LU TP 17-17
May 2017

Explaining $B \to D^{(*)}\tau\nu$ Decays with a Two Higgs Doublet Model using the Froggatt-Nielsen Framework

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Abstract

In recent years, the BaBar, Belle and LHCb experiments have observed an excess of $B \to D^{(*)}\tau\nu$ decays compared to Standard Model predictions. In this thesis, we investigate if it is possible to explain this excess with a two Higgs doublet model using the Froggatt-Nielsen framework. Two Higgs doublet models allow new decays at tree-level and the Froggatt-Nielsen mechanism gives an explanation to the large mass hierarchy amongst fermions. New particles and concepts are introduced with the Froggatt-Nielsen mechanism, there among a new type of $U(1)$ charge, called flavon charge, and a symmetry breaking parameter $\varepsilon \approx 0.2$. We express all Yukawa couplings, fermion masses and elements of the Cabibbo-Kobayashi-Maskawa matrix in terms of ε and the flavon charges. By considering physical constraints, such as limits from flavour changing neutral currents, we determine how the flavon charges for the Higgs fields and the Standard Model fermions can be chosen while satisfying these constraints. We investigate how the $B \to D^{(*)}\tau\nu$ decays depends on ε and the flavon charges, and for valid sets of flavon charges, we check if the observed excess can be explained. We find some sets where this could be possible and conclude that further and more detailed studies are motivated.

Populärvetenskaplig sammanfattning

Teoretiska fysiker tycker verkligen om att bygga modeller av sin omvärld för att kunna göra förutsägelser om den. En av de mest lyckade teoretiska modellerna är standardmodellen inom partikelfysik. Med den har fysiker lyckats förklara och förstå hur universums allra minsta beståndsdelar fungerar. Den har till och med vid ett flertal tillfällen lyckats att förutsäga existensen av nya partiklar, som sedan har hittats experimentellt med stora partikelacceleratorer. Det mest kända exemplet är såklart Higgspartikeln som förutsades 1964 och sedan hittades 2012.

Däremot, har det vid allt fler tillfällen hänt att experimentella resultat avviker från vad som kan förväntas utifrån standardmodellen. Ett sådant exempel som vi ska studera närmare är sönderfallet av den så kallade B-mesonen. Mesoner är partiklar som är uppbyggda av två kvarkar. Kvarkar i sin tur är en typ av elementarpartiklar, de allra minsta av naturens byggstenar. B-mesonen, liksom alla andra mesoner, är instabil vilket innebär att den måste sönderfalla. När den sönderfaller omvandlas en av kvarkarna till en lättare kvark med hjälp av en så kallad boson, en annan typ av elementarpartikel. Sönderfall kan ske på olika sätt, via olika sönderfallskanaler, vilket resulterar i olika slutprodukter. Med standardmodellen går det att räkna ut hur vanligt ett sönderfall via en specifik sönderfallskanal bör vara. För det särskilda sönderfallet av B-mesonen som vi är intresserade av har det uppmätts en större mängd sönderfall än vad som har förutsagts med standardmodellen.

Det här uppmätta överskottet av B-mesonsönderfallet ska vi försöka förklara genom att bygga ut standardmodellen. Detta ska vi göra genom att använda oss av en Tvåhiggsdubblettmodell (2HDM). I standardmodellen finns det *en* Higgsdubblett vilket resulterar i *en* Higgspartikel. I en 2HDM finns det, vilket hörs på namnet, *två* Higgsdubbletter, vilket istället visar sig resultera i *fem* Higgspartiklar. En av de här nya partiklarna skulle i teorin även den orsaka vårt B-mesonsönderfall utöver den mängd som redan sker enligt standardmodellen. Genom att anpassa Higgspartiklarnas kopplingar till kvarkarna skulle det här extra bidraget kunna få förutsägelser gjorda med 2HDM att stämma överens med experimentella data.

I arbetet använder vi oss av en speciell version av 2HDM som använder sig av den så kallade Froggatt-Nielsen-mekanismen. Den introducerar en ny typ av laddning som partiklar kan ha, precis som att de kan ha elektrisk laddning, som heter flavonladdning. Beroende på vilka flavonladdningar de olika partiklarna har skulle de eventuellt kunna förklara varför kvarkarnas massor är så pass olika. Det nya 2HDM-bidraget till B-mesonsönderfallet skulle också påverkas av flavonladdningarna. Vi undersöker om det finns uppsättningar av flavonladdningar för de olika partiklarna som kan återskapa både rätt massrelationer bland kvarkarna och samtidigt förutsäga samma mängd av sönderfallet som experimenten visar. Vi kommer i slutändan fram till att detta faktiskt verkar vara möjligt.

Abbreviations

2HDM Two Higgs Doublet Model

CKM Cabibbo-Kobayashi-Maskawa

CP Charge-Parity

FCNC Flavour Changing Neutral Currents

FN Froggatt-Nielsen

RGE Renormalisation Group Equation

SM Standard Model

VEV Vacuum Expectation Value

Acknowledgements

I would like to extend my great gratitude to my supervisor J. Rathsman for his enthusiasm and support while supervising my Bachelor thesis. I would also like to thank my colleague J. Sandberg for many giving discussions about the model.

Contents

1 Introduction

The Standard Model (SM) of particle physics has since it was developed during the second half of the 20th century been very successful [1]. One quality of the SM, which is part of its success, is that particles have been proposed on theoretical ground and then later been found experimentally. The prime example of this is of course the Higgs particle, which was first proposed in the early 60's and was eventually discovered at CERN in 2012 [1]. However, despite its many great achievements, there are some phenomena within particle physics which cannot be explained with the SM and also some experimental data which do not agree with predictions. One such problem is that many predictions regarding the B-meson do not agree with experimental data. Therefore, decays of the B-meson is often used as a probe for new physics. Another such problem is that there might be more charge-parity (CP) violation needed than allowed by the SM.

One attempt to solve some of the problems of the SM, including B-meson physics, is to add one extra Higgs-doublet, i.e using a two Higgs doublet model (2HDM). Such a model results in five Higgs bosons (as opposed to the one included in the SM): two CP-even neutral scalars (h and H), one CP-odd neutral scalar (A) and two charged scalars ($H^{\pm}$) [2]. These extra Higgs particles can mediate more decays at tree-level, which possibly could explain the data that do not correspond with the SM prediction, such as various B-meson decays. However, one possible danger which follows is that flavour changing neutral currents (FCNC) may occur at tree-level. From experiments, these are known to be severely constrained.

One motivation to why an extra Higgs doublet might be needed is because super-symmetric theories require two Higgs doublets. Further, new sources of CP violation would be introduced. We do, however, focus on a CP-conserving version of 2HDM.

In this thesis, we investigate one type of B-meson decays more closely, namely $B \to D^{(*)}\tau\nu$. Data from BaBar, Belle and LHCb suggest higher decay rates than the SM predictions [3–5]. The decays combined give a $3.4\,\sigma$ deviation from the SM prediction [3]. In particle physics, a deviation of $5\,\sigma$ is considered to imply a discovery of new physics beyond the SM. However, a deviation of $3.4\,\sigma$ is large enough for it to be interesting to look for explanations in new physics. We investigate the possibility of explaining the collected data with a 2HDM.

There are a few different types of 2HDMs depending on which symmetries one chooses to impose. Various attempts to explain the data with models of the different standard types of 2HDMs (these are explained in section 3.2) have been made, see for example [6]. We do not use any of the four standard types, but instead use the Froggatt-Nielsen (FN) mechanism which was first proposed by C.D. Froggatt and H.B. Nielsen in 1978 [7]. The concept was introduced as an attempt to explain the large mass hierarchy amongst the quarks. It is also interesting since it at the same time gives a theoretical explanation to the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes how strongly different quarks mix.

In order for the FN mechanism to work, new heavy fermions must be introduced. Also, a scalar field called flavon, with a corresponding flavon charge originating from a $U(1)$ gauge symmetry, must be introduced. Each of the SM fermions and the two Higgs doublets of the 2HDM all have flavon charges, which affect the couplings in decays. We investigate in what manner these flavon charges can be chosen while still satisfying various physical constraints which are imposed. Finally, we examine how these charges affect the couplings that enter the $B \to D^{(*)}\tau\nu$-decays. We mainly explore how the couplings are affected at order of magnitude to find out if it is at all possible to explain the decays with this model. Both the quark-sector and the lepton-sector are affected by the Froggatt-Nielsen mechanism, but we mainly focus on the quark-sector.

The report is structured as follows: First, the Standard Model Brout-Englert-Higgs mechanism is briefly revised in section 2. Then the general two Higgs doublet model is explained in section 3 followed by an account of the Froggatt-Nielsen mechanism in the SM and how it is extended to 2HDM in section 4. This is followed by an discussion about which constraints that need to be imposed in section 5 and in section 6 we explore if there exists any sets of flavon charges which satisfy the restrictions. After this, in section 7, the $B \to D^{(*)}\tau\nu$-decays are investigated and we will see if they may be explained with a 2HDM in the FN framework (FN-2HDM).

2 The Standard Model Brout-Englert-Higgs mechanism

The Brout-Englert-Higgs mechanism[1] is the way which bosons and fermions of the SM acquire mass. The mechanism is described in detail in various advanced undergraduate textbooks as for example [1] and [8]. Here, we will only give a brief review of the essential parts.

We start with a single Higgs field Φ, which is a doublet under the $SU(2)_L$ gauge symmetry and which we assign hypercharge +1,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \tag{2.1}$$

The Lagrangian can be expressed as

$$\mathcal{L} = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - V(\Phi), \tag{2.2}$$

where the first term is the kinetic term and the second is the potential

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2. \tag{2.3}$$

In order to obtain finite minima for the potential, λ has to be positive. On the other hand, μ^2 may be either positive or negative. There will be a single minimum at $\phi^+ = \phi^0 = 0$ if $\mu^2 > 0$. If instead $\mu^2 < 0$, there will be an infinite set of non-zero minima fulfilling

$$\Phi^\dagger \Phi = \frac{1}{2}\left(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2\right) = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}. \tag{2.4}$$

All the minima are equally valid but we have to choose one to continue. We choose

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{2.5}$$

Choosing one minimum as the vacuum expectation value (VEV) leads to that the symmetry is broken, something which is called spontaneous symmetry breaking. One can find that $v \approx 246\,\text{GeV}$ from experiments. Particles correspond to excitations of the field and can thus be investigated by considering a perturbation around the minimum

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ v + \eta(x) + i\phi_4(x) \end{pmatrix}. \tag{2.6}$$

If we were to write out the Lagrangian of equation (2.2) using (2.3) and (2.6), we would find that ϕ_1, ϕ_2 and ϕ_4 would be the massless Goldstone bosons for a global symmetry. Choosing to make a gauge transformation into unitary gauge, the Goldstone bosons will be absorbed and give the longitudinal degrees of freedoms to the physical gauge fields $W^\pm$ and Z^0, which sometimes is called that the Goldstone bosons get eaten by the physical fields. Finally, we are left with

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \tag{2.7}$$

[1]Most commonly only called just the Higgs mechanism but Robert Brout and François Englert together proposed it independently from Peter Higgs and should thus be included in the name. We will, however, only call it the Higgs mechanism for compactness. Only Englert and Higgs shared the Nobel prise in 2013 since Brout passed way in 2011.

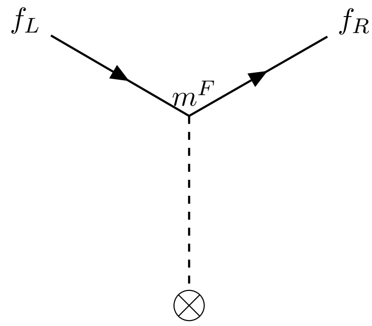

Figure 1: The process where fermions acquire mass. The fermions switch chirality upon interacting with the Higgs VEV. The coupling strength is the fermion mass m_f. All the Feynman diagrams in this work are drawn with Ti*k*Z-Feynman [9].

where $\eta(x)$ is now called $h(x)$, which is the physical Higgs boson.

The fermion masses can be found by considering the so-called Yukawa Lagrangian.[2] We initially only consider the Yukawa Lagrangian for the first generation of fermions

$$-\mathcal{L}_Y = \bar{Q}_L g_Y^d \Phi d_R + \bar{Q}_L g_Y^u \tilde{\Phi} u_R + \bar{L}_L g_Y^\ell \Phi e_R + \text{ h.c.} \quad , \tag{2.8}$$

where Q_L is the left-handed $SU(2)$ quark doublet, L_L is the left-handed lepton doublet and d_R, u_R and e_R are the right-handed singlets. $g_Y^{u,d,\ell}$ are coupling constants called Yukawa couplings. In order to give mass to the up quark, the conjugate is also introduced such that

$$\tilde{\Phi} = i\sigma_2 \Phi^*, \tag{2.9}$$

which also is an $SU(2)$ doublet but with hypercharge -1 and where σ_2 is the second Pauli matrix. Writing out the Yukawa Lagrangian (2.8) using the doublet (2.7) yields

$$\begin{aligned} -\mathcal{L}_y = \; & \frac{g_Y^d}{\sqrt{2}} v(\bar{d}_L d_R + \bar{d}_R d_L) + \frac{g_Y^u}{\sqrt{2}} v(\bar{u}_L u_R + \bar{u}_R u_L) + \frac{g_Y^e}{\sqrt{2}} v(\bar{e}_L e_R + \bar{e}_R e_L) \\ & + \frac{g_Y^d}{\sqrt{2}} h(\bar{d}_L d_R + \bar{d}_R d_L) + \frac{g_Y^u}{\sqrt{2}} h(\bar{u}_L u_R + \bar{u}_R u_L) + \frac{g_Y^e}{\sqrt{2}} h(\bar{e}_L e_R + \bar{e}_R e_L). \end{aligned} \tag{2.10}$$

The terms with two fields, i.e. those on the first line of (2.10), corresponds to the mass terms where the constants in front of the fields are the mass. Hence, fermions gain mass when they change chirality by interacting by a non-zero VEV, as illustrated in figure 1, where the mass is

$$m^F = \frac{v}{\sqrt{2}} g_Y^F, \tag{2.11}$$

where F is used to denote either of u, d and ℓ. The terms with three fields, i.e. those on the second line of equation (2.10), corresponds to interactions with the Higgs field which are proportional to the mass m^F.

This reasoning can of course also be extended to include the second and third generation of fermions as well. For all three generations, the Lagrangian in equation (2.8) can be written in a similar manner but letting Q_L, d_R, u_R, L_L and e_R be three-dimensional vectors in generation

[2] We assume that the neutrinos gain their mass from another mechanism and they are considered massless in this framework.

space instead (these are properly defined in section 3.1). Then the Yukawa couplings g_y^F would instead need to be 3×3 matrices Y^F. In order to obtain the mass eigenstates for the quarks, one would need to diagonalise $Y^{u,d}$ by a bi-unitary transformation using the unitary matrices $V_{L,R}^{u,d}$, see appendix A for details about the transformation. The quark masses are then given by

$$m_i^{u,d} = \frac{v}{\sqrt{2}} \left(V_L^{u,d} Y^{u,d} V_R^{u,d\dagger} \right)_{ii}, \tag{2.12}$$

where $i = \{1, 2, 3\}$ corresponds to $\{u, c, t\}$ for the up-sector and $\{d, s, b\}$ for the down-sector. As a result of the bi-unitary transformation, interactions between up-type quarks and down-type quarks will include a factor $V_L^u V_L^{d\dagger} \hat{=} V^{CKM}$ which is the definition of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [10]. This is explored more in later sections. The same reasoning can be extended to the lepon-sector, but since the neutrinos are assumed to be massless, the PMNS matrix does not enter.

3 The two Higgs doublet model

In order to find answers which the SM cannot provide, it is natural to endeavour to obtain these by modifying the SM. One of the simplest extensions to the SM which can be made is to add one more Higgs doublet, identical to the original. This extension is called the two Higgs doublet model and a comprehensive review of it can be found in [2]. Both the Higgs doublets are complex $SU(2)$ doublets which are singlets under $U(1)$ with hypercharge $+1$ and also singlets in $SU(3)$ colour space. After adding this extra doublet, the Brout-Englert-Higgs mechanism will work in the same way as in the SM as described in section 2 but extended to both the doublets. The potential will become more complicated. The simplest form of the general potential becomes with the Higgs doublet Φ_1 and Φ_2

$$\begin{aligned} V = \quad & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - (m_{12}^2\Phi_1^\dagger\Phi_2 + \text{ h.c.}) \\ & + \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \frac{1}{2}\lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \frac{1}{2}\lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\ & + \left\{\frac{1}{2}\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \left[\lambda_6\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{ h.c.}\right\}, \end{aligned} \tag{3.1}$$

where m_{11}^2, m_{22}^2 and λ_{1-4} are real numbers and the other parameters, m_{12}^2 and λ_{5-7}, are complex. By minimising the potential, the VEVs can be obtained. After spontaneous symmetry breaking, we choose VEVs for each doublet such that

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}e^{i\theta_1}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}e^{i\theta_2}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \tag{3.2}$$

The complex phases are the *CP*-violating parameters but since we are not including *CP*-violation, we set $\theta_1 = \theta_2 = 0$.

All bases are equivalent, hence we are free to use whichever basis we like. This basis, where both the Higgs doublets obtain VEVs, is called the generic basis. Another useful basis is the one where only one of the doublets obtains a VEV, which is known as the Higgs basis. One obtains this basis by a rotation of the generic basis with an angle β:

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \tag{3.3}$$

with corresponding VEVs

$$\langle H_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \langle H_2\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{3.4}$$

where

$$\tan\beta = \frac{v_2}{v_1}, \qquad v^2 = v_1^2 + v_2^2. \tag{3.5}$$

Just as in the SM, $v \approx 246\,\text{GeV}$ and by convention $\beta \in [0, \pi/2]$ is chosen.

Each of the two doublets consists of four independent fields, which gives a total of eight degrees of freedom. The doublets can be written in a number of different ways but they will of course all render in the same physical fields in the end. One way to write them is

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}(G^+c_\beta - H^+s_\beta) \\ v_1 + \phi_1 + i(G^0c_\beta - As_\beta) \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}(G^+s_\beta + H^+c_\beta) \\ v_2 + \phi_2 + i(G^0s_\beta + Ac_\beta) \end{pmatrix}, \tag{3.6}$$

where $c_\beta \hat{=} \cos\beta$ and $s_\beta \hat{=} \sin\beta$. Three of the eight fields, $G^\pm$ and G^0, would have been massless Goldstone bosons but they can be gauged away. The fields ϕ_1 and ϕ_2 are not the physical fields. The two *CP*-even physical Higgs field (the lighter h and heavier H) can be obtained by the linear combination

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{3.7}$$

of ϕ_1 and ϕ_2 where α is the angle between the states. Finally, we can express the Higgs doublets in the Higgs basis with only the physical fields by inserting equation (3.6) and (3.7) into (3.3) and gauging away the Goldstone bosons, which yields

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + Hc_{\beta-\alpha} + hs_{\beta-\alpha} \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ -Hs_{\beta-\alpha} + hc_{\beta-\alpha} + iA \end{pmatrix}. \tag{3.8}$$

The SM Higgs boson is identified as the linear combination $H^{SM} = Hc_{\beta-\alpha} + hs_{\beta-\alpha}$. If either $c_{\beta-\alpha} \approx 1$ or $s_{\beta-\alpha} \approx 1$, then H respectively h can be associated with the SM Higgs boson.

To summarise, we have by introducing one extra Higgs doublet to the SM obtained five Higgs fields instead of one. These are: two neutral *CP*-even scalar fields h and H, one neutral *CP*-odd scalar field A and two charged scalar fields $H^\pm$.

3.1 The Yukawa sector

In the last section, we arrived at the five Higgs fields. In this section we will see how fermions obtain masses and what new interactions are possible with the extra fields. Since the mass differences between neutrinos and other fermions are vast, we assume that the neutrinos obtain their mass from another mechanism and they are considered to be massless in this framework, just as we did in the SM.

In order to get a compact notation which is easy to manage, we define the three-dimensional vectors in generation space, where we put the left-handed flavour $SU(2)$ doublets in one vector each for quarks and leptons:

$$Q_L = \left(\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right)^T, \quad L_L = \left(\begin{pmatrix} \nu_{e\,L} \\ e_L^- \end{pmatrix}, \begin{pmatrix} \nu_{\mu\,L} \\ \mu_L^- \end{pmatrix}, \begin{pmatrix} \nu_{\tau\,L} \\ \tau_L^- \end{pmatrix} \right)^T. \tag{3.9}$$

The right-handed singlets are put in three different vectors but without any right-handed neutrinos since they are assumed not to be present:

$$U_R = (u_R, c_R, t_R)^T, \quad D_R = (d_R, s_R, b_R)^T, \quad E_R = \left(e_R^-, \mu_R^-, \tau_R^-\right)^T. \tag{3.10}$$

To be able to give mass to the up-type quarks, we need the complex conjugate of Φ which is defined the same way as in the SM, see equation (2.9).

Now, we can express the Yukawa Lagrangian in the generic basis where Y_a^F is the Yukawa coupling matrix for each sector ($F = u, d, \ell$) that couples to Higgs doublet Φ_a for $a = 1, 2$

$$\begin{aligned} -\mathcal{L}_Y^{gen} = \; & \bar{Q}_L Y_1^d \Phi_1 D_R + \bar{Q}_L Y_1^u \tilde{\Phi}_1 U_R + \bar{L}_L Y_1^\ell \Phi_1 E_R \\ & + \bar{Q}_L Y_2^d \Phi_2 D_R + \bar{Q}_L Y_2^u \tilde{\Phi}_2 U_R + \bar{L}_L Y_2^\ell \Phi_2 E_R + \text{ h.c.} \quad . \end{aligned} \tag{3.11}$$

The Yukawa Lagrangian is equally valid if it is expressed in another basis. By constructing the linear combination

$$\begin{pmatrix} \kappa_0^F \\ \rho_0^F \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} Y_1^F \\ Y_2^F \end{pmatrix} \tag{3.12}$$

for the Yukawa matrices, the Yukawa Lagrangian can be expressed in the Higgs basis as

$$\begin{aligned} -\mathcal{L}_Y^{Higgs} = \ & \bar{Q}_L \kappa_0^d H_1 D_R + \bar{Q}_L \kappa_0^u \tilde{H}_1 u_R + \bar{L}_L \kappa_0^\ell H_1 L_R \\ & + \bar{Q}_L \rho_0^d H_2 D_R + \bar{Q}_L \rho_0^u \tilde{H}_2 U_R + \bar{L}_L \rho_0^\ell H_2 L_R + \text{ h.c.} \quad . \end{aligned} \tag{3.13}$$

In its current form, in equation (3.13), the Yukawa Lagrangian is expressed in the fermion's weak eigenstates, but to generate the fermion masses it needs to be written in the mass eigenstates. This can be done by a bi-unitary transform of κ_0^F with the unitary matrices V_L^F and V_R^F just as in the SM:

$$\kappa^F = V_L^F \kappa_0^F V_R^{F\dagger}, \tag{3.14}$$

$$\rho^F = V_L^F \rho_0^F V_R^{F\dagger}. \tag{3.15}$$

The matrices κ^F are now diagonal and its elements correspond to the fermion masses:

$$m_i^F = M_{ii}^F = \frac{v}{\sqrt{2}} \kappa_{ii}^F, \tag{3.16}$$

where M^F are the mass matrices and $i = \{1, 2, 3\}$ are the generation indices which map in the same way as introduced in section 2 with the extension $\{e, \mu, \tau\}$ for the lepton-sector. However, the same transformation does generally not diagonalise two different matrices and thus ρ^F will not be diagonal in general. If this is the case, then the non-zero off-diagonal elements will cause FCNC which need to be constrained, something we will investigate closer later. For everything to be consistent, also the weak currents need to be transformed by the same matrices, i.e. they need to be rewritten in the mass eigenstates. In an attempt to keep the expressions as clean as possible we redefine the notation

$$V_{L,R}^F F_{L,R} \to F_{L,R}, \qquad \bar{F}_{L,R} V_{L,R}^{F\dagger} \to \bar{F}_{L,R}. \tag{3.17}$$

Before we attack the Lagrangian in equation (3.13) in full detail, we explore what happens for a simplified field $H_i = (A_i, B_i)^T$ in the down-sector. First, we expand Q_L and define D_L and U_L in corresponding way as D_R and U_L in (3.10), i.e. $D_L = (d_L, s_L, b_L)^T$ and $U_L = (U_L, c_L, t_L)^T$, which yields

$$-\mathcal{L}^d = \bar{D}_L(\kappa_0^d B_1 + \rho_0^d B_2) D_R + \bar{U}_L(\kappa_0^d A_1 + \rho_0^d A_2) D_R + \text{ h.c.} \quad . \tag{3.18}$$

The next step is to insert identity matrices $V_{L,R}^{u,d} V_{L,R}^{u,d\dagger}$ such that all weak eigenstates transforms into mass eigenstates, such as

$$\begin{aligned} -\mathcal{L}^d &= \bar{D}_L V_L^{d\dagger} V_L^d (\kappa_0^d B_1 + \rho_0^d B_2) V_R^{d\dagger} V_R^d D_R + \bar{U}_L V_L^{u\dagger} V_L^u V_L^{d\dagger} V_L^d (\kappa_0^d A_1 + \rho_0^d A_2) V_R^{d\dagger} V_R^d D_R + \text{ h.c.} \\ &\to \bar{D}_L(\kappa^d B_1 + \rho^d B_2) D_R + \bar{U}_L V_L^u V_L^{d\dagger} (\kappa^d A_1 + \rho^d A_2) D_R + \text{ h.c.} \quad , \end{aligned} \tag{3.19}$$

where we use equation (3.17) to obtain the second line. All the inserted matrices were used to transform the weak eigenstates into the mass eigenstates except for the remnant $V_L^u V_L^{d\dagger}$, from which we define

$$V^{CKM} \hat{=} V_L^u V_L^{d\dagger}, \tag{3.20}$$

which we recognise from section 2 as the CKM matrix.

If we now attack the complete Yukawa Lagrangian (3.13) in the same manner but with the actual Higgs fields (3.8), and we use the projection operators $P_{R/L} = (1 \pm \gamma_5)/2$, we finally obtain

$$
\begin{aligned}
-\mathcal{L}_Y = \; & \frac{1}{\sqrt{2}}\bar{D}\Big[\kappa^d c_{\beta-\alpha} - \big(\rho^d P_R + \rho^{d\dagger} P_L\big) s_{\beta-\alpha}\Big] DH \\
& + \frac{1}{\sqrt{2}}\bar{D}\Big[\kappa^d s_{\beta-\alpha} + \big(\rho^d P_R + \rho^{d\dagger} P_L\big) c_{\beta-\alpha}\Big] Dh + \frac{i}{\sqrt{2}}\bar{D}\Big[\rho^d P_R - \rho^{d\dagger} P_L\Big] DA \\
& + \frac{1}{\sqrt{2}}\bar{U}\Big[\kappa^u c_{\beta-\alpha} - \big(\rho^u P_R + \rho^{u\dagger} P_L\big) s_{\beta-\alpha}\Big] UH \\
& + \frac{1}{\sqrt{2}}\bar{U}\Big[\kappa^u s_{\beta-\alpha} + \big(\rho^u P_R + \rho^{u\dagger} P_L\big) c_{\beta-\alpha}\Big] Uh - \frac{i}{\sqrt{2}}\bar{U}\Big[\rho^u P_R - \rho^{u\dagger} P_L\Big] UA \\
& + \frac{1}{\sqrt{2}}\bar{L}\Big[\kappa^\ell c_{\beta-\alpha} - \big(\rho^\ell P_R + \rho^{\ell\dagger} P_L\big) s_{\beta-\alpha}\Big] LH \\
& + \frac{1}{\sqrt{2}}\bar{L}\Big[\kappa^\ell s_{\beta-\alpha} + \big(\rho^\ell P_R + \rho^{\ell\dagger} P_L\big) c_{\beta-\alpha}\Big] Lh + \frac{i}{\sqrt{2}}\bar{L}\Big[\rho^\ell P_R - \rho^{\ell\dagger} P_L\Big] LA \\
& + \Big(\bar{U}\Big[V^{CKM}\rho^d P_R - \rho^{u\dagger} V^{CKM} P_L\Big] DH^+ + \bar{\nu}\rho^\ell P_R L H^+ + \text{ h.c.}\Big) .
\end{aligned}
\tag{3.21}
$$

Note that since we have assumed massless neutrinos there is no mixing amongst them and their weak states and mass states are the same and thus we do not get the PMNS matrix.

3.2 Different types of 2HDMs

From the Yukawa Lagrangian in equation (3.21), it is evident that FCNC will arise at tree-level if ρ^F is not diagonal. These could be a problem if they are larger than experiments allow. There exists a few different approaches to avoid too large FCNC. One way is to use the Glashow-Weinberg criterion [11]. The criterion is that if particles from one fermion-sector (up, down and lepton) only couple to one of the Higgs doublets, then the FCNC at tree-level vanish. This can be achieved by imposing a Z_2-symmetry, i.e. that the potential in equation (3.1) and the Lagrangian in equation (3.9) are invariant under transformations $\Phi_1 \to -\Phi_1$, $\Phi_2 \to \Phi_2$, $Q_L \to Q_L$, $L_L \to L_L$ and $F_R \to \pm F_R$. For this to hold for the potential, then $\lambda_6 = \lambda_7 = m_{12} = 0$. However, if $m_{12} \neq 0$, the symmetry will still be recovered for large Φ which is called that the Z_2-symmetry is softly broken.

The Z_2 symmetry must as mentioned, also be conserved for the Yukawa Lagrangian. By convention, one most often chooses the up-type quarks to couple to Φ_2 and thus $U_R \to +U_R$. That leaves four possible combinations of how the down-type quarks and leptons couple and this results in that ρ^F are proportional to κ^F. Since these Z_2-symmetry types of 2HDMs are not the focus of this thesis, this will not be shown here but it can be read about in [12]. The four types are summarised in table 1.

It is also possible to use generic models where fermions from the different sectors all may couple to both Φ_1 and Φ_2. The most common general model is the so-called type III. [3] A 2HDM where the so-called Froggatt-Nielsen mechanism is imposed is one example of a type III model and it is the approach we use. Type III models allow FCNC at tree-level, which

[3] Type Y is sometimes also called type III which should not be confused with the generic type III.

Table 1: The four standard types of 2HDMs with Z_2-symmetry. For each type, it is stated which doublet fermions of each sector couples to and if F_R are odd (-1) or even (+1) under Z_2-transformation when Φ_1 is chosen to be odd and Φ_2 even. The table also shows how ρ^F depend on κ^F.

Type	U	D	L	ρ^u	ρ^d	ρ^ℓ
I	$\Phi_2/+$	$\Phi_2/+$	$\Phi_2/+$	$\kappa^u \cot\beta$	$\kappa^d \cot\beta$	$\kappa^\ell \cot\beta$
II	$\Phi_2/+$	$\Phi_1/-$	$\Phi_1/-$	$\kappa^u \cot\beta$	$-\kappa^d \tan\beta$	$-\kappa^\ell \tan\beta$
Y	$\Phi_2/+$	$\Phi_1/-$	$\Phi_2/+$	$\kappa^u \cot\beta$	$-\kappa^d \tan\beta$	$\kappa^\ell \cot\beta$
X	$\Phi_2/+$	$\Phi_2/+$	$\Phi_1/-$	$\kappa^u \cot\beta$	$\kappa^d \cot\beta$	$-\kappa^\ell \tan\beta$

means that the off-diagonal elements of ρ^F must be constrained. When constraining ρ^F, it is convenient to quantise the elements with dimensionless parameters. One way to do this is to use the Cheng-Sher ansatz [13]. This ansatz parametrises the elements of ρ^F according to

$$\rho_{ij}^F = \lambda_{ij}^F \sqrt{\frac{2m_i^F m_j^F}{v^2}}, \tag{3.22}$$

where m_i^F and m_j^F are the masses of the of fermion i and j in sector F. The original proposition by Cheng and Sher was that λ_{ij} are dimensionless coefficients of $\mathcal{O}(1)$ for all fermion-sectors. However, in later years experiments have put more severe constraints on the off-diagonal elements in the down sector such that $\lambda_{i\neq j}^d \lesssim 0.1$ [14]. This limitation of λ_{ij} suppress the off-diagonal elements of ρ^F such that the FCNC become small enough.

4 The Froggatt-Nielsen mechanism

The Froggatt-Nielsen mechanism was first proposed by C.D. Froggatt and H.B. Nielsen in 1978 and was an attempt to explain the large mass ratios between fermions [7]. The mechanism is based on the assumption that it would be natural if all couplings are of order one. In order to achieve this, a new property, flavon charge, is introduced in such a way that the mass ratios between fermions depends on the difference of the flavon charges. If this can be achieved with small flavon charge differences, it can be considered to be natural. If large differences are required one has simply moved the problem of unnaturalness from the fermion masses to the flavon charges. In this section, we first explain how the FN mechanism works in the SM and then we describe how it is extended to fit within the 2HDM.

4.1 The Froggatt-Nielsen mechanism in the Standard Model

The FN mechanism works through new concepts which are introduced beyond the SM. First a new scalar field S, a so-called flavon, which is a SM singlet with no weak hypercharge, is introduced. An $U(1)$ symmetry, flavon symmetry, with its associated flavon charge follows and it is defined that S has the flavon charge -1. This symmetry also affects the SM particles, which means that all of them obtain flavon charge. In a similar manner as for the Higgs mechanism, the flavour symmetry can be spontaneously broken, which gives the flavon a non-zero VEV $\langle S \rangle$.

Further, Froggatt and Nielsen assumed the existence of a set of heavy fermions, which we call Froggatt-Nielsen fermions. These are all assumed to have masses at the same energy scale M^{FN} which makes the SM fermions to appear massless in comparison. All the SM fermions have their corresponding sets of FN fermions with the same quantum number except mass and flavon charge.[4] The FN fermions acquire their mass through interactions with some unidentified Higgs scalar $\hat{H}$ that has zero flavon charge. Just as in the SM Higgs mechanism, the FN fermions switch chirality when interacting with the non-zero VEV of $\hat{H}$. The FN fermions can only interact with SM fermions through couplings to either the flavon or the SM Higgs doublet according to the diagram in figure 2.

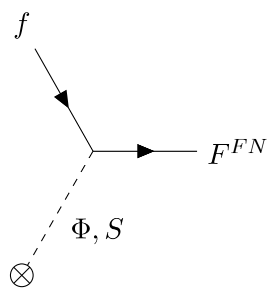

Figure 2: Vertex between a SM fermion and a FN fermion. They can interact either via the Higgs doublet or the flavon.

A crucial point of the FN mechanism is that left-handed fermions and their right-handed counterparts can have different flavon charges. When a FN fermion interacts with $\hat{H}$, it changes chirality and when it interacts with the flavon, it changes both chirality and flavon

[4]This means that FN fermions could be incorporated in some super-symmetric theories which is one reason to why the FN mechanism is interesting to investigate.

charge. Since S has flavon charge -1, the outgoing FN fermion will have one unit lower flavon charge than the incoming. Hence, the process that gives mass to a FN fermion and conserves its chirality must be a FN fermion interacting with both $\hat{H}$ and S, as shown in figure 3.

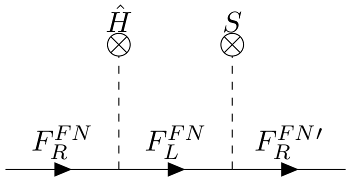

Figure 3: An incoming right-handed FN fermion first switches chirality via $\hat{H}$ and then again via S which also lower its flavon charge by one unit.

For energies much smaller that M^{FN}, we can from this diagram (figure 3) define a symmetry breaking parameter ε

$$\varepsilon \hat{=} \frac{\langle S \rangle}{M^{FN}}, \tag{4.1}$$

which will enter diagrams later. For a SM fermion that has different flavon charge for the left-handed and right-handed versions we can, by using the processes of figures 2 and 3, construct a diagram which gives mass to the SM fermion and conserves the flavon charge as is illustrated in figure 4.

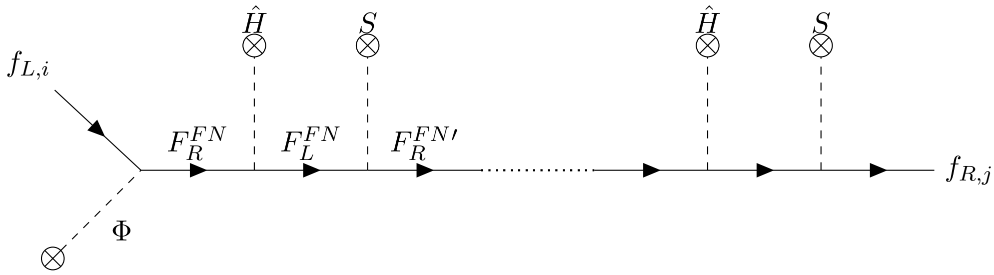

Figure 4: The Froggatt-Nielsen mechanism. An extended version of the Higgs mechanism where a left-handed SM fermion switches chirality after interaction with the Higgs field. In order to conserve flavon charge, this happens via a chain of FN fermions interacting with $\hat{H}$ and S.

The diagram in figure 4 can for an incoming left-handed fermion $f_{L,i}$ and outgoing right-handed fermion $f_{R,j}$ be expressed as the effective operator

$$\bar{f}_{R,j} Y_{ij}^F \Phi_{SM} f_{L,i}, \qquad Y_{ij}^F = g_{ij}^F \varepsilon^{n_{ij}}, \tag{4.2}$$

where n_{ij} is the number of times the process of figure 3 is repeated and g_{ij}^F are random complex numbers of order 1 [7].

In order to determine n_{ij}, suppose that an incoming left-handed quark $q_{i,L}$ has flavon charge $c + b_i$ and that an outgoing right-handed quark $q_{j,R}$ has flavon charge $c - a_j$ where $c, b_i, a_j \in \mathbb{N}$. The mechanism needs to conserve the flavon charge, hence, n_{ij} must depend on the flavon charge difference between $q_{i,L}$ and $q_{j,R}$. However, since the Higgs field does not specifically have zero flavon charge, it also affects n_{ij}. The dependence on the Higgs field is different for the down- and up-sector since it is the complex conjugate of the Higgs doublet which couples to the up-sector and the non-conjugate that couples to the down-sector. This means that the flavon charge of the Higgs doublet enters with opposite signs. Additionally, due to that the left-handed quarks of the same generation are in the same $SU(2)$ doublet, they will have the same flavon charge while the right-handed may have different. If we assign the Higgs doublet flavon charge R then, to conserve the flavon charge, n_{ij} must be

$$n_{ij} = |c + b_i^q - (c - a_j^d) - R| = |b_i^q + a_j^d - R|, \tag{4.3}$$

for the down-sector and

$$n_{ij} = |c + b_i^q - (c - a_j^u) + R| = |b_i^q + a_j^u + R|, \tag{4.4}$$

for the up-sector. The absolute signs are included because n_{ij} is the number of iterations, which cannot be negative. The same thing can be done for the lepton-sector. We assign charge $d + b_i^\ell$ for left-handed leptons and $d - a_j^\ell$ for right-handed which gives

$$n_{ij} = |d + b_i^\ell - (d - a_j^\ell) - R| = |b_i^\ell + a_j^\ell - R|. \tag{4.5}$$

From equation (4.2)-(4.5), we can now formulate the Yukawa matrices as

$$\begin{aligned} Y_{ij}^u &= g_{ij}^u \varepsilon^{|b_i^q + a_j^u - R|}, \\ Y_{ij}^d &= g_{ij}^d \varepsilon^{|b_i^q + a_j^d + R|}, \\ Y_{ij}^\ell &= g_{ij}^\ell \varepsilon^{|b_i^\ell + a_j^\ell + R|}. \end{aligned} \tag{4.6}$$

4.2 The Froggatt-Nielsen mechanism extended to 2HDM

It does not take much modification to extend the Froggatt-Nielsen mechanism to work for a 2HDM. In this framework, the FN fermions can couple to both the Higgs doublets Φ_1 and Φ_2 which are assigned flavon charges R_1 and R_2 respectively. Thus the Yukawa matrices can be parametrised as

$$\begin{aligned} (Y_{1,2}^u)_{ij} &\sim \varepsilon^{|b_i^q + a_j^u + R_{1,2}|}, \\ (Y_{1,2}^d)_{ij} &\sim \varepsilon^{|b_i^q + a_j^d - R_{1,2}|}, \\ (Y_{1,2}^\ell)_{ij} &\sim \varepsilon^{|b_i^\ell + a_j^\ell - R_{1,2}|}, \end{aligned} \tag{4.7}$$

where $\sim$ is used to denote the dependence of orders in epsilon without regarding the coefficients of $\mathcal{O}(1)$ [15]. Throughout this work, we only investigate how the coupling depends on leading order of magnitude in ε. As a consequence, the Yukawa matrices are considered and dealt with as they were real.

The value of ε may be determined such that the FN mechanism reproduces the CKM matrix. The numerical absolute value representation of the CKM matrix is [10]

$$V^{CKM} = \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.974 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{pmatrix}. \tag{4.8}$$

As we work with orders in ε, we want to find the value of ε which allows for (4.8) to be expressed as integer exponents of ε, up to factors of $\mathcal{O}(1)$. We note that this is the case for

$\varepsilon \approx 0.2$, which yields

$$V^{CKM} \sim \begin{pmatrix} \varepsilon^0 & \varepsilon^1 & \varepsilon^3 \\ \varepsilon^1 & \varepsilon^0 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^0 \end{pmatrix}. \tag{4.9}$$

We can also express the Yukawa matrices as κ_0^F and ρ_0^F by considering the linear combination in equation (3.12) which gives them in the Higgs basis. Since we are only interested in orders in ε, we also want to express $\tan\beta$ as a power of ε and thus we define $\tan\beta = \varepsilon^{-\eta}$ where $\eta \in \mathbb{N}$. The value of $\tan\beta$ is preferably large [6]. For large values of $\tan\beta$ we can then approximate $\sin\beta \sim 1$ and $\cos\beta \sim \varepsilon^{\eta}$. Finally, by performing the rotation of equation (3.12), κ_0^F and ρ_0^F can be expressed as

$$\begin{aligned} (\kappa_0^F)_{ij} &\sim \varepsilon^{\eta+|b_i^{q,\ell}+a_j^F \pm R_1|} + \varepsilon^{|b_i^{q,\ell}+a_j^F \pm R_2|} \sim \max\left\{\varepsilon^{\eta+|b_i^{q,\ell}+a_j^F \pm R_1|},\ \varepsilon^{|b_i^{q,\ell}+a_j^F \pm R_2|}\right\}, \\ (\rho_0^F)_{ij} &\sim \varepsilon^{|b_i^{q,\ell}+a_j^F \pm R_1|} + \varepsilon^{\eta+|b_i^{q,\ell}+a_j^F \pm R_2|} \sim \max\left\{\varepsilon^{|b_i^{q,\ell}+a_j^F \pm R_1|},\ \varepsilon^{\eta+|b_i^{q,\ell}+a_j^F \pm R_2|}\right\}, \end{aligned} \tag{4.10}$$

where only leading order in epsilon are considered in the last step and thus also the signs are neglected. Again, plus $R_{1,2}$ is for the up-sector and minus for the down- and lepton-sector.

5 Constraining the flavon charges

After the introduction of the Froggatt-Nielsen mechanism in the last section, we have now gained 17 degrees of freedom with the flavon charges. To review, we have: the three $b^q_{1,2,3}$, the three $b^\ell_{1,2,3}$, the nine $a^F_{1,2,3}$ and the two $R_{1,2}$. The only constraints inflicted on them as yet, are that they are all integers and the differences between them should be small. We will in this section investigate which constraints which need to be imposed due to physical reasons.

5.1 Speculation about naturalness

The main point of the Froggatt-Nielsen mechanism is to explain the large mass differences between the fermions in a natural way. Froggatt and Nielsen speculate in their article that it is most natural if the flavon charges fulfil the requirements that $b^q_i \geq 0$ and $a^{u,d}_j \geq 1$ and that they are ordered such that $b^q_i \geq b^q_{i+1}$ and $a^{u,d}_j \geq a^{u,d}_{j+1}$ [7]. Further, Dery and Nir argue in [15] that it is most natural with small differences between the flavon charges, just as we have discussed before. If the differences between the flavon charges is large, we have merely moved the problem of the large mass hierarchy amongst the fermion masses to the flavon charges. The most natural way to obtain small differences should arguably be to have small flavon charges to begin with.

Considering these arguments, we adopt a first limitation that the flavon charges must be contained within the integer intervals: $b^q_i \in [0, 5]$, $a^{u,d}_j \in [1, 5]$ and $R_a \in [-5, 5]$. These intervals are referred to as the *natural* intervals. We do not restrict ourself as much regarding the leptons; our main focus lies on the quark-sector. Froggatt and Nielsen did not speculate about the leptons, but we extend the criterion of ordered flavon charges also to the lepton-sector, such that $b^\ell_i \geq b^\ell_{i+1}$ and $a^\ell_j \geq a^\ell_{j+1}$. However, we do not impose any more fixed limits on the lepton-sector as for now but we will keep in mind that they probably ought to be similar to the limits on the quark-sector.

5.2 Recovering the CKM matrix

An important property which limits possible charge combinations severely is that the experimentally known CKM matrix must be reproduced. We saw earlier that the CKM matrix can to leading order in $\varepsilon \approx 0.2$ be expressed as in equation (4.9). In order to find how the CKM matrix depends on the flavon charges, one must construct the unitary matrices V^u_L and V^d_L from equation (3.20). The main principles of diagonalisation by bi-unitary transformations are covered in appendix A. It is the same derivation for V^u_L and V^d_L and therefore we drop the index for now. Throughout this work, we have disregarded the coefficients of order one, but we include them here for the sake of completeness. Since V_L is unitary, we can to leading order in ε write it as

$$V_L = \begin{pmatrix} 1 & c_{12}\varepsilon^{n_{12}} & c_{13}\varepsilon^{n_{13}} \\ c_{21}\varepsilon^{n_{21}} & 1 & c_{23}\varepsilon^{n_{23}} \\ c_{31}\varepsilon^{n_{31}} & c_{32}\varepsilon^{n_{32}} & 1 \end{pmatrix}, \qquad \begin{cases} c_{ij} = & -c^*_{ji} \\ n_{ij} = & n_{ji} \end{cases} . \tag{5.1}$$

Next, we need to consider $H = \kappa_0 \kappa_0^\dagger$. Here we reach a complication compared to Froggatt and Nielsen in [7]. Due to that they only use one Higgs doublet, their corresponding κ_0 matrix is ordered such that $\kappa_{0,\,i1} \leq \kappa_{0,\,i2} \leq \kappa_{0,\,i3}$ and $\kappa_{0,\,1j} \leq \kappa_{0,\,2j} \leq \kappa_{0,\,3j}$ since the flavon charges are ordered.[5] In our case, however, when we use two Higgs doublets, it is not guaranteed that the elements in κ_0 are ordered since it is generated from equation (4.10). Hence, we introduce as a requirement that a set of flavon charges must yield an ordered κ_0 to be valid. We require this for us to keep the analysis as simple as possible and to be able to follow the same procedure as Froggatt and Nielsen in [7].

Now when we know that κ_0 is ordered, we also know that H is ordered. However, it is not possible to know in general which flavon charges that contribute to the leading order. Thus we express κ_0 to leading order in the more general form

$$\kappa_0 = \begin{pmatrix} g_{11}\varepsilon^{x_{11}} & g_{12}\varepsilon^{x_{12}} & g_{13}\varepsilon^{x_{13}} \\ g_{21}\varepsilon^{x_{21}} & g_{22}\varepsilon^{x_{22}} & g_{23}\varepsilon^{x_{23}} \\ g_{31}\varepsilon^{x_{31}} & g_{32}\varepsilon^{x_{32}} & g_{33}\varepsilon^{x_{33}} \end{pmatrix}, \tag{5.2}$$

where g_{ij} are complex coefficients of $\mathcal{O}(1)$ and x_{ij} are the resulting exponents ordered such that $x_{i1} \geq x_{i2} \geq x_{i3}$ and $x_{1j} \geq x_{2j} \geq x_{3j}$ for $i, j = 1, 2, 3$. Constructing the matrix $H = \kappa_0\kappa_0^\dagger$ and carrying out the multiplication result in that H to leading order in ε can be expressed as

$$H \approx g_{i3}g_{j3}^*\varepsilon^{x_{i3}+x_{j3}} \hat{=} h_{ij}\varepsilon^{x_{i3}+x_{j3}}, \tag{5.3}$$

where the coefficients have been redefined to another complex coefficient of order 1. H is of course Hermitian which means that $h_{ij} = h_{ji}^*$ and it is diagonalised by $V_L H V_L^\dagger = D^2$. The off-diagonal elements of D^2 need of course to be zero to leading order. By carrying out the diagonalisation, the diagonal elements of D^2 become

$$\begin{cases} D_{11}^2 \approx h_{11}\varepsilon^{x_{13}+x_{13}} \\ D_{22}^2 \approx h_{22}\varepsilon^{x_{23}+x_{23}} \\ D_{33}^2 \approx h_{33}\varepsilon^{x_{33}+x_{33}} \end{cases}, \tag{5.4}$$

which follows from the ordering of x_{ij}. The leading order of the off-diagonal elements $D_{23,32}^2$ and $D_{13,31}^2$ requires one more term:

$$\begin{cases} D_{23}^2 \approx h_{23}\varepsilon^{x_{23}+x_{33}} + h_{33}c_{23}\varepsilon^{x_{33}+x_{33}+n_{23}} \overset{!}{=} 0 \\ D_{32}^2 \approx h_{32}\varepsilon^{x_{23}+x_{33}} + h_{33}c_{23}^*\varepsilon^{x_{33}+x_{33}+n_{23}} \overset{!}{=} 0 \end{cases} \Rightarrow \begin{cases} c_{23} = -\dfrac{h_{23}}{h_{33}} \\ n_{23} = x_{23} - x_{33} \end{cases}, \tag{5.5}$$

$$\begin{cases} D_{13}^2 \approx h_{13}\varepsilon^{x_{13}+x_{33}} + h_{33}c_{13}\varepsilon^{x_{33}+x_{33}+n_{13}} \overset{!}{=} 0 \\ D_{31}^2 \approx h_{31}\varepsilon^{x_{13}+x_{33}} + h_{33}c_{13}^*\varepsilon^{x_{33}+x_{33}+n_{13}} \overset{!}{=} 0 \end{cases} \Rightarrow \begin{cases} c_{13} = -\dfrac{h_{13}}{h_{33}} \\ n_{13} = x_{13} - x_{33} \end{cases}. \tag{5.6}$$

For $D_{12,21}^2$, one needs to regard a few more terms to obtain the leading order:

$$\begin{cases} D_{13}^2 \approx & h_{12}\varepsilon^{x_{13}+x_{23}} + h_{13}c_{23}^*\varepsilon^{x_{13}+x_{33}+n_{23}} + h_{32}c_{13}\varepsilon^{x_{23}+x_{33}+n_{13}} + h_{22}c_{12}\varepsilon^{x_{23}+x_{23}+n_{12}} \\ & + h_{33}c_{23}^*c_{13}\varepsilon^{x_{33}+x_{33}+n_{23}+n_{13}} + h_{23}c_{23}^*c_{12}\varepsilon^{x_{23}+x_{33}+n_{23}+n_{12}} \overset{!}{=} 0 \\ D_{31}^2 \approx & h_{21}\varepsilon^{x_{13}+x_{23}} + h_{31}c_{23}\varepsilon^{x_{13}+x_{33}+n_{23}} + h_{23}c_{13}^*\varepsilon^{x_{23}+x_{33}+n_{13}} + h_{22}c_{12}^*\varepsilon^{x_{23}+x_{23}+n_{12}} \\ & + h_{33}c_{13}^*c_{23}\varepsilon^{x_{33}+x_{33}+n_{13}+n_{23}} + h_{32}c_{12}^*c_{23}\varepsilon^{x_{23}+x_{33}+n_{21}+n_{23}} \overset{!}{=} 0 \end{cases}, \tag{5.7}$$

[5]Froggatt and Nielsen have ordered the flavon charges in the other way but here that has been changed to correspond with our notation.

which gives

$$\begin{cases} c_{12} = -\dfrac{h_{33}h_{12} - h_{13}h_{32}}{h_{23}h_{32} - h_{33}h_{22}} \\ n_{12} = x_{13} - x_{23} \end{cases} . \tag{5.8}$$

A similar derivation has been done before by Book in [16]. Now, we can conclude that the left transformation matrix behaves like

$$V_L \sim \begin{pmatrix} 1 & \varepsilon^{x_{13}-x_{23}} & \varepsilon^{x_{13}-x_{33}} \\ \varepsilon^{x_{13}-x_{23}} & 1 & \varepsilon^{x_{23}-x_{33}} \\ \varepsilon^{x_{13}-x_{33}} & \varepsilon^{x_{23}-x_{33}} & 1 \end{pmatrix}, \tag{5.9}$$

where we once again disregard the coefficients. From here, it is straightforward to multiply V_L^u and $V_L^{d\dagger}$ to leading order. If the product is identical to the CKM matrix in (4.9) for a set of flavon charges, then the set is considered to be a good set regarding this limit. From considering the CKM matrix we have obtained three constraints.

5.3 Recovering the mass spectrum

Since κ^F is proportional to the mass matrix M^F, see equation (3.16), its elements obviously need to be determined uniquely. Using the current-quark masses and lepton masses [10] summarised in table 2 and that $\varepsilon \approx 0.2$ yield the following approximations of κ^F:

$$\kappa^u \sim \begin{pmatrix} \varepsilon^7 & & \\ & \varepsilon^3 & \\ & & \varepsilon^0 \end{pmatrix}, \quad \kappa^d \sim \begin{pmatrix} \varepsilon^6 & & \\ & \varepsilon^4 & \\ & & \varepsilon^2 \end{pmatrix}, \quad \kappa^\ell \sim \begin{pmatrix} \varepsilon^8 & & \\ & \varepsilon^5 & \\ & & \varepsilon^3 \end{pmatrix}. \tag{5.10}$$

The empty elements represents zeroes. This is the same power of ε approximation as used in [15], but other papers, such as [17], use slightly different values. This depends on that somewhat different values and approximation have been used.

Table 2: The current-quark masses and lepton masses [10].

Up-sector	Down-sector	Lepton-sector
$m_u = 2.2\,\text{MeV}$	$m_d = 4.7\,\text{MeV}$	$m_e = 0.51\,\text{MeV}$
$m_c = 1.27\,\text{GeV}$	$m_s = 96\,\text{MeV}$	$m_\mu = 106\,\text{MeV}$
$m_t = 173\,\text{GeV}$	$m_b = 4.66\,\text{GeV}$	$m_\tau = 1.78\,\text{GeV}$

To find how the elements of the mass matrices depend on the flavon charges, we need to find the connection between the diagonal elements of the undiagonalised κ_0^F and the elements of the diagonalised κ^F. Since we in section 5.2 introduced the requirement that κ_0^F are ordered, it is possible to follow the steps of Froggatt and Nielsen. They showed that the diagonal elements of κ_0^F are proportional to the elements of κ^F. Thus, applying the same notation as in equation (5.2), we can conclude that we have obtained nine new constraints

$$\begin{cases} x_{11}^u = 7 \\ x_{22}^u = 3 \\ x_{33}^u = 0 \end{cases}, \quad \begin{cases} x_{11}^d = 6 \\ x_{22}^d = 4 \\ x_{33}^d = 2 \end{cases}, \quad \begin{cases} x_{11}^\ell = 8 \\ x_{22}^\ell = 5 \\ x_{33}^\ell = 2 \end{cases}. \tag{5.11}$$

5.4 Limits on flavour changing neutral currents

We have so far applied the constraints that a set of flavon charges needs to generate the correct mass matrix and the correct CKM matrix. Now we also impose constraints on ρ^F. As discussed in section 3, non-zero off-diagonal elements of ρ^F results in FCNC. From experiments it is known that these are severely constrained, which can be included in the model by using the Cheng-Sher ansatz where ρ^F are parametrised as in equation (3.22). However, the experimental constraints have mostly been found for the down-sector where they are even more severe, rendering coefficients $\lambda^d_{i\neq j} \lesssim 0.1$ [14]. The up- and lepton-sector, on the other hand, have barely been constrained at all experimentally [18]. Hence, we only apply the Cheng-Sher ansatz for the down-sector. Since we are working with orders in ε, we approximate the limit to be $\lambda^d_{i\neq j} \lesssim \varepsilon$. By using equation (3.22) and the constraint above, the corresponding limits in the off-diagonal values of ρ^d in terms of powers of ε can be found to be

$$\rho^d_{12,21} \lesssim \varepsilon^6 \approx 6.4 \cdot 10^{-5}, \qquad \rho^d_{13,31} \lesssim \varepsilon^5 \approx 3.2 \cdot 10^{-4}, \qquad \rho^d_{23,32} \lesssim \varepsilon^4 \approx 1.6 \cdot 10^{-3}. \tag{5.12}$$

5.5 Bounds on $\tan\beta$

The last constraint which we impose is on $\tan\beta$. In [12], it is shown that the value $\tan\beta$ needs to be larger than 1 and smaller than ~ 50 at $m_{H^\pm} \approx 500\,\text{GeV}$ in order to satisfy conditions for all four standard types of 2HDMs. We will return to the mass of the charged Higgs boson later. Also in [6], the largest value used for $\tan\beta$ is 50. Hence, it seems as a reasonable interval to adopt, even though none of the standard types are used. For the parametrisation $\tan\beta = \varepsilon^{-\eta}$, the possible values of η which correspond to $\tan\beta \in [1, 50]$ are $\eta \in \{0, 1, 2\} \Leftrightarrow \tan\beta \in \{1, 5, 25\}$.

6 Search for valid sets of flavon charges

Now when all the constraints have been established, sets of flavon charges which satisfy them all can be searched for. This was done by writing a simple Java program. For a set of flavon charges, κ^F and ρ^F were constructed according to equation (4.10) and from these, the CKM matrix was calculated via equation (5.9). The calculations were only performed to leading order in ε. The program then checked whether the constraints discussed in sections 5.2, 5.3 and 5.4 were satisfied or not. As a summary of the constraints, each set of flavon charges must satisfy the following to be valid:

- The exponents of κ_0^F must be ordered as $x_{i1}^F \geq x_{i2}^F \geq x_{i3}^F$ and $x_{1j}^F \geq x_{2j}^F \geq x_{3j}^F$ for $i, j = 1, 2, 3$.
- The mass matrices must be recovered correctly according to equation (5.10).
- The CKM matrix must be recovered correctly according to equation (4.9).
- The off-diagonal elements of ρ^d must be constrained due to FCNC according to equation (5.12).

All sets of combinations of flavon charges that can be constructed from the *natural* intervals – which fulfil the requirements of being ordered discussed in section 5.1 – were checked whether they satisfied the above constraints or not. This was done for the values of η considered in section 5.5, i.e. $\eta = 0, 1, 2$. Since $\tan\beta \in [1, 50]$ is not a strictly determined interval, values just outside the interval may occur. Thus the neighbouring values $\eta = -1$ and $\eta = 3$ were also tested. If all requirements were met for a set of flavon charges (including a specific η) it was considered a valid set. The program also determined which flavon charges that contributed to leading order, i.e. if $x_{ij}^F = \eta + |b_i^{q,\ell} + a_j^F \pm R_1|$ or $x_{ij}^F = |b_i^{q,\ell} + a_j^F \pm R_2|$ for κ^F and correspondingly for ρ^F.

Since we do not have as severe constraints on the lepton-sector, it was dealt with separately. The only parameters that connect the quark-sector with the lepton-sector are the flavon charges of the Higgs fields R_1 and R_2. Thus, only values on R_1 and R_2 which are part of valid sets of flavon charges for the quark sectors were used in the search for acceptable sets of lepton flavon charges.

The study was divided into two cases depending on the relation between R_1 and R_2. First, valid sets of flavon charges were searched for in the case when $R_1 \geq R_2$. No combinations of charges chosen from the *natural* intervals satisfied all the requirements for any of the values of η. In order to find any valid combinations, the least motivated constraint – that the charges need to be chosen from the *natural* interval – was discarded. For the expanded intervals $b_i^q \in [0, 7]$, $a_j^{u,d} \in [-6, 5]$ and $R_a \in [-10, 10]$ instead 30 valid combinations were found for $\eta = 2$ and the same 30 combinations for $\eta = 3$. These are presented in appendix B. The matrix κ^u has the same flavon charge dependence $\kappa^u \sim \varepsilon^{|b_i^q + a_j^u + R_2|}$ for each of the 30 combinations with both values of η. The same is true for $\kappa^d \sim \varepsilon^{|b_i^q + a_j^d + R_2|}$ and $\rho^u \sim \varepsilon^{\eta + |b_i^q + a_j^u + R_2|}$. Numerically, κ^u and κ^d are obviously the same as in (5.10) since that was a requirement. For the two different η, ρ^u becomes for all combinations

$$\eta = 2: \quad \rho^u \sim \begin{pmatrix} \varepsilon^9 & \varepsilon^6 & \varepsilon^5 \\ \varepsilon^8 & \varepsilon^5 & \varepsilon^4 \\ \varepsilon^6 & \varepsilon^3 & \varepsilon^2 \end{pmatrix}, \qquad \eta = 3: \quad \rho^u \sim \begin{pmatrix} \varepsilon^{10} & \varepsilon^7 & \varepsilon^6 \\ \varepsilon^9 & \varepsilon^6 & \varepsilon^5 \\ \varepsilon^7 & \varepsilon^4 & \varepsilon^3 \end{pmatrix}. \tag{6.1}$$

However, all the elements of ρ^d do not have the same flavon charge dependence, neither within one specific matrix, nor for the different sets. Two examples of valid sets for the different η are presented in table 3.

Table 3: Two examples of valid sets of quark and Higgs field flavon charges for the case $R_1 \geq R_2$.

η	(b_1^q, b_2^q, b_3^q)	(a_1^u, a_2^u, a_3^u)	(a_1^d, a_2^d, a_3^d)	(R_1, R_2)
2	$(6, 5, 3)$	$(5, 2, 1)$	$(-4, -5, -5)$	$(10, -4)$
3	$(4, 3, 1)$	$(5, 2, 1)$	$(0, -1, -1)$	$(9, -2)$

The obtained ρ^d for the example with $\eta = 2$ is

$$\rho^d \sim \begin{pmatrix} \varepsilon^8(R_1, R_2) & \varepsilon^7(R_2) & \varepsilon^7(R_2) \\ \varepsilon^7(R_2) & \varepsilon^6(R_2) & \varepsilon^6(R_2) \\ \varepsilon^5(R_2) & \varepsilon^4(R_2) & \varepsilon^4(R_2) \end{pmatrix}, \tag{6.2}$$

and for the example with $\eta = 3$

$$\rho^d \sim \begin{pmatrix} \varepsilon^5(R_1) & \varepsilon^6(R_1) & \varepsilon^6(R_1) \\ \varepsilon^6(R_1) & \varepsilon^7(R_1, R_2) & \varepsilon^7(R_1, R_2) \\ \varepsilon^6(R_2) & \varepsilon^5(R_2) & \varepsilon^5(R_2) \end{pmatrix}. \tag{6.3}$$

$(R_{1,2})$ denotes if the leading order is obtained from the R_1 or R_2 contribution or both. It is interesting to note that the five elements $\rho^d_{13}, \rho^d_{22}, \rho^d_{23,32}$ and ρ^d_{33} all depend on at least R_2, which gives the same numerical values (but differs by one unit for $\eta = 2$ and $\eta = 3$) for all combinations while it may vary for the other elements. For the example with $\eta = 2$, κ^F and ρ^F show an approximate type I relation. Even though these valid sets are chosen from outside the *natural* interval, that does not make them uninteresting to investigate by themselves. However, it will turn out that none of them can explain the B-meson decays we are interested in.

In the other case, when $R_2 > R_1$ was considered, five valid sets from the *natural* intervals were found: one for $\eta = 2$ and four for $\eta = 3$. The set for $\eta = 2$ and one example of the sets for $\eta = 3$ are presented in table 4. The three remaining sets can be found in appendix B.

Table 4: Two examples of valid sets of quark and Higgs field flavon charges for the case $R_2 > R_1$.

η	(b_1^q, b_2^q, b_3^q)	(a_1^u, a_2^u, a_3^u)	(a_1^d, a_2^d, a_3^d)	(R_1, R_2)
2	$(3, 2, 0)$	$(5, 2, 1)$	$(2, 1, 1)$	$(-3, -1)$
3	$(3, 2, 0)$	$(5, 2, 1)$	$(2, 1, 1)$	$(-3, -1)$

We will see that these two sets are going to be interesting when the B-meson decays are considered. The generated matrices for the set with $\eta = 2$ are

$$\kappa^u \sim \begin{pmatrix} \varepsilon^7(R_1,R_2) & & \\ & \varepsilon^3(R_1,R_2) & \\ & & \varepsilon^0(R_2) \end{pmatrix}, \quad \kappa^d \sim \begin{pmatrix} \varepsilon^6(R_2) & & \\ & \varepsilon^4(R_2) & \\ & & \varepsilon^2(R_2) \end{pmatrix}, \tag{6.4}$$

$$\rho^u \sim \begin{pmatrix} \varepsilon^5(R_1) & \varepsilon^2(R_1) & \varepsilon^1(R_1) \\ \varepsilon^4(R_1) & \varepsilon^1(R_1) & \varepsilon^0(R_1) \\ \varepsilon^2(R_1) & \varepsilon^1(R_1) & \varepsilon^2(R_1,R_2) \end{pmatrix}, \quad \rho^d \sim \begin{pmatrix} \varepsilon^8(R_1,R_2) & \varepsilon^7(R_1,R_2) & \varepsilon^7(R_1,R_2) \\ \varepsilon^7(R_1,R_2) & \varepsilon^6(R_1,R_2) & \varepsilon^6(R_1,R_2) \\ \varepsilon^5(R_1,R_2) & \varepsilon^4(R_1,R_2) & \varepsilon^4(R_1,R_2) \end{pmatrix},$$

and for $\eta = 3$

$$\kappa^u \sim \begin{pmatrix} \varepsilon^7(R_2) & & \\ & \varepsilon^3(R_2) & \\ & & \varepsilon^0(R_2) \end{pmatrix}, \quad \kappa^d \sim \begin{pmatrix} \varepsilon^6(R_2) & & \\ & \varepsilon^4(R_2) & \\ & & \varepsilon^2(R_2) \end{pmatrix},$$

$$\rho^u \sim \begin{pmatrix} \varepsilon^5(R_1) & \varepsilon^2(R_1) & \varepsilon^1(R_1) \\ \varepsilon^4(R_1) & \varepsilon^1(R_1) & \varepsilon^0(R_1) \\ \varepsilon^2(R_1) & \varepsilon^1(R_1) & \varepsilon^2(R_1) \end{pmatrix}, \quad \rho^d \sim \begin{pmatrix} \varepsilon^8(R_1) & \varepsilon^7(R_1) & \varepsilon^7(R_1) \\ \varepsilon^7(R_1) & \varepsilon^6(R_1) & \varepsilon^6(R_1) \\ \varepsilon^5(R_1) & \varepsilon^4(R_1) & \varepsilon^4(R_1) \end{pmatrix}. \tag{6.5}$$

The two sets generate identical matrices, although they get the contribution to leading order differently. Both the up- and down-sector gain mass from Φ_2 for the example with $\eta = 3$, while the up-sector in the example with $\eta = 2$ gains mass both from Φ_1 and Φ_2. When $\eta = 2$, this can be explained with that the difference between R_1 and R_2 is equal to η. When $\eta = 3$, the difference is smaller and the leading order gets contribution only from one of the Higgs fields. Arguably, the set with $\eta = 2$ can be considered more preferable since that corresponds to $\tan\beta = 25$ which lies inside the discussed interval as opposed to $\eta = 3$ which corresponds to $\tan\beta = 125$ which is far outside the interval. The fact that only valid sets were found for $\eta = 2, 3$ agrees well with the statement that $\tan\beta$ preferably is large.

Since the main focus of this work lies on the quark-sector and the since the lepton-sector is not as constrained as the quark-sector, we also loosen the *natural* interval for lepton flavon charges. $R_1 = -3$ and $R_2 = -1$ were used to match the found sets of quark flavon charges. For $b_i^\ell \in [0, 5]$, $a_j^\ell \in [-7, 6]$, 44 valid sets were found. These are valid and yield the same matrices both for $\eta = 2$ and $\eta = 3$. The only difference is once again which Higgs flavon charge that gives contribution to the leading order. Two different examples will be presented. The first one has what could be considered natural flavon charge differences, while the second has larger differences, see table 5.

Table 5: Two examples of valid sets of lepton flavon charges for $R_1 = -3$ and $R_2 = -1$.

η	$(b_1^\ell, b_2^\ell, b_3^\ell)$	$(a_1^\ell, a_2^\ell, a_3^\ell)$
$2, 3$	$(4, 2, 1)$	$(3, 2, 1)$
$2, 3$	$(2, 2, 2)$	$(5, 2, -6)$

The first set gives the matrices

$$\kappa^\ell \sim \begin{pmatrix} \varepsilon^8 & & \\ & \varepsilon^5 & \\ & & \varepsilon^3 \end{pmatrix}, \qquad \rho^\ell \sim \begin{pmatrix} \varepsilon^{10} & \varepsilon^9 & \varepsilon^8 \\ \varepsilon^8 & \varepsilon^7 & \varepsilon^6 \\ \varepsilon^7 & \varepsilon^6 & \varepsilon^5 \end{pmatrix}, \tag{6.6}$$

and the second set gives

$$\kappa^\ell \sim \begin{pmatrix} \varepsilon^8 & & \\ & \varepsilon^5 & \\ & & \varepsilon^3 \end{pmatrix}, \qquad \rho^\ell \sim \begin{pmatrix} \varepsilon^{10} & \varepsilon^7 & \varepsilon^1 \\ \varepsilon^{10} & \varepsilon^7 & \varepsilon^1 \\ \varepsilon^{10} & \varepsilon^7 & \varepsilon^1 \end{pmatrix}. \tag{6.7}$$

The first example would be preferable since it has smaller differences between the flavon charges, but it will turn out that only the second example can be used when explaining the $B \to D^{(*)}\tau\nu$ decays.

7 The $B \to D^{(*)}\tau\nu$ decays

Various decays of B-mesons are interesting when it comes to signs of new physics. Data from many of them do not agree with predictions from the SM. Here we investigate the semi-leptonic decays where the neutral B-meson decays into either the positive or negative D-meson (it can either be the pseudoscalar version (D) or the vector version (D^*), where either of them is denoted $D^{(*)}$), a tau lepton and a tau neutrino, i.e. $\bar{B}^0 \to D^{(*)+}\tau^-\bar{\nu}_\tau$ and $B^0 \to D^{(*)-}\tau^+\nu_\tau$, see figure 5. We use the notation $B \to D^{(*)}\tau\nu$ which include all these decays. In this section, we investigate if these decays may be explained by the FN-2HDM.

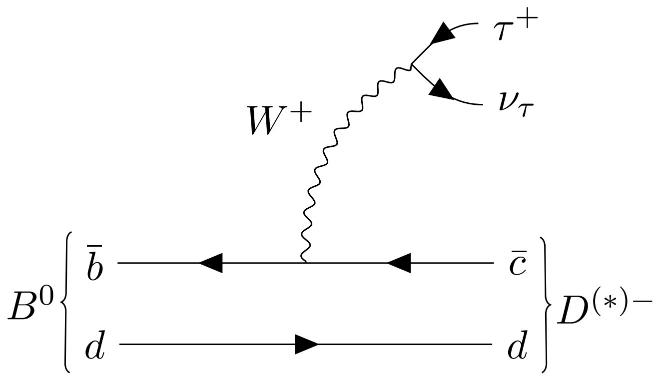

Figure 5: The $B^0 \to D^{(*)-}\tau^+\nu_\tau$ decays mediated by W^+ in the SM. Corresponding decays of $\bar{B}^0$ are mediated by W^-.

When investigating these decays, it is convenient to work with the ratios between the branching ratios

$$\mathcal{R}(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}l\nu)}, \tag{7.1}$$

where $l = e, \mu$. The advantage with using the ratios $\mathcal{R}(D^{(*)})$, instead of using only the branching ratios $\mathcal{B}(B \to D^{(*)}\tau\nu)$, is that the form factors cancel. These decays have been seen in a few different experiments and results from BaBar [3], Belle [4] and LHCb [5], together with SM predictions [3], are summarised in table 6.

The results from BaBar corresponds to a deviation from the SM predictions by $2.2\,\sigma$ and $2.7\,\sigma$ for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ respectively. These two can be combined into a total $3.4\,\sigma$ deviation from the SM prediction [3]. This shows a clear excess of the tauonic decays compared to SM

Table 6: Values on the ratios $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ from BaBar [3], Belle [4] and LHCb [5]. The first error is statistical and the second is systematic. The SM predictions are also from [3].

	$\mathcal{R}(\mathcal{D})$	$\mathcal{R}(\mathcal{D}^*)$
BaBar	$0.440 \pm 0.058 \pm 0.042$	$0.332 \pm 0.024 \pm 0.018$
Belle	$0.375 \pm 0.064 \pm 0.026$	$0.293 \pm 0.038 \pm 0.015$
LHCb	-	$0.336 \pm 0.027 \pm 0.030$
SM	0.297 ± 0.017	0.252 ± 0.003

predictions, even though it would require a $5\,\sigma$ deviation to be considered a discovery of new physics.

In order to explain the excess of $B \to D^{(*)}\tau\nu$, one most often prefers to use a model which allows additional decay channels at tree-level to obtain a large enough effect. Thus various types of 2HDMs are often used since they have this quality, which can be seen in the last line of the Yukawa Lagrangian in equation (3.21). The charged Higgs particles $H^{\pm}$ can mediate the decay in a similar way as $W^{\pm}$ in the SM, as illustrated in figure 6. 2HDM does also offer an explanation to why only excess in $B \to D^{(*)}\tau\nu$ decays have been seen and never $B \to D^{(*)}e\nu$ nor $B \to D^{(*)}\mu\nu$. This is because the charged Higgs particles coupling strengths to fermions are proportional to the fermion mass and the mass of the tau lepton is much larger than the masses for the electron and muon.

The 2HDM type II was during long time the preferred model since it is used in supersymmetric models and it does not allow FCNC at tree-level. However, Crivellin et al. showed that 2HDM type II is not able to describe both $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ simultaneously [6]. They instead favoured to use a generic 2HDM of type III. We will in the following section present the set-up of their model and then translate it into the FN framework.

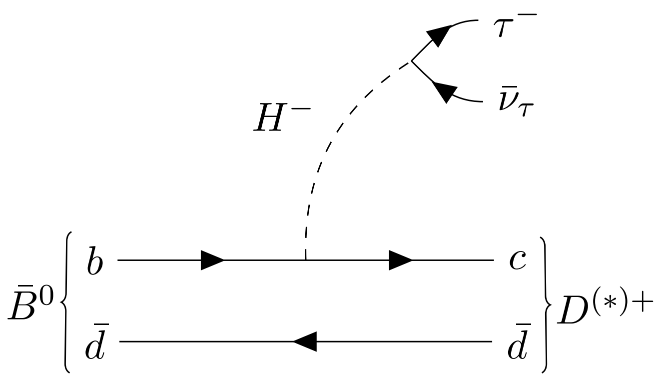

Figure 6: The extra decay channel for the $\bar{B}^0$-meson in a 2HDM. $H^{\pm}$ takes the place of $W^{\pm}$ in the SM.

7.1 $B \to D^{(*)}\tau\nu$ in a generic 2HDM type III and FN-2HDM

An alternative way of expressing the Yukawa Lagrangian compared to equation (3.11) is to use the so-called holomorphic coupling matrices Y^F and non-holomorphic couplings ϵ^F [6]. For the quark-sector, it becomes [6]

$$\mathcal{L}_Y = -\bar{Q}_L Y^u \tilde{\Phi}_u U_R - \bar{Q}_L \epsilon^u \tilde{\Phi}_d U_R - \bar{Q}_L Y^d \Phi_d D_R - \bar{Q}_L \epsilon^d \Phi_u D_R + \text{ h.c.} \quad . \tag{7.2}$$

The Lagrangian is constructed in such a way that in the limit where the non-holomorphic corrections $\epsilon^{u,d} \to 0$, the 2HDM of type II is recovered. This set-up is used in [6] since a type II model is required in the Minimal Supersymmetric Standard Model.

The Lagrangian in equation (7.2) can be treated in the same way as in section 3.1 and, hence, be expressed in the mass eigenstates and the physical fields. In [6], the authors conclude

[6]Beware of the somewhat confusing notation. This ϵ^F is not the same as the symmetry breaking parameter ε in equation (4.1).

that $\tan\beta$ needs to be large and then arrive at the relevant couplings between fermions and Higgs bosons

$$i\left(\Gamma^{LRH^\pm}_{u_f d_i} P_R + \Gamma^{RLH^\pm}_{u_f d_i} P_L\right), \tag{7.3}$$

with[7]

$$\Gamma^{LRH^\pm}_{u_f d_i} = \sum_{j=1}^{3} \sin\beta\, V^{CKM}_{fj} \left(\frac{\sqrt{2}m_{d_i}}{v\cos\beta}\delta_{ji} - \epsilon^d_{ji}\tan\beta\right), \tag{7.4}$$

$$\Gamma^{RLH^\pm}_{u_f d_i} = \sum_{j=1}^{3} \cos\beta \left(\frac{\sqrt{2}m_{u_f}}{v\sin\beta}\delta_{jf} - \epsilon^{u*}_{jf}\tan\beta\right) V^{CKM}_{ji}, \tag{7.5}$$

where $\Gamma^{RLH^\pm}_{q_f q_i} = \Gamma^{LRH^\pm *}_{q_i q_f}$. The notation LR and RL has to do with the chirality of the particles. The total coupling in equation (7.3) contains contributions both from when the initial particle is left-handed and the final right-handed $\left(\Gamma^{LRH^\pm}_{u_f d_i}\right)$ and the other way around $\left(\Gamma^{RLH^\pm}_{u_f d_i}\right)$.

The lepton-sector can be treated in a similar way which yields the same relation between the couplings but with[8]

$$\Gamma^{LRH^\pm}_{\nu_f \ell_i} = \sum_{j=1}^{3} \sin\beta \left(\frac{\sqrt{2}m_{\ell_i}}{v\cos\beta}\delta_{ji} - \epsilon^\ell_{ji}\tan\beta\right). \tag{7.6}$$

Since the Lagrangian in equations (7.2) and (3.21) are equivalent to each other, we can express the couplings in terms of ρ^F by comparing the last line of equation (3.21) with equations (7.4)-(7.6). We also note that $\frac{\sqrt{2}m_{Fi}}{v}\delta_{ji} = \kappa^F_{ji}$. The relations are thus

$$\Gamma^{LRH^\pm}_{u_f d_i} = \tan\beta \sum_{j=1}^{3} V^{CKM}_{fj}\left(\kappa^d_{ji} - \epsilon^d_{ji}\sin\beta\right) = -\left(V^{CKM}\rho^d\right)_{fi}, \tag{7.7}$$

$$\Gamma^{RLH^\pm}_{u_f d_i} = \sum_{j=1}^{3}\left(\cot\beta\,\kappa^u_{jf} - \epsilon^{u*}_{jf}\sin\beta\right) V^{CKM}_{ji} = \left(\rho^{u\dagger}V^{CKM}\right)_{fi}, \tag{7.8}$$

$$\Gamma^{LRH^\pm}_{\nu_f \ell_i} = \tan\beta \sum_{j=1}^{3}\left(\kappa^\ell_{ji} - \epsilon^\ell_{ji}\sin\beta\right) = -\left(\rho^\ell\right)_{fi}. \tag{7.9}$$

By using the parametrisation of $\tan\beta$ in terms of η from the end of section 4.2, the relation between ϵ^F, κ^F and ρ^F can be expressed within the FN framework as

$$\begin{aligned} \epsilon^d &\sim \kappa^d + \varepsilon^\eta \rho^d, \\ \epsilon^u &\sim \varepsilon^\eta \kappa^u - \rho^u, \\ \epsilon^\ell &\sim \kappa^\ell + \varepsilon^\eta \rho^\ell. \end{aligned} \tag{7.10}$$

[7] Note that in [6] the convention $v \approx 174\,\text{GeV}$ is used but we use the convention $v \approx 246\,\text{GeV}$ throughout the work.

[8] Here the PMNS matrix is omitted due to the same reasons as in section 3.1.

In order to calculate the ratios $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$, one uses the two Wilson coefficients C_R^{cb} and C_L^{cb} [6]. In the calculations, one normalises C_R^{cb} and C_L^{cb} against the SM coefficient $C_{SM}^{cb} = 4G_F V_{cb}^{CKM}/\sqrt{2}$. These coefficients are in general given by

$$C_{R(L)}^{u_f d_i, \ell_j} = -\frac{1}{m_{H^\pm}^2} \Gamma_{u_f d_i}^{LR(RL)H^\pm} \Gamma_{\nu \ell_j}^{LRH^\pm *}, \tag{7.11}$$

where $1/m_{H^\pm}^2$ comes from the Higgs propagator. Inserting the couplings from equations (7.4)-(7.6) gives

$$C_R^{u_f d_i, \ell_j} = -\frac{\tan^2\beta}{m_{H^\pm}^2} \left(V_{fi}^{CKM} \kappa_{ii}^d - \sin\beta \sum_{j=1}^{3} V_{fj}^{CKM} \epsilon_{ji}^d \right) \left(\kappa_{jj}^\ell - \sin\beta \sum_{k=1}^{3} \epsilon_{kj}^{\ell *} \right), \tag{7.12}$$

respectively

$$C_L^{u_f d_i, \ell_j} = -\frac{\tan\beta}{m_{H^\pm}^2} \left(\cot\beta \kappa_{ff}^u V_{fi}^{CKM} - \sin\beta \sum_{j=1}^{3} \epsilon_{jf}^{u*} V_{ji}^{CKM} \right) \left(\kappa_{jj}^\ell - \sin\beta \sum_{k=1}^{3} \epsilon_{kj}^{\ell *} \right). \tag{7.13}$$

For the $B \to D^{(*)}\tau\nu$ decays specifically, $u_f = c$, $d_i = b$ and $\ell_j = \tau$. Now, we can compare the decays' Wilson coefficients for 2HDM type III and for FN-2HDM. Applying equations (7.10) and (4.9) to equation (7.12) and (7.13) and remembering that the κ^F are diagonal yield the Wilson coefficients in the FN framework:

$$\begin{aligned} C_R^{cb,\tau} &= -\frac{\tan^2\beta}{m_{H^\pm}^2} \left(V_{23}^{CKM} \kappa_{33}^d - \sin\beta \sum_{j=1}^{3} V_{2j}^{CKM} \epsilon_{j3}^d \right) \left(\kappa_{33}^\ell - \sin\beta \sum_{k=1}^{3} \epsilon_{3j}^{\ell *} \right) \\ &\sim -\frac{1}{m_{H^\pm}^2} \left(\varepsilon^1 \rho_{13}^d + \varepsilon^0 \rho_{23}^d + \varepsilon^2 \rho_{33}^d \right) \left(\rho_{13}^\ell + \rho_{23}^\ell + \rho_{33}^\ell \right), \end{aligned} \tag{7.14}$$

$$\begin{aligned} C_L^{cb,\tau} &= -\frac{\tan\beta}{m_{H^\pm}^2} \left(\cot\beta \kappa_{22}^u V_{23}^{CKM} - \sin\beta \sum_{j=1}^{3} \epsilon_{j2}^{u*} V_{j3}^{CKM} \right) \left(\kappa_{33}^\ell - \sin\beta \sum_{k=1}^{3} \epsilon_{k3}^{\ell *} \right) \\ &\sim \frac{1}{m_{H^\pm}^2} \left(\varepsilon^3 \rho_{12}^u + \varepsilon^2 \rho_{22}^u + \varepsilon^0 \rho_{32}^u \right) \left(\rho_{13}^\ell + \rho_{23}^\ell + \rho_{33}^\ell \right). \end{aligned} \tag{7.15}$$

Note, however, that they are given at the scale of the mass of the charged Higgs boson $m_{H^\pm}$ but they enter the calculations $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ at the scale of the B-meson. Hence, one would need to perform Renormalisation Group Equation (RGE) evolution on them to obtain the Wilson coefficients at the right scale. This is however beyond the scope of this thesis. Instead of calculating the ratios directly, we compare with pre-existing calculations. The authors of [6,18] reason that it is only ϵ_{33}^d that can contribute significantly to $C_R^{cb,\tau}$ since both ϵ_{13}^d and ϵ_{23}^d are heavily constrained by FCNC. Further, they similarly reason that only ϵ_{32}^u has a sizeable contribution to $C_L^{cb,\tau}$ since both ϵ_{12}^u and ϵ_{22}^u are suppressed by the CKM matrix.

Regarding the contributions from the leptons, they assume that the non-holomorphic coupling ϵ^ℓ is negligible and thus only κ_{33}^ℓ contributes. Additionally, they show that C_R^{cb} cannot explain $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ simultaneously on its own which is possible for C_L^{cb}. Thus, the only

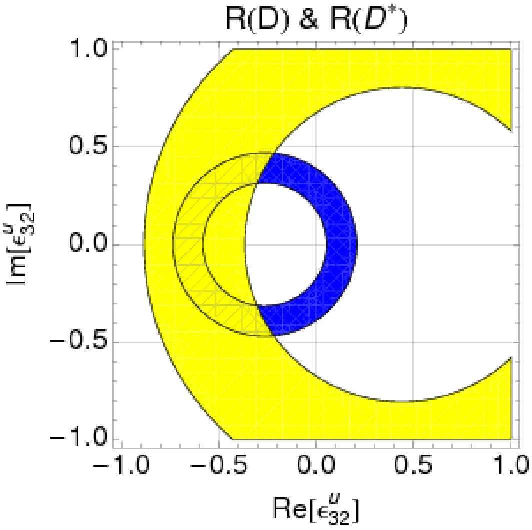

Figure 7: Regions in the complex plane for ϵ_{32}^u where correct values of $\mathcal{R}(D)$ (blue) and $\mathcal{R}(D^*)$ (yellow) are obtained within $1\,\sigma$ deviation for $m_{H^\pm} = 500\,\text{GeV}$ and $\tan\beta = 50$. The plot is taken from [6].

free parameter that can be adjusted to explain both $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ is ϵ_{32}^u. Considering these simplifications, C_L^{cb} can be reduced to

$$C_L^{cb} \approx \frac{\tan\beta}{m_{H^\pm}^2}\epsilon_{32}^u\kappa_{33}^\ell. \tag{7.16}$$

Comparing equation (7.16) and the last line of (7.15), we can conclude that ϵ_{32}^u corresponds to $(\varepsilon^3\rho_{12}^u + \varepsilon^2\rho_{22}^u + \varepsilon^0\rho_{32}^u)$ and similarly does $\tan\beta\kappa_{33}^\ell$ corresponds to $(\rho_{13}^\ell + \rho_{23}^\ell + \rho_{33}^\ell)$.

7.2 Comparison to data

In order to determine if any of the obtained sets of flavon charges from section 6 could explain the $B \to D^{(*)}\tau\nu$ decays, we must first investigate ϵ_{32}^u closer. Since we have only been working with orders of magnitude, and hence only have real matrices, we can only compare elements of ρ^u with real values of ϵ_{32}^u. Figure 7 shows a plot over the regions in the complex plane for ϵ_{32}^u where correct values of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are obtained within $1\,\sigma$ for a charged Higgs mass $m_{H^\pm}^2 = 500\,\text{GeV}$ and $\tan\beta = 50$. From figure 7 one can read off that the only real value that satisfies both $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ is $\epsilon_{32}^u \approx -0.65$. However, this is with $\tan\beta = 50$ while we either have $\tan\beta = \varepsilon^{-2} = 25$ or $\tan\beta = \varepsilon^{-3} = 125$. Considering that the Wilson coefficient C_L^{cb} needs to have its specific value, and remembering the simplified version C_L^{cb} in equation (7.16), we can conclude that ϵ_{32}^u must scale inversely proportional against other values of $\tan\beta$. Hence, $\epsilon_{32}^u \approx -1.3 \sim \varepsilon^0$ for $\eta = 2$ and $\epsilon_{32}^u \approx -0.26 \sim \varepsilon^1$ for $\eta = 3$. Therefore, the leading term of $(\varepsilon^3\rho_{12}^u + \varepsilon^2\rho_{22}^u + \varepsilon^0\rho_{32}^u)$ must be at least either ε^0 or ε^1 depending on η. Given the two ρ^u matrices in equations (6.4) and (6.5), $\varepsilon^0\rho_{32}^u$ is the leading term for both cases. For both the examples, $\rho_{32}^u \sim \varepsilon^1$, which means that it is only in the example with $\eta = 3$ that the element is large enough. Further, we note that none of the sets with $R_1 \geq R_2$ in table 8 in appendix B yield ρ_{32}^u-elements that are sufficiently large.

If we shortly also consider the other Wilson coefficient C_R^{cb} in equation (7.14), we see that no matter which ρ^d matrix we consider from section 6, the leading term in $(\varepsilon^1\rho_{13}^d + \varepsilon^0\rho_{23}^d +$

$\varepsilon^2\rho^d_{33}$) is at least $\sim \varepsilon^6$. This means that C^{cb}_R is negligible compared to C^{cb}_L and does not contribute to $\mathcal{R}(D)$ and $\mathcal{R}(D^{(*)})$, just as assumed in [6].

Shifting the attention to the lepton-sector and comparing with the simplified version of C^{cb}_L, it is evident that the leading term of $(\rho^\ell_{13} + \rho^\ell_{23} + \rho^\ell_{33})$ needs to be approximately equal to $\tan\beta\kappa^\ell_{33}$. For $\eta = 2$ this gives that $\varepsilon^{-2}\sqrt{2}m_\tau/v \sim \varepsilon^1$ and for $\eta = 3$ $\varepsilon^{-3}\sqrt{2}m_\tau/v \sim \varepsilon^0$. The second example of lepton flavon charges in section 6 (equation (6.7)) satisfies the requirement for $\eta = 2$.[9] However, none of the 44 sets generates matrices that satisfies the requirement for $\eta = 3$.

Hence, for the two cases $\eta = 2$ and $\eta = 3$, either ρ^u_{32} or $\tan\beta\kappa^\ell_{33}$ is one order in ε too high to fit with the results in [6]. Nonetheless, considering the precision in the approximations used, it is too early to discard the model for not being able to explain the decays. Approximations have been made for the parametrisation of the quark masses and the elements of the CKM matrix and the exact coefficients have been neglected. Further, the charged Higgs mass is unknown, $m_{H^\pm} = 500\,\text{GeV}$ is only an assumed value. Therefore, the uncertainty can be assumed to be at least one order in ε. Thus, it could still be possible to explain the $B \to D^{(*)}\tau\nu$ decays with the model but that would require a more detailed study.

[9] Two other of the 44 sets also generates the same matrices.

8 Conclusion and outlook

In this thesis, a 2HDM in the FN framework has been used to try to explain the $B \to D^{(*)}\tau\nu$ decays. This work can be considered to consists of two parts. The first part was to constrain the model in order to find sets of physically valid flavon charges and the second part was to consider how the $B \to D^{(*)}\tau\nu$ decays are affected by the FN mechanism and see if the valid sets of flavon charges can explain the excess of $B \to D^{(*)}\tau\nu$ decays.

The main point with the FN mechanism is to explain the seemingly unnaturally large fermion mass ratios in a natural way. Thus, we required that the flavon charges and the difference between them ought to be small; otherwise the problem of unnaturalness would just have transferred to the flavon charges instead of fermion masses. Further, we found nine constraints which ensured that the flavon charges gave the correct fermion masses and three constraints that gave the correct CKM matrix. Additionally, we imposed limits on the off-diagonal elements of ρ^d to avoid larger FCNC than observed. A fairly large number of sets of flavon charges that satisfied the constraints were found. Out of these, two preferred candidates with small charge differences were selected for further study.

In the second part, we found that the $B \to D^{(*)}\tau\nu$ decays are mainly affected by ρ^u_{32} and the leading term of $(\rho^\ell_{13} + \rho^\ell_{23} + \rho^\ell_{33})$ for the two preferred sets. In order to explain the excess, $\rho^u_{32} \sim \varepsilon^0$ for $\eta = 2$ and $\rho^u_{32} \sim \varepsilon^1$ for $\eta = 3$. Similarly, the leading term of $(\rho^\ell_{13} + \rho^\ell_{23} + \rho^\ell_{33})$ must satisfy $\sim \varepsilon^1$ for $\eta = 2$ and $\sim \varepsilon^0$ for $\eta = 3$. Using the second set of lepton flavon charges in table 5, we saw that either ρ^u_{32} or the leading term of $(\rho^\ell_{13} + \rho^\ell_{23} + \rho^\ell_{33})$ is one order in ε too high for both $\eta = 2$ and $\eta = 3$. However, since all the calculations only were carried out to leading order and since exact coefficients were omitted, the uncertainty is at least one order of magnitude. Hence, our main conclusion is that a 2HDM in the FN framework could possibly explain the excess of $B \to D^{(*)}\tau\nu$ decays and it motivates to study this model further in greater detail.

The study could be expanded in a few different areas. Within the Froggatt-Nielsen framework, one extension is to find the complex coefficients which have been omitted. Another extension is to investigate other symmetry groups. We have only been working with Froggatt's and Nielsen's original model with an $U(1)$ symmetry. However, the model could be extended to a $U(1) \times \ldots \times U(1)$ symmetry with $N \in \mathbb{N}$ $U(1)$ symmetries. This means that the model would include N flavons and N symmetry breaking parameters $\varepsilon_{1,\ldots,N}$.

Further, it could be interesting to perform the RGE evolution of the Wilson coefficients. Then it would be possible to calculate the ratios $\mathcal{R}(D)$ and $\mathcal{R}(D^{(*)})$ directly for the model instead of just comparing the couplings with existing calculations. There are also other experimental constraints that could be considered. Radiative B-meson decays such as $b \to s\gamma$ and $b \to d\gamma$, radiative lepton decays such as $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ and electric dipole moments could all be considered and would constrain other elements than the ones constrained by the FCNC. Also another tauonic B-meson decay, $B \to \tau\nu$, could be interesting to include.

Finally, one could take a closer look at the implications of this model. For the found sets of flavon charges which comply with the B-meson decays, the element $\rho^u_{23} = \varepsilon^0$ is large. This would result in sizeable FCNC. Some decays where the ρ^u_{23} element enters are the top quark decays into the charm quark via one of the neutral Higgs, $t \to c\phi$ ($\phi = h, H, A$).

A Bi-unitary transformation

The mass matrices are diagonalised into the mass eigenstates by bi-unitary transformations [19]. A general complex square $n \times n$ matrix M can be diagonalised by the bi-unitary transformation

$$U_1 M U_2^\dagger = D, \tag{A.1}$$

where U_1 and U_2 are unitary matrices and D is diagonal. Multiplying equation (A.5) by its Hermitian conjugate from the right respectively from the left yields

$$U_1 M M^\dagger U_1^\dagger = D^2, \tag{A.2}$$

$$U_2 M^\dagger M U_2^\dagger = D^2. \tag{A.3}$$

Now, U_1 and U_2 can be found by defining $MM^\dagger = H$ and $M^\dagger M = \tilde{H}$ and considering equation (A.2) and (A.3) as normal diagonalisations of H and $\tilde{H}$. U_1 can be determined by solving the eigenvalue equation

$$Hx_j = \lambda_j x_j, \tag{A.4}$$

for eigenvalues λ_j and eigenvectors x_j and then construct U_1 such that the columns are the eigenvectors $x_1, \dots x_n$ [20]. U_2 can of course be obtained by solving the corresponding eigenvector equation for $\tilde{H}$.

There is also the possibility to find U_2 by solving equation (A.5) instead as it in some cases might be simpler. In order to find U_2 this way one must determine the inverse of D^2, since

$$U_2 = D^{-1} U_1 M. \tag{A.5}$$

The diagonal matrix D is constructed with the eigenvalues λ_j as its elements $D = \text{diag}(\lambda_1, \dots \lambda_n)$. Hence, the inverse of D is $D^{-1} = \text{diag}(1/\lambda_1, \dots 1/\lambda_n)$ [20].

B Valid sets of flavon charges

The five valid sets of flavon charges chosen from $b_i^q \in [0,5]$, $a_j^{u,d} \in [1,5]$ and $R_a \in [-5,5]$ in the case $R_2 > R1$ are presented in table 7. For flavon charges chosen from the intervals $a_j^{u,d} \in [-6,5]$ and $R_a \in [-10,10]$ in the case $R_1 \geq R_2$, 30 different valid sets were found, see table 8. All these 30 sets are valid both for $\eta = 2$ and $\eta = 3$.

Table 7: Valid sets of quark and Higgs field flavon charges for the case $R_2 > R_1$.

η	(b_1^q, b_2^q, b_3^q)	(a_1^u, a_2^u, a_3^u)	(a_1^d, a_2^d, a_3^d)	(R_1, R_2)
2	$(3,2,0)$	$(5,2,1)$	$(2,1,1)$	$(-3,-1)$
3	$(3,2,0)$	$(5,2,1)$	$(2,1,1)$	$(-4,-1)$
	$(3,2,0)$	$(5,2,1)$	$(2,1,1)$	$(-3,-1)$
	$(4,2,0)$	$(5,2,1)$	$(2,1,1)$	$(-5,-1)$
	$(4,2,0)$	$(5,3,1)$	$(2,1,1)$	$(-5,-1)$

Table 8: Valid sets of quark and Higgs field flavon charges for the case $R_1 \geq R_2$. All combinations are valid for $\eta = 2$ and $\eta = 3$.

(b_1, b_2, b_3)	(a_1^u, a_2^u, a_3^u)	(a_1^d, a_2^d, a_3^d)	(R_1, R_2)
$(3, 2, 0)$	$(5, 2, 1)$	$(2, 1, 1)$	$(10, -1)$
$(4, 3, 1)$	$(5, 2, 1)$	$(0, -1, -1)$	$(9, -2)$
$(4, 3, 1)$	$(5, 2, 1)$	$(0, -1, -1)$	$(10, -2)$
$(4, 3, 1)$	$(4, 1, 0)$	$(1, 0, 0)$	$(10, -1)$
$(5, 4, 2)$	$(5, 2, 1)$	$(-2, -3, -3)$	$(8, -3)$
$(5, 4, 2)$	$(5, 2, 1)$	$(-2, -3, -3)$	$(9, -3)$
$(5, 4, 2)$	$(4, 1, 0)$	$(-1, -2, -2)$	$(9, -2)$
$(5, 4, 2)$	$(5, 2, 1)$	$(-2, -3, -3)$	$(10, -3)$
$(5, 4, 2)$	$(4, 1, 0)$	$(-1, -2, -2)$	$(10, -2)$
$(5, 4, 2)$	$(3, 0, -1)$	$(0, -1, -1)$	$(10, -1)$
$(6, 5, 3)$	$(5, 2, 1)$	$(-4, -5, -5)$	$(7, -4)$
$(6, 5, 3)$	$(5, 2, 1)$	$(-4, -5, -5)$	$(8, -4)$
$(6, 5, 3)$	$(4, 1, 0)$	$(-3, -4, -4)$	$(8, -3)$
$(6, 5, 3)$	$(5, 2, 1)$	$(-4, -5, -5)$	$(9, -4)$
$(6, 5, 3)$	$(4, 1, 0)$	$(-3, -4, -4)$	$(9, -3)$
$(6, 5, 3)$	$(3, 0, -1)$	$(-2, -3, -3)$	$(9, -2)$
$(6, 5, 3)$	$(5, 2, 1)$	$(-4, -5, -5)$	$(10, -4)$
$(6, 5, 3)$	$(4, 1, 0)$	$(-3, -4, -4)$	$(10, -3)$
$(6, 5, 3)$	$(3, 0, -1)$	$(-2, -3, -3)$	$(10, -2)$
$(6, 5, 3)$	$(2, -1, -2)$	$(-1, -2, -2)$	$(10, -1)$
$(7, 6, 4)$	$(4, 1, 0)$	$(-5, -6, -6)$	$(7, -4)$
$(7, 6, 4)$	$(4, 1, 0)$	$(-5, -6, -6)$	$(8, -4)$
$(7, 6, 4)$	$(3, 0, -1)$	$(-4, -5, -5)$	$(8, -3)$
$(7, 6, 4)$	$(4, 1, 0)$	$(-5, -6, -6)$	$(9, -4)$
$(7, 6, 4)$	$(3, 0, -1)$	$(-4, -5, -5)$	$(9, -3)$
$(7, 6, 4)$	$(2, -1, -2)$	$(-3, -4, -4)$	$(9, -2)$
$(7, 6, 4)$	$(4, 1, 0)$	$(-5, -6, -6)$	$(10, -4)$
$(7, 6, 4)$	$(3, 0, -1)$	$(-4, -5, -5)$	$(10, -3)$
$(7, 6, 4)$	$(2, -1, -2)$	$(-3, -4, -4)$	$(10, -2)$
$(7, 6, 4)$	$(1, -2, -3)$	$(-2, -3, -3)$	$(10, -1)$

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