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PREDICITING BULL AND BEAR IN S&P500

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Abstract

This paper seeks to investigate if factors extracted from macroeconomic and financial variables can improve the forecast accuracy of the bull and bear market in the S&P500 stock index. The study extended the models constructed by Nyberg(2012) and Chen (2009) by augmenting their model with factors. Very little, if any research has been done in modelling the bull and bear using this approach. After using the Bry-Boschan method to identify the two regimes in the stock market, eleven models were constructed using a static probit or dynamic probit model framework. The out of sample forecast results indicates that, probit models augmented with factors have a relatively lower Quadratic Probability Score (QPS) than the corresponding models without factors. Among all the models employed, the dynamic probit model outperformed all the models, while the static probit model without factors is the least performing model. The results also showed that returns on assets and money stock are among the key leading indicators of the S&P500 stock index. Thus, there is evidence that factors can improve the forecast accuracy of the bull and bear market in the S&P500 stock index.

Keywords: Stock indices, Bear stock markets, State, Probability, Probit

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1 Introduction

The equity markets all over the world endure periods, when stock prices are rising (bull market) and corresponding periods in which stock prices are falling (bear markets). Each of these cycles have ramifications on the economy and most importantly on the behavior of private economic agents. The US stock market endures periods of bull and bear markets which dates back to as early as the 1770s while the effects of the recent 2008 financial crisis still being felt. S&P 500 stock index which is the largest and most diverse stock market in the world with stocks from all sectors of the US economy, is one of the most important US economic development indicators. The investing and consumption decisions of the agents acting in the economy is affected by the state of the equity market. Most importantly, it dictates the future state of the US economy, thus affecting the policies of the Federal Reserve and the Government.

The S&P 500 has stock representation from all sectors of the US economy –- Technology, Financial, Energy, Industrial, Consumer staples, Consumer discretionary, Materials, Utilities and Telecom. It is therefore very diverse. It represents about 80% of the total value of US stock market. For a company to be included, it must be a US company with a market value at $4billion and half of their stock should be publicly traded, its earnings most have four consecutive quarters of positive growth, the stock prices must be at $1 per share (U.S. Securities and Exchange Commission). This makes the index very important for investors and consumers' confidence. As a change in the index force investors to change their earnings projections and the riskiness of investing in the large company's stocks. Thus, changes to the index that correspond to bull or bear market typically indicates the overall directions of the economy.

This has attracted a lot of studies in forecasting the state of the equity market. For example, Chen (2009) and Nyberg (2012) both predicted the bull and bear markets of the S&P 500 using a dynamic probit model, and found that the model improves the forecast accuracy of the static probit model. Successful prediction of the state of the equity market will help investor in their timing and pricing strategies, as investors gain buying stocks when the bull market is at early stages but sell at the early stages of the bear market. This strategy was emphasized by Graham (1949) in his book the intelligent investor when he wrote "Buy cheap sell dear".

This motivates us to extend the studies done by Chen (2009), and Nyberg (2012) in forecasting the S&P 500 stock index by augmenting their models with factors extracted from a group of variables assumed to have effect on the stock market. Factor analysis summarizes the information in a large group of correlated variables into few factors, capturing numerous shocks that could affect the stock market without over parameterizing the model. Fossati (2015), and Baetje and Menkhoff (2013) both used factors in a probit model to predict the business cycle and stock market respectively.

The McFadden R-squared of the binary time series models constructed by Guidolin and Timmermann (2005) and Chen (2009) were all below 50%. This creates an incentive to use information in more variables. According to stock and Watson (2002) and Forni et al (2005) factors improve the forecast accuracy of macroeconomic variables. However, several studies have presented different results on the forecasting performance of static and dynamic factors. Example Ginters (2010) and Forni et al (2014) found that the Root Mean Square Error of the model incorporating the dynamic factor model is lower than the model with static factors. While Cheung and Demers (2007) in determining the forecast performance of both factors, found that static factors predict GDP growth rate and inflation of Canada better than the Dynamic factor. Thus, both factors will be employed to determine whether they can improve the forecast accuracy of the S&P500 stock index.

The goal of this study is to extend the studies done by Chen (2009) and Nyberg (2012), by investigating whether factors can improve the predictive accuracy of the probit model in forecasting US stock market. This research is motivated by the fact there is little or no study done on predicting the US stock market with factors. Thus, we seek to answer the question: Does factors increase the accuracy of the probability of detecting a bull and bear market? Eleven forecasting models are considered, including the static probit model, the dynamic probit and the dynamic autoregressive probit model. The Performance of the models are evaluated using AIC, BIC, QPS and Pseudo R-squared and the statistical significant of the leading indicators.

The rest of the paper is organized as follows, chapter 2 deals with the empirical literature. Chapter 3 deals with methodology of the study. Chapter 4 presents the interpretations and analysis of the results and chapter 5 concludes.

2 Literature Review

Stock markets over the world are one of the leading indicators of the future state of a country's economic growth. During the past decades, numerous studies were conducted in predicting the state of the equity markets. These studies mostly employ binary time series models, in which the bull and bear regimes are typically determined by the Bry-Boschan method or the Markov switching model. Binary time's series models have an advantage in forecasting the stock market regimes as it represents the necessary information in the variables by giving the probability of a particular regime occurring Fornari and Lemke (2010) and the estimation and forecasting is straight forward.

Baetje and Menkhoff (2013) in studying the risk premia in the US stock market, used macroeconomic factors to predict the US stock market risk premia in the bull and bear markets. These factors were used to predict four standard US stock market risk premia, namely market excess returns, size, value and momentum of the stock market. The in-sample forecasting accuracy of the factors, especially at a one year horizon was impressive. A related study was done by Taulbee (2001) and Stock and Watson (2005) who used both dynamic and static factors extracted from both macroeconomic and financial time series to predict the S&P 500 stock index. The authors argued that their study will help investors understand how these factors are affecting the stock market.

As already mentioned the bull and bear market, requires the identification of the state of the stock market into regimes. These regimes are represented by a binary variable, identified by the Markov switching model or the Bry-Boschan method. Although bull and bear market are common words in an investors dictionary there does not exist any academic consensus on the definition. Instead researchers have been using different ways in which they capture the bull and bear movements in the market. The Bry-Boschan method has been extensively used in the business cycle literature to determine the turning points in real economic activity Nyberg (2012). Studies conducted by Nyberg (2012), Chen (2009), Pagan and Sossounov (2003), Baetje and Menkhoff (2013) all used the Bry-Boschan method to identify the bull and bear market. According to Chen Chen (2009) who used different methods to determine the turning point in the bull and bear market, found that the model that uses the information from the Bry-Boschan method has more significant variables than the other methods. Cespedes, Chauvet and Lima (2006) in forecasting Brazilian output and its turning points in the presence of breaks, found that the forecasting model that used the Bry-Boschan method outperforms the other models that did not use it. Ahking (2014) found that the Bry-Boschan method's identification of the US business cycle turning point is in line with National Bureau of

Economic Research (NBER's) chronology. He said that the Bry-Boschan method is developed for detecting business cycles in monthly time series data.

Several studies have been conducted in modeling the state of the equity market using different types of probit models. The goal of those studies were to compare the models and to determine which one is the best. These studies were done by Fossati (2015), Chen (2009), Nyberg (2012) to name a few, all of whom were predicting the state of the equity market. For example, Chen (2008) used both static and dynamic probit models to predict the business cycle. Kauppi and Saikkonen (2008) argued that the dynamic probit model augmented with lags of the binary variables capture the state of the equity market and the autocorrelation structure. Most of the results from these studies indicate that the dynamic binary model outperforms the static probit model. However, Fossati (2015) found that the static probit model outperform the other models extended from it and is robust to additional variables.

Several studies used a group of both financial and macroeconomic time series variables to determine forecasting model for the bull and bear market. Chen (2009) and Nyberg (2012) ran a static probit model for each variable on the state of equity market and use the statistical significance and Pseudo R-square to determine which variables to use in constructing the forecasting model. These studies indicate that the McFadden R-squared of the all models are consistently below 50%. To determine the out of sample performance of their forecasting models Chen (2009), and Nyberg(2012) used the Quadratic Probability score to evaluate their models.

3 Methodology

In this section, the methodology used in predicting the S&P500 stock index is discussed. First, the strategy used to confirm the existence of regimes in the stock markets is discussed, this helps in determining the two states assumed to exist in the stock market at any point in time. Second**,** a discussed of factors analysis is conducted, because the goal of this study is to determine if factors can improve the forecast accuracy of the S&P500 stock index. Third, factor augmented probit model is presented, as the econometric method used in predicting the stock market. Fourth, the forecasting models employed in this study were presented. And finally, the criterion used to determine the forecast accuracy of all the models is discussed.

3.1 Regime Switching dynamics

The stock markets go through prolong periods of rising or falling stock prices just like the business cycle. Before we look at the models used for forecasting the state of the equity market, it is important to determine the bear and bull market periods first. Several methods have been employed by researchers in identifying the turning points in the bull and bear markets such as the Markov switching method, Bry-Boschan methods, the Naïve moving average method to name a few. However, there doesn't seem to be a clear-cut consensus on which method identifies the regimes better.

Before going further, in this study we assume the existence of two regimes in the stock market which are define as follows:

$$r_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_t = s_t(\mu_1 + \sigma_1\varepsilon_t) + (1 - s_t)(\mu_0 + \sigma_0\varepsilon_t) \qquad (1)$$

Where $r_t = \Delta log P_t$ is the log difference of the stock price index P_t , S_t is the observed binary time series and ε_t is the independent and identically distributed error term with mean zero and a unit variance. In this study, the state of the equity market is define as:

$$s_t = \begin{cases} 1, & a\ bull\ market\ at\ time\ t \\ 0, & a\ bear\ market\ at\ time\ t \end{cases} \qquad (2)$$

Thus, when the market is enduring rising stock prices ($s_t = 1$) the market is in the bull market, the mean return is generated by (1) is μ_1 and in the bear market state mean return is μ_0 . Given the definition of bull and bear markets in this model it is important to notice that one cannot rule out the possibility that during a bear (bull) market an individual monthly stock return may be positive (negative). The idea is that, if we can predict the regimes in (2), we should be able to predict stock returns in equation (1).

In this study, just like in Nyberg (2012) and Chen (2009) the Bry-Boschan method(B&B) is employed to determine the bull and bear market in the stock market. According to Pagan and Sossounov (2003) the Bry-Boschan method is easy to use and is more practical than other methods. Seidl (2012) found that the regime switching algorithm B&B improves the performance of the optimal portfolio.

Bry-Boschan method is an algorithm consisting of a set of filters and rules to locate the turning points. The minimum duration spent in either a bull or bear market is set to 6 months. There can be daily fluctuation but the goal is to capture the main movements in which the trend is set over a period of months. The period of the complete cycle which is the time between a peak and a trough and back to a peak is set at 15 months following Nyberg (2012). It is important to note that the method has a drawback in real time lag, this is because the dating rules that the Bry and Boschan uses is based on a two-sided moving average filter, that requires information on the future values of the price data. This is done by setting the rule at $\{r_t<r_{t\pm l}\}$ and $\{r_t>r_{t\pm l}\}$. In this study, l=6, therefore, as the future stock returns are unknown at time t, there will be few months delay before the algorithm can identify a real turning point.

3.2 Factor Augmented Probit Model

Stock markets all over the world are influenced by many factors in the country they reside and beyond. The S&P 500 stock index is not an exception as the index represent the stock of companies from all sectors in the US economy. Therefore, predicting the bull and bear market with only the key leading indicators might not capture all the factors that could be affecting the stock index. This is evident in the studies done by Pagan and Sossounov (2003), Chen (2009) and Nyberg (2012) all of whom's models have a R-squared below 50%. However, adding all the variables individually to the model that could affect the stock market is not feasible. But a Factor Augmented probit model could exploit the information in these variables, without over parameterizing the model. The assumption is that the covariance between a group of financial and macroeconomic variables can be summarized into few common factors, without losing too much information from the original variables. These factors are used as leading indicators in forecasting the S&P 500 stock index. The Factor augmented probit model is a two-step estimation procedure in which two equations are estimated, the factor model and the probit model augmented with factors which serves as the forecasting model.

3.3 Factor model

Factor models exploits the co-movement in a group of correlated time series variables into few common variables. In factor analysis, the observed variables are divided into two parts: the common component, this captures the covariance between the variables and the idiosyncratic error component, which captures unique shocks of each variables. In factors analysis, the unique shocks are the information that is lost while the common component which is relatively larger, is what is used to construct the factors. Principal component analysis is used to estimate the common components in the observed time series variables.

Let the observed time series variables be, X_1,X_2, X_3,...., X_N, the unobserved factors are F_1, F_2, F_3,..... F_r and unique factors are ξ_1, ξ_2, $\xi_{3,}$.....,ξ_n. The factor analysis equation is stated as follows:

$$X_t = \Lambda F_t + \xi_t \tag{3}$$

X_t is an $Nx1$ vector of time series variables. F_t is an $r\ x1$ vector of latent factors, Λ is an $N\ x\ r$ matrix of factor loadings, ΛF_t the product of factor loadings and latent factor is the common component and ξ_t is $Nx1$ vector of unique idiosyncratic error term. The proportion of the variance of the time series variables is represented by the square of the factor loadings Λ^2. Factor analysis is desirable the closer the proportion is closer to 1. Hence, factor analysis condenses the dimension of the N time series variables into r factors. The factors can be extracted as static dynamic or factor factors.

Static factors allow the group of variables to be contemporaneously related to the factors. The factors and the idiosyncratic errors are orthogonal at all time periods. The equation (3) is a static factor model, as the factors are contemporaneously related with the observed variables. According to Boivina and Ng (2005) static factors are easy to extract and are practically preferred.

The corresponding dynamic form of the static factor model in which the factors depend on their previous shocks by evolving over time is given by:

$$X_{it} = \Lambda_i(L)F_t + \zeta_{it} \tag{4}$$

X_{it}, is a $N\ x\ 1$ vector of time series variables, F_t is a $M\ x\ q$ matrix of latent factors, $\Lambda(L)$ is the factor loading, $\Lambda(L)F$ is the common component, with a distributive lag structure and ζ_t is a $M\ x\ 1$ vector of idiosyncratic error. The dynamic forms of the static factors have a static representation given by, $\Lambda F_t = \lambda_i(L)f_t$. Thus, a dynamic factor model with q factors have $r = q(s+1)$ static factors. These estimated factors have a time series dimension as they are extracted from time series variables.

3.4 Forecasting Model

In this study, both the static and dynamic probit models are used in predicting the S&P500 stock index. The inclusion of the extracted factors from observed time series variables in a probit model as additional predictors is what is called a factor augmented probit model. This study predicts the state of the S&P500 stock index with binary time series models. In a binary time series models, where the dependent variable s_t, (t=1, 2… T) is a realization of a stochastic process that takes on the values 1 or 0 at time t. As defined in equation (2), the value one (s_t=1) represents the bull market and the value zero (s_t=0) indicates a bear market. In this model, the goal is to model s_t using X_t which is a matrix of explanatory variables including the factors.

The conditional expectation of s_t, on the information set Ω_{t-1} at time t-1 given by $E_{t-1}(s_t)$, thus the conditional probability of the bull market at time t can be stated as:

$$p_t = E_{t-1}(s_t) = P_{t-1}(s_t = 1) = \Phi(\pi_t) \qquad (5)$$

In practice, researchers assume that $\Phi(.)$ is the cumulative distribution function (CDF) of either the standard normal distribution or the logistic distribution. The former is the probit model and the latter is the logit model. As mention above, in this paper the probit model is adopted as the empirical model. This ensures that $\Phi(\pi_t)$ take values in the unit interval (0, 1), giving us the probability of the occurrence of both the two states in the markets. In equation (5) π_t is a linear function of variables included in the information set. The **static probit model** is given by:

$$\pi_t = \omega + x'_{t-k}\beta \qquad (6)$$

Where π_t is a linear function of the variables in the information set and x_{t-h} is a matrix of explanatory variables. The static model captures the contemporaneous relationship between the explanatory variables and the state of the equity market. However, it does not allow for the possible autocorrelation in s_t. This model can be extended in various ways as shown by Kauppi and Saikkonen (2008).

The dynamic probit model is specified as:

$$\pi_t = \omega + \delta s_{t-j} + x'_{t-k}\beta \qquad (7)$$

This allows for the inclusion of the lagged state variable as a predictor variable (x_{t-k}). However, the model has a limitation of being dependent on the assumed real-time information lag of the stock market indicator equation (2). To solve this problem the autoregressive model presented below is employed.

$$\pi_t = \omega + \alpha\pi_{t-1} + x'_{t-k}\beta \qquad (8)$$

Where $|\alpha| < 1$. Using recursive substitution in equation (8), the model can be represented by an infinite order static probit model of equation (6), where the entire history of the explanatory variables in x_{t-h} are assumed to influence the conditional probability. Unlike that static probit model, the dynamic probit model accounts for the potential autocorrelation in π_t which provides strong dynamics between success units of π_t. A key advantage of this model and its special cases is that, the one period and multi period forecasts can be estimated by an explicit formula Kauppi and Saikkonen (2008).This is ideal in forecasting, as it normally produces good forecast relative to other models.

Merging of equation (7) and (8) results in the dynamic autoregressive probit model.

$$\pi_t = \omega + \alpha\pi_{t-1} + \delta s_{t-j} + x'_{t-k}\beta \qquad (9)$$

This model allows for richer dynamics in the process of π_t and hence on the conditional probability equation (5). Adding π_{t-1} to the model you can get a more parsimonious model if many lagged values of the explanatory and binary variable are needed Kauppi and Saikkonen (2008).

The model can be further extended as done by Kauppi and Saikkonen (2008) were the effect of the explanatory variables is dependent on the lagged value of the state of the equity market s_t. For example, the impact of the term spread on the bull and bear markets may differ and this model captures this by interacting the lagged state variable with the explanatory variables. This model is given by:

$$\pi_t = \omega + \alpha\pi_{t-1} + \delta s_{t-j} + x'_{t-k}\beta + \gamma s_{t-1}x_{t-h} \qquad (10)$$

3.5 Technique of estimation

The state of the equity market is modeled with different combinations of both static and dynamic probit models as discussed above. Since the goal of this study is to determine whether factors extracted from macroeconomics and financial variables can improve the forecast accuracy of the bull and bear market in the S&P500 stock index. First, each model discussed above is estimated with the observed time series variables assumed to be the leading indicators of the S&P500 stock index. Second, all the model estimated without the factors are augmented with the factors. This technique helps in decomposing the effect of factors in the forecasting models. Thus, making it easy to compare the forecasting performance of all the models against each other.

3.6 Forecast evaluation

The out of-sample and in-sample forecast performance of the models, are evaluated using the Quadratic Probability Score (QPS) and the Pseudo R-square. QPS is the probability equivalent of the root mean squared error in probability forecasting. First the probability forecast is transformed into categorical prediction of binary outcomes, and evaluating the number of times the predicted binary outcomes correctly predicted the event. Like the root mean square error the smaller the value the better the probability forecast as it penalizes bigger mistakes more.

The QPS is defined as:

$$QPS = \frac{2}{N}\sum(f_t - y_t)^2 \qquad (11)$$

Where N is the sample size, f_t is the predicted probabilities of the state and y_t is the observed state. The QPS is used to evaluate the probability forecasting errors of binary response models.

3.7 Forecasting model specification

This section presents how factors are introduced in the estimation of all the models employed in this study. This is done by augmenting the models employed by Chen (2009) and Nyberg (2013) with factors extracted from macroeconomic and financial variables.

3.8 Factor Augmented Probit Regression

This study seeks to determine if the addition of factors as extra leading indicators of the stock market can improve the forecast accuracy of the models employed by Chen (2009) and Nyberg (2013). These models are a generalized version of the Fishers equation. The Fisher's equation describes the relationship between inflation rate and the real and nominal interest rate. In this study, the inflation rate which is the general price level, is replaced by the stock price index. This model is widely used by stock market investors, policy makers and researchers to model the state of the stock markets all over the world. The addition of factors extracted from macroeconomic and financial time series variables is an augmentation of previous models used in this topic. The use of factors creates a new approach to forecasting the state of the equity market. These factors summarize information in time series variables that represent the broader macroeconomic activity of the US. Each of the models considered in this study are estimated with and without the factors to decompose the impact of the factors. The study considers the static, dynamic and the dynamic autoregressive augmented probit models. The empirical models are specified as:

$$\pi_{t+h} = \omega + x'_{t-j}\beta + F'_t \tag{12}$$

$$\pi_{t+h} = \omega + x'_{t-k}\delta \tag{13}$$

$$\pi_t = \omega + \delta s_{t-j} + x'_{t-k}\beta + F'_t\theta \tag{14}$$

$$\pi_t = \omega + \delta s_{t-j} + x'_{t-k}\beta \tag{15}$$

$$\pi_t = \omega + \alpha\pi_{t-1} + x'_{t-k}\beta + F'_t\theta \tag{16}$$

$$\pi_t = \omega + \alpha\pi_{t-1} + x'_{t-k}\beta \tag{17}$$

$$\pi_t = \omega + \alpha\pi_{t-1} + \delta s_{t-j} + x'_{t-k}\beta + F'_t\theta \tag{18}$$

$$\pi_t = \omega + \alpha\pi_{t-1} + \delta s_{t-j} + x'_{t-k}\beta \tag{19}$$

Where in all the models π_t is a linear function of the variables in the information set. This contains the lagged value of s_t in the dynamic model, ω is a constant term, x'_t is the vector of explanatory variables assumed to be the key leading indicators of the stock market in this study (Broad money supply, Term spread, Dividend yield and returns on assets) and F is the vector of the factors extracted from macroeconomic and financial variables see table 10 & 11. Thus, the model estimated with only these variables is the benchmark model. The introduction of the two factors augment the models, this makes it easy to evaluate the contribution of the factors in forecasting the bull and bear market of the S&P500 stock index. Thus equations 12, 14, 16 and 18 the are augmented with factors, while equation 13, 15, 17 and 19 are estimated without the factors. The use of all these models will help in determining the contribution of factors, since a consistent effect of the factors in all the models will make it easy to make conclusions on the performance of the factors in forecasting the bull and bear market.

To determine if the degree of the effects of the variables on the two states assumed in the market are different, the explanatory variables are interacted with the state variable in the model below.

$$\pi_t = \omega + \alpha\pi_{t-1} + \delta s_{t-j} + x'_{t-k}\beta + \gamma s_{t-1}x_{t-h} \qquad (20)$$

This will further help in determining if some variables are more influential in the bull market than in the bear market and vice versa. Thus, depending on which state of the market is occurring those variables would serve as the key indicators of that state.

3.9 Forecasting procedure

The forecasting procedure follows the techniques outlined in Nyberg (2012) and Kauppi and Saikkonen (2008). Looking at the general model, an optimal h-month forecast of s_t based on set $\Omega_{t-h,}$ is the conditional expectation $E_{t-h}(s_t)$. From model (5), relying on the law of iterated expectation the expected value is:

$$E_{t-h}(s_t) = E_{t-h}(P_{t-1}(s_t = 1)) = E_{t-h}(\Phi(\pi_t)) \qquad (21)$$

Where all the parameters have been previously discussed. The forecasting procedure to obtain the h-steps ahead forecast depends on the model employed.

3.10 Static Forecasting

For the static model, one only need to insert the linear function (6) into (11). Where k represents the employed lag order of the explanatory variables, in our case $k \geq h$, where again h is the forecast horizon. Because the value of the explanatory variables are known at time $t - h$. Many previous studies in forecasting the state of the equity market set k=h for example Nyberg (2012). With this, the employed lag of x_t is chosen to match the forecast horizon. However, Kauppi and Saikkonen (2008) argued that a rule based on lag order selection may lead poor forecasts, because good results are hard to get when choosing lags that are supported by statistical model selection criterion, this is the procedure employed in this paper.

Forecasting with model (8) the procedure is similar to the static model (6) since when h=2, by recursive substitution we get:

$$\pi_t = \omega + \alpha\pi_{t-1} + x'_{t-k}\beta = (1+\alpha)\omega + \alpha^2\pi_{t-2} + \alpha x'_{t-k-1}\beta + x'_{t-k}\beta \quad (22)$$

This again shows that π_t only depends on the information available at time t-k. So again, we can simply obtain the h period forecasts by directly inserting the autoregressive function into model (11).

3.11 Dynamic forecasting

The forecast gets more complicated when the lagged state variable (s_{t-1}) is introduced in model (9). Thus, it leads to an iterative multi period forecasting approach. Looking at the two periods ahead forecast (h=2) again, set j=1 in model (9) and inserting it into model (11) leads to:

$$E_{t-2}(s_t) = E_{t-2}(\Phi(\omega + \alpha\pi_{t-1} + \delta s_{t-1} + x'_{t-k}\beta)) \quad (23)$$

This expression has the unknown value of s_{t-1} on the right-hand side. The binary nature of s_t makes it possible to compute the forecast of model (12) by using the relation that account for two possible paths between s_{t-2} and s_t. With this, we have model (12) stated as:

$$E_{t-2}(s_t) = \begin{cases} \widetilde{\Phi}(0), = if\ s_{t-1} = 0 \\ \widetilde{\Phi}(1), = if\ s_{t-1} = 1 \end{cases} \quad (24)$$

Now $\widetilde{\Phi}(0)$ and $\widetilde{\Phi}(1)$ denotes the two possible paths depending on the value of s_{t-1}:

$$\Phi(0) = \Phi((1+\alpha)\omega + \alpha^2\pi_{t-2} + \alpha(\delta s_{t-2} + x'_{t-k-1}\beta) + x'_{t-k}\beta) \quad (25)$$

$$\Phi(1) = \Phi((1+\alpha)\omega + \alpha^2\pi_{t-2} + \alpha(\delta s_{t-2} + x'_{t-k-1}\beta) + \delta + x'_{t-k}\beta) \quad (26)$$

The conditional probability of the bear market at time t-1 is:

$$p_{t-1} = P_{t-2}(s_t = 1) = \Phi(\omega + \alpha\pi_{t-2} + \delta s_{t-2} + x'_{t-k-1}\beta) \quad (27)$$

The two period ahead forecast is:

$$E_{t-2}(s_t) = (1 - p_{t-1})\Phi(0) + p_{t-1}\Phi(1) \quad (28)$$

This is the solution derived iteratively by accounting for two possible values of s_{t-1} and their conditional probabilites, using the same one period model (9). Using h>2 the expression above explodes since the number of paths between t-h and t is larger and the situation gets more complicated. Using h=2 is not always practical because of the real time lags of s_t. Since the last few months are not known in real time when estimating the forecasts . Therefore, even though s_{t-1} is a good predictor it is not a good option for using in out of sample forecasting. This paper only look at h=1, which is the most interesting and practical horizon for investors in the market. For model (10), assuming again that $j = 1$, the forecast procedure made at $t - h$ requires the evaluation of:

$$\begin{aligned} E_{t-h}(s_t) &= E_{t-h}\big(\Phi(\omega + \alpha\pi_{t-1} + \delta s_{t-1} + x'_{t-k}\beta + \gamma s_{t-1}x_{t-h})\big) \\ &= E_{t-h}\big(\Phi(\omega + \alpha\pi_{t-1} + (\delta + \gamma x_{t-h})s_{t-1} + x'_{t-k}\beta)\big) \end{aligned} \quad (29)$$

Note that this conditional expectation differs from model (10) because the value of s_{t-1} depends on the value of x_{t-h}. Modifying the example from above when h=2 one replaces δ with the time variant coefficient $\delta + \gamma x_{t-h}$.

Therefore, the out-of sample forecasts probabilities of the bull and bear market were predicted by recursively estimating the models and forecasting at every step. The data used in estimating the models range from 1973:M3 for static probit model and 1973:M5 for dynamic probit model to 2011:M12.

4 Results

This section presents the interpretation and analysis of the findings of this paper. A summary of the data and its specifications are also presented.

4.1 Data

The objective of this study is to determine if principal components can improve the forecast accuracy of the S&P 500 stock index, using macroeconomic and financial variables. The study uses US monthly data ranging from 1973 to 2016. In line with studies done by Nyberg (2012) and Chen (2009), the bull and bear markets of the S&P 500 stock is derived from the S&P500 stock index. The following five variables are use as the key leading indicators: Dividend yield, unemployment rate, money stock, returns on asset and term spread. Thus, these leading indicators were initially excluded from the factor extraction since they were directly used in the estimation of the empirical models, this is in line with Kauppi and Saikkonen (2008) and Chen (2009). After the initial test to determine which variables to use as leading indicators money stock, returns on asset and Term spread were retained, while dividend yield and unemployment rate were included in the group of variables from which factors were extracted. Two principal components were extracted, one from a group of finance variables and another from macroeconomic variables see Table 10 &11. The macroeconomic variables are the variables related to the broad sections of the market, while the finance variables are related to the ways and means in which money is created and managed.

A unit root test on all the variables used in this study were conducted to avoid using nonstationary variables as shown in Table 1. The two term spreads were calculated by taking the spread between the 10 years government bond and the 3 months Treasury bill rate (TS10) and the spread between the 5 years government bond and the 3 months Treasury bill rate (TS5).

The null hypothesis of a unit root in all the variables is rejected for all the variables at 10%, except for M2 in DF-GLS test, but given that ADF and PP both indicates stationarity the DF-GLS results is disregarded. Furthermore, all the variables used in extracting the factors were standardized with mean zero and a unit variance. This is ideally in factor analysis as it reduces the influence of variables with larger variance.

4.2 The Dependent Variable

The method for creating the dependent variable i.e. the binary time series variable is discussed in section 3.1. Thus, here only the important statistical results of the state of the stock market is presented. The result from using the B&B is shown in Figure 1.

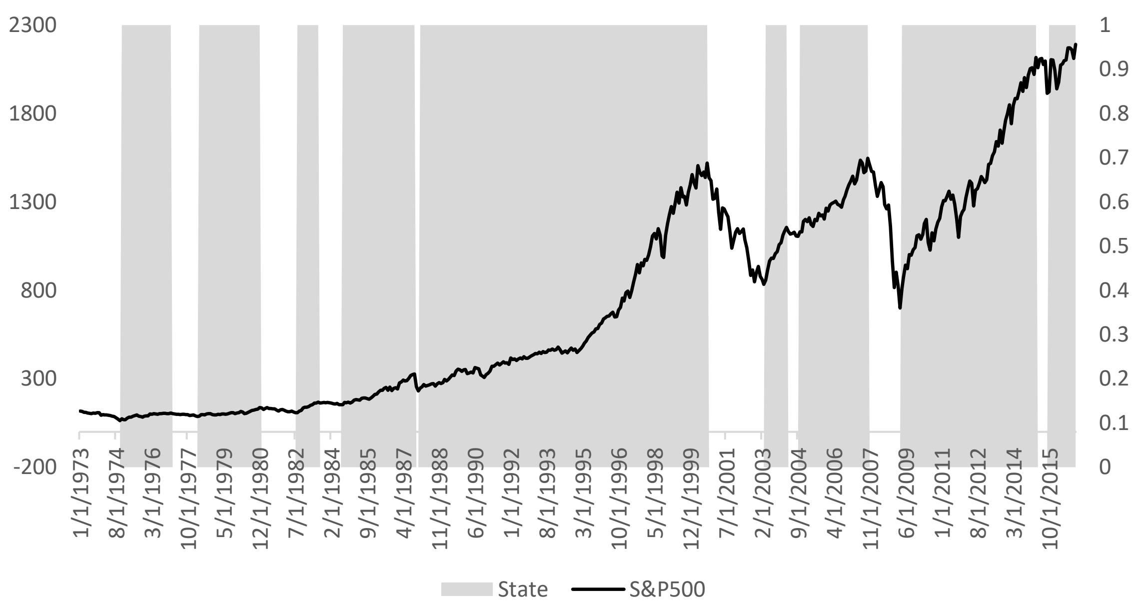
Table 1

Unit root tests

Variable	ADF	PP	DF-GLS
Changes in dividend yield	-21.60	-21.62	-19.40
Changes in unemployment rate	-6.75	-22.84	-5.30
Growth rate of broad money (M2)	-6.35	-14.51	-2.32
Nominal returns	-22.20	-22.21	-4.41
Term Spreads (10Y-3M)	-4.04	-3.82	-2.87
Term Spreads(5Y-3M)	-4.50	-4.27	-3.73

NOTE: ADF, PP, DF-GLS are Augmented Dickey-Fuller, Phillips-Perron and Elliott Rothenberg-Stock DF-GLS test statistics, respectively. The null hypothesis is that the series has a unit root. The critical values for ADF and PP are -3.45 (1%), -2.87 (-5%) and -2.57 (10%). The critical values for DF-GLS are -2.57 (1%), -1.94 (-5%) and
-1.62 (10%).

The state variable is the dependent variable which only take on the value 1 for bull and 0 for bear i.e. the greys areas are the bull regimes while the white areas are bear regimes. This result, reveals that B&B method replicates the regimes as in the S&P 500 stock index.



The chronology of stock market turning points and the corresponding bear and bull market periods are presented in Table 2. In deciding the troughs and peaks the study follows the convention used in previous literatures, the peak month is classified as the last month of a bull market and the trough month is the last month of a bear market. From figure 1, between 1987:10 and 1987:12 there is a 2-month contraction in the S&P500 index, this might look like a brief time but given that it is such a large movement (-34.42%)

it is reasonable that the B&B classify it as a bear market. The turning points are similar to Chauvet and Potter (1997), Pagan and Sossounov (2003) and Nyberg (2012).

The maximum number of months in the bull market is 153 months with a mean of 44.44 months and minimum of 12. While the bear market has a maximum of 30 consecutive months with a mean of 14.11 and a minimum of 2 months. This is expected, as in periods of bull market, the confident level in the stock market accelerates and continue to go up until a shock occurs. These shocks rarely last long as policy are normally devises policies to combat them.

Table 2

Linear and Regime switching dynamics in stock returns

Peaks	Troughs	Bull duration	Bull Change%	Bear duration	Bear Change%
	(1974:10)				
1977:01	1978:03	27	52.35	14	-20.75
1980:12	1982:07	33	45.34	19	-23.28
1983:07	1984:07	12	44.07	12	-09.76
1987:10	1987:12	39	75.92	2	-34.42
2000:09	2003:03	153	188.00	30	-59.98
2004:03	2004:09	12	32.55	6	-04.43
2007:10	2009:03	37	33.57	17	-79.18
2015:03	2015:09	72	110.57	6	-10.11
(2016:12)		15	13.53		

Notes: The first (second) column gives the peak (troughs) turning points of the S&P 500 index determined by the Bry and Boschan (1971) dating method. The sample period is 1973:01-2016:12. A bear market starts after the peak month and ends at the trough and vice versa with a bull market. Bull (bear) duration shows the time in months from the last trough (peak) to the next peak (trough). The percentage change in the S&P500 index during the bear and bull is denoted by "Change%".

Table 3 presents the result from model (1) in which r_t is regressed on the state variable to determine the existence of regimes. The mean of the bull market is 1.49 while that of the bear market is -2.39, the two values are significantly different, which is an indication that the S&P500 stock market can be decomposed into two regimes (bull and bear market). The standard error of the bear markets is 5.22 and that of the bull market is 3.87. This is an indication that the bear market is relatively more volatile than the bull markets, as it is characterized with periods of high volatility and loss of confidence in the stock market. The bull market durations have relatively higher persistent than the bear market.

Table 3

Mean and standard deviation of returns given the two regimes.

Model	*Linear*	*Regime switching*
μ	0.56**	
μ_0		-2.39***
μ_1		1.49***
σ	4.54	
σ_0		5.22
σ_1		3.87

4.3 In-sample forecast

After identification of the two regimes, an in-sample predictability test is conducted to determine which variables and at what lags should be included in the multivariate forecasting model as done by Nyberg (2012) and Chen (2009). The state of the equity market, which represents the bull and bear market is regressed on each of the observed variables and factors using a static probit model. The predictive power of each variable is estimated at horizons of 1 to 12 months.

4.3.1 Univariate model

From Table 4 the in-sample result indicates that broad money is statistically significant at all lags(K). Both term spreads become statistically significant when k>2. While dividend yield and returns data is statistically significant at lags 1-6. According to pseudo R^2 term spreads and broad money has better goodness-of-fit beyond 5 months, while dividend yield and return data has better fit within 5 months.

The results indicate that broad money supply has the highest pseudo R-squared with 13% at the twelfth lag, similarly term spreads also had their highest value of Pseudo R-squared at the twelfth lag while dividend yield and returns recorded their highest value of Pseudo R-squared at the first lag. The unemployment rate does not have any explanatory power on the bull and bear market at all the forecast horizons. Thus, it is moved to the group of variables used in extracting the principal components. The term

Table 4

In sample predictability test results for predicting stock markets

Standard probit model: $\Pr(S_t = 1) = \Phi(\omega + \beta X_{t-k})$

Data for:1973:05-2011:12

	DY = First difference of dividend yield					RT = return of the stock market			
	$\hat{\beta}$ (SE)	t-stat	p-value	R^2		$\hat{\beta}$ (SE)	t-stat	p-value	R^2
k=1	-2.32 (0.49)	-4.77	0.000***	**0.0678**	k=1	8.80 (1.60)	5.51	0.000***	**0.0791**
k=2	-1.60 (0.44)	-3.62	0.000***	0.0385	k=2	6.44 (1.58)	4.05	0.000***	0.0521
k=3	-1.51 (0.43)	-3.54	0.000***	0.0405	k=3	6.20 (1.55)	4.00	0.000***	0.0538
k=4	-1.42 (0.42)	-3.42	0.001***	0.0428	k=4	5.78 (1.53)	3.76	0.000***	0.0540
k=5	-0.98 (0.38)	-2.60	0.009**	0.0368	k=5	4.49 (1.44)	3.13	0.002**	0.0462
k=6	-0.72 (0.36)	-2.00	0.046*	0.0373	k=6	3.13 (1.34)	2.34	0.020*	0.0413
k=7	-0.47 (0.35)	-1.34	0.181	0.0395	k=7	2.68 (1.33)	2.02	0.044*	0.0441
k=8	-0.35 (0.35)	-0.988	0.323	0.0437	k=8	2.39 (1.31)	1.82	0.070	0.0480

	M2 = growth of Broad Money					TS5 = Term Spread (5Y-3M)			
	$\hat{\beta}$ (SE)	t-stat	p-value	R^2		$\hat{\beta}$ (SE)	t-stat	p-value	R^2
k=1	-57.5 (17.8)	-3.23	0.001**	0.0282	k=1	0.08 (0.06)	1.33	0.184	0.0091
k=2	-63.6 (18.5)	-3.43	0.000***	0.0386	k=2	0.12 (0.06)	1.92	0.055	0.0188
k=3	-65.2 (18.8)	-3.46	0.000***	0.0453	k=3	0.15 (0.06)	2.39	0.007*	0.0282
k=4	-57.8 (18.5)	-3.12	0.002**	0.0443	k=4	0.17 (0.06)	2.71	0.007*	0.0367
k=5	-53.7 (19.0)	-2.82	0.005**	0.0457	k=5	0.18 (0.06)	2.88	0.004**	0.0438
k=10	-78.1 (20.6)	-3.79	0.000***	0.0922	k=10	0.26 (0.06)	4.16	0.000***	0.0894
k=11	-92.4 (22.0)	-4.19	0.000***	0.1118	k=11	0.26 (0.07)	4.03	0.000***	0.0972
k=12	-102 (22.7)	-4.52	0.000***	**0.1276**	k=12	0.27 (0.07)	3.86	0.000***	**0.1033**

	TS10 = Term Spread (10Y-3M)			
	$\hat{\beta}$ (SE)	t-stat	p-value	R^2
k=1	0.07 (0.05)	1.45	0.147	0.0093
k=2	0.10 (0.05)	1.93	0.054	0.0188
k=3	0.11 (0.05)	2.29	0.022*	0.0270
k=4	0.12 (0.05)	2.55	0.011*	0.0348
k=5	0.13 (0.05)	2.75	0.006**	0.0419
k=11	0.21 (0.05)	4.18	0.000***	0.0958
k=12	0.21 (0.05)	4.18	0.000***	**0.1025**

unmp = First difference of unemployment rate showed no statistical significance at any lag lengths from 1:12 and was left out of the table.
R^2 is a Pseudo R-squared measure. The one used in this table is Mcfaddens R^2. $R^2 = 1 - (\ln \hat{L}(M_{full)}/ \ln \hat{L}(M_{intercept)})$

spread and returns all have a positive effect on the state of the equity market while broad money supply and dividend have a negative effect on the state of the equity market.

The in-sample predictable of both dynamic and static factors is also assessed at horizons 1 to 12 to determine which factors to use, the results are presented in Table 5. The Static factor extracted from the interest rates is significant at all lags, but the macroeconomic factor is statistically significant only at lags four and above. However, both factors become more influential as k increase. The interest rate factor has more influence on the state of the equity market at horizons 12 with a pseudo R-squared of 10%, similarly the macroeconomic factor also has its highest effect at horizon 12 with a Pseudo R-square of 7%. The two price index factors both have similar dynamics, as they become more significant and influential at shorter horizons. The first index factor registered its highest effect on the state of the equity market at horizon 1, while the influence fades away the longer the horizon.

For the dynamic version of the static model only two factors are extracted one from a groups of interest variables and another from a group of macroeconomic variables. The in- sample performance of the dynamic factor are significant at all horizons and they become more significant at longer horizons. The dynamic interest rate factor has it largest influence at horizon 12 with a pseudo R-square of 10.8%, similarly the dynamic macroeconomic factor also has its biggest influence on the state of the equity at horizons 12 with a pseudo R-squared of 8.14%. The dynamic factor extracted from price indexes and commodity were not significant, thus were dropped from the model.

4.3.2 Multi variate models

Table 6 and Table 7 reports the in sample forecast results for the multivariate models considered. After conducting the individual in-sample test for all the variables, a multivariate model is constructed. The in-sample forecasting model, uses sample data from 1973M5 to 2011M12 to estimate the parameters in all the models considered.

Looking at the Pseudo R-squares from Table 6 and Table 7 the models that include factors consistently outperform the corresponding models without factors. The dynamic autoregressive model 8 with static factors outperforms all the models with a pseudo R-squared of 0.771, while the static probit model 2 without the factors is the least performing model with a pseudo R-squared of 0.175. This result is in line with studies done by Chen (2009) and Nyberg (2012) all of whom predicted the US stock market and found that dynamic autoregressive model outperforms the static probit model. One has to remember though that using model 8 in real life isn't realistic, because of the real-time lag mentioned in section 3.1.

Table 5

In sample predictability test results for predicting stock markets

Standard probit model: $\Pr(S_t = 1) = \Phi(\omega + \beta F_{t-k})$ F notation is used to indicate fators variables.

Data for:1973:05-2011:12

	rates = static first factor made up of different interest rates					econ = static first factor made up different economic variables			
	$\hat{\beta}$ (SE)	t-stat	p-value	R^2		$\hat{\beta}$ (SE)	t-stat	p-value	R^2
k=1	0.26 (0.06)	4.17	0.000***	0.4280	k=1	-0.03 (0.06)	-0.43	0.669	0.0055
k=2	0.26 (0.06)	4.22	0.000***	0.0485	k=2	-0.05 (0.06)	-0.82	0.412	0.0116
k=3	0.25 (0.06)	3.97	0.000***	0.0501	k=3	-0.06 (0.06)	-1.02	0.310	0.0176
k=4	0.24 (0.06)	3.84	0.000***	0.0524	k=4	-0.09 (0.06)	-1.43	0.153	0.0248
k=5	0.23 (0.06)	3.72	0.000***	0.0556	k=5	-0.13 (0.06)	-2.21	0.027*	0.0341
k=10	0.29 (0.06)	4.70	0.000***	0.0975	k=10	-0.18 (0.06)	-2.79	0.005**	0.0669
k=11	0.28 (0.06)	4.65	0.000***	0.1011	k=11	-0.19 (0.07)	-2.82	0.005**	0.0734
k=12	0.27 (0.06)	4.40	0.000***	**0.1018**	k=12	-0.18 (0.07)	-2.75	0.006**	**0.0784**
	Ind1 = static factor made up different index prices					Ind2 = static factor made up different inde yield, PE ratios			
	$\hat{\beta}$ (SE)	t-stat	p-value	R^2		$\hat{\beta}$ (SE)	t-stat	p-value	R^2
k=1	0.35 (0.07)	4.89	0.000***	**0.0604**	k=1	0.37 (0.07)	5.38	0.000***	**0.0698**
k=2	0.24 (0.07)	3.53	0.000***	0.0383	k=2	0.27 (0.07)	3.83	0.000***	0.0451
k=3	0.25 (0.07)	3.65	0.000***	0.0454	k=3	0.26 (0.07)	3.78	0.000***	0.0479
k=4	0.22 (0.07)	3.36	0.001***	0.0448	k=4	0.25 (0.07)	3.61	0.000***	0.0498
k=5	0.17 (0.06)	2.71	0.001***	0.0399	k=5	0.20 (0.07)	3.01	0.002*	0.0449
k=10	0.03 (0.06)	0.58	0.560	0.0413	k=10	0.03 (0.06)	0.51	0.608	0.0538
k=11	-0.00 (0.06)	-0.29	0.977	0.0588	k=11	-0.02 (0.06)	-0.37	0.715	0.0590
k=12	-0.03 (0.06)	-0.55	0.582	0.0647	k=12	-0.04 (0.06)	-0.57	0.570	0.0648
	ar1rates= dynamic first factor of rates, AR1					ar1econ= dynamic first factor of economic, AR1			
	$\hat{\beta}$ (SE)	t-stat	p-value	R^2		$\hat{\beta}$ (SE)	t-stat	p-value	R^2
k=1	0.25 (0.07)	3.78	000***	0.0355	k=1	0.51 (0.43)	1.88	0.235	0.0088
k=2	0.25 (0.07)	3.74	000***	0.0394	k=2	0.58 (0.42)	1.38	0.168	0.0149
k=3	0.24 (0.07)	3.64	000***	0.0429	k=3	0.54 (0.42)	1.30	0.193	0.0197
k=4	0.24 (0.07)	3.61	000***	0.0478	k=4	0.39 (0.42)	0.92	0.358	0.0230
k=5	0.25 (0.07)	3.76	000***	0.0557	k=5	0.17 (0.42)	0.40	0.693	0.0267
k=10	0.32 (0.07)	4.89	000***	0.0981	k=10	-0.86 (0.41)	-2.09	0.037*	0.0632
k=11	0.31 (0.07)	4.75	000***	0.1016	k=11	-1.01 (0.41)	-2.43	0.015*	0.0724
k=12	0.32 (0.07)	4.82	000***	**0.1080**	k=12	-1.13 (0.42)	-2.71	0.007**	**0.0814**

com = static commodity factor showed no statistical significance at any lag lengths from 1:12 and was left out of the table.
rates2= static second factor made up of different interest rates showed no statistical significance at any lag lengths from 1:12 and was left out of the table.
R^2 is a Pseudo R-squared measure. The one used in this table is Mcfaddens R^2. $R^2 = 1 - (\ln \hat{L}(M_{full}) / \ln \hat{L}(M_{intercept}))$

According to the quadratic probability score, which evaluates the forecast into binary categories by looking at the times the model predicts the actual forecast correctly. Like the pseudo R-squared, the dynamic autoregressive model 8 with static factors had the lowest QPS with 0.065 while the static model without factors again registered the highest QPS.

Returns on assets and money stock are significant and influential in all the models. Returns have a positive effect on the state of the equity market while money stock had a negative effect on the state of the equity market. Similarly, the factor extracted from the interest rates are highly significant and is positively associated with the state of the equity market. The macroeconomic factor is also significant in all the models but has a negative effect on the state of the equity market. Dividend yield does not improve the the forecasting accuracy of the models as the performance of the models increase without it. According to the Pseudo R-squared in all the models, the in-sample forecast accuracy increases at higher horizons, this is not expected as forecasting at longer horizon is difficult and uncertain as the pseudo R-squared increases at higher horizons. Overall the results indicate that factors increase the forecast accuracy of the S&P 500 stock index and the results are robust to all the different types of probit model employed. This is in order with Fossati (2015) who uses dynamic factor extracted from macroeconomic variables in forecasting the US recession and found that factors improve the forecast accuracy of the model than the observed variables.

4.4 Out of sample forecast

Figure 2, 3 and 4 presents the out of sample forecast results of model 5, 6, 7, 8, 9, 10 and 11. The results of the four static probit model are not presented due to poor forecasting results relative to the other models. The variables used in each model can be seen in Table 6 and Table 7. The out of sample period was set at the sample period January 2102 to December 2016. Returns on assets, money stock and the two factors provides a good fit of the state of the equity market. Similar to the in-sample forecast results, the models including the factors outperform the other models without the factors. While the dynamic autoregressive model 8 is the best forecasting model, this result are in line with Kauppi and Saikkonen (2008) who also found that dynamic autoregressive model outperforming the static probit model in forecasting US recession. Like the in-sample forecast results, the QPS in Table 8 shows that the models including the factors outperforms the other models without the factors. Clearly model 8 outperforms all the models with a forecast accuracy of 97% while the least performing models are models 5 and model 6 with 88% forecast accuracy see Table 8. The autoregressive component is influential as it improve the

Table 6
In sample predictability test results for predicting stock markets, using different models from method section

Data for:1973:05-2011:12

	Static model (1)	Static model (2)	Static model (3)	Static model (4)	Auto. Model (5)	Auto. Model (6)	Auto. Model (7)
ω constant	0.996	1.001	1.447	1.314	0.408	0.607	0.453
	(0.148)***	(0.148)***	(0.149)***	(0.137)***	(0.254)*	(0.247)*	(0.243)*
π_{t-1}					0.800	0.788	0.785
					(0.143)***	(0.141)***	(0.145)***
s_{t-j}							
r_{t-1}	10.384	8.158	8.396	8.439	6.893	7.051	8.336
	(3.271)***	(1.551)***	(1.561)***	(1.547)***	(1.332)***	(1.333)***	(1.518)***
$ts5_{t-12}$	0.275	0.275			0.084		
	(0.063)***	(0.063)***			(0.128)		
$m2_{t-12}$	-110.328	-109.129	-112.005	-91.725	-63.903	-73.030	-50.129
	(19.939)***	(19.908)***	(20.706)***	(19.940)***	(26.097)**	(25.599)**	(25.488)*
dy_{t-1}	0.731						
	(0.940)						
$rates_{t-12}$			0.269			0.076	
			(0.065)***			(0.120)	
$econ_{t-12}$			0.240			-0.0861	
			(0.080)**			(0.154)	
$dyrates_{t-12}$				0.277			0.067
				(0.073)***			(0.135)
$dyecon_{t-12}$				-0.871			-0.359
				(0.406)*			(0.716)
QPS					0.221	0.221	0.223
Pseudo-R^2	0.176	0.175	0.189	0.178	0.362	0.365	0.362
AIC	0.926	0.923	0.912	0.917	0.713	0.714	0.717
BIC	0.971	0.959	0.957	0.963	0.750	0.760	0.763

Standard errors are given in parentheses are computed using the procedures suggested by Kauppi and Saikkonen (2008). In the table, the values of the Pseudo-R^2, is calculated using Mcfadden´s.
Signif. codes: 0 '***' 0.001 '**' 0.01 '*'

forecast accuracy of model 6 and 7. The study does not found any difference in the degree to which the explanatory variables affect the state of the S&P500 stock index, since the interaction between the variables and the state dummy were all insignificant. Thus, impact each explanatory variable has on either the bull market period or bear market period are similar. Therefore, this study found that factors can improve the forecast accuracy of the models.

Table 7
In sample predictability test results for predicting stock markets, using different models from method section
Data for:1973:05-2011:12

	Dyn.auto. model (8) J=1	Dyn.auto. model (9) J=5	Dyn.auto. model (10) J=5	Dyn.auto. model (11) J=5
ω constant	-1.215	0.492	0.305	0.318
	(0.316)***	(0.242)*	(0.239)*	(0.246)*
π_{t-1}	-0.059	0.425	0.426	0.405
	(0.085)	(0.123)***	(0.131)***	(0.121)***
s_{t-j}	4.034	0.781	0.827	0.872
	(0.411)***	(0.278)***	(0.306)***	(0.284)***
r_{t-1}	-6.298	7.591	8.375	8.140
	(2.113)***	(1.451)***	(1.612)***	(1.548)***
$ts5_{t-12}$				
$m2_{t-12}$	-83.364	-96.334	-72.081	-83.031
	(27.399)***	(22.788)***	(25.145)***	(22.935)***
dy_{t-1}				
$rates_{t-12}$	0.051	0.104		
	(0.128)	(0.129)		
$econ_{t-12}$	-0.098	-0.144		
	(0.122)	(0.149)		
$dyrates_{t-12}$			0.094	
			(0.147)	
$dyecon_{t-12}$			-0.791	
			(0.747)	
QPS	0.065	0.201	0.203	0.208
Pseudo-R^2	0.771	0.397	0.400	0.334
AIC	0.277	0.684	0.681	0.698
BIC	0.331	0.738	0.735	0.735

Standard errors are given in parentheses are computed using the procedures suggested by Kauppi and Saikkonen (2008). In the table, the values of the Pseudo-R^2, is calculated using Mcfadden´s.
Signif. codes: 0 '***' 0.001 '**' 0.01 '*'

4.5 ROBUSTNESS CHECKS

To examine whether the performance of the models are not specific to a particular model or dataset. The models are augmented with dummy variables to capture the seasonal effects in the stock market. The in-sample fit of the individual variables selected in the study were all introduced either at lag 1 or lag 12. It is also said by many followers of the stock markets that, stock prices usually rise during the end of the year and fall around August and September. This behavior could be attributed to the end of year interest payments, holiday effect which is assumed to have an effect of the mood of investors etc. Thus, two dummy variables were introduced: The end of year dummy and the August September dummy. The end of year dummy is equal to unity if in December or January zero otherwise while August September dummy is equal to zero if August or September and one otherwise.

Table 8
In sample predictability test results for predicting stock markets, using different models from method section

	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11
Predicted Bull months	59	60	60	54	59	60	58
Predicted Bear months	1	0	0	6	1	0	2
Correct prediction in %	0.88	0.9	0.9	0.97	0.88	0.9	0.87
QPS	0.255	0.209	0.208	0.060	0.260	0.248	0.278

Figure 2: Results for out of sample prediction using h=1.

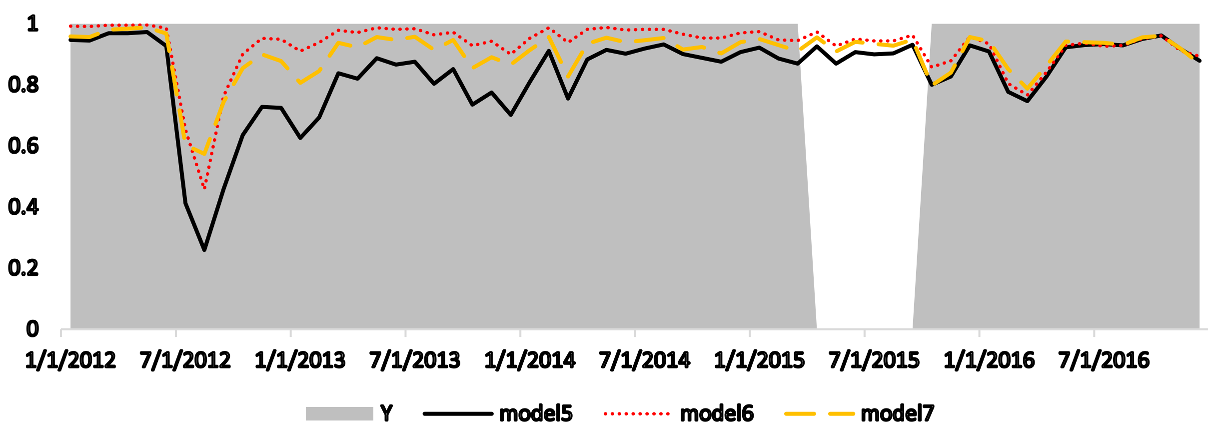


Figure 3: Out of sample forecast of Model 8

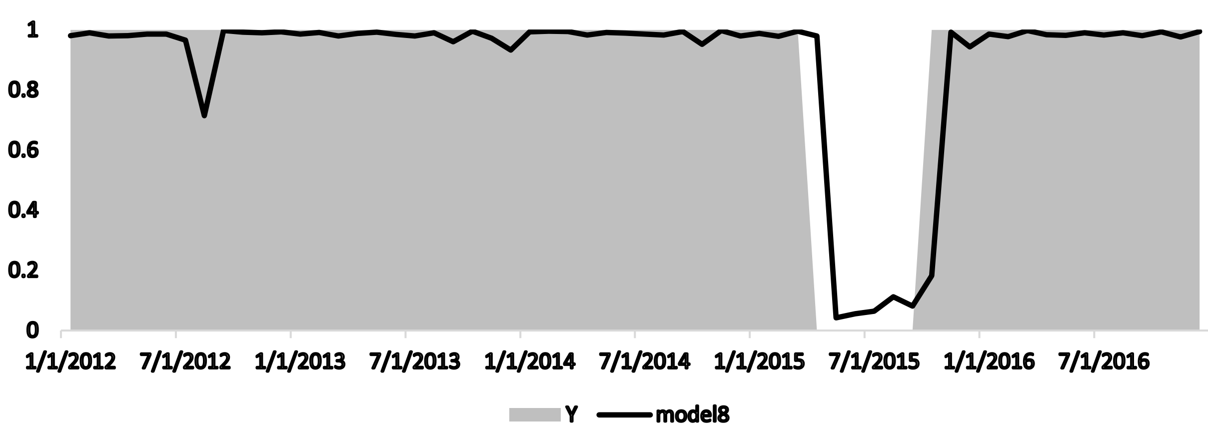
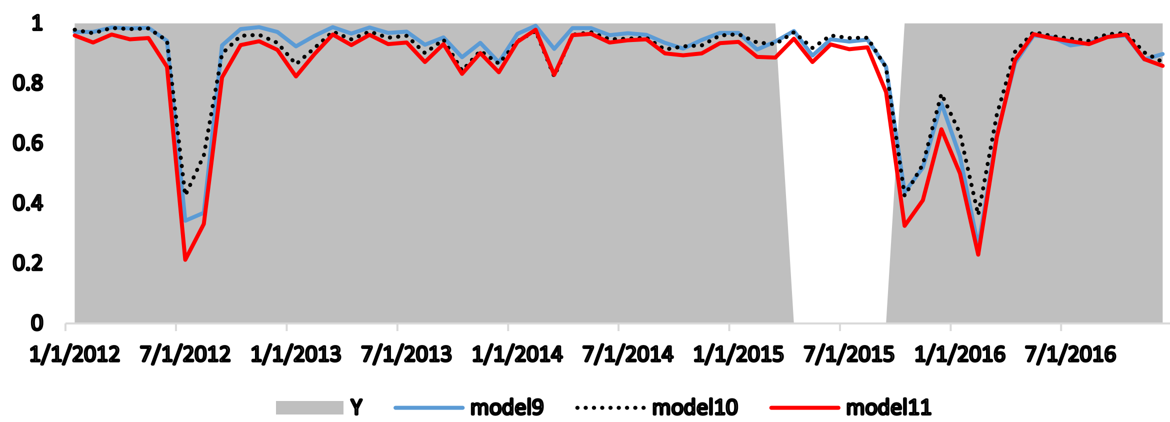


Figure 4: Out of sample forecast of Model 9, 10 & 11

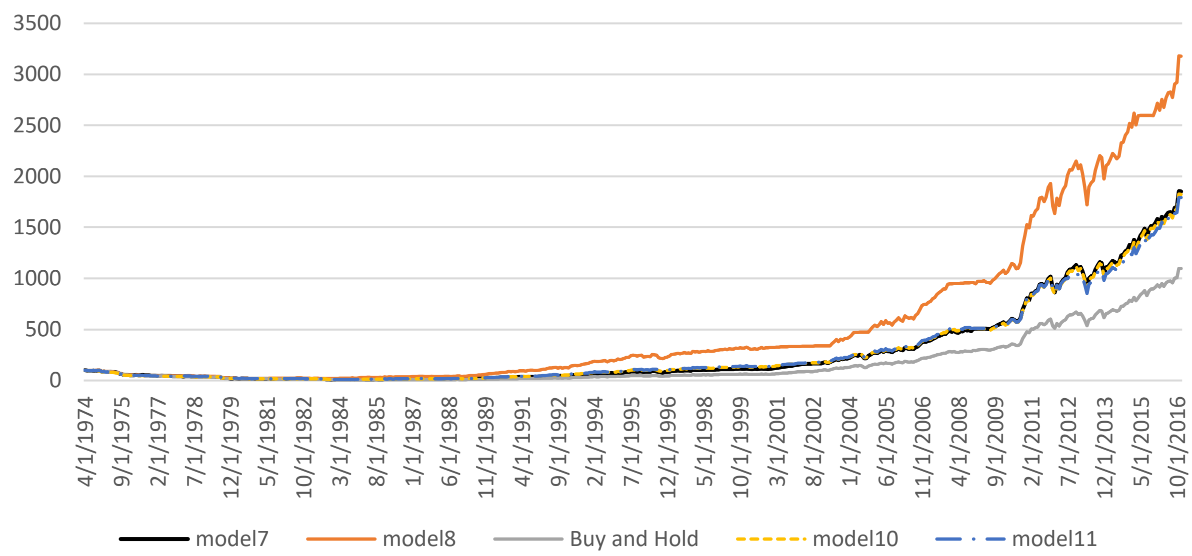


The introduction of the dummies does not change the results of the findings. The QPS of the models with factors are still lower than the models without factors and it does not even change. Both returns and money stock are still significant at lag 1 and 12 respectively. Again, the introduction of dummies does not change the performance of static factors over dynamic factors. Similarly, the Pseudo R-squared does not change in all models with the introduction of the seasonal dummies but AIC and BIC information criteria changed. This might be an indication that the seasonal dummies do not capture the true seasonal effect in the stock market see Table 9.

4.6 Economic Value

Following Chen (2009) we investigated whether using factors in predicting bear markets are useful for market participants trying to time the turning points. This is done by checking the models used versus the buy and hold strategy, we start by investing Sek 100.The sample period is set for the entire period at 1974:05-2016:12. The threshold is set at 30% that is if the probability of a bull market is less than 30% we switch to a 3 month Treasury Bill Rate. We compare the best models with the Buy and Hold strategy. Looking at Figure 5 we clearly see that the switching strategies with forecasting information about the bear market probability, outperforms the Buy-and-Hold strategy. Note that we haven't accounted for transactions cost. This is in line with Chen (2009), Nyberg (2012), Seidl (2012) and Erdogan, Bennett and Ozyildirim (2015). This exercise illustrates the reason for why these models need to be considered, for their usefulness of predicting bear markets.

Figure 5 : Economic value of the models



5 Conclusion

The goal of this study is to investigate whether factors extracted from macroeconomic and financial variables can improve the forecast accuracy of the bull and bear market in the S&P500 stock market. This is done by comparing probit models augmented with both static and dynamic factors. Each model is estimated three times, two with static factors or dynamic factors and one without the factors. According to the QPS the models that has factors perform better compared to models without factors. Likewise, comparing the performance of the two types of factors, the models that used the static factors has higher prediction accuracy than models with dynamic factors. The importance of returns on assets and money supply as predictors of the bull and bear market was confirmed in this study.

According to AIC, BIC and Pseudo R-squared the factors add value to the model, relative to the models without factors. The static probit model with factors outperformed the corresponding model without factors. Equally, the dynamic probit model with factors outperformed the model without factors. The dynamic probit models with factors had a superior forecast accuracy than all the other models.

Giving the QPS as the main indicator for forecast accuracy, the factors can improve the prediction accuracy of the bull and bear market in the S&P500 stock index. Thus, this study reveals that factors can improve the forecast accuracy of the bull and bear market of the S&P500. Contrary to Nyberg (2012) and Chen (2009) term spread and unemployment are not important predictors of the bull and bear market in S&P500 stock index.

Notwithstanding, the results showed that factors can improve the forecast accuracy of the bull and bear market of the S&P500. The study can be extended by increasing the number of regimes to four as done by Song (2011). Because, looking at the results of the Bry-Boschan Algorithm the number of cycles in the stock market seems to have different features. Model 8 is the best model in this study but there are limitation with regards to real time value, as the value of the dependent variable one period back is not observed.

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7 Appendix

Table 9

In sample predictability test results for predicting stock markets, using different models from method section

Robustness test using dummy variables

Data for:1973:05-2011:12

	Auto. Model (5) with d1a12	Auto. Model (6) with d1a12	Auto. Model (7) with d1a12	Auto. Model (5) with d8a12	Auto. Model (6) with d8a12	Auto. Model (7) with d8a12
QPS	0.222	0.222	0.223	0.221	0.221	0.223
Pseudo-R^2	0.362	0.365	0.362	0.362	0.365	0.362
AIC	0.717	0.719	0.722	0.717	0.718	0.721
BIC	0.763	0.773	0.776	0.763	0.773	0.776

Standard errors are given in parentheses are computed using the procedures suggested by Kauppi and Saikkonen (2008). In the table, the values of the Pseudo-R^2, is calculated using Mcfadden´s.

Signif. codes: 0 '***' 0.001 '**' 0.01 '*'

D1a12 = dummy variable with Jan and Dec = 1 and 0 everywhere else

D8a9 = dummy variable with Aug and Sept = 1 and 0 everywhere else

Table 10: Finance variables

No.	Variables	Source
1	Federal fund rate	Federal Reserve Bank of St. Louis
2	3-Month treasury bill minus federal funds rate	Federal Reserve Bank of St. Louis
3	Moody's seasoned Baa corporate bond minus federal funds	Federal Reserve Bank of St. Louis
4	10 year treasury constant maturity minus federal funds rate	Federal Reserve Bank of St. Louis
5	1 year treasury constant maturity minus federal funds rate	Federal Reserve Bank of St. Louis
6	5 year treasury constant maturity minus federal funds rate	Federal Reserve Bank of St. Louis
7	Moody's seasoned Aaa corporate bond minus federal funds rate	Federal Reserve Bank of St. Louis
8	6- months treasury bill minus federal funds rate	Federal Reserve Bank of St. Louis
9	Risk free rates	Datastream
10	US government 3 months treasury bills rate	Datastream

Table 10 reports the number of finance variables used to extract the finance factor and source of the variables.

Table 11: Macroeconomic varaibles

No.	Variables	Source
1	Unemployment rate	Datastream
2	Industrial production	Datastream
3	Total public debt	Datastream
4	Inflation rate	Datastream
5	Money supply M2	Datastream
6	Industrial production – manufacturing	Datastream
7	Money supply M1	Datastream
8	Personal saving as % of disposable personal income	Datastream
9	Total treasury securities outstanding	Datastream
10	Civilian labor force participation rate	Datastream
11	Commercial bank assets - loans & leases in bank credit	Datastream
12	Gross Domestic product	Datastream

Table 11 reports the number of macroeconomic variables used to extract the macroeconomic factor and source of the variables.