Evaluation of an Inventory Policy in a Divergent Multi-Echelon System with Upstream Demand

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ABSTRACT

The purpose of this master thesis is to evaluate the performance of an inventory policy in a one warehouse, multiple retailer inventory system with end customer demand at all stock locations. The objective is to compare the performance of a multi-echelon model with stock rationing, compared to real inventory data which is based on uncoordinated single-echelon optimization. The comparison is carried out by simulations, where the focus is put on expected service levels and expected inventory levels.

The multi-echelon model with service constraints in Berling and Marklund (2013) is used with the inclusion of a virtual retailer which only serves the end customer demand at the central warehouse (upstream demand). The virtual retailer approach is used to approximate a critical level policy at the central warehouse. This means that when the stock on hand at the warehouse falls to or below the critical level, only customer orders are satisfied while retailer orders are backordered.

The results show that the multi-echelon model greatly outperform the uncoordinated solution in terms of the ability to reach target service levels. This is particularly evident when customer order sizes are large. Furthermore, the virtual retailer approach is shown to overestimate the critical level which leads to excess stock. However, the multi-echelon model still holds on average 10% less inventory at the central warehouse when both models achieve the target service level. Finally, the sensistivity analysis illustrates that a critical level policy has the potential to reduce the total inventory with up to 25% but the potential reductions diminish as the fraction of upstream demand increases.

Keywords: Multi-Echelon, Critical level, Stock rationing, Upstream demand, Virtual retailer

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CONTENTS

1 INTRODUCTION

This chapter describes the background for the thesis. It also presents a problem definition, the purpose of the work, its delimitations, Syncron International and finally it discusses the target audiences and provides a chapter overview.

1.1 BACKGROUND

For many companies a single warehouse is not sufficient to satisfy their network of customers. A common solution is to create a network of multiple central and regional warehouses, a so called multi-echelon inventory system. These systems have a flow of goods which enter at the central warehouse or warehouses that supply the retailers, which in turn deliver to the end customer.

When considering a multi-echelon system it is relatively simple to optimize each installation's inventory levels separately, solely based on the demand they experience. However, from a cost perspective this approach is many times inefficient as the system as a whole might carry excess stock. There are large potential savings to be realized by instead coordinating the inventory control decisions for the system. That is, to optimize the overall cost and inventory levels in the entire network, while focusing on the service requirements of the end customers. This is the main purpose of multi-echelon modeling.

Because of geographical or business model reasons the central warehouse can be allowed to serve end customers, causing the phenomenon this thesis refer to as upstream demand. An example of a divergent multi-echelon system with upstream demand is depicted in Figure 1.

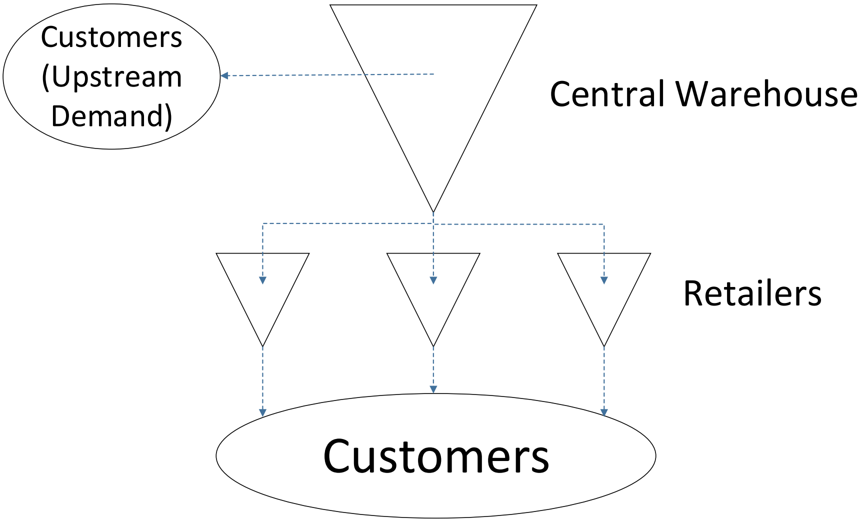


Figure 1- Divergent Multi-Echelon Inventory System with Upstream Demand

One important aspect which makes modelling multi-echelon systems with upstream demand complex is that retailers and end customers usually have very different service requirements. This is problematic because when optimizing multi-echelon systems, stock is usually pushed from the central warehouse to the retailers. Consequently, the central warehouse gets a quite low measured service (Axsäter, et al., 2007), compared to the very high measured service at retailers.

Therefore, when both end customers and retailers are served from a central warehouse, either both demand classes will get a high service level and excess stock is carried in the system, or both will get a lower service and the end customer will not be satisfied.

One approach to meet the challenge of differentiating the service at the central warehouse to different demand classes, is to reserve stock for the high priority upstream demand. This can be accomplished by implementing a critical level policy. That is, when the stock at the warehouse is less than or equal to the critical level, only upstream demand is satisfied directly from stock while retailer orders are backordered (Axsäter, et al., 2007).

The global software provider, Syncron International, offer solutions that can help organizations manage their global supply chains. Syncron realize that a critical level policy has the potential to assist their customers in their objective to reduce stock while still maintaining excellent service. As a result, Syncron is interested in evaluating the critical level policy by comparing its performance to real inventory data.

In previous cooperation between Syncron and Production Management at LTH, Berling and Marklund (2013, 2014) developed a multi-echelon model for a divergent system, the BM-model. In this model end customer demand only occur at the retailers. However, with slight modification, the model can also handle end customer demand at the central warehouse.

In this thesis the BM-model will incorporate a critical level policy and its performance will be compared to inventory data, provided by one of Syncron's customers.

1.2 PROBLEM DESCRIPTION

This thesis will evaluate the BM-model presented by Berling & Marklund (2013; 2014) in a divergent multi-echelon setting with end customer demand at all locations. This model is chosen because of its flexibility in handling different demand distributions as well as its computational and conceptual simplicity. Parts of it has also been implemented in Syncron's inventory management software.

The main challenge is that the BM-model has to be able to differentiate between the service requirements for end customers and retailers at the central warehouse. Focus is put on evaluating how well the BM-model performs, in terms of reaching target service levels (TSL) with as little expected inventory as possible.

This leads to the following questions that the thesis aim to answer:

- What impact does reserving stock have on the precision of achieving TSLs?
- What are the potential reductions in total inventory compared to the real inventory data, when reserving stock at the central warehouse for end customer demand?
- How does the fraction of upstream demand affect the potential reductions in inventory?

1.3 PURPOSE

The purpose of this thesis is to evaluate the performance of the BM-model in a one warehouse, multiple retailer system with upstream demand. This will be achieved by performing a comparison between empirical data, provided by one of Syncron's customers, and analytical computations based on the BM-model.

The comparison will be performed with the help of simulation models, which are created to represent the real inventory system. The performance will be evaluated with respect to expected service levels and expected inventory levels.

1.4 SYNCRON INTERNATIONAL

The empirical data is provided by Syncron International which is a global "Software as a Service" (SaaS) provider that specializes in managing complex global supply chains. They have customers in over 100 countries and offices in a handful countries over the world. In Sweden they have offices in Stockholm, the global headquarters, and in Malmö.

There are four main products in Syncron's software; Inventory Management, Pricing Management, Order Management and Master Data Management. They also offer services in advanced analytics. Figure 2 presents a conceptual model showing how all of Syncron's offers ties in to each other. This thesis is performed in connection to their Inventory Management Software, IM.

Figure 2 - Conceptual model of Syncron's service offerings. Source: Syncron (2015)

The IM software offers companies the potential to reduce stock and increase end-customer stock availability by differentiating products, reallocating inventory and eliminating excess and obsolete stock. They offer both single- and multi-echelon modeling and tailor solutions to the customer.

1.5 DELIMITATIONS

This thesis will cover a divergent multi-echelon inventory system with one central warehouse and several retailers. Empirical data have been collected from one of Syncron's customers, who wish to be known only as the case company. The data is restricted to items with customer demand at both retailer and central warehouse. All items are spare parts, or share characteristics with them. The results will be limited to measured service levels and expected inventory levels, no costs will be covered. The service measure used in this thesis is the demand fill-rate[1], also referred to as the service level throughout the thesis.

1.6 TARGET AUDIENCE

The target audience for this thesis are Syncron, people working for the case company as well as inventory management students and professionals with an interest in inventory control.

1.7 DISPOSITION

The disposition in this thesis is based on a separated model for master theses by Blomkvist & Hallin (2014). It moves logically through the thesis based on a chronical setup, even if the thesis process in itself was iterative. The most important parts of this thesis, for all readers, are the adopted method in Section 2.1.2, the results in Chapter 7 and the conclusions in Chapter 8.

Chapter 1 – Introduction

This chapter describes the background for the thesis. It presents a problem definition, the purpose of the work, its delimitations and Syncron International. It also discusses the target audience.

[1] Fraction of demand satisfied directly from stock

Chapter 2 – Methodology

This chapter begins with a description of a general approach for operations research projects from the literature. This is followed by an adapted framework which is modified to align with the purpose of this thesis. The chapter ends with a description of some general scientific approaches and concepts as well as the positioning of the thesis in this context.

Chapter 3 – Theoretical Framework

This chapter covers the theory on which this thesis is based. First a literature review is presented which describes existing research conducted in the same field of study. This is followed by a description of the heuristic used in the main model for this thesis. Finally a section on different service level classifications will end this chapter.

Chapter 4 – Data Collection and Analysis of Input Data

This chapter describes the collection of data from the case company as well as the process of converting that data into useful information for this thesis. How the selection procedure of items to study were performed is also described.

Chapter 5 – Analytical Calculations

This chapter will present the procedure of using the BM-model in order to obtain reorder points. Furthermore the modifications to the BM-model are described. The chapter ends with a description of the recalculation of reorder points.

Chapter 6 – Simulation

This chapter explains the simulation modeling and analysis used in this work. Simulation is used to assess the performance of the analytical method used for obtaining reorder points for all the different stock points in the system.

Chapter 7 – Results and Analysis

In this chapter the results from studying the simulation will be presented. Focus is put on measured service levels and expected inventory levels. Then the results as well as assumptions will be discussed and analyzed in order to help the reader understand the results. Finally the results of the sensitivity analysis are presented.

Chapter 8 – Conclusions

This chapter will present a short summary of the key components of the analysis. Followed by a remark of what future research may be undertaken to further validate the results of this study.

2 METHODOLOGY

This chapter begins with a description of a general approach for operations research projects from the literature. This is followed by an adapted framework which is modified to align with the purpose of this thesis. The chapter ends with a description of some general scientific approaches and concepts as well as the positioning of the thesis in this context.

2.1 OPERATIONS RESEARCH

As organizations grow, the complexity of its operations increases and new challenges and problems arise. One common problem is that when specialization and complexity in organization increases, it gets more difficult to allocate available resources to various activities in a way that benefits the organization as a whole (Hillier & Lieberman, 2001). As a result, individual units in an organization might grow into autonomous entities with their own goals and lose sight of the overall objective (Lieberman and Hiller, 2001).

Problems and challenges like these paved the way for the emergence of Operations Research (OR). OR uses mathematical modelling to provide near-optimal solutions for complex decision-making problems i.e. provide the best course of action (Hillier & Lieberman, 2001).

In the following sections, an overview of the generic Operations Research modeling approach described in Lieberman and Hillier (2001) will be presented, followed by the adapted framework which is more suited for the purpose of this thesis.

2.1.1 The Generic Operations Research Modeling Approach

The purpose of this section is to present an overview of the major phases of an Operations Research study, as presented by Lieberman and Hillier (2001). Even though mathematical modelling plays a major role in OR, it often represents a relatively small part of the total effort required (Hillier & Lieberman, 2001), as the following section will demonstrate.

The generic modeling approach can be divided into six steps:

1. **Define the problem of interest and gather relevant data.**
   Formulate a clear statement that describes the problem of interest. Gather relevant data to gain understanding of the problem and to provide input to the mathematical model.
2. **Formulate a mathematical model to represent the problem.**
   Translate the studied problem into a form that is convenient for analysis. Mathematical modeling simplifies the problem with the help of assumptions and approximations, while still capturing the essential challenges and reveal cause-and-effect correlations.
3. **Develop a computer-based procedure for deriving solutions to the problem from the model.**
   Develop a procedure to be able to execute the mathematical model and determine near-optimal solutions. A well formulated and tested model should provide a good approximation of the best course of action.
4. **Test the model and refine it as needed.**
   Model verification to find and remove errors and model validation to establish that the model captures the ideas and concepts of the studied problem.
5. **Prepare for the ongoing application of the model as prescribed by management.**
   Install a system for applying the model as prescribed by management. The system includes the model, the solution procedure and operating procedures for implementation.
6. **Implement.**
   Implementation of the system, including the mathematical model.

2.1.2 The Adapted Operations Research Modeling Approach

The modelling approach described in the previous section has to be adjusted to fit the purpose of this thesis. The most notable difference is that the fifth and sixth steps will be disregarded as implementation is outside the scope of this thesis. Furthermore, time will not be spent on developing a new analytical model. Instead we adapt an existing model that has been proposed for handling upstream demand, but which has never been tested in this respect. More details regarding the existing model can be found in Section 3.2.

The adapted modeling approach contains the following steps:

1. **Define the problem of interest and gather relevant data**
2. **Perform data analysis and select items to study**
3. **Modify the existing model and derive solutions**
4. **Evaluate the model by comparison through simulations**
5. **Analyze the results and refine if needed**

Each of these steps are further explained below.

1. Define the problem and gather relevant data

The problem definition originated from discussions with representatives from Syncron and Production Management at LTH. Syncron were interested in evaluation of an inventory policy that can be applied to an inventory system with upstream demand. In previous cooperation, Berling and Marklund (2013, 2014) developed a multi-echelon model, the BM-model, for a divergent system where end customer demand only occur at the retailers.

However, with slight modification, the model can also handle end customer demand at the central warehouse. This aspect of the model had so far not been evaluated. Consequently the purpose of the master thesis was created to evaluate the performance of this model when allowing customer demand at the central warehouse as well. The evaluation will be carried out through simulations, by comparing the BM-model with N+1 uncoordinated single-echelon models. Real data from a case company will be used as input and the performance will be assessed based on the two models' ability to reach target service levels and the total expected inventory.

To gather relevant data, a literature study was performed which can be found in Section 3.1. The purpose of this study was to acquire an understanding of the academic research that is available on the topic of upstream demand, in both single- and multi-echelon settings. Later, this study was extended to include research on stock rationing and multiple demand classes as these areas also cover inventory policies which involve demand streams with different service requirements or priority.

The data from practice was provided by the case company. Data analysis and manipulation will be covered in the next section.

2. Perform data analysis and select items to study

The purpose of using data from practice as input is twofold, using data from the case company facilitates the comparison as parameters such as order quantities, target service levels and lead-times are readily available. Furthermore, fictional data would be difficult to construct in a way that captures the sometimes elusive nature of demand patterns. Hence, by using fictional data some of the challenges of inventory control in practice may be lost in the process. More details regarding the empirical data can be found in Section 4.1.

The items for the study are selected based on a list of criteria that ensures that appropriate items are used. The sample size is set to be large enough to sufficiently capture a wide range of item characteristics but not too large as the study is rather time consuming. About 100 items is be considered enough for this purpose. The execution of this selection is described in Section 4.2.

For each of the selected items the mean and standard deviations of the demand per day were determined together with the distributions of the customer order sizes. These parameters can be derived from the extracted demand history, provided by the case company. This data analysis is further explained in Section 4.3.

3. Modify the existing model and derive solutions

The BM-model is originally constructed to model a pure multi-echelon system where customer demand only takes place at the retailers. However, it is also possible to let the model handle upstream demand by setting the transportation time to one of the retailers to zero. This retailer will be referred to as the virtual retailer and its inventory is reserved to only satisfy the upstream demand, see Figure 3. Furthermore, the virtual retailer replenishes its stock from the central warehouse using a continuous review (S-1, S) policy. Conceptually the central warehouse and the virtual retailer are modeled as separate stock points that are linked by the lead-time which depends on R_0. However, in reality they may both be part of the same stock.

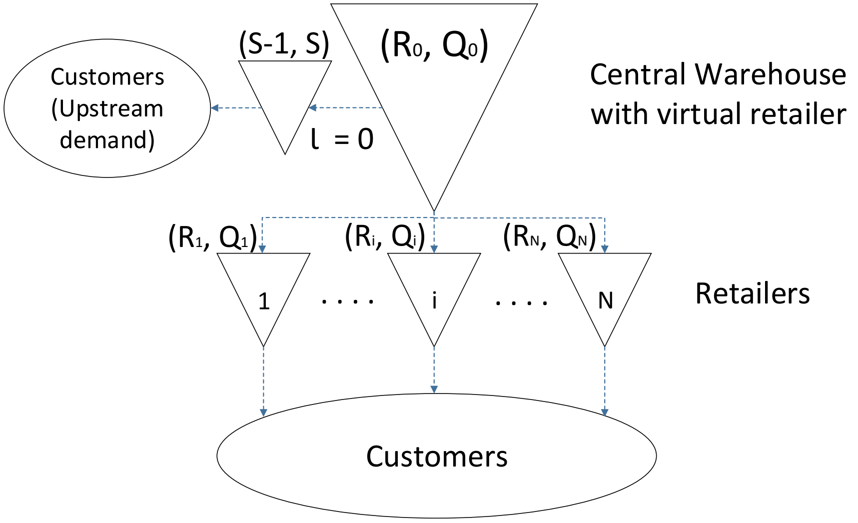


Figure 3- Divergent Multi-Echelon System with Upstream Demand and a Virtual Retailer

The approach to model with a virtual retailer can be viewed as an approximation of a so called critical level policy. This policy defines a nonnegative critical level c for stock on hand at the central warehouse. If the stock on hand is less than or equal to c, only customer demand is satisfied while retailer orders are backordered (Axsäter, et al., 2007).

As there are two stock locations in the approximation, the central warehouse reorder point of a critical level policy can be approximated as $S+R_0$. Furthermore, the critical level c can be approximated by S, as depicted in Figure 4.

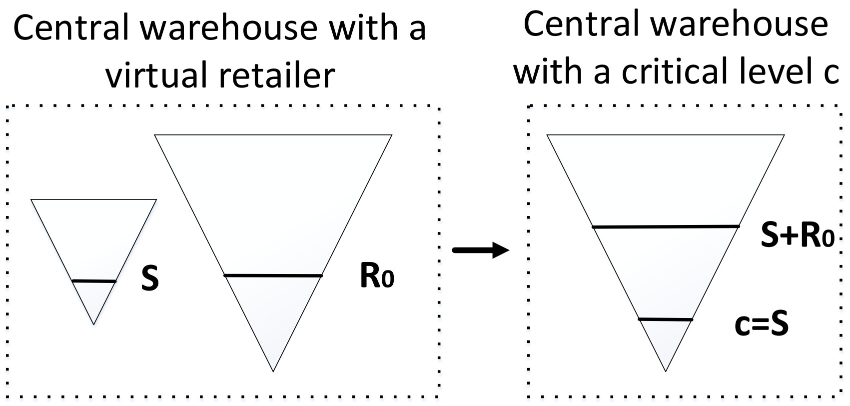


Figure 4 - Illustration of the Relationship Between a Central Warehouse with a Virtual Retailer and a Central Warehouse with a Critical Level c.

Based on the input parameters derived from the empirical data, the BM-model is used to determine near-optimal reorder points for all stock locations, including the S-level at the virtual retailer. More details regarding deriving solutions from the BM-model can be found in Section 5.1.2.

4. Evaluate the model performance using simulation

In order to evaluate the performance of the model, simulations are used where simulation models are created for each individual item to emulate the real inventory system. The comparison consists of simulating each item two times, where the reorder points are the only input parameters that are changed between simulations. The studied reorder points are the ones determined with the BM-model as well as reorder points from N+1 uncoordinated single-echelon models. The results from the simulations include expected service levels and expected inventory levels for each stock location including the virtual retailer.

5. Analyze the results and refine as needed

As this study consists of a number of steps where data is manually inserted and all these steps are repeated for all items in the study, there is a risk of making a mistake somewhere. To reduce this risk a check list was used which systematically explained the procedure step by step. In addition to the check list, the results in each step were analyzed to detect any unreasonable values and to get a chance to correct any errors that could occur.

2.2 SCIENTIFIC APPROACH

There are a vast array of scientific approaches that are relevant when conducting an operations research study. This include, but is not limited to, what the study is trying to accomplish, what the process looks like and what data analysis method that will be used.

2.2.1 Explorative, Descriptive, Explanatory and Normative

The type of study that is used depend on the existing body of knowledge in the particular area (Björklund & Paulsson, 2014).

An explorative or investigatory study is used when there are little knowledge in the area and the purpose is to attain fundamental knowledge. A descriptive study builds on the existing knowledge by describing the existing correlations without explaining. Explanatory studies strive for deeper knowledge in an area and seek to both describe and explain. Finally, normative studies are used when there are already knowledge and understanding in the area and the objective is to provide further guidance and prescribe what to do (Björklund & Paulsson, 2014).

Based on these classifications this thesis will contain elements of an explanatory study, as the purpose is to gain deeper knowledge in a specific area which has already been described in previous research.

2.2.2 Deductive and Inductive

Research approaches may also be classified as deductive or inductive[2]. The deductive process, also known as theory testing process, starts with a known theory and aims to test if it applies in a certain context (Spens & Kovács, 2006). The inductive process does the opposite and starts with an observed phenomenon and seek to generalize it into a new theory (Spens & Kovács, 2006).

This thesis will use both the deductive and inductive approach. It is deductive due to the fact that the BM-model can be classified as known and ratified theory and the objective is to test its performance in another

[2] Some argue for a third approach, abductive, as a mix of the deductive and the inductive approaches.

context. However, the study is inductive when it comes to understanding and explaining the observed results.

2.2.3 Quantitative and Qualitative

Research can also be classified as qualitative or quantitative. According to (Spens & Kovács, 2006) this classification should be decided based on which data analysis technique is used and not the means of gathering data. Qualitative research is all research that does not use statistical measures or try to quantify the problem (Golafshani, 2003). Quantitative research aim to generalize findings and to find a casual determination (Golafshani, 2003). Within logistics, quantitative research is most often conducted with numerical data analysis (Spens & Kovács, 2006).

This study will mainly use quantitative analysis and conduct numerical data analysis. There will be qualitative aspects to the final results but those are not the main focus but rather a result of data anomalies that need further investigation.

2.3 Quality Dimensions

There are several different dimensions when assessing quality. Näslund (2002) classifies the quantitative research as part of the Positivist paradigm. To this paradigm he pairs four criteria for good research; Internal validity, external validity, reliability and objectivity. This section will explain these criteria with respect to the thesis and its operations research approach.

2.3.1 Validity

Validity is a way to assess if the observations in a study in a meaningful way capture the ideas and concepts it studies (Adcock & Collier, 2001). Potential validity issues can be caused by delimitations or assumptions in the model. Näslund (2002) divides validity into two parts, external and internal. Where internal is how well the model represents the reality and external concerns how transferable the results are to other similar situations as those studied.

As this study does not include the development of a new model but instead utilize the existing BM-model, validity is not an issue. The BM-model has been thoroughly tested and used in previous academic research and theses, its internal validity is well-grounded. Furthermore, regarding its external validity, it is constructed with few restrictions but many options which makes it valid for many divergent multi-echelon systems.

2.3.2 Reliability

Reliability concerns how replicable an observation or result is (Golafshani, 2003). This is important because if the results cannot be established as reliable then they are of little use. A reliable result will be the same, within specified limits, when the test is repeated in order for the results to be trusted. It is important to remember that reliability does not say anything about the correctness of the result just the ability of the method or tool used to consistently produce the same result.

The main aspect that affects the reliability of the results in this study is not the model itself but the human factor. This is due to the fact that data is manually inserted in several steps. To reduce the risk of creating errors a check list is created in combination with careful analysis of the results in each step in order to expose anomalies.

Furthermore, the simulation model, that is used to evaluate the performance in this study, has been thoroughly tested in previous research. The minor changes that are made are tested to make sure that the model operates as it is intended.

2.3.3 Objectivity

This quality dimension concerns how free the results are from bias (Näslund, 2002). By clarifying and motivating the different choices made in the study the reader is given the possibility to reflect on the course of action as well as the results of the study (Björklund & Paulsson, 2014).

When using synopses of other author's papers, objectivity problems can arise (Björklund & Paulsson, 2014). It is a matter of recounting the content in an unbiased manner by, first and foremost, reciting statements without any factual errors. Secondly, there should be no distorted selection of facts or arguments, with the purpose to support your own point of view. Finally, one should avoid the using a negative vocabulary which can give the impression that the original author has an erroneous perception.

The main purpose of the thesis is not to promote nor try to prove the efficiency of a method. The purpose is to observe the results and try to understand and explain why the results are as they are, without an agenda. Furthermore, the papers included in the literature review (see Section 3.1) are not selected to prove a certain point. Instead they are selected to illustrate what academic research that has been conducted in the area of upstream demand, multiple demand classes and stock rationing.

3 THEORETICAL FRAMEWORK

This chapter covers the theory on which this thesis is based. First a literature review is presented which describes other academic research conducted in the same field of study. This is followed by a description of the heuristic used in the main model for this thesis. Finally a section on different service level classifications will end this chapter.

3.1 LITERATURE REVIEW

The purpose of the literature review is to position this thesis in relation to the existing literature. Following are a number of inventory models that are developed for handling multiple demand classes with different service requirements. This typically means the use of critical level policies or general stock rationing policies. As mentioned in Section 2.1.2, a critical level policy essentially means that some inventory is reserved for a demand class which have higher service requirements. The first section covers models for single-echelon systems, followed by a section for multi-echelon systems.

For a single-echelon inventory model the objective is to optimize the inventory decisions at a single stock location, independent of the other stock locations. Consequently, when using single-echelon optimization in a system with several stock locations, the inventory at each stock location is sub-optimized.

In multi-echelon optimization the objective is to optimize the inventory decisions for the entire system according to certain objectives. Usually these objectives involve minimizing the overall expected costs. When modelling with service constraints this means that only stock locations which face customer demand need to reach a certain target service. Furthermore, the lead-time between stock locations depends on the risk of stock-out at the supplying location.

3.1.1 Single-echelon models with critical level policy

Dekker et al. (2002) use an (S-1, S) policy with lost sales and can show that a critical level policy outperforms the first-come, first-serve policy (FCFS). The total cost savings when using service constraints have an average of 33.3% and show the largest savings when the majority of demand is in the lower service level class (Dekker, et al., 2002).

Ha (1997) investigated the use of a critical level in combination with an (s, S) policy, several demand classes and lost sales. This means that the objective is to minimize the total expected holding and lost sales costs. He finds that the potential cost savings are strongly related to the ratio between the high priority demand rate (λ_1) and the low priority demand rate (λ_2), Figure 5. The largest cost reduction occur when the high priority demand is equal to or slightly smaller than the low priority demand (Ha, 1997). Furthermore Ha discusses that when arrivals of customers of one demand class is rare, the cost reductions are very small. This is because the system in those cases is close to a system with a single demand class.

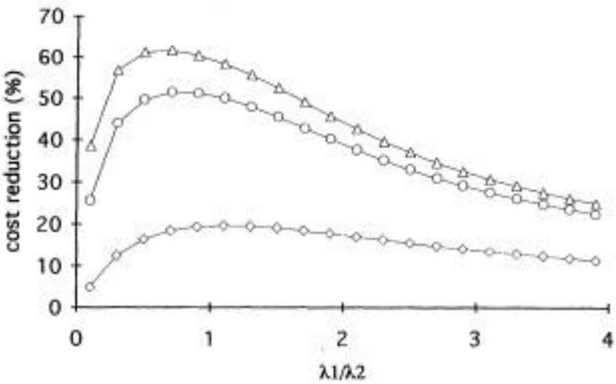


Figure 5 - Cost Reduction when Using versus Demand Ratio for Three Examples with Different Lost Sales Cost Structures (Ha, 1997)

Melchiors et al. (2000) investigates a combination of a critical level and a (R, Q) policy with lost sales and two demand classes. They find that the cost reductions vary depending on whether the critical level is larger or smaller than the reorder point. When the critical level is larger, cost reduction of up to 50% were recorded. When it was instead smaller than the reorder point, the maximum cost reduction were only 10% and that was in a situation where the cost of lost sales were extremely high (Melchiors, et al., 2000).

They also found that the greatest cost reductions occurred when the high priority demand were between 10 to 25 percent of total demand. Finally the authors also conclude that it might be cost efficient to let the policy be lead-time dependent. That is, low priority demand can be satisfied even though the inventory is below the critical level if there is a certain time left before a replenishment order from the supplier arrives.

Wang and Tang (2014) study a system with a mix of demand classes with backorders as well as lost sales. Furthermore, it consists of multiple periods where the inventory level is replenished at the start of each period with a lead-time of zero. The critical levels are dynamic and changes over time. They conclude that the average cost reductions are slightly above 10 percent and they also conclude that rationing policies are close to useless when one of the demand classes is dominant (Wang & Tang, 2014).

Nahmias and Demmy (1981) analyze the effects of using a critical level in combination with a (R, Q) policy with both continuous and periodic review. Given a certain combination of reorder points and order quantities, they construct tables with resulting fill-rates for both with and without stock rationing. They conclude that stock rationing can achieve higher fill-rates than without stock rationing, given a certain reorder point (Nahmias & Demmy, 1981).

Moon and Kang (1981) use an (R, Q) policy with compound Poisson and a critical level to be able to differentiate between demand classes. They also add several critical levels and study how the expected number of backorders for the different demand classes changes (Moon & Kang, 1998). They conclude that by using stock rationing it is possible to efficiently satisfy different demand classes.

A conclusion that many of these papers have in common is that a critical level policy is most valuable when a minority (≈10-40%) of the total demand is of higher priority. However, if one demand class is dominant (>90%), a critical level policy yields no better result than a regular FCFS policy.

3.1.2 Multi-echelon models with critical level policies

In a Multi-Echelon setting there are very few studies made on continuous review systems with stock rationing. One problem seems to be that there are no known exact mathematical solutions, instead numerical studies using simulation is required. (Axsäter, et al., 2007).

Axsäter et al. (2004) use a (S-1, S) policy with lost sales and multiple demand classes. They also find that a critical level policy leads to cost savings, in this case often around 5%. They also state that in more extreme cases, substantially larger cost reductions are possible (Axsäter, et al., 2004).

Numerical results shows that the critical level policy performs better than both standard FCFS, as expected, and the so called separate stock point policy (Axsäter, et al., 2007). The separate stock point policy uses a virtual retailer which carry stock and only serve the high priority demand. This is the same approach that is used in this thesis and is described in Section 2.1.2.

The advantage of a critical level policy over separate stock policies is not clearly stated in the article but could be due to the fact that the results for the critical level policy are obtained by simulation while the results for the separate stock point policies are analytically calculated. The separate stock policy has the advantage that it can be evaluated and optimized without special treatment for direct demand (Axsäter, et al., 2007). Using simulation to determine policy parameters is not feasible for large real world systems.

3.2 THE BM-MODEL

This section will describe the models presented in Berling and Marklund (2013; 2014). The models are similar with the important exception that the retailer demand is modelled by a normal distribution in Berling and Marklund (2014), and a compound Poisson distribution in Berling and Marklund (2013). Therefore the models will be treated as one, here referred to as the BM-model, with two options on retailer demand. The authors also conclude that it is possible to combine these models (Berling & Marklund, 2014).

Figure 6 depicts the considered one-warehouse, N-retailer system. All stock-points use continuous review (R_i, nQ_i) policies to replenish their inventory. This means that when the inventory position[3] reaches or falls below R, an order is triggered to get the inventory position back above R. Orders are placed as integer multiples, n, of the fixed order quantity Q_i at each installation. The central warehouse replenishes from a supplier that is assumed to have an infinite amount of stock. (Berling & Marklund, 2014)

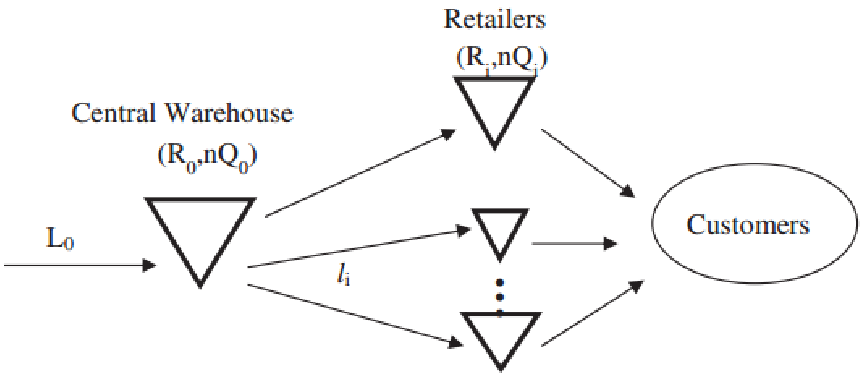


Figure 6 – Multi-Echelon Inventory System with a Central Warehouse and N Non-Identical Retailers. Source: (Berling & Marklund, 2014)

The BM-model assumes that complete backordering and partial deliveries are used. Furthermore, FCFS policies are employed throughout the system. The lead-time, L_0, from a supplier to the central warehouse as well as the transportation times to the retailers from the central warehouse are assumed to be constant. However, lead-times to retailers are stochastic due to the possibility of stock outs at the central warehouse. The notation used in this chapter can be found in Table 1.

[3] Inventory position is defined as: Stock on hand + outstanding orders - backorders

Table 1- Notations used in the BM-Model

Notation	*Description*
R_i	***Reorder Point at retailer i (0 denoting CW)***
β	***Induced backorder Cost at Central Warehouse***
β_i	***Induced back order cost for retailer i***
μ_i	***Mean demand for retailer i (0 denoting CW)***
σ_i^2	***Variance at retailer i (0 denoting CW)***
p_i	***Shortage cost at retailer i***
h_i	***Holding cost at retailer i (0 Denoting CW)***
L_0	***Lead-Time to supplier***
l_i	***Transportation time to retailer i***
γ_i	***Fill-rate at retailer i***
TF_i	***Target Fill-rate at retailer i***
C_0	***Cost at total warehouse, function of R0***
C_i	***Cost at retailer i , function of Ri***
Q_0	***Batch size to Central Warehouse***
Q_i	***Batch size to retailer i***
IL_0^+	***Stock on hand at central warehouse***
IL_i^+	***Stock on hand at retailer i***
B_0	***Backorder at Central Warehouse***
B_i	***Backorder at Retailer i***

The BM-model incorporates two options for service requirements (Berling & Marklund, 2013). The first one is the service level model, where the objective is to minimize the expected holding costs while meeting the specified target service levels. The second option is the backorder cost model, where the objective is to minimize the expected backorder and holding costs, without adding any service level constraints. This master thesis only uses the service level model, therefore the backorder cost model will not be explained any further.

The BM-model makes use of an induced backorder cost, β, which should capture the costs at the retailers due to the delivery delays at the central warehouse. Using this induced backorder costs makes it possible to decompose the multi-echelon system into N coordinated single-echelon systems (Berling & Marklund, 2006) which are possible to solve using installation stock policies.

This approach was first presented in Andersson et al. (1998) where β_i is defined as the expected marginal cost with respect to retailer i's lead-time. Later this method was used in Berling and Marklund (2006), to estimate closed form estimates of a near-optimal β-value, which are conceptually and computationally simple to use.

3.2.1 How to Use the Heuristic

The BM-Model can be divided in five steps. These steps are performed in sequence once, which explains why the model is computationally possible to use for large problems. The steps are (Berling & Marklund, 2014):

1. Determination of near optimal induced backorder cost
2. Determination of lead-time demand at the central warehouse
3. Determination of reorder point at central warehouse
4. Determination of lead-time demand at each retailer
5. Determination of reorder points at each retailer

1. Determination of near optimal induced backorder cost

The induced backorder cost, β, is an approximation of the cost caused by the central warehouse when it fails to deliver as ordered. This includes both the cost for safety stock and the shortage cost at the retailers (Berling & Marklund, 2014).

To estimate the induced backorder cost associated with retailer i, the approach in Berling and Marklund (2006) is used. This approach first normalizes the system parameters with respect to the tranportation time (l_i), the mean demand (μ_i) and the holding cost (h_i). The table which shows how to move between the original and normalized parameters can be found in Appendix A – table for determining normalized parameters.

One of the system parameters is the shortage cost (p_i), which is not directly available in a model with service level constraint. However, it can be estimated with the help of the corresponding target fill-rate by using the optimality condition (Axsäter, 2006) which is described as a relation between fill-rate, holding- and shortage costs in (1). The optimality condition is exact for normally distributed demand but has shown to work as a good approximation for other distributions as well in the context of the BM-model (Berling & Marklund, 2014).

$$TF_i = \frac{p_i}{h_i + p_i} \tag{1}$$

Each retailer's induced backorder cost, β_i, is calculated according to (2) which uses the closed form estimate from Berling and Marklund (2006).

$$\beta_i = h_i * g(Q_{i,n}, p_{i,n}) * \sigma_{i,n}^{k(Q_{i,n}, p_{i,n})} \tag{2}$$

An alternative to the closed form expressions is to use tabulated values of $g(Q_{i,n}, p_{i,n})$ and $k(Q_{i,n}, p_{i,n})$, which can be found in Appendix A. Between parameter values in the tables, linear interpolation may be used, and outside of the table range the closed form expressions for $g(Q_{i,n}, p_{i,n})$ and $\sigma_{i,n}^{k(Q_{i,n}, p_{i,n})}$ need to be used.

The induced backorder cost for the central warehouse is then estimated as a demand weighted average of the different $\beta_i's$. The weighting method for the induced backorder cost at the central warehouse can be found in (3) and its performance is established in (Berling & Marklund, 2006). The main reason for selecting this weighting formula is that it has shown near-optimal results and not performing significantly worse than other plausible but more complex weighting schemes (Berling & Marklund, 2014). It is therefore the best option based on its simplicity.

$$\beta = \frac{\sum_{i=1}^{N} \mu_i \beta_i}{\sum_{i=1}^{N} \mu_i} \tag{3}$$

2. Determination of lead-time demand at the central warehouse

The BM-model approximates the true lead-time demand distribution by using three standard distributions based on the variance-to-mean ratios (Berling & Marklund, 2014). The three standard distributions are;

- Negative Binomial, when $\frac{\sigma_0^2}{\mu_0} \geq 1$
- Discrete normal approximation, if $\frac{\sigma_0^2}{\mu_0} < 1 \; and \; \frac{\sigma_0}{\mu_0} < 0.25$
- Discrete gamma approximation for all other cases.

This approximation scheme is used to improve the computational performance of the model which would have decreased considerably if the true lead-time distribution is used. Berling and Marklund (2014) show that this approximation typically render the same reorder points as the true distribution. Consequently, the lead-time demand at the central warehouse is approximated by fitting distributions to the correct mean (4) and variance (5).

$$\mu_o = \mu_0^1 + \mu_0^2 + \cdots + \mu_0^N \quad where \quad \mu_0^i = \frac{\mu_i L_o}{Q} \tag{4}$$

$$\sigma_0^2 = (\sigma_0^1)^2 + (\sigma_o^2)^2 + \cdots + (\sigma_o^N)^2 \tag{5}$$

$$where \;\; \left(\sigma_o^i\right)^2 = \sum_{n=0}^{\infty} (\mu_0^i - nq_i)^2 g_0^i(nq_i)$$

The variance of the lead-time demand at the central warehouse depends on the number of retailer orders that are triggered during L_0. Defining, $D_0^i(L_0)$, as the sub-batch demand from retailer i during L_0 time units and its probability mass function $g_0^i(u)$ according to (6), we have:

$$g_0^i(u) = P\left(D_0^i(L_0) = u\right) =$$

$$= \begin{cases} \delta_i(0) & if \;\; u = 0 \\ \delta_i(n) - \delta_i(n-1) & if \;\; u = nq_{i,} \;\; n = 1,2,\ldots \\ 0 & otherwise \end{cases} \tag{6}$$

3. Determination of the reorder point at central warehouse, R_0

When the induced backorder cost and the distribution of the lead-time demand is known, the reorder point at the central warehouse (R_0) can be determined by minimizing the expected holding and induced backorder costs. (Berling & Marklund, 2014). This cost function can be found in (7).

$$C_0 = h_0 E[IL_0^+(R_0)] + \beta E[B_0(R_0)] \tag{7}$$

Since the cost function is convex in R_0 the optimal reorder point for the central warehouse can be found by a simple search while applying the condition in (8). (Berling & Marklund, 2014)

$$R_0 = \max\{R_0 : C_0(R_0) - C_0(R_0 - 1) \leq 0\} \tag{8}$$

4. Determination of lead-time demand at each retailer

The replenishment lead-time to retailer i, and the associated lead-time demand, are functions of R_0. In this step Berling and Marklund (2014) use two different methods for calculating the variability of the lead-time. Both methods use an approximated mean for the lead-time (9) by applying Little's law to find the expected warehouse delay (Berling & Marklund, 2014).

$$\bar{L}_i(R_0^*) = \mu_\delta(R_0^*) + l_i = \frac{L_0}{\mu_0 Q} E[B_0(R_0^*)] + l_i \tag{9}$$

When determining the standard deviation of the lead-time, the methods differ. The first method uses the METRIC approximation (Sherbrooke, 1968) which sets the lead-time variance to zero, basically disregarding it. The second method estimates the lead-time variability by adapting the method in Axsäter (2003). This method will not be used in this thesis as it is more computationally demanding and do not give better results (Berling & Marklund, 2014). Using the first approach, the mean and standard deviation of the retailer lead-time demand are then defined according to (10) and (11) respectively.

$$\mu_{D_i(L_i)} = \mu_i \bar{L}_i \tag{10}$$

$$\sigma_{D_i(L_i)} = \sqrt{\sigma_i^2 \bar{L}_i} \tag{11}$$

5. Determination of reorder points at each retailer

After the lead-time demand for each retailer is determined, the multi-echelon system is decomposed into N coordinated single-echelon systems that can be optimized using single-echelon methods. The objective is to find the smallest reorder point that still satisfies the target service level. The optimal reorder point is easily found by performing a search until (12) is satisfied, starting from $R_i = -Q_i$ and increasing R_i with integer steps.

$$R_i = \min\{R_i : \gamma_i \geq TF_i\}\, for\ i = 1,2,\dots,N \tag{12}$$

The service level requirements are calculated according to (13), fill-rate calculation under the assumption of normal demand, and (14), fill-rate calculations for compound Poisson demand (Berling & Marklund, 2013, 2014).

$$\gamma_i = 1 - P(IL_i \leq 0) =$$

$$= 1 \ - \left(\frac{\sigma_{D_i(L_i)}}{Q_i}\left[G\left(\frac{R_i - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}}\right) - \ G\left(\frac{R_i + Q_i - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}}\right)\right]\right) \tag{13}$$

$$\gamma_i = \frac{\sum_{d=1}^{\infty}\sum_{j=1}^{\infty}\min(j,d) f_i(d) P(IL_i = j|R_i)}{\sum_{d=1}^{\infty} d f_i(d)} \tag{14}$$

The underlying assumption in the normal distribution is that all customer orders arrive continuously which create problems in this approach. If the actual demand consists of customers with different order quantities it means that the inventory position falls below the reorder point before a replenishment order is triggered. The normal demand model on the other hand assumes that a replenishment order is triggered at the exact moment when the inventory level hits the reorder point. (Berling & Marklund, 2014)

Berling and Marklund (2014) uses two different undershoot adjustment methods to compensate for this property of the normal distribution. One based on realized reorder points and the other based on the mean and variance of the undershoot.

The method based on realized reorder points uses the assumption that the inventory position is uniformly distributed on [R_i+1, R_i+Q_i], which is a good approximation for normally distributed demand as long as the probability of negative demand is small (Berling & Marklund, 2014). With the probability of an undershoot of size u, as presented in (15), it is possible to calculate the expected fill-rate for the following realized reorder point, as can be seen in (16).

$$U_i(u) = \frac{1}{Q_i}\sum_{k=u+1}^{u+Q_i} O_i(k) \quad (15)$$

$$\gamma_i = \sum_{u=0}^{\hat{u}} SERV_1(R_i - u)U_i(u) \quad (16)$$

3.3 SERVICE LEVELS

There are many ways to measure service levels. However, there are three main categories that covers most of the service level measurements; Probability of no stock out during an order cycle, Fill-rate and Ready Rate[4]. In this thesis Fill-rate will be used as the service level measurement and it will be defined as:

> *Fraction of demand that can be satisfied immediately from stock on hand – (Axsäter, 2006)*

The reason fill-rate is used is that, except that it is easy to evaluate both theoretically and in practice, it only decreases when customers demand an item that is not in stock. Ready rate on the other hand decreases during stock outs, whether or not there are any current customer demand.

Probability of no stock out during an order cycle, $SERV_1$, is easy to conceptually understand and use in practice. However, $SERV_1$ has one great disadvantage, it does not take the order size into account. This means that when the order quantity is large this measurement will underestimate the service experienced by the customer. Consequently when the replenishment order quantity is small the experienced service (e.g. fill-rate) can be low even if $SERV_1$ is high. (Axsäter, 2006)

[4] Also known as $SERV_1$, $SERV_2$ and $SERV_3$

4 DATA COLLECTION AND ANALYSIS OF INPUT DATA

This chapter describes the collection and analysis of the data from the case company as well as the process of converting that data into useful information for this thesis. How the selection procedure for items to study is also described.

4.1 DATA RECEIVED

In order to use the BM-model as well as compare the results to the empirical data several parameters are needed. This material was provided by the case company and extracted from Syncron´s system. The data material was received as two excel files and contained item information as well as demand history for the studied inventory system. The parameters in the data material can be seen in Table 2. In response to a request from the case company, only the results from using this information will be presented in this thesis, and not the data itself.

Table 2 - Case Company Data Extracted from IM

Excel file 1 (Item information)	Excel file 2 (Demand history)
Item code	Item code
Warehouse code and name	Receiving Warehouse name
Order level (Reorder point)	Customer code
Order quantity	Requested date
Inventory policy	Requested quantity
Pick class	Sales order number
VAU class	Sales order line number
Target service level	
Stocking policy	
Supplier ID	
Lead-time (Days)	
Demand type	

Some of the parameters in Table 2 are only of practical use for keeping track of and sorting items, suppliers, customers and warehouses. The following parameters are needed for the numerical study:

Demand history
This will be used to determine a statistical distribution to represent the item's demand pattern. The demand covers three years back from October 2015 and an assumption was made that all items were introduced before the start of this time period. The most important parameters are the requested date and requested quantity which are needed to calculate the mean and standard deviation of the demand per day, for all selected items. In total the demand history consisted of over 300 000 transactions.

Lead-times
In order to determine the safety stock levels properly, the replenishment times are essential. The replenishment times from the case company are the agreed times between order and delivery. The historic data does not account for stock out delays at the warehouse. This is however calculated in the BM-model.

Target service level
As the real inventory system is optimized using single-echelon methods with service constraints, all stock locations including the central warehouse has a target service level. However, the target service level for the central warehouse is not used in the BM-model as it uses multi-echelon optimization.

Order quantities
This thesis will use the fixed order quantities from the real data. This is because even though it is possible to calculate the Economic Order Quantity, often restricted by other factors. Optimizing the order quantities also requires information regarding the order set up costs which are not available from the case company.

Order level (Reorder point)
The reorder points in Syncron´s system are calculated as uncoordinated single-echelon installations with normal, Poisson or negative binomial distribution to approximate the demand. The reorder points are important because they are needed in the situation for comparison with the analytical model's reorder points to. As explained in Section 5.2 some of the extracted reorder points were later recalculated in order to get a more fair comparison of the performances of the models.

One parameter which would intuitively seem important but have been disregarded in this thesis is the holding cost rate at central warehouse and retailers. After discussion with Syncron it has been concluded that the holding cost rates are the same at all stock-points since this is generally how most of the customers have their setup, including the case company. As a result, minimizing the total expected inventory is equivalent to minimizing the expected holding costs.

4.2 ITEM SELECTION

There are several ways of selecting the data for a quantitative study. The broadest distinction is between random and non-random samples. Non-random samples include comfortability selection[5] and yes-sayer selection[6] (Blomkvist & Hallin, 2014). The random sampling methods include complete random selection, systematic random selection, cluster selection and proportional stratified selection (Blomkvist & Hallin, 2014). If the aim for the study is to be able to extrapolate the results to a larger group than the sample, the random selections are preferable. This is because of the fact that non-random selection methods can end up with a large portion of bias.

Initially the plan was to make the selection of items in two steps, where the first step included setting up a number of criteria in order to remove the items that did not fit the scope of the thesis. The second step would then be to perform a stratified selection to select a range of items that represented the overall characteristics of the entire product assortment. Even though the data consisted of more than 4000 stocked items, only a few hundred items fulfilled the list of criteria that was set up. Therefore the stratified selection were deemed unnecessary and the items were instead selected solely based on the following list of criteria.

1. Items stocked in central warehouse
2. Items stocked at no less than two retailers
3. Items with at least 10 transactions per stock location

With the help of these criteria 92 items were chosen to represent the case company's article range in this study.

[5] Using the data which is easiest to obtain
[6] Use those that want to be a part of the study

4.3 DETERMINING INPUT PARAMETERS

In order to be able to determine optimal reorder points in the BM-model, several input parameters and the order size distribution are required. These are as follows:

- Mean demand per day
- Standard deviation of demand per day
- Customer order size distribution
- Lead-time
- Order quantity
- Target service level
- Holding cost

All parameters are needed for each item and stock location, including the virtual retailer facing the upstream demand at the central warehouse. Lead-time, order quantity and target service level were provided by the case company, the holding cost rate is assumed to be the same for all stock locations and is therefore, without loss of quality, set to one. Consequently, the first three input data on the list need to be calculated.

4.3.1 Distribution of demand at retailers

The mean demand is simply an average of the demand experienced at each stock location per day. The variance of the demand per day is estimated using (17). $\bar{x}$ denotes the mean demand of n observations. Subsequently, the standard deviation of the demand per day is calculated as the square root of the variance.

$$Var = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \tag{17}$$

The customer order size distribution i.e. the probability of customers ordering a certain number of units, are estimated from the data using relative frequencies. This mean that the number of orders of a certain unit size at each retailer is divided by the total number of orders at that retailer.

4.3.2 Mean and standard deviation for upstream demand

From the data received it was not possible to distinguish the upstream demand from retailer orders at the central warehouse. In order to estimate the upstream demand the following approach was therefore used.

The approach consisted of comparing the mean and standard deviation of the demand experienced at the central warehouse from the empirical data, with the aggregated demand from the retailers as calculated in the BM-model (see Section 0 for calculations of μ_0^i and $(\sigma_0^i)^2$ for retailer i). If the aggregated demand from the retailers is subtracted from the total demand at the central warehouse, the remainder will equal the mean (μ_{UD}) and standard deviation (σ_{UD}) of the upstream demand, as presented in (18) and (19).

$$\mu_{UD} = \mu_0 - \sum_{i=1}^{n} \mu_0^i \tag{18}$$

$$\sigma_{UD} = \sqrt{(\sigma_0)^2 - \sum_{i=1}^{n} (\sigma_0^i)^2} \tag{19}$$

When the virtual retailer is added to the BM-model and new reorder points are calculated, the resulting mean and standard deviation of the demand experienced by the central warehouse should be equal to the values of these same parameters obtained from the empirical data. This worked for the mean demand but the calculated deviation from the BM-model were usually lower than the respective standard deviation from the data.

The reason for the difference is that the BM-model assumes that retailers only use the fixed order quantity, Q_i. In reality this is not the case. From the data it is clear that, in the real system, manual adjustments to the order quantity are common. This will increase the variance of the demand expected at the central warehouse and increase the need for safety stock at this location.

In order to compare the performance of the BM-model to the uncoordinated single-echelon solution, it is essential that they are based on the same values of mean and standard deviation of the demand. As a result it was decided to recalculate the reorder points at the central warehouse for the uncoordinated solution, based on the limited information we had about the IM software. Therefore the normal distribution was used to approximate the demand and the reorder points were calculated using single-echelon methods, as described in Section 5.2.

4.3.3 **Customer order size distribution for upstream demand**

Regarding the order size distribution an assumption was made that customers that orders directly from the central warehouse request orders of the same sizes as the customers at the retailers. Therefore the probability of a customer ordering k units from the central warehouse ($O_{UD}(k)$) was calculated as a weighted average (20). Where the probability that an arriving customer chooses retailer i is denoted ρ_i, and the probability that this customer orders k units is denoted $O_i(k)$. ρ_i was determined by dividing the number of orders at each retailer by the total number of orders.

$$O_{UD}(k) = \sum_{i=1}^{n} O_i(k)\rho_i\,, \; k = 1, 2, 3 \ldots \tag{20}$$

5 ANALYTICAL CALCULATIONS

This chapter will present the procedure of using the BM-model in order to obtain reorder points. Furthermore the modifications to the BM-model are described. The chapter ends with a description of how the extracted reorder points at the central warehouse were recalculated.

5.1 ANALYTICAL MODEL

The analytical model as used in this thesis is constructed to calculate optimal reorder points, based on fill-rate constraints, with the BM-method described in Section 3.2. The input parameters in the model are listed in Section 4.3. The distribution and approaches that are included in the model are:

- Central warehouse demand distribution
- Retailer demand distribution
- Compounding distribution
- Retailer lead-time approach
- Induced backorder cost approach
- Undershoot adjustment approach

Some of these distributions and approaches have already been discussed in Section 3.2 but for completeness we will recapitulate the choices made in the following description of the analytical model. As a starting point the negative binomial distribution was chosen to approximate the central warehouse demand. If the variance-to-mean ratio of this demand is below one the normal distribution is used instead.

Compound Poisson with empirical compounding distribution was chosen to approximate the retailer demand. This distribution is deemed the best choice because it works well for lumpy demand. The reason it works well is because it takes the customer order sizes explicitly into account with the help of a compounding distribution. If the variance-to-mean ratio is below one, the normal distribution is used instead.

The METRIC type approximation (see Section 0) was used to calculate the expected retailer lead-time and tabulated values were used for the g and k – functions to calculate the induced backorder cost (see Appendix B - Tabulated Values of $\boldsymbol{g}$ and $\boldsymbol{k}$ Functions). If the normal distribution is used, an undershoot adjustment of the reorder point is needed in order to be able to reach the TSL. The undershoot method in this thesis uses the realized reorder points to calculate the expected fill-rate.

5.1.1 Modifications to the analytical model

The BM-model is originally constructed and evaluated as a pure multi-echelon model where customer demand only occur at the retailers. However, as Berling and Marklund (2013) mention, it is possible to deal with upstream demand by introducing a virtual retailer with transportation time zero and base stock ordering. This approach is referred to as the separate stock policy in Axsäter et al. (2007). The virtual retailer uses an (S-1, S) policy to replenish its stock from the central warehouse and the stock at the virtual retailer is reserved to only satisfy upstream demand.

A complication of using a transportation time of zero is that the calculation of the induced backorder cost is not defined. Instead the induced backorder cost, for the virtual retailer, is assumed to be equal to the shortage cost corresponding to the target fill-rate of the upstream demand (21). The relationship between shortage cost, target fill-rate and holding cost is obtained from (1). In our study the target fill-rate for the upstream demand was estimated as an average of the target fill-rates at the retailers. This is because of the assumption that customers at the upstream demand have the same characteristics as the customers at the retailers.

$$\beta_{UD} = p = \frac{TF * h}{1 - TF} \tag{21}$$

In the BM-model the virtual retailer and central warehouse stock are optimized as separate stock points that are connected by the expected lead-time to the virtual retailer. This means that in the BM-model the fill-rate of the virtual retailer is calculated based on the stock at this location, independent of the central warehouse stock. However, when simulating this system, the virtual retailer will also use the available central warehouse stock to satisfy upstream demand.

Consequently, the analytically calculated S-level at the virtual retailer is expected to be overestimated and exceed the TSL. This overshoot is also expected to increase as the customer order sizes increase. The impact of approximating a critical level using the virtual retailer approach is discussed in Section 7.3.3

5.1.2 Procedure to derive near-optimal reorder points

The procedure of obtaining near-optimal reorder points with the BM-model consists of two steps in this thesis.

In the first step only the regular retailers are included and all their input parameters. The purpose of this first step is to let the model calculate the mean and standard deviation of the demand experienced by the central warehouse. Since the virtual retailer is excluded in this step, this demand only consists of replenishment orders from the retailers. The mean and standard deviation are needed to calculate the mean and standard deviation of the upstream demand, according to (18) and (19) in Section 4.3.2.

In the second step the virtual retailer is added to the model, with all its input parameters as well as the order size distribution. The BM-model is then used for calculating the reorder points at the warehouse, the retailers and the virtual retailer.

5.2 RECALCULATION OF REORDER POINTS EXTRACTED FROM IM

When calculating the reorder points in the BM-model it was noticed that the resulting standard deviation of the demand experienced by the central warehouse did not correspond to the standard deviation obtained from the empirical data. In order to make a fair comparison of the performance of the reorder points, it is important that they are calculated based on the same mean and standard deviation of the demand. For this reason the originally obtained central warehouse reorder points from the IM software were recalculated with the standard deviation obtained from the BM-model.

For the recalculations the normal distribution was used to approximate the demand at the central warehouse. The motivation for this is that it is the normal distribution that the IM software uses for the items in the study. Hence the new reorder points at the central warehouse were calculated using the standard single-echelon method in Axsäter (2006) for continouos normally distributed demand with fill-rate constraint. Here the target fill-rate for the central warehouse was obtained from the extracted data from the case company.

As a result of the recalculation the comparison consists of the BM-model, with a virtual retailer to estimate a critical level, compared to N+1 uncoordinated single-echelon solutions. In the uncoordinated setup the reorder points at the central warehouse were calculated while the reorder points at the retailers were obtained from Syncron's IM software.

5.2.1 Central warehouse TSL and upstream demand TSL

It is important to emphasize that in this thesis we separate between target service levels at the central warehouse and for the upstream demand. Therefore this section will clarify the difference between them, describe how they are used and finally what the consequences are.

The central warehouse TSL is obtained from the extracted data. For the real inventory system that the thesis studies, Syncron's IM software uses single-echelon optimization with service constraints, which requires a TSL for the central warehouse. Therefore, this measure is used when the reorder points at the central warehouse are recalculated. However, this measure is not used in the BM-model which uses multi-echelon optimization.

The upstream demand TSL is an estimation of what service requirements upstream demand customers have. This estimation is needed because the data from the case company did not contain any information about the TSL for upstream demand. Remember that the TSL for upstream demand was calculated as an average of the target service levels at the retailers. This is motivated by the assumption that upstream demand customers have the same characteristics as customers at the retailers. The upstream demand TSL is used to determine the S-level in the BM-model. It is also a benchmarking tool when comparing the performance of the BM-model to the uncoordinated N+1 single-echelon models.

It should be noted that the central warehouse TSL and the upstream demand TSL are not necessarily the same for the items in the study. Figure 7 depicts the difference in percentage points between the upstream Demand TSL compared to the central warehouse TSL.

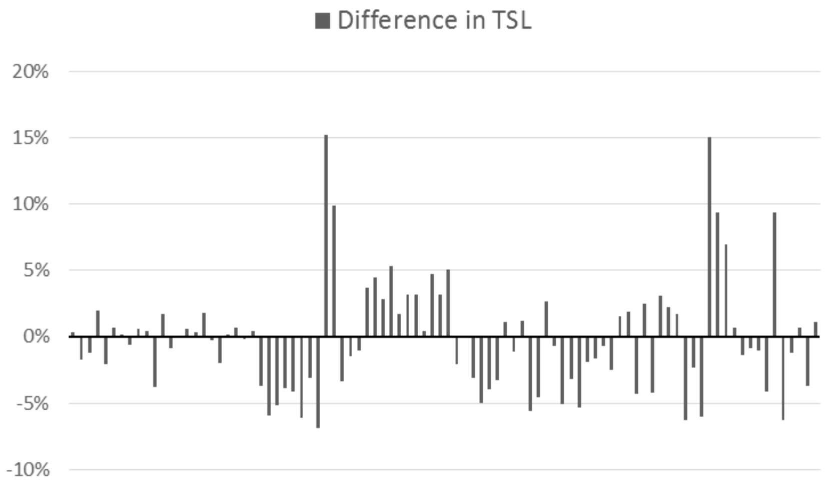


Figure 7 - Difference between TSL for Upstream Demand compared to the TSL for the Central Warehouse Measured in Percentage Points and Sorted by Item Number

When comparing the BM-model's and the uncoordinated solution's abilities to reach the upstream demand TSL, it is important to remember that the central warehouse reorder points for the uncoordinated solution were calculated based on the central warehouse TSL.

An alternative approach would be not to treat these two measures separately. One could assume that the central warehouse TSL, extracted from the IM software, is equal to the TSL of the upstream demand customers. This approach was rejected in this thesis in order to stay consistent with the assumption that all customers in the system have the same characteristics. Furthermore, representatives from Syncron agreed that this is a reasonable assumption with respect to the studied inventory system.

6 SIMULATION

This chapter explains the simulation modeling and analysis used in this work. Simulation is used to assess the performance of the analytical method used for obtaining reorder points for all the different stock points in the system.

6.1 EXTEND

The software used in this thesis is called Extend V6, henceforth referred to as Extend, and is developed by Imagine That Inc. Extend use a graphical interface to build models where complex system can be easily overviewed and at the same time offers large possibilities for tailoring the system.

6.2 THE SIMULATION MODEL

The simulation model have previously been used in related research, for example Berling and Marklund (2013, 2014) and is therefore carefully verified and has high internal validity. The only modification made is a change in the fill-rate calculations for the virtual retailer due to the zero transportation time. Consequently, the modification to the model will be presented while the original model will only be graphically displayed.

In Figure 8 an overview over the entire model is shown. The leftmost side of the model displays the actual flow of orders and inventory. The remainder of the model contains input and output blocks as well as calculations of total cost, expected inventory and expected service levels. The model is set up for one central warehouse and ten retailers where only the retailers used, for the simulated item, are connected. One retailer will in this thesis be dedicated to the direct customer demand to the central warehouse, namely retailer 10.

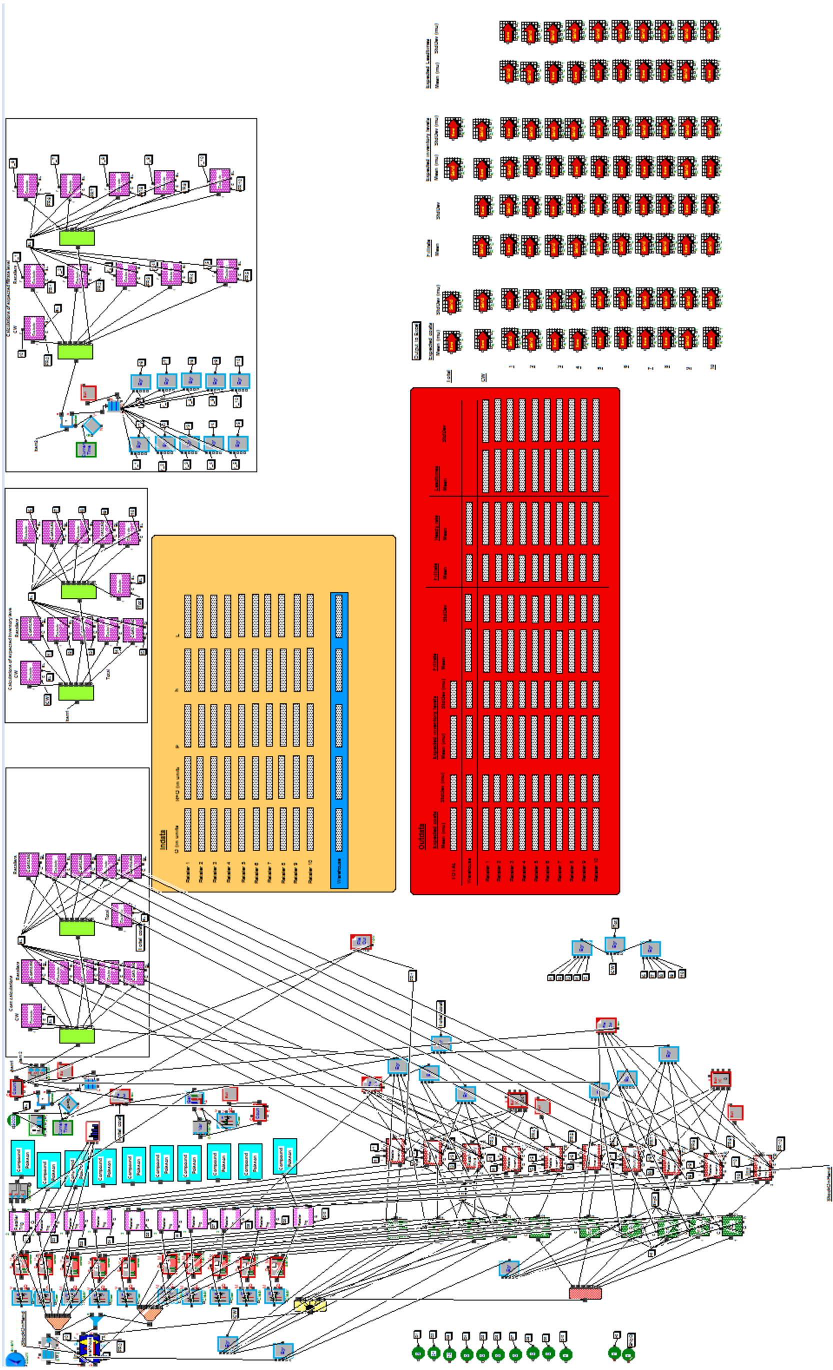


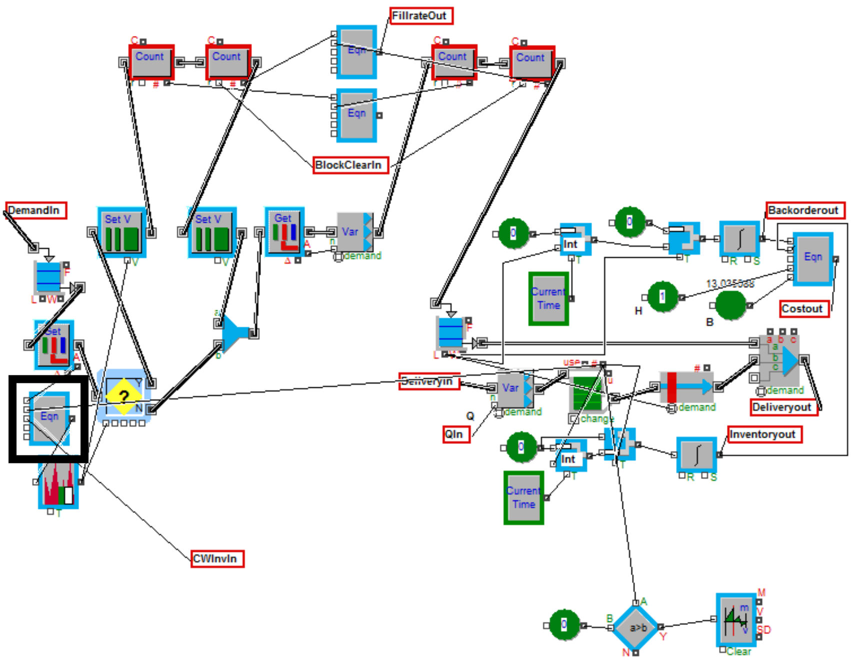
Figure 8- The Simulation Model Used in this Thesis

As the virtual retailer is modelled with zero transportation time from the central warehouse, the fill-rate calculations for this retailer has to be modified. This is because orders to the virtual retailer can not only be satisfied immediately from the stock on hand at the virtual retailer, but also immediately from stock on hand at the central warehouse.

As orders arrives to the virtual retailer, the equation block marked with a black bolded black bolded frame in

Figure 9, compares the number of units in the customer order with the amount of units in stock. The difference in this retailer inventory block compared to the regular retailer inventory blocks is how stock on hand is calculated. In this block the amount of units in stock for the virtual retailer is calculated as the total amount of stock at the virtual retailer and the central warehouse. While regular retailer inventory blocks only account for the stock at that specific retailer.

Figure 9 - Retailer Inventory Block for the Virtual Retailer



6.3 SIMULATION APPROACH

Each item is simulated two times, first using the reorder points from the BM-model and the second time using the reorder points from the uncoordinated single-echelon solutions. The length of the simulations are 150 000 days divided into 30 blocks of 5000 days. The motivation for using this long simulation time and block length is to attain independent observations. For each one of the 30 blocks, fill-rate and mean inventory are calculated. Based on these observations the overall mean and standard deviation of the mean are estimated.

When a simulation is completed the results are exported to Excel where they are collected in one sheet per article with all information and results concerning that specific item.

7 RESULTS AND ANALYSIS

In this chapter the results from studying the simulation will be presented. Focus is put on measured service levels and expected inventory levels. Then the results as well as assumptions will be discussed and analyzed in order to help the reader understand the results. Finally the results of the sensitivity analysis are presented.

Results are at the core of the thesis and can be divided into a lot of different categories. We will primarily focus on results connected to measured service levels and expected inventory as these are the performance measures chosen for evaluating the models. Furthermore, data concerning costs have not been available and therefore we measure relative expected inventory instead. The majority of the results will be displayed in graphs, more detailed numbers can be found in Appendix C – . Furthermore, if nothing else is stated in the captions of the graphs, the x-axis displays all 92 items in the study.

7.1 EXPECTED SERVICE LEVELS

The target service level (TSL) is the constraint under which we aim to optimize the reorder points. The objective is to achieve the TSL without exceeding it too much, which would mean that excess stock is carried. At the same time, not being able to achieve the TSL is also highly undesirable.

Table 3 presents the results of deviations from TSL for the two models. A description of the different measures can be found in Appendix D – Description of measures in Table 3. The BM-model with a virtual retailer has a more even distribution of results and does not have the extreme deviations that the single-echelon model shows. This means that the BM model have a higher degree of accuracy and can be trusted to deliver the fill-rate that are desired when calculating reorder points.

The average deviations are calculated as a simple average over all regular retailers for each item, then the mean is calculated over all 92 items. Since the measured fill-rates might exceed the TSL in some cases while not reaching it in others, the actual deviations might even out when calculating the average. Therefore these measures are complemented with the absolute deviations as well as maximum result over and under the TSL.

Table 3 - Summary of Results of Deviations from TSL in Percentage Points

Measure	BM with a Virtual Retailer	Single-Echelon
Mean Deviation	1.67	-10.66
Mean Absolute Deviation	1.88	12.98
Weighted Mean Deviation (Weighted by μ_0)	0.63	-13.23
Weighted Mean Absolute Deviation (Weighted by μ_0)	1.02	15.04
Largest Positive Deviation	9.45	10.00
Largest Negative Deviation	-2.42	-79.22
Mean Deviation at Virtual Retailer	3.67	-1.41

7.1.1 Retailers

When focusing on the retailer performance, Figure 10 shows that the BM-model with a virtual retailer hovers right around zero percent average deviation from TSL. In many cases where the items seem to overshoot the TSL quite a bit, the mean demand is very low resulting in a low reorder point. In these cases it is not uncommon that the measured fill-rate is a few percentage points over the TSL, as each incremental change in reorder point affects the fill-rate drastically.

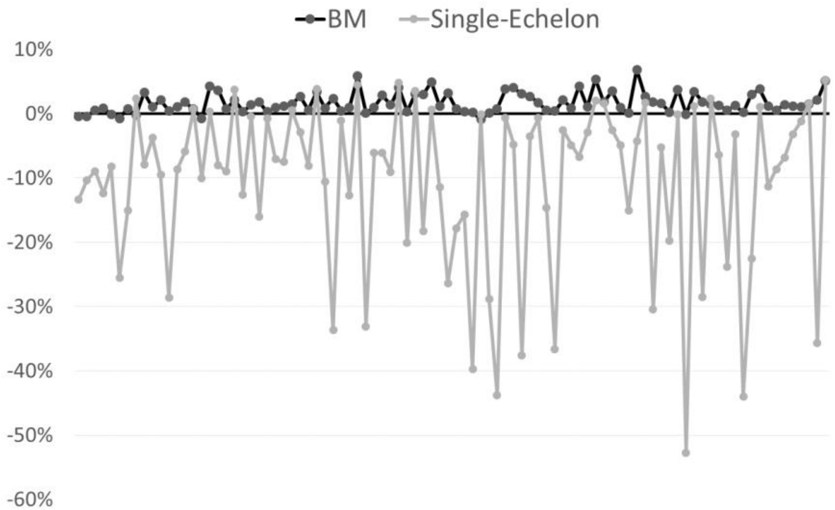


Figure 10 – Average Deviation from retailer TSL in Percentage Points Sorted by Increasing Fraction of Upstream Demand. The Virtual Retailer is not Included in these Results.

The single echelon model on the other hand seems to drastically undershoot the TSL in many cases. This can partly be explained by the distributions used to approximate the retailer demand. Some distributions are sensitive to large order sizes and this is further explained in Section 7.1.3. However, the distributions alone does not explain all extreme deviations.

Both the BM-model and the IM software use the same demand history when determining reorder points in this study. The time frame used might differ but this should not have an impact on the result unless there a major positive or negative trends. As the purpose of this thesis does not include the objective of analyzing the technicalities of the IM software, this analysis will simply include some possible explanations to the deviations.

One explanation is that IM software uses a different forecasting method. This may lead to smaller discrepancies between the mean and standard deviations used in this thesis compared to what is used in the calculations in the IM software. However, this should not have a major impact on the results.

Another explanation migh be that operators at the case company have manually adjusted the reorder points of some of the items, for reasons unknown in this study. Manual adjustments can have a great impact on the results.

Finally, in this study all orders are included when calculating mean and standard deviation of the retailer demand. It is possible that the IM software, or the case company, remove orders that are much larger than the average order size before calculating mean and standard deviation. These large orders might be satisfied from stock location higher up in the inventory system. If this is the case it would explain some of the extreme deviations.

7.1.2 Upstream demand

As discussed in Section 5.2.1, in this thesis we make a distinction between upstream demand TSL and the central warehouse TSL. The difference in percentage points between these service requirements are illustrated in Figure 11. This difference only affects the results of the single-echelon model as the BM-model is not optimized with a central warehouse TSL but only with respect to the retailer TSL and the upstream demand TSL.

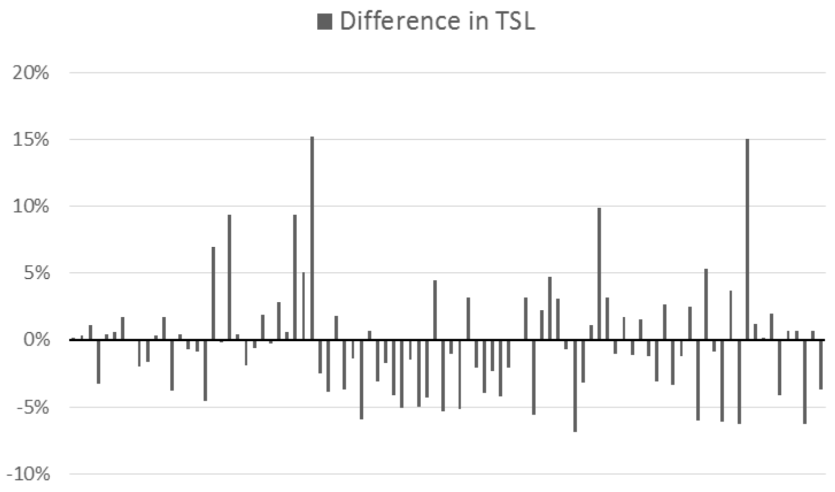


Figure 11 - Difference between Upstream Demand TSL compared to the Central Warehouse TSL for all items. Measured in Percentage Points and Sorted by Increasing Fraction of Upstream Demand.

Comparison with both these service requirements are interesting but show different things. Figure 12 illustrates the deviation from the upstream demand TSL in percentage points, for the BM-model as well as single-echelon model. The BM-model exceeds its TSL with an average of 3.67 percentage points. This probably means that it is possible to find a lower S-level and still be able to achieve the TSL for the upstream demand. This is further discussed in Section 7.3.3 and a local search for optimal S-levels for a selected group of items is presented in Section 7.4.1.

The single-echelon model has extremely varying results where the majority of items do not achieve its TSL. The three items with more than 10 percentage points below the TSL for the single-echelon model are items with customer demand which includes extremely large customer order sizes compared to the mean order size.

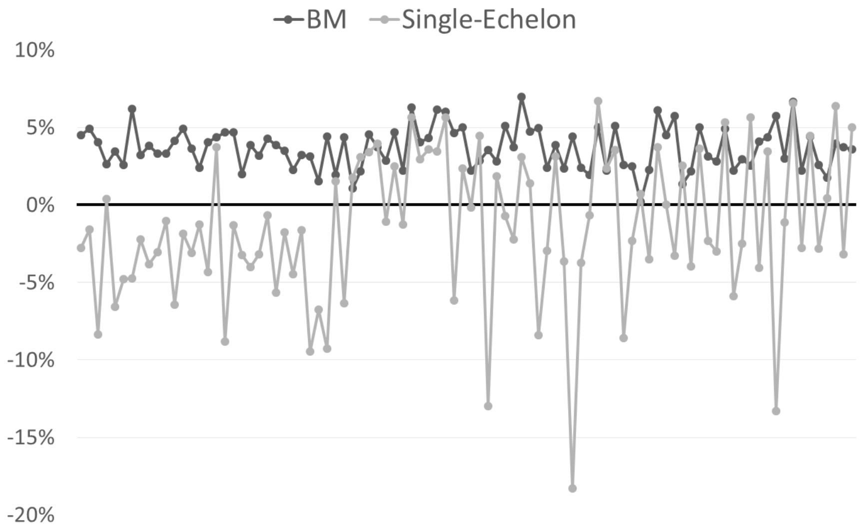


Figure 12- Deviation from Upstream Demand TSL in Percentage Points Sorted by Increasing Fraction of Upstream Demand.

Remember that the central warehouse reorder points in the single-echelon model were calculated using the central warehouse TSL. However, in Figure 12 the expected service level is compared to the upstream demand TSL. Consequently, this figure illustrates the single-echelon model's ability to reach the estimated service requirements of the upstream demand customers. However, it does not reflect a single-echelon model's ability of reaching TSLs in general due to the difference between upstream demand TSL and central warehouse TSL (Figure 11).

Figure 13 illustrates the deviation of the single-echelon model's expected service level from the central warehouse TSL, which the reorder points in this model were calculated with respect to. It also shows the BM-model's deviation from the upstream demand, as in the previous figure.

The single-echelon model has a tendency to fall below the central warehouse TSL, which primarily can be explained by the normal distributions sensitivity to larger order sizes. The items where the single-echelon model seems to exceed the TSL by more than 5 points are items where the reorder points are very small.

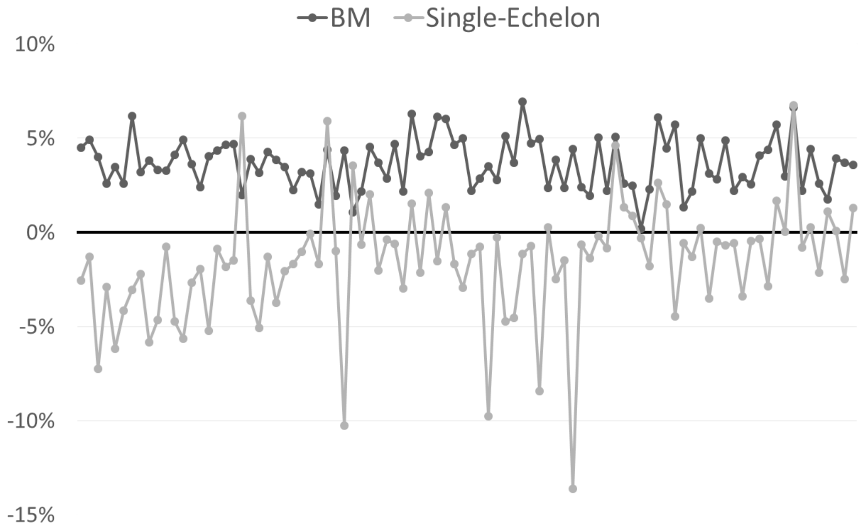


Figure 13 - Deviation from Upstream Demand TSL for the BM-Model and Deviation from Central Warehouse TSL for the Single-Echelon Model. Measured in Percentage Points and Sorted by Increasing Fraction of Upstream Demand.

Both Figure 12 and Figure 13 are sorted by increasing fraction of upstream demand. This shows that fraction of upstream demand does not have any major effect on the ability to reach target service levels.

7.1.3 Effect of increasing customer order sizes

The normal and Poisson distributions are built on the assumptions of continuous demand for the former and unit demand for the latter. As these distributions disregard the fact that in reality customers can usually order many units at the same time, thus these distributions will have a hard time reaching their TSL when the order sizes are large.

This is because when order sizes are larger than one, there is a probability of undershooting the reorder point, i.e. the inventory level decreasing below the reorder point before a replenishment order is created. And of course, the larger the mean order sizes are, the larger the expected undershoot will be. This dilemma is shown in Figure 14 where the single-echelon model has an evident trend away from achieving TSL when mean order size increases.

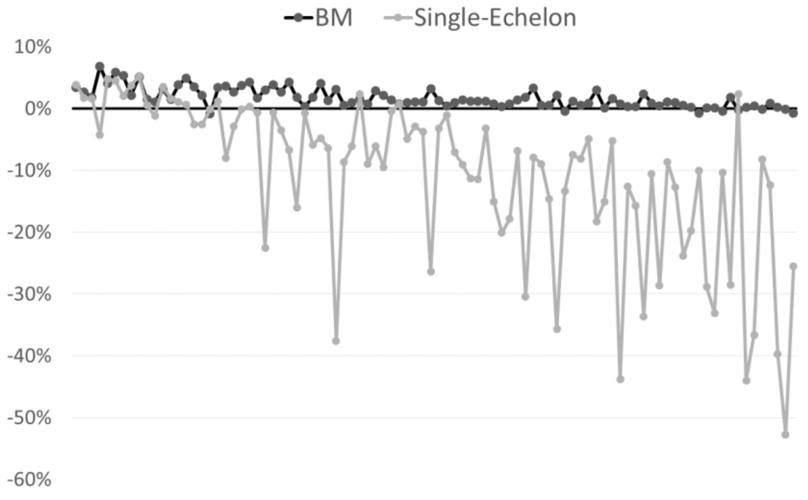


Figure 14 - Average Deviation from Retailer TSL in Percentage Points, Excluding the Virtual Retailer. Sorted by Mean Customer Order Size.

In Figure 15 the deviation from upstream demand TSL for the BM-model and the deviation from the central warehouse TSL for the single-echelon model are depicted. Once again there is an evident trend where the single-echelon model falls below the TSL as the order sizes increase.

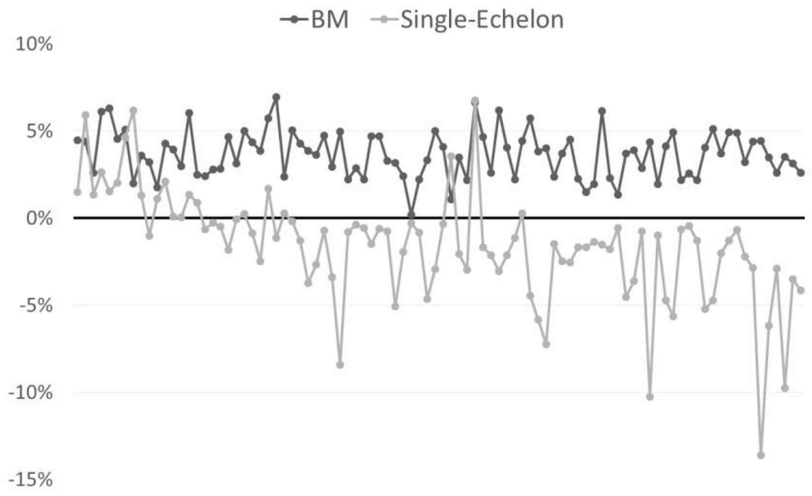


Figure 15 - Average Deviation from Upstream Demand TSL for the BM-Model and Deviation from the Central Warehouse TSL for the Single-Echelon Model. Measured in Percentage Points and Sorted by increasing Mean Customer Order Size.

To counter the problem of falling below the TSL when undershooting the reorder point, it is preferable to approximate the demand with a distribution that takes the order sizes into account. One such distribution is the compound Poisson. Another approach is to compensate the undershoot by adjusting the reorder point, as explained in Section 0.

7.1.4 Effect of fraction of upstream demand on CW fill-rate

One major expectation when modeling a multi-echelon system is that the fill-rate at the central warehouse will go down due to the fact that the end customer fill-rate is the important measurement and consequently stock is pushed out to the retailers. When adding upstream demand the expectation is that as the fraction of upstream demand increases, the BM-model will move towards a single-echelon system and hence the fill-rate at the central warehouse will be higher.

In Figure 16 it is illustrated that when fraction of upstream demand are low the fill-rate is low as expected from a multi-echelon model, and as the fraction of upstream demand increases the fill-rate moves towards what is expected of a single-echelon system. This suggests that the benefits obtained by multi-echelon optimization will diminish as the fraction of upstream demand increases.

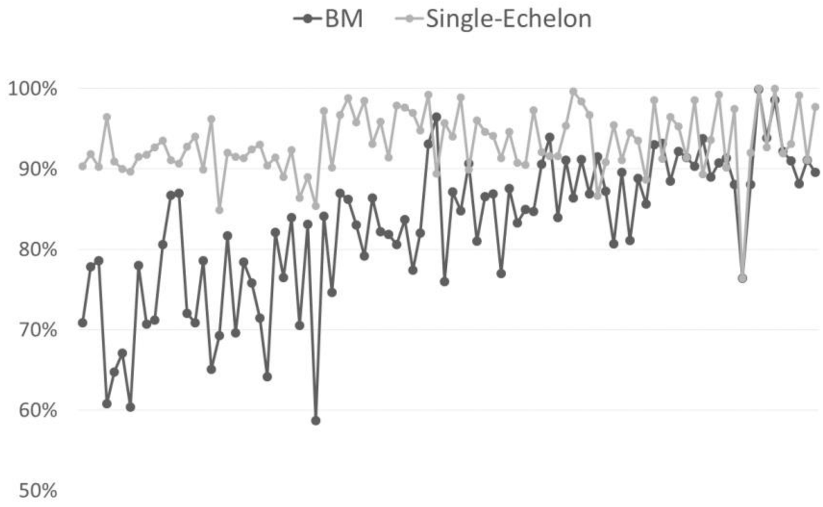


Figure 16 – Fill-Rate at the Central Warehouse of the Total Demand Including the Upstream Demand. Sorted by Increasing Fraction of Upstream Demand.

7.2 EXPECTED INVENTORY LEVELS

As holding cost rate and unit cost are not known in this thesis, total expected system inventory is used instead of expected holding cost. The actual values of the inventory is not that interesting but the relative difference between the models are.

7.2.1 Total inventory

Figure 17 presents the difference in total expected inventory, at the central warehouse and all the retailers, as the relative increase in inventory held with the BM-model. The average increase over all 92 items is 15.69%. The reason that there is more inventory, on average, held with the BM-model is that the single-echelon model more often than not fail to reach the TSL and in many cases undershoot it drastically. The average deviation from retailer TSL, illustrated in Figure 18, is closely connected to the difference in total inventory. Therefore it is difficult to compare total inventory when there are such large differences in the models' ability to reach TSLs. However, note that when both models deliver approximately the same service level, the BM-model often carry less inventory.

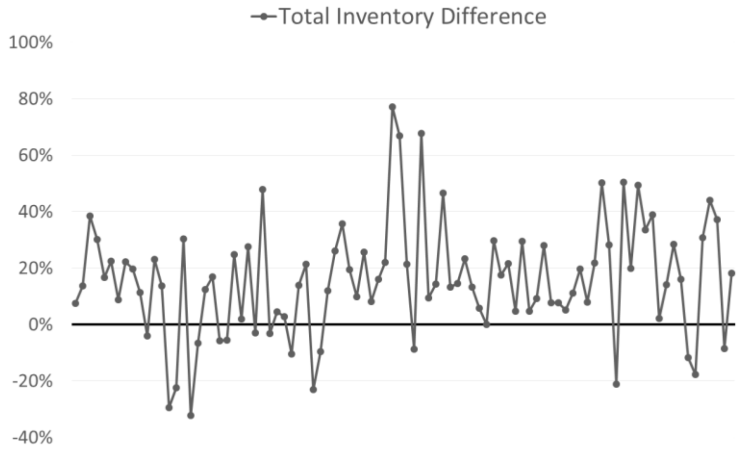


Figure 17 - The Relative Difference of the Total Inventory, Including the Virtual Retailer, for the BM-Model Compared to the Single-Echelon Model. Sorted by Item Number.

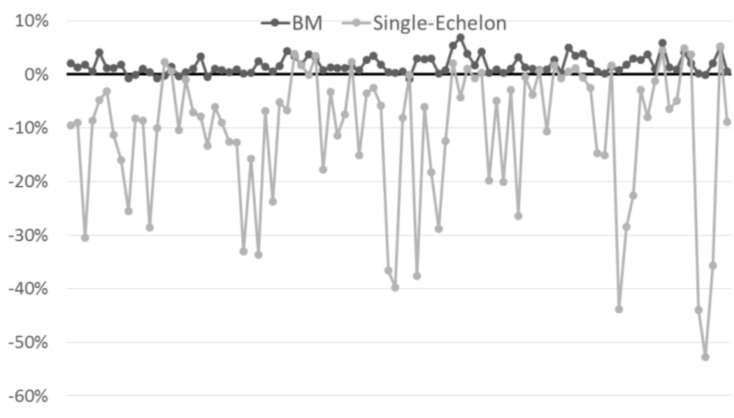


Figure 18 - Average Deviation from retailer TSL in Percentage Points Sorted by Item Number. The Virtual Retailer is not Included in these Results.

7.2.2 Central warehouse

Figure 19 shows the difference in expected inventory at the central warehouse. On average the BM-model holds 10.53% more inventory, which ones again mainly can be explained by the fact that the single-echelon model often fails to reach the TSL. The items where the BM-model holds more than twice as much inventory are items where the single-echelon model is far below the TSL, as illustrated in Figure 20.

Another important factor is that the BM-model seems to exceed the TSL for the upstream demand as a consequence of reserving too much stock at the virtual retailer.

Figure 19 - The Relative Difference of the Central Warehouse Inventory, Including the Virtual Retailer, for the BM-Model compared to the Single-Echelon Model. Sorted by Item Number.

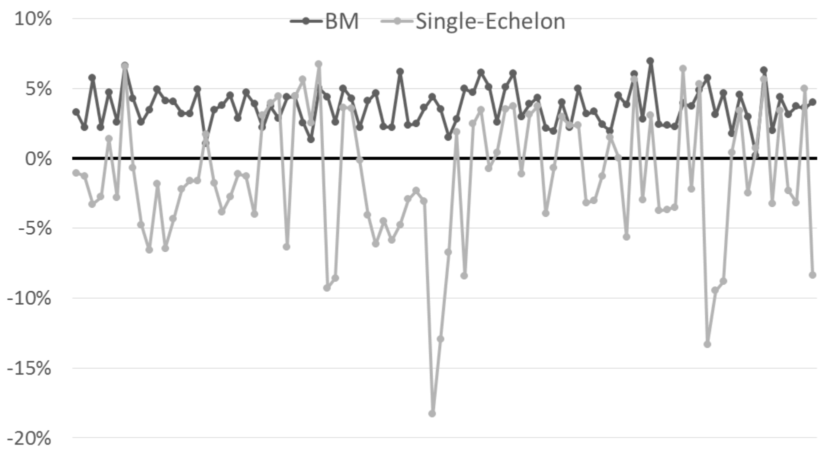


Figure 20 - Deviation from Upstream Demand TSL in Percentage Points for the BM-Model and the Single-Echelon Model. Sorted by Item Number.

7.2.3 Effect of fraction of upstream demand on CW inventory

Figure 21 demonstrates that in the cases where both models achieve their TSL for upstream demand, the BM-model with a virtual retailer holds significantly lower inventory in the central warehouse. On average the BM-model holds 10.15% less inventory in the central warehouse, and in individual cases this number can be as great as 30%. The reason that total expected inventory is not used for this comparison is that there a very few items where the single-echelon model is able to reach the TSL for upstream demand and at the same time have a positive average deviation at the retailers.

From the literature review in Section 3.1 we learned that in other academic research conclusions have been made that a critical level policy is most effective when the fraction of upstream demand is somewhere between 10 and 40 percent. Our results are inconclusive in this matter even though they might hint that the greatest potential reductions can be found when the fraction of upstream demand is small. In order to support this theory a larger number of observations is needed in combination with a local search of optimal reorder points for both models.

Figure 21 – The Relative Difference of BM Central Warehouse Inventory compared to Single-Echelon, Sorted by Increasing Fraction of Upstream Demand. Only Items Where Both Models Reach TSL for Upstream Demand are Included.

7.3 IMPACT OF ASSUMPTIONS

In order to keep the complexity of the observed system down to a level that is manageable, only a certain number of parameters are used to describe the real inventory system. The chosen parameters are closely related to the underlying assumptions. In the following section some of the assumptions of this study are discussed in terms of their impact on the results.

7.3.1 The BM-model

From the BM-model there are two assumptions that need to be discussed, namely the assumption that the central warehouse replenishes from a supplier with infinite stock, and the assumption that retailers only create orders of a fixed order quantity.

The result of the former assumption is that the lead-time from supplier to central warehouse is constant. In reality there are no such thing as constant lead-times, any order can be delayed. A consequence of this assumption may be that the calculated safety stock at the central warehouse is not large enough to cover the uncertainty in the real inventory system. However, if this would become a problem when applying the BM-model on a real system, it would not be hard to include uncertainty to the central warehouse lead-time.

Regarding the second assumption it seems reasonable enough that retailers replenish from the central warehouse with a fixed order quantity. However, in this study it became clear that the fixed order quantities were often manually adjusted by the personnel at the case company. For the comparison it would have been useful to calculate new order quantities based on the empirical data. However, in the data it was not possible to identify from which retailer a certain replenishment order had emanated. By using the fixed order quantities which are not used in reality, we unintentionally tamper with the standard deviation of the demand experienced by the central warehouse. This was the main reason to why the reorder points extracted from IM had to be recalculated.

7.3.2 Upstream demand in the empirical data

As already mentioned, some information in the data was missing which made it impossible to distinguish from which retailer the orders to the central warehouse came from. This forced a number of assumptions to be made in order to study the aspects of upstream demand.

First, the mean and standard deviation of the upstream demand needed to be calculated somehow, and we chose the approach described in Sections 4.3.2 and 4.3.3. This works well for the mean demand but may be more questionable when it comes to the standard deviation. This is closely related to the discussion about the fixed order quantities in the previous section.

As a consequence of using order quantities that does not seem to cohere with what is used in reality, the standard deviation of the demand at the central warehouse is tampered with. In turn, this variance is used to calculate the standard deviation of the upstream demand and thereby the discrepancy will be transferred to the upstream demand. Again, this was countered by recalculating the extracted reorder points at the central warehouse.

Another assumption concerned the customer order sizes at the upstream demand. The assumption was made that the upstream demand customers share characteristics with retailer customers and therefore use the same order sizes. In cases where the upstream demand order sizes are much larger in reality, our estimations of the time between customer arrivals will be overestimated and vice versa. This does not affect the performance in terms of measured service levels in this study. However, in terms of analysis and discussion it would have been preferable to have correct data on the order sizes as the upstream demand customers might have completely different characteristics than retailer customers.

7.3.3 The use of a virtual retailer with lead-time zero

The results indicate that the BM-model with a virtual retailer overestimates the critical level, resulting in an overshoot of the target fill-rate. This is likely to be the result of two assumptions connected to the virtual retailer.

First, the virtual retailer and the central warehouse stock were optimized as two separate stock points that are connected by the expected lead-time to the virtual retailer. When using this approach to approximate the critical level, it is expected to overestimate the S-level at the virtual retailer. This is because the fill-rate calculation at the virtual retailer only depend on the virtual retailer stock and not the central warehouse stock. In reality the upstream demand can be satisfied from stock at both the virtual retailer and the central warehouse. Since the transportation time is zero, this is not a problem as long as the orders to the virtual retailer are small. However, as the order sizes increase, the overestimation of the critical level is expected to increase. In Section 7.4.1 a sensitivity analysis is performed with a local search for near-optimal S-levels.

Secondly, when modeling the upstream demand with the help of a virtual retailer with lead-time zero, it requires an alternative approach to calculate the induced backorder cost. This study used the approach to let the induced backorder cost at the virtual retailer be equal to the shortage cost, as calculated in (21) in Section 5.1.1. Clearly there are other possible approaches to approximate this induced backorder cost. However there are no obvious indications in the results that leads us to question the chosen approach.

7.4 SENSITIVITY ANALYSIS

The sensitivity analysis proposed in this Section was performed to see if the results with simple measures can be made better. Focus is put on performing local searches, with the help of simulations, to find reorder points that perform better than the ones from the study.

7.4.1 Local search for near-optimal S-levels

The results show that S-levels determined by the BM-model tend to render a fill-rate for the upstream demand that exceeds the target. However, this does not necessarily mean that there are lower S-levels that still achieve the target. When modeling with service constraints and integer values of the reorder points, the fill-rate is expected to exceed the target, or at least not be exactly equal to the target.

To investigate if there are in fact lower S-levels that still reach the TSL, 20 items were randomly selected to be included in a sensitivity analysis. The analysis was carried out by successively lowering the S-levels obtained from the BM-model and then use the simulation model to determine the corresponding fill-rates. The investigation stopped when the first S-level did not manage to reach the TSL and consequently the previous S-level was noted.

The results are presented in Figure 22 where the calculated S-levels are compared to the lower simulated values. It is evident that the S-levels are overestimated, the average reduction was almost 23 percent. The maximum reduction were as much as 82 percent for an item where the mean order size were large, which forced a significant increase of the S-level in the BM-model. At the same time the central warehouse carried a lot of stock which helped satisfy the upstream demand. Therefore it was possible to reduce the S-level from 21 to 3 in the simulations.

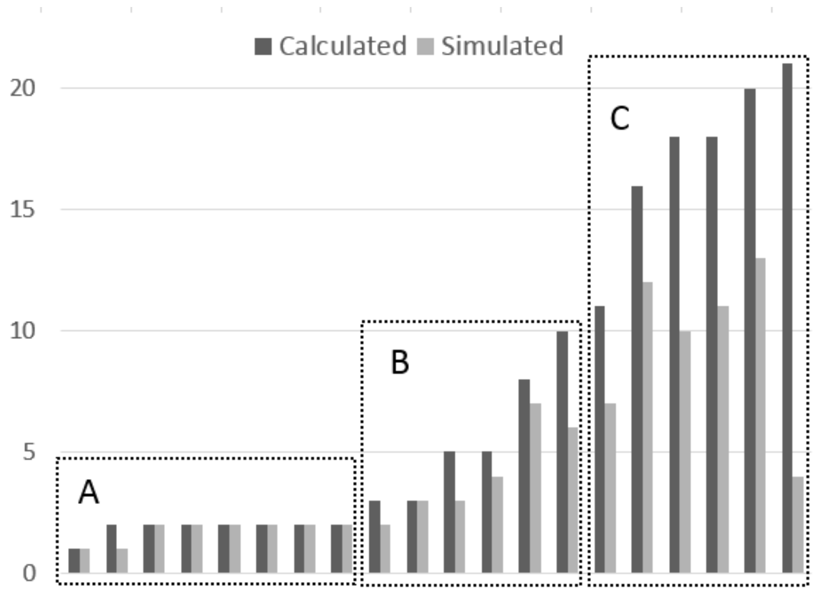


Figure 22 - Comparison between S-Levels Obtained from the BM-model and Simulated S-Levels for 20 Randomly Selected Items, Sorted by Increasing S-Level from the BM-model

The items in Figure 22 are also divided into three groups. In the group denoted A, the calculated S-levels are no larger than two and the mean customer order sizes are close to one unit. In this group the there was only one item where it was possible to lower the S-level and still reach the target which means that for these items the S-levels obtained from the BM-model worked very well.

In the second group the mean order sizes are somewhere between two and four units. The calculated S-levels for these items are only slightly overestimated so they work fairly well.

In the last group the mean order sizes are somewhere between four and eleven units. The S-levels for these items are significantly overestimated and can therefore carry excess stock.

Even though the S-levels are overestimated at times, Figure 21 shows that the inventory at the central warehouse is most of the time still lower with the BM-model when both models reach the TSL for upstream demand.

7.4.2 Local search for near-optimal reorder points

In order to find the potential of a critical level policy regardless of which model used to attain the reorder points, a local search by simulation were performed to find near-optimal reorder points for the system, both with and without a critical level. Since it is very time consuming to perform a local search through simulations, only twenty randomly selected items were used in this search.

For the system without a critical level, the search was performed by increasing the reorder point at the central warehouse until the fill-rate for the upstream demand reached its target. After this point was found the same procedure was performed for the retailer reorder points.

For the critical level setup the procedure is a little more complex since not only the reorder point at the central warehouse but also the one at the virtual retailer have to be lowered in the first step. Consequently, to find the optimal setup a vast number of iterations are needed. Instead an approach were used where the reorder point for the central warehouse calculated with BM were initially assumed to be optimal. Then the reorder points for all retailers including the virtual retailer were adjusted until the smallest reorder points which reached the TSLs were chosen. Finally the central warehouse reorder point was lowered to investigate if the retailers still were able to reach their TSLs.

Figure 23 depicts the results of this heuristic search when the items are sorted by increasing fraction of upstream demand. The results illustrates that a critical level policy in most cases have a significantly lower expected total inventory. Over the 30 items the average reduction is 8.43 % with a maximum reduction of 25%. In this small sample size it seems that the greatest potential savings occur when the fraction of upstream demand is below 50 percent which is also concluded in several papers discussed in Section 3.1. Furthermore the potential reductions increases when the mean demand is rather high which results in large reorder points.

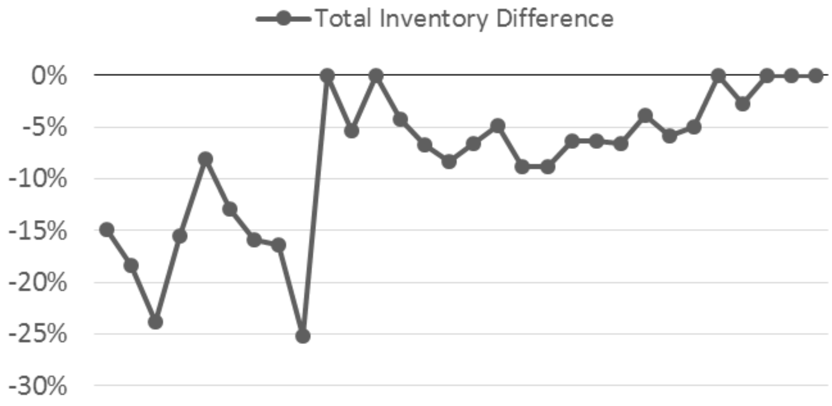


Figure 23 – The Relative Difference in Total Inventory of a Critical Level Policy Compared to Policy Without Critical Level. The Items are Sorted by Increasing Fraction of Upstream Demand.

The six items that show marginal or no improvements can be explained by two reasons. Either they have very small reorder points to begin with which leaves no room for improvement, or the fraction of upstream demand is very high (>90% of total demand). For an item with a very high fraction of upstream demand, that dominates the lower demand class, a critical level policy attain no advantage.

8 CONCLUSIONS

This chapter will present a short summary of the key components of the analysis. Followed by a remark of what future research may be undertaken to further validate the results of this study.

8.1 ACHIEVING TARGET FILL-RATES AND REDUCING INVENTORY

The results show that the BM-model with a virtual retailer significantly outperform the single-echelon model when it comes to reaching target fill-rates. However, the calculated S-level at the virtual retailer are in general slightly overestimated, and as a result the central warehouse carry some excess stock. This is because of the fill-rate calculation and can be improved by simple adjustments. Nonetheless, when comparing the expected inventory in the central warehouse for items where both models achieve the TSL for upstream demand, the BM-model still holds on average 10% less inventory than the single-echelon model.

When it comes to target fill-rates at the retailers, the BM-model is very precise with an average weighted deviation of 0.63 percentage points, when weighted by mean demand. One other conclusion that was expected is that when the mean order size increases, the single-echelon model do not keep enough stock to compensate for the assumption of continuous or unit demand. This is because of undershoot of the reorder point and is known for the normal and Poisson distributions and can be adjusted for.

8.2 POTENTIAL OF A CRITICAL LEVEL POLICY

The results of the sensitivity analysis suggests that a model with a critical level policy, in this thesis modeled as a virtual retailer, consistently outperform a model without a critical level. In some cases a near-optimal critical level policy can reduce the total inventory with up to 25 percent while still managing to fulfill the service commitments.

A critical level policy seems to have the greatest potential when the mean demand is high and fraction of upstream demand is less than 50 percent. When this fraction gets to large and the upstream demand dominates, the potential reduction in inventory diminishes.

When combining a critical level and multi-echelon optimization the largest potential reductions are expected for items with small fraction of upstream demand. This is because the benefit of coordinated control is to reduce the central warehouse inventory, which will mainly be accomplished when fraction of upstream demand is small. As the fraction of demand increases, the benefits of multi-echelon optimization diminishes.

The potential of a critical level needs to be validated in a study with less uncertainty in the data and with a larger sample size. The results obtained in the sensitivity analysis are promising but the sample size is too small to draw any general conclusions.

8.3 UNCERTAINTY IN THE EXTRACTED DATA

During the numerical study it was obvious that the extracted reorder points were calculated based on another set of mean and standard deviation of the demand compared to what was derived from the demand history. The extracted reorder points for the central warehouse were recalculated but the ones for the retailers were left as they were. However, as the recalculated reorder points at the central warehouse are independent of the reorder points at the retailers, the comparison of fill-rates for the upstream demand, as well as the central warehouse inventory, were unaffected by this.

Not being able to distinguish between replenishment orders and upstream demand resulted in an alternative approach to calculate the mean and standard deviation as well as the distribution of customer order sizes for the upstream demand. This does not affect the performance measures but the study failed to fully capture the characteristics of upstream demand customers at the case company.

8.4 FUTURE RESEARCH

Future numerical studies on the potential of critical level policies are suggested to further validate the results of this thesis. It could bring great value if a study were able to pinpoint more exactly how different item characteristics influence the potential inventory reductions of a critical level.

Secondly it would be interesting to be able to model a dynamic critical level that changes depending on the remaining time of an outstanding order. If an outstanding order is about to arrive at the warehouse in a few days, replenishment orders could be satisfied even though the inventory level is at or below the critical level.

Finally, an interesting topic for another thesis would be to evaluate some different ways to adjust the virtual retailer approach to prevent it from overestimating the critical level. More specifically, the fill-rate calculation at the virtual retailer in the BM-model need to be adjusted so that it also consider the expected inventory level at the central warehouse.

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APPENDICES

APPENDIX A – TABLE FOR DETERMINING NORMALIZED PARAMETERS

Table 4 present how to move between original and normalized parameters when estimating the induced backorder cost at retailer I in the BM-model.

Table 4 - Transfer Table between Original and Normalized Parameters.

Original System Parameters	Normalized System Parameters
$\boldsymbol{Q_i}$	$Q_{i,n} = 100\ Q_i/(\mu_i l_i)$
$\boldsymbol{Q_0}$	$Q_{0,n} = Q_0$
$\boldsymbol{h_i}$	$h_i = 1$
$\boldsymbol{h_0}$	$h_{0,n} = h_0/h_i$
$\boldsymbol{p_i}$	$p_{i,n} = p_i/h_i$
$\boldsymbol{L_0}$	$L_{0,n} = L_0/l_i$
$\boldsymbol{l_i}$	$l_{i,n} = 1$
$\boldsymbol{\mu_i}$	$\mu_{i,n} = 100$
$\boldsymbol{\sigma_i}$	$\sigma_{i,n} = 100\sigma_i/(\mu_i\sqrt{l_i})$
$\boldsymbol{\beta^* = \beta_n^* h_i}$	$\boldsymbol{\beta_n^*}$

APPENDIX B - TABULATED VALUES OF g AND k FUNCTIONS

Appendix B

Table B1 Tabulated Values of $g(p_{i,n}, Q_{i,n}) \cdot 10^4$, i.e., if x is an entry in the table then $g(p_{i,n}, Q_{i,n}) = x \cdot 10^{-4}$

$Q_{i,n}$	$p_{i,n} = 1$	5	10	20	30	40	50	60	70	80	90	100	150	200	250
1	40.8	75.7	90.5	104	112	117	121	125	127	129	131	133	140	144	148
10	20.4	45.1	59.0	72.8	80.7	85.9	90.0	93.3	96.0	98.3	100	102	109	114	117
20	9.74	26.7	39.8	54.0	62.2	68.2	72.4	75.9	78.8	81.3	83.5	85.4	92.4	97.3	101
30	5.87	17.8	29.4	43.0	51.0	56.5	60.7	64.2	67.0	69.8	71.9	73.8	81.0	85.9	89.7
40	4.03	12.8	22.5	35.4	43.1	49.0	53.2	56.7	59.6	62.0	64.2	66.1	72.9	78.2	81.9
50	2.98	9.91	18.3	30.5	38.2	43.9	48.0	51.5	54.3	56.7	58.9	60.8	67.7	72.6	76.5
60	2.35	7.91	15.0	26.1	33.5	38.8	42.8	46.1	48.9	51.3	53.4	55.2	62.0	67.3	70.6
70	1.91	6.53	12.8	23.2	30.5	35.9	40.1	43.5	46.4	48.8	51.0	52.9	59.7	64.6	68.3
80	1.60	5.51	10.9	20.3	27.1	32.2	36.2	39.5	42.2	44.5	46.6	48.4	55.2	59.9	63.5
90	1.38	4.80	9.54	18.0	24.4	29.2	33.0	36.3	38.7	40.9	42.8	44.6	51.1	55.5	58.9
100	1.21	4.18	8.48	16.5	22.7	27.6	31.5	34.7	37.4	39.7	41.7	43.8	50.3	55.5	59.0
150	0.734	2.56	5.27	10.8	15.7	19.6	22.6	25.3	27.6	29.6	31.3	33.3	39.2	43.3	46.4
200	0.527	1.81	3.78	7.86	11.8	15.1	17.8	20.1	22.4	23.8	25.4	26.7	32.0	36.0	39.5
250	0.415	1.40	2.92	6.21	9.56	12.6	15.5	17.9	20.0	22.0	23.7	25.2	31.7	36.8	40.2
300	0.340	1.15	2.34	5.05	7.68	10.1	12.3	14.2	15.9	17.3	19.0	20.2	24.8	28.1	29.7
400	0.252	0.828	1.71	3.66	5.72	7.73	9.61	11.2	12.7	14.1	15.4	16.6	21.1	24.3	26.7
500	0.201	0.644	1.31	2.88	4.49	6.21	7.84	9.49	11.0	12.3	13.8	15.0	20.2	23.9	26.3

Table B2 Tabulated Values of $k(p_{i,n}, Q_{i,n})$

$Q_{i,n}$	$p_{i,n} = 1$	5	10	20	30	40	50	60	70	80	90	100	150	200	250
1	0.943	0.957	0.961	0.965	0.966	0.967	0.968	0.969	0.969	0.970	0.970	0.970	0.971	0.972	0.973
10	1.163	1.119	1.094	1.076	1.066	1.062	1.059	1.056	1.054	1.053	1.051	1.050	1.046	1.043	1.042
20	1.381	1.271	1.207	1.158	1.138	1.124	1.117	1.111	1.107	1.103	1.099	1.097	1.088	1.084	1.079
30	1.519	1.383	1.287	1.217	1.189	1.172	1.162	1.153	1.147	1.141	1.136	1.132	1.119	1.111	1.106
40	1.613	1.469	1.357	1.264	1.228	1.205	1.194	1.183	1.175	1.168	1.163	1.158	1.145	1.134	1.128
50	1.686	1.533	1.410	1.299	1.255	1.228	1.213	1.200	1.193	1.185	1.179	1.174	1.159	1.148	1.141
60	1.736	1.584	1.458	1.337	1.287	1.257	1.240	1.226	1.216	1.207	1.200	1.194	1.177	1.163	1.158
70	1.779	1.630	1.491	1.363	1.306	1.271	1.251	1.235	1.223	1.214	1.206	1.199	1.183	1.170	1.161
80	1.814	1.666	1.529	1.392	1.333	1.296	1.272	1.255	1.242	1.235	1.227	1.220	1.197	1.183	1.174
90	1.838	1.691	1.556	1.419	1.355	1.316	1.291	1.271	1.265	1.254	1.246	1.238	1.214	1.200	1.190
100	1.861	1.721	1.582	1.437	1.370	1.327	1.300	1.280	1.266	1.254	1.244	1.235	1.214	1.196	1.186
150	1.931	1.803	1.673	1.520	1.441	1.390	1.365	1.341	1.323	1.309	1.297	1.285	1.257	1.240	1.228
200	1.963	1.852	1.725	1.579	1.492	1.437	1.400	1.379	1.355	1.349	1.336	1.325	1.291	1.270	1.250
250	1.974	1.880	1.764	1.626	1.533	1.471	1.425	1.393	1.369	1.354	1.338	1.325	1.276	1.244	1.227
300	1.987	1.896	1.796	1.650	1.567	1.506	1.464	1.432	1.408	1.389	1.371	1.359	1.318	1.294	1.300
400	1.995	1.926	1.829	1.698	1.608	1.543	1.494	1.468	1.439	1.417	1.398	1.384	1.333	1.306	1.287
500	1.997	1.948	1.865	1.733	1.655	1.587	1.536	1.493	1.461	1.435	1.409	1.390	1.323	1.287	1.278

APPENDIX C – TABLE AND DESCRIPTION OF RESULTS

Following is a description of the result categories for the tables on the following pages. The result in each category are based on each individual item from the study. BM refers to the BM-model with a critical level and SE refers to the uncoordinated single-echelon models.

- ***TSL Dev. BM/SE (pp)* –** The average deviation from target service level at all retailers not including the virtual retailer. Measured in percentage points.
- ***TSL Upstream Demand BM/SE (pp)* –** Deviation from target service level for upstream demand. Measured in percentage points.
- ***Mean Total Inv. BM/SE (units)* –** The expected total inventory. Measured in units.
- ***Inv. Diff. (%)* –** The difference in total expected inventory. Measured in percentage as the increase/decrease when modeling with the BM-model.
- ***Mean CW Inv. BM/SE (units)*** – The expected inventory at the central warehouse. Measured in units.
- ***CW Inv*. *Diff. (%)* –** The difference in expected central warehouse inventory. Measured in percentage the as increase/decrease when modeling with the BM-model.

Item No.	TSL Dev. BM (pp)	TSL Dev. SE (pp)	TSL Upstream Demand BM (pp)	TSL Upstream Demand SE (pp)	Mean Total Inv. BM (units)	Mean Total Inv. SE (units)	Inv. Diff. (%)	Mean CW Inv. BM (units)	Mean CW Inv. SE (units)	CW Inv. Diff. (%)
1	2.11	− 9.56	3.29	− 1.06	49.99	46.23	7.51	17.78	20.99	− 15.30
2	1.32	− 9.07	2.19	− 1.26	59.58	51.50	13.57	28.33	29.77	− 4.84
3	1.77	− 30.48	5.74	− 3.28	31.80	19.58	38.45	21.63	15.85	36.45
4	0.50	− 8.62	2.23	− 2.78	61.08	42.67	30.13	40.68	31.09	30.82
5	4.02	− 4.80	4.72	1.39	23.57	19.66	16.61	14.52	13.44	8.01
6	1.11	− 3.19	2.59	− 2.82	144.68	112.19	22.46	93.65	73.70	27.08
7	1.19	− 11.32	6.63	6.55	200.72	182.98	8.84	158.46	145.47	8.93
8	1.75	− 15.99	4.27	− 0.69	13.18	10.24	22.32	6.00	6.65	− 9.72
9	− 0.75	− 25.55	2.58	− 4.77	365.90	294.21	19.59	150.94	193.36	− 21.93
10	− 0.08	− 8.26	3.47	− 6.57	128.74	114.16	11.33	58.25	71.58	− 18.63
11	1.01	− 8.67	4.91	− 1.85	102.30	106.48	− 4.08	50.55	48.00	5.33
12	0.43	− 28.60	4.13	− 6.43	89.71	69.05	23.03	41.55	32.15	29.24
13	− 0.74	− 10.08	4.06	− 4.34	205.87	177.76	13.65	110.96	112.15	− 1.06
14	− 0.27	2.31	3.20	− 2.21	583.27	754.79	− 29.41	215.04	272.02	− 20.95
15	1.46	0.52	3.21	− 1.63	58.36	71.38	− 22.32	21.80	23.33	− 6.54
16	− 0.41	− 10.38	4.91	− 1.61	534.80	372.74	30.30	130.47	153.77	− 15.15
17	0.42	− 1.07	1.08	1.74	118.63	156.85	− 32.22	48.90	63.73	− 23.27
18	0.98	− 7.07	3.47	− 1.76	94.40	100.72	− 6.70	35.62	38.61	− 7.75
19	3.26	− 7.91	3.80	− 3.83	30.02	26.31	12.36	13.32	13.45	− 0.95
20	− 0.49	− 13.36	4.51	− 2.76	32.92	27.34	16.92	15.06	17.66	− 14.68

Item No.	TSL Dev. BM (pp)	TSL Dev. SE (pp)	TSL Upstream Demand BM (pp)	TSL Upstream Demand SE (pp)	Mean Total Inv. BM (units)	Mean Total Inv. SE (units)	Inv. Diff. (%)	Mean CW Inv. BM (units)	Mean CW Inv. SE (units)	CW Inv. Diff. (%)
21	0.97	− 6.12	2.86	− 1.10	39.72	42.04	− 5.83	19.52	19.32	1.07
22	0.73	− 9.02	4.68	− 1.30	38.96	41.12	− 5.53	16.74	17.25	− 2.99
23	0.35	− 12.58	3.88	− 4.01	49.95	37.53	24.87	24.90	23.00	8.25
24	0.96	− 12.76	2.18	3.06	144.42	141.79	1.82	68.29	93.80	− 27.20
25	0.09	− 33.10	3.71	3.94	100.94	73.08	27.59	39.27	49.16	− 20.12
26	0.28	− 15.77	2.85	4.46	79.60	81.93	− 2.93	35.78	51.91	− 31.08
27	2.37	− 33.68	4.36	− 6.35	40.64	21.19	47.86	16.33	11.95	36.67
28	1.39	− 6.86	4.43	4.43	126.94	131.11	− 3.28	92.06	108.02	− 14.77
29	0.50	− 23.79	2.54	5.64	223.15	212.97	4.56	126.72	162.76	− 22.15
30	1.56	− 5.24	1.33	2.54	190.84	185.66	2.72	106.00	125.43	− 15.49
31	4.28	− 6.74	5.02	6.71	19.47	21.52	− 10.51	12.45	17.32	− 28.13
32	3.38	3.85	4.41	− 9.28	7.13	6.14	13.87	2.74	1.65	66.39
33	1.76	1.63	2.59	− 8.58	13.59	10.68	21.39	7.87	5.00	57.43
34	3.74	− 0.08	5.00	3.63	22.04	27.12	− 23.02	10.09	10.05	0.43
35	3.10	3.51	4.29	3.59	10.46	11.46	− 9.56	4.49	5.37	− 16.44
36	0.75	− 17.85	2.23	− 0.16	62.69	55.14	12.04	27.65	28.42	− 2.71
37	1.31	− 3.25	4.09	− 4.05	80.96	59.92	25.99	54.01	38.75	39.39
38	1.17	− 11.45	4.66	− 6.16	54.61	35.12	35.69	34.70	19.26	80.20
39	1.12	− 7.52	2.24	− 4.48	43.83	35.35	19.35	15.44	15.91	− 2.90
40	1.45	2.29	2.22	− 5.87	38.23	34.44	9.90	21.29	15.58	36.61

Item No.	TSL Dev. BM (pp)	TSL Dev. SE (pp)	TSL Upstream Demand BM (pp)	TSL Upstream Demand SE (pp)	Mean Total Inv. BM (units)	Mean Total Inv. SE (units)	Inv. Diff. (%)	Mean CW Inv. BM (units)	Mean CW Inv. SE (units)	CW Inv. Diff. (%)
41	0.76	− 15.08	6.19	− 4.76	29.56	21.97	25.69	13.02	11.70	11.35
42	2.68	− 3.50	2.39	− 2.94	23.39	21.50	8.08	11.66	10.64	9.64
43	3.47	− 2.55	2.47	− 2.34	18.10	15.20	16.00	8.61	7.63	12.78
44	1.78	− 5.85	3.62	− 3.09	17.98	14.03	21.95	6.01	5.76	4.26
45	0.40	− 36.63	4.41	− 18.31	995.27	227.08	77.18	577.73	164.55	251.10
46	0.23	− 39.76	3.52	− 12.96	1 198.21	397.63	66.81	737.35	310.96	137.12
47	0.53	− 8.17	1.50	− 6.76	70.28	55.22	21.43	27.82	25.56	8.84
48	− 0.91	− 0.17	2.80	1.85	11.21	12.20	− 8.85	5.57	6.46	− 13.69
49	3.00	− 37.63	4.97	− 8.43	15.86	5.12	67.69	6.35	3.38	88.03
50	2.83	− 6.09	4.69	2.47	30.60	27.70	9.48	15.84	17.57	− 9.87
51	2.96	− 18.32	6.14	3.47	45.09	38.66	14.26	30.04	30.89	− 2.76
52	0.13	− 28.83	5.10	− 0.73	210.72	112.50	46.61	101.31	78.70	28.73
53	0.79	− 12.41	2.61	0.39	193.06	167.48	13.25	88.43	118.56	− 25.41
54	5.33	2.04	5.08	3.53	13.63	11.65	14.54	5.28	5.25	0.63
55	6.83	− 4.33	6.09	3.74	12.64	9.69	23.31	4.81	4.76	1.01
56	3.81	1.03	2.99	− 1.11	14.52	12.60	13.20	7.95	6.98	13.96
57	1.67	− 0.71	3.87	3.13	16.78	15.80	5.82	7.89	8.79	− 10.20
58	4.26	0.27	4.36	3.73	12.35	12.36	− 0.05	4.69	6.43	− 26.95
59	0.21	− 19.76	2.16	− 3.98	413.87	290.69	29.76	218.71	178.81	22.31
60	0.88	− 4.91	1.94	− 0.66	126.69	104.39	17.60	64.36	61.23	5.10

Item No.	TSL Dev. BM (pp)	TSL Dev. SE (pp)	TSL Upstream Demand BM (pp)	TSL Upstream Demand SE (pp)	Mean Total Inv. BM (units)	Mean Total Inv. SE (units)	Inv. Diff. (%)	Mean CW Inv. BM (units)	Mean CW Inv. SE (units)	CW Inv. Diff. (%)
61	0.31	– 20.10	4.03	2.95	76.08	59.60	21.66	34.77	38.13	– 8.81
62	1.02	– 2.91	2.20	2.36	80.39	76.66	4.63	43.26	48.99	– 11.69
63	3.23	– 26.45	4.99	2.35	16.58	11.70	29.39	7.38	7.23	2.13
64	1.33	– 0.44	3.18	– 3.18	20.60	19.64	4.67	9.18	8.87	3.53
65	1.01	– 3.78	3.33	– 3.04	21.04	19.11	9.19	8.61	9.12	– 5.57
66	0.72	0.70	2.41	– 1.26	54.00	38.88	28.00	16.48	21.43	– 23.09
67	0.86	– 10.63	1.94	1.50	129.47	119.61	7.61	60.38	76.71	– 21.29
68	2.69	1.73	4.48	0.00	12.35	11.40	7.70	6.40	5.40	18.40
69	0.29	– 0.79	3.84	– 5.64	18.33	17.38	5.19	9.26	8.69	6.57
70	4.93	0.58	6.02	5.62	8.92	7.92	11.16	5.18	5.17	0.22
71	3.39	1.14	2.83	– 2.99	19.94	16.03	19.61	9.02	7.08	27.29
72	3.80	– 0.65	6.96	3.07	12.09	11.14	7.83	4.86	4.77	1.80
73	2.08	– 2.54	2.40	– 3.75	13.55	10.61	21.72	4.93	3.93	25.52
74	0.49	– 14.69	2.37	– 3.67	226.40	112.63	50.25	76.43	62.85	21.60
75	0.09	– 15.13	2.27	– 3.51	151.23	108.64	28.16	77.42	64.96	19.19
76	1.45	1.63	3.94	6.38	9.35	11.33	– 21.15	6.06	8.03	– 24.49
77	0.76	– 43.83	3.72	– 2.23	55.60	27.56	50.43	20.84	16.74	24.48
78	1.82	– 28.51	4.90	5.33	117.55	94.08	19.97	61.41	69.06	– 11.07
79	2.98	– 22.56	5.74	– 13.32	7.68	3.89	49.40	4.06	2.17	87.04
80	2.68	– 2.92	3.11	– 9.47	8.88	5.91	33.47	3.40	2.33	45.52

Item No.	TSL Dev. BM (pp)	TSL Dev. SE (pp)	TSL Upstream Demand BM (pp)	TSL Upstream Demand SE (pp)	Mean Total Inv. BM (units)	Mean Total Inv. SE (units)	Inv. Diff. (%)	Mean CW Inv. BM (units)	Mean CW Inv. SE (units)	CW Inv. Diff. (%)
81	3.66	− 7.98	4.67	− 8.81	10.20	6.24	38.83	3.73	2.63	41.93
82	1.03	− 1.24	1.77	0.41	43.34	42.43	2.11	33.33	32.38	2.94
83	5.84	4.47	4.54	3.41	10.44	8.96	14.17	3.12	3.36	− 7.28
84	1.30	− 6.42	2.94	− 2.49	13.82	9.89	28.45	7.27	5.28	37.62
85	0.89	− 4.90	0.20	0.71	30.72	25.78	16.06	11.93	12.75	− 6.39
86	4.02	4.81	6.28	5.64	8.43	9.43	− 11.83	4.53	5.43	− 16.65
87	2.11	3.74	1.98	− 3.23	5.61	6.61	− 17.81	1.96	1.90	3.38
88	0.17	− 43.96	4.37	3.43	122.99	85.10	30.81	65.90	66.52	− 0.93
89	− 0.08	− 52.73	3.13	− 2.32	153.38	85.99	43.94	88.41	68.54	28.99
90	2.07	− 35.69	3.71	− 3.17	39.01	24.50	37.19	25.72	19.01	35.28
91	5.11	5.17	3.60	5.00	11.63	12.61	− 8.45	7.94	8.91	− 10.90
92	0.52	− 8.94	4.02	− 8.36	43.52	35.64	18.12	19.82	13.22	49.94

APPENDIX D – DESCRIPTION OF MEASURES IN TABLE 3

Following is a description of the measures presented in Table 3. The first four measures in the list are excluding the virtual retailer. All measures' units are in percentage points.

- ***Mean Deviation* –** The mean of all item's average deviations from TSL. The average deviation for an individual item is calculated as an average of the deviations from TSL at all retailers.
- ***Mean Absolute Deviation* -** The mean of all item's absolute average deviations from TSL. The absolute average deviation for an individual item is calculated as the average of the absolute values of the deviations from TSL at all retailers.
- ***Weighted Mean Deviation* –** The weighted mean deviations from TSL for all items. This measure is weighted based on the mean demand at the central warehouse for each item.
- ***Weighted Mean Absolute Deviation* –** The weighted mean absolute deviations from TSL for all items. This measure is weighted based on the mean demand at the central warehouse for each item.
- ***Largest Positive Deviation* –** The maximum overshoot of the TSL for all items and retailers.
- ***Largest Negative Deviation* –** The maximum undershoot of the TSL for all items and retailers.
- ***Mean Deviation at Virtual Retailer* –** The mean of the deviation from TSL for all items.