

# Car modelling and simulation of an electronic stability control

Thesis

Presented in partial fulfillment of the requirements for the degree of Bachelor of Science

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Bachelor program in electrical engineering with automation

Lund University, Helsingborg 2017

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Printed in Sweden  
E-husets tryckeri  
Lund 2017

# Abstract

Modern cars have a wide range of safety features to ensure that driving is as safe as possible. One of these safety features is called electronic stability control, often abbreviated as E.S.C. The goal of the E.S.C. is to make sure that the car stays on the road even under dangerous circumstances such as a slippery road or the driver making mistakes. During this project a simulation program is made to help with the development of an E.S.C. for a new car.

The purpose of this thesis work was to create a vehicle model and a simulation program that could be used to study a vehicle with and without electronic stability control. The program is made for the company Uniti Sweden AB that is currently developing an electric car. The goal is to give the user of the simulation program the ability to simulate a car making a turn under various driving conditions and to test an electronic stability control program while it is under development.

During the project a three degrees of freedom vehicle dynamics model is created together with two automatic control systems to represent E.S.C.s. and a simulation program in Scilab. The program is able to accurately describe the behaviour of the vehicle during various driving conditions and vehicle parameters. Of the two controllers one is a sliding mode controller and the other is an On/Off controller. Simulations are run to test the controllers and the car model to see how they behave under situations resulting in oversteer or understeer.

## Keywords

Electronic stability control  
Vehicle dynamics model  
Scilab  
On/off controller  
Sliding mode controller

# Sammanfattning

Moderna bilar idag har en mängd säkerhetsfunktioner för att göra körningen så säker som möjligt. En av dessa säkerhetsfunktioner kallas electronic stability control och förkortas ofta E.S.C. Målet med E.S.C:n är att säkerställa att bilen stannar kvar på vägen även under farliga förhållanden så som halt vägunderlag eller förarmisstag. Under projektet skapades ett simuleringsprogram som hjälper vid utvecklandet av en E.S.C. för en ny bil.

Syftet med detta examensarbete är att utveckla en fordonsmodell och ett simuleringsprogram som ska kunna användas för att studera beteendet av ett fordon med, och utan electronic stability control. Programmet är skapat till företaget Uniti Sverige AB som för närvarande utvecklar en elbil. Målet är att ge användaren av simuleringsprogrammet möjlighet att simulera en bil som gör en sväng under olika körförhållanden och testa electronic stability control-programmet medans det utvecklas.

Under projektet har en dynamisk fordonsmodell med tre frihetsgrader skapats tillsammans med två regulatorer som representerar E.S.C. och ett simuleringsprogram i Scilab. Programmet kan på ett bra sätt beskriva uppförandet av fordonet under olika förhållanden och fordonsparametrar.

Av de två olika regulatorerna är den ena en sliding mode regulator och den andra är en av/på-regulator. Simuleringarna körs för att testa regulatorerna och bilmodellen för att se hur dessa uppför sig i situationer som resulterat i överstyrning eller understyrning.

## Nyckelord

Antisladdsystem  
Fordonsdynamisk modell  
Scilab  
På/Av regulator  
Sliding mode reglering

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# Preface

An object stays at rest or in constant motion unless acted on by an unbalanced force.

--Newton's first law of motion





# 1. Introduction

Every car on the market today have a lot of safety features. One of these features is the electronic stability control (E.S.C.) which is a feature that prevents the car from driving off the road by counteracting understeering or oversteering.

Most companies wants to be able to create simulation environments to test the behaviour for their products under different conditions. The car industry wants to be able to simulate the behaviour of their cars on different road conditions to be able to create safety features. To develop these features a lot of testings have to be made which might be both dangerous and expensive since the tests must be made with real vehicles and perhaps even with real people. The use of a simulation environment makes these tests both cheaper and safer since most tests can be made in a computer instead of in a real vehicle on a real road. To be able to reduce the amount of real world testing, the model and the simulation environment must behave as close as possible to the real thing.

## 1.1 Background

This thesis work will be done at Uniti Sweden AB. They are currently developing a high-tech electric car aimed at urban transportation. This car will, like most modern cars, have an E.S.C. (electronic stability control) system as a safety feature that prevents potential accidents from oversteering or understeering. This E.S.C. system is currently in the very early stages of development and it will have to be carefully tested during development to ensure that it is working as desired. This testing will have to be done both by actual physical tests but also through computer simulation. It is with these computer simulations that the thesis workers will work to give Uniti the ability to test their E.S.C. systems as it is developed. Simulation programs already exist but they are usually aimed to be used for specific cars and are expensive. In order to use as little resources as possible and to develop a program specifically for the Uniti car the program is to be developed using Scilab with Xcos and with the possibility of doing certain features in python if they prove difficult to do in either of these environments. These are free so this brings down the development costs substantially. This is new since no E.S.C. simulation system have ever been developed in Scilab and Xcos and never for this particular car.

## 1.2 Purpose

The purpose is to make a vehicle model and an automatic control system that can be used in a simulation environment to test an E.S.C. system for the Uniti car.

## 1.3 Goal

The goal is to have a detailed vehicle model for the car and be able to use this to make an automatic control system that represents the E.S.C. in a given situation. Two examples of situations are to control lateral position when oversteering and lateral position when understeering. Tests will be done in a simulation program that is also to be developed during the project.

## 1.4 Problems

The project goes through a number of phases and for all these phases there are problems that needs to be solved in order for the project to continue ahead. Problems that were faced includes what tools to use, how to make the vehicle model that is to be simulated and to decide on the type of controllers. This all had to be done in a specific order, see section 3.1 for more information. Generally, the problems can be listed as four questions that had to be answered during the different phases of the project. These are listed below.

- 1.Which simulation environment is most appropriate to use?
- 2.Which type of model should be used in the simulation and how should the parameters in this model be estimated?
- 3.Which type of controller is most suitable to use?
- 4.Are the tools in Scilab appropriate to create a graphical user interface for the simulation program or will another program have to be used?

## 1.5 Motivating the choice of thesis work

Our motivation for choosing this thesis work was mostly that the company have very exciting goals and ambitions. They face many interesting technological problems which was the main reason we choose to do our thesis work here.

## 1.6 Demarcation

The thesis work is limited to simulations and not physical tests. A complete E.S.C. system uses very complex algorithms that have to be developed over time for the specific vehicle with all the exact mechanical properties of this vehicle. So the thesis workers will not developed a full E.S.C. system but rather a smaller system that can be used to test the simulation environment and the vehicle model. The project will also focus on yaw electronic stability and not rollover electronic stability. Due to many unknown parameters the project also don't look at wheel rotational dynamics.

## 2. Technical Background

### 2.1 Understeer and Oversteer

When going into a turn a car can experience two undesired types of behaviour: Understeering and oversteering. Understeering typically happens when the car is driving on a slippery road where there is too little traction between the tire and the road. When the driver tries to turn they don't have enough grip on the road and so gets a much larger turn radius than desired.

Oversteering happens when the car turns too much. This typically occurs when the car loses traction on the rear tires so the rear of the car starts to turn out, this increases the yaw angle of the car and causes it to turn more sharply. See figure 1 below:

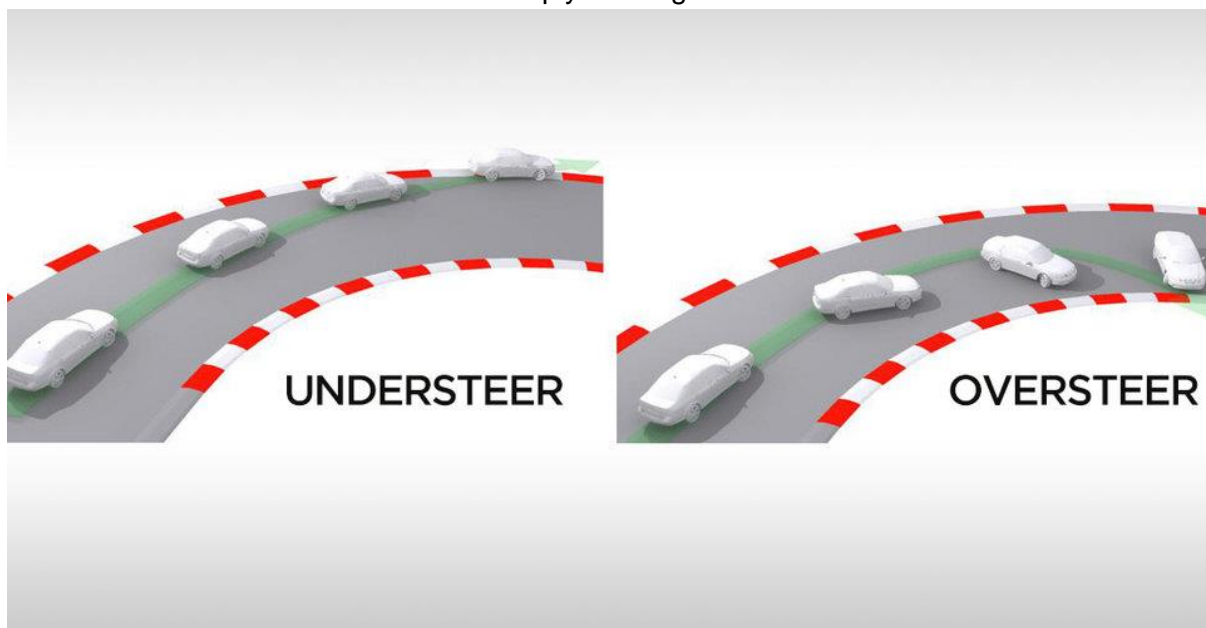


Figure 1: Understeer and Oversteer. Source: [3] Caradvice. Courtesy: Alborz Fallah

As can be seen in figure 1 on the left the car is not turning enough and continues to drive off the road. On the right in figure 1, depicting an oversteer, the car is turning too much and also drives off the road.

### 2.2 E.S.C.

An E.S.C. is based on the principle that there is a computer on the car that constantly gets information about wheel angle and vehicle speed and from this calculates the expected yaw rate and side slip angle of the car. This is then compared to the actual yaw rate and side slip angle of the car as measured by sensors onboard. If the difference between the actual and expected values are small the E.S.C. will do nothing but if the differences are greater than a certain threshold it will draw the conclusion that there is a problem with the yaw stability of the vehicle and act accordingly. This is done by the system applying brakes to the individual

wheels to create a correcting torque on the vehicle body. For example if the vehicle is in understeer then the E.S.C. can brake the rear inner wheel to create some extra torque on the car so that it hopefully stays on the road.

E.S.C. programs are usually very complex algorithms for determining whether a car is unstable or not. Determining their exact parameters is difficult and requires a lot of testing. To save money and resources some of this testing is carried out in simulations rather than in the real world and during this thesis work such a simulation program was made.

### 2.3 Vehicle dynamics model

A vehicle dynamics model uses the forces acting on a vehicle to calculate its behaviour over time. In this section a mathematical description of a vehicle dynamics model is provided. The vehicle model is based on the single track model by Gerdts [2] with force calculations and some additions to the equations of motions from the vehicle dynamics and control book by Rajamani [1].

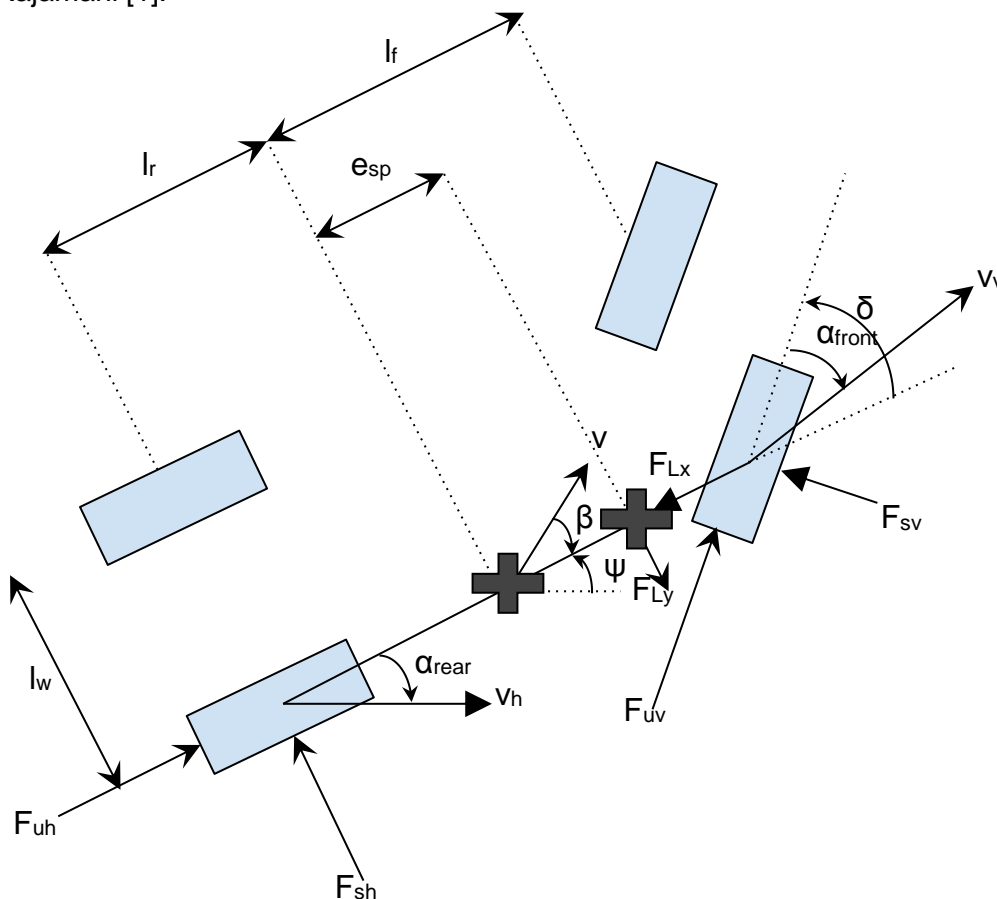


Figure 2: Car model where the symbols are shown ([2] Gerdts. 2003 p. 1).

The first step towards making the simulation program was to have a vehicle model that could describe the behaviour of the vehicle during various situations where the E.S.C. could be involved. Essentially the vehicle model is a mathematical description of how the vehicle behaves. It calculates the forces on the car at a given moment and feed these into a set of differential equations that calculate the second derivative of position and yaw angle. The

side slip angle is also calculated this way. A depiction of the vehicle model is shown in figure 2 that can be seen above. This picture shows the different forces, velocities, distances and angles that are important to the model. For information about the different symbols shown in the figure see chapter 7 of this report.

First the tire slip angle for the different wheels are given by the following expressions ([2] Gerdt. 2003 p. 2).

$$\alpha_{front} = \delta - \tan^{-1} \left( \frac{l_f \dot{\psi} - v \sin \beta}{v \cos \beta} \right) \quad (1)$$

$$\alpha_{rear} = \tan^{-1} \left( \frac{l_r \dot{\psi} - v \sin \beta}{v \cos \beta} \right) \quad (2)$$

To calculate the forces it is also required to have the slip for each tire. First the longitudinal slips are defined by ([1] Rajamani, 2012. p. 384):

$$\sigma_x = \frac{r_{eff} \omega_w - V}{V} \quad (3)$$

during braking and

$$\sigma_x = \frac{r_{eff} \omega_w - V}{r_{eff} \omega_w} \quad (4)$$

during acceleration. Then the lateral slip is defined by:

$$\sigma_y = \frac{V_x \tan \alpha}{r_{eff} \omega_w} \quad (5)$$

Forces on the tires are then calculated with Dugoff's tire model, see Chapter 13 in [1]. Important factors in this tire model are two properties of a tire: the longitudinal stiffness  $C_\sigma$  and the cornering stiffness  $C_\alpha$ . With the help of these two constants and the values for slip and angles of slipping above the longitudinal and lateral forces on a tire can be calculated.

The longitudinal force is given by ([1] Rajamani, 2012. p. 206):

$$F_x = \frac{C_\sigma \sigma f(\lambda)}{1 + \sigma} \quad (6)$$

and the lateral force is calculated with ([1] Rajamani, 2012. p. 206):

$$F_y = \frac{C_\sigma f(\lambda) \tan \alpha}{1 + \sigma} \quad (7)$$

Before these equations can be used the lambda function has to be calculated. For this the lambda value has to be calculated first. This is done with the following equation ([1] Rajamani, 2012. p. 206):

$$\lambda = \frac{\mu F_z (1 + \sigma)}{2((C_\sigma \sigma)^2 + (C_\alpha \tan \alpha)^2)^{1/2}} \quad (8)$$

The lambda function is defined by ([1] Rajamani, 2012. p. 206):

$$f(\lambda) = \begin{cases} 1 & \lambda \geq 1 \\ (2 - \lambda)\lambda & \lambda < 1 \end{cases} \quad (9)$$

The graph of the lambda function is shown in Figure 3:

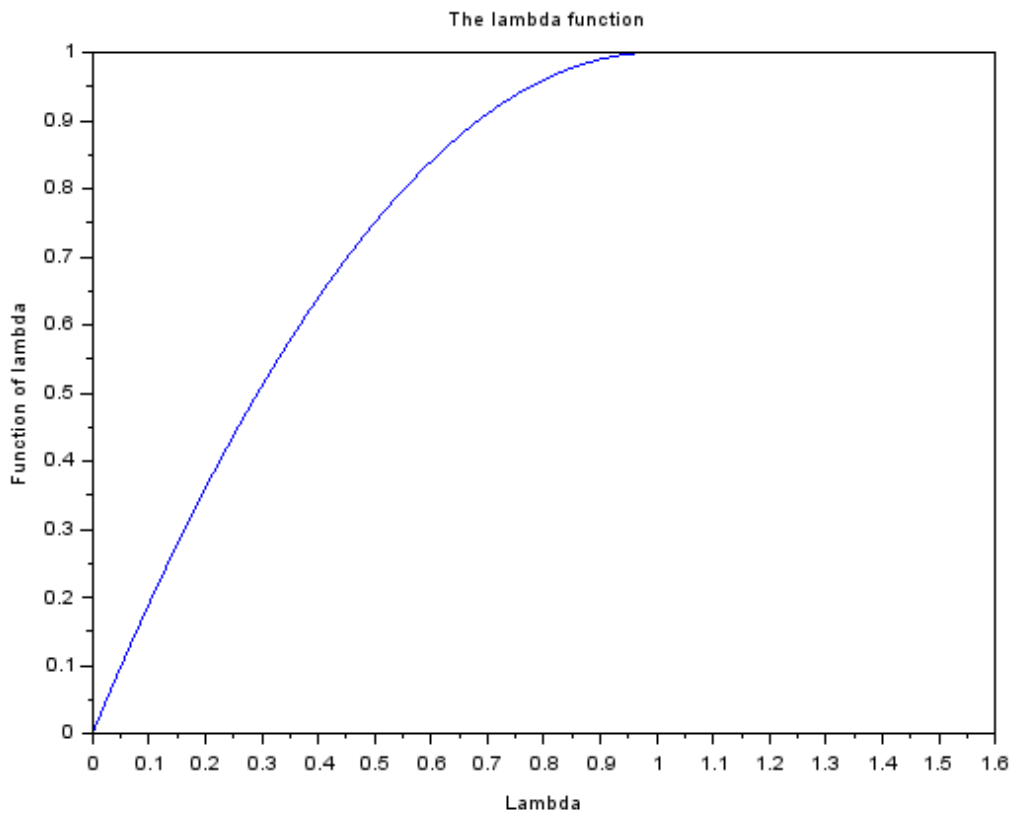


Figure 3: The lambda function.

As can be seen in figure 3 the lambda function has one curvy part when lambda is small and is simply one when lambda is larger than one. This is connected to the fact that the tire is losing grip when the denominator in equation (8) is larger than the nominator.

In order to make the simulation program more useful and to give the user more control over inserting loss of traction accidents during the simulation a loss of traction coefficient is multiplied into equation (7). This gives the following expression:

$$F_y = T_{loss} \frac{C_\sigma f(\lambda) \tan \alpha}{1 + \sigma} \quad (10)$$

When the forces on the car are known Newton's second law of motion can be used to calculate the acceleration on the surface that the car is moving on and to calculate the yaw rate change. From Gerdts we have for the acceleration in the x and y direction ([2] Gerdts. 2003. p. 5):

$$\ddot{x} = \frac{1}{m} [(F_{uh} - F_{Lx}) \cos \psi + F_{uv} \cos(\delta + \psi) - F_{sv} \sin(\delta + \psi) - (F_{sh} - F_{Ly}) \sin \psi] \quad (11)$$

$$\ddot{y} = \frac{1}{m} [(F_{uh} - F_{Lx}) \sin \psi + F_{uv} \sin(\delta + \psi) + F_{sv} \cos(\delta + \psi) + (F_{sh} - F_{Ly}) \cos \psi] \quad (12)$$

The equation for the yaw rate change is a bit more complicated, it also have to take into account that there can be differences between the forces on tires in the same wheel pair. This means that a single track model is not sufficient to describe the behaviour of the yaw rate change so the single track model equation for the yaw rate change needs to be expanded in order to fix this. From Gerdts is as below ([2] Gerdts. 2003. p. 5):

$$\ddot{\psi}_{Gerdts} = \frac{1}{I_{zz}} [F_{sv} l_f \cos \delta - F_{sh} l_r - F_{Ly} e_{sp} + F_{uv} l_f \sin \delta] \quad (13)$$

From Rajamani we have that the total torque on the vehicle produced by differences in forces between wheels in the same wheel pair is given by ([1] Rajamani, 2012. p. 205):

$$M = \frac{l_w}{2} ((F_{xfr} - F_{xfl}) \cos \delta + (F_{xrr} - F_{xrl}) + (F_{yfl} - F_{yfr}) \sin \delta) \quad (14)$$

By adding the torque from the differences in the forces on wheels in a wheelpair to the torque in equation (13) an equation of motion for yaw rate change with the inclusion of differences of forces in a wheel pair is derived. From Newton's second law for rotation the following condition can be used:

$$\ddot{\psi} = \ddot{\psi}_{Gerdts} + \frac{M}{I_{zz}} \quad (15)$$

If  $M$  and  $\ddot{\psi}_{Gerdts}$  is inserted into equation (15) and  $I_{zz}^{-1}$  is factored out this leads to the equation of motion below:

$$\ddot{\psi} = \frac{1}{I_{zz}} \left[ F_{sv} l_f \cos \delta - F_{sh} l_r - F_{Ly} e_{sp} + F_{uv} l_f \sin \delta + \frac{l_w}{2} ((F_{xfr} - F_{xfl}) \cos \delta + (F_{xrr} - F_{xrl}) + (F_{yfl} - F_{yfr}) \sin \delta) \right] \quad (16)$$

The equations above uses a number of forces. They are explained below.

The specific force on a tire has the following notation:  $F_{abc}$  where index a describes if the force is longitudinal (x) or lateral (y). The b index says if the tire is front (f) or rear (r) and the c index tells if it is on the right (r) or the left (l) side of the car. For example  $F_{yrl}$  denotes the

lateral force on the rear left tire, see section 7.2 for more information. It is these forces that can be calculated with Dugoff's tire model.

Most of the forces in the above equations of motion are sums of some of these tire forces. These are described below.

$$F_{uh} = F_{xrl} + F_{xrr} \quad (17)$$

$$F_{uv} = F_{xfl} + F_{xfr} \quad (18)$$

$$F_{sv} = F_{yfl} + F_{yfr} \quad (19)$$

$$F_{sh} = F_{yrl} + F_{yrr} \quad (20)$$

Apart from these there are also the air resistance forces, denoted  $F_{Lx}$  and  $F_{Ly}$  that represent longitudinal and lateral air resistance respectively. These are calculated as follows from the general formula for air drag at non-supersonic speeds:

$$F_{Lx} = 0.5c_w\rho Av^2 \quad (21)$$

$$F_{Ly} = 0.5c_y\rho A(e_{sp}\psi)^2 \quad (22)$$

Since the model will be used to simulate E.S.C. and because of the formulas used to calculate tire slip angle it is necessary to also calculate the side slip angle for the entire car. This can be done by this differential equation ([2] Gerdts. 2003. p. 4):

$$\dot{\beta} = \dot{\psi} + \frac{1}{mv} [(F_{uh} - F_{Lx}) \sin \beta + F_{uv} \sin(\delta + \beta) + (F_{sh} - F_{Ly}) \cos \beta + F_{sv} \cos(\delta + \beta)] \quad (23)$$

These are the equations needed to make a dynamics model for a vehicle. For a list of the symbols used and their meaning see section 7.2.

## 2.4 Programming language and IDE: Scilab

For the project it was decided to use the Scilab with Xcos program to make the simulation program. Scilab and Xcos are similar to Matlab and Simulink although there are differences. Just like Matlab Scilab is a numerically oriented programming language that is inspired by matlab but is open source and free. There are some difference in syntax between matlab and simulink but people with experience of one of these languages generally learns the other one quickly. The main difference between them is that with matlab there are a lot of toolboxes and other helpful functions and programs readily available to the programmer, this is not the case with Scilab. In Scilab the programmer is more dependent upon the basic functions and building blocks of the language. The available online documentation of Scilab is also more limited and harder to find so the one using it has to figure a lot out for themselves.

Still Scilab is powerful. All the necessary basic functions and commands are there and although it may not be possible to take advantage of others work and help the same way as matlab, with some time and effort it is possible to do the same things as in matlab. There is



also a growing list of people and companies using it due to the low cost so the support and available toolboxes is likely to grow over time. ([6] Baudin (2010).).

## 2.5 Sliding mode control

For the project a controller had to be made to work as an E.S.C. One of the controllers used was a sliding mode (also known as a sliding surface) controller. Sliding mode control was first developed by Russian scientists in the sixties. It is a nonlinear control method that uses discontinuous feedback to force the system to reach and stay on the desired surface. This is done by letting the system slide along a cross-section of the systems normal behaviour. The controller consists of one part that forces the state variable towards a control surface. On this surface the dynamics is such that the state is attracted to zero

The main reasons to use sliding mode control are that the controller is fast and robust which means that the controller isn't sensitive to disturbances. This makes the controller suitable for usage in E.S.C. where the controller need to be fast and stable. The controller is efficient even if the system parameters would change.



## 3. Method

### 3.1 Development process

The project went through a couple of phases that had to be done in a specific order since starting a phase required the previous phase to be completed. The project has more or less been following the waterfall model. The exception was the GUI that was developed in parallel to the vehicle model and the two controllers, this is since it doesn't require that the E.S.C. controllers or the vehicle models are finished before it is made. Otherwise the project needs to follow this specific schedule since to develop the controllers the vehicle model first have to be made. See the flowchart in figure 4 for an overview of the development process.

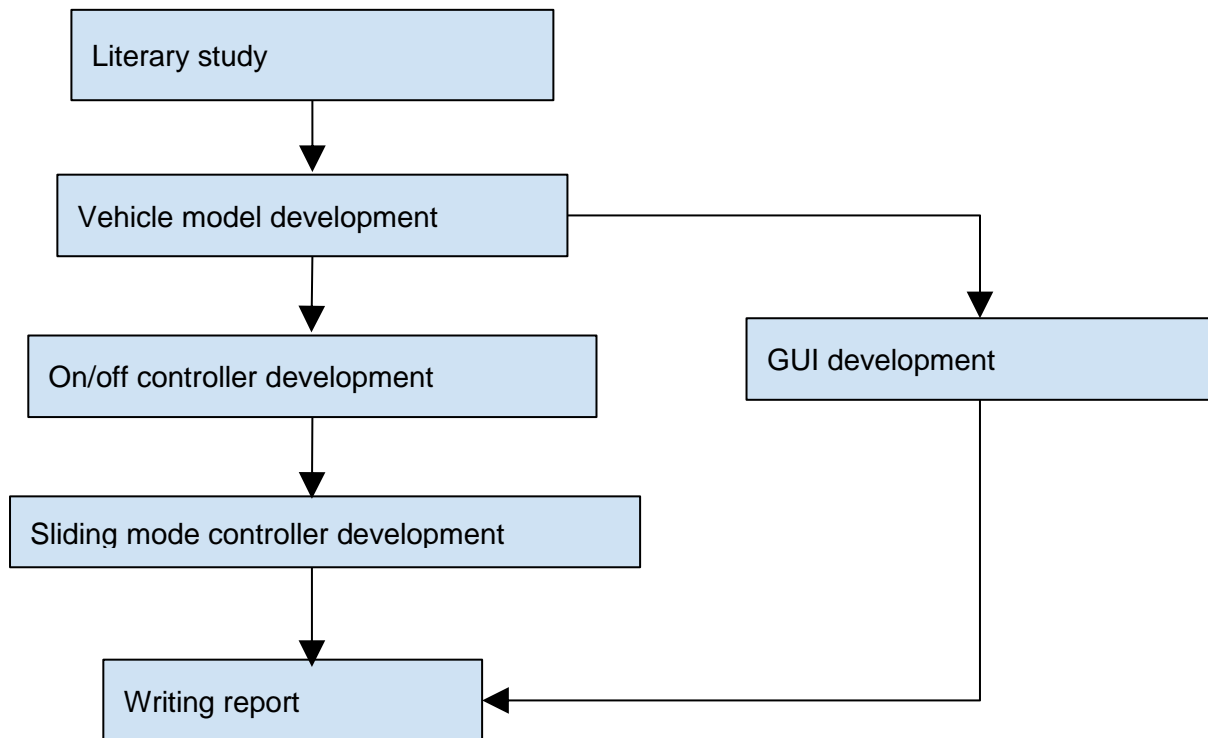


Figure 4 Overview of project

All of the work was carried out at the company where the thesis was done. The two authors usually worked together and since they were located at the company all communications with the company were made face to face for the most part or through the app used for internal communications at the Uniti office: slack.

### 3.1.1 Vehicle model development

The project started with some literature study and finding the mathematical base for the vehicle model. Also a lot of theoretical knowledge had to be found and understood before the actual work could begin. When this literature study was over the next step was to make the vehicle model. This step involved both programming, finding or estimating parameters and writing the code that simulated the vehicle behaviour.

Making the vehicle model was done in a number of steps. First two models were found to describe vehicle behaviour. One was in a book ([1] Rajamani, 2012.) and the other one was a paper describing a single track vehicle model ([2] Gerds 2003). After this a case with only longitudinal forces were studied and a program was made that calculated braking distance on surfaces with given friction coefficient. This was then compared to actual braking distance data to control that this one-dimensional model worked as intended.

Next a two dimensional model was made. This model also calculated lateral tire forces but not longitudinal ones. This was done partly in order to ensure that the lateral forces were calculated correctly but also so that the equations of motions behaved as desired. A significant amount of bug elimination had to be done. The decision was made to use the equations of motion from the single track model paper but modified with some terms in the

yaw rate change equation from the vehicle dynamics control book. These extra terms described how the yaw rate change is affected by two different wheels in a wheel pair having different torque.

The third step was to add the longitudinal forces to the two dimensional model and to calculate the longitudinal and lateral tire forces for each wheel individually. At this point the program followed the mathematics described in section 2.3 of this report. It was now a three degrees of freedom model, consisting of x and y position and yaw angle. First tire side slip angle is calculated by using equations (1) and (2) and from this the tire forces can be calculated using equations (3) and (10). Air resistance forces are calculated last with equation (21) and (22). With the forces known it is possible to solve the equations of motion (11), (12) and (16) and thereby find the components of the acceleration vector and the yaw rate change. From this the velocity and position can be found using the following formulas:

$$V_x = \dot{x}h + V_{x\_old} \quad (24)$$

$$V_y = \dot{y}h + V_{y\_old} \quad (25)$$

$$P_x = V_x h + P_{x\_old} \quad (26)$$

$$P_y = V_y h + P_{y\_old} \quad (27)$$

And yaw rate together with yaw angle can be similarly calculated with the following equations:

$$\dot{\psi} = \dot{\psi}h + \dot{\psi}_{old} \quad (28)$$

$$\psi = \dot{\psi}h + \psi_{old} \quad (29)$$

Where the variables with the "old" in the index means that the value is the value calculated during the earlier timestep. The value for the side slip angle can be found in a similar way from equation (23) and then using the formula below:

$$\beta = \dot{\beta}h + \beta_{old} \quad (30)$$

After this testing was conducted by comparing the vehicle behaviour during various driver behaviour with the desired values of side slip angle and yaw rate. These were calculated with the formulas below ([1] Rajamani, 2012. p. 209-211).

$$\dot{\psi}_{des} = \frac{v\delta}{l_f + l_r + \frac{mv^2(l_r - l_f)}{2C_\alpha(l_f + l_r)}} \quad (31)$$

$$\beta_{des} = \frac{l_r - \frac{l_f mv^2}{2C_\alpha(l_f + l_r)}}{l_f + l_r + \frac{mv^2(l_r - l_f)}{2C_\alpha(l_f + l_r)}} \delta \quad (32)$$

Under good road conditions and driving behaviour these desired values should be attainable by the car without any special intervention but when conditions are bad or the driver makes mistakes the yaw rate and side slip angle of the car will be significantly different from them.

The final important factor of the vehicle dynamics model that were of interest and also very useful for debugging is the kinetic energy of the system. This can be calculated with the following equation.

$$E = \frac{mV^2}{2} \quad (33)$$

### 3.1.2 On/Off controller development

When the vehicle model was done the next step was to make an E.S.C. controller. The first step was to make an On/Off controller and test this with understeering and oversteering.

The basis for the on/off regulator was that it looked at the difference between the desired and the actual yaw rate. If the difference between the two were small then the controller would do nothing. If it was over a certain threshold the controller started braking the front outer and rear inner wheel respectively to stabilise the vehicle. The controller can decide if the vehicle is in oversteer or understeer by looking at the sign of the desired value minus the actual value:  $\dot{\psi}_{des} - \dot{\psi}$ . If this is negative the car is in oversteer otherwise it is in understeer. The E.S.C. should not be active all the time since that would result in the driving being rather uncomfortable. To avoid this a sensitivity parameter of the controller have to be used, in other words, this means that we want a value  $a$  so that

$$\dot{\psi}_{des} - \dot{\psi} > a \quad (34)$$

Through testing this a value was chosen as

$$a = \frac{\dot{\psi}_{des}}{8} \quad (35)$$

When the E.S.C. system know that the car is unstable and should be stabilised it needs to determine if it is in understeer or oversteer. This can be done in accordance with the table below:

Turn and incident	Oversteer	Understeer
left	$\dot{\psi}_{des} < \dot{\psi}$	$\dot{\psi}_{des} > \dot{\psi}$
right	$\dot{\psi}_{des} > \dot{\psi}$	$\dot{\psi}_{des} < \dot{\psi}$

Table 1. Turn direction and type of incident.

Once the system knows what the situation looks like the next step is to manipulate the system so that it becomes more stable. This is done following a simple rule: if the car is in understeer brake the inner rear wheel, if it is in oversteer brake the outer front wheel. This braking creates a torque on the car that makes the yaw rate and side slip come closer to the desired values.

After this testing was made with both the vehicle model and the E.S.C. controller to make sure that they both worked as they should. An E.S.C. controller always has limitations where it will no longer be able to correct the car and these were studied.

### 3.1.3 Graphical user interface

During the developing of the E.S.C. and the model a GUI was also created. The purpose of the GUI is to create an opportunity for a general car simulation. The user is able to put in parameter values not only for the unit car but from any car that the user wants to simulate.

The GUI has one screen with editboxes to type in parameter values and checkboxes to simulate whatever the user want to test. Examples of simulations could be left or right turns on different road conditions e.g dry tarmac or ice. The simulation give the user the opportunity to see the car behaviour in a safe environment on the computer instead of on a real road where the test could result in a crash.

To be able to create the GUI an add-on program was needed to the Scilab program. The name of the program is called GUIbuilder and it let the user create textboxes, buttons and checkboxes (and a bunch of other boxes not needed in this GUI). These can later be linked to the Scilab program so the user can use all the buttons and boxes to type the values of the parameters in Scilab from the GUI.

### 3.1.4 Sliding mode controller

At this point a sliding mode controller was made. The sliding mode controller is typically what is used as the E.S.C. in an actual car. This controller looked at both the slip angle and the yaw rate to determine if the car was unstable. This sliding mode controller is based on the description for making a sliding surface controller found in Rajamani p 208-218. The first step in making this controller was to find the upper values for side slip angle and yaw rate with the equations below ([1] Rajamani, 2012. p. 212-213).

$$\dot{\psi}_{upper\_bound} = \frac{0.85\mu g}{v} \quad (36)$$

$$\beta_{upper\_bound} = \tan^{-1}(0.02\mu g) \quad (37)$$

With the help of these values the values for the targeted side slip angle and yaw rate can be found with the following formulas ([1] Rajamani, 2012. p. 212-213).

$$\dot{\psi}_{target} = \dot{\psi}_{des} \text{ if } |\dot{\psi}_{des}| \leq \dot{\psi}_{upper\_bound} \quad (38)$$

$$\dot{\psi}_{target} = \dot{\psi}_{upper\_bound} \text{ sign}(\dot{\psi}_{des}) \text{ if } |\dot{\psi}_{des}| > \dot{\psi}_{upper\_bound} \quad (39)$$

$$\beta_{target} = \beta_{des} \text{ if } |\beta_{des}| \leq \beta_{upper\_bound} \quad (40)$$

$$\beta_{target} = \beta_{upper\_bound} \text{ sign}(\beta_{des}) \text{ if } |\beta_{des}| > \beta_{upper\_bound} \quad (41)$$

Next a sliding surface was chosen. A recommended type of sliding surface is the one seen below ([1] Rajamani, 2012. p. 214).

$$s = \psi - \psi_{target} + \xi(\beta - \beta_{target}) \quad (42)$$

For the sliding mode controller not only the surface is required but also the derivative of the surface. If  $s$  is differentiated the following equation is acquired.

$$\dot{s} = \ddot{\psi} - \ddot{\psi}_{target} + \xi(\dot{\beta} - \dot{\beta}_{target}) \quad (43)$$

The negative fraction between the differential of the surface and the surface can be calculated the following way.

$$\eta = -\frac{\dot{s}}{s} \quad (44)$$

This  $\eta$  parameter needs to have a value. It decides how fast  $s$  is forced towards 0. Through testing this value was chosen as  $\eta = -2$ .

The sliding mode controller has to produce a correcting torque on the vehicle in order for the target values and the desired values to get close enough. In other words the controller tries to achieve  $s=0$ . The correcting torque is represented in the following way.

$$M_{\psi b} = \frac{l_w}{2} (F_{xfr} - F_{xfl}) \quad (45)$$

For the most part braking on the front rear tires of a car is controlled by a valve that distributes the force. How this force is distributed is given by a constant  $\rho$ . In this project this proportioning constant is considered to be equal to 1 so  $\rho = 1$ . This means that  $F_{xfr} = F_{xfl}$  and  $F_{xrr} = F_{xrl}$ . This results in (from equation (16)):

$$M_{\psi b}(\rho + \cos \delta) = \frac{l_w}{2} \left( (F_{xfr} - F_{xfl}) \cos \delta + (F_{xfr} - F_{xfl}) \right) \quad (46)$$

The correcting torque have to be used in the yaw rate change calculations (i.e. equation (16)) and if  $M_{\psi b}(\cos \delta + \rho)$  is substituted into this equation the following one is acquired:

$$\ddot{\psi} = \frac{1}{I_{zz}} \left[ F_{sv} l_f \cos \delta - F_{sh} l_r - F_{Ly} e_{sp} + F_{uv} l_f \sin \delta + \frac{l_w}{2} (M_{\psi b}(\cos \delta + \rho) + (F_{yfl} - F_{yfr}) \sin \delta) \right] \quad (47)$$

With the help of equations (43) and (44) an expression for the yaw rate change can be found:

$$\ddot{\psi} = -\eta s + \ddot{\psi}_{target} - \xi(\dot{\beta} - \dot{\beta}_{target}) \quad (48)$$

Substituting this into equation (47) and factoring out  $M_{\psi b}(\rho + \cos \delta)$  we arrive at the following controller equation:



$$\begin{aligned}
M_{\psi b}(\rho + \cos \delta) &= F_{sv}l_f \cos \delta - F_{sh}l_r - F_{Ly}e_{sp} + F_{uv}l_f \sin \delta + \frac{l_w}{2}(F_{yfl} - F_{yfr}) \sin \delta \\
&+ I_{zz}(-\eta s + \ddot{\psi}_{target} - \xi(\dot{\beta} - \dot{\beta}_{target})) \quad (49)
\end{aligned}$$

In order to be able to use this a value for the  $\xi$  parameter needs to be decided. This can be done in a number of ways but here the value was chosen as shown by the equations below ([4] Liqiang 2014).

$$\left\{ \begin{array}{ll} \xi = 0 & \beta < \tan^{-1}(0.02\mu g) \\ \xi = \frac{\beta}{\tan^{-1}(0.025\mu g)} & \tan^{-1}(0.02\mu g) \leq \beta < \tan^{-1}(0.025\mu g) \\ \xi = 1 & \beta \geq \tan^{-1}(0.025\mu g) \end{array} \right. \quad (50)$$

To solve equation (49) it is also necessary to know the  $\dot{\beta}_{target}$  and  $\ddot{\psi}_{target}$  values. The side slip angle rate target can be calculated from equation (23) and using  $\beta_{target}$ . The yaw rate change target is a feedforward term and can be calculated from the  $\dot{\psi}_{target}$  using the definition of the derivative. If  $\dot{\psi}_{target,t}$  is the yaw rate at time t and  $\dot{\psi}_{target,t-h}$  is the yaw rate one timestep before t then  $\ddot{\psi}_{target}$  is given by the formula below.

$$\ddot{\psi}_{target} = \frac{\dot{\psi}_{target,t} - \dot{\psi}_{target,t-h}}{h} \quad (51)$$

When all the values for solving equation (49) are known it can be solved. Once the control torque is known the individual wheel forces are calculated from the equation between the control torque and the individual wheel forces according to the following equation.

$$\Delta F_{xf} = \frac{2M_{\psi b}}{l_w} \quad (52)$$

## 3.2 Source criticism

There are seven references used in this work. They are described successively below.

[1] Rajamani (2012) Vehicle dynamics and control:

This is a book about vehicle dynamics and describes vehicle behaviour under many different circumstances and situations. It covers a wide range of topics within this subject of which E.S.C. is only a small part.

The book seems like a very reliable source of information. It is one of the more popular books on the subject of vehicle dynamics and it contains references to published peer reviewed papers on all statements within it which makes it easy to find the source of the information. It is also published by Springer that generally makes sure that the contents of the books should keep a high quality.

[2] Gerdts (2003) The single track model:

This article of single track models contains basic information on the mathematical basis for a single track model.

The article is not published in a paper and so likely not peer reviewed but written by a doctor on applied mathematics at the Bayreuth university. It is also published by the university which adds to the reliability.

[3] Fallah (2006) What is understeer and oversteer?:

This is a source describing the basic terminology of understeer and oversteer. It is published on Caradvice which is one of the biggest sites in Australia for general information of cars. The author is the founder of the site. The information is not from a scientific journal but still rather reliable since it is from one of the larger sites for information about cars in the world.

[4] Liqiang, j et al. (2014). Study on adaptive slid mode controller for improving handling stability of motorized electric vehicles.

This is a paper on developing sliding mode controllers for electrical vehicles. It is considered reliable since it is peer reviewed.

[5] Dang, J (2004) *Preliminary results analyzing the effectiveness of electronic stability control (ESC) systems.*

This is a report made after an investigation made by the national highway safety administration in the U.S.A. It is considered reliable since it is published by an administrative branch of the american government.

[6] Baudin, M. (2010). *Introduction to Scilab.*

This article contains basic information about scilab made by the scilab company themselves. It is targeted to people who wants an introduction to the language. It is considered reliable since it is made by the same organization that made the language.

[7] Thomas, B (2008). *Modern reglerteknik*

This book contains basic information about controllers and the theory for them. It is considered a reliable source since it is one of the most used books about controllers in Sweden for university students.

## 4. Analysis

This section of the report contains information about the different choices and problems faced during the project. These are sometimes connected to each other since encountered problems led to decisions that pushed the project in a certain direction. For example problems when making the vehicle model resulted in choices having to be done on what references to base the vehicle models mathematical description on. Below there are two sections dealing with both the choices that have been done and the problems and how they were solved.

### 4.1 Choices

During the project many problems were faced and many choices had to be made. The first one was the choice of tools and language for the simulation program. Earlier E.S.C. simulation programs used matlab and simulink, and often with matlab based tools that simplified the work. In this project the same was attempted without using these tools and try to use only free and open source software and tools. Early the choice fell on Scilab as the language to use since it is an open source numerically oriented programming language that was perfect for this type of task. In the beginning it was not certain that the Scilab language would be enough to do all that was supposed to be done and the idea was that if the functions in Scilab was not enough python would be used for those parts instead. However Scilab proved to contain all the required functions and hence Scilab was used for all the programming.

There were also two parts of the Scilab IDE that could be used for the project. The Scilab code itself and the graphical editor Xcos. In the beginning it was thought that Xcos would be used since it is made for constructing system dynamical models and could prove very useful when making the different automatic control systems that were required for the project. However in the end it was decided to do all the coding in the Scilab editor using only the Scilab code since this proved to be a more straightforward way to incorporate all the mathematics into the code.

Another choice was how many degrees of freedom that should be used in the simulation. Typical numbers of degrees of freedom are 3 or 7. These are the x-position, y-position, yaw angle and in the case with 7 degrees also four extra degrees of freedom which are the rotational speeds for each wheel. During the project it was decided to use a three degrees of freedom model. This is because most of the factors required to accurately describe wheel dynamics for the uniti car are unknown and a sufficiently good simulation could be done anyway without knowing the exact dynamics of the wheels.

Another choice was on how the forces on the wheels should be calculated. There are two very common ways to do this. One is the Dugoff's tire model and the other is with Pacejka's magic formula. For this project the Dugoff's tire model was used. The main reason for using Dugoff's formula was that it accurately describes tire traction loss in cornering. The other reason was that tire parameters are easier to find using Dugoff's formula than from using Pacejka's. This is since Pacejka's formula require four tire constants out of which three are

difficult to find. The Dugoff's formula uses only two constants and these are somewhat easier to find data for.

Later in the project it was also decided what type of controllers to use. The recommended is to use a sliding surface controller so a program using a sliding mode controller was developed. This type of controller is very good to use for this type of application since it is easy to control many variables of the system with a single controller and it is also relatively fast. An on/off controller was also developed to compare with the slide mode controller and this gave a straightforward way to study the vehicle model under typical E.S.C. intervention. The on/off controller is relatively simple mathematically and so makes it easy to study the behaviour of the system. There are of course many other types of controllers that could have been used for the project and it was considered for a while to try and use a PID controller but in the end sliding mode and on/off was chosen. The reason for not choosing the PID controller was mainly because the system is so non-linear that it's unlikely that a PID controller would be able to control the system.

## 4.2 Problems and their solutions

Many of the problems faced during this project have been coupled to testing. Testing is necessary to ensure that the program is working as intended and does not contain any bugs.

The first problem was to ensure that the vehicle model worked. That the tire force calculations worked could be decided by running simulations on braking cars and compare these with actual braking distances from physical tests. The rest of the dynamics model were more difficult to test since vehicle behaviour data for a car with the same parameters as the uniti car was unavailable. Many tests using different driving behaviour and with changes to the friction coefficient were conducted. The position, kinetic energy, yaw rate and side slip angle was used to determine if the vehicle was behaving as it should. A great advantage here was that the desired yaw rate and side slip angle could be calculated from equations (31) and (32) and then the actual yaw rate and side slip angle could be compared to these to see that the model behaved as expected.

Another problem was that the model dynamics model used (see section 2.3) didn't take into consideration what would happen if the side slip angle became very large. If the side slip angle was larger than  $0.5\pi$  but smaller than  $1.5\pi$  the equations of motion (11), (12) and (16) treated the braking forces from the E.S.C. as accelerating forces instead. This was fixed by extending the code calculating the tire forces so that they were inverting all forces if side slip angle was within this interval.

Another problem was the difficulty of finding the vehicle parameters. Some of them could be found from documentation at Uniti but the majority had to be estimated. To aid in the estimation it was necessary to use information found about other cars and make reasonable assumptions about the Uniti car based on these.

A few times it was not possible to make the simulation program running correctly with the formulas found from various sources. For example when using the equations of motion found in the Vehicle dynamics control book ([1] Rajamani, 2012 p. 205) the program did not

calculate correct yaw angle or position. This was solved by using the equations of motion from The single track model instead ([2] Gerds. 2003) but modified to take into account that there are four wheels that can provide torque on the entire car if braked individually.



## 5. Results

The result of the thesis work is a simulation program with a vehicle dynamics model and two automatic controllers that functions as E.S.C.s in the simulation. This chapter describes some tests and their resulting outputs for some given inputs using the program to study both oversteering and understeering with the E.S.C. either on or off.

Tests are carried out using the steady state cornering test to find situations where a vehicle might experience understeer or oversteer. In this section results from some such tests are shown. First for some stable conditions of the vehicle dynamics model and then from unstable conditions without an E.S.C. After that there are two sections dealing with the results from the On/Off controller and the sliding mode controller respectively.

The steady state cornering test can be done in a few different ways. The test is conducted with the vehicle entering a turn with a given velocity and direction, the vehicle then steers with a specific angle on the front wheels. The vehicle will then go into a trajectory resembling a circle if the road conditions are good and nothing unexpected happens. If oversteering or understeering occurs then the car will not move in the intended way and the effectiveness of the E.S.C. can be studied by seeing if it can correct this or not.

## 5.1 The vehicle dynamics model

### 5.1.1 Vehicle parameters

The vehicle parameters were found or estimated according to the following table:

Parameter	Symbol	Value	Unit	How value was found
Mass	$m$	450	kg	Uniti car specification sheet
Moment of inertia	$I_{zz}$	338	$\text{kgm}^2$	$I_{zz} = \frac{m}{12} (l^2 + w^2)$
Distance C.O.G. to front axis	$l_f$	0.9	m	Estimated
Distance C.O.G. to rear axis	$l_r$	0.9	m	Estimated
Distance between wheels on the same axis	$l_w$	1.0	m	Measured
Cornering tire stiffness	$c_\alpha$	20000	N/rad	Estimated
Longitudinal tire stiffness	$c_\sigma$	100000	N/unit slip	Estimated
Drag mount point	$esp$	0.5	m	Estimated
Air density	$\rho$	1.2754	$\text{kg/m}^3$	Normal air density
Longitudinal air drag coefficient	$c_w$	0.3		Normal air drag coefficient
Lateral air drag coefficient	$c_y$	0.3		Normal air drag coefficient
Air drag area	$A$	1.1	$\text{m}^2$	Estimated

Table 3. Vehicle parameters



### 5.1.2 Good driving conditions

For all simulations the car starts in position (0,0) and is pointing along the x-axis in the positive direction, i.e. yaw angle is zero. side slip angle is set to zero degrees and the yaw rate and steering wheel angle is also zero.

The following graphs are from a steady state cornering test with  $V=10$  m/s,  $\mu = 0.9$  (dry tarmac) and  $\delta = 0.2$  rad. The simulation lasts for 7 seconds with  $h = 0.01$  seconds and the steering is initiated at 0.2 seconds.

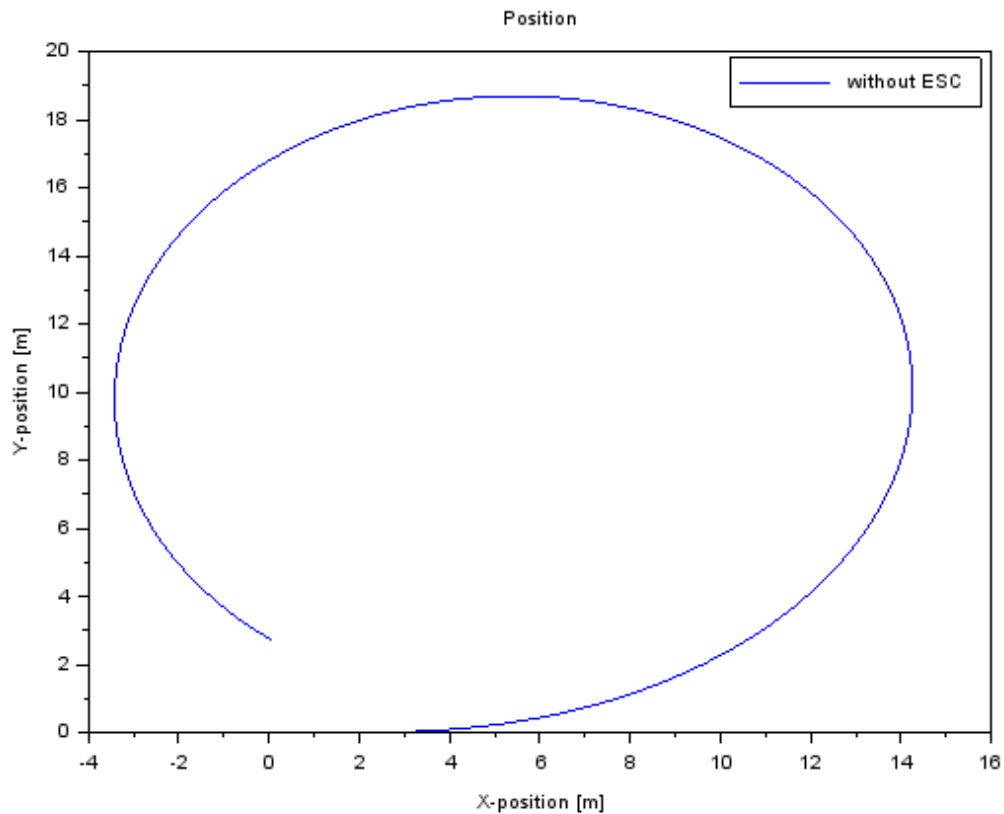


Figure 5: A left turn without E.S.C. on dry tarmac where the driver has the same wheel angle during the whole simulation

Figure 5 shows the trajectory of the car in the situation described above. As can be seen it is approximately a circle. Important is also the side slip angle and yaw rate, which are shown in Figure 6 and Figure 7 respectively:

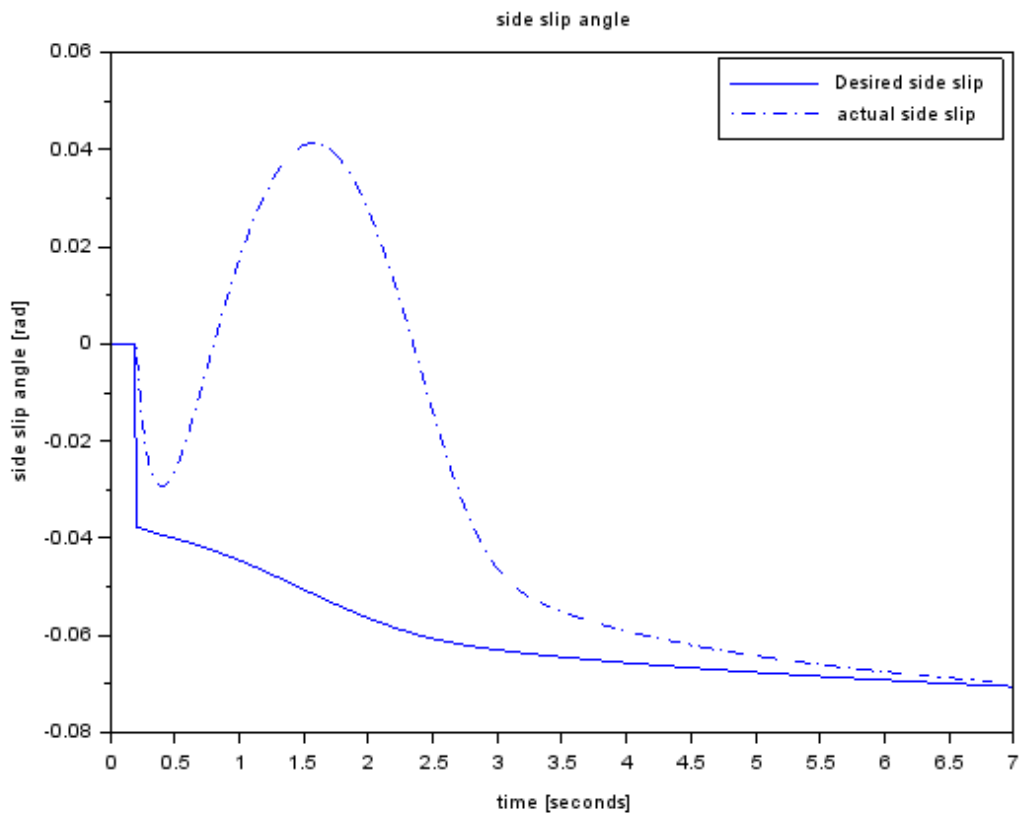


Figure 6: The side slip angle of the car when turning left without E.S.C on dry tarmac where the driver has the same wheel angle during the whole simulation

Here in figure 6 can be seen a certain bump in the side slip angle. This is commonly seen in vehicle dynamics and depends on the fact that the car changes the velocity angle at a different pace than the yaw angle when a turn is initiated.

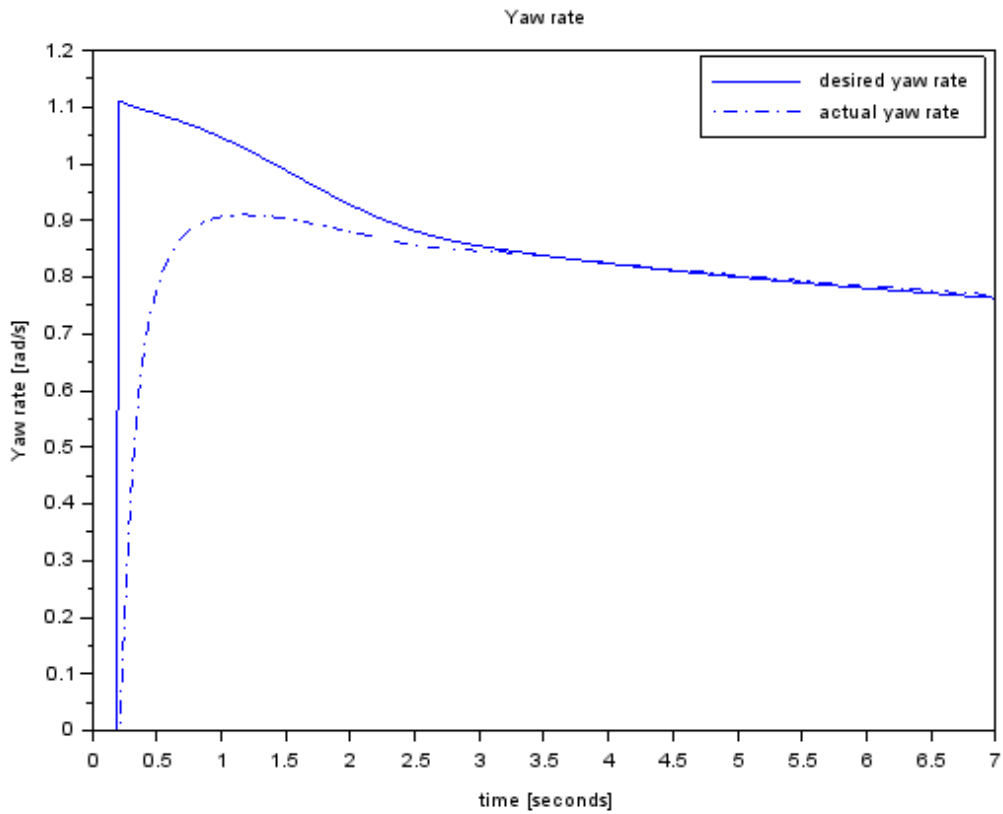


Figure 7: The yaw rate of the car when turning left without E.S.C. on dry tarmac where the driver has the same wheel angle during the whole simulation

As expected the actual values for side slip angle and yaw rate goes towards the desired values. Under good conditions the vehicle system regulates itself and strives to achieve the desired values. In figure 7 it can be seen that the yaw rate starts out at zero but quickly rises to the desired values and reaches this after about 3 seconds.

The simulation program also calculates and plots the kinetic energy of the system. The kinetic energy for this test can be seen below:

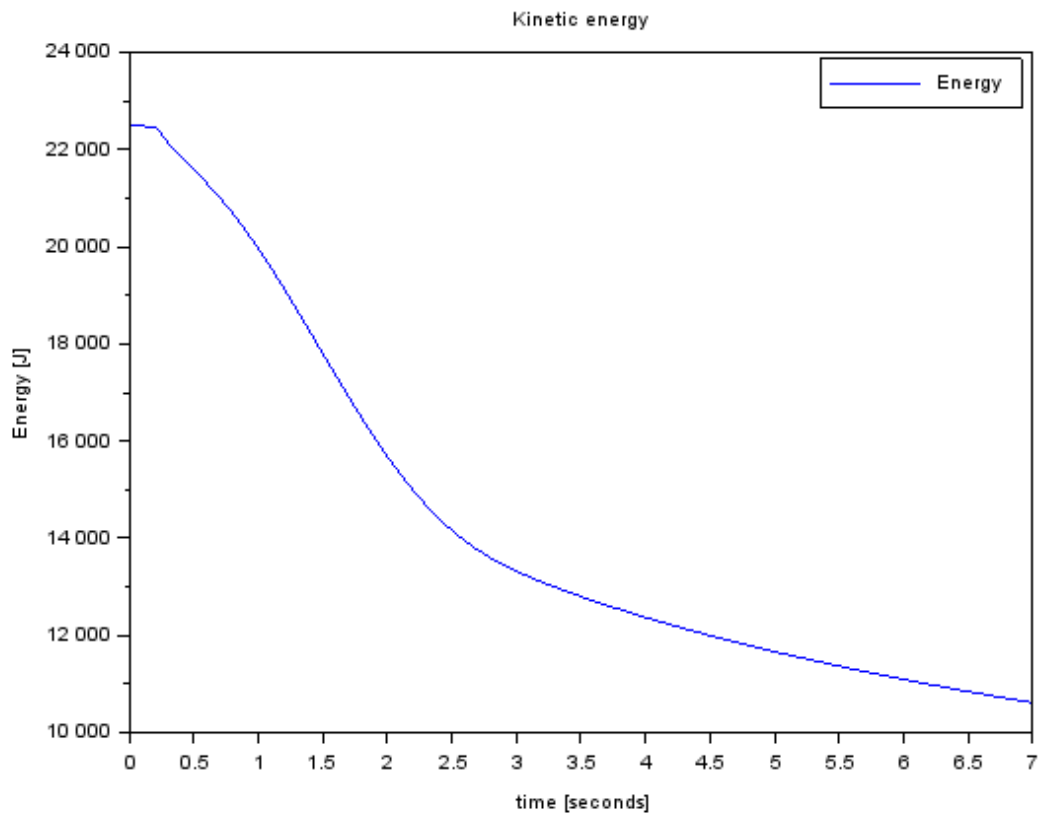


Figure 8: The kinetic energy of the car when turning left without E.S.C on dry tarmac where the driver has the same wheel angle during the whole simulation

As can be seen in figure 8 the Kinetic energy is steadily decreasing. This is because energy is leaving the system through resistance and also some kinetic energy is converted to rotational energy of the vehicle.

### 5.1.3 Understeer without E.S.C.

An understeer situation is created by setting  $\mu = 0.2$  (icy road) and  $\delta = 0.1$  rad. Other variables unchanged.

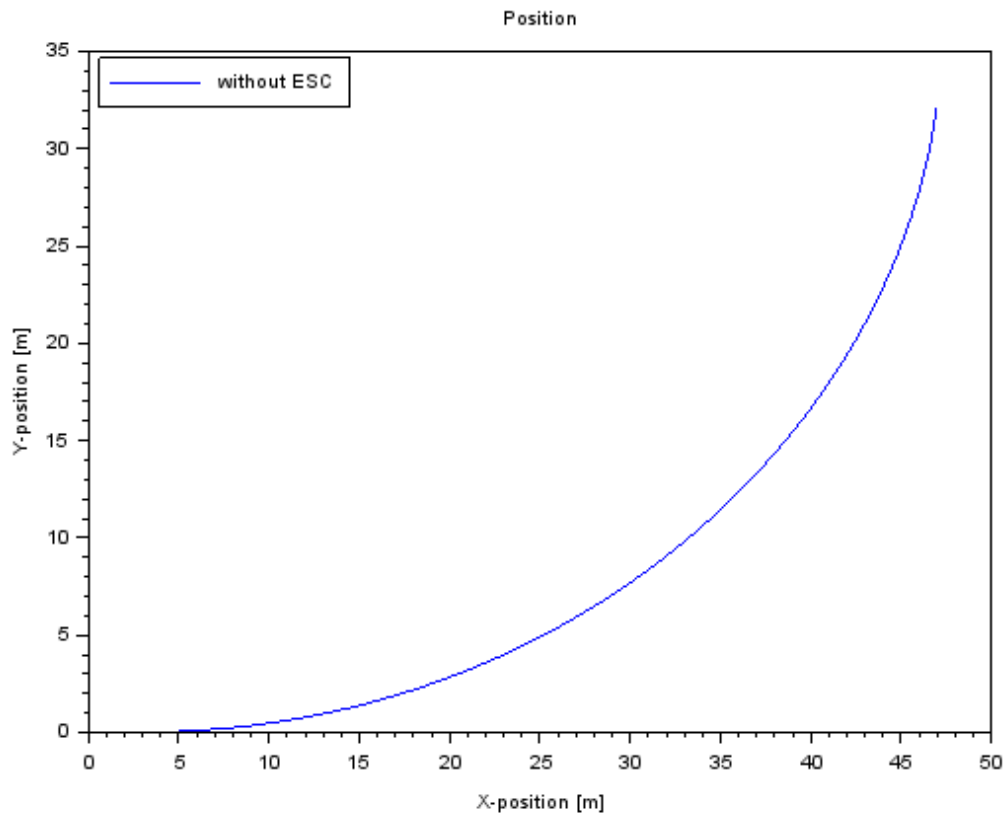


Figure 9: A left turn without E.S.C. where the wheels are in the same angle the whole simulation but loses grip to the road and understeer due to icy conditions

Figure 9 shows the trajectory of the car under these conditions. The curvature is large since the vehicle can't get a good grip to turn.

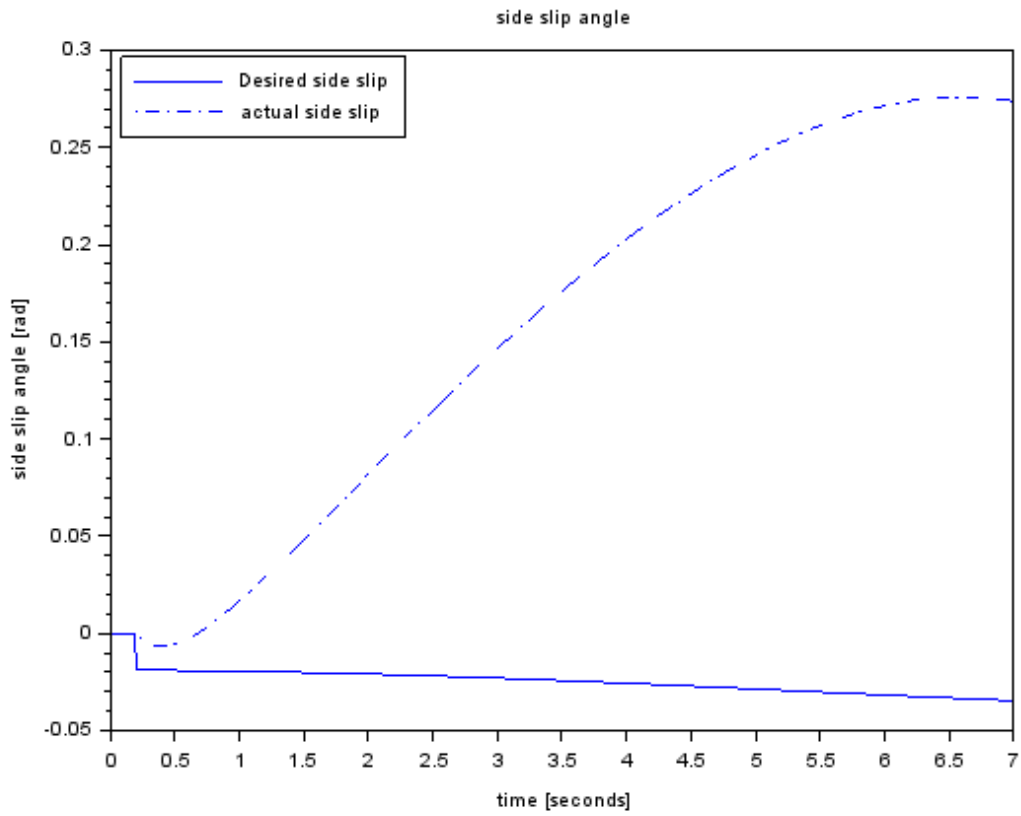


Figure 10: The side slip angle of the car when turning left without E.S.C and understeer due to icy road conditions. The driver has the same wheel angle during the whole simulation

In figure 10 large positive values for the side slip angle can be seen. This is a typical result when driving on a slippery road in a small car. It is very difficult for the car to change the angle of the velocity but easier to change the yaw angle.

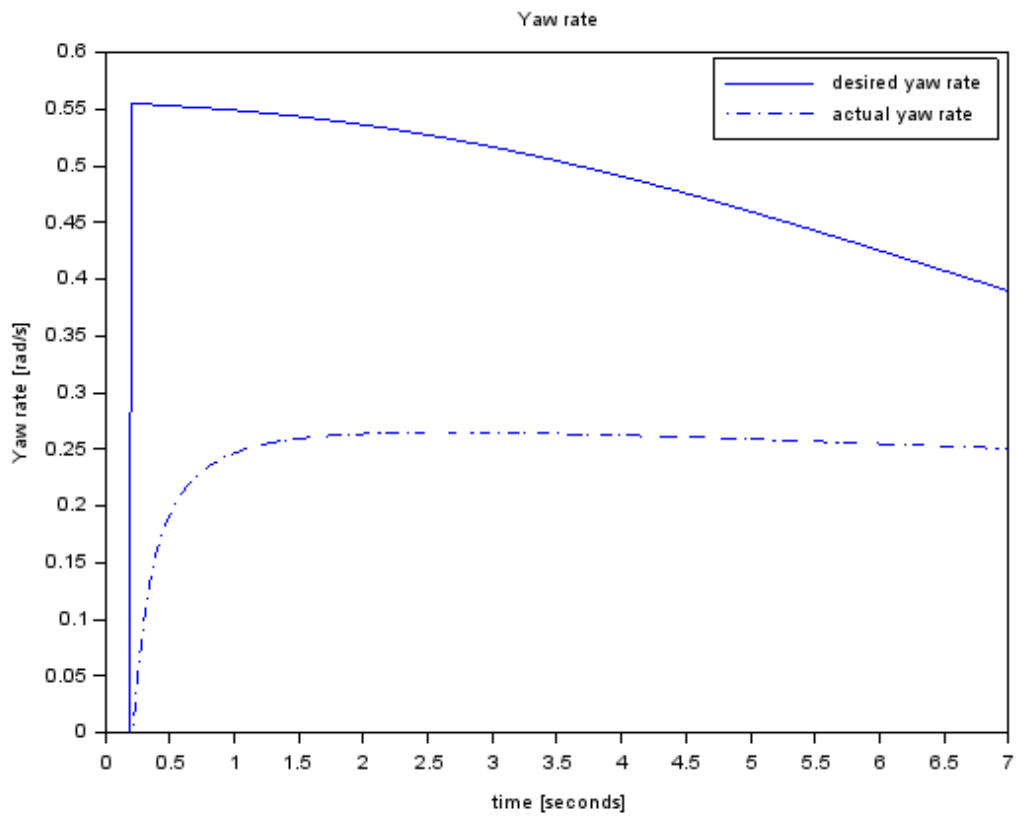


Figure 11: The yaw rate of the car when turning left without E.S.C. and understeer due to icy road conditions. The driver has the same wheel angle during the whole simulation

In figure 11 the side slip angle becomes very large since the vehicle turns but the velocity angle doesn't change that much. When it comes to the yaw rate it is low compared to the desired values, the vehicle doesn't get enough yaw rate.

### 5.1.4 Oversteer without E.S.C.

An oversteer situation is created with  $\mu = 0.9$  (dry tarmac)  $\delta = 0.2$  rad and a loss of 10% traction on the rear tires ( $T_{loss} = 0.9$ ). To make matters worse the traction loss coefficient is reset to  $T_{loss} = 1$  when  $\psi = \pi$  i.e. the car regains grip when it has turned 180 degrees. Other variables are unchanged. This results in the following trajectory, side slip angle and yaw rate:

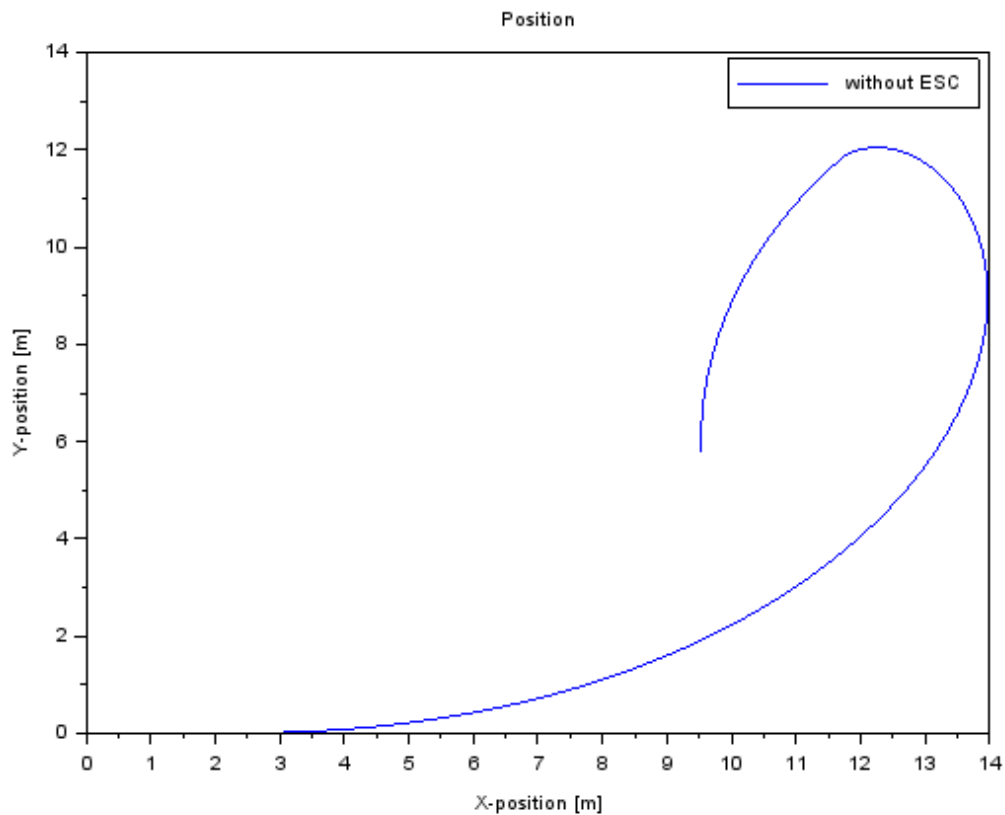


Figure 12: A left turn without E.S.C. where the car has the same wheel angle during the whole simulation. The car is oversteering

Figure 12 shows an oversteer. The car loses grip on rear tires and turns in a sharp bend.



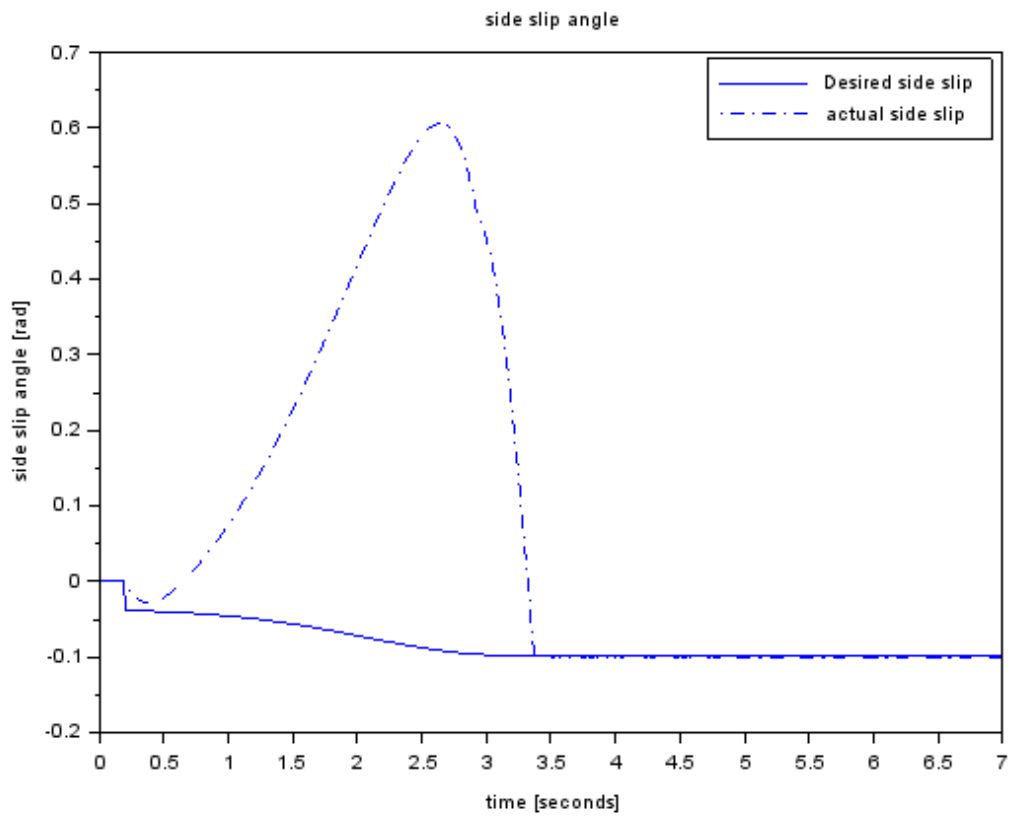


Figure 13: The side slip angle of the car when turning left without E.S.C and oversteer

Figure 13 shows a large bump in the side slip angle, the car is changing yaw angle very fast and then the angle of velocity catches up to bring the car into a sharp bend.

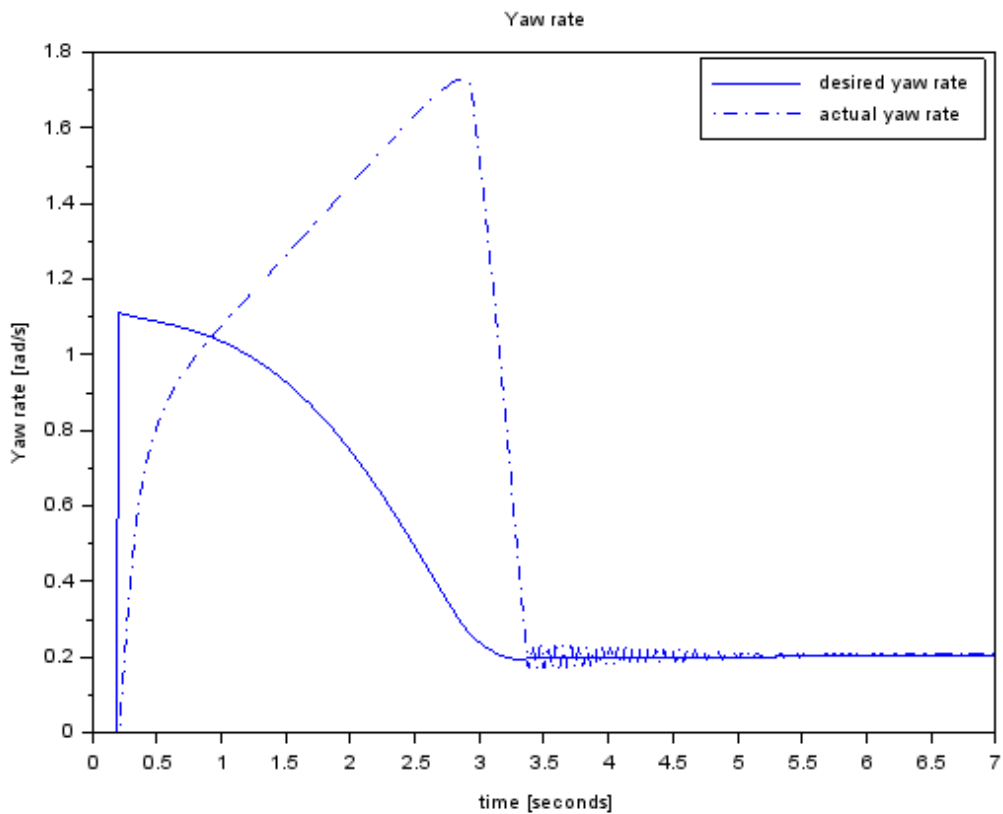


Figure 14: The yaw rate of the car when turning left without E.S.C. and oversteer

In figure 14 the yaw rate can be seen to start low but after approximately 1 second it is larger than the desired yaw rate, meaning that the car is turning too quickly. In both graphs it can be seen that both yaw rate and side slip angle eventually reach their desired values (3-3,5 seconds) but this happens when the car is already moving in the wrong direction so that doesn't solve the problem.

These are the results of the tested situations without an E.S.C. intervention. Next will be the results when the two types of E.S.C. are active in the same scenarios as above.

## 5.2 The on/off controller E.S.C.

The pictures below are understeer and oversteer turns controlled with the on/off controller working as an E.S.C.

### 5.2.1 Understeer

Understeer situation with the exact same parameters as in section 5.1.3.

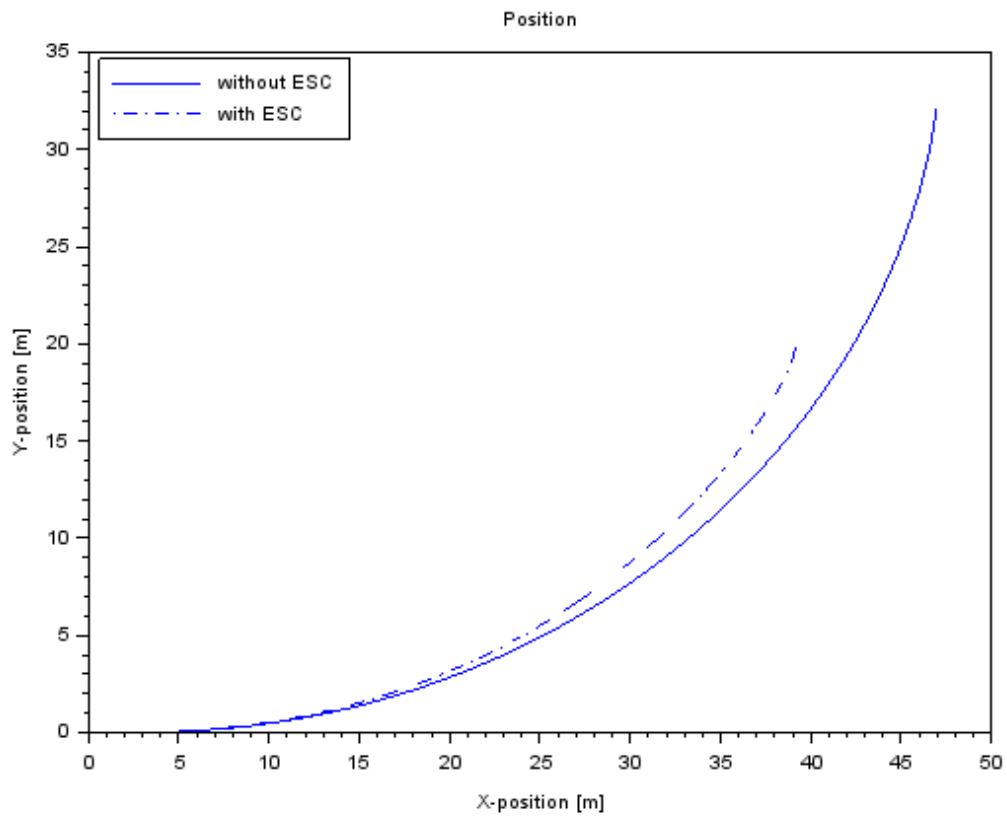


Figure 15: A left turn with and without the on/off controller where the wheels are in the same angle the whole simulation and the car is understeering due to icy conditions

In figure 15 it can be seen that the car is turning somewhat sharper and doesn't move as far as the car without the E.S.C.

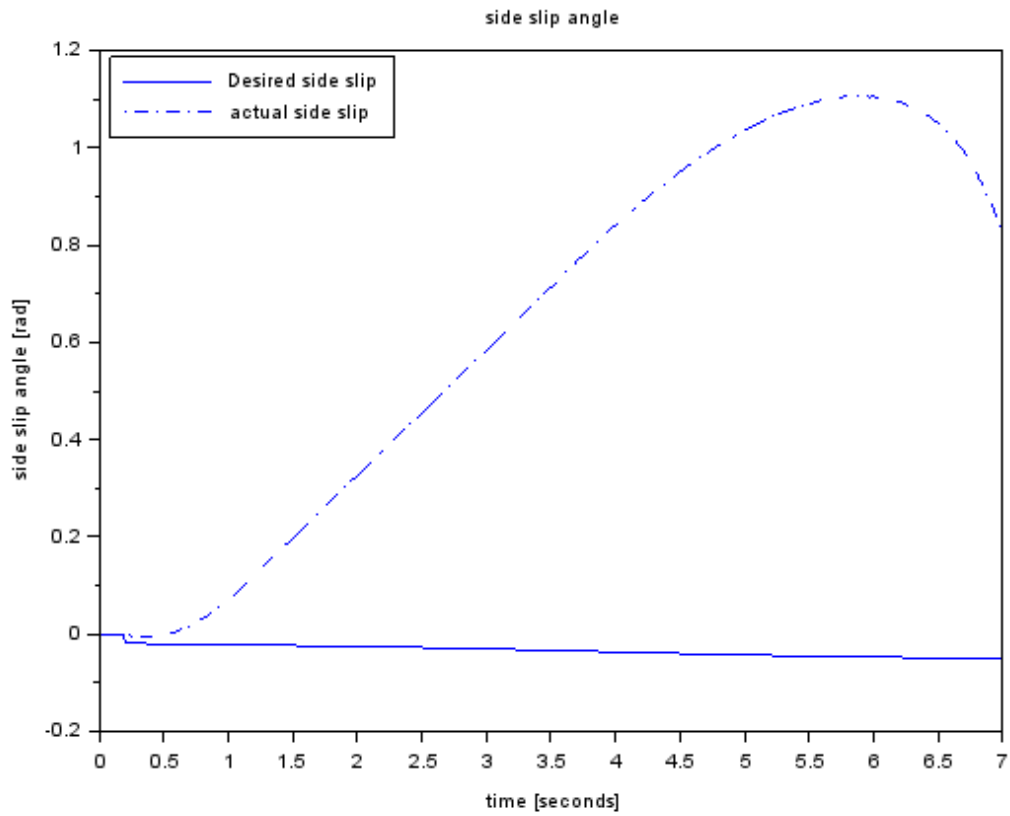


Figure 16: The side slip angle of the car when turning left with the on/off controller and the car is understeering

In figure 16 a larger side slip angle can be seen since the E.S.C. turns the car but doesn't regulate with respect to the side slip angle, only the yaw rate. This is one of the main differences between the on/off controller and the sliding mode controller.

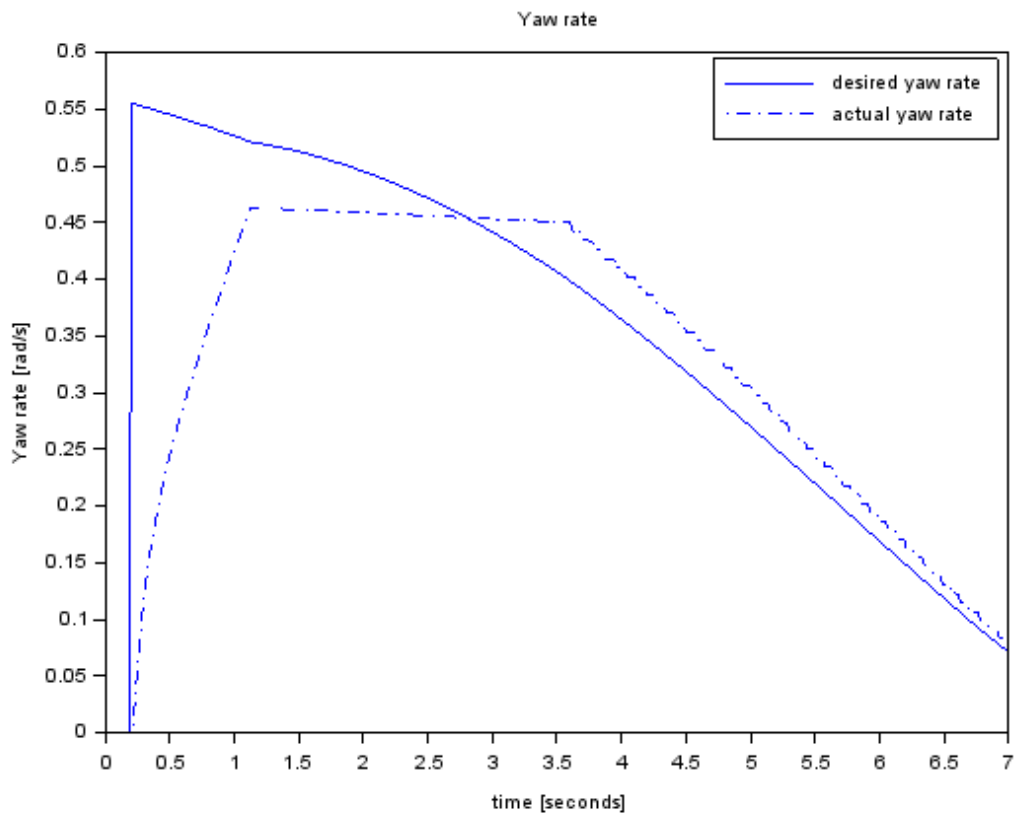


Figure 17: The yaw rate of the car when turning left with the on/off controller and the car is understeering

As can be seen in figure 17 the on/off controller brings the desired yaw rate up to the desired value and keeps it close to it. compare to figure 11. The desired value is sharply reduced though since the E.S.C. causes the vehicle to lose velocity.

## 5.2.2 Oversteer

Oversteer situation with the exact same parameters as in section 5.1.4.

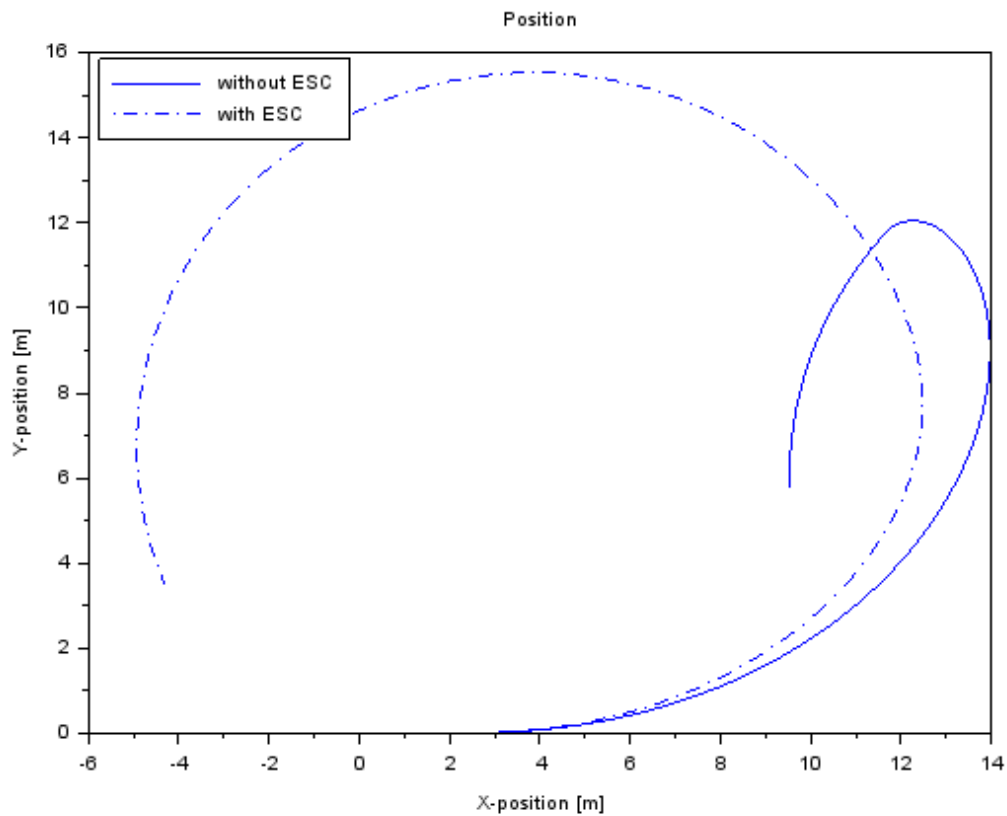


Figure 18: A left turn with and without the on/off controller where the wheels are in the same angle the whole simulation and the car is oversteering

In figure 18 it can be seen that the E.S.C. keeps the car on a safer trajectory, without the sharp bend. Compare to figure 5.

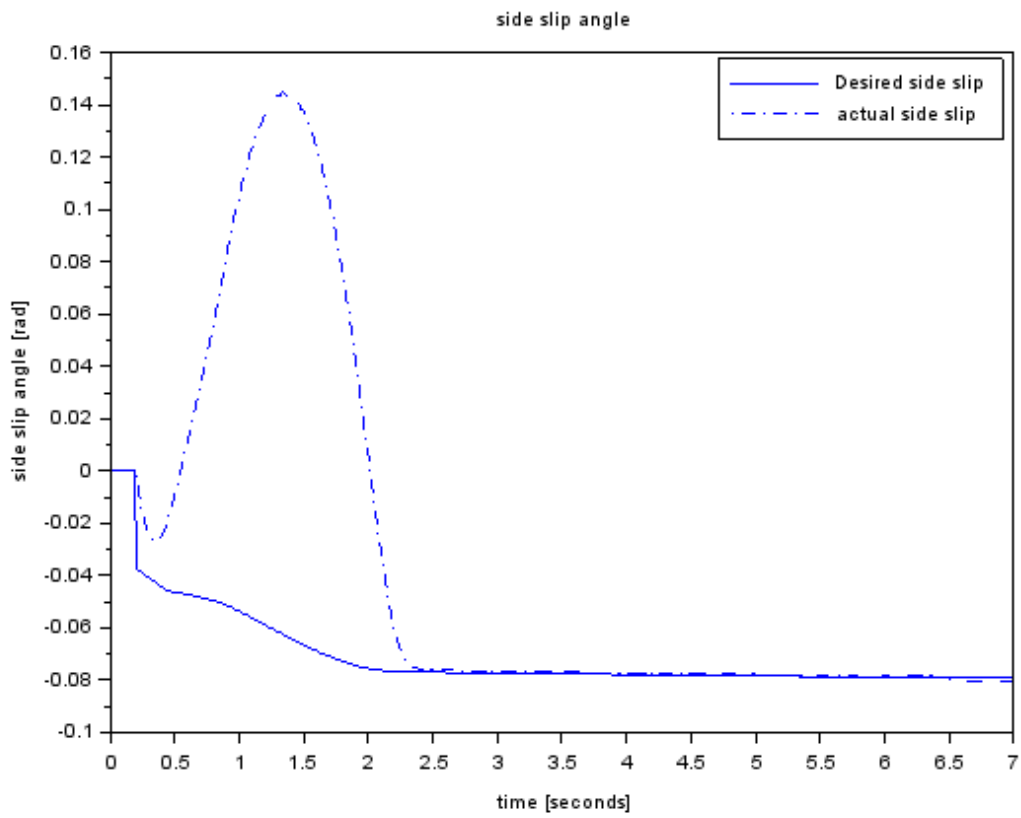


Figure 19: The side slip angle of the car when turning left with on/off controller and the car is oversteering

Figure 19 shows that the peak of the side slip angle is reduced when compared to the situation without E.S.C.

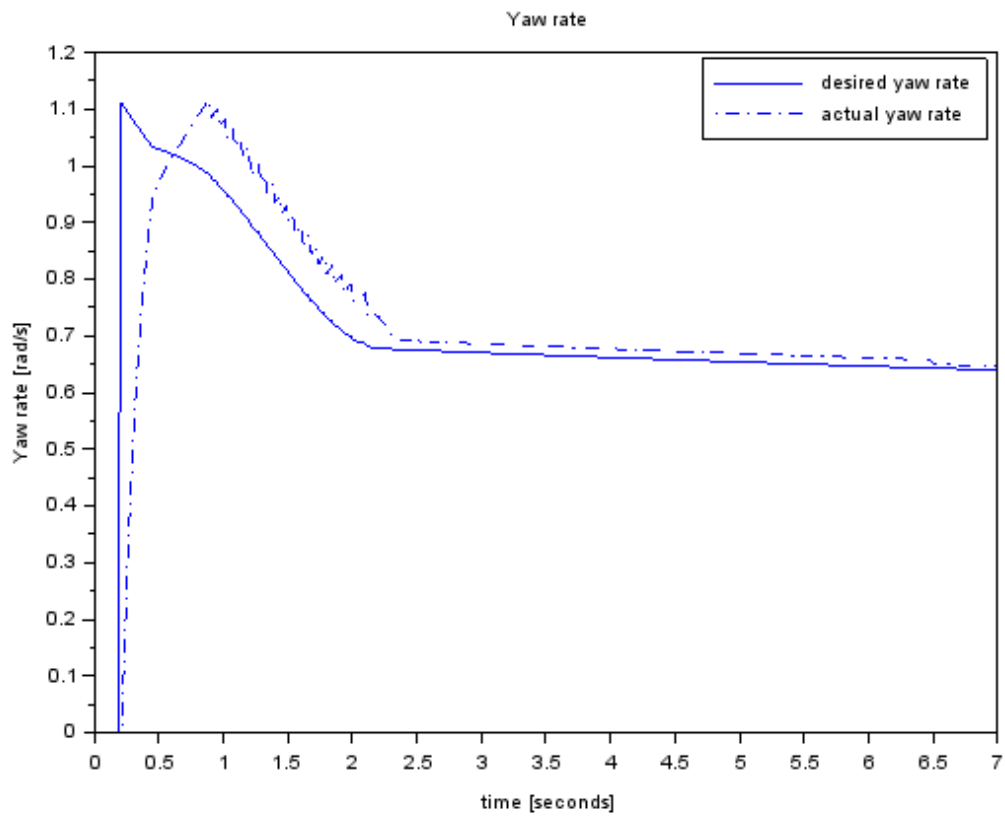


Figure 20: The yaw rate of the car when turning left with the on/off controller and the car is oversteering

In figure 20 it can be seen that the yaw rate goes much closer to the desired value.



## 5.3 The sliding mode controller E.S.C.

The pictures in this section are understeer and oversteer turns with the sliding mode controller working as an E.S.C.

### 5.3.1 Understeer

Understeer situation with the exact same parameters as in section 5.1.3.

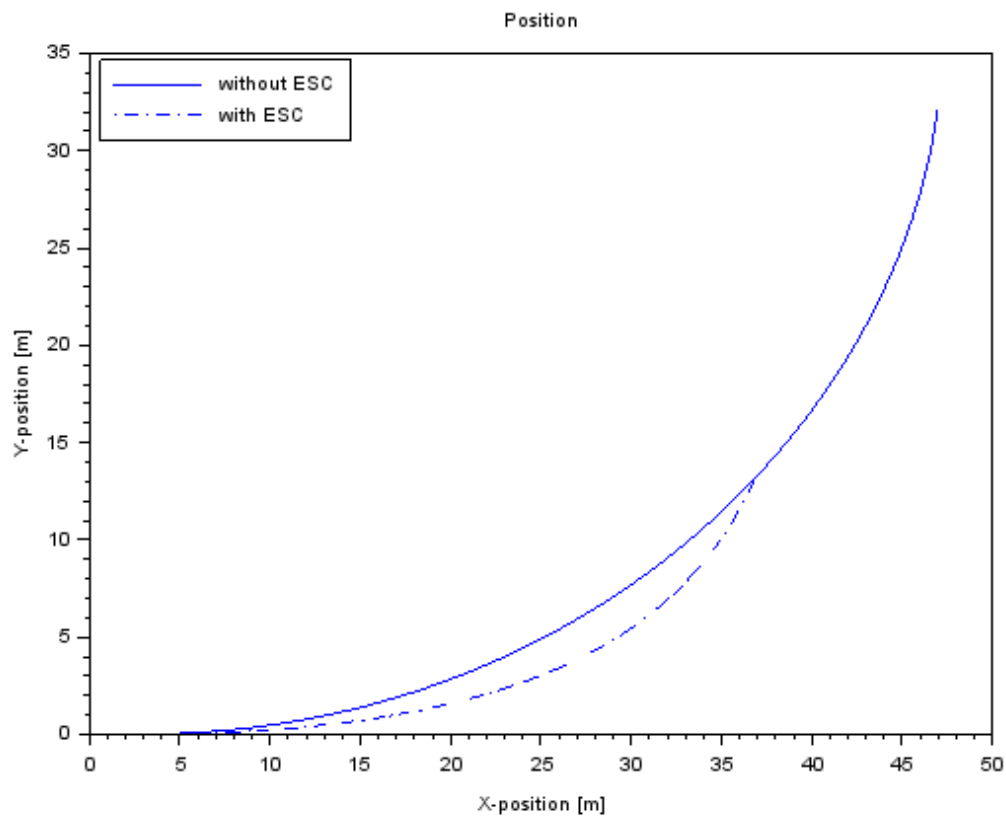


Figure 21: A left turn with and without the sliding mode controller where the wheels are in the same angle the whole simulation and the car is understeering due to icy conditions

As with the on/off controller the sliding mode controller gives a shorter trajectory due to braking and it turns somewhat sharper as is shown in figure 21 above.

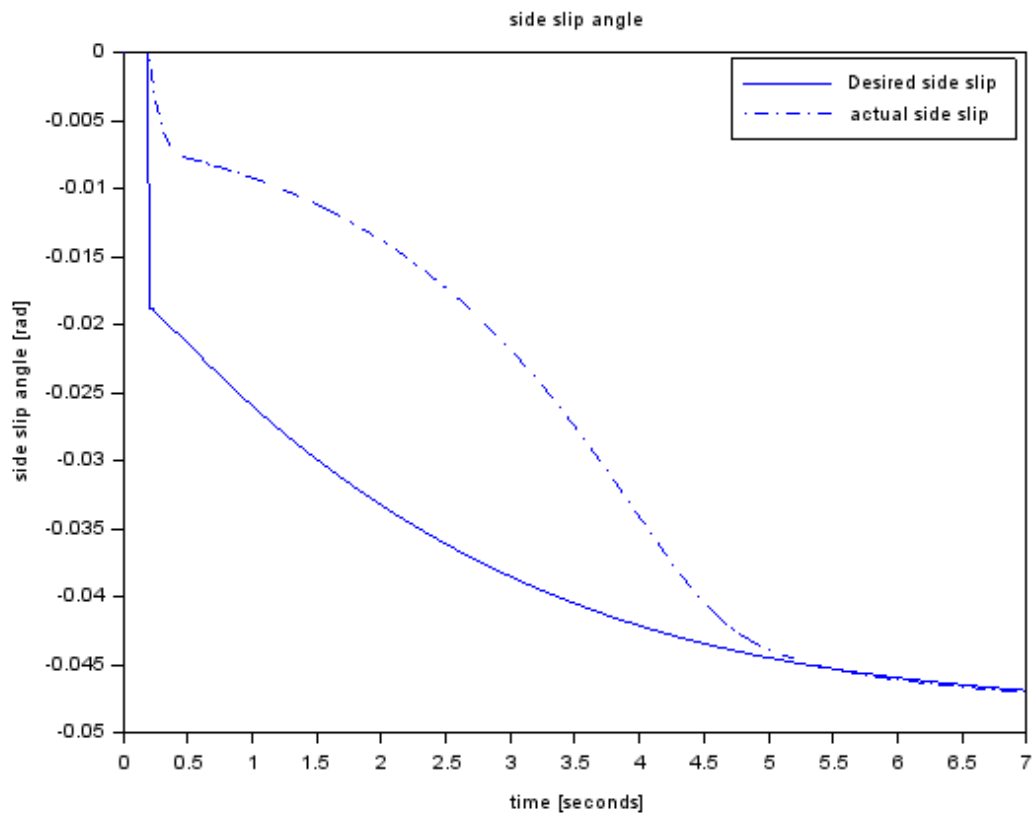


Figure 22: The side slip angle of the car when turning left with the sliding mode controller and the car is understeering due to icy conditions

Here in figure 22 is shown the side slip when the car understeer with the sliding mode controller active. The sliding mode controller regulates both yaw rate and side slip angle simultaneously as can be seen by comparing figure 22 with figure 16. With the sliding mode controller no large positive peak is produced in the side slip angle.

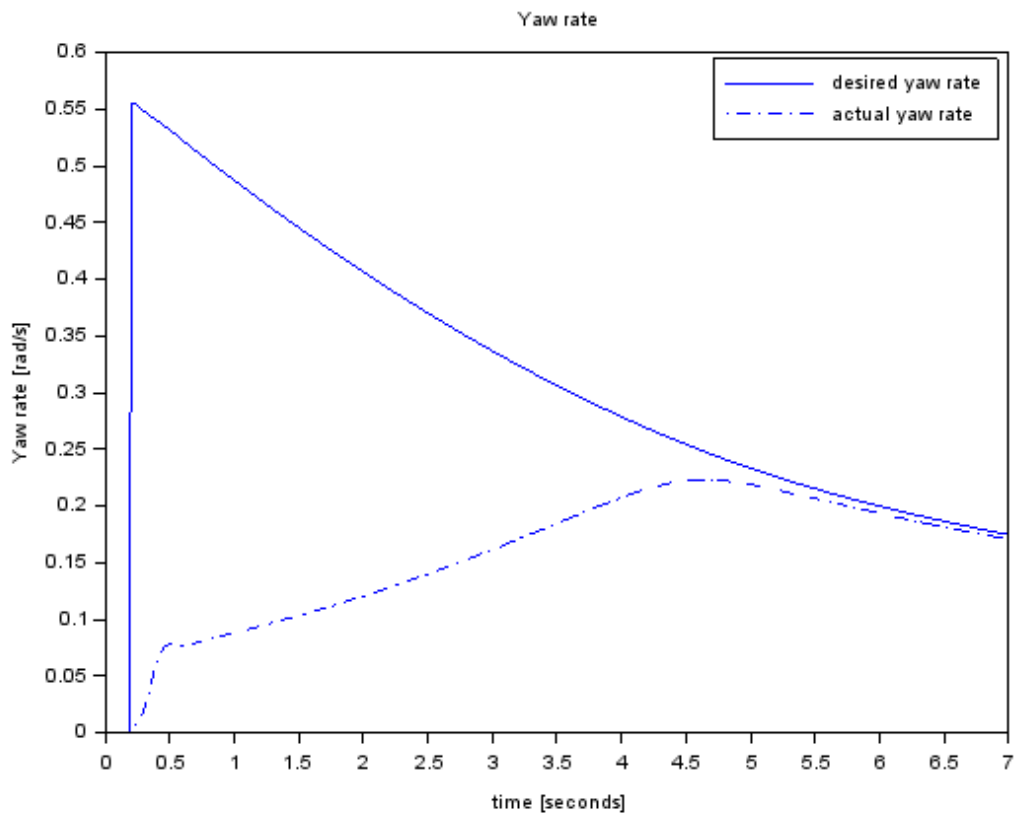


Figure 23: The yaw rate of the car when turning left with on/off controller and the car is understeering due to icy conditions

In figure 23 it can be seen that the yaw rate reaches the desired value but not as fast as with the on/off controller.

### 5.3.2 Oversteer

Oversteer situation with the exact same parameters as in section 5.1.4.

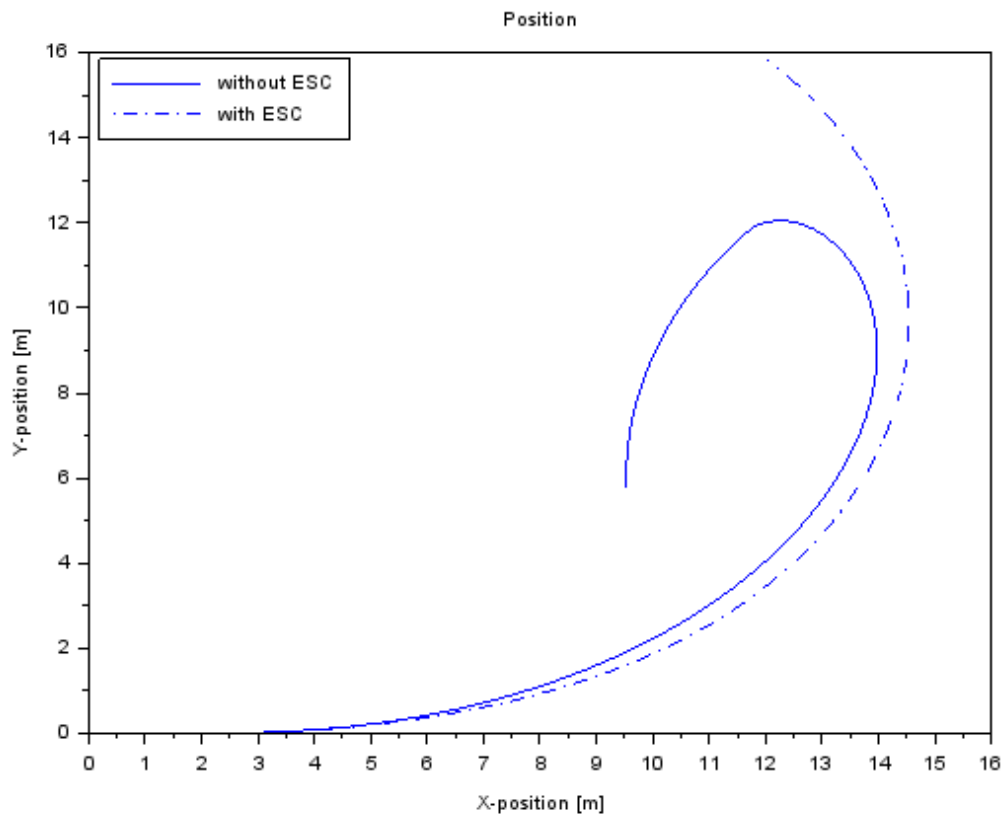


Figure 24: A left turn with and without the sliding mode controller where the wheels are in the same angle the whole simulation and the car is oversteering

As can be seen in figure 24 the E.S.C. keeps the car on a stable trajectory and prevents it from going into a sharp bend.

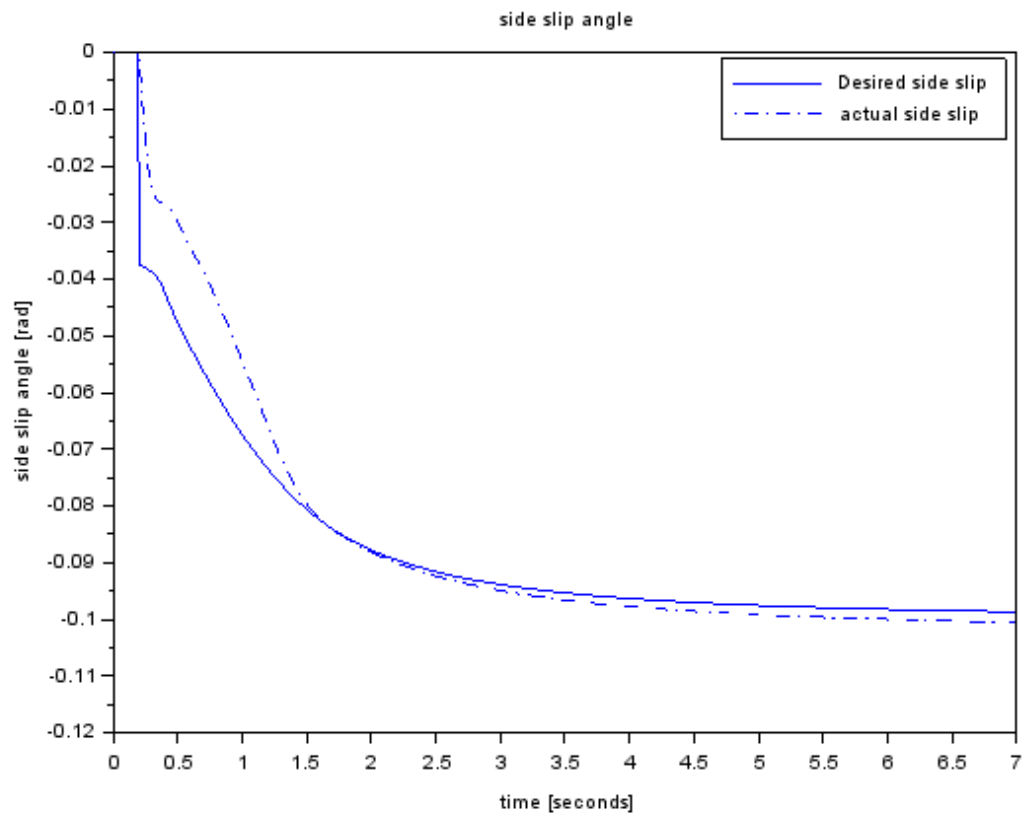


Figure 25: The side slip angle of the car when turning left with the sliding mode controller and the car is oversteering.

In oversteer the E.S.C. again keeps the side slip angle close to the desired value as shown in figure 25 above.

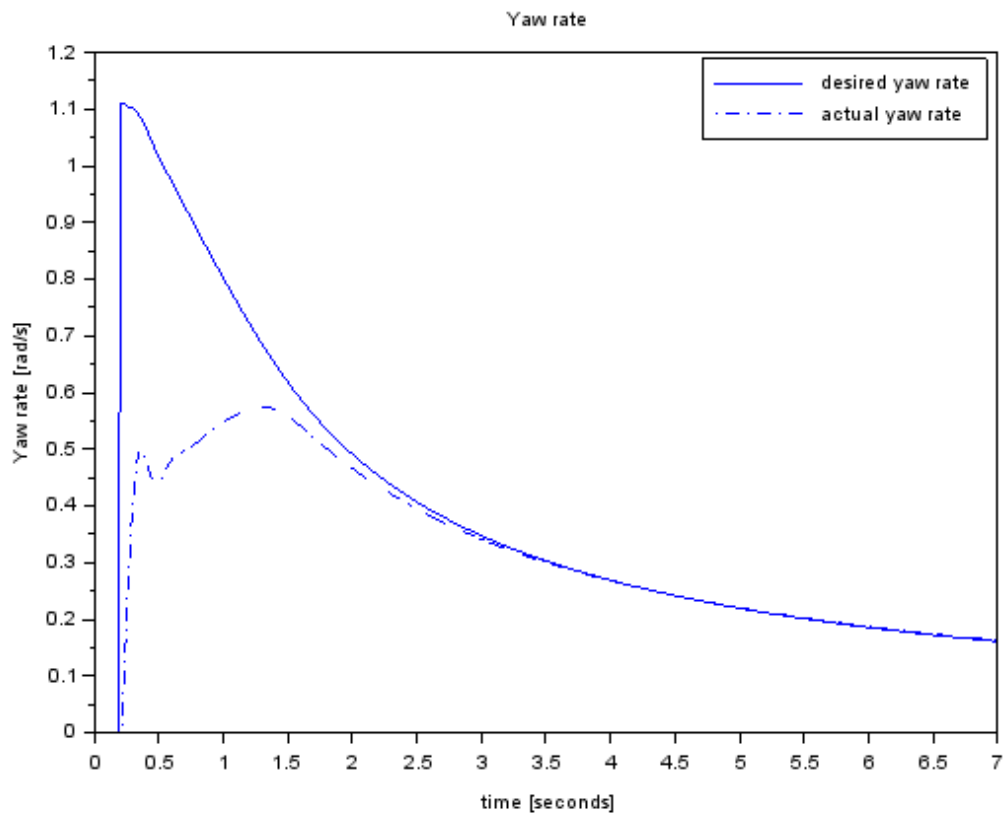


Figure 26: The yaw rate of the car when turning left with the sliding mode controller and the car is oversteering as can be seen in figure 26 the sliding mode E.S.C. keeps the yaw rate close to the desired value from about 1,5 seconds and onwards.

## 5.4 The GUI

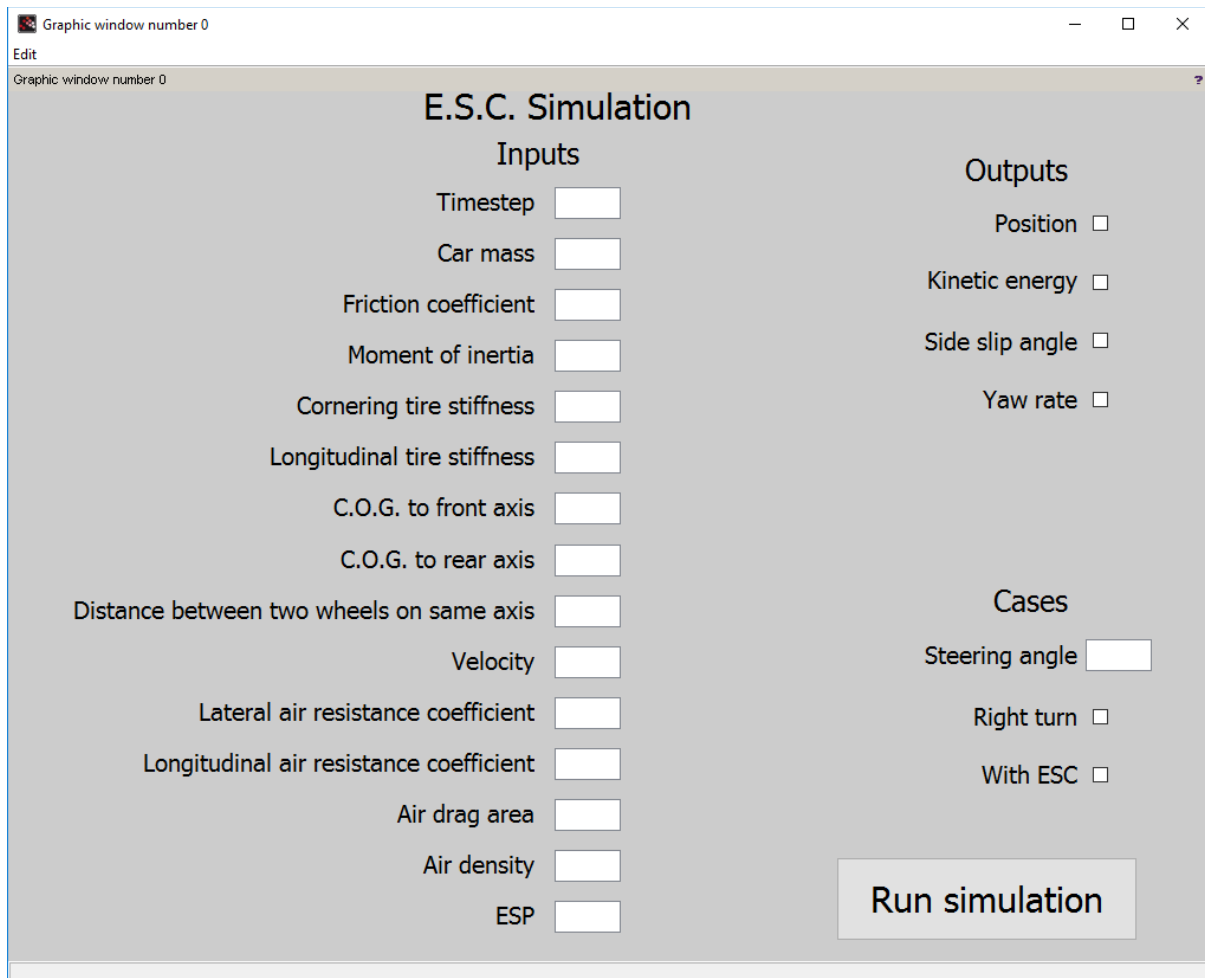


Figure 27: The GUI where the user can type in the parameters and road conditions to simulate

Figure 27 shows the GUI window. The editboxes in the row on the left allows the user to type in parameter values, the checkboxes under outputs allow the user to chose which graphs that will be displayed and the checkboxes under cases allow the user to make a left/right turn and also if the vehicle will make the turn with or without the E.S.C. The editbox under cases allow the user to type in the angle at which the front wheels should be set.





## 6. Conclusion

From the results it is clear that the two controllers help keep the vehicle on a more stable trajectory in all of the cases. With the on/off controller the E.S.C. is focused only on the yaw rate and not the side slip angle when it controls the vehicle. This can result in a rather large side slip angle error as can be seen in figure 16. However it can still control the yaw rate well as can be seen from figure 17. The reason it does not reach the desired value perfectly is that there was a certain sensitivity parameter and if the vehicle is within this non sensitivity area then there will be no action from the controller. When it comes to oversteer the on/off controller manages to not only control the yaw rate but also to reduce the side slip angle substantially. This is since the longitudinal force on the front tire that brakes is counteracting the larger lateral force on the front wheels. Thereby ensuring that the yaw angle of the car is closer to the angle of the velocity of the car.

The sliding mode controller not only controls the yaw rate during understeer but it also manages to control the side slip angle at the same time, as can be seen on figure 22 and 23. The same thing can be seen during oversteer, both the yaw rate and side slip angle reach their desired values quickly. It is very important for the vehicle to achieve the desired values as soon as possible since these are the conditions that the driver is most used to dealing with.

An interesting fact to observe is that in all cases the desired value for yaw rate and side slip angle changes during the tests. This is because we have a situation where the energy, and so the velocity, of the car always decreases. This is causing the desired value to change in accordance with equations (31) and (32). This can be even more noticeable when the E.S.C. is active and this is because the braking of the wheels of the car causes a large decrease in velocity which causes larger changes in the desired values.

Another interesting thing to observe is the remaining error that the sliding mode controller produces with respect to side slip angle as can be seen in figure 25. This is due to how the surface parameter  $\xi$  is chosen. It can be seen from equation (50) that if the side slip angle is small enough then the surface parameter will be zero and that means that the controller only takes the yaw rate into account when controlling the vehicle behaviour. This can result in a small error in the sliding mode controller but as long as it is small enough this doesn't make any problems. Generally, the E.S.C. always prioritizes the yaw rate above the side slip angle, this is the reason why  $\xi = 0$  for small side slip angles.

Another important factor in the sliding mode controller is the  $\eta$  parameter. Through testing this was chosen to have the value -2. For values closer to zero of this parameter the main difference is that the system is somewhat slower at reaching the desired values especially when it comes to understeering but not so much when it comes to oversteering. This is since the controller creates a larger torque on the car for more negative values of  $\eta$ . In the understeering situation there is a certain limitation in that the slippery road conditions usually present during understeering prevents the controller from achieving more than the maximum traction that's possible for the road conditions. This results in a limitation in how fast it can

stabilise the vehicle on slippery roads. Therefore it does not help to have values of  $\eta$  higher than -2 for the controller.

Another interesting phenomenon is the many small interventions made by the on/off controller. This can be observed in figure 17 and figure 20. This is because the actual values of the vehicle's yaw rate lies on the verge of the distance they can be from the desired values. When the yaw rate changes a bit and moves out of the zone where no controller intervention is made the controller immediately steps in and forces the system back in towards the desired value. Then the yaw rate slips out of the zone again and the process repeats. This is a common trait seen with on/off controllers.

The uniti car have a few unusual properties with respect to E.S.C. The car have a rather low moment of inertia relative to its mass compared to other cars and this means that it more easily changes the yaw angle than the angle of the velocity. This means it easily can get a large side slip angle. This can be especially seen in the results from the On/Off controller when controlling understeer. The controller can easily turn the car but have a more difficult time changing the angle of the velocity as can be seen in figure 16. This low ratio between the moment of inertia and mass also affects the yaw rate. In figure 17 and 20 the On/off controller control the yaw rate at the upper bound instead of the lower i.e. the car changes angle a little faster than the desired values. When the same controller is used for cars with a higher moment of inertia to mass ratio, such as the ratio usually found in a normal sized car, the On/Off controller usually controls on the lower bound of the yaw rate.

During the initial phase of the project four problems were formulated. The answer to them will be discussed here.

### 1. Which simulation environment is most appropriate to use?

It was decided to use the Scilabs basic simulation environment for the project and without using the Xcos add-on. Also no programming in any other programming language had to be used. The entire simulation program was written in the Scilab editor and compiled using the Scilab compiler.

The main reason for only using the Scilab language was that it was soon apparent that all the necessary commands for writing the simulation program was available. The simulation could properly have been made in Xcos as well but Xcos proved more difficult to use compared to just writing the code directly into Scilab. This was mostly because the Xcos interface proved tricky to use.

### 2. Which type of model should be used in the simulation and how should the parameters in this model be estimated?

It was decided to use a 3 degrees of freedom vehicle dynamics model with tire forces calculated with Dugoff's tire model and the equations of motion as the equations (11), (12) and (16).

The uniti car does not exist yet and many of the parameters are unknown. Therefore most of them currently used in the simulation are guesses or made from rough measurements or

found in internal documentation. The GUI also allows for easy changes of parameters when they become more well known in the future. Some vehicle parameters will also change from time to time, for example vehicle mass and moment of inertia depends not only on the car but on luggage and weight of passengers.

To see the complete list of parameters found and estimated in this project see table 3 in the result section.

### 3.Which type of controller is most suitable to use?

A controller had to be made to work as an E.S.C. for the simulation program. During the project it was decided to make two different kinds of controllers. One on/off controller and one sliding mode controller. The on/off controller allows the user of the program to in more detail study how the car behaves under differential braking. It also allows the user to build other types of controllers based on what type of differential braking proves most effective.

The sliding mode controller is more similar to how an actual E.S.C. found in modern cars look like. It can be used to study how the Uniti car behaves when influenced by something similar to the real E.S.C. that's going to be in the car.

### 4.Are the tools in Scilab appropriate to create a graphic user interface for the simulation program or will another program have to be used?

It was possible to make a functioning GUI using only the tools available in the Scilab language however to simplify the work an add-on called GUIbuilder was used. The GUI making commands doesn't give much support for making GUIs with good design but for the purpose of this project it was mostly so that relevant parameters could be sent into the program and to allow some choices for the user as to how the simulations should be run and what outputs were of interest. It should be noted that changing driver behaviour (i.e. multiple changes in the steering angle, accelerating or braking torque) and to add unexpected occurrences (such as loss of traction) is difficult to include in a GUI. So for more complex behaviours of the car or the driver it is necessary to use the Scilab editor to gain full control over all factors.

### Does the results fulfilled the purpose of the thesis work?

The purpose of this thesis work was to create a vehicle model and a simulation program that could be used to test an E.S.C. while it is developed. In order to do so a vehicle dynamics model, a simulation program and two automatic control systems that works as two simple E.S.C.s. were made. This simulation software can be used in a couple of ways to help with the development of an E.S.C. for the Uniti car. It can be used to test how the Uniti car behaves under the influence of differential braking and also how it behaves with different types of E.S.C. active in the simulation program. The two E.S.C. controllers made during this project can also be used as comparisons for when the actual E.S.C. is developed to ensure that it works as intended. The parameters of the E.S.C. can be tweaked using the simulation program to study how it affects the entire car. Finally the sliding mode controller could be used as the mathematical basis to construct a real E.S.C. so the results fulfilled the purpose of the thesis work.

## 6.1 Reflection of ethical aspects

The public welfare advantage from functioning E.S.C. can be important. According to an investigation by the U.S. department of transportation ([5] Dang, 2004) 30% of fatal accidents could be prevented by the use of the technology for passenger cars. In SUVs the number is even higher, at around 63%. In the investigation it was also found that E.S.C. prevented around 35% of single vehicle crashes for passenger cars and up to 67% for SUVs. So the E.S.C. have an important role to play when it comes to minimizing damage and loss of life in traffic.

E.S.C. also raises some ethical aspects. Making the E.S.C. see the difference between situations when it prevents an accident and situations when the driver puts the car in an intentional unstable situation to prevent an even worse accident is difficult.

An example of such a situation is if the driver needs to avoid an obstacle further along on the road. If a truck comes towards the car on the wrong side of the road the driver in the car might be able to prevent an accident by swiftly steering out onto the other side of the road or even drive off the road. The E.S.C. can decide that this is an oversteer situation and try to keep the car on the old trajectory causing a collision between the car and the truck. How is the E.S.C. supposed to solve these types of problems? A full answer is not completely available and there is a certain risk that the E.S.C. will cause trouble in some situations. The ethical question is how to make the E.S.C. so that it can make a decision in these situations? Should it be better at avoiding understeer and oversteer or should it be somewhat worse at this but more tolerant to the driver making potentially dangerous maneuvers? Technology might also have the potential to make improvements here.

## 6.2 Future aspects

The E.S.C. is supposed to be a safety feature for cars. The developed sliding mode E.S.C. could in the future be implemented in a test car. However some improvements are needed in order to get an E.S.C. safe enough to be used in a real car. The E.S.C.s. are not tested in reality, so physical tests must be made to get all the correct parameters and to see the behaviour of the car under different conditions. The E.S.C. also only focus on the yaw stability control, a real E.S.C. should also have rollover stability control to prevent the car from rolling over as well as to keep the car on the road.

Another potential future improvement is to verify the vehicle model with physical tests and to include more physical variables into the simulation if they can be found and quantified in a reliable way. Examples of such variables are longitudinal rolling resistance, looking at the car suspension and the wheel dynamics. In order to develop rollover prevention it is also necessary to include more degrees of freedom to describe the car in three dimensional space rather than on a surface as it is right now.

In time the closest improvement that can be done however is to more accurately find the vehicle parameters once the actual design and the properties of the car have been decided. That way the estimations in table 3 can be replaced by the uniti car's actual values.



# 7. Terminology

## 7.1 List of words

C.O.G.	Center of gravity
Dugoff's tire model	An algorithm for calculating forces on tires
E.S.C.	Electronic stability control, see section 2.2.
Longitudinal force	Force parallel to the direction of a tire
Lateral force	Force perpendicular to the direction of a tire
Pacejka's magic formula	A formula for calculating forces on tires
Sliding mode controller	See section 2.5
side slip angle	The angle of the actual moving direction of the vehicle Compared to the direction the vehicle is pointing
Vehicle dynamics model	See section 2.3.
Yaw angle	This is the angle that the car is pointing.
Yaw rate	The rate at which the yaw angle changes.
Yaw rate change	The rate at which the yaw rate changes.

## 7.2 List of symbols

$\alpha_{front}$	Slip angle front wheels
$\alpha_{rear}$	Slip angle rear wheels
$\delta$	Front wheel angle
$h$	Timestep
$l$	Car length
$l_f$	Distance C.O.G. to front wheel pair
$l_r$	Distance C.O.G. to rear wheel pair
$l_w$	Distance between wheels in a wheel pair
$w$	Car width
$\psi$	Yaw angle
$\psi_{old}$	Yaw angle one timestep before the current time.
$\dot{\psi}$	Yaw rate
$\dot{\psi}_{old}$	Yaw rate one timestep before the current time
$\ddot{\psi}$	Yaw rate change
$\ddot{\psi}_{Gerats}$	Yaw rate change from Gerdt's
$v$	Velocity
$\beta$	side slip angle
$\beta_{old}$	side slip angle one timestep before the current time
$\dot{\beta}$	side slip angle change
$\sigma_x$	Longitudinal slip
$\sigma_y$	Lateral slip
$r_{eff}$	Wheel radius
$\omega_w$	Wheel angular velocity
V	Velocity in slip and energy formula

$p_x$	Car position on x-axis
$p_{x\_old}$	Car position on x-axis one timestep before the current time
$p_y$	Car position on y-axis
$p_{y\_old}$	Car position on y-axis one timestep before the current time
$V_x$	Car velocity in x-direction
$V_y$	Car velocity in y-direction
$v_h$	Velocity rear tires av the car
$v_v$	Velocity front tires av the car
$C_\sigma$	Tire longitudinal stiffness
$C_\alpha$	Tire cornering stiffness
$e_{sp}$	Air drag mount point
$T_{loss}$	A loss of lateral force coefficient
$\lambda$	Lambda factor in Dugoff's tire model
$\mu$	Friction coefficient
$F_z$	Normal force
$\ddot{x}$	Acceleration in x-direction
$\ddot{y}$	Acceleration in y-direction
$F_{uh}$	Combined longitudinal force rear wheels
$F_{uv}$	Combined longitudinal force front wheels
$F_{sv}$	Combined lateral force front wheels
$F_{sh}$	Combined lateral force rear wheels
$F_{Lx}$	Air resistance force x-direction
$F_{Ly}$	Air resistance force y-direction
$F_{yfl}$	Lateral force front left tire
$F_{yfr}$	Lateral force front right tire
$F_{yrl}$	Lateral force rear left tire
$F_{yrr}$	Lateral force rear right tire
$F_{xfl}$	Longitudinal force front left tire
$F_{xfr}$	Longitudinal force front right tire
$F_{xrl}$	Longitudinal force rear left tire
$F_{xrr}$	Longitudinal force rear right tire
$M$	Torque on the car from individual wheel forces in the same wheel pair
$a$	On/Off controller sensitivity parameter.
$I_{zz}$	Car moment of inertia
$\dot{\psi}_{des}$	Desired yaw rate
$g$	Constant of gravity.
$\dot{\psi}_{upper\_bound}$	An upper value of the yaw rate
$\beta_{upper\_bound}$	An upper value of the side slip angle
$\beta_{target}$	The side slip angle target
$\dot{\psi}_{target}$	The targeted yaw rate
$\ddot{\psi}_{target}$	The target yaw rate change
$\dot{\psi}_{target\_t}$	The target yaw rate at time t
$\dot{\psi}_{target\_t-h}$	The target yaw rate at one timestep before time t
$s$	The sliding surface
$\xi$	A parameter in the sliding surface equation



$\dot{s}$	The differentiated sliding surface
$\eta$	The negative ratio between the surface and its differential
$M_{\psi b}$	The correcting torque from the sliding mode controller
$\Delta F_{xf}$	The difference in longitudinal force between front tires



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