Using Cooperative Game Theory to Analyse Allocations of Costs Related to Connecting Renewable Energy to the Power Grid

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A thesis submitted for the degree of

B.sc. Economics

2016

Abstract

Building new wind farms is an important part in the transition to renewable energy. However these projects often require large investments, and the cost of upgrading the existing infrastructure - to allow for the connection of new parks - is a component that can not be ignored. When several wind farms seek interconnections, they are forced to collaborate and financing the connection. This leads to the question of how these costs should be shared between different agents. We investigate whether the currently used allocation method can and should be replaced with a fairer method, by formulating the problem as a cooperative game and using some well established fairness axioms. Furthermore we apply our findings on the real case example Havsnäs wind farm in Sweden, as well as some fictive examples. Our findings show that in most cases, the current method where costs are divided proportionally to the total effect that the wind farm produces, is best suited. However, in some particular examples a more intricate mathematical method of cost sharing would be preferred.

Keywords: *renewable energy, grid upgrade, grid extension, grid connection, wind farm projects, cost sharing game, cost allocations, cooperative game theory, shapley value, nucleolus, the core, transition to renewable energy*

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Nomenclature

Roman Symbols

A The grand coalition of players

C The cost function

$I(v)$ The set of all imputations

L_i A proper subset of A i.e. a sub coalition of the players in A

v The value function

$\mathbf{x} = (x_1, x_2, ..., x_N)^T$ A payoff vector for N players

Greek Symbols

P Vector of all the players allocation with the proportional split rule

ρ_i The cost allocated to player i when using a proportional split

$\mathbf{\Phi}$ Shapley vector containing all players shapley value

ϕ_i Shapley value for player i

$\mathbf{\Theta}$ Vector of all the players allocation using the nucleolus as allocation rule

θ_i The cost allocated to player i when using the nucleolus

$\mathscr{P}(A)$ The set of all possible subsets in A

Chapter 1

Introduction

The world is facing growing energy demand. Meanwhile, an increased understanding of energy production's impact on the environment, and in particular the impact of non-renewable energy, are calling for a transition from the current energy system. This is urging governments and legislators to facilitate for an expansion of *RES-E* (renewable energy source - electricity), to have have electricity produced with less impact on the environment, with competitive cost and high delivery reliability.

Wind turbines have a significant role in this conversion, with an exponential increase in installed wind power capacity globally (see for example Bloomberg's New Energy Outlook, 2016). This is also the case for many northern European countries, and the Swedish government has given *Energimyndigheten (Ei)* the mission to actively simplify and facilitate planning and permitting processes in wind power projects by identifying and eliminating obstacles [ER 2007:33].

The costs of connecting wind turbines to the existing grid and grid upgrades can have an effect on whether wind power projects are realised, and is a relevant aspects for integrating renewable energy sources into the energy system (Swider et al. [2008]). Wind power plants rely on the electricity grid to supply produced energy. The distribution of electricity and grid networks forms a *natural monopoly*[1]. For the electricity transmission network, this means that produc-

[1] A natural monopoly refers to the relationship between the cost of supply, and demand. If total demand is supplied at its lowest cost by one actor, rather then by several, the specific market is a natural monopoly (Posner [1969]). A natural monopoly with more than one actor

ers are dependent on a natural monopoly, where network owners are generally granted concession by the government, in order to supply their production. To ensure that tariffs are objective and non-discriminating (as stated by law in Sweden, electricity law (1997:857)), governments control the actions of the network through regulations. The law (chapter 3. 7 § in the electricity law (1997:857)) also states that the network concession owner is obliged, unless there are specific reasons, to connect a facility within its area.

The development of wind power require geographical locations where wind patterns are adequate in order for it to be cost- and environmental efficient. Due to specific geographical requirements for wind power, individual wind power turbines together form groups of wind turbines or *wind farms*. Often, attractive areas consists of several wind power developers with planned wind farms adjacent to each other, forming *wind power clusters*. As a result, power network companies receives frequent requests for grid connection from different wind power projects in the same area. The specific dependence on on geographical location and the technical restrictions for grid upgrades creates a situation where grid upgrade costs are not easily divided between the parties involved. The cost of connecting a power generator or group of generators is highly dependent on the point of connection to the existing grid and the existing grids current configuration (Knight et al. [2005]). There are *economies of scale* in planning the network extensions and capacity increase with the entire cluster in mind, where the final configurations have implications on individual parties direct and indirect connection costs.

The natural monopoly create a situation where generated power is restricted to a grid configuration that is determined by the network operator. With the Swedish laws stating reasonable conditions and objective and non-discrimination connection costs, the question of how to divide producers connection costs for a given grid configuration arise. A fair cost allocation allow competition between producer, where no party gets an advantage or disadvantage from the connection costs it needs to pay in order to supply produced electricity to the market (Swedish electricity law (1997:857)).

will either consolidate to one, or be inefficient. Thus, competition is not a viable market mechanism to control profits, specific rates, extensions, usage, and permits.

The thesis explore this problem by comparing the current commonly used allocation method with allocation methods derived in cooperative game theory. This is described further in the subsequent sections of the introduction.

1.1 Current allocation method

In Sweden, the grid owner can stipulate that the first wind developer seeking a connection pays for the entire upgrade cost, even when the capacity increase is dimensioned for future interconnections from other wind developers. This creates a *threshold* effect, as wind developers are reluctant to carry the full investment which often makes projects financially unattractive.

When developing new electricity production sites, the producer has to pay for their connection to the regional or national power grids (Ellagen 1997:857). Often, there is a lack of capacity in the grid, requiring costly upgrades which has to be paid by the producer through a connection fee. The most long-term economical rational upgrade (due to technical restrictions and socioeconomic efficiency) is generally to increase the capacity above the connecting producers need. For example, the grid owner could charge a connection fee covering the entire upgrade, where the added capacity allows for additional connection of electricity production. Not only is it often too costly for the single energy producer to afford, creating a barrier for introducing wind power. It is also perceived as unfair as the first producer is basically subsidising additional connections from competitors. This creates incentives for prospectors to postpone greenfield renewable production investments until the grid is upgraded. If no producer can carry the cost of the grid upgrade, no projects will be realised.

The Swedish government has identified connection issues, and to overcome these barriers Ei has proposed a financing solution for large project areas (above 100 MW) so that the grid owners can be confident that they will not be affected by a a priori cost allocation where it is unknown if additional producers will enter at a later stage and fund the remaining costs of the upgrade. At the moment there is a transition legislation regarding the cost allocation where costs are divided solely based on the *fraction of the upgraded capacity each producer is using* (Prop. 2013/2014:156). As the proposed permanent solution is solely focusing on the financing, the cost allocation is unlikely to change from the temporary solution. Furthermore, Ei has ruled that a proportional rule should be used to determine the cost allocation, if the grid owner decides to allocate costs between multiple actor seeking connection and a grid upgraded is required (see 'Mjölbyfallet', Ei, ref.nr 2009-101533).

As a result of the inefficiencies from the threshold effect and Ei's ruling, the most common allocation method a priori is based on the proportional rule (interviews with: R. Husblad, Ei, [2015]; A. Agervik, Vattenfall, [2015]). Vattenfall state in their document "Tillämpningsbestämmelser för anslutningsavgift för anslutning av elproduktionsanläggning till elnät tillhörande Vattenfall Eldistribution AB" the following:

"If the assessment is that existing customers or expected new connections will use the new facilities, the costs of the new facilities should be allocated between concerned existing customers and known/planned new connections within the next few years. The connection's estimated share of the cost for the facilities is to be included in the real charge of the cost. Estimated share refers to the new electricity plant's **proportionate share of dimensioned effect** among existing and known / planned new connections in the coming years, which will utilise the new facilities. There will be no future reimbursement of connection fees divided in this way." [Authors' translation, emphasis added]

This means that areas where the planed development of wind power effect is less than 100 MW, grid owners often allocate the costs of the connection solution so that each player carry the cost where it is the only user, and the costs of shared facilities are divided either based on the proportion compared to the other agents, or the proportion compared to the new upgrades capacity.

1.2 Problem formulation and thesis question

1.2.1 Problem formulation

As the interconnection tariffs are out of control for power producers seeking connection, with the law explicitly stating that tariffs and forced costs should be objective and non-discriminatory, there is a cost allocation problem. The current allocation method, a proportional split rule, can at first glance be perceived as fair. However, it can be questioned whether it is in fact fair. To our knowledge, there has been no evaluation to this point in whether the proportional rule is indeed a fair method of allocating interconnection costs. Based on this, the question for the thesis to investigate is formulated.

1.2.2 Research question

This thesis will give an answer to the question:
Is the current proportional allocation method fair from an cooperative game-theoretical perspective? Are there alternative allocation methods that would yield outcomes that could be considered fairer?

In the process of answering these questions we investigate:

- How the cost sharing problem can be analysed using cooperative game-theory. *With the aim to apply game- and cost sharing theory to investigate potential allocation methods and their implications.*
- If the current allocation method satisfy generally accepted axiom and criteria for fairness in cooperative game theory. *Aiming to give insights on how costs are currently allocated, and its implications from a game theoretical perspective.*

This will provide insight on how the cost allocation game can be solved in such way that it is considered fair by the players.

1.3 Method

When more than two agents are competing to maximise utility in a situation, the agents can form coalitions to get better outcomes among themselves. This can be studied with cooperative game theory. The reasons for forming coalitions is usually to maximise the pay-off for the player, e.g. by reducing investment costs, reducing life cycle costs or increase life cycle profits, but the collaboration can also be forced upon the players by an outsider.

We explore and compare the proportional allocation with cooperative game-theoretical allocation methods, and evaluate whether the current method satisfy generally accepted fairness principles from cooperative game theory. Thus, this is not a study on what constitutes fairness. The thesis has a theoretical focus - where modelling is utilised to answer the underlying thesis question.

Evaluating the cost sharing problem from a fairness standpoint requires a defined set of concepts. This thesis uses methods from the cooperative game-theoretic field, which provides a set of concepts with specified rules for decision making in competitive and cooperative situations.

The method can be described with four distinct steps:

1. Formulating the allocation problem within the context of game theory
2. Using game theory to define axioms for fairness, and adequate allocation methods, given these axioms.
3. Applying the allocation methods and the proportional rule method on:
   (a) A case study of a real-world interconnection allocation problems
   (b) Illustrative hypothetical examples of interconnection allocation problems
4. Analysis of results

1.3.1 Formulating the allocation problem

This allows for us to define the characteristics of the game, and define the the problem as a specific type of game.

As a result of the theoretical study, the Shapley value, nucleolus, the core concept, and imputations became the method of analysis.

1.3.2 Defining axioms for fairness

With the problem formulated and defined within cooperative game theory, the analysis is extended by including axioms that define and create objective criteria for fairness when allocating interconnection costs.

1.3.3 Determine allocation methods

With the fairness axioms, the method of allocating the costs are derived. The cooperative game-theoretical methods utilized are ones where the resulting allocations adhere to certain fairness axioms.

1.3.4 Case study

In the case study, the research is focused on a real-world setting, with a defined set of boundaries that are rational for this research: There are economies of scale, and financially and socioeconomically efficiency in a collaborative interconnection solution. The case represent a setting in which the different allocation methods' results can be scrutinized and compared.

1.3.5 Illustrative examples

To complement the case study, and make the conclusions more nuanced and precised, we use a few illustrative examples. These are hypothetical allocation problems where we apply the related cost data for the grid interconnections to the hypothetical wind areas and farms. The illustrative example allows us to find situation where the resulting allocations can differ substantially between the allocation methods.

1.3.6 Analysis of allocation results

For the thesis to provide insights that are valuable, the resulting allocations are put into a larger context. By comparing the differences with other costs for a wind farm project, future revenues and the complexity of the different methods, the thesis provides valuable insights into which allocations methods that are realistic to use in real-world situations, and whether they seem to be fair from a theoretical perspective.

1.3.7 Data

1.3.7.1 Cost data

The Swedish Energy Markets Inspectorate, *Energimarknadsinspektionen (Ei)*, has compiled a list of the investment costs a network license holder, *nätkoncessionsinnehavare*, would expect to have if they were to acquire or manufacture a specific asset needed for the grid network.

When an expansion or reinforcement of the grid is performed the incremental costs incurred by the license holder are to be calculated by the principals in the regulation *intäktsramförordningen*. This regulation express that these costs should primarily be calculated with *normvärdeslistan* that Ei has compiled. All parts of the investments that are partly or fully used are to be included. In this thesis we use the list to calculate hypothetical costs for each coalition. Ei prefers this list for grid upgrade costs compared to using the actual investment costs [Energimarknadsinspektionen, 2015]. The interested reader is referred to Grontmij [2015] to learn more.

1.3.7.2 Case study - Havsnäs

Havsnäs wind area consists of 48 turbines deployed in three separate areas, with a total effect of 95.4 MW and annual production of about 260 GWh. The turbines are placed at three heights: Area D (Ritjelberget) with 21 turbines, Area E (Ursåsen) with 11 turbines and Area F (Järvsand) with 16 turbines. The distance between the areas are a few kilometres. It is located in Strömsund, Jämtland County, Sweden. The project has been developed, finance and built by Nordic Windpower in collaboration with its British parent corporation RES. The wind farm is also operated, serviced and maintained by Nordic Windpower.

The reasons for choosing Havsnäs as a case study to illustrate the opportunities of cooperative game theory to share costs in simultaneous connection to wind farms are:

- The wind farm has been built and is up and running today. Thus, we know that the actual connection took place, and that the connection was considered worthwhile and possible.

- Even though there is only one project company, the connection problem include three distinct and separate areas, making it representative for connection problems where several companies could be involved.

- The area's total effect is close to, but less than, 100 MW. It is a rather large wind farm, but still not covered by the new financing solution put forward by Ei. If the areas would have had separate owners, it is plausible that a connection cost allocation would have been necessary for the projects be realised.

- The data from Ei's 'normvärdestabell' are suitable to use as cost assumptions

- The most rational connection for each coalition can be assumed with a large degree of confidence. Given the restrictions due to intrusion in the nature and from a cost perspective, the alternatives are presented in Figure 5.1.

Chapter 2

Cooperative game theory

In the economic context, games in game theory are mathematical models of rational agents' interactions. A rational agent's goal is to achieve the best possible outcome, measured e.g. by the agent's utility from different outcomes. In non-cooperative game theory, this means maximising its specific pay-off function for the game, where the pay-off is dependent on the actions undertaken by all other agents in the game. Cooperative game theory allow the formation of coalitions between agents to increase their utility. The field mainly studies when these coalitions will be formed and how to distribute costs alt. profits among the agents in the coalition.

In this thesis the agents will be called players of the game.

2.1 Classification of games

It may be useful to classify games with respect to whether they are Transferable Utility (TU) games or not. A subset of the class of TU games is the set of Characteristic Function Games (CFG). (Saad et al. [2009]). The problems discussed in this thesis will be studied through the concept of CFG.

2.1.1 Transferable Utility Games

The most widely studied type of cooperative game is the so called Transferable Utility Game, also called (TU-game) (p.214 Diaz [2010]). A TU-game is characterised by that the benefits awarded to the players are not unique to a specific player. Hence it is possible to freely distribute the utility among all players (p.214, Diaz [2010]). The most intuitive example would be that the players are competing for money and that all players have constant and equal marginal utility of one monetary unit.

2.1.2 CFG

The *Characteristic Function Games* is a subset of the *TU-games* where there is a fundamental assumption that all players are cooperating and forming a *grand coalition* (p.127, Gibbons [2001]). Since we are assuming that the grand coalition is formed, the primary interest would naturally be to find out how the pay-off should be allocated. The strive for the pay-off allocation would be to make it fair to the concerned players. The reason for full cooperation can sometimes be a voluntary participation in the grand coalition or forced upon by an outsider (p.127, Gibbons [2001]). The game is sometimes called a purely cooperative game among n players (p.127, Gibbons [2001]).

2.2 Coalition form

2.2.1 The value function

Let A be the set of all players $A = \{1, 2, ..., N\}$ Then the value function v is a function of all possible subsets of A to a real number.

$$v : \mathscr{P}(A) \rightarrow \mathbb{R} \tag{2.1}$$

The notation $v(L)$ can be used to represent the value of coalition $L \subseteq A$. For example $v(\{1, 2\})$ is the value that a coalition formed by Player 1 and Player 2 can obtain.

A nice way to get an overview of the value function in a three player game is shown in figure 2.1

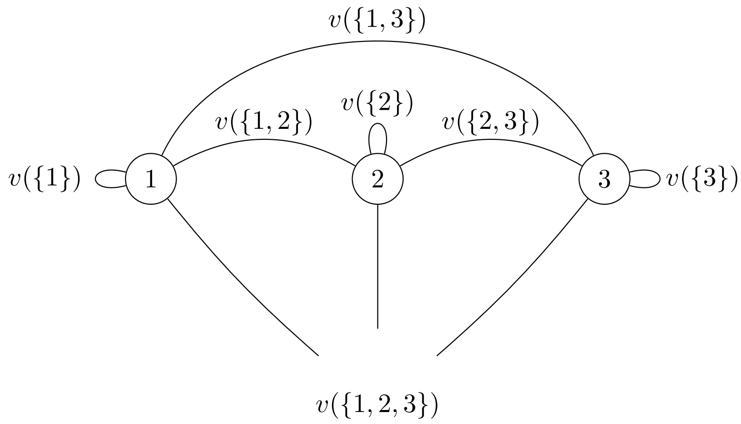

Figure 2.1: The above graphic representation of a value function in a three player cooperative game gives a nice overview of the value of different coalitions of the players 1,2 and 3

2.2.2 A cost function

In a cost sharing game there are several ways of defining the value function. In this thesis a cost function will be formed. If the reader finds it intuitive, one can think of the cost function as a negative valued value function. In the games that will be discussed in this thesis the cost function is an abstraction of the underlying optimisation problem that minimises the costs for that coalition. In fact the players are not them selves in position to choose the final solution that induces the total costs. This is done by the grid owner and can be a consequence of other parameters than the investment cost (which is the subject of this thesis).

The notation $C(L)$ will be used in this thesis to represent the cost of coalition $L \subseteq A$. For example $C(\{1,2\})$ is the Cost that a coalition formed by Player 1 and Player 2 will obtain.

In this type of games an allocation vector can be formed allocating the total cost to all the players. This will be denoted with the vector $\mathbf{x} = (x_1, ..., x_n)$ where $\sum x_i = C(A)$, and x_i is the cost allocated to player i. Different *cost allocation methods* will generally generate different unique allocation vectors to every game.

2.2.3 Null player

Player i is called a **null player** if:

$$v\big(L \cup \{i\}\big) = v(L) \tag{2.2}$$

That is, the marginal contribution to a coalition L of player i is zero i.e. it does not matter if i is in the coalition or not as the payoff/cost will be the same for all other players regardless (Feltkamp [1995]).

2.2.4 Substitutes

Two players i and j are called substitutes (p.74 Chakravarty et al. [2015]) if:

$$v\big(L \cup \{i\}\big) = v\big(L \cup \{j\}\big) \qquad \forall\ L \ \ \textit{such that} \ \ \{i\} \nsubseteq L \tag{2.3}$$

In words this means that the value/cost is the same if i or j joins a coalition L neither containing i nor j. Hence it does not matter which one of i and j who join the coalition

2.2.5 Superadditivity

A value function v is said to be Superadditive (p.21 Chakravarty et al. [2015]) if:

$$v(L_1 \cup L_2) \geq v(L_1) + v(L_2) \tag{2.4}$$

for any disjoint coalitions $L_1, L_2 \subseteq A$. This means that two coalition forming a grand coalitions can guarantee to get at least the payoff (maximum get the costs) they could guarantee to get if not collaborating.

2.2.6 Essential game

An game is said to be **essential** (p.22 Chakravarty et al. [2015]) if it in coalition form holds that the grand coalition A can get a lower cost than if each player were not to be collaborating i.e.:

$$v(A) > v(\{1\}) + v(\{2\}) + \cdots + v(\{N\}) \tag{2.5}$$

If a game is not essential, i.e. inessential, the players do not have incentives to cooperate since the maximum payoff (minimum cost) for the grand coalition is achieved without any cooperation.

2.2.7 Imputation

Let $\mathbf{x} \in \mathbb{R}^N$ $\mathbf{x} = (x_1, x_2, ..., x_N)^T$ denote the vector of the pay-offs. An imputation is a distribution of the pay off that satisfies efficiency and rationality (see 2.2.7.1 and 2.2.7.2). The set of all imputations, for a game with the value function v, is in this thesis denoted $I(v)$.

2.2.7.1 Efficiency/Group rationality

In order for a payoff vector to be *group rational* the sum of the payoff vector $\mathbf{x}$ should always be equal to the value of the grand coalition (p.13 Chakravarty et al. [2015]). Which basically means that all the value obtained by all the players in the game will be distributed among the players.

$$\sum_{i=1}^{N} x_i = v(A) \tag{2.6}$$

2.2.7.2 Individual Rationality

Individual rationality means that each player will gain at least as much from the total pay-off that this player would get if the player were not cooperating with anyone else (p.11 Chakravarty et al. [2015]).

$$x_i \geq v(\{i\}) \quad \forall \quad 1 \leq i \leq N \tag{2.7}$$

2.2.7.3 Domination of imputations

Let $\mathbf{x}$ and $\mathbf{y}$ be two imputations i.e. $\mathbf{x}, \mathbf{y} \in I(v)$. Then, $\mathbf{x}$ dominates $\mathbf{y}$ through a coalition L (p.12 Chakravarty et al. [2015]), if:

$$\begin{aligned} x_i > y_i \qquad & \forall \;\; i \in L \\ \sum_{i \in L} x_i \leq v(L) & \end{aligned} \tag{2.8}$$

This simply means that all of the players in the coalition L will get a strictly better payoff $\mathbf{x}$ than before if collaborating in the coalition L and that the payoff is feasible. We will denote that $\mathbf{x}$ dominates $\mathbf{y}$ through coalition L by:

$$\mathbf{x} \succ_L \mathbf{y} \tag{2.9}$$

2.2.8 The Core

Having the above definitions and concepts it is possible to define the concept the core as it was defined in a paper by Lloyd Shapley and Martin Shubik 1969 (Shapley and Shubik [1969]).

Definition 2.1. The core of a game is the set of outcomes where no sub-coalitions can achieve a more profitably allocation.

Hence the core is the set of imputations that can not be dominated by any other imputations through any possible coalition.

This can be formalised with the following equations: A vector $\mathbf{x} \in \mathbb{R}^N$ is in the core if and only if:

$$x_1 + x_2 + \cdots + x_N = v(A) \tag{2.10}$$

and

$$\sum_{i \in L} x_i \geq v(L) \tag{2.11}$$

for all possible coalitions L such that $L \subseteq A$.

2.2.8.1 The core in a three player cost sharing game

As in Maschler et al. [1979], the core for a three player cost sharing game can be graphically illustrated in a triangle plotted in *Barycentric coordinates*. Figure 2.2 on page 17 illustrates the core where $C_{\{1,2,3\}}$ is the cost for the grand coalition and $C_{\{S\}}$ the cost for the sub-coalition $S \subseteq A$. $\mathbf{x} = (x_1, x_2, x_3)$ is the cost allocation vector where x_i is the cost allocated to player i. For example in the allocation at the top corner of the triangle, player 1 will obtain the whole cost. At the opposite side of the triangle, the same player wont need to pay anything since in the allocations along that side represents allocations where the whole cost is split among player 2 and player 3. For any point between the corner and the opposite line the player pays the same proportion of the total cost as the proportion of the length between the corner and the line.

The whole triangle is within the $x_1+x_2+x_3 = C_{\{1,2,3\}}$ plane, i.e. all allocations in the figure satisfies group rationality which is one of the preconditions for a solution to be in the core.

2.2.9 The nucleolus

The nucleolus is yet another allocation rule for a TU-game first proposed by Schmeidler [1969]. It is not as commonly used as the Shapley value, which will be described later in this chapter, but not far behind (p. 231, Diaz [2010]).

If assuming that a coalition S would be dissatisfied if they receive less when cooperating in a greater coalition than the coalition would be able to obtain by only cooperating among the players in S (p. 139, Gibbons [2001]), the nucleolus is the imputation that minimises the maximum dissatisfaction among the coalitions.This can be used to motivate the nucleolus as an fair allocation rule.

To formalise the nucleolus we define the excess of a coalition L as with respect to an imputation x as:

$$e(L, x) = v(L) - \sum_{i \in L} x_i \tag{2.12}$$

In words, $e(L, x)$ is a quantity measuring the difference of the value generated by the players in a sub-coalition L and the total pay-off the players in this coalition

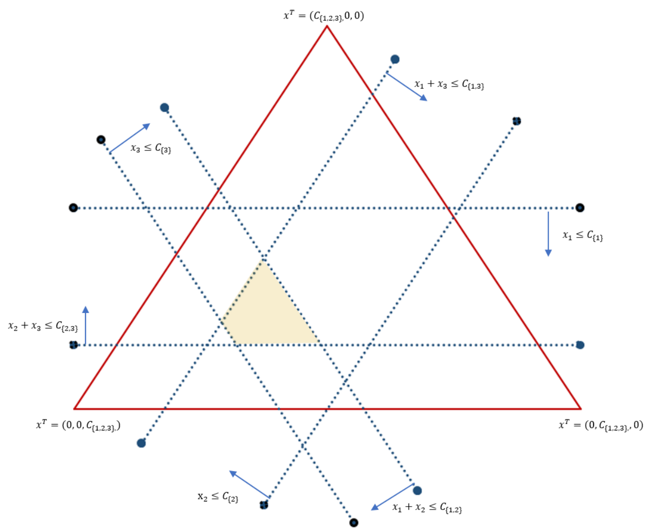

Figure 2.2: The figure shows cost allocations of the grand coalition cost illustrated in a triangle, where the lines represent the boundaries where the sub-coalitions' are indifferent to the allocation. In the direction of the arrows, the sub-coalitions' are better of with the grand coalition's allocation. The yellow area represents the core. Allocations are plotted in barycentric coordinates, where each point represents a cost allocation (x_1, x_2, x_3)

get. In some sense it is a measure of dissatisfaction of the coalition when the pay-off is distributed (231, Diaz [2010]).

Now by varying x we can minimise the maximum of all the excesses. This can be done by linear optimisation by formulating the problem into a linear programming (LP-problem) and solve it with e.g. the simplex method. See for example Boiers [2010].

The solution obtained from this *minimax* problem is called *the least core* or ϵ^1. If it has an unique solution this is also what is called *the nucleolus*, however we are not guaranteed that the least core only consists of a single point. We call X_1 a set of imputation in *the least core*. To find a single allocation among these imputation that form *the least core* we continue to minimise the second largest of the excesses out of all $x \in X_1$. Again there is no guarantee to have an unique solution and we need to define the solution as a set X_2. The process continue until X_k consist of a single imputation. This unique imputation is the nucleolus (Young et al. [1982]).

The nucleolus can be a bit technical to calculate by hand. In a discussion paper by Guajardo and Jörnsten [2015] numerous publications is identified where the nucleolus has been miscalculated. With this in mind the calculations in this thesis are made through a MATLAB ToolBox, an application programmed to get some of the well knows solutions to cooperative *TU-games*. [1]

2.2.10 The Shapley value

2.2.10.1 Definition

For games represented in value or cost functions, the Shapley value is the most commonly used solution

Assume a game given in coalition form as described above. In such game the Shapley value of each player i in the game can be defined as:

$$\phi_i = \sum_{\{L:i\in L\}} \frac{(N-|L|)!\cdot(|L|-1)!}{N!}\cdot\delta(i,L) \tag{2.13}$$

[1] http://www.mathworks.com/matlabcentral/fileexchange/35933-mattugames

where,

$$\delta(i, L) = v(L) - v(L \setminus \{i\}) \tag{2.14}$$

The right hand side of equation 2.14 above is to be read as the value of coalition L minus the value of coalition L without player i. Note that if i were not in L from the beginning $\delta(i, L) = 0$. Hence, $\delta(i, L)$ is simply to be viewed as the marginal contribution of player i to coalition L.

In order to get an intuition of what the Shapley Value really is we do the following observation: The binomial coefficient $\binom{n}{k}$ can be interpreted in combinatorial mathematics as the number of possible ways, disregarding order, to choose k objects among n objects without replacement. By using knowledge from combinatorial mathematics we can rewrite,

$$\binom{N}{|L|} = \frac{N!}{(N - |L|)!|L|!} \tag{2.15}$$

By dividing both the denominator and the numerator, in equation 2.15, with $|L|$ we get:

$$\binom{N}{|L|} = \frac{N!/|L|}{(N - |L|)!|L|!/|L|} = \frac{1}{|L|} \cdot \frac{N!}{(N - |L|))!(|L| - 1)!} \tag{2.16}$$

It should now be clear that the first product in each term of the Shapley value of player i, in eqiation 2.13 can be rewritten as:

$$\frac{(N - |L|)! \cdot (|L| - 1)!}{N!} = \frac{1}{|L| \cdot \binom{N}{|L|}} \tag{2.17}$$

And the final step is to rewrite equation 2.13 as:

$$\phi_i = \sum_{\{L: i \in L\}} \frac{\delta(i, L)}{|L| \cdot \binom{N}{|L|}} \tag{2.18}$$

The gain from rewriting 2.13 into 2.18 is that it makes more sense when reading it out so that it easier to understand what is actually going on in the formula. Formula 2.18 is read: The Shapley value of player i is equal to the sum, over all coalitions L in which the the player i is a member, of the marginal contribution of player i in that coalition divided by the number of players in the coalition times the number of possible ways to form that coalition out of the grand coalition.

We also define the Shapley Vector of a game with N players as the vector:

$$\mathbf{\Phi} = (\phi_1, \phi_2, ..., \phi_N) \tag{2.19}$$

For a intuitive understanding of the Shapley value, it can be interpreted as the following. The grand coalition is formed in a sequence where one player is added to the coalition at a time. The sequence is formed randomly, and thus the marginal contribution from a player entering the coalition is in general dependent on its place in the queue. When a player enters the sub-coalition (the coalition that has been formed at that point, i.e., it consists of all the players that were before the entering player in the queue and the entering player) the entering player is given a payoff equal to his marginal contribution of the earnings (savings) of the sub-coalition. All possible sequence (queues) of forming the coalition for the players are then considered. The Shapley value for a player is the mean of all marginal contribution in joining the different sub-coalitions.

The Shapley vector is also the **only** vector always satisfying the following conditions:

(1) **Symmetry** - if player i and j are *substitutes* as described in section 2.2.4 then $\phi_i = \phi_j$. Players who contribute equally to all possible coalitions are allocated the same payoff from the savings.

(2) **Null player condition** - if player i is a *null player* as described in section 2.2.3 then $\phi_i = 0$, i.e. players without contribution to any coalition will have zero payoff from the game.

(2) **Efficiency** - $\phi_1 + \phi_2 + \cdots + \phi_N = v(A)$. This condition ensure that the Shapley value method allocates the total value of the coalition to its players, and that players outside of the coalition receives zero payoff.

(4) **Additivity** - if v and v' are two value functions then $(\phi_{v+v'})_i = (\phi_v)_i + (\phi_{v'})_i$. The distributed payoffs should correspond to the payoffs derived from v and the payoffs derived from v'

.

2.3 Cost sharing

Cooperative game theory has been widely used in various different cost-sharing games. The various solution concepts have appealing features e.g. principles of "fairness" when dividing costs among players. Osborne and Rubinstein [1994] shows how the concepts of the solutions discussed and explained in the this thesis are valid also when defining a cost function, in the thesis denoted C_L, containing the costs of the possible coalitions $L \subseteq A$.

2.3.1 Classification of the cost sharing game

In order to know where to look for solutions of a game it is an important first step to classify it. A cost-sharing game is intuitively a TU-game if all players have constant and equal marginal utility of not having a cost. The utility, *money*, is of course transferable among the players, see section 2.1.1.

In the cost-sharing game that will be studied in this thesis the agents of the game are forced to cooperate and form the grand coalition by the grid owner, this makes the game a CFG-game according to 2.1.2. As seen before a characteristic function game can be studied through it's sub-coalitions.

Chapter 3

Concept of fairness

3.1 Fairness axioms and allocation methods

The definition of the concept of *fairness* is a controversial issue. First we introduce different ideas of fairness that we find sensible and related concepts. It is not an extensive list, and we acknowledge that there are differing views and what should be considered fair:

- Costs are shared equally
- Costs are allocated proportionally based on capacity need
- Individual rationality - no player is allocated a cost larger than its stand-alone cost (see 2.2.7.2)
- Coalition rationality (essential game) - no sub-coalition is allocated costs larger than its stand-alone cost (see 2.2.6)
- Group rationality - the grand coalition's cost is lower than the sum of all players' stand-alone costs (see 2.2.7.1)
- Symmetry - substitute players are allocated the same costs (see 2.2.10)
- Costs are allocated based on each players contribution to the efficiency improvements in the forced collaboration (see the definition of the Shapley value)

- Minimised dissatisfaction - minimise the maximum dissatisfaction among the players (see the definition of the nucleolus)

However, since the natural monopoly and forced collaboration does not reflect a competitive market situation (which is striven after, based on the law), we need to introduce concepts that take this into account. The major difference from a forced collaboration is that players would not chose a certain collaboration if they would be better of else-how

With individual rationality, the possible allocations for the grand coalition is the set of *imputations* ($I(V)$), see section 2.2.7.

For coalition rationality, which by its definition also includes individual rationality, the set of allocations make up *the core*, see section 2.2.8. The core, as a concepts, is a method to determine if the players would be willing to form the grand coalition given the way the costs are divided. It allows us to understand if players could be better of by forming sub-coalitions, even if the grand coalition provides the highest total savings. In the monopoly market, it is not possible to deter from the connection provided by the grid owner, but it is important to understand if players would be willing to do so if they had the option. The reason of introducing the concept of the core is to answer whether players would prefer a certain allocation in a hypothetical non-monopoly setting.

The use of the concepts imputations and the core from a fairness perspective is evident, however it does not provide an allocation, but rather a evaluation tool. Thus, we introduce the The Shapley value, and the nucleolus that provide an precise allocation, as they are commonly considered to result in fair allocations based on given assumptions and axioms (see for example Young [1985a], Myerson [1980], Roth [1988]). Furthermore, it is suggested that the Shapley value and the nucleoulus are the most appropriate allocation methods on problems of the type that is covered in this thesis (Young [1985b]).

- The Shapley Value

  The Shapley value allocation method is a normative allocation concept that is generally considered as a fair way of dividing gains from collaboration. As a consequence of the definition four appealing axioms hold: Symmetry, Null player condition, Efficiency and Additivity (Shapley [1967]). Efficiency is defined by Pareto efficiency, which guarantees that players cannot benefit from a better allocation without another player being worse of. Additivity specifies a certain way in which the values of different sub-coalitions must relate to each other. The condition for symmetry means that the allocation is not dependent on when players enters the game. The null player condition means that a player with zero marginal contribution to any coalition is given a value of zero.

- Nucleolus

  The nucleolus allocates the costs based on the principle to "minimise the maximum regret". It provides a allocation within the core (if the core is non-empty).

We now have four concepts for our analysis, that are based on axioms, creating an objective standpoint to determine whether and allocation can be considered as fair:

1. Imputations
2. The core
3. The Shapley value
4. Nucleoulus

These concepts complement each other well. The core defines all possible allocations where no player can benefit from from forming a different coalition than the grand coalition. Thus, the core include all allocations giving each player costs not exceeding their alternatives providing stable allocations. The core is a sub-set of the set of imputations, as it has stricter boundaries (coalition rationality is included in the core, whereas imputations only need to adhere to individual rationality). The Shapley value allocates costs based on the players marginal contributions to the costs savings made possible by the grand coalition (De Clippel and Rozen [2013]). The concepts each capture different levels of allocation fairness: The core capture a solution space, and the Shapley value and nucleolus actual allocations. There is a solid theoretical foundation, arguing they can be considered fair methods.

Chapter 4

Defining the general grid upgrade game

4.1 Introduction

Suppose three wind power suppliers, *1, 2* and *3*, are to connect to the regional grid in the same area. The grid owner has calculated the most efficient way to complement and upgrade the grid so that the electricity generated from the wind parks can be used in the main grid. However since it is the power suppliers that should fund the upgrade the grid owner has to allocate the costs to the three suppliers. If this is not done fairly, the suppliers might appeal in court which could be both inconvenient and costly. In order to make the optimisation of this upgrade easier to follow this example assumes that the transformation station is built in direct access to the regional grid, i.e. the transformation station is the connection point to the existing grid. It is also assumed that all the players has to connect to the transformation station separately.

4.2 The cost structure of the problem

Figure 4.1 on page 26 shows an overview of the cost structure. Costs are divided into 6 parts. One for each electricity producer to connect to the transformation station (C^1, C^2 *and* C^3), one cost for the transformation station (C^T), and one for the necessary upgrade in the regional grid (C^U). By stating some definitions and assumptions we will derive some facts about the different costs.

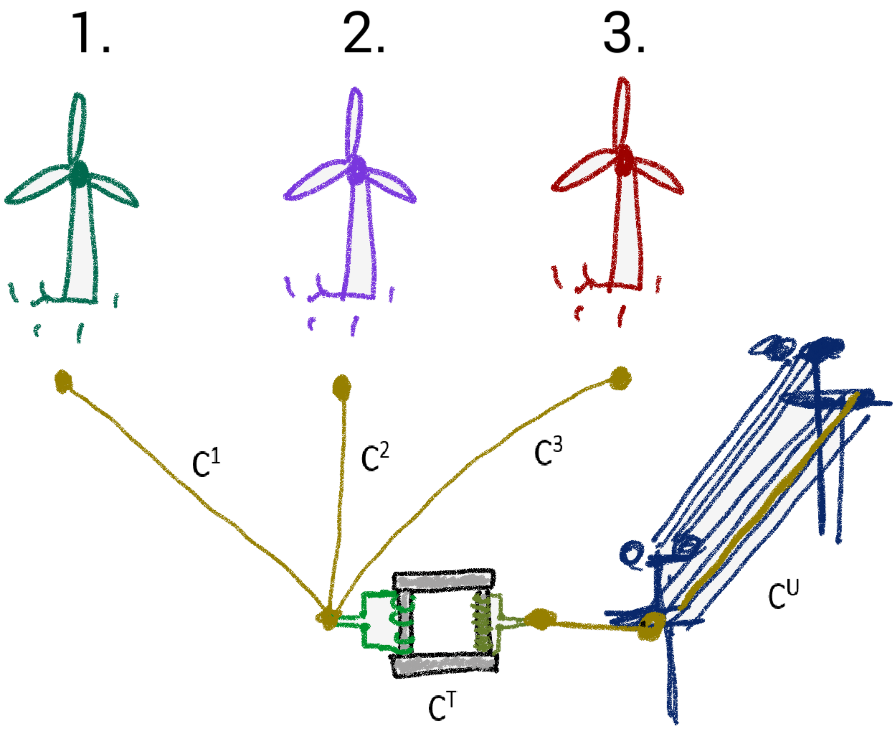

Figure 4.1: The figure shows the necessary investments for a grid upgrade for the grand coalition. The costs are divided into 6 parts. One for each electricity producer to connect to the transformation station (C^1, C^2 *and* C^3), one cost for the transformation station (C^T) and one for the necessary upgrade in the regional grid (C^U).

All upgrades and new facilities has *discontinuous* costs in at least one cost-driver, the effect a facility can handle. Transformation stations are generally designed to handle effects in certain intervals. For example the smallest transformation station of a specific type might be designed to handle effects up to 4MW while the second smallest transformations station handles effects between 4 MW and 6.3 MW and so on. This means that the cost of such upgrade isn't a continuous function of the effect. The grid cost is however continuous in the length dimension, which is its most significant cost driver.

4.2.1 Notations

- $C_L^i \quad (i = 1, 2, 3, ...)$ - the cost of connecting player i to the transformation station when coalition L is formed
- C_L^T - the cost of transforming the electricity generated by coalition L to the right voltage
- C_L^U is the cost of the the necessary upgrades on the regional grid when the players in coalition L are connecting
- C_L^{tot} is the total cost of the upgrade and connection of coalition L i.e

$$C_L^{tot} = C_L^T + C_L^U + \sum_{i \in L} C_L^i \tag{4.1}$$

- P_L is the effect (in watt) that power is generated from by coalition L

4.2.2 Assumptions

4.2.2.1 Full cooperation

The players are forced to cooperate if they wish to connect, as a result of the natural monopoly. This means that the game is a characteristic function game, (CFG), as discussed in section 2.1.

4.2.2.2 Monetary utility

The utility of each player will be measured in monetary units and all players have the same constant marginal utility. This means that the game will is a TU-game. In this study, for simplicity's sake, we will only look at the investment costs. An interesting broadening of the scope could be to look at the life-cycle profit for the grand coalition plus the grid owner.

4.2.2.3 One transformation station is optimal

For simplicity, optimality for all coalitions is assumed with one transformation station. This is reasonable as the players are assumed to be located in the same area and building several transformation stations can have a long term effect on profitability. However since the long term effects are invisible when looking at the investment costs the assumption can in some cases remove superadditivity from the cost function. Especially when the player are located far from each other, they may be forced to build a lot of extra grid to connect to the joint connection point which may be quite costly.

To make the optimisation of the upgrade easier to follow, this example assumes that the transformation station is built in direct access to the regional grid, and that all players connects to the transformation station separately.

4.2.2.4 Relationship between power generated and transformation costs

The cost of the equipment to transform the power from the players in a coalition is only depending on the amount of power the players are generating. Further more it holds that

$$P_{L_1} < P_{L_2} \implies C^T_{L_1} \leq C^T_{L_2} \tag{4.2}$$

$$P_{L_1} > P_{L_2} \implies C^T_{L_1} \geq C^T_{L_2} \tag{4.3}$$

$$P_{L_1} = P_{L_2} \implies C^T_{L_1} = C^T_{L_2} \tag{4.4}$$

Generally an increase in the amount of power produced would lead to an increase in transformation costs, however since there might be fixed steps to which the transformation station can be upgraded this might not be a strict inequality.

4.2.2.5 Relationship between power generated and regional grid upgrade cost

The same reasoning as above in section 4.2.2.4 the cost of upgrading the existing regional grid C^β_L does only depend on the amount of power produced by the players in coalition L. It holds that:

$$P_{L_1} < P_{L_2} \implies C^\beta_{L_1} \leq C^U_{L_2} \tag{4.5}$$

$$P_{L_1} > P_{L_2} \implies C^\beta_{L_1} \geq C^U_{L_2} \tag{4.6}$$

$$P_{L_1} = P_{L_2} \implies C^\beta_{L_1} = C^U_{L_2} \tag{4.7}$$

Again generally an increase in the amount of power produced would lead to an increase in upgrade cost of the existing regional grid. However again upgrades might be done in fixed steps and there might be free capacity for some of the players hence there is not a strict inequality.

4.2.3 Superadditive transformation costs

Adding a player to the coalition will inevitably increase the power production of that coalition. Hence we can for player i and coalitions L_2 and L_1 say:

$$L_2 = L_1 \cup \{i\} \implies C^T_{L_1} \leq C^T_{L_2} \tag{4.8}$$

4.3 Details of the problem

The players, i.e. the power suppliers, are forced to cooperate by the grid owner hence the game is a coalition form game. There are $2^3 - 1 = 7$ possible non-empty sets, in order to characterise the game we need to calculate the value of all these subsets i.e. the value function.

4.3.1 The position of the transformation station

The transformation station will for each sub coalition have an optimal position i.e. the position that minimises the costs of connecting the players in that coalition to the station. The following matrix describes this cost structure:

4.3.2 Optimal grid upgrade of different coalitions

For the sub-coalitions the optimal grid upgrades are just theoretical as these upgrades will not be realised. Thus, the actual cost the upgrade would incur cannot be used - instead they are calculated. This is however not an issue, as Ei use the normvärdestabell instead of actual costs if there is any conflict regarding what a wind developer is charged for the connection. Note that the costs with notations as in figure 4.1 on page 26 can be completely different from one coalition to another. E.g., for C^i, it should be evident how this cost changes by looking at figure 4.2 on page 36 which describes how the optimal placement of the transformation station changes depending on the coalition.

All costs can be summarised in a matrix like the one in table 4.1.

Table 4.1: Cost structure of the grid upgrade game

L	C_L^1	C_L^2	C_L^3	C_L^T	C_L^U	C_L^{tot}
∅	0	0	0	0	0	0
{1}	$C_{\{1\}}^1$	0	0	$C_{\{1\}}^T$	$C_{\{1\}}^U$	$\sum$
{2}	0	$C_{\{2\}}^2$	0	$C_{\{2\}}^T$	$C_{\{2\}}^U$	$\sum$
{3}	0	0	$C_{\{3\}}^3$	$C_{\{3\}}^T$	$C_{\{3\}}^U$	$\sum$
{1,2}	$C_{\{1,2\}}^1$	$C_{\{1,2\}}^2$	0	$C_{\{1,2\}}^T$	$C_{\{1,2\}}^U$	$\sum$
{1,3}	$C_{\{1,3\}}^1$	0	$C_{\{1,3\}}^3$	$C_{\{1,3\}}^T$	$C_{\{1.3\}}^U$	$\sum$
{2,3}	0	$C_{\{2,3\}}^2$	$C_{\{2,3\}}^3$	$C_{\{2,3\}}^T$	$C_{\{2,3\}}^U$	$\sum$
{1,2,3}	$C_{\{1,2,3\}}^1$	$C_{\{1,2,3\}}^2$	$C_{\{1,2,3\}}^3$	$C_{\{1,2,3\}}^T$	$C_{\{1,2,3\}}^U$	$\sum$

The total cost of the grand coalition, that is the overall optimal cost for all players, is the actual cost that will occur all other costs are just calculated theoretically in order to describe the situation as a characteristic function game.

In the game described in this chapter we look at 8 different optimal upgrades of the power grid, one for each possible sub-coalition.

4.4 Example

4.4.1 Problem scope

Consider the situation previously described in this chapter where three wind power suppliers are establishing new wind parks in an area. All players are using wind turbines generating 2 MW a piece. The wind parks they are planning to build are of different sizes. The total power generated can be seen in table 4.2

The players in this game, the wind power suppliers, are all using underground cables to connect the park to the existing grid via a transformation station that is not yet built. Each player wants to minimise the distance to the planned transformations system in order to have as little cost for the underground cable as possible. If a coalition of players are formed the coalition strives to minimise the total cost of cable. In this example we will assume that all players are using

Player	# of turbines	Power generated
1	5	10 MW
2	4	8 MW
3	8	16 MW
$\sum$	17	34 MW

Table 4.2: The table shows the power generated from each players wind farm in the example described in section 4.4

the same type of cable this makes the cost minimisation of cables equivalent to minimisation of the sum of the distances from the transformation station to the players in to coalition.

4.4.1.1 Minimising cable length

The principle of the placement of the transformation station optimal for a coalition is to minimise the length of the cables that are connecting the players in the coalition to the station. An example of this can be seen in picture 4.2 on page 36. For each active coalition there is a different optimal position. Depending on how the ingoing effect is to the transformation station the cost will be different. If different cables are used from different wind parks this costs can be added as weights in the minimisation problem. In, Appdx A - Minimisation of Cable Length, the principles of the optimisation is described.

For each coalition the optimal solution of grid connection is calculated. Optimal is here defined as the solution with lowest cost. Some assumptions are made to simplify the cost calculation. However the overall results would still be valid of more details where taken into account when minimising the costs of each coalition.

4.4.2 Assumptions

1. Each agent is connecting directly to a transformation station located somewhere along the regional grid

2. The regional grid is a straight power line passing by all agents

3. The optimal solution is always to connect all agents to the same transformation station
4. All agents are using the same type of power line as a straight line all the way to the transformation station.

4.4.3 Costs

The underground cables needed for each player can be assumed to cost 300'000 SEK per km (assumption from normvärdeslista jordkabel landsbyggd).

The transformation station has different costs (also assumed from normvärdeslista) depending on how many MW it is handling and between what voltage it is transforming. The different stations that are available for this project are delimited to transformation from 12-24 kV to 52 kV. The different effects and the cost associated are presented in table 4.3. The possible effects in the coalitions are 8, 10, 16, 18, 24, 26, 34 with the associated costs presented in table 4.3.

Table 4.3: Costs of transformation stations with different effects

MW	4	6,3	10	16	20	25	40	63
Cost (MSEK)	3.3	3.8	4.1	4.5	5.6	6.1	7.2	8.7

4.4.4 Characteristic function (Cost function)

Based on the assumptions above, the resulting cost function of the example is given in table 4.4.4

4.4.5 Applying solution concepts to the game

4.4.5.1 Imputations

The set of all imputations is defined by individual rationality and group rationality (also called efficiency). The allocation vector, $\mathbf{x} = (x_1, x_2, x_3)$ that is a imputation, is in this example in the set:

$$\{\mathbf{x} : x_1 + x_2 + x_3 = 8.4, x_1 \leq 4.4, x_2 \leq 4.2, x_3 \leq 5\} \tag{4.9}$$

Table 4.4: Cost structure of the grid upgrade game in the above example

L	C_L^{tot} (MSEK)
∅	
{1}	4.4
{2}	4.2
{3}	5.0
{1,2}	4.8
{1,3}	6.9
{2,3}	7.0
{1,2,3}	8.4

The set is a triangle within the plane $x_1+x_2+x_3 = 8.4$ with corners in the points: $(4.4, 4.2, -0.2), (-0.8, 4.2, 5.0), (4.4, -1.0, 5)$, see Figure 4.4. Notable is that the set of imputation also includes a possibility to have negative costs for the players. This means that one of the player actually can receive money from the other two players in an allocation in the set of imputations.

4.4.5.2 Superadditivity

The game is superadditive since the following inequality holds for all coalitions L_1, L_2 in the set of all coalitions.

$$C(L_1 \cup L_2) \leq C(L_1) + C(L_2) \tag{4.10}$$

4.4.5.3 The Core

To form the core of the game one have to set up the inequalities from Equation 2.10 and 2.11 on page 15. The core of this game is formed by the following inequalities within the plane $x_1 + x_2 + x_3 = 8.4$:

$$x_1 \leq 4.4 \quad , \quad x_2 \leq 4.2 \quad , \quad x_3 \leq 5 \tag{4.11}$$

$$x_1 + x_2 \leq 4.8 \quad , \quad x_1 + x_3 \leq 6.9 \quad , \quad x_2 + x_3 \leq 7 \tag{4.12}$$

$$x_1 + x_2 + x_3 = 8.4 \tag{4.13}$$

This means that the geometric body of the core in the game is the convex hull of the four points $(3.3, 1.5, 3.6), (1.4, , 3.6), (1.4, 2, 5), (1.9, 1.5, 5)$.

A graphical representation of the core can be seen in Figure 4.5 on page 38.

4.4.5.4 Shapley Value

The Shapley vector $\mathbf{\Phi}$ for the game has been calculated:

$$\mathbf{\Phi} = (\phi_1, \phi_2, \phi_3) = (2.4, 2.25, 3.75) \tag{4.14}$$

Notice that the Shapley vector, in this example, is within the core. The Shapley value is calculated according to section 2.2.10 by a program written and available by the authors.

4.4.5.5 Nucleolus

The nucleolus of the game is calculated:

$$\mathbf{\Theta} = (\theta_1, \theta_2, \theta_3) = (2.03, 2.13, 4.23) \tag{4.15}$$

Notice that the nucleolus is also within the core in this example. The calculations have been performed with the MatLab routine mentioned in section 2.2.9.

4.4.5.6 Proportional split

The proportional split were each player pays for their own connection to the transformation station and the cost of the transformation station is divided by the players by the proportion of the total power production each player stands for is calculated for the example:

$$\mathrm{P} = (\rho_1, \rho_2, \rho_3) = (2.43, 2.07, 3.93) \tag{4.16}$$

Again notice that also a proportional split of the costs is in the core in this particular example.

4.4.5.7 Summary of the results

From the analysis of the different solution concepts of the example in this section we have seen that all proposed allocations are within the core. A solution inside the core is stable in the sense that no sub coalition can profitably block the allocation. As we will see later on, the existence of a core in "The grid upgrade game" is not inherent.

Figure 4.5 shows a graphical representation of all the allocations and the core (the yellow field) of the example.

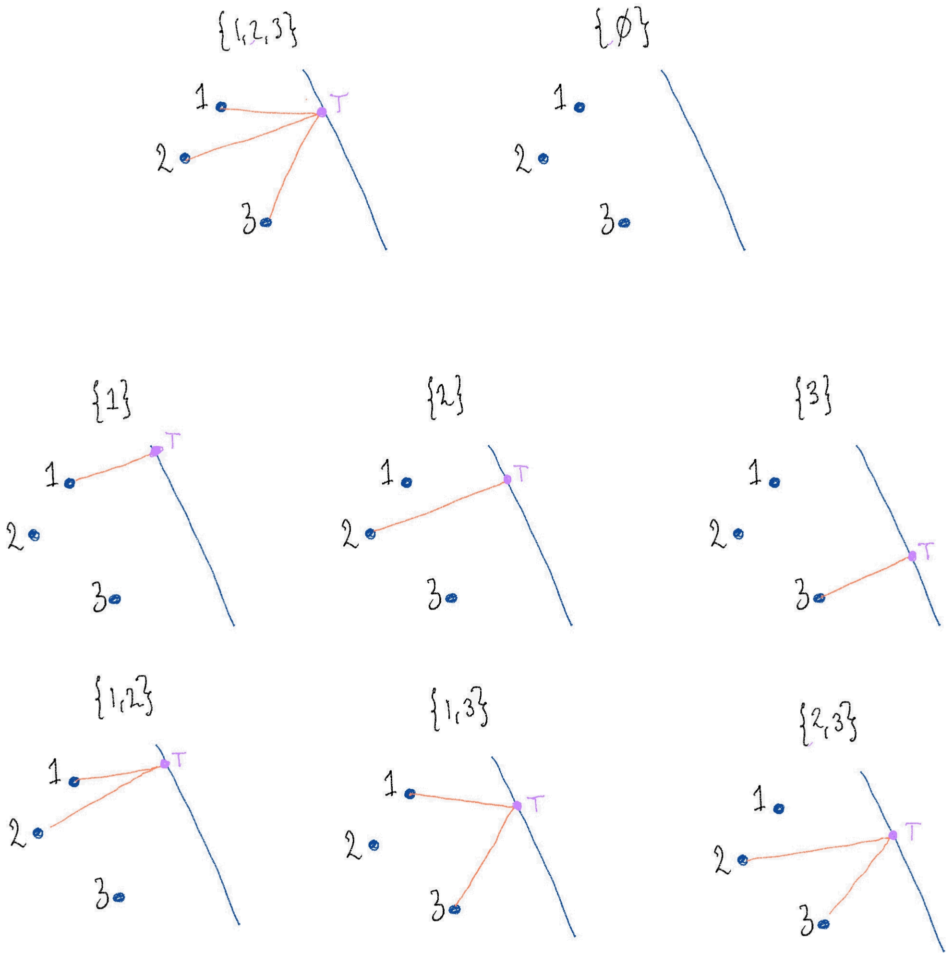

Figure 4.2: The figure shows the optimal upgrade of the power line when the 8 possible coalitions are active. The dots marked with 1,2 and 3 are the locations of the wind parks. The dot marked with T is the optimal location of the transformation station for the active transformation, the orange lines are the connection power lines from each player to the regional grid and the blue line represents the regional grid.

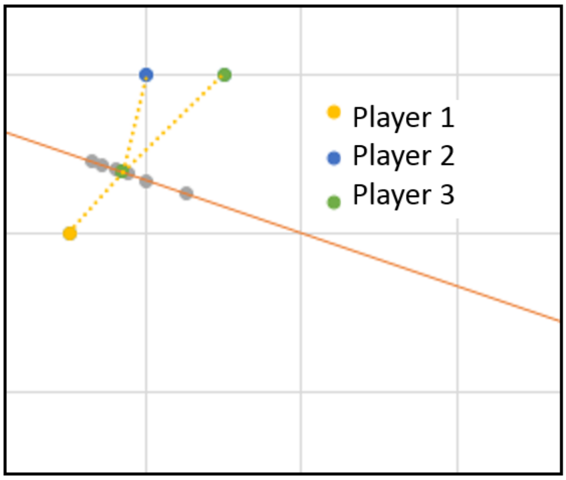

Figure 4.3: The figure shows optimal locations for a transformation station when seven different coalitions are formed. The yellow dotted lines connect each player to the optimal location when the grand coalition is formed.

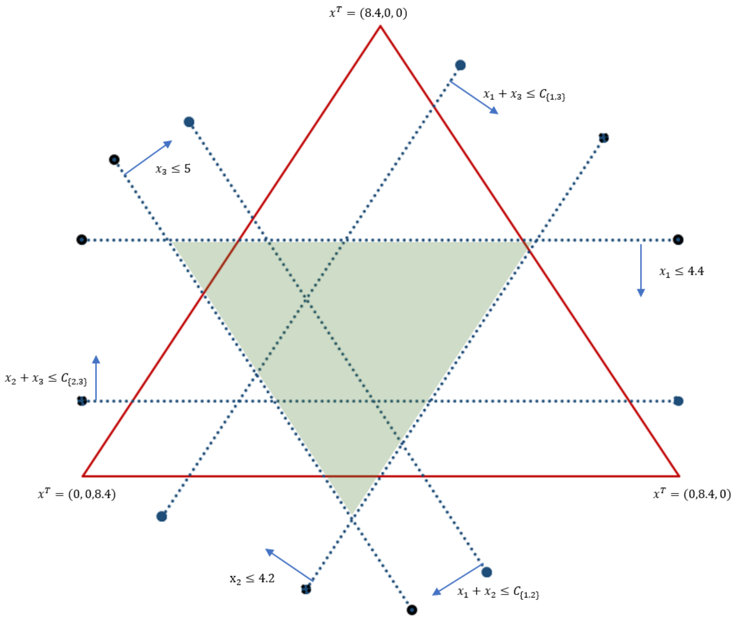

Figure 4.4: The figure shows a graphical representation of the set of all imputation as a green triangle. The figure is plotted in Barycentric coordinates.

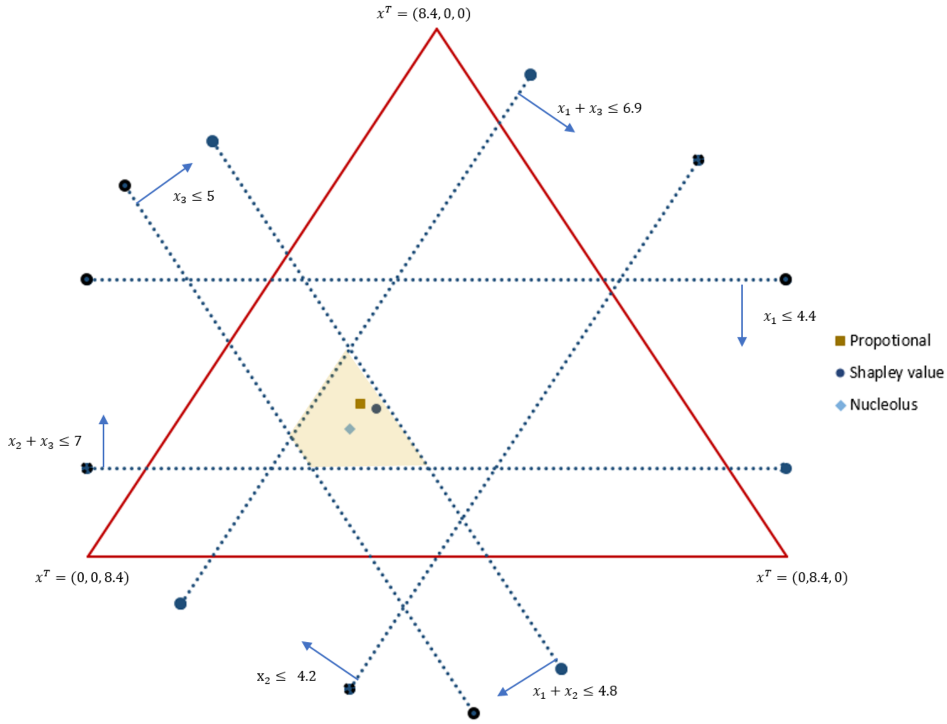

Figure 4.5: A graphical representation of all the proposed solution concepts of the game as well as the core (the yellow field). The triangle is plotted in Barycentric coordinates.

Chapter 5

Real application - a case study

5.1 The Havsnäs wind farm project

The wind farm is connected to Svenska Kraftnäts national 220 kV line 'AL1', whose trajectory is well-suited for the project. It runs right through the project and had unused capacity to handle the new wind power. The linking of the separate areas was made with a 33 kV overhead system distributing energy of the respective areas; 1, 2 and 3, to a 220/33 kV transformation station, where the energy is transformed and feed onto the AL1 line. Each area has beneath surface cables connecting each turbine to the 33 kV system. Approximately five kilometres of the new 33 kV overhead lines has been built next to the existing 220 kV line to minimise the intrusions in the nature. This is considered when determining the sub coalitions optimal solutions so that the hypothetical grid connection would be with relatively little impact on nature.

The wind farms are the players and are numerated in the following way:

- Player 1 - Ritjelberget
- Player 2 - Ursåsen
- Player 3 - Järvsand

5.1.1 Costs

Each player generates power according to table 5.1, the required transformation (trafo) station according to the Normvärdestaball is seen together with the trafo cost is seen in Table 5.2. Table 5.3 shows the needed investment in overhead lines for each coalition in order to connect each wind farm in the coalition to the trafo station. The cost per km of the transmission lines are assumed to be 500'000 SEK based on Ei's normvärdestabell and the total coalition cost of these transmission lines are seen in the same Table 5.3.

Player	# of turbines & Power generated
1	42 MW
2	22 MW
3	32 MW
$\sum$	96 MW

Table 5.1: The effects for the three players

Cost of trafo station for all coalitions are seen in table 5.2 below.

Table 5.2: Effect, transformation MVA required, and the cost of the transformation station, for each sub-coalition

Coalition	{2}	{3}	{1}	{2,3}	{1,2}	{1,3}	{1,2,3}
MW	22	32	42	54	64	74	96
Trafo MVA	25	40	63	63	80	80	100
Cost (MSEK)	14.3	15.1	16.3	16.3	17.4	17.4	18.4

Figure 5.1 shows a topological map of the infrastructure project in Havsnäs with all its proposed upgrades. From the map the distances that new cable needs to be drawn has been measured to calculate the total cable cost for each coalition. These can be found in table 5.3. The real map of the area, from which the topological map is derived, can be found in Appdx B - Map of Havsnäs, Figure 1 on page 55.

5.1.2 Characteristic function

The total cost of each coalition can be seen in table 5.4 on page 42.

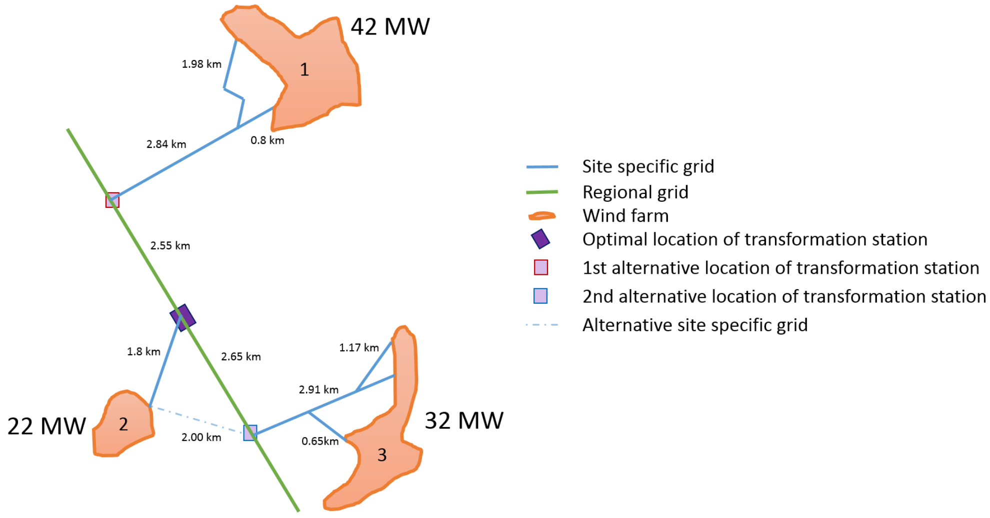

Figure 5.1: The topological map shows the wind farms and how they connect to the regional grid in different coalitions. Player 1 would in its singleton coalition connect to a transformation at the *1st alternative location of transformation station*. Player 3 would in its singleton coalition connect to the *2nd alternative location* and player 2 to the optimal location. Coalition {2,3} i.e. the coalition of player 2 and 3 would connect at the *2nd alternative location*, this is the only coalition in which the site specific grid of a player is not the same as in the optimal location (*player 2 connect to the 2nd alternative location through the alternative site specific grid*. All other coalitions uses the optimal location of transformation station

5.1.3 Imputations

The set of all imputations if formed by the following triangle in the $x_1+x_2+x_3 = 27.1$ plane:

$$\{\mathbf{x} : x_1 + x_2 + x_3 = 27.1, x_1 \leq 19.1, x_2 \leq 15.2, x_3 \leq 17.8\} \tag{5.1}$$

Figure 5.2 shows the set of all imputations in barycentric coordinates. The triangle has its corners in the three points $(19.1, 15.2, -7.2)$, $(19.1, -9.8, 17.8)$ and $(-5.9, 15.2, 17.8)$.

Table 5.3: Required total grid extension, length and cost, for each coalition. Based on the measurements in Figure 5.1

Coalition	{2}	{3}	{1}	{2,3}	{1,2}	{1,3}	{1,2,3}
Length (km)	1.8	4.73	5.62	6.73	9.97	15.55	17.35
Cost (MSEK)	0.9	2.4	2.8	3.4	5.0	7.8	8.7

Table 5.4: Total cost of each coalition. Based on the measurements in Figure 5.1

L	C_L^{tot} (MSEK)	C_L^{trafo} (MSEK)	C_L^{grid} (MSEK)
{1}	19.1	16.3	2.8
{2}	15.2	14.3	0.9
{3}	17.8	15.4	2.4
{1,2}	22.4	17.4	5
{1,3}	25.2	17.4	7.8
{2,3}	19.7	16.3	3.4
{1,2,3}	27.1	18.4	8.7

5.1.3.1 Superadditivity

The game is super additive since the following inequality holds for all coalitions L_1, L_2 in the set of all coalitions.

$$C(L_1 \cup L_2) \leq C(L_1) + C(L_2) \tag{5.2}$$

5.1.3.2 The Core

The core of the game is formed by the following inequalities within the plane $x_1 + x_2 + x_3 = 27.1$:

$$x_1 \leq 19.1 \quad , \quad x_2 \leq 15.2 \quad , \quad x_3 \leq 17.8 \tag{5.3}$$

$$x_1 + x_2 \leq 22.4 \quad , \quad x_1 + x_3 \leq 25.2 \quad , \quad x_2 + x_3 \leq 19.7 \tag{5.4}$$

$$x_1 + x_2 + x_3 = 27.1 \tag{5.5}$$

A graphical representation in Barycentric coordinates of the core can be seen in Figure 5.3 on page 45. The core of the game is formed by the convex hull (in this case an irregular pentagon) of the five points: (19.1,3.3,4.7), (19.1,1.9,6.1), (7.2,15.2,4.7), (7.4,15.2,4.5) and (7.4,1.9,17.8).

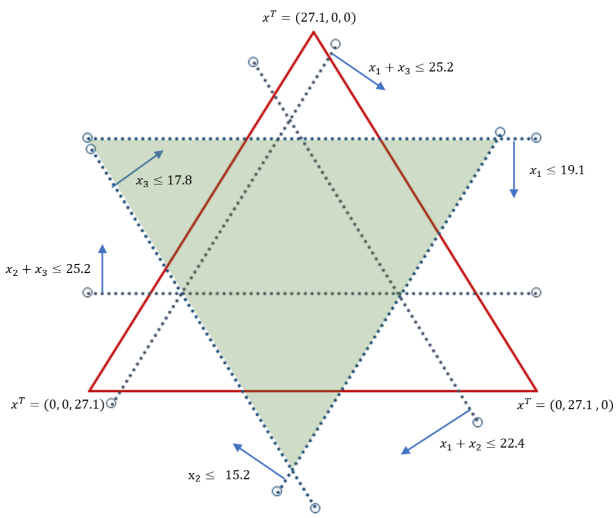

Figure 5.2: Havsnäs - A Graphical representation of the game in barycentric coordinates. The set of all imputations is the green triangle

5.1.3.3 Shapley Value

The Shapley vector $\mathbf{\Phi}$ for the game has been calculated:

$$\mathbf{\Phi} = (\phi_1, \phi_2, \phi_3) = (11.27, 6.57, 9.27) \tag{5.6}$$

The Shapley vector has been plotted together with the core and the other allocation rules in figure 5.3. Notice that the Shapley vector, in this example, is within the core.

5.1.3.4 Nucleolus

The nucleolus of the game is calculated:

$$\mathbf{\Theta} = (\theta_1, \theta_2, \theta_3) = (11.77, 6.27, 9.07) \tag{5.7}$$

The nucleolus has been plotted together with the core and the other allocation rules in figure 5.3.

5.1.3.5 Proportional split

The proportional split were each player pays for their own connection to the transformation station and the cost of the transformation station is divided by the players by the proportion of the total power production each player stands for is calculated for the example:

$$\mathrm{P} = (\rho_1, \rho_2, \rho_3) = (12.14, 5.12, 9.82) \tag{5.8}$$

The nucleolus has been plotted together with the core and the other allocation rules in Figure 5.3. Again notice that the proportional split is within the core in this example.

5.1.4 Summary of results

Given the total coalition costs in table 5.4 the Shapley value, the nucleolus and the proportional allocation is calculated and presented in table 5.5. A graphical representation of the core and the three allocations can be seen in Figure 5.3.

Table 5.5: Cost allocations (MSEK) for the three players depending on allocation method in the Havsnäs example.

Player	1	2	3
Proportional	12.14	5.12	9.82
Shapley Value	11.27	6.57	9.27
Nucleolus	11.77	6.27	9.07

5.1.5 Discussion

In terms of cost and percentages, there are substantial differences between the allocation methods. With the proportional method, player 2 benefits as it has the shortest path to the location of the final transformation station. As such, it is benefiting from being in the middle in relation to the other players and the power line. With the Shapely value and the nucleolus - the costs for the lines to the transformation station is also allocated among the player's, and thus player 2 has to contribute to the costs player 1 and 3 has.

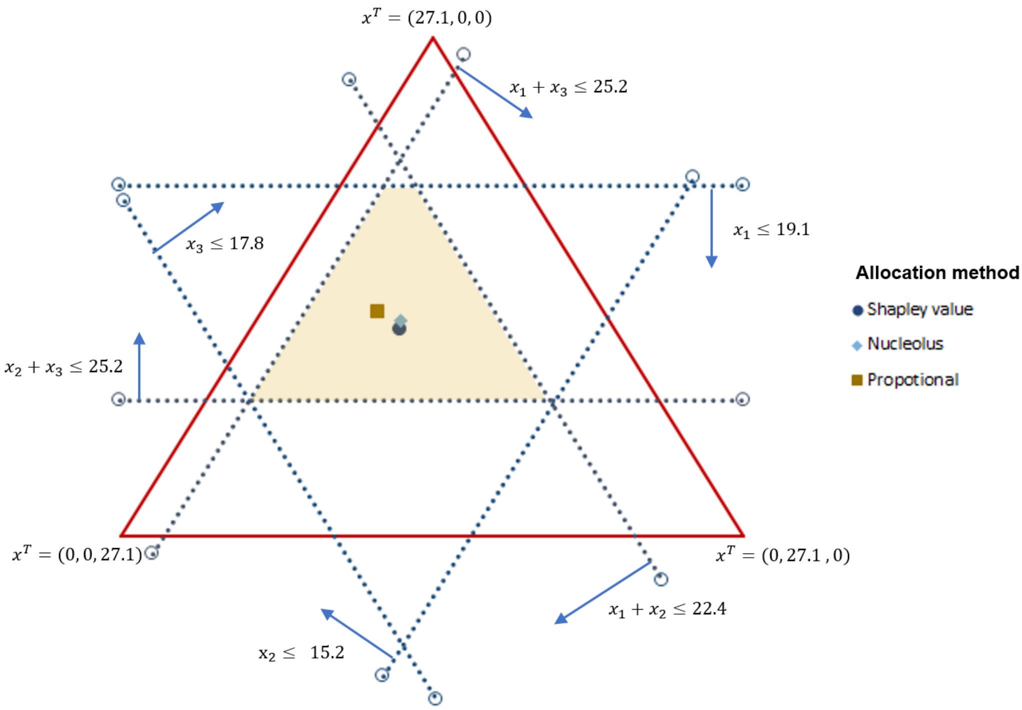

Figure 5.3: Havsnäs - A Graphical representation of the core (the yellow irregular pentagon) and the three unique allocations

Still, cost are small in comparison to the entire costs for each wind farm project, and it is unlikely that any of the individual players would lobby, or take other actions, to change the proportional allocation.

Chapter 6

Conclusions

6.1 The solution concepts

We have compared three different allocation concepts in this thesis:

6.1.1 Proportional split

The main benefits with the proportional split seems to be that calculations can be done in a way that is understandable for a person without a background in economics or mathematics and that one only needs the actual costs of the upgrade, no estimation of costs for sub-coalitions etc. has to be made. This means that the allocation will be accurate a priori connections, and that there is no need to know the specifics of future projects seeking connection.

From an cooperate game theory perspective, the counter arguments are numerous. First of all, when scrutinising the allocation method it is difficult to find objective measures indicating that it is a fair method, other than the intuitive feeling of fairness. Considering the players' non enforceable alternatives, a rational player would likely not see the logic in the method if it would have been better outside of the coalition. One can create axioms that would imply that the proportional rule is a fair allocation method - however, we find no general acceptance nor arguments that this is an appropriate rule within the field of cooperative game theory.

The fact that the solution concept does not necessarily give an allocation within the core even though it exists is worrying. If a player would have been better of in a hypothetical *free market situation*, where everyone played by them selves to maximise their own utility, the risks of this player objecting to the allocation is likely to increase.

For the grid upgrade problem, under the assumptions in this thesis, the main disadvantage of the method is the following: It does not account for when a player need to increase the length of transmission lines from the power production to the transformation station as a result of optimising the entire problem. This is a cost that the wind project developer has to carry on their own, as it is not utilised by other players. This has a major impact on games where one or several players are located far away from the transformation station's optimal location in the grand coalition.

6.1.2 Shapley Value

The Shapley value is one of the most used concept from cooperative game theory in cost allocation problems. It fulfils fairness axioms, and is generally considered one of the best methods of distributing costs in a fair way as it accounts for the players marginal contribution to the cost savings achieved by cooperation. The Shapley value however has two substantial weaknesses:

1. It is difficult to calculate and comprehend, especially when there are several players involved. The marginal contribution to cost savings are not as easy to estimate or relate to as the proportion of the usage of the upgrade
2. The allocation require full knowledge of future projects connecting and utilising the increased capacity. This could however be solved by retroactive cost allocations and transfer of money between the players.

6.1.3 Nucleolus

The nucleolus has the appealing property of always being in the core, if the core exists (thus, it is also an imputation in such case). Furthermore, when there is no core - the concept of minimising the maximum dissatisfaction can be seen as a reasonable compromise. In our case, where there is forced cooperation - it is a concept that should be acceptable by players worse of than in an alternative coalition.

The method however has the same drawbacks as the Shapley value and has even larger computational complexity.

6.1.4 Generalisability of conclusions

Throughout the creation of this thesis, we have modelled a large number (> 20) of fictive examples, ranging from extreme (large differences between the players and their geographical position in relation the the existing grid) and unlikely connection problems, to realistic situations. Except for the situations discussed previously in the thesis, the results are very similar to that of Havsnäs. As such, the following conclusions are based on that our assumptions hold true. In such case we are confident that they are valid for the vast majority of real-world connection problems.

However, we are humble to the fact that in reality, not all of our assumptions necessarily hold true. We list potential shortfalls in generalising our results:

- Connecting players:

  In terms of the number of connecting players, it is not impossible that there could be more than three players connecting simultaneously. However, we have not seen examples of four or more players in a coalition in a real-world case.

- Costs not accounted for:

  There could also be cost elements involved that we have not taken into account which could potentially skew the allocations. These cost would in such case be unusual, as they would not be covered by Ei's *normvärdeslista*.

- Wider perspective with the allocated:

  Minimising the investment cost is not the only consideration when optimising the grid extension. Factors such as total life-cycle cost, quality and future earnings are also important inputs that should be considered. A model accounting for this would likely yield a different outcome - and the differences between allocation methods could potentially have a much larger impact. The absence of superadditivity in some of the games we have discussed and analysed in this thesis might feel unintuitive. If e.g. $C_{\{1,2,3\}} > C_{\{1,2\}} + C_{\{3\}}$ it would be reasonable to question why not the player can replicate how they act as separate coalitions in the grand coalition. However as discussed in this section the grid owner is not forcing collaboration and a joint connection to the grid without reason. It could be of interest to incorporate this into the model. Today, only investments costs are allocated, and the thesis is based on the same premises. It would be interesting for future research to look at how the connection solution effects the life-cycle cost for a wind farm. Taking into consideration that costs for transfer tariffs, operating costs, maintenance etc. could be different, there is a much larger amount of costs involved. As seen be our analysis, there is quite a large difference in percentage depending on the allocation method.

- Joint connection grid:

  We assume that the players have independent lines from the wind farm to the existing grid. In reality there is the option to interconnect wind farms to a joint power line that connects to the grid. The model would still be the same, but with more complexity when computing the optimal solution for connecting each sub-coalitions. This would only be applicable when distances from the wind farms and the existing grid is very far. In such a case the difficulties of allocating the costs would be even more profound, policy makers are recommended to explore whether this could have a substantial impact on the allocations.

6.1.5 Conclusions on allocation differences

The difference between the allocated cost depending on method can be in SEK millions. This is of course a substantial amount of money, but in order to understand its impact one must put it into context. An onshore wind project generally cost 10-20 SEK millions per installed MW. Thus for projects that are rather large scale - the difference in allocated cost is comparably irrelevant to the total costs. Also considering the standard internal rate of return of 8-9 percent in the industry, the cost difference is rather negligible in comparison with expected revenues. Thus, the players main focus is to shorten development time rather then entering a long process to alter the cost allocation.

For the transformation stations, which have small or non-existing economies of scale, the proportional method will yield an allocation within the core. The main problem is with the distance of the individual connection lines and the special case when a null player is involved. As such, the driver behind differences is not the size of the wind farms connecting, as costs are rather linear. It is the distance to the connection point that has a effect on the costs, as with the current method the players has to carry all separable costs.

The proportional method generally yield allocations that are within the core (when there is one), except for in special cases as presented in a few of the examples. Furthermore, a case study on forest transportation in Sweden highlights that the easiest allocation method to accept for the players is the proportional method, even if some players are worse of than operating on their own (Frisk et al. [2010]). Although there are obvious differences with the allocation methods and the analysed methods does in fact yield a result that can be considered more fair from a non-monopoly perspective - in reality they must be feasible to use and accepted by the stakeholders. The study found that all though it was rather clear for the researchers that using cooperative game theory would substantially increase the fairness compared to "simpler" methods. However when presenting these concepts for the involved companies the researchers failed to convince the lumbering companies of its benefits. As such, we find it unlikely that the wind power developers would be in favour of the allocation methods proposed in this thesis given its comparative complexity. Moreover, we find no evidence that the methods would impact specific and over-all wind investments.

6.1.6 Policy recommendations

Our recommendations is to keep the proportional rule as the general allocation method - but that this can be reconsidered when the placement of the transformation station has a serious effect on one or several players' connection line length. In such case, we recommend that grid owners and regulators are responsive and has an open discussion with the different players, when requested before hand by any one player. If the regulators see that the potential disadvantage to any player could be substantial and outweighs the added cost of allocation complexity - we recommend an reviewing the proportional allocation with the analytical methods proposed in the thesis, to ensure that no player is disproportionally disadvantaged from the optimal placement of the transformation station.

This recommendation is conditioned on that our assumptions hold.

Appdx A - Minimisation of Cable Length

For each coalition the optimal solution of grid connection is calculated. Optimal is here defined as the solution with lowest cost and hence the shortest amount of new power grid. Some assumptions are made to simplify the cost calculation however the overall results would still be valid of more details were taken into account when minimising the costs of each coalition.

.1 Assumptions

1. Each agent is connecting directly to a transformation station located somewhere along the regional grid
2. The regional grid is a straight power line passing by all agents
3. The optimal solution is always to connect all agents to the same transformation station
4. All agents are using the same type of power line as a straight line all the way to the transformation station.

.2 Connecting several nodes to a single point at a line

The problem of finding the a straight line that minimizes the distance from one point to another line is trivial in linear algebra, the two lines has to be right-angled. When connecting two points to a line at the same node with minimal length of the connecting lines one can form a function of the length of the two connections and minimise it.

Without loss of generality we can assume that the connection is to be made along the line $y = 0$ for all x. The points that are to be connected will be called (x_1, y_1) and (x_2, y_2). A point at the line is expressed by $(x_c, 0)$ hence we can formulate the distance from the two points to a joint connection point at the line as a function of x_c

$$f(x_c) = \sqrt{(x_c - x_1)^2 + y_1^2} + \sqrt{(x_c - x_2)^2 + y_2^2} \tag{1}$$

Generalized to n points we get:

$$f(x_c) = \sum_{i=1}^{n} \sqrt{(x_c - x_i)^2 + y_i^2} \tag{2}$$

Differentiation gives:

$$\frac{\partial f(x_c)}{\partial x_c} = \sum_{i=1}^{n} \frac{x_i - x_c}{\sqrt{(x_c - x_i)^2 + y_i^2}} \tag{3}$$

By finding all solutions to $\frac{\partial f(x_c)}{\partial x_c} = 0$ and pick the solution that minimises $f(x_c)$ the global minimum is found.

Appdx B - Map of Havsnäs

In figure 1 on page 55 a map of the real project site can be seen.

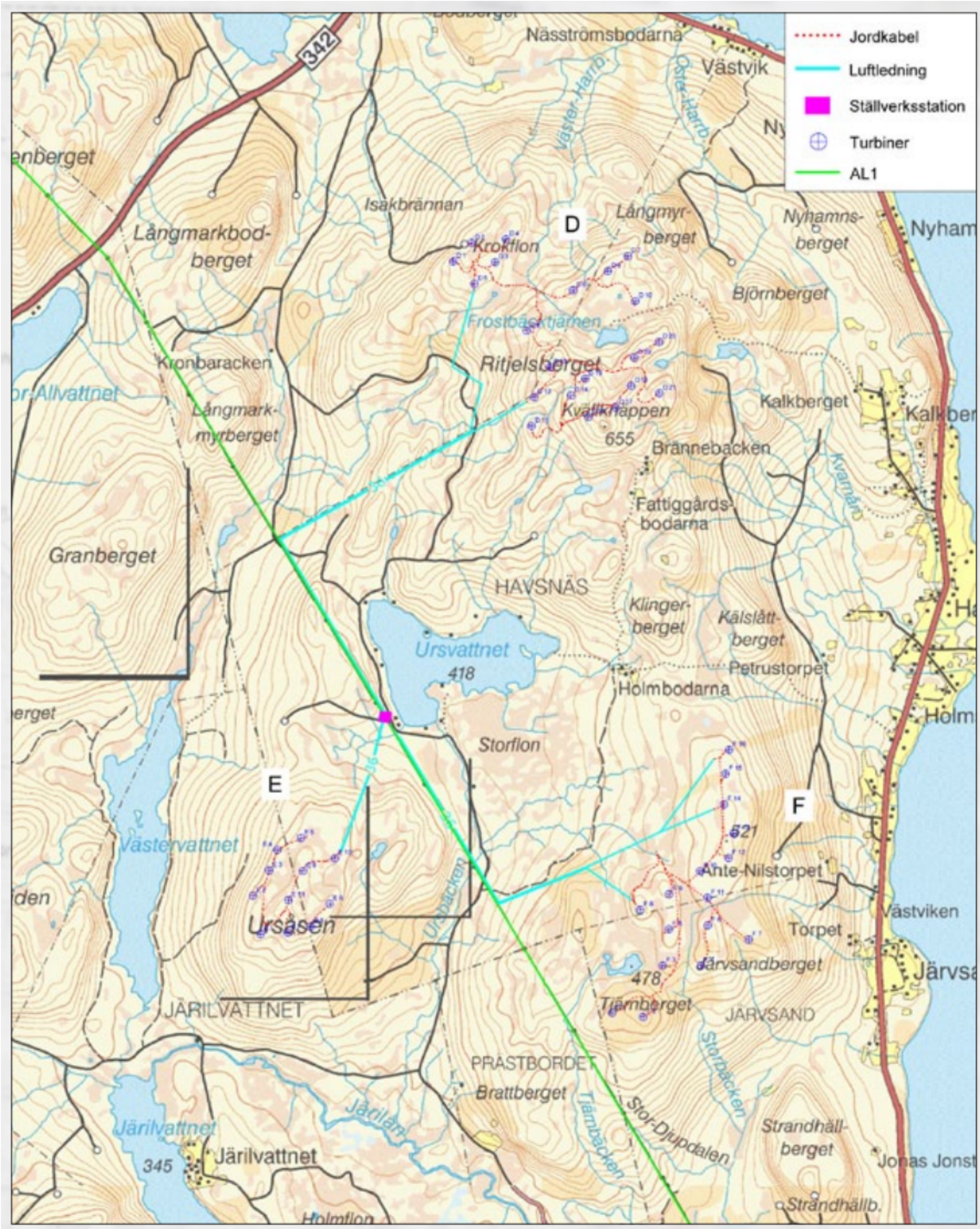

Figure 1: A map of the area for the Havsnäs project

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