Structural Modelling of Credit Spreads on the European Bond Market: An Empirical Study

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Abstract

This thesis empirically tests the explanatory power of structural models on the European corporate bond market. Using new evaluation methods, including LASSO and gradient boosting regression, we can provide an in-depth assessment of the models' shortcomings. With these tools we show that the structural models tend to systematically overstate or understate the spread due to an oversensitivity to leverage ratio and asset volatility. We introduce a novel extension to the Black Cox model in order to mitigate the observed weaknesses. Our extension is calibrated to match historical default probabilities with an additional baseline default risk component attributable to all firms. This approach manages to increase the R-squared from 39 % to 47 % and at the same time reduce the residual dependencies of leverage ratio and asset volatility.

Keywords: Structural models, Merton model, Black Cox model, European corporate bond spreads

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1 Introduction

1.1 Background

Companies in need of financing can besides from raising capital through bank loans, issue bonds to investors. A bond is a contractual agreement, in which a firm promises investors to receive future payments in exchange for an upfront payment at the date of issue. The general structure of a bond contract is in many ways similar to a bank loan. However, there are important differences between the two ways of financing that affect the value of the contracts and the incentives of stakeholders. From the issuer's point of view, a bond can finance projects and activities that are too large and risky for a single bank to fund alone. Instead capital and risk is pooled among more investors owning a smaller share of the debt. From the investors' perspective, the upside of a bond investment is that the returns of promised future cash flows often exceed the interest rate of a risk-free position. The investors also have the possibility of quickly re-gaining cash, by selling the bond contract on the secondary market. Moreover, there are various bond types and contractual specifications that introduce an additional freedom for the bond issuer and investor. This contractual flexibility imply that the stakeholders can agree to mutually optimal conditions, circumventing the usually stricter requirements posed by banks.

Obviously, corporate bonds have additional risks compared to government bonds, which are usually considered risk-free. The most central part in the assessment of a corporate bond is to understand and quantify the risks associated with the firm and the specific contract. More specifically, this means that the investor needs to investigate whether the borrower will be able to meet its obligations defined in the bond contract. If a firm fails to meet its repayment obligations when due, a legal process takes place in which investors reclaim their legislative assets as defined in the contract. The recovery of a defaulted bond varies depending on the outcome of the legal process, the bankruptcy costs and the liquidation of assets, causing additional downside risk faced by the investor. Other considerable risk components include for example the risk of the investor not being able to sell the bond contract when desired or the risk of inflation denomination of the contracted amount. Naturally, these risk components depend strongly on numerous characteristics of the issuer, the bond contract and the state of the economy. For example, the issuer's financial leverage should intuitively affect the firm's probability of default over time. Moreover, an infrequently traded bond contract is likely harder to sell upon desire, suggesting that trade frequency should affect the liquidity risk. All of these factors - that affect the risk exposure of the investor - ought to be reflected in the market's pricing of a bond contract. The price of a bond is commonly quoted in terms of its yield spread, which is the difference in yield between the bond and its benchmark government bond. With higher risk, investors demand additional risk premium in terms of increased yield spreads and thereby inducing a higher potential pay-off.

Due to the complex nature of corporate debt, the translation of bond characteristics to yield spread is not straightforward and research has therefore historically focused on trying to describe yields on the market through different theoretically founded models. There are primarily two classes of models used to value defaultable bonds, namely the structural and the reduced form models. The structural approach was first pioneered by Black and Scholes (1973) and Merton (1974), who view equity and debt of a firm as contingent claims of the firm's asset value. In this setting it is assumed that a firm's total debt is financed by a single zero-coupon bond. Upon maturity the bond holders are first paid the face value and the equity holders will receive the remaining amount of the firm value. If the firm value is less than the nominal amount of debt at maturity, the debt holders will only be partially reimbursed, while the equity holders will receive nothing. Thereby the equity is valued as an European call option with the firm value as underlying instrument. Analogously the debt is valued as a short position in a European put option combined with a long risk-free position. These concepts will be explained in-depth later on in this report.

The fundamental idea of the structural model, referred to as the Merton model, has further been extended and modified to incorporate features observed on the market. To name a few, Black and Cox (1976) allow the firm to default prior to the debt's maturity if the firm value falls below a exogenous threshold. Geske (1977) introduces the possibility for the firm to raise new funds in order to finance its payment obligations. Longstaff and Schwartz (1995) extend the Merton framework into a two-factor model, which has stochastic interest rates. Leland and Toft (1996) take tax and bankruptcy costs into consideration when defining their modifications to the Merton model. The reduced form models on the other hand regard the event of a default as a Poisson process with time and state dependent intensity of default. The main benefit of this class of models is their mathematical tractability. However, in contrast to the structural models the Poisson process of default lack an intuitive interpretation, which explains why the success of reduced form models is relatively limited (Arora, Bohn, & Zhu, 2005).

Neither of the two model families have been recognised to fully explain the true yield spreads observed on the markets. The most common explanation to the models' shortcoming is the fact that yield spreads should constitute of a default component and a non-default component, of which credit spread models allegedly only account for the former. However, the composition of yield spreads is broadly debated and different research papers show different results depending on the scope and time horizon of the study.

Traditionally, structural models are widely recognised to underestimate corporate yield spreads. This inability to predict true empirical results is commonly referred to as the Credit Spread Puzzle. Huang & Huang (2012) show that only a small fraction of the investment grade yield spread is due to model implied credit risk, while for speculative bonds, credit risk accounts for a somewhat larger fraction of the yield spread. In contrast, more recent studies by Chen, Collin-Dufresne, and Goldstein (2009) and Feldhütter and Schaefer (2016) question the existence of the Credit Spread Puzzle. Schaefer & Feldhütter argue that structural models in fact are able to match empirical data for all ratings when calibrated to a longer history of default rates. Evidently, there is no general consensus on the performance and adequacy of the structural approach of modelling corporate yield spreads.

Previous research on the field of structural models have mainly been based on US bond data provided by open databases such as the Mergent Fixed Investment Securities Database (FISD), the Trade Reporting and Compliance Engine (TRACE) and COMPUSTAT. There are at least two reasons causing this skewed research scope. Firstly, the US bond market is significantly larger in terms of the total amount outstanding and can thereby be regarded as more developed than other markets (Blackrock, 2016). The implication is that there is greater interest among investors to fully understand the market dynamics, and that researchers have reason to believe that the market is well-functioning. Secondly, there is an availability bias, which arises from the ease of obtaining US bond data. Data for European corporate bond trades and the issuing firms' accounting data is not packaged and readily available, resulting in a less studied sample.

Since the financial crisis in 2009, the European corporate bond market has experienced a steady expansion in terms of its size, thus becoming increasingly interesting for a broader scale of investors. The growth has been boosted by factors such as the European Central Bank's Corporate Sector Purchase Program (CSPP), record low interest rates and the banking sector's stricter capital requirements. Furthermore, as the market is maturing, European corporate data is available to a greater extent. In the beginning of 2018 the Financial Instruments Directive II (MiFID II), which is a compulsory reporting system for public European bond trades, will take effect. The American equivalent of centralised bond reporting, named TRACE, has been in place since 2002. Consequently, the MiFID II environment is expected to bring more academic attention to the European bond market (Blackrock, 2016).

To the authors' best knowledge, previous academic research on determinants of yield spreads on the European bond market is limited and inconclusive with respect to the success of credit spread models. In addition, the European bond market is growing and data is becoming accessible to a greater extent. With these arguments we motivate that there is a need for further exploration of structural models applied on European corporate bonds and this is the research gap we intend to investigate in this thesis.

1.2 Problem Formulation

Clearly, there are several open and debated topics within the area of yield spread modelling. Proceeding from the discussion above, this thesis will address the question whether structural models have explanatory power when applied on the European fixed income markets. Regarding the explanatory power of structural models, we will attempt to answer the following questions

- To what extent can yield spreads for European corporate bonds be explained by structural models with respect to
  - cross sectional average yield spreads?
  - time series variation in yield spreads?
  - individual bond yield spreads?
- Which structural and non-structural parameters affect unexplained yield spreads and how large are their corresponding influences?
- Is it possible to remove all dependence of the input parameters of the structural models?

1.3 Thesis Outline

Chapter 2 explains the mathematical, statistical and financial concepts used in this thesis. Chapter 3 discusses the data gathering, data preparation and removal of outliers and deficiencies in the data. Chapter 4 involves the details of the methodology of the empirical study. We also explain how the data and input parameters are structured in order to evaluate the model implied spreads. Chapter 5 presents the results of the study and Chapter 6 discusses the results and presents a novel extension to structural models of yield spreads. Chapter 7 concludes.

2 Theory & Concepts

2.1 Mathematical and Statistical Theory

2.1.1 The Brownian Motion

The Brownian motion is named after Robert Brown (1773–1858) who studied the motion of pollen seeds suspended in liquids. Brown's observations laid the ground for the discovery and explanation of the random movements of particles due to collisions on molecular level (Mazo, 2008). Since the discovery of the Brownian motion, the phenomenon has been subject to extensive research and proven to be of great importance in several academic disciplines such as finance and the valuation of derivatives.

Definition of Brownian Motion

Let $\{W_t\}_{t\geq 0}$ be a stochastic process defined on $\mathbb{R}$. Then $\{W_t\}_{t\geq 0}$ is a Brownian motion if

- $W_0 = x$
- For all times $t_1 \leq t_2 \leq \cdots \leq t_n$ we have that $W_{t_n} - W_{t_{n-1}} \perp\!\!\!\perp W_{t_{n-1}} - W_{t_{n-2}} \perp\!\!\!\perp \cdots \perp\!\!\!\perp W_{t_2} - W_{t_1}$
- For all times $0 \leq s < t$ we have that $W_t - W_s \sim N(0, t-s)$
- The function $t \mapsto W_{t_t}$ is almost surely continuous.

The standard Brownian motion satisfy all conditions above, with the exception that its initial value is $W_0 = 0$.

Stochastic Integral and Stochastic Differential Equations

Let W_t be a Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and adapted to the complete filtration $\{\mathcal{F}_t : t \geq 0\}$. For a function $f(t,x) \in \mathcal{L}^2$ it is now possible to define the stochastic integral as

$$\int_0^T f(t, W_t) \mathrm{d}W_t \tag{2.1}$$

where $f(t, W_t)$ is a stochastic process driven by W_t. Since the Brownian motion is almost surely of infinite variation, the Lebesgue integral approach to Equation 2.1 is not well defined (Mörters & Peres, 2010). However, since the Brownian motion is bounded in quadratic variation in probability, it is possible to define an integral with respect to W_t (Åberg, 2010). This is performed in the Itô formula for the standard Brownian motion wich states that

$$\begin{aligned} f(T, W_T) &= f(0,0) + \int_0^T \frac{\partial f(t, W_t)}{\partial t} \mathrm{d}t + \int_0^T \frac{\partial f(t, W_t)}{\partial x} \mathrm{d}W_t + \frac{1}{2} \int_0^T \frac{\partial^2 f(t, W_t)}{\partial x^2} \mathrm{d}t && (2.2) \\ &= f(0,0) + \int_0^T \mu(t, W_t)\, \mathrm{d}t + \int_0^T \sigma(t, W_t)\, \mathrm{d}W_t && (2.3) \end{aligned}$$

Due to notational convenience the relation in Equation 2.3 is often stated in the stochastic differential equation (SDE) form below

$$\mathrm{d}f(t, W_t) = \mu(t, W_t)\, \mathrm{d}t + \sigma(t, W_t)\, \mathrm{d}W_t \tag{2.4}$$

with

$$\begin{aligned} \mu(t, W_t) &= \frac{\partial f(t, W_t)}{\partial t} + \frac{1}{2} \frac{\partial^2 f(t, W_t)}{\partial x^2} && (2.5) \\ \sigma &= \frac{\partial f(t, W_t)}{\partial x} && (2.6) \end{aligned}$$

Geometric Brownian Motion

The stochastic process $X_t = f(t, W_t)$ is said to be a geometric Brownian motion (GBM) if it satisfies the following stochastic differential form

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t \tag{2.7}$$

where μ and σ are fixed constants. To solve the SDE for a given initial value of $X_0 = x_0$, the transform $Z_t = \ln(X_t)$ is applied. Using the Itô formula (Equation 2.3) with this transformation and rearranging the drift and diffusion terms, we arrive at the stochastic differential expression

$$\mathrm{d}Z_t = (\mu - \frac{\sigma^2}{2})\mathrm{d}t + \sigma \mathrm{d}W_t \tag{2.8}$$

$$Z_0 = ln(x_0) \tag{2.9}$$

which has the solution $Z_t \overset{d}{=} \ln(x_0) + (\mu - \frac{\sigma^2}{2})t + \sigma\sqrt{t}G$ where G is a standard normal distributed random variable. Taking exponential of the solution for Z_t, we return to the X_t domain and arrive at the GBM solution

$$X_t \overset{d}{=} x_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma\sqrt{t}G} \tag{2.10}$$

The expected value of the solution is given by $\mathbb{E}\, X_t = x_0 e^{\mu t}$ and its variance is $\mathrm{Var}\, X_t = x_0^2\, e^{2\mu t}(e^{\sigma^2 t} - 1)$. A simulation of 1000 identically distributed and independent geometric Brownian motions is summarised by Figure 2.1. The purpose is to let the reader familiarise with the concept of a stochastic process and statistical measures associated the process. (Björk, 2004)

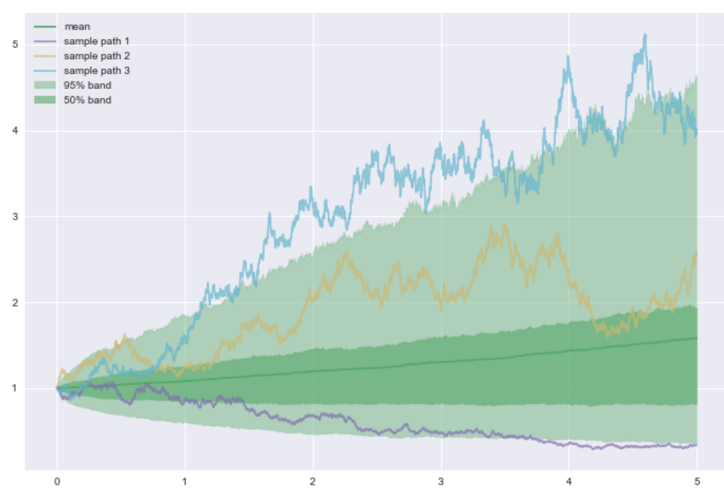


Figure 2.1: A visualisation of the trajectory for a GBM with parameters $\mu = 0.1$ and $\sigma = 0.3$, generated from 1000 Euler-Maruyama simulations. The figure shows the simulated mean, the 50 %- and 95 %-quantile bands and 3 sample paths from the simulation set.

The maximum likelihood parameter estimates of an observed geometric Brownian motion, $\{x_{t_0}, ..., x_{t_n}\}$, on an equidistant time grid are

$$\hat{\mu} = \frac{1}{n\,\Delta}\sum_{i=1}^{n} z_i \tag{2.11}$$

$$\hat{\sigma}^2 = \frac{1}{(n-1)\Delta}\sum_{i=1}^{n}(z_i - \hat{\mu}\,\Delta)^2 \tag{2.12}$$

where $z_i = \ln(x_{t_i}/x_{t_{i-1}})$ and $\Delta = t_i - t_{i-1}$. (Lindström, Madsen, & Nygaard Nielsen, 2015)

2.1.2 EM Algorithm

The EM algorithm provides a method of generating maximum likelihood parameter estimates in cases where data is incomplete, meaning that there are either missing or hidden data. In the general setting we assume observed data X and unobserved data Y and a set of parameters θ connected through a joint density function

$$p(X, Y \mid \theta) = p(Y \mid X, \theta)\, p(X \mid \theta) \tag{2.13}$$

The overall objective is to find a maximum likelihood estimate of the complete log-likelihood function $\ell(\theta \mid X, Y) = \log p(X, Y \mid \theta)$. However, finding the optimal parameters is often hard and analytic solutions may be unavailable. In this setting the EM algorithm provides a tractable and efficient method of iteratively optimising the log-likelihood function above. First, the E-step finds the expected value with respect to Y of the log-likelihood function given the observed X and the current value of the parameters $\theta^{(p-1)}$. This expected value of the log-likelihood function, denoted $Q(\theta \mid \theta^{(p-1)})$, is calculated as

$$Q(\theta, \theta^{(p-1)}) = \mathbb{E}_Y[\ell(\theta \mid X, Y) \mid X, \theta^{(p-1)}] = \int_Y \ell(\theta \mid X, y) p(y \mid X, \theta^{(p-1)}) \mathrm{d}y \tag{2.14}$$

where $p(y \mid X, \theta^{(p-1)})$ is the marginal distribution of Y given X and $\theta^{(p-1)}$. Second, the M-step finds the parameter value that maximises the expected log-likelihood such that

$$\theta^{(p)} = \arg\max_{\theta} Q(\theta, \theta^{(p-1)}) \tag{2.15}$$

The E and M-steps are iterated until a convergence in $\theta^{(p)}$ is reached. Each iteration is guaranteed to increase the log-likelihood and the algorithm is guaranteed to converge to a local maximum of the likelihood function. (Bilmes, 1998)

2.1.3 Linear Regression

We jump directly to review the multivariate linear regression model. The additive linear model for relating a dependent variable to p independent variables is

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_1 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i \quad \forall i = 1 \ldots n \tag{2.16}$$

where all ε_i are assumed to be independent identically distributed Guassian random variables with zero-mean and variance σ_ε^2. A more conventional form of expressing the multivariate linear model in 2.16, is by using matrix notations instead

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \tag{2.17}$$

In order to find parameters for the model we consider the ordinary least square optimisation problem as follows

$$\hat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}}{\arg\min} \; || \; \boldsymbol{Y} - \boldsymbol{X\beta} \; ||_2^2 \tag{2.18}$$

If an inverse to $\boldsymbol{X^T X}$ exists, the least square estimate of the coefficients is unique and given by $\hat{\boldsymbol{\beta}} = (\boldsymbol{X^T X})^{-1}(\boldsymbol{X^T Y})$. Moreover, it can be shown that the estimate is unbiased and distributed as $\hat{\boldsymbol{\beta}} \sim N(\boldsymbol{\beta}, (\boldsymbol{X^T X})^{-1}\sigma_\varepsilon^2)$, if the chosen model is correct. (Rawlings, Pantula, & Dickey, 2001)

The model proposed in Equation 2.16 can deal with categorical covariates, by introducing dummy variables. Given a categorical feature C with F-factors, these are encoded by the F independent variables $x_{C,1}, \ldots x_{C,F}$, each corresponding to one of the factors. An observation in the sample activates the independent variable corresponding to its categorical factor. The activated independent variable is assigned the value 1, while the other dummy variables are 0. It shall be noted that this representation can also be reduced to $F - 1$ variables, where the baseline factor is embedded in the intercept and activated when all dummy variables are zero.

2.1.4 LASSO

Least absolute shrinkage and selection operator (LASSO) is a method to estimate a linear regression model, which was first proposed by Tibshirani (1996). The LASSO model introduces a regularising

L_1-penalty term to the objective function, in order to constrain the size of the coefficients. Because of the nature of this constraint it tends to produce coefficients that are exactly 0 for less contributing covariates. Hence, the LASSO gives interpretable models containing only a selection of the most influencing covariates. The objective function in the LASSO method is set as

$$\hat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}}{\arg\min} \mid\mid \boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} \mid\mid_2^2 \; + \alpha \mid\mid \boldsymbol{\beta} \mid\mid_1 \tag{2.19}$$

This setting differs just slightly from the Ridge regression, which has L_2-penalty term instead. While the Ridge regression has a closed form solution, the solution to the LASSO model is a quadratic programming problem that can be solved with standard numerical methods. (J. Friedman, Hastie, & Tibshirani, 2001)

2.1.5 Gradient Boosting Regression

Gradient boosting regression is a machine learning algorithm used in this thesis to find a regression model that minimises a pre-determined penalty function. The main idea of the algorithm is to build a strong prediction model by iteratively combining weaker simple models, called learners. That is, given a set of observations $(x_i, y_i)_{1\leq i \leq n}$, where x_i is the input vector of an observation and y_i the corresponding scalar output, we want to find a model F_M such that the loss function $L(\boldsymbol{y}, F(\boldsymbol{x})) = \sum_{i=1}^{n} L(y_i, F(x_i))$ is minimised. Each consecutive stage, $m \in [1, M]$, of the regression model is constructed as an ensemble of functions

$$F_m = F_0 + \sum_{i=1}^{m} h_i \tag{2.20}$$

where F_0 is some initial model and $h_i \in \mathcal{H}$ are learners constricted to some set of functions. The learners are updated based on the present value of F_m such that

$$F_m = F_{m-1} + \underset{h \in \mathcal{H}}{\arg\min}\, L(\boldsymbol{y}, F_{m-1}(\boldsymbol{x}) + h(\boldsymbol{x})) \tag{2.21}$$

In general the optimisation problem in 2.21 is computationally demanding. This problem is addressed by considering the steepest descent step $\boldsymbol{g}_m \in \mathbb{R}^N$ defined by

$$g_{i,m} = -\left.\frac{\partial L(y_i, F(x_i))}{\partial F(x_i)}\right|_{F(x_i)=F_{m-1}(x_i)} \qquad \forall i \in [1, N] \tag{2.22}$$

Assigning $h_m = \rho_m \, \boldsymbol{g}_m$ where ρ_m is defined as $\rho_m = \arg\min_{\rho} L(\boldsymbol{y}, F_{m-1}(\boldsymbol{x}) - \rho \, \boldsymbol{g}_m)$ would minimise Equation 2.21 in an efficient manner. However, since the gradient is only defined on the training set, this optimisation would not be robust out of sample. Instead a tree regression is performed such that a tree T_m fits the negative gradients defined in Equation 2.22. This fitted tree regression is assumed to have J_m terminal regions $R_{j,m}$ for $j \in [1, J_m]$ in which T_m is constant. The constants are updated in each terminal region by

$$\gamma_{j,m} = \arg\min_{\gamma} \sum_{x_i \in R_{j,m}} L(y_i, F_{m-1}(x_i) + \gamma) \quad \forall j \in [1, J_m] \tag{2.23}$$

The learner is now defined using the terminal regions from the negative gradient tree regression and the constants derived in Equation 2.23 as

$$h_m(x) = \sum_{j=1}^{J_m} \gamma_{j,m} \mathbb{1}_{x \in R_{j,m}} \tag{2.24}$$

Lastly, the model is updated as in Equation 2.21 above

$$F_m(x) = F_{m-1}(x) + \sum_{j=1}^{J_m} \gamma_{j,m} \mathbb{1}_{x \in R_{j,m}} \tag{2.25}$$

This procedure is iterated until a pre-specified convergence condition is achieved. In order to constraint M and mitigate the risk of overfitting, such a conditions involve both minimising the prediction error in a test sample and introducing a regularising term. (J. Friedman et al., 2001)

2.2 Financial Theory

Now that the most central mathematical concepts for this thesis have been introduced to the reader, we shift focus to review the financial concepts and models applicable for this thesis.

2.2.1 Corporate Bonds

Corporate bonds are debt securities that are issued by corporations and sold to investors. Selling bonds to the primary market is a method for corporations to raise money and finance its investments today, in exchange for promised future payments to the bond holders. The future payments consist of a principal amount paid on the bond's maturity date T and most commonly also periodical payments, named coupons. The coupons are defined by a yearly coupon rate, which is a fraction of the principal amount, and a frequency (e.g. quarterly, semi-annually or annually) for which they are paid until maturity. A bond that doesn't pay any coupons is termed a zero-coupon bond (ZCB). The terms and conditions for the future payments are specified in the bond certificate.

Investors of corporate bonds face the risk that the issuer does not honour the payments as contracted. The rating agency Moody's Investors Service, often referred to as Moody's, define four scenarios that trigger a debt default:

1. a missed or delayed disbursement of a contractually-obligated interest or principal payment (excluding missed payments cured within a contractually allowed grace period), as defined in credit agreements and indentures;

2. a bankruptcy filing or legal receivership by the debt issuer or obligor that will likely cause a miss or delay in future contractually-obligated debt service payments;

3. a distressed exchange whereby 1) an issuer offers creditors a new or restructured debt, or a new package of securities, cash or assets, that amount to a diminished value relative to the debt obligation's original promise and 2) the exchange has the effect of allowing the issuer to avoid a likely eventual default;

4. a change in the payment terms of a credit agreement or indenture imposed by the sovereign that results in a diminished financial obligation, such as a forced currency re-denomination or

a forced change in some other aspect of the original promise, such as indexation or maturity. Compared to bank loans, the risk of not receiving the future payments is pooled among a larger group of participants. This opens up the possibility for firms to raise larger amounts of capital and find investors that accept increased risks.

2.2.2 Structural Models

In this thesis we will focus only on the Merton and Black Cox framework to model credit spreads. Notwithstanding the fact that involved extensions of structural models might be more realistic, Huang & Huang (2012)show that a wide class of structural models and extensions perform similarly when calibrating to historical default loss experience and equity returns. Consequently, Merton and Black Cox provide a framework for pricing of corporate debt which is intuitive, analytically tractable and robust over a wide class of structural models. The general setting of Merton and Black Cox is to assume that a firm is financed by a single zero-coupon bond with maturity T and equity. The bond value and equity make up the firm's asset value, which is assumed to evolve in the physical measure as the following

$$\mathrm{d}V_t = (\pi_t^{\mathbb{P}} + r_t - \delta_t)V_t\mathrm{d}t + \sigma V_t \mathrm{d}W_t^{\mathbb{P}} \tag{2.26}$$

where V_t is the firm value at time t and $W_t^{\mathbb{P}}$ is the standard Brownian motion under the physical measure. Furthermore, $\pi_t^{\mathbb{P}}$ is the asset risk premium, r_t risk-free rate and δ_t the firm's continuous payout of dividend and interests as a ratio of the firm value. Under the risk-neutral measure $\mathbb{Q}$ the dynamic of the firm value becomes

$$\mathrm{d}V_t = (r_t - \delta_t)V_t\mathrm{d}t + \sigma V_t \mathrm{d}W_t^{\mathbb{Q}} \tag{2.27}$$

where $W_t^{\mathbb{Q}}$ is the standard Brownian motion under the risk-neutral measure. Given the above dynamic of the firm value process corporate debt and equity is prised under the Black & Scholes framework of option pricing. The analogy to option pricing derives from the fact that debt holders are prioritised higher than equity holders in terms of repayment. Imagine a firm active only during one year and liquidated at the end of that year. If the firm remains solvent until liquidation, debt holders are repaid the nominal amount of debt and equity holders receive the residual firm value after debt repayment. However, in the event of default during the active year, debt holders are repaid the residual firm value after bankruptcy costs, and equity holders receive nothing. With this repayment structure, the firm equity dynamics are equivalent to that of a European call option with the nominal amount of debt as strike price. With similar arguments, debt dynamics are equivalent to a long risk-free position amounting the discounted nominal debt and a short position in a European put option with the nominal amount of debt as strike price. This simplified but intuitive interpretation of corporate debt and equity is the essence of structural models and the foundation to all extensions thereof.

Obviously, the default trigger is central for valuing corporate debt through the structural approach. Within the family of structural models, a default event will occur when the firm fail to meet the solvency conditions specified in each model. When the issuer of a bond defaults, the bond holders receive a fraction of the predetermined face value. Within the structural framework this fraction is often referred to as the recovery rate (RR) and it reflects the expected shortfall of firm value and the expected costs of bankruptcy. The RR is incorporated in the structural models through additional downside risk exposure yielding the following pay-off to the bond holder at maturity

$$\phi(\{V_t\}_{0\leq t\leq T}) = \begin{cases} K & \text{Issuing firm remains solvent} \\ f(K, \{V_t\}_{0\leq t\leq T}, RR, \Theta) & \text{Issuing firm defaults} \end{cases} \tag{2.28}$$

where $f(\cdot)$ represents a model specific pay-off at default and Θ holds model specific parameters. Note that in some models Θ may contain unobservable parameters. For a given variety of structural models, the probability of default (PD) and value of debt (P) can be calculated on a closed form using the following set of input parameters

$$PD(t, T, V_t, K, \sigma, r_{t,T}, \delta, \pi_t^{\mathbb{P}}, \Theta) \tag{2.29}$$

$$P(t, T, V_t, K, \sigma, r_{t,T}, \delta, RR, \Theta) \tag{2.30}$$

where Θ is defined as in Equation 2.28. An important note is that PD is calculated in the physical measure while P is derived through risk-neutral option pricing. This allows us to calibrate model parameters to real world observed default frequencies. Having defined the price function the model implied spreads are calculated by using the yield to maturity relation

$$e^{-(r+s)(T-t)}K = P(t, T, V_t, K, \sigma, r_{t,T}, \delta, RR, \Theta) \tag{2.31}$$

Solving for s in Equation 2.31 we get

$$s = -\frac{1}{(T-t)} \ln \Big[\frac{P(t, T, V_t, K, \sigma, r_{t,T}, \delta, RR, \Theta)}{K} \Big] - r \tag{2.32}$$

Merton

The idea of the original Merton model, is that the issuing firm can default only at the time of maturity and the default boundary is the nominal amount of debt. That is, the recovery rate is set to one, implying the following pay-off to bond holders at maturity

$$\phi(V_T) = \begin{cases} K & V_T \geq K \\ V_T & V_T < K \end{cases} \tag{2.33}$$

Under the Merton assumptions, the value at time t for a bond with maturity T, face value K, underlying asset process $\{V_t\}_{0 \leq t \leq T}$, asset volatility σ and payout ratio δ is described by

$$P_t = e^{-(T-t)r} \, \mathbb{E}^{\mathbb{Q}} \Big[K - (K - V_T)^+ | \mathcal{F}_t \Big] \tag{2.34}$$

where $\mathcal{F}_t$ is the natural filtration of the firm value process up to time t. Standard arguments in risk-neutral derivative pricing lead to

$$P_t = e^{-(T-t)r} K(1 - N(-d_2)) + e^{(T-t)\delta} N(-d_1) \tag{2.35}$$

where

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T-t}} \left[\ln\left(\frac{V_t}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)(T-t) \right] \\ d_2 &= d_1 - \sigma\sqrt{T-t} \end{aligned} \tag{2.36}$$

The probability of default for bond at a given time is, as mentioned above, equivalent to the probability that the underlying firm value will fall below the face value of debt at maturity. That is, the probability of default at time t is

$$PD(t, T) = \mathbb{P}(V_T < K | \mathcal{F}_t) = \mathbb{P}(V_t \; e^{(r + \pi^{\mathbb{P}} - \delta - \sigma^2/2)(T-t) + \sigma\sqrt{T-t} \; G} < K | \mathcal{F}_t) \tag{2.37}$$

where G is a normally distributed random variable such that $G \sim N(0, 1)$. The solution to Equation 2.37 is

$$PD^{\mathbb{P}}(t, T) = \mathbb{P}\Big(G < \frac{1}{\sigma\sqrt{T-t}} \left[\ln\left(\frac{K}{V_t}\right) + \left(r + \pi^{\mathbb{P}} - \delta - \frac{\sigma^2}{2}\right)(T-t) \right] \Big| \mathcal{F}_t \Big) \tag{2.38}$$

Binary Merton

The binary Merton model is closely related to the original version. The important difference between the models is that the binary Merton has an exogenously given recovery rate. That is, at maturity the bond holder receives the following pay-off

$$\phi(V_T) = \begin{cases} K & V_T \geq K \\ RR \cdot K & V_T < K \end{cases} \tag{2.39}$$

Since Merton and binary Merton model have identical solvency conditions we can deduce that the probability of default for given input parameters are equal for the two models. That is, for the binary Merton model the physical measure probability of default at time t for given structural input parameters is defined as

$$PD^{\mathbb{P}}(t,T) = N\Big(\frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{V_t}{K}\right) + \left(r + \pi^{\mathbb{P}} - \delta + \frac{\sigma^2}{2}\right)(T-t)\right]\Big) \tag{2.40}$$

Given the structural input parameters and the default probability function derived above, the risk-neutral price of a bond is described by

$$P_t = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\Big(\phi(V_T)\Big) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\Big(\mathbb{1}_{V_T \leq K} RR \cdot K + \mathbb{1}_{V_T > K} K\Big) \tag{2.41}$$

Since we calculate the price under the risk-neutral measure, Equation 2.40 is transformed into risk-neutral probabilities by removing the credit risk premium $\pi^{\mathbb{P}}$. The resulting price function becomes

$$P_t = e^{-r(T-t)}\Big(PD^{\mathbb{Q}}(t,T)RR \cdot K + (1 - PD^{\mathbb{Q}})K\Big) \tag{2.42}$$

Black Cox

The Black Cox model values corporate debt as a barrier option with down and out structure. Under the Black Cox framework, the firm can default at any time on or before maturity if the corresponding value process falls below a predetermined fraction, d, of debt. That is, the firm defaults at time τ defined as $\tau = \inf\{t : V_t < dK\}$. By the event of default the bond holder receives a fixed amount, $RR \cdot K$ yielding the following pay-off function at maturity

$$\phi(\{V_t\}_{0\le t\le T}) = \begin{cases} RR \cdot K & \tau \le T \\ K & \text{otherwise} \end{cases} \tag{2.43}$$

As shown in Equation 2.43 the expected pay-off conditioned on default is deterministic and thus independent of the firm value process. This characteristic of the pay-off function is unwanted but necessary to achieve analytical tractability. The cumulative default probability at time t, for a bond with maturity T, leverage ratio $L = K/V_0$, underlying asset process $\{V_i\}_{0\le i\le T}$, asset volatility σ, payout ratio δ and default boundary d is derived in Bao (2009) as

$$PD^{\mathbb{P}}(t,T) = N\Big[\frac{\log(dL) - (r + \pi^{\mathbb{P}} - \delta - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\Big] + \tag{2.44}$$

$$\exp\Big[\frac{2\log(dL)(r + \pi^{\mathbb{P}} - \delta - \frac{\sigma^2}{2})}{\sigma^2}\Big] N\Big[\frac{\log(dL) + (r + \pi^{\mathbb{P}} - \delta - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\Big] \tag{2.45}$$

This default probability is calculated in the physical measure when calibrating to historical default rates. As familiar from conventional bond pricing theory, the price of a bond with pay-off function as in Equation 2.44 can be written as

$$P_t = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\Big(\phi(\{V_t\}_{0\le t\le T})\Big) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\Big(\mathbb{1}_{\tau\le T} RR\cdot K + \mathbb{1}_{(\tau\le T)^{\complement}} K\Big) \tag{2.46}$$

Using Equations 2.46 and 2.44, it is possible to construct an analytic expression for the theoretical bond price under the Black Cox framework. The resulting price equation is

$$\begin{aligned} P_t &= e^{-r(T-t)}\mathbb{E}^{Q}\Big[\phi(\{V_t\}_{0\le t\le T})\Big] \\ &= e^{-r(T-t)}\Big(PD^{\mathbb{Q}}(t,T)RR\cdot K + (1 - PD^{\mathbb{Q}}(t,T))K\Big) \\ &= e^{-r(T-t)}\Big(K(1-(1-RR)PD^{\mathbb{Q}}(t,T))\Big) \end{aligned} \tag{2.47}$$

Using the yield to maturity relation and Equation 2.47, the bond spread is calculated as

$$e^{-(r+s)(T-t)}K = e^{-r(T-t)}K(1-(1-RR)PD^{\mathbb{Q}}(t,T)) \tag{2.48}$$

Finally, as implied by Equation 2.48, we get the following expression to calculate bond spreads in the Black Cox setting

$$s = -\frac{1}{T-t}\log(1-(1-RR)PD^{\mathbb{Q}}(t,T)) \tag{2.49}$$

3 Data

As mentioned above, the structural approach to credit spread modelling requires both time series data and technical metadata for all bonds within the scope of this thesis. As of today there is no centralised and complete reporting environment for the European bonds. Fortunately, the required data is made available to an acceptable extent by combining data from the sources: Bloomberg Professional API, Compustat - Capital IQ, Reuters EIKON, ECB Statistical Warehouse and the German Bundesbank. However, using data which is aggregated and combined from different sources with varying reporting standards, requires great forethought and extensive preparatory data modifications. This section aims to explain and motivate the details of the data gathering and the preparatory modifications performed on the data before modelling.

3.1 Selection of Bonds

The bond sample used for the empirical modelling is a subset of all available bonds monitored by Bloomberg's fixed income security database called *SRCH*. A bond is included if and only if all the following criteria are satisfied

- The bond is either active or inactive
- The bond issuer has rating data from at least one of S&P, Moody's or Fitch
- The bond issuer is not a financial corporation or a government
- The bond issuer is not a private company[1]
- The bond issuer's country of domicile is any European country
- The bond type is either fixed coupon, zero coupon or defaulted

[1] This is the search criteria used to distinguish public companies in Bloomberg *SRCH*.

- The bond matures on or after 2000-01-01

With these restrictions, the dataset constitutes of both active, inactive and defaulted bullet bonds issued by European firms. Defaulted bonds are included in order to minimise the effects of a survival biased sample. The survival bias is reduced but still prevalent, since we require that the issuer's equity is public. Thus the defaulted bonds in our sample are issued by firms that have had insolvency problems, but managed to continue to its operations through a reconstruction process or debt write-down. Financial corporations, such as insurance companies and banks, are excluded due to their distinctiveness in capital structure and regulatory environment. Lastly, the issuing firms are required to be public due to the simple fact that equity data is necessary as model inputs. With these filter restrictions the Bloomberg SRCH database generates a sample of 3992 bonds from 702 distinct firms.

3.1.1 Merge of Bloomberg and Compustat data

The Bloomberg API is used to gather bond level metadata and historical market data of equity and bonds, while Compustat is used to obtain accounting data for the issuing firms. To be able to merge data from the Bloomberg API with Compustat, it is necessary that the issuing firms are monitored by both databases. The linkage between the database services is enabled by identifying issuers through their ISIN (International Securities Identification Number). In order to deal with complex subsidiary structures, the issuing firm for each bond is identified using the Bloomberg field *ISSUER_PARENT_EQY_TICKER*. In this way it is possible to aggregate bonds issued by local branches to the parent entity and regard the parent company's balance sheet in Compustat.

The currency used in this study is chosen to be euro, due to convenience following characteristics of the sample set. The daily equity quotes from the Bloomberg API are all quoted in euro, thanks to its internal currency conversion engine. On the other hand the Compustat accounting data is quoted quarterly in the issuer's accounting currency. Quarterly historical exchange rates are gathered from ECB's Statistical Data Warehouse and used to convert the accounting currency to euro when needed. Bonds from firms with an accounting currency that is not included in ECB's Statistical Data Warehouse are removed.

The Compustat database is comprehensive and covers the majority of the firms in the first draft of the sample. However, since the firms originate from different industries and countries of domicile, the accounting standards lack general consistency through the whole dataset. This problem was addressed in 2005 when the International Financial Reporting Standards (IFRS) became mandatory for all companies listed in Europe (European Commission, 2016). Another issue with the Compustat database is that there is a delay between the publishing of financial statements and when the data is available at Compustat. The consequence of this is that accounting data after the second quarter of 2016 is scarce. In order to ensure comparability and a consistency of sample depth over time, the time span for the dataset narrowed to include bonds that are active in some part of the period between 2005-01-01 and 2016-06-30. The number of firms that meet the above requirements is reduced to 570, that collectively have 2995 bonds.

3.1.2 Liquidity Requirements and Missing Quotes

The time series of equity and bond quotes are central components to our study, as they are input respectively target measures for the models. Therefore we require that the data quality of these observations are reliable, comparable and robust. An important assumption in the Black Scholes framework for option pricing is that the underlying asset is perfectly liquid and implicitly the contingent claim as well (Black & Scholes, 1973). Therefore it is reasonable to disregard firms and bonds with inferior trading frequency.

The equity is traded on public stock exchanges across Europe. These market places provide high transparency of trades and are generally considered liquid markets. Different stock exchanges may have different business days and bank holidays, causing missing data points in the time series. Besides market holidays, a missing data point could also be explained by the fact that no deals are closed in a given day. To mitigate the potential illiquidity problem, an issuer's stock must have quotes on at least 200 of the trailing 252 business days to be included as a observation. A few missing data points is not considered a problem, since the issuer's market capitalisation is aggregated to a monthly mean. The firm's interday market capitalisation is used to calculate its equity and asset volatility, which is a calculation considered acceptably robust when missing up to 52 quotes of the trailing 252 business days.

The bonds on the other hand are in general traded over-the-counter (OTC) between a limited number of market participants, causing an issue of quote opacity and reduced liquidity. In comparison to equity, bond deals are characterised by less frequent trades and each trade involving a high volume of bonds. As a consequence of bond trades not needing to comply with the transparent reporting standards of public stock exchanges, there is a possibility that OTC deals are never reported to a pricing source. The Bloomberg database is to our best knowledge the most fair inter-day pricing of bonds that we can obtain. Since Bloomberg aggregates trade data from several pricing sources, it is a good attempt to provide an as complete market valuation as possible. However, within the active period of a bond, it is hard to know whether a missing quote is due to trades done on a non-covered exchange or the nonexistence of any completed trades. Since the API don't give access to the daily volumes traded, we can only assume that the daily quotes listed by Bloomberg are backed by an acceptable number of trades.

The dataset of daily bond quotes that are retrieved from the Bloomberg API, reveal that a noteworthy fraction, 1141 of 3995 bonds, don't have any trades registered. In total 2521 bonds have more than 100 registered end of day quotes in Bloomberg. Yet some of these bonds could still suffer from liquidity issues, since these trades do not necessarily occur on consecutive business days. For a bond to be included as a monthly observation, we require that it has at least 15 quotes reported in Bloomberg for the given months.

When studying the dataset in more depth some inconsistencies and extreme outliers were detected and removed. For example, this manual cleaning task included removal of errant price quotes [2] and one bond trading at spreads far below zero.

To summarise the restrictions posed in this section concerning bond and equity liquidity, a bonds and its issuing firm need to fulfil the following requirements to qualify as a bond month observation.

- The issuing firm has equity price quotes on at least 200 of the trailing 365 days from the month's end date.

- The bond has price quotes on at least 15 of the business days within the current month.

[2] A bond was traded at levels more than 10 times higher than its nominal value of 100.

When all modifications and restrictions are implemented, the dataset consists of 402 firms and 2 171 bonds and 91 510 end-of-month observations.

3.1.3 Data limitations

Issue Currency

In order to avoid unnecessary complexity, we chose to exclude bonds with other issue currency than EUR. This choice of sample reduction is motivated by the fact that the bond yields have systematic differences depending on the currency it is issued in. For example the average yield for EUR denominated bonds is 2.94 %, while the average yield in GBP is 4.51 % and CHF is 1.14 %. The natural explanation for these differences is the variations in federal interest rates for different currencies. The 12-month LIBOR rate during the period 2005-01-01 to 2016-06-30 was 1.88 % for EUR, 2.63 % for GBP and 0.87 % for CHF respectively (ICE Benchmark Administration Limited (IBA), 2017). Clearly, the bond yields are closely related to a premium on top of the risk-free interest rate denominated in the same currency. Other explanations for these variations include varying views of political and inflation risks, as well as market discrepancies with respect to demand, supply and risk appetite.

Time to maturity

In other empirical studies of structural models, it has been shown that bonds with very short or very long residual time to maturity are difficult to model. Therefore these observations are usually disregarded from the sample. Eom, Helwege, and Huang (2004) include only bonds that mature within the span of 1 to 30 year, while Feldhütter and Schaefer (2016) look at bonds with 3 to 30 years of residual time to maturity. We have chosen to restrict our sample to bonds with maturies between 1 and 20 years, in order to match well with Moody's table for expected default frequencies.

After these final reduction operations we arrive at a sample set that consists of 1 116 bonds (289 firms) observed over 138 months (2005-01-01 to 2016-06-30). In total the dataset has 50 222 bond-month observations.

3.1.4 Selection Bias

Since we perform an extensive cleaning of the initial bond sample, there is a mentionable and unavoidable risk that the sample is no longer entirely representative of the true population. When bonds with unsatisfactory level of data quality are removed, this selection bias becomes even more relevant. This possible selection bias is unavoidable in order to maintain a consistent and generalised model approach. The consequence of the selection bias is that our model results will only be applicable to bonds that meet the same requirements to qualify for the sample set.

3.2 Interest Rate Parameters

The historical yield curves of European risk-free interest rates are retrieved from the German Bundesbank. These yield cureves are based on market quotes of listed German federal securities denominated in euro. Many investors regard the German Bunds as the most reliable federal security in the Euro-zone, which therefore functions as a good benchmark for the risk-free rate. The yield curves are compiled and published on a monthly basis by the German Bundesbank using the Svensson method (Svensson, 1994), to approximate the term structure. At the end of each month the yields are reported for residual maturities of 0.5 years and integer years from 1 to 30. Given a bond observation, we match its residual maturity with the same yield quoted by the German Bundesbank in that specific month. When a bond has residual maturity between two integer years in the term structure, the risk-free rate is obtained by a linear interpolation.

3.3 Rating

As a constraint in the bond data retrieval from Bloomberg, described in Section 3.1, we require that the bond issuer has rating data from at least one of the top three rating agencies: Standard & Poor, Moody's or Fitch. Unfortunately, as it turned out, this query suffered from two weaknesses. First, the rating agencies use different criteria in their rating assessments and different labelling. Therefore comparability across rating agencies is neither straightforward nor unambiguous. Furthermore, it was later on noticed that Bloomberg regards a withdrawn rating as a non-empty rating field in our query, implying that such bonds are included in our sample. To avoid additional complexity in this matter, we use the ratings reported by Moody's to form three rating groups: 'Investment Grade' (IG), 'Sub-Investment Grade' (SG) and 'Without Rating' (WR). All Aaa-Baa rated bonds are labelled 'Investment Grade' while Ba-C are labelled 'Sub-Investment Grade'. The bonds without rating from Moody's are grouped together into 'Without Rating' (WR). The latter group also includes bonds in which Moody's have withdrawn rating. Secondly, the rating data in Bloomberg is an instantaneous snapshot taken on the date of the data request[3]. Despite great efforts to obtain historical rating changes, we did not manage to compile this to an acceptable extent. Instead, the current rating for the issuers is back-filled and assigned to all historic observations of the same issuer.

[3] The data from Bloomberg was retrieved on 2017-03-01.

4 Method

"Successful modelling of a complex dataset is part science, part statistical methods, and part experience and common sense." (Hosmer Jr, Lemeshow, & Sturdivant, 2013)

When assessing the overall explanatory power of structural models, the objective is to understand to what extent the fundamental ideas of Merton and Black Cox are incorporated in the market's valuation of corporate debt. Departing from a large set of observed bond data there are numerous ways to translate the input data to comparable model output. The limited academic research done on the European bond market, brings an uncertainty (and freedom) regarding how to construct input variables that are reasonable for the model assumptions and that are comparable to the conducted research on American bonds. Furthermore, the method of evaluating explanatory ability from model output to observed data is neither straightforward nor univocal. Different choices of input data structures and assessment structures will generate varying results - each requiring its own interpretation - and it is therefore important to understand the methodology in detail. This section concerns, (1) the construction of input data, (2) model calibration and (3) the structure of the model assessment. The reader should be familiar with the main concepts explained in the theory chapter, as these concepts will now be applied to a practical setting.

4.1 Structural Models Input

As mentioned above, the bond data consists of a snapshot of the basic bond meta data such as issuing date, maturity date, coupon rate, rating and coupon frequency. In addition, we have access to time series data for the firm's equity and balance sheet, as well as the observed bond quotes. In line with Feldhütter and Schaefer (2016) and Duffee (1998) a sample set of monthly bond observations is constructed as the foundation to the assessment of structural models. More specifically, this means that time series data is averaged on a monthly basis, while balance sheet data is set such that all months within a given quarter have the same accounting data as reported for the quarter a posteriori. The bond meta data is not time dependent and remains constant within the active period of each bond. The remaining features and model inputs are generated and estimated as functions of the raw data and, where applicable, put together on the monthly averaged bond observation form. In Sections 4.1.1 to 4.1.5 the feature estimation methods and corresponding assumptions are explained in detail.

4.1.1 Asset Volatility

As familiar, the firm value process is unobservable thus making it hard to estimate the volatility. There are a broad range of alternatives that have been used in the literature, these are further reviewed and discussed in Appendix A.1. We choose to implement the KMV[1] method to obtain the asset volatility as it is closely aligned to the Merton framework. The KMV estimation of asset volatility is based on the one-to-one mapping, from firm value to equity value which is observable on the market. Given $k+1$ historically observed daily equity quotes $\{E_{t_0}, E_{t_1} ... E_{t_k}\}$ for a certain firm and an initial asset volatility estimate $\hat{\sigma}_V^{(0)}$ it is possible to imply the asset value by using the Black-Scholes call option formula relationship. This corresponds to the E-step in the EM-algorithm described in Section 2.1.2. At iteration p in the EM-algorithm, the implied firm value is calculated by numerically finding the root to the Black-Scholes call option formula. This procedure is denoted as

$$V_{t_i}^{(p)} = \mathrm{BSCall}^{-1}(E_{t_i} | \hat{\sigma}_V^{(p)}, \Theta) \quad \forall t_i \in \{t_0 ... t_k\} \tag{4.1}$$

where Θ contains the firm specific variables needed to compute the Black Scholes call option formula[2]. Next the M-step of the EM-algorithm is performed, which corresponds to finding the pa-

[1]The firm KMV is named after Kealhofer, McQuown and Vasicek, the founders of the company in 2002. It has since been sold to Moody's.

[2]If a company has several outstanding bonds at time t, the company's maturity T is set to the average of the bonds. The risk-free rate is also approximated by the average risk-free rate between t and the individual the bonds' maturities. The firm's leverage ratio is specified in the next section (4.1.2). However, since it relies on reported balance sheet data from Compustat there will almost surely be quarterly jumps in the leverage ratio. The jumps in debt can be regarded as a partially observable stochastic process. Consequently, the jumps in debt are also prevalent in the implied asset value process, which contradicts the assumption of a continuous geometric Brownian motion. As a simple solution to mitigate unwanted jumps, we linearly interpolated the debt for time points between the

rameters that maximises the log-likelihood function for the sequence of implied asset values.

$$\underset{\{\mu_V,\sigma_V\}}{\arg\max} \sum_{i=0}^{k} \ln(f(V_{t_i}^{(p)}|\mu_V,\sigma_V)) \tag{4.2}$$

Given the assumption that the firm value process is a geometric Brownian motion, the maximum-likelihood estimate of the diffusion parameter σ, can be calculated as shown in Section 2.1.1

$$\sigma_V^{(p+1)} = \frac{1}{k-1}\sum_{i=1}^{k}(r_{t_i} - \bar{r})^2 \quad \text{where } r_{t_i} = \ln\left(\frac{V_{t_i}^{(p)}}{V_{t_{i-1}}^{(p)}}\right) \tag{4.3}$$

The E- and M-steps are repeated until the parameter estimate of σ has converged. The condition for convergence is chosen to be when the absolute relative change falls below a predetermined threshold, that is $|\sigma_V^{(p+1)} - \sigma_V^{(p)}|_1 < \varepsilon$.

Due to notational challenges we have left out a subtle detail in the description of our implementation. Instead of calculating a constant asset volatility for each firm, $\sigma_{V,f}$, as the formulas above suggest, our implementation allows a time varying asset volatility. For each day t_i the asset volatility is estimated using the trailing 252 business days of implied firm value. What motivates the time-varying asset volatility is that a constant volatility estimate contains an inconsistent mixture of both future and historic information, depending on which point in time is regarded. Consequently the early time points in the sample, will have an asset volatility estimate dependent on a high fraction of future information, while the opposite applies for more recent time points.

4.1.2 Leverage

Leverage ratio measures the relation between a corporation's debt and equity and is therefore closely related to credit risk. The definition of the measure varies within different applications, but will in this thesis be defined as

$$L = \frac{D_b}{D_b + E_m}$$

where E_m is the market value of equity and D_b is the book value of debt. The market value of equity is calculated on a daily basis and on firm level as the product of shares outstanding and price per share. The book value of debt is estimated as the sum of long term debt (dlttq)[3] and debt in

quarterly reporting dates.

[3] The text codes in parenthesis are field codes for the Compustat - Capital IQ data. Full variable descriptions are available in Appendix A.6

current liabilities (dlcq) from the subsequent quarter end in relation to the data date. For a given month, t, and firm, f, the monthly averaged leverage observation is defined as

$$L = \frac{D_{b,f,t}}{D_{b,f,t} + \overline{E}_{m,f,t}} \tag{4.4}$$

where $\overline{E}_{m,f,t}$ is the t-month average of daily market cap observations for firm f. The combining of market value of equity and book value of debt is a noteworthy simplification in the model input. Again this is a simplification made due to the limited observability of market value of debt. However, most bonds are traded close to par and the debt book value is therefore believed to constitute a good proxy for the market value of debt. This view of using book value as a proxy of market value is in line with Eom et al. (2004) and Feldhütter and Schaefer (2016).

4.1.3 Payout Ratio

Financial cash flows such as dividends and interest rate payments affect the firm value process and are incorporated in the structural models through the payout ratio. The yearly outflow of cash to financial stakeholders is estimated as the sum of yearly total dividend payments (dvty), yearly interest and related expenses (xinty) and yearly purchase of common and preferred stock (prstkcy). For a given firm f and month t in year y, the payout ratio is calculated as

$$\delta_{f,t} = \frac{\mathrm{FCF}_{y-1,f}}{V_{t,f}} \tag{4.5}$$

where $V_{t,f}$ is the monthly averaged firm value defined by $V_{t,f} = D_{b,f,t} + \overline{E}_{m,f,t}$ with the same notation as in 4.1.2 and $FCF_{y-1,f}$ is the sum of dvty, xinty and prstkcy for year $y-1$ and firm f.

4.1.4 Recovery Rate

Our implementations of binary Merton and Black Cox both require an exogenously given estimate of the recovery rates at default. Moody's Investors Service (2017) estimate of the long term average recovery rate for senior unsecured bonds amounts to 37.5 %. In line with this historical average, we set the recovery rate to be 40 % for all bonds in the sample.

4.1.5 Summary Structural Models Input

The observed monthly structural inputs are summarised in Table 4.1 below. The data contains meta data of European bullet bonds actively traded within the period from 2005-01-01 to 2016-06-30. In total, the dataset contains 50 222 bond month observations generated by 1 116 bonds and 289 unique firms. The data sources are Bloomberg professional and Compustat - Capital IQ.

Table 4.1: Summary statistics. The actual spread is expressed in basis points calculated as the difference between yield to maturity and the spot risk-free rate with corresponding maturity. The equity volatility is the standard deviation of the issuing firm's equity log-returns, based on the trailing 252 business days. Time to maturity is given in years. The details of our derivations of asset volatility, leverage ratio and payout ratio are described in Sections 4.1.1, 4.1.2 and 4.1.3 respectively. A full description of the rating encodings IG, SG and WR is available in Section 3.3.

		mean	std	min	5%	50%	95%	max
Actual Spread	IG	113.217	72.064	-84.147	35.187	97.287	248.040	1426.347
	SG	262.167	249.232	2.062	58.659	215.090	594.400	4828.649
	WR	239.386	436.576	-9.633	47.844	167.381	576.289	25194.930
Equity Volatility	IG	0.280	0.105	0.062	0.167	0.257	0.485	1.067
	SG	0.364	0.127	0.152	0.198	0.344	0.613	1.007
	WR	0.311	0.129	0.062	0.178	0.279	0.561	1.482
Asset Volatility	IG	0.187	0.086	0.046	0.102	0.167	0.351	0.960
	SG	0.211	0.104	0.056	0.096	0.178	0.434	0.784
	WR	0.206	0.102	0.022	0.086	0.188	0.393	0.957
Leverage Ratio	IG	0.379	0.165	0.025	0.140	0.359	0.661	0.884
	SG	0.487	0.216	0.002	0.140	0.494	0.830	0.951
	WR	0.390	0.206	0.014	0.114	0.358	0.812	0.963
Payout Ratio	IG	0.033	0.017	0.000	0.007	0.032	0.062	0.098
	SG	0.036	0.017	0.000	0.009	0.035	0.063	0.096
	WR	0.029	0.017	-0.001	0.005	0.029	0.056	0.353
Time to Maturity	IG	5.969	3.789	1.003	1.468	5.153	13.752	19.995
	SG	4.762	3.006	1.003	1.382	4.111	10.749	19.995
	WR	4.850	3.168	1.003	1.397	4.173	11.216	19.942

4.2 Model Calibration

Compared to the original Merton model, the Black Cox framework provides an additional degree of freedom through its down and out structure. As the firm's default boundaries are unobservable the variables are estimated by calibrating the model implied default probabilities to match historical averages. The calibration target is the reported average issuer weighted default frequencies tracked by Moody's within the period between 1920 and 2016. In order to understand the logic behind our calibration method, one first needs to understand the methodology of Moody's reporting of expected default frequencies. At the beginning of each year (1920-2016) rated firms within the same rating category form a cohort group. For each consecutive year Moody's track the fraction of firms that have defaulted within each cohort and time horizon. More specifically, for a given year, y, rating group z, and time interval, t, the marginal default probability $d_y^z(t)$ is calculated as

$$d_y^z(t) = \frac{x_y^z(t)}{n_y^z(t)} \tag{4.6}$$

where $x_y^z(t)$ is the number of defaulted firms within the cohort and $n_y^z(t)$ is the size of the cohort. The cumulative default probability for the same cohort as above, and a given investment horizon T is calculated as

$$D_y^z(T) = 1 - \Pi_{t=1}^{T}(1 - d_y^z(t)) \tag{4.7}$$

The average cumulative default probability for the investment horizon T over a set of years Y is defines as

$$\overline{D}^z(T) = 1 - \Pi_{t=1}^{T}(1 - \overline{d}^z(t)) \tag{4.8}$$

where

$$\overline{d}^z(t) = \frac{\sum_{y \in Y} x_y^z(t)}{\sum_{y \in Y} n_y^z(t)} \tag{4.9}$$

Moody's cumulative average issuer weighted default probabilities are shown in Table A.1. Corollary, depending on the state of the economy default probabilities are subject to change but should correspond to Table A.1 on average over time. This perspective of the over time average default probabilities is hereafter referred to as through the cycle. Since the Moody's default probabilities are target variables when estimating the default boundary, it is reasonable to imitate Moody's methodology as close as possible. This is achieved by calibrating implied default probabilities through the cycle. In Section 2.2.2 the Black Cox implied physical cumulative default probability is derived as

$$PD^{\mathbb{P}}(d,\Theta) = N\Big[\frac{-\log(dL)-(r+\pi^{\mathbb{P}}-\delta-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\Big] + \exp\Big[\frac{-2\log(dL)(r+\pi^{\mathbb{P}}-\delta-\frac{\sigma^2}{2})}{\sigma^2}\Big] N\Big[\frac{-\log(dL)+(r+\pi^{\mathbb{P}}-\delta-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\Big] \tag{4.10}$$

where d is the percentage default boundary expressed as a fraction of the face value of debt and Θ represents the remaining input variables to the model. Calibrating towards real world default probabilities, we use the physical default probability implied by the Black Cox framework. This important detail adds the credit risk premium $\pi^{\mathbb{P}}$ to the input parameter set Θ. In line with Chen et al. (2009) we use a constant Sharpe ratio of $\theta_s = 0.22$ in order to calculate the credit risk premium as

$$\pi^{\mathbb{P}} = \theta_s \cdot \sigma_V \tag{4.11}$$

For a given cohort with ratings z and time horizon interval τ we calibrate the default boundary d_τ^z such that it minimises

$$\arg\min_{d_{z,\tau}} \mid \frac{1}{N}\sum_{y=1}^{N} \overline{PD}_y(d_{z,\tau}) - PD_{z,\tau} \mid^2 \tag{4.12}$$

where $\overline{PD}_y(d_{z,\tau})$ is the average model implied physical default probability on year y and cohort group $\{z,\tau\}$. $PD_{z,\tau}$ is the corresponding target default probability for the cohort $\{z,\tau\}$ given by Moody's (Appendix A.1). With this methodology, the yearly default rate is allowed to vary within each cohort, but corresponds to the observed target default rates on average over time. Moody's Investors Service (2006)

4.3 Model Assessment

Our overall objective in this thesis is to investigate the structural credit risk models' ability to explain European bond yield spreads. In order to evaluate the descriptive power of the structural models tested in this thesis, we need a fair and comparable method of model assessment. A well performing descriptive model generates model output consistently close to the observed data. However, another equally important but not as obvious aspect of the model's descriptive power is to what extent the input data depends on the actual yield spread observations and model residuals. The break down of the model assessment involves (1) an dependency analysis of the relation between all available input features and the output data and (2) model prediction and residual analysis.

4.3.1 Dependency Analysis

The initial phase aims to identify dependencies between input data and output data before modelling yield spreads within the structural frameworks. To identify the most influencing input parameters a LASSO regression is performed with all available data as explanatory variables. As the input data is not completely comprehensive, missing values are imputed with its corresponding mean value over time. To obtain comparable regression coefficients, the input data is standardised such that each feature has zero mean and unit variance. The most influencing features are found by identifying non-trivial covariates with respect to absolute coefficient value after the LASSO regression. In order to include as much descriptive information as possible in the dataset, additional auxiliary features are generated. Below is a summary of the additional features included in the regression input

De Facto Seniority

In Bao and Hou (2016) they show that de facto seniority has a non-trivial influence on market yield spreads of corporate bonds. De facto seniority is a measure on the amount of debt that is due prior to a given bond's maturity. Intuitively, if most of the firm debt is due prior to a bond's maturity, the bond is considered more risky in relation to earlier maturing bonds. Analogously, if a bond matures before the majority of the firm debt, the bond is considered less risky. Given a firm with n loans and bonds outstanding $\{K_1, K_2, ..., K_n\}$ with increasing time to maturity, such that

$TTM(K_1) \leq TTM(K_2) \leq ... \leq TTM(K_n)$, the de facto seniority for bond K_t is defined as

$$DFS(K_t) = \begin{cases} 0 & t = 1 \\ \sum_{i=1}^{t-1} K_i / \sum_{i=1}^{n} K_i & t = 2 \dots n \end{cases} \tag{4.13}$$

Amount Issued Relative

The amount issued relative is a measure of an individual bond's issued amount in comparison to the firm's total debt. The feature holds information about whether the bond constitutes for a large or small fraction of the total book value of debt. This measure brings additional bond level information since the models assume that the face value each bond is equal to the issuing firm's total debt. The amount issued relative is calculated as

$$AIR = \frac{K_i}{D_b} \tag{4.14}$$

EBITDA to firm value ratio

Structural models do not include measures on the profitability of the issuing firms. However, a well performing firm with high profitability might be considered less risky than a non-profitable peer. In order to take profitability in consideration, we introduce the EBITDA to firm value ratio defined as

$$E_R = \frac{EBITDA}{D_b + E_m} \tag{4.15}$$

FX Converted Balance Sheet

Issuing firms from several different countries are included in the bond month dataset. As a consequence the accounting currencies for these firms are not completely uniform. While a majority of the firms report in EUR, a mentionable set of firms have other accounting currencies such as GBP, CHF or SEK. All financial fields in quoted in non-EUR are converted to EUR using data from the European Central Bank. The FX converted balance sheet feature is a factor variable that states weather a firm's accounting data has been converted to EUR or not.

4.3.2 Descriptive Power of Structural Models

The second phase aims to explore the descriptive power of the structural models and calibration methods discussed in this thesis. The reader should be aware that the are a vast number of ways of evaluating and interpreting a model's performance, which may lead to different conclusions. Therefore our intention is to view each model from several aspects in order to get a comprehensive assessment of the performance. Each structural model will be examined on the three levels:

1. Group averages and median spread
2. Individual bond spread
3. Time series spread

We believe that these three perspectives are collectively exhaustive to understand the descriptiveness of the models. As a further motivation for these choices we will outline how they have been implemented by other researchers.

1) The majority of the previous empirical studies conducted on structural models have mainly evaluated the performance on bond groups, generally based on rating and residual time to maturity. For example, in the seminal paper by Huang & Huang (2012), they construct a representative firm for each group and compare the model results to the actual average spreads within the groups. Their evaluation methodology corresponds to a one-to-one comparison on an average level. In the way the assessment is constructed, heterogeneity among firms is vanished, and thus the model is robust against potential outliers. Furthermore, due to the convex characteristics that structural models exhibit, Jensen's inequality suggests that the representative firm approach will undershoot compared to applying the model on several firms and then averaging. This topic is addressed in Feldhütter and Schaefer (2016), in which they mitigate the convexity bias by applying the model on individual firms and then comparing the group averages. As expected Feldhütter and Schaefer's methodology showed higher on average spread than Huang and Huang, suggesting that there is no credit spread puzzle.

Regarding the choice of partitioning by time to maturity and rating categories, there is a trade off between sample size and homogeneity. With increasing group sizes the idiosyncratic errors will decrease causing stable and more reliable model outputs. However, if the subset is not homogeneous, one may be averaging out important differences in underlying risk and misestimating spot rates because they are estimated for a group of bonds where subsets of the group have different yield curves.

2) What both Huang & Huang (2012) and Feldhütter and Schaefer omit in their empirical studies is a deeper comparison on the bond level performance. By solely comparing model spreads on average levels, it does not necessarily mean that the model performs well. The average spreads will likely mask pricing errors on individual bonds. Examining the model's performance on individual bonds is a more natural measure of performance in the sense that the results could be applicable to real investment strategies. A bond level analysis is conducted in Eom et al. (2004), where they investigate the performance of five structural model extensions on firms with simple capital structure. For each year and bond within their sample the authors match the December average bond prices with the year end accounting data in order to compute bond level spreads. The bond level analysis is more sensitive to outliers and data anomalies, but the result is more likely to reveal information on how and where structural models perform well or underperform.

3) As an additional way of assessing the model we will view the model and actual spreads in a time series perspective. This analysis will be done by comparing the model and actual average spreads as well as the of quantiles in each point in time for the sample period. In this setting it is possible to identify how the performance may vary through different cycles of the industry.

By assuming that the model is able to capture the default risk factors of the yield spread, the residuals of the model should in that case not be dependent to the model input. To test this idea we will conduct a new LASSO regression on the residuals to obtain the most influencing parameters. If there is a high dependence between the model residuals and input variables, it would indicate that the model fail to correctly use the information embedded in the data. Similarly, if the dependence is low, the model succeeds to capture structures in the data to explain the dependent variable. In other words, the ideal describing model generates output satisfactory close to the observed data in combination with low residual dependence of the input variables. The influencing LASSO coefficients will be compared to results from the initial dependency analysis, which will be seen as a proxy.

5 Results

Knowledge about the process being modelled starts fairly low, then increases as understanding is obtained and tapers off to a high value at the end. (Chestnut, 1965)

5.1 Dependency Analysis

5.1.1 LASSO Regression

A LASSO regression is performed - as described in Section 4.3.1 above - with the standardised input dataset as independent variables and all of the bond month observations as dependent. From a 10-fold cross validation[1] the punishment coefficient turned out to be optimal for $\alpha = 1.44$. In this model the intercept was 163.8 and 61 of 289 available features are identified as contributing. The 10 most influencing positive feature coefficients and 10 most influencing negative feature coefficients are presented in Figure 5.1. A full variable code description is available in Appendix A.6.

From the figure we can deduce that a selection of the structural model input variables, namely leverage ratio, asset volatility and time to maturity, are included in the set of influencing features. Both leverage ratio and asset volatility have positive coefficients and therefore positively correlated to observed yield spreads according to the LASSO model. Time to maturity on the other hand, is less than zero indicating that yield curve is downward sloping. The fact that these parameters are included in the top 20 most influencing features indicates that structural models have potential to describe the dynamics of market yield spreads to some extent.

We note that there are several other dependencies prevalent, not the least common liquidity risk factors, such as trailing bond trade count (1 month) and bid ask spread. The trailing bond trade count measures the number of days that a bond trade has occurred the last 30 days while bid

[1]The cross validation chooses the α that gives the lowest mean square error on average for the 10 folds.

ask spread measures the relative difference of bid and ask prices quoted for the bonds. Other important influencing features are firm level parameters such as industry, total liabilities, equity drift and market cap. The fact that non-structural features are influential weakens the hypothesis that structural models alone can describe credit spreads observed on the market.

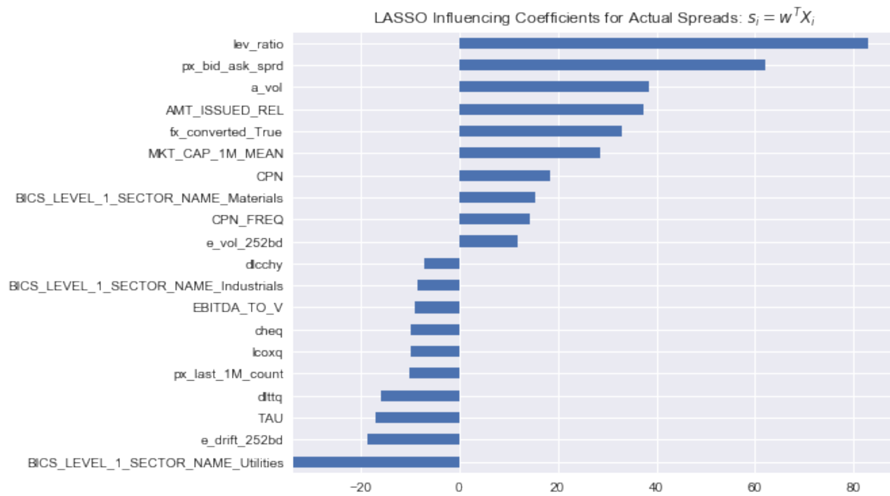


Figure 5.1: The figure displays 20 of the most influencing coefficients in a LASSO model trained to predict the actual spreads with all available bond month data as independent variables. Note that the model implied spreads are not included as covariates. Full variable descriptions are available in Appendix A.6

5.1.2 Analysis of Influential Features

As mentioned above, the LASSO regression identified a set of influential features when trained to predict actual market spreads. To further investigate how the influential input relate to observed spreads, their dependencies are reviewed visually through compound scatter plots. Figure 5.2 below shows the relation between a selection of the most influencing input variables identified in the LASSO model and the observed spreads. The figure is organised in such way that the top row presents input variables for the structural models, while the bottom row displays other dependencies. The observed data divided into three subgroups depending on bond rating. As described in Section 3.3, all Aaa-Baa are labelled 'Investment Grade' (IG) while Ba-C are labelled 'Sub-Investment Grade' (SG). The unrated bonds are grouped together into 'Without Rating' (WR).

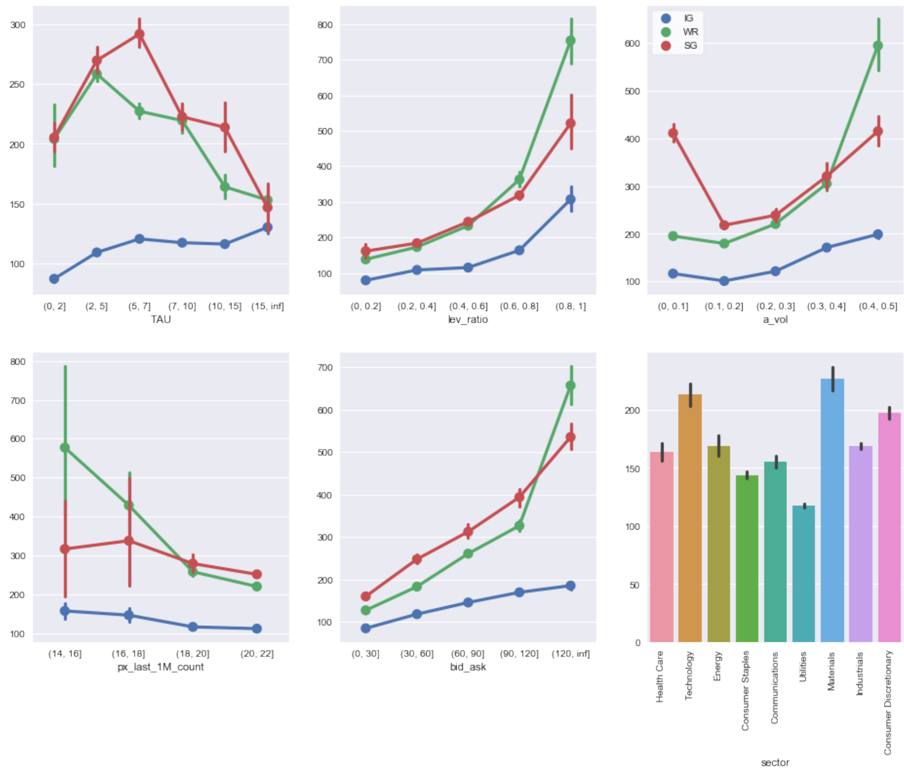


Figure 5.2: The compounded scatter plots show the relation between time to maturity, leverage ratio, asset volatility, monthly number of trades, bid-ask spread and industry sector to the observed spreads.

Interestingly, the term structures of time to maturity and leverage ratio are similar to those implied by structural models (Figure A.1). On the contrary, the relation between asset volatility and market spreads seems to deviate from the structural model predictions. Specifically, the Sub-Investment Grade group shows deviant behaviour in that the average spreads are similar for the two extreme groups $(0, 0.1]$ and $(0.4, 0.5]$. For the factors in between, $(0.1, 0.2]$ to $(0.3, 0.4]$, the average spreads are lower but increasing with asset volatility. Note that the number of constituents in each group vary, possibly causing deviant behaviour in the figure, such as in the case mentioned above.

The features shown in the bottom row in Figure 5.2, are not taken into consideration in the structural models. Nonetheless, it is possible to detect clear patterns and differences within these non-structural features. The most distinguished feature is the bid ask spread which seems to have a strong linear-like dependence to observed spreads. When a bond is traded at sub-par levels the bid ask spread widens, due to increased uncertainty among buyers and holders. However, since both the yield spread and bid ask spread are strongly interlinked with the traded price of a bond, the causality in this relationship is unclear. For the trailing bond trade count, it is harder to detect a clear dependence. What we can deduce however, is that the variability of observed spreads seems to increase within the factors with less frequently traded bonds.

5.2 Descriptive Power of Structural Models

5.2.1 Merton

Group spreads

The group comparisons presented in Appendix A.4 show a systematic underprediction on both mean and median levels. The median values of the spreads indicate that a large fraction of the spreads are nearly zero. Combining this knowledge with the observation of slightly higher levels of mean spread, hints that there are spreads within the groups that are significantly higher, thus bringing

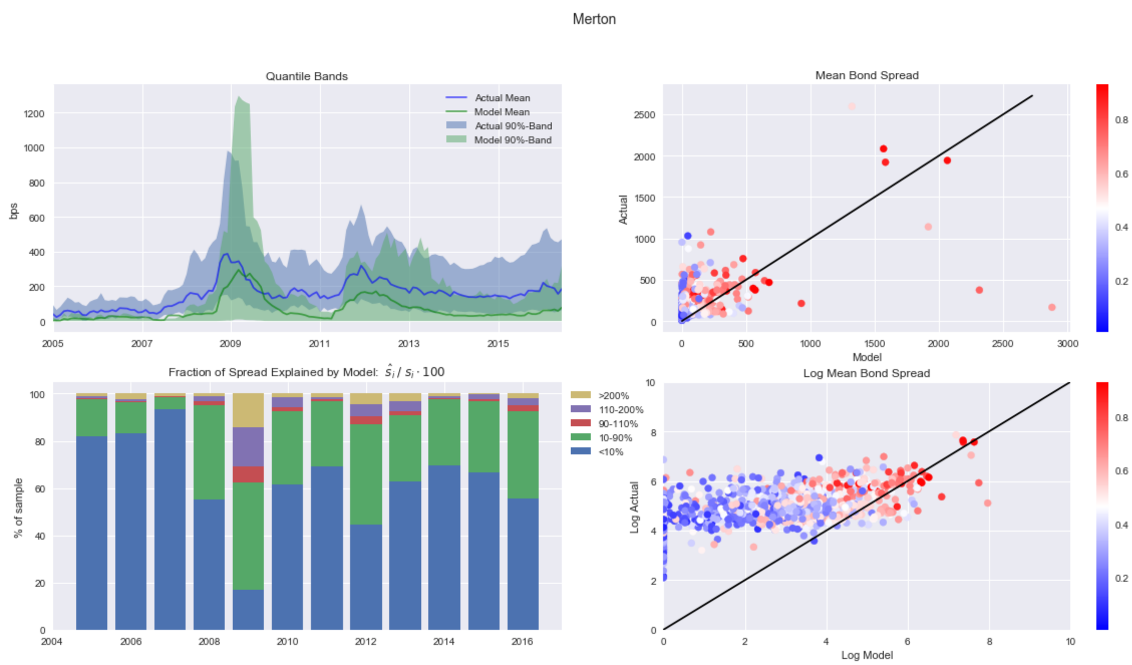


Figure 5.3: ***Merton:*** *The two figures to the left show the time series performance on a bond month level. The upper left shows averages and 90%-quantile bands for the model spreads and actual spreads in each month. The lower left figure shows the distribution of the model errors in each year. The model errors measured by the ratio between model spread and actual spread. The scatter plots to the right display the relation between average actual and model spread for each bond. The colour encoding in the scatter plots represents the average leverage ratio of the bond's issuing firm.*

the mean up. Interestingly the groups along the time to maturity axis, show some similarities with respect to the shape of the yield curve. There is an increase from short to medium term maturities and a decrease from medium to long term maturities in yield spreads. In that aspect, even if the absolute prediction errors are very wrong, the model is capable of capturing the innovations between different maturities on a mean level to some extent.

Time Series

The time series plot of the Merton model shows that there is an apparent and systematic underprediction of yield spreads in the Merton model. Apart from a single period during the financial crisis the mean prediction spread is strictly below the actual mean spread. For a few periods we notice that not even the 90%-quantile bands overlap, which is caused by the fact that a greater part of the model spreads are close to zero. During the period 2005-2007 before the financial crisis, 86 % of the model implied spreads explained 10 % or less than their actual values. The same measure decreased to 61 % in the period 2010-2016, after the financial crisis. It is notable that the Merton model manages to predict the spreads relatively well in the financially distressed period of 2008 and 2009. The reasons behind the improved performance lies in the drastic changes of input parameters to the model. As the stock markets fell, the leverage ratios of the firms in our sample increased, leading to a high risk of default according to the model. In addition, the volatile equity markets are reflected into the EM-estimate of asset volatility, which also increased during this period.

Bond level

Regarding the individual bond performance the scatter plot shows that there is a large cluster of model spreads equal or close to zero. (19 414 out of 50 079 samples). This indicates that in the model framework these bonds are considered equally as risky as the benchmark government bond. In other words the closed form solution of the probability of default, expressed by Equation 2.38, is very small for a majority of the bonds. As the colour encoding of leverage ratio indicates, the underprediction of spreads is mainly prevalent for low levered firms. The model seems to do relatively well for high levered firms, as the residuals for this subset are both positive and negative. An explanation for these observed phenomena may be found in the way the spread is derived in the

Merton model. Recall from Section 2.2.2 that there is no exogenously fixed loss given default as parameter input to the original Merton model. Instead the recovery rate experienced by the bond investors is stochastic and could be time varying. One might argue that this setting is realistic, since the recovery rate for bonds traded on the market typically have stochastic elements. However the stochastic recovery rate in the Merton model setting is highly dependent of the firm's leverage. With the Merton model the expected loss given default is higher for firms with a large fraction of debt compared to lower levered firms. As we in the next section shift focus to the binary Merton model, the recovery rate is fixed, which in particular will have effect on the low levered firms.

5.2.2 Binary Merton

Group spreads

The binary Merton model demonstrates a systematic underprediction of yield spreads on average. However, the model generates higher spreads on average for short and medium maturities ranging from 1 to 10 years compared to the original Merton. For longer maturities on the other hand, binary Merton generates lower spreads on average compared to Merton. Median spreads are higher likely due to the fact that the number of model spreads less than 1 BPS is decreased to 14 332. The reduction of model implied zero spreads follows from that the binary Merton has an exogenous

Figure 5.4: ***Binary Merton:*** *The two figures to the left show the time series performance on a bond month level. The upper left shows averages and 90%-quantile bands for the model spreads and actual spreads in each month. The lower left figure shows the distribution of the model errors in each year. The model errors measured by the ratio between model spread and actual spread. The scatter plots to the right display the relation between average actual and model spread for each bond. The colour encoding in the scatter plots represents the average leverage ratio of the bond's issuing firm.*

and deterministic loss given default. Consequently, bonds with short to medium term maturities, as well as bonds with lower leverage ratio, are more risky compared to Merton.

Time series

The time series aspect further prove the systematic underprediction in the binary Merton setting. It is noteworthy that under a short time period in 2009 the average implied spreads are higher than the real spreads. In line with Merton the 90 % confidence band overlaps poorly before 2009 and overlap better after 2009. Comparing the time series yearly distribution plots for Merton and the binary version, the fraction of samples that explain below 10 % of the actual spread have decreased from 86 % to 75 % in the pre financial crisis period (2005 to 2007). In addition, the corresponding reduction after the financial crisis was from 61 % to 44 %.

Bond Level

As mentioned above the clustering around zero for model implied spreads is slightly improved in the binary Merton model. Looking at the scatter plots in Figure 5.4 it is possible to detect systematicity in the relation between model error and leverage. For highly levered bonds (dark red) the model seems to overshoot systematically, subject to a few exceptions. The explanation to this is the fact that the model implied bankruptcy costs for a majority of the defaulted bonds are significantly increased implying an augmented risk and thus higher model spreads. On the contrary, for low levered bonds (dark blue) the model seems to undershoot systematically.

5.2.3 Black Cox

Calibration

The default boundary calibration is implemented as described in Section 4.2. For each cohort a unique default boundary is calibrated to match the corresponding target default frequency obtained from Moody's. The results of the calibration are presented below in Table 5.1.

Table 5.1: The table shows the calibrated default boundaries, d, for each cohort group. The default boundaries are calibrated such that the average model implied probability of default matches Moody's reported default frequencies for each group.

	(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]
Investment Grade	0.765083	0.694399	0.712877	0.665603	0.661463	0.8773
Sub-investment Grade	0.926038	0.816824	0.790879	0.809423	0.710546	1.28644
Without Rating	0.91173	0.8513	0.745958	0.737812	0.93235	0.734807

The average calibrated default boundary amounts to 0.81 while the corresponding weighted average is 0.75. Our average of 0.81 consort well with the default boundary calibrated in Feldhütter and Schaefer (2016) which they found to be 0.87. However, in (Feldhütter & Schaefer, 2016) they use this averaged boundary as a constant for all bonds independent of rating and time to maturity. In contrast, our approach is to evaluate the model groupwise, using the calibrated default boundary for each cohort. That is, for a given monthly bond observation the bond's cohort is identified and the model implied spread is calculated using the cohort's calibrated default boundary. Allocating a default boundary to each cohort, we allow for heterogeneity and group level differences that could otherwise be averaged out. Therefore this additional model dynamic is in many ways more natural than the method used by Feldhütter and Schaefer, but it brings the disadvantage of loss of intuition. One can question the plausibility of two companies having different default boundaries solely due to differences in bond rating or time to maturity. Looking to the purpose of the model calibration, we want to evaluate the model spreads having similar model implied default frequencies as historically realised, using as much information as possible from Moody's reported expected default frequency Appendix A.1. For this purpose we believe that our calibration method is more appropriate despite its diminutive deficiency.

Group Spreads

For the calibrated Black Cox model there is a systematic underprediction of bond spreads within the investment grade cohort. Across the group, model implied average spreads explained 69 % of the real yield spreads. The median spreads are significantly lower than the mean values indicating that the modelled spreads are skewed and that a large share of the spreads are close to zero. The without rating groups have similar results with consistent underprediction explaining 78 % of the observed spreads. The median model spreads are again significantly smaller than the mean levels. Within the sub-investment grade groups the underprediction is not equally present. For the groups (1,2] and (7,10] to (15, 20] the model overshoots whereas the medium term maturities (2,5] and

Figure 5.5: ***Calibrated Black Cox*** *The two figures to the left show the time series performance on a bond month level. The upper left shows averages and 90%-quantile bands for the model spreads and actual spreads in each month. The lower left figure shows the distribution of the model errors in each year. The model errors measured by the ratio between model spread and actual spread. The scatter plots to the right display the relation between average actual and model spread for each bond. The colour encoding in the scatter plots represents the average leverage ratio of the bond's issuing firm.*

(5,7] are below the actual spreads. On average the model overshoots the actual spreads of the sub-investment grade group by 9 %. Interestingly, the sub-investment grade median spreads are distinctly larger compared to the investment grade and without rating groups. On median level, the model generates spreads that correspond to 65 % of the realised spreads on average, compared to 23 % and 25 % for investment grade and without rating respectively. This difference show that the problem with model implied zero-spreads is improved for the sub-investment grade group when calibrating to historical default frequencies.

Time Series

As stated above, the group level result reveals a systematic underprediction for the investment grade (IG) and without rating (WR) group and a slight overprediction within the sub-investment group. This is reflected in the time series plot in Figure 5.5 by the 90 % band being significantly widened compared to both Merton and binary Merton models. The model overshoot during the financial crisis is further amplified indicating a greater model sensitivity to changes in model inputs such as leverage ratio and asset volatility. The yearly distributions of model explained spreads show that the share of observations with 10 % or less explanatory ability is 81 % during the pre-financial crisis. The same measure after the financial crisis amounts to 58 %.

Bond Level

The calibrated Black Cox framework demonstrates a wider interval of model implied spreads with a maximum average bond spread of 3862 bps, compared to 2879 bps for Merton and 1693 for binary Merton. In Figure 5.5 the scatter plot shows a preserved systematicity with respect to leverage and model implied spreads, where highly levered firms are overpredicted and low levered firms are underpredicted in the model.

5.2.4 Summary

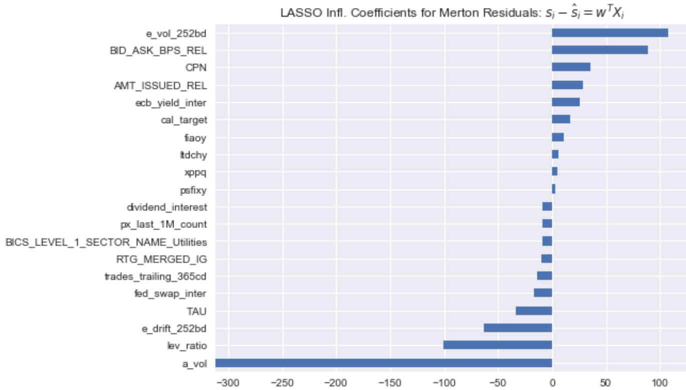
An overview of the structural models' explanatory abilities is presented in Table 5.2 below. To summarise the results we note that Merton, binary Merton and calibrated Black Cox generate model spreads that on average amount to 42 %, 57 % and 85 % respectively of the observed spreads. These results are based on an average level spanning over all rating groups and maturities. One should note that the Black Cox model is calibrated towards historical default frequency data while the remaining models rely only on observed structural inputs. In order to convey a complete picture of the models' performance the following sections will focus on evaluating the model residuals and applying conventional and comparable statistical inference on the models.

Table 5.2: The table presents the fraction (in percent) of the actual credit spread that the three models capture in each cohort group. The absolute model implied group level spreads are found Appendix A.4. A full description of the rating encodings IG, SG and WR is available in Section 3.3.

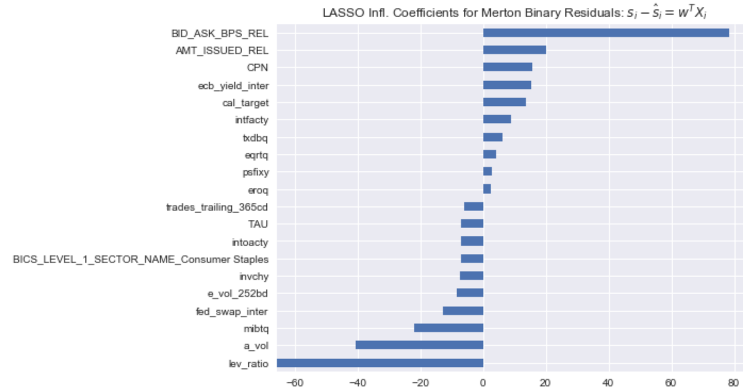
	IG						SG						WR					
TAU	(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]	(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]	(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]
Merton	19	31	39	55	62	42	18	31	45	56	69	47	30	39	49	49	29	43
Binary Merton	61	62	63	69	60	35	93	82	69	64	50	28	72	61	50	42	35	33
Calibrated Black Cox	41	47	60	75	83	108	104	76	97	120	104	154	94	78	63	67	92	74

5.2.5 Residual Analysis with LASSO

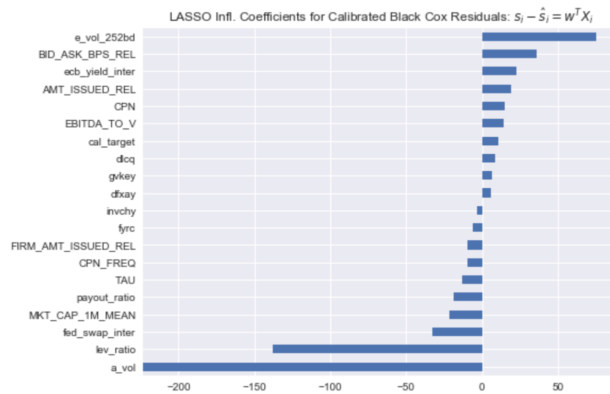
We now conduct a comparison between the models by regarding how well they have captured the credit risk components in the LASSO model. Recall the initial dependency analysis conducted with a LASSO regression at the beginning of the results section. We noted that the actual spreads have a positive dependency of the firm's leverage ratio and asset volatility, while there was a negative dependency of the residual time to maturity. Given the model results presented above, we now want to check whether these dependencies have been incorporated by the model. Therefore new LASSO regressions are conducted on the model residuals, in order to extract the most influencing covariates in the dataset. The results from these regressions are presented in Figure 5.6, by graphically displaying the twenty most influencing covariates in each of the residual regressions. A noteworthy observation is that in all of the three regressions, the residual dependencies to leverage ratio and asset volatility have opposite signs compared to the initial dependency analysis. In addition, the absolute values of these two coefficients have increased in relation to the other covariates, thus considered strong contributors to the LASSO model's explanatory power. Together these two effects imply that a small increment in asset volatility or leverage ratio corresponds to a larger negative residual, which means that our implemented structural models overstate the credit spread. Our interpretation of these results are that the structural models have too sensitive characteristics along the dimensions asset volatility and leverage compared to the market's valuation.



(a) Merton



(b) Binary Merton



(c) Calibrated Black Cox

Figure 5.6: Three LASSO regressions are performed with the model residuals as dependent variables. All of the bond month observations, including numerical and categorical features, are included as independent variables. First a LASSO regression was performed for each model generating three model specific punishment terms. In order to preserve comparability each regression is performed again with α set to the average of the individual punishment terms. The average punishment term turned out to be $\alpha = 3.15$. The 10 most influencing positive feature coefficients and 10 most influencing negative feature coefficients generated with the average punishment term are presented for each model in the figures. A full variable code description is available in Appendix A.6

5.2.6 Evaluation Metrics

In the previous sections we have analysed the results from the models based on their group, time series and bond-level performance. Moreover, the residual dependencies have been analysed by means of a LASSO regression. All of these aspects bring insights on the performance and deficiencies of the models but does not fully include conventional model validation techniques. In order to achieve more formal and comparable results, a simple linear regression is performed according to Equation 5.1 below

$$s_{obs} - \bar{s}_{obs} = \beta(s_{model} - \bar{s}_{model}) + e \tag{5.1}$$

where the dependent variable is observed yield spread and the the regressor is the modelled spread. For each structural model the observed spread is predicted with its corresponding linear regression generating the sets $\hat{S}_M$, $\hat{S}_{BM}$ and $\hat{S}_{BC}$ of predicted values. The structural models are then evaluated through computing mean square errors, median absolute errors and R-squared on the sets $\hat{S}_M$, $\hat{S}_{BM}$ and $\hat{S}_{BC}$. The innovation correlation, ρ_Δ is calculated as the correlation between the differentiated time series of modelled bond month spreads, s_{model}, and differentiated observed monthly spreads s_{obs}. In Table 5.3 a summary of the linear regressions and evaluation metrics explained above are presented. Note that the β column is supplemented with the regression parameter t-stat values.

Table 5.3: The table shows summary statistics for the tree implemented structural models.

	β	MSE	MAE	R^2	ρ_Δ
Merton	0.69063 (158)	28 421	55.321	0.33300	0.19375
Binary Merton	0.48169 (141)	30 516	52.127	0.28383	0.27972
Calibrated Black Cox	0.44785 (165)	25 978	51.102	0.39032	0.28198

The slopes from the linear regressions are all significantly different from both zero and one indicating that the modelled model spreads to some extent have explanatory power to the observed spreads. Regarding slope the Merton model has largest β and calibrated Black Cox has smallest slope value. With respect to MSE, MAE and R-squared the calibrated Black Cox model consistently performs best and the model seems to be able to explain about 40 % of the variation in observed spreads. One should bear in mind that MSE, MAE and R-squared are measures of the overall model performance with respect to total variability. That is, the measures do not indicate to what extent changes in the modelled spreads relate to changes in observed spreads. This aspect is interesting under the assumption that yield spreads are generated by default and non-default components. In such case the structural models would not generate spreads on the same level as the observed, but changes in modelled spreads would correspond to changes in observed spreads. To investigate how innovations in model spreads correspond to changes in observed spreads we compute the innovation correlation. As shown in Table 5.3, the innovation correlation (ρ_Δ) is on the same level for binary Merton and Black Cox while significantly lower for the Merton model. The interpretation of this is that binary Merton and Black Cox absorb information in changes in input variables better than the Merton model.

6 Discussion

Complete realism is clearly unattainable, and the question whether a theory is realistic enough can be settled only by seeing whether it yields predictions that are good enough for the purpose in hand or that are better than predictions from alternative theories. (M. Friedman, 1953)

6.1 General Results

The common denominator for the structural models evaluated in this thesis is a consistent under-prediction of the average observed yield spreads on the European bond markets. This result is in line with previous research, not the least the seminal paper by Huang & Huang (2012). In this broad point of view, the structural models seem to behave similarly, and the applicability of the models appear to be limited. However, analysing the model results from other aspects it is possible to detect significant discrepancies and differences within the models. This knowledge helps to indicate where and how structural models perform well and how they might be improved in order to increase their explanatory power and thus applicability in practice. Without definitive conclusions, previous research explore the possibility of real spreads being built up not only by structural default components but a combination of these default components and non default components. If the structural models succeeded to model out the dependence of structural input variables, we could deduce that we have correctly used all the default risk information in the input data. Therefore the remaining unexplained yield should be related to non default components. In Section 5.2.5 we conclude that all the evaluated structural models have significant residual dependence to asset volatility and leverage ratio. In this aspect the binary Merton have least residual dependence to these structural input parameters and could for this reason be considered to be better in capturing the credit risk components than the original Merton and Black Cox models.

Looking at the evaluation metrics in Section 5.2.6, the MSE, MAE and R-squared indicate that Black Cox has the highest explanatory power to the variations in the observed spreads. In addition Black Cox has the highest innovation correlation reflecting that innovations in model spread are correlated to innovations in observed spreads to a greater extent for Black Cox than Merton and binary Merton. For Black Cox and binary Merton we can detect clear systematic prediction errors on bond level with respect to leverage ratio (see the bottom right scatter plots in Figures 5.4 and 5.5). For high levered firms the spreads are overpredicted, while low levered firms are underpredicted. This feature, common to binary Merton and Black Cox, can be interpreted as an oversensitivity to leverage ratio which is likely due to discrepancies between the models' and market's view of credit risk as a function of leverage.

Neither of the aspects discussed above can alone resolve which model performs best or explains real yield spreads best. When analysing the results from different point of views, we can not appoint a model that consistently performs superior to the others. The binary Merton manages to remove the influence of input parameters most efficiently making the result a cleaner measure of default risk. On the other hand Black Cox is superior with respect to explanatory power of variability of observed spreads. What the two models have in common is the clear over sensitivity to leverage ratio asset volatility as evidenced by bond level analysis and LASSO residual regressions above. The high level of sensitivity in these two dimensions is further visualised in Figure A.1.

Moreover, our findings are in line with the the empirical studies on structural models conducted by Bao (2009) and Eom et al. (2004). Bao concludes that in the cross section, Black Cox can explain approximately 45 % of observed yield spreads on the US bond market and that future research should focus on finding theoretically founded models that explain observed yield spreads better. In similar spirit, Eom et al. concludes that structural model spreads are often either close to zero or extremely large. In order to further improve the understanding and performance of structural models we believe that addressing this over-sensitivity is of great importance. Recirculating back to our problem formulation, we asked whether the structural models can absorb the observed dependencies of its input parameters. Based on our findings and backed by these results from previous research, will now attempt to explore ways of improving structural models' performance with respect to residual dependencies. Due mainly to three reasons we have chosen to continue to evaluate explanatory ability of structural models through the Black Cox model. First, the

Black Cox framework is the most realistic model containing the important feature of default before maturity. Secondly, Black Cox is the most commonly used model in the literature, making our results comparable to a greater extent. Lastly, the model involves more degrees of freedom implying broader aspects of improvement compared to Merton and binary Merton.

6.2 Black Cox Extensions

6.2.1 Targeting Over Influencing Input Parameters

As our objective in the following extensions is not to match model spreads with actual spread, but to evaluate explanatory abilities, this further exploration aims rather to continue the removal of credit risk factors from the residuals. The two dependencies that we will to address in this further investigation are leverage ratio and asset volatility. In order to better understand their impact we begin by looking at their dependencies isolated from other factors. First, we imply out what the leverage ratio and asset volatility should be given the remaining input parameters and the observed yields spreads in a structural setting. Secondly, we use gradient boosting regression to find how the model residuals depend on leverage ratio and asset volatility separately.

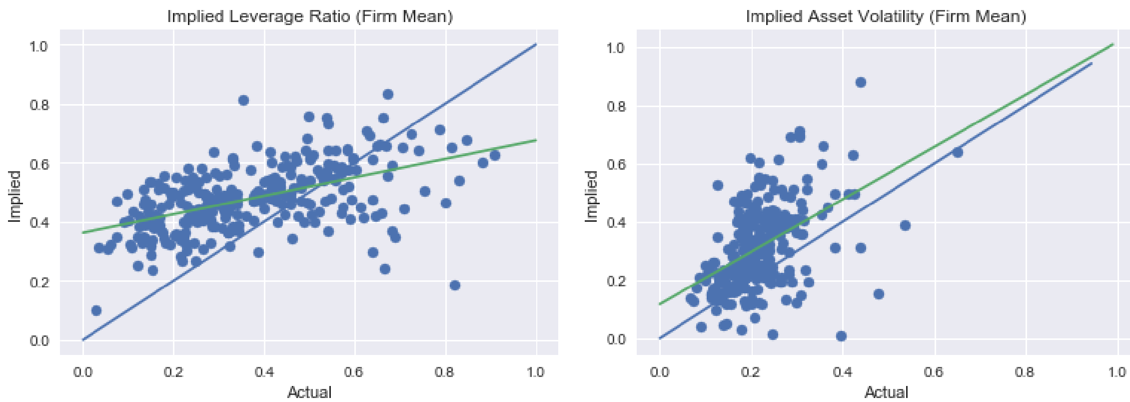
To imply out the theoretical levels of leverage ratio and asset volatility, the Black Cox framework is used. In Section 2.2.2 we derived an expression for the model implied spread given all input parameters. With a slight modification in our notations, we disaggregate the leverage ratio and get the following expression for the model spread

$$\hat{s}(\Theta_{obs}, L) = -\frac{1}{T-t} \log(1 - (1 - RR)PD^{\mathbb{Q}}(t, T, \Theta_{obs}, L)) \tag{6.1}$$

where Θ_{obs} is the set of observed structural inputs excluding the leverage ratio. Given the spread formula above and a specific monthly bond observation we then imply out the theoretical leverage ratio by optimising

$$L^* = \arg\min_{L} \ | \ s_{obs} - \hat{s}(\Theta_{obs}, L) \ | \tag{6.2}$$

where s_{obs} is the observed yield spread. With analogous methodology we imply out the theoretical asset volatility. The results of the implied leverage ratio and asset volatility are presented in Figures 6.1a and 6.1b, where the bond month observations are aggregated to firm level averages. Each point represents a specific firm's average actual value on the x-axis and the corresponding model implied value on the y-axis. The scatter plots are complemented with two lines, one which represents the $y = x$ curve (blue line) and one which represents a linear regression from actual parameter values to implied (green line).



(a) Regression: $L_{imp} = 0.363 + 0.313\, L$

(b) Regression: $\sigma_{v,imp} = 0.117 + 0.900\, \sigma_v$

Figure 6.1: The left figure shows the relation between model implied leverage ratio and observed leverage ratio on firm level average. The right figure shows the relation between model asset volatility and observed asset volatility on firm level average. The blue line is represents $y = x$, while the green line is a linear regression from actual to implied values.

As seen in Figures 6.1a and 6.1b above, the implied leverage ratios are overstated for the low levered firms and understated for high levered firms. This result confirms that the model is oversensitive to leverage ratios. This oversensitivity could be due to limitations in the model structure but it can also be an indication that the market has a different view of credit risk regarding leverage ratio. For the asset volatility, the results are not as clear. The regression line is slightly above the 45 degree line for all asset volatilities. However, looking at the scatter plot it is possible to distinguish possible outliers and leverage points that might affect the regression significantly. The scatter plot indicates that the low actual asset volatilities seem to correspond fairly well to the theoretical, while the higher actual volatilities have a more diffused relation to the theoretical.

Moving on to the gradient boosting regression analysis, we want to investigate the relation between model residuals and the identified over-influential input parameters. A 50 % quantile regression is performed with gradient boosting where the number of leaves is limited to 100 and the number of tree estimators is fixed to 20. More specifically, this means that leverage ratio and asset volatility are partitioned by the gradient boosting algorithm into a maximum of $100 \cdot 20$ sub-intervals. Each interval is assigned a value, corresponding to the median model residuals within each interval. In Figure 6.2 below, the results of the regressions are presented.

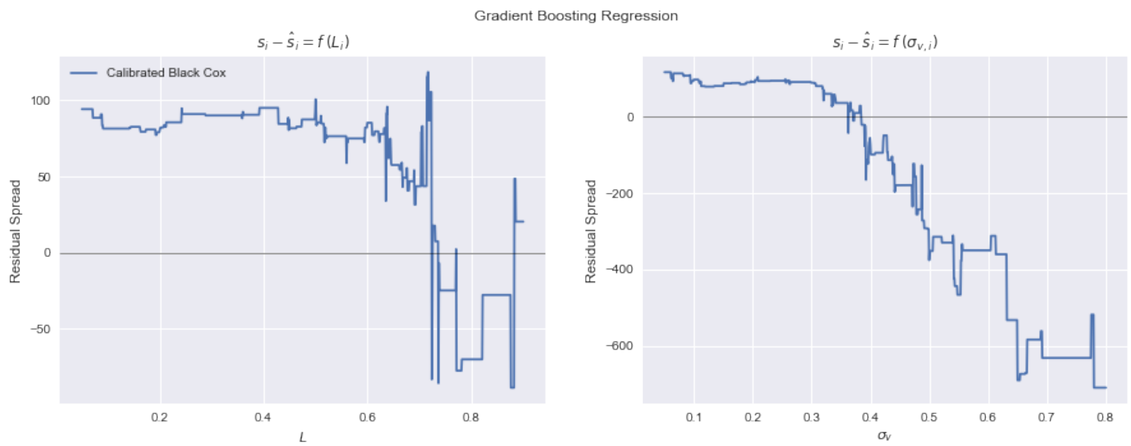


Figure 6.2: The figures show gradient boosting regressions with leverage ratio (left) and asset volatility (right) as independent variables. The calibrated Black Cox model residuals are set as dependent variable. The gradient boosting regression is performed as a quantile regression in order to stabilise the resulting model and making it more tractable and easily interpreted. The figures show that the residuals are positive for small values of the independent variables and become negative as they increase. The break point from underprediction to overprediction seems to occur at 0.7 for leverage ratio and 0.4 for asset volatility.

The gradient boosting regressions once again show an underprediction associated with low leverage ratios and the contrary for high leverage ratios. This result is completely in line with previous conclusions and therefore little unexpected. In contrast to the theoretical approach above, the regression model for asset volatility demonstrate a clear - almost linear - relationship to the model residuals.

6.2.2 Intercept Calibrated Leverage Ratio

To begin with we target the dependence of leverage ratio and find inspiration from the conventional calibration methods used in Huang & Huang (2012) and Feldhütter and Schaefer (2016). The default boundary for the Black Cox model is calibrated against historical expected default probabilities by adjusting d in Equation 2.44. The adjustments to d correspond to fractional changes in the default boundary. In effect high levered firms will still remain more likely to default on their debt than low levered firms. In reality the amount of debt to equity is not necessarily a good proxy for the default risk, as it may vary widely between different industries and firms. For example, an IT-company with 20 % leverage ratio may have bonds traded at higher spreads than a highly levered real-estate company. The issue with the fractional default boundary is that the systematic over- and underprediction dependence of leverage ratio remains unsolved. We propose a novel approach, which we name intercept calibrated leverage ratio (ICLR), set out to target these systematic errors in leverage ratio.

The idea behind the ICLR approach is that the exogenous default boundary for a firm is constructed by a fixed and a fractional component, instead of solely a fraction of the debt. The fixed component, or the intercept, corresponds to a baseline level of risk of default. In some sense it can be regarded as a systematic market risk of default, which can be caused by for example force majeure or a major scandal. Therefore a systematic default risk should be assigned to every firm no matter its leverage ratio, which is the intuition behind the intercept term. The fractional component, or the slope, represents investors interpretation of additional risk as a function of leverage ratio. If we assume that the intercept is positive and the slope is greater than zero and less than one, the ICLR approach corresponds to a linear transform of the default boundary. This transform shifts the default boundary up for low levered firms and down for high levered firms. These are the characteristics that we found when looking at the implied leverage ratio and asset volatility in the previous section.

To formalise the ICLR approach we first need to recall how the default time, first-passage time, is defined in the original Black Cox model. The model assumes that a firm will be forced into default when the firm value process falls below a fraction of the debt. The stochastic time when a default occurs is τ, defined as $\tau = \inf\{t : V_t/V_0 < dK/V_0\}$, where $L = K/V_0$ is the leverage ratio and d is an exogenous model parameter. In the ICLR approach we modify this first-passage time slightly by adding an intercept term to the right-hand side. The resulting expression for the default time is denoted $\bar{\tau}$ and defined by $\bar{\tau} = \inf\{t : V_t/V_0 < \alpha + \beta K/V_0\}$. With this minor modification the probability of default in the ICLR is higher for firms with low leverage and the contrary applies for high levered firms. These effects are visualised in Figure 6.3, where the probability of default is compared for the two default boundary definitions. We can deduce that the introduction of an intercept to the default boundary achieves the desired effect, which we discussed earlier.

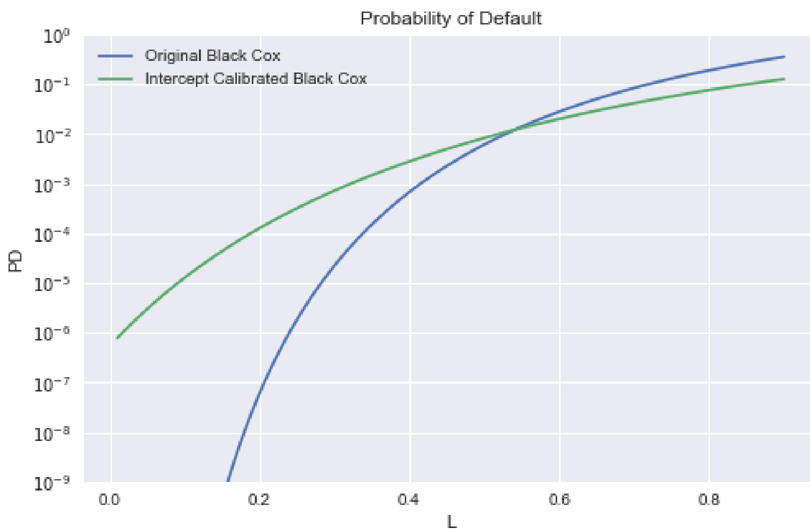


Figure 6.3: The figure shows two physical probability of default measures as a function of leverage ratio, L, for the same firm ($\sigma = 0.2$, $\theta = 0.22$ $\delta = 0.02$ $r = 0.03$). The original Black Cox has its probability of default given by Equation 2.44, where $d = 0.87$ in this example. The intercept calibrated leverage ratio has the probability of default given by Equation 6.3, where we have set $\alpha = 0.2$ and $\beta = 0.5$. Note that the probability of default axis is log-scaled.

$$\begin{aligned} PD^{\mathbb{P}}(\alpha,\beta,\Theta) =& N\Big[\frac{-\log(\alpha+\beta L) - (r+\pi^{\mathbb{P}} - \delta - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\Big] \\ &+ \exp\Big[\frac{-2\log(\alpha+\beta L)(r+\pi^{\mathbb{P}} - \delta - \frac{\sigma^2}{2})}{\sigma^2}\Big] \\ &\cdot N\Big[\frac{-\log(\alpha+\beta L) + (r+\pi^{\mathbb{P}} - \delta - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\Big] \end{aligned} \tag{6.3}$$

Next we need to find estimates for the newly introduced parameters α and β by calibrating the model to the historical default probabilities.

Calibration Results

Analogously to the calibrated Black Cox model, presented earlier in this report, we obtain estimates for exogenous model parameters by calibrating cohort groups to match their corresponding expected default frequencies through the cycle. In Appendix A.2 the calibration targets are summarised for each cohort group. The objective function in the calibration procedure is a slight modification to the method presented in Section 4.2, where we now have to optimise α and β for each group as

$$\underset{\alpha_{z,\tau},\beta_{z,\tau}}{\arg\min} \mid \frac{1}{N}\sum_{y=1}^{N}\overline{PD}_y(\alpha_{z,\tau},\beta_{z,\tau}) - PD_{z,\tau} \mid^2 \tag{6.4}$$

where y denotes a year in our sample ranging from 1 to N. The combination $\{z,\tau\}$ groups a bond by its rating category and residual time to maturity according to the Appendix A.2.

With two degrees of freedom in the optimisation problem, there is a greater possibility for multiple solutions. The solution to the optimisation problem is summarised by Table 6.1 below[1]. The average α and β of the 18 cohort groups turn out to be 0.1087 respectively 0.6038. These results are in line with the discussion of model implied leverage ratios a few paragraphs above. Furthermore the results indicate that the convergence is consistent for each group, which becomes evident by looking at the minimum, mean and maximum as well as the quantiles of the group estimates.

[1] We find that the convergence of the minimisation problem is sensitive to the initial guess. The solution presented is the most robust with respect to convergence in different initial points and generating homogeneous results across cohort groups.

Table 6.1: The table shows a summary of the calibrated default boundary variables $\hat{\alpha}$ and $\hat{\beta}$ for the 18 cohort groups. The default boundary variables are calibrated such that the average model implied probability of default matches Moody's reported default frequencies for each group.

	Count	Mean	Std	Min	25%	50%	75%	Max
$\hat{\alpha}$	18	0.108737	0.053455	0.041980	0.069001	0.101433	0.133484	0.250801
$\hat{\beta}$	18	0.603763	0.024256	0.573065	0.585620	0.600758	0.616683	0.666962

In order to evaluate the potential improvements achieved by the intercept calibrated leverage ratio model, we compare it using the same measures as in the results section. The R-squared for the ICLR model is 0.3793, which is directly comparable to 0.39032 for the calibrated Black Cox. Apart from the intercept term these two models are identical with respect to input variables, model assumptions and calibration techniques. From this perspective the intercept modifications did not help to increase the explanatory power. The decrease in R-squared is so small that we can still can consider them equally powerful in this aspect.

The interesting assessment of the ICLR model is to test its residual dependence of leverage ratio. In the results section we have used the LASSO regression model as proxy measure of the models' dependencies. With the same methodology as in Section 6.2.1, we complete the LASSO analysis with a gradient boosting regression on the influencing input parameters to the residuals. To begin with Figure 6.4 shows the residual LASSO dependencies for the ICLR approach. We can deduce that the leverage ratio dependency has reduced compared to the calibrated Black Cox, even though it is still a influencing parameter in the model. The results from the gradient boosting in Figure 6.5 show that the break point of leverage ratio overprediction is shifted from 0.7 to 0.9. As a consequence, the median prediction error with respect to leverage ratio is more uniform. Therefore we can conclude that the intercept term is a realistic extension to the Black Cox model, as we can mitigate some of the original model's systematic errors in leverage ratio. However, we find a clear systematic dependency with respect to the asset volatility, which we will attempt to remove in the next section.

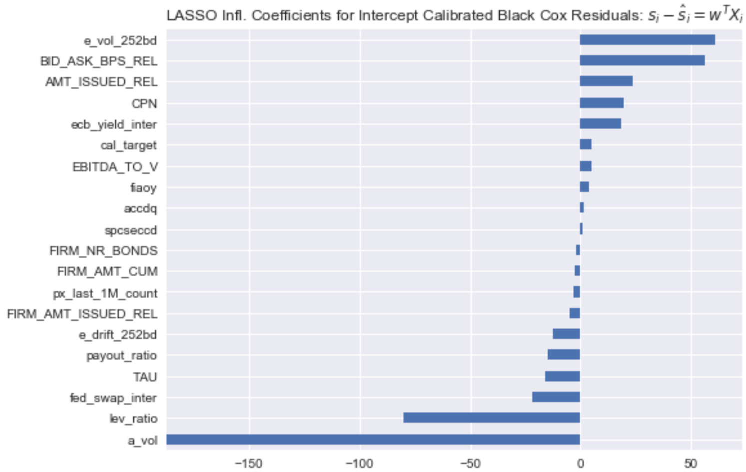


Figure 6.4: A LASSO regression is performed with the ICLR model residuals as dependent variables. All of the bond month observations, including numerical and categorical features, are included as independent variables. In order to preserve comparability the regression is performed once again with $\alpha = 3.15$. *The 10 most influencing positive feature coefficients and 10 most influencing negative feature coefficients generated with the average punishment term are presented for each model in the figures. A full variable code description is available in Appendix A.6*

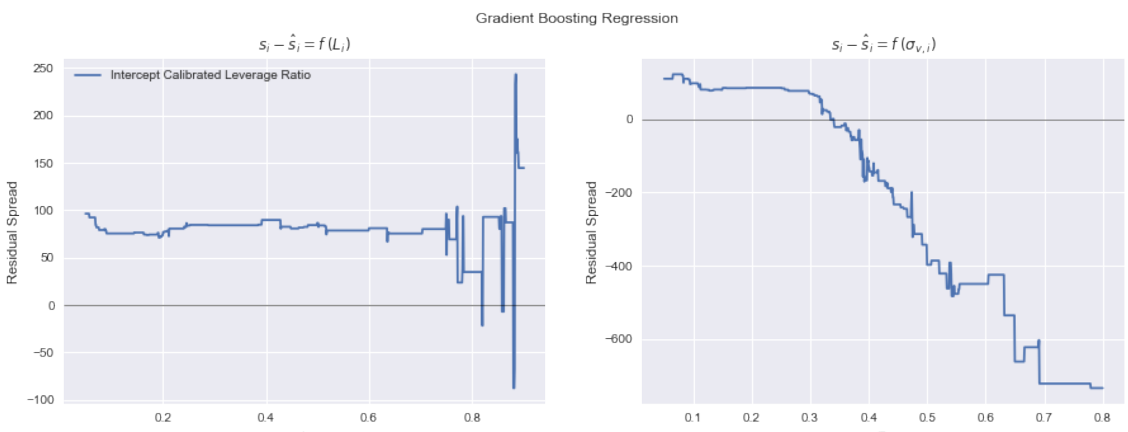


Figure 6.5: The figures show gradient boosting regressions with leverage ratio (left) and asset volatility (right) as independent variables. The ICLR model residuals are set as dependent variable. The gradient boosting regression is performed as a quantile regression in order to stabilise the resulting model and making it more tractable and easily interpreted. The figures show that the residuals are positive for small values of the independent variables and become negative as they increase. The break point from underprediction to overprediction has shifted from 0.7 to 0.9 for leverage ratio. The residual dependence with respect to asset volatility is unchanged in comparison to Figure 6.2.

6.2.3 Asset Volatility Extensions

In order to reduce the Black Cox model's oversensitivity to asset volatility, we have considered and implemented a wide range of approaches. An initial approach to remove the systematicity, shown in Figure 6.2, involved estimating an intercept model for the asset volatility in the same fashion as in ICLR. The calibration procedure involved a minimisation problem in four variables including intercept and slope for both leverage ratio and asset volatility. Due to an enlarged search space and more degrees of freedom, we could not find robust and univocal solutions. In addition the right graph in Figure 6.2 hints that the correction is not necessarily linear. Therefore we discontinued the search for linear transformation parameters for the asset volatility.

Our next attempt to address the volatility dependence was to cap the time varying asset volatility to 0.3. The cap level was chosen to 0.3, as it appears to be the upper limit of the well-behaving region in Figure 6.2. In this setting the ICLR parameters were re-estimated resulting in an R-squared of 0.402, which is close to the R-squared of calibrated Black Cox (0.390). With respect to residual LASSO dependence (Appendix A.5) the capped asset volatility model managed to reduce dependence to asset volatility, while the leverage ratio dependence increased slightly. Not surprisingly the asset volatility dependence turned out to be more uniform in the gradient boosting aspect as seen in Figure 6.6.

Yet another adjustment that we implemented, was a towards mean regression of the asset volatility. The economic intuition for this approach is that investors do not have an instantaneous perspective of a firm's asset volatility as our implementation of structural models assume. When a certain firm or the market in general is in a high volatility state, it's possible that investors are aware of the contingency of the extreme period. As a consequence the volatility should be adjusted to reflect the investment horizon. In high volatility states this implies a downward adjustment of the asset volatility, while the opposite applies for low volatility states. In summary this economically tractable mechanism motivate a toward mean regression for the asset volatility. To implement this regression we define the investment horizon adjusted asset volatility as a linear combination of instantaneous volatility and the firm level mean volatility. In order to find the optimal weights, which are aligned with the market's valuation of debt, we conducted a least square optimisation to match the model spreads with the actual spreads. In this new setting the asset volatility for a given firm at a certain time t, is calculated as

$$\hat{\sigma}_{V,t} = \sigma_{V,t}\, w + \bar{\sigma}_V\, (1-w), \qquad w \in [0,1] \tag{6.5}$$

where $\bar{\sigma}_V$ is the firm's average volatility in our sample period. The optimal weight was $w = 0.13$, meaning that 13 % of the instantaneous asset volatility is attributable to the valuation of the bond. The R-squared for the mean regressed model was 0.437 which is the highest noted for the models included in this study so far. The reader should bear in mind that information from the actual spreads is used to arrive at this result. An important observation is however that the instantaneous asset volatility has little influence on the modelled spreads in comparison to the firms' average asset volatility.

As a final model approach, that doesn't rely on the observed spreads, we let every bond have a constant firm asset volatility and conduct the ICLR calibration once again. The intercept and the slope was re-estimated as 0.13 respectively 0.63, which is consistent with our previous estimates. The R-squared for this final model increased to 0.467. Compared to the initial ICLR model with time varying asset volatility, which had an R-squared of 0.3793, the firm constant asset volatility is better in these terms. The correlation between the model and actual spread's innovation, which

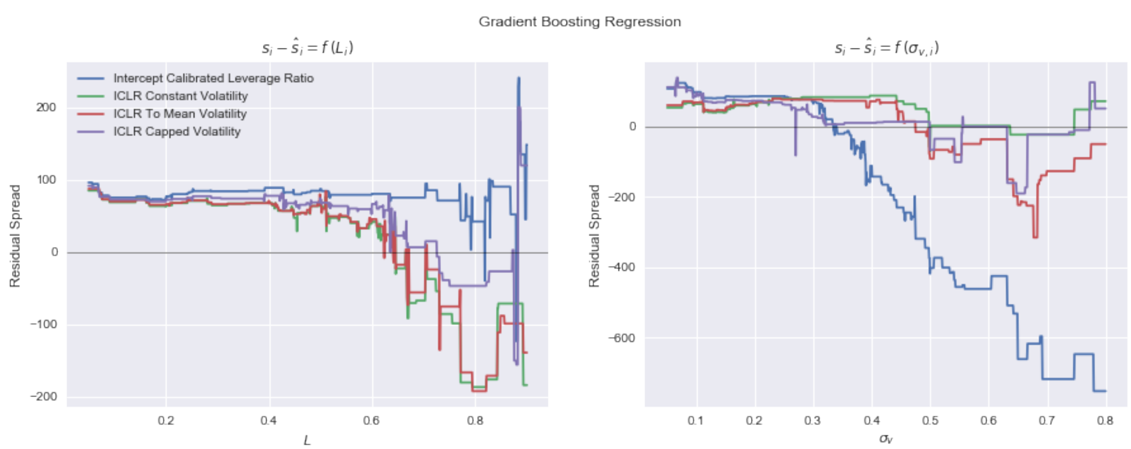


Figure 6.6: The figures show gradient boosting regressions with leverage ratio (left) and asset volatility (right) as independent variables. The four ICLR extensions' model residuals are set as dependent variable. The gradient boosting regressions are performed as a quantile regression in order to stabilise the resulting model and making it more tractable and easily interpreted.

we have termed innovation correlation, increased from 0.327 (ICLR) to 0.356. The interpretation is that the model with firm constant asset volatility has an improved ability of explaining changes in observed yield spreads. In Figure 6.6 gradient boosting regressions for the approaches introduced in this section are presented.

From Figure 6.6 we can conclude that the original ICLR model performs best with respect to residual dependence of leverage ratio in the gradient boosting models. On the other hand, regarding residual dependence of asset volatility for the original ICLR it performs the worst. In the gradient boosting aspect the overall best performing extensions seems to be ICLR with capped volatility or ICLR with constant volatility. The results for the residual LASSO regressions for all the ICLR extensions are presented in Appendix A.5. The LASSO analysis shows similar dependency patterns for all of the ICLR extensions. Despite our effort to remove dependencies of leverage ratio and asset volatility, we can deduce that the variables' residual dependence are still prevalent. Our interpretation of this result is that the structural model do not manage to incorporate these variables correctly. However, since our ICLR extensions have reduced the the systematic errors in terms of decrease LASSO coefficients and stabilised gradient boosting regressions, we can conclude that these improvements manage to mitigate some of the structural models' weaknesses.

7 Conclusions

In this thesis we have conducted an empirical study of structural models' ability to explain corporate treasury yield spreads for European bonds. Our results have shown that structural models substantially and systematically underpredict the spreads both on a cross-sectional level and for individual bonds. The systematic underpredictions occur mainly due to the input parameters: leverage ratio and asset volatility, which prove to be highly dependent in the residual analysis for all evaluated models. Targeting these drawbacks of the structural models we introduce a novel approach of the Black Cox framework, which manages to reduce the systematic errors caused by leverage ratio. The improved model is still in line with economic intuition as its unique feature adds a base level of default risk for all firms. This model manages to explain 38 % of the variability in the sample set.

While reducing the residual dependence to leverage ratio compared to the calibrated Black Cox model, asset volatility dependence remains substantial. In our search for methods to remove the asset volatility dependence, we concluded that time varying asset volatility is neither evident in the observed spreads nor realistic from an investor's perspective. According to our results a constant asset volatility model generates spreads that explain about half (47 %) of the variations in observed spreads.

The residuals from our final model show decreased dependence of leverage ratio and asset volatility. However, these input parameters along with the risk-free interest rate and payout ratio remain influential. This result hints that the structural models are not fully capable of absorbing the default risk components of European corporate yield spreads. Other remaining dependencies are found to originate from the bid ask spread, the size of the bond issue in relation to total debt, the US federal swap rate and the market capitalisation as a few examples. Ultimately, the dependency analysis show that other factors which are not considered in the structural models affect the level of yield spreads on the European bond market. These factors are considered as non default components, which rather relate to liquidity, political, inflation and supply risks. Further research should therefore focus on finding theoretically backed models of disaggregating default and non-default components in the observed yield spreads. This will result in other model evaluation frameworks, which can better assess and compare the explanatory ability of structural models.

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Appendix

A.1 Asset Volatility Estimation

In the literature researchers have discussed a wide range of different possible approaches to obtain reasonable estimates of the asset volatility. To the authors knowledge there is no consensus concerning which estimate is the most appropriate. That gives a freedom of choice, but for the completeness we will briefly review the most common methods found in published articles.

A.1.1 De-levered Equity Volatility

In the Merton setting the firm value process is the sum of the equity and debt value processes. From this relationship it can be shown that the two combined GBM processes have the following diffusion parameter

$$\sigma_{V,t} = (1 - L_t)^2 \sigma_{E,t}^2 + L_t \sigma_{D,t}^2 + 2L_t(1 - L_t)\sigma_{ED,t} \tag{A.1}$$

In S. M. Schaefer and Strebulaev (2008) they propose a lower bound for the asset volatility by assuming that the debt volatility is zero. From their proposal it follows that the expression above is reduced to $\sigma_{V,t} = (1 - L_t)\sigma_{E,t}$ Since the equity volatility $\sigma_{E,t}$ is easy to obtain, this is a simple and transparent method of estimating the asset volatility.

A.1.2 Itô Relationship

In line with the Merton framework we assume that a firm's equity is a call option of the underlying asset value process. Further assuming the the equity is a geometric Brownian motion and that the contract can be replicated by the delta hedge, that is $E_t = \partial E_t/\partial V_t V_t + (1 - \partial E_t/\partial V_t)B_t$, we arrive at the following relation to obtain the asset volatility

$$\sigma_{V,t} = \sigma_{E,t} \frac{E_t}{V_t} \frac{\partial E_t}{\partial V_t} = \sigma_{E,t} \frac{E_t}{V_t} N(d_1) \tag{A.2}$$

A.2 Variable Sensitivity Black Cox

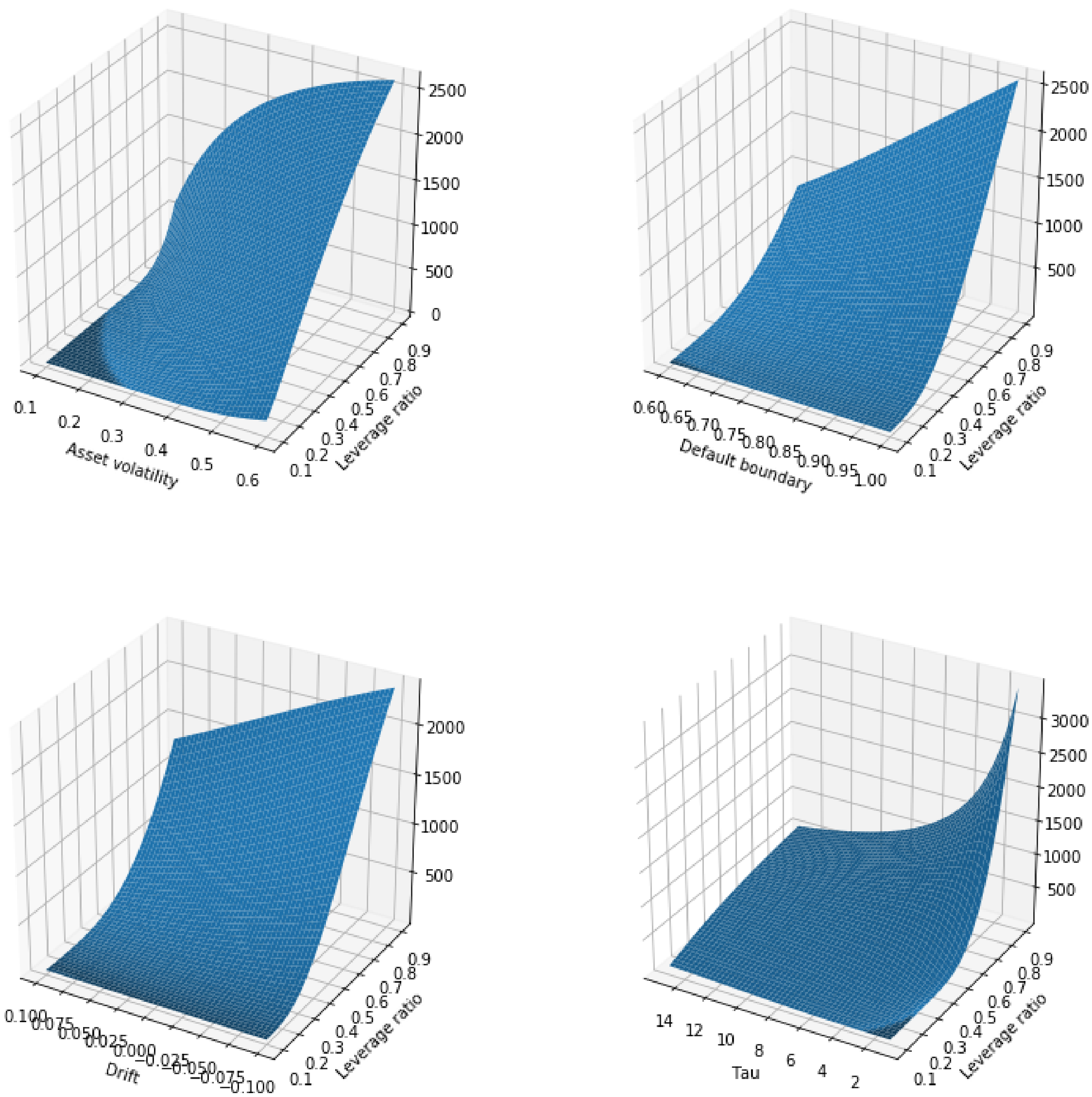


Figure A.1: This figure shows the the model implied spreads as a function of leverage ratio and asset volatility, default boundary, value process drift (drift) and time to maturity (tau). The spread functions are calculated holding all other structural inputs constant.

A.3 Moody's Expected Default Frequency (EDF)

Table A.1: Moody's cumulative expected default probabilities in percent, grouped by letter rating. The data is based on yearly cohort studies from 1920 to 2016. Further details about the methodology used by Moody's to generate this table are described in Section 4.2

Rating/Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Aaa	0	0.008	0.027	0.074	0.143	0.216	0.309	0.436	0.568	0.723	0.856	0.965	1.08	1.114	1.144	1.206	1.268	1.318	1.376	1.415
Aa	0.063	0.18	0.285	0.438	0.67	0.939	1.212	1.47	1.708	1.975	2.282	2.616	2.947	3.263	3.498	3.679	3.843	4.038	4.278	4.479
A	0.086	0.259	0.528	0.826	1.151	1.502	1.871	2.243	2.652	3.067	3.496	3.917	4.302	4.69	5.136	5.532	5.854	6.172	6.473	6.776
Baa	0.265	0.748	1.308	1.927	2.572	3.216	3.831	4.465	5.125	5.782	6.444	7.124	7.808	8.422	8.984	9.578	10.158	10.687	11.191	11.702
Ba	1.231	2.917	4.783	6.736	8.618	10.41	12.049	13.626	15.162	16.784	18.19	19.59	20.934	22.135	23.276	24.382	25.476	26.538	27.498	28.373
B	3.507	7.962	12.41	16.43	20.027	23.157	25.996	28.406	30.535	32.352	33.968	35.416	36.845	38.297	39.679	41.017	42.222	43.175	43.869	44.421
Caa-C	10.423	18.193	24.401	29.372	33.394	36.598	39.335	41.811	44.222	46.32	48.295	50.134	51.716	53.317	55.003	56.653	58.183	59.646	61.107	62.591
Inv Grade	0.145	0.414	0.742	1.112	1.515	1.934	2.349	2.769	3.207	3.652	4.11	4.574	5.022	5.439	5.842	6.224	6.566	6.897	7.223	7.54
Spec Grade	3.739	7.5	11.009	14.148	16.918	19.337	21.492	23.413	25.192	26.874	28.355	29.763	31.108	32.378	33.595	34.775	35.899	36.927	37.823	38.636
All	1.5	3.022	4.435	5.698	6.822	7.814	8.701	9.509	10.278	11.016	11.706	12.375	13.013	13.602	14.165	14.7	15.19	15.647	16.071	16.47

Table A.2: Target default frequencies for the time to maturity and rating cohorts.A full description of the rating encodings IG, SG and WR is available in Section 3.3.

TAU	(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]
IG	0.41	1.37	2.69	4.04	6.43	9.51
SG	2.51	6.32	10.32	14.23	19.46	25.94
WR	2.29	5.08	7.73	9.86	12.52	15.53

A.4 Cross Sectional Tables

Table A.3: Count of the number of bond-month constituents in each cross-sectional group. A full description of the rating encodings IG, SG and WR is available in Section 3.3.

TAU	(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]
IG	3226	10870	6014	5561	2485	975
SG	610	2214	851	416	199	60
WR	2215	8579	3471	1420	727	329

Table A.4: A summary of the model spread and actual spread within each cross-section for the Merton model. The left part of the table presents mean spreads while the right part presents median spreads. A full description of the rating encodings IG, SG and WR is available in Section 3.3.

Merton		mean						median					
		(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]	(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]
IG	Actual	87	110	121	118	117	131	69	91	104	104	107	124
	Model	17	34	47	64	72	54	0	1	7	16	27	35
SG	Actual	205	270	292	223	214	147	162	214	267	197	167	111
	Model	37	85	131	124	148	70	4	28	73	85	76	10
WR	Actual	199	253	224	219	164	153	120	186	180	155	122	139
	Model	57	73	76	96	47	65	0	2	7	14	21	44

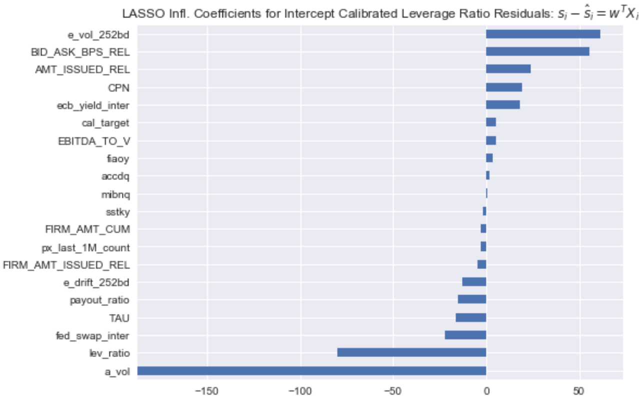
Table A.5: A summary of the model spread and actual spread within each cross-section for the Merton model. The left part of the table presents mean spreads while the right part presents median spreads. A full description of the rating encodings IG, SG and WR is available in Section 3.3.

Binary Merton		mean						median					
		(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]	(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]
IG	Actual	87	110	121	118	117	131	69	91	104	104	107	124
	Model	53	67	76	81	70	45	0	4	25	42	47	42
SG	Actual	205	270	292	223	214	147	162	214	267	197	167	111
	Model	191	222	202	141	108	42	36	115	179	138	87	13
WR	Actual	199	253	224	219	164	153	120	186	180	155	122	139
	Model	145	151	110	91	58	51	0	9	24	38	36	44

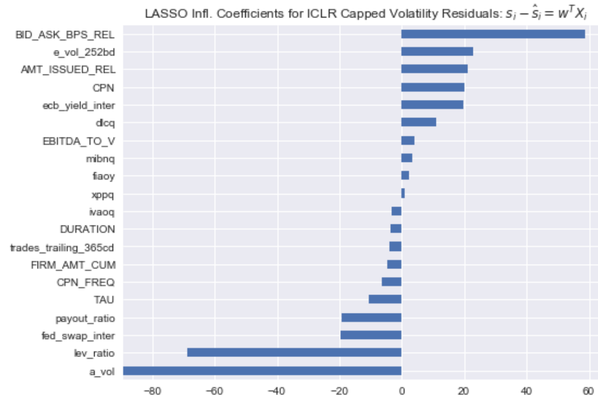
Table A.6: A summary of the model spread and actual spread within each cross-section for the Merton model. The left part of the table presents mean spreads while the right part presents median spreads. A full description of the rating encodings IG, SG and WR is available in Section 3.3.

Calibrated Black Cox		mean						median					
		(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]	(1, 2]	(2, 5]	(5, 7]	(7, 10]	(10, 15]	(15, 20]
IG	Actual	87	110	121	118	117	131	69	91	104	104	107	124
	Model	30	43	61	73	80	116	0	0	5	12	26	96
SG	Actual	205	270	292	223	214	147	162	214	267	197	167	111
	Model	182	172	235	220	181	182	23	46	141	185	119	83
WR	Actual	199	253	224	219	164	153	120	186	180	155	122	139
	Model	154	165	120	121	124	94	0	4	10	20	74	68

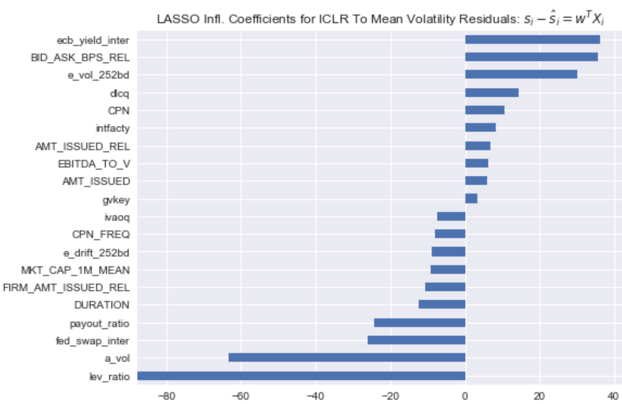
A.5 LASSO Regression for ICLR Extensions



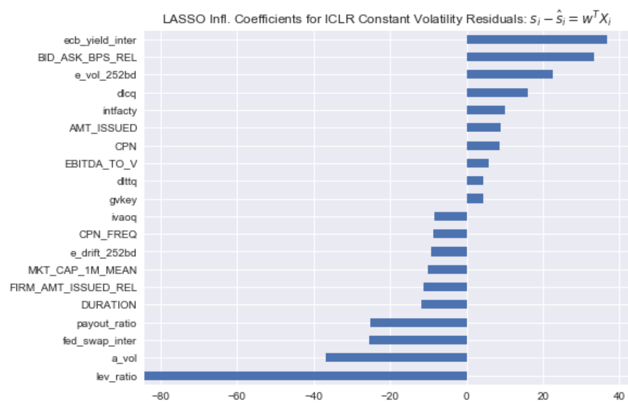
(a) ICLR



(b) ICLR Capped Asset Volatility



(c) ICLR To Mean Asset Volatility



(d) ICLR Constant Asset Volatility

Figure A.2: LASSO regressions are performed with the four ICLR model extension residuals as dependent variables. All of the bond month observations, including numerical and categorical features, are included as independent variables. In order to preserve comparability the regressions are performed once again with $\alpha =$ 3.15. *The 10 most influencing positive feature coefficients and 10 most influencing negative feature coefficients generated with the average punishment term are presented for each model in the figures. A full variable code description is available in Appendix A.6*

A.6 Compustat fields

Table A.7: Compustat field descriptions pt.1

Compustat Field	Field type	Description	Compustat Field	Field type	Description
accdq	NUM	Accrued Expenses and Deferred Income	cfpdoq	NUM	Commissions and Fees Paid - Other
accliy	NUM	Accrued Liabilities - Increase/Decrease	cfpdoy	NUM	Commissions and Fees Paid - Other
accoq	NUM	Acceptances Outstanding	chechy	NUM	Cash and Cash Equivalents - Increase (Decrease)
acctstdq	CHAR	Accounting Standard	chenfdy	NUM	Cash/Cash Equivalents/Net Funds - Increase/Decrease
acoq	NUM	Current Assets - Other - Total	cheq	NUM	Cash and Short-Term Investments
acoxq	NUM	Other Current Assets - Sundry	chq	NUM	Cash
acqdisny	NUM	Acquisitions and Disposals - Net Cash Flow	chsq	NUM	Cash and Deposits - Segregated
acqdisoy	NUM	Acquisitions and Disposals - Other	cik	CHAR	CIK Number
actq	NUM	Current Assets - Total	city	CHAR	City
add1	CHAR	Address Line 1	cltq	NUM	Contingent Liabilities- Total
add2	CHAR	Address Line 2	cogsq	NUM	Cost of Goods Sold
add3	CHAR	Address Line 3	cogsy	NUM	Cost of Goods Sold
add4	CHAR	Address Line 4	compstq	CHAR	Comparability Status
addzip	CHAR	Postal Code	conm	CHAR	Company Name
adpacq	NUM	Amortization of Deferred Policy Acquisition Costs	conml	CHAR	Company Legal Name
adpacy	NUM	Amortization of Deferred Policy Acquisition Costs	county	CHAR	County Code
amq	NUM	Amortization of Intangibles	cstkq	NUM	Common/Ordinary Stock (Capital)
amy	NUM	Amortization of Intangibles	curcdq	CHAR	ISO Currency Code
ancq	NUM	Non-Current Assets - Total	datacqtr	CHAR	Calendar Data Year and Quarter
aolochy	NUM	Assets and Liabilities - Other (Net Change)	datafqtr	CHAR	Fiscal Data Year and Quarter
aoq	NUM	Assets - Other - Total	dcsfdy	NUM	Current Debt - Source of Funds
aotq	NUM	Assets- Other- Total	dcufdy	NUM	Current Debt - Use of Funds
apalchy	NUM	Accounts Payable and Accrued Liabilities - Increase (Decrease)	dfpacq	NUM	Deferred Policy Acquisition Costs
apchy	NUM	Accounts Payable/Creditors - Increase(Decrease)	dfxaq	NUM	Depreciation of Fixed Assets (Tangible)
apoq	NUM	Accounts Payable - Other	dfxay	NUM	Depreciation of Fixed Assets (Tangible)
apq	NUM	Account Payable/Creditors - Trade	dispochy	NUM	Disposals - Other - (Gain)/Loss
aqcy	NUM	Acquisitions	ditq	NUM	Dividend Income
artfsq	NUM	Accounts Receivable/Debtors - Total	dity	NUM	Dividend Income
asdisy	NUM	Associated Undertakings - Disposal	dlcchy	NUM	Changes in Current Debt
asinvy	NUM	Associated Undertakings - Investment	dlcq	NUM	Debt in Current Liabilities
atochy	NUM	Assets - Other - Change	dldte	DATE	Research Company Deletion Date
atq	NUM	Assets - Total	dlrsn	CHAR	Research Co Reason for Deletion
autxrq	NUM	Appropriations to Untaxed Reserves	dltisy	NUM	Long-Term Debt - Issuance
autxry	NUM	Appropriations to Untaxed Reserves	dltry	NUM	Long-Term Debt - Reduction
bcefq	NUM	Brokerage, Clearing and Exchange Fees	dlttq	NUM	Long-Term Debt - Total
bcefy	NUM	Brokerage, Clearing and Exchange Fees	docy	NUM	Discontinued Operations (FOF) - Memo
bctq	NUM	Benefits and Claims - Total (Insurance)	dpactq	NUM	Depreciation, Depletion and Amortization (Accumulated)
bcty	NUM	Benefits and Claims - Total (Insurance)	dpcy	NUM	Depreciation and Amortization - Statement of Cash Flows
bdiq	NUM	Broker / Dealer Income - Total	dpq	NUM	Depreciation and Amortization - Total
bdiy	NUM	Broker / Dealer Income - Total	dptbq	NUM	Deposits - Total - Banks
bsprq	CHAR	Balance Sheet Presentation	dptcq	NUM	Deposits - Total - Customer
busdesc	CHAR	SandP Business Description	dpy	NUM	Depreciation and Amortization - Total
capcstq	NUM	Capitalized Costs	dvpdpq	NUM	Dividends and Bonuses Paid Policyholders
capcsty	NUM	Capitalized Costs	dvpdpy	NUM	Dividends and Bonuses Paid Policyholders
capfly	NUM	Capital Element of Finance Lease Rental Payments	dvrecy	NUM	Dividends Received
capr1q	NUM	Risk-Adjusted Capital Ratio - Tier 1	dvrreq	NUM	Development Revenue (Real Estate)
capr2q	NUM	Risk-Adjusted Capital Ratio - Tier 2	dvrrey	NUM	Development Revenue (Real Estate)
capr3q	NUM	Risk-Adjusted Capital Ratio - Combined	dvtq	NUM	Dividends - Total
capsq	NUM	Capital Surplus/Share Premium Reserve	dvty	NUM	Dividends - Total
capxfiy	NUM	Capital Expenditures and Financial Investment - Net Cash Flow	dvy	NUM	Cash Dividends
capxy	NUM	Capital Expenditures	eieacy	NUM	Equity Interest in Earnings of Associated Companies
caq	NUM	Customers' Acceptance	ein	CHAR	Employer Identification Number
ceqq	NUM	Common/Ordinary Equity - Total	eqdivpy	NUM	Equity Dividend Paid
cfbdq	NUM	Commissions and Fees (Broker Dealer)	eqrtq	NUM	Equity Reserves - Total
cfbdy	NUM	Commissions and Fees (Broker Dealer)	eroq	NUM	Equity Reserves - Other
cfereq	NUM	Commissions and Fees (Real Estate)	esubq	NUM	Equity in Earnings (I/S) - Unconsolidated Subsidiaries
cferey	NUM	Commissions and Fees (Real Estate)	esuby	NUM	Equity in Earnings (I/S)- Unconsolidated Subsidiaries
cflaothy	NUM	Cash Flow Adjustments - Other	exchg	NUM	Stock Exchange Code
cfoq	NUM	Commissions and Fees - Other	exresy	NUM	Exchange Rate Effect - Source of Funds
cfoy	NUM	Commissions and Fees - Other	exreuy	NUM	Exchange Rate Effect - Use of Funds

Table A.8: Compustat field descriptions pt.2

Compustat Field	Field type	Description	Compustat Field	Field type	Description
exrey	NUM	Exchange Rate Effect	intpdy	NUM	Interest Paid
fax	CHAR	Fax Number	intrcy	NUM	Interest Received
fcaq	NUM	Foreign Exchange Income (Loss)	invchy	NUM	Inventory - Decrease (Increase)
fcay	NUM	Foreign Exchange Income (Loss)	invdspy	NUM	Investments - Disposal
fdateq	DATE	Final Date	invsvcy	NUM	Investments and Servicing of Finance - Net Cash Flow
feaq	NUM	Foreign Exchange Assets	invtq	NUM	Inventories - Total
felq	NUM	Foreign Exchange Liabilities	iobdq	NUM	Income - Other (Broker Dealer)
fiaoy	NUM	Financing Activities - Other	iobdy	NUM	Income - Other (Broker Dealer)
fincfy	NUM	Financing Activities - Net Cash Flow	ioiq	NUM	Income - Other (Insurance)
finincy	NUM	Financing Increase- Total	ioiy	NUM	Income - Other (Insurance)
finley	NUM	Finance Lease Increases	ioreq	NUM	Income - Other (Real Estate)
finrey	NUM	Financing Repayments/Reductions- Total	iorey	NUM	Income - Other (Real Estate)
finvaoy	NUM	Funds from Investment and Finance Activities - Other	ipodate	DATE	Company Initial Public Offering Date
fopoy	NUM	Funds from Operations - Other	ipq	NUM	Investment Property
fqtr	NUM	Fiscal Quarter	iptiq	NUM	Insurance Premiums - Total (Insurance)
fsrcopoy	NUM	Sources of Operating Funds - Other	iptiy	NUM	Insurance Premiums - Total (Insurance)
fsrcopty	NUM	Source of Funds From Operations - Total	isgtq	NUM	Investment Securities - Gain (Loss) - Total
fsrcoy	NUM	Sources of Funds - Other	isgty	NUM	Investment Securities - Gain (Loss) - Total
fsrcty	NUM	Sources of Funds - Total	isin	CHAR	International Security ID
fuseoy	NUM	Uses of Funds - Other	istq	NUM	Investment Securities -Total
fusety	NUM	Uses of Funds - Total	ivacoy	NUM	Investing Activities - Other
fyearq	NUM	Fiscal Year	ivaeqq	NUM	Investment and Advances - Equity
fyr	NUM	Fiscal Year-end	ivaoq	NUM	Investment and Advances - Other
fyr	NUM	Fiscal Year-end Month	ivchy	NUM	Increase in Investments
fyr	NUM	Fiscal Year-end Month	iviq	NUM	Investment Income - Total (Insurance)
fyrc	NUM	Current Fiscal Year End Month	iviy	NUM	Investment Income - Total (Insurance)
gdwlamq	NUM	Amortization of Goodwill	ivncfy	NUM	Investing Activities - Net Cash Flow
gdwlamy	NUM	Amortization of Goodwill	ivptq	NUM	Investments - Permanent - Total
gdwlq	NUM	Goodwill (net)	ivstchy	NUM	Short-Term Investments - Change
ggroup	CHAR	GIC Groups	ivstq	NUM	Short-Term Investments- Total
gind	CHAR	GIC Industries	ivtfsq	NUM	Financial Services Investment Assets- Total
gpq	NUM	Gross Profit (Loss)	lcabgq	NUM	Loans/Claims/Advances - Banks and Government - Total
gpy	NUM	Gross Profit (Loss)	lcacuq	NUM	Loans/Claims/Advances - Customers- Total
gsector	CHAR	GIC Sectors	lcoq	NUM	Current Liabilities - Other - Total
gsubind	CHAR	GIC Sub-Industries	lcoxq	NUM	Current Liabilities - Other (Sundry)
iatiq	NUM	Investment Assets - Total (Insurance)	lctq	NUM	Current Liabilities - Total
ibcy	NUM	Income Before Extraordinary Items - Statement of Cash Flows	liqresny	NUM	Management of Liquid Resources - Net Cash Flow
ibkiq	NUM	Investment Banking Income	liqresoy	NUM	Liquid Resources - Other Movements
ibkiy	NUM	Investment Banking Income	lltq	NUM	Long-Term Liabilities (Total)
ibmiiq	NUM	Income before Extraordinary Items and Noncontrolling Interests	lndepy	NUM	Loans and Deposits - (Increase)/Decrease
ibmiiy	NUM	Income before Extraordinary Items and Noncontrolling Interests	lnincy	NUM	Loan Increase/Additions
ibq	NUM	Income Before Extraordinary Items	lnmdy	NUM	Loans (Made)/Repaid
iby	NUM	Income Before Extraordinary Items	lnrepy	NUM	Loan Repayments/Reductions
idbflag	CHAR	International, Domestic, Both Indicator	loc	CHAR	Current ISO Country Code - Headquarters
iditq	NUM	Interest Income - Total	loq	NUM	Liabilities - Other
idity	NUM	Interest Income - Total	lseq	NUM	Liabilities and Stockholders Equity - Total
iireq	NUM	Investment Income (Real Estate)	lsq	NUM	Liabilities - Other
iirey	NUM	Investment Income (Real Estate)	ltdchy	NUM	Long-Term Debt - Change
iitq	NUM	Insurance Income - Total	ltdlchy	NUM	Long-Term Debt/Liabilities - Change
iity	NUM	Insurance Income - Total	ltloy	NUM	Long-Term Liabilities - Other - Increase/(Decrease)
incorp	CHAR	Current State/Province of Incorporation Code	ltmibq	NUM	Liabilities - Total and Noncontrolling Interest
intandy	NUM	Intangible Assets - Disposal	ltq	NUM	Liabilities - Total
intanpy	NUM	Intangible Assets - Purchase	mibnq	NUM	Noncontrolling Interests - Nonredeemable - Balance Sheet
intanq	NUM	Intangible Assets - Total	mibq	NUM	Noncontrolling Interest - Redeemable - Balance Sheet
intcq	NUM	Interest Capitalized	mibtq	NUM	Noncontrolling Interests - Total - Balance Sheet
intcy	NUM	Interest Capitalized	micy	NUM	Noncontrolling Interest (FOF)
intfacty	NUM	Interest and Dividend Adjustments - Financing Activities	miiq	NUM	Noncontrolling Interest - Income Account
intfly	NUM	Interest Element of Finance Leases	miiy	NUM	Noncontrolling Interest - Income Account
intiacty	NUM	Interest and Dividend Adjustments - Investing Activities	miseqy	NUM	Noncontrolling Interest In Stockholders Equity - Change
intoacty	NUM	Interest and Dividend Adjustments - Operating Activities	mtlq	NUM	Loans From Securities Finance Companies for Margin Transactions

Table A.9: Compustat field descriptions pt.3

Compustat Field	Field type	Description
naics	CHAR	North American Industry Classification Code
ncfliqy	NUM	Net Cash Flow Before Management of Liquid Resources and Financing
neqmiy	NUM	Non-Equity and Noncontrolling Interest Dividends Paid
nitq	NUM	Net Item - Total
nity	NUM	Net Item - Total
noasuby	NUM	Net Overdrafts Acquired with Subsidiaries
nopioq	NUM	Other Non-Operating Inc/Expense
nopioy	NUM	Other Non-Operating Inc/Expense
nopiq	NUM	Non-Operating Income (Expense) - Total
nopiy	NUM	Non-Operating Income (Expense) - Total
oancfcy	NUM	Related to Continuing Operations
oancfdy	NUM	Related to Discontinued Operations
oancfy	NUM	Operating Activities - Net Cash Flow
oiadpq	NUM	Operating Income After Depreciation - Quarterly
oiadpy	NUM	Operating Income After Depreciation - Year-to-Date
oibdpq	NUM	Operating Income Before Depreciation - Quarterly
oibdpy	NUM	Operating Income Before Depreciation
opprfty	NUM	Operating Profit
oproq	NUM	Operating Revenues - Other
oproy	NUM	Operating Revenues - Other
pclq	NUM	Provision - Credit Losses (Income Account)
pcly	NUM	Provision - Credit Losses (Income Account)
pdateq	DATE	Preliminary Date
pdq	NUM	Months in Period - Quarterly
pdsa	NUM	Months in Period - Semi-annual
pdytd	NUM	Months in Period - YTD
phone	CHAR	Phone Number
piq	NUM	Pretax Income
piy	NUM	Pretax Income
pliachy	NUM	Pension Liabilities - Change
ppentq	NUM	Property Plant and Equipment - Total (Net)
prcq	NUM	Participation Rights Certificates
prican	CHAR	Current Primary Issue Tag - Canada
prirow	CHAR	Primary Issue Tag - Rest of World
priusa	CHAR	Current Primary Issue Tag - US
prosaiy	NUM	Proceeds From Sale of Fixed Assets and Sale of Investments
prstkcy	NUM	Purchase of Common and Preferred Stock
prvy	NUM	Provisions (FOF)
psfixy	NUM	Proceeds- Sale of Fixed Assets
pstkq	NUM	Preferred/Preference Stock (Capital) - Total
ptranq	NUM	Principal Transactions
ptrany	NUM	Principal Transactions
purtshry	NUM	Purchase of Treasury Shares
pvoq	NUM	Provisions - Other (Net)
pvoy	NUM	Provisions - Other (Net)
pvtq	NUM	Provisions - Total
ratiq	NUM	Reinsurance Assets - Total (Insurance)
rawmsmq	NUM	Raw Materials, Supplies, and Merchandise
rawmsmy	NUM	Raw Materials, Supplies, and Merchandise
recchy	NUM	Accounts Receivable - Decrease (Increase)
reccoq	NUM	Receivables - Current - Other
rectoq	NUM	Receivables - Current Other incl Tax Refunds
rectq	NUM	Receivables - Total
rectrq	NUM	Receivables - Trade
reitq	NUM	Real Estate Income - Total
reity	NUM	Real Estate Income - Total
req	NUM	Retained Earnings
revtq	NUM	Revenue - Total
revty	NUM	Revenue - Total
risq	NUM	Revenue/Income - Sundry
risy	NUM	Revenue/Income - Sundry
rltq	NUM	Reinsurance Liabilities - Total
rp	CHAR	Reporting Periodicity
rvlrvq	NUM	Revaluation Reserve
rvtiq	NUM	Reserves - Total (Insurance)
rvutxq	NUM	Reserves - Untaxed
rvy	NUM	Reserves
saaq	NUM	Separate Account Assets
saleq	NUM	Sales/Turnover (Net)
saley	NUM	Sales/Turnover (Net)
salq	NUM	Separate Account Liabilities
sbdcq	NUM	Securities Borrowed and Deposited by Customers
scfq	NUM	Cash Flow Model
scoq	NUM	Share Capital - Other
scq	NUM	Securities In Custody
sctq	NUM	Total Share Capital
sedol	CHAR	SEDOL
seqq	NUM	Stockholders Equity >Parent >Index Fundamental >Quarterly
shrcapy	NUM	Share Capital Transactions - Other
sic	CHAR	Standard Industry Classification Code
sivy	NUM	Sale of Investments
spcindcd	NUM	SandP Industry Sector Code
spcseccd	NUM	SandP Economic Sector Code
spcsrc	CHAR	SandP Quality Ranking - Current
spiq	NUM	Special Items
spiy	NUM	Special Items
sppchy	NUM	Sale of Fixed Assets - (Gain)/Loss
sppivy	NUM	Sale of PPandE and Investments - (Gain) Loss
srcq	NUM	Source Code
ssnpq	NUM	Securities Sold Not Yet Purchased
sstky	NUM	Sale of Common and Preferred Stock
staltq	CHAR	Status Alert
state	CHAR	State/Province
stfixay	NUM	Sale of Tangible Fixed Assets
stinvy	NUM	Short Term Investments - (Increase)/Decrease
stkchq	NUM	Change in Stocks
stkchy	NUM	Change in Stocks
stko	NUM	Stock Ownership Code
subdisy	NUM	Subsidiary Undertakings - Disposal
subpury	NUM	Subsidiary Undertakings - Purchase
tdsgq	NUM	Trading/Dealing Securities - Gain (Loss)
tdsgy	NUM	Trading/Dealing Securities - Gain (Loss)
tdstq	NUM	Trading/Dealing Account Securities - Total
teqq	NUM	Stockholders Equity - Total
transaq	NUM	Cumulative Translation Adjustment
tstkq	NUM	Treasury Stock - Total (All Capital)
txdbq	NUM	Deferred Taxes - Balance Sheet
txdcy	NUM	Deferred Taxes (Statement of Cash Flows)
txopy	NUM	Taxation - Operating Activities
txtq	NUM	Income Taxes - Total
txty	NUM	Income Taxes - Total
txy	NUM	Taxation
unnpq	NUM	Unappropriated Net Profit (Shareholders" Equity)
updq	NUM	Update Code
wcapchcy	NUM	Working Capital - Change
wcapchy	NUM	Working Capital Changes - Total
wcapopcy	NUM	Working Capital/Net Operating Assets - Change
wcapsay	NUM	Working Capital Change (Separate Account)
wcapsuy	NUM	Source and Use of Funds/Working Capital Adjustments - Other
wcapsy	NUM	Working Capital Change - Source of Funds
wcapty	NUM	Working Capital/Cash/Net Funds Change - Total
wcapuy	NUM	Working Capital Change - Use of Funds
weburl	CHAR	Web URL
xagtq	NUM	Administrative and General Expense - Total
xagty	NUM	Administrative and General Expense - Total
xbdtq	NUM	Broker / Dealer Expense - Total
xbdty	NUM	Broker / Dealer Expense - Total
xcomiq	NUM	Commissions Expense (Insurance)
xcomiy	NUM	Commissions Expense (Insurance)
xcomq	NUM	Communications Expense (Broker/Dealer)
xcomy	NUM	Communications Expense (Broker/Dealer)
xdvreq	NUM	Expense - Development (Real Estate)
xdvrey	NUM	Expense - Development (Real Estate)
xidocy	NUM	Extraordinary Items and Discontinued Operations (Statement of Cash Flows)
xintq	NUM	Interest and Related Expense- Total
xinty	NUM	Interest and Related Expense- Total
xioq	NUM	Insurance Expense - Other - Total
xioy	NUM	Insurance Expense - Other - Total
xiq	NUM	Extraordinary Items
xiviq	NUM	Investment Expense (Insurance)
xiviy	NUM	Investment Expense (Insurance)
xivreq	NUM	Expense - Investment (Real Estate)
xivrey	NUM	Expense - Investment (Real Estate)
xiy	NUM	Extraordinary Items
xobdq	NUM	Expense - Other (Broker/Dealer)
xobdy	NUM	Expense - Other (Broker/Dealer)
xoiq	NUM	Expenses - Other (Insurance)
xoiy	NUM	Expenses - Other (Insurance)
xoproq	NUM	Operating Expense - Other
xoproy	NUM	Operating Expense - Other
xoprq	NUM	Operating Expense- Total
xopry	NUM	Operating Expense- Total
xoreq	NUM	Expense - Other (Real Estate)
xorey	NUM	Expense - Other (Real Estate)
xppq	NUM	Prepaid Expenses and Accrued Income
xretq	NUM	Real Estate Expense - Total
xrety	NUM	Real Estate Expense - Total
xsgaq	NUM	Selling, General and Administrative Expenses
xsgay	NUM	Selling, General and Administrative Expenses
xsq	NUM	Expense - Sundry
xstoq	NUM	Staff Expense - Other
xstoy	NUM	Staff Expense - Other
xstq	NUM	Staff Expense - Wages/Salaries
xsty	NUM	Staff Expense - Wages/Salaries
xsy	NUM	Expense - Sundry
xtq	NUM	Expense - Total
xty	NUM	Expense - Total