Modelling and Control of the Crazyflie Quadrotor for Aggressive and Autonomous Flight by Optical Flow Driven State Estimation

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I. Introduction

Autonomous aircraft control is an interesting and widely researched problem, as it poses great theoretical challenges and may be considered for a vast amount of applications. For instance, small unmanned aerial vehicles (*UAV*) have previously been used to build temporary rope bridges between gorges, with the intention of eventually aiding aid fire brigades and rescue workers [1]. Other, more distant prospects include *UAV*s that navigate public spaces safely to perform basic tasks such as inventorying and refilling shelves in supermarkets. With the rapid development of machine learning and innovations in embedded systems, applications concerning human interaction that were unimaginable ten years ago are almost possible today and will be readily implementable in the near future.

Development of any such application, two points are of key importance. The first is is robust autonomous control on the embedded systems level, enabling operation free from a host computer, enabling applications with warms of drones. The second point second is cheap and accurate positional state estimation maximising the flyable volume. This work seeks to provide basic building blocks for researches and hobbyists, making advanced *UAV* applications possible at a low monetary cost. For this purpose, the *Crazylie 2.0 UAV* is used as a control object. In the modelling section, two different rigid-body models are presented to discuss on the phenomenon of dynamical unwinding and controllability. The corresponding differential flatness equations are derived for both systems, which combined with task scheduling algorithms are create powerful means of obstacle avoiding motion-planning. These algorithms will hopefully serve as a starting point for any *UAV* implementation, but are modular and can be implemented as required.

With respect to autonomous control, an adaptive control system is presented in theoretical terms to improve system robustness to time varying parameters using rotor-current feedback. Furthermore, various rigid-body controllers are considered, and a promising geometric tracking controller is implemented in the embedded system enabling aggressive autonomous flight on the fringe of dynamical.

Finally, the problem of state estimation is tackled by implementing an *IMU*-driven scalar update extended Kalman filter, capable of fusing motion capture data (*MOCAP*), ultra-wideband information (*UWB*), optical flow measurements and laser ranging. Useful algorithms are contributed to the *MOCAP* and *UWB* estimation, and new methods of optical flow and laser ranging are developed from scratch to enable flight in a virtually unbounded flyable volume at a fraction of the cost associated with the *MOCAP* and *UWB* systems.

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II. Modelling

In order to predict and simulate the time evolution of the *UAV*, the rigid-body dynamics are modelled in terms translation $\mathbf{p}(t) \in \mathbb{R}^3$ $[m]$ in the global coordinate system, a quaternion attitude $\mathbf{q}(t) \in \mathbb{R}^4$ and an angular rate vector in the body frame $\boldsymbol{\omega}_{\mathcal{B}}(t) \in \mathbb{R}^3$ $[rad/s]$. Letting $\mathbf{T}_{\mathcal{B}}(t) \in \mathbb{R}^3$ $[N]$ denote forces acting on the body and $\boldsymbol{\tau}_{\mathcal{B}}(t) \in \mathbb{R}^3$ $[N\cdot m]$ denote the torques rotating the body around it's centre of mass, the *UAV* system may be expressed

$$\begin{cases} \ddot{\mathbf{p}}_{\mathcal{G}} = \frac{1}{m}\Im\left\{\mathbf{q}\otimes\begin{bmatrix}0\\ \mathbf{T}_{\mathcal{B}}\end{bmatrix}\otimes\mathbf{q}^*\right\} - g\hat{\mathbf{z}}_{\mathcal{G}} - \frac{1}{m}\mathbf{D}_{\mathcal{G}}\dot{\mathbf{p}}_{\mathcal{G}} \\ \dot{\mathbf{q}} = \frac{1}{2}\mathbf{q}\otimes\begin{bmatrix}0\\ \boldsymbol{\omega}_{\mathcal{B}}\end{bmatrix} \\ \dot{\boldsymbol{\omega}}_{\mathcal{B}} = \mathbf{I}_{\mathcal{B}}^{-1}(\boldsymbol{\tau}_{\mathcal{B}} - [\boldsymbol{\omega}_{\mathcal{B}}]_{\times}\mathbf{I}_{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}}) \end{cases} \tag{1}$$

with the Newton-Euler equations [2]. This quaternion model is superior to common approaches with the Euler- or Tait-Bryan angles in terms of conditioning [3], as it contains no dynamical singularities and is capable of performing aggressive manoeuvres including rolls and loops.

The maps from the generated thrust T_i of each i rotor to body thrusts and torques were presented in the unconventional rotor configuration of the *Crazylie*, and contrary to popular approaches [4] [5], the rotors were modelled as non-linear second-order systems in order to more accurately simulate the system. These models have been distributed open-source to facilitate further development of the *Crazyflie* control system [6].

III. Motion Planning

The motion-planning is a central component of any *UAV* application. With a good architecture, the algorithms can accommodate the needs of many and find use in projects all over the world. For this purpose, we seek to (i) implement dynamically feasible trajectories in minimal information, (ii) minimise energy consumption and (iii) enable obstacle avoidance through quadratic programming.

For the first problem, the notion of differential flatness [7][8] is used, which had previously been proven for a *UAV* dynamics with XYZ Euler-Angles [9]. As a consequence of this work, it is possible to find a set of flat outputs $\boldsymbol{\gamma} \in \mathbb{R}^{N \times 1}$,

$$\boldsymbol{\gamma} = \mathbf{h}(\mathbf{x}, \mathbf{u}, \dot{\mathbf{u}}, \cdots, \mathbf{u}^{(r)}) \tag{2}$$

such that

$$\mathbf{x} = \mathbf{f}_\alpha(\boldsymbol{\gamma}, \dot{\boldsymbol{\gamma}}, \cdots, \boldsymbol{\gamma}^{(q)}), \qquad \mathbf{u} = \mathbf{f}_\beta(\boldsymbol{\gamma}, \dot{\boldsymbol{\gamma}}, \cdots, \boldsymbol{\gamma}^{(q)}). \tag{3}$$

which allows the identification of all system states, including control signals from the $\boldsymbol{\gamma}$ parameters without integration. Having chosen a flat output space of $\boldsymbol{\gamma} = \begin{bmatrix} x(t) & y(t) & z(t) & \psi(t) \end{bmatrix}^T$, the flatness equations were derived and implemented for cheap computation of feed-forward terms in attitude (rotation matrix and quaternion), angular rates, and control signals $\{\mathbf{q}_r, \mathbf{R}_r, \boldsymbol{\omega}_r, \mathbf{T}_r, \boldsymbol{\tau}_r\}$, in the embedded system.

For the second problem of power consumption, consider a *UAV* "mission" where several points need to be traversed in $\mathbb{R}^3$. The power consumed during the mission may be equated to the distance travelled, and, as the flat outputs are differentially independent, we may find the optimal path by solving the travelling salesman problem (*TSP*) in $\mathbb{R}^3$ [10][11][12]. A a genetic algorithm is proposed for this purpose, capable of generating solutions with respect to obstacles and priorities, enabling the *UAV* to reach all high priority points before proceeding through those with lower priority (see Figure 1).

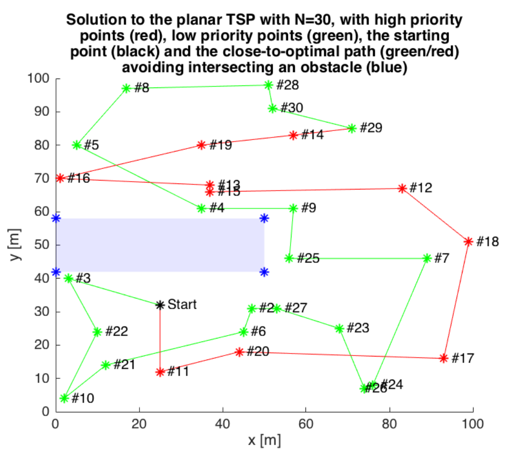

Fig. 1. TSP solution with point priority subsets, with the starting point (black), high priority points $\#11-20$ (red), priority points (green) avoiding an obstacle (blue).

In order to fly along the path generated by the *TSP*-solver, common approaches of quadratic programming [13] are implemented for polynomial path planning with constraints to accommodate generation of obstacle avoiding trajectories compatible with the system dynamics.

With these tools of obstacle avoidance, task scheduling and feedforward computation from the differential flatness, the trajectories may parametrised as sinusoids, Bezier-curves, polynomials or simple linear segments and sent to the *UAV* via radio for on-line autonomous evaluation. With this theory, trajectories may be modified on-line to accommodate complete autonomy allowing the *UAV* to be disconnected from the host computer.

IV. Control

With good motion planning in place, several controllers were considered and compared in theory including *PID*, *MRAC* and variations of the *LQ* controllers. One of the more promising is a the geometric tracking controller [14], in which an attitude error function is defined using the references from the flatness equations

$$\Psi(\mathbf{R}_{\mathcal{BG}}, \mathbf{R}_r) = \frac{1}{2}\mathrm{tr}[\mathbf{I} - \mathbf{R}_r^T \mathbf{R}_{\mathcal{BG}}]. \tag{4}$$

Minimising the error function by driving a control error

$$\mathbf{e}_R(t) = -\frac{1}{2}[\mathbf{R}_r^T \mathbf{R}_{\mathcal{BG}} - \mathbf{R}_{\mathcal{BG}}^T \mathbf{R}_r]^\vee \tag{5}$$

to zero, with the *vee* map $[\cdot]^\vee : SO(3) \to \mathbb{R}^3$ being the inverse of the skew-symmetric map $[\cdot]_\times$ satisfying $[\mathbf{u}]_\times \mathbf{v} = \mathbf{u} \times \mathbf{v}$ for some $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$. Similarly, a control error related to the time derivative of the rotation matrix

$$\mathbf{e}_\omega(t) = \mathbf{R}_{\mathcal{BG}}^T \mathbf{R}_r \boldsymbol{\omega}_r - \boldsymbol{\omega}_{\mathcal{B}} \tag{6}$$

is to be minimised. This is accomplished by computing the control signal torques

$$\begin{bmatrix} \tau_x \\ \tau_y \\ \tau_x \end{bmatrix} = \mathbf{K}_R \mathbf{e}_R(t) + \mathbf{K}_\omega \mathbf{e}_\omega(t) + \boldsymbol{\tau}_r(t). \tag{7}$$

Using a similar approach to the quadcopter translation, with *PD* control and feed-forward terms from the flatness equations, a robust control system is formed with guarantees on robustness and stability [14].

Surprisingly, this control system less robust in reality than in the simulations, likely due to noise induced by asymmetries in the rotors. Running this computationally simple feedback law at a rate of 500 $[Hz]$ in the firmware enables aggressive autonomous trajectory following, here showcased with the system avoiding two tables using *IMU* and *UWB* state estimation (see Figure 2).

V. State Estimation

To reach the goals of cheap autonomous control, good state estimation is of vital importance. If relying solely on *IMU* measurements, biases in the accelerometer measurements will be integrated twice causing the positional estimate to diverge in time. This phenomenon is commonly referred to as dead reckoning control, and illustrates why positional sensory feedback is needed when attempting autonomous flight of the *UAV*. Prior to this thesis, an extended Kalman filter *EKF* had been contributed by Michael Hamer for *UWB* positioning [15][16][17], using a scalar updates and the Loco Positioning System (*LPS*). While being approximately a factor 10 cheaper then it's *MOCAP* ricals, the *LPS* provides far less precise measurements (± 10 $[cm]$) than high performance camera systems used in research labs, typically yielding sub-millimetre accuracy. However, both suffer on account of

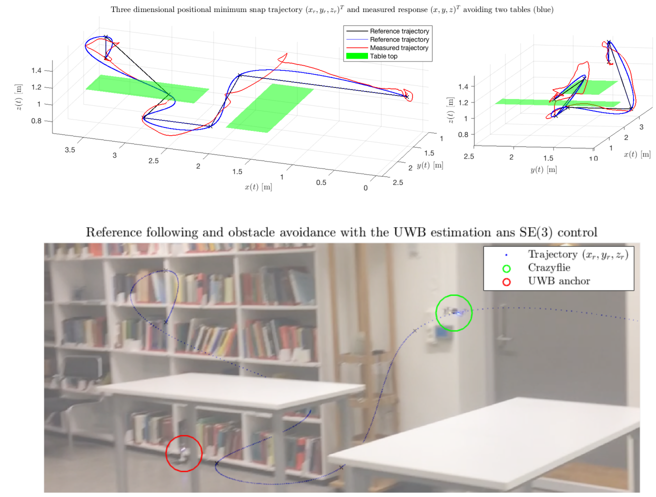

Fig. 2. *Top:* Linear path (black) and generated reference trajectory (blue) as evaluated in the *UAV* firmware and the estimated position (red) avoiding two tables (green).

being external motion capture systems, implicitly putting a bound on the flyable volume of the *UAV*.

While this work has contributed methods of *MOCAP*-positioning and methods of multi path compensation in the *UWB* estimation, a separate goal was to enable positioning free of any external system. For this purpose a cheap method of optical flow and laser ranging was considered, made possibly by developed expansion boards implementing an alpha camera prototype (details of which is currently covered by an *NDA*) and the *vl53l0x* laser sensor (see Figure 3).

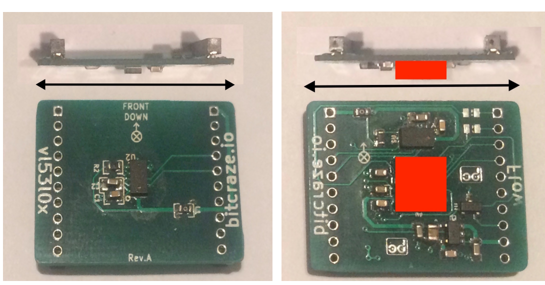

Fig. 3. *Left*: *PCB* for laser ranging in the negative $\hat{\mathbf{z}}_{\mathcal{B}}$-direction using the *vl053x* sensor (black, center) with a 30 [*mm*] arrow. *Right*: *PCB* for laser ranging (black, top) and optical flow (covered red, center) with a 30 [*mm*] arrow.

With the above expansion boards, stable autonomous flight was accomplished at a fraction of the cost of the *LPS*. The method operates without any external system for motion capture, but only provides information on attitude and translational velocity. As such, the problem of dead reckoning and drifting positional estimates persists, but the drift is small enough for the quad-rotor to stay close to the intended target.

VI. Conclusion and Future Work

The implemented control system on the *Crazyflie UAV* is at the time of writing capable of autonomous and aggressive flight using a wide range of sensory information. It provides good on-line motion planning, an embedded implementation of the differential flatness equations, advanced geometric tracking control operating on the rotation matrix and a scalar update *EKF*. The power of the system lies in it's ability to fuse multiple sources of information depending on the available sensors. For instance, using the *UWB* system typically results in a poor altitude estimation which can be improved by simply plugging in the laser ranging, which automatically configures the embedded system to include the measurements in the *EKF*.

Much work remains to be done, and moving forward three topics are of particular interest. The first is to implement make better use of the channel impulse response in the *UWB* estimation and track multi-path components in time using Rao-Blackwellized particle filters. The second is to enable methods of real-time drift removal in the optical flow state estimation by synchronising with known features. The third aspect is to investigate and implement a non-linear feedback quaternion controller based on Lyapunov theory as an alternative to the geometric controller, as it has potential to be more robust in the face of motor noise.

On a final note, much has been done so far, but even more remains to be done. If there is interest in contributing to the project, all code is available open source on-line and collaboration is welcomed [6].

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