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Chiral Perturbation Theory for Neutron-Antineutron Oscillations

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Abstract

This thesis treats neutron-antineutron oscillations in the framework of chiral perturbation theory. An effective Lagrangian, using mesons and nucleons as degrees of freedom, is constructed to capture the low-energy behavior of the effective six-quark operators, which is the set of higher dimensional operators that can induce neutron to antineutron transitions at the quark level. These operators have been used previously to model the oscillations using lattice QCD simulations. This Lagrangian is used to compute the neutron to antineutron transition amplitude at $O(p^2)$ in the chiral perturbation expansion. The resulting amplitude is given as a function of the pion mass and could be of use for matching lattice simulations at unphysical quark masses to the physical situation. Furthermore, the effect that finite lattice volume has is estimated by using the difference between the known finite and infinite volume versions of chiral perturbation theory.

Populärvetenskaplig sammanfattning

I partikelfysik beskrivs en partikel av en samling tal kallade *kvanttal*. Exempel på sådana inkluderar elektrisk laddning men även andra som exempelvis så kallade smaker of färger. Bevarade kvanttal är en speciellt viktig typ som går under det kollektiva namnet laddningar. Den viktigaste egenskapen som sådana laddningar har är att de är bevarade, alltså att de inte får förändras med tiden. Ibland förekommer kvanttal som endast approximativt är bevarade. Med approximativt menas att de processer som kan ändra på den givna laddningen är ovanligt förekommande.

Ett viktigt kvanttal i partikelfysik är det så kallade baryontalet vilket är definierat så att protoner och neutroner har värdet ett medan deras antipartiklar har värdet minus ett. Det finns anledningar att tro att baryontalet inte är exakt bevarat och att det således finns sällsynta processer där det ändras. En experimentell anledning att tro att det borde finnas sådana förlopp är att vårt universum verkar uppvisa i princip uteslutande materia och ingen antimateria. Detta verkar bara vara förenligt med fysikaliska modeller om baryontal får ändras. En möjlig konsekvens av baryontalsändring i naturen skulle vara så kallade neutron-antineutronoscillationer vilket är att neutroner kan övergå i att bli antineutroner, vilket sänker baryontalet med två enheter.

Denna uppsats undersöker hur neutron-antineutronoscillationer kan modelleras matematiskt på låga energier under premissen att baryontalet får ändras med två enheter. Metoden som används är en, i andra sammanhang, vältestad teori för hur protoner, neutroner och pioner växelverkar vid låga energier, och går under namnet *kiral störningsteori*.

Contents

Conventions and Notation

In this section some conventions that are used throughout the thesis are introduced. This thesis uses natural units where $\hbar = c = 1$ and it employs the standard Einstein summation convention for Lorentz as well as internal space indices, meaning that every index that is repeated in an equation is summed over. A fundamental representation, ψ, of a unitary group is denoted with a lower index ψ_i and transforms under the group as $\psi_i \to U_i^j \psi_j$. Its complex conjugate, $\psi^\dagger$, has an upper index $\psi^{\dagger i} \equiv (\psi_i)^*$ and transforms as $\psi^{\dagger i} \to (U_i^j)^* \psi^{\dagger j} = (U^{\dagger T})_i^j \psi^{\dagger j} = (U^\dagger)_j^i \psi^{\dagger j}$.

Representation (rep) is taken to mean a homomorphism from abstract group elements to operators on some linear space. Homomorphisms to non-linear transformations, such as exhibited by the meson and nucleon fields will be referred to as (non-linear) *realizations*.

The Pauli matrices are denoted τ^a and the $SU(2)$ generators are normalized $\frac{1}{2}\tau^a$. General $SU(N)$ generators are denoted T^a and the general structure constants are denoted f^{abc} making the Lie algebra $[T^a, T^c] = if^{abc}T^c$. The general normalization is $\mathrm{Tr}(T^a T^b) = \delta^{ab}/2$.

The Minkowski metric is denoted $\eta^{\mu\nu}$ and the signature is chosen to be $(+ - --)$. The number of spacetime dimensions is denoted by $d = 4 - \epsilon$ when doing dimensional regularization. Representations of a group are denoted by their dimensionality[1] e.g. 2 and 3 for the fundamental and adjoint representations of $SU(2)$. Indices also get subscripts to denote in which space they live. The subscripts are c, F L, R and B (for color, flavor, left, right and baryon).

For future convenience here follows a few conventions for notions that are defined further on. Under group transformations $(L, R) \in SU(2)_L \times SU(2)_R$ lower indices i_L and i_R transform by $L_{i_L}^{j_L}$ and $R_{i_R}^{j_R}$ respectively. For instance the meson exponential transforms as $U_{i_R}^{i_L} \to R_{i_R}^{j_R} U_{j_R}^{j_L} L_{j_L}^{\dagger i_L}$. Whenever χ appears as a subscript it stands for either L or R and is never summed over. Since baryons transform by a non-linearly related unitary matrix, the lower B-type indices transform according to $\psi_{i_B} \to h(L, R, U)_{i_B}^{j_B} \psi_{j_B}$. More on group theory, generators and upper/lower indices can be found in e.g. ref [3].

[1] In the quantum mechanics of spin the representations are classified by their angular momentum quantum number j. This classification describes them by $2j + 1$ instead.

List of Abbreviations

BNV	Baryon Number Violation	QCD	Quantum Chromodynamics
ChPT	Chiral Perturbation Theory	QED	Quantum Electrodynamics
EFT	Effective Field Theory	QFT	Quantum Field Theory
HBChPT	Heavy Baryon ChPT	SM	Standard Model
ESS	European Spallation Source	UV	Ultraviolet
LSZ	Lehmann-Symanzik-Zimmermann	VEV	Vacuum Expectation Value
app.	appendix	(ir)rep	(irreducible) representation
eq.	equation	ref.	reference
fig.	figure	sec.	section

1 Introduction

There are multiple reasons to investigate models in which the baryon number is broken. For instance, one of the Sakharov conditions for baryogenesis - the origin of the observed matter-antimatter asymmetry of the universe - requires[2] Baryon Number Violation (BNV). Another reason is that the Standard Model (SM) has baryon conservation only because of an accidental symmetry since no renormalizable interactions can be formed from its fields which break it. Finally, as a remark, all grand unified theories that put quarks and leptons in the same gauge multiplet will violate baryon number due to the exchange of gauge bosons. It should also be noted that baryon number seems to be broken in the SM as well, due to non-perturbative transitions in the electroweak sector. This does, however, conserve baryon minus lepton number ($B - L$). For neutron-antineutron oscillations to occur the baryon number must be changed by two units (while lepton number remains unchanged) and thus they are absent in theories that conserve $B - L$.

To the best of our understanding the theory for strong interactions is Quantum Chromodynamics (QCD) which is a non-Abelian gauge theory consisting of quarks and gluons. Due to a feature called asymptotic freedom the QCD gauge coupling decreases with energy which means that the theory is weakly coupled at high energies. Running the argument the other way, the coupling strength increases as the energy is lowered, and thus there is a scale, Λ_{QCD}, where the coupling becomes sufficiently large that QCD is non-perturbative.

At low energies the particle spectrum of QCD will not be the quarks and gluons because all bound states, hadrons, are gauge singlets (colorless). The asymptotic particle states consist of mesons (quark and antiquark states)

[2] Asymmetric initial conditions is an alternative explanation.

and baryons (three quark states). One way to understand the physics of QCD at these scales is to make an Effective Field Theory (EFT) where the degrees of freedom are taken to be mesons and baryons instead. The EFT studied here is called Chiral Perturbation Theory (ChPT) which is based on the approximate flavor and chiral symmetries of QCD.

This thesis begins with an introduction to particle-antiparticle oscillations and higher dimensional operators, which constitutes sec. 2. In sec. 3 the approximate chiral structure of QCD, the technique of external field spurions, and the higher dimensional six-quark operators inducing transitions $\mathrm{n} \to \bar{\mathrm{n}}$ are discussed. Those topics lay the theoretical foundation of both ChPT and effective neutron oscillations. This is followed, in sec. 4, by an introduction to ChPT, the main tool of this thesis, including a short treatment of finite volume effects. An effective Lagrangian for $\mathrm{n}\bar{\mathrm{n}}$ oscillation is constructed in sec. 5. The relevant tree and loop order processes contributing to oscillations, at $O(p^2)$ (which is next to leading order), are performed in sec. 6. The full functional dependence of the neutron-antineutron transition amplitude on the pion mass is given in sec. 7. The findings are summerized and conclusions drawn in sec. 8.

2 Neutron-Antineutron Oscillations and Higher Dimensional Operators

2.1 Neutron-Antineutron Oscillations

Neutron-antineutron oscillations can occur for models where the baryon symmetry is not exact, i.e. when its corresponding operator does not commute with the Hamiltonian. As for e.g. kaon or neutrino oscillations, the oscillatory behaviour originates from the mass eigenstates not coinciding with the eigenbasis of some other operator (strangeness and lepton flavor in the examples given). The other operator of interest here is the baryon number operator with the eigenbasis $|\mathrm{n}\rangle$ and $|\bar{\mathrm{n}}\rangle$. If $\mathrm{n} \to \bar{\mathrm{n}}$ transitions are allowed the states of definite baryon number, $|\mathrm{n}\rangle$ and $|\bar{\mathrm{n}}\rangle$, are not mass eigenstates (due to the effective Majorana mass terms in the Lagrangian). To first approximation the mass eigenstates can be written [1] $|\mathrm{n}_1\rangle = \cos(\varphi)|\mathrm{n}\rangle + \sin(\varphi)|\bar{\mathrm{n}}\rangle$ and $|\mathrm{n}_2\rangle = -\sin(\varphi)|\mathrm{n}\rangle + \cos(\varphi)|\bar{\mathrm{n}}\rangle$ for mixing angle φ. For particles at rest $H|\mathrm{n}_1\rangle = m_1|\mathrm{n}_1\rangle$ and $H|\mathrm{n}_2\rangle = m_2|\mathrm{n}_2\rangle$. By expanding $|\mathrm{n}\rangle$ in the mass states and time evolving (by e^{-iHt}) one gets the result that

$$e^{-iHt}|\mathrm{n}\rangle = e^{-im_1t}[\cos^2(\varphi)+e^{-i\Delta mt}\sin^2(\varphi)]|\mathrm{n}\rangle+e^{-im_1t}\sin(\varphi)\cos(\varphi)[1-e^{-i\Delta mt}]|\bar{\mathrm{n}}\rangle,$$

where $\Delta m = m_2 - m_1$ is the oscillation frequency. The amplitude for having an antineutron at time t, if the initial state was a neutron at time 0, is

$$\langle \bar{\mathrm{n}} | e^{-iHt} | \mathrm{n} \rangle = e^{-im_1 t}[1 - e^{-i\Delta m t}] \tag{2.1}$$

For further background see ref. [1]. An interesting discussion on how to make this model slightly more sophisticated by including changes of the vacuum state can be found in ref. [2]. The effect of this is to add a momentum dependent term to eq. (2.1).

2.2 Higher Dimensional Operators

As stated previously baryon number is an accidental symmetry of the SM. This means that there are no gauge invariant, Lorentz invariant product of fields, of mass dimension ≤ 4 (renormalizable), that can be written down, using only SM fields, which violate baryon number[3]. However, there is nothing prohibiting that terms which allow for baryon number breaking processes to be introduced, in theories extending the SM, when more fields come into play at higher energies. If there exits a more ultraviolet (UV) complete theory that extends the SM, and that has baryon violation, the effect will show up in the low-energy limit as higher dimensional operators, which are field products of mass dimension > 4.

To illustrate how this works consider a theory containing higher dimensional operators, with a known extension into a renormalizable theory (the SM): Fermi's theory of beta decay. In 4-Fermi theory we have (among others) a vertex $\bar{e}_L \gamma^\mu \nu_{eL} \bar{\nu}_{\mu L} \gamma_\mu \mu_L$ where the fields are in order (all left-handed) electron, electron neutrino, muon neutrino and muon. This is a mass dimension six operator that allows for muon decay and in order to make this into a vertex one must multiply with a coupling of mass dimension -2, denoted Λ^{-2}.

In the SM this process is described by the propagation of a new field, the W gauge field, which was absent in the low scale EFT. At energies far below the mass of the W, M_W, this propagator can be approximated by $\frac{1}{M_W^2}$ and two gauge couplings, one at emission and one at absorption of the W, enter. At these energies the process of muon decay looks exactly like the above four-fermion vertex with the prefactor[4] $\frac{\alpha_W}{M_W^2}$ where α_W is the weak fine

[3] There are non-perturbative electroweak transitions inducing BNV predicted by the SM, called sphalerons. These conserve $B - L$ which means that $\mathrm{n}\bar{\mathrm{n}}$ oscillations are not induced by these.

[4] There are factors of 2, $\sqrt{2}$ and π omitted to put focus on the overall reasoning.

structure constant. This allows as to identify the phenomenological scale Λ of the EFT with

$$\Lambda^2 = \frac{M_W^2}{\alpha_W}. \tag{2.2}$$

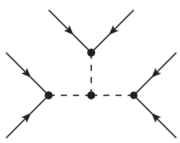
The muon decay rate is by dimensional analysis[5] $\frac{m_\mu^5}{\Lambda^4}$ where m_μ is the muon mass. This allows for the interpretation of Λ as a scale of new physics and also explains why the process is rare at low energies: it is suppressed by ratios of a low energy scale to an UV scale.

In this thesis the assumption is made that there exists a process that culminates in a higher dimensional operator capable of inducing $\mathrm{n\bar{n}}$ transitions which consists of six-quark fields of correct quantum numbers. The coupling of such a vertex can be written as $\Lambda_{\mathrm{n\bar{n}}}^{-5}$. Dimensional analysis predicts an oscillation frequency of[6]

$$\omega_{\mathrm{n\bar{n}}} = \frac{m_{\mathrm{n}}^6}{\Lambda_{\mathrm{n\bar{n}}}^5}, \tag{2.3}$$

which gives a way to measure the scale of new physics if $\mathrm{n\bar{n}}$ oscillations are observed. The appearance of the (hypothetical) suppressing scale, and other scales for BNV processes like proton decay, can therefore be related to the accidental symmetries of the SM: the scale hierarchy makes them *irrelevant operators* (or rather: much less relevant), in the language of EFT, at low energies.

An example of a grand unified theory in which $\mathrm{n\bar{n}}$ transitions occurs is Georgi-Glashow's $SU(5)$ (see e.g. ref. [3]) extended to include an extra 15 dimensional Higgs multiplet [4]. The important feature is how the oscillations appear in this model, namely through the diagram



, (2.4)

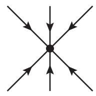
where the solid lines are quarks and the dotted are Higgses. In a close analogy with the 4-Fermi case the amplitude for this process at low energies is $y^3\mu/M_H^6$ where y is a Yukawa coupling, μ is a Higgs cubic coupling and M_H is the Higgs mass. One can then identify the scale as

$$\Lambda_{\mathrm{n\bar{n}}} = \left(\frac{M_H^6}{y^3\mu}\right)^{1/5} \tag{2.5}$$

[5]Squaring the amplitude for decay and using that the energy scale of phase space integration etc. is the muon mass.

[6]Oscillations take place at the amplitude level and therefore the matrix element is not squared.

and replace the process by the effective vertex



$$. \qquad (2.6) $$

Note that this effective vertex looks the same regardless of the underlying UV scale theory.

In order to put limits on what the scale $\Lambda_{\mathrm{n\bar{n}}}$ can be, experimental bounds on the mean neutron-antineutron oscillation time, $\tau_{\mathrm{n\bar{n}}}$, are needed. The best data on this is currently [5] $\tau_{\mathrm{n\bar{n}}} > 2.7 \cdot 10^8$s (Super-Kamiokande) from bound neutron measurement and $\tau_{\mathrm{n\bar{n}}} > 0.86 \cdot 10^8$s (Institute Laue Langevin) from free neutrons, yielding $\Lambda_{\mathrm{n\bar{n}}} \gtrsim 10^6$GeV. There is a proposal to improve the experimental limits on neutron-antineutron oscillations at the European Spallation Source (ESS) currently under construction in Lund. This experiment could improve the limits by up to three orders of magnitude [6]. The stability of oxygen nuclei ($\gtrsim 10^{31}$years) from proton decay experiments seems to put rather strong constraints on $\tau_{\mathrm{n\bar{n}}}$. This is, however, not the case since there is a strong suppression mechanism due to the fact that the $\mathrm{\bar{n}}$ state has a rather large annihilation width, $\Gamma_{\mathrm{\bar{n}}} \sim 100$MeV, with the neutrons in the nuclear environment while the n state does not [7].

3 QCD, Spurions and Six-Quark Operators

This section deals with QCD and in particular the global, chiral flavor symmetries of the light quark sector which is the foundation of ChPT. The concept of spurions, external source fields with symmetry group properties, is introduced. This concept plays an important role in this thesis. Finally the effective higher dimensional operators that are added to QCD in order to allow for neutron-antineutron oscillations are presented and the spurions allowing for treatment with ChPT techniques are introduced.

3.1 Chiral Symmetries of QCD

As mentioned previously, chiral perturbation theory relies on (approximate) global flavor symmetries of QCD which are discussed briefly in this section. Each quark field $q_{i_F i_c}$ has an index, i_c, in the triplet representation (rep) of the gauge group $SU(3)_C$. The index i_F is a label for the flavor of the quark. Here and onward "quark" (and "flavor") is taken to mean those quarks (and flavors) which are light compared to the QCD scale so for the purposes of this thesis (u,d). The three flavors (u,d,s) can also be taken to be light. The

Lagrangian of QCD is given by

$$\begin{aligned}\mathcal{L}_{\text{QCD}} \quad &= \bar{q}^{i_F i_c} i\gamma^\mu \left[\delta^{j_c}_{i_c}\partial_\mu + igG^{a_c}_\mu (T^{a_c}_C)^{j_c}_{i_c}\right] q_{i_F j_c} \\ &-\bar{q}^{i_F i_c}(M_q)^{j_F}_{i_F} q_{j_F j_c} - \frac{1}{4}G^{a_c}_{\mu\nu}G^{a_c\mu\nu}, \end{aligned} \tag{3.1}$$

where g is the gauge coupling, M_q is the (diagonal) quark mass matrix and $G^{a_c}_\mu$ are the gluon gauge fields and $G^{a_c}_{\mu\nu}$ the corresponding field strength.

As can be seen from eq. (3.1) the QCD gauge interaction (and the kinetic term) treats all flavors identically, which is something that is not true for the mass term. If the light quarks were massless, $M_q = 0$, one would have an exact continuous symmetry between the flavors. For massless fermions the two chiralites $q_L = \frac{1}{2}(1-\gamma^5)q$ and $q_R = \frac{1}{2}(1+\gamma^5)q$ become independent fields. The matrix[7] q is defined by $q = (q_{1_F} \cdots q_{N_F})^T$ where N is the number of light flavors. After defining the gauge covariant derivative, $D_\mu = \partial_\mu + igG^{a_c}_\mu T^{a_c}$, the Lagrangian for massless QCD is

$$\mathcal{L}_{\text{QCD}} = \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}. \tag{3.2}$$

This equation defines QCD in the *chiral limit* and it has an invariance under group transformations, in $U(1)_L \times SU(N)_L \times U(1)_R \times SU(N)_R$, of the flavors into each other:

$$\begin{aligned} q_L \quad &\mapsto \mathcal{U}_L q_L \quad = e^{i\theta_L} e^{i\theta^a_L T^a_L} q_L \\ q_R \quad &\mapsto \mathcal{U}_R q_R \quad = e^{i\theta_R} e^{i\theta^a_R T^a_R} q_R, \end{aligned} \tag{3.3}$$

where $e^{i\theta_{L/R}}$ is the transformation under $U(1)_{L/R}$ and where $T^a_{L/R}$ are the generators of $SU(N)_{L/R}$. This invariance is called the chiral symmetry of QCD. An important subgroup is the $SU(N)_V$ vector subgroup, that is defined by $\theta^a_L = \theta^a_R$, which can be associated with the (approximate) symmetry of the hadron spectrum. If $SU(N)_L \times SU(N)_R$ would be a symmetry of the spectrum one would see a parity doubling in the hadron mass states, and such doublings are absent in nature. This vector subgroup is the (strong) isospin group in our two flavor case and it will play an important role in this thesis. $U(1)_L \times U(1)_R$ is a symmetry of the classical theory only since loop effects break this in the so-called axial anomaly, leaving only $U(1)_V$, where $\theta_L = \theta_R \equiv -\theta/3$, a symmetry. The conserved charge associated with $U(1)_V$ is the baryon number.

Since the mass spectrum only shows symmetry under the vector $SU(N)_V$ subgroup, while the Lagrangian is invariant under $SU(N)_L \times SU(N)_R$ this

[7] Indices in color space are left implicit from here on.

suggests that a spontaneous symmetry breaking has occurred. Calculations using lattice QCD in the chiral limit support this notion [8]. The assumption is that non-perturbative QCD dynamics generates a quark condensate, i.e. a non-zero vacuum expectation value (VEV) for the field operator $\bar{q}^{i_F} q_{j_F}$. This VEV is invariant under $SU(N)_V$ but not under the full group and is thus a sufficient (but not necessary [9]) condition for a spontaneous breaking of $SU(N)_L \times SU(N)_R \to SU(N)_V$, which is discussed further on.

The presence of quark masses breaks chiral symmetry explicitly, but the energy scale for spontaneous chiral symmetry breaking (which is an effect of QCD dynamics) is large compared to the quark masses. This allows for a perturbative expansion around exact chiral symmetry in the symmetry breaking parameter, i.e. the quark masses, which is called chiral perturbation theory.

3.2 Chiral Ward Identities and Spurions

For our purposes it will be convenient to study the Lagrangian of chiral QCD in the presence of the external source fields v^μ, a^μ, s, p which are all Hermitian matrices acting on flavor space, where v and a are, in addition, traceless. These sources couple to quark currents and densities in the following way:

$$\begin{aligned}\mathcal{L} = \quad & \bar{q} i\gamma^\mu D_\mu q - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \\ & + \bar{q}\gamma_\mu (v^\mu + a^\mu \gamma^5) q + \bar{q}(-s + ip\gamma^5) q \end{aligned} \tag{3.4}$$

which, with $l = v - a$ and $r = v + a$ can be written

$$\begin{aligned}\mathcal{L} = \quad & \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \\ & + \bar{q}_L \gamma_\mu l^\mu q_L + \bar{q}_R \gamma_\mu r^\mu q_R - \bar{q}_R (s + ip) q_L - \bar{q}_L (s - ip) q_R. \end{aligned} \tag{3.5}$$

Chiral symmetry implies the existence of so-called chiral Ward identities analogous to how gauge invariance implies the Ward identity of quantum electrodynamics (QED). The chiral Ward identities are relations between different Green's functions composed of chiral quark currents. It is shown in ref. [10] that the collection of Ward identities is equivalent to imposing a *local* symmetry in such a way that the external sources absorb the extra terms arising from the gradients in the kinetic terms, i.e. under the transformation $(L, R) \in SU(N)_L \times SU(N)_R$ the sources are changed by

$$\begin{aligned} r_\mu \to R r_\mu R^\dagger + iR\partial_\mu R^\dagger; \quad & l_\mu \to L l_\mu L^\dagger + iL\partial_\mu L^\dagger; \\ s + ip \to R(s + ip)L^\dagger; \quad & s - ip \to L(s - ip)R^\dagger. \end{aligned} \tag{3.6}$$

External source fields obeying group transformations, like the ones above, are called *spurions*.

Note that if $s+ip$ is set equal to the quark mass matrix M_q then QCD with real, non-zero, quark masses is recovered. Furthermore, when the effective field theory ChPT is constructed, if the same spurions are included in the Lagrangian one can retain the chiral symmetries in the EFT (which means that all Ward identities hold). Hence, when an invariant Lagrangian has been constructed, all that has to be done to introduce the explicit chiral symmetry breaking is setting $s+ip = M_q$, which means that the corresponding explicit breaking for adding quark masses appear both at the quark and hadron levels automatically. In a very similar way the knowledge of how gauge fields couple to the quark currents will allow for a direct coupling at the level of hadrons by setting suitable combinations of v and a equal to the gauge field. This is important e.g. for pion decay through the charged current of the weak interaction.

3.3 Lattice QCD, Artifacts and Motivation

An alternative to making an effective field theory of the strong interaction, is to attempt to solve QCD non-perturbatively. This can done numerically using a discretized version of the QCD action using the path integral formulation of QFT (see chapter 9 in ref [11]). For an introduction to lattice QCD see e.g. chapter 13 of ref. [12]. The simulations are often performed with unphysical quark masses, which will lead to unphysical pion masses[8]. The main aim of this thesis is to guide such simulations by studying the dependence of the $n\bar{n}$ oscillation amplitude on the pion mass using ChPT.

Another artifact of lattice QCD is that the simulations by necessity take place at finite volume. There is no practical way of knowing exactly what effect this finite volume has, since increasing the volume is incredibly computationally costly. What can be done, however, is to take ChPT at finite volume (since its infinite volume limit is known) and estimate the effect of finite volume.

The matrix elements for the $n\bar{n}$ transition have been calculated using effective higher dimensional operators in ref. [13]. There the pion mass was set to the value $m_\pi = 390$ MeV and the lattice spacing was given by $m_\pi L = 7.8$. The effect of the unphysical pion mass and the finite volume is investigated in this thesis.

[8]The pion mass squared is proportional to the quark masses as will be discussed below.

3.4 Charge Conjugation on Quarks and Nucleons

In order to make a Lorentz scalar six-quark operator inducing particle to antiparticle transitions, charge conjugated spinors are needed. The convention taken here is that for any spinor ξ its charge conjugate is defined by $\xi^c = -i\gamma^2\xi^*$. All spinors will have a flavor group index like e.g. $(q_L)_{i_L}$. Charge conjugation on such a multiplet will be defined by $(q_L^c)^{i_L} = -i\gamma^2[(q_L)_{i_L}]^*$ so that it has an upper index (i.e. like a left space *row* matrix) making $(\bar{q_L^c})_{i_L} = [(q_L^c)^{i_L}]^\dagger\gamma^0$ have a lower index due to an extra complex conjugation (i.e. a *column* matrix). In matrix notation, for the nucleon doublet

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}, \tag{3.7}$$

this can be written

$$\psi^c = \begin{pmatrix} p^c & n^c \end{pmatrix} \tag{3.8}$$

and

$$\bar{\psi}^c = \begin{pmatrix} \bar{p}^c \\ \bar{n}^c \end{pmatrix}. \tag{3.9}$$

3.5 Six-Quark Operators

At the level of quarks the transition from n to n̄ is given by a multi quark field operator, $\mathcal{O}$. The operator should have a nonzero matrix element between an initial neutron state and a final antineutron state, $\langle \bar{\mathrm{n}}|\mathcal{O}|\mathrm{n}\rangle$. It will therefore consist of six-quark fields, $\mathcal{O} = \bar{q}^c\bar{q}^c\bar{q}^c qqq$ with appropriate spinor, color and flavor contractions, in order to annihilate the three valence quarks of n and create the three valence antiquarks of n̄.

These six-quark operators can be expressed as terms which are all irreducible representations (irreps) of the chiral group $SU(2)_L \times SU(2)_R$. The gauge group properties of the operators are chosen in such a way that they transform as singlets under $SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{em}}$. The full vertices, on the other hand, will have to be singlets under the entire $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{W}} \times U(1)_{\mathrm{Y}}$ group of the SM but nothing prevents the effective couplings to encode for high scale fields (which are integrated out) that are not singlets under $SU(2)_{\mathrm{W}} \times U(1)_{\mathrm{Y}}$. However, since nn̄ oscillations takes place below the electroweak breaking scale, an EFT is created with six-quark operators that are invariant under only $SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{em}}$. This EFT is matched to the $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{W}} \times U(1)_{\mathrm{Y}}$ invariant operators at the electroweak scale and then run down to the neutron mass using the renormalization group [14].

The properties of these operators are taken from ref. [14]. The full set of six-quark operators was first written down in ref. [15] and they are

$$\begin{aligned}
Q_1 &= (\bar{q^c_R} i\tau^2_R q_R)(\bar{q^c_R} i\tau^2_R q_R)(\bar{q^c_R} i\tau^2_R \tau^+_R q_R)T^{AAS} \\
Q_2 &= (\bar{q^c_L} i\tau^2_L q_L)(\bar{q^c_R} i\tau^2_R q_R)(\bar{q^c_R} i\tau^2_R \tau^+_R q_R)T^{AAS} \\
Q_3 &= (\bar{q^c_L} i\tau^2_L q_L)(\bar{q^c_L} i\tau^2_R q_R)(\bar{q^c_R} i\tau^2_R \tau^+_R q_R)T^{AAS} \\
Q_4 &= (\bar{q^c_R} i\tau^2_R i\tau^3_R q_R)(\bar{q^c_L} i\tau^2_R \tau^3_R q_R)(\bar{q^c_R} i\tau^2_R \tau^+_R q_R)T^{SSS} \\
&\quad -\frac{1}{5}(\bar{q^c_R} i\tau^2_R \tau^a_R q_R)(\bar{q^c_R} i\tau^2_R \tau^a_R q_R)(\bar{q^c_R} i\tau^2_R \tau^+_R q_R)T^{SSS} \\
Q_5 &= (\bar{q^c_R} i\tau^2_R \tau^-_R q_R)(\bar{q^c_L} i\tau^2_R \tau^+_R q_R)(\bar{L^c_R} i\tau^2_L \tau^+_L q_L)T^{SSS} \\
Q_6 &= (\bar{q^c_R} i\tau^2_R \tau^3_R q_R)(\bar{q^c_L} i\tau^2_L \tau^3_L q_L)(\bar{q^c_L} i\tau^2_L \tau^+_L q_L)T^{SSS} \\
Q_7 &= (\bar{q^c_L} i\tau^2_L \tau^3_L q_L)(\bar{q^c_L} i\tau^2_L \tau^3_L q_L)(\bar{q^c_R} i\tau^2_R \tau^+_R q_R)T^{SSS} \\
&\quad -\frac{1}{3}(\bar{q^c_L} i\tau^2_L \tau^a_L q_L)(\bar{q^c_L} i\tau^2_L \tau^a_L q_L)(\bar{q^c_R} i\tau^2_R \tau^+_R q_R)T^{SSS}
\end{aligned} \tag{3.10}$$

where $\tau^\pm = (\tau^1 \pm \tau^2)/\sqrt{2}$, and T^{SSS} and T^{AAS} are color $SU(3)$ tensor that symmetrize (and/or antisymmetrize) the quark color indices so that the operator is a color singlet. In the above expression $(\bar{q^c}_\chi i\tau^2_\chi q_\chi)$ is shorthand for $(\bar{q^c}_\chi)_{i_\chi}(i\tau^2_\chi)^{i_\chi j_\chi}(q_\chi)_{j_\chi}$. The full model for neutron oscillations treated here is given by the Lagrangian

$$\mathcal{L}_{\text{eff,n}\bar{\text{n}}} = \mathcal{L}_{QCD} + \sum_m \varsigma_m Q_m + \text{h.c.} \tag{3.11}$$

It is of this model that an EFT limit will be constructed, using ChPT as a basis. Note that these operators manifestly break chiral (and isospin) symmetry by picking out special directions, like τ^+. This is analogous to the quark masses and will be treated the same way: spurion fields will be coupled to the six-quark operators, which is discussed in detail below.

3.6 Isospin of States, Operators and Spurions

The transformation properties of quantum states and field operators are important to the logic of this thesis, so here a quick review, focused on the isospin of nucleon states and field operators, is given. Let t^a be the generators of some symmetry group on the Hilbert space of states. Under a transformation, $|\Psi\rangle \to |\Psi'\rangle = e^{i\theta^a t^a}|\Psi\rangle$ the operator matrix elements transform as $\langle\Phi|\mathcal{O}|\Psi\rangle = \langle\Phi'|e^{i\theta^a t^a}\mathcal{O}e^{-i\theta^a t^a}|\Psi'\rangle = \langle\Phi'|\mathcal{O}'|\Psi'\rangle$. With infinitesimal θ^a the operator transformation properties are $\mathcal{O}' = \mathcal{O} + i\theta^a[t^a, \mathcal{O}]$. If the operator $\mathcal{O}$ is in a representation (with generators X^a and generic indices I and J) of the symmetry group then

$$\mathcal{O}'_I = \mathcal{O}_I + i\theta^a[t^a, \mathcal{O}_I] = \mathcal{O}_I + i\theta^a (X^a)^J_I \mathcal{O}_J. \tag{3.12}$$

Here the t^a are taken to be the isospin group generators resulting in the following transformation properties for the nucleon field operators, ψ_i and $\bar{\psi}^i$,

$$[t^a, \psi_i] = (T^a)_i^j \psi_j \quad \text{and} \quad [t^a, \bar{\psi}^i] = -(T^a)_j^i \bar{\psi}^j. \tag{3.13}$$

A slightly subtle fact is that the nucleon states transform in a conjugate way to the operators since a nucleon state can be written $|\mathrm{N}^i\rangle = \bar{\psi}^i|0\rangle$ and an antinucleon $|\bar{\mathrm{N}}_i\rangle = \psi_i|0\rangle$ which gives the following relation[9]

$$t^a|\mathrm{N}^i\rangle = t^a\bar{\psi}^i|0\rangle = [t^a, \bar{\psi}^i]|0\rangle = -(T^a)_j^i\bar{\psi}^j|0\rangle = -(T^a)_j^i|\mathrm{N}^j\rangle \tag{3.14}$$

and (analogously)

$$t^a|\bar{\mathrm{N}}_i\rangle = (T^a)_i^j|\bar{\mathrm{N}}_j\rangle. \tag{3.15}$$

The isospin properties of the six-quark transition were an important consideration when the operators were constructed [4, 15, 16]. The isospin 3-component, I^3, is the eigenvalue of the t^3 generator which makes the neutron field operator, $n = \psi_{-1/2}$, an $I^3 = -1/2$ object and its Dirac conjugate $\bar{n} = \bar{\psi}^{-1/2}$ a $+1/2$. Thus, a neutron state $|\mathrm{n}\rangle = |\mathrm{N}^{-1/2}\rangle$ has isospin $+1/2$ while $|\bar{\mathrm{n}}\rangle$ has isospin $-1/2$. If there exists an operator, $\mathcal{Q}$, such that $\langle\bar{\mathrm{n}}|\mathcal{Q}|\mathrm{n}\rangle$ is non-zero then $\mathcal{Q}$ has to be an isospin $I^3 = -1$ operator (so that the tensor operator $\mathcal{Q}$ decreases the isospin and thus it can have overlap with the isospin $-1/2$ state $|\bar{\mathrm{n}}\rangle$).

An important case, for the six-quark operators and their spurions, will be operators of the form $\Theta^a = \phi^i(T^a)_i^j\psi_j$. Acting with t^a yields

$$\begin{aligned} [t^a, \phi^i(T^b)_i^j\psi_j] &= [t^a, \phi^i](T^b)_i^j\psi_j + \phi^i(T^b)_i^j[t^a, \psi_j] \\ &= -\phi^k(T^a)_k^i(T^b)_i^j\psi_j + \phi^i(T^b)_i^j(T^a)_j^k\psi_k \end{aligned} \tag{3.16}$$

which is exactly the transformation law of the adjoint rep since

$$[t^a, \phi^i(T^b)_i^j\psi_j] = -\phi^i[T^a, T^b]_i^j\psi_j = -if^{abc}\phi^i(T^c)_i^j\psi_j. \tag{3.17}$$

In the $SU(2)$ case $f^{abc} = \epsilon^{abc}$, and in this representation the isospin $+1$, 0 and -1 states are Θ^-, Θ^3 and Θ^+ respectively, where $\Theta^\pm = \frac{1}{\sqrt{2}}(\Theta^1 \pm i\Theta^2)$. (The deduction of the spin states for the adjoint is performed in app. B.1.)

The tensor contraction of two adjoint reps, $\Theta^a\Omega^a$, is

$$\Theta^a\Omega^a = \frac{1}{\sqrt{2}}(\Theta^+ + \Theta^-)\Omega^1 + \frac{-i}{\sqrt{2}}(\Theta^+ - \Theta^-)\Omega^2 + \Theta^3\Omega^3 = \Theta^+\Omega^- + \Theta^-\Omega^+ + \Theta^3\Omega^3. \tag{3.18}$$

[9] Assuming the vacuum is a singlet i.e. $t^a|0\rangle = 0$ (no spontaneous symmetry breaking).

This will be used in the construction of the spurions that will be coupled to the six-quark operators in the following. If one defines the index A running over $+, -, 3$ and defines the lower version of these indices by $\Theta_\pm \equiv \Theta^\mp$ then the contraction can be written

$$\Theta^a \Omega^a = \Theta_A \Omega^A = \Theta_- \Omega^- + \Theta_+ \Omega^+ + \Theta_3 \Omega^3. \tag{3.19}$$

This type of contractions will be useful when determining which components of spurions to set to non-zero value.

3.7 Spurion Construction and Six-Quark Operators

In order to extract the spurions needed for the construction of the effective vertices, some further study of the chiral transformation properties of the six-quark operators is needed. Those properties are extracted below and chiral group spurions are introduced.

The building blocks when constructing the six-quark operators are the $SU(2)_\chi$ spin zero and spin one quark bilinears

$$\mathcal{D}_\chi = (\bar{q^c}_\chi)_{i_\chi} \epsilon^{i_\chi j_\chi} (q_\chi)_{j_\chi} \quad \text{and} \quad \mathcal{D}^a_\chi = (\bar{q^c}_\chi)_{i_\chi} \epsilon^{i_\chi j_\chi} (\tau^a_\chi)^{k_\chi}_{j_\chi} (q_\chi)_{k_\chi}. \tag{3.20}$$

$\mathcal{D}^a_\chi$ transforms in the adjoint 3_χ but the states $a = 1, 2, 3$ are not the $SU(2)_\chi$ 3-component spin states. Those are given by $\mathcal{D}^-_\chi$, $\mathcal{D}^3_\chi$, $\mathcal{D}^+_\chi$. Higher isospin operators are formed by Clebsch-Gordan decomposition[10] of the product operators, which was deduced in ref [14]:

$$\begin{aligned}
\mathcal{D}^{ab}_\chi &= \mathcal{D}^{\{a}_\chi \mathcal{D}^{b\}}_\chi - \frac{1}{3}\delta^{ab} \mathcal{D}^c_\chi \mathcal{D}^c_\chi, \qquad (3.21)\\
\mathcal{D}^{abc}_\chi &= \mathcal{D}^{\{a}_\chi \mathcal{D}^b_\chi \mathcal{D}^{c\}}_\chi - \frac{1}{5}[\delta^{ab} \mathcal{D}^{\{c}_\chi \mathcal{D}^d_\chi \mathcal{D}^{d\}}_\chi + \delta^{ac} \mathcal{D}^{\{b}_\chi \mathcal{D}^d_\chi \mathcal{D}^{d\}}_\chi + \delta^{bc} \mathcal{D}^{\{a}_\chi \mathcal{D}^d_\chi \mathcal{D}^{d\}}_\chi],
\end{aligned}$$

which form the 5 and 7 reps of $SU(2)_\chi$, respectively. In this language the six-quark operators can be written

$$\begin{array}{llllll}
Q_1 & = \mathcal{D}_R \mathcal{D}_R \mathcal{D}^+_R, & Q_2 & = \mathcal{D}_L \mathcal{D}_L \mathcal{D}^+_R, & Q_3 & = \mathcal{D}_L \mathcal{D}_R \mathcal{D}^+_R, \\
Q_4 & = \mathcal{D}^{33+}_R, & Q_5 & = \mathcal{D}^-_R \mathcal{D}^{++}_L, & Q_6 & = \mathcal{D}^3_R \mathcal{D}^{3+}_L, \\
Q_7 & = \mathcal{D}^+_R \mathcal{D}^{33}_L & & \text{and} & & (L \leftrightarrow R).
\end{array}$$

Using this, $\mathrm{n\bar{n}}$ transition sector of the Lagrangian is

$$\mathcal{L}_{\mathrm{n\bar{n}}} = \omega_R Q_1 + \theta_R Q_2 + \varphi_R Q_3 + \Omega Q_4 + \zeta_{R(1)} Q_5 + \zeta_{R(2)} Q_6 + \zeta_{R(3)} Q_7 + (R \leftrightarrow L), \tag{3.22}$$

[10] Here braces around indices denoted total symmetrization of those indices e.g. $\phi^{\{i}\psi^{j\}} = \frac{1}{2!}(\phi^i\psi^j + \phi^j\psi^i)$ and $\phi^{\{i}\psi^j\xi^{k\}} = \frac{1}{3!}(\phi^i\psi^j\xi^k + \phi^k\psi^i\xi^j + \phi^j\psi^k\xi^i + \phi^j\psi^i\xi^k + \phi^k\psi^j\xi^i + \phi^i\psi^k\xi^j)$.

where the coefficients are effective interactions strengths.

As can be seen from this representation the six-quark operators are not singlets under chiral transformations. In order to treat these operators with the techniques of ChPT, which relies heavily on chiral symmetry, one will need chirally symmetric interaction operators. The solution is analogous to the introduction of the spurion $s + ip$ in order to get the quark masses - one introduces new spurions transforming under the chiral group in order to form chiral singlet six-quark operators. That procedure allows us to add the invariant operators to the Lagrangian of chiral QCD without removing any chiral symmetries. This, in turn, allows for the use of ChPT at low energies if one extends the existing ChPT Lagrangian to also include the most general terms involving the same spurions.

Introduce five spurions ω_R^a, θ_R^a, φ_R^a, Ω_R^{abc} and ζ_R^{abc} and their parity conjugates ($L \leftrightarrow R$), transforming under the chiral group[11]. With those it is possible to write down a set of chiral singlet six-quark operators with spurions:

$$\begin{aligned} \mathcal{Q}_1 &= \omega^a \mathcal{D}_R \mathcal{D}_R \mathcal{D}_R^a, & \mathcal{Q}_2 &= \theta^a \mathcal{D}_L \mathcal{D}_R \mathcal{D}_R^a, & \mathcal{Q}_3 &= \varphi^a \mathcal{D}_L \mathcal{D}_L \mathcal{D}_R^a \\ \mathcal{Q}_4 &= \Omega^{abc} \mathcal{D}_R^{abc}, & \mathcal{Q}_5 &= \zeta^{abc} \mathcal{D}_R^a \mathcal{D}_L^{bc} & &\text{and } (L \leftrightarrow R). \end{aligned} \tag{3.23}$$

Here the important point is that if the spurions are set to suitable non-transforming values then the $\mathcal{Q}$ operators reduce to the Q operators. If one starts with the chirally invariant Lagrangian with spurions,

$$\mathcal{L}_{\mathrm{n\bar{n}}}^{(\mathrm{spurions})} = \mathcal{Q}_1 + \mathcal{Q}_2 + \mathcal{Q}_3 + \mathcal{Q}_4 + \mathcal{Q}_5 + (L \leftrightarrow R), \tag{3.24}$$

and set the (only non-zero) spurion components to $\omega_R^- = \omega_R$, $\theta_R^- = \theta_R$, $\varphi_R^- = \varphi_R$, $\Omega_R^{33-} = \Omega_R$, $\zeta_R^{+--} = \zeta_{R(1)}$, $\zeta_R^{33-} = \zeta_{R(2)}$, and $\zeta_R^{-33} = \zeta_{R(3)}$ then the desired Lagrangian, eq. (3.22), is recovered. As an example consider the operator $\mathcal{Q}_1$ and write it in the form of eq (3.19): $\mathcal{Q}_1 = \omega_A \mathcal{D}_R \mathcal{D}_R \mathcal{D}_R^A$. If ω^A is set to ω for $A = -$, and 0 otherwise, then $\mathcal{Q}_1$ is identical to term ωQ_1 term in eq. (3.22).

The fundamental building blocks at the hadron level will all be objects with upper or lower 2_χ indices. This makes it desirable to express the higher spin spurions as (properly symmetrized) multi-2_χ index objects e.g. $\theta^{i_L j_L}$ instead of θ^a, and $\zeta^{i_R j_R k_R l_R k_L l_L}$ instead of ζ^{abc}, which is done by a Clebsch-Gordan expansion of a direct product of 2_χ reps (see app. B.2).

[11] ω_R^a, θ_R^a, φ_R^a all transform as $(1_L, 3_R)$, Ω_R^{abc} as $(1_L, 7_R)$, and ζ_R^{abc} as $(5_L, 3_R)$.

4 A Short Introduction to ChPT

A brief introduction to the field of ChPT is given since this thesis relies on it as a foundation for the model of neutron-antineutron oscillations. Good and more complete introductions are found in refs. [9, 17] (approximately equivalent content) and ref. [18].

4.1 Light Mesons as Pseudo-Goldstone Bosons

This section introduces the notion of mesons being the pseudo-Goldstone bosons from a spontaneous chiral symmetry breaking. Since the axial generators are broken, one expects a Goldstone boson for each of them living in the adjoint representation of $SU(N)_V$ (see app. A.1). In the isospin case, $N = 2$, these are the pion triplet $(\pi^{\pm}, \pi^0)$ and in the $N = 3$ case the meson octet $(\pi^{\pm}, \pi^0, K^{\pm}K^0\bar{K}^0, \eta)$. This thesis works with $N = 2$.

The spontaneous chiral symmetry breaking should leave the $SU(N)_V$ group as a symmetry of the vacuum. The breaking is through a quark condensate, $\langle \bar{q}^{i_F} q_{j_F} \rangle$, which transforms under the vector subgroup, $V \in SU(N)_V$, into $(V^\dagger)^{i_F}_{k_F} V^{l_F}_{j_F} \langle \bar{q}^{k_F} q_{l_F} \rangle$. There is only one rank two, $SU(N)$ invariant tensor (up to normalization), the Kronecker delta[12], which means that $\langle \bar{q}^{i_F} q_{j_F} \rangle = \langle \bar{q}q \rangle \delta^{i_F}_{j_F}$ where $\langle \bar{q}q \rangle$ is the VEV for any specific flavor. Expanding the vacuum in terms of chiral quarks yield $\langle \bar{q}^{i_F} q_{j_F} \rangle = \langle \bar{q}^{i_F}_L q_{Rj_F} \rangle + \langle \bar{q}^{i_F}_R q_{Lj_F} \rangle$ and (non-breaking of) parity symmetry implies that both terms give the contribution $\frac{1}{2}\langle \bar{q}q \rangle \delta^{i_F}_{j_F}$.

The spontaneous breaking of chiral symmetry is now manifest since

$$\langle \bar{q}^{i_F}_L q_{Rj_F} \rangle \to (L^\dagger)^{i_F}_{k_F} R^{l_F}_{j_F} \langle \bar{q}^{k_F}_L q_{Rl_F} \rangle \tag{4.1}$$

which implies

$$\langle \bar{q}^{i_F}_L q_{Rj_F} \rangle \to (L^\dagger)^{i_F}_{k_F} R^{l_F}_{j_F} \delta^{k_F}_{l_F} \frac{1}{2}\langle \bar{q}q \rangle = (L^\dagger)^{i_F}_{k_F} R^{k_F}_{j_F} \frac{1}{2}\langle \bar{q}q \rangle = (RL^\dagger)^{i_F}_{j_F} \frac{1}{2}\langle \bar{q}q \rangle \tag{4.2}$$

and $RL^\dagger$ is not 1 unless $R = L$ which is an $SU(N)_V$ transformation. As the discussion in app. (A.1) suggests, doing a transformation in the broken group on the vacuum will yield a configuration of Goldstone boson fields. This means that the Goldstone fields can be characterized by the $SU(N)$ matrix field[13] $U(x) = \tilde{R}(x)\tilde{L}^\dagger(x)$ for some $(\tilde{L}, \tilde{R}) \in SU(N)_R \times SU(N)_R$. The

[12]In $SU(2)$ there is ϵ^{ij} and ϵ_{ij} as well but the index placements (and thus transformation laws) do not fit our condensate.

[13]A more rigorous argument for the parametrization and transformation properties of the Goldstone configurations can be found in [9, 17]. In those, a one-to-one correspondence between cosets of the unbroken subgroup with respect to the broken group elements is established, resulting in the same conclusion as here.

transformation properties for U should be the same as $\langle \bar{q}_L^{i_F} q_{Rj_F} \rangle$ i.e.

$$U(x) \to RU(x)L^\dagger. \tag{4.3}$$

Like any special unitary matrix, U can be put in exponential parametrization

$$U(x) = \exp\left(\frac{i}{F}\phi(x)\right) = \exp\left(\frac{i2}{F}\phi^a(x)T^a\right) \tag{4.4}$$

where $\phi^a(x)$ are the Goldstone fields and the mass dimension one constant F is called the pion decay constant since it is the parameter determining the pion decay rate to leading order[14]. Under the vector subgroup the fields ϕ transform as $\phi \to V\phi V^\dagger$ which is the (anticipated) adjoint representation.

For the effective Goldstone Lagrangian chiral invariant terms are needed. Such terms can be constructed from U, $U^\dagger$ and their derivatives. The invariants have all indices contracted: $(\cdot)^{i_L}_{i_R}(*)^{i_R}_{i_L} = \mathrm{Tr}[(\cdot)(*)]$ which gives the building blocks $\mathrm{Tr}(U^\dagger U)$, $\mathrm{Tr}(U^\dagger\partial_\mu U)$ and $\mathrm{Tr}(\partial_\nu U^\dagger \partial_\mu U)$ if derivatives higher than two are ignored. Only $\mathrm{Tr}(\partial_\nu U^\dagger\partial_\mu U)$ is non-trivial since $U^\dagger U = \mathbf{1}$ has a constant trace, and since $\mathrm{Tr}(U^\dagger\partial_\mu U) = [\partial_\mu \det(U)]/\det(U) = 0$, because $\det(U) = 1$ for $SU(N)$. The Lagrangian with the lowest number of derivatives is thus

$$\begin{aligned}\mathcal{L} \quad &= \frac{F^2}{4}\mathrm{Tr}[\partial_\mu U^\dagger \partial^\mu U] = \frac{F^2}{4}\mathrm{Tr}\left[\frac{4}{F^2}\partial_\mu\phi^a T^a \partial^\mu \phi^b T^b + \dots\right] \\ &= \partial_\mu\phi^a\partial^\mu\phi^b \mathrm{Tr}[T^aT^b + \dots] = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a + \dots \end{aligned} \tag{4.5}$$

In order to keep the chiral Ward identities (through the local invariance) ∂_μ should be replaced by D_μ in eq. (4.5) where $D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu$ is the chirally covariant derivative.

Using the spurions s and p there are two further invariants namely $\mathrm{Tr}[(s+ip)^\dagger U]$ and $\mathrm{Tr}[U^\dagger(s+ip)]$. Adding terms proportional to these to eq. (4.5) will allow the study of what happens to the Goldstone bosons in the presence of explicit symmetry breaking when $s+ip$ is set to M_q. The new terms come with a new coupling, B, which becomes a parameter in the Lagrangian and it is convenient to define the combination $\chi = 2B(s+ip)$. With said definitions the Lagrangian for the Goldstone bosons in the presence of symmetry breaking is

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U + \chi U^\dagger + \chi^\dagger U\right]. \tag{4.6}$$

Expanding out the exponential U and inserting into the $\chi U^\dagger + U\chi^\dagger$ trace, one finds quadratic terms of the form $\frac{1}{2}\phi^a(m^2)_{ab}\phi^b$, i.e. a mass term, in the

[14]The charged current that couples to the $W^\pm$ boson is $\frac{F}{2\sqrt{2}}\partial^\mu(\phi^1 \pm i\phi^2)$ to lowest order.

presence of quark masses, which makes the mesons into pseudo-Goldstone bosons. The easiest case to show how this comes about is in the isospin limit, where all quarks have identical mass: $M_q = m_q\mathbf{1}$. The non-transforming value of χ is then $2Bm_q\mathbf{1}$ and, using $\mathrm{Tr}(T^a) = 0$ and $\mathrm{Tr}(T^aT^b) = \delta^{ab}/2$, one obtains

$$\begin{aligned}\mathrm{Tr}[\chi U^\dagger + \chi^\dagger U] = \;& 2Bm_q\mathrm{Tr}[U] + \text{c.c.} = 2Bm_q\mathrm{Tr}\left[\mathbf{1} + \frac{i}{F}\phi - \frac{1}{2F^2}\phi^2 + \cdots\right] \\ & +\text{c.c.} = -\frac{2Bm_q}{F^2}\mathrm{Tr}\left[\phi^2 + \cdots\right] = -\frac{4Bm_q}{F^2}\phi^a\phi^a + \cdots \end{aligned} \quad (4.7)$$

and thus one can identify the lowest order meson mass as $m_\pi^2 = 2Bm_q$. This thesis deals exclusively with the isospin limit, and hence the non-transforming value is $\chi = m_\pi^2\mathbf{1}$.

4.2 Perturbative Expansion and Power Counting

Weinberg's "folk theorem" states that if one writes down the most general Lorentz invariant, local Lagrangian that respects assumed symmetry principles and computes diagrams from it then one gets the most general S matrix obeying unitarity, Lorentz invariance, cluster decomposition and assumed symmetries [19]. This is a cornerstone of EFT. The most general Lagrangian in meson ChPT has the form $\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$ where the subscript either denotes the number of derivatives or half the powers of χ occurring in the term. With exact chiral symmetry all interactions are momentum dependent (since eq. (4.5) only has gradients of U) which means that the effective coupling vanish, for sufficiently small momenta[15]. So, to what extent does this hold true in the presence of small explicit breaking?

In order to have a predictive model in an EFT that includes all vertices compatible with symmetry principles, there must be some way of picking out the most important ones. This selection is performed by the use of Weinberg's Power Counting Scheme [19] in which a perturbative expansion in meson masses and momenta is established.

ChPT is valid in the limits of small momentum scale and small explicit chiral breaking. Thus the most important processes are those that go to zero the slowest as the meson masses and momenta approach zero. To study this consider the matrix element, $\mathcal{M}(p_i, M^2)$, for some process (e.g. meson-meson scattering) with p_i being the external momenta and M the meson mass. If a rescaling, $p_i \mapsto tp_i$ and $M^2 \mapsto t^2M^2$, of the parameters is performed then

[15]In the chiral limit the pions are massless which means that $p^\mu \to 0$ as $\mathbf{p} \to 0$.

the matrix element is rescaled by

$$\mathcal{M}(tp_i, t^2M^2) = t^D \mathcal{M}(p_i, M^2), \tag{4.8}$$

where D is called the *chiral dimension* of the diagram. Diagrams of dimension D scale down more significantly (for small t) as D becomes larger which means that the most significant processes are those of lowest D, assuming sufficiently low energies and small explicit breaking. An equation for the chiral dimension is [9]

$$D = 2 + (d-2)N_L + \sum_{k=1}^{\infty} 2(k-1)N_{2k}, \tag{4.9}$$

where d is the number of spacetime dimensions, N_L is the number of loops (independent undetermined momenta) and N_{2k} is the number of vertices from $\mathcal{L}_{2k}$.

Each derivative of a meson field in a vertex counts as meson momentum and is said to be at order p, $O(p)$. Since $p^2 = M^2 \sim m_q$ each quark mass counts at $O(p^2)$. The philosophy of ChPT is that if one wishes to know a process at $O(p^n)$ one constructs the most general Lagrangian with terms of $O(p^k)$, where $k \leq n$, and compute all diagrams with $D \leq n$ from it.

4.3 Baryonic ChPT and Heavy Baryon ChPT

As is stated in ref. [20] the baryon fields can be chosen to have any transformation properties under the chiral group as long as they have the correct transformation properties under the vector subgroup, i.e. being an isospin doublet for the $SU(2)$. A further complication with baryons compared to the pseudoscalar mesons is that baryons are not massless in the chiral limit of QCD. But since ChPT is an expansion in meson momenta and masses divided by the chiral symmetry breaking scale ($\sim$ 1 GeV) and the nucleon momentum contains a mass ($\sim$ 1 GeV) the nucleon momenta do not behave as meson momenta in power counting. This means that the mass has to be extracted somehow from the momentum in order to establish a perturbative expansion. This is done here by the application of Heavy Baryon ChPT (HBChPT) [21, 22].

Now the nucleon transformation properties and their Lagrangian will be treated briefly. It is here convenient to introduce the unitary matrix[16] $u = \sqrt{U}$. Since $u^2 \to Ru^2L^\dagger = Ruh^{-1}huL^\dagger$ we have that u transforms either

[16]Using the exponential parametrization $u = \exp(i\phi/2F)$ it becomes apparent that the square root of a unitary matrix exists and is unitary.

as $u \to Ruh^{-1}$ or $u \to huL^\dagger$ where h is some nonlinerar matrix function of L, R and u such that the two transformations yield equal results. That such a matrix is unique and existing is non-trivial, but it is deduced in the general theory of non-linear realizations in ref. [23]. Since $1 = uu^\dagger$ and $uu^\dagger \to huL^\dagger Lu^\dagger h^\dagger = hh^\dagger = 1$ the matrix h is unitary. Note that under an isospin transformation $L = R = V$ we have $h = V$. The nucleons are described by a spinor doublet ψ which means that it is possible to pick the nucleon transformation law

$$\psi \to e^{-i\theta} h(L, R, u)\psi, \tag{4.10}$$

where θ is the parameter of $U(1)_V$. The transformation properties lead to a covariant derivative of the form $D_\mu\psi = (\partial_\mu + \Gamma_\mu)\psi$ where $\Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger]$. The lowest order Lagrangian is [9]

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\psi}\left(i\not{D} - m + \frac{1}{2}g\gamma^\mu\gamma^5 u_\mu\right)\psi, \tag{4.11}$$

where $u_\mu = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger]$ and g is the axial-vector coupling. The field u_μ transforms under the chiral groups as $u_\mu \to hu_\mu h^\dagger$. The matrix u will also be useful in constructing vertices that transform as irreducible representations of the chiral group since $u\psi \to Ruh^\dagger h\psi = Ru\psi$ and $u^\dagger\psi \to Lu^\dagger h^\dagger h\psi = Lu^\dagger\psi$.

In Heavy Baryon Chiral Perturbation Theory (HBChPT) the large baryon mass is extracted from the four momentum of the nucleon. In this limit a forward, timelike vector v^μ is chosen (in practice this is the four-velocity of the frame in which the baryon is slow) and a small momentum k is introduced, defined by $p^\mu = mv^\mu + k^\mu$. This k behaves as a meson momentum in power counting. When taking the nonrelativistic limit the spinors for the particle and antiparticles should be separated and therefore the orthogonal projectors $P_{v\pm} = \frac{1}{2}(1 \pm \not{v})$ are constructed as to separate the four degrees of freedom in ψ into two pairs. The light and heavy field respectively are defined by

$$\mathcal{N}(x) = e^{imv\cdot x}P_{v+}\psi(x) \quad \text{and} \quad \mathcal{H}(x) = e^{imv\cdot x}P_{v-}\psi(x). \tag{4.12}$$

Because of the derivative term in eq. (4.11) the mass dependence will vanish from the equations of motion for $\mathcal{N}$ while the solution of $\mathcal{H}$ is suppressed by a factor $1/m$ compared to $\mathcal{N}$ (see app. A.2). This allows us to recast eq. (4.11) to its HBChPT analogue

$$\widehat{\mathcal{L}}^{(1)}_{\pi N} = \bar{\mathcal{N}}\left(iv\cdot D + gS\cdot u\right)\mathcal{N} + O\left(m^{-1}\right) \tag{4.13}$$

where $S^\mu = \frac{i}{2}\gamma^5\sigma^{\mu\nu}v_\nu$ is the spin four-vector, with $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$.

In this formalism the nucleon propagator becomes

$$\frac{iP_{v+}}{v \cdot k + i\epsilon}, \tag{4.14}$$

which is most easily verified by expanding $p = mv + k$ in the Dirac propagator[17]

$$\frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} = \frac{i(\not{k} + m(1 + \not{v}))}{2mv \cdot k + k^2 + i\epsilon} = \frac{i(\not{k}/m + 2P_{v+})}{2v \cdot k + k^2/m + i\epsilon} = \frac{iP_{v+}}{v \cdot k + i\epsilon} \tag{4.15}$$

assuming that k is a small deviation and dropping all terms suppressed by powers of m.

In this thesis two separate heavy baryon expansions are needed since the initial state is a non-relativistic neutron and the final state a non-relativistic antineutron. This can be seen as an expansion for small k for both $p^\mu = mv^\mu + k^\mu$ and $p^\mu = -mv^\mu - k^\mu$ of the Dirac field[18]. The double expansions are, however, not a double counting since both modes are present in the Dirac field. The low energy limit will have two sectors: a neutron sector for $p^\mu = mv^\mu + k^\mu$, and an antineutron sector for $p^\mu = -mv^\mu - k^\mu$ where both k are small. Those regions are well separated in momentum space. The neutron sector is expanded in a heavy baryon limit around $p^\mu = mv^\mu$ of ψ yielding the terms in eq. (4.13). The antineutron sector is created with $p^\mu = -mv^\mu - k^\mu$ for ψ, or equivalently $p^\mu = mv^\mu + k^\mu$ for ψ^c which enables a HBChPT expansion for ψ^c. This is easiest to do by rewriting $\bar{\psi} i \not{D} \psi = \bar{\psi}^c i \not{D} \psi^c$ and this yields terms identical to eq. (4.13) but with $\mathcal{N}$ replaced by $\mathcal{N}^c$. The Lagrangian for this double HBChPT expansion is thus

$$\widehat{\mathcal{L}}^{(1)}_{\pi N \bar{N}} = \bar{\mathcal{N}} \left(iv \cdot D + gS \cdot u \right) \mathcal{N} + \bar{\mathcal{N}}^c \left(iv \cdot D + gS \cdot u \right) \mathcal{N}^c + O\left(m^{-1}\right). \tag{4.16}$$

Note that even though the two kinetic terms originate from the same term in the relativistic Lagrangian there is no double counting having both terms normalized to have a coefficient of 1. This is because the expansion is performed, of the same Dirac kinetic term, around points in two disjoint regions in p^μ space. Saying the same thing another way, when integrating over all on-shell momenta in the Dirac kinetic term there exists two spots in momentum space, around $\pm m$, where the kinetic term simplifies to a non-relativistic fermion kinetic term, see fig. 1.

[17] There are more rigorous ways to derive this but they have the same conclusion.

[18] The sign convention for the negative mode, $p^\mu = -mv^\mu - k^\mu$, is so that k (rather than $-k$) comes out as the residual momentum of the antineutron.

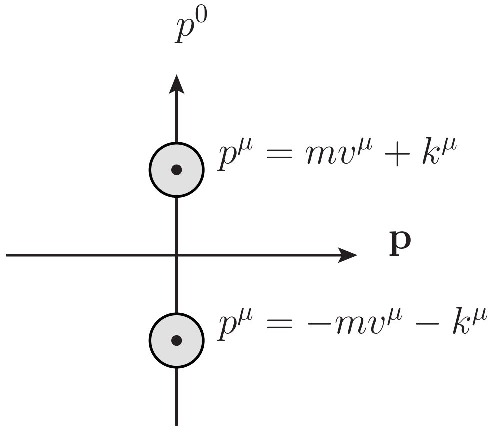


Figure 1: The momentum space plane of the heavy baryon expansions. All on-shell momenta are integrated over in the Dirac kinetic term. Each of the gray circles corresponds to a HBChPT expansion.

4.4 Renormalization in ChPT

This thesis will treat loop diagram contributions to the neutron-antineutron transition due to the exchange of virtual pions. These diagrams are divergent and thus require renormalization. The specific scheme chosen here is *dimensional regularization*. A short discussion treating renormalization and dimensional regularization can be found in app. C.1. A more complete analysis can be found in e.g. chapters 6, 7 and 10 of ref. [11].

It is worth mentioning that the renormalization of ChPT differs from the one used in e.g. QED. QED is a renormalizable theory which means that all divergences can be renormalized to finite values by absorbing all the infinities occurring in diagrams by suitable definitions of the (bare) charge and mass. ChPT, on the other hand, is a non-renormalizable theory which means that all infinities from diagrams cannot be absorbed into a finite number of parameters. This is no problem in the practical case since one may cancel all infinities from loop diagrams at $O(p^n)$ by a redefinition of the (finite number of) coupling constants of the vertices at $O(p^n)$.

4.5 Loop Integrals at Finite Volume

In order to estimate the effects arising from the finite volume of lattice QCD simulations, one can perform ChPT at finite volume and determine the differences from the infinite volume calculation. Loop integrals at finite volume are treated in ref. [24] and the integrals used here are taken from this reference.

At finite volume, of side length L, with periodic boundary conditions momentum is quantized, $p_n = \frac{2\pi n}{L}$ for integer n, and thus loop integrals are replaced by Riemann sums according to[19]

$$\int \frac{d^d p}{(2\pi)^d} F(p) \to \frac{1}{L^{d-1}} \sum_{\mathbf{p}} \int \frac{dp^0}{(2\pi)} F(p) \equiv \int_V \frac{d^d p}{(2\pi)^d} F(p). \tag{4.17}$$

This section is using the Euclidean metric (i.e. after Wick rotation, see e.g. app. C.1), where the metric $g^{\mu\nu} = \delta^{\mu\nu}$ rather than $g^{\mu\nu} = \eta^{\mu\nu}$. With $t^\mu \equiv \delta_0^\mu$ we can define $t^{\mu\nu} = \delta^{\mu\nu} - t^\mu t^\nu$, the space part of the metric.

In app. D.2 and in ref. [24] it is shown that

$$\begin{aligned} \int_V \frac{d^d p_E}{(2\pi)^d} F(p_E) &= \sum_{l_E} \int \frac{d^d p_E}{(2\pi)^d} F(p_E) e^{ip_E \cdot l_E} \\ &= \int \frac{d^d p_E}{(2\pi)^d} F(p_E) + \sum_{l_E : l_E \neq 0} \int \frac{d^d p_E}{(2\pi)^d} F(p_E) e^{ip_E \cdot l_E}, \end{aligned}$$

where $l_{E\mu}$ is a four-vector of only spatial entries, each being an integer times L. The first term, the $l_E = 0$ mode, can be recognized as the infinite volume version of the loop integral and thus the second term is the finite volume correction. The Euclidean integrals at finite volume are denoted

$$[X]^n(M^2) = \int_V \frac{d^d p_E}{(2\pi)^d} \frac{X}{(p_E^2 + M^2)^n} = \sum_{l_E} \int \frac{d^d p_E}{(2\pi)^d} \frac{X}{(p_E^2 + M^2)^n} e^{ip_E \cdot l_E}, \tag{4.18}$$

where X is either 1, p^μ or $p^\mu p^\nu$. The $l = 0$ term corresponds to the infinite volume version, $[X]^n_\infty(M^2)$. The full integral in finite volume can be written $[X]^n(M^2) = [X]^n_\infty(M^2) + [X]^n_V(M^2)$ where the sum excluding $l = 0$, denoted by primed summation, is the finite volume correction

$$[X]^n_V(M^2) = \sum_{l_E}{}' \int \frac{d^d p_E}{(2\pi)^d} \frac{X}{(p_E^2 + M^2)^n} e^{ip_E \cdot l_E}. \tag{4.19}$$

The integrals in eq. (4.19) can be expressed as integrals over special functions, which was done in ref. [24] with the (for this thesis relevant)

[19]Note that the volume of *space* is finite but time is kept infinite. That the time dimension is larger is done in the lattice simulations as well. For the lattice used in ref. [13] the dimensions are $32^3 \times 256$.

results: the tadpole integral $[1]^n_V(M^2)$ and the tensor integral $[p^\mu p^\nu]^n_V(M^2)$. The result is that the tadpole integral is [24]

$$[1]^n_V(M^2) = \frac{(L^2/4)^{n-d/2}}{\Gamma(n)(4\pi)^{d/2}} \int_0^\infty d\lambda \lambda^{n-1-d/2} e^{-\lambda M^2 L^2/4} \left[\theta_{30}\left(e^{-1/\lambda}\right)^3 - 1\right], \tag{4.20}$$

and the tensor integral is

$$\begin{aligned} [p^\mu p^\nu]^n_V(M^2) &= \frac{(L^2/4)^{n-d/2}}{\Gamma(n)(4\pi)^{d/2}} \int_0^\infty d\lambda \lambda^{n-2-d/2} e^{-\lambda M^2 L^2/4} \\ &\cdot \left\{ \frac{\delta^{\mu\nu}}{2}\left[\theta_{30}\left(e^{-1/\lambda}\right)^3 - 1\right] - \frac{t^{\mu\nu}}{\lambda}\left[\theta_{30}\left(e^{-1/\lambda}\right)^2 \theta_{32}\left(e^{-1/\lambda}\right)\right] \right\}, \end{aligned} \tag{4.21}$$

where θ_{30} and θ_{30} are Jacobi theta related functions, and are given in app. D.4. For a vector, S_μ, with zero time component the contraction can be written $S_\mu S_\nu [p^\mu p^\nu]^n_V(M^2) = S^2 [p^2]^n_V(M^2)$ where

$$\begin{aligned} [p^2]^n_V(M^2) &= \frac{(L^2/4)^{n-d/2}}{\Gamma(n)(4\pi)^{d/2}} \int_0^\infty d\lambda \lambda^{n-2-d/2} e^{-\lambda M^2 L^2/4} \\ &\cdot \left\{ \frac{1}{2}\left[\theta_{30}\left(e^{-1/\lambda}\right)^3 - 1\right] - \frac{1}{\lambda}\left[\theta_{30}\left(e^{-1/\lambda}\right)^2 \theta_{32}\left(e^{-1/\lambda}\right)\right] \right\}. \end{aligned} \tag{4.22}$$

In order to translate from this section to Minkowski signature one can use the substitutions

$$d^d q = i d^d q_E, \quad \text{and} \quad q \cdot k = -q_E \cdot k_E. \tag{4.23}$$

5 Neutron-Antineutron Transition Terms

Below the construction of the Lagrangian of the n$\bar{\text{n}}$ oscillation sector in ChPT is discussed. The degrees of freedom used are the nucleons, ψ, the antinucleons ψ^c, the mesons, u, all spurions of chiral QCD, and the spurions of the six-quark operator sector, and using those the most general Lagrangian is constructed. This EFT, extending ChPT, should reproduce the same physics at low energies as the theory of QCD with six-quark operators added. Furthermore, the interactions that give relevant contributions to the oscillation processes are listed and a power counting of the transition is performed.

5.1 The Oscillation Lagrangian at $O(p^2)$

The general Lagrangian allowing for neutron-antineutron oscillations at $O(p^2)$ is of the form

$$\mathcal{L}^{(2)}_{\pi n\bar{n}} = \mathcal{L}_2 + \mathcal{L}^{(1)}_{\pi N\bar{N}} + \sum_{k=0}^{2} \mathcal{L}^{(k)}_{n\bar{n}} \tag{5.1}$$

where $\mathcal{L}^{(k)}_{n\bar{n}}$, the oscillation sector of $O(p^k)$, is the sum of all two unit-baryon violating, Dirac bilinear vertices that can be coupled to the spurions, that were introduced at the quark level, to form chiral singlets. In the following the vertices that can enter will be discussed.

5.2 Transition Vertex Structure

When working on the level of nucleons, the operator that turns a neutron into an antineutron is a bilinear of the form $(\bar{\psi}^c)_{i_B}(\psi)_{j_B}$. Since we wish to couple chiral irrep spurions to these terms, the operators $(u\psi)_{i_R} = u^{i_B}_{i_R}\psi_{i_B}$ and analogously $(u^\dagger\psi)_{i_L}$, $(u\bar{\psi}^c)_{i_R}$ and $(u^\dagger\bar{\psi}^c)_{i_L}$ are more useful building blocks. Further building blocks are U and u_μ and all possible derivatives D_μ. In order to have a power counting scheme even when D_μ acts on the nucleon field, we write down the vertices using HBChPT. In the HBChPT formulation the vertex $(\bar{\psi}^c)_{i_B}(\psi)_{j_B}$ is replaced by $(\bar{\mathcal{N}}^c)_{i_B}(\mathcal{N})_{j_B}$. Finally, any vertex can include the quark mass spurion χ or the field strength spurions $l_{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu]$, $r_{\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]$ and $f^{\mu\nu}_{\pm} = u l_{\mu\nu} u^\dagger \pm u^\dagger r_{\mu\nu} u$.

In the construction of the terms, the lowest order equation of motion can be substituted into the Lagrangian to eliminate some terms. This is essentially done by non-linear field redefinition, as is done in ref. [25], in the functional formulation of QFT, $\phi \to \phi + \delta\phi$. This redefinition induces the addition of a new term, $\displaystyle\int d^4x\,\delta\phi(x)\frac{\delta S[\phi]}{\delta\phi(x)}$, to the action. The equations of motion are $0 = \frac{\delta S}{\delta\phi(x)}$ and thus terms proportional to e.g. $\frac{\delta \mathcal{L}_2}{\delta\phi(x)}$, the lowest order equations of motion, can be removed by a redefinition of the fields and a change of the higher order Lagrangian. For details see ref. [26] and app. A of ref. [27]. The types of terms that are eliminated this way are those proportional to $v\cdot D\mathcal{N}$, $v\cdot D\mathcal{N}^c$ and $D_\mu u^\mu$.

5.3 Mesons and Spurions: Independent Tensors

In order to form the higher isospin vertices, e.g. the $(3_L, 5_R)$ or $(7_L, 1_R)$, more open indices than the ones of $(u^\dagger\bar{\psi}^c)_{i_L}(u^\dagger\psi)_{j_L}$ will be needed. Those must be supplied by either a U, a χ or their hermitian conjugates. To simplify the procedure, all $SU(2)$ indicies are lowered, so that the U is implemented as $(Ui\tau^2)_{i_R i_L} = U_{i_R}^{j_L}\epsilon_{j_L i_L}$. One might be tempted to think that $(Ui\tau^2)_{i_R i_L}$ and $(U^\dagger i\tau^2)_{i_L i_R}$ are independent, but they are, in fact, linked by the identity $\tau^a\tau^2 = -\tau^2(\tau^a)^*$. This is easiest shown by iteratively using said identity in the exponential parametrization:

$$Ui\tau^2 = \sum_{N=0}^{\infty}\frac{1}{N!}(i\phi^a\tau^a/F)^N i\tau^2 = i\tau^2\sum_{N=0}^{\infty}\frac{1}{N!}(-i\phi^a\tau^{a*}/F)^N, \tag{5.2}$$

and since $(-i\phi^a\tau^{a*}/F)^N = [(i\phi^a\tau^a/F)^N]^*$ this can be written as

$$Ui\tau^2 = i\tau^2\sum_{N=0}^{\infty}\frac{1}{N!}[(i\phi^a\tau^a/F)^N]^* = i\tau^2 U^* = i\tau^2[U^\dagger]^T = -[U^\dagger i\tau^2]^T. \tag{5.3}$$

Hence, $(Ui\tau^2)_{i_R i_L} = -(U^\dagger i\tau^2)_{i_L i_R}$ and only one of them is needed.

In the discussion above the unitarity of U played an important role. This suggests that $(\chi i\tau^2)_{i_R i_L}$ and $(\chi^\dagger i\tau^2)_{i_L i_R}$ might be independent objects and in fact they are. To show this note that χ is a general 2×2 complex matrix and can thus be written as $\chi = (a^0 + ib^0)\mathbf{1} + (a^a + ib^a)\tau^a$ which leads to $\chi^\dagger = (a^0 - ib^0)\mathbf{1} + (a^a - ib^a)\tau^a$. But

$$\begin{aligned}
[\chi i\tau^2]^T &= [i\tau^2(a^0 + ib^0)\mathbf{1} - i\tau^2(a^a + ib^a)\tau^{a*}]^T && (5.4)\\
&= -(a^0 + ib^0)\mathbf{1}i\tau^2 + (a^a + ib^a)\tau^a i\tau^2, && (5.5)
\end{aligned}$$

which is completely independent of $\chi^\dagger i\tau^2$. Therefore the new spurion $(\tilde{\chi})_{i_L}^{i_R} = (\tau^2\chi^T\tau^2)_{i_L}^{i_R} = \epsilon_{i_L j_L}\chi_{j_R}^{j_L}\epsilon^{i_R j_R}$, and its hermitian conjugate, are introduced. The chiral transformation properties of $\tilde{\chi}$ is as the index structure suggests, i.e. like $U^\dagger$. Thus, when constructing the vertices χ, $\chi^\dagger$, $\tilde{\chi}$, and $\tilde{\chi}^\dagger$ must be treated as independent spurions[20]. Note that the $\tilde{\chi}$ and $\tilde{\chi}^\dagger$ do not give new independent traces with U and $U^\dagger$ since e.g.

$$\mathrm{Tr}[\tilde{\chi}U] = \mathrm{Tr}[\tau^2\chi^T\tau^2 U] = \mathrm{Tr}[\chi^T\tau^2 U\tau^2] = -\mathrm{Tr}[\chi^T U^*] = -\mathrm{Tr}[U^\dagger\chi]. \tag{5.6}$$

5.4 The Vertices of the Oscillation Sector

A list of compatible vertices, constructed using the arguments above, can be found in app. E. The list contains some 130 terms (depending on the counting procedure since all terms in the $(3_L, 1_R)$ sector are coupled to all of the three spurions ω_L, θ_L and φ_L). Furthermore the list given is only half of

[20]When χ is set to $2BM_q$, which is real and symmetric, then all spurions acquire the same value so the independence is only during the construction of the Lagrangian.

the occurring vertices since there is an identical list that can be obtained by the exchange $L \leftrightarrow R$. However, as will be seen below, when power counting and calculations are performed only a few of those vertices contribute to the oscillation at $O(p^2)$ and the contributing ones often turn out to yield identical results. All terms are not completely independent from each other because of the fact that the model should be the limit of a relativistically covariant theory, implying some relations between the couplings. This can be dealt with by either writing down all vertices in the relativistic formalism (which does not have a power counting) before taking the HBChPT limit, or by reparametrization invariance [28].

5.5 Contributing Transition Vertices

As mentioned previously, not all the interaction terms, in the Lagrangian for neutron oscillations up to $O(p^2)$, contribute to the oscillation, when it is calculated at $O(p^2)$. Almost all vertices give vanishing contributions because they either have the derivative of a meson field or a nucleon field. A derivative of a meson field will cause a momentum dependent emission of a pion (which is at least $O(p)$) and since the pion cannot be in the out state, it must be part of a loop which will add an extra two powers of p. The nucleon field derivative, on the other hand, brings down a residual nucleon momentum, $k^\mu = p^\mu - mv^\mu$, which is zero in the nucleon rest frame (where the amplitude is evaluated).

Therefore the only contributing interaction terms to the $O(p^2)$ oscillation will be the vertices of $O(p^0)$ (with $O(p^2)$ loops and field renormalization) and the χ type terms.

In the $(3_L, 1_R)$ spurion sector one has the following $O(p^0)$ terms

$$\begin{aligned} &\lambda_{(1)}\omega_L^{i_L j_L}(u^\dagger \bar{\mathcal{N}}^c)_{i_L}(u^\dagger \mathcal{N})_{j_L} \\ &\lambda_{(2)}\theta_L^{i_L j_L}(u^\dagger \bar{\mathcal{N}}^c)_{i_L}(u^\dagger \mathcal{N})_{j_L} \\ &\lambda_{(3)}\varphi_L^{i_L j_L}(u^\dagger \bar{\mathcal{N}}^c)_{i_L}(u^\dagger \mathcal{N})_{j_L}, \end{aligned} \tag{5.7}$$

and at $O(p^2)$ the following terms contribute:

$$
\begin{aligned}
&(\rho_{(1)}\omega_L^{i_Lj_L}+\upsilon_{(1)}\theta_L^{i_Lj_L}+\tau_{(1)}\varphi_L^{i_Lj_L})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(\chi^\dagger u\mathcal{N})_{j_L},\\
&(\rho_{(2)}\omega_L^{i_Lj_L}+\upsilon_{(2)}\theta_L^{i_Lj_L}+\tau_{(2)}\varphi_L^{i_Lj_L})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(\tilde{\chi} u\mathcal{N})_{j_L},\\
&(\rho_{(3)}\omega_L^{i_Lj_L}+\upsilon_{(3)}\theta_L^{i_Lj_L}+\tau_{(3)}\varphi_L^{i_Lj_L})(\chi^\dagger u\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L}\\
&(\rho_{(4)}\omega_L^{i_Lj_L}+\upsilon_{(4)}\theta_L^{i_Lj_L}+\tau_{(4)}\varphi_L^{i_Lj_L})(\tilde{\chi} u\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L}\\
&(\rho_{(5)}\omega_L^{i_Lj_L}+\upsilon_{(5)}\theta_L^{i_Lj_L}+\tau_{(5)}\varphi_L^{i_Lj_L})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(U^\dagger\chi u^\dagger\mathcal{N})_{j_L},\\
&(\rho_{(6)}\omega_L^{i_Lj_L}+\upsilon_{(6)}\theta_L^{i_Lj_L}+\tau_{(6)}\varphi_L^{i_Lj_L})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(U^\dagger\tilde{\chi}^\dagger u^\dagger\mathcal{N})_{j_L},\\
&(\rho_{(7)}\omega_L^{i_Lj_L}+\upsilon_{(7)}\theta_L^{i_Lj_L}+\tau_{(7)}\varphi_L^{i_Lj_L})(U^\dagger\chi u^\dagger\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L},\\
&(\rho_{(8)}\omega_L^{i_Lj_L}+\upsilon_{(8)}\theta_L^{i_Lj_L}+\tau_{(8)}\varphi_L^{i_Lj_L})(U^\dagger\tilde{\chi}^\dagger u^\dagger\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L},\\
&(\rho_{(9)}\omega_L^{i_Lj_L}+\upsilon_{(9)}\theta_L^{i_Lj_L}+\tau_{(9)}\varphi_L^{i_Lj_L})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L}\frac{1}{2}\mathrm{Tr}[\chi^\dagger U],\\
&(\rho_{(10)}\omega_L^{i_Lj_L}+\upsilon_{(10)}\theta_L^{i_Lj_L}+\tau_{(10)}\varphi_L^{i_Lj_L})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L}\frac{1}{2}\mathrm{Tr}[U^\dagger\chi].
\end{aligned}
\tag{5.8}
$$

In the $(3_L, 5_R)$ sector the $O(p^0)$ term is given by

$$
\kappa\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}
\tag{5.9}
$$

and at $O(p^2)$ the following terms contribute:

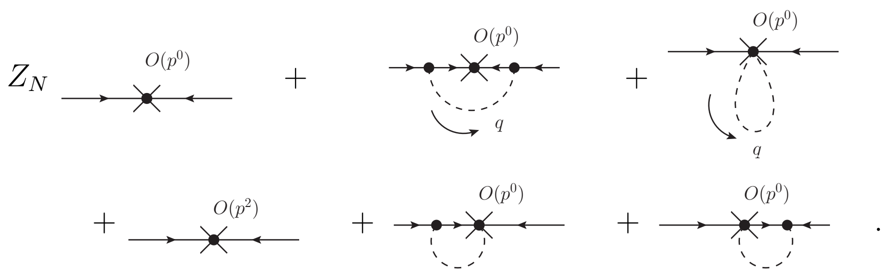
$$
\begin{aligned}
&\nu_{(1)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(u\bar{\mathcal{N}}^c)_{i_R}(\chi u^\dagger\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\nu_{(2)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(\chi u^\dagger\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L},\\
&\nu_{(3)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(u\bar{\mathcal{N}}^c)_{i_R}(\tilde{\chi}^\dagger u^\dagger\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L},\\
&\nu_{(4)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(\tilde{\chi}^\dagger u^\dagger\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\nu_{(5)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(u\bar{\mathcal{N}}^c)_{i_R}(U\chi^\dagger u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\nu_{(6)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(U\chi^\dagger u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\nu_{(7)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(u\bar{\mathcal{N}}^c)_{i_R}(U\tilde{\chi} u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\nu_{(8)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(U\tilde{\chi} u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L},\\
&\nu_{(9)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(u\bar{\mathcal{N}}^c)_{i_R}(\mathcal{N})_{j_R}(\chi i\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L},\\
&\nu_{(10)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(u\bar{\mathcal{N}}^c)_{i_R}(\mathcal{N})_{j_R}(\tilde{\chi}^\dagger i\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L},\\
&\nu_{(11)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\frac{1}{2}\mathrm{Tr}[\chi^\dagger U],\\
&\nu_{(12)}\zeta_L^{i_Rj_Rk_Rl_Rk_Ll_L}(u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\frac{1}{2}\mathrm{Tr}[U^\dagger\chi]
\end{aligned}
\tag{5.10}
$$

Note that at this stage, the parameters ρ, υ, τ and ν all contain both finite pieces (i.e. new low energy constants) and infinite pieces to cancel the one-loop infinities of the $O(p^0)$ sector.

In the isospin symmetry limit ($m_u = m_d$) the isospin 3 operator Q_4 does not contribute. Q_4 transforms as a 7 and thus $Q_4|\text{n}\rangle$ transforms as a $7 \otimes 2 = 8 \oplus 6$. Since this Clebsch-Gordan decomposition does not contain a 2 there can be no overlap between $Q_4|\text{n}\rangle$ and $|\bar{\text{n}}\rangle$. This is a special case of the Wigner-Eckart theorem, see chapter 4 of ref. [3]. Considering isospin symmetry breaking this argument no longer holds and the operator can contribute. The contribution is, however, still suppressed since the isospin breaking parameter is the deviation of χ from its isospin limit. This part transforms under isospin as a 3 (since it has a 2_L and 2_R index, so under isospin it is a $2 \otimes 2 = 3 \oplus 1$, where 1 is the isospin limit version of χ) and thus two powers of χ, i.e. $O(p^4)$, is required for a contribution of Q_4. Therefore no terms from the $(7_L, 1_R)$ sector can contribute since this thesis uses the isospin symmetry approximation.

5.6 Power Counting for the $\text{n}\bar{\text{n}}$ Transition

Here a power counting for the $\text{n}\bar{\text{n}}$ transition process is performed using the vertices from sec. 5.5. With an $O(p^2)$ calculation it is possible to have tree order transition using $O(p^0)$ vertices, a transition using an $O(p^0)$ vertex together with a p^2 meson loop, and finally the vertices at $O(p^2)$. One further modification comes in when computing the $O(p^0)$ tree level transition namely the field renormalization of the external legs in the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula as can be seen through eq. (C.10). Field renormalization and the LSZ formula are briefly introduced in apps. C.2 and C.3. The nucleon self-energy is an $O(p^3)$ process but it adds a nucleon propagator, eq. (4.14), at $O(p^{-1})$, making field renormalization, Z_N, appear at $O(p^2)$. The contributing processes are best summarized by the following diagrams (where each graph is a class of diagrams rather than a process):



. (5.11)

The crossed vertices represent $\text{n}\bar{\text{n}}$ transition vertices and the non-crossed are axial vector vertices from eq. (4.13). For future reference, the second term in eq. (5.11) will be called a "loop across" diagram and the third term a "vertex tadpole" diagram.

6 Amplitude Calculations for $\mathrm{n\bar{n}}$ Transitions

In this section the constructed Lagrangian is used to calculate the $\mathrm{n} \to \bar{\mathrm{n}}$ transition amplitude at $O(p^2)$. The loop integral calculations done in this section rely on previously known one loop integrals in ChPT, and the source used is appendix C of ref. [17]. The standard integrals used are listed in app. D.1, for reference.

6.1 General Structure in Spinor and Isospin space

The Wick contraction of a field operator on the initial one neutron state is

$$\underbracket{\psi_{i_B}(x)|\mathrm{n}:}\,p,s\rangle = u^s(p)e^{-ip\cdot x}\delta_{i_B}^{-1/2}|0\rangle. \tag{6.1}$$

The same equation holds for $\bar{\mathrm{n}}$ with ψ replaced by ψ^c:

$$\underbracket{\psi^{c\ i_B}(x)|\bar{\mathrm{n}}:}\,p,s\rangle = u^s(p)e^{-ip\cdot x}\delta^{i_B}_{-1/2}|0\rangle, \tag{6.2}$$

which turns into

$$\langle\bar{\mathrm{n}}\underbracket{:\,p,s|\bar{\psi}^c_{i_B}}(x) = \langle 0|\bar{u}^s(p)e^{ip\cdot x}\delta_{i_B}^{-1/2}. \tag{6.3}$$

In every transition amplitude the structure

$$\langle\bar{\mathrm{n}}:p',s'|\bar{\psi}^c_{a\ i_B}(x)\mathcal{O}_{ab}\psi_{b\ j_B}(x)|\mathrm{n}:p,s\rangle, \tag{6.4}$$

will appear, where a and b are Dirac spinor indices and $\mathcal{O}$ is some operator in spinor space (usually δ_{ab}). Using the fact that momentum and spin conserving deltas will always be multiplicative factors, the matrix element for the transition can be written as[21]

$$i\mathcal{M} = -i\mathcal{A}^{i_Bj_B}\delta_{i_B}^{-1/2}\delta_{j_B}^{-1/2}\bar{u}^s_a(p)\mathcal{O}_{ab}u^s_b(p), \tag{6.5}$$

where $\mathcal{A}^{i_Bj_B}$ encapsulates the remaining dynamic of the amplitude. In the frequently occurring special case $\mathcal{O}_{ab} = \delta_{ab}$ this reduces to

$$i\mathcal{M} = -i\mathcal{A}^{i_Bj_B}2m\delta_{i_B}^{-1/2}\delta_{j_B}^{-1/2}. \tag{6.6}$$

In HBChPT the initial state contraction is

$$\underbracket{\mathcal{N}_{i_B}(x)|\mathrm{n}:}\,p,s\rangle = e^{imv\cdot x}P_{v+}\underbracket{\psi_{i_B}(x)|\mathrm{n}:}\,p,s\rangle = P_{v+}u^s(p)e^{-ik\cdot x}\delta_{i_B}^{-1/2}|0\rangle. \tag{6.7}$$

[21]No summation on s is intended here.

By the choice $v^\mu = \delta_0^\mu$, i.e choosing the rest frame of the initial n, and the Dirac representation of the gamma matrices, the projected basis spinor can be written as

$$P_{v+}u^s(p) = \frac{1}{2}(1+\gamma^0)\sqrt{E+m}\begin{pmatrix} \xi^s \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\xi^s \end{pmatrix}, \tag{6.8}$$

which, in the explicit Dirac basis becomes

$$P_{v+}u^s(p) = \sqrt{E+m}\begin{pmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \xi^s \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\xi^s \end{pmatrix} \simeq \sqrt{2m}\begin{pmatrix} \xi^s \\ 0 \end{pmatrix}, \tag{6.9}$$

leading to the intuitive conclusion that the external on-shell spinor in the non-relativistic limit is the two-component Pauli spinor (the factor $\sqrt{2m}$ is present only due to the relativistic normalization of momentum states).

6.2 Axial Vertex Factors for Various Fields

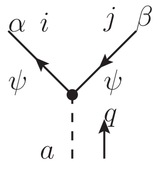
In order to compute the loop diagrams, one will need the axial vector coupling expressed for both $\mathcal{N}$ and $\mathcal{N}^c$. The axial interaction comes from the term

$$\frac{1}{2}g\bar{\psi}^i\gamma^\mu\gamma^5(u_\mu)_i^j\psi_j. \tag{6.10}$$

Since (for $l = r = 0$) $u_\mu = i[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger]$, and u can be expanded in its exponential parametrization yielding $\partial_\mu u = \partial_\mu\left(1 + \frac{i}{2F}\phi + \cdots\right) = \frac{i}{2F}\partial_\mu\phi + \cdots$ which in turn gives

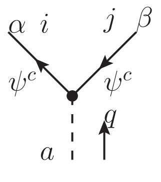
$$u_\mu = -\frac{1}{F}\partial_\mu\phi + \cdots = -\frac{1}{F}\partial_\mu\phi^a\tau^a + \cdots . \tag{6.11}$$

The vertex factor of this interaction can be figured out, e.g. by computing a momentum space three point function, and it is



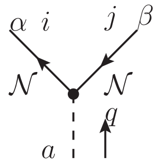
$$= -\frac{g}{2F}q_\mu(\tau^a)_i^j(\gamma^\mu\gamma^5)_{\alpha\beta}, \tag{6.12}$$

where α and β are spinor indices. Using the charge conjugation identity $\bar{\psi}^i\gamma^\mu\gamma^5\psi_j = (\bar{\psi}^c)_j\gamma^\mu\gamma^5(\psi^c)^i$ the "antinucleon" version of this vertex is



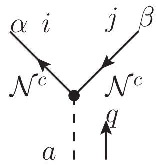
$$= -\frac{g}{2F}q_\mu(\tau^a)_j^i(\gamma^\mu\gamma^5)_{\alpha\beta}. \tag{6.13}$$

In HBChPT the corresponding vertex factors can be obtained from the above factors by multiplying with the projection operator P_{v+} from the left and right and using $P_{v+}\gamma^\mu\gamma^5 P_{v+} = 2S^\mu$ yielding



$$= -\frac{g}{F} q_\mu (\tau^a)_i^j (S^\mu)_{\alpha\beta} \tag{6.14}$$

and



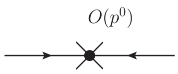
$$= -\frac{g}{F} q_\mu (\tau^a)_j^i (S^\mu)_{\alpha\beta}. \tag{6.15}$$

6.3 Tree Order Transitions with $(3_L, 1_R)$

The simplest vertex that can induce transitions is

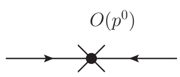
$$\lambda_{(1)}\omega_L^{i_L j_L}(u^\dagger \bar{\mathcal{N}}^c)_{i_L}(u^\dagger \mathcal{N})_{j_L}. \tag{6.16}$$

Using the general form of eq. (6.6), and that to lowest order $(u^\dagger)_{i_L}^{i_B} = \delta_{i_L}^{i_B}$, yields a matrix element of the form



$$= i\lambda_{(1)}2m(\omega_L)^{i_L j_L}\delta_{i_L}^{-1/2}\delta_{j_L}^{-1/2} = -i\lambda_{(1)}2m\omega_L. \tag{6.17}$$

The two further θ_L and φ_L vertices have identical structure and the sum of them can immediately be written down as

$\sum$ 

$$= i2m(\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L). \tag{6.18}$$

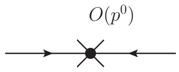
All that remains now is to multiply with the field renormalization for the nucleons in HBChPT [17]:

$$Z_N = 1 + \frac{4a_3 m_\pi^2}{m^2} - \frac{9g^2 m_\pi^2}{4(4\pi F)^2}\left[R + \ln\left(\frac{m_\pi^2}{\mu^2}\right) + \frac{2}{3}\right], \tag{6.19}$$

where a_3 is a low energy constant of the $O(p^3)$ Lagrangian and

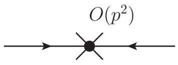
$$R = \frac{2}{d-4} + \gamma_E - \ln(4\pi) - 1. \tag{6.20}$$

The full result is

$$Z_N \sum$$ 

$$= i2m(\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L)$$
$$\cdot\left(1 + \frac{4a_3 m_\pi^2}{m^2} - \frac{9g^2 m_\pi^2}{4(4\pi F)^2}\left[R + \ln\left(\frac{m_\pi^2}{\mu^2}\right) + \frac{2}{3}\right]\right). \qquad (6.21)$$

For the χ type vertices one proceeds analogously with the result that

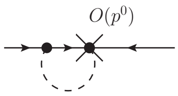


$$= i2mm_\pi^2 \sum_{A=1}^{10} (\rho_{(A)}\omega_L + \upsilon_{(A)}\theta_L + \tau_{(A)}\varphi_L). \qquad (6.22)$$

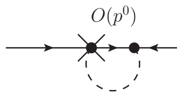
Notice that in this case, the tree order transitions are not multiplied by a field renormalization, since it comes from a sector where the vertices are at $O(p^2)$, and thus field renormalization enters only at higher orders for those.

6.4 Momentum Independent Pion Emission in a $\mathrm{n\bar{n}}$ Vertex

Every vertex has a factor $(u^\dagger \bar{\psi}^c)_{i_L}(u^\dagger \psi)_{j_L}$, or $(u\bar{\psi}^c)_{i_R}(u\psi)_{j_R}$, which means that if one expands out the u as $(1 + i\phi/2F - \phi^2/8F^2 + \cdots)$ it becomes apparent that each vertex has the possibility of a momentum independent pion emission. The diagrams arising from those emissions are



and



. (6.23)

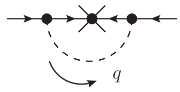
The loop part of these processes includes an integral [17]

$$\int \frac{d^4q}{(2\pi)^4} q^\mu f(q^2, v\cdot q) \sim v^\mu. \qquad (6.24)$$

The other end of the pion line will be contracted into an axial vertex, which means that it is multiplied by a S_μ giving $v \cdot S$ which is zero. Thus the diagrams discussed in this section will have a vanishing contribution.

6.5 Loop Across the Transition Vertex in $(3_L, 1_R)$

The first loop diagram that is evaluated here is the loop across type diagram in the $(3_L, 1_R)$ sector. The pion emitting vertices here are the axial vertices, and thus the vertex factors in eqs. (6.14) and (6.15) will be used. For simplicity, the amplitude is calculated using only one of the 3_L spurions but since they all contribute in an identical fashion they can immediately be summed up if one knows the amplitude for one of them. One of the loop across diagrams has the amplitude



$$
\begin{aligned}
= \\
&\sqrt{2m}\xi^\dagger \mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \left(\frac{g}{F} q^\mu S_\mu\right) (\tau^a)^{-1/2}_{i_L} \frac{i}{(v\cdot q + i\epsilon)} \frac{i}{(q^2 - m_\pi^2 + i\epsilon)} \\
&\cdot (i\lambda_{(1)})\omega^{i_L j_L} \frac{i}{(v\cdot q + i\epsilon)} \left(-\frac{g}{F} q^\nu S_\nu\right) (\tau^a)^{-1/2}_{j_L} \sqrt{2m}\xi \\
&= 2m\xi^\dagger S_\mu S_\nu \xi \frac{g^2}{F^2} (\tau^a)^{-1/2}_{i_L} (\tau^a)^{-1/2}_{j_L} \lambda_{(1)} \omega^{i_L j_L} \\
&\cdot \mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \frac{1}{(v\cdot q + i\epsilon)^2} \frac{q^\mu q^\nu}{(q^2 - m_\pi^2 + i\epsilon)} \\
&\equiv 2m\xi^\dagger S_\mu S_\nu \xi \frac{g^2}{F^2} (\tau^a)^{-1/2}_{i_L} (\tau^a)^{-1/2}_{j_L} \lambda_{(1)} \omega^{i_L j_L} (-i) K^{\mu\nu}_{\pi NN}(0), \qquad (6.25)
\end{aligned}
$$

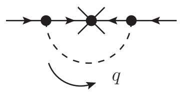
where $K^{\mu\nu}_{\pi NN}$ is an integral defined in eq. (D.21). The value of this integral is found in app. D.1. It has a piece proportional to $\eta^{\mu\nu}$ and one to $v^\mu v^\nu$. Because the spin S^μ is orthogonal to v^μ, only the $\eta^{\mu\nu}$ piece survises the contraction. That piece is proportional to $S_\mu S^\mu = (1-d)/4$ which can be verified by the identity $\{S^\mu, S^\nu\} = (v^\mu v^\nu - \eta^{\mu\nu})/2$ inserted into

$$
S^\mu \eta_{\mu\nu} S^\nu = \eta_{\mu\nu} \frac{1}{2} \{S^\mu, S^\nu\} = \frac{1}{4}(v^2 - \eta_{\mu\nu}\eta^{\mu\nu}) = \frac{1}{4}(1-d) = \frac{\epsilon}{4} - \frac{3}{4}. \qquad (6.26)
$$

Note that the extra ϵ term will give a contribution since $\epsilon R = \epsilon(-2/\epsilon + O(1)) = (-2 + O(\epsilon))$. The Pauli matrices can be simplified by the Fierz identity

$$
(\tau^a)^k_i (\tau^a)^l_j = 2\delta^l_i \delta^k_j - \delta^k_i \delta^l_j, \qquad (6.27)
$$

and these simplifications yield



$$
\begin{aligned}
&= -i\frac{3mg^2}{32\pi^2 F^2} (\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L) m_\pi^2 \\
&\cdot \left[\frac{2}{3} + R + \ln\left(\frac{m_\pi^2}{\mu^2}\right)\right], \qquad (6.28)
\end{aligned}
$$

where the contributions from all three spurions have been added. Note that the value of this diagram is proportional to the field renormalization. This is due to the fact that they are both derivatives of $J^{\mu\nu}$ type integrals, see app. D.1. The difference of a factor of three between Z_N and the loop across diagram can be understood by the fact that for external legs $\pi^{\pm}$ and π^0 can all run in the loop while only π^0 can run in the loop across diagram since only neutrons can make transitions to antineutrons (and not protons to antiprotons). The fact that these two contributions are proportional is used when determining Z_N in finite volume.

6.6 Vertex Tadpole in $(3_L, 1_R)$

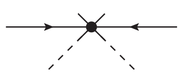
For diagrams of vertex tadpole type one needs the vertex factor for the emission of two momentum independent pions. This can be found by expanding all factors of u to second order in ϕ. In the $(3_L, 1_R)$ case, the factor is $(u^\dagger)^k_i (u^\dagger)^l_j$ and it can be expanded as

$$\begin{aligned} (u^\dagger)^k_i (u^\dagger)^l_j &= \left(1 - \frac{i}{2F}\phi - \frac{1}{8F^2}\phi^2\right)^k_i \left(1 - \frac{i}{2F}\phi - \frac{1}{8F^2}\phi^2\right)^l_j \\ &= -\frac{1}{4F^2}\phi^k_i \phi^l_j - \frac{1}{8F^2}(\phi^2)^k_i \delta^l_j - \frac{1}{8F^2}(\phi^2)^l_j \delta^k_i + \cdots, \end{aligned} \tag{6.29}$$

which can be further simplified using $\phi = \phi^a \tau^a$ and $\tau^a \tau^b = \frac{1}{2}\{\tau^a, \tau^b\} + \frac{1}{2}[\tau^a, \tau^b] = \delta^{ab}\mathbf{1} + i\epsilon^{abc}\tau^c$. Hence

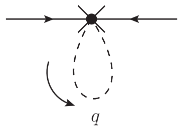
$$\begin{aligned} (u^\dagger)^k_i (u^\dagger)^l_j &= -\frac{1}{8F^2}\phi^a \phi^b [2(\tau^a)^k_i (\tau^b)^l_j + 2\delta^{ab}\delta^k_i \delta^l_j \\ &\quad + i\epsilon^{abc}(\tau^c)^k_i \delta^l_j + i\epsilon^{abc}(\tau^c)^l_j \delta^k_i] + \cdots, \end{aligned} \tag{6.30}$$

and the vertex factor for two pion emissions (for one of three identical spurions) is



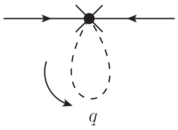
$$\begin{aligned} &= -i\frac{\lambda_{(1)}}{8F^2}\omega^{i_L j_L}[2(\tau^a)^{i_B}_{i_L}(\tau^b)^{j_B}_{j_L} + 2\delta^{ab}\delta^{i_B}_{i_L}\delta^{j_B}_{j_L} \\ &\quad + i\epsilon^{abc}(\tau^c)^{i_B}_{i_L}\delta^{j_B}_{j_L} + i\epsilon^{abc}(\tau^c)^{j_B}_{j_L}\delta^{i_B}_{i_L}]. \end{aligned} \tag{6.31}$$

The vertex tadpole diagram can be evaluated, using this vertex factor, to be



$$
\begin{aligned}
&= 2m\xi^\dagger\xi \frac{-i\lambda_{(1)}\omega_L^{i_l j_L}}{8F^2}[2(\tau^a)_{i_L}^{-1/2}(\tau^a)_{j_L}^{-1/2} \\
&+2\delta^{aa}\delta_{i_L}^{i_B}\delta_{j_L}^{j_B}]\mu^{4-d}\int\frac{d^dq}{(2\pi)^d}\frac{i}{q^2-m_\pi^2+i\epsilon} \\
&= \frac{-im\lambda_{(1)}\omega^{i_L j_L}}{2F^2}[\delta_{i_L}^{-1/2}\delta_{j_L}^{-1/2}+3\delta_{i_L}^{-1/2}\delta_{j_L}^{-1/2}]I_\pi(0) \\
&= \frac{-i2m\lambda_{(1)}\omega_L}{F^2}I_\pi(0) \\
&= \frac{-i2m\lambda_{(1)}\omega_L}{F^2}\frac{m_\pi^2}{16\pi^2}\left[R+\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right], \qquad (6.32)
\end{aligned}
$$

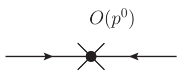
where I_π is an integral defined in app. D.1. All contributions to the transition through a vertex tadpole can now be added up to be



$$
= -\frac{im(\lambda_{(1)}\omega_L+\lambda_{(2)}\theta_L+\lambda_{(3)}\varphi_L)}{8\pi^2F^2}m_\pi^2\left[R+\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right].
$$

6.7 Tree Order Transitions with $(3_L, 5_R)$

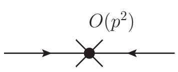
The amplitudes for $(3_L, 5_R)$ are calculated exactly as in the $(3_L, 1_R)$ case but the calculation is more involved due to the six indices of the ζ_R spurion (instead of the previous two):

Z_N 

$$
\begin{aligned}
&= i\kappa 2m\epsilon_{k_L k_R}\epsilon_{l_L l_R}\zeta^{-1/2_R,-1/2_R k_R l_R k_L l_L} \\
&= i\kappa 2m\left[\zeta_{R(1)}-\frac{1}{\sqrt{2}}\zeta_{R(2)}+\frac{1}{\sqrt{6}}\zeta_{R(3)}\right]Z_N. \qquad (6.33)
\end{aligned}
$$

The value of $\epsilon_{k_L k_R}\epsilon_{l_L l_R}\zeta^{-1/2_R,-1/2_R k_R l_R k_L l_L}$ has been determined from the decomposition in app. B.2.

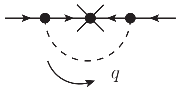
There is no further complication in calculating tree order χ type vertices, and they become

$\sum$ 

$$
= i\sum_{A=1}^{12}\nu_{(A)}2mm_\pi^2\left[\zeta_{R(1)}-\frac{1}{\sqrt{2}}\zeta_{R(2)}+\frac{1}{\sqrt{6}}\zeta_{R(3)}\right].
$$

6.8 Loop Across in $(3_L, 5_R)$

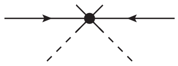
Completely analogously with the previous calculation in the $(3_L, 1_R)$ case the amplitude can be written down:



$$= -i\frac{3mg^2}{32\pi^2 F^2}\kappa\left[\zeta_{R(1)} - \frac{1}{\sqrt{2}}\zeta_{R(2)} + \frac{1}{\sqrt{6}}\zeta_{R(3)}\right]$$
$$\cdot m_\pi^2\left[\frac{2}{3} + R + \ln\left(\frac{m_\pi^2}{\mu^2}\right)\right]. \qquad (6.34)$$

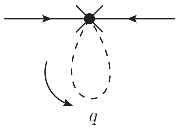
6.9 Vertex Tadpole in $(3_L, 5_R)$

The calculation here follows the pervious vertex tadpole calculation except that the two pion emission vertex, the quadratic term of $(u)^{i_B}_{i_R}(u)^{j_B}_{j_R}(U)^{m_L}_{k_R}(U)^{n_L}_{l_R}$, is more complicated. It is expanded in an analogous way as in sec. 6.6, yielding the vertex factor



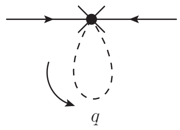
$$= -\frac{i\kappa}{F^2}\epsilon_{k_L m_L}\epsilon_{l_L n_L}\zeta^{i_R j_R k_R l_R k_L l_L}\Bigg\{\frac{5}{4}\delta^{ab}\delta^{i_B}_{i_R}\delta^{j_B}_{j_R}\delta^{m_L}_{k_R}\delta^{n_L}_{l_R}$$
$$+(\tau^a)^{m_L}_{k_R}(\tau^b)^{n_L}_{l_R}\delta^{i_B}_{i_R}\delta^{j_B}_{j_R} + \frac{1}{2}(\tau^a)^{j_B}_{j_R}(\tau^b)^{n_L}_{l_R}\delta^{i_B}_{i_R}\delta^{m_L}_{k_R} + \frac{1}{2}(\tau^a)^{i_B}_{i_R}(\tau^b)^{n_L}_{l_R}\delta^{j_B}_{j_R}\delta^{m_L}_{k_R}$$
$$+\frac{1}{2}(\tau^a)^{i_B}_{i_R}(\tau^b)^{m_L}_{k_R}\delta^{j_B}_{j_R}\delta^{n_L}_{l_R} + \frac{1}{2}(\tau^a)^{j_B}_{j_R}(\tau^b)^{m_L}_{k_R}\delta^{i_B}_{i_R}\delta^{n_L}_{l_R} + \frac{1}{4}(\tau^a)^{i_B}_{i_R}(\tau^b)^{j_B}_{j_R}\delta^{m_L}_{k_R}\delta^{n_L}_{l_R}$$
$$+\frac{1}{2}\epsilon^{abc}(\tau^c)^{n_L}_{l_R}\delta^{i_B}_{i_R}\delta^{j_B}_{j_R}\delta^{m_L}_{k_R}\delta^{n_L}_{l_R} + \frac{1}{2}\epsilon^{abc}(\tau^c)^{m_L}_{k_R}\delta^{i_B}_{i_R}\delta^{j_B}_{j_R}\delta^{l_R}_{n_L}\delta^{n_L}_{l_R}$$
$$+\frac{1}{8}\epsilon^{abc}(\tau^c)^{j_B}_{j_R}\delta^{i_B}_{i_R}\delta^{m_L}_{k_R}\delta^{n_L}_{l_R} + \frac{1}{8}\epsilon^{abc}(\tau^c)^{i_B}_{i_R}\delta^{j_B}_{j_R}\delta^{m_L}_{k_R}\delta^{n_L}_{l_R}\Bigg\}. \qquad (6.35)$$

Forming a tadpole of this vertex yields



$$= \frac{-i2m\lambda\epsilon_{k_L m_L}\epsilon_{l_L n_L}\zeta^{i_R j_R k_R l_R k_L l_L}}{F^2}I_\pi(0)$$
$$\cdot\Bigg\{\delta^{-1/2}_{i_R}\delta^{-1/2}_{j_R}\delta^{m_L}_{k_R}\delta^{n_L}_{l_R} + 2\delta^{-1/2}_{i_R}\delta^{-1/2}_{j_R}\delta^{n_L}_{k_R}\delta^{m_L}_{l_R} + \delta^{-1/2}_{i_R}\delta^{n_L}_{j_R}\delta^{m_L}_{k_R}\delta^{-1/2}_{l_R}$$
$$+\delta^{n_L}_{i_R}\delta^{-1/2}_{j_R}\delta^{m_L}_{k_R}\delta^{-1/2}_{l_R} + \delta^{m_L}_{i_R}\delta^{-1/2}_{j_R}\delta^{-1/2}_{k_R}\delta^{n_L}_{l_R} + \delta^{-1/2}_{i_R}\delta^{m_L}_{j_R}\delta^{-1/2}_{k_R}\delta^{n_L}_{l_R}\Bigg\}, \qquad (6.36)$$

which reduces the expression to



$$= -\frac{im7\kappa}{8\pi^2 F^2} m_\pi^2 \left[\zeta_{(1)} + \frac{1}{\sqrt{2}}\zeta_{(2)} + \frac{1}{\sqrt{6}}\zeta_{(3)}\right] \cdot \left[R + \ln\left(\frac{m_\pi^2}{\mu^2}\right)\right]. \tag{6.37}$$

This concludes the computation of diagrams that contribute to the n$\bar{\text{n}}$ oscillations. Now all the individual results can be summed up and the full contribution can be obtained.

6.10 Finite Volume Corrections and Numerics

Here the finite volume corrections to the loop integrals are deduced. First we must incorporate the nucleon propagator into the form of the standard integrals of sec. 4.5. The procedure used here is similar to the infinite volume analogue of appendix C in ref. [17].

The finite volume loop integrals of this thesis can be written, in the notation of sec. 4.5, as

$$\begin{aligned}
&\int_V \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2 - m_\pi^2)} = i \int_V \frac{d^d q_E}{(2\pi)^d} \frac{1}{(-q_E^2 - m_\pi^2)} = -i[1]^1(m_\pi^2),\\
&\int_V \frac{d^d q}{(2\pi)^d} \frac{q^\mu}{(q^2 - m_\pi^2)} \frac{1}{(v \cdot q)} = -i2 \int_0^\infty dy\ y v^\mu [1]^2(m_\pi^2 + y^2),\\
&\int_V \frac{d^d q}{(2\pi)^d} \frac{q^\mu q^\nu}{(q^2 - m_\pi^2)} \frac{1}{(v \cdot q)^2} = -i8 \int_0^\infty dy\ y\Big([q^\mu q^\nu]^3(m_\pi^2 + y^2) + y^2 v^\mu v^\nu [1]^3(m_\pi^2 + y^2)\Big).
\end{aligned} \tag{6.38}$$

These integrals are deduced in app. D.3. Note that the middle line of eq. (6.38) is still proportional to v^μ so the integrals of the type of sec. 6.4 are still zero due to $v \cdot S = 0$.

The finite volume corrections are

$$I_\pi^V(0) = [1]_V^1(m_\pi^2), \tag{6.39}$$

$$S_\mu S_\nu K_{\pi NN}^{V\ \mu\nu}(0) = 8S^2 \int_0^\infty dy\ y[q^2]^3(m_\pi^2 + y^2) \equiv K_{\pi NN}^V(0). \tag{6.40}$$

I_π^V gives a finite volume modification contribution to the vertex tadpoles while $K_{\pi NN}^V$ modifies the loop across diagrams and the field renormalization. The reason that it modifies the latter is that the nucleon self energy is, like

$S_\mu S_\nu K^{\mu\nu}_{\pi NN}$, proportional to the derivative of $S_\mu S_\nu J^{\mu\nu}_{\pi N}$ (where the derivative comes from taking the residue).

Numerical evaluation of loop integrals, at finite and infinite volume, were performed by implementing the C++ library CHIRON [29]. CHIRON is designed to compute loop integrals (up to two loops) in ChPT. Nucleonic integrals are not contained in the library but they can be implemented by using the integral form in eq. 6.40, and evaluated using the numerical quadrature supplied by CHIRON.

7 The Dependence of the Transition Amplitude on the Pion Mass and on the Volume

The amplitude for $\mathrm{n}\bar{\mathrm{n}}$ transitions can be found by combining all the previously calculated contributions. Especially interesting for guiding lattice simulations will be the functional dependence on the value of the pion mass. Three different types of dependencies on m_π occur: independent of it, proportional to m_π^2, and proportional to the *chiral logarithm* $m_\pi^2 \ln(m_\pi^2)$. Therefore the amplitude can be written as

$$i\mathcal{M}(\mathrm{n} \to \bar{\mathrm{n}}) = i2m \left[a_L(\mathcal{G}) + m_\pi^2 f_L(\mathcal{G}) + m_\pi^2 \ln\left(\frac{m_\pi^2}{\mu^2}\right) h_L(\mathcal{G}) \right] + (L \leftrightarrow R), \tag{7.1}$$

where $\mathcal{G}$ is the set of coupling constants and spurion values in the model. In this amplitude, the infinities arising from loop momenta, the R terms, are canceled against infinite pieces of the $O(p^2)$ couplings $\rho_{(A)}$, $\upsilon_{(A)}$, $\tau_{(A)}$ and $\nu_{(A)}$. Finite pieces of those coupling constants remain in the final amplitude and they are denoted $\rho^r_{(A)}$, $\upsilon^r_{(A)}$, $\tau^r_{(A)}$ and $\nu^r_{(A)}$. The functional dependence found is

$$\begin{aligned} a_L(\mathcal{G}) &= (\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L) \\ &\quad + \kappa \left[\zeta_{R(1)} - \frac{1}{\sqrt{2}}\zeta_{R(2)} + \frac{1}{\sqrt{6}}\zeta_{R(3)} \right], \end{aligned} \tag{7.2}$$

$$\begin{aligned} f_L(\mathcal{G}) &= \left(\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L + \kappa \left[\zeta_{R(1)} - \frac{1}{\sqrt{2}}\zeta_{R(2)} + \frac{1}{\sqrt{6}}\zeta_{R(3)} \right] \right) \\ &\quad \cdot \left(\frac{4a_3}{m^2} - \frac{g^2}{8\pi^2 F^2} \right) + \sum_{A=1}^{10} (\rho^r_{(A)}\omega_L + \upsilon^r_{(A)}\theta_L + \tau^r_{(A)}\varphi_L) \\ &\quad + \sum_{A=1}^{12} \nu^r_{(A)} \left[\zeta_{R(1)} - \frac{1}{\sqrt{2}}\zeta_{R(2)} + \frac{1}{\sqrt{6}}\zeta_{R(3)} \right] \end{aligned} \tag{7.3}$$

and

$$
\begin{aligned}
h_L(\mathcal{G}) &= -\frac{3g^2}{16\pi^2 F^2}\left(\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L + \kappa\left[\zeta_{R(1)} - \frac{1}{\sqrt{2}}\zeta_{R(2)} + \frac{1}{\sqrt{6}}\zeta_{R(3)}\right]\right) \\
&\quad -\frac{(\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L)}{16\pi^2 F^2} - \frac{7\kappa}{16\pi^2 F^2}\left[\zeta_{(1)} + \frac{1}{\sqrt{2}}\zeta_{(2)} + \frac{1}{\sqrt{6}}\zeta_{(3)}\right]. \quad (7.4)
\end{aligned}
$$

Note that, while a and f are free parameters, h is completely determined by the lower order parameters. This makes the form of the chiral logarithm coefficient a prediction of the effective ChPT model employed here. The finite volume correction to eq. (7.1) is given by adding the extra term

$$
\begin{aligned}
b_L^V(\mathcal{G}) &= \frac{3g^2}{F^2}\left(\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L + \kappa\left[\zeta_{R(1)} - \frac{1}{\sqrt{2}}\zeta_{R(2)} + \frac{1}{\sqrt{6}}\zeta_{R(3)}\right]\right) K^V_{\pi NN} \\
&\quad -\frac{\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L + 7\kappa\left[\zeta_{R(1)} - \frac{1}{\sqrt{2}}\zeta_{R(2)} + \frac{1}{\sqrt{6}}\zeta_{R(3)}\right]}{F^2} I^V_\pi. \quad (7.5)
\end{aligned}
$$

Alternatively, the amplitude can be presented as

$$
\begin{aligned}
i\mathcal{M}(\mathrm{n} \to \bar{\mathrm{n}}) = \quad & i2m\Big[(\lambda_{(1)}\omega_L + \lambda_{(2)}\theta_L + \lambda_{(3)}\varphi_L)(1 + \Delta_V^{(1)} + \Delta_m^{(1)}) \quad (7.6) \\
& +\kappa\left[\zeta_{R(1)} - \frac{1}{\sqrt{2}}\zeta_{R(2)} + \frac{1}{\sqrt{6}}\zeta_{R(3)}\right](1 + \Delta_V^{(5)} + \Delta_m^{(5)}) + m_\pi^2 f_L(\mathcal{G})\Big]
\end{aligned}
$$

where Δ_m is the chiral logarithm dependence, Δ_V is the finite volume correction and the superscript indicate which class of six-quark operator that gives the contribution (1 for $Q_{1/2/3}$ and 5 for $Q_{5/6/7}$). The dependence of Δ_m on m_π^2 can be found in fig. 2. Δ_V in as a function of volume is plotted in fig. 3.

8 Conclusions and Outlook

The main result of this thesis is the form of the a, f and h coefficients in the amplitude in eq. (7.1), which can be of interest for improvement of lattice QCD simulations, at unphysical quark masses, of the same matrix element. By comparing the lattice calculated amplitude to the functional dependence of the amplitude on m_π which is deduced here using ChPT, the effect of the quark mass scheme can be found and the results can be extrapolated to the physical masses.

As can be seen in fig. 3 the finite volume effect at $m_\pi L = 7.8$ is around the percent level. This means that the finite volume effects are quite small

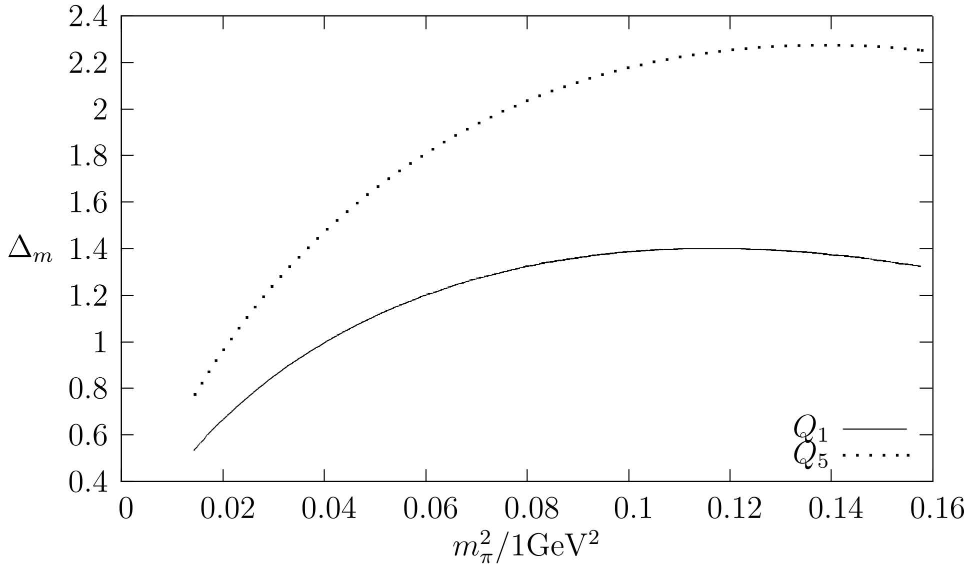


Figure 2: Infinite volume loop integrals as a function of the pion mass. Q_1 stands for the contribution from operators of type $Q_{1/2/3}$ and Q_5 for $Q_{5/6/7}$. The numerical values used are $F = 92.2$ MeV, $g = 1.2694$, and $\mu = 770$ MeV.

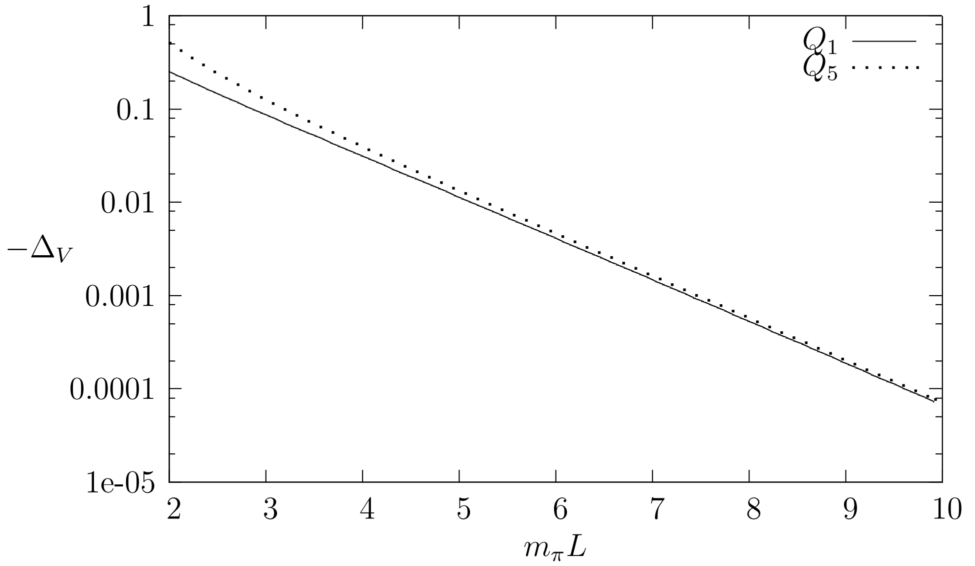


Figure 3: Finite volume correction to the infinite volume results for different volumes. Q_1 stands for the contribution from operators of type $Q_{1/2/3}$ and Q_5 for $Q_{5/6/7}$. The numerical values used are $m_\pi = 135$ MeV, $F = 92.2$ MeV, and $g = 1.2694$. Note that the actual finite volume correction is negative so what is plotted is the negative correction.

at this box size. The unphysical pion mass, on the other hand, does appear to have a more significant effect on the transition. The finding of this thesis is that there is roughly a factor of two between the transition amplitude at a pion mass of 390 MeV and the physical at 135 MeV, which can be seen in fig. 2.

Ensuring that no vertices in the oscillation sector have been overlooked is a formidable task and there is no guarantee that the list supplied here is complete. For the purposes of this thesis, however, only the $O(p^0)$ (which are all guaranteed to be found) and the χ type vertices contribute. All the χ type vertices contribute identically, which means that for eq. (7.1) it is only relevant to have sufficiently many χ vertices for f to be an independent parameter. This has certainly been achieved by the included vertices.

Furthermore, as an outlook, it is always possible to perform the ChPT calculation at a higher order of perturbative expansion, e.g. $O(p^3)$ or $O(p^4)$. However, the number of contributing terms in the Lagrangian grows fast with the order of perturbative expansion. The calculation can also be performed using some other nucleon description (e.g. relativistic baryon ChPT) in order to ensure the consistency of the results.

Acknowledgements

I would like to thank Johan Bijnens for taking his time to supervise me and for explaining the things I did not understand. I have learned a lot from this project thanks to him. Furthermore, I would like to express my gratitude towards the persons that I got the opportunity to share an office with during this thesis work for being such good company.

A Further Background on ChPT

A.1 Symmetry, Charges and Goldstone Bosons

Noether's theorem states that every continuous symmetry transformation that leaves the Lagrangian invariant leads to a current, j^μ, that is conserved $\partial_\mu j^\mu = 0$ (see e.g. chapter 2 of [11]). For the chiral symmetries of QCD these currents are $j_\chi^{a\ \mu} = \bar{q}_\chi \gamma^\mu T_\chi^a q_\chi$ where χ is L or R. The conserved charges are

$$Q_\chi^a = \int d^3x\ j_\chi^{a\ 0}(\mathbf{x}) = \int d^3x\ q_\chi^\dagger(\mathbf{x}) T_\chi^a q_\chi(\mathbf{x}) \tag{A.1}$$

and since the charges are conserved by Heisenberg's equations of motion (in the Heisenberg picture) $\dot{Q}_\chi^a(t) = i[H, Q_\chi^a(t)] = 0$ so $[H, Q_\chi^a] = 0$. But this commutation also implies that Q_χ^a is a symmetry of the Hamiltonian. Furthermore, it can be checked that the charges form a representation of the Lie algebra, i.e. $[Q_\chi^a, Q_\chi^b] = if^{abc}Q_\chi^c$. In fact, the charges are the representation of the group generators acting on the Hilbert space of quantum states.

The Lie algebra for $SU(N)_L \times SU(N)_R$ is

$$[T_L^a, T_L^b] = if^{abc}T_L^c, \quad [T_R^a, T_R^b] = if^{abc}T_R^c, \quad \text{and } [T_L^a, T_R^b] = 0 \tag{A.2}$$

which implies in the axial basis, $T_A^a = T_R^a - T_L^a$ and , $T_V^a = T_R^a + T_L^a$, that

$$[T_V^a, T_V^b] = if^{abc}T_V^c, \quad [T_V^a, T_A^b] = if^{abc}T_A^c, \quad \text{and } [T_A^a, T_A^b] = if^{abc}T_V^c. \tag{A.3}$$

with exactly the same equations holding for T replaced by Q. From the assumption of a quark condensate, it follows that the vacuum, $|0\rangle$, is not invariant under axial transformations $|0\rangle \to (1 + i\theta_A^a Q_A^a)|0\rangle \neq |0\rangle$ so $Q_A^a|0\rangle \neq 0$. Since the charge operator Q_A^a still commutes with the Hamiltonian, the state $Q_A^a|0\rangle$ will be degenerate with the vacuum but is another independent state. The only type of non-vacuum state that $Q_A^a|0\rangle$ can have a non-zero overlap with is a zero-momentum limit of a massless state. Hence the spontaneous symmetry breaking implies the existence of massless Goldstone bosons with the same quantum numbers as Q_A^a, and this is the essence of Goldstone's theorem. That the massless states are scalar bosons can be understood from the fact that Q_A^a is a Lorentz scalar operator. The bosons will be pseudoscalar since a parity transformation interchanges left and right so $Q_A^a \to -Q_A^a$. The final conclusion about Goldstone bosons is that they live in the adjoint representation of $SU(N)_V$ since they transform as T_A^a under vector transformations and the middle equation of (A.3) is precisely the change of an adjoint representation under infinitesimal transformations by T_V^a.

A.2 The Lagrangian of HBChPT

In order to deduce the Lagrangian of HBChPT some properties of $P_{v\pm} = \frac{1}{2}(1 \pm \not{v})$ are needed. It is easy to show that $\not{v}P_{v\pm} = P_{v\pm}\not{v} = \pm P_{v\pm}$. From this one can deduce that $P_{v\pm}\not{A}P_{v\pm} = \pm P_{v\pm}\not{v}\not{A}P_{v\pm}$ which can be anticommuted using the Clifford algebra yielding $\pm P_{v\pm}v_\mu A_\nu(2\eta^{\mu\nu} - \gamma^\nu\gamma^\mu)P_{v\pm} = \pm P_{v\pm}(2v \cdot A - \not{A}\not{v})P_{v\pm}$ which in turn equals $\pm P_{v\pm}2(v \cdot A) - P_{v\pm}\not{A}P_{v\pm}$ so

$$P_{v\pm}\not{A}P_{v\pm} = \pm P_{v\pm}(v \cdot A). \tag{A.4}$$

Starting from the Lagrangian in eq. (4.11) one can expand the ψ field in terms of the light and heavy components $\mathcal{N}$ and $\mathcal{H}$ as

$$\psi(x) = e^{-imv\cdot x}[\mathcal{N}(x) + \mathcal{H}(x)] \quad , \quad \bar{\psi}(x) = e^{imv\cdot x}[\bar{\mathcal{N}}(x) + \bar{\mathcal{H}}(x)]. \tag{A.5}$$

In this expansion the kinetic term becomes

$$i\partial\!\!\!/\psi = i\partial\!\!\!/\left\{e^{-imv\cdot x}[\mathcal{N}(x)+\mathcal{H}(x)]\right\} = e^{-imv\cdot x}(i\partial\!\!\!/ + m v\!\!\!/)[\mathcal{N}(x)+\mathcal{H}(x)] \tag{A.6}$$

which can be further simplified to

$$e^{-imv\cdot x}[(i\partial\!\!\!/ + m)\mathcal{N}(x) + (i\partial\!\!\!/ - m)\mathcal{H}(x)]. \tag{A.7}$$

Using that $P_{v\pm}P_{v\mp} = 0$ (which means no cross terms between $\mathcal{N}$ and $\mathcal{H}$) the Lagrangian can be rewritten as

$$\bar{\psi}(iD\!\!\!\!/ - m)\psi = \bar{\mathcal{N}}iD\!\!\!\!/\mathcal{N} + \bar{\mathcal{H}}(iD\!\!\!\!/ - 2m)\mathcal{H}. \tag{A.8}$$

The fields $\mathcal{N}$ and $\mathcal{H}$ are coupled by the Dirac equation

$$0 = \left(iD\!\!\!\!/ - m + \frac{1}{2}g\gamma^\mu\gamma^5 u_\mu\right)\psi \tag{A.9}$$

or

$$0 = iD\!\!\!\!/\mathcal{N} + (iD\!\!\!\!/ - 2m)\mathcal{H}. \tag{A.10}$$

Multiplying this equation with P_{v-} yields

$$0 = iD\!\!\!\!/_\perp\mathcal{N} + (-iv\cdot D - 2m)\mathcal{H}, \tag{A.11}$$

which has a solution that can be written formally as

$$\mathcal{H} = (iv\cdot D + 2m)^{-1}iD\!\!\!\!/_\perp\mathcal{N}. \tag{A.12}$$

This equation shows that $\mathcal{H}$ is suppressed by a factor $1/m$ compared to $\mathcal{N}$ which means that it can be ignored in the Lagrangian. Using eq. (A.4) the kinetic term can be simplified to

$$\bar{\mathcal{N}}iD\!\!\!\!/\mathcal{N} = \bar{\mathcal{N}}iP_{v+}D\!\!\!\!/P_{v+}\mathcal{N} = \bar{\mathcal{N}}iv\cdot D\mathcal{N}. \tag{A.13}$$

In an analogous fashion it can be shown that $P_{v+}\gamma^\mu\gamma^5 P_{v+} = 2S^\mu$ from this and the above deduction eq. (4.13) follows.

B Group Properties and Clebsch-Gordan Coefficients for Spurions

B.1 Adjoint representation and its Weights in $SU(2)$

Here the transformation properties of the adjoint rep, and in particular its weights (spin projections) are discussed. Let $\delta\xi^i = -i\varphi^a(T^a)^i_k\xi^k$ and $\delta\psi_j =$

$i\varphi^a(T^a)^k_j\psi_k$. Then the contraction $\xi^i(T^b)^j_i\Theta^b\psi_j$ is invariant if the field Θ^a transforms as the adjoint rep. This yields the condition

$$0 = \delta(\xi^i(T^b)^j_i\Theta^b\psi_j) = \xi^i[\delta\Theta^b(T^b)^j_i + i\varphi^a\Theta^b(T^a)^j_k(T^b)^k_i - i\varphi^a\Theta^b(T^a)^k_i(T^b)^j_k]\psi_j,$$

so

$$\delta\Theta^b(T^b)^j_i = i\varphi^a[T^a,T^b]^j_i\Theta^b = i\varphi^a if^{abc}\Theta^b(T^c)^j_i = i\varphi^a if^{acb}\Theta^c(T^b)^j_i. \quad \text{(B.1)}$$

Identifying the coefficients of T^b and using the antisymmetry of f results in

$$\delta\Theta^b = i\varphi^a[-if^{abc}]\Theta^c, \quad \text{(B.2)}$$

i.e. the structure constants are the (matrix elements of the) generators in the adjoint rep. That those generators obey the Lie algebra can be deduced from the fact that commutators obey the Jacobi identity, see e.g. ref [3].

In $SU(2)$, due to $i\tau^2\tau^a = -(\tau^a)^* i\tau^2$, there is an isomorphism between the 2 and $\bar{2}$ reps. If $\delta\psi_i = i\varphi^a(\frac{1}{2}\tau^a)^j_i\psi_j$ then $\psi^i \equiv (i\tau^2)^{ij}\psi_j = \epsilon^{ij}\psi_j$ so

$$\begin{aligned}\delta\psi^i \quad &= (i\tau^2)^{ij} i\varphi^a\frac{1}{2}(\tau^a)^k_j\psi_k = i\varphi^a\frac{1}{2}\left(i\tau^2\tau^a\right)^k_i\psi_k = i\varphi^a\frac{1}{2}\left(-\tau^{a*}i\tau^2\right)^{ik}\psi_k \\ &= -i\varphi^a\frac{1}{2}(\tau^{a*})^j_i\psi^j = -i\varphi^a\frac{1}{2}\left(\tau^{a\dagger}\right)^i_j\psi^j = -i\varphi^a\frac{1}{2}(\tau^a)^i_j\psi^j, \quad \text{(B.3)}\end{aligned}$$

that is ψ^i transforms as ${\psi^\dagger}^i$ which motivates the index placement.

Equipped with this one can deduce the spin of the adjoint rep using the fact that $\xi^i(T^b)^j_i\Theta^b\psi_j$ transforms as a singlet, the spins must add up to zero. Choosing both spins to be up, $\xi_i = \psi_i = \delta_i^{+1/2}$, results in $\xi^i = -\delta^i_{-1/2}$, and thus the adjoint Θ should be in its spin -1 configuration, leading to $\xi^i(T^b)^j_i\Theta^b\psi_j = -\delta^i_{-1/2}(T^b)^j_i\Theta^b\delta_i^{+1/2} = (T^b)^{1/2}_{-1/2}\Theta^b$. Looking at the explicit matrix elements of the Pauli matrices one finds that the spin down component of Θ is proportional to

$$\Theta^{-1} = (T^b)^{1/2}_{-1/2}\Theta^b \sim \Theta^1 + i\Theta^2. \quad \text{(B.4)}$$

B.2 Clebsch-Gordan Coefficients

Instead of writing the spurions in the form of irreducible $SU(2)$ tensors it is convenient to write them as product tensors of 2:s with proper symmetrizations. The translation between irreducible and product tensors is done by Clebsch-Gordan decomposition. Let θ be a 3 and Ξ be a 5 rep. Then the

Clebsch-Gordan decomposition is

$$\begin{aligned}
(\theta^-)^{ij} &= \delta^i_{-1/2}\delta^j_{-1/2} \\
(\theta^3)^{ij} &= \frac{1}{\sqrt{2}}\left[\delta^i_{1/2}\delta^j_{-1/2} + \delta^i_{-1/2}\delta^j_{1/2}\right] \\
(\theta^+)^{ij} &= \delta^i_{1/2}\delta^j_{1/2}
\end{aligned} \tag{B.5}$$

for the 3 and

$$\begin{aligned}
(\Xi^{--})^{ijkl} &= \delta^i_{-1/2}\delta^j_{-1/2}\delta^k_{-1/2}\delta^l_{-1/2} \\
(\Xi^{\{-3\}})^{ijkl} &= \frac{1}{2}[\delta^i_{1/2}\delta^j_{-1/2}\delta^k_{-1/2}\delta^l_{-1/2} + \delta^i_{-1/2}\delta^j_{1/2}\delta^k_{-1/2}\delta^l_{-1/2} \\
&\quad +\delta^i_{-1/2}\delta^j_{-1/2}\delta^k_{1/2}\delta^l_{-1/2} + \delta^i_{-1/2}\delta^j_{-1/2}\delta^k_{-1/2}\delta^l_{1/2}] \\
(\Xi^{33})^{ijkl} &= \frac{1}{\sqrt{6}}[\delta^i_{-1/2}\delta^j_{-1/2}\delta^k_{1/2}\delta^l_{1/2} + \delta^i_{-1/2}\delta^j_{1/2}\delta^k_{-1/2}\delta^l_{1/2} \\
&\quad +\delta^i_{-1/2}\delta^j_{1/2}\delta^k_{1/2}\delta^l_{-1/2} + \delta^i_{1/2}\delta^j_{-1/2}\delta^k_{-1/2}\delta^l_{1/2} \\
&\quad +\delta^i_{1/2}\delta^j_{-1/2}\delta^k_{1/2}\delta^l_{-1/2} + \delta^i_{1/2}\delta^j_{1/2}\delta^k_{-1/2}\delta^l_{-1/2}]\ldots
\end{aligned} \tag{B.6}$$

for the 5.

C Renormalization and the LSZ formula

C.1 Renormalization and Dimensional Regularization

Loop corrections in QFT almost always lead to divergent integrals contributing to physical observables. To make sense of these quantum corrections one can use the techniques of *renormalization*. A good starting point for the procedure is chapters 6, 7 and 10 of ref [11]. Here a quick overview of the craft is given.

The first step of renormalization is a modification of the integral to make it finite in such a way that low energy[22] physics remains unmodified. This is called *regularization*. The second step is to use a so-called renormalization condition which is when the loop corrected amplitude is set equal to a finite (and experimentally measured) quantity. In this stage the parameters appearing in the Lagrangian will have to be infinite in order to cancel the loop divergence but this is no trouble because the Lagrangian does not have to be physical; the only requirement on it is that it makes physical predictions.

[22] This is for so-called UV divergences when propagators at high energies cause infinities. Infrared divergences, associated with low energies are a bit more subtle, see e.g. chapter 6 of ref. [11].

In gauge theories it is important that the regularization procedure respects the gauge symmetry of the theory so that quantum corrections will not break gauge invariance. In ChPT the analogue symmetries that should be preserved are the chiral Ward identities. Both those are respected by a scheme called *dimensional regularization.*

To illustrate how dimensional regularization works consider the integral

$$I(a,b,\Delta)=\int\frac{d^4k}{(2\pi)^4}\frac{(k^2)^a}{(k^2-\Delta+i\epsilon)^b},\tag{C.1}$$

which is a form that any one-loop integral can be written as a linear combination of, using a trick called Feynman parameters. Take for simplicity $I(0,1,\Delta)$. Since the poles of the propagator at $\pm\Delta$ are shifted slightly into the complex plane, the integration can be viewed as a complex line integral along the real axis and can thus be deformed to lie on the imaginary axis without crossing any pole. This is called a Wick rotation and is a general procedure used in most regularization schemes. The Wick rotation turns $k^0=ik_E^0$ and $k^i=k_E^i$ and in particular it changes the Minkowski product $k^2=(k^0)^2-\mathbf{k}^2$ into the Euclidean $-k_E^2=-(k_E^0)^2-\mathbf{k}_E^2$ hence the subscript E. The reason for the Wick rotation is that it allows for evaluation using d-dimensional spherical coordinates which is not possible in a Minkowski signature. The integral is now

$$I(0,1,\Delta)=i\int\frac{d^4k_E}{(2\pi)^4}\frac{1}{(-k_E^2-\Delta)}.\tag{C.2}$$

The idea of dimensional regularization is to analytically continue the integral as a function of (non-integer) spacetime dimension[23], $d=4-\epsilon$. This will induce a modification of the integral to

$$I(0,1,\Delta)=i\mu^{4-d}\int\frac{d^dk_E}{(2\pi)^d}\frac{1}{(-k_E^2-\Delta)}=\frac{i\mu^{4-d}}{(2\pi)^d}\int_0^\infty dk_E\frac{k_E^{d-1}}{(-k_E^2-\Delta)}\int d\Omega_d\tag{C.3}$$

where the change of variables to spherical coordinates has been performed and a new arbitrary mass dimension one parameter, μ, has been introduced in order to preserve the mass dimension of the integral in all d.

The same procedure of Wick rotating can be performed in the more general case of eq. (C.1). By studying the Jacobian under a change of variables from Cartesian to spherical coordinate in Gaussian integrals one finds that

[23] It is not obvious that changing to fractal spacetime dimension is a modification of high energy physics that leaves low energy phenomena invariant but a good justification that this is the case is given in appendix A of ref [20].

the integral over Ω_d is $\frac{2\pi^{d/2}}{\Gamma(d/2)}$. The radial part can be evaluated using clever transformations and the Beta function, see e.g. ref [20] appendix A, and the result is

$$I(a,b,\Delta) = i(-1)^b \frac{\mu^{4-d}\Delta^{d/2+a-b}}{(4\pi)^{d/2}} \frac{\Gamma(a+d/2)\Gamma(b-a-d/2)}{\Gamma(d/2)\Gamma(b)}. \tag{C.4}$$

One of the most important cases is when the divergence is logarithmic which is when $4+2a-2b=0$. In that case $\Gamma(b-a-d/2) = \Gamma(2-d/2) = \Gamma(\epsilon/2)$. In order to evaluate one can use the "factorial identity" and Taylor expand

$$x\Gamma(x) = \Gamma(1+x) = \Gamma(1) + \Gamma'(1)x + O(x^2) = 1 - \gamma_E x + O(x^2) \tag{C.5}$$

where γ_E is the Euler-Mascheroni constant. Then

$$\Gamma(\epsilon/2) = \frac{2}{\epsilon} - \gamma_E + O(\epsilon). \tag{C.6}$$

In the dimensionful prefactor the dependence on ϵ can be taken care of the following way:

$$\mu^\epsilon \Delta^{-\epsilon/2}(4\pi)^{\epsilon/2} = \exp\left[\frac{\epsilon}{2}\ln\left(\frac{\mu^2}{\Delta}4\pi\right)\right] = 1 + \frac{\epsilon}{2}\ln\left(\frac{\mu^2}{\Delta}4\pi\right) + O(\epsilon^2). \tag{C.7}$$

Multiplying eqs. (C.7) and (C.6) yields a factor of

$$\frac{2}{\epsilon} - \gamma_E - \ln(4\pi) + \ln\left(\frac{\mu^2}{\Delta}\right) + O(\epsilon). \tag{C.8}$$

Next one must choose a subtraction scheme which is a way to absorb the $\frac{1}{\epsilon}$ pole into a redefinition of some coupling constant. The most common scheme is modified minimal subtraction, ($\overline{\text{MS}}$), in which the first three terms in eq. (C.8) are absorbed. ChPT uses a slightly different scheme called $\widetilde{\text{MS}}$ where these terms and an extra -1 are absorbed. This factor shows up frequently and is given the name R:

$$-R \equiv \frac{2}{\epsilon} - \gamma_E + \ln(4\pi) + 1. \tag{C.9}$$

The point of this procedure is that after the so-called subtraction, where the couplings are redefined to absorb the infinities, every observable is finite in the limit $\epsilon \to 0$ and the model makes finite predictions. Note that the arbitrary scale μ is not a cutoff scale[24] that should be taken to be very large or infinite.

[24] Cutoff regularization is a more intuitive but also a less useful regularization scheme. It consists of simply integrating up to some maximum length of Euclidean four-momentum $k_E^2 \leq \Lambda^2$.

C.2 The LSZ reduction formula

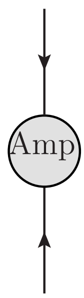
The LSZ (Lehmann-Symanzik-Zimmermann) reduction formula is one of the cornerstones of quantum field theory. It relates momentum space S-matrix elements (which are easy to measure) to time ordered field correlation functions in momentum space (which are easy to compute). A discussion of the formula can be found in e.g. chapter 7 of ref [11]. Here the result is just stated

$$\left(\prod_{i=1}^{n}\int d^4x_i e^{ip_i\cdot x}\right)\left(\prod_{j=1}^{m}\int d^4y_j e^{-ik_j\cdot y_j}\right)\langle\Omega|T\phi(x_1)\ldots\phi(x_n)\phi(y_1)\ldots\phi(y_m)|\Omega\rangle$$
$$\simeq\left(\prod_{i=1}^{n}\frac{i\sqrt{Z}}{p_i^2-m^2+i\epsilon}\right)\left(\prod_{j=1}^{n}\frac{i\sqrt{Z}}{k_j^2-m^2+i\epsilon}\right)\langle p_1\ldots p_n|S|k_1\ldots k_m\rangle$$

where $\simeq$ means that the poles of both sides are identical. The constant Z is called the field renormalization and is the residue at the mass pole of the exact propagator. One of the strengths of this formula is that ϕ can be any field operator, fundamental or composite, that has a nonzero overlap with an asymptotic momentum state of the theory $\langle p|\phi(x)|\Omega\rangle = \sqrt{Z}e^{ip\cdot x}$. This means that it holds equally well for e.g. electron fields as it does for meson fields, which would be represented by $q^\dagger(x)\gamma^5T_A^aq(x)$ in the more fundamental degrees of freedom.

The special guise of the LSZ reduction formula that will be needed in this thesis is the case of a nucleon in the in state and an antinucleon in the out state.

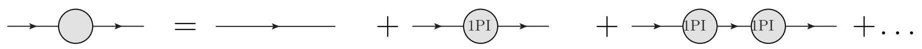
$$\langle\bar{n}:p',s';\text{out}|n:p,s;\text{in}\rangle = Z_N\bar{u}^{s'}_{\mathbf{p}'}u^s_{\mathbf{p}} \tag{C.10}$$



where the blob labeled "Amp" represents a sum of all amputated diagrams and Z_N is the field renormalization of the nucleon.

C.3 Field Renormalization

The neutron-antineutron transition element is calculated in bare perturbation theory and in order to use the LSZ reduction formula for amputated matrix elements (see eq. (C.10)), the self-energy, $-i\Sigma(p)$, of the nucleons [30] will be needed. First a relativistic fermion is treated. The field renornomalization can be calculated perturbatively by



(C.11)

which evaluates to

$$\frac{i}{\not{p} - m_0 + i\epsilon} + \frac{i}{\not{p} - m_0 + i\epsilon}[-i\Sigma(p)]\frac{i}{\not{p} - m_0 + i\epsilon} + \dots . \tag{C.12}$$

These diagrams form a geometric progression and can be summed up (see e.g. chapter 7 of [11])

$$\frac{i}{\not{p} - m_0 + i\epsilon}\left[1 + \Sigma(\not{p})\frac{1}{\not{p} - m_0 + i\epsilon} + \left(\Sigma(\not{p})\frac{i}{\not{p} - m_0 + i\epsilon}\right)^2 + \dots\right] \tag{C.13}$$

which is

$$\frac{i}{\not{p} - m_0 + i\epsilon}\frac{1}{1 + \Sigma(\not{p})\frac{1}{\not{p} - m_0 + i\epsilon}} = \frac{i}{\not{p} - m_0 - \Sigma(\not{p}) + i\epsilon}. \tag{C.14}$$

The pole mass, m, can now be defined by $m = m_0 + \Sigma(m)$. The field strength renormalization, Z, is the residue of the propagator at the pole mass, which is found by Taylor expanding Σ around m: $\Sigma(\not{p}) = \Sigma(m) + \Sigma'(m)(\not{p} - m) + O((\not{p} - m)^2)$. Thus

$$Z = \frac{1}{1 - \Sigma'(m)}. \tag{C.15}$$

For nucleons in ChPT the self-energy correction to the propagator can be deduced the same way from eq. (4.14) and is

$$\frac{i}{v \cdot k - \Sigma(k) + i\epsilon}. \tag{C.16}$$

In the power counting of ChPT, $\Sigma(p)$ is at $O(p^3)$, but acting on an external leg (which is how it enters the LSZ formula), a new propagator, which is $O(p^{-1})$, is created. Thus, the field renormalization enters at $O(p^2)$.

D Integrals in Finite and Infinite Volumes

D.1 Standard Integrals

The standard integrals used are taken from appendix C of ref [17], where more information and derivations can be found. The most fundamental ones of ChPT are the pion integral

$$I_\pi(k) = I_\pi(0) = i\mu^{4-d}\int\frac{d^dq}{(2\pi)^d}\frac{1}{((k+q)^2-m_\pi^2+i\epsilon)}, \tag{D.17}$$

where

$$I_\pi(0) = \frac{m_\pi^2}{16\pi^2}\left[R+\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right]+O(d-4), \tag{D.18}$$

and the nucleon integral

$$\begin{aligned} J_{\pi N}(k,\omega) \quad &= J_{\pi N}(0,\omega-v\cdot k) \\ &= i\mu^{4-d}\int\frac{d^dq}{(2\pi)^d}\frac{1}{((k+q)^2-m_\pi^2+i\epsilon)}\frac{1}{(v\cdot q+\omega+i\epsilon)}. \end{aligned} \tag{D.19}$$

For $\omega^2 < m_\pi^2$ it can be evaluated to be

$$\begin{aligned} J_{\pi N}(0,\omega) \quad &= \frac{\omega}{8\pi^2}\left[R+\ln\left(\frac{m_\pi^2}{\mu^2}\right)-1\right] \\ &+\frac{1}{4\pi^2}\sqrt{m_\pi^2-\omega^2}\arccos\left(-\frac{\omega}{m_\pi}\right). \end{aligned} \tag{D.20}$$

For this thesis we will need the πNN tensor integral

$$K^{\mu\nu}_{\pi NN}(\omega) \equiv i\mu^{4-d}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(v\cdot q+\omega+i\epsilon)^2}\frac{q^\mu q^\nu}{(q^2-m_\pi^2+i\epsilon)}. \tag{D.21}$$

this integral is not listed in ref [17] but can be computed by taking a derivative of the listed standard integral

$$J^{\mu\nu}_{\pi N}(k,\omega) = i\mu^{4-d}\int\frac{d^dq}{(2\pi)^d}\frac{q^\mu q^\nu}{((k+q)^2-m_\pi^2+i\epsilon)}\frac{1}{(v\cdot q+\omega+i\epsilon)}.$$

This integral can depend only on the rank two tensors $v^\mu v^\nu$ and $\eta^{\mu\nu}$ so

$$J^{\mu\nu}_{\pi N}(k,\omega) = v^\mu v^\nu C_{20}(\omega)+\eta^{\mu\nu}C_{21}(\omega)$$

For the purposes of this thesis only the C_{21} is needed (since $J^{\mu\nu}$ is contracted with an S_μ and $S\cdot v=0$) which turns out to be

$$C_{21}(\omega) = \frac{1}{3}\left[(m_\pi^2-\omega^2)J_{\pi N}(0,\omega)+\omega I_\pi(0)\right]-\frac{\omega}{12\pi^2}\left(\frac{m_\pi^2}{2}-\frac{\omega}{3}\right). \tag{D.22}$$

In order to evaluate $K^{\mu\nu}_{\pi NN}$ one needs to compute $-dJ^{\mu\nu}(\omega)/d\omega$. This leads to an expression of the form

$$K^{\mu\nu}_{\pi NN}(0) = -\eta^{\mu\nu}\left[\frac{m_\pi^2}{16\pi^2}\left(R+\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right)\right]-v^\mu v^\nu C'_{20}(0). \tag{D.23}$$

D.2 Loop Integrals at Finite Volume

Here some properties of loop integrals at finite volume are given as a quick reference. The subject is treated more thoroughly in ref. [24]. As a proxy for a loop integral consider the one dimensional integral

$$\int \frac{dp}{2\pi} \tilde{F}(p). \tag{D.24}$$

In the finite volume, length L, this integral is replaced by a sum over all momenta allowed by boundary conditions, which are chosen to be periodical, restricting the momenta to $p_n = \frac{n2\pi}{L}$ for integer n. Then the integral is replaced by the Riemann sum

$$\int \frac{dp}{2\pi} \tilde{F}(p) \rightarrow \sum_{n\in\mathbb{Z}} \frac{\Delta p}{2\pi} \tilde{F}(p_n) = \frac{1}{L} \sum_{n\in\mathbb{Z}} \tilde{F}(p_n) \equiv \int_V \frac{dp}{2\pi} \tilde{F}(p). \tag{D.25}$$

In order to proceed we use Poisson's summation formula, which can be shown by

$$\sum_{n\in\mathbb{Z}} f(n) = \int dx f(x) \sum_n \delta(x-n) = \int dx f(x) \sum_{m\in\mathbb{Z}} e^{im2\pi} = \sum_{m\in\mathbb{Z}} \tilde{f}(-m2\pi)$$

where $\tilde{f}$ denotes the Fourier transform. This can be rewritten, by the scaling rule, $f(ax) \mapsto \tilde{f}(k)/|a|$, and the sign flip symmetry of the sum as

$$\sum_{n\in\mathbb{Z}} f(n) = \frac{1}{2\pi} \sum_{m\in\mathbb{Z}} \tilde{f}(m), \tag{D.26}$$

which is the Poisson formula. Using $F(x) = f(\frac{x}{L})$, which implies $\tilde{F}(k) = L\tilde{f}(Lk)$, in this formula yields

$$\frac{1}{L} \sum_n \tilde{F}\left(\frac{2\pi n}{L}\right) = \sum_m f(m) = \sum_m \int \frac{d\kappa}{2\pi} \tilde{f}(\kappa) e^{i\kappa m} = \sum_m \int \frac{d\kappa}{2\pi} \frac{1}{L} \tilde{F}\left(\frac{\kappa}{L}\right) e^{i\kappa m}$$

and by making the change of variables $k = \kappa/L$ this yields

$$\frac{1}{L} \sum_n \tilde{F}\left(\frac{2\pi n}{L}\right) = \sum_m \int \frac{dk}{2\pi} \tilde{F}(k)\, e^{ikLm}. \tag{D.27}$$

By defining $l_m = mL$ the finite volume integral can be written

$$\int_V \frac{dp}{2\pi} \tilde{F}(p) = \frac{1}{L} \sum_{n\in\mathbb{Z}} \tilde{F}(p_n) = \sum_{l_m} \int \frac{dk}{2\pi} \tilde{F}(k)\, e^{ikl_m}. \tag{D.28}$$

Note that the $l = 0$ mode of this expression is exactly eq. (D.24), i.e. the integral in infinite volume. Therefor the primed sum, without this mode, is the correction to the momentum integral due to finite volume

$$\sum_{l_m}{}' \int \frac{dk}{2\pi} \tilde{F}(k)\, e^{ikl_m}. \tag{D.29}$$

That this expression vanishes in the infinite volume limit can be guessed from a naive application of the Riemann-Lebesgue lemma.

D.3 Nucleon Loop Integrals at Finite Volume

Here the nucleon integrals appearing in this thesis are evaluated at finite volume. The procedure is analogous to what is done in appendix C of ref. [17]. Start with the integral

$$\int_V \frac{d^d q}{(2\pi)^d} \frac{q^\mu q^\nu}{(q^2 - \Delta)} \frac{1}{(v \cdot q + \omega)} \tag{D.30}$$

and use the Feynman trick (where $y = \frac{x}{2(1-x)}$)

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{[xa + (1-x)b]^2} = 2\int_0^\infty dy \frac{1}{(2ya + b)^2}. \tag{D.31}$$

The integral can then be written on the form

$$\int_V \frac{d^d q}{(2\pi)^d} \frac{q^\mu q^\nu}{(q^2 - \Delta)} \frac{1}{(v \cdot q + \omega)} = 2\int_0^\infty dy \int_V \frac{d^d q}{(2\pi)^d} \frac{q^\mu q^\nu}{(q^2 - \Delta + 2yv \cdot q + 2y\omega)^2}$$

and by shifting the the integration variable by $q'^\mu = q^\mu + yv^\mu$ (and then renaming the integration variable from q' to q) we get

$$2\int_0^\infty dy \int_V \frac{d^d q}{(2\pi)^d} \frac{q^\mu q^\nu - yv^\mu q^\nu - yv^\nu q^\mu + y^2 v^\mu v^\nu}{(q^2 - \Delta - y^2 + 2y\omega)^2} \tag{D.32}$$

and since the summation is over a symmetric interval the linear terms in q vanish yielding

$$2\int_0^\infty dy \int_V \frac{d^d q}{(2\pi)^d} \frac{q^\mu q^\nu + y^2 v^\mu v^\nu}{(q^2 - \Delta - y^2 + 2y\omega)^2}. \tag{D.33}$$

The desired integrals of the expressions in eq. (6.38) can be found by applying $-d/d\omega$ to eq. (D.30) and evaluating in $\omega = 0$. This yields

$$\int_V \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2 - \Delta)} \frac{1}{(v \cdot q)^2} = 8\int_0^\infty dy \int_V \frac{d^d q}{(2\pi)^d} \frac{y(q^\mu q^\nu + y^2 v^\mu v^\nu)}{(q^2 - \Delta - y^2)^3} \tag{D.34}$$

which, when evaluated at $\Delta = m_\pi^2$, changed to Euclidean signature and written in the notation of sec. 4.5 gives the expression

$$\int_V \frac{d^dq}{(2\pi)^d} \frac{q^\mu q^\nu}{(q^2 - m_\pi^2)} \frac{1}{(v \cdot q)^2} = -i8 \int_0^\infty dyy \Big([q^\mu q^\nu]^3 (m_\pi^2 + y^2) + y^2 v^\mu v^\nu [1]^3 (m_\pi^2 + y^2) \Big).$$

The integral linear in q is evaluated analogously.

D.4 The Jacobi Theta function and Related Functions

In the expressions for the finite volume corrections the third Jacobi theta function, θ_3, and related functions show up. This function is given by

$$\theta_3(u, q) = \sum_{n \in \mathbb{Z}} q^{n^2} e^{i2\pi nu}. \tag{D.35}$$

The related functions used here are

$$\theta_{30}(q) \equiv \theta_3(0, q) \quad \text{and} \quad \theta_{32}(q) \equiv q \frac{\partial}{\partial q} \theta_3(0, q). \tag{D.36}$$

E Vertices of the Oscillation Lagrangian

E.1 $(3_L, 1_R)$ vertices using HBChPT

Here follows a list of the types of terms that show up in the $(3_L, 1_R)$ sector. Each term here couples to the spurions ω_L, θ_L and φ_L with independent coupling constants. At $O(p^0)$ we have

$$(u^\dagger \bar{\mathcal{N}}^c)_{i_L} (u^\dagger \mathcal{N})_{j_L}$$

and a $O(p^1)$, using derivatives

$$\begin{gathered}
(u^\dagger \bar{\mathcal{N}}^c)_{i_L} S^\mu (u^\dagger D_\mu \mathcal{N})_{j_L} \\
(u^\dagger D_\mu \bar{\mathcal{N}}^c)_{i_L} S^\mu (u^\dagger \mathcal{N})_{j_L} \\
(u^\dagger \bar{\mathcal{N}}^c)_{i_L} v^\mu (u^\dagger u_\mu \mathcal{N})_{j_L} \\
(u^\dagger \bar{\mathcal{N}}^c)_{i_L} S^\mu (u^\dagger u_\mu \mathcal{N})_{j_L} \\
(u^\dagger u_\mu \bar{\mathcal{N}}^c)_{i_L} v^\mu (u^\dagger \mathcal{N})_{j_L} \\
(u^\dagger u_\mu \bar{\mathcal{N}}^c)_{i_L} S^\mu (u^\dagger \mathcal{N})_{j_L}
\end{gathered}$$

and at $O(p^2)$ using the χ spurion

$$
\begin{aligned}
&(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(\chi^\dagger u\mathcal{N})_{j_L}\\
&(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(\tilde{\chi} u\mathcal{N})_{j_L}\\
&(\chi^\dagger u\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L}\\
&(\tilde{\chi} u\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L}\\
&(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(\chi^\dagger u\mathcal{N})_{j_L}\\
&(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(U^\dagger\chi u^\dagger\mathcal{N})_{j_L}\\
&(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(U^\dagger\tilde{\chi}^\dagger u^\dagger\mathcal{N})_{j_L}\\
&(U^\dagger\chi u^\dagger\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L}\\
&(U^\dagger\tilde{\chi}^\dagger u^\dagger\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_L}
\end{aligned}
$$

and with the field strength spurion

$$
\begin{aligned}
&\epsilon^{\alpha\beta\mu\nu}\mathrm{Tr}(f_{+\mu\nu})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}S_\alpha S_\beta(u^\dagger\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}\mathrm{Tr}(f_{-\mu\nu})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}S_\alpha S_\beta(u^\dagger\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger\bar{\mathcal{N}}^c)_{i_L}S_\alpha S_\beta(u^\dagger f_{+\mu\nu}\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger\bar{\mathcal{N}}^c)_{i_L}S_\alpha S_\beta(u^\dagger f_{-\mu\nu}\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger f_{+\mu\nu}\bar{\mathcal{N}}^c)_{i_L}S_\alpha S_\beta(u^\dagger\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger f_{-\mu\nu}\bar{\mathcal{N}}^c)_{i_L}S_\alpha S_\beta(u^\dagger\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}\mathrm{Tr}(f_{+\mu\nu})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}\mathrm{Tr}(f_{-\mu\nu})(u^\dagger\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger f_{+\mu\nu}\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger f_{-\mu\nu}\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger f_{+\mu\nu}\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger f_{-\mu\nu}\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger\mathcal{N})_{j_L}
\end{aligned}
$$

and derivatives at $O(p^2)$ with u_μ as the derivative of u

$$
\begin{aligned}
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger D_\mu\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger D_\nu\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger u_\mu D_\nu\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger u_\mu\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger D_\nu\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger u_\mu D_\nu\mathcal{N})_{j_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u^\dagger\bar{\mathcal{N}}^c)_{i_L}v_\alpha S_\beta(u^\dagger u_\mu D_\nu\mathcal{N})_{j_L}
\end{aligned}
$$

$$
\begin{aligned}
&(u^\dagger D_{\{\alpha}u_{\beta\}}\bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger \mathcal{N})_{j_L}\\
&(u^\dagger u_\beta D_\alpha \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger \mathcal{N})_{j_L}\\
&(u^\dagger u_\beta \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger D_\alpha \mathcal{N})_{j_L}\\
&(u^\dagger \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger D_{\{\alpha}u_{\beta\}} \mathcal{N})_{j_L}\\
&(u^\dagger \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger u_\beta D_\alpha \mathcal{N})_{j_L}\\
&(u^\dagger D_\alpha \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger u_\beta \mathcal{N})_{j_L}
\end{aligned}
$$

$$
\begin{aligned}
&(u^\dagger u_\alpha u_\beta \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger \mathcal{N})_{j_L}\\
&(u^\dagger u_\beta u_\alpha \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger \mathcal{N})_{j_L}\\
&(u^\dagger u_\alpha \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger u_\beta \mathcal{N})_{j_L}\\
&(u^\dagger u_\beta \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger u_\alpha \mathcal{N})_{j_L}\\
&(u^\dagger \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger u_\alpha u_\beta \mathcal{N})_{j_L}\\
&(u^\dagger \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger u_\alpha u_\beta \mathcal{N})_{j_L}\\
&(u^\dagger \bar{\mathcal{N}}^c)_{i_L} v^\beta S^\alpha (u^\dagger \mathcal{N})_{j_L} \mathrm{Tr}(u_\alpha u_\beta)
\end{aligned}
$$

$$
\begin{aligned}
&(u^\dagger u_\mu D^\mu \bar{\mathcal{N}}^c)_{i_L} (u^\dagger \mathcal{N})_{j_L}\\
&(u^\dagger D^\mu \bar{\mathcal{N}}^c)_{i_L} (u^\dagger u_\mu \mathcal{N})_{j_L}\\
&(u^\dagger u_\mu \bar{\mathcal{N}}^c)_{i_L} (u^\dagger D^\mu \mathcal{N})_{j_L}\\
&(u^\dagger \bar{\mathcal{N}}^c)_{i_L} (u^\dagger u_\mu D^\mu \mathcal{N})_{j_L}
\end{aligned}
$$

$$
\begin{aligned}
&(u^\dagger u_\mu u^\mu \bar{\mathcal{N}}^c)_{i_L} (u^\dagger \mathcal{N})_{j_L}\\
&(u^\dagger u^\mu \bar{\mathcal{N}}^c)_{i_L} (u^\dagger u_\mu \mathcal{N})_{j_L}\\
&(u^\dagger \bar{\mathcal{N}}^c)_{i_L} (u^\dagger u_\mu u^\mu \mathcal{N})_{j_L}\\
&(u^\dagger \bar{\mathcal{N}}^c)_{i_L} (u^\dagger \mathcal{N})_{j_L} \mathrm{Tr}(u_\mu u^\mu)
\end{aligned}
$$

$$
\begin{aligned}
&(u^\dagger D^2 \bar{\mathcal{N}}^c)_{i_L} (u^\dagger \mathcal{N})_{j_L}\\
&(u^\dagger D_\mu \bar{\mathcal{N}}^c)_{i_L} (u^\dagger D^\mu \mathcal{N})_{j_L}\\
&(u^\dagger \bar{\mathcal{N}}^c)_{i_L} (u^\dagger D^2 \mathcal{N})_{j_L}
\end{aligned}
$$

E.2 $(3_L, 5_R)$ vertices using HBChPT

At $O(p^0)$ we have the term

$$(u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$

and at $O(p)$ using one derivative

$$\begin{gathered}
(u\bar{\mathcal{N}}^c)_{i_R}S^\mu(uD_\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(uD_\mu\bar{\mathcal{N}}^c)_{i_R}S^\mu(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(u\bar{\mathcal{N}}^c)_{i_R}S^\mu(u\mathcal{N})_{j_R}(D_\mu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(u\bar{\mathcal{N}}^c)_{i_R}S^\mu(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(D_\mu Ui\tau_L^2)_{l_Rl_L}\\
(u\bar{\mathcal{N}}^c)_{i_R}v^\mu(u\mathcal{N})_{j_R}(D_\mu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(u\bar{\mathcal{N}}^c)_{i_R}v^\mu(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(D_\mu Ui\tau_L^2)_{l_Rl_L}
\end{gathered}$$

$$\begin{gathered}
(u\bar{\mathcal{N}}^c)_{i_R}S^\mu(uu_\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(uu_\mu\bar{\mathcal{N}}^c)_{i_R}S^\mu(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(u\bar{\mathcal{N}}^c)_{i_R}v\mu(uu_\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(uu_\mu\bar{\mathcal{N}}^c)_{i_R}v^\mu(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}
\end{gathered}$$

$$\begin{gathered}
(u\bar{\mathcal{N}}^c)_{i_R}S^\mu(u^\dagger\mathcal{N})_{k_L}(UD_\mu U^\dagger i\tau_R^2)_{k_Rj_R}(Ui\tau_L^2)_{l_Rl_L}\\
(u^\dagger\bar{\mathcal{N}}^c)_{k_L}S^\mu(u\mathcal{N})_{j_R}(UD_\mu U^\dagger i\tau_R^2)_{k_Ri_R}(Ui\tau_L^2)_{l_Rl_L}\\
(u\bar{\mathcal{N}}^c)_{i_R}v^\mu(u^\dagger\mathcal{N})_{k_L}(UD_\mu U^\dagger i\tau_R^2)_{k_Rj_R}(Ui\tau_L^2)_{l_Rl_L}\\
(u^\dagger\bar{\mathcal{N}}^c)_{k_L}v^\mu(u\mathcal{N})_{j_R}(UD_\mu U^\dagger i\tau_R^2)_{k_Ri_R}(Ui\tau_L^2)_{l_Rl_L}
\end{gathered}$$

at $O(p^2)$ with derivatives

$$\begin{gathered}
(u\bar{\mathcal{N}}^c)_{i_R}(uu_\mu D^\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(uu_\mu\bar{\mathcal{N}}^c)_{i_R}(uD^\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(uD^\mu\bar{\mathcal{N}}^c)_{i_R}(uu_\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(uu_\mu D^\mu\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(u\bar{\mathcal{N}}^c)_{i_R}(uu_\mu\mathcal{N})_{j_R}(D^\mu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
(uu_\mu\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(D^\mu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}
\end{gathered}$$

and the $O(p^2)$ terms using χ

$$
\begin{aligned}
&(u\bar{\mathcal{N}}^c)_{i_R}(\chi u^\dagger \mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(\chi u^\dagger\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(\tilde{\chi}^\dagger u^\dagger \mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(\tilde{\chi}^\dagger u^\dagger\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(U\chi^\dagger u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(U\chi^\dagger u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(U\tilde{\chi}u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(U\tilde{\chi}u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(\mathcal{N})_{j_R}(\chi i\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(\chi i\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(\mathcal{N})_{j_R}(\tilde{\chi}^\dagger i\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(\tilde{\chi}^\dagger i\tau_L^2)_{l_Rl_L}
\end{aligned}
$$

and some further derivatives at $O(p^2)$

$$
\begin{aligned}
&(uD_\mu\bar{\mathcal{N}}^c)_{i_R}(uD^\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(uD^2\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(uD^2\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(uD^\mu\mathcal{N})_{j_R}(D_\mu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(uD_\mu\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(D^\mu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(D^2 Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&(u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(D^\mu Ui\tau_L^2)_{k_Rk_L}(D_\mu Ui\tau_L^2)_{l_Rl_L}
\end{aligned}
$$

$$
\begin{aligned}
&\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(uu_\mu D_\nu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\epsilon^{\alpha\beta\mu\nu}(uu_\mu\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(uD_\nu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\epsilon^{\alpha\beta\mu\nu}(uD_\nu\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(uu_\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\epsilon^{\alpha\beta\mu\nu}(uu_\mu D_\nu\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(uu_\mu\mathcal{N})_{j_R}(D_\nu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\\
&\epsilon^{\alpha\beta\mu\nu}(uu_\mu\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(u\mathcal{N})_{j_R}(D_\nu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}
\end{aligned}
$$

and $O(p^2)$ using field strength spurions

$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(uf_{+\mu\nu}\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(uf_{-\mu\nu}\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(uf_{+\mu\nu}\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(uf_{-\mu\nu}\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\mathrm{Tr}(f_{+\mu\nu})$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\mathrm{Tr}(f_{-\mu\nu})$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ul_{\mu\nu}i\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha v_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(r_{\mu\nu U}i\tau_L^2)_{l_Rl_L}$$

$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha S_\beta(uf_{+\mu\nu}\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha S_\beta(uf_{-\mu\nu}\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(uf_{+\mu\nu}\bar{\mathcal{N}}^c)_{i_R}S_\alpha S_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(uf_{-\mu\nu}\bar{\mathcal{N}}^c)_{i_R}S_\alpha S_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha S_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\mathrm{Tr}(f_{+\mu\nu})$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha S_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\mathrm{Tr}(f_{-\mu\nu})$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha S_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ul_{\mu\nu}i\tau_L^2)_{l_Rl_L}$$
$$\epsilon^{\alpha\beta\mu\nu}(u\bar{\mathcal{N}}^c)_{i_R}S_\alpha S_\beta(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(r_{\mu\nu U}i\tau_L^2)_{l_Rl_L}$$
$$(u\bar{\mathcal{N}}^c)_{i_R}v^\mu S^\nu(uD_\nu\mathcal{N})_{j_R}(D_\mu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(uD_\nu\bar{\mathcal{N}}^c)_{i_R}v^\mu S^\nu(u\mathcal{N})_{j_R}(D_\mu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(u\bar{\mathcal{N}}^c)_{i_R}v^\mu S^\nu(u\mathcal{N})_{j_R}(D_\mu D_\nu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(u\bar{\mathcal{N}}^c)_{i_R}v^\mu v^\nu(u\mathcal{N})_{j_R}(D_\mu D_\nu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(u\bar{\mathcal{N}}^c)_{i_R}v^\mu v^\nu(u\mathcal{N})_{j_R}(D_\mu D_\nu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$

and some further derivative terms at $O(p^2)$

$$(uu_\mu\bar{\mathcal{N}}^c)_{i_R}(uu^\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(u\bar{\mathcal{N}}^c)_{i_R}(uu_\mu u^\mu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(uu_\mu u^\mu\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(u\bar{\mathcal{N}}^c)_{i_R}(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}\,\mathrm{Tr}(u_\mu u^\mu)$$
$$(uu_\mu\bar{\mathcal{N}}^c)_{i_R}v^\mu v^\nu(uu_\nu\mathcal{N})_{j_R}(D_\mu Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(uu_\mu u_\nu\bar{\mathcal{N}}^c)_{i_R}v^\mu v^\nu(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(u\bar{\mathcal{N}}^c)_{i_R}v^\mu v^\nu(uu_\mu u_\nu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(uu_\mu\bar{\mathcal{N}}^c)_{i_R}v^\mu S^\nu(uu_\nu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(uu_\mu u_\nu\bar{\mathcal{N}}^c)_{i_R}S^\mu v^\nu(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(u\bar{\mathcal{N}}^c)_{i_R}v^\mu S^\nu(uu_\mu u_\nu\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$

$$(uD_{\{\mu}u_{\nu\}}\bar{\mathcal{N}}^c)_{i_R}v^\mu S^\nu(u\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$
$$(u\bar{\mathcal{N}}^c)_{i_R}v^\mu S^\nu(uD_{\{\mu}u_{\nu\}}\mathcal{N})_{j_R}(Ui\tau_L^2)_{k_Rk_L}(Ui\tau_L^2)_{l_Rl_L}$$

E.3 $(7_L, 1_R)$ vertices using HBChPT

At $O(p^0)$ we have

$$(u^\dagger\bar{\mathcal{N}}^c)_{i_L}(u^\dagger\mathcal{N})_{j_R}(U^\dagger D_\mu Ui\tau_L^2)_{k_Rk_L}(U^\dagger D^\mu Ui\tau_L^2)_{l_Rl_L}.$$

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