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Managing Risk with Energy Commodities using Value-at-Risk and Extreme Value Theory

Master Thesis

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Abstract

Today's society requires an endless supply of energy resources to keep functioning properly. The fluctuations in the prices of energy commodities are always a concern as it affects not only investors, but regular households as well. With the general turmoil the market is experiencing, the necessity for risk management has become of utmost importance. Thus, this paper provides an empirical study of the determination of the risk for four different energy commodities.

The focus is on crude oil (WTI), gasoline, natural gas and coal, as they represent most of the world's energy consumption of today. The time periods from 2010 to 2016 will be analyzed as it represents a new period of increased volatility.

The empirical research presented consists of calculating Value-at-Risk (*VaR*) for Value Weighted Historical Simulation (VWHS), Student's t-distribution and the EVT Conditional Peaks over Threshold (POT) approaches together with three different volatility estimates, Generalized Autoregressive Conditional Heteroskedasticity (GARCH (1.1)), Exponential GARCH (EGARCH) and Threshold GARCH (TGARCH). The different modeling approaches of volatility estimations will account for price asymmetries in the distributions.

Based of the empirical results, the study indicates that the Student's t-distribution with the EGARCH volatility process is the preferred method for *VaR* estimation.

Key Words: Energy Commodities, *VaR*, EVT, POT, VWHS, GARCH, EGARCH, TGARCH

Acknowledgement

We would like to thank our respective families and everyone who helped us and supported us throughout the completion of our Master degree in Finance.

Our achievements and determination are both shaped by the people surrounding us, and this accomplishment would not have been possible without them.

Alexander Noshkov – Zafer Demirtas

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1. INTRODUCTION

Energy is one of the fundamental needs of today's economic environment since it is one of the most important inputs for not only production, but also transportation. The world's energy need is enormous, which results in significant energy expenditures. For example, according to U.S. Energy Information Administration's data from 1990 to 2014, the energy expenditures as a share of GDP for the United-States fluctuate from 5.8% to as much as 9.6% (EIA Monthly Energy Review, 2017), which is considerable. The focus of the paper is on four main energy commodities, which are coal, crude oil, gasoline, and natural gas, and to assess them for risk management purposes. We are interested in the aforementioned commodities because of their importance, and the relatively dynamic nature of the energy industry.

Traditionally, since centuries, the Earth's energy needs were mainly satisfied with fossil fuels. Coal is one of the oldest commodities in use. In the beginning of the industrial revolution in the United Kingdom in the late 18th century, the coal's high calorie nature and industrial applications have brought tremendous changes to the human life. The second big paradigm in the energy industry was petroleum. In the modern history, the first ever drill with percussion took place in Baku. Starting from the early 20th century, crude oil production boomed tremendously and took the throne away from coal. Although, coal's weight in consumption decreased since then, its importance prevailed so far by an increasing usage. Gasoline is a byproduct of crude oil. On average it is possible to produce 72 liters of gasoline from 159 liters of crude oil (Automotive Fuels, 2007). On the other hand, it is believed that crude oil prices and gasoline prices have some asymmetries. Retail gasoline prices are believed to respond quicker to increases than to decreases in crude oil prices due to possible sources of asymmetry such as production, market adjustment lags and market power of some sellers.

Natural gas is another hydrocarbon fuel used in various purposes such as heating, electricity production and for industrial usage as well. After WWII, more advanced and efficient pipeline technologies facilitated transportation and increased the importance of natural gas.

According to the International Energy Commission (Key World Energy Statistics, 2016), the total world's primary energy supply in 2014, was composed of 31.3% oil, 28.6% coal, 21.2% natural gas, 10.3% biofuels and waste, 4.8% nuclear, 2.4% Hydro and 1.4% other types (solar, wind, heat, etc.). It can be easily seen that world consumption is highly dominated by hydrocarbon fuels.

Energy commodities are extremely important in today's society. The price changes and the volatility changes in the energy sector have great repercussions on the financial markets and on any country's macro-economic indicators. It also directly affects regular people and households since the prices for almost all the goods and services they consume are reflected from commodities' price changes. This makes risk management in energy commodities a very important and also very interesting topic area.

The society today is very dependent on fossil fuels. The four energy commodities analyzed in this paper are chosen because they constitute the biggest part of the world's energy consumption, they represent the overall energy market. Looking at history, it can easily be seen that periods of major political or economical events are always associated with high volatility. Figure 1 in the Appendix represents a long-term loss graph of WTI Crude Oil from the year 1986 to 2017. The effects of the Gulf war in the early 90's, the Asian Crisis in the late 90's, as well as the 2008 Global Financial crisis can all be seen by high volatility periods. However, it is interesting to notice that there is another high volatility period starting from 2014 and persisting to the more recent observations without there being any major stress events. This volatile period is the main point of interest of the paper.

It is believed that the high volatility for the 2014 period is a result of market restructuring. There is a mismatch between demand and supply, the demand for fossil fuels is not increasing, but the supply is increasing tremendously. The main factor influencing the increase in supply is the improvement in technology. However, nowadays fossil fuels are viewed as negative sources of energy. Environmental concerns and sustainability considerations have made people more reluctant to the use of fossil fuels, thus the demand side of the equation does not increase and tends to shift slowly towards renewable energy.

The December 2015, COP21 Paris conference resulted with the decision to hold global warming level at +2 degree Celsius. By increasing concerns of global warming, traditional fossil fuels fell subject to many controversies. The COP21's goal requires building a more carbon-free economy, which leads to a paradigm shift of expectations in the energy sector. The European Union aims to cut greenhouse emissions to 80% below the 1990's level, and the main way to manage this change is through the power-generation sector (European Commission 2017). The world is putting notorious efforts for a new area in the energy equation. Renewable energy sources role in the world energy sector has been increasing dramatically. According to the world energy outlook report, in 2014, a capacity of 128 GW was generated through renewable energy, with a worldwide investment of \$270 billion. For the first time in history, it is believed that renewables can be alternatives to conventional carbon producing power plants.

On the other hand, the world still heavily depends on fossil fuels. Also, carbon emission concerns, geopolitics, energy security issues, and supply and demand imbalances in the market make the energy equation even more complex. Therefore, the energy market has become increasingly volatile and risky, and for this reason risk management has become extremely important. The aim of this paper is to investigate market risks of different energy commodities and come up with a well-functioning model that can be applied in risk management.

The paper is organized as follows. In Section 2, the literature review provides a description of previous research papers about the development of *VaR*. Section 3, explains the main methods and the concepts of the calculations of *VaR*, and presents also three different GARCH models, namely GARCH (1.1), EGARCH and the TGARCH model, all under the Student T-Distribution. Section 4 consists of the presentation of the empirical results of the models with in-depth analysis, and lastly, the final section summarizes the results of the empirical study and draws conclusions and recommendations based on those results.

2. LITTERATURE REVIEW

The financial market, as it has become increasingly volatile and risky over the years, raises a big concern to institutions. The never-ending shift in market risk has made risk management indispensable for the firms' survival and prosperity.

Therefore, to help management understand and interpret risk in terms of money, in 1994, J.P Morgan introduced Risk Metrics. They explain the term *VaR* and quantify the risk, which resulted in a very simple and convenient system to use. The *VaR* method became very popular and widely used for risk management.

However, *VaR* has several unfortunate limitations and further adjustments to the Risk Metrics's paper are needed. It is a commonly accepted fact that financial returns are heavy-tailed. Brendan O. Bradley and Murad S. Taqqu (2001), describe some of the methods that can be used with heavy-tailed distributions.

Kevin Dowd (2002) provides very good feedback on the accuracy of the use of *VaR* in risk management, as well as many quantitative methods for applying different *VaR* models. He also points out different drawbacks and limitations to *VaR*, but since it is simple and convenient, it is still a widely used method in risk management.

Moreover, the traditional *VaR* does not capture very well the shifts in volatility, or volatility clustering. As showed in previous research, volatility is non-constant in financial returns. Engle (1982), proposed the Autoregressive Conditional Heteroskedasticity (ARCH) to estimate volatility. In his paper he estimates the variance of the United-Kingdom's Inflation.

To add to that research, later on Bollerslev (1986), proposed the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model as an improvement to the previous ARCH model. It has been shown that the model reflects all of the volatility shifts in financial data.

So far the standard GARCH model assumes a symmetric effect on volatility, and it works, but in practice it is not always the case. It has been shown that in financial data, volatility acts asymmetrically depending on the sign of the shocks according to the *Leverage Effect* mentioned by Black (1976). To account for that, Nelson (1991), proposed the Exponential (EGARCH) model. The model is able to distinguish the sign and magnitude of the shocks.

The last main model analyzed is the Threshold GARCH (TGARCH) model proposed by Zakoian (1994). It is another model that is able to distinguish between positive and negatives effects and adds a Threshold parameter to the model.

Finally, Kupiec P. (1995) proposes a technique to verify the accuracy of the above-mentioned models. It estimates the tail values of the distribution and shows the number of *VaR* violations produced by the models. This is the test that will be used to verify the accuracy of all the constructed empirical models.

This paper is inspired by previous research of *VaR* in the energy market. Notably, Hung, Lee, and Liu (2008), they estimate *VaR* by using different GARCH models for five different energy commodities, namely WTI Crude Oil, Brent Crude Oil, Heating Oil #2, propane and New-York Harbor Conventional Gasoline, using a data period

from 1996 to 2006. The highlight of their research suggests that using heavy-tailed distributions for estimating *VaR* is the most suitable method for energy commodities. This study has been one of the main guides to applying *VaR* in this paper.

Similarly, Aloui and Mabrouk (2010) estimate *VaR* and Expected Shortfall with different GARCH models for energy commodities using different distributions. Their conclusion was that fat-tailed distributions are better and that asymmetric models outperform the symmetrical GARCH models for energy commodities, which coincide with the empirical results obtained in this paper.

Marimoutou, Raggad and Trabelsi (2009), investigated the Conditional and Unconditional EVT models on energy commodities to forecast *VaR*. Their conclusion was that the Conditional EVT models offer a major improvement over the conventional methods, and parallel as other research, the use of fat-tailed distributions is suggested.

The focus of this paper is to use previous empirical research to provide a basis for the selection of the best models to be used to analyze the four chosen energy commodities. The empirical study provided here differs from previous research due to the selection of a more reflective time period to generate the most representative results possible. The time period analyzed is 2010 to 2016, which is a post-crash time period, and therefore it represents a new wave in the economic cycle. As it can be seen in the data, it is a period of newly stressed volatility. Starting from the second period of 2014, the energy commodity market sustains increased volatility due to different political and economical factors. It is still a very recent time period, and there are not many studies available yet. This paper will provide a complete empirical study on *VaR* estimates of the four aforementioned energy commodities for the recent stressed volatility time period. As this behavior is expected to continue in the near future, it makes it a very interesting area to analyze.

3. METHODOLOGY

3.1. An Introduction to Value-at-Risk

Rational investors always prefer a higher return with a lower risk, they are considered to be risk averse. It has always been the case, and now, with the increase of the general market volatility, it has become more and more risky to invest, thus giving rise to the need for risk management. A rising volatility means a higher risk for an investor, it is a simple concept but also very limited. This is where Value-at-Risk (*VaR*) comes in play, it is much more intuitive as it quantifies the risk in terms of an amount of loss.

VaR is the smallest loss ℓ so that the probability of a future portfolio loss L that is larger than the loss ℓ , has to be less than or equal to $1 - \alpha$. We can define *VaR* mathematically with the following equation:

$$VaR_{\alpha}(L) = \min\{\ell: \Pr(L > \ell) \leq 1 - \alpha\} \quad (1)$$

Under the assumption of a continuous loss distribution, the equation for *VaR* can be rewritten as follows:

$$\Pr(L > VaR_{\alpha}(L)) = 1 - \alpha \quad (2)$$

From a statistical point of view, *VaR* is the quantile of the loss distribution. The α represents the confidence levels, usually it is set to $\alpha = 0.95$ or $\alpha = 0.99$, typically for more accurate results on financial data, the latter is preferred.

To put it simply, *VaR* is the amount you expect to loose at a certain confidence level. Since *VaR* is based on losses, it means that the data on the energy commodities has to be converted to daily losses before starting any calculations. The daily losses will

take on positive values and the gains will have negative values to be able to calculate VaR.

3.2 Value-at-Risk under the Student t-Distribution

It is a well-known fact that financial returns have excess kurtosis, fat tails. A normal distribution has a kurtosis that is equal to 3, the t-distribution accommodates for fatter tails (kurtosis larger than 3), which is the main motivation for using it. The results should theoretically be more accurately represented.

The probability density function for a Student t-distribution with mean μ , volatility σ , and degrees of freedom ν can be written as follows:

$$f(x) = \frac{\Gamma\left[\frac{\nu+1}{2}\right]}{\sigma\sqrt{(\nu-2)\pi}\Gamma\left(\frac{\nu}{2}\right)} \left[1 + \frac{1}{\nu-2} \left(\frac{x-\mu}{\sigma}\right)^2\right]^{-(\nu+1)/2} \quad \text{for } x \in (-\infty, \infty) \quad (3)$$

The degrees of freedom parameter ν is a little complicated to estimate, normally it can be calculated it with a Maximum Likelihood (ML) function based on the probability density formula, however, if ν is larger than 4, then can be used the relationship between kurtosis k and the parameter ν to derive ν with the following formula:

$$\nu = \frac{4k - 6}{k - 3} \quad (4)$$

To arrive at the VaR model calculation under the t-distribution, the equation below can be used:

$$VaR_{\alpha}(L) = \mu + \sqrt{\frac{v-2}{v}} \sigma_{T+1} t_{\alpha, v} \quad (5)$$

Which allows for time varying volatility, where σ_{T+1} is either a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model or an Exponentially Weighted Moving Average (EWMA) estimate of volatility for the next period (one period out of sample).

3.3 Volatility Weighted Historical Simulation

Volatility Weighted Historical Simulation (VWHS), is a time-series method, it applies basic historical simulation to a rescaled sample of T observed losses. The motivation for choosing this method is that it allows accounting for volatility clustering. Volatility clustering is a principle that explains an aggregation of high (low) volatilities in time. If volatility is higher (lower) than average for a certain holding period, then volatility is likely to be higher (lower) for the following holding period as well. It is clear that financial instruments tend to exhibit such a phenomenon, as does the energy market, which means that this volatility pattern should be reflected in the estimates of VaR for the next holding period.

The main attraction for the model is that instead of taking an overall average of the data period, it uses a decay factor to reflect the current market conditions with volatility scaling of observed losses.

The samples of rescaled losses are calculated as follows:

$$\begin{aligned}
\ell_1^* &= \frac{\sigma_{T+1}}{\sigma_1} \ell_1 \\
\ell_2^* &= \frac{\sigma_{T+1}}{\sigma_2} \ell_2 \\
&\cdot \\
&\cdot \\
&\cdot \\
\ell_T^* &= \frac{\sigma_{T+1}}{\sigma_T} \ell_T
\end{aligned} \tag{6}$$

Where σ_T are the volatilities of the observed losses and σ_{T+1} is a volatility forecast for the next period. To estimate *VaR*, the basic Historical Simulation (HS) is applied to the rescaled losses. The Historical Simulation relies directly on the observed losses, according to the *VaR* definition $VaR_\alpha(L) = \min\{\ell: \Pr(L > \ell) \leq 1 - \alpha\}$, explained in the introduction to *VaR* section. Value-at-Risk is the smallest loss ℓ such the $\Pr(L > \ell) \leq 1 - \alpha$, according to this logic we can therefore use the $(1 - \alpha)N + 1$ largest loss in the empirical sample data, this equation will give the estimate of $VaR_\alpha(L)$.

To be able to implement the model, an estimation of the volatility is needed. Once again a time series model will be used to account for time varying volatility, which consists of different GARCH models, or the EWMA approach, both will be explained more in details in a further section.

3.4 Extreme Value Theory

The main aim to this paper is to explain the risk recurring on energy commodities, and Extreme Value Theory (EVT) will be used as a complement to *VaR* for that purpose. Value-at-Risk has some limitations that will be alleviated by the Extreme Value theory. *VaR* typically states an amount of loss that could incur at a designated

significance level, it gives information about the probability of losses that are larger than VaR , but the problem is that it is silent on the size of those losses. It does not consider the tail of the distribution, the only thing it will say, is that if a tail-event occurs, then the loss in that case is larger than VaR .

EVT is therefore more valuable in estimating Value-at-Risk, as it takes into consideration the tail, the extreme values of the losses. It takes into account all the losses that are higher than VaR .

The Peaks over Threshold (POT) model is the preferred Extreme Value approach in finance, hence this is the model that will be used. The reason is that with the traditional EVT, some information from the tail will be thrown away, with the POT model all losses larger than a pre-specified threshold are used. This of course raises the problem of choosing a proper threshold.

Indeed, there is some arbitrariness in the choice of the threshold value. There is also a trade-off in the model that has to be considered. The complication arises in the fact that for the underlying theory to go through, a high threshold value is needed, but for the estimation of the parameters, it is necessary to have as many observations as possible, thus in practice a lower threshold will be more satisfactory. A standard way to deal with this issue is to set the threshold u such that 4% of the original losses are considered as extreme losses.

To dig into the model, the theory underlying the POT approach consists at modeling excess losses $L - u$, where u is like mentioned above, a predetermined threshold value. Assuming that L is a stochastic loss variable, we can define a cumulative density function $F_u(\ell)$ for excess losses $L - u$ given that $L > u$, the expression taken from McNeil (1999) can be written as follows:

$$F_u(\ell) = \Pr(L - u \leq \ell | L > u) = \Pr(L \leq \ell + u | L > u) \quad (7)$$

To explain further, the definition of conditional probability $\Pr(A|B)$ is used to obtain an expression for $F_u(\ell)$:

$$F_u(\ell - u) = \frac{F(\ell) - F(u)}{1 - F(u)} \quad (8)$$

This equation represents the basis of the Peaks over Threshold Extreme Value Theory. Keeping focus on $F_u(\ell - u)$, we can further solve for $F(\ell)$ and derive the following expression:

$$F(\ell) = [1 - F(u)]F_u(\ell - u) + F(u) \quad (9)$$

Now the idea is to put $F(VaR_\alpha) = \alpha$ and to solve for the α -quantile VaR_α , but the problem is that $F_u(\ell - u)$ is an unknown distribution. This is where the extreme value theorem by Pickands (1975) Balkema-deHaan (1974) comes in handy. Their theorem states that $F_u(\ell - u)$ can be approximated using a Generalized Pareto Distribution (GPD) under certain (weak) assumptions. Under the limiting distribution of $F_u(\ell - u)$ as $u \rightarrow \infty$ is always a Generalized Pareto distribution (GPD):

$$G(\ell - u) = \begin{cases} 1 - \left(1 + \xi \frac{\ell - u}{\beta}\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - \exp\left(-\frac{\ell - u}{\beta}\right) & \text{if } \xi = 0 \end{cases} \quad (10)$$

This implies that $F_u(\ell - u) \approx G(\ell - u)$ with high values of the threshold u . In the model, the parameter β represents a scale parameter, and ξ is a shape parameter, it governs the tail behavior of the distribution, a larger ξ implies fatter tails.

The parameters β and ξ are estimated by using the Maximum Likelihood (ML) function. The calculation goes by taking the Logs of the respective probability

density functions and summing over m observations that are larger than the threshold u , it then follows that the corresponding log-likelihood functions are written as follows:

$$\text{Log}L(\beta, \xi) = -mLN\beta - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^m LN\left(1 + \xi \frac{\ell_u^i - u}{\beta}\right) \quad (11)$$

$$\text{Log}L(\beta) = -mLN\beta - \frac{1}{\beta} \sum_{i=1}^m (\ell_u^i - u) \quad (12)$$

The ML estimates of the parameters are obtained by maximizing the log-likelihood functions with a maximizing algorithm, then solving for VaR_α will give a Value-at-Risk estimate based on POT Extreme Value Theory.

This is where the two branches of the POT model split, deriving the equations to solve for VaR_α will result in an estimate of the Unconditional POT model. The other branch is the Conditional Peaks over Threshold EVT (Conditional POT) model. This method accounts for time-varying volatility, which makes the POT model more dynamic and allows to account for the current market conditions. Which is why this paper will focus only on the Conditional POT.

The Conditional POT has some more steps to it, first, the standardized loss residuals are applied to the POT analysis, and then it is combined with a GARCH or an EWMA volatility model.

The standardized residuals which to apply to the POT analysis are defined as follows:

$$\begin{aligned}
\varepsilon_1^* &= \frac{\ell_1 - \bar{\ell}}{\sigma_1} \\
\varepsilon_2^* &= \frac{\ell_2 - \bar{\ell}}{\sigma_2} \\
&\cdot \\
&\cdot \\
&\cdot \\
\varepsilon_T^* &= \frac{\ell_T - \bar{\ell}}{\sigma_T}
\end{aligned} \tag{13}$$

In this method, q_α , which is the α -quantile of the Generalized Pareto Distribution (GPD) of the standardized residuals, is used to calculate VaR_α :

$$VaR_\alpha = \mu + \sigma_{T+1} q_\alpha \tag{14}$$

The α -quantiles, q_α , for the standardized residuals based on the POT model can then be calculated using the following mathematical expressions:

$$q_\alpha = u^* + \frac{\beta^*}{\xi^*} \left[\left(\frac{N}{N_{u^*}} (1 - \alpha) \right)^{-\xi^*} - 1 \right] \quad \text{if } \xi^* \neq 0 \tag{15}$$

$$q_\alpha = u^* - \beta^* LN \left(\frac{N}{N_{u^*}} (1 - \alpha) \right) \quad \text{if } \xi^* = 0 \tag{16}$$

The parameters with the star-notation indicate that they are not the original parameters, but GPD parameters. The $\bar{\ell}$ parameter is simply the sample averages of the observed losses. The only thing left to be able to use the model and to estimate VaR_α , is the volatility estimate, σ_{T+1} , one day out of sample, this can of course be calculated with one of the GARCH models or the EWMA model.

3.5 GARCH Model

Finally, with all the above theorems defined, the only parameter missing to be able to forecast VaR_α under different assumptions are the volatilities of the data. Since volatility is not directly observable, a time-series model will be needed. The Generalized Autoregressive Conditional Heteroscecasticity (GARCH) model will be used, it allows to account for time-varying conditional volatility. There are several variations of this model, which means that several GARCH models will be analyzed in this paper with different specifications, to establish which one will best fit the dataset.

The GARCH (p,q) model introduced by Bollerslev (1986), using the notation of the original paper is expressed as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 \quad (17)$$

Where ϵ_{t-i}^2 represents the squared residuals and σ_{t-i}^2 represents the variance from previous time periods. The p parameter is the order of the GARCH terms σ^2 and q is the order of the ARCH terms ϵ^2 .

The standard GARCH (1,1) model can be expressed as:

$$\sigma_{T+1}^2 = \omega + \alpha_1 \epsilon_T^2 + \beta_1 \sigma_T^2 \quad (18)$$

The σ_{T+1}^2 is the next holding period, one holding period out of sample variance. The α and β terms are estimated by Maximum likelihood (ML) and these parameter estimates can be directly used in the GARCH recursions. The expression above can therefore be used to forecast the one period ahead volatility, which is used to estimate the one period ahead forecasted VaR_α .

A very popular alternative to the GARCH model to estimate volatility that is worth mentioning, is the Exponentially Weighted Moving Average (EWMA):

$$\sigma_{T+1}^2 = (1 - \lambda)\epsilon_T^2 + \lambda\sigma_T^2 \quad (19)$$

Where λ is a fixed constant, according to RiskMetrics, the optimal value for daily data is $\lambda = 0.94$, of course it can be easily adjusted, the higher the λ the slower the decay, the lower the λ the faster the decay. It shows that EWMA can be interpreted as a special case of the GARCH model. The reasons why it is a popular approach is that EWMA takes the stylized fact of volatility clustering directly into account, puts a greater weight upon more recent observations, and it is convenient as it bypasses the parameters estimation issue, which means that compared to GARCH, it is easier to implement.

This means that obviously both models capture the volatility-clustering phenomenon, however, there is of course a drawback to the simplicity of the EWMA model. Indeed, the λ is a very arbitrary decay factor, which introduces subjectivity into the estimation, thus making the model weak and limited with focus on explanatory power. Those are the reasons why in this paper, the GARCH models will be used for the estimation of data volatility.

The GARCH model represents a combination of past squared residuals and past variances to estimate the out of sample variance, which means that it is a symmetrical model. In other words, due to the quadratic past residuals, the impact of positive and negative shocks in the returns is equivalent for both. This is of course a problem, as it can be seen empirically that there is an asymmetry in how volatility reacts to different types of shocks. To account for this, the symmetrical GARCH model will be compared against two other important asymmetrical models to see which one has the best fit for the data analyzed.

3.6 Exponential GARCH Model

The Standard GARCH assumes that positive and negative error terms have a symmetric effect on volatility, but in practice this assumption is frequently violated. In the financial market, and this includes the energy market, it is easy to see that volatility of returns increase more after bad news than after good news. This principal is called the *Leverage Effect*, firstly mentioned by Black (1976). It states that when asset prices fall, the debt-to-equity ratio will increase, which leads to a higher volatility. Empirically it has been proven that the volatility reacts asymmetrically to the sign of the shocks, thus resulting in a number of parameterized extensions of the regular GARCH model. The Exponential GARCH (EGARCH) is one of the two most important asymmetrical models.

The EGARCH is a model specified by Nelson (1991), it is given by the following expression:

$$\text{LOG} \sigma_t^2 = \omega + \sum_{k=1}^q \alpha_k g(Z_{t-k}) + \sum_{k=1}^p \beta_k \text{LOG} \sigma_{t-k}^2 \quad (20)$$

The terms ω_t and β_k are deterministic coefficients, where:

$$g(Z_t) = \theta Z_t + \gamma(|Z_t| - E(|Z_t|)) \quad (21)$$

The parameters $\omega, \alpha_k, \beta_k, \theta$ and γ are coefficients; σ_t^2 is the conditional variance. The expression $g(Z_t)$ represents the sign and the magnitude, and it allows Z_t to have asymmetrical effects on the volatility. It is particularly useful for financial instruments, and since the EGARCH model reflects this asymmetry, more reliable estimates could be obtained compared to using the standard GARCH. Another positive point is that there is no nonnegative limitation in the EGARCH model, the

logarithmic form makes sure that the variance will be positive, thus there are no parameter restrictions in the model.

3.7 Threshold GARCH Model

The second most important asymmetrical model is the Threshold GARCH (TGARCH), it is based on the principal that shocks greater than the threshold have different effects than shocks below the threshold. The threshold value will be the point where the market is not affected neither by good news, nor by bad news, it is the optimal value that the debt-to-equity ratio will take based on the *Leverage Effect* theorem.

The TGARCH uses the conditional standard deviation for estimation instead of the usual conditional variance as seen before, therefore, since the term residuals are not squared, they can take positive or negative values, thus the model captures all the asymmetrical effects on volatility.

The TGARCH model is a model created by Zakoian (1994), it can be expressed as follows:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha_1^+ \epsilon_{t-1}^+ + \alpha_1^- \epsilon_{t-1}^- \quad (22)$$

Where $\epsilon_{t-1}^+ = \epsilon_{t-1}$ if $\epsilon_{t-1} > 0$, and $\epsilon_{t-1}^+ = 0$ if $\epsilon_{t-1} \leq 0$. Likewise, $\epsilon_{t-1}^- = \epsilon_{t-1}$ if $\epsilon_{t-1} \leq 0$ and $\epsilon_{t-1}^- = 0$ if $\epsilon_{t-1} > 0$.

The difference between the TGARCH and the EGARCH is that the TGARCH has a more relaxed structure for the lags. In the model, the asymmetric effects can be different for each lag, which is not the case for EGARCH.

3.8 Backtesting

For the last step, to ensure the validity of the models and to be able to compare them, backtesting will be used. The standard Kupiec test will be applied, which is a standard frequency test that compares the frequency of the number of observed *VaR* violations with the expected or predicted frequency of *VaR* violations during the test period. A *VaR* violation occurs when the loss observed for a given day exceeds the Value-at-Risk estimated.

The Kupiec test is a binomial test; it can either be a violation, either a non-violation. A violation is coded as 1, and a non-violation is coded as 0. A stochastic variable that can take only those two values is called a Bernoulli variable. From a statistics point of view, the sum of Bernoulli stochastic variables is a binomially distributed variable.

If the number of observations for a test period is denoted as N observations and the expected frequency of *VaR* violations is denoted as $p = 1 - \alpha$, then the following probability formula can be used to estimate the probability of observing exactly X , where X is the number of violations:

$$\Pr(X = x) = \binom{N}{x} p^x (1 - p)^{N-x} \quad (23)$$

Cumulative probabilities can be calculated as shown below:

$$\Pr(X \leq x) = \sum_{i=0}^x \binom{N}{i} p^i (1 - p)^{N-i} \quad (24)$$

Cumulative probabilities can be calculated using the above formula. The Kupiec test consists of a one-sided or a two-sided test, for the two-sided test, the cumulative probability formula is used. It requires the construction of a confidence interval for

the observed frequency violations, if the actual number of violations falls outside the confidence interval, then the model is rejected, which means it is not a valid model.

The confidence interval is defined by the lower bound x_{low} and the upper bound x_{high} , these bounds have an equal probability of observing fewer than x_{low} and observing more than x_{high} number of violations. The standard test can be estimated with a 95% confidence interval. This test can give an overall say about the validity and the performance of the *VaR* estimates resulted from the models described above.

4. EMPIRICAL RESULTS

4.1 Data Analysis

In this paper, daily data of the overall price level of four commodities, from January 1, 2010, to December 31 2016 is used. It is taken from the U.S Energy Information Administration (EIA). The data sources are as follows, WTI Crude oil Spot Price FOB (Dollars per barrel), Conventional Gasoline Regular Spot Price FOB (Dollars per Gallon), Henry Hub Natural Gas Spot Price (Dollars per Million BTU), and finally International Coal Exchange (Dollar per Metric Tonne).

The data will be analyzed and the tests will be performed using Eviews and Excel as the main data analysis tools. The unit root test will be performed to test if the data is stationary and possess a unit root. The Values-at Risk will be calculated with VWHS, Student's t-distribution, and EVT conditional POT models, each with three different GARCH-type models as volatility parameters. The regular GARCH (1.1), the EGARCH and the TGARCH models will be applied. Finally at the end, to verify the accuracy of the data, the Kupiec test will be applied to test the statistical significance of the empirical results obtained.

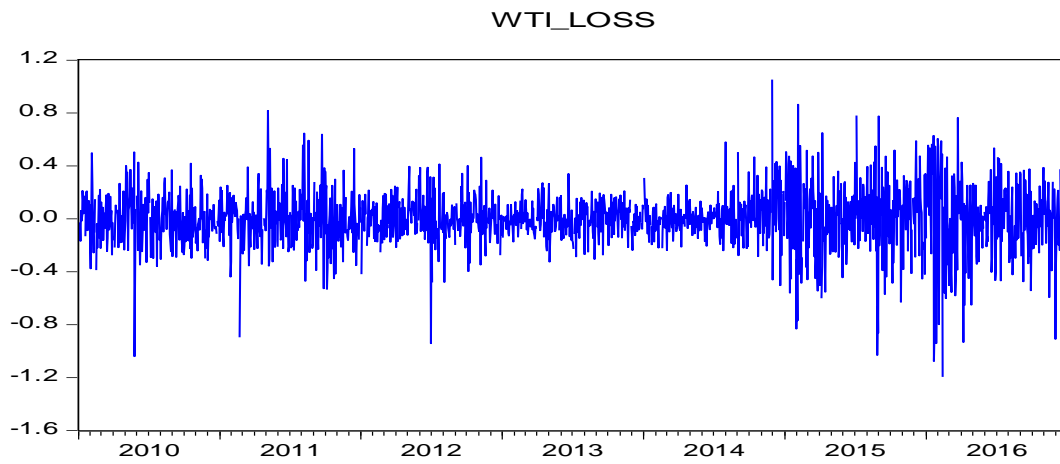


Figure 2. Daily Loss of WTI Crude oil

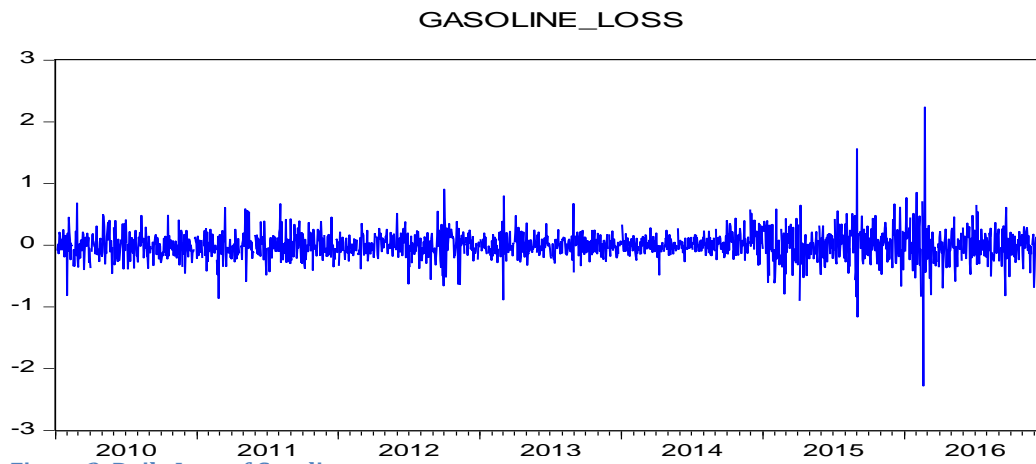


Figure 3. Daily Loss of Gasoline

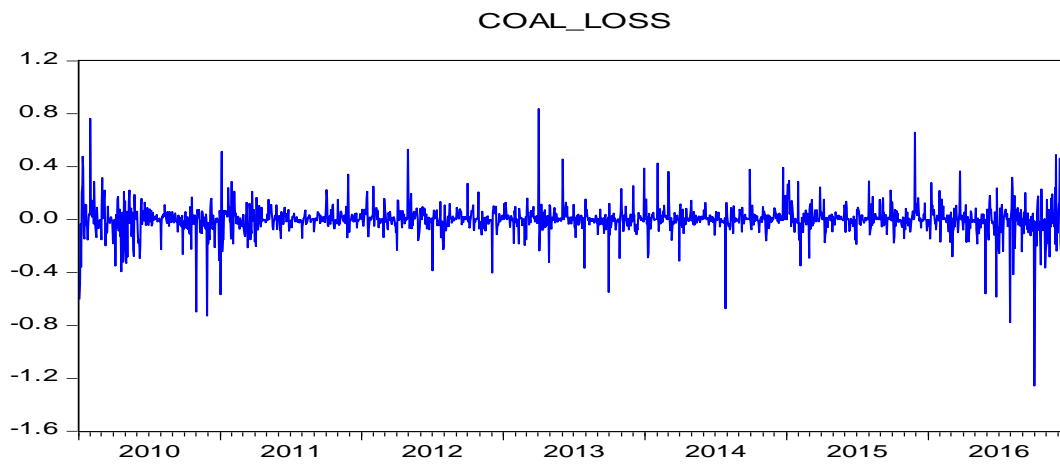


Figure 4. Daily Loss of International Coal Exchange

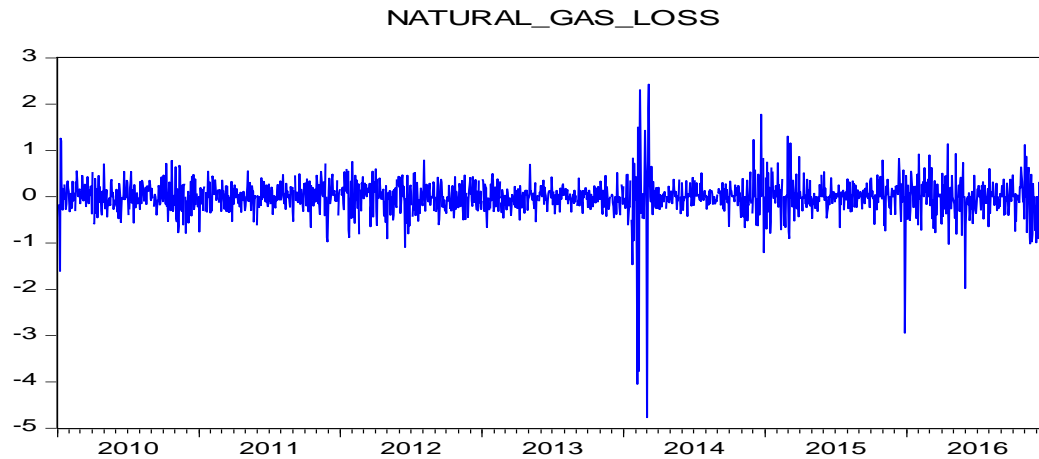


Figure 5. Daily Loss of Henry Hub Natural Gas

The above graphs represent returns transformed into losses by assuming a portfolio with a value of 1000. It can clearly be seen that the fluctuations presented by those graphs are clustered. If volatility is higher (lower) at a certain period of time, it will be higher (lower) in the following period as well, this effect can be observed in the underlying data. The phenomenon is called volatility clustering.

	Estimation Period	# of Obs.	Test Period	# of Obs.
Sample 1	1.01.2010 31.12.2013	1043	1.01.2014 31.12.2014	261
Sample 2	3.01.2011 31.12.2014	1043	2.01.2015 31.12.2015	260
Sample 3	2.01.2012 31.12.2015	1044	1.01.2016 31.12.2016	260

Table 1. Data

As shown in the table above, a rolling 4 years of data is used to calculate each 1-year out of sample for the last 3 years, 2014, 2015 and 2016. This is to ensure the accuracy of the model and for better estimation results. Those particular dates have been chosen because it represents a time frame with an increased volatility.

It is not a period of crisis, but yet the volatility is still very high compared to other previous stable periods. This makes the topic very interesting to analyze, and there has not been many researches since the losses are not dramatic, however it is still alarming. Volatility is at high levels and seems to be persistent as there is mismatch with supply and demand of the aforementioned commodities.

	WTI Crude Oil Loss	Gasoline Loss
Sample	1.01.2010 to 12.30.2016	1.01.2010 to 12.30.2016
Number of observations	1826	1826
Mean	-0.000136	-0.001561
Median	0.000000	0.000000
Maximum	1.052917	2.240605
Minimum	-1.195113	-2.280355
Std. Dev.	0.213573	0.223816
Skewness	-0.336468	0.010242
Kurtosis	6.422645	16.77193
Jarque-Bera	925.7320	14430.46

Table 2. Descriptive Statistics of Daily Loss of WTI Crude Oil and Gasoline

	Coal Loss	Natural Gas Loss
Sample	1.01.2010 to 12.30.2016	1.01.2010 to 12.30.2016
Number of observations	1826	1826
Mean	-0.000809	-0.003699
Median	0.000000	0.000000
Maximum	0,837237	2.367601
Minimum	-1.256961	-4.595745
Std. Dev.	0.112251	0.359527
Skewness	-1.093179	-2.350900
Kurtosis	22.43795	34.88127
Jarque-Bera	29110.54	79014.25

Table 3. Descriptive Statistics of Daily Loss of International Coal Exchange and Natural Gas

As it can be seen from Tables 2 and 3, the values of skewness are not 0 and the kurtosis values are all larger than 3, therefore all the commodities indicate a non-normal distribution. As mentioned before, financial data has fat-tails and presents asymmetry, which means that the Student t-distribution, the Generalized Error Distribution (GED) or any other fat-tailed distribution is preferred to the normal distribution. In this paper, the Student t-distribution is applied throughout all of the empirical calculations.

4.2 Unit Root

The unit root test is a test to see whether a time series is non-stationary and possess a unit root or not. Following the Augmented Dickey-Fuller test, the null hypothesis is defined as the presence of a unit root, and the alternative is stationarity, trend stationarity or explosive root. Since energy commodity prices are affected by many factors, it means that the series are non-stationary, however once transformed into losses the data then becomes stationary. The results of the Dickey-Fuller test are shown in the Figures 6 to 9 in the Appendix. As it can be seen from the tests, all of the underlying datasets reject the null, which means that the data is stationary and therefore can be used for the purpose of the empirical research presented.

4.3 GARCH Estimates

Throughout the empirical research, all the models for *VaR* calculations use three different estimates of volatility, GARCH (1.1), TGARCH, and EGARCH. The reason behind this is to see the impact that those three different volatility estimates have on *VaR*, and decide which one produces the best results.

Eviews has been used to generate the different GARCH models' parameters, which have then been transposed to Excel to calculate the volatility. All the models have been calculated using the Student's t-distribution.

In Figure 10, presented below, can be seen a graphical representation of the volatility estimates of the three different GARCH models. The ordinary GARCH model has much higher estimates of volatility over time since it doesn't account for asymmetries. This makes the TGARCH and the EGARCH models much smoother and faster to adapt to volatility changes.

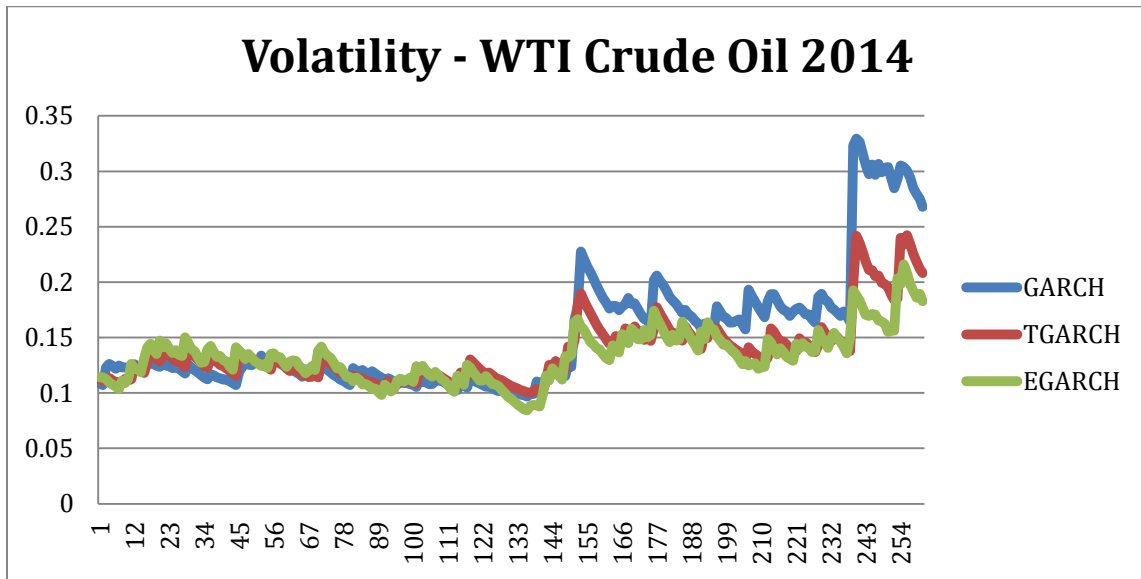


Figure 10. GARCH Volatilities WTI Crude Oil 2014

The Figure above represents the 2014 out of sample volatility for WTI crude oil. For further analysis, Figures 11 to 21 are shown in the Appendix as visual representations of all the GARCH models for each out of sample year for the four commodities analyzed.

As it can be seen, the symmetrical ordinary GARCH model produces higher volatilities. The TGARCH and EGARCH models put different values on positive and negative shocks, which produces lower estimates and makes them more adaptable to different circumstances. However, an important consideration is the in-sample volatility compared to the out of sample volatility. For example, if the in-sample data has low volatility and for some unforeseen factors, the out of sample data has high volatility, the symmetrical GARCH will perform better.

Those are special circumstances, the out of sample period being a period of crisis. Jánský and Rippel (2011) performed similar research. They tested GARCH, EGARCH, and TARCH on six stock indices with a time period from 2004 to 2009, which included the 2008 crash.

Their conclusion is that the symmetrical GARCH model performs better in times of increased volatility. However, under other circumstances, the research suggests that the EGARCH model produces the most adequate results. Such was the research of Angelidis, Benos and Degiannakis (2004), in their paper, five stock indices were analyzed for modeling *VaR*, and their conclusion was that the EGARCH model generates the most adequate *VaR* forecasts for the majority of the markets.

In this paper the period analyzed is a post-crash period, the volatility should not be of the highest, therefore the more adequate model of volatility is expected to be the EGARCH model.

4.4 Backtesting

To be able to establish which of the models perform the best, the Kupiec test will be used. Since the one-sided and the two-sided Kupiec tests result in the same conclusion, the two-sided Kupiec test will be used in this paper for simplicity reasons.

The lower and higher bounds are represented as integers, thus making it easier to visualize if the different *VaR* models should be rejected or not. The upper and lower bounds of the test are the maximum and minimum allowed number of *VaR* violations for every model.

It is calculated in excel and the result for the lower bound is, $x_{low} = 0$, and the higher bound, $x_{high} = 8$.

For more accurate results, a level of significance of 99% has been used to obtain the above-mentioned confidence interval. In other words, if the model produces more than 8 violations, then it means that it is not a good *VaR* predictor and is therefore rejected.

4.5 Volatility Weighted Historical Simulation

The volatility Weighted Historical Simulation is a time-series that applies basic Historical Simulation to a rescaled sample of observed losses. To construct the scaled losses, it means that a new volatility estimate has to be forecasted for each day in the test period, however, this is implausible. It will give too many estimates to be feasible. Therefore, the losses have been rescaled to calculate a new σ_{T+1} for every month, giving a total of 12 series of rescaled losses per sample. This means that the volatility is updated once a month instead of everyday.

This will cause the model to be less reflective of new market behaviors. Hence, it is expected not to produce the best results compared to the other models.

The *VaR* violations results are shown in the table below:

WTI Crude Oil	VWHS-GARCH	VWHS-TGARCH	VWHS- EGARCH
Sample 1	3	4	5
Sample 2	4	5	10
Sample 3	8	6	6
Gasoline	VWHS-GARCH	VWHS-TGARCH	VWHS- EGARCH
Sample 1	1	1	1
Sample 2	9	8	13
Sample 3	9	7	10

Natural Gas	VWHS-GARCH	VWHS-TGARCH	VWHS- EGARCH
Sample 1	9	9	10
Sample 2	3	3	8
Sample 3	6	7	16
Coal	VWHS-GARCH	VWHS-TGARCH	VWHS- EGARCH
Sample 1	2	2	2
Sample 2	2	2	3
Sample 3	5	3	0

Table 4. Number of VaR Violations under VWHS

It can be seen that the model produces too many violations for most of the years and also for most of the commodities. Strangely coal has no violations that exceed the upper bound, this means that the VWHS can be used as an adequate predictor of *VaR* for the coal commodity.

The other surprising result is that the EGARCH process generates the most violations of all.

4.6 Student t-Distribution

The Student's t-distribution has been chosen in this paper as it allows for fat-tails and kurtosis above 3. It is a common fact that financial data presents those characteristics.

The Student's t-distribution is a very popular parametrical approach. Used together with the three GARCH processes, the t-distribution will be able to account for time varying volatility as well as heavy-tails.

The following table shows the results of the t-distribution approach:

WTI Crude Oil	t-Dist. GARCH	t-Dist. TGARCH	t-Dist. EGARCH
Sample 1	0	0	0
Sample 2	1	4	2
Sample 3	3	3	3
Gasoline	t-Dist. GARCH	t-Dist. TGARCH	t-Dist. EGARCH
Sample 1	1	1	0
Sample 2	5	4	1
Sample 3	4	3	2
Natural Gas	t-Dist. GARCH	t-Dist. TGARCH	t-Dist. EGARCH
Sample 1	6	6	5
Sample 2	1	2	1
Sample 3	1	2	0
Coal	t-Dist. GARCH	t-Dist. TGARCH	t-Dist. EGARCH
Sample 1	3	3	0
Sample 2	2	2	0
Sample 3	5	0	0

Table 5. Number of VaR Violations under the t-Distribution

The results with this model are very satisfying, the model works very well for all the above commodities. All the violations are within the confidence interval established. The EGARCH volatility parameter produces a number of violations that are closer to the expected number of *VaR* violations ($260 * 0.01 = 2.6$). Therefore it is reasonable to assume that the EGARCH volatility estimate has the best fit.

This is a very robust and stable approach to *VaR* calculations, it is also a less laborious model to construct. There is no need for rescaling or maximizing variable parameters, which makes it an extremely good approach to use for risk management.

4.7 Extreme Value Theory

The EVT conditional POT approach is the most complex *VaR* model of all the models applied in this paper. The construction requires many additional steps. There are two branches to the POT model, the unconditional POT and the conditional POT. The former one is more dynamic because it takes into account the current market conditions.

The conditional POT is applied to rescaled losses or standardized loss residuals together with the GARCH volatility models. Usually the POT is used to estimate VaR_α at very high confidence levels.

Echaust and Just (2013) compare in their paper conditional and unconditional models using backtesting procedures. Their conclusion is that the conditional models perform much better than the unconditional models.

Many previous research support this conclusion as well, hence only the conditional POT model will be analyzed in this paper.

The next step after the estimation of the residuals is the choice of the threshold, u . Beirlant, Teugels and Vynckier (1996) propose different methods of estimating u , however as a general concept in this paper, 4% of the original loss residuals are regarded as extreme. The choice of u of 4 % is because it is not too small and not too big as a threshold. For 1043 in-sample observations, the 41th largest residual loss will be the threshold value. The 40 observations above the threshold represent m , and will be used to calculate β and ξ .

The parameters β and ξ can be estimated with the Maximum Likelihood (ML) function, the equations used are displayed in the EVT part in the Methodology section, and are solved by maximizing the Log-Likelihood.

	β^*	$\xi^* \neq 0$	u^*	$\frac{N}{N_{u^*}}$	β^*	$\xi^* = 0$
WTI Crude oil						
Sample 1	0.3923	0.3562	1.7248	24.9521	0.5992	0
Sample 2	0.5479	0.1379	1.6416	24.9521	0.6356	0
Sample 3	0.8020	-0.0315	1.7002	24.9521	0.7775	0
Gasoline						
Sample 1	0.5278	0.1251	1.7014	24.9521	0.6006	0
Sample 2	0.7763	0	1.5398	24.9521	0.7772	0
Sample 3	0.8112	0	1.6021	24.9521	0.8112	0
Natural Gas						
Sample 1	0.5726	0	1.7750	24.9521	0.5711	0
Sample 2	0.5369	0.5325	1.2965	24.9521	1.0469	0
Sample 3	0.5934	0.5380	1.2906	24.9521	1.1402	0
Coal						
Sample 1	1.0129	0.1065	1.6852	24.9521	1.1301	0
Sample 2	0.7799	0.2564	1.4731	24.9521	1.0307	0
Sample 3	0.9350	0.1713	1.2698	24.9521	1.1199	0

Table 6. EVT POT Parameter Estimates

The table above shows the estimation results of all the parameters required. The β represents a scale parameter and ξ is a shape parameter. It can be seen that two situations occur, one where $\xi^* = 0$ and one where $\xi^* \neq 0$. If $\xi^* > 0$, it means that the distribution has fat tails and is called a Frechet Distribution. As mentioned before, financial data usually has fat tails.

However, in some situations ξ^* tend to 0, this is called a Gumbel Distribution. In one case, $\xi^* < 0$, this is a Weibull Distribution, it means that the tails are thinner than the normal distribution and it is very uncommon.

Now that all the parameters have been calculated, only the α -quantiles, q_α , are left to calculate and be able to finally estimate VaR_α . The mathematical equations for the quantiles are shown in the Methodology part of the paper.

Finally, the following table displays the EVT POT VaR violations obtained:

	EVT POT $\xi^* \neq 0$			EVT POT $\xi^* = 0$		
WTI Crude Oil	GARCH	TGARCH	EGARCH	GARCH	TGARCH	EGARCH
Sample 1	0	1	0	0	1	1
Sample 2	2	4	2	1	4	4
Sample 3	2	3	2	2	3	3
Gasoline	GARCH	TGARCH	EGARCH	GARCH	TGARCH	EGARCH
Sample 1	1	1	0	1	1	1
Sample 2	3	4	1	3	4	4
Sample 3	4	3	2	4	3	3
Natural Gas	GARCH	TGARCH	EGARCH	GARCH	TGARCH	EGARCH
Sample 1	6	6	5	6	6	6
Sample 2	4	2	1	1	2	2
Sample 3	2	2	0	1	1	1
Coal	GARCH	TGARCH	EGARCH	GARCH	TGARCH	EGARCH
Sample 1	1	1	0	1	1	1
Sample 2	2	2	0	2	2	2
Sample 3	5	0	0	5	0	0

Table 7. Number of VaR Violations under EVT

Once again the model is great, no VaR violations above the upper bound of the Kupiec test. The EVT POT both with $\xi^* \neq 0$ and $\xi^* = 0$ seem to have performed slightly better overall with EGARCH for volatility parameter, as it shows a number of violations closer to the expected number of violations.

4.8 Overall Results

It is difficult to decide which method is the most appropriate to implement, most of the models analyzed are adequate. It is no surprise that the results are good, since the choice of the models is based on previous studies.

That being said, the analysis shows that the EGARCH volatility process generates a good number of violations for any model. This means that the EGARCH has the best estimates of volatility for all the commodities analyzed during the time period.

Also, the parametrical Student's t-distribution with EGARCH produces almost the same number of violations as the EVT conditional POT $\xi^* \neq 0$ with EGARCH. Both models are almost identical and are both great predictors of *VaR* estimates.

However, the EVT POT model presents a limitation, the choice of the threshold, u , is very subjective. For this paper, a constant of 4% of the original losses has been selected. This is the case because as mentioned in the Methodology section, there is a trade-off between a large u and a large m . Hence, the 4% constant, it is not too low, and not too high, this is a very standard way to deal with this issue.

A different threshold selection might improve the results and even surpass the t-distribution model, but the selection of this threshold is very complex and subjective.

Therefore, from an economically resounding point of view and for simplicity reasons, the Student's t-distribution with EGARCH would definitely be the preferred model to use for risk management purposes.

5. CONCLUSION

The main purpose of this paper is to provide an adequate empirical research on Value-at-Risk and Extreme Value models in light of the recent new market conditions that are causing high volatility in the energy sector.

The results obtained coincide with the general expectations, the GARCH, TGARCH, and EGARCH processes perform well for energy commodities. The EGARCH with the Student's t-distribution seems to be performing the best. It is consistent with the nature of the data, since based on analysis it can be seen that the data does not follow a normal distribution and presents heavy-tails. Therefore, the GARCH models with the Student's t-distribution are the most reasonable choices for such time series.

The results show that the Volatility Weighted Historical Simulation (VWHS) models underperform the other *VaR* models in most cases. This is due to the scaling of the losses, if the losses were rescaled for shorter time periods, such as weekly periods, it would translate into a faster volatility update for the dataset and the results could improve drastically. The problem with this method is that it requires a considerable amount of time and calculations.

The EVT conditional POT model and the Student's t-distribution present the best results, they both describe the loss data adequately and both models are sound risk management tools to be used for the energy commodities analyzed.

There are no significant differences between the $\xi^* \neq 0$ and $\xi^* = 0$ cases of the EVT conditional POT model. It works well in most of the commodities with the three GARCH models, except for some cases with coal. The *VaR*'s generated with the EGARCH model seem to be unnecessarily high to avoid violations.

This has happened due to the in-sample dataset, there seems to be very high persisting volatility causing the EGARCH model to adapt, thus generating a higher *VaR* for all the subsequent out of sample periods. This means that for coal, it might not be the method of choice to use, or perhaps a different in-sample dataset will suffice to reduce the gap of the *VaR* estimates.

In the Appendix, Figures 22 to 45 show all the *VaR* estimations obtained in a graphical representation. The *VaR* behaviors can be assessed and interpreted visually for a more convenient outlook of the empirical results.

The research provided in this paper is related to previous studies on Value-at-Risk for energy commodities with different fat-tailed GARCH models. The results obtained in those previous studies are similar to the results obtained here. The difference is that this paper provides a more recent time period analysis, a post crisis period, which is more relevant to the current economic situation.

Overall, the Student's t-distribution with the EGARCH volatility process, and the EVT conditional POT model, again with the EGARCH volatility process, provide both the most reliable estimates of *VaR* for Crude Oil, gasoline, natural gas, and coal commodities. Both methods are very reliable risk management tools that can be used as good *VaR* predictors. A preference is however pointed out towards the parametrical approach with the Student's t-distribution, as it is easier and simpler to implement.

References

Aloui, C., and Mabrouk, S. (2010). Value-at-Risk Estimations of Energy Commodities via Long-Memory, Asymmetry and Fat-Tailed GARCH models. *Energy Policy* 38, pp. 23026-2339.

Angelidis, T., Benos, A., and Degiannakis S. (2004). The Use of GARCH Models in VaR Estimation. *Statistical Methodology*, Vol.1, No. 2, pp. 105-128.

Balkema, A., and de Haan, L. (1974). Residual Life Time at Great Age. *Annals of Probability*, 2, pp. 792-804.

Beirlant, J., Teugels, J., and Vynckier, P. (1996). Practical Analysis of Extreme Values. Leuven University Press. Mathematics and Economics, vol. 21, issue 1, pp. 91-92.

Black F. (1976). Studies in Stock Price Volatility Changes, Meeting of the Business and Economic Statistics Section, *American Statistical Association*, pp. 177-181.

Bollerslev, Tim (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* 31, pp. 307-327.

Borenstein S., Cameron A., and Gilbert R. (1997). Do Gasoline Prices Respond Asymmetrically to Crude Oil Price Changes. *The Quarterly Journal of Economics*.

Brendan O. Bradley and Murad S. Taqqu (2001). Financial Risk and Heavy Tails. Boston University.

Echaust, K., Just, M. (2013). Conditional versus Unconditional Models for VaR Measurement. *SSRN Electronic Journal*. Poznan University of Economics.

Engle, Robert F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, Vol. 50, Issue 4, pp. 987-1008.

European Commission (2017), 2050 Low-Carbon Economy – Climate Action. Available from: https://ec.europa.eu/clima/policies/strategies/2050_en [Accessed May 19, 2017]

Hung, J., Lee, M., and Liu H. (2008). Estimation of Value-at-Risk for Energy Commodities via Fat-Tailed GARCH Models. *Energy Economics* 30, pp. 1173-1191.

International Energy Agency (2016). Key World Energy Statistics.

Jánský, Ivo, and Rippel, Milan (2011). Value at Risk Forecasting with the ARMA-GARCH Family of Models in Times of Increasing Volatility. *Institute of Economic Studies*. Charles university, IES Working paper: 27/2011.

Kevin Dowd (2002). Measuring Market Risk. *John Wiley & Sons*.

Kupiec, P. (1995). Techniques for Verifying the Accuracy of Risk Management Models. *Journal of Derivatives*, Vol. 3, pp. 73-84.

Marimoutou, V., Raggad, B., and Trabelsi, A. (2009). Extreme Value Theory and Value at Risk: Application to Oil Market. *Energy Economics*, vol. 31, issue 4, pp. 519-530.

McKain, David L., and Bernard L. Allen (1994). Where it all began: The Story of the People and Places Where the Oil Industry Began.

McNeil, A. (1999). Extreme Value Theory for Risk Managers. *In international Modelling and CAD II*, Risk Waters Group, London, pp. 93-113

Nelson, Daniel B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, Vol. 59, No.2, pp. 347-370.

Pickands, J. (1975). Statistical Inference Using Extreme Order Statistics. *Annals of Statistics*, 3, pp. 119-131.

Rabemananjara R., and Zakoian J. M. (1993). Threshold ARCH Models and Asymmetries in Volatility. *Journal of Applied Econometrics* 8, pp. 31-49.

U.S. Energy Information Administration (2017). Monthly Energy Review. Available from <https://www.eia.gov/totalenergy/data/monthly> [Accessed May 1, 2017]

Werner Dabelstein, Arno Reglitzky, Andrea Schütze and Klaus Reders (2007). Automotive Fuels in Ullmann's Encyclopedia of Industrial Chemistry.

Zakoian, Jean-Michel (1994). Threshold Heteroskedastic Models. *Journal of Economic Dynamics and Control*, Volume 18, Issue 5, pp. 931-955.

APPENDIX

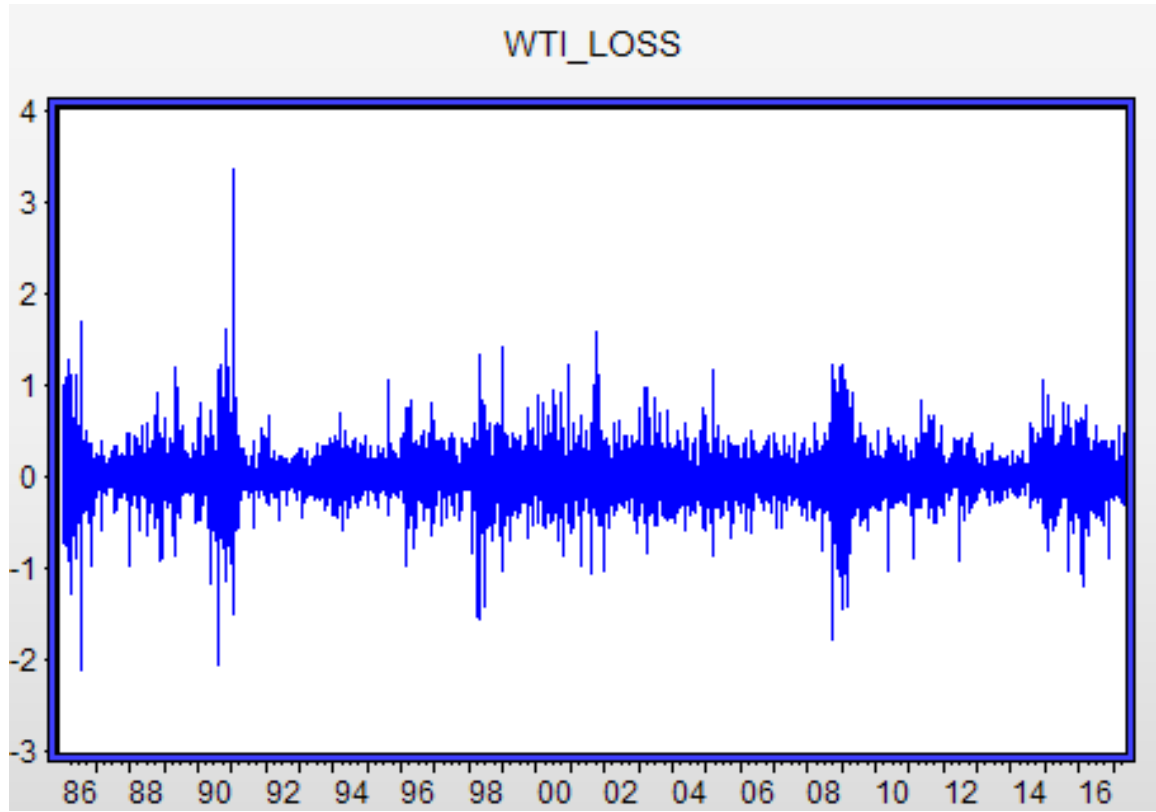


Figure 1. WTI Crude Oil long-term loss

Null Hypothesis: WTI_LOSS has a unit root
 Exogenous: Constant
 Lag Length: 0 (Automatic - based on SIC, maxlag=24)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-44.67425	0.0001
Test critical values:		
1% level	-3.433732	
5% level	-2.862920	
10% level	-2.567552	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
 Dependent Variable: D(WTI_LOSS)
 Method: Least Squares
 Date: 05/20/17 Time: 11:23
 Sample (adjusted): 1/04/2010 12/30/2016
 Included observations: 1825 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
WTI_LOSS(-1)	-1.045248	0.023397	-44.67425	0.0000
C	-0.000142	0.004997	-0.028455	0.9773
R-squared	0.522624	Mean dependent var		5.09E-06
Adjusted R-squared	0.522362	S.D. dependent var		0.308880
S.E. of regression	0.213471	Akaike info criterion		-0.249534
Sum squared resid	83.07407	Schwarz criterion		-0.243496
Log likelihood	229.6995	Hannan-Quinn criter.		-0.247307
F-statistic	1995.789	Durbin-Watson stat		2.001741
Prob(F-statistic)	0.000000			

Figure 6. Unit Root test WTI Crude oil

Null Hypothesis: GASOLINE_LOSS has a unit root
 Exogenous: Constant
 Lag Length: 0 (Automatic - based on SIC, maxlag=24)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-44.81978	0.0001
Test critical values:		
1% level	-3.433732	
5% level	-2.862920	
10% level	-2.567552	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
 Dependent Variable: D(GASOLINE_LOSS)
 Method: Least Squares
 Date: 05/20/17 Time: 11:26
 Sample (adjusted): 1/04/2010 12/30/2016
 Included observations: 1825 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GASOLINE_LOSS(-1)	-1.048502	0.023394	-44.81978	0.0000
C	-0.001639	0.005236	-0.313052	0.7543
R-squared	0.524246	Mean dependent var		2.28E-05
Adjusted R-squared	0.523985	S.D. dependent var		0.324195
S.E. of regression	0.223675	Akaike info criterion		-0.156152
Sum squared resid	91.20547	Schwarz criterion		-0.150114
Log likelihood	144.4883	Hannan-Quinn criter.		-0.153924
F-statistic	2008.813	Durbin-Watson stat		2.003458
Prob(F-statistic)	0.000000			

Figure 7. Unit Root Test Gasoline

Null Hypothesis: COAL_LOSS has a unit root
 Exogenous: Constant
 Lag Length: 0 (Automatic - based on SIC, maxlag=24)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-37.27878	0.0000
Test critical values:		
1% level	-3.433732	
5% level	-2.862920	
10% level	-2.567552	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
 Dependent Variable: D(COAL_LOSS)
 Method: Least Squares
 Date: 05/20/17 Time: 11:26
 Sample (adjusted): 1/04/2010 12/30/2016
 Included observations: 1825 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COAL_LOSS(-1)	-0.869730	0.023330	-37.27878	0.0000
C	-0.000671	0.002607	-0.257468	0.7968
R-squared	0.432566	Mean dependent var		0.000251
Adjusted R-squared	0.432255	S.D. dependent var		0.147798
S.E. of regression	0.111364	Akaike info criterion		-1.550932
Sum squared resid	22.60869	Schwarz criterion		-1.544895
Log likelihood	1417.226	Hannan-Quinn criter.		-1.548705
F-statistic	1389.708	Durbin-Watson stat		1.975090
Prob(F-statistic)	0.000000			

Figure 8. Unit Root Test International Coal Exchange

Null Hypothesis: NG_LOSS has a unit root
 Exogenous: Constant
 Lag Length: 1 (Automatic - based on SIC, maxlag=24)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-35.33059	0.0000
Test critical values:		
1% level	-3.433734	
5% level	-2.862921	
10% level	-2.567552	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
 Dependent Variable: D(NG_LOSS)
 Method: Least Squares
 Date: 05/20/17 Time: 11:27
 Sample (adjusted): 1/05/2010 12/30/2016
 Included observations: 1824 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
NG_LOSS(-1)	-1.070225	0.030292	-35.33059	0.0000
D(NG_LOSS(-1))	0.218615	0.022856	9.564774	0.0000
C	-0.003702	0.008159	-0.453734	0.6501
R-squared	0.466159	Mean dependent var		0.000254
Adjusted R-squared	0.465573	S.D. dependent var		0.476626
S.E. of regression	0.348435	Akaike info criterion		0.730915
Sum squared resid	221.0824	Schwarz criterion		0.739975
Log likelihood	-663.5944	Hannan-Quinn criter.		0.734257
F-statistic	795.0644	Durbin-Watson stat		2.015907
Prob(F-statistic)	0.000000			

Figure 9. Unit Root Test Henry Hub Natural Gas

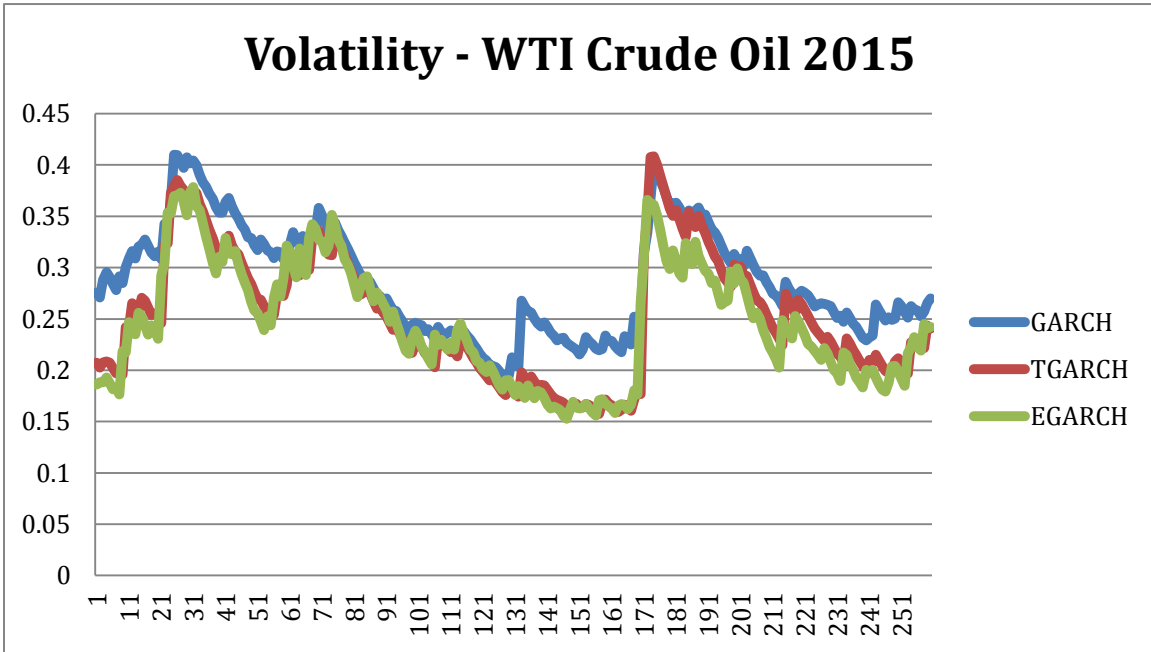


Figure 11. GARCH Volatilities WTI Crude Oil 2015

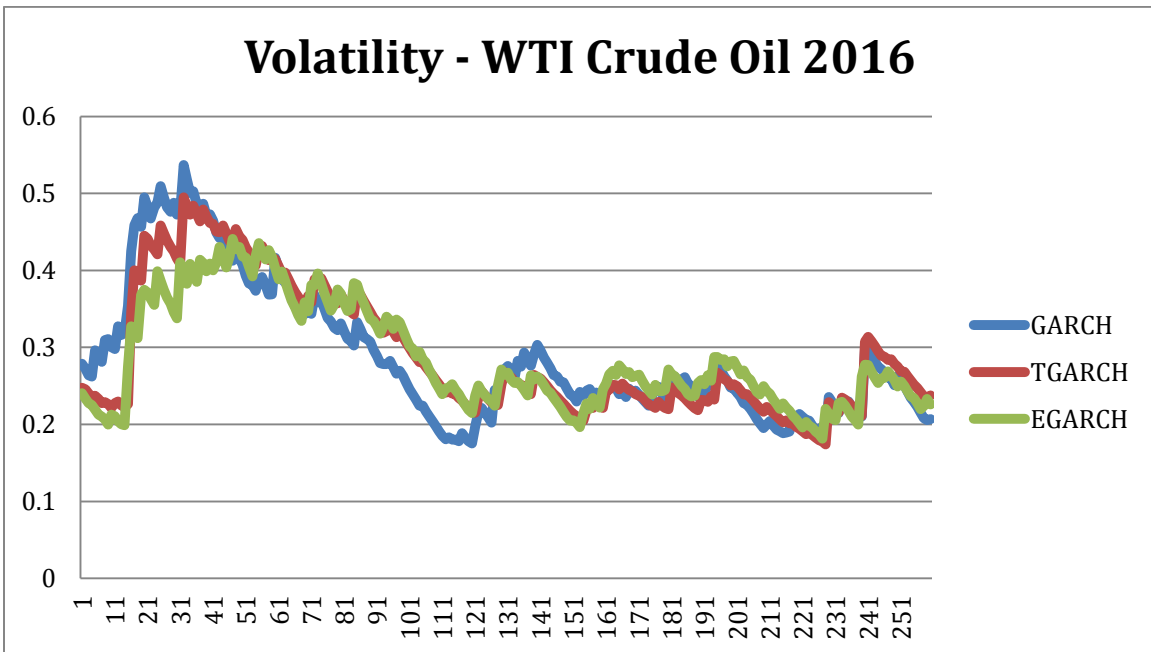


Figure 12. GARCH Volatilities WTI Crude Oil 2016

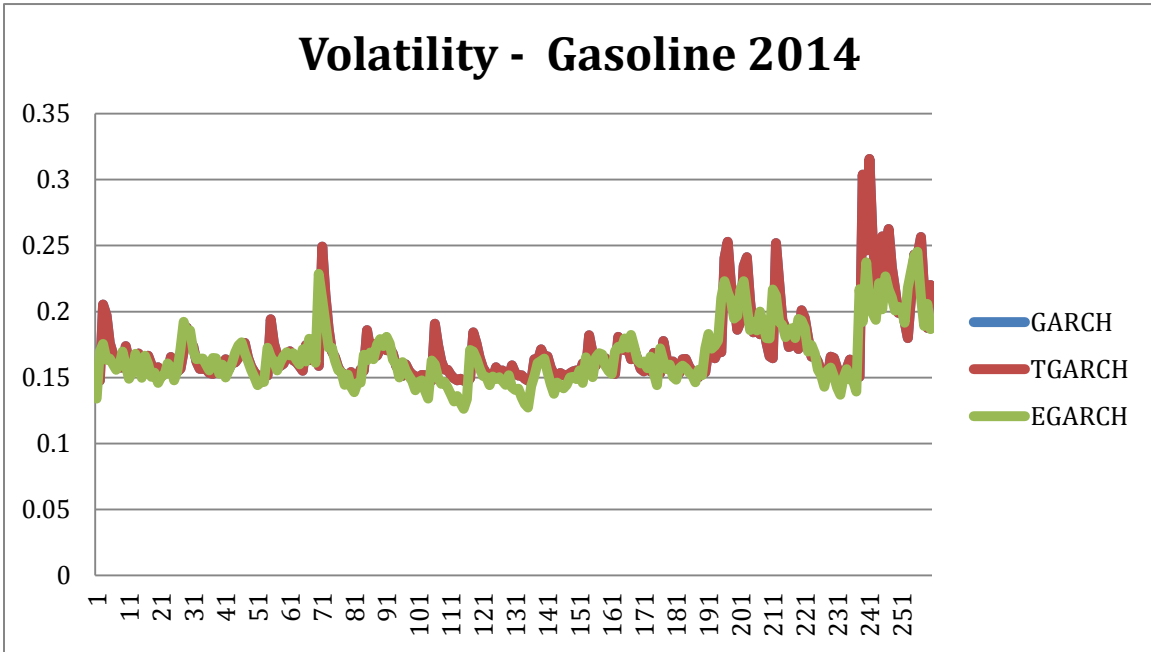


Figure 13. GARCH Volatilities Gasoline 2014

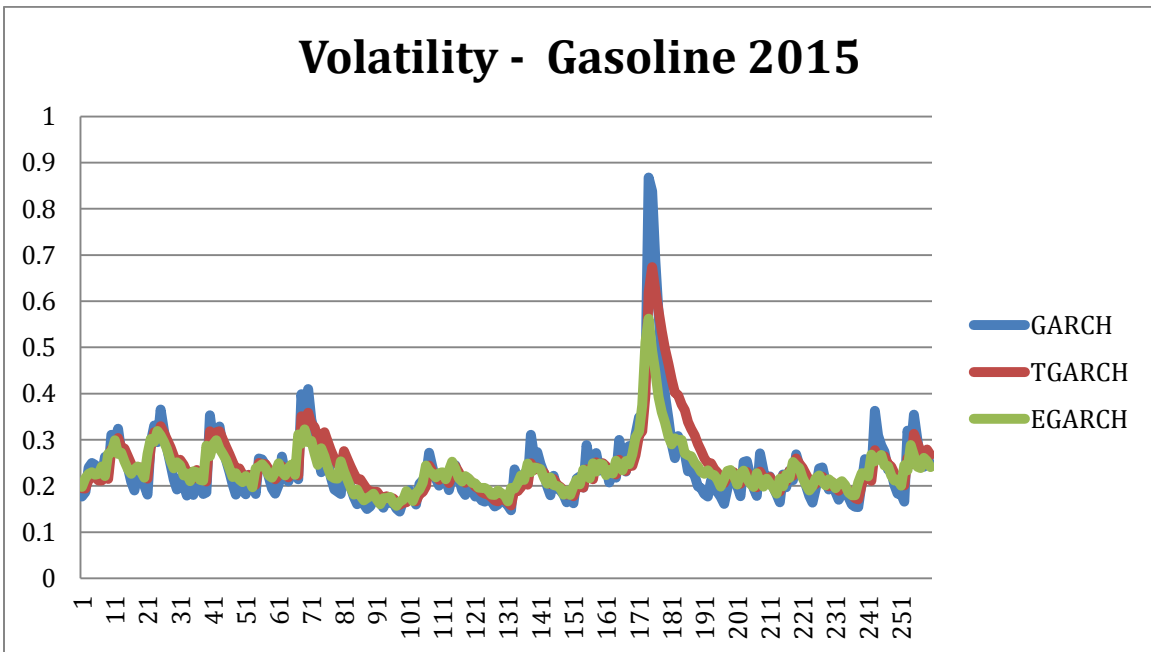


Figure 14. GARCH Volatilities Gasoline 2015

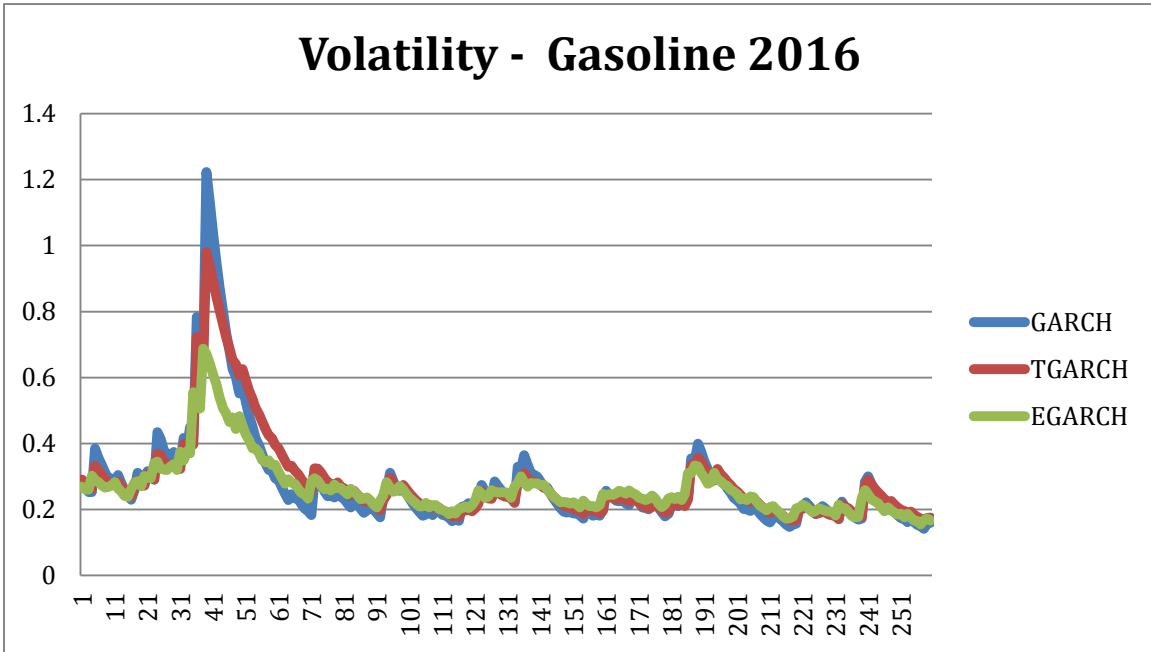


Figure 15. GARCH Volatilities Gasoline 2016

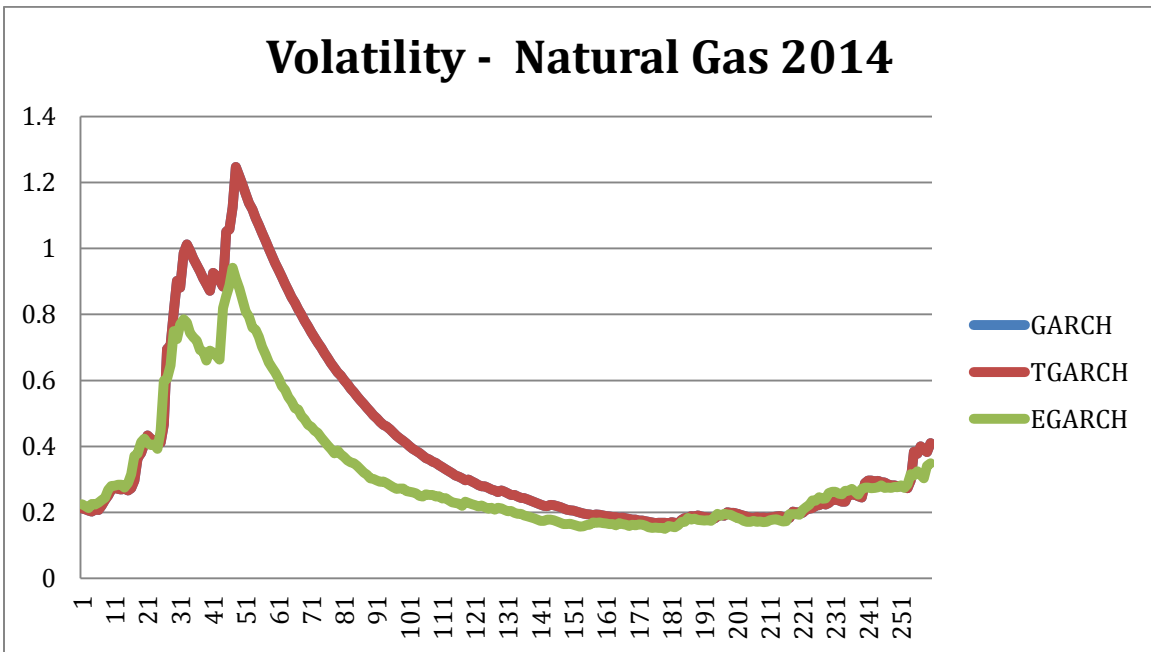


Figure 16. GARCH Volatilities Henry Hub Natural Gas 2014

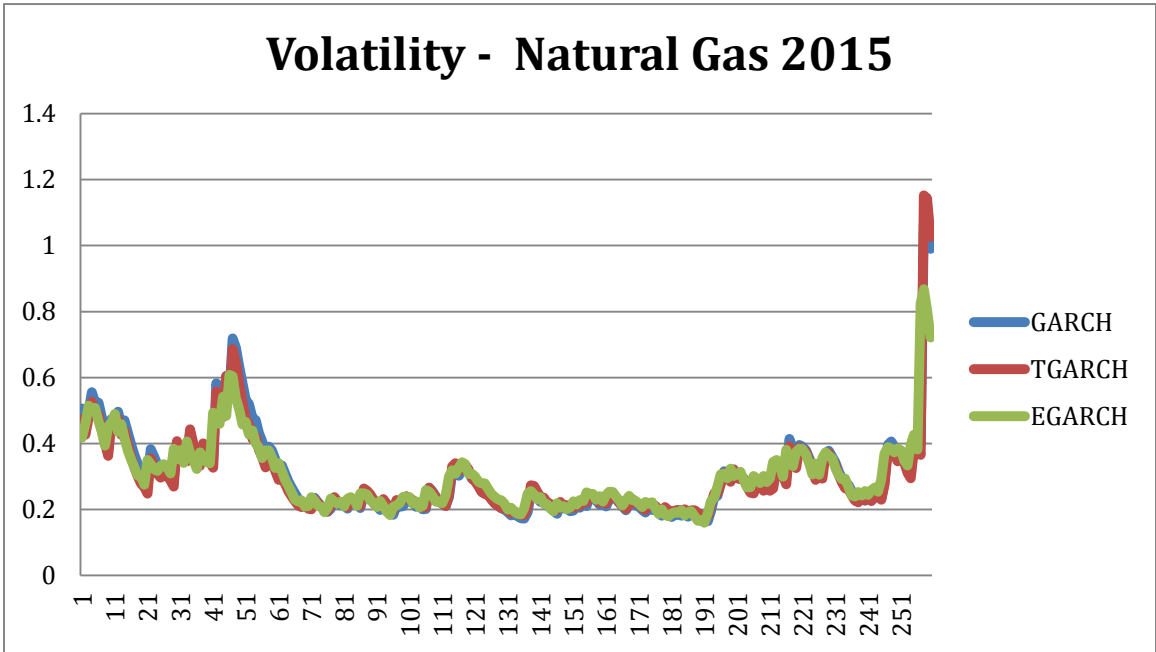


Figure 17. GARCH Volatilities Henry Hub Natural Gas 2015

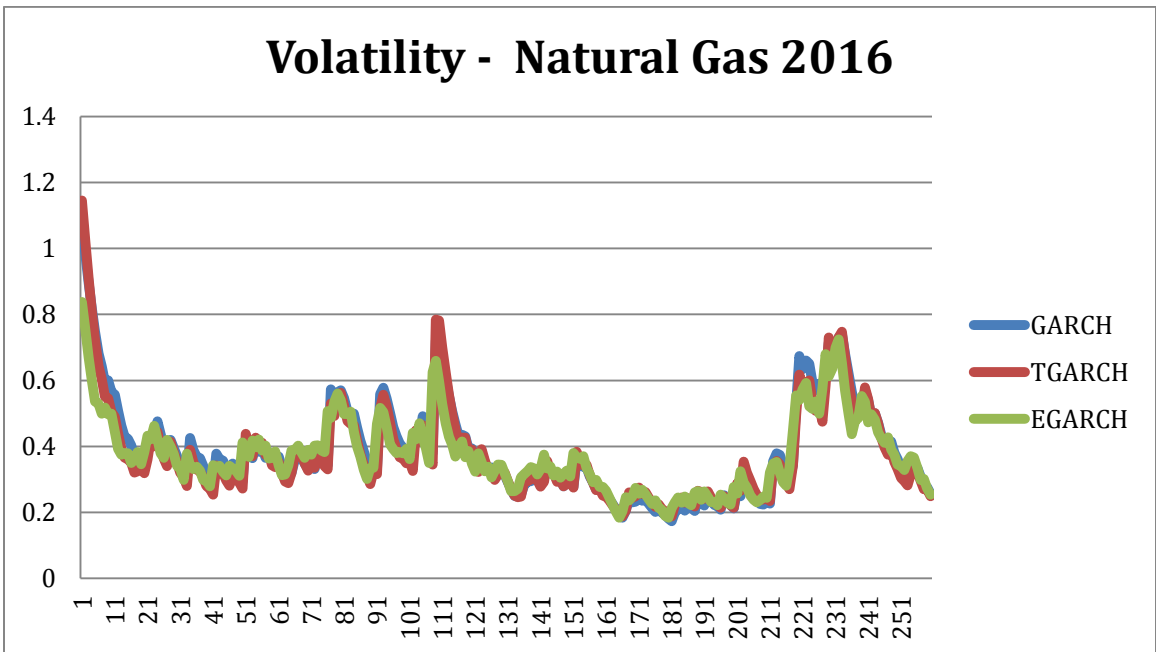


Figure 18. GARCH Volatilities Henry Hub Natural Gas 2016

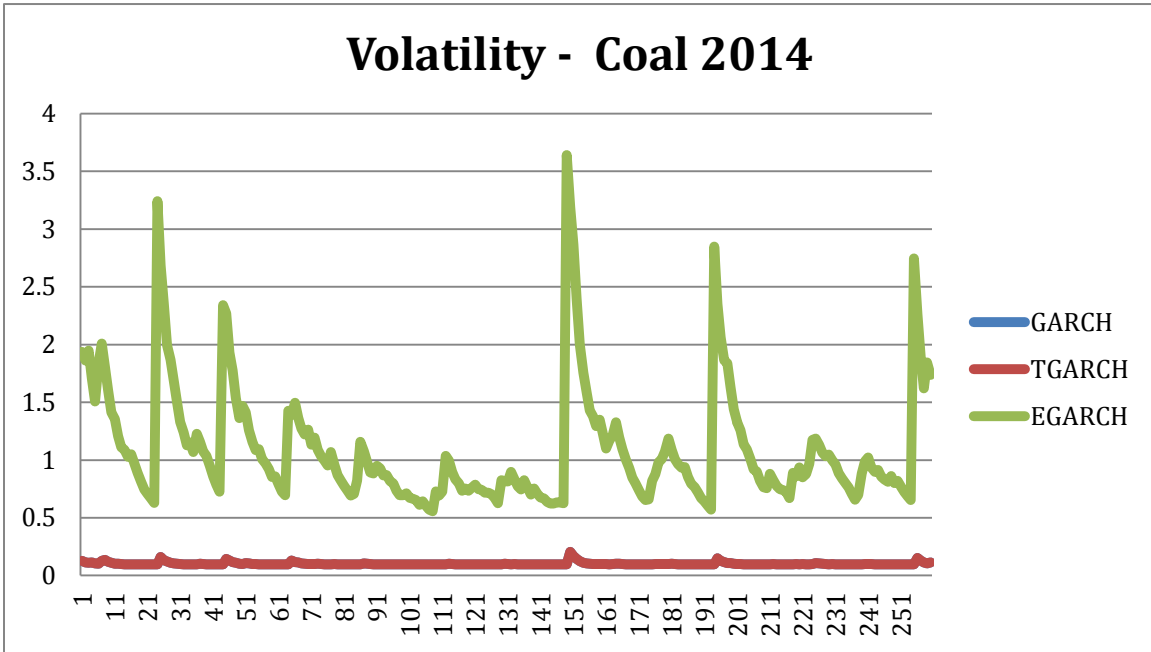


Figure 19. GARCH Volatilities International Coal Exchange 2014

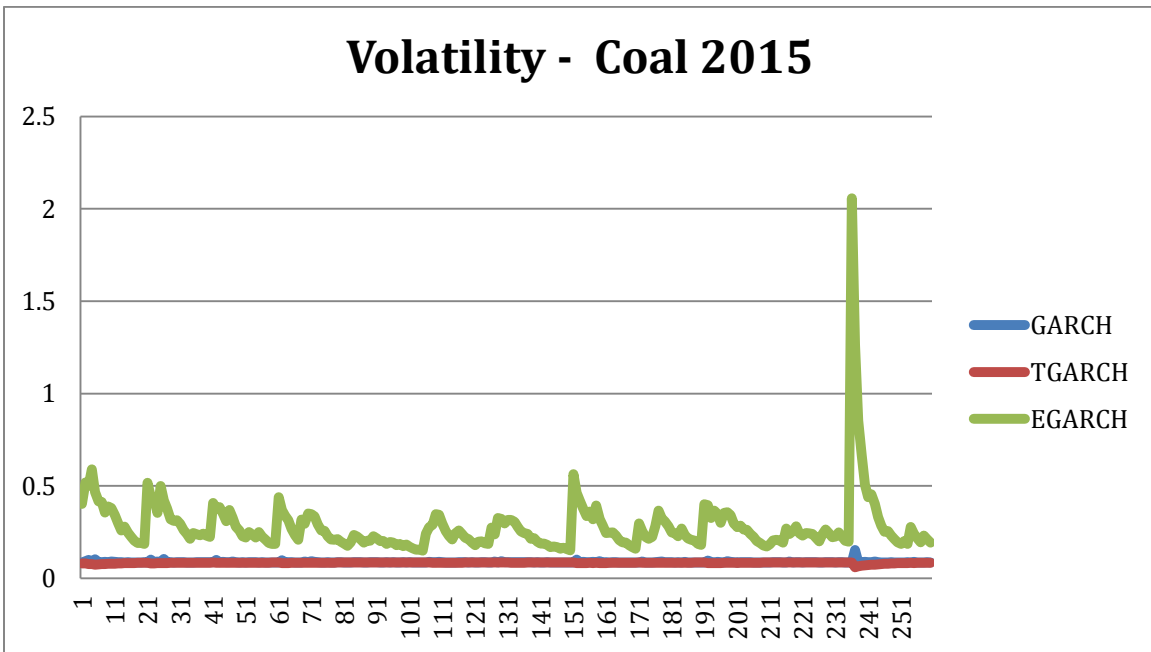


Figure 20. GARCH Volatilities International Coal Exchange 2015

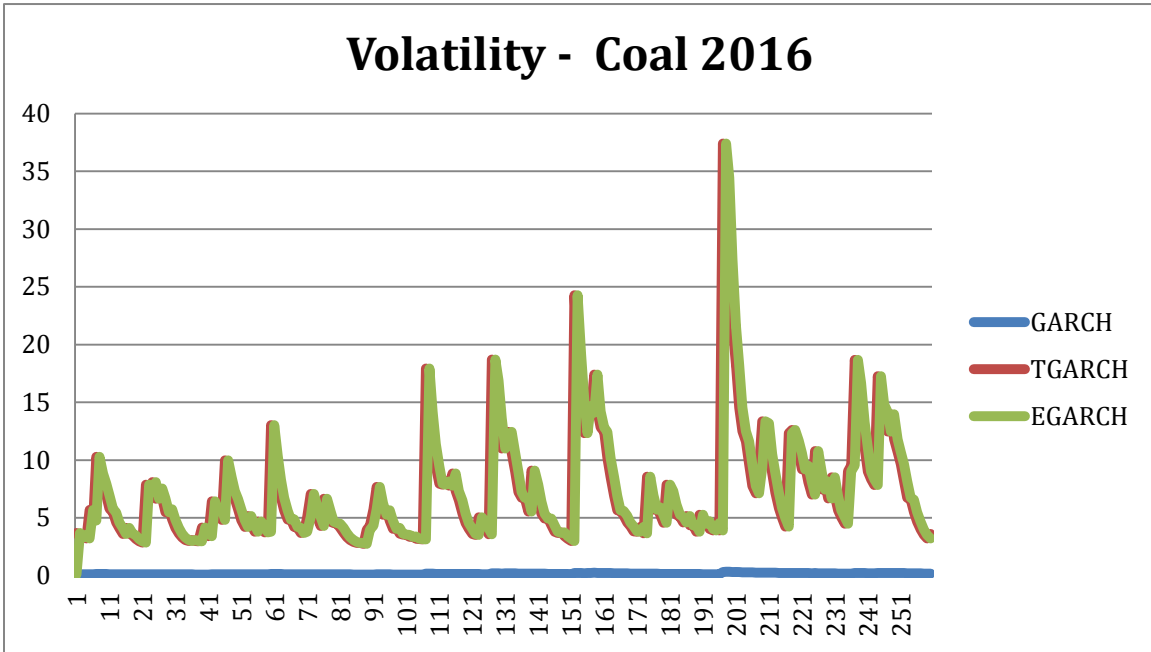


Figure 21. GARCH Volatilities International Coal Exchange 2016

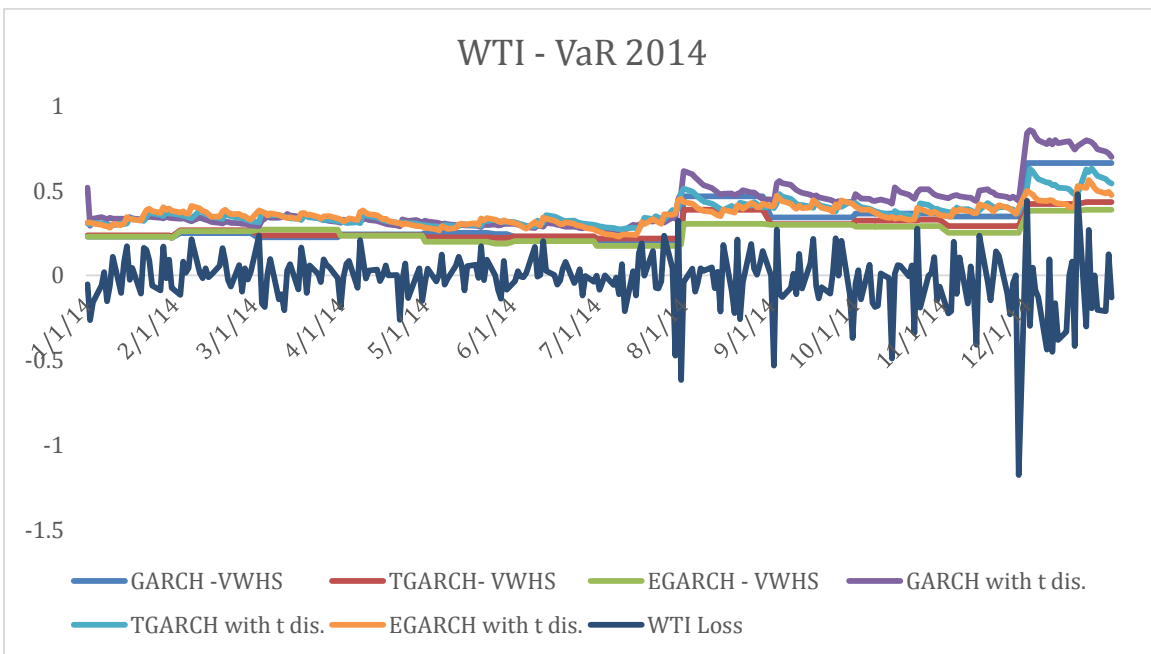


Figure 22. VaR Estimates WTI Crude Oil 2014

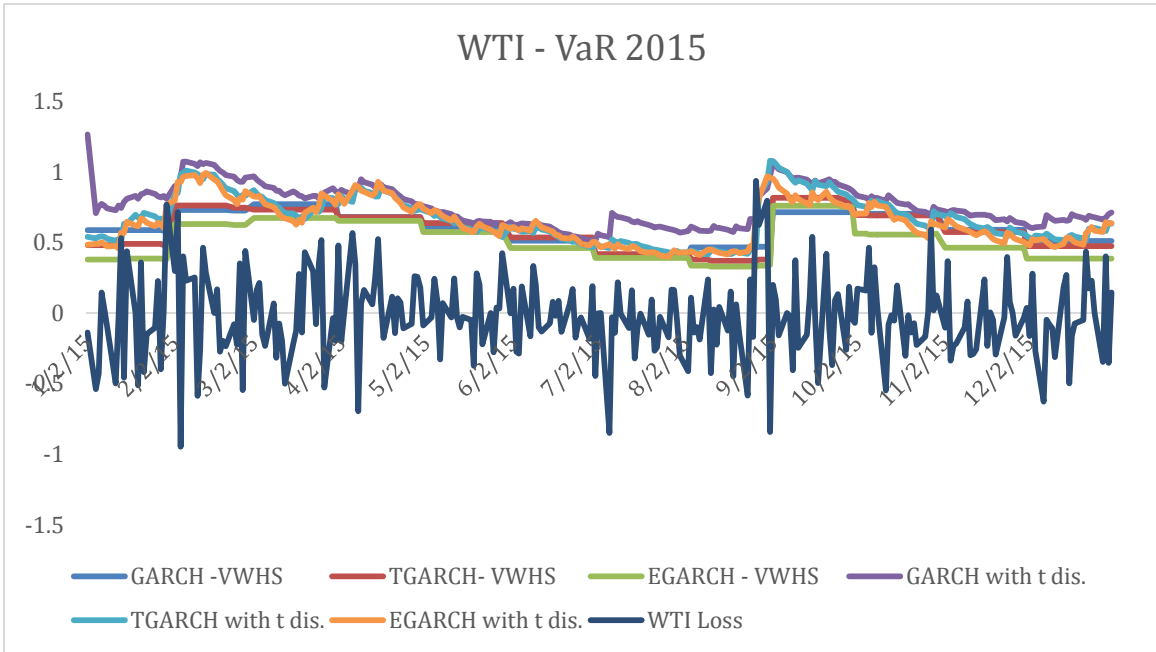


Figure 23. VaR Estimates WTI Crude Oil 2015

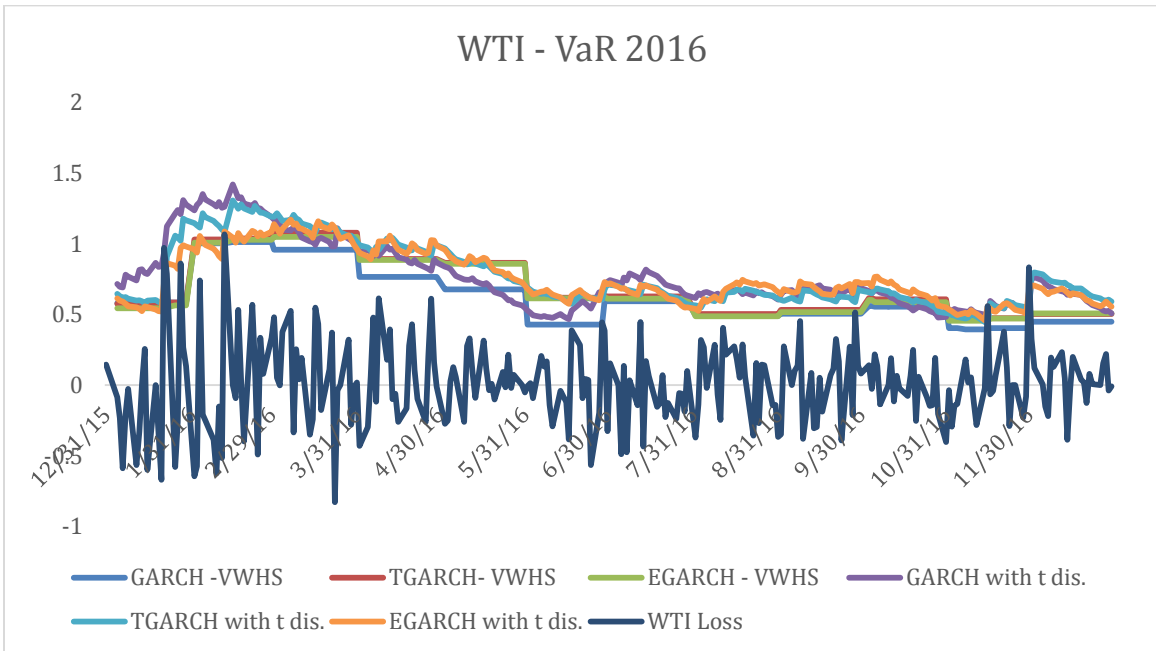


Figure 24. VaR Estimates WTI Crude Oil 2016

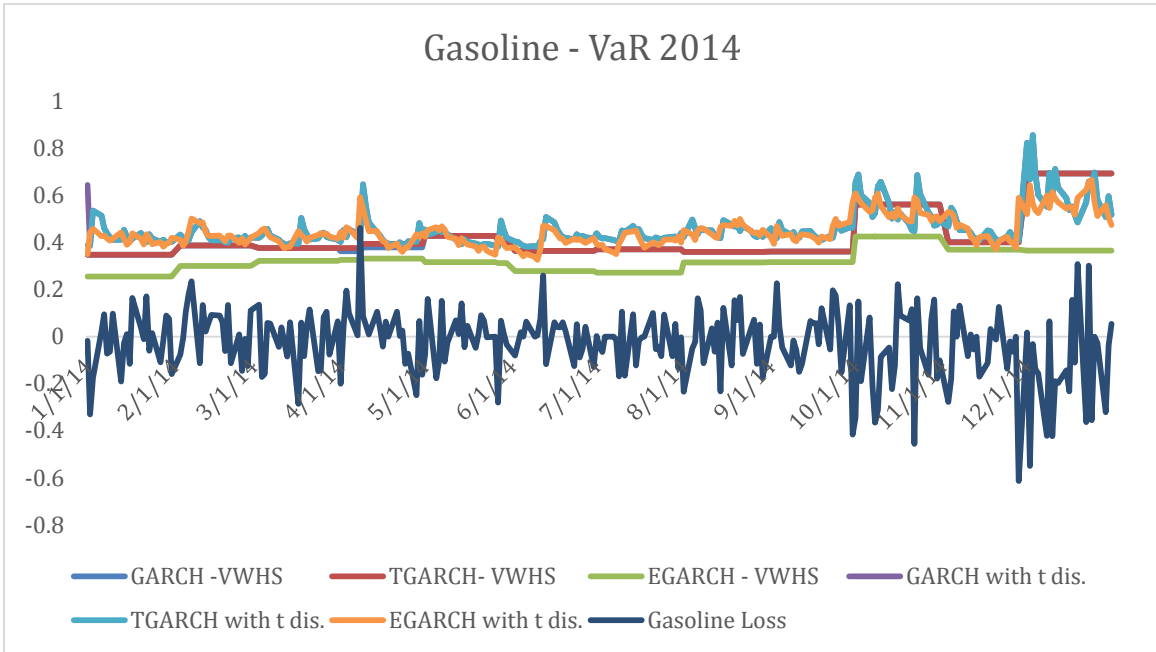


Figure 25. VaR Estimates Gasoline 2014

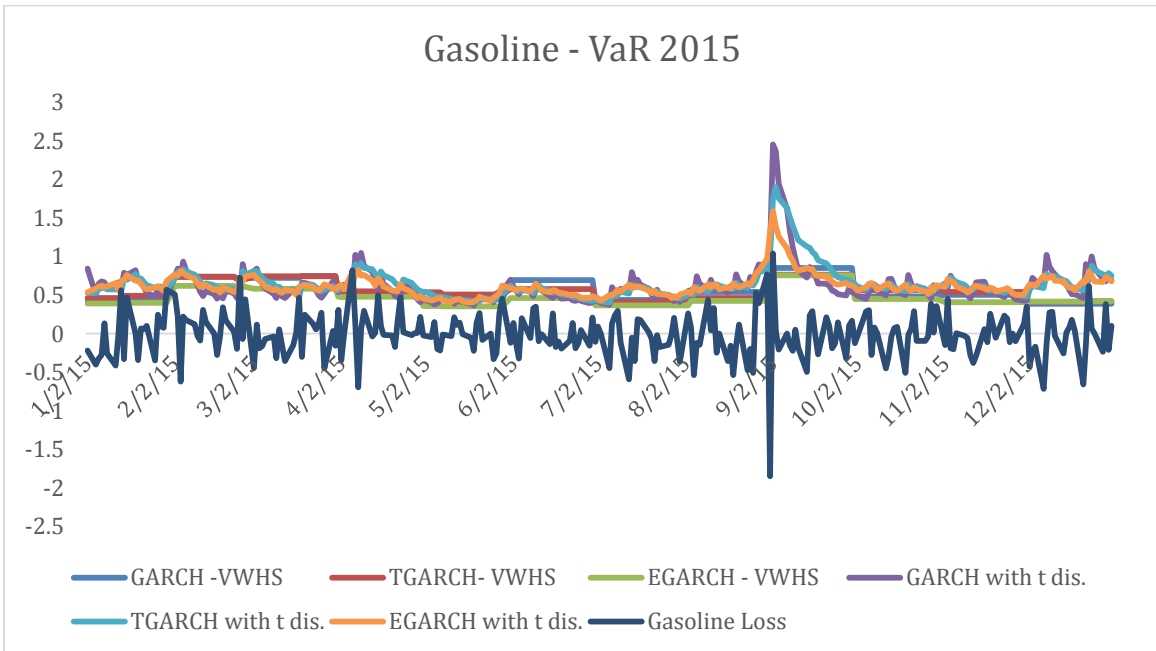


Figure 26. VaR Estimates Gasoline 2015

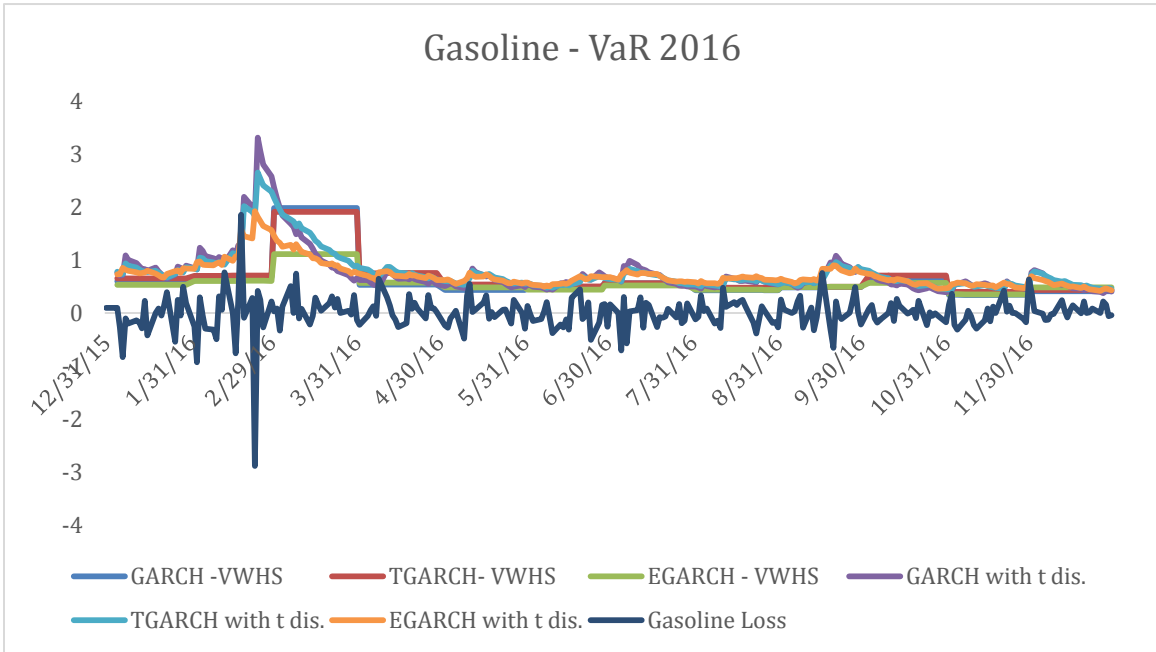


Figure 27. VaR Estimates Gasoline 2016

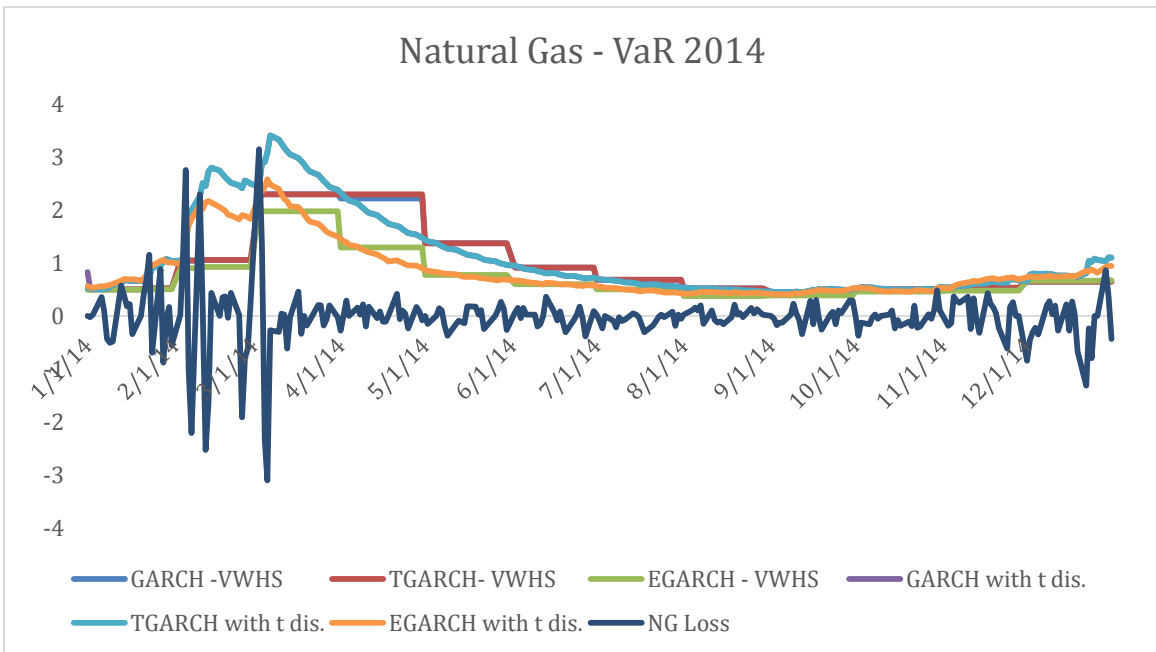


Figure 28. VaR Estimates Henry Hub Natural Gas 2014

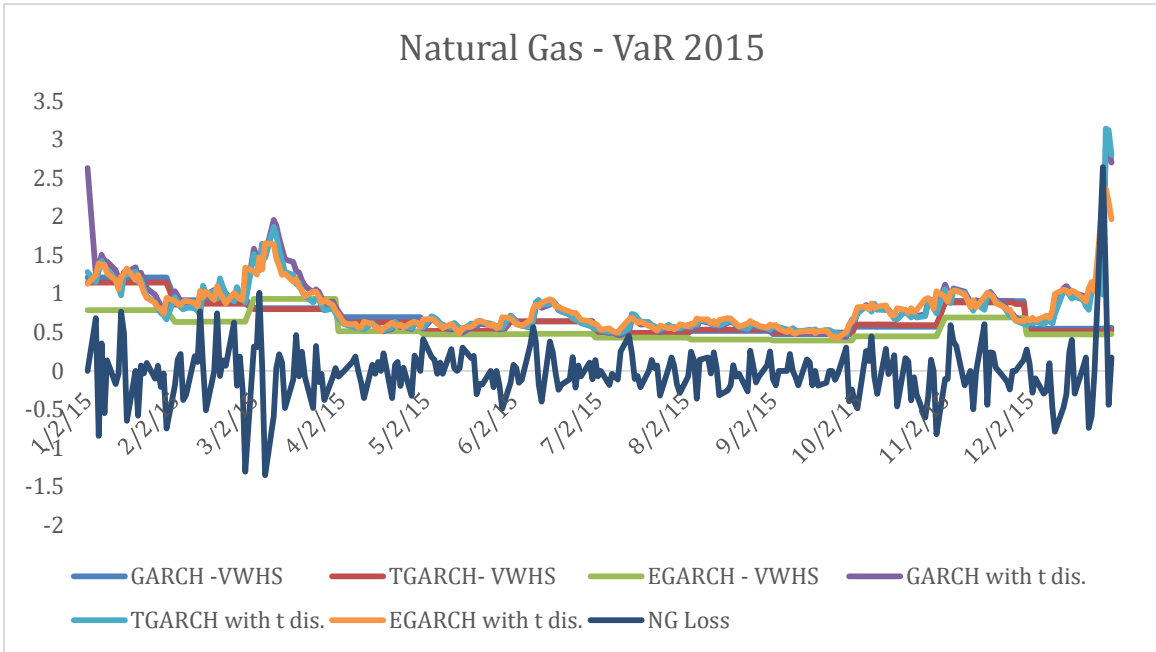


Figure 29. VaR Estimates Henry Hub Natural Gas 2015

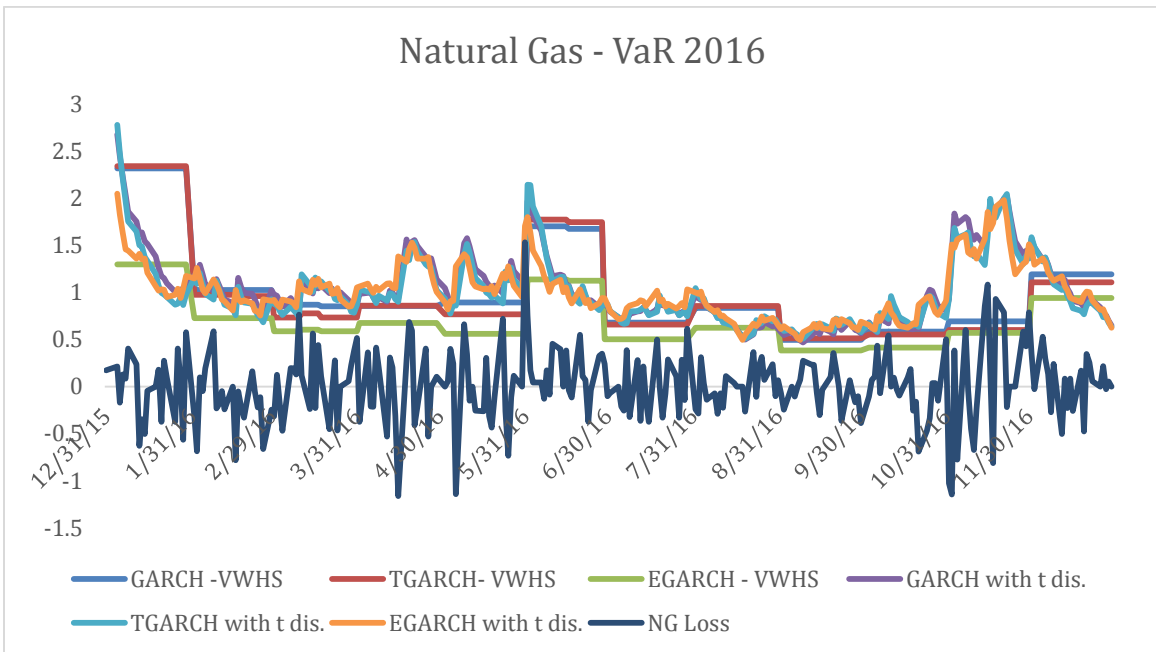


Figure 30. VaR Estimates Henry Hub Natural Gas 2016

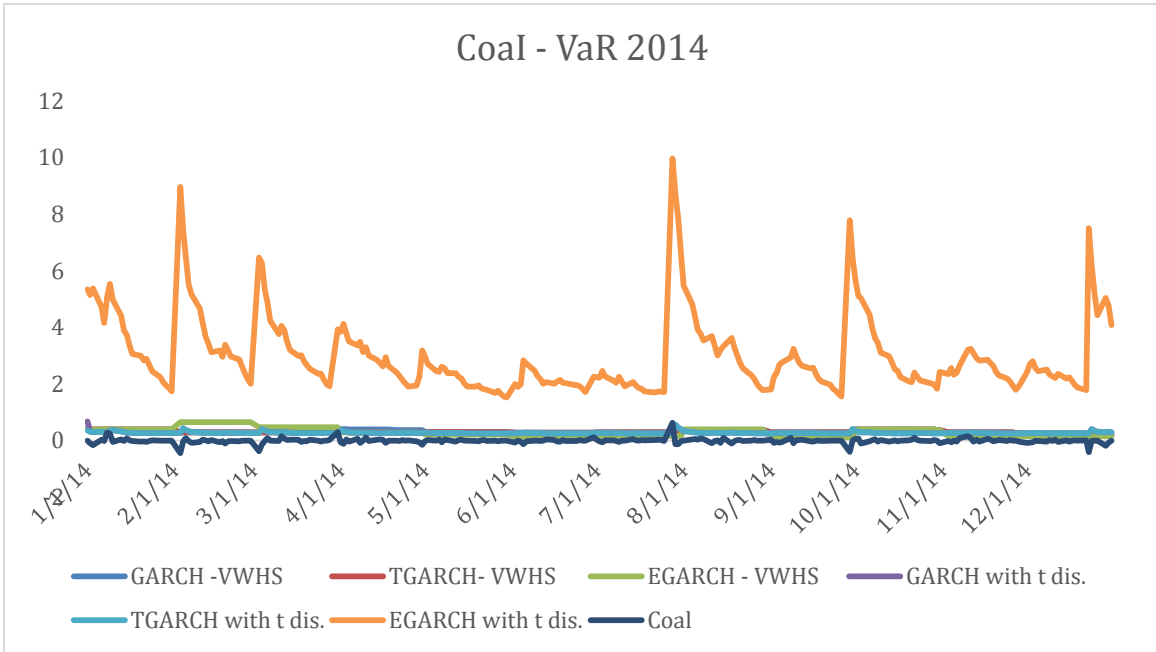


Figure 31. VaR Estimates Coal International Exchange 2014

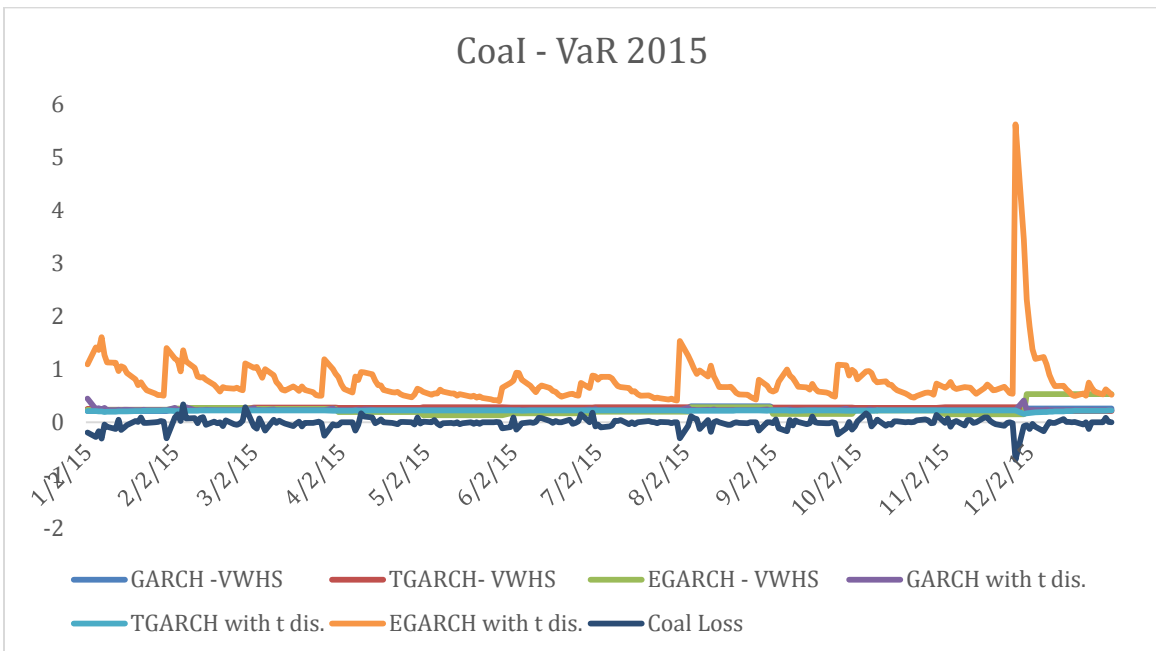


Figure 32. VaR Estimates Coal International Exchange 2015

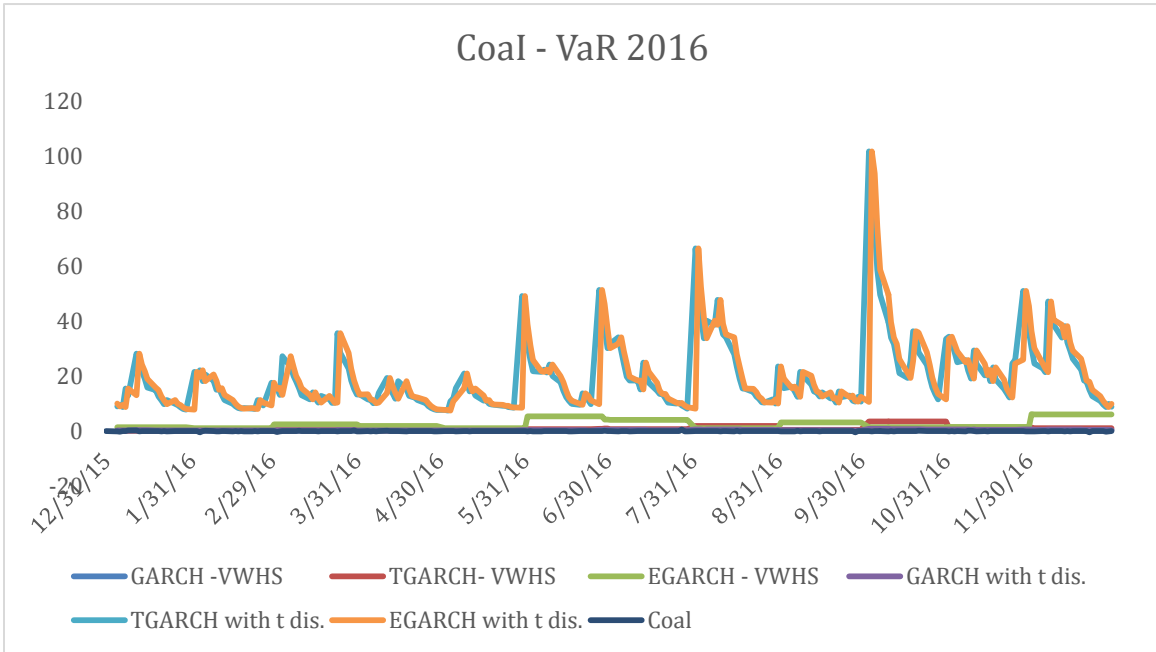


Figure 33. VaR Estimates Coal International Exchange 2016

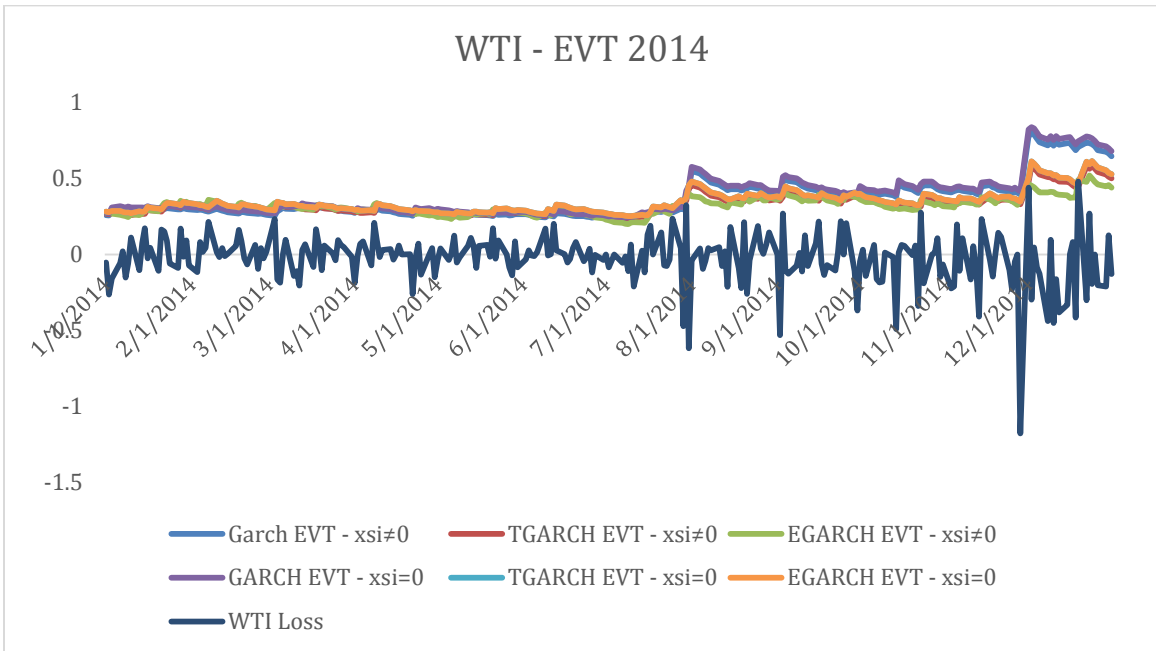


Figure 34. EVT VaR Estimates WTI Crude Oil 2014

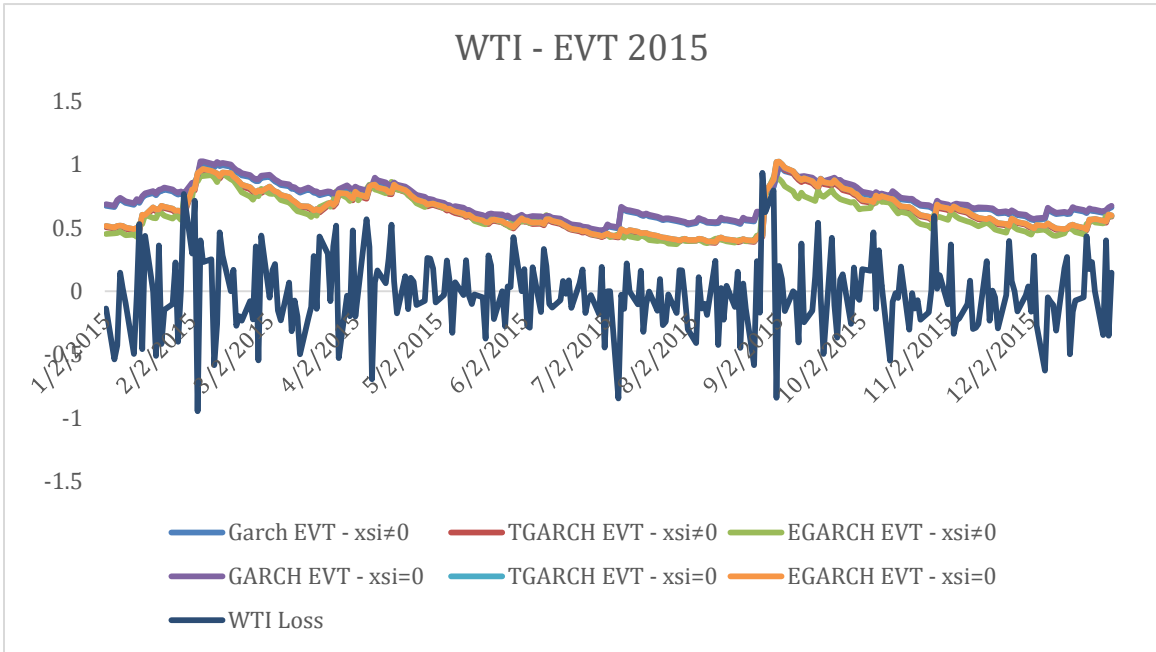


Figure 35. EVT VaR Estimates WTI Crude Oil 2015

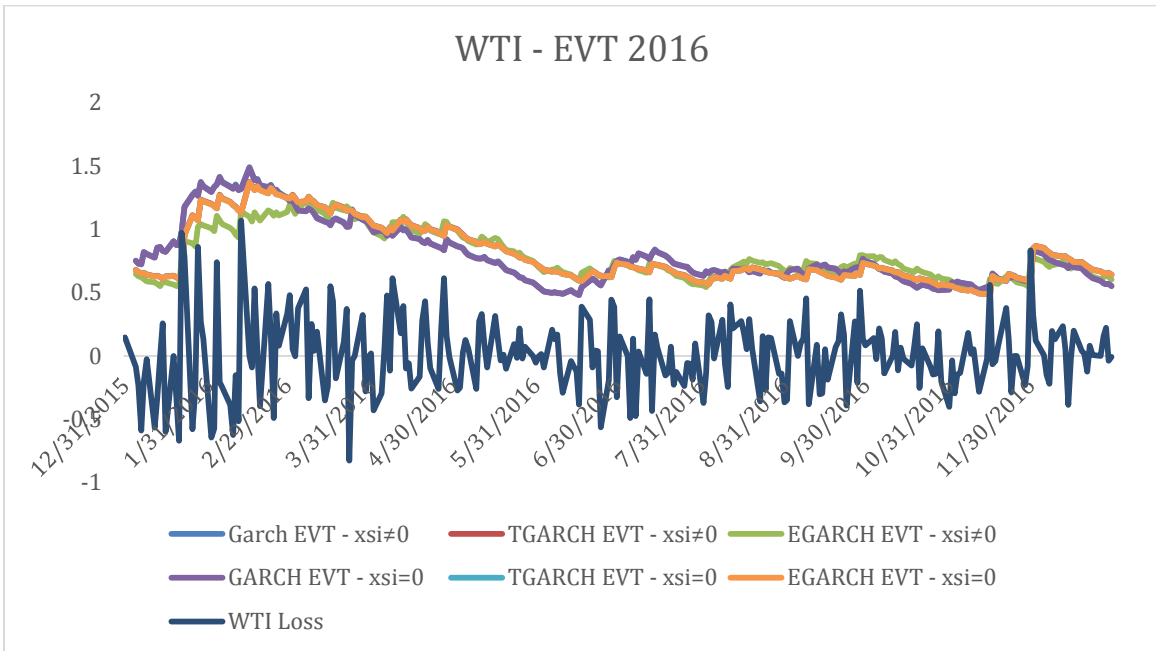


Figure 36. EVT VaR Estimates WTI Crude Oil 2016

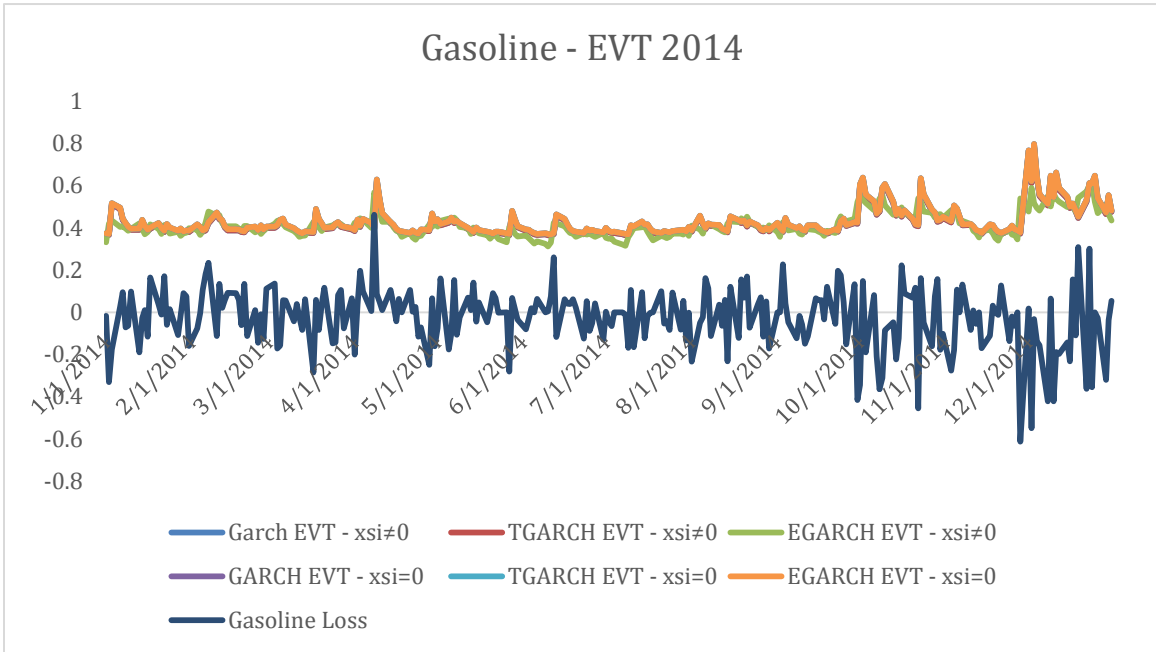


Figure 37. EVT VaR Estimates Gasoline 2014

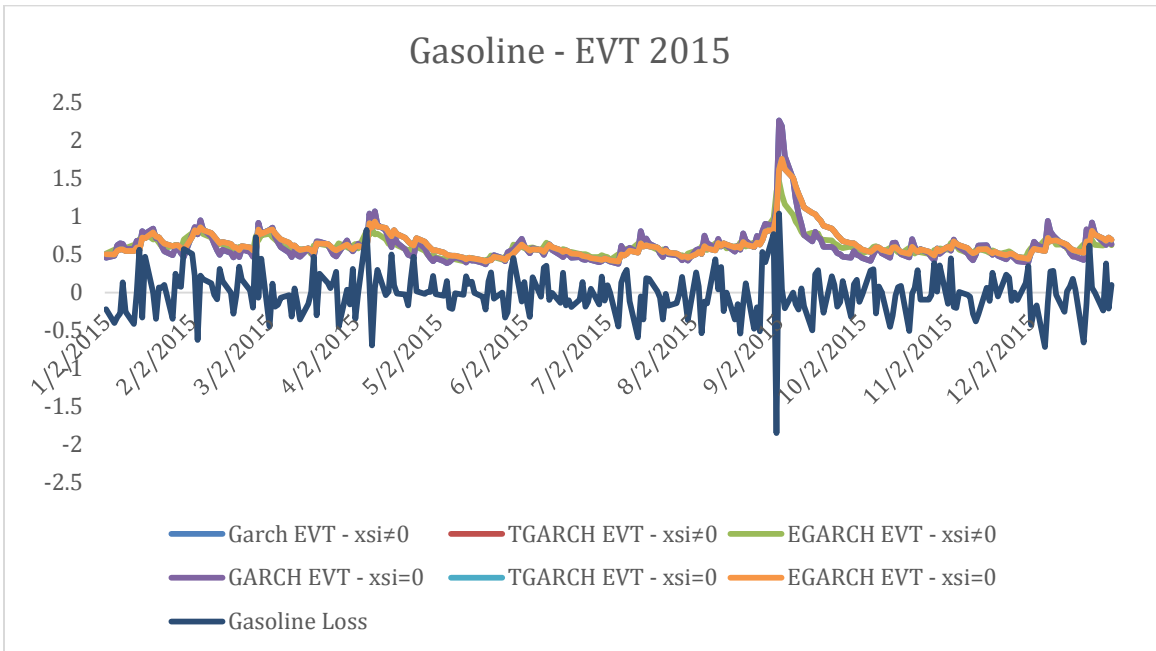


Figure 38. EVT VaR Estimates Gasoline 2015

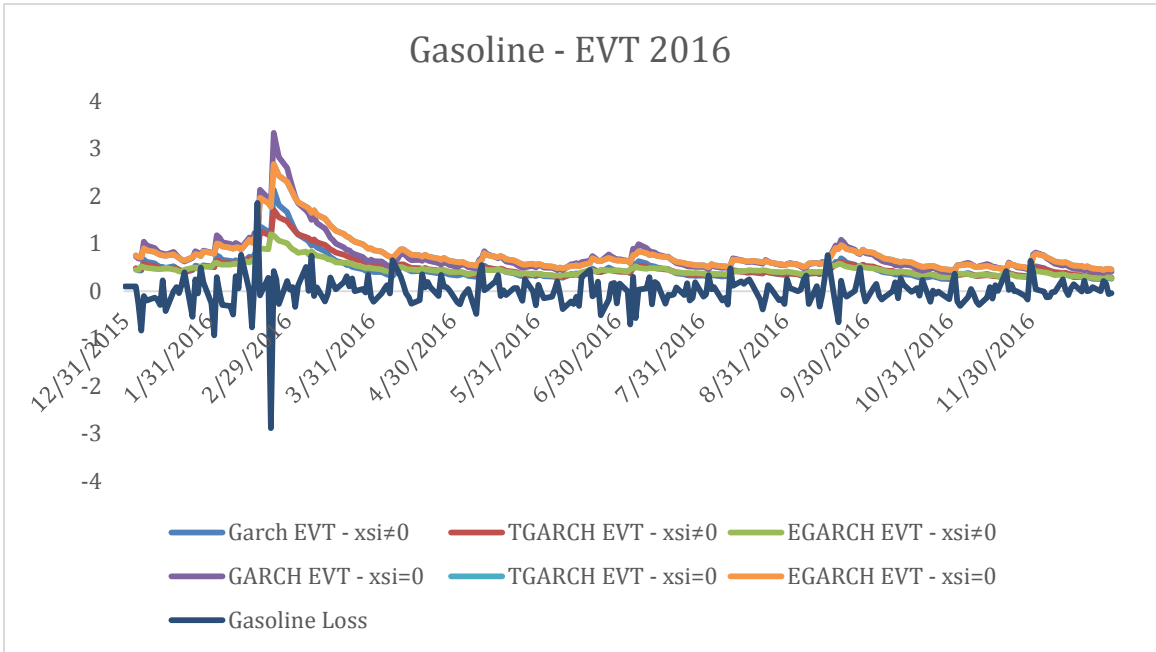


Figure 39. EVT VaR Estimates Gasoline 2016

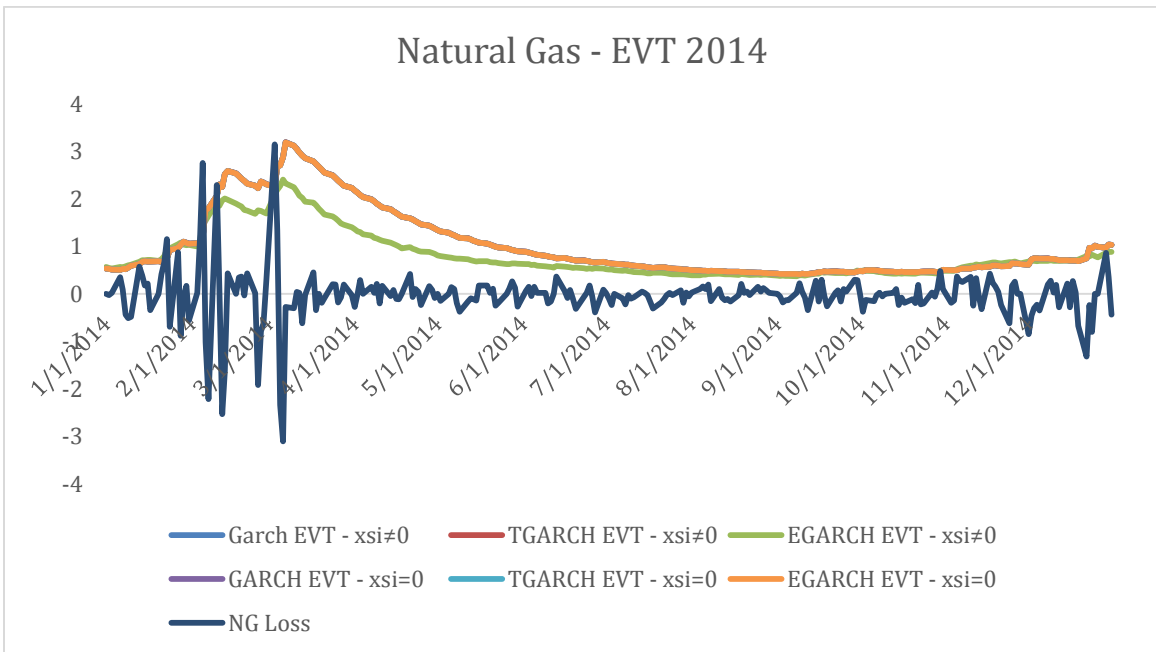


Figure 40. EVT VaR Estimates Henry Hub Natural Gas 2014

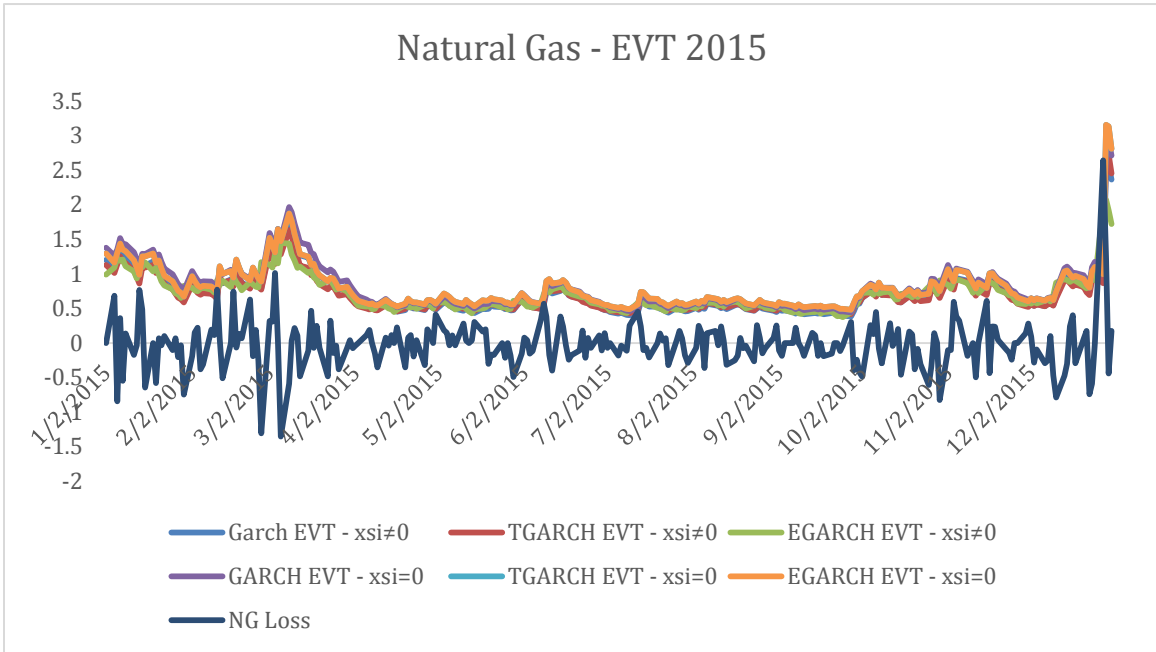


Figure 41. EVT VaR Estimates Henry Hub Natural Gas 2015

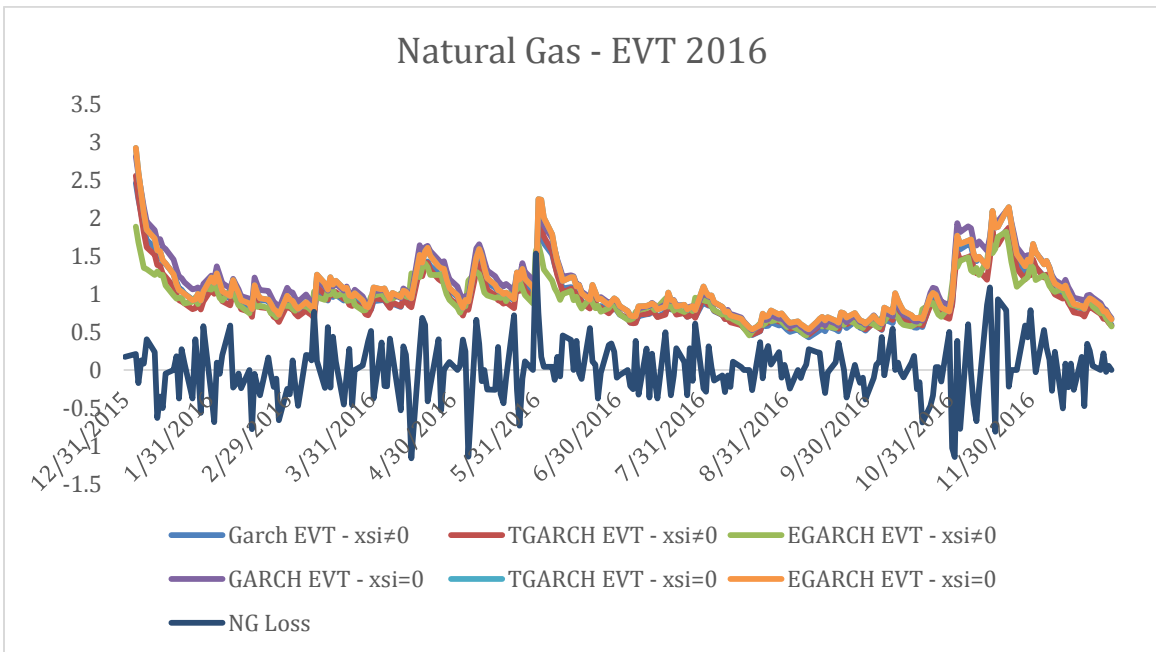


Figure 42. EVT VaR Estimates Henry Hub Natural Gas 2016

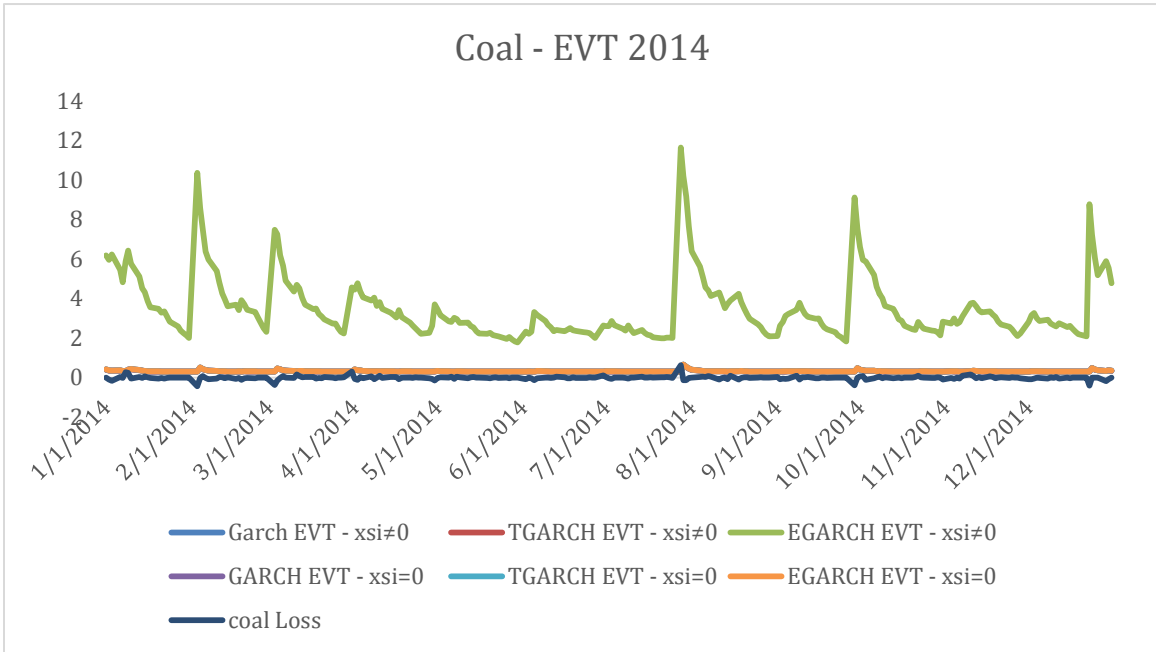


Figure 43. EVT VaR Estimates Coal International Exchange 2014

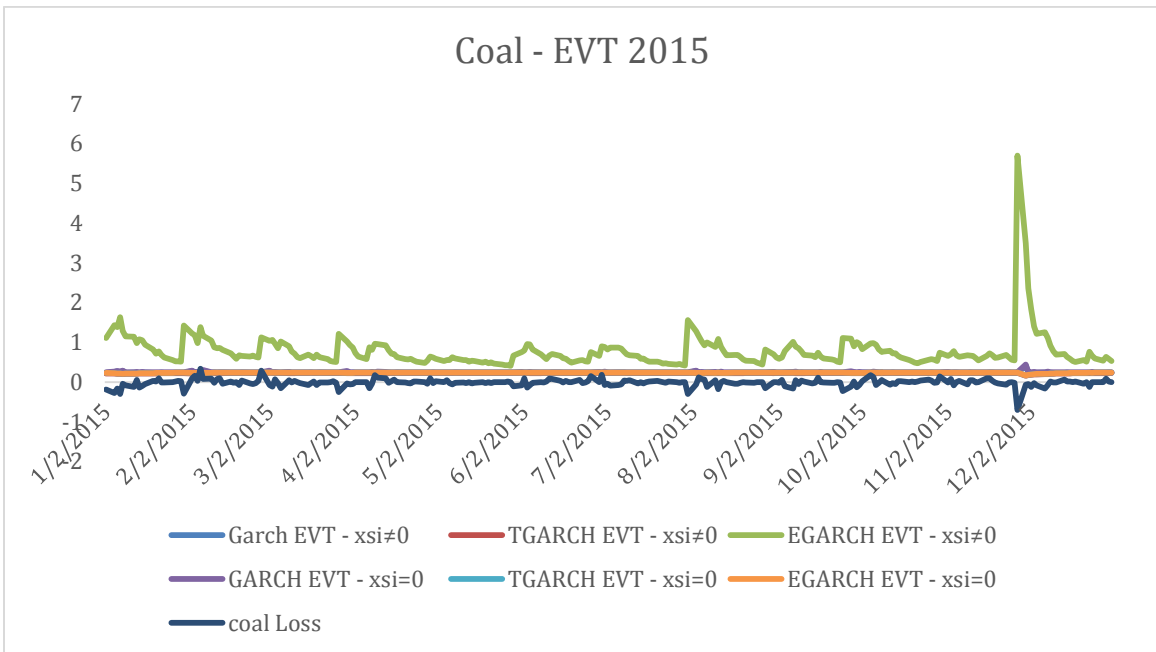


Figure 44. EVT VaR Estimates Coal International Exchange 2015

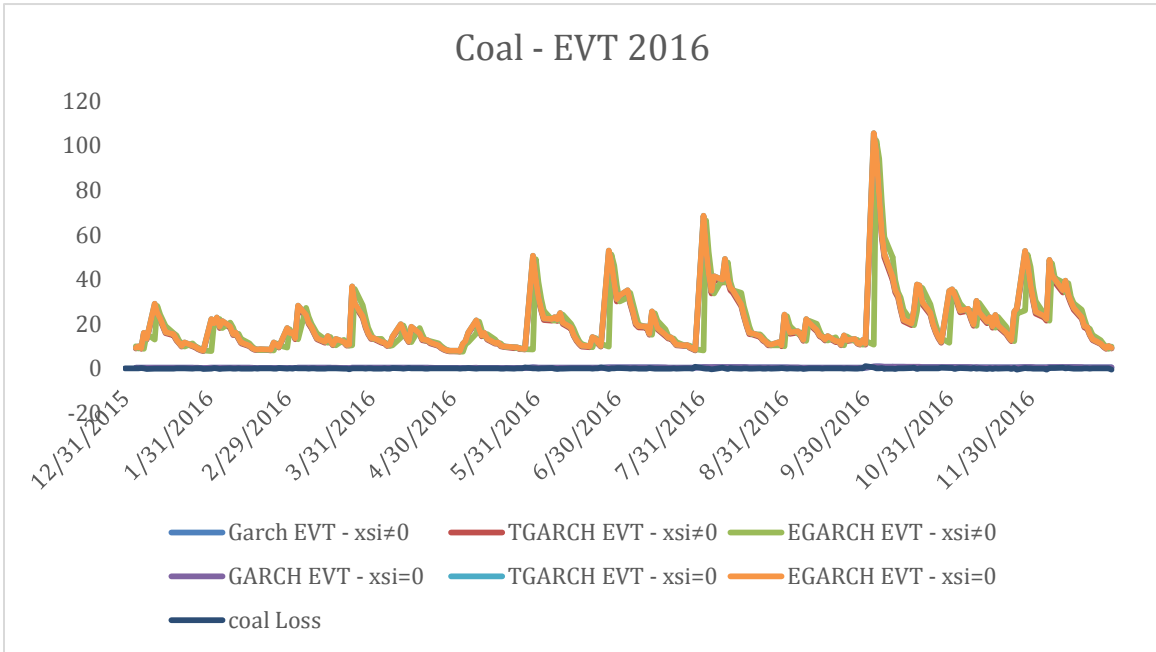


Figure 45. EVT VaR Estimates Coal International Exchange 2016