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Serial Dependence and Portfolio Performance in the Swedish Stock Market

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Abstract

This paper studies the possibility to exploit linear dependence in stock returns of the Swedish OMX 30 index. The main model studied in the paper is a Vector autoregressive (VAR) model. Ten years of data from the OMX 30 index is used, consisting of 27 stocks for the period 2006-2015 which is transformed into daily, weekly and monthly returns. First the significance of the models is tested and it is found that for the daily and weekly data all of the models are significant. For monthly data 77 % of the estimated models are significant. After this the performance of a VAR arbitrage portfolio is evaluated and compared to the performance of a contrarian and an unconditional sample mean arbitrage portfolio. For daily data the VAR model works quite well, yielding a higher Sharpe ratio than the two other arbitrage portfolios. For weekly and monthly data, the performance is not as good. In the weekly case the mean return is negative and in the monthly case the mean return is close to zero. After this a conditional VAR investment portfolio is formed and compared with the benchmark minimum-variance and mean variance portfolios. For daily data the VAR investment portfolio yields a good return, but also exhibits a high variance and very high turnover. For weekly data both the variance and the mean return is very high with a slightly better Sharpe ratio than for daily data. In the case with monthly data the model has a very high negative return, which can best be explained by the fact that the model is not well behaved in this case. The short sale constraints introduced seem inadequate at reducing the turnover of the VAR portfolios in the sense that even though the turnover is reduced so is the Sharpe ratios. Norm constraints was a better alternative to reducing turnover while also improving the Sharpe ratios. However, the tradeoff between mean return and turnover in the presence of transaction cost made it unclear whether the VAR portfolio strategy is a better alternative than using the traditional minimum-variance or mean-variance portfolios.

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1. Introduction

An often debated and quoted hypothesis in connection to financial markets is the random walk hypothesis. If movements in price between two times do not follow a random walk, then that implies that they are at least partly forecastable. If they are, then with what certainty? What are the risks involved and what are the transaction costs associated with pursuing a perhaps profitable strategy based on dependence in stock returns?

There has been extensive research on the topic of serial dependence in stock returns. This concept can be utilized both for serial dependence in the same stocks, but also dependence between different stocks or assets. Portfolios can then be formed that abuse this serial dependence in different ways. For example, one can devise a portfolio where one takes a long position in assets that had high return during the last month and a short in ones with a negative return. If one has found that a certain stock usually leads another stock, then one can use the return of that stock today to decide what position to take in the other stock tomorrow.

1.1 Purpose

The purpose of this thesis was to estimate and evaluate a vector autoregressive model on several stocks from the Swedish OMX 30 index. The model was first tested for its significance. Conditional on the estimated model a VAR arbitrage portfolio was formed and the performance of this portfolio out of sample was compared to that of other arbitrage portfolios. Furthermore, the VAR model was used in investment portfolios and its performance compared with traditional investment portfolios. To assess the practical application, transaction costs was also introduced to assess if the potential profit of the different models could be achieved in a real world scenario.

1.2 Previous research

The subject of serial dependence in stock returns has been studied extensively and for a long time. As DeBondt & Thaler (1985) note, individuals tend to overreact to information. In their paper they study whether the same applies to the stock market. Based on monthly return data from the CRSP they note that their results are “consistent with the overreaction hypothesis”. They also link these to the well know January market anomaly, where past “losers” show abnormally large returns in January. Jegadeesh & Sheridan (1993) document how buying past winners and selling past losers generate positive returns for holding periods up to 12 months. They also find that the profitability of these strategies are “not due to their systematic risk or to delayed stock price reactions to common factors”. Lo & MacKinlay (1990) find that past returns on large stocks influence the future return on small stocks and use this to form contrarian portfolios that exhibit positive returns. They also note that this is evidence against the belief that only overreaction is the source of the apparent profitability of certain contrarian strategies.

VAR models have also been used in other problems with strategic asset allocation problems. For example Campbell, Chan & Viciara (2001) study how investors with different levels of risk aversion and preferences should invest in stocks, bills & bonds. They use a VAR system of these assets as well as other variables which should be able to predict the return of risky assets, such as short-term nominal interest rate, yield spread etc. DeMiguel, Nogales & Uppal (2010) analyze whether portfolios formed from a vector autoregressive model can be used to generate positive returns on the stock market, out of sample and net of transaction costs. They use individual portfolios of stocks formed on size and book-to-market, industry and also individual stocks from the CRSP database. This paper will to a large extent be based on the methods and models of DeMiguel, Nogales & Uppal (2010) but applied to individual stocks of the Swedish OMX 30 index.

Yang, Kolari & Min (2002) use a vector autoregression to study cointegration among the US and several Asian markets with focus on the financial crisis in Asia during 1997-1998. They find that the degree of integration increased during the financial crisis and also that the degree of integration between different markets varies over time.

1.3 Disposition

In chapter 2 the data and estimation methodology is discussed. Chapter 3 gives the theoretical background of the models used in the evaluation. Chapter 4 holds the empirical results and chapter 5 holds analysis of the results while chapter 6 holds the conclusion and suggestions for further research.

2. Data & Methodology

2.1 Software

The estimation and forecasting was done in Matlab. It was chosen because it is a flexible and widely used software. Matlab's built in routines was used to convert the stock prices into returns, to estimate the models and to calculate the portfolio weights and the resulting portfolio returns.

2.2 Data

The data used consists of daily closing prices of 27 stocks on the Stockholm OMX 30 index. 10 years of data was used for the entire years of 2006-2015. The stocks were chosen as they are the ones who has been in the index the whole period. In total 2500 daily observations from the 10 years of data was available for each stock. A rolling window technique was used to produce the weights of the different stocks in the different portfolios. The first 1000 observations were used to determine the weights for day 1001. Then observation 2-1001 observation was used to determine the weight for day 1002 and so on. That leaves a total of 1500 estimated portfolio weights per model. For the weekly data 260 observations was used for the estimation of each model and for monthly data 60 observations.

The price observations were transformed into returns according to

$$r_t = \ln_t\left(\frac{P_t}{P_{t-1}}\right) \quad (1)$$

Where the index $t - 1$ represents the period in question. So for daily data it is from one day to the next, for weekly it is the differences in price from one week to the next and for monthly it is the difference in price from one month to the next.

2.3 Performance Measures

The different strategies of portfolio selection were ranked on mean return, variance, Sharpe ratio and turnover.

The mean is defined in the following way and is simply the average of the return for each strategy every day:

$$\mu = \frac{1}{T} \sum_{t=1}^T r_t \quad (2)$$

T is to the total number of forecasted days and r_t is the return vector for day t which is formed by multiplying the estimated portfolio weight for that day with the corresponding realized return for every stock that day.

The sample variance is defined in the following way:

$$\sigma^2 = \frac{1}{T-1} \sum_{t=1}^T (r_{t+1} - \mu)^2 \quad (3)$$

Where μ is the mean return defined in equation (2).

The Sharpe ratio is a widely used measure of portfolio performance, defined in the following way as the mean divided by the standard deviation.

$$\text{Sharpe Ratio} = \frac{\mu}{\sigma} \quad (4)$$

A higher Sharpe ratio is better than a lower. The idea behind this is that the mean return of course is good for the portfolio, whereas the standard deviation represents total risk and is “bad” for the portfolio. In the tables in the result section what is reported is the annualized Sharpe ratio. This is a somewhat blunt measure as Lo (2002) shows but nevertheless a widely used measure of portfolio performance. In this case the resulting annualized Sharpe ratio for daily data will be.

$$\text{Annualized Sharpe Ratio} = \frac{\mu}{\sigma} \sqrt{252} \quad (5)$$

For weekly and monthly data, the Sharpe ratio is correspondingly multiplied by $\sqrt{52}$ & $\sqrt{12}$ to yield the annualized Sharpe ratios.

The turnover is defined as:

$$Turnover = \frac{1}{T-1} \sum_{t=1}^T \sum_{k=1}^N (|w_{k,t+1} - w_{k,t}|) \quad (6)$$

Where $(|w_{k,t+1} - w_{k,t}|)$ represents the difference in weights between two time periods for a certain stock in for a certain strategy.

Having defined the turnover, we can define the return of a strategy minus the transaction costs as:

$$r_{t+1} = 1 - \kappa \sum_{k=1}^N (|w_{k,t} - w_{k,t-1}|) (w_t)^T r_{t+1} \quad (7)$$

Where κ represents an arbitrary proportional transaction cost. This equation can be used to solve for the maximum value of κ that still lets the portfolio make money. This is reported in the tables in the result section for those portfolios that exhibit a positive mean return when transaction costs are not taken into account.

3. Theoretical Framework

3.1 The VAR Model

The following model was introduced to capture the serial dependence amongst the stock returns. The model and the different portfolios used are from DeMiguel, Nogales & Uppal (2010). This is the vector autoregression (VAR) model which is the base of the VAR portfolios whose performance was evaluated.

$$r_{t+1} = a + Br_t + \epsilon_{t+1} \quad (8)$$

Where r_t is the array of stock returns for day t , a is an array of intercepts, B a matrix of slopes and ϵ is an *iid* array of errors.

The matrix B gives us an estimation of how each stock influence each other stock and itself. In this way the return on every stock tomorrow may depend on the return of all the other stocks the day before. To give an example, the element B_{12} in the matrix represents how the return on $r_{2,t}$ today affects the return of $r_{1,t+1}$ tomorrow. The model is of the first order, which was found by DeMiguel, Nogales & Uppal (2010) to be sufficient. No information criteria test was performed in this paper to see how a model with more lags performs, instead the 1-lag model was used.

3.2 Test for significance

To test for the significance of the VAR model a test from Tiao and Box (1981) was used. The null hypothesis for this test is

$$H_0: B = 0 \quad (9)$$

The test statistic is given by

$$M = -(\tau - N - 2.5) \ln\left(\frac{|\Omega_1|}{|\Gamma_0|}\right) \sim \chi^2(N^2) \quad (10)$$

Where τ is the number of observations used in the estimation of the VAR model, $N = 27$ is the number of stocks and $\Omega_1 = \frac{1}{\tau-3} \epsilon^T \epsilon$ is the residual covariance matrix. ϵ are the residuals from eq. (8) after the fitting of the model and Γ_0 is the covariance matrix of the returns. This test was performed for each of the estimated models to assess how many of them were significant. The test statistic under the null-hypothesis follows a chi-squared distribution with N^2 degrees of freedom. As 27 stocks were studied it means that $M \sim \chi^2(729)$.

3.3 Arbitrage Portfolios

3.3.1 VAR arbitrage portfolio

To study the extent to which the VAR model can be used in practice a VAR arbitrage portfolio was formed in the following way according to DeMiguel, Nogales & Uppal (2010).

$$w_{v,t+1} = \frac{1}{N} (a + Br_t - r_{vt}e) \quad (11)$$

Where $r_{vt} = (a + Br_t)^T e / N$ is the conditional forecast of the return of the equally weighted VAR portfolio and e is a matrix of ones introduced for dimensional purposes. $a + Br_t$ is the predicted conditional return for every stock based on the estimated coefficients in the matrices a & B . The weights sum to zero, and therefore it is called an arbitrage portfolio. A positive weight is assigned to stocks with conditional return higher than that of the equally weighted portfolio and a negative weight to stocks with a conditional return smaller than the equally weighted portfolio.

The expected return of the VAR arbitrage portfolio can be decomposed in the following way according to DeMiguel, Nogales & Uppal (2010).

$$E[w_{vt}^T r_t] = G + \sigma^2(\mu) \geq 0 \quad (12)$$

Where

$$G = \frac{\text{tr}(\Gamma_1^T \Gamma_0^{-1} \Gamma_1)}{N} - \frac{e^T \Gamma_1^T \Gamma_0^{-1} \Gamma_1 e}{N^2} \quad (13)$$

$$\sigma^2(\mu) = \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2 \quad (14)$$

In theory, as can be seen, the expected return of the VAR arbitrage portfolio is positive. However, this assumes that the returns are jointly covariance-stationary and that Γ_0 is positive definite. Further it is assumed that the software manages to estimate the VAR models perfectly so that $B = \Gamma_1^T \Gamma_0^{-1}$ and $a = (I - B)\mu$ where I is the identity matrix. This will be kept in memory when we evaluate the performance of the VAR arbitrage portfolio.

3.3.2 Contrarian arbitrage portfolio

The VAR arbitrage portfolio will be compared to the return of the contrarian arbitrage portfolio of Lo & MacKinlay (1990). This is to assess to which extent the VAR portfolio performance can be explained by overreaction. The contrarian portfolio is weighted in the following way

$$w_{c,t+1} = -\frac{1}{N} (r_t - r_{et} e) \quad (15)$$

Where $r_{et} = e^T r_t / N$ is the return of an equally weighted portfolio of the stocks and e is again a matrix of ones. The weight assigned to a certain stock for a certain day is the difference between the return of that stock and the return of the equally weighted portfolio the day before. A smaller return of a certain stock today means a positive weight tomorrow. The weights sum to zero, and hence it is also an arbitrage portfolio.

The expected return of this portfolio can be decomposed in the following way

$$E[w_{ct}^T r_t] = C + O - \sigma^2(\mu) \quad (16)$$

Where

$$C = \frac{1}{N^2} (e^T \Gamma_1 e - \text{tr}(\Gamma_1)) \quad (17)$$

$$O = -\frac{N-1}{N^2} \text{tr}(\Gamma_1) \quad (18)$$

$$\sigma^2(\mu) = \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2 \quad (19)$$

As can be seen from the equations above the expected return of the contrarian portfolio is increased for positive cross covariances, negative autocovariances and decreased by the variance of the stock returns.

3.3.3 Unconditional sample mean portfolio

A further arbitrage portfolio that was compared to the VAR portfolio is the unconditional sample mean portfolio defined as:

$$w_{s,t+1} = \frac{1}{N} \left(\mu - \frac{\mu^T e}{N} e \right) \quad (20)$$

Where μ is an array of mean returns in the estimation period, $\frac{\mu^T e}{N}$ is the mean return of the equally weighted portfolio with e again being a matrix of ones. This portfolio assigns positive weights to stocks with higher mean return in the sample than the mean return of the equally weighted portfolio.

3.4 Investment Portfolios

In addition to arbitrage portfolios with zero cost, investment portfolios with a non-zero cost was studied as well. Following DeMiguel, Nogales & Uppal (2010) The VAR investment portfolios with different constraints was compared to benchmark portfolios that do not take serial dependence into account. First three portfolios ignoring serial dependence will be outlined in below.

3.4.1 Portfolios ignoring serial dependence

1. The equally weighted portfolio

This portfolio simply assigns an equal weight $\frac{1}{N}$ to each of the N stocks studied. The total weight of the portfolio every day will naturally sum to one.

2. The minimum variance portfolio

The famous minimum variance portfolio is the one that solves the following equation

$$\begin{aligned} \min_w \quad & w^T \Sigma w \\ \text{s. t.} \quad & w^T e = 1 \end{aligned} \tag{21}$$

Where Σ is the covariance matrix of the returns and the weights are summed to one in the constraint.

3. The mean variance portfolio

The mean-variance portfolio is the one that solves the following equation

$$\begin{aligned} \min_w \quad & w^T \Sigma w - \frac{1}{\gamma} w^T \mu \\ \text{s. t.} \quad & w^T e = 1 \end{aligned} \tag{22}$$

The γ -parameter represent the risk aversion of the investor in question. This constant was set to $\gamma = 1$ in most cases.

3.4.2 VAR portfolios

Below three VAR investment portfolios are introduced. These take linear dependence into account and are all based on the VAR equation of returns in equation (8).

1. Conditional mean-variance portfolio VAR

The mean-variance portfolio in equation (22) is estimated, but the mean and covariance matrix are replaced with their respective conditional estimates from the VAR model in equation (8).

$$\mu_{VAR} = a + Br_t \quad (23)$$

$$\sum_{VAR} = 1/\tau \sum_{i=1}^T (r_i - a - Br_{i-1})(r_i - a - Br_{i-1})^T \quad (24)$$

Where τ is the number of observations used in the estimation of each VAR model.

2. Conditional mean-variance VAR with short sale constraints

Again the mean-variance portfolios are estimated according to equation (22) with the mean and covariance matrix replaced with the estimates from equation (23) and equation (24). However, the additional constraint of $w \geq 0$ is imposed as well. This means that all weights must be positive, which as Jagannathan & Ma 2(2003) find should decrease the portfolio turnover. Since the weights are still required to sum to 1 it leaves the software with a lot less room in the estimation of the weights, and one can understand why the turnover should be decreased.

3.4.3 Norm-constraints

The second method of controlling the turnover is a norm constraint. The equation for the mean-variance portfolio in equation (8) is used for the optimization, again with the estimates from equation (23) and equation (24). An additional norm-constraint is imposed as well according to equation (25) below.

$$\|w - w_0\|_1 = \sum_{i=1}^N |w_i - (w_0)_i| \leq \delta \quad (25)$$

Where w_0 is the corresponding estimated weights from the unconditional minimum-variance portfolio and δ is an arbitrary threshold constant. This constraint prevents the weights of the estimated mean-variance portfolio to deviate too much from the weights of the unconditional minimum variance portfolio, which according to DeMiguel, Nogales & Uppal (2010), exhibits relative stability in its weights. The estimation will be run with values of $\delta = 0.05, 0.5$.

4. Results

4.1 Significance Tests

Following eq. (9) and eq. (10) the significance test was performed for each of the models. The critical value for the $N = 27$ variables on a 95 % significance level yields a value of $\chi^2(0.95, 27^2) = 793$. The mean value of the test statistics for daily data in equation (10) was 1184 with the smallest value being 873. This tells us that we can reject the null hypothesis of $B = 0$ and all our estimated models should be significant in this case.

For weekly data the mean was 965 with the smallest being 797 and again we can reject the null hypothesis for every estimated model. For monthly data the mean value was 845 and the minimum value was 746. 13 of the 57 estimated models in the case of monthly data had corresponding t-statistics smaller than 793. This tells us that we cannot reject the null hypothesis and that these models are not significant.

4.2 Portfolio Performance

In the chapters that follows the empirical performance of the different portfolios is reported. The results are reported for daily, weekly and monthly data respectively. For each kind of data, the result is split into arbitrage and investment portfolios. The *Kappa* column represents the maximum proportional transaction cost for which the strategy can still make money. This is for obvious reasons only reported for strategies with a positive mean return. Varying the parameter γ in the estimation of the mean variance portfolio for the VAR model was only done for monthly data due to lack of time, but also because it did not seem to have a great effect on the Sharpe ratio of the monthly portfolios.

4.2.1 Daily Data

Arbitrage Portfolios

	Mean	Variance	Sharpe ratio	Turnover	Kappa
VAR	0.00090	8.2e-05	1.58	0.36	0.0025
Contrarian	-6.3e-07	2.6e-09	-0.20	0.012	
UNC	9.4e-09	2.1e-12	0.10	1.4e-05	0.00069

Table 1 Reports the results of the performance measures for daily data on arbitrage portfolios.

Investment Portfolios Ignoring Linear Dependence

	Mean	Variance	Sharpe ratio	Turnover	Kappa
1/N	0.00024	0.00015	0.31	0	
Min-variance	0.00033	7.6e-05	0.61	0.0094	0.036
Mean-variance	0.0033	0.023	0.35	1.36	0.0024
SL-Min	0.00033	8.2e-05	0.58	0.063	0.0053
SL-Mean	-0.00015	0.00052	-0.10	0.0027	

Table 2 Reports the results of the performance measures for daily data on investment portfolios ignoring linear dependence.

VAR Investment Portfolios

	Mean	Variance	Sharpe ratio	Turnover	Kappa
VAR	0.056	1.27	0.79	307	0.00018
SL-VAR	0.00060	0.00035	0.51	2.0	0.00031
VAR Norm 0.05	0.00054	8.9e-05	0.91	1.3	0.00040
VAR Norm 0.5	0.0025	0.0013	1.1	13	0.00019

Table 3 Reports the results of the performance measures for daily data on VAR investment portfolios.

4.2.2 Weekly Data

Arbitrage Portfolios

Portfolio	Mean	Variance	Sharpe ratio	Turnover	Kappa
VAR	-0.00049	9.9*e ⁻⁰⁵	-0.35	0.34	
Contrarian	2.1*e ⁻⁰⁵	5.2*e ⁻⁰⁸	0.65	0.029	0.00072
UNC	-1.2*e ⁻⁰⁷	2.1*e ⁻¹⁰	-0.058	0.00013	

Table 4 Reports the results of the performance measures for weekly data on arbitrage portfolios.

Investment Portfolios Ignoring Linear Dependence

Portfolio	Mean	Variance	Sharpe ratio	Turnover	Kappa
1/N	0.00085	0.00067	0.24	0	
Min-variance	0.00064	0.00036	0.24	0.091	0.0071
Mean-variance	0.032	0.14	0.63	5.4	0.0059
SL-Min	0.0015	0.00037	0.56	0.026	0.57
SL-Mean	-0.0025	0.0028	-0.34	0.13	

Table 5 Reports the results of the performance measures for weekly data on investment portfolios ignoring linear dependence.

VAR Investment Portfolios

	Mean	Variance	Sharpe ratio	Turnover	Kappa
VAR	0.36	9.1	0.86	573	0.00063
SL-VAR	0.0016	0.0018	0.27	1.8	0.00087
VAR Norm 0.05	0.0016	0.00044	0.56	1.4	0.0012
VAR Norm 0.5	0.0110	0.0069	0.95	13	0.00082

Table 6 Reports the results of the performance measures for weekly data on VAR investment portfolios

4.2.3 Monthly Data

Arbitrage Portfolios

Portfolio	Mean	Variance	Sharpe ratio	Turnover	Kappa
VAR	0.00088	0.0029	0.056	1.5	0.00058
Contrarian	-6.8e-05	7.3e-07	-0.28	0.062	
UNC	-5.8e-07	1.5e-08	-0.016	0.0012	

Table 7 Reports the results of the performance measures for monthly data on arbitrage portfolios.

Investment Portfolios Ignoring Linear Dependence

	Mean	Variance	Sharpe ratio	Turnover	Kappa
1/N	0.0030	0.0024	0.21	0	
Min-variance	0.0034	0.0022	0.25	0.63	0.0054
Mean-variance	0.42	3.5	0.78	31	0.014
SL-Min	0.0082	0.0015	0.72	0.097	0.085
SL-Mean	-0.0095	0.0119	-0.30	0.27	

Table 8 Reports the results of the performance measures for monthly data on investment portfolios ignoring linear dependence

VAR Investment Portfolios

VAR	Mean	Variance	Sharpe	Turnover	Kappa
$\gamma=1$	-7.6	18400	-0.20	16200	
$\gamma=2$	-3.8	4590	-0.20	8100	
$\gamma=5$	-1.5	740	-0.19	3200	
$\gamma=10$	-0.78	184	-0.19	1600	
SL $\gamma=1$	-0.0059	0.0093	-0.21	1.8	
$\gamma=2$	-0.0051	0.0090	-0.19	1.8	
$\gamma=5$	-0.0048	0.0087	-0.18	1.8	
$\gamma=10$	-0.0038	0.0080	-0.15	1.8	
Norm 0.05	0.0016	0.0027	0.11	1.7	0.00096
Norm 0.5	-0.017	0.037	-0.31	14	

Table 9 Reports the results of the performance measures for monthly data on VAR investment portfolios.

5. Discussion

A discussion of the results follows below. This section is split into a discussion of the results of the significance tests and a discussion of the results of the portfolio performance estimation.

5.1 Significance Tests

In the daily and weekly cases the null hypothesis of $B = 0$ was rejected in every case. This tells us that taken as a whole this matrix of intercepts was significantly different from zero on the 95 % significance level. It does not tell us about the significance of individual elements in the matrix, and surely some of them are insignificant or straight up incorrect. In the monthly case 77% of the models were significant. This might be a result of the rather few (60) observations used for estimation of each model. It may also be the case that the monthly linear dependence is not as strong as the daily or weekly linear dependence among the stock returns studied.

5.2 Portfolio Performance

5.2.1 Daily Data

We can see that of the three arbitrage portfolios the “best” performing is indeed the VAR arbitrage portfolio. It has the highest mean return by far, but also the highest variance and turnover. The annualized Sharpe ratio for the VAR arbitrage portfolio is about 1.6 which is a pretty good result indeed. However, in the case of proportional trading costs the maximum proportional transaction cost that still lets the strategy make money is 0.25 % which might not be available to every investor.

Of the investment portfolios without restrictions the conditional VAR is again the best performing in terms of Sharpe ratio. We see now that the average daily return is about 5% which can be considered very good for sure. However, the variance is over 120 % and the turnover about 300 per day on average. Since the weights in each stock every day were summed to one in the optimization problem this means that the trader would have to trade 300 times their daily wealth on average, which of course is a significant amount.

The minimum variance portfolio yields a higher Sharpe ratio than the mean variance portfolio, however we can see that both the return and the variance is substantially lower for the minimum variance portfolio.

We note that imposing short sale constraints have a significant impact on the conditional portfolio from the VAR model. Both the mean return, the variance of the return and the turnover is significantly decreased with several orders. The Sharpe ratio is decreased by about 35 %. However, this time the Sharpe ratio is lower than the minimum-variance portfolio and also the turnover of the VAR portfolio is higher than that of the minimum-variance portfolio. So even though the short sale constraints take care of the problem of high turnover one might as well invest with the minimum-variance portfolio, which, although its widely used, is not that sophisticated.

In the case of the mean variance portfolio imposing short sale constraints means substantially decreasing the return, in fact a negative mean return and as a result negative Sharpe ratio is the result. In the case of the minimum variance portfolio nothing much happens to the mean return or the variance, however the turnover is increased.

Imposing norm constraints both with $\gamma = 0.05$ and $\gamma = 0.5$ improves the Sharpe ratio with respect to the unrestricted conditional VAR portfolio. The mean return, variance and turnover is significantly decreased in both cases. However, it is still the case that if there exist proportional transaction costs they could not be higher than a couple of hundreds of a percent for the strategies to continue making money.

5.2.2 Weekly Data

For the VAR arbitrage portfolio in the case of weekly data the results are very poor. The mean return and the Sharpe ratio are both negative. As was outlined in equations (12-14) the expected return of the VAR arbitrage portfolio is positive which tells us that either the model is not estimated perfectly or any of the other assumptions are incorrect. The unconditional sample mean return portfolio also exhibits a negative mean return and Sharpe ratio. The contrarian portfolio is the only one with a positive mean return and a decent Sharpe ratio of about 0.65.

For the investment portfolios we can see that the VAR arbitrage portfolio again manages an impressive mean return of 36 % on a weekly basis. However, the variance is over 900 % and the turnover of over 500 means a trader would have to trade 500 times his wealth daily. Looking at the critical value of κ we can see that it is again just a couple of hundreds of a percent.

Imposing short sale constraints on the VAR portfolio shows the same patterns as the case with daily data. The turnover is significantly decreased, but so is the Sharpe ratio. The conditional VAR portfolio with short sale constraints fail to beat the unconstrained mean variance portfolio or the minimum variance portfolio with short sale constraints.

The norm constraints produce a more satisfying result. Norm constraints with $\delta = 0.5$ gives the best Sharpe ratio of all the weekly portfolios. However, the critical value of κ is small and the turnover is still 13 which means a trader would have to trade 13 times his wealth daily which might be difficult in a real world scenario.

5.2.3 Monthly Data

With monthly data none of the arbitrage portfolios perform well. The VAR arbitrage portfolio is the only one with a positive Sharpe ratio, however it is very small.

The VAR investment portfolio without constraint produces an abnormally large negative return an extreme variance and a very high turnover. In this case it is safe to say that the model works very poorly. One should remember that here the model was not significant in all of the cases. The short sale constraints introduced does nothing for the Sharpe ratio, which is still highly negative, however all of the other measures are decreased significantly.

The mean variance portfolio outperforms the minimum variance portfolio in the case with no constraints. However, when short sale constraints are introduced the minimum variance portfolio performs better.

The norm constraints with $\delta = 0.5$ still gives a negative mean return, with $\delta = 0.05$ a positive but not very impressive result is produced. In this case it is safe none of the variations of the VAR model beats the traditional mean variance and minimum variance portfolios. It might very well be the case that these kinds of models should not be used with monthly data.

6. Conclusions & Suggestion for Further Research

It should be mentioned that the 27 stocks selected all come from the OMX index. Some of them are from similar sectors or industries. The idea behind the vector autoregressive model is that the linear dependence does not necessarily need a strong theoretical background as to why certain stocks influence others. The idea behind this paper was to see whether there was any dependence and to what extent this could be exploited in the presence of transaction costs. The models were in the case of daily and weekly data found to be significant. In these cases, a substantial mean return was also predicted by most of the conditional VAR models. However, the returns exhibited a large variance and turnover. In the monthly case the VAR model performed poorly which might be a result of the relatively few observations used in the estimation of each model. Included in the years studied was the financial crisis that started in 2008. This might have made the models behave strangely as the OMX index exhibited large variance and many days of negative return during this period. As was discussed in the section on previous research, financial crisis can have a big impact on dependence among different markets and might have the same effect on dependence in the same market as well.

Further it should be noted that varying the risk aversion parameters and threshold constants can have a big impact on the performance of the model. While it was found in the tests that was run that varying the risk aversion parameter did not change the Sharpe ratio of the VAR portfolio with monthly data, it should be noted that this might not have been the case with weekly or daily data. As could be seen, varying the value of the threshold parameter δ in the norm-constraints optimization problem had quite a big impact and could change a negative Sharpe ratio to a positive. Furthermore, since the theory behind the VAR model is quite weak it is hard to know if the results would be the same if the performance evaluation was performed on data from other years, or for that sake in the years to come.

Even though in the case of daily and weekly data the portfolio with the highest Sharpe ratio was a VAR portfolio with norm constraints in both cases there was always one case of the minimum-variance or mean-variance portfolio that let a trader make money if subject to a higher proportional trading cost. Considering the critical value of $Kappa$, the limit on proportional trading costs where the VAR portfolio strategy makes money, one can draw the conclusion that for a regular investor it would be hard to make money off of that strategy. The maximum allowed proportional transaction cost for any of the VAR portfolios was the 0.25 % allowed in the case of arbitrage portfolio on daily data. So even though there might be serial dependence among the stock returns of the OMX index it would probably be hard to use these models to make money as there is always a tradeoff between turnover and mean return. For further research it would be interesting to see if the VAR models could be

manipulated in another way to produce good mean returns while still being practically applicable. This means reducing the turnover even more while keeping the mean return and Sharpe ratio high.

It would also be interesting to use a VAR model on intraday data. Considering the pattern that monthly data performed poorly, weekly decent but daily the best there might be an interesting case in studying hourly data or even data sampled every 5 minutes.

7. References

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