table 7 shows the regression for the goods market equilibrium in (58) above. All the regressions indicate the parameter  $\alpha_r$  (corresponding to the effect of the real interest rate) being approximately equal to  $-1$  and lower than  $\alpha_2$  in the original goods market equilibrium (54) in absolute value. The significance of the parameters vary greatly. The least significant parameter is the debt  $\Delta D$ , indicating that it is not a good explanatory variable for the dynamics of the goods market equilibrium. This lack of significance is in line with the findings of Nickell (2004). Also the effects of the construction costs  $\Delta P$  seems to be far less significant than the other two remaining regressors. Moreover, the significance of the effect of the housing price inflation on GDP varies, which in turn is also in line with the results of Nickell (2004). Nevertheless, the author (ibid., p. 388) argues that the "current and future expected level of house price inflation will have a direct impact on monetary policy because of the effect on general inflation via household consumption growth."

Another issue of the regression is collinearity between the regressors. As observed in Table 6, the debt difference is highly correlated with both construction costs and inflation of housing prices. Omitting this variable, however, does not change the results significantly.<sup>23</sup> Nonetheless, a fairly satisfactory fit is observed with the coefficient of determination for e.g. the regression in the PEM (I) model being above 50%.

In all of the regressions, few leverage points of the residuals surpass the significance threshold indicating potential influence.<sup>24</sup> However, either one or none of the corresponding measures of Cook's distance surpass any

<sup>22</sup>For details see section (7.2) in the Appendix. Figures 12, 13, 14 and 15 show the diagnostics of the OLS, PEM (I), (II) and (III) regressions respectively.

<sup>23</sup>There is also other significant correlations observed between the construction costs  $\Delta P$  and the housing price inflations  $\pi^H$ , which is expected.

<sup>24</sup>For details see section (7.2) in the Appendix. Figures 16, 17 and 18 show the diagnostics of the OLS, PEM (I) and PEM (II) regressions respectively.

of the significance thresholds. Removing the outliers does not influence the results. With respect to the cross-correlation between the different inputs and the noise, in all of the regressions less than four significant lags are observed. This is deemed acceptable as the confidence interval in the figures has 95% significance allowing for at most that number of exceedances. Moreover, these exceeding instances are also small in size. Hence, one can argue for the independence of the input and noise in these regressions.

<b>Corr.</b>					<b>p-val.</b>				
	$\hat{r}_t$	$\Delta \ln p_t^H$	$\Delta P_t$	$\Delta D_t$		$\hat{r}_t$	$\Delta \ln p_t^H$	$\Delta P_t$	$\Delta D_t$
$\hat{r}_t$	1.0000	-0.2281	0.0246	-0.0843	$\hat{r}_t$	0.0000	0.0287	0.8156	0.4245
$\Delta \ln p_t^H$	-0.2281	1.0000	0.3254	0.4458	$\Delta \ln p_t^H$	0.0000	0.0000	0.0016	0.0000
$\Delta P_t$	0.0246	0.3254	1.0000	0.2618	$\Delta P_t$	0.8156	0.0016	0.0000	0.0117
$\Delta D_t$	-0.0843	0.4458	0.2618	1.0000	$\Delta D_t$	0.4245	0.0000	0.0117	0.0000

Table 6: The Spearman correlation coefficients (left) and the corresponding p-values (right) corresponding to the inputs of the alternative goods market equilibrium in (58). Low p-values indicate significant correlations.

Method Submethod	OLS			PEM	
	Standard	NW-HAC	Excl. Out.	(I)	(II)
$\alpha_r$	-1.0759***	-1.0759**	-0.9752***	-0.8622**	-0.8916**
S.E.	(0.2881)	(0.5018)	(0.2783)	(0.3847)	(0.3808)
p-val.	[0.0003]	[0.0349]	[0.0007]	[0.0277]	[0.0216]
$\alpha_H$	0.4271***	0.4271**	0.4350***	0.2374	0.2470*
S.E.	(0.1517)	(0.1759)	(0.1455)	(0.1452)	(0.1454)
p-val.	[0.0061]	[0.0173]	[0.0037]	[0.1058]	[0.0930]
$\alpha_p$	-0.7725**	-0.7725	-0.7364**	-0.4727	-0.3826
S.E.	(0.3555)	(0.5110)	(0.3411)	(0.3824)	(0.3628)
p-val.	[0.0326]	[0.1344]	[0.0337]	[0.2199]	[0.2947]
$\alpha_D$	0.0014	0.0014	0.0138	0.1330	
S.E.	(0.1684)	(0.2083)	(0.1615)	(0.1489)	
p-val.	[0.9934]	[0.9947]	[0.9323]	[0.3742]	
$\phi_1^\theta$	0.5739***	0.5739***	0.5682***	0.6839***	0.6681***
S.E.	(0.0867)	(0.0954)	(0.0929)	(0.0835)	(0.0858)
p-val.	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
<b>Fit</b>					
$R^2$ (%)	27.88		27.92	56.06	55.49
	33.74		30.64		
Adj. $R^2$ (%)	25.31		25.31	55.55	53.90
	33.73		30.64		
RMSE	0.0206		0.0197	0.0188	0.0189
	0.0161		0.0158		
<b>Whiteness test</b>					
Ljung-Box-Pierce	Y [0.0436]		Y [0.1842]	N [0.0006]	N [0.0004]
McLeod-Li	Y [0.0270]		Y [0.0867]	Y [0.6104]	Y [0.5000]
Monti	N [0.0058]		Y [0.0766]	N [0.0009]	N [0.0008]
Sign change	Y [0.8292]		Y [1.0000]	Y [0.3454]	Y [0.3454]
Cumul. periodogram	Y		Y	Y	Y
<b>Normality test</b>					
Jarque-Bera	Y > [0.5]		Y > [0.5]	Y [0.3639]	Y [0.3759]
D'AP- $K^2$	Y [0.9867]		Y [0.9297]	Y [0.4651]	Y [0.4765]

Table 7: Regression results of the goods market equilibrium in (58). The estimations of  $\phi_1^\theta$  using OLS is done *ex post*, while all the estimations using PEM is performed simultaneously. In the fit section of the OLS method, the value at the top and bottom of each measure of fit corresponds to the *ex ante* and *ex post* regressions respectively. The whiteness and normality tests of the OLS regression is only reported for the *ex post* regressions as the *ex ante* regressions rarely passed any of the tests. Values in parentheses indicate standard errors and those in the brackets indicate p-values. The p-values of the whiteness and normality tests should be - contrary to the p-values of the parameters in the regression - a high value in order to pass the tests. The letters **Y** and **N** - indicating *Yes* and *No* respectively - summarize whether whiteness or normality can be concluded at 99% significance. (\*: 90% significance; \*\*: 95% significance; \*\*\*: 99% significance.)

## 4 Simulations of the Alternative Rate-Setting Rule

In this section, we will produce the counterfactual simulations of the progression of the monetary variables according to the alternative goods market equilibrium and its corresponding optimal rate-setting rule. In section 3 it was illustrated that the alternative parameter  $\tilde{R}$  in of the corresponding rate-setting rule (52) is retrieved in accordance with  $\tilde{R} = \alpha_2 \cdot R / \alpha_r$ , where  $\alpha_2$  and  $\alpha_r$  are respectively the original and alternative parameters of the real interest rate in the different versions of the goods market equilibrium in (54) and (58), and  $R$  is the original parameter in the rate-setting rule (17), derived from the regression in (57). The regression results for PEM (I) models in tables 2, 5 and 7 provide the values  $\alpha_2 = -1.0368$ ,  $R = 0.3394$  and  $\alpha_r = -0.8622$  respectively. This yields the approximation  $\tilde{R} \approx 0.4081$ . Already we can see that this value is well within the 95% confidence interval of  $R$  in the original rate-setting rule (Table 5). Thus, one can already suspect that the results will not be conclusive.

In order to perform the simulations, the regression results which have the structure closest to the theoretical ones are chosen.<sup>25</sup> These coincide with the regression results for PEM (I) models in tables 2-7. In most cases, these models are also the ones with the best statistical qualities. For the simulations, trend values along with housing prices, construction costs and debt dynamics are assumed to be exogenous to the model. Moreover, between one to eight data points are assumed as initial values for the different parameters.<sup>26</sup> Furthermore, the standard deviations of the processes are set equal to the residual deviations from the corresponding regressions.<sup>27</sup> These simulations are conducted  $10^5$  times and then the mean and standard deviation over the simulation path is averaged (see Table 8). Moreover, 95% confidence interval of the simulation paths are produced using simple bootstraps (see Figure 1).

The simulated paths of repo and inflation rates via the alternative regime in Figure 1 are more jagged than the original one. There are two main reason for this phenomenon. First, according to the model used in this study, the central bank reacts continuously to the changes of the relevant factors. In reality, most central banks adjust the monetary policy instrument along certain discrete increments (Taylor, 2007, p. 466). Indeed the Riksbank usually has been debating over increments of 0.25 percentage points (Goodfriend & King, 2016). Incremental shifts will of course dampen the jaggedness. Moreover, the model used here puts a lot of weight on expectations, specifically that of the inflation rate with a one-year lag. Hence, this factor will emphasize the periodicity in the paths pertaining to the rates of repo and inflation as well. Furthermore, the models used in the simulations have fit of about 30 to 60%, so they do not explain all the dynamics of the processes. The low fit may generally be due to the noisiness of quarterly data.

Contrary to the paths of the monetary instrument and inflation rate in Figure 1, the simulated path of the output is smoother than the original one. This is partly due to the effects of the new inputs into the model, i.e. housing price inflation, debt and construction costs. Furthermore, recall that output (GDP) incorporated into the model is transformed several times, namely logarithmic transform, cyclical components extracted through the HP-filter and finally one-year differentiated. The last transformation removes a lot of the periodicity of the model. Moreover, the output gap, when examined, did indeed manifest an autoregressive nature of order one, while our regressions use moving average frameworks. As the reader knows one moving average component of order one is equivalent to an infinite number of autoregressive ones, which tend to have a smoothing effect (see e.g. Jakobsson, 2013). Also, as mentioned earlier, the models used in the simulations have fit of about 30 to 60%, so they do not explain all the dynamics of the processes.

Viewing Table 8, one sees that as expected, including the dynamics of private debt and housing prices will on average induce a more contractionary monetary policy with slightly higher repo rate with mean 2.9%, compared to the true value 2.4%. Furthermore, this repo-rate path is more volatile having higher standard deviation on average, indicating a more active and unpredictable rate-setting regime. On average, the path suggested by the alternative monetary policy would result in higher GDP (7.2E5 vs. 7E5 Million SEK), higher inflation

<sup>25</sup>By the theoretical models it is meant the ones mentioned in the box titled Model 1, with  $n_p = 4$ .

<sup>26</sup>One data point for inflation expectations and real interest rate, four data points for output gap and unemployment gap, and eight data points for inflation.

<sup>27</sup> $\sigma_\theta = 0.0194$ ,  $\sigma_s = 0.0088$  and  $\sigma_\theta = 0.0091$  corresponding to regressions (58), (55) and (56) respectively.

(2.5% vs. 1.5%) and lower unemployment rate (7% vs. 8%). Moreover, the volatility of the inflation path is increased significantly, which is a serious downside given the fact that a stable inflation path is arguably the most crucial of the Riksbank's goals. Indeed, while a 70% confidence interval<sup>28</sup> for the true inflation path lies approximately between one to three percent - the Riksbank's chief explicit goal until mid-2010 (Goodfriend & King 2016, p. 100) - the corresponding interval under the alternative regime is about zero to five percent. It is worth noting that some authors (e.g. Andersson & Jonung, 2016; Heikensten & Vredin, 1998, pp. 22-23) have argued that the policy interval 1-3 % is too narrow. Proponents of this idea might take issue with the line of argument above. Indeed, Andersson and Jonung, (2016) for instance suggest a tolerance interval of 0-4%, and might not mind the interval mentioned above. However, this author has to point out that the interval mentioned above (0-5%) has a measly significance of 70%. Requiring higher significances (e.g. 95% or 99%) would result in much larger intervals - including even negative inflation rates. While the width of the tolerance level could be debated, no author - to my knowledge - has suggested that an interval resulting in values below zero - i.e. deflation - is satisfactory. Hence, regardless of whether a widened tolerance interval is desirable or not, including private indebtedness and housing prices would render said interval far too wide for being prudent macroeconomic policy.

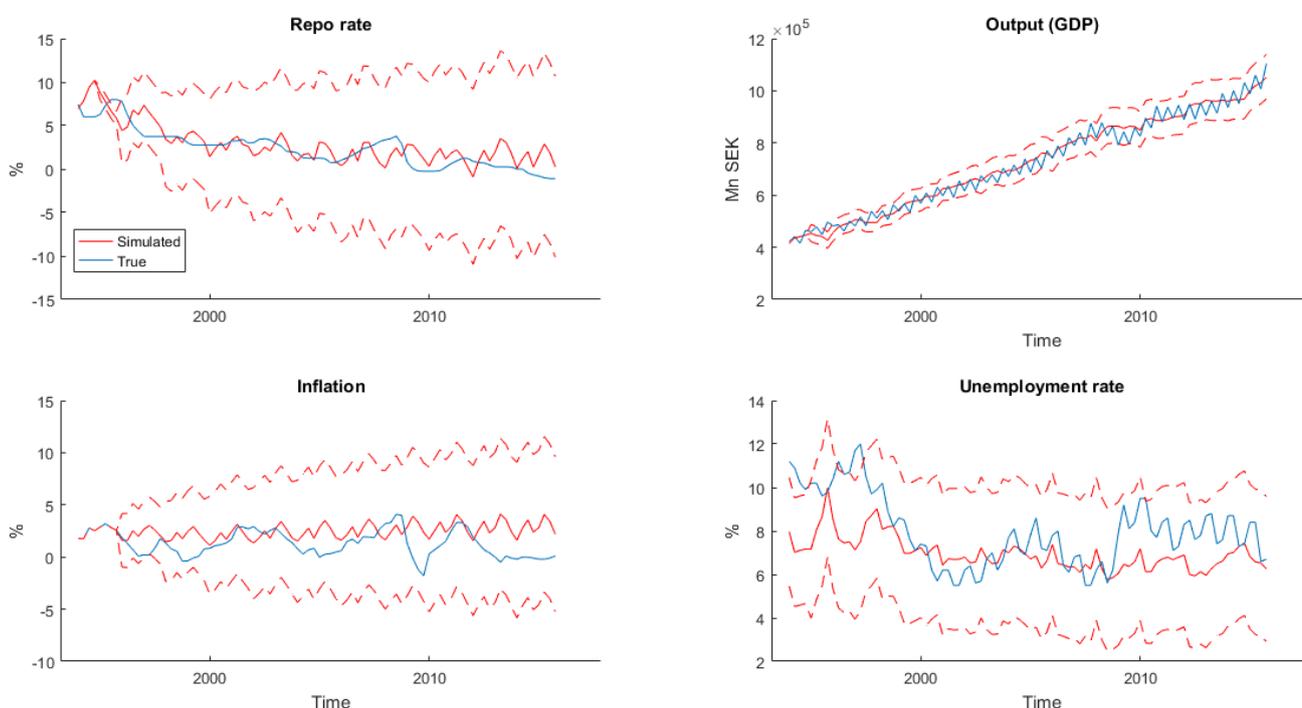


Figure 1: Simulations of the economic development under of the alternative optimal rate-setting regime using calibrated values from of the PEM (I) regression parameters from Tables 2-7 for the period 1993Q1-2015Q4. The red line in the subfigures is the mean of the  $10^5$  simulated paths, while the blue line is the true data path. The dashed red lines indicate the 95% significance interval.

		Repo rate (%)	GDP output (Mn SEK)	Inflation (%)	Unemployment rate (%)
Mean	True	2.3710	6.96E5	1.4816	8.0208
	Sim.	2.9146	7.24E5	2.4870	6.9642
Std.	True	(2.2109)	(1.95E5)	(1.4447)	(1.6602)
	Sim.	(3.8340)	(1.83E5)	(2.3281)	(0.7599)

Table 8: The mean and standard deviation of the monetary variables according to the simulation in comparison to the true values of the time-series data (1994Q1-2015Q4).

It should be noted, however, that almost the entire true paths<sup>29</sup> lie within the 95% confidence interval of the

<sup>28</sup>The mean plus and minus one standard deviation in Table 8.

<sup>29</sup>Except for a couple of points in the unemployment rate data.

simulated ones (Figure 1), which implies that the inclusion of these parameters into the decision-making process either would not on average make a significant impact on the outcome, or - at least - the data does not rule out the aforementioned prospect. Moreover, the unpredictable nature of the rate-setting regime creates more volatility in the path of inflation rate. Since a low and predictable inflation rate is arguably the most important goal of the Riksbank, this consequence is indeed a significant downside.

While the results above provide some insight, two aspects reduce their impact. The first one is that the period of the simulation is far too long. The second one, which is in a way a consequence of the first, is the width of the confidence interval rendering the significance of the impact of the new factors questionable.

It would thus be interesting to see which situation would arise over a shorter and more recent period of time, say 2010Q1-2015Q4. The result of such simulations is presented in Figure 2 and Table 9. These results show what would have happened if at the beginning of the year 2010, the Riksbank would *fully* commit to the inclusion of private indebtedness, housing price inflation and construction costs in accordance with the alternative model presented in this study. This period is interesting as it was around this junction that the Riksbank seems to have become concerned with the development of private indebtedness and housing prices and the implication of the development of these factors for their goals and the Swedish economy as a whole (Riksbank, 2010a; Goodfriend and King, 2016).

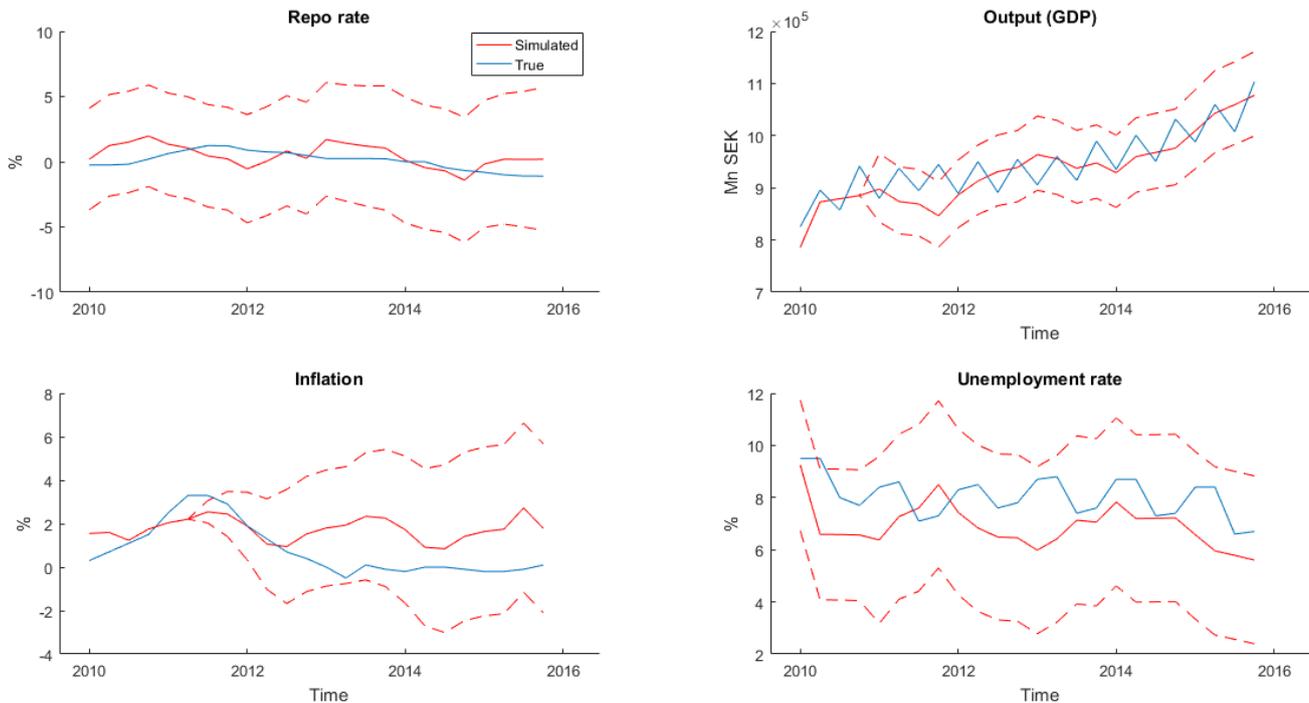


Figure 2: Simulations of the economic development under of the alternative optimal rate-setting regime using calibrated values from of the PEM (I) regression parameters from Tables 2-7 for the period 2010Q1-2015Q4. The red line in the subfigures is the mean of the  $10^5$  simulated paths, while the blue line is the true data path. The dashed red lines indicate the 95% significance interval.

		Repo rate (%)	GDP output (Mn SEK)	Inflation (%)	Unemployment rate (%)
Mean	True	0.0920	9.46E5	0.7792	8.0417
	Sim.	0.4950	9.34E5	1.7424	6.9129
Std.	True	(0.7016)	(6.49E4)	(1.1847)	(0.7901)
	Sim.	(0.9542)	(6.90E4)	(1.7597)	(0.8374)

Table 9: The mean and standard deviation of the monetary variables according to the simulation in comparison to the true values of the time-series data (2010Q1-2015Q4).

The results are really interesting. Let us first focus on Figure 2. The figure shows that if the Riksbank had fully incorporated the effects of the housing and debt markets at the beginning of the year 2010, it would have conducted a much more contractionary monetary policy, i.e. a higher repo rate (see also Table 9). As a result of this contraction, on average a significant dip in output and a surge in unemployment is observed at the end of the year 2011 and beginning of the year 2012. These factors are indicators of a contraction in the economy borderline to a crisis. Indeed, while the inflation rate is close to the target at this period of time in the simulations, having this level of inflation while the output is significantly contracting and unemployment surging could spiral into stagflation, especially if the model were to expand to include the reaction of credit and stock market dynamics as well. Such contraction is furthermore detrimental to the goals of the Riksbank.

As illustrated in Table 9, output will be generally lower for this period under the alternative regime, due to the more contractionary monetary policy, i.e. higher repo rate. The inflation rate has a higher level on average and closer to the target while the rate of unemployment is lower for the most part. Nevertheless, the original paths of the rates of inflation and unemployment lie within the 95% confidence interval under the alternative regime making the figures once again non-conclusive.

One should also note that the volatility of the inflation rate is once again higher under the alternative regime (1.76% vs. 1.18%). Indeed, a 95% confidence interval lies between  $(-1.7, 5.2)\%$  as opposed to  $(-1.5, 3.8)\%$  under the original regime. The latter is completely contained within the first. The volatility of unemployment and output is also marginally increased under the alternative regime. These issues further suggest the factors of housing prices, private debt and construction costs not to be prudent indicators for formulating monetary policy, especially with respect to the credibility of the Riksbanks goals.

These findings seem to nonetheless overall confirm the recommendation of Goodfriend and King (2016) to give macro-prudential powers to another agency - e.g. Finansinspektionen - when setting the repo rate, and subsequently the concurring nods of the several other authors and agencies (Svensson, 2016; Swedish National Audit Service (Riksrevisionen), 2016; Swedish National Debt Office (Riksgälden), 2016). Indeed, the results suggest that these factors - housing price inflation, household indebtedness and construction costs - be given little weight in routine adjustments of the repo rate.

## 5 Concluding Remarks

The results are a bit ambivalent, even though a larger abundance of the results point more clearly towards a certain conclusion. On the one hand, the mean of variables in the counterfactual simulations indicate that the alternative rate-setting regime would better suit the Riksbank's goals. It results in higher inflation rate (close to 2%), higher GDP path, and lower unemployment. Additionally, the volatility of the inflation path is increased significantly, which is a significant downside given the fact that a stable inflation path is arguably the most crucial of the Riksbank's goals. Indeed, while a 70% confidence interval for the true inflation path lies approximately between one to three percent, the corresponding interval in the alternative path is about zero to five percent.

However, the true simulation path lies within the 95% bootstrapped confidence interval of the simulated paths implying that adopting the alternative rate-setting regime may not have resulted in a path similar to the true one. Another point to make here is regarding the fit and significance of the parameters in the original and alternative regressions of the goods market equilibrium in (54) and (58). First, the alternative and original regression results do not differ significantly with respect to fit despite the alternative goods market equilibrium having three more explanatory variables (compare results in Tables 2 and 7). Indeed, the other variables (debt, housing prices and constructions costs) do not seem to be significant variables in explaining the dynamics of the demand side, especially when the regression is done simultaneously (e.g. in PEM (I)). The most extreme case is of course private debt, which has very little explanatory power. Moreover, the fit of the models are only moderate (between 30 to 60%), indicating that there is still a fair amount of dynamics not explained the model utilized.

In short, the answer to the question whether the dynamics of private debt and housing prices should be taken into account when the Riksbank sets the repo rate is a cautious no. The negative response is motivated by the high volatility of the simulated inflation path and the lack of significance that the housing market and private debt demonstrate in explaining the dynamics of the goods market equilibrium. Indeed, as the dynamics of these variables are more indicative of financial stability, their monitoring is better left to *Finansinspektionen* - Sweden's financial supervisory authority (Goodfriend & King; 2016; Svensson, 2016; Swedish National Audit Service (Riksrevisionen), 2016; Swedish National Debt Office (Riksgälden), 2016). A more formalized relationship between the *Finansinspektionen* and Riksbank is nevertheless needed so as to facilitate decision-making, if *Finansinspektionen* deems monetary policy to be detrimental to financial stability (NIER, 2016; Riksbank, 2016). Caution in the conclusion above is nonetheless motivated due to the fact that the true path lies indeed within the 95% confidence interval of the simulated path and moderate fit rendering any conclusion indecisive.

In fact, a decisive verdict in this respect requires probably more data - preferably panel data from different countries which could serve as a topic for future studies. Such a study should focus first on further determining the level of significant impact which the housing market and the private debt dynamics have on the goods market equilibrium. Second, such study should look into the volatility that the alternative regime has on the inflation path. Another approach suitable for future studies is rendering the assumption regarding forward-looking households. Gelain, Lansing and Mendicino (2012), for instance, allow for a subset of agents to employ simple moving-average forecast rules. This approach together with the loan-to-value model of Lambertini, Mendicino and Punzi (2013) could internalize the dynamics of housing prices, private indebtedness and credit markets which in the model of this study - for the case of simplicity - is assumed to be exogenous.

Moreover, the models utilized in this study pertain to an event of general equilibrium, excluding financial crises. Hence our conclusion exclusively envelop routine rate-setting deliberations during normal and stable periods. Hence, another approach for future studies could be expanding the model to include crises. Such expansion could be done for instance by modifying the no-Ponzi-game rule in (22) to be credible only within certain threshold. This threshold, in turn, could be dependent on some criteria for credibility, for instance the ratio of debt to income or their predicted values within certain periods in near future.

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## 7 Appendix

### 7.1 Detailed Calculations

#### 7.1.1 Optimal Interest Rate Rule

Consider the expression  $Q$  in (16). Of course, in order to minimize this expression we look at the first-order condition:

$$\frac{d}{d i_t} \mathbb{E}_t [Q] = \mathbb{E}_t \left[ \frac{d}{d i_t} Q \right] = 0 \quad (59)$$

The optimal point here is furthermore a minimum due to the quadratic nature of  $Q$ . The expression in  $Q$  consists of three main components: the output gap  $y_{t+1} - \bar{y}$ , the unemployment gap  $u_{t+1} - \bar{u}$  and the inflation gap two periods ahead  $\pi_{t+2} - \pi^*$ . The first component, i.e. the output gap, is replaced by its expression given in the goods market equilibrium (7). In doing so, the derivative of the square of this component becomes the following:

$$\mathbb{E}_t \left[ \frac{d}{d i_t} (y_{t+1} - \bar{y})^2 \right] = \mathbb{E}_t [-2\alpha_2(\Phi_z(B)z_{t+1} - \alpha_2(i_t - \pi_t^e))] = 2\alpha_2^2(i_t - \pi_t^e) \quad (60)$$

Since,  $\mathbb{E}_t[\Phi_z(B)z_{t+1}] = 0$ . Similarly, one can replace the unemployment gap component in  $Q$  with an expression on the output gap using Okun's law in (13), and subsequently replacing the the expression for the output gap with that of the inflation gap using the goods market equilibrium in (7) as done in the previous step. In doing so, the derivative of the square of this component becomes the following:

$$\mathbb{E}_t \left[ \frac{d}{d i_t} (u_{t+1} - \bar{u})^2 \right] = \mathbb{E}_t \left[ -2 \frac{\alpha_2}{(1-\alpha)^2} (\Phi_z(B)z_{t+1} - \alpha_2(i_t - \pi_t^e) + \vartheta_{t+1}) \right] = 2 \frac{\alpha_2^2}{(1-\alpha)^2} (i_t - \pi_t^e) \quad (61)$$

Expressing the inflation gap two periods ahead in terms of the known parameters is slightly more complicated. First, we express  $\pi_{t+2}$  using the SRAS curve in (9):

$$\pi_{t+2} - \pi^* = \pi_{t+1}^e - \pi^* + \gamma(y_{t+1} - \bar{y}) + \Phi_s(B)s_{t+2} + \frac{\vartheta_{t+1}}{1-\alpha} \quad (62)$$

The reader should know that the inflation expectations are unknown at period  $t+1$ . As the inflation expectations are assumed to be exogenous to the model, the central bank needs to estimate the expectations. Inflation expectations in the next period could be estimated by employing SRAS for  $t+1$  where  $\pi_t^e$  is set equal to current inflation  $\pi_t$ .<sup>30</sup>

$$\pi_{t+1} = \pi_t + \gamma(y_t - \bar{y}) + \Phi_s(B)s_{t+1} + \frac{\vartheta_t}{1-\alpha} \quad (63)$$

which leads to

$$\pi_{t+1}^e = \pi_{t+1|t}^e = \mathbb{E}_t [\pi_{t+1}] = \pi_t + \gamma(y_t - \bar{y}). \quad (64)$$

since  $\mathbb{E}[\Phi_s(B)s_{t+1}] = \mathbb{E}[\vartheta_t] = 0$  by assumption. Replacing (64) and the goods market equilibrium (7) into (62) results in the following expression:

$$\pi_{t+2} - \pi^* = \pi_t - \pi^* + \gamma(y_t - \bar{y}) + \gamma[\Phi_z(B)z_{t+1} - \alpha_2(i_t - \pi_t^e)] + \Phi_s(B)s_{t+2} + \frac{\vartheta_{t+1}}{1-\alpha}$$

---

<sup>30</sup>The reason  $\pi_t^e$  is set equal to current inflation  $\pi_t$ , is that the households and the central bank has information regarding the actual level of inflation at time  $t$ , and hence does not need to predict it.

which in turn yields the following:

$$\mathbb{E}_t \left[ \frac{d}{d i_t} (\pi_{t+2} - \pi^*)^2 \right] = -2\alpha_2 \gamma (\pi_t - \pi^* + \gamma(y_t - \bar{y})) + 2\alpha_2^2 \gamma^2 (i_t - \pi_t^e). \quad (65)$$

since  $\mathbb{E}[\Phi_s(B)_{s_{t+2}}] = \mathbb{E}[\vartheta_{t+1}] = 0$  by assumption. Employing (59) together with expressions for  $Q$ 's components in (60), (61) and (65), the the first order condition results in the equation below:

$$\frac{d}{d i_t} \mathbb{E}_t [Q] = \left( \kappa_y \alpha_2^2 + \frac{\kappa_u \alpha_2^2}{(1-\alpha)^2} + \alpha_2^2 \gamma^2 \phi \right) (i_t - \pi_t^e) - \alpha_2 \gamma \phi [\pi_t - \pi^* + \gamma(y_t - \bar{y})] = 0.$$

Rewriting the expression above, the central bank rule for setting the optimal interest rate is given by the following expression:

$$\begin{cases} i_t - \pi_t^e = R (\pi_t - \pi^* + \gamma(y_t - \bar{y})) \\ R \equiv \frac{\gamma \phi}{\alpha_2 (\kappa_y + \kappa_u / (1-\alpha)^2 + \gamma^2 \phi)} \end{cases} \quad (66)$$

### 7.1.2 Alternative Optimal Interest Rate Regime

Consider the expression  $Q$  in (51). Minimization requires that:

$$\frac{d}{d i_t} \mathbb{E}_t [Q] = \mathbb{E}_t \left[ \frac{d}{d i_t} Q \right] = 0 \quad (67)$$

The rest of the calculations are identical to the line of argument in (60)-(65) with two changes: First, all  $\alpha_2$  are replaced by  $\alpha_r$ , the new coefficient of  $i_t$  in the alternative goods market equilibrium (49). Second, every expression  $z_{t+1} - \alpha_2(i_t - \pi_t^e)$  is replaced by its new counterpart in (49),  $\theta_{t+1} - \alpha_r(i_t - \pi_t^e) + \alpha_\pi \pi_t^H + \alpha_D \Delta D_t + \alpha_P P_t$ . Indeed, equivalents to the (60), (61) and (65) are the following three equations respectively:

$$\mathbb{E}_t \left[ \frac{d}{d i_t} (y_{t+1} - \bar{y})^2 \right] = 2\alpha_r^2 (i_t - \pi_t^e) - 2\alpha_r K_t, \quad (68)$$

$$\mathbb{E}_t \left[ \frac{d}{d i_t} (u_{t+1} - \bar{u})^2 \right] = \frac{2\alpha_r^2}{(1-\alpha)^2} (i_t - \pi_t^e) - \frac{2\alpha_r}{(1-\alpha)^2} K_t, \quad (69)$$

$$\mathbb{E}_t \left[ \frac{d}{d i_t} (\pi_{t+2} - \pi^*)^2 \right] = 2\alpha_r^2 \gamma^2 (i_t - \pi_t^e) - 2\alpha_r \gamma (\pi_t - \pi^* + \gamma(y_t - \bar{y}) + \gamma K_t), \quad (70)$$

where  $K_t \equiv \alpha_H \pi_t^H + \alpha_D \Delta D_t + \alpha_P \Delta P_t$ . The resulting first-order condition is the following:

$$\frac{d}{d i_t} \mathbb{E}_t [Q] = \alpha_r^2 \left( \kappa_y + \frac{\kappa_u}{(1-\alpha)^2} + \gamma^2 \phi \right) (i_t - \pi_t^e) - \alpha_r \left( \kappa_y + \frac{\kappa_u}{(1-\alpha)^2} + \gamma^2 \phi \right) K_t - \alpha_r \gamma \phi [\pi_t - \pi^* + \gamma(y_t - \bar{y})] = 0.$$

Rewriting the expression above, the alternative central bank rule for setting the optimal interest rate is given by the following expression:

$$\begin{cases} i_t - \pi_t^e = \tilde{R} [\pi_t - \pi^* + \gamma(y_t - \bar{y})] + \frac{1}{\alpha_r} K_t \\ K_t \equiv \alpha_H \pi_t^H + \alpha_D \Delta D_t + \alpha_P \Delta P_t \\ \tilde{R} \equiv \frac{\gamma \phi}{\alpha_r (\kappa_y + \kappa_u / (1-\alpha)^2 + \gamma^2 \phi)} \end{cases} \quad (71)$$

## 7.2 Diagnostic Figures

### 7.2.1 Goods Market Equilibrium

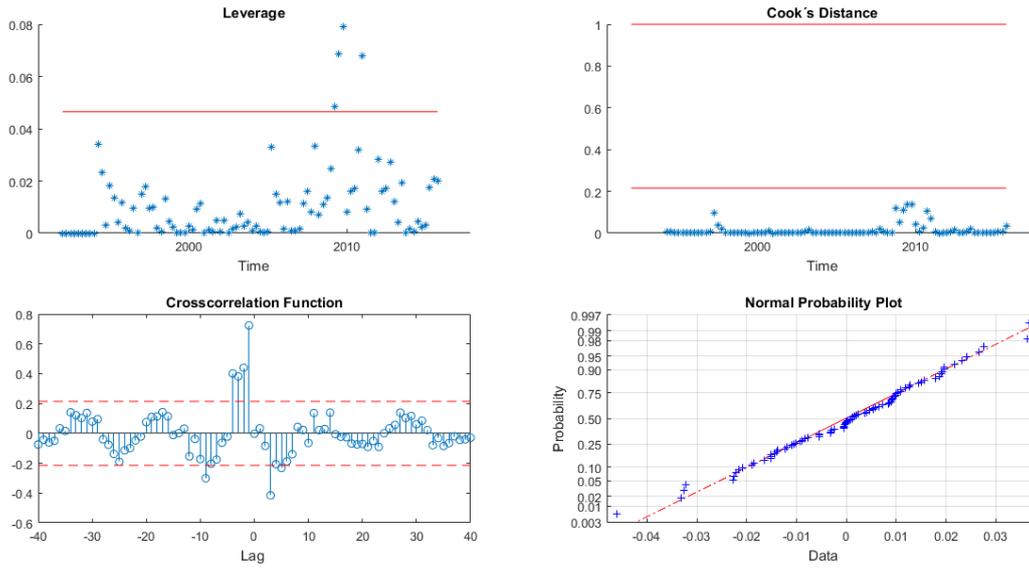


Figure 3: Diagnostics of the OLS regression of the goods market equilibrium in (54) with results in Table 2. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. No outliers are observed.

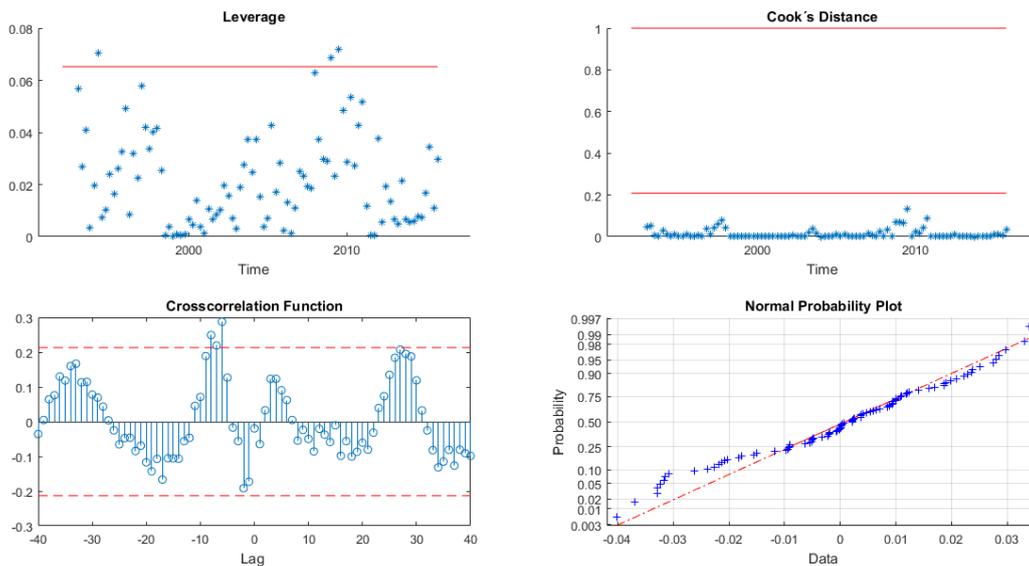


Figure 4: Diagnostics of the PEM (I) regression of the goods market equilibrium in (54) with results in Table 2. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. No outliers are observed.

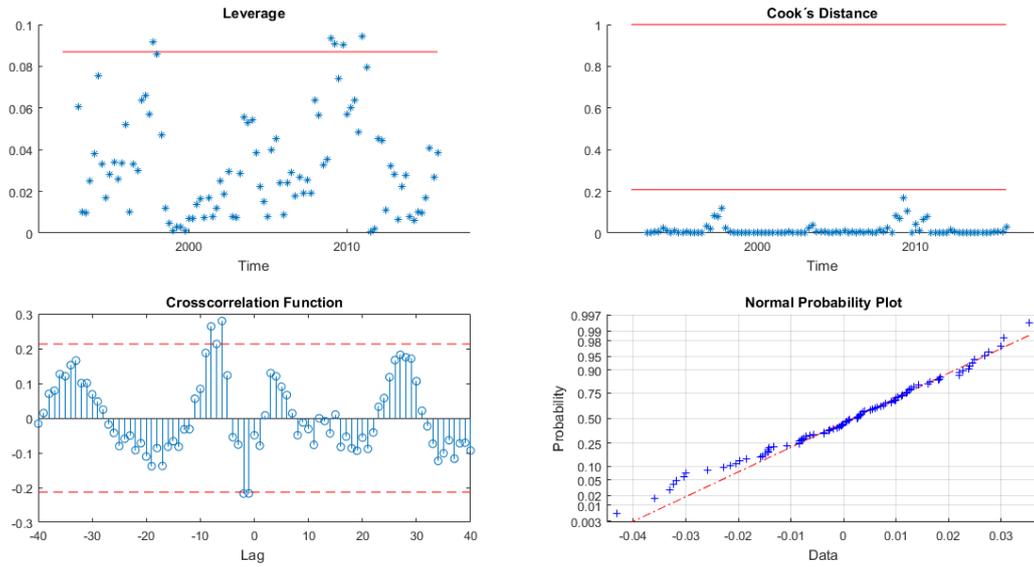


Figure 5: Diagnostics of the PEM (II) regression of the goods market equilibrium in (54) with results in Table 2. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. No outliers are observed.

### 7.2.2 Short-Run Aggregate Supply

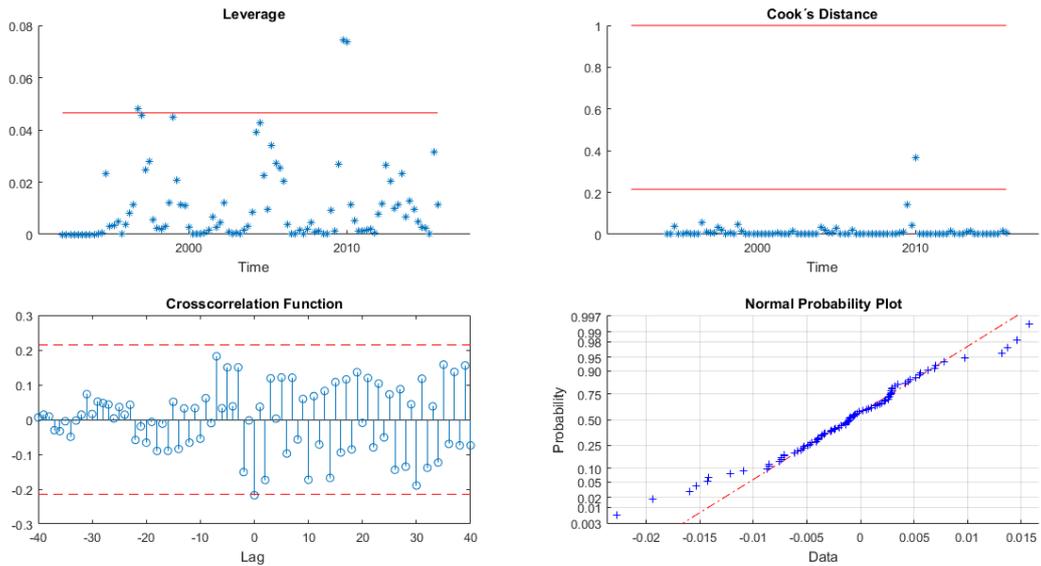


Figure 6: Diagnostics of the OLS regression of the SRAS curve in (55) with results in Table 3. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outliers observed corresponds to 2010Q1.

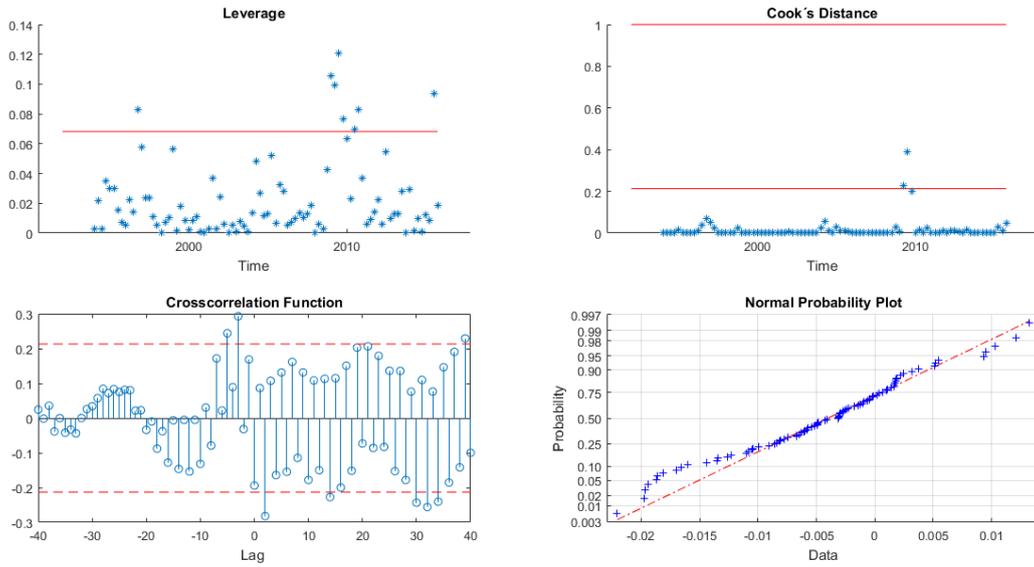


Figure 7: Diagnostics of the PEM (I) regression of the SRAS curve in (55) with results in Table 3. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outliers observed correspond to 2009Q2 and Q3.

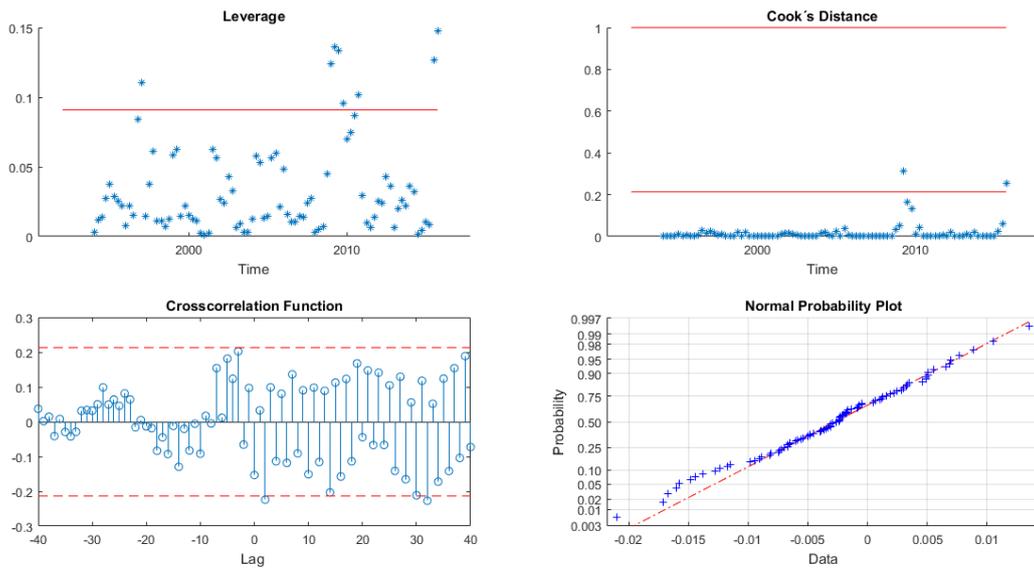


Figure 8: Diagnostics of the PEM (II) regression of the SRAS curve in (55) with results in Table 3. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outliers observed correspond to 2009Q2 and 2015Q4.

### 7.2.3 Okun's Law

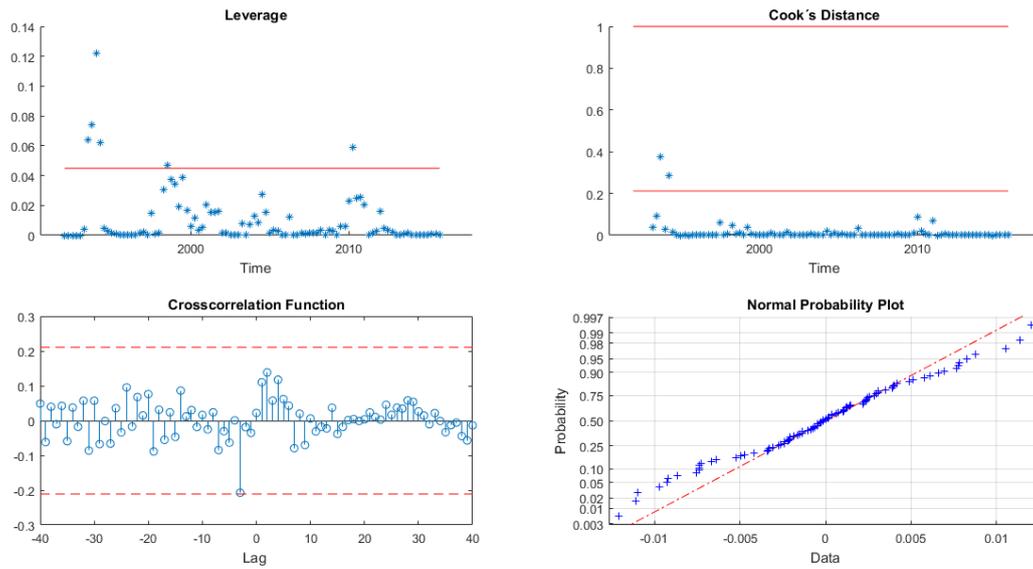


Figure 9: Diagnostics of the OLS regression of Okun's Law in (56) with results in Table 4. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outliers observed corresponds to 1993Q4 and 1994Q2.

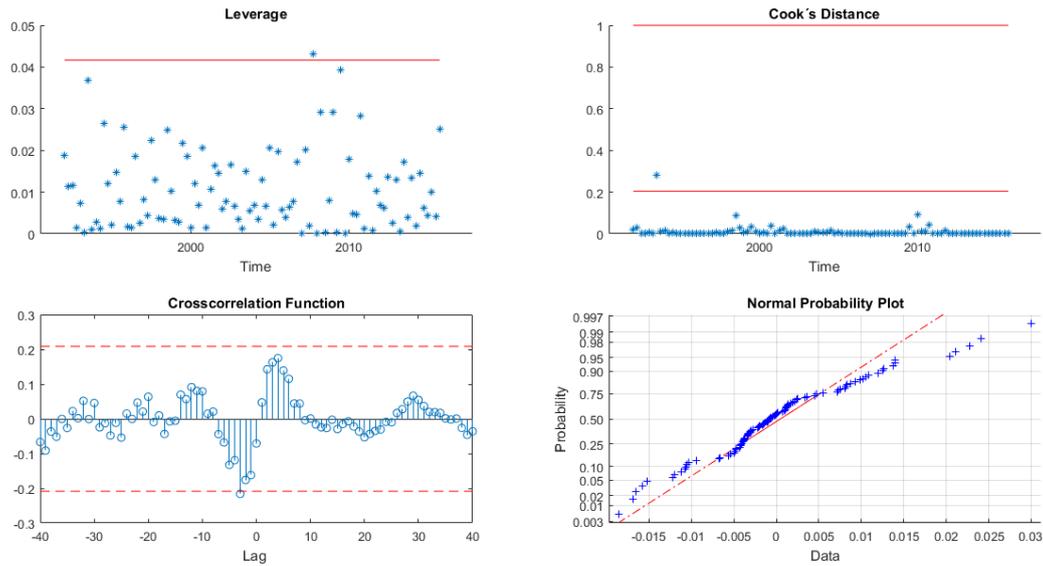


Figure 10: Diagnostics of the PEM (I) regression of Okun's Law in (56) with results in Table 4. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outlier observed corresponds to 1993Q3.

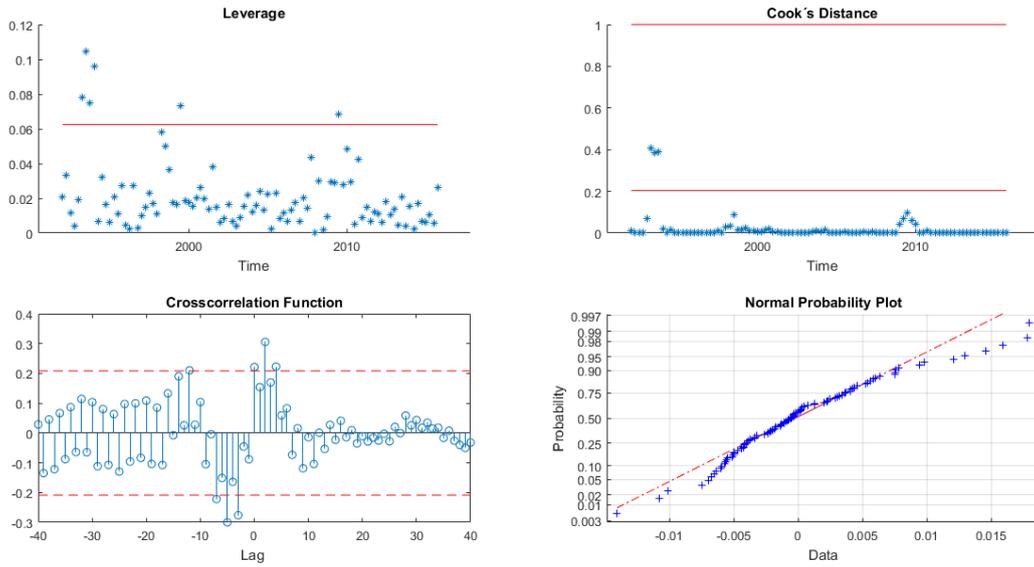


Figure 11: Diagnostics of the PEM (II) regression of Okun's Law in (56) with results in Table 4. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outliers observed correspond to 1993Q2, Q3 and Q4.

### 7.2.4 Optimal Rate-Setting Rule

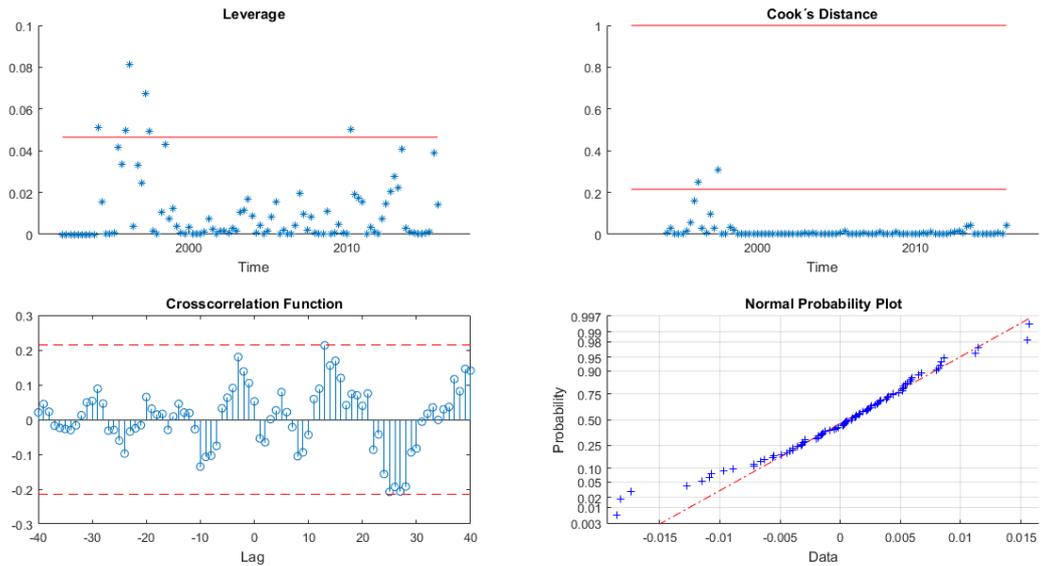


Figure 12: Diagnostics of the OLS regression of the optimal rate-setting rule in (57) with results in Table 5. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outliers observed corresponds to 1996Q2 and 1997Q3.

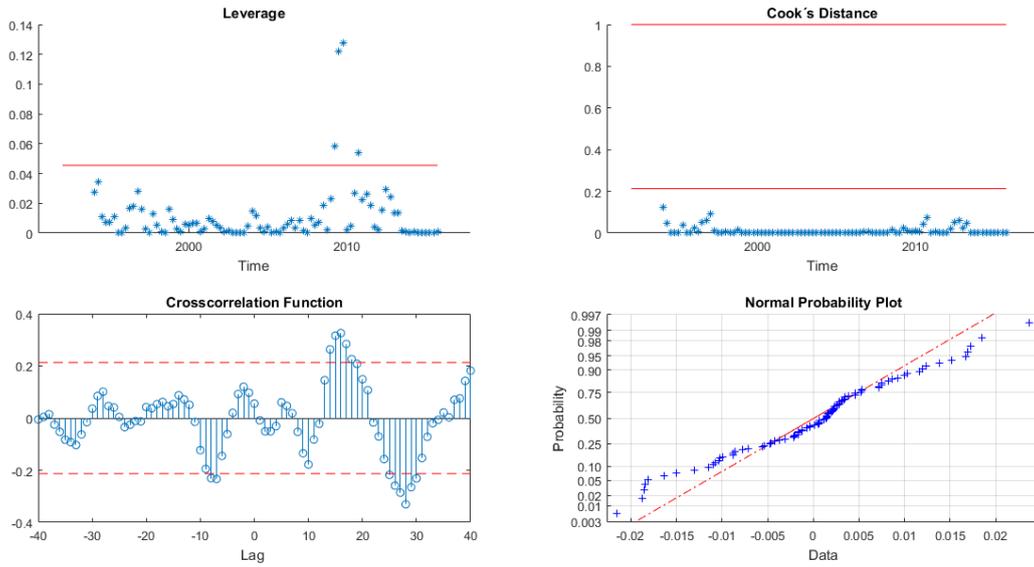


Figure 13: Diagnostics of the PEM (I) regression of the optimal rate-setting rule in (57) with results in Table 5. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. No outliers observed.

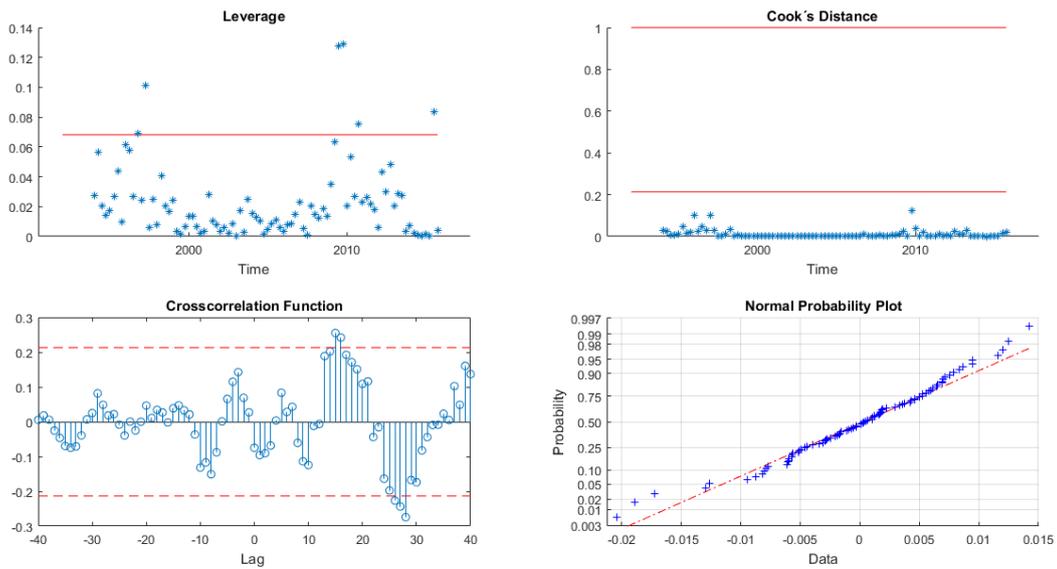


Figure 14: Diagnostics of the PEM (II) regression of the optimal rate-setting rule in (57) with results in Table 4. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. No outliers observed.

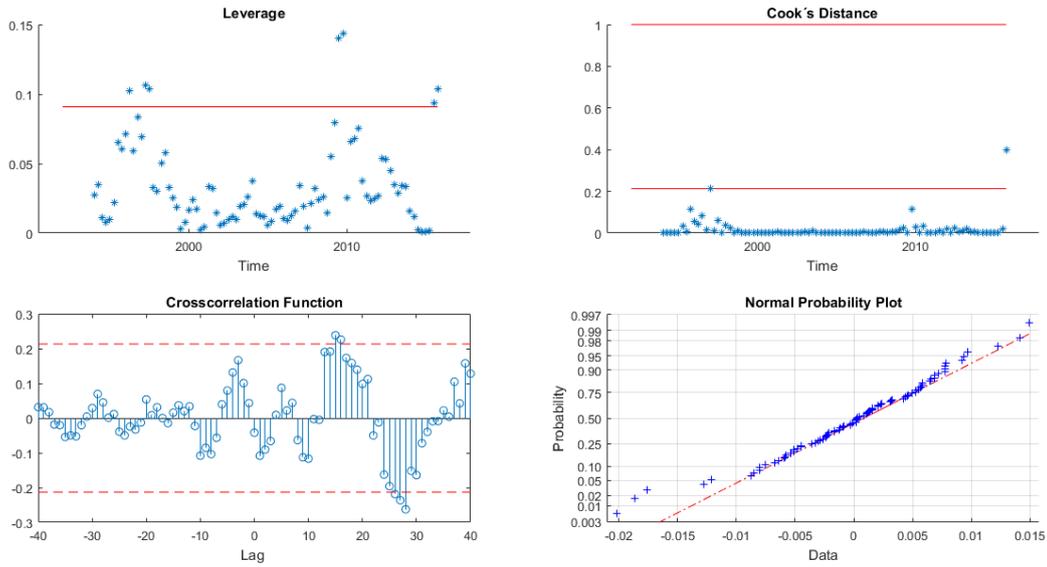


Figure 15: Diagnostics of the PEM (III) regression of the optimal rate-setting rule in (57) with results in Table 5. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outlier observed corresponds to 2015Q4.

### 7.2.5 Alternative Goods Market Equilibrium

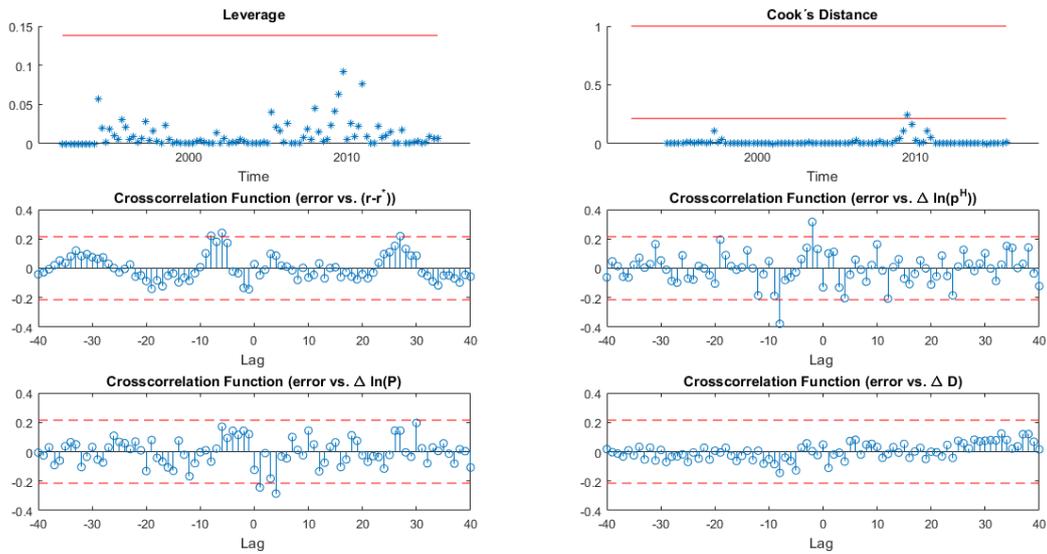


Figure 16: Diagnostics of the OLS regression of the goods market equilibrium in (58) with results in Table 7. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outlier observed corresponds to 2009Q3.

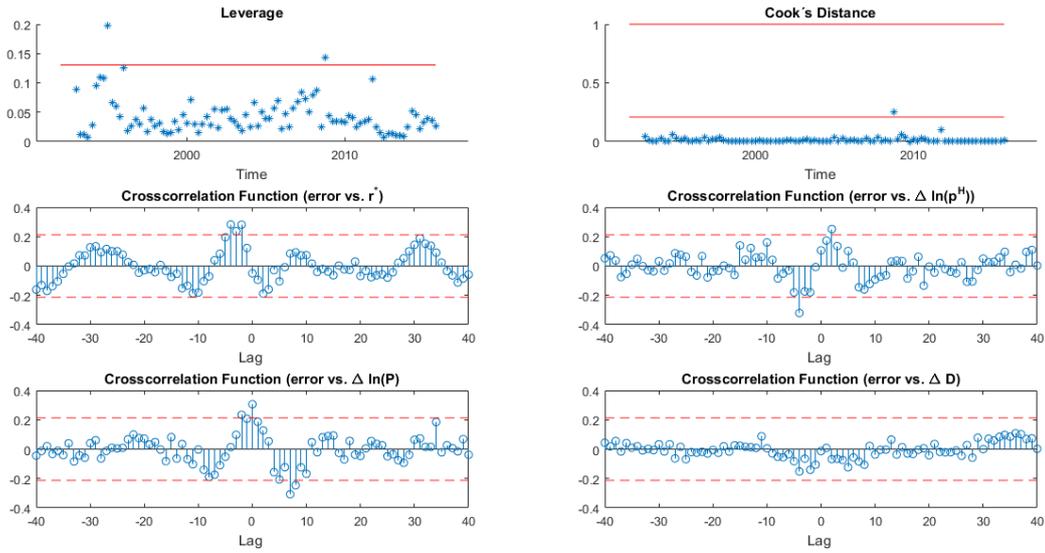


Figure 17: Diagnostics of the PEM (I) regression of the goods market equilibrium in (58) with results in Table 7. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. The outlier observed corresponds to 2010Q1.

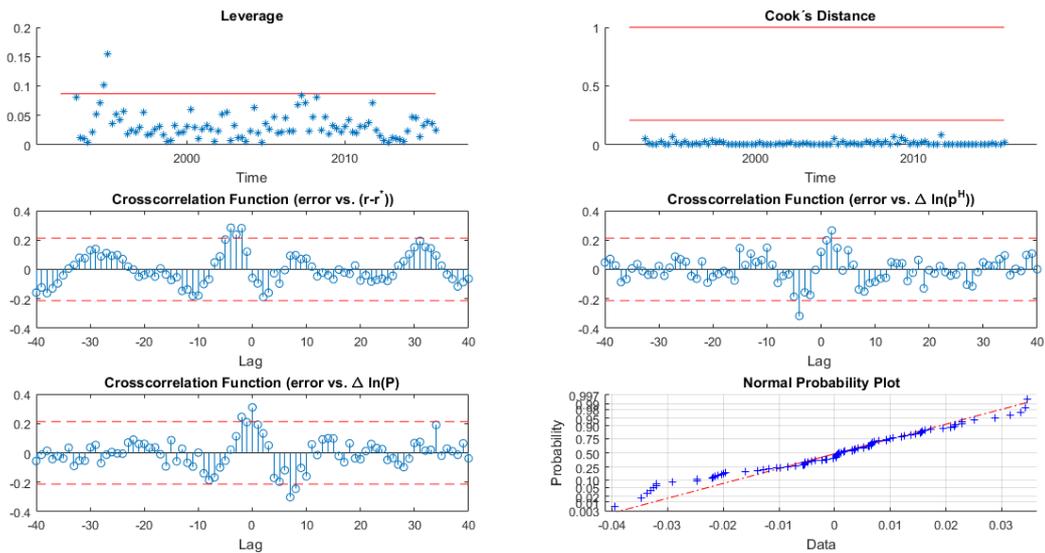


Figure 18: Diagnostics of the PEM (II) regression of the goods market equilibrium in (58) with results in Table 7. The red line in the subfigures relating to leverage and Cook's Distance are significance thresholds. The dashed red line in the cross-correlation subfigure indicates the 95% significance interval. No outliers are observed.