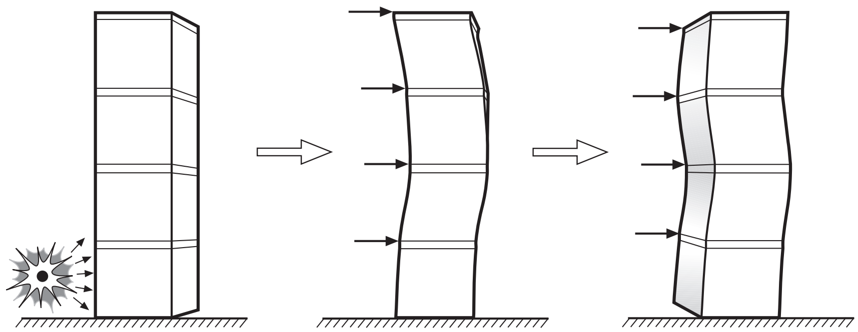

RESPONSE OF BUILDINGS EXPOSED TO BLAST LOAD

Method Evaluation

CARL LÖFQUIST

Structural Mechanics

Master's Dissertation

DEPARTMENT OF CONSTRUCTION SCIENCES

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MASTER'S DISSERTATION

RESPONSE OF BUILDINGS EXPOSED TO BLAST LOAD

Method Evaluation

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Abstract

Due to accidental external explosions ,which is not handle in the Eurocodes, are structural designers looking for other ways to design loads for external loads due to explosions. This combined with lighter building structures is a reason to address the issues of analysing the dynamic behavior of buildings subjected to air blasts from these extreme situations. The structural designers analyze the structures dynamic effects from these attacks with advanced finite element programs. These analysis due to long designing and computational time are normally costly. To minimize this and still acquire adequate accuracy in the results, reduced models of the structure system could be used. The reduction method according to Rayleigh-Ritz could be a useful alternative.

The purpose of this Master thesis is to investigate and develop dynamic models that combines sufficient accuracy with computational efficiency for structures effected by blast wave loads due to explosions. This is achieved by analysing two cases were the blast wave load is handled differently. First case handles the load as a triangular pulse load and the second case handles the load by transforming it into velocities according to the impulse and moment laws, and set as initial values when solving it as a free vibration problem. These cases are modeled in a full model and a model reduced with Ritz vectors.

The results showed that the method of designing Ritz vectors according to deflections similar to the eigenshapes gave a satisfying output. The reduction of the equation system with the use of the chosen Ritz vectors showed satisfying values for frequencies and responses for the analyzed structure. The computing time were reduced to lower than 10% with the use of 2 Ritz vectors for both analyzed cases.

Keywords: Explosion, Blast Wave, Impulse, Rayleigh-Ritz metod, Ritz vectors, Computational effeciency, Modal Reduction, Eurocode

Preface

This Master thesis was carried out during April to October 2016 at the Division of Structural Mechanics at Lund University Faculty of Engineering (LTH). This thesis is the final assignment of my Masters degree in Structural and Civil engineering.

I would like to express my gratitude to my supervisor Prof. Per-Erik Austrell who gave me support and guidance throughout this work.

This thesis is the final of five years of studies at LTH and I would like to thank my family, girlfriend and friends for your support during all my years of education.

Lund, October 2016.

Carl Löfquist

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Chapter 1

Introduction

1.1 Background

Structures are designed according to Eurocode Standards to withstand accidental loads from i.e. derailed trains, oncoming vehicle and explosions. The Eurocode Standards are focusing on accidental explosions when the structure is used for storing or handling with flammable gases, liquids and explosives. These explosions are assumed to occur internally and not externally which is not handle in the standards [1]. Due to this and the use of explosives in urban areas targeting civilian buildings, which have become a growing treat during the last decade, are structural designers looking for other ways to design loads for external loads due to explosions i.e. a truck filled with explosive cargo which is ignited.

The mentioned scenarios combined with lighter building structures is a reason to address the issues of securing structural integrity and analysing the dynamic behavior of buildings subjected to air blasts from these extreme situations. The physical dynamic behavior of structures are different from static loading when subjected to impulse loads. The structural engineers analyze the structures dynamic response from these explosions with advanced finite element programs. The analyzed models could usually be constructed as a system of three-dimensional elements to represent the building [4]. This gives large multi degree of freedom systems with huge number of degrees of freedom to analyze. These analysis are normally costly due to long modeling and computational time to get sufficient accuracy. To minimize the computational time and still acquire adequate accuracy in the results, reduced models of the designed structure system could be used. To reduce the equation systems build up using finite element method, the Rayleigh-Ritz method could be a useful alternative.

The main purpose with the Rayleigh-Ritz method is to reduce a multi degree of freedom system with minimum loss of information about the dynamic behavior of the system. This method was successfully used when analysing glass panes subjected to impact as Maria Fröling showed in her Ph.D thesis [3]. Frölings results showed that the use of two Ritz vectors is a good approximation which gives sufficient results and reduces the degrees of freedom into just two generalized coordinates.

1.2 Aim and Objective

The purpose of this Master thesis is to investigate and develop dynamic models that combines sufficient accuracy with computational efficiency for structures affected by blast wave loads due to an external explosion. The method will be based on using one or more static deflection modes for the structure that still fulfills the kinematic boundary conditions. By using Rayleigh-Ritz method a simpler static finite element program could be used instead of more advanced dynamic finite element software.
Issues to be investigated:

- How should the impulse from the explosion be distributed in space over the building?
- Does the difference in arrival time for the blast wave to reach different parts of the building affect the result?
- Could initial velocities be set at the front facade nearest the charge? How is it compared to a pressure pulse approach?
- The existing FE-model of the building considered is static so a mass matrix needs to be obtained. Could a lumped mass at translational degrees of freedom with small values for rotational degrees of freedom be used?
- How much computing time could be saved with a model reduced by Ritz vectors?
- Is the Rayleigh-Ritz method useful when analyzing a structure subjected to blast waves?

1.3 Scope of the thesis

The scope of the thesis is to investigate an approximate approach so some simplifications are done. The analyzed finite element model resembles a structure with symmetric cross-section with centre mass and moment of inertia coinciding on each floor. The structure will be analyzed as if there is no surrounding structures and with 2D beam elements. The facade of the structure is assumed to stay intact during the loading. The equation system will be reduced with maximum of the four first modes of vibration, other modes will be neglected.

Detonation, as when igniting TriNitroToluene (TNT) or similar, is the type of explosion analyzed and not a deflagration type, as a gas explosion. The fragmentation effect of the explosion on the facade and the structural elements integrity, are not handled and focus lies on the global response of the structure due to blast wave. The impulse load by the explosion will be modeled as four different scenarios and simplified in to a 2D model. The normal reflected pressure, P_r, will be used as value for pressure load. The effect of the incident angle will not be relevant because of the chosen reference building and it will be neglected. The negative phase of the blast wave load and the diffraction phenomena are also neglected.

Chapter 2

Theory

2.1 Explosions and Blast waves

A brief introduction to the basics of explosions and loads from blast waves that are created at the detonation, will be given. The material is based on reports from Electronic Journal of Structural Engineering, Myndigheten för Samhällskydd och Beredskap (MSB), and JRC Technical reports from European Commission [6] [7] [2].

2.1.1 Explosion Types and Blast Phenomenon

An explosion can be defined as a quick chemical reaction in solids, dust or gases where an expansion of the matter occurs. This type of explosion is divided into two types, deflagration and detonation. Deflagration is when a subsonic combustion proceeds because of the heat transfer, in other words when hot burning material heats and ignites material in its surrounding. The supersonic combustion, that often has a duration of microseconds, is called detonation and is the type of explosion which occurs when for example the explosive TNT is ignited. Further on in this thesis when referred to explosions the detonation type is the one that is implied. An ideal explosion is illustrated in Figure 2.1.

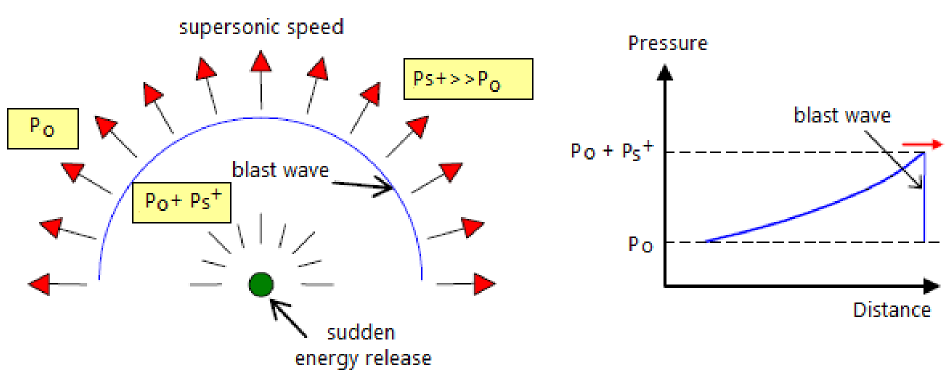


Figure 2.1: *Ideal explosion and propagating supersonic blast wave as presented in [7].*

The Blast Wave & Loading Phases

When an explosion occurs it will result in a production of exceedingly high temperatures and pressures due to the expanding hot gases that are created. The expansion leads to wave type propagation in the surrounding medium in a spherical form and a shock front or blast wave is produced as the hot gases compresses the surrounding air, shown in Figure 2.1. The blast wave is instantly increased to higher pressure levels than the ambient atmospheric pressure. This high energy intensity decreases as the blast wave moves further away from the explosion source and may after a short time drop below ambient pressure before evening out. This ideal pressure time history is illustrated in Figure 2.2 from Ngo et. als report on blast effects [6]. As seen in this figure there are a portion above and a one

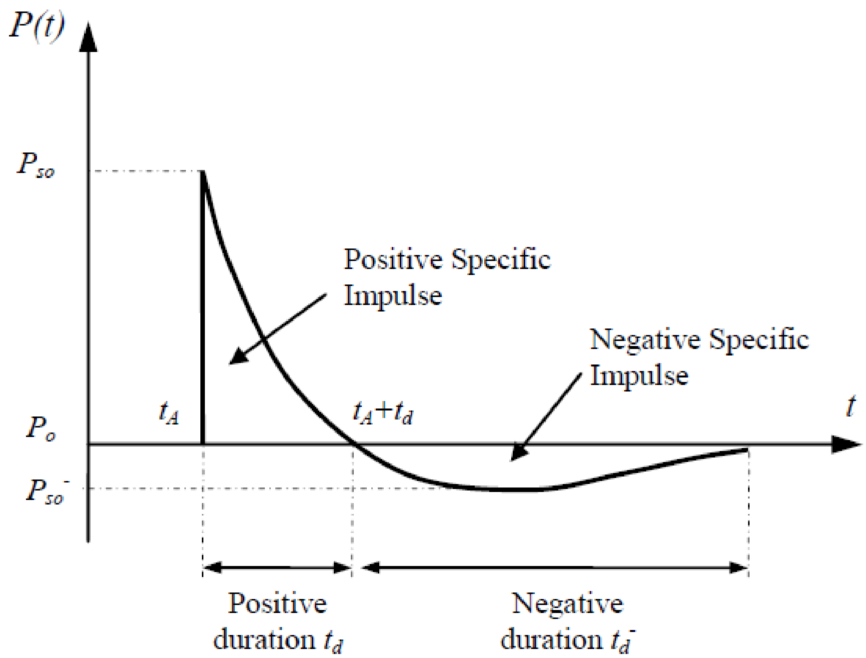


Figure 2.2: *Time history for Blast wave pressure [6].*

below the ambient pressure value, P_o, of the pressure-time profile. These two portions is usually referred to as the positive phase of duration, t_d, and negative phase of duration, t_d^-.

When structural engineers are designing for blast wave loads the general practice is to neglect the negative phase due to the longer duration and lower intensity, compared to the dominating positive phase [7]. The positive phase has a much higher energy intensity and a very short duration which have been verified to be the main reason to which structural damage occurs when investigating the integrity of the structure [2]. When modeling the blast wave loads in this thesis, the negative phase is also neglected according to the general practice. Figure 2.2 shows time history for the blast wave where the greatest magnitude, P_{so}, is the overpressure over the ambient pressure, P_o. This overpressure occurs at arrival time, t_A, and decreases over the duration time, t_d, which may only last for millior microseconds. When analyzing and designing a structure affected by a blast wave, the acting pressure load could be idealized as an equivalent triangular pulse with maximum peak pressure and the duration, t_d. The impulse per surface area i_r is calculated as in the Equation 2.1 due to a triangular shape of the acting impulse and is illustrated in Figure 2.3.

$$i_r = \frac{1}{2} P_r t_d \qquad [Ns/m^2] \tag{2.1}$$

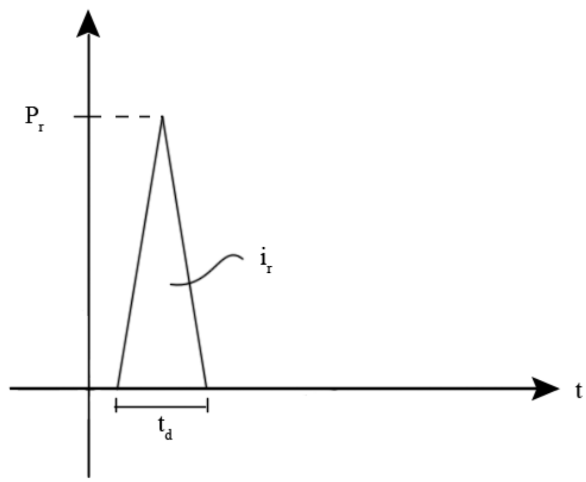


Figure 2.3: *Idealized triangular impulse.*

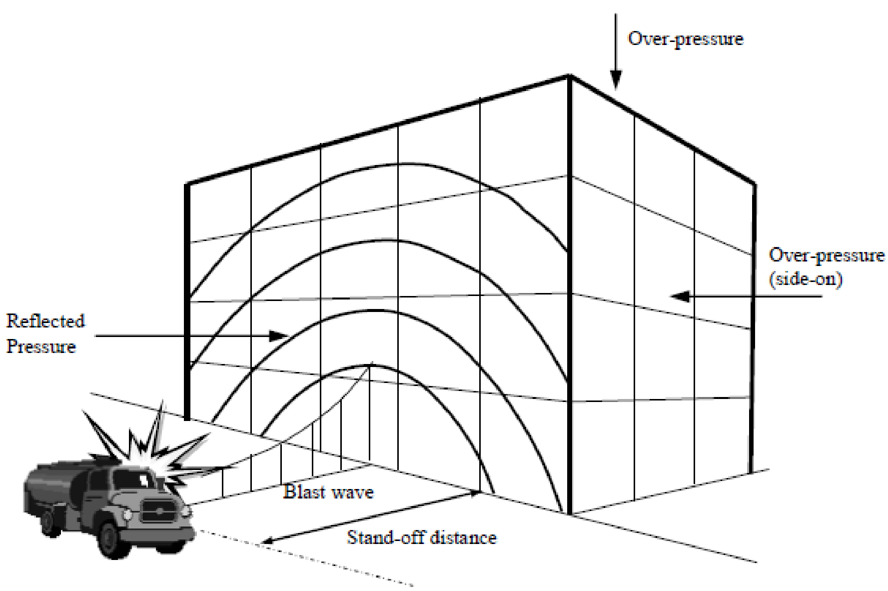


Figure 2.4: *How a Blast Wave affectes a building from Ngo et al [6].*

Stand-off Distance and Charge weight

The two most relevant variables that are crucial when mentioning threats from a conventional bomb are the charge weight W, and the stand-off distance R, between the detonation and the target, as seen in Figure 2.4. As Ngo et al.[6] mentioned, the range of explosive attacks from terrorist varies from a small letter bomb to a large truck bomb as used in the Oklahoma City bombing. Table 2.1 shows some estimates of the quantity of explosives that could be transported in different sets of vehicles.

Carrier	Explosive weight $[kg]$
Suitcase	10
Medium car	200
Large car	300
Pick-up truck	1400
Van	3000
Truck	5000
Truck with trailer	10000

Table 2.1: *Upper limits of charge weight per means of transportation [6]*

2.1.2 Scaling Laws for Blast Waves

The stand-off distance R, from detonation to subject, is as mentioned one of the most crucial parameters when computing blast loading. The peak pressure value and the blast wave velocity are diminishing quickly when this distance is increased. This is illustrated in Figure 2.5 as presented in JRCs Technical Report [2]. As the characteristics of the blast are so strongly affected by the distance, a way of taking it into account were developed. The introduction of scaling laws were made and the idea of these scaling laws is that charges with different weight and distances but equal scaled distance Z, produces an equally large blast wave load. These laws are based on experimental observations and theoretical studies and made it possible to compare the effect of different explosions at varying distances.

This blast effect is generalized and could be described by scaling the distance relative to the released energy E, and ambient pressure P_o, as $(E/P_o)^{1/3}$ and the pressure scaled relative to ambient pressure P_o. To get a more convenient expression the general practice is to express the energy input of the explosion E, as a charge weight W, which is an equivalent mass of TNT. This gives the dimensional distance parameter called scaling distance, Z from Hopkinson-Cranz scaling law which is illustrated in Figure 2.6. As the equation 2.2 shows, Z is based on the actual effective distance from the explosion centre in meters R and W that generally is expressed in kilograms of TNT.

$$Z = \frac{R}{\sqrt[3]{W}} \tag{2.2}$$

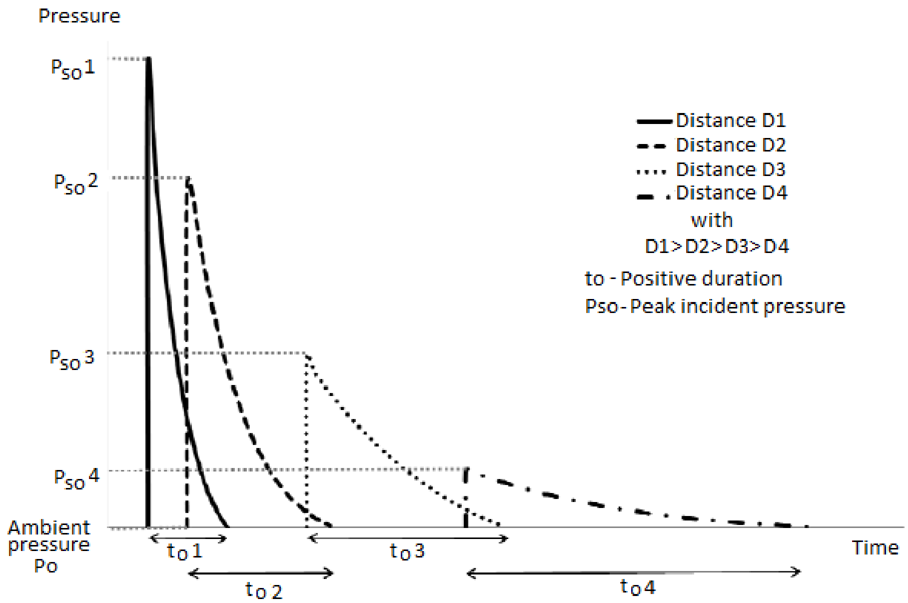


Figure 2.5: *Influence of distance on the blast wave positive pressure phase [2].*

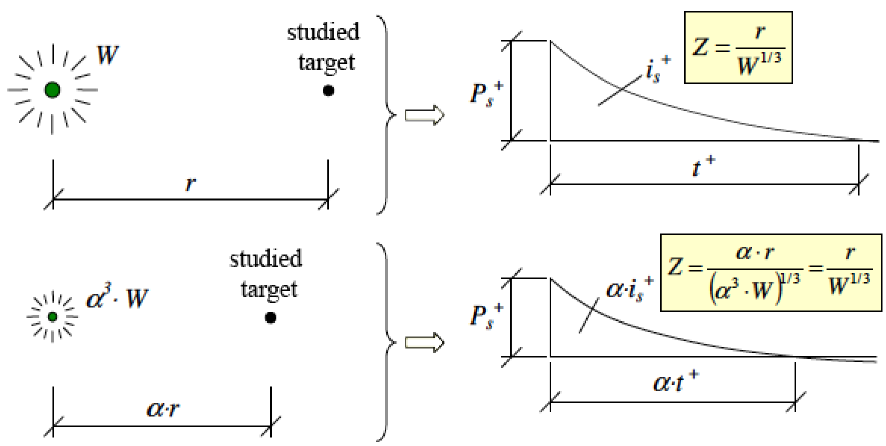


Figure 2.6: *Hopkinson-Cranz scaling law grafically illustrated [7].*

This scaling is based on an explosion in a surrounding where free expansion in every direction is possible and the amount of energy released is distributed within a spherical volume. If a free spherical expansion occurs, the volume within the blast wave is proportional to the expanded distance r, in cubic, i.e. the volume $V = V(r^3)$ [7]. When analyzing and modeling the blast waves in plane 2D or 1D the scaling is proportional to cylindrical and linear distance which are illustrated in Figure 2.7. The cylindrical expanded volume is then proportional to $V = V(r^2)$ and the linear to $V = V(r)$. The scaling laws are then as shown in Equation 2.3:

$$
\begin{aligned}
Z_{3D} &= \frac{R}{W^{1/3}} \\
Z_{2D} &= \frac{R}{W^{1/2}} \\
Z_{1D} &= \frac{R}{W}
\end{aligned}
\tag{2.3}
$$

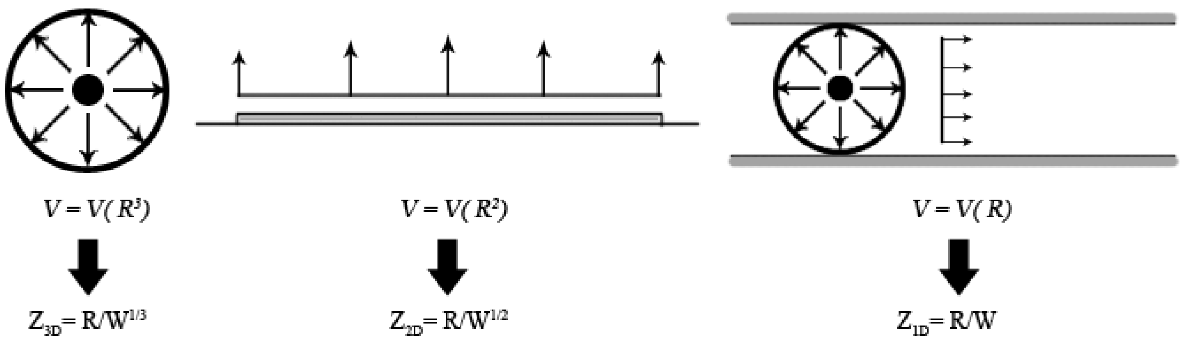


Figure 2.7: *Scaling laws in 3D, 2D and 1D [7].*

This concludes that different scaling laws are applicable depending on the environment surrounding the explosion. For example an explosion in a tunnel as illustrated in Figure 2.7, the scaling for Z_{1D} is used and not the free expansion in 3D, Z_{3D} [7]. In the analysis in this thesis the scaling based on the free propagation in 3D is assumed.

2.1.3 Blast Loading Types

As an explosive charge detonates in urban areas the surroundings, and relative placement of the explosion, effects the loading on the structures. The type of explosion that is considered in this thesis is unconfined, non-contact explosions which is external to the building structure. As Karlos et al. [2] writes, the type is divided into three subtypes such as a) free-air bursts, b) air bursts and c) surface bursts.

The Free-air burst is an explosion detonated in the air where the blast wave propagate spherically outwards and hits the structure without prior interaction with the surrounding. This gives a load acting ontop of the structure and on the facade. The Air burst is basically the same as a free-air burst with the difference that the blast wave hits the ground first which gives rise to a Mach wave front which then impinge onto the structure. While the other two types of bursts detonates in mid-air the surface burst detonates almost at the ground surface. The blast waves from the surface burst immediately interacts with the ground and then propagates hemispherical outwards and impinge onto the structures facade [2]. The first two load types could originate from explosives like granates. The surface burst type could originate from a truck with explosives that accidentally detonates which will be used when modeling blast loads in this thesis. In Figure 2.8 the three burst types are shown.

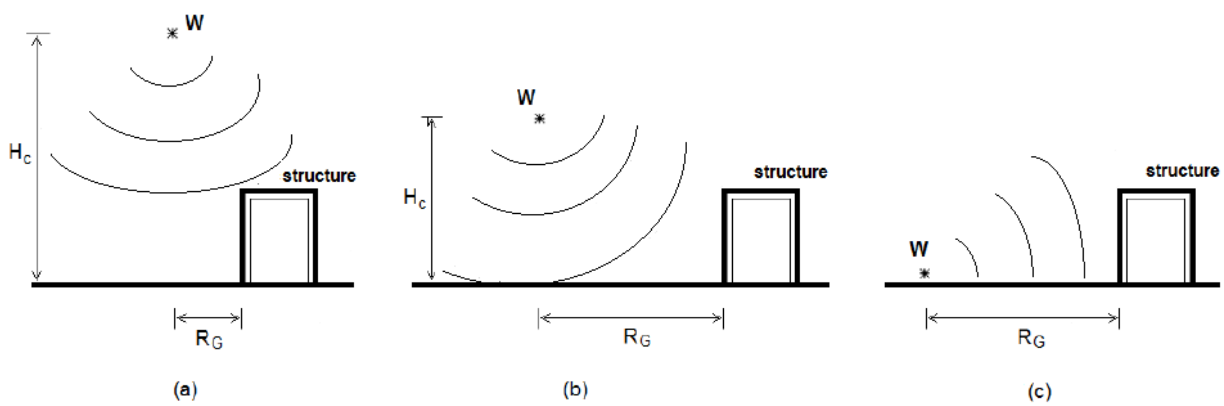


Figure 2.8: *Blast loading from explosion types [2].*

Reflection, Diffraction and the Angle of Incidence

In the case where the blast waves encounters an obstacle perpendicular to the propagation direction, the reflection increases the overpressure and gains the maximum reflected pressure P_r. According to Ngo et al. [6] the reflected pressure is calculated by Equation 2.4. This maximum reflected pressure P_r is always greater than the incident pressure P_{so}, and therefor often used for the design [2].

$$P_r = 2P_{so}(\frac{7P_o + 4P_{so}}{7P_s + P_{so}}) \tag{2.4}$$

The reflected overpressure maximum value, as stated, occur when the blast wave hits an obstacle perpendicular. When the angle between wave propagation direction and the affected surface, α, (angle of incidence) increases, the reflection process may be different. This angle affects the value of P_r and for small to moderate values with α between $40^o - 55^o$ the risk of underestimating is immanent if normal reflection is assumed. Within the range of $40^o - 55^o$, a Mach stem could be created and this coalescent wave can in some cases be much larger than the normal reflected overpressure P_r [2]. The normal reflected overpressure will be the modeled pressure used in the analysis and the reference building model is chosen so the effect of the angle is not relevant.

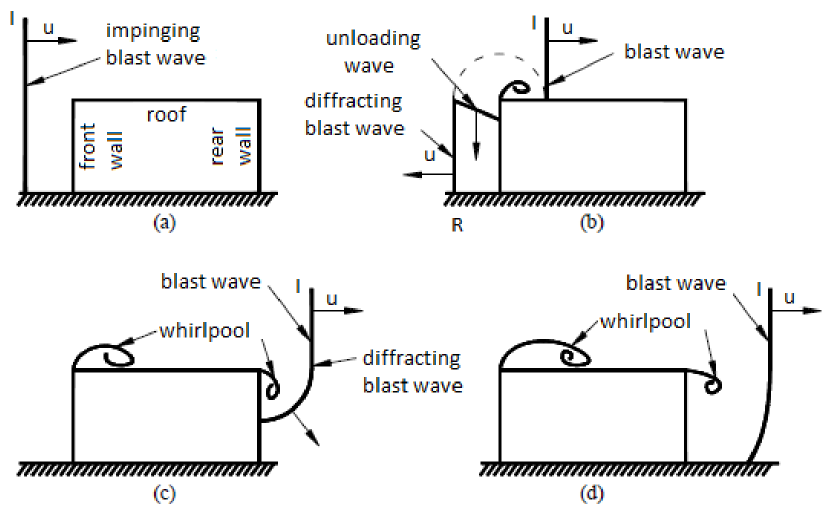


Figure 2.9: *The diffraction phenomenon as illustrated in MSBs rapport [7].*

When a blast wave reaches a structure, a complex phenomenon appears which is called diffraction. Basically, the diffraction is when the blast wave propagates past, behind and engulfs the subjected structure. This may have a great importance as for how the structure is affected by a blast wave. The diffraction is primary important when blast waves with longer duration are affecting a structure. Due to this the diffraction phenomenon is neglected in this analysis as the duration of the blast wave will be quite short.

Loading from Surface Bursts

A common way for terrorist-groups to attack structures in a bombing scenario is by using a vehicle loaded with explosives and detonate it with a remote control or a time trigger. This gives the blast loading type c) in Figure 2.9 the surface burst where the incident wave is reflected immediately from the ground surface which leads to higher pressure levels. The hemispherical wave that is propagating from this type of explosion has characteristics that resembles those of a Mach front wave. If the detonation is close to the structure then it will be loaded as illustrated in Figure 2.10 a) and for an adequately large stand-off distance it could be modeled as uniform or plane as shown in Figure 2.10 b). When the blast wave interacts with the a structure, it first hits the front wall and then engulfs and diffracts around the rest of the building and apply loading on every side. This effect is neglected in this thesis due to the use of a 2D model and the surface burst blast type will be used.

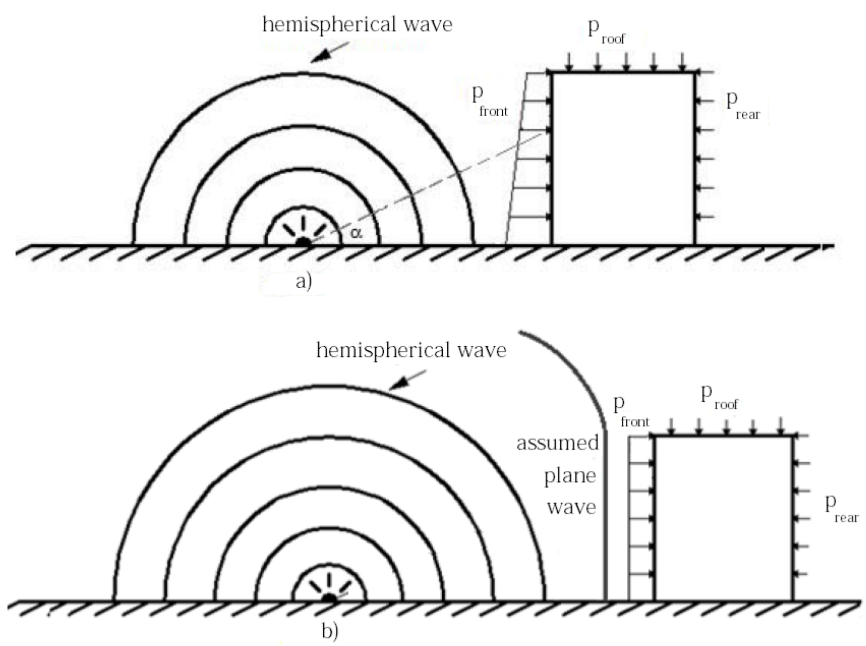


Figure 2.10: *Blast loading at close distance a) and sufficiently large distance b) [2].*

2.2 Rayleigh-Ritz Method

This section gives a brief introduction to the use of Rayleigh-Ritz method and Ritz vectors where the theory is retrieved from and further described in Chopra [5].

Method for Reducing Equation Systems

When designing and analyzing multi degree of freedom models, it is for some applications desirable to reduce the model with minimal loss of information in the dynamic behavior of the system. This reduction is done to give quicker but yet accurate results when analysing the structure. One method to do this reduction is to use the Rayleigh-Ritz method.

This method was originally developed for systems with distributed mass and elasticity. It is a method for reducing the number of degrees of freedom and finding approximations to the lower natural frequencies and modes. The equation of motion for a multi degree of freedom system with N degrees of freedom, which is subjected to forces could be formulated as shown in equation 2.5 below.

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}(t) \tag{2.5}$$

The mass-, damping-and stiffness-matrices is represented by $\mathbf{m}$, $\mathbf{c}$ and $\mathbf{k}$. Further on is $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$ and $\mathbf{u}$ representing the accelerations, velocities and displacements. $\mathbf{p}(t)$ is the vector that contains the external force applied in each degree of freedom. In the Rayleigh-Ritz method the displacements are expressed as a linear combination of several shape vectors, ψ:

$$\mathbf{u}(t) = \Sigma \mathbf{z_j}(t) \cdot \psi_\mathbf{j} = \psi \cdot \mathbf{z}(t) \tag{2.6}$$

, where $z_j(t)$ is the generalized coordinates, and ψ are the Ritz vectors that must be linearly independent vectors satisfying the geometric boundary conditions. The Ritz vectors in ψ are chosen appropriate to the system to be analyzed. With this approach the equation of motion will be given the form:

$$\mathbf{m}\psi\ddot{\mathbf{z}} + \mathbf{c}\psi\dot{\mathbf{z}} + \mathbf{k}\psi\mathbf{z} = \mathbf{p}(t) \tag{2.7}$$

and then the terms are premultiplied with the transpose of the Ritz vectors, ψ^T, to obtain:

$$\tilde{\mathbf{m}}\ddot{\mathbf{z}} + \tilde{\mathbf{c}}\dot{\mathbf{z}} + \tilde{\mathbf{k}}\mathbf{z} = \tilde{\mathbf{p}}(t) \tag{2.8}$$

The Ritz transformation of the displacements $\mathbf{u}(\mathbf{t})$ make it possible to reduce the original set of equations to a smaller set in generalized coordinates $\mathbf{z}$. This reduces the computing time. Once the system is reduced the dynamic analysis could be performed using a regular time-stepping scheme.The physical displacements, velocities and accelerations are found as:

$$\begin{cases} \ddot{\mathbf{u}} = \psi \cdot \ddot{\mathbf{z}} \\ \dot{\mathbf{u}} = \psi \cdot \dot{\mathbf{z}} \\ \mathbf{u} = \psi \cdot \mathbf{z} \end{cases} \tag{2.9}$$

The reduced systems variables are transformed with the same Ritz vectors ψ as before. This will give the vectors the same size as the initial system and this allows the multi degree of freedom system to be analyzed as a lower degree of freedom system. When the analysis is finished, the low degree of freedom system can be transformed back and give results for any degree of freedom in the original full system. The accuracy of the result is depending on the how well the choice of Ritz vectors is done [5].

Selection of Ritz Vectors

The Ritz vectors, (shape vectors), are most wisely determined based on the physical insight of the building's behavior. While this may mostly be suitable for simpler systems, such as shear buildings, estimating mode shape for buildings with a higher number of degrees of freedom may be more difficult. If visualisation of the first few eigenvectors or eigenshapes for the targeted structure could be made, the Ritz vectors could be selected according to these shapes. This could be difficult for complex systems because of the difficulty to visualize their mode shapes if no similar structures have been analyzed before. This visualization could especially be difficult if two or three-dimensional motions is included in the natural modes. If more Ritz vectors than the number of modes desired are used, the greater the accuracy of the approximated result will be. The accuracy of the approximated result is usually better for the lower modes than for higher [5].

Another way of determining the Ritz vectors is by applying a static load and using the calculated deflection as a Ritz vector. This method was successfully used when analyzing a glass structure subjected to dynamic impact load [3]. This method is used when constructing the Ritz vectors for the analyzed buildings in this thesis.

Solving the Reduced System With Initial Velocities

As the motion of equation is reduced by chosen Ritz vectors, the equation system have a lower number of unknown variables. To solve the system some initial values have to be invoked. One approach is to multiply the initial values in Equation 2.10 with the transposed Ritz vectors ψ^T.

$$\begin{cases} \mathbf{u}(0) = \psi \cdot \mathbf{z}(0) \\ \dot{\mathbf{u}}(0) = \psi \cdot \dot{\mathbf{z}}(0) \end{cases} \tag{2.10}$$

$$\begin{cases} \psi^{\mathbf{T}} \cdot \mathbf{u}(0) = \psi^{\mathbf{T}} \psi \mathbf{z}(0) \\ \psi^{\mathbf{T}} \cdot \dot{\mathbf{u}}(0) = \psi^{\mathbf{T}} \psi \dot{\mathbf{z}}(0) \end{cases} \tag{2.11}$$

The physical initial values can be optained by using the impulse created by the explosion load acting on the building structure shown as in Figure 2.11 a). This could be done due to the pulse duration is considered to be significantly smaller than the natural period of the system, $t_{d(imp)} << T_n$. This is based on the law of impulse and momentum where it is stated that if the pulse duration is short enough the velocity of the mass can be derived. This means that the mass has no displacement at time $t = 0$, but gets an initial velocity

$v(0)$, instead [5].

$$\int_0^{t_d} \mathbf{p}(\mathbf{t})dt = m\mathbf{v}(0) \Longrightarrow \mathbf{v}(0) = \dot{\mathbf{u}}(0) = \frac{\mathbf{I}}{m} \tag{2.12}$$

Equation 2.12 shows how the initial velocity $\dot{u}(0)$, is calculated, where **I** is the impulse and m is mass. This will induce a vibration in the structure as a free vibration response, with a maximum displacement u_{max}. In Figure 2.11 b) the time history for the displacement $u(t)$ is shown.

2.3 Free Vibrations

As the eigenvalue problem is solved and the eigenmodes and eigenfrequencies are determined the free vibration response has a general form for a N degree of freedom system which is given by superposition of the response of the individual modes [5].

$$\mathbf{u} = \Sigma\Phi_i(\mathbf{A}_i cos(\omega_i \cdot t) + \mathbf{B}_i sin(\omega_i \cdot t)) \tag{2.13}$$

The general form for the velocities is similar to the response:

$$\dot{\mathbf{u}} = \Sigma\Phi_i\omega_i(-\mathbf{A}_i sin(\omega_i \cdot t) + \mathbf{B}_i cos(\omega_i \cdot t)) \tag{2.14}$$

The initial values, that are assumed to be known, are formulated by putting, $t = 0$.

$$\begin{cases} \mathbf{u}(0) = \Sigma\Phi_i\mathbf{A}_i \\ \dot{\mathbf{u}}(0) = \Sigma\Phi_i\omega_i\mathbf{B}_i \end{cases} \tag{2.15}$$

$\mathbf{u}(0)$ and $\dot{\mathbf{u}}(0)$ are identified as modal expansions which gives expressions for $\mathbf{A}_n$ and $\mathbf{B}_n$.

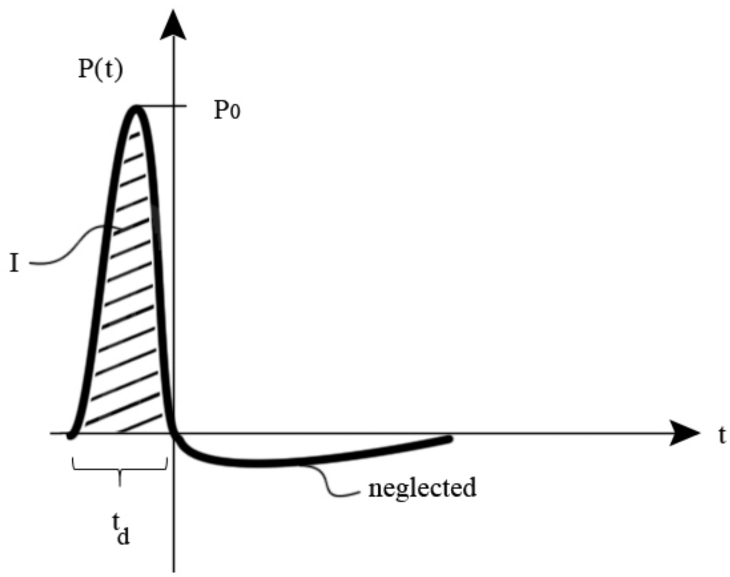
$$\mathbf{A}_n = \frac{\Phi_n^T m \cdot \mathbf{u}(0)}{\Phi_n^T m \cdot \Phi_n} \quad , \quad \mathbf{B}_n = \frac{1}{\omega_n} \cdot \frac{\Phi_n^T m \cdot \dot{\mathbf{u}}(0)}{\Phi_n^T m \cdot \Phi_n} \tag{2.16}$$

And if the initial displacement, $\mathbf{u}(0)$, is zero the initial value of $\mathbf{A}_n$ is also zero. This shows that only $\mathbf{B}_n$ is needed to be solved, in other words it is the initial velocities that are needed in order to solve the equation system.

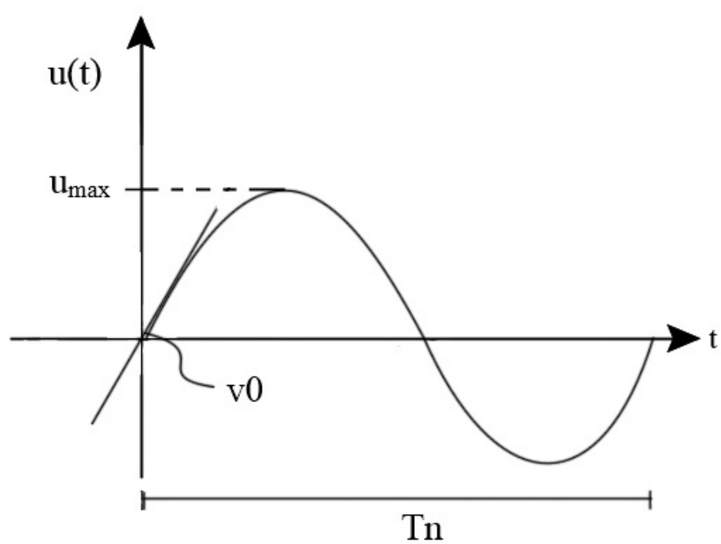
$$\mathbf{u}(0) = 0 \rightarrow \Sigma\mathbf{A}_i = 0\dot{\mathbf{u}}(0) = \Sigma\boldsymbol{\psi}_\mathbf{i}\boldsymbol{\omega}_\mathbf{i}\mathbf{B}_\mathbf{i} \tag{2.17}$$

The calculated response or the displacements **u** are:

$$\mathbf{u} = \Sigma\psi_i B_i sin(\omega_i \cdot t) \tag{2.18}$$



(a) *Typical impulse due to an explosion.*



(b) *Initial velocity in theory.*

Figure 2.11: *Impulse and initial velocity*

Chapter 3

Method and Analysis

In this section two methods for analyzing a structure affected by an impulse load from an explosion will be used on a 2D structural finite element model. The methods are based on a short duration of the impulse and a reduction of the analyzed system with the Rayleigh-Ritz method. The use of this method give a shorter computing time with sufficient results compared to a regular full scale model analysis. The model is constructed and analyzed using the MATLAB plug-in CALFEM developed at Department of Construction Sciences at Lund University [8]. This section will also present the assumptions being made in the analysis.

3.1 Analysis of Reference Structure

To investigate how a building is affected by an impulse load from a blast wave, a 2D finite element model was created using CALFEM. The blast wave affecting the model is then modeled in two different load cases where each case includes a full model and a reduced model. The reduction of the models is done accordingly to the Rayleigh-Ritz method. The different cases are analyzed to investigate if they are useful for varying types of external explosion scenarios and four scenarios are simulated to show this. The cases are shown in Figure 3.1 and will be more developed further on in this chapter.

3.1.1 Model Set-up

The reference building model is constructed as a 6 storey building with square cross section having 7 meter long sides, a building height of 21 meters and a storey height of 3.5 meters. The 2D finite element model of the reference building is illustrated in Figure 3.2. This model can be represented as a multi degree of freedom system with one degree of freedom for every Ritz vector. The model is analyzed as a 2D finite element model with a supporting structure which is based on steel pillars with concrete slabs as system of joists. The choice of this structure is made based on the simplicity of the model and that this is a common supporting structure for multi storey buildings such as offices [10].
The pillars are modeled as HEA 340 steel pillars with strength S355 which are carrying concrete joists with characteristics as of C30 concrete [9]. The structure is build up by 66

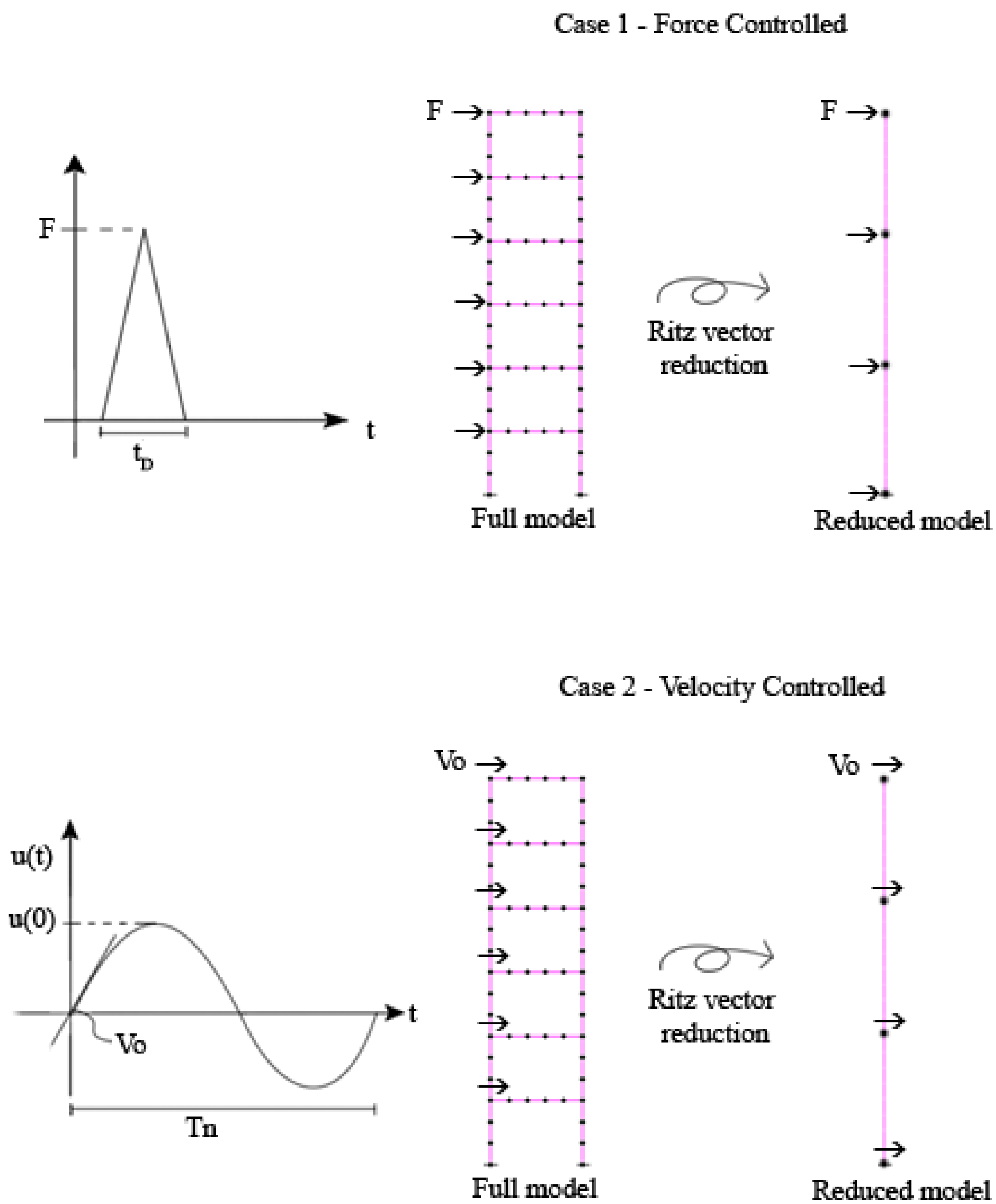


Figure 3.1: *Analyzed Cases 1 and 2.*

2D beam elements with 6 degrees of freedom in every element, which are displacement in horizontal and vertical directions and one rotational degree of freedom in every node this is illustrated in Figure 3.3. The total number of degrees of freedom is 186 in the model. No damping is invoked in the model and the base of the model is modeled as fixed to the ground. As this analysis is done as an undamped system the equations of motion are:

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{0} \tag{3.1}$$

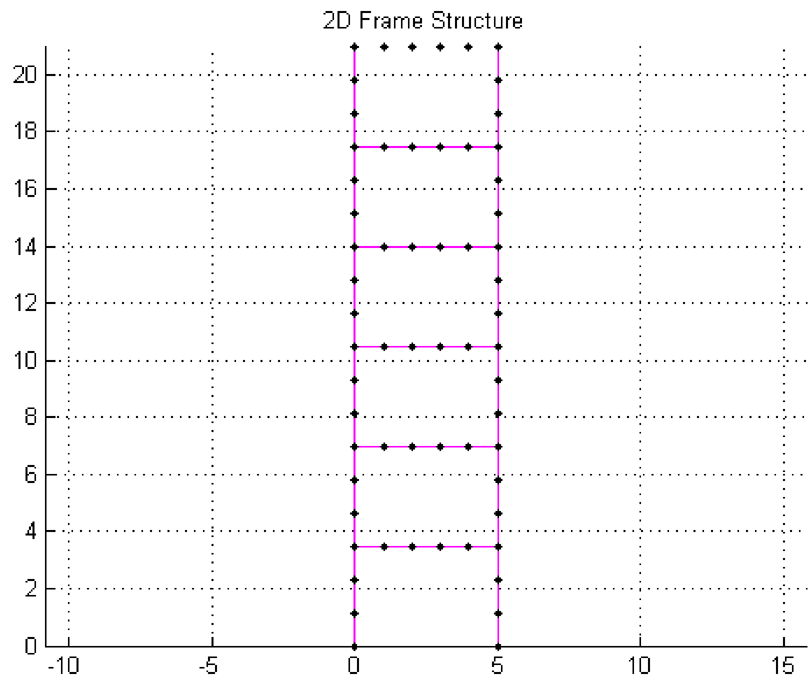


Figure 3.2: *2D Structure Model.*

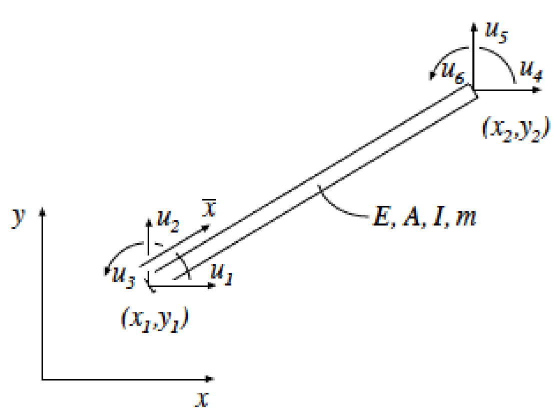


Figure 3.3: *2D beam element "beam2d" from CALFEM [8].*

Stiffness & Mass

The stiffness is modeled with the finite element method and the CALFEM command for dynamic elements, *beam2d* [8] giving a symmetric stiffness matrix in the size of the total number of degrees of freedom. The *beam2d* command also creates a symmetric mass matrix but due to negligible influence of the rotational inertia, the mass in the rotational DOFs are usually equal to zero. The mass matrix is therefore arranged as a diagonal lumped mass matrix which is a general approach when analysing the dynamics of a building structure. The lumped mass matrix is constructed by adding the total weight of the structural elements for the whole building and distribute it evenly to vertical and horizontal degrees of freedom. In the rotational degrees of freedom a value of 1/1000 of the nodal mass are placed to prevent numerical errors when solving the equation system. The eigenmodes and the eigenfrequencies are calculated by solving the eigenvalue problem of the equation system [5].

Blast Load Distribution

The reference building is subjected to an impulse load due to the explosion. This impulse load or blast wave is modeled to impinge on the left side of the model and is assumed to act in the storey heights. This gives six points where the modeled forces are acting as in Figure 3.4. The distribution of the pressure over the affected side of the building is varying as shown in Figure 3.5 where Case D, -B are stand of distance 15 m and Case A, -C are 5 m. In Figure 3.6 the difference in arrival time over the left side of the building is shown. The explosion is assumed to be situated in a vehicle and therefor elevated from the ground. This gives a detonation height, h_d , which is approximated to 1.5 m above ground, and the distances from the center of detonation to the structure nodes are calculated as:

$$R_h = \sqrt{R^2 + h^2} \tag{3.2}$$

Where h are the storey heights subtracted with the detonation height h_d and R is the stand off distance from the structure which is illustrated in Figure 3.4. These distances are then scaled according to the scaling law presented in Equation 2.3, i.e. $Z = \frac{R}{\sqrt[3]{W}}$.

The blast parameters are derived using the calculated values and the diagram in Appendix A which is presented in JRCs Technical Report [2]. The arrival time, t_A, duration, t_o and the impulse of the positive phase are scaled with $W^{1/3}$ so the presented values in these cases are multiplied with this factor. From these values the loads in the analysis model are designed. More detailed information on the calculations is found in JRCs Technical report [2].

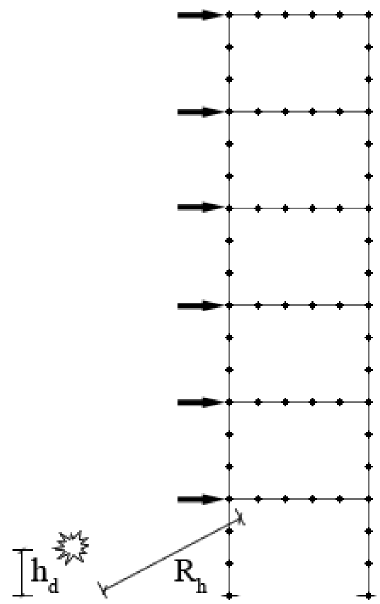


Figure 3.4: *Loading in model, detonation height h_d and distance R_h.*

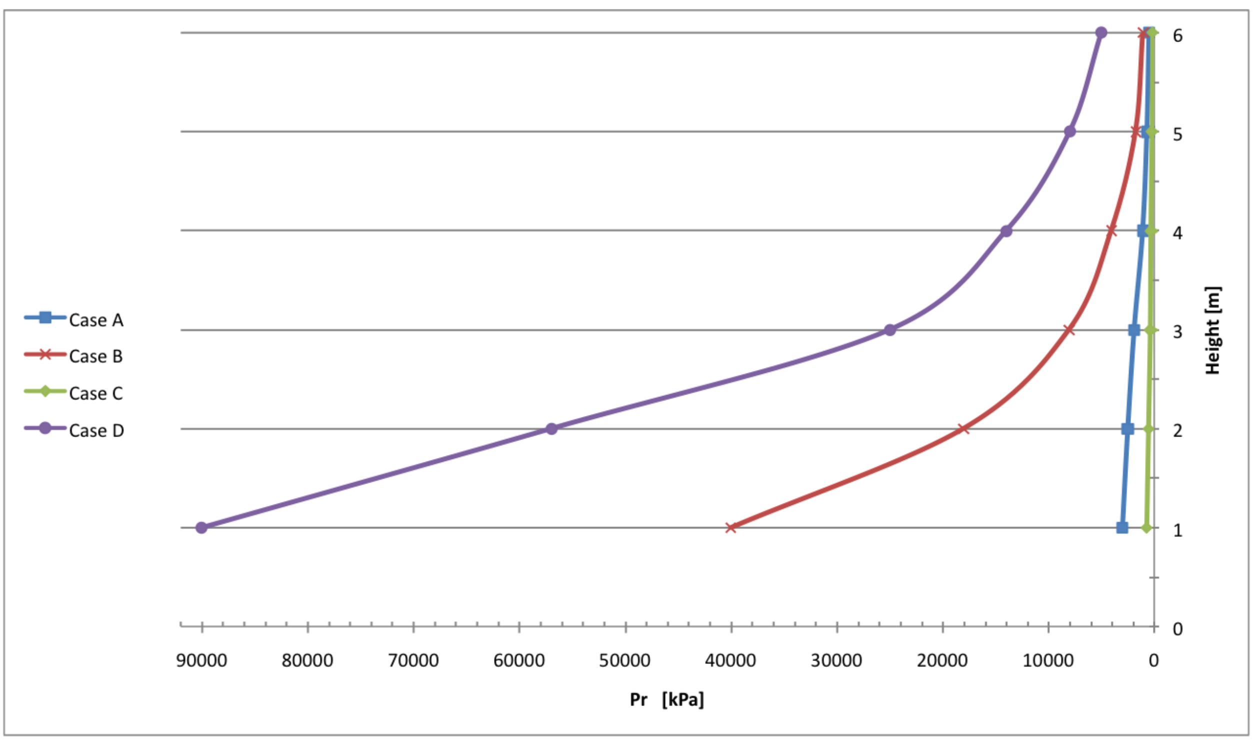


Figure 3.5: *Pressure over the 2D structures left side.*

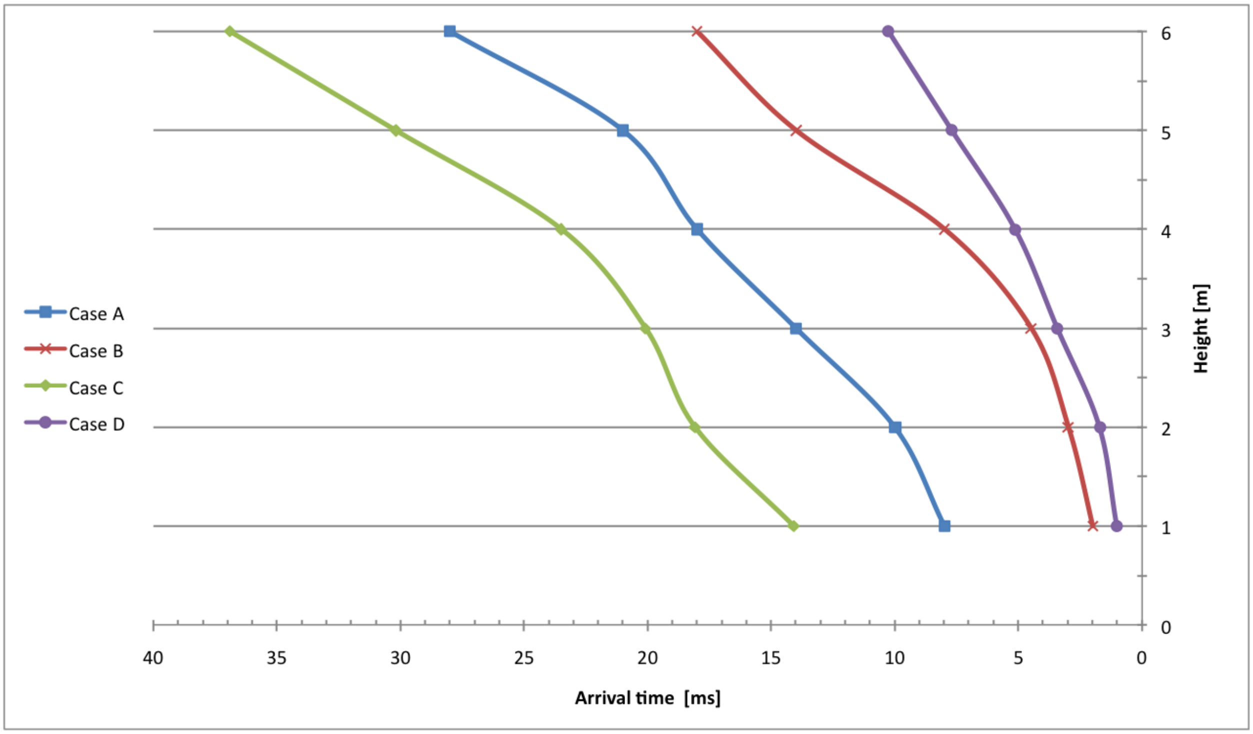


Figure 3.6: *Arrival time, tA, over the 2D Building Structures left side.*

Explosion Scenarios

In the evaluation of explosion situations that may occur, the most common situation of terror attacks are similar to the surface burst type mentioned earlier i.e. an explosion close to the ground [2]. There are four different load scenarios chosen where stand off distance R and explosives equivalent TNT weight W are varied. The four scenarios are shown in Table 3.1. The varying load scenarios are selected to check whether or not the different analysed cases give accurate results for large or small explosive weight respective close or far stand-off distance to the targeted object due to varying arrival times and pressure values. The calculated load scenarios are based on the case studies performed in JRC's technical report [2], however in a simpler 2D version. Values and description of the scenarios are found in Appendix B.

Load Scenario $A_{W1000R15}$ - Pick-up truck far from building	(Load weight 1400 kg Distance 15 m)
Load Scenario $B_{W1000R5}$ - Pick-up truck close to building	(Load weight 1400 kg Distance 5 m)
Load Scenario $C_{W300R15}$ - Large car bomb far from building	(Load weight 300 kg Distance 15 m)
Load Scenario $D_{W5000R5}$ - Truck bomb close to building	(Load weight 5000 kg Distance 15 m)

Table 3.1: *Four explosion scenarios analyzed*

3.2 Case Study

In these analyzed cases the reference building is subjected to an impulse load on the left side of the structure. The modeled cases are analyzed with the command $step2$ in CALFEM and the response from the system is saved and plotted. $step2$ is an algorithm for dynamic solution of second-order finite element equations considering boundary conditions. It is based on the Newmark's method and is a time-stepping method used in MATLAB [8]. The reduced models, shown in Figure 3.1, in both cases are analyzed with regard of one to four active Ritz vectors for the four different load scenarios, (A - D). While using different number of Ritz vectors, the eigenfrequencies for the reduced system saved and compared to the full model in Case 1_{Force}.

The studied variables are the maximum displacement at the top of the structure u_{max} response in the six storey levels of the structure and elapsed computing time. If the computing time for the reduced model is lower than 20% of the normal computing time the result is accepted and seen as a significant reduction. This is also applied for the accuracy when focusing on the comparison of the displacements in the models.

3.2.1 Case 1 - Force Impulse Model

To handle the load a triangular load is constructed as a linear load vector $\mathbf{f}$ which is varying as the time integration analysis is performed. This is a simplification of the real acting force loads positive phase. The time dependent load vector $\mathbf{f}(t)$ is created with the function $gfunc$ in CALFEM, where the maximum value is the maximum reflected pressure value Pr and the duration time t_d from the calculated explosion scenarios. The model is analyzed with numerical time-stepping.

Reduced Model

The system is transformed into a reduced generalized system with the Rayleigh-Ritz method presented in section 2.2. The time dependent load vector $\mathbf{f}(t)$ is also transformed with the same Ritz vectors to be compatible in the reduced system as:

$$\tilde{\mathbf{f}}(t) = \psi^T \cdot \mathbf{f}(t) \tag{3.3}$$

The reduced equation system eigenvalue problem is also solved and eigenmodes together with corresponding eigenfrequencies are determined. When analyzing the reduced system affected by the transformed time dependent load $\tilde{\mathbf{f}}(t)$ the same numerical procedure as presented for the full system is used. The response from the reduced system is then transformed back in to the original full system with the same Ritz vectors as used to reduce the system. This transformation is shown in Equation 2.9. The time dependent displacements $\mathbf{u}(t)$ and eigenfrequencies are compared to the full dynamic model.

3.2.2 Case 2 - Initial Velocities Model

In Case $2_{Velocity}$ the impulse load is handled according to an initial velocity problem. The same impinging triangular impulse load as in Case 1_{Force} is transformed accordingly to the impulse and momentum law relation described in section 2.2. This gives initial velocity values to insert in the numerical time stepping analysis instead of the time dependent load vector used in Case 1_{Force}. These initial velocities are placed in the same nodes as the affecting loads in Case 1_{Force}. The response gives a maximum displacement $\mathbf{u}_{max}$ in the six storeys and these are compared to the full dynamic model in Case 1_{Force}.

Reduced Model

As in Case 1_{Force}, this model is also transformed into a reduced generalized system with the same Ritz vectors. The initial velocities which are arranged as a vector $\dot{\mathbf{u}}(0)$ are also transformed with the chosen Ritz vectors to be used in the analysis of the reduced system. The response is then transformed back in to the original system as in Case 1_{Force} and is compared with the results from the other case.

3.3 Selecting the Ritz vectors

As mentioned in section 2.2, the Ritz vectors are quite difficult to select and the accuracy of the approximated results are depending on them. The way of determining Ritz vectors in this thesis are done by applying equivalent static loads and using the calculated deflections as Ritz vectors. A similar method were used by Maria Fröling in her Ph.D thesis with satisfying results [3].

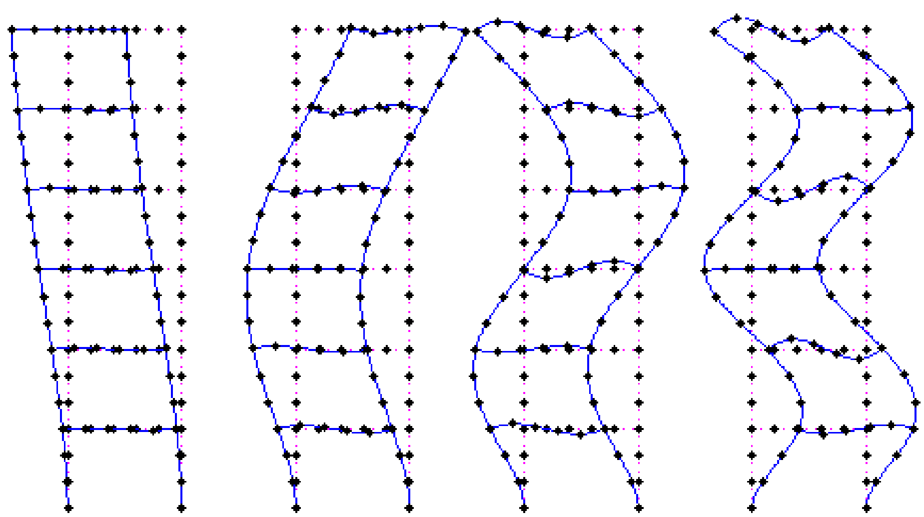


Figure 3.7: *The four first eigenmodes for the Reference Building.*

Freq 1	Freq 2	Freq 3	Freq 4
1.02	3.32	5.79	8.7

Table 3.2: *First four eigenfrequencies in the system*

Methodology

The full reference building model is first analyzed numerically and the eigenfrequencies and eigenmodes are determined and shown in Table 3.2 and Figure 3.7. With the knowledge of the lowest eigenmodes the Ritz vectors can be constructed and the four first shapes are described below. The first vector is constructed as the first eigenmode, the second vector is constructed as the second eigenmode, and so on.

Vector 1 is designed by applying an evenly distributed load on the left side of the model. The load applied in the nodes is 1 kN and this seems to be enough to give desired shape to use as a Ritz vector. This load results in a deflected form as shown in Figure 3.8.

Vector 2 is constructed by applying an evenly distributed load on the lower half of the model on one side and a point load that acts in the opposite direction on the other side. The load applied is as in Vector 1, 1 kN in the nodes. This is shown as arrows in Figure 3.8 where the shape is also shown.

Vector 3 is similar to Vector 2 designed by applying point loads. The point loads are placed in the top of the model and $\frac{1}{6}$ of the building height. The load values are 1 kN for the top and 3 kN for the lower. To achieve the S-shaped form, as the corresponding eigenmode, a point load acting in the opposite direction is also used. This opposite acting load is placed $\frac{2}{3}$ of the building height with load value of 2 [kN]. This is shown as arrows in Figure 3.8.

Vector 4 is chosen as the fourth eigenmode. The double S-shape is achieved by applying point loads in $\frac{5}{6}$ and $\frac{1}{6}$ of the building height. The load values are 1.5 and 3 kN. The opposite point loads are placed in the top of the building and at $\frac{1}{3}$ of the building height with 0.75 respective 2 kN as load values.

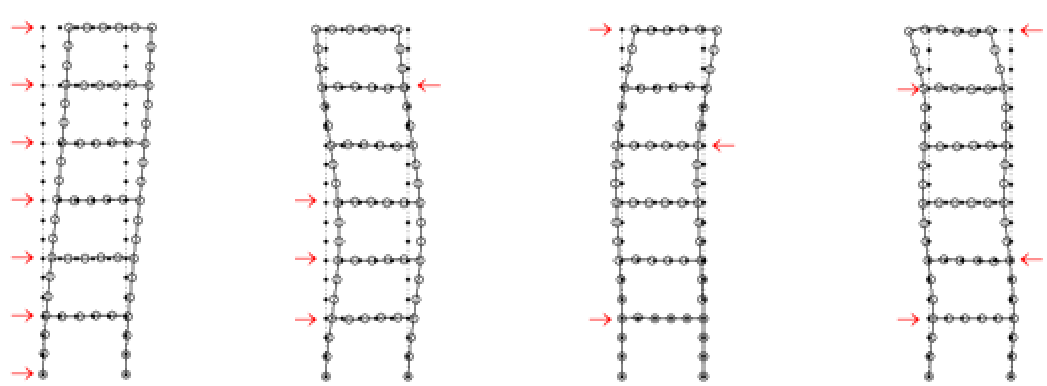


Figure 3.8: *The designed Ritz vectors 1 - 4.*

Chapter 4

Results and Discussion

4.1 Reduction With Rayleigh-Ritz method

The complete output data are collected from the analyzed structure with load scenarios and can be found in the appendices.

4.1.1 The Ritz Vector Choice

As shown in section 3.3 the Ritz vectors are chosen to mimic the eigenmodes of the structure. These modes were first calculated from the reference building model then visualized and used as preference when creating the Ritz vectors. This step is time consuming if it were to be done every time a new structure were to be analyzed i.e for a 3D model. The shapes are typical for a 2D model of a structure, similar to a shearbuilding, and with common knowledge of these modes the Ritz vectors could be created by using similar static deflection as these eigenmode shapes.

This approach could be used to reduce the model to much lower degrees of freedom system before the analysis is performed. This could serve as a first quick way of analysing a structure when dealing with explosion loads. A full analysis could then be performed as a complement to verify the quicker initial analysis. This approach worked well for a slender symmetric building as analyzed in this thesis.

4.1.2 Eigenfrequencies and Number of Ritz Vectors

In the reduced models the number of eigenfrequencies are bound to the number of chosen Ritz vectors. Therefore there is only one frequency to compare with when only one Ritz vector is used and so on.

As shown in Table 4.1 the eigenfrequencies are close to the compared values in the full model. This shows that the first eigenfrequency is well represented by the first Ritz vector based on the evenly distributed load over the structure. When two Ritz vectors are used the frequencies are still quite close to the full dynamic model, however the second eigenfrequency differs with 0.51 Hz and could be more accurate. This is achieved when a third vector is used and the difference is only 0.1 Hz for the second frequency.

Nr of Ritz vectors	Freq 1	Freq 2	Freq 3	Freq 4
1	1.82	-	-	-
2	1.81	7.02	-	-
3	1.81	6.60	14.8	-
4	1.81	6.59	14.7	27.6
Full Model	1.81	6.53	14.09	24.75

Table 4.1: *Eigenfrequencies with different amount of Ritz vectors used*

The third frequency differs with 0.71 Hz when three vectors are used. As a fourth Ritz vector is implemented the frequencies does not get substantially lower and the fourth eigenfrequency in the reduced system differs with about 3 Hz and this is not an accurate value. Usually the lower eigenfrequencies are the desired values to be known and could be achieved with two, or three Ritz vectors with sufficient accuracy. With four vectors the effect on the results is barely noticeable and the reason for this could mean that the shape is not representing the fourth eigenmode as well as desired. The higher frequencies and modes are thus more difficult to represent with the constructed Ritz vectors. As Maria Fröling concluded in her thesis[3], two Ritz vectors give sufficient results to be used as a approximation. A conclusion that can be drawn from this analysis aswell.

4.2 Analyzed Cases

4.2.1 Case $1_{\mathbf{Force}}$ Full and Reduced model

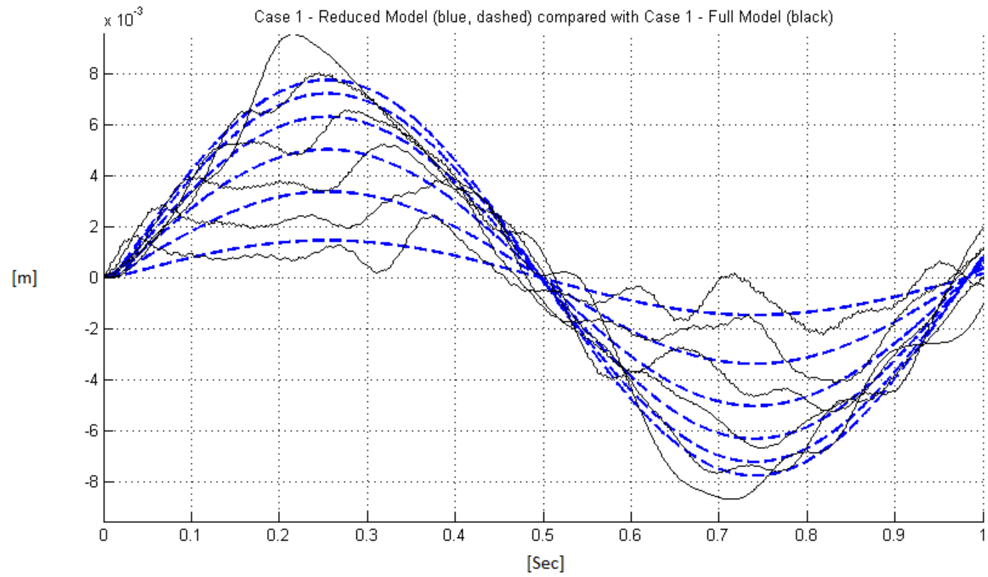
When the reduced model in Case 1_{Force}, the force impulse model, is analyzed and compared to the full model with only one Ritz vector the maximum displacement for the top of the structure were in the span of 15-28 % lower value than the full model. This could be seen as a distinct deviation from the full dynamic model. As increasing numbers of Ritz vectors are used the difference is decreasing to below 5 %, which is seen in Load Scenario $D_{W5000R5}$ analyzed with four Ritz vectors. When the same load scenario is used, with only one Ritz vector, the largest difference in values for the displacements are close to 2 mm. It is shown from the results, that at least two Ritz vectors are preferable for an accuracy about 5 % that could seem to be acceptable. Three vectors would be desirable but this could be difficult to achieve when modeling a more complicated structure design. More than three Ritz vectors did not affect the result substantially and it seems unnecessary to try to find more than three, at least when modeling in 2D. To conclude, the use of two Ritz vectors is giving sufficient accuracy of the reduced modeled system. The results can be seen in Table 4.2.

Values shown in Table 4.2 show the maximum displacement at the top of the structure and could be deceiving if we only were to study them. In Figure 4.1 response diagrams for the six storeys are shown and distinct resemblance is detected. Maximum values together with the resemblance in the diagram show that the results for the model reduced with Ritz vectors are quite sufficient. When using two Ritz vectors the results are close to the

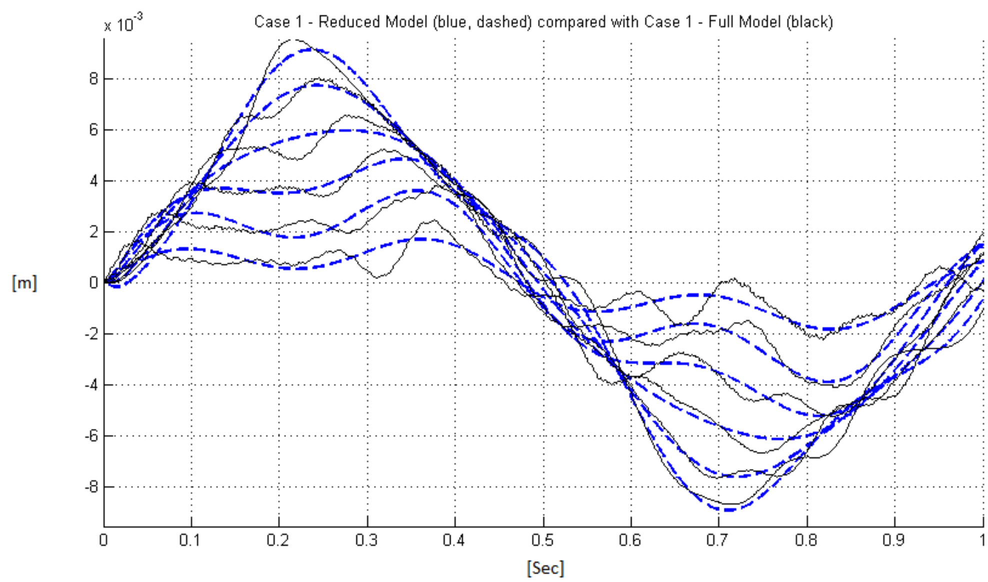
		Load Scenarios							
		$A_{W1000R15}$		$B_{W1000R5}$		$C_{W300R15}$		$D_{W5000R5}$	
Ritz vectors		**Full**	**Red.**	**Full**	**Red.**	**Full**	**Red.**	**Full**	**Red.**
1	Disp. [mm]	1.24	0.90	3.58	2.66	0.26	0.22	9.58	7.78
	Difference[%]	-27.4		-25.9		-15.4		-18.7	
2	Disp. [mm]	-	1.19	-	3.57	-	0.26	-	9.17
	Difference[%]	-3.7		-0.3		0		-4.3	
3	Disp. [mm]	-	1.22	-	3.58	-	0.26	-	9.35
	Difference[%]	-1.1		-0.2		0		-2.4	
4	Disp. [mm]	-	1.20	-	3.56	-	0.26	-	9.15
	Difference[%]	-2.6		-0.6		0		-4.5	

Table 4.2: *Difference between Full & Reduced model in Case* 1_{Force}

original values, and this could be an efficient and quick way of making a first evaluation of the structures dynamic behaviour when modelling explosion loads. Figure 4.1 shows Load Scenario $D_{W5000R5}$, Truck close to building, with one and two Ritz vectors and the other cases can be found in Appendix C.



(a) *1 Ritz vector*



(b) *2 Ritz vectors*

Figure 4.1: *Response for the reduced model in Case* 1_{Force} *with 1 & 2 Ritz vectors*

4.2.2 Case $2_{\mathbf{Velocity}}$ Full model and Reduced Model

The results show that the difference between the full models in Case 1_{Force} and Case $2_{Velocity}$, the inital velocity model, are lower than 11 %, the values are shown in table 4.3. It is shown that the structure have similar response in both cases and this can be seen in Figure 4.2 which is from Load Scenario $D_{W5000R5}$, the other cases can be found in Appendix C. The figure shows the response for storeys 1, 3 and 6. This approach gives better results when the pressure impulse impinge into the structure with same arrival time, like an evenly distributed load, which happens when the stand-off distance, *R*, is longer. The conclusion is that transforming the blast wave from an explosion to an initial velocity and solving it like a numerical time stepping free vibration problem could be a way to analyze a structure affected by impulse loads. This without losing any substantial amount of information and get sufficient results.

Load scenario	$A_{W1000R15}$	$B_{W1000R5}$	$C_{W300R15}$	$D_{W5000R5}$
Difference [%]	-3	-8.5	-3.8	-10.6

Table 4.3: *Difference between Full Model in Case* 1_{Force} *& 2*

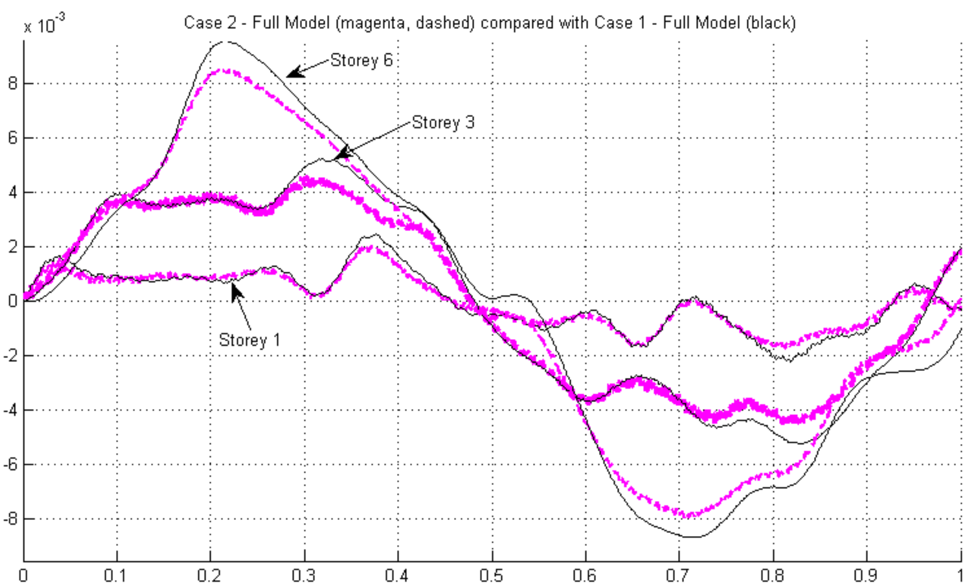


Figure 4.2: *Displacements for Load Scenario* $D_{W5000R5}$ *- Case* $2_{Velocity}$ *Full model*

Reduced model

The results for the reduced models in Case 1_{Force} and 2 are quite similar when different number of Ritz vectors are used. As shown in Table 4.4 the differences between reduced model and full model are decreasing when more Ritz vectors are used. These results are not as close as for the reduced model in Case 1_{Force} but when studying the displacement values something else is shown. The displacements in the full model, for some of the scenarios, are less then 1 mm and this gives larger differences when comparing the differing values. In Load Scenario $D_{W5000R5}$, which is the one with largest displacement, the reduced model shows 1 mm lower value then the full model which have 9.58 mm maximum displacement. This is seen as a quite sufficient result value even if the ratio is 15 %. The conclusion drawn is that the reduced initial velocity model approach gives good approximation when analyzing a structure affected by explosions. The use of two Ritz vectors is a good choice to get sufficient approximation for this case. The results for the reduced model in Case $2_{Velocity}$ can be seen in Table 4.4.

		Load Scenarios							
		$A_{W1000R15}$		$B_{W1000R5}$		$C_{W300R15}$		$D_{W5000R5}$	
Ritz vectors		**Full**	**Red.**	**Full**	**Red.**	**Full**	**Red.**	**Full**	**Red.**
1	Disp. [mm]	1.24	0.86	3.58	2.43	0.26	0.20	9.58	7.09
	Difference[%]	-30.8		-32.3		-22.6		-26.0	
2	Disp. [mm]	-	1.13	-	3.22	-	0.25	-	8.14
	Difference[%]	-8.6		-10.2		-7.2		-15.0	
3	Disp. [mm]	-	1.16	-	3.25	-	0.25	-	8.24
	Difference[%]	-5.8		-9.4		-6.0		-13.9	
4	Disp. [mm]	-	1.15	-	3.24	-	0.25	-	8.19
	Difference[%]	-6.9		-9.6		-6.0		-14.5	

Table 4.4: *Difference between the Full model in Case 1_{Force} & the Reduced model in Case $2_{Velocity}$*

When analyzing the response from Load Scenario $D_{W5000R5}$ with one and two Ritz vectors one could detect a convergence between the models. This is shown in Figure 4.3. In this case the maximum values and convergence in the diagram shows that the model reduced with Ritz vectors are quite satisfying. Using two Ritz vectors would give a result close to the original values and could be a sufficient and quick way of making a first evaluation of the structures dynamic behaviour when modelling explosion resonpse in 2D.

4.2.3 Arrival time, Pressure distribution and Mass

The pressure distribution over the left side of the structure when the explosion is close and have a great weight, shows a large difference in pressure levels. The pressure distribution for a larger stand-off distance between the explosion and the building is more even as shown in Figure 3.5. When focusing on the arrival time in the different cases, the results

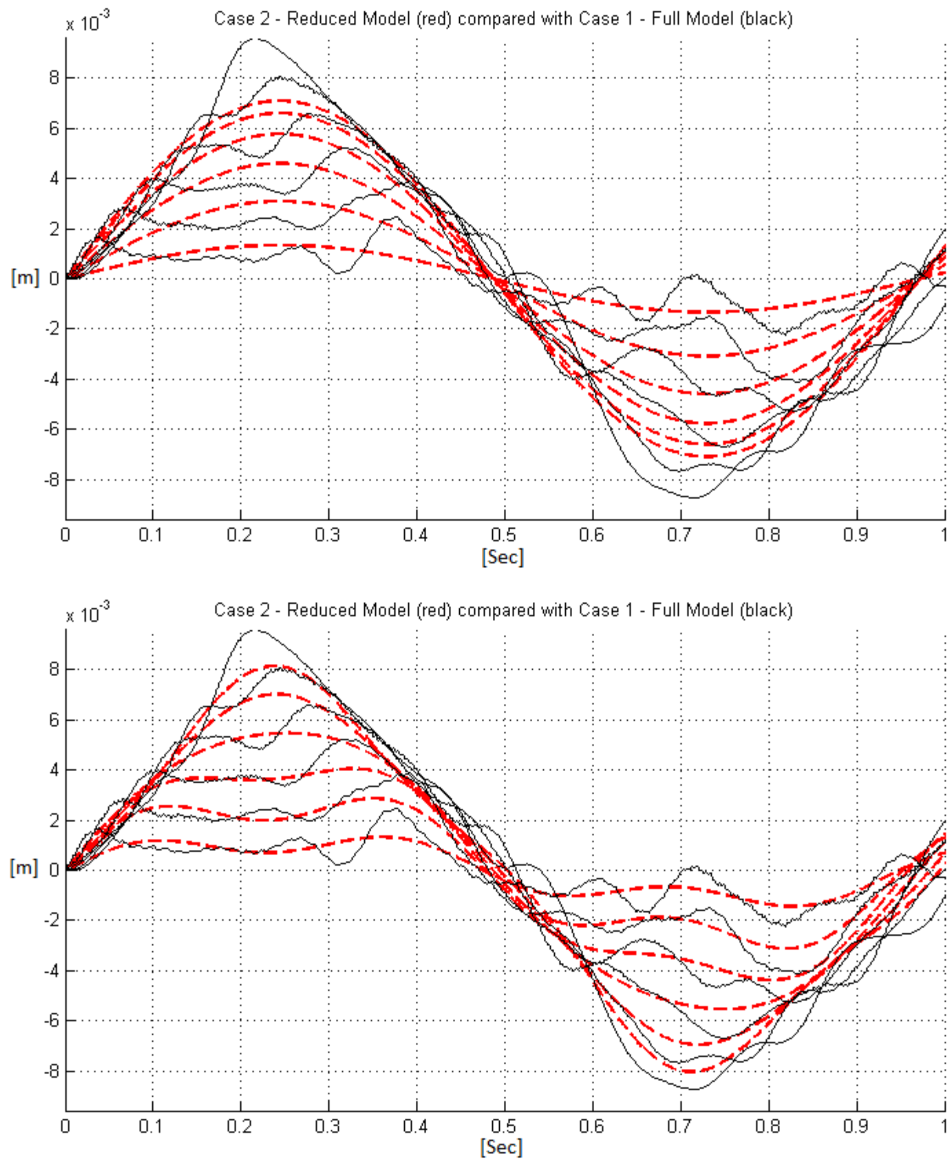


Figure 4.3: *Response for the reduced model in Case* $2_{Velocity}$ *with 1 & 2 Ritz vectors*

show that the model is not that sensitive to the varying arrival time over the structure. Even though some small effect could be seen when using the initial velocity approach in Case $2_{Velocity}$. The models show similar displacements and behaviour when comparing between the load scenarios. The arrival time over the left side of the structure varied as seen in Figure 3.6 earlier.

The use of numerical calculations when analysing the structure made it crucial to use a lumped mass matrix to avoid numerical errors. The masses were distributed event to all of the degrees of freedom except for the rotational degrees of freedom were one thousandth of the nodal mass where assigned. This worked out very well but a comparison with a actual explosion testing and a more advanced 3D model analysis of the building would be of interest so the assumption can be verified.

4.2.4 Computing Time

When recording the computing time for the different models the MATLAB program where run on a stationary computer in V-huset at LTH and this could affected the results. However, the values are presented showing a significant saving in computing time for the reduced models using the Rayleigh-Ritz method. As shown in Tables 4.5-4.6 the computing time for the reduced model in Case 1_{Force} is just 3-9% of the full model. For the reduced model in Case $2_{Velocity}$ the time where between 7-9% of the full model. This shows clearly that a great reduction in computing time in both models are made and that the force impulse reduced model in Case 1_{Force} was slightly more efficient when fewer Ritz vectors were used. This computing time for these small systems were round 2 seconds for the full model and as low as 0.07 seconds for the reduced models. If greater building structures were to be analyzed the gained time could be enormous.

		Case $1_{\mathbf{Force}}$		**Case $2_{\mathbf{Velocity}}$**
Nr of Ritz vectors		**Full**	**Reduced**	**Reduced**
1	Time elapsed [sec] (Average)	1.95	0.07	0.14
	Precentage of [%]	3.6		7.2
2	Time elapsed [sec] (Average)	-	0.11	0.16
	Precentage of [%]	5.6		8.2
3	Time elapsed [sec] (Average)	-	0.18	0.17
	Precentage of [%]	9.2		8.7
4	Time elapsed [sec] (Average)	-	0.14	0.16
	Precentage of [%]	7.2		8.2

Table 4.5: *Computing time compared Full model Case 1_{Force}, Reduced models Case 1_{Force}& 2*

To see if there were some time saving when using the full model in Case $2_{Velocity}$, it were compared to the full model in Case 1_{Force}. The results showed that the full model in Case $2_{Velocity}$ used 34% of the computing time compared to Case 1_{Force}. This show that the approach of using the initial velocities, as done in Case $2_{Velocity}$, could be a useful approach. To verify this method more tests would be appropriate.

	Case $1_{\mathbf{Force}}$	**Case $2_{\mathbf{Velocity}}$**
	Full	**Full**
Time elapsed [sec] (Average)	1.95	0.66
Precentage of[%]	34	

Table 4.6: *Computing time compared Full models in Cases 1_{Force} & $2_{Velocity}$*

4.3 Summary and conclusion

Reducing the computing time when analysing structures dynamic response, is favorably done with a reduced model according to the Rayleigh-Ritz method. The use of Ritz vectors based on static deflections give sufficient accuracy and significant shorter computing time. The computing time were greatly reduced to just a few percent of the full system.

The choice of Ritz vectors gave similar eigenfrequencies and eigenmodes on both systems, even though the fourth vector did not affect the results as much as hoped. The use of two Ritz vectors gave sufficient accuracy in both maximum displacements and correlating responses, in the 2D model. When the third and the fourth vector were used the results were not affected and the use of just two Ritz vectors is enough.

A lumped mass distribution was satisfying for both the initial velocity- and force pulse model. The distribution was simple yet efficient but an other approach of defining the different values for each node could be investigated further. 1/1000 of the value of the distributed mass in rotational nodes helped avoiding numerical errors and gave desired results.

Transforming the acting impulse load on the structure to initial velocities and solving it with numerical time stepping could be a useful approach. This was done without losing any substantial amount of information of the dynamic behavior. The approach worked well in this 2D structure model with the given assumptions.

Varying arrival time of the blast wave did not affect any of the cases significantly and this could be due to the small differences in the arrival time. The distribution of the load concentrated into the 6 storeys also gave satisfying results when comparing the models.

Future Work

A method to create standardized deflection modes as Ritz vectors, which correlates with the lowest eigenmodes in different structure types could be developed, similar to Frölings [3] developed approach, which could be flexible to use in different scenarios and structures. This could then be used when specific building structures are analyzed to gain a primary perception of the dynamic behavior. Further tests of the Rayleigh-Ritz method in 3D models could be performed which then could be compared to actual explosion tests values. This could then lead to a development of programs to MATLAB and CALFEM which could design Ritz vectors due to the typical eigenmodes, reduce and analyze the structure, then finally transform it back to full model and show the desired values.

Other ideas could be to use the typical deflection shapes, which correspond to the lower eigenmodes of the model, as Ritz vectors and standardize them for specific type of structures. The structures could be reduced and analyzed with regard to the special set of standardized deflection shapes i.e the four lowest eigenmodes used in this thesis.

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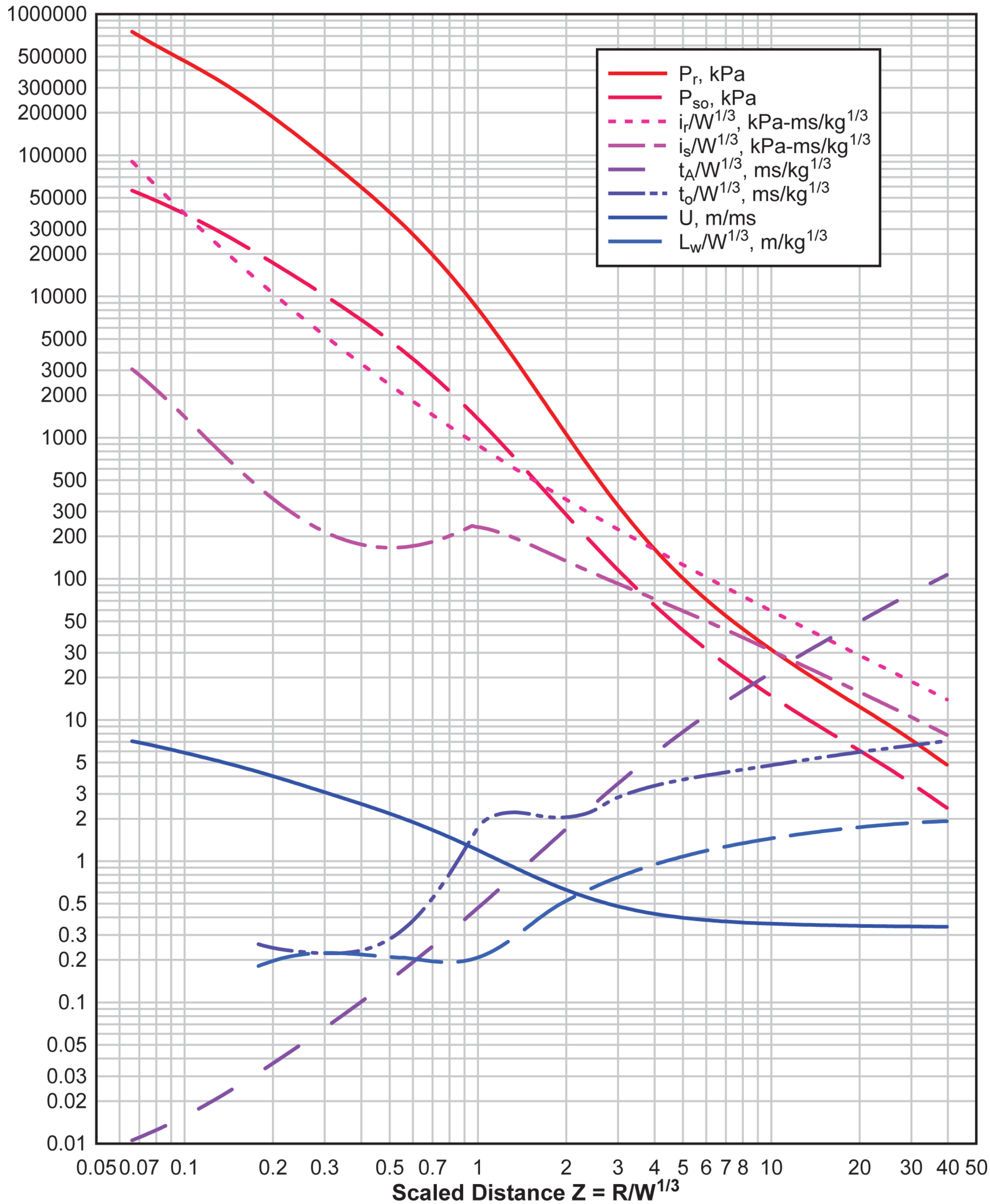
Appendices

Appendix A

Scaled Distance Diagram



1000000
500000
300000
200000
100000
50000
30000
20000
10000
5000
3000
2000
1000
500
300
200
100
50
30
20
10
5
3
2
1
0.5
0.3
0.2
0.1
0.05
0.03
0.02
0.01
0.05 0.07 0.1 0.2 0.3 0.5 0.7 1 2 3 4 5 6 7 8 10 20 30 40 50
Scaled Distance Z = R/W^1/3
P_r, kPa
P_so, kPa
i_r/W^1/3, kPa-ms/kg^1/3
i_s/W^1/3, kPa-ms/kg^1/3
t_A/W^1/3, ms/kg^1/3
t_o/W^1/3, ms/kg^1/3
U, m/ms
L_w/W^1/3, m/kg^1/3

Appendix B

Blast Wave Parameters For Load Scenarios

The blast parameters are derived from the calculated values and using the diagram in Appendix A which is presented in JRCs Technical Report [2]. The arrival time, t_A, duration, t_o and the impulse of the positive phase are scaled with $W^{1/3}$ so the presented values in the cases are multiplied with this factor. The different explosion situations are as presented below. The pressure distribution over the left side of the structure is shown in Figure 3.5.

Z	scaled distance
P_r	peak reflected pressure
P_{so}	incident peak overpressure
i_r	positive reflected impulse
i_s	positive incident impulse
t_A	arrival time
t_o	positive duration time
W	mass of explosive in kg
R	distance from detonation source to object

Table B.1: *Legend to the tables*

Load Scenario $A_{W1000R15}$ - Pick-up truck 15 m from the building

W $[kg]$	R $[m]$	$W^{1/3}$ $[kg^{1/3}]$
1000	15	10

$Storey$	Z $[\frac{m}{kg^{1/3}}]$	P_r $[kPa]$	P_{so} $[kPa]$	i_r $[kPa \cdot ms]$	i_s $[kPa \cdot ms]$	t_A $[ms]$	t_o $[ms]$
1	1.51	3000	600	6000	2000	8	20
2	1.60	2500	550	5000	1900	10	21
3	1.75	1900	400	4200	1700	14	20
4	1.95	1100	300	3900	1500	18	20
5	2.20	700	200	3000	1200	21	21
6	2.46	500	180	2800	1000	28	24

Table B.2: *Blast parameters for Load Scenario* $A_{W1000R15}$

Load Scenario $B_{W1000R5}$ - Pick-up truck 5 m to building

W $[kg]$	R $[m]$	$W^{1/3}$ $[kg^{1/3}]$
1000	5	10

$Storey$	Z $[\frac{m}{kg^{1/3}}]$	P_r $[kPa]$	P_{so} $[kPa]$	i_r $[kPa \cdot ms]$	i_s $[kPa \cdot ms]$	t_A $[ms]$	t_o $[ms]$
1	0.54	40000	4500	20000	1700	2	3.2
2	0.74	18000	2400	14000	2000	3	7
3	1.03	8000	1500	9000	2100	4.5	18
4	1.34	4000	750	6500	2000	8	21
5	1.68	1700	420	4000	1600	14	20
6	2.01	1000	300	3800	1500	18	20

Table B.3: *Blast parameters for Load Scenario* $B_{W1000R5}$

Load Scenario $C_{W300R15}$ - Large car 15 m from the building

W $[kg]$	R $[m]$	$W^{1/3}$ $[kg^{1/3}]$
300	15	6.7

Storey	Z $[\frac{m}{kg^{1/3}}]$	P_r $[kPa]$	P_{so} $[kPa]$	i_r $[kPa \cdot ms]$	i_s $[kPa \cdot ms]$	t_A $[ms]$	t_o $[ms]$
1	2.26	700	200	2077	804	14	14
2	2.40	550	180	1876	737	18	15
3	2.60	400	150	1809	670	30	17
4	2.90	350	120	1541	603	23	19
5	3.30	250	90	1340	570	30	21
6	3.70	190	75	1206	536	37	23

Table B.4: *Blast parameters for Load Scenario* $C_{W300R15}$

Load Scenario $D_{W5000R5}$ - Truck bomb 5 m to building

W $[kg]$	R $[m]$	$W^{1/3}$ $[kg^{1/3}]$
5000	5	17.1

Storey	Z $[\frac{m}{kg^{1/3}}]$	P_r $[kPa]$	P_{so} $[kPa]$	i_r $[kPa \cdot ms]$	i_s $[kPa \cdot ms]$	t_A $[ms]$	t_o $[ms]$
1	0.31	90000	10000	94050	3760	1.0	4
2	0.41	57000	6700	51300	3080	1.7	4
3	0.60	25000	3600	32490	3080	3.4	7
4	0.79	14000	2100	20520	3420	5.1	14
5	0.98	8000	1500	14535	4100	7.7	33
6	1.18	5000	1000	11970	3420	10.3	34

Table B.5: *Blast parameters for Load Scenario* $D_{W5000R5}$

Appendix C

Calculated Results and Figures

The results from the mentioned load scenarios A - D for Case 1_{Force} and $2_{Velocity}$ are presented in this section. The time elapsed, the difference between the displacements for the models are the focus of these results.

To answer if the dynamic models have sufficient accuracy and computational efficiency are the computing time recorded, the difference in time dependent displacements and eigenfrequencies compared for every model. The time elapsed is recorded with MATLAB commando *tic* and the analyzed displacements which are compared are the nodes in the left side of every storey in the 2D model, see Figure C.1. The cases are first analyzed with only the first Ritz vector and then with first and second Ritz vectors combined and so on. This is done for every Load Scenario $A_{W1000R15}$-$D_{W5000R5}$ and are presented below. The eigenfrequencies for the full dynamic analysis are displayed in Table C.1 which also includes the reduced models eigenfrequencies for varying numbers of Ritz vectors used.

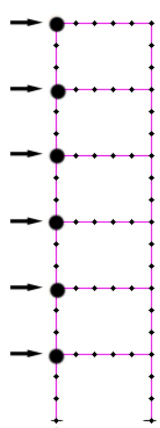


Figure C.1: *In which nodes the displacements is recorded.*

Nr of Ritz vectors	Freq 1	Freq 2	Freq 3	Freq 4
1	1.03	-	-	-
2	1.03	3.24	-	-
3	1.02	3.24	6.16	-
4	1.02	3.22	5.93	9.92
Full model	1.02	3.21	5.79	8.7

Table C.1: *Eigenfrequency for the full model and reduced models in Hz*

C.1 Load Scenario $A_{W1000R15}$ - Pick-up truck 15 m from building

The pulses shown in Figure C.2 are the derived values from Appendix B for the Load Scenario $A_{W1000R15}$, represented and placed in the respective storey. This is done for every load scenario which are shown in Figures C.9, C.16 and C.23.

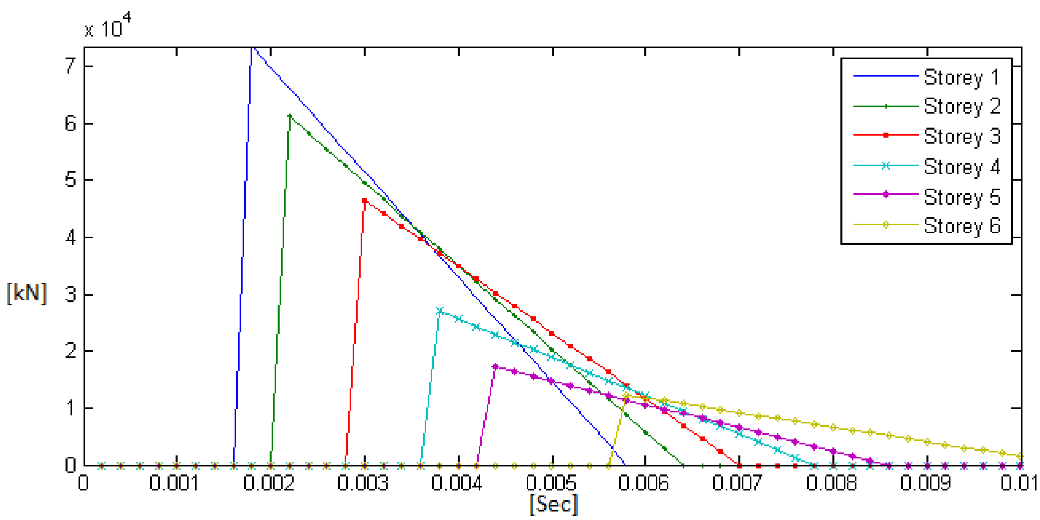


Figure C.2: *Pulses acting in resp. storey - Load Scenario $A_{W1000R15}$.*

The shown values in Table C.2 are computing time, the maximum displacement in the model and the Difference ratio between maximum displacements in the reduced- and full model in Case 1_{Force}. This is also done for Load Scenarios B-D and displayed i Table C.3, C.4 and C.5. The following Figures C.3 - C.8 shows the resulting response compared with the full model at the six chosen node points.

		Case 1		Case 2	
Nr of Ritz vectors		**Full**	**Reduced**	**Full**	**Reduced**
1	Time elapsed [sec]	1.87	0.07	0.65	0.15
	Disp. at top [mm]	1.24	0.90	1.20	0.86
	Difference[%]	-	-27.4	3.0	-30.8
2	Time elapsed [sec]	1.88	0.12	0.66	0.16
	Disp. at top [mm]	-	1.19	-	1.13
	Difference[%]	-	-3.7	-	-8.6
3	Time elapsed [sec]	1.89	0.13	0.64	0.16
	Disp. at top [mm]	-	1.22	-	1.16
	Difference[%]	-	-1.11	-	-5.88
4	Time elapsed [sec]	2.03	0.10	0.64	0.16
	Disp. at top [mm]	-	1.20	-	1.151
	Difference[%]	-	-2.6	-	-6.93

Table C.2: *Results with differing amount of Ritz vector - Load Scenario* $A_{W1000R15}$

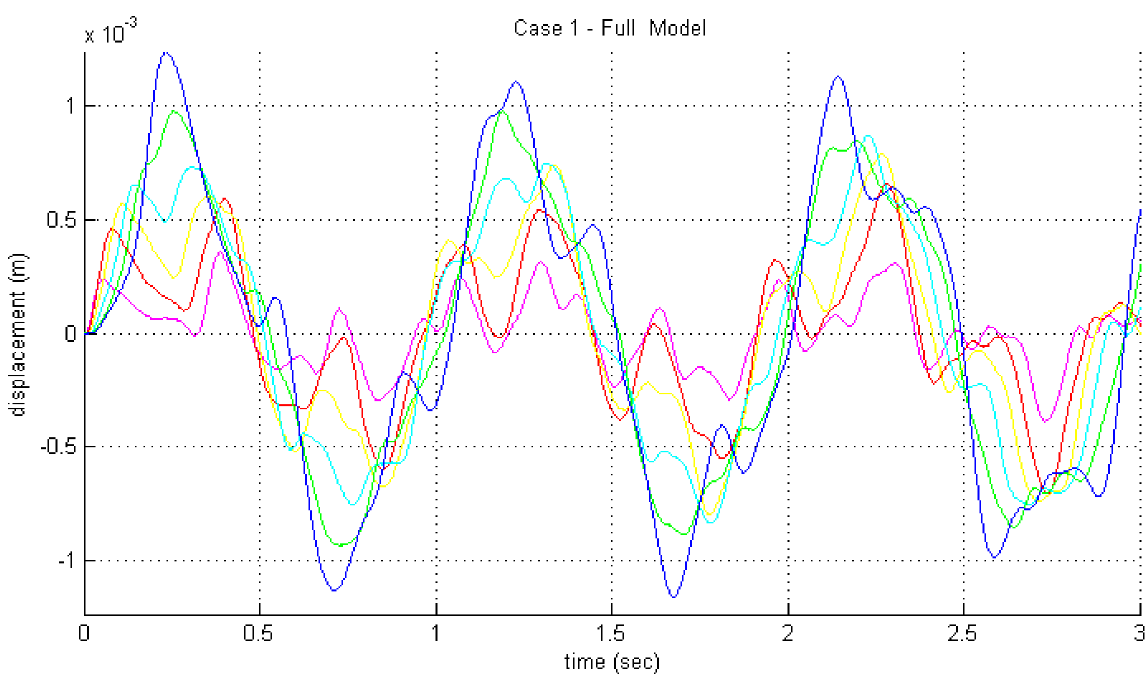


Figure C.3: *Displacements for Load Scenario* $A_{W1000R15}$ *- Case* 1_{Force} *Full model*

-

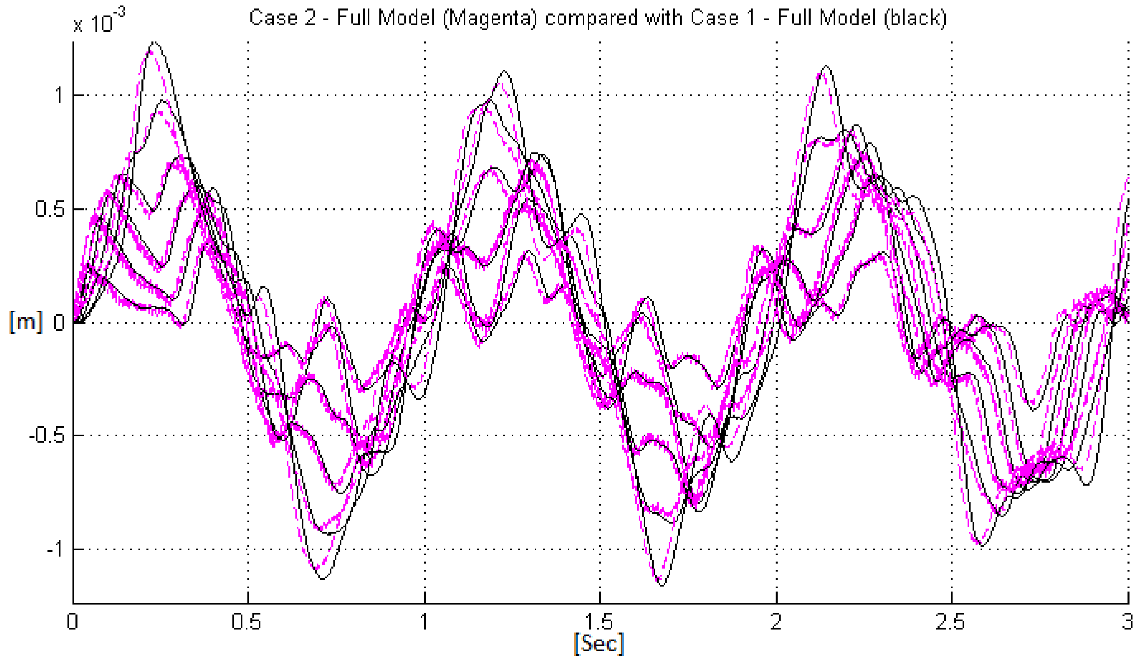
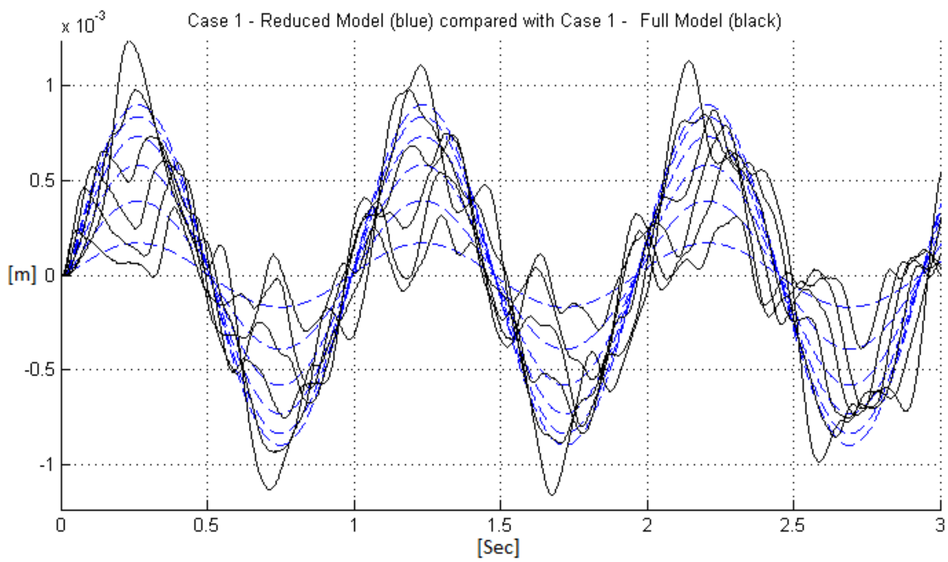
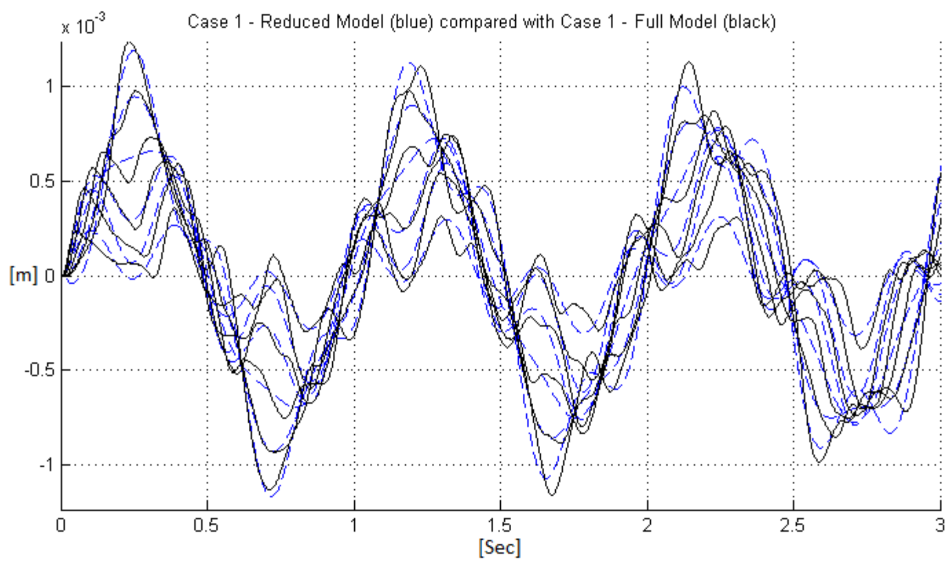


Figure C.4: *Displacements for Load Scenario* $A_{W1000R15}$ - *Case* $2_{Velocity}$ *Full model*

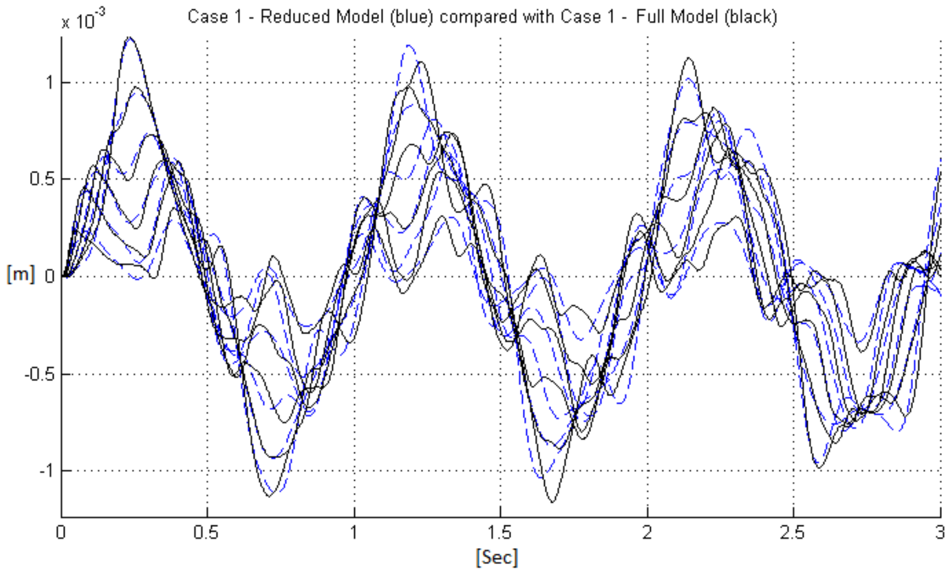


(a) *Case 1_{Force} Reduced model - one Ritz vector*

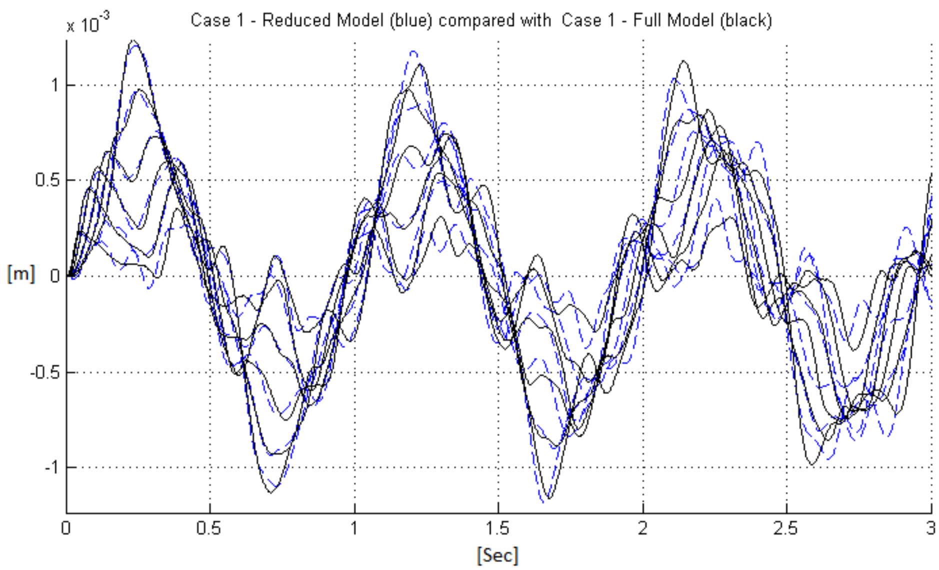


(b) *Case 1_{Force} Reduced model - two Ritz vectors*

Figure C.5: *Response for one and two Ritz vectors - Load Scenario $A_{W1000R15}$ - Case 1_{Force} Reduced model*

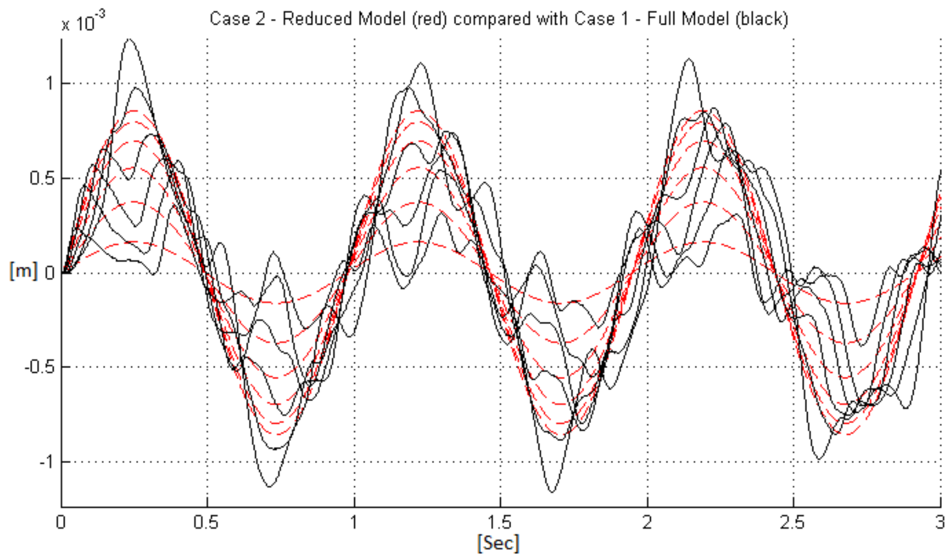


(a) *Case* 1_{Force} *Reduced model - three Ritz vectors*

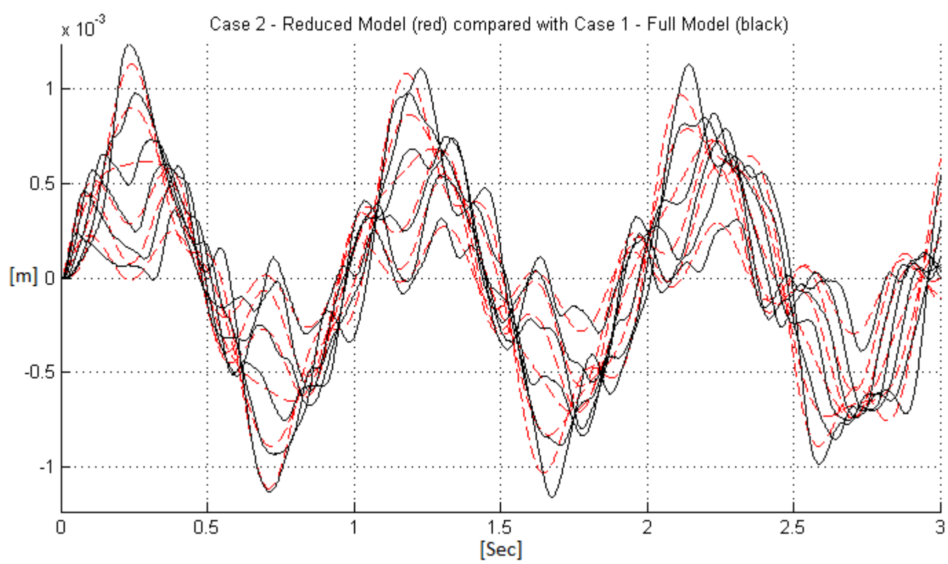


(b) *Case* 1_{Force} *Reduced model - four Ritz vectors*

Figure C.6: *Response for three & four Ritz vectors - Load Scenario* $A_{W1000R15}$ *- Case* 1_{Force} *Reduced model*

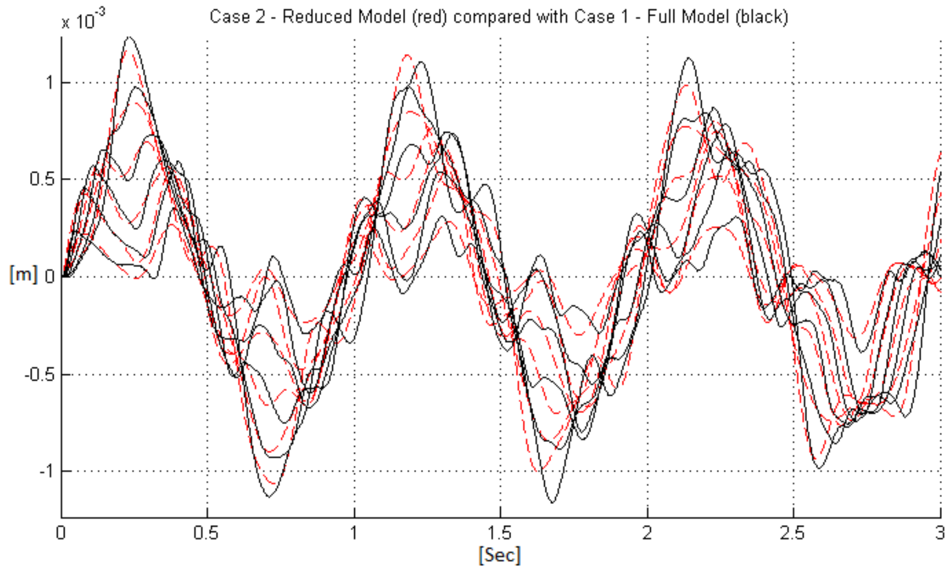


(a) *Case* $2_{Velocity}$ *Reduced model - one Ritz vector*

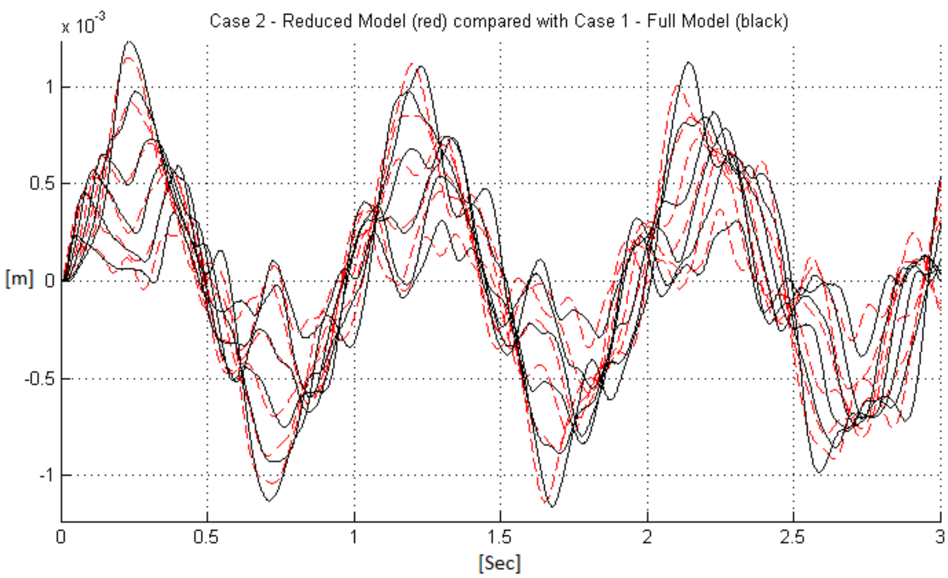


(b) *Case* $2_{Velocity}$ *Reduced model - two Ritz vectors*

Figure C.7: *Response for one and two Ritz vectors - Load Scenario* $A_{W1000R15}$ *- Case* $2_{Velocity}$ *Reduced model*



(a) *Case* $2_{Velocity}$ *Reduced model - three Ritz vectors*



(b) *Case* $2_{Velocity}$ *Reduced model - four Ritz vectors*

Figure C.8: *Response for three and four Ritz vectors - Load Scenario* $A_{W1000R15}$ *- Case* $2_{Velocity}$ *Reduced model*

C.2 Load Scenario $\mathbf{B_{W1000R5}}$ - Pick-up truck 5 m from the building

The following Figures C.10 - C.15 shows resulting response, the pulses are shown in Figure C.9 and the computing time, max displacement and Difference ratio is shown in Table C.3.

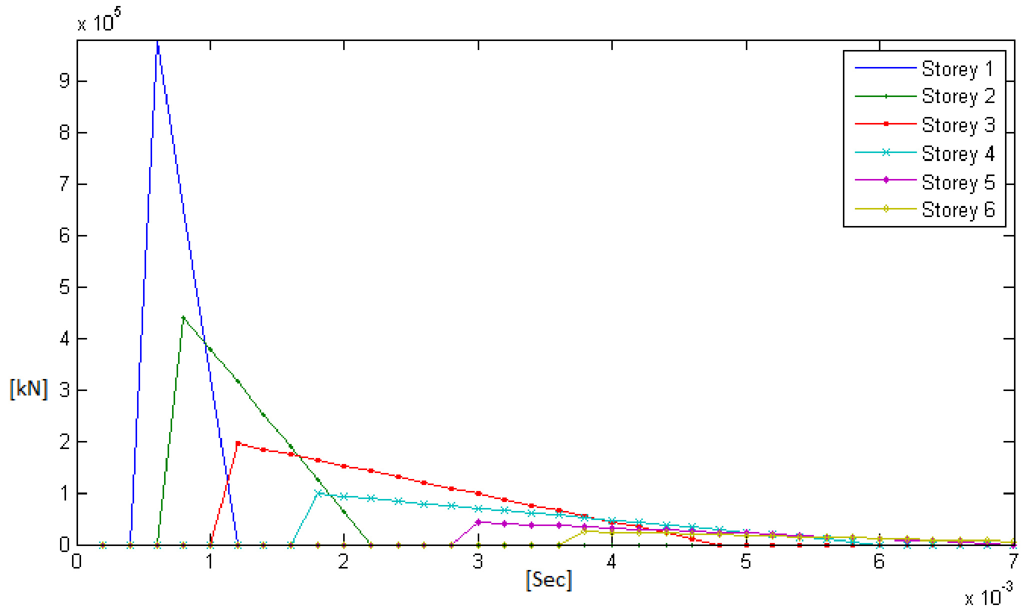


Figure C.9: *Pulses acting in resp. storey - Load Scenario $B_{W1000R5}$.*

		Case 1		Case 2	
Nr of Ritz vectors		**Full**	**Reduced**	**Full**	**Reduced**
1	Time elapsed [sec]	1.85	0.07	0.68	0.13
	Disp. at top [mm]	3.58	2.66	3.28	2.43
	Disp. Difference[%]	-	-25.9	8.5	-32.2
2	Time elapsed [sec]	1.92	0.12	0.64	0.16
	Disp. at top [mm]	-	3.57	-	3.22
	Disp. Difference[%]	-	-0.318	-	-10.2
3	Time elapsed [sec]	1.98	0.14	0.65	0.17
	Disp. at top [mm]	-	3.578	-	3.245
	Disp. Difference[%]	-	-0.15	-	-9.44
4	Time elapsed [sec]	1.93	0.11	0.65	0.16
	Disp. at top [mm]	-	3.56	-	3.24
	Disp. Difference[%]	-	-0.64	-	-9.66

Table C.3: *Values to compare with differing amount of Ritz vector - Load Scenario $B_{W1000R5}$*

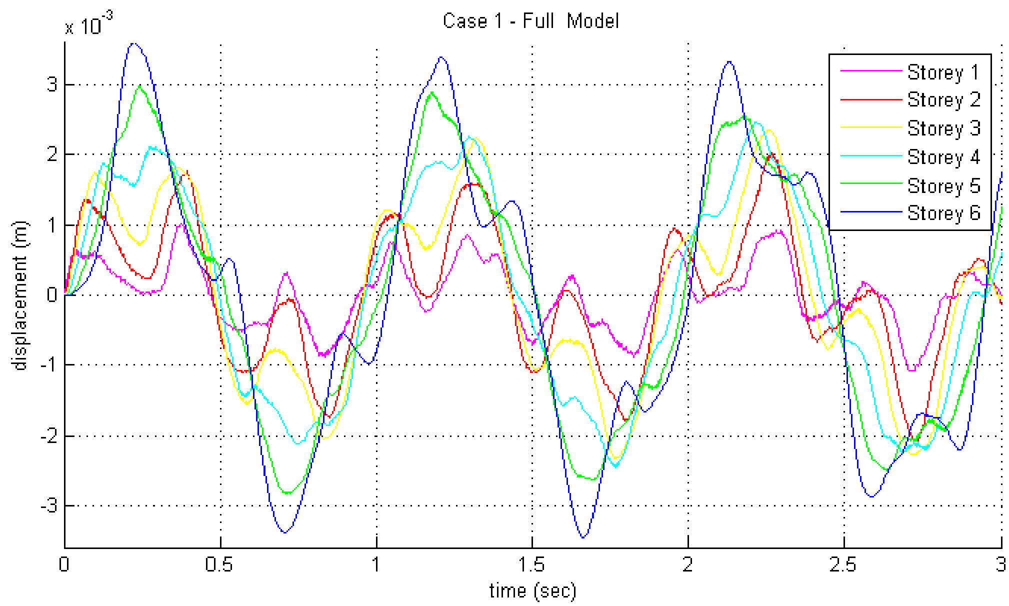


Figure C.10: *Displacements for Load Scenario* $B_{W1000R5}$ - *Case* 1_{Force} *Full model*

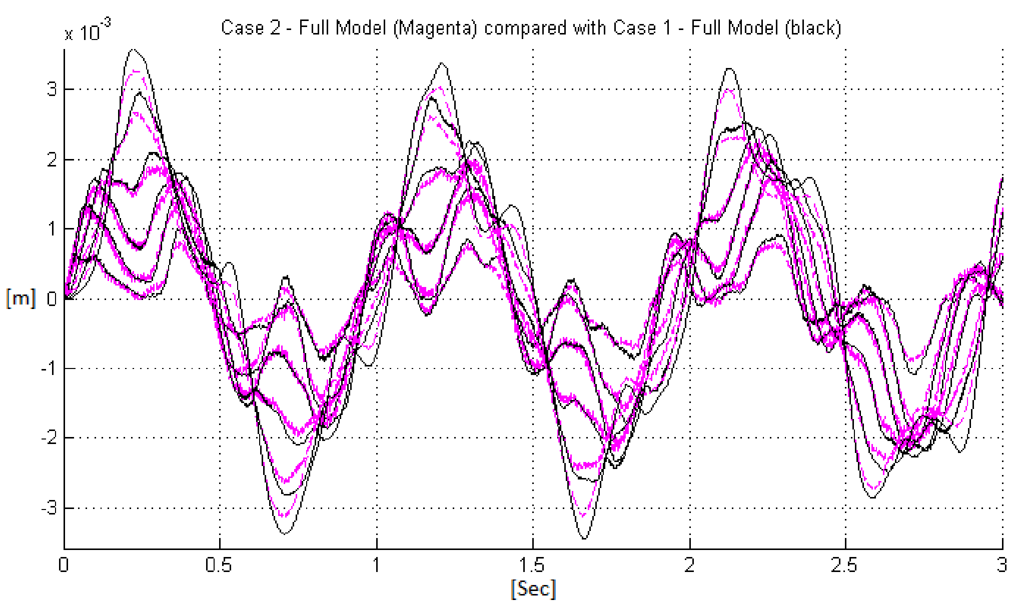
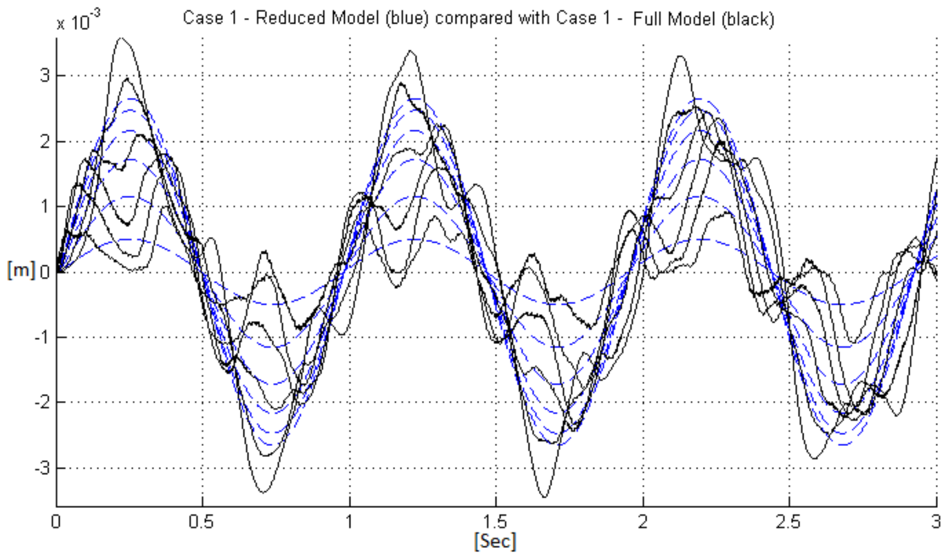
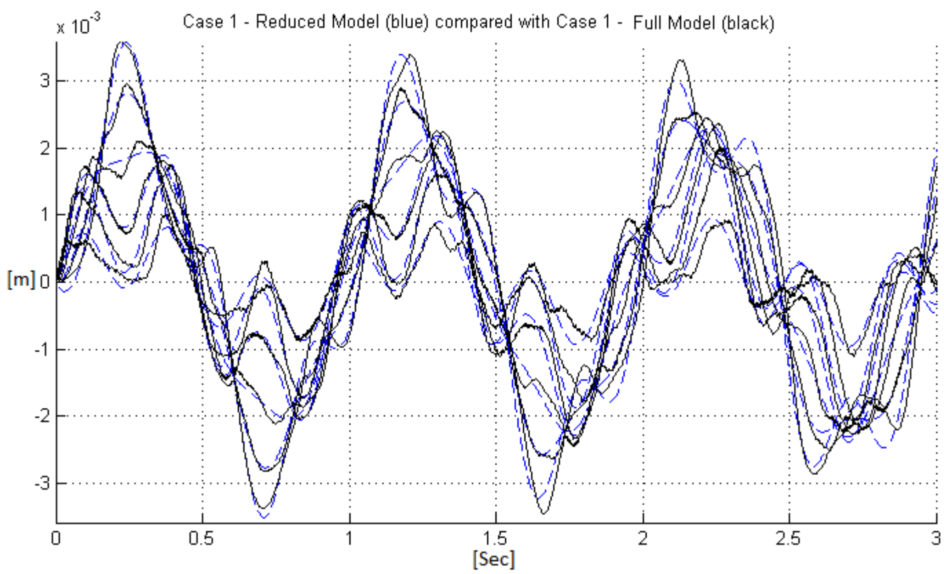


Figure C.11: *Displacements for Load Scenario* $B_{W1000R5}$ - *Case* $2_{Velocity}$ *Full model*

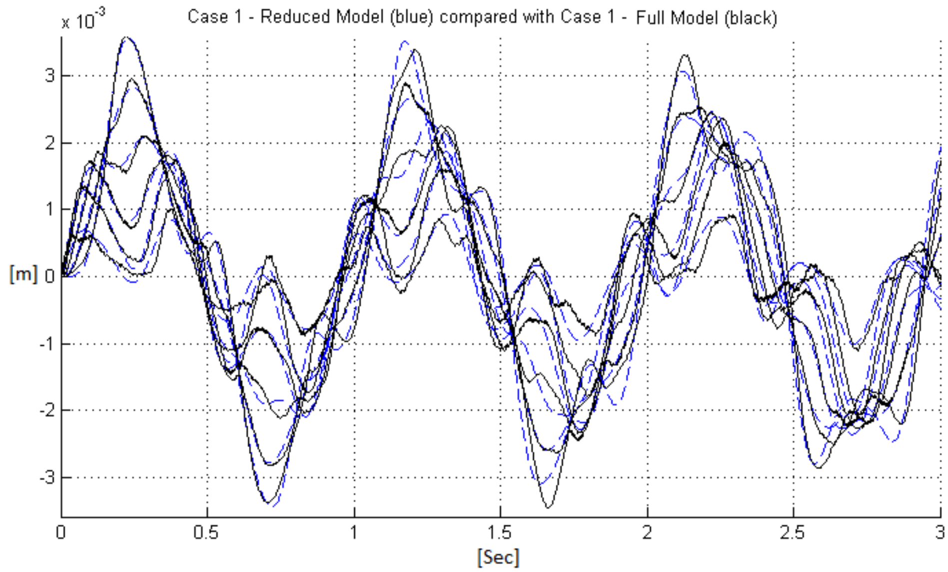


(a) *Case* 1_{Force} *Reduced model - one Ritz vector*

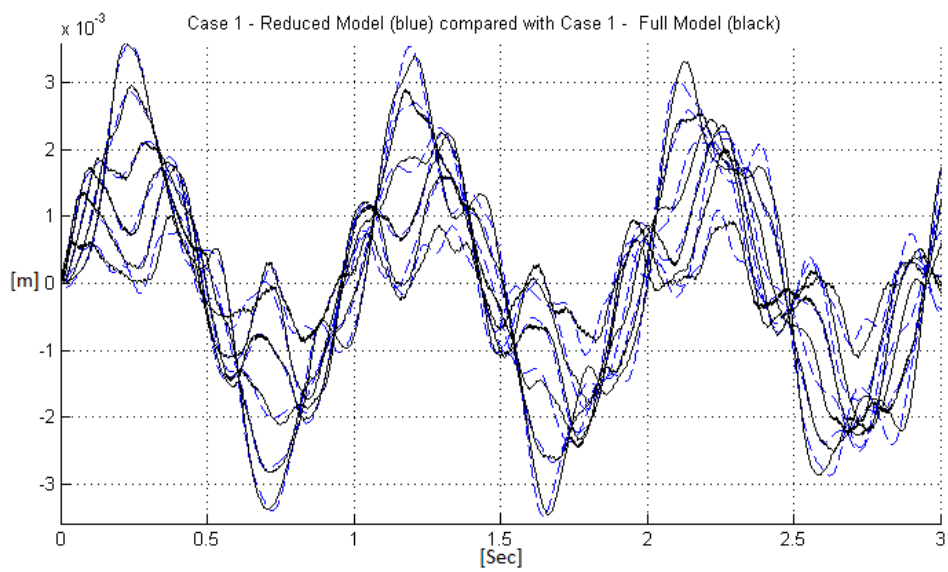


(b) *Case* 1_{Force} *Reduced model - two Ritz vectors*

Figure C.12: *Response for one and two Ritz vectors - Load Scenario* $B_{W1000R5}$ *- Case* 1_{Force} *Reduced model*

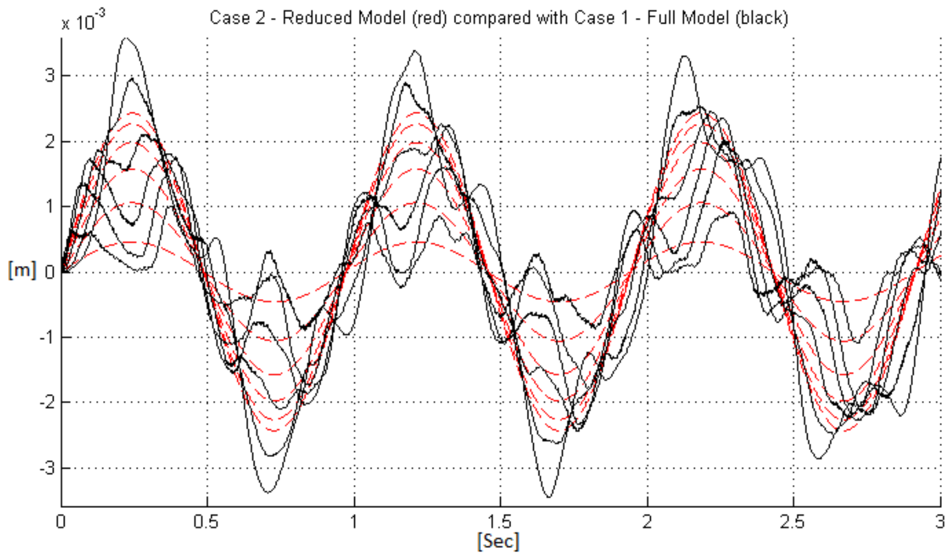


(a) *Case* 1_{Force} *Reduced model - three Ritz vectors*

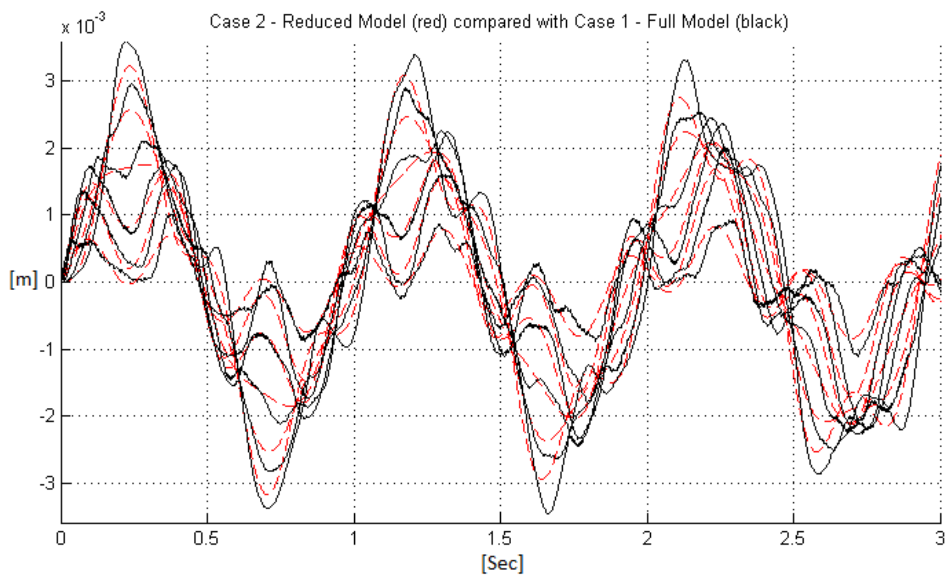


(b) *Case* 1_{Force} *Reduced model - four Ritz vectors*

Figure C.13: *Response for three and four Ritz vectors - Load Scenario* $B_{W1000R5}$ *- Case* 1_{Force} *Reduced model*

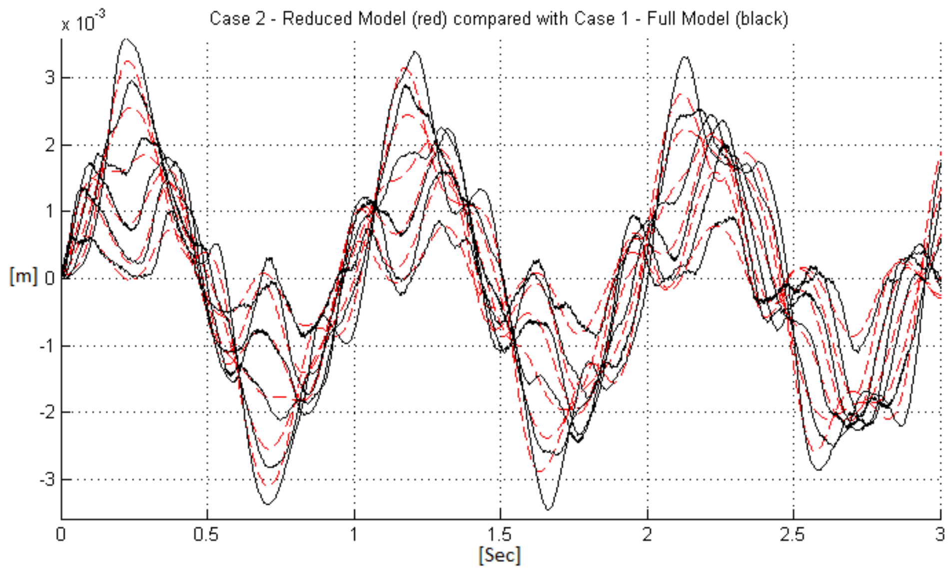


(a) *Case* $2_{Velocity}$ *Reduced model - one Ritz vector*

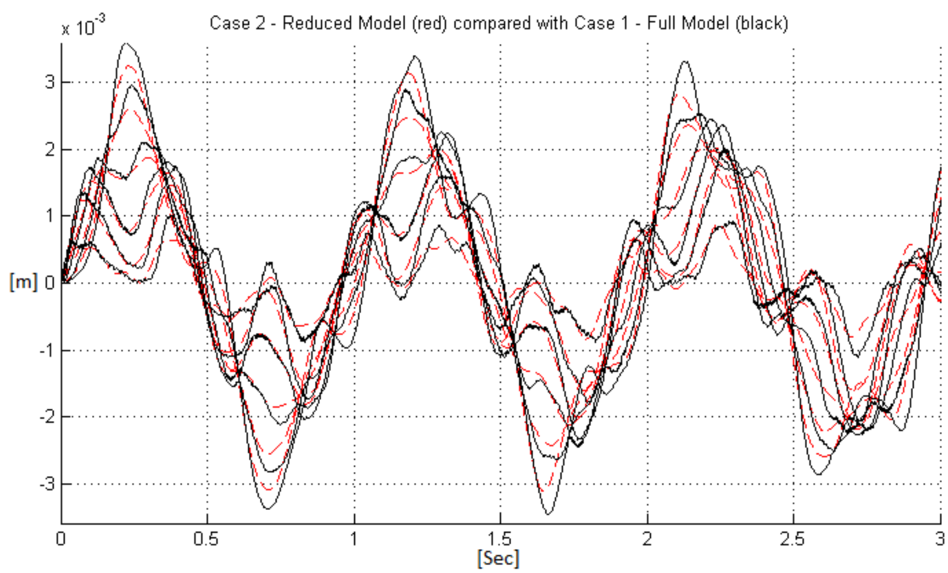


(b) *Case* $2_{Velocity}$ *Reduced model - two Ritz vectors*

Figure C.14: *Response for one and two Ritz vectors - Load Scenario* $B_{W1000R5}$ *- Case* $2_{Velocity}$ *Reduced model*



(a) *Case $2_{Velocity}$ Reduced model - three Ritz vectors*



(b) *Case $2_{Velocity}$ Reduced model - four Ritz vectors*

Figure C.15: *Response for three and four Ritz vectors - Load Scenario $B_{W1000R5}$ - Case $2_{Velocity}$ Reduced model*

C.3 Load Scenario $C_{W300R15}$ - Large Car 15 m from the building

The following Figures C.17 - C.22 shows resulting response, the pulses are shown in Figure C.16 and the computing time, max displacement and Difference ratio is shown in Table C.4.

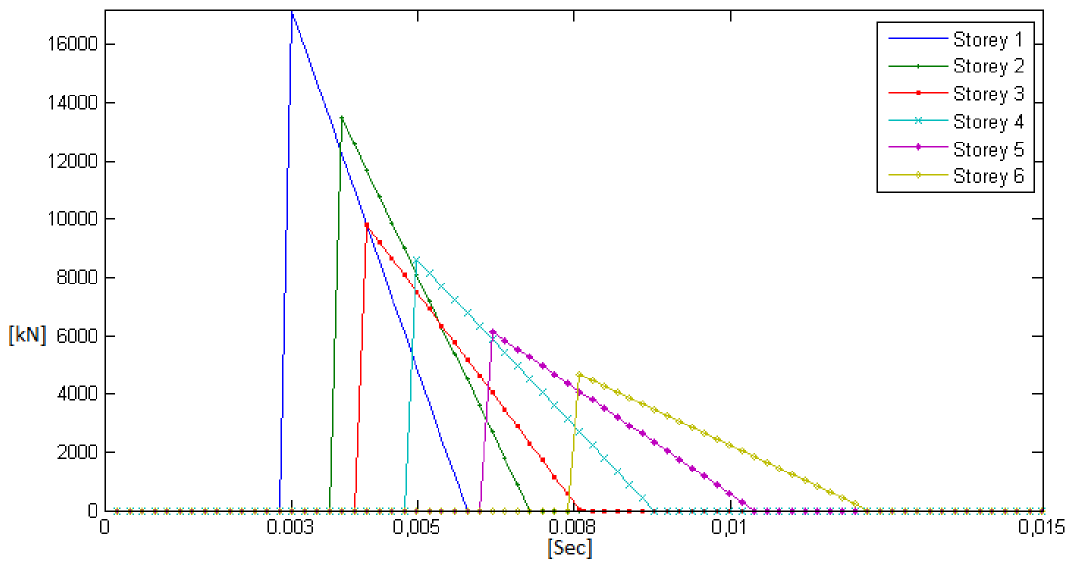


Figure C.16: *Pulses acting in resp. storey - Load Scenario $C_{W300R15}$.*

		Case 1		**Case $2_{Velocity}$**	
Nr of Ritz vectors		**Full**	**Reduced**	**Full**	**Reduced**
1	Time elapsed [sec]	2.03	0.08	0.65	0.14
	Disp. at top [mm]	0.26	0.22	0.25	0.20
	Disp. Difference [%]	-	-18.2	-3.7	-22.6
2	Time elapsed [sec]	1.99	0.11	0.66	0.16
	Disp. at top [mm]	-	0.26	-	0.25
	Disp. Difference [%]	-	-1.5	-	-7.15
3	Time elapsed [sec]	2.06	0.23	0.63	0.17
	Disp. at top [mm]	-	0.26	-	0.25
	Disp. Difference [%]	-	-0.4	-	-6.0
4	Time elapsed [sec]	1.97	0.17	0.67	0.17
	Disp. at top [mm]	-	0.26	-	0.25
	Disp. Difference [%]	-	-1.7	-	-6.0

Table C.4: *Values to compare with differing amount of Ritz vector - Load Scenario $C_{W300R15}$*

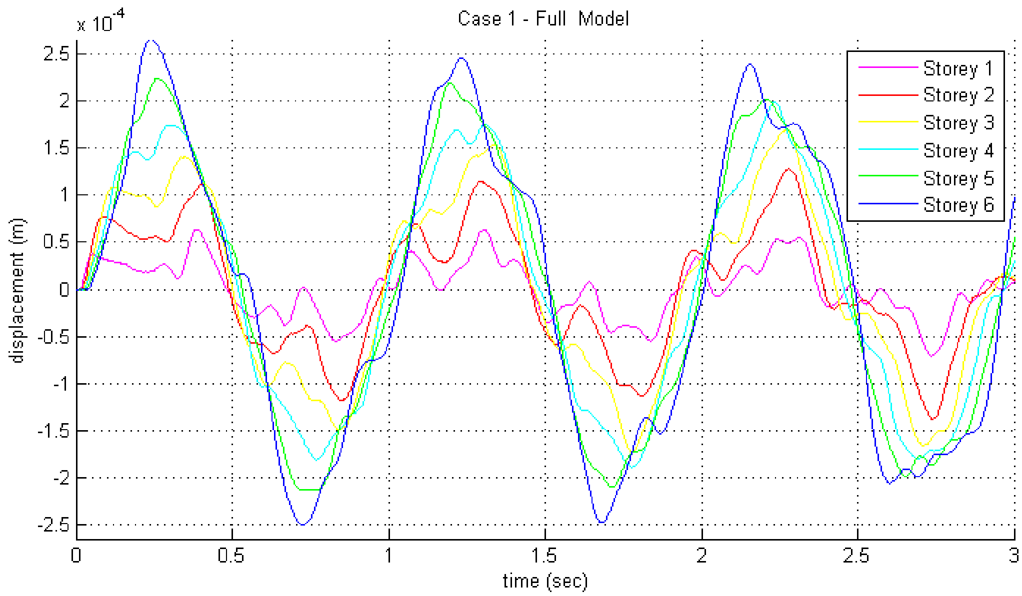


Figure C.17: *Displacements for Load Scenario* $C_{W300R15}$ - *Case* 1_{Force} *Full model*

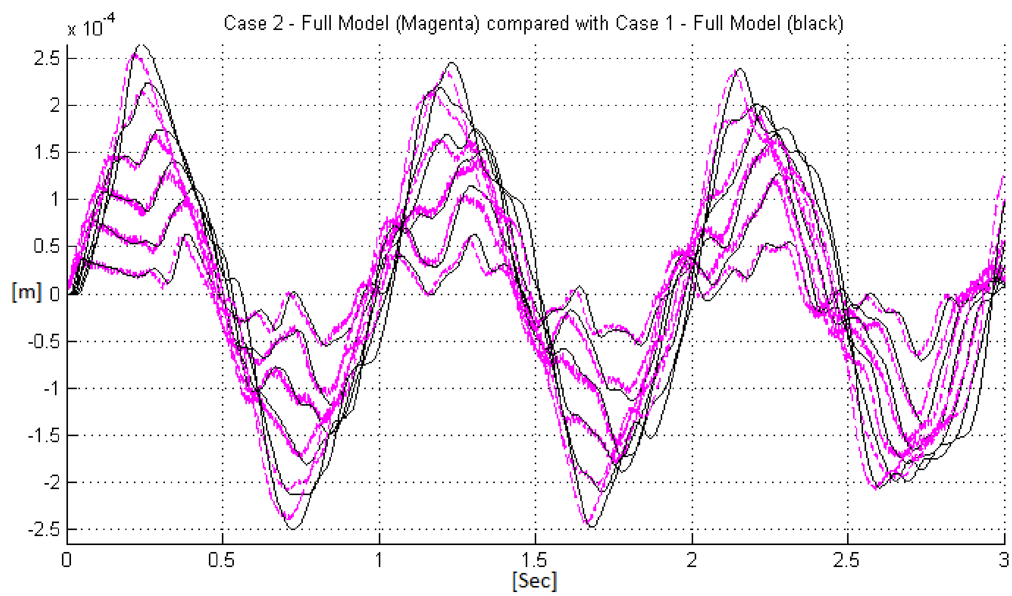
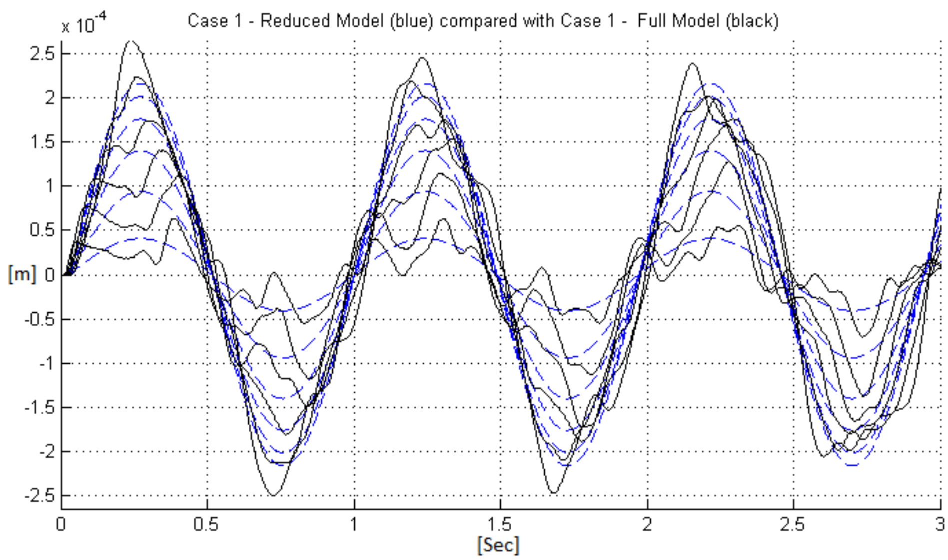
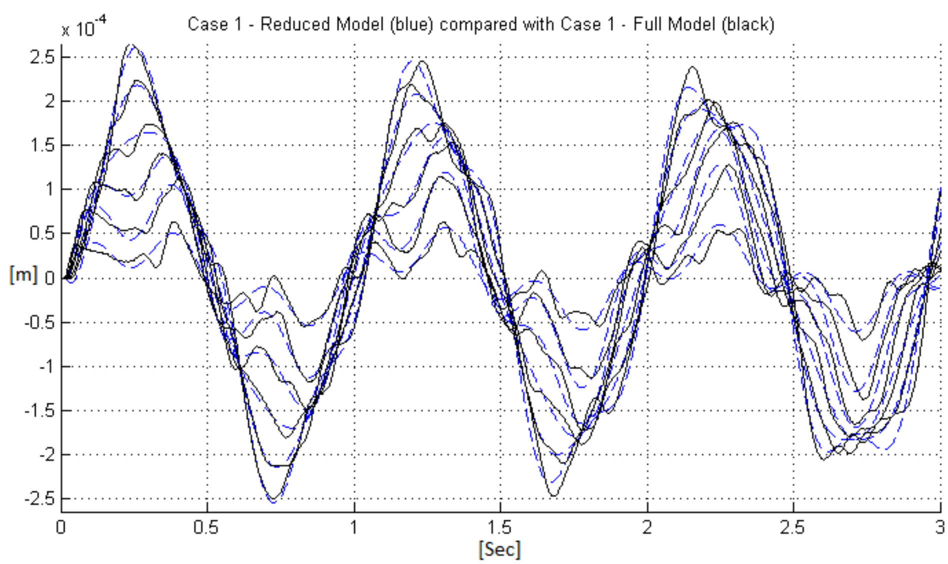


Figure C.18: *Displacements for Load Scenario* $C_{W300R15}$ - *Case* $2_{Velocity}$ *Full model*

-

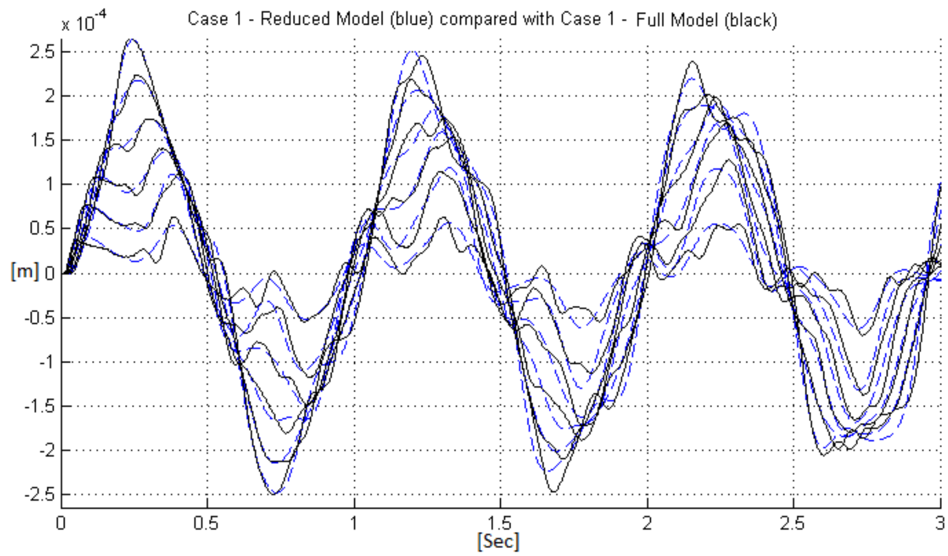


(a) *Case* 1_{Force} *Reduced model - one Ritz vector*

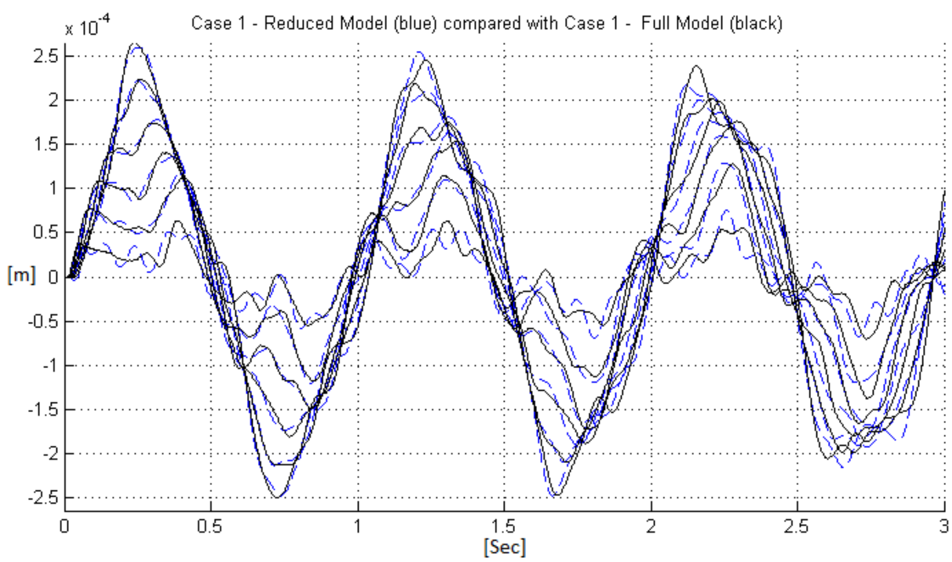


(b) *Case* 1_{Force} *Reduced model - two Ritz vectors*

Figure C.19: *Response for one and two Ritz vectors - Load Scenario* $C_{W300R15}$ *- Case* 1_{Force} *Reduced model*

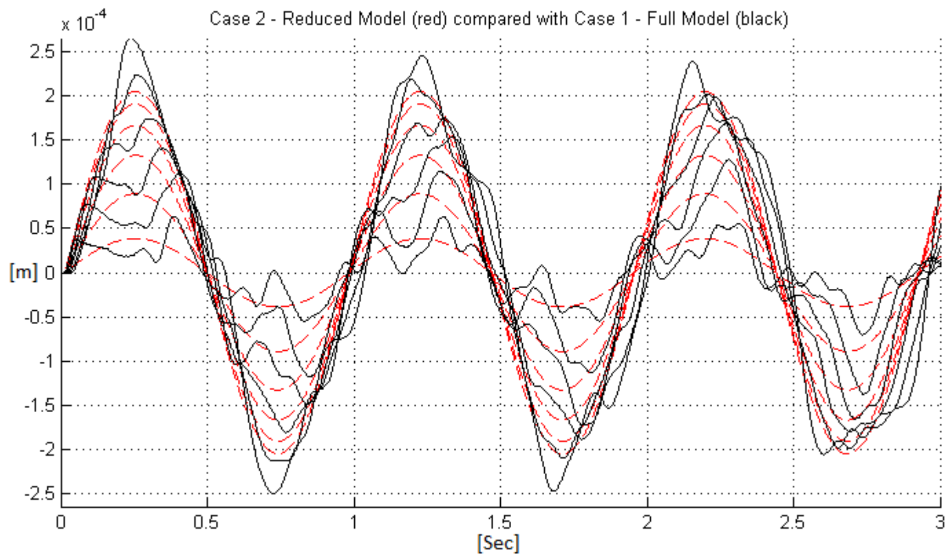


(a) *Case* 1_{Force} *Reduced model - three Ritz vectors*

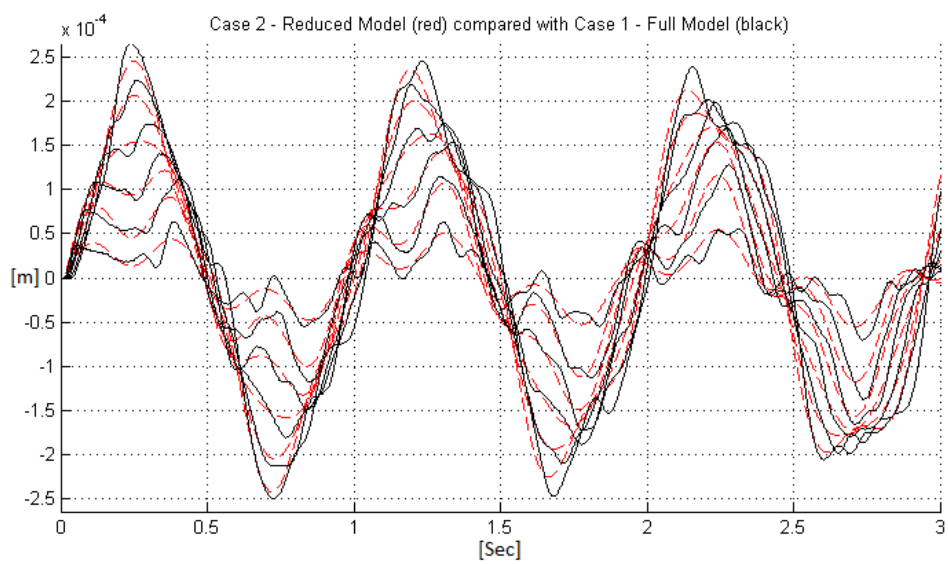


(b) *Case* 1_{Force} *Reduced model - four Ritz vectors*

Figure C.20: *Response for three and four Ritz vectors - Load Scenario* $C_{W300R15}$ *- Case* 1_{Force} *Reduced model*

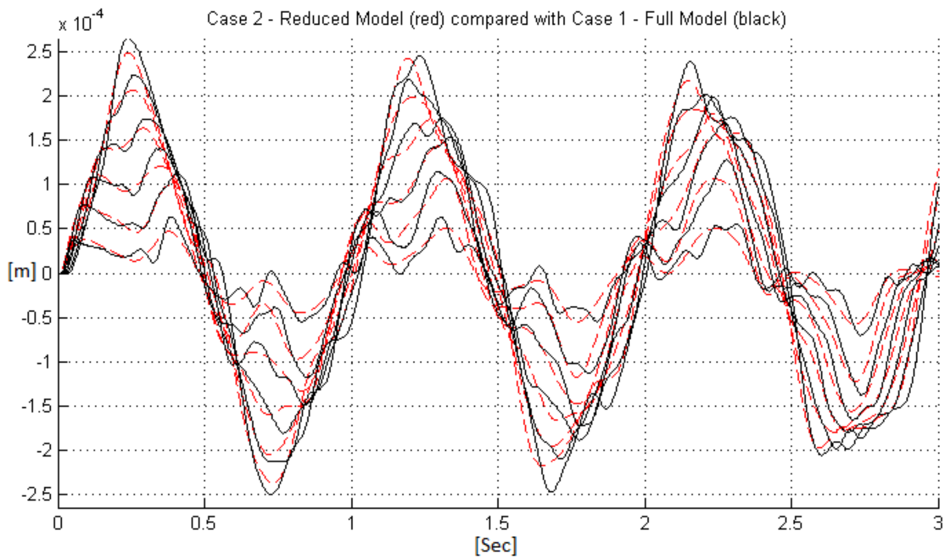


(a) *Case* $2_{Velocity}$ *Reduced model - one Ritz vector*

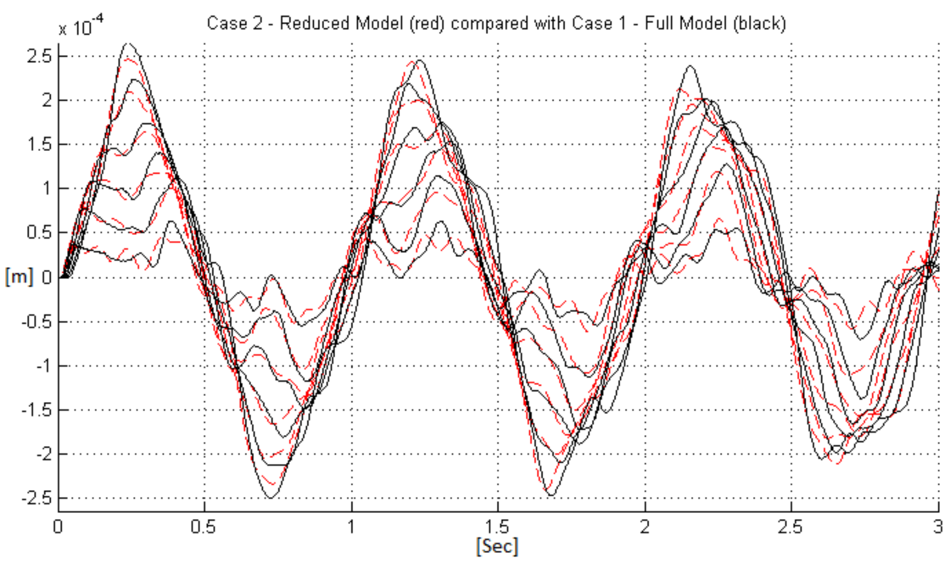


(b) *Case* $2_{Velocity}$ *Reduced model - two Ritz vectors*

Figure C.21: *Response for one and two Ritz vectors - Load Scenario* $C_{W300R15}$ *- Case* $2_{Velocity}$ *Reduced model*



(a) *Case* $2_{Velocity}$ *Reduced model - three Ritz vectors*



(b) *Case* $2_{Velocity}$ *Reduced model - four Ritz vectors*

Figure C.22: *Response for three and four Ritz vectors - Load Scenario* $C_{W300R15}$ *- Case* $2_{Velocity}$ *Reduced model*

C.4 Load Scenario $\mathbf{D_{W5000R5}}$ - Truck 5 m from the building

The following Figures C.24 - C.29 shows resulting response, the pulses are shown in Figure C.23 and the computing time, max displacement and Difference ratio is shown in Table C.5.

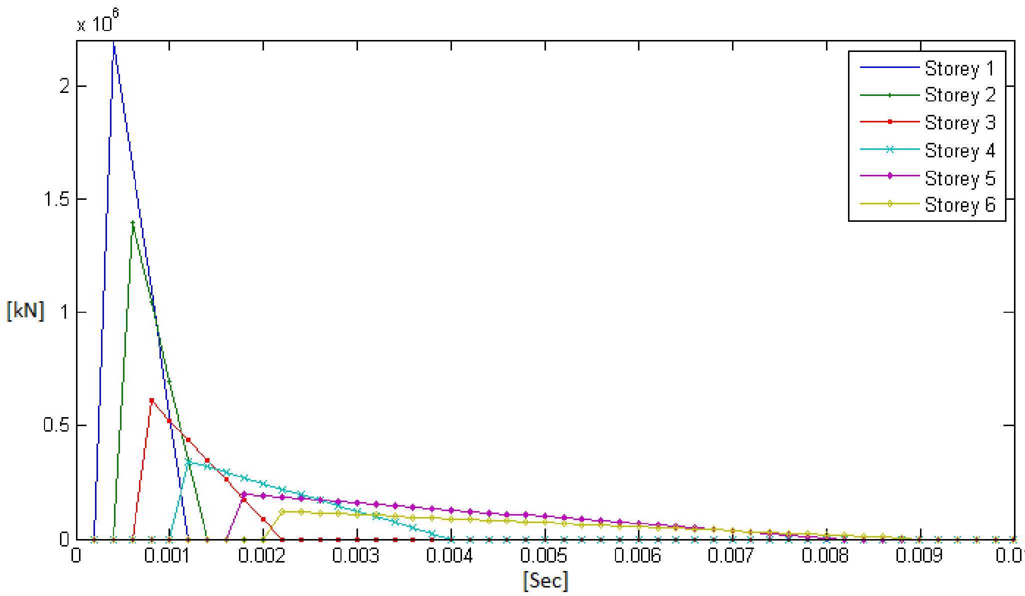


Figure C.23: *Pulses acting in resp. storey - Load Scenario $D_{W5000R5}$.*

		Case 1		Case 2	
Nr of Ritz vectors		**Full**	**Reduced**	**Full**	**Reduced**
1	Time elapsed [sec]	2.02	0.07	0.66	0.13
	Disp. at top [mm]	9.58	7.78	8.56	7.09
	Disp. Difference[%]	-	-18.7	-10.6	-26.0
2	Time elapsed [sec]	1.86	0.10	0.66	0.17
	Disp. at top [mm]	-	9.17	-	8.14
	Disp. Difference[%]	-	-4.3	-	-15
3	Time elapsed [sec]	1.87	0.14	0.66	0.17
	Disp. at top [mm]	-	9.35	-	8.24
	Disp. Difference[%]	-	-2.36	-	-13.9
4	Time elapsed [sec]	1.84	0.11	0.65	0.16
	Disp. at top [mm]	-	9.15	-	8.19
	Disp. Difference[%]	-	-4.5	-	-14.5

Table C.5: *Values to compare with differing amount of Ritz vector - Load Scenario $D_{W5000R5}$*

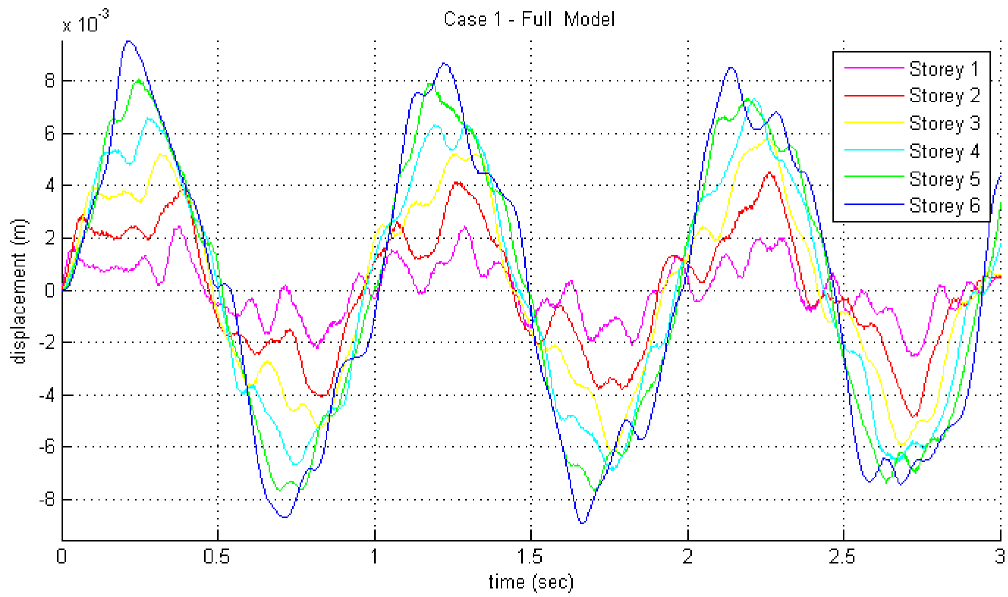


Figure C.24: *Displacements for Load Scenario* $D_{W5000R5}$ *- Case* 1_{Force} *Full model*

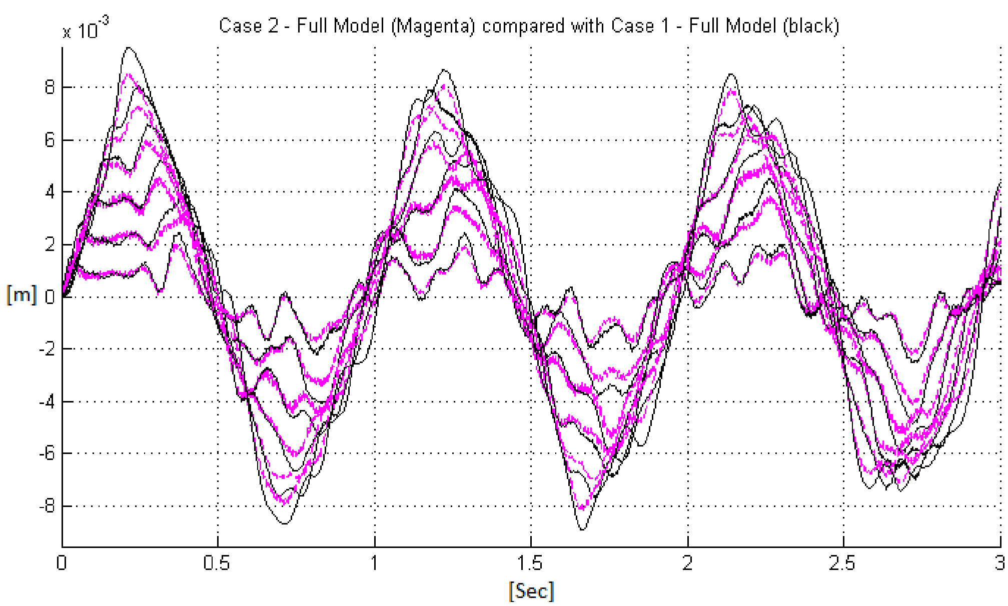
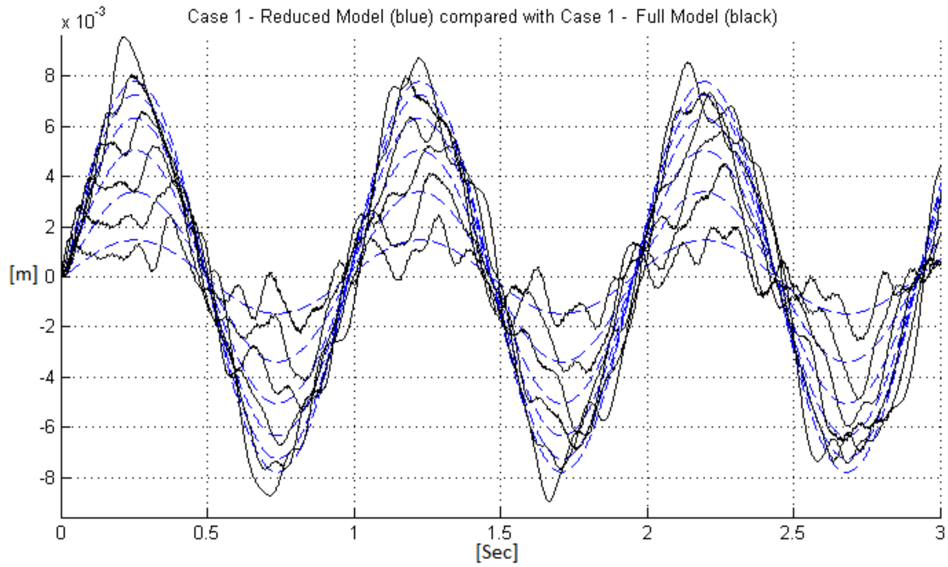
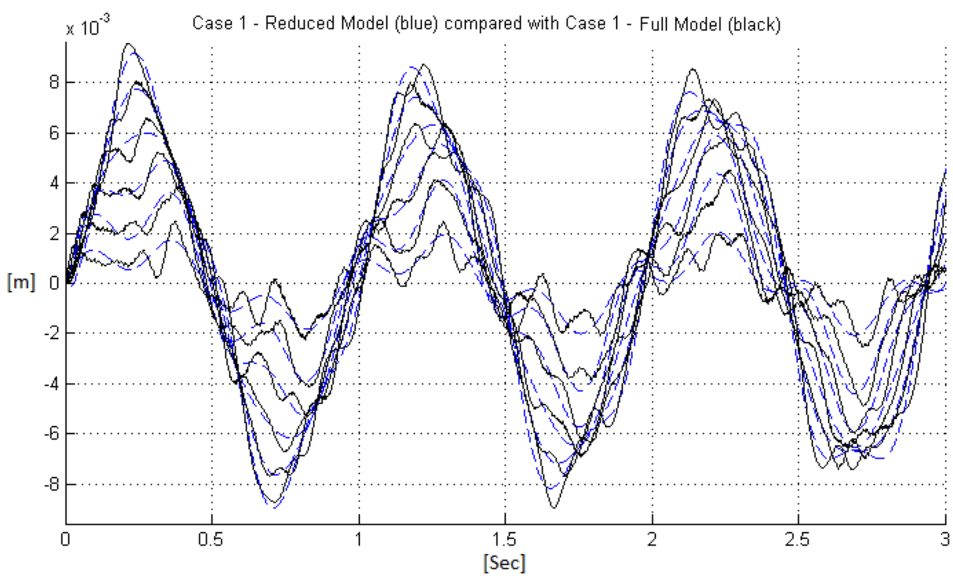


Figure C.25: *Displacements for Load Scenario* $D_{W5000R5}$ *- Case* $2_{Velocity}$ *Full model*

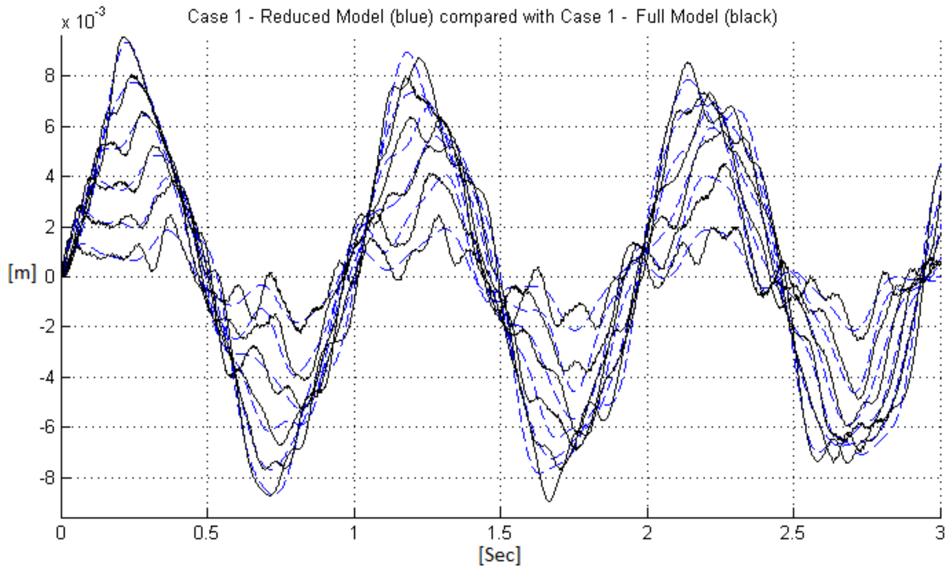


(a) *Case* 1_{Force} *Reduced model - one Ritz vector*

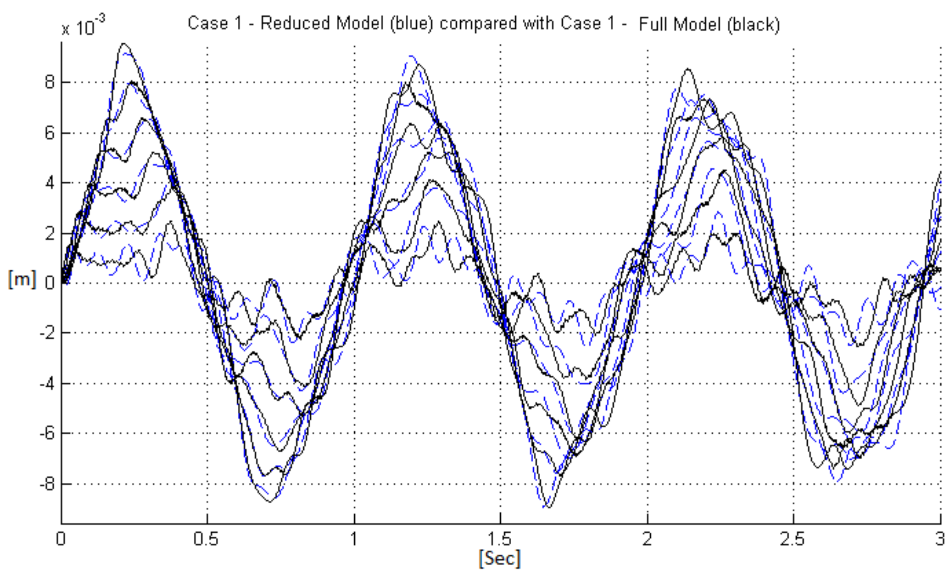


(b) *Case* 1_{Force} *Reduced model - two Ritz vectors*

Figure C.26: *Response for one and two Ritz vectors - Load Scenario* $D_{W5000R5}$ *- Case* 1_{Force} *Reduced model*

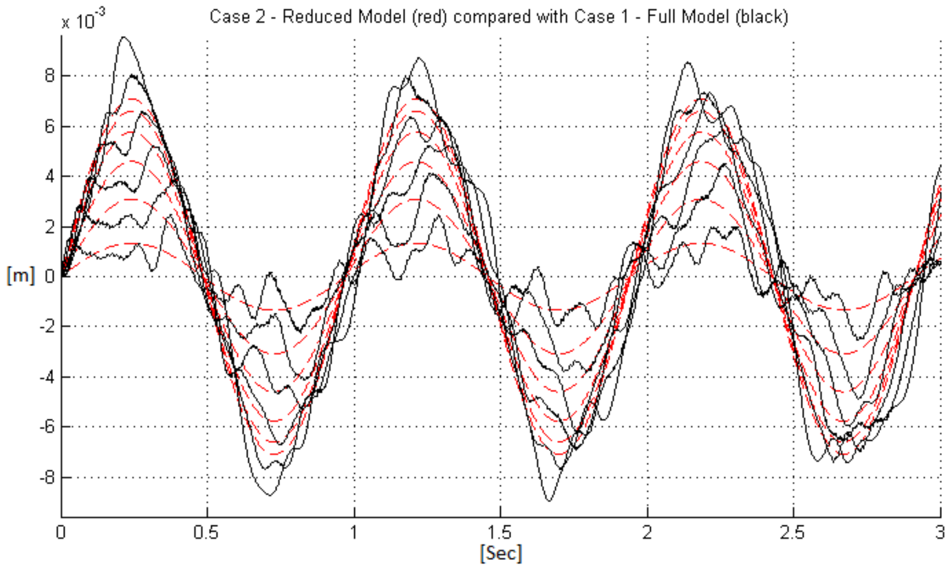


(a) *Case 1_{Force} Reduced model - three Ritz vectors*

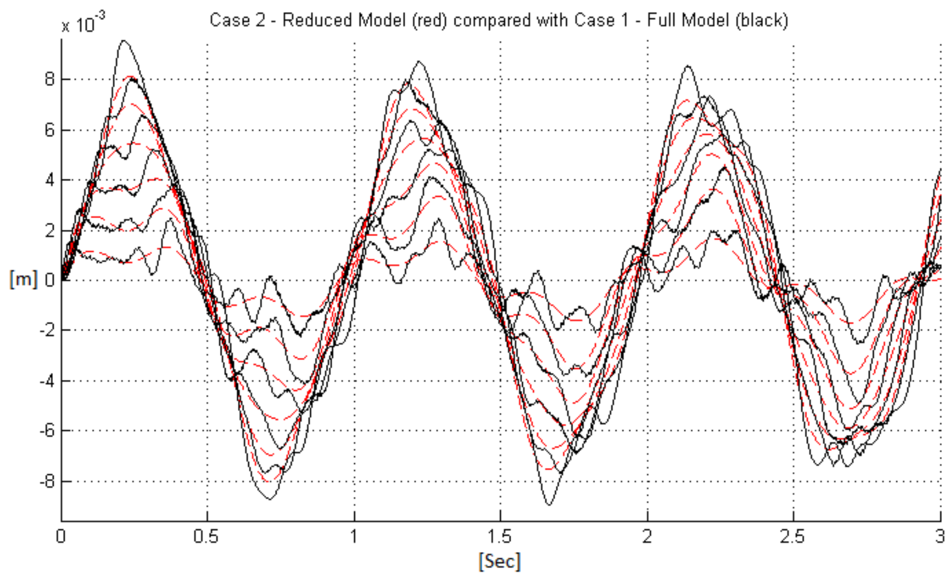


(b) *Case 1_{Force} Reduced model - four Ritz vectors*

Figure C.27: *Response for three and four Ritz vectors - Load Scenario $D_{W5000R5}$ - Case 1_{Force} Reduced model*

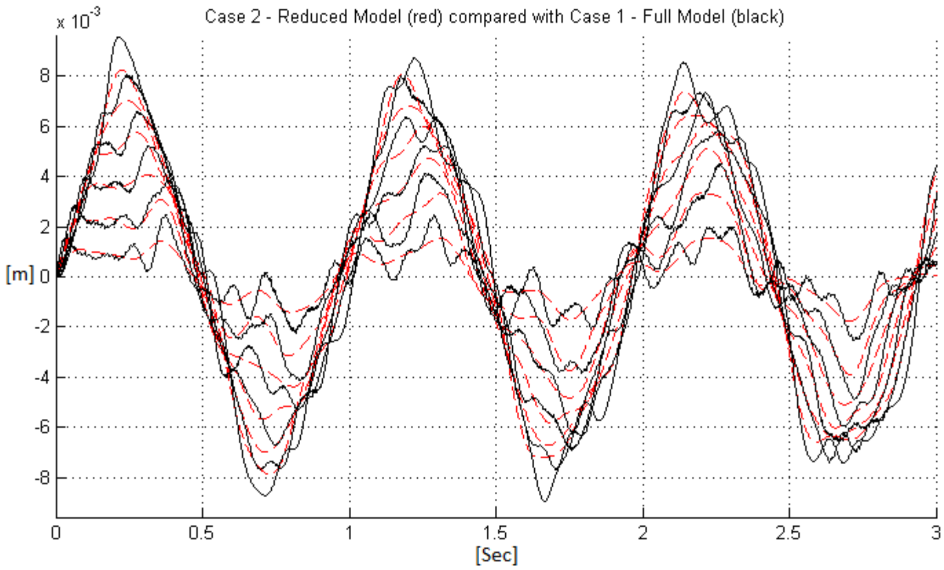


(a) *Case* $2_{Velocity}$ *Reduced model - one Ritz vector*

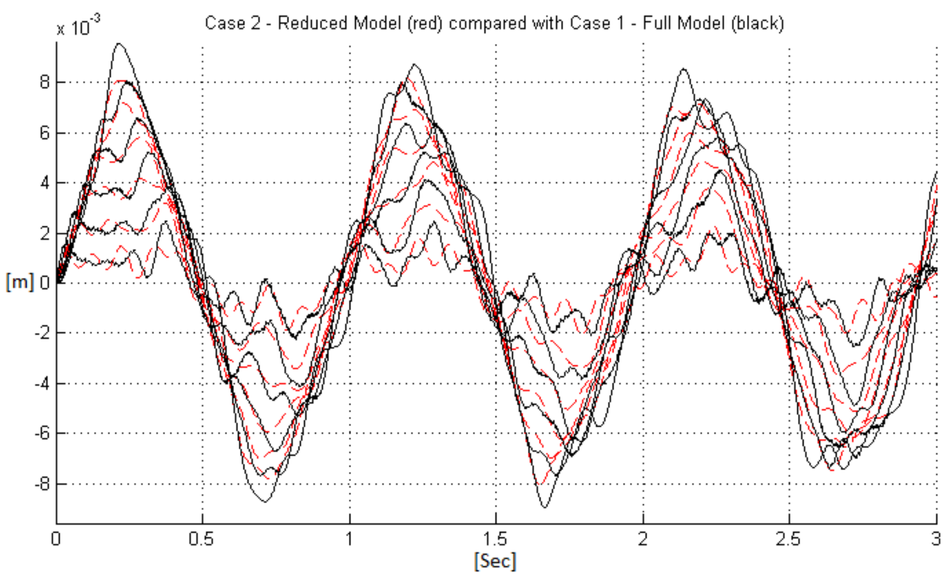


(b) *Case* $2_{Velocity}$ *Reduced model - two Ritz vectors*

Figure C.28: *Response for one and two Ritz vectors - Load Scenario* $D_{W5000R5}$ *- Case* $2_{Velocity}$ *Reduced model*



(a) *Case* $2_{Velocity}$ *Reduced model - three Ritz vectors*



(b) *Case* $2_{Velocity}$ *Reduced model - four Ritz vectors*

Figure C.29: *Response for three and four Ritz vectors - Load Scenario* $D_{W5000R5}$ *- Case* $2_{Velocity}$ *Reduced model*

Appendix D

MATLAB CODE

D.1 2D Model with Full Model for Case 1_{Force} & $2_{Velocity}$

2Dmodel script

```
% --- Ramverksmodell pÂ ett flervÂningshus ---
% --- utsatt fˆr en impulslast.
% ... Uppbyggd av stÂlpelare med betong bj‰lklag ...
clear all;close all; clc; load Blastparameters;
format compact;
% Inga Rayleigh v‰rden ao och a1 ‰r inkluderade
% vilket inneb‰r ett d‰mpat system.
% --- ... --- ... --- ... --- ... --- ... --- ... ---
% 6 VÂningar
% --- ... --- ... --- ... --- ... --- ... --- ... ---
% --- Material data -- Baserat pÂ C30 betong
% --- och HEA 340 stÂlpelare
tic
% H‰r skapas byggnadsmodellen upp
nelh=3;nelw=5;nstorey=6;
width=7;height=3.5;nodestorey=(nelh+1)*2+nelw-1;
theight=height*nstorey;wlength=width/nelw;
hlength=height/nelh;Edof=[];ex=[]';ey=[]';Edofs=[1:6];
for j=1:nstorey
x=(0:nstorey+1)*height;
for i=2:nodestorey-1
Edofs=[Edofs;Edofs(i-1,4):(Edofs(i-1,4)+5)];
exs=[zeros(1,nelh) linspace(0,(nelw-1)*wlength,nelw)
ones(1,nelh)*width;
zeros(1,nelh) linspace(wlength,(nelw)*wlength,nelw)
ones(1,nelh)*width];
eys=[linspace(x(j),x(j+1)-hlength,nelh)
ones(1,nelw)*x(j+1)
linspace(x(j),x(j+1)-hlength,nelh);
linspace(x(j)+hlength,x(j+1),nelh) ones(1,nelw)*x(j+1)
linspace(x(j)+hlength,x(j+1),nelh) ];
end
Edofs(9:nodestorey-1,:)=Edofs(fliplr(9:nodestorey-
1),[4:6 1:3]);
ex=[ex exs];
ey=[ey eys];
Edof=[Edof;Edofs];
if j>1
Edof(end-2,1:3)=Edof(8+(nelh+nelh+nelw)*(j-2),4:6);
end
Edofs=[Edof(3+(nelh+nelh+nelw)*(j-1),4:6)
max(max(Edof))+1:max(max(Edof))+3];
end
[a,b]=size(Edof);plus=[1:a]';
Edof=[plus Edof];
NDOF=max(max(Edof));[ooo,NOE]=size(ex);ex=ex';ey=ey';
% --- FEM-mesh plottas ---
clf;
eldraw2(ex,ey,[1 3 0],Edof)
grid off; title('2D Frame Structure');
% StÂlpelare: S355 ---
% ------
Es=210e9; % Pa
As=4*0.01335; % m2
Is=4*0.0002769; % m4
rhos=7850; % kg/m3
mas=rhos*As; % kg/m
eps=[Es As Is mas];
% Betongbj‰lklag: C30
```

```
% ------
Eb=33e9; % Pa
b=width; h=3.5;d=0.3; % m
Ab=b*d;% m2; tv%rsnittsarea
Ib=(b*d^3)/12; % m4
rhob=2500; % kg/m3
mab=rhob*Ab; % kg/m
epb=[Eb Ab Ib mab];
% -- Generar element matriser och assemblerar
% -- i globala matriser
K=zeros(NDOF);f=zeros(NDOF,1); M=zeros(NDOF);
%----
pillars=[];floors=[];
for i=[1:nelh nelh+nelw+1:nelh+nelw+nelh]
pillars=sort([pillars i:(nelh+nelh+nelw):NOE]);
end
for i=[nelh+1:nelh+nelw]
floors=sort([floors i:(nelh+nelh+nelw):NOE]);
end
% --- Genererar stÂl pelare
for i=pillars
[Ke,Me]=beam2d(ex(i,:),ey(i,:),eps);
K=assem(Edof(i,:),K,Ke); Mk=assem(Edof(i,:),M,Me);
end
% --- Genererar betong bj%lklag
for i=floors
[Ke,Me]=beam2d(ex(i,:),ey(i,:),epb);
K=assem(Edof(i,:),K,Ke); Mk=assem(Edof(i,:),M,Me);
end
% --- Massmatris
M=zeros(NDOF,1);
for k=1:3:NDOF
M(k:(k+1))=1;
end
for k=3:3:NDOF
M(k:(k))=0.0001;
end
vikt=1800*21*5*5/(2/3*NDOF);
% 1800 %r teoretisk densitet för byggnaden
M=vikt*diag(M);
Ml=M; % --- Ml Lumpad massmatris
%---------------------
% --- Randvillkor ----
% --------------------
vupplag=Edof(1,[2 3 4 ]); hupplag=Edof(2*nelh+nelw-2,[2
3 4]);
bc=sort([vupplag hupplag]);%
%--------
[La,Egv]=eigen(K,Ml,bc);
Freq=sqrt(abs(La))/(2*pi);
% --- Visar de 8 f^rsta egenmoderna ---
figure
clf, axis('equal'), hold on, axis off
sfac=1000;
title('The first 8 eigenmodes (Hz)' )
for i=1:4;
Ext=ex+(i-1)*width*2; eldraw2(Ext,ey,[3 3 0]);
Edb=extract(Edof,Egv(:,i));
eldisp2(Ext,ey,Edb,[1 2 0],sfac);
FreqText=num2str(Freq(i),3); text(width*2*(i-1)+1,-
2,FreqText);
```

```
end;
Eyt=ey-nstorey*height-7;
for i=5:8;
Ext=ex+(i-5)*width*2; eldraw2(Ext,Eyt,[3 3 0]);
Edb=extract(Edof,Egv(:,i));
eldisp2(Ext,Eyt,Edb,[1 2 0],sfac);
FreqText=num2str(Freq(i),3);
text(width*2*(i-5)+1,-nstorey*height-10,FreqText);
end
% --- *** --- *** --- *** ---
% --- Transient last ---
% --- *** --- *** --- *** ---
% Tidssteg
% ----------------------- Case 1 - Full modell --------
------
dt=1/1000; T=5; % Tidssteg och Total tid
% --- Inh%mtning av last frÂn excel-dokument ---
load blastexcel;gi=[];tD=tDa;tA=tAa;
% <- V%lj tDa,tAa om Last Scenario A vill anv%ndas
tDb,tAb om B.osv.
for i=1:6
G=[0 0;(tA(i)-tA(i)/1000) 0;tA(i) 1; tA(i)+tD(i) 0; T
0];
[t,g]=gfunc(G,dt);
b=width;gi=[gi; g];
end
f=zeros(NDOF, length(g)); F=width*height*Pra; % kPa*m2
= kN,
leftside=[3:11:66];ju=[0];
% <- V%nstra sidans belastade horisontella DOFs
for i=[Edof(leftside, 5)'] % ---
ju=[ju+i/i];
f(i,:)=F(ju)*gi(ju,:);
end
% --------------------------
figure
title('Pulse')
plot((1:length(f))/length(f)*1,f(Edof([leftside],
5),:))
axis([0 0.01 0 max(max(F))])
legend('Storey 1','Storey 2','Storey 3','Storey
4','Storey 5',
'Storey 6')
% --- Randvillkor , Initialv%rden------------------
bc=[bc' zeros(length(bc),1)];
d0=zeros(NDOF,1); v0=zeros(NDOF,1); hist=[
Edof([leftside],5)'];
% hist= vilka DOFs i tidsserien som ska sparas
ntimes=[0.1:0.1:T]; nhist=[hist];
% --- Tidsintegrations parametrar ---------------------
------
ip=[dt T 0.25 0.5 length(ntimes) 2 ntimes nhist];
% --- Tidsintegration ---------------------------------
------
k=sparse(K); m=sparse(Ml);
[Dsnap,D,V,A]=step2(k,[],m,d0,v0,ip,f,bc); % [] %r C
matrisen
figure
hold on
for i=1:length(nhist)
plot(t,D(i,:))
```

```
end
[n,m]=size(D);
maxdisp=max(abs(D(n,:)));
maxdisp_snaptime=find(D(6,:)>(maxdisp-0.0000001));
hold off
grid, xlabel(' time (sec)'), ylabel('displacement (m)')
title('Case 1 - Full Model ')
legend('Storey 1','Storey 2','Storey 3','Storey
4','Storey 5','Storey 6')
axis([0 T -maxdisp maxdisp])
time_model_A=toc
% ----- Skapar impulsv‰rden fˆr Initalhastighets
modellen -----
imp=[zeros(NDOF,1)];
for i=[Edof(leftside,5)] %
imp(i)=(F.*(tD)/2); % Impulsen, ‰r triangul‰r och
% r‰knad som arean under grafen
end
% ---- ****** -------- ****** -------- ****** --------
******
% Case 2 - Full modell, initialv‰rden enl. uoprick=I/m
% ---- ****** -------- ****** -------- ****** --------
******
tic
uoprick=imp./diag(Ml); % Initialhastigheter frÂn
Impulslast/Massa
t=(0:(5*1000))/1000;uzt=[zeros(NDOF,length(t))];
% Tidssteg
dt=1/1000; T=5;
% --- Lasten ------------------------------------------
-------
fe=[]; % ingen last endast initialhastigheter
% --- Randvillkor , Initialv‰rden----------------------
-------
d0=zeros(NDOF,1); v0=zeros(NDOF,1);
v0=uoprick;
% --- Utparametrar ------------------------------------
--
ntimes=[0.1:0.1:T]; nhist=[hist];
% --- Tidsintegrationsparametrar ----------------------
------
ip=[dt T 0.25 0.5 length(ntimes) 2 ntimes nhist];
% --- Tidsintegration ---------------------------------
------
k=sparse(K); m=sparse(Ml);
[Dsnape,De,Ve,Ae]=step2(k,[],m,d0,v0,ip,fe,bc); % [] är
C matris
figure
hold on
% --------- Case 2 - Full Modell --------
plot(t,zeros(1,1),'k--')
for i=1:length(nhist)
plot(t,De(i,:),'m--')
plot(t,D(i,:),'k')
end
% --------- Case 1 - Full Modell -------
grid on
hold off
axis([0 T -maxdisp maxdisp])
title('Modell C (magenta) compared with Model A
(black)')
```

```
time_model_C=toc
[n,m]=size(De);
umaximp_D=max(De(n,:));
ratio_model_C_A=umaximp_D/maxdisp
% Sparar v‰rden fˆr 2D modell och laster fˆr
% att anv‰ndas i matlabscript
% reducedmodel_6van_smalsymmetrisk_fallA
save 6van_2d_smalsym_fallA
% Kˆr script som skapar Ritzvektorerna
disp('Run Ritzvektor 1')
run ritzvektor1_6van_smalsymmetrisk.m
disp('Run Ritzvektor 2')
run ritzvektor2_6van_smalsymmetrisk.m
disp('Run Ritzvektor 3')
run ritzvektor3_6van_smalsymmetrisk.m
disp('Run Ritzvektor 4')
run ritzvektor4_6van_smalsymmetrisk.m
disp('Run Reducedmodel')
% Kˆr script som reducerar den fulla Modell, fˆr bÂde
% Case 1 & 2,lˆser egenv‰rdesproblemet i reducerad
modell,
% analyserar med bÂda Casen och transformerar tillbaka
till
% full modell. H‰r visas ocksÂ plottar och j‰mfˆrda
v‰rden
% fˆr de olika modellerna i responsplot.
run reducedmodel_6van_smalsymmetrisk_fallA
```

D.2 Reduced Models

Reducedmodel script

```
% --- H‰mtar v‰rden frÂn filen "6van_2d_smalsym_fallA.m"
% --- sedan reduceras det med ritzvektorerna frÂn Ritz-filerna -
---
clear all;format long;format compact;
tic
load 6van_2d_smalsym_fallA;DD=D;tt=t;
% Importerar ritzvektorer
load ritz1_smalsym; load ritz2_smalsym;load ritz3_smalsym;load
ritz4_smalsym;
% uoprick ‰r h‰mtade hastigheter frÂn impulslasten, uoprick=I/m
psi=[ ritz1_smalsym ritz2_smalsym ritz3_smalsym
ritz4_smalsym];%ritz2_smalsym
ritz3_smalsym ritz4_smalsym% S‰tt in antal ritzvektorer
"ritz1_6van","ritz2_6van" osv...
% Antal element ;
Mlr=[]; % Lumpad Massmatris, som gˆrs till en vektor
for i=1:NDOF
Mlr=[Mlr Ml(i,i)];
end
Kr=psi'*K*psi; % Reducerad Styvhetsmatris
Mr=psi'*Ml*psi; % Reducerad Mass-matris, Ml lumpad
massmatris
[Lar,Egvr]=eigen(Kr,Mr); % Egenv‰rdesanalys av reducerat
system
Freqr=sqrt(abs(Lar))/(2*pi); % Fiktiv egenfrekvens Freqr
format short
frekvr=num2str(Freqr',3);frekv=num2str(Freq([1:5])',3);
disp(['Frekvenser i oreducerat system: '] )
disp([frekv,'[Hz]'])%Fem fˆrsta frekvenserna frÂn oreducerat
system
disp([' '] )
disp(['Frekvenser i reducerat system: '] )
disp([frekvr,'[Hz]'])
disp([' '] )
% ------ Ritzvektor reduktion med modellerad puls -------------
% --------------------------Case 1 - Reducerad modell ---------
fr=psi'*f; % Reducerad kraftvektor ,fr
d0=zeros(length(psi(1,:)),1); v0=zeros(length(psi(1,:)),1);
ntimes=[0.1:0.1:T]; nhist=[1:length(psi(1,:))];
ip=[dt T 0.25 0.5 length(ntimes) 2 ntimes nhist];bc=[];
% % --- tidsintegration ---------------------------------------
[Dsnapr,Dr,Vr,Ar]=step2(Kr,[],Mr,d0,v0,ip,fr,bc); % [] ‰r C
matris
% --- Transformeras tillbaka ---------------------------------
u=psi*Dr;
umaxred=max(max(u(hist(length(hist)),:)));
% --- Plottar ----
figure
hold on
for i=[hist]
plot(t,u(i,:),'b--')
end
for j=1:length(hist)
plot(tt,D(j,:),'k')
end
% ----- Case 1 - Full modell -----------
grid on
hold off
```

```
axis([0 T -maxdisp maxdisp])
title(' Modell B (blue) compared with Model A (black)')
time_model_B=toc
% ---- ****** -------- ****** -------- ****** -------- ******
% Ritzvektor reduktion med initialv‰rden enl. uoprick=I/m
% ---- ****** -------- ****** -------- ****** -------- ******
tic
uoprick=imp./Mlr';% Initialhastigheter frÂn Impulslast/Massa
[m/s]
zoprick=inv(psi'*psi)*psi'*uoprick;
% Fiktiv initialhastighet [m/s] zoprick
t=(0:(5*1000))/1000;
% ---- ****** -------- ****** -------- ****** -------- ******
% Case 2 - Reducerad Modell
% ---- ****** -------- ****** -------- ****** -------- ******
dt=1/1000;T=5;
fre=[]; % Reducerad kraftvektor fr
d0=zeros(length(psi(1,:)),1); v0=zoprick;ntimes=[0.1:0.1:T];
nhist=[1:length(psi(1,:))];
ip=[dt T 0.25 0.5 length(ntimes) 2 ntimes nhist];bc=[];
% % --- tidsintegration -----------------------------------
[Dsnapre,Dre,Vre,Are]=step2(Kr,[],Mr,d0,v0,ip,fre,bc);
% [] är C matris
t=t+tA(1);
figure
hold on
Dre=psi*Dre;
for i=[hist]
plot(t,Dre(i,:),'r--')
end
for j=1:length(hist)
plot(tt,D(j,:),'k')
end
% ----- Case 1 - Full Modell ------------
grid on
hold off
axis([0 T -maxdisp maxdisp])
title(' Modell D (red) compared with Model A (black) ')
time_model_D=toc
% --------------------------------------
% Max. fˆrkjutningar fˆr de olika fallen j‰mflˆrs och
presenteras
umaximp=max(max(Dre(hist(length(hist)),:)));
maxdisplacement=num2str(maxdisp*1000,4);
maxred=num2str(umaxred*1000,4);
maximp=num2str(umaximp*1000,4);maximp_D=num2str(umaximp_D*1000,4
);
proc1=num2str(100*((umaxred-maxdisp)/maxdisp),3);
proc2=num2str(100*((umaximp-maxdisp)/maxdisp),3);
proc3=num2str(100*((umaximp_D-maxdisp)/maxdisp),3);
disp(' ')
disp(['Max fˆrskjutning i toppen av byggnaden'])
disp(['Oreducerat system ( Kraftimpuls ): ',maxdisplacement,'
[mm]'])
disp(['Reducerat system ( Kraftimpuls ): ',maxred,' [mm]
Differans:
', proc1, ' %'])
disp(['Oreducerat system ( Initial hastighet ): ',maximp_D,'
[mm]
Differans: ', proc3, ' %'])
```

D.3 Ritz vectors

Ritz vector 1 script

```
% --- Statisk utbˆjning av referensbyggnad ----
% --- Ramverksmodell pÂ ett 6 vÂningshus ---
% --- utsatt fˆr statisk last
% --- ::: ... --- ... ::: ---
% Ritz vektor 1
format compact;clear all;close all; clc;
load 6van_2d_smalsym_fallA
% % --- FEM-mesh plot ---
figure(1)
eldraw2(ex,ey,[1 3 0],Edof);
grid; title('2D Frame Structure');
% --- *** --- *** --- *** ---
% --- Static load ---
% --- *** --- *** --- *** ---
% Lasten tr‰ffar i plan 3 vid v‰nstra kanten
fri=zeros(NDOF,1); last=[3:11:NOE];
fri(Edof([last], 5))=1e3;
format short; format compact
[a,r]=solveq(K,fri,bc);
%----- Section forces -----------------------------
--------------
Ed=extract(Edof,a);
%----- Draw deformed frame ------------------------
---------------
figure(2)
for i=1:NOE
plotpar=[3 1 0];
eldraw2(ex(i,:),ey(i,:),plotpar);
sfac=1000;
plotpar=[1 2 1];
eldisp2(ex(i,:),ey(i,:),Ed(i,:),plotpar,sfac);
%title('displacements')
axis off
end
text(-
2*ones(length(last),1),[(1:length(last))*height]','
\color{red}\rightarrow',
'FontSize',18)
xleddisp=Ed(1:NOE,[1 4]);
ritz1_smalsym=a;
save ritz1_smalsym ritz1_smalsym
```

Ritz vector 2 script

```
% --- Statisk utbˆjning av referensbyggnad ----
% --- Ramverksmodell pÂ ett 6 vÂningshus ---
% --- utsatt fˆr statisk last
% --- ::: ... --- ... ::: ---
% Ritz vektor 2
format compact;clear all;close all; clc;
load 6van_2d_smalsym_fallA
% --- FEM-mesh plot ---
figure(1)
eldraw2(ex,ey,[1 3 0],Edof);
grid; title('2D Frame Structure');
% --- *** --- *** --- *** ---
% --- Static load ---
% --- *** --- *** --- *** ---
% Lasten tr‰ffar i plan 3 vid v‰nstra kanten
fri=zeros(NDOF,1); last=[3:11:NOE*(1/2)];
fri(Edof([last], 5))=1e3; fri(Edof([NOE-nelh ],
5))=-1e3;
```

```
format short; format compact
[a,r]=solveq(K,fri,bc);
%----- Section forces -----------------------------
--------------
Ed=extract(Edof,a);
%----- Draw deformed frame ------------------------
---------------
figure(2)
for i=1:NOE
plotpar=[2 1 0];
eldraw2(ex(i,:),ey(i,:),plotpar);
sfac=10000;
plotpar=[1 2 1];
eldisp2(ex(i,:),ey(i,:),Ed(i,:),plotpar,sfac);
title('displacements')
end
text(-
2*ones(length(last),1),[1:length(last)]*height,'\co
lor{red}\rightarrow','Fo
ntSize',18);
text(5.5*ones(1,1),[nstorey]*height,'\color{red}\le
ftarrow','FontSize',18)
xleddisp=Ed(1:NOE,[1 4]);
ritz2_smalsym=a;
save ritz2_smalsym ritz2_smalsym
```

Ritz vector 3 script

```
% --- Statisk utb^jning av referensbyggnad ----
% --- Ramverksmodell pÂ ett 6 vÂningshus ---
% --- utsatt f^r statisk last
% --- ::: ... --- ... ::: ---
% Ritz vektor 3
format compact;clear all;close all; clc;
load 6van_2d_smalsym_fallA
% --- FEM-mesh plot ---
figure(1)
eldraw2(ex,ey,[1 3 0],Edof);
grid; title('2D Frame Structure');
% --- *** --- *** --- *** ---
% --- Static load ---
% --- *** --- *** --- *** ---
% Lasten tr‰ffar i plan 3 vid v‰nstra kanten
fri=zeros(NDOF,1); % last=[3:11:NOE*(1/2)];
fri(Edof([ 3+floor(nstorey/5)*11], 5))=3e3;
fri(Edof([ NOE-(nelh+nelw) ], 5))=1e3;
fri(Edof([ floor(nstorey*2/3)*11], 5))=-2e3;
format short; format compact
[a,r]=solveq(K,fri,bc);
%----- Section forces -----------------------------
--------------
Ed=extract(Edof,a);
%----- Draw deformed frame ------------------------
---------------
figure(2)
for i=1:NOE
plotpar=[3 1 0];
eldraw2(ex(i,:),ey(i,:),plotpar);
sfac=10000;
plotpar=[1 2 1];
eldisp2(ex(i,:),ey(i,:),Ed(i,:),plotpar,sfac);
%title('displacements')
axis off
```