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# Age Structure and Economic Growth

*The Case of Sri Lanka*

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## ABSTRACT

The age structure and its impact on economic growth is a current issue that concerns all countries in the world. For Sri Lanka, the issue lies within an ageing population and the economic implications of the growing share of elderly. Thus, this paper strives to investigate how the age structure in Sri Lanka affects the economic growth, mainly looking at the ageing population. A time series regression analysis will be conducted with age shares as explanatory variables to investigate the age structure's effect on the economic growth. The regression analysis is based on the Solow model with human capital that has been extended to include variables for age structure. The result shows that only the prime working age group has a significantly positive effect on the economy.

Key words: Economic growth, age structure, Sri Lanka, demography

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# 1. INTRODUCTION

Ageing population is a key demographic global challenge that all countries eventually will have to meet, whether being a developed country or a developing country. United Nations (henceforth UN) reports that the global population share of people over 60 years of age, has increased from 9.2 % in 1990 to 12.5 % in 2012, and is projected to reach a staggering 21.1 % by 2050 (UN, 2015). This means that the number of aged people will exceed the number of children for the first time ever by 2047. The distribution of elderly will be the largest in the developing countries, where 66 % of the population in Asia alone is projected to be a person over 60 years old by 2030.

Ageing population is the result of a rapid demographic transition involving decreasing mortality and fertility, and increasing life expectancy. In Sri Lanka there were in 2012, 12.4 % above the age of 60 years old and the percentage will continue to increase as the country moves through the demographic transition. The change in age structure and an ever-increasing number of elderly in Sri Lanka is putting pressure on the working population to support the non-working population. Between the years of 1981−2012, the elderly population in Sri Lanka has increased from 6.6 % to 12.4 %, thus shrinking the support base from an average seven persons (age 20-59 years old) per elderly in 1981, to four persons per elderly in 2012 (the World Bank, 2012). Therefore, it is imperative to face the demographic challenge with seriousness and understand the multifaceted nature of the problem. In the case of Sri Lanka, the socioeconomic implications of an elderly population will need to be met in order to adapt and implement the adequate policies.

## 1.1 Purpose

The purpose of this paper is to investigate whether age structure affects the economic growth in Sri Lanka. This is an interesting case because the age structure has changed dramatically during the 20’th century and will continue to change over the coming decades, the most pending question being how the large increase in elderly people will affect the country’s economic growth. Thus, this paper aims to answer the following question:

- How does age structure affect the economic growth in Sri Lanka?

Sub query; do the elderly have a positive or negative impact on the economy?

## 1.2 Method

The method used to answer the stated question is time series analysis using a regression model with yearly data from Sri Lanka. The empirical model has a solid ground in theory in the neoclassical growth model first presented by Solow in 1956 (Solow, 1956). However, the original Solow model has been modified to include age structure variables as explanatory variables for the economic growth. The age structure variables have been chosen to roughly reflect the different stages of an individual's lifecycle, in terms of labor productivity and savings.

## 1.3 Limitations

The paper will focus on Sri Lanka because of the country's unique situation in terms of elderly population, but also for the position in the demographic transition. The aim is to answer the stated question by testing the theory empirically on Sri Lanka. When studying economic growth, it is common and often necessary to look at a very long period of time to be able to detect a country's transition to steady state. The transition is usually taking place over a period of 30−50 years (Hansson, 2006). The data in this paper ranges from 1960−2014 due to lack of data before 1960, but the data set should indeed be able to present some information about Sri Lanka's economic growth.

## 1.4 Disposition

The second chapter will present the background of the study. The demographic transition model will be explained, as well as the dependency ratio and the lifecycle deficit. In light of that, the Sri Lankan demographics will be presented, both the situation historically and the situation today thus making clear why Sri Lanka is an exceptional case. An international comparison of the demographic situation in other parts of the world will be presented and to conclude chapter two, previous studies in the same field will be described.

In the third chapter, the theoretical framework of the Solow model will be presented. In the fourth chapter the data used for the regression model will be presented, as well as necessary computations made, along with the variables included in the regression. The chapter concludes by presenting the range of tests conducted on the regression and the results of these tests. The fifth chapter will present the results of the regression and in the final section, chapter six, the paper is summarized with a conclusion of the primary findings.

# 2. BACKGROUND

The theory in this section will offer an explanatory background for the demographic changes that can be observed in all counties, and how they affect economic growth. The theory of the demographic transition model (Thomson, 1929) describes in three steps how fertility rate, death rate and replacement level change in a country through time. This three-step model is the most commonly known and studied demographic transition model. Lee (2003) however, states that there is a fourth step in the transition, which consists of shifts in age distribution. The steps of the basic demographic transition model will be described in more detail but first, it is important to understand the terminology of the process.

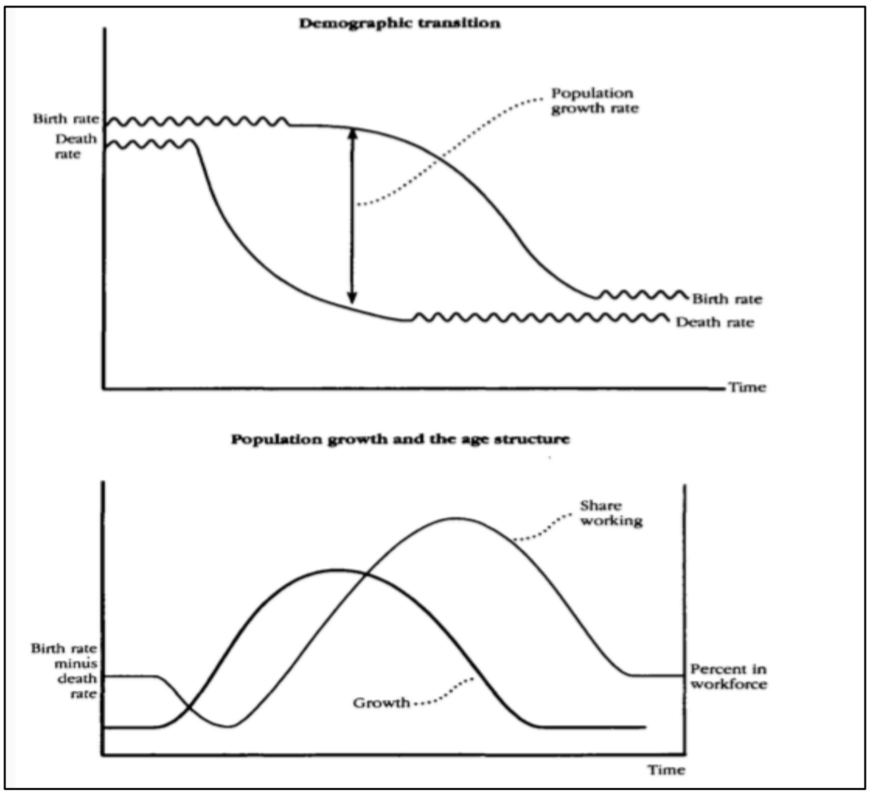
## 2.1 Terminology

*Fertility rate* is the average number of children born per woman in a country. *Replacement level* describes how many children a woman needs to give birth to in order for the population to reproduce, meaning that the population remains at a constant level and does not increase or decrease. At the replacement level, the fertility rate and the *death rate* should be equal. Death rate is simply how many people that die in a specific period divided by the total population during the same period, and is usually stated as a ratio for a specific age group or for the population as whole. *Dependency ratio* is invented to measure the share of the population outside the working force as opposed to those in the working force. Another way to measure dependency is through the *lifecycle deficit,* which is defined as consumption (both private and public) minus labor income. Dependency ratio and lifecycle deficit are two common ways to measure age structure effects on economic growth.

## 2.2 The demographic transition model

The beginning of the world's demographic transition took place in Europe around the 1900's. The first stage is high fertility and high death rates, the second stage is high fertility and lower death rates and the third stage is low fertility and low death rates (Thomson, 1929). Figure 1 shows the demographic transition model.

**Figure 1: The demographic transition model**




*Source: Bloom, D. E. and Williamson, J. G (1988), page 423, figure 1*

The transition into the second stage is initiated by general improvements in health, reduction of famine and contagious diseases, which leads to a decrease in mortality and an increase in life expectancy. Many of the world's developing countries are in stage 2 (Lee, 2003).

The transition into the third stage is characterized by lower fertility rates. When fertility declines, it declines mostly amongst the youngest (because of introduction of contraceptives) and the oldest, which means that the childbearing age becomes concentrated in the twenties and early thirties. Today, 60 countries (43 % of the world's population) currently have fertility rates under the replacement level, most of them being developed countries (Lee, 2003).

In the third stage the rates are often decrease gradually and are the result of improved economic conditions, women empowerment and access to contraceptives. The final step applies to most of the developed countries (Todaro & Smith, 2003).

The demographic transition has a large impact on the age structure of a country. As shown in figure 1, there is a time delay between the decline in mortality and the decline in fertility in the second stage of the transition, which will lead to a generation that is larger than previous generations. When that generation reaches fertile age they will produce yet another generation that is larger than the average. This effect is called *population momentum* (Bloom, Canning & Malaney, 2001) and can go on for decades until the new fertility rate reaches replacement level and the population stabilizes at a new level of equilibrium. Also, when the larger than average generation is young, the dependency ratio will be high since they cannot support themselves. This will demand great resources in terms of education and health care. However, when the young cohort transitions into working age, it will grow faster than the population as whole, so that the total dependency ratio declines. This will offer great potential through the *demographic dividend* (Bloom, Canning & Sevilla, 2003) that will spur the economic growth in the country because of the great increase of the labor force. Ideally, if the country adapts well to the generation in its young years, they might reap the benefits of the demographic dividend. However, if the country fails to adapt to the larger generation in its young years, it might lead to high unemployment rates, general discontent and even political instability (Bloom *et al*. 2003).

The population momentum and the demographic dividend give a fair understanding of why the demographic transition has such a big impact on the age structure in a country. The same is true for the inverse relationship i.e. that the age structure impacts the demographic transition of a country, although this will not be treated in greater detail in this paper.

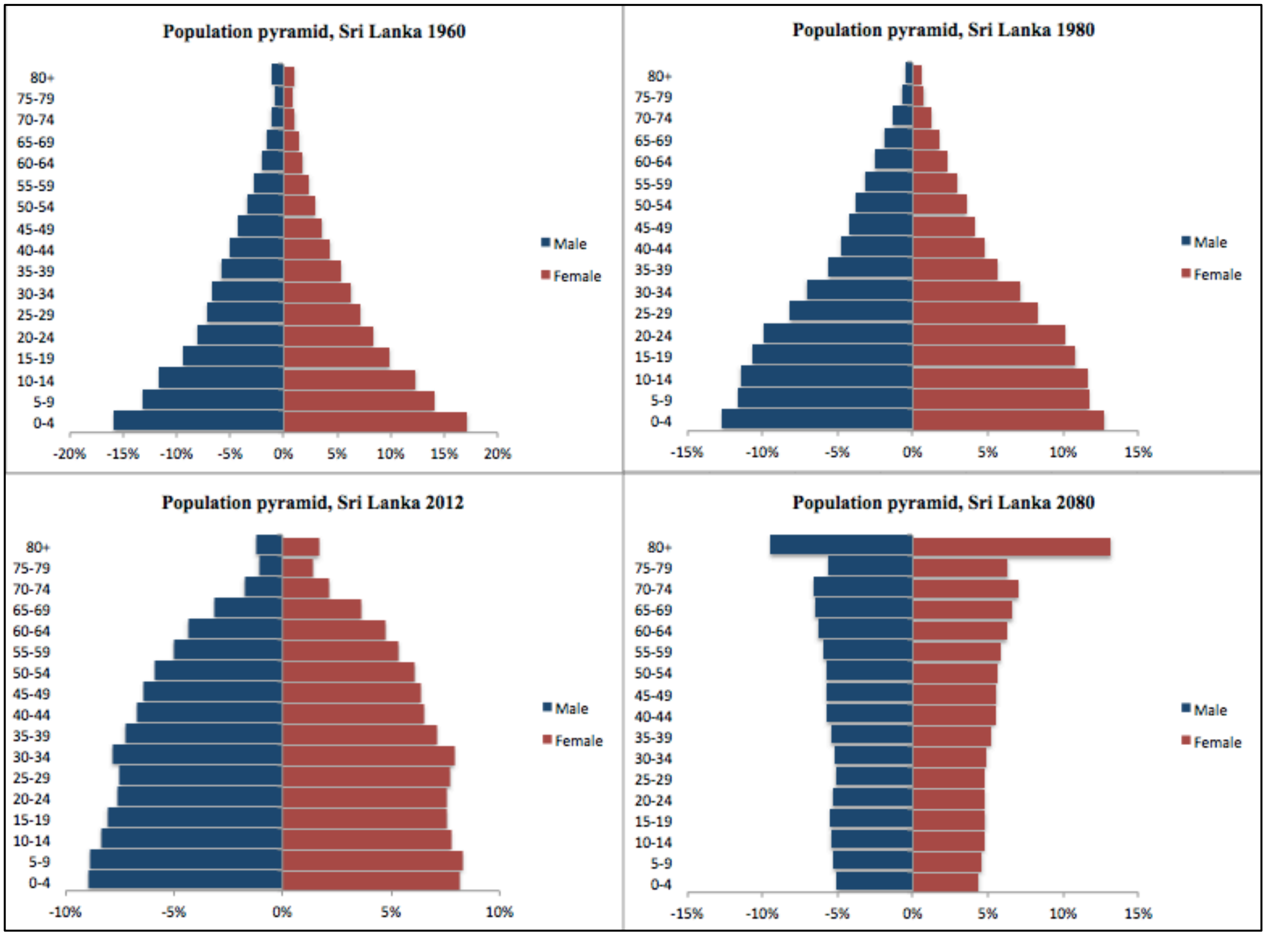
## 2.3 Demographic changes in Sri Lanka

During the last decades the total size, sex and age structure of Sri Lanka's population have changed significantly. Post World War II, the death rates started to decline, in large due to the eradication of malaria in combination with better health care as well as a general improvement in food supplies, which were welcome effects of the ameliorated economic wellbeing and development in the social infrastructure (the World Bank, 2012). These factors combined led to an increase of life expectancy in Sri Lanka, which is currently 74.9 years and one of the highest in Asia (UNFPA, 2012).

In the early 1930's, when Sri Lanka's population was in the first stage of the transition, the Sri Lankan government was not late to realize the great potential of the demographic dividend to come, which is why they started to invest a great deal in education and health services (the World Bank, 2012). Death rates started to decline already in 1947, but the birth rates did not start to decline until 1960, forcing the Sri Lankan government to drive family planning efforts to integrate maternal- and child health care programs and implement a five-year plan to ameliorate the situation. Between the years of 1960-1990 Sri Lanka experienced the highest decline in fertility rate, compared to other South Asian countries (the World Bank. 2012). Despite the efforts of the Sri Lankan government, fertility rates continued to drastically fall from 5.3 births per women in 1953 to 4.2 in 1971, and by 2007 as low as 2.3 births per woman (UN, 2015). The decline in fertility was also a result of the increased use of contraceptives, which was as high as 68 % in 2007 (the World Bank, 2012). According to UN (2015), the fertility rate was 2.06 in 2015, which is lower than the replacement level of 2.1 births per woman.

A direct result of the efforts of the Sri Lankan government is that the country today has a very large working age population, 67 % and a large share of young and old people. In the population pyramids in Figure 2, the transition of a young cohort can be seen from 1960 to 2012. The population pyramid for 2080 is a projection made by the UN, 2017. These pyramids offer a great illustration of the demographic changes in Sri Lanka and it is clear that age structure and the demographic dividend offer great possibilities, but as the cohort ages, the challenges of an increasing dependency ratio will follow.

**Figure 2: Sri Lanka population pyramids, actual and projected, 1960 to 2080**



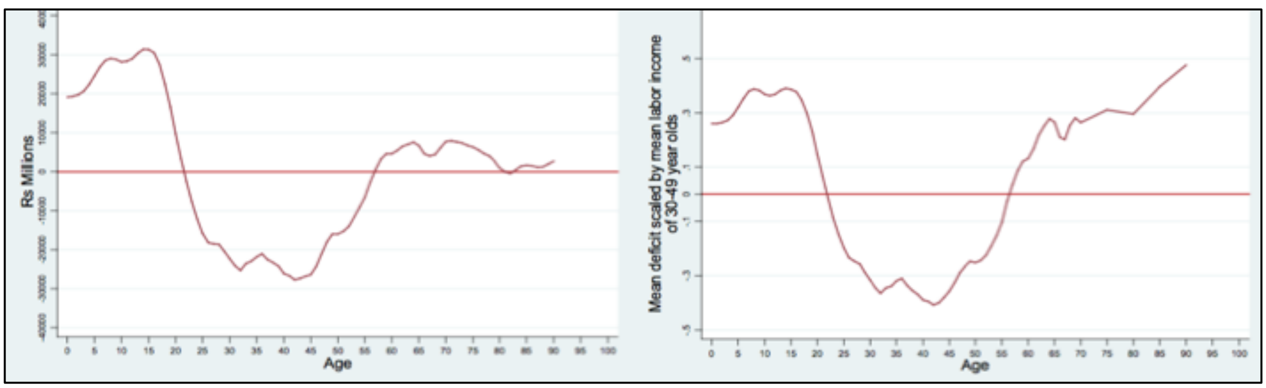

*Source: Figure prepared for this paper with data from the UN, 2017.*

### *2.3.1 Dependency ratio and lifecycle deficit*

In Sri Lanka, the majority of the labor income is earned by people in the ages between 19-59 years old, which currently are attributed nearly 85 % of all labor income. On an aggregate level, consumption however is heavily skewed towards the younger age group, which account for 26 % whilst only contributing to labor income by 2.5 % (the World Bank, 2012). The large share of resources going to the younger individuals of the population indicates the large proportion of young in Sri Lanka, and is also consistent with Sri Lanka being in a relatively early stage of the demographic transition. Countries that have reached a more developed stage in the transition also put more efforts into the elderly, for example Japan and the United States, where the proportion of elderly are large and demands a lot of resources (Lee, 2003).

Currently, the lifecycle deficit and the dependency ratio for elderly are relatively small in Sri Lanka in the aggregate view since they account for a small proportion of the population. However, the share of elderly will grow as fertility rates decline and life expectancy rises, and the children's share in the lifecycle deficit will decline. Figure 3, illustrates that the per capita view (right side) of the lifecycle deficit for the elderly are much higher than the aggregate view (left side), which is due to the low numbers of elderly relative to total population. In Sri Lanka, it is not uncommon for elderly to work well up into their seventies or even in their eighties since they cannot afford to stop. The official retirement age in the formal sector is between the ages of 57−62, but many of the elderly work in the informal sector and even though they might earn some income from activities, the consumption (private and pubic) far exceeds that income (the World Bank, 2012). The older they are the greater the deficit, which is to bear in mind considering the increasingly high life expectancy.

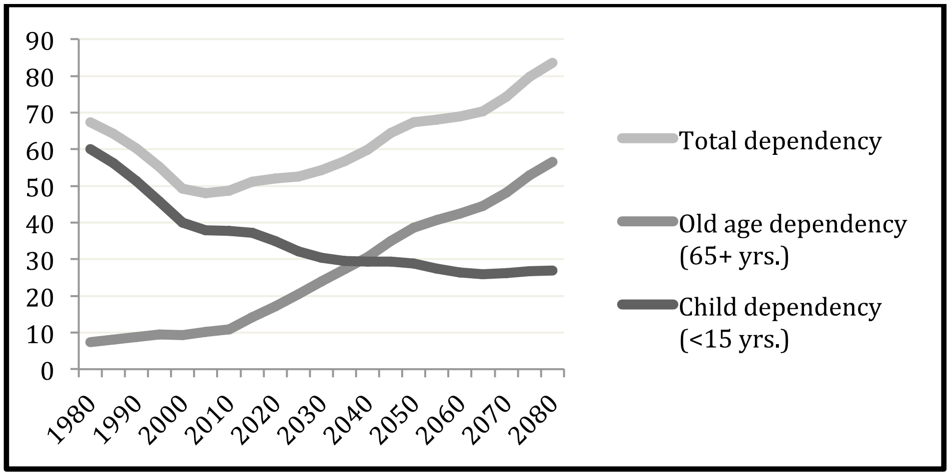
**Figure 3: Life-Cycle deficit, Sri Lanka; aggregate view and per capita view**




*Source: The World Bank, 2012, page 8, figure 3 and 4*

In 2001, there were 55 dependent on every 100 working people and that figure is estimated to increase to 58.3 % by year 2031 (the World Bank, 2012). As shown in Figure 4, the younger cohort is decreasing and the elderly population is increasing due to the demographic transition. The dependency ratio for elderly will eventually outpace the decline in youth dependency. The share of working individuals in proportion to dependents in Sri Lanka was going to remain large until 2017, at which point the share of dependents would start surpassing the working population (the World Bank, 2012).

**Figure 4: Projected dependency ratio 2017 to 2080**




*Source: Figure prepared for this paper with data from the UN, 2017.*

As demonstrated by Figure 4, it is apparent that Sri Lanka will have to meet the challenges of an increase in the total dependency ratio, but also of an increase in the dependency of elderly. Dependency for people aged 60 years old and above is expected to rise from 12.5 % to 16.7 % in 2021, and by 2041, one out of four is expected to be an elderly (the World Bank, 2012).

## 2.4 International comparison

The regions of the world are each in different stages of the demographic transition model. After the industrial revolution in the 1900's, each country has developed in different paces through the transition due to different prerequisites and economic conditions. In the first stage we find the least developed countries in regions like Africa, who has a median age of only 18 (Lee, 2003).

In the third stage of the demographic transition model we find countries in Europe that has a large share of elderly in the population, and low fertility rates. The median age in Europe is 37.7, which is 10 years higher than the median age of the world, and the fertility rate is only 1.4 children per woman as opposed to the world fertility rate of 2.6 children per woman (UN, 2017). They face unique and difficult challenges in the way that they are unprecedented. Europe is the only continent that has a forecasted population decrease (Lee, 2003).

In Asia, the working age population is growing at around 1.5 % per year, which is slightly over the average of the world. The region has already experienced the "youth bulge" that will eventually result in population ageing as the generation passes through the age structure (Bloom

& Williamson, 1998). However, the structure does not look the same for all of the countries in Asia because there are significant differences both in timing and magnitude of the change. For example, some parts of East Asia are now experiencing a decline in population growth, whilst South Asia and Southeast Asia is experiencing a rapid population growth. The vast differences within the region are also why it is interesting to study from a demographic point of view. It is calculated that by 2055, the Asian region will inhabit 55 % of the world's working age group; a generation that is also higher educated and integrated globally economically and socially than past generations (the World Bank, 2012).

Sri Lanka, being in the second stage of the demographic transition, actually stands out in the Southeast region with its considerable advancements made in human development. The early investments in health and education has resulted in a reduction in child mortality, an increase in life expectancy and decreased fertility rates, which has influenced Sri Lanka's demographic cycle. The population is projected to peak in 2046 and as mentioned, one out of four is expected to be an elderly person by 2041, making Sri Lankans the oldest population in South Asia (the World Bank, 2012).

## 2.5 Previous studies

The neoclassical growth model of Solow has been used as basis for a number of studies to measure the effect of population growth on the economy. The basic model states that population growth has a negative impact on economic growth but Bloom and Williams (1997; 1998) and Bloom et al. (2001) came to the conclusion that population growth cannot explain economic growth, growth rate of total population being the only variable. They do show however, that demography does play a significant role for economic growth, once the age structure is considered and the underlying assumption that the population has a constant age composition is dropped. By using regression analysis, they showed that the growth rate of the working age population and various other control variables, have a significantly positive effect on the growth rate of GDP per capita. Moreover, they found a significantly negative effect for the growth rate of the total population, and that the dependents (age groups 0-14 and 65+) slow down the economic growth. The results using the basic Solow model as basis for determining population growth's effect on economic growth varies however, and there is still no consensus whether it is positive or negative.

McMillan and Baesel presented the first study using lifecycle theory to measure the effects of age structure on economic growth in 1990. The study investigated the impact of the baby boom generation using post war data from the US. They found that age distribution of the population affects a number of macroeconomic variables such as economic growth, unemployment and real interest rates. A similar study to that of McMillan and Baesel was conducted by Fair and Dominguez (1991), who also found significant results for age distributions and the age group's effect on money demand, labor force participation, consumption and housing-investments. Other studies (e.g. Lee, Lee & Mason, 2005; Bloom & Williamson, 1997) that use the lifecycle theory and cross-country analysis data have found significant impacts of savings and investment on economic growth.

Some more recent studies that include population age structure in their models to measure the impact for each age group separately on economic growth, are Brander and Dowrick (1994), Lindh and Malmberg (1995; 1996) and Lenehan (1996). Following Mankiw, Romer and Weil (1992), Lindh and Malmberg presented the article "Age structure effects and growth in the OECD" in 1999, which is based on the Solow model with human capital. Using regression analysis, they found significant results for age structure's effect on economic growth, inflation and savings. They base their study on cross-country analysis using data over five-year periods between the years of 1950 to 1990. They divide the population into five age groups, according to the classification of lifecycle theory and the age groups behavior in the economy: 0-14 year olds (young), 15-29 year olds (youth), 30-49 year olds (prime working), 50-64 year olds (middle-aged) and 65 year olds and above (old). They found a strong positive correlation between middle-aged people and economic growth, and a strong negative correlation between old aged people and economic growth.

Thus far, the mentioned studies have used cross-sectional analysis at one point in time, or cross-country panel data analysis. More recently, Andersson (2001) studied the age structure's effect on economic growth using time series analysis. He studied the effect for the Scandinavian countries on a yearly basis between 1950-1992. Because of the data being measured yearly, he includes additional variables to come to terms with the problem of otherwise measuring the business cycles. He classified the age groups the same as Lindh and Malmberg (1999) and found that the middle-aged population share had a positive impact on the economic growth.

This paper will rely on the previously mentioned studies, in particular following the model of Lindh and Malmberg (1999) for the theoretical framework. However, no earlier study has been conducted to measure the impact of age structure on economic growth in Sri Lanka during the demographic transition, using time series analysis. This will be a contribution to the field in that it combines the model of Lindh and Malmberg (1999) and the model of Andersson (2001).

## 2.6 Hypothesis

This paper strives to investigate how and if age structure affects the economic growth in Sri Lanka. The hypothesis is that the elderly (age 65+) has a negative impact on the economic growth since they no longer contribute to productivity but instead live off savings or are supported by society. Since the age groups are measured as shares of total population, it should implicate that an increase in the age share "*old*" leads to lower growth in GDP per capita. The age group "*prime*" is assumed to have a positive impact on economic growth since they contribute to the labor force and to productivity. The paper also wishes to investigate whether the assumption of the Solow model stands for Sri Lanka; that higher population growth has a negative impact on economic growth.

# 3. THEORETICAL FRAMEWORK

As previously mentioned, this paper is based upon the neoclassical economic growth model created by Solow in 1956. The Solow model has been modified to include variables for age structure, in large following the model of Lindh and Malmberg (1999) in regards to the division of age shares.

## 3.1 The Solow model

The Solow model exists in more or less extended versions. According to the Solow model with human capital, the economic growth for a country depends upon savings, population and the proximity to steady state i.e. when all the variables included in the model is in growth equilibrium (Jones, 2002). The further away a country is from its steady state, the faster growth they will experience. When it finally reaches its steady state, there is no change in physical capital per capita because the savings is equaled out by the depreciation rate of capital and population growth (Jones, 2002). A country with high savings rate should have a higher steady state, while high growth in population on the other hand decreases a country's steady state as more people share the existing resources (Jones, 2002). The Solow model with human capital has been extended to include technology in an attempt to explain the long run economic growth. Technological progress means the best technology available and it is assumed to grow at a constant, exogenously given rate. This means that the country can grow even in its steady state (Jones, 2002). In this model, physical capital per capita and technology grow at the same exogenous pace, and changes in savings rate or population growth will affect the GDP per capita, but not the long run growth (Jones, 2002). Aggregated human capital is another important factor in the Solow model with human capital. It is also important when determining age structure's effect on economic growth (Mankiw *et al.*, 1992). People accumulate the most work life experience in the early stages of the life cycle, and the human capital peaks when reaching the middle age. Therefore, the country's aggregated human capital should be large if they have more experienced people in working age. Human capital is assumed to produce better having undergone formal education and the empirical evidence show that the model fit reality very well, depicting that physical capital and human capital each contributed to a third of a country's economic growth. This model has later been further developed by several others for example Lindh and Malmberg (1999) and Andersson (2001).

## 3.2 The model of Lindh and Malmberg

In this section the model of Lindh and Malmberg (1999) will be presented as a theoretical background for the empirical test in later chapter (see Appendix 10.1 for a full mathematical derivation). The model relies on the solid micro economic foundation that an experienced labor force is more productive than an inexperienced labor force, as has been shown empirically in several studies (Lindh & Malmberg, 1999). The labor force is divided into age groups to represent the different levels of work experience. This way it is possible to measure the different age group's effect on the economic growth. Although this paper does not use the model exactly as intended by Lindh and Malmberg for the empirical testing[1], it is still gives good foundation of the theoretical background of how one can include age structure in the studying of economic growth.

Lindh and Malmberg use the following Cobb-Douglas index to divide the population into age groups:

$$N = \prod_i n_i^{a_i} \quad (3:1)$$

Where:

- $N$ is the Cobb-Douglas index
- $n_i$ is the age group's share of the total population
- $a_i$ is work life experience

The production function is also based upon a Cobb-Douglas with two inputs: physical capital, $K$ that is accumulated by savings and human capital, $L$ that grows exogenously through population growth (Jones 2002). Output per worker, $y$ can be written as:

$$y = Ak^{\alpha}(hN)^{\beta} \quad (3:2)$$

Where:

- $0<\alpha<1$, $0<\beta<1$ and $0<\alpha+\beta<1$
- $\alpha$ is the GDP share paid to physical capital
- $\beta$ is the GDP share paid to human capital
- $A$ denotes technology level
- $k$ is physical capital per worker
- $h$ is educational capital per worker

[1]The regression model in this paper, as opposed to the model of Lindh & Malmberg, has been extended with additional variables

The production function states that GDP per worker depends on the technology level in the country, multiplied by the physical capital and human capital. The human capital has been divided into age groups.

The physical- and human capital accumulation $\dot{k}$ and $\dot{h}$, is governed by the standard dynamic equation, taking the depreciation rates $\delta_k$ and $\delta_h$ and the savings rates $s_k$ and $s_h$ as exogenous:

$$\dot{k} = s_k y - (\delta_k + w)k \quad (3{:}\ 3)$$

$$\dot{h} = s_h y - (\delta_h + w)h \quad (3{:}\ 4)$$

Where;

- $s_k$ and $s_h$ are investments in physical and human capital
- $\delta_k$ and $\delta_h$ are constant depreciation factors of physical and human capital
- $w$ is the exogenous growth rate of the population

The growth in physical capital and human capital is the difference between the savings rate multiplied by GDP per worker, minus the depreciation rate and the population growth. One of the basic assumptions of the Solow model with human capital is that the depreciation rates are the same for the physical- and human capital. Lindh and Malmberg also assume that the savings rates are the same for the two; $\delta=\delta_k=\delta_h$ and $s=s_k=s_h$

With these assumptions, we arrive at the expressions for the steady state growth in physical- and human capital, which looks exactly the same:

$$k^* = \left(\frac{s}{(\delta+w)} A * N^{\beta}\right)^{\frac{1}{(1-\alpha-\beta)}} \quad (3{:}\ 5)$$

$$h^* = \left(\frac{s}{(\delta+w)} A * N^{\beta}\right)^{\frac{1}{(1-\alpha-\beta)}} \quad (3{:}\ 6)$$

The steady state growth for physical capital (and human capital) depends on the savings rate divided by the depreciation rate and population growth, multiplied by the technology level of the country and the age groups share of the population.

Lindh and Malmberg also make the assumption that the technology level of a country is converging towards the world technology i.e. the best available technology that is known. This is slightly different from the Solow model, where technology is the same for all countries.

$$\dot{A} = \gamma(A^* - A) \tag{3: 7}$$

Where;

- $\gamma$ is adaptation rate
- $A^*$ is world technology
- $A$ is technology level in the country

Rewrite the steady state for the growth of GDP per worker $ln\ y^* - ln\ y$ by using $k^*$ and $h^*$ (3:5; 3: 6):

$$\ln y^* - \ln y = \frac{1}{1-\alpha-\beta}\ln A * \frac{\alpha+\beta}{1+\alpha+\beta}\ln\frac{s}{\delta+w} + \frac{\beta}{1-\alpha-\beta}\ln N - \ln y \tag{3: 8}$$

By using the equations for technology (3: 7) and the equations for physical capital accumulation and human capital accumulation (3: 3, 3: 4), we can compute and simplify (for a full derivation, see Appendix 10.1) and arrive at the equation for the transitional growth rate of GDP per worker towards steady state, which is:

$$g_y = \frac{\delta lny}{\delta t} = \lambda(lny^* - lny) + u \tag{3: 9}$$

$$\text{and } \lambda = \tilde{\gamma}(\delta + w)(1 - \alpha - \beta) \tag{3: 10}$$

Where;

- $u$ is an error term
- $\lambda$ is proportional to $\tilde{\gamma}$

The transitional growth rate for an economy depends on the technological adjustment rate multiplied by the depreciation rate and the population growth.

When you insert the values for steady state stocks and divide by $\Gamma = \tilde{\gamma}(\delta + w)$, you get the basic growth equation, which can be written as:

$$\frac{g}{\Gamma} = ln\ A^*(\alpha+\beta)\ ((ln\ s - ln\ (\delta + w)) - (1-\alpha-\beta)\ ln\ y\ +\ \beta\ ln\ N\ +\ \frac{u}{\Gamma} \tag{3: 11}$$

The interpretation of this equation is that if there is no change in age structure, saving rate, depreciation, work force growth or potential technology level, the economy will eventually come so close to steady state that there will be no growth. Although, if any of the parameters should change it would also shift the steady state income, as well as the transitional growth rate.

Variations in age structure will also imply variations in the short-run growth rate (Lindh & Malmberg, 1999).

From equation (3:11) Lindh and Malmberg continue to derive the following expression that they use to test their empirical model, which consists of panel data from the OECD-countries with the average of five-year periods. The theoretical model of Lindh and Malmberg is also the basis for the empirical specification for the regression model used for the purpose of this paper. The empirical model thus far can be written as::

$$\frac{y_{g,t}}{\Gamma_t} = \beta_0 + \beta_1 \Delta inv_t + \beta_2 \ln y_{t-1} + \beta_3 \Delta(\delta + w_t) + \beta_4 \ln youth_t +$$

$$\beta_5 \ln prime_t + \beta_6 \, ln \, middle_t + \beta_7 \, ln \, old_t + \beta_8 \frac{1}{\Gamma_t} + \varepsilon_{it} \quad (3{:}\ 12)$$

The growth rate of GDP per worker depends on the growth rate in investments, initial GDP per worker, the growth rate of total population and the age variables *youth, prime, middle* and *old*. The measurements of these variables will be described in section 3.3. The inverse of the convergence term is added to take care of a non-zero mean in the approximation error, *u.* The technology gap, *ln A*, is estimated in the constant and cannot be separately identified.

However, Lindh and Malmberg do raise the concern that their empirical specification is too simple to explain the volatility of yearly data, since business cycles will be measured and impact the result, thus making it difficult to separate the effects.

Since this paper strives to investigate yearly data using time series analysis, the model of Andersson (2001) will be incorporated, adding additional variables to the regression model 3:12. Andersson bases his study upon the previous mentioned study by Mankiw, Romer and Weil from 1992. The additional variables are government expenditures, trade balance and inflation rate and they have been added as a way of "clearing out" the effect of the business cycles, and have been found significant in other papers that studies economic growth with regression analysis, for example Levine and Renelt (1992). Andersson states that the measurement of yearly data for individual countries makes it harder to detect any demographic impact because of the business cycles that are present in the short term, as opposed to its impact in the long run economy. This is a solid argument, but Andersson does not make any further attempts to catch the extensive dynamics of a business cycle since the task would be too great for a "single regression level"-analysis (Andersson, 2001, p.3). What he does find important to mention is the inclusion of savings rate instead of the usual variable specification of investment growth rate, which is due to lowering the risk of the data being non-stationary. Since Andersson does not make any other argument for the inclusion of the variables, neither will I, but simply follow his model.

## 3.3 The regression model

With the model of Lindh and Malmberg (1999) and the model of Andersson (2001) combined, we arrive at the regression model used in this paper to test the data empirically[2]:

$$\frac{y_{g,t}}{\Gamma_t} = \beta_0 + \beta_1 \Delta inv_t + \beta_2 \Delta govex_t + \beta_3 \Delta nfb_t + \beta_4 \pi_t + \beta_5 \ln y_{t-1} + \beta_6(\delta + w_t) + \beta_7 \, ln\, youth_t + \beta_8 \, ln\, prime_t + \beta_9 \, ln\, middle_t + \beta_{10} \, ln\, old_t + \beta_{11} \frac{1}{\Gamma_t} + \varepsilon_{it} \quad (3{:}\ 13)$$

Where:
- $y_{g,t}$ = growth rate in GDP per worker
- $\Gamma_t$ = time-specific convergence term
- $\Delta inv_t$ = growth rate in investments
- $\Delta govex_t$ = growth rate in government expenditures
- $\Delta nfb_t$ = growth rate in net foreign balance
- $\pi_t$ = inflation rate
- *$ln\ y_{t-1}$ = initial GDP per worker*
- $(\delta + w_t)$ = growth rate in total population
- $ln\ youth_t$ = age share of youths
- $ln\ prime_t$ = age share of prime working age
- $ln\ middle_t$ = age share of middle aged
- $ln\ old_t$ = age share of elderly

The growth rate of real GDP per worker divided by the time specific convergence term $\Gamma_t$, is influenced by *(i)* the growth rate of investments, measured as the growth rate of total investments divided by GDP, *(ii)* the growth rate of government expenditures divided by GDP, *(iii)* the growth rate of the net foreign balance, *(iv)* the inflation rate, *(v)* the initial GDP per worker in the period, *(vi)* the growth rate of total population and a fixed rate of depreciation, $\delta$ (constant with a

[2] The statistical program used for the regression analysis is Stata, version 13.0.

stylized value of 0.03), *(vii)* the age share youth, measured as the population share of 15-24 year olds, *(viii)* the age share prime working, measured as the population share of 25-49 year olds, *(ix)* the age share middle-aged, measured as the population share of 50-64 year olds, *(x)* the age share old aged, measured as the population share of 65+ year olds.

## 4. METHODOLOGY

The variables used in the regression model are presented in Table 1, along variable description and the source of the data. As mentioned in section 3, the independent variables government expenditure, trade balance and inflation stems from the model of Andersson, which will be explained in this section.

The data used in the regression model is measured on a yearly basis, and the time period ranges from 1960 to 2014. The specific time period is limited by availability of data for the consumer price index (CPI henceforth) in Sri Lanka. When studying economic growth, it is important to have data for a long enough period to be able to see any results, according to macroeconomic theory (Hansson, 2006).

The economic variables for Sri Lanka are all in constant national prices (LKR). For the exception of the technological adjustment rate, no computation was done since the focus of this paper is to measure growth rates, and not absolute values. To calculate the technological adjustment rate, real GDP per capita in Sri Lanka has been transformed into USD with the exchange rate of 2011 year's value. When presented in fixed prices, the variables have been cleaned from price changes due to inflation and the real change in the variable can be observed. The computation is done by dividing the real GDP per capita in LKR with the exchange rate (LKR/USD) for the year of 2011.

### 4.1 Technological adaptation for the world technology

As previously stated, the technology in this model grows at an exogenous rate and converges toward the world leading technology, i.e. the best technology known. Technology consists of ideas, which are transferable so although the cost of innovation is great, it is only the technological leading countries that need to make the initial investment in research and development. When the ideas are out in the market, the technology is free to use for other countries without any larger cost, although it takes time to implement. That is why Sri Lanka's economy will gradually converge towards the world technology, an assumption also made by Lindh and Malmberg. Since there is no general measurement for world technology, the US GDP per capita will serve as a proxy for the world technology since the assumption is made that they are the leaders in the field due to their possibilities of developing new technology. Hence, the rate of technological adjustment is the difference between Sri Lanka's GDP per capita and the US GDP per capital, multiplied with a proportional constant which will be given the value of 1 (Lindh & Malmberg, 1999). The technological adaptation rate is written as follows:

$$\dot{A} = \gamma(GDP_{US} - GDP_{Sri\ Lanka}) \quad (4{:}\ 1)$$

## 4.2 Economic variables for Sri Lanka

The dependent variable in the regression i.e. the economic growth for Sri Lanka will be measured as growth in real GDP per worker, divided by the time specific convergence term. The real GDP per worker has been calculated by natural logarithms and differentiating as follows:

$$y_{g,t} = \ln GDP_t - \ln GDP_{t-1} \quad (4{:}\ 2)$$

### *4.2.1 Investments*

Investments are a part of the output side of GDP and here it serves as a basic component in the Solow model. Investments are positive for the economic growth since it attributes to the capital stock (Jones, 2002). In this paper, the approach of Lindh and Malmberg will be followed in regards to investments. They state that savings rate and investment rate do not have to be equally large because foreign investors can invest in the country in question, and vice versa. However, they continue to state that the difference is miniscule, which has been shown in previous studies (Lindh & Malmberg, 1999). The growth rate of the investment share has been calculated by dividing investments with GDP, and then taking natural logarithms and differentiating as follows:

$$Investment\ rate = \frac{Real\ investments}{Real\ GDP}$$

$$\Delta inv_t = \ln inv_t - \ln inv_{t-1} \quad (4{:}\ 3)$$

#### *4.2.2 Government expenditure*

As part of the output side of GDP, government expenditure is included in the model to catch the effects of business cycles, following the model of Andersson. However, previous studies generally find mixed empirical evidence of the relationship between government expenditure and economic growth. Wu, Tang and Lin recently showed in a study involving 182 countries that government spending is helpful to economic growth regardless of the size of the government or the economic growth (Wu, Tang & Lin, 2010). Others (e.g. Barro, 1991) state, on the other hand state that government expenditure disrupts the economy and decrease investment, which in turns affects economic growth in a negative manner. The growth rate of government expenditure share has been calculated by taking natural logarithms, and then differentiating as follows:

$$Government\ expenditures = \frac{Real\ government\ expenditures}{Real\ GDP}$$

$$\Delta govex_t = ln\ govex_t - ln\ govex_{t-1} \qquad (4:4)$$

#### *4.2.3 Trade balance*

Net foreign balance (export minus imports) is a part of the expenditure side of GDP and is included to catch the effects of business cycles in the regression, following the model of Andersson. A positive net export should indicate a positive effect on economic growth since more export is commonly known as good for an economy and serves as an indication that it works well. The growth rate of net foreign balance has been calculated in a slightly different manner than the other economic variables, due to the fact that Sri Lanka's trade balance have been negative for almost every year in question. Since it is not possible to take the natural logarithm of a negative number, the computation has been made in the following way:

$$\Delta nfb_t = \frac{(trb_t - tb_{t-1})}{trb_{t-1}} \qquad (4:5)$$

#### *4.2.4 Inflation*

In theory, inflation may affect an economy in several ways. High inflation can lead to uncertainty about the future profitability of investment because of the varying price, which in turn can lead to more restrictive strategies when investing. It also has an impact on the trade balance since the exports are made relatively more expensive and therefore reduces a country's competitiveness and reducing economic growth. The inflation rate has been calculated by taking the natural logarithm of CPI for Sri Lanka, and then differentiating as follows:

$$\pi_t = \ln CPI_t - \ln CPI_{t-1} \qquad (4{:}\ 6)$$

## 4.3 Demographic variables for Sri Lanka

Population growth, which is one of the assumptions of the basic Solow model, states that higher growth in population is negative for the economic growth since both physical and human capital has to be divided amongst more people, thus lowering productivity and capital per capita (Jones 2002). The growth rate of population growth has been calculated by taking the natural logarithm, and then differentiating as follows:

$$(\delta + w_t) = \ln(\delta + w_t) - \ln(\delta + w_{t-1}) \qquad (4{:}\ 7)$$

### *4.3.1 Age structure*

Sri Lanka's population is divided into five age groups, roughly following the model of Lindh and Malmberg[3]. The different groups are chosen to roughly reflect the stages of a life cycle. The first group includes people between the ages of 0-14. They are supported by their parents and do not contribute to the economic growth in large, but is rather negative for the economic growth since their spending exceeds their income. This group will be called *young* henceforth. The second age group consists of people between the ages of 15-24. They are also assumed not to work but rather go to university and therefore they do not contribute to the economic growth in a positive manner either. They acquire education and still need to be supported by their parents and have no savings of their own since they do not make any money. This group will be called *youth* henceforth. The third group consists of the working age population, ranging from 25−49 years of age, thus consisting of younger adults that work. They will acquire working experience throughout their working life and since they work and can afford to save, they are assumed to affect the economy in a positive manner. This group of younger adults will be called *prime* henceforth. The fourth age group consists of older adults between the ages of 50-64, and they also make up the working age population. This group has gathered a lot of working experience and savings. They are assumed to have a family to support but still have a positive impact on the economy. The older

[3] In this paper, the age groups *youth* and *prime* differ from Lindh and Malmberg's to better suit Sri Lanka's population.

adults will be called *middle* henceforth. The fifth and last group will consist of people that are 65 years of age, and above. By now they are assumed to have left the labor market and will live off of savings and be supported by the society. In the model they will be addressed as *old*.

In the regression, each age group's share will be an independent variable and they have all been computed by dividing the number of people in each age group by total population, and then taking the natural logarithm as follows:

$$\ln(age\ share_t) = ln\ (\frac{number\ of\ people\ in\ age\ group_t}{total\ population_t}) \qquad (4:8)$$

The following table summarizes and describes the variables[4]:

**Table 1: Variable description table**

| Variable | Variable name | Variable description | Source |
|---|---|---|---|
| Growth in GDP per worker | $y_{g,t}$ | Growth rate of GDP per worker, at constant 2011 LKR | PWT, 9.0, NA |
| Growth in investments | $\Delta inv_t$ | Growth rate of investment share of GDP, at constant 2011 LKR | PWT, 9.0, NA |
| Growth in government expenditure | $\Delta govex_t$ | Growth rate of government expenditure share of GDP, at constant 2011 LKR | PWT, 9.0, NA |
| Growth in net foreign balance | $\Delta nfb_t$ | Growth rate of net foreign balance, at constant 2011 LKR | PWT, 9.0, NA |
| Inflation rate | $\pi_t$ | Inflation rate in SL | WDI |
| Initial GDP | $ln\ y_{t-1}$ | Initial GDP per worker | PWT, 9.0, NA |
| Population growth | $(\delta+w_t)$ | Growth rate of total population | UN |
| Age share young | $ln\ young_t$ | Share of total population aged 0-14 | UN |
| Age share youth | $ln\ youth_t$ | Share of total population aged 15-24 | UN |
| Age share prime working | $ln\ prime_t$ | Share of total population aged 25-49 | UN |
| Age share middle | $ln\ middle_t$ | Share of total population aged 50-64 | UN |
| Age share old | $ln\ old_t$ | Share of total population aged 65 + | UN |
| Technology convergence term | $\frac{1}{\Gamma_t}$ | The technological adaptation rate to world technology | PWT, 9.0, NA |

*Note: SL means Sri Lanka, CPI means Consumer price Index, PWT means Penn World Tables, NA means National Accounts, UN means United Nations, WDI means World Development Indicators*

[4] For source codes, see Appendix 8.3

## 4.4 Tests conducted on the regression

Before the results are presented, it seems suitable to present the series of tests conducted to make certain the model does not suffer from any of the common problems when dealing with regression analysis. The Wald test is important in this case to establish the significance of the age structure variables. The test result shows join significance for the age share variables, which means that the hypothesis that age structure has an effect on the economic growth is not rejected by the data (see Appendix 8.2.5). Further, when dealing with time series analysis, the independent variable is often constant over time, as for example GDP per capita, which often leads to a problem with non-stationary variables. This is not a problem in this case because the regression model is already differentiated, which is otherwise the common treatment for non-stationarity (Westerlund, 2005).

#### *4.4.1 Normality test*

The first test is performed to see if the residuals are normally distributed. Should this not be the case, you cannot go on to create a confidence interval, nor make inference or draw any conclusions or hypothesis testing on the parameters of the model (Westerlund, 2005). The regression in this paper has normally distributed residuals, which is shown in a histogram (see Appendix, 8.2.1).

#### *4.4.2 Multicollinearity*

The second test is important if you estimate a model with more than one independent variable because of the risk that they depend on each other in a systematic manner i.e. they are collinear. If the variables are collinear, the effects of the individual parameters are difficult to separate. One way to detect multicollinearity is to study the correlation between the independent variables. If the correlation is above 0.8 the variables are too highly correlated and one should take affirmative action (Westerlund, 2005). The result is that the regression model suffers from multicollinearity in the variables that include population or age structure (see Appendix, 8.2.2). The most common way to correct collinearity is to remove variables, which has been done by removing the age group *young*. However, collinearity remains, and in this case the variables causing the collinearity are the very variables that we intend to study. Therefore, removing them is not an option and the reader should be aware of collinearity in the regression.

#### *4.4.3 Heteroscedasticity*

The third test to conduct also concerns the residuals. One of the problems in the case of heteroscedasticity is that the estimator no longer has the lowest variance, and therefore is no longer the best estimator. The second problem is that we will make inference on a false variance covariance matrix, which means that we are not able to calculate confidence interval and test hypotheses (Westerlund, 2005). Heteroscedasticity was detected when running the regression with the residuals as dependent variable and the independent variables in regression (3:13) as independent variables. The heteroscedasticity was corrected with White's robust standards errors (see Appendix, 8.2.3).

#### *4.4.4 Autocorrelation*

The fourth and last test conducted is to make sure that the residuals are not auto-correlated. The problem with autocorrelation is that observations are no longer independent, but have a covariance different from zero (Westerlund, 2005). This problem is particularly common when dealing with time series analysis because the observations are chronological over time. The problems arising with autocorrelation are the same as in the case with heteroscedasticity. That is, the estimator used no longer has the lowest variance, and therefore is no longer the best estimator. And we will make inference on a false variance covariance matrix, which means that we are not able to calculate confidence interval and test hypotheses. A regression was run with $\hat{e}$ as dependent variable and $\hat{e}$ *lag (1)* along with the independent variables in regression (3:13) as independent variables. The result shows that the residuals do not suffer from autocorrelation (see Appendix, 8.2.4).

## 5. EMPIRICAL FINDINGS

As previously stated, the variable *young* has been excluded from the regression in order to measure the other age group's effect on economic growth relative to the young, and to avoid high degrees of linear dependency amongst the age variables. Since the age groups are measured as shares of total population, the sum of them will always be equal to 1, causing the regression to suffer from perfect collinearity. As mentioned however, the regression still suffers from collinearity amongst the age share variables, with the exception of the age share *youth*.

**Table 2: Regression results, growth in GDP per worker divided by the time specific convergence term as dependent variable**

| Independent variables | Coefficients |
|---|---|
| Growth in investments | 0.10**<br>(0.04) |
| Growth in government expenditures | -0.10***<br>(0.05) |
| Growth in net foreign balance | 0.03**<br>(0.01) |
| Inflation rate | 0.00<br>(0.00) |
| Initial GDP per worker | -0.14**<br>(0.06) |
| Population growth | 0.42**<br>(0.16) |
| Age share young (0-14 year olds) | N.I |
| Age share youth (15-24 year olds) | 0.30<br>(0.20) |
| Age share prime (25−49 year olds) | 0.73*<br>(0.19) |
| Age share middle aged (50−64 year olds) | 0.01<br>(0.08) |
| Age share old (65+ year olds) | -0.00<br>(0.08) |
| Inverse technology convergence term $\left(\frac{1}{r}\right)$ | 5.38*<br>(1.93) |
| Constant | -0.95<br>(1.41) |
| Number of observations | 54 |
| $R^2$ | 0.5207 |

*Note: White's robust standard errors in parenthesis, N.I means not included,*
** p<0,01 %; ** p<0,05 %; *** p<0,1 %*

Table 2 contains the result of the regression. The relatively high $R^2$ suggest that the independent variables included can explain 52.07 % of the economic growth. Seven of the twelve variables show significant effect on the economic growth and these are growth in investments, the control variables government expenditures and net foreign balance, initial GDP per worker, population growth, the age share of prime working people and the technological adaptation rate.

The remaining six independent variables that do not show any significance are inflation rate, the age share youth, the age share middle, the age share old and the constant, which is known as the intercept of the regression.

## 6. DISCUSSION AND CONCLUSION

When looking at the significant non-age variables, we note that growth in investments show the expected positive sign, which is well in line with theoretical background of the Solow model, as well as the empirical findings of Lindh and Malmberg (1999). The control variable growth in government expenditure displays the expected negative sign, which is consistent with the results of Andersson (2001), where growth in government expenditure also showed a significantly negative effect for three of the four Scandinavian countries. The net foreign balance is generally considered having a positive effect on the economy and shows the expected positive result, which this is consistent with empirical findings for at least one of the Scandinavian countries in Andersson's study. Initial GDP per worker is commonly expected to show a negative sign because of convergence and this is in line with the result of Lindh and Malmberg (999). The inversed convergence term shows a significantly positive effect on the economy, which is expected. The further away from its steady state, the higher the technological adjustment rate will be. The estimated mean approximation error is significantly different from zero and the point estimate is 1.1 %, which seems reasonable since the mean of the error mainly should catch the growth in the US (see Appendix for details), which is also in line with the result of Lindh and Malmberg (1999). Population growth shows a significantly positive effect in Sri Lanka. The result is not surprising since opinions still differ widely in terms of population growth's effect on the economy, and no consensus has been reached whether it has a positive or negative effect. According to the Solow model population growth affects the economy in a negative manner since, the more people that need to share the resources, the smaller the portion of resource for each individual will be. However, the positive sign in the case of Sri Lanka can be explained by the high literacy in the country and free education up to the age of 15, thus leading to a greater proportion of educated workforce when entering the working age.

In regards to the age structure variables, the only age variable that shows a significantly positive effect on the economic growth is the age share *prime* working. This is an expected result, according to theoretical and empirical evidence and it is also in accordance with the hypothesis. The significant result of this age group means that people in prime working age have a significantly different effect on the economic growth than the excluded age share *young*. The age shares *youth* and *middle* do not have a significant effect on the economy, but the variables do show the expected positive sign since both age groups are expected to have a more positive impact on the economy than *young*.

According to the hypothesis, the age share *old* would have a negative effect on the economy. Although not significant, it does show the expected negative sign, which is consistent with previous studies such as the findings of Lindh and Malmberg (1999). They found a significantly negative effect on the economy for the same age group. One reason for the insignificant result in the case of Sri Lanka might be that the retirement age actually is at 62 years old in the formal sector. As mentioned however, many elderly continue to work well up in their seventies and eighties. Also, the informal sector is very large in Sri Lanka, which is in no way measured in the regression, thus not reflecting the true value of elderly as a dependent age group. Another reason might be that the increase in the age share elderly has not been large enough the last decades to see any effect yet, since Sri Lanka is just about to transition into the last stage of the demographic transition model. It is likely that the increase in elderly will show more significant effect in the near future, as the Sri Lankan population continues to grow older.

### *Conclusion*

The objective of the paper was to investigate how changes in the age structure affects the economic growth in Sri Lanka, mainly focusing on whether the elderly have a negative impact on the economy, considering Sri Lanka's ageing population.

A time series analysis approach has been used to answer the stated question and the result from the regression differ somewhat from the expected. Results show that the only age group that has a significant effect on the economy in Sri Lanka is a change in the prime working age group. This result however, is in accordance with many others in the field (e.g. Bloom & Williams, 1997;1998, Bloom et al., 2001). However, the result differs from Lindh and Malmberg (1999) in regards to the elderly population, where the age group does show a significantly negative effect, whereas the result of this paper show that the elderly do not have a significant effect on the Sri Lankan economy, which might be due to the early stage in the demographic transition.

Based on the results of the regression model, it is impossible to say what effect the elderly have for the Sri Lankan economy. In my opinion however, it is important to continue to map and investigate the demographic changes. My suggestion is to implement adequate policies to increase the official retirement age so that people might work longer. Another very important focus is to work on women empowerment, so that more women may join the work force. Lastly, I suggest looking at other countries further ahead in the demographic transition model in order to develop strategies to mitigate the negative effects of a continuously growing dependency ratio for the elderly in the future.

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# 8. APPENDIX

## 8.1 Derivation of the extended Solow model by Lindh and Malmberg

GDP per capita, $y$ is written with following production function:

$$y = Ak^{\alpha}(hN)^{\beta}; \tag{A1: 1}$$

where:
- $0<\alpha<1$, $0<\beta<1$ and $0<\alpha+\beta<1$
- $\alpha$ is the GDP share paid to capital
- $\beta$ is the GDP share paid to human capital
- $A$ denotes technology level
- $k$ is physical capital per worker
- $h$ is educational capital per worker

The production function states that GDP per capita depends on the technology level in the country multiplied by the physical capital and human capital, which have been divided into age groups.

In the model, the population of a country is divided into different age groups in a Cobb-Douglas index:

$$N = \prod_i n_i^{a_i} \tag{A1: 2}$$

where;
- $N$ is the Cobb-Douglas index
- $n_i$ is the age group's share of the total population
- $a_i$ is work life experience

The physical capital accumulation and human capital accumulation are given exogenously:

$$\dot{k} = s_k y - (\delta_k + w)k \tag{A1: 3}$$

$$\dot{h} = s_h y - (\delta_h + w)h \tag{A1: 4}$$

where:
- $s_k$ and $s_h$ is investments in real and human capital
- $\delta_k$ and $\delta_h$ is depreciation rate of real and human capital
- $w$ is population growth

The change in physical capital and human capital is the difference between the savings rate multiplied by GDP minus the depreciation rate and the population growth.

Basic assumptions of the Solow model are that the depreciation rates are the same for physical capital and human capital. Also, the savings rates are the same:

$$\delta = \delta_k = \delta_h \quad \text{(A1: 5)}$$

$$s = s_k = s_h \quad \text{(A1: 6)}$$

Another assumption of the model is that the technology of the country is converging towards the world technology:

$$\dot{A} = \gamma(A^* - A) \quad \text{(A1: 7)}$$

where:

- $\gamma$ is adaptation rate
- $A^*$ is world technology and
- $A$ is technology level in the country

To get the steady state levels for physical capital and human capital, the following computations are made:

In steady state $\dot{k} = \dot{h} = 0$, which leads to:

$$s_k y = (\delta_k + w)k \text{ and} \quad \text{(A1: 8)}$$

$$s_h s y = (\delta_h + w)h \quad \text{(A1: 9)}$$

In the next step, only the computation for physical capital will be shown since the computation for human capital looks exactly the same due to the assumptions of the model.

So, put (A1: 8) into the production function (A1: 1) and get:

$$k = \frac{s}{\delta + w} * (Ak^{\alpha}(hN))^{\beta} \quad \text{(A1: 10)}$$

Rewrite the expression:

$$k = \left(\frac{s}{\delta + w} * A(hN)^{\beta}\right)^{\frac{1}{1-\alpha}} \quad \text{(A1: 11)}$$

$$h = \left(\frac{s}{\delta + w} * A * k^{\alpha} N^{\beta}\right)^{\frac{1}{1-\beta}} \quad \text{(A1: 12)}$$

Put (A1: 12) into (A1: 11) and solve for *k*:

$$k = \left(\frac{s}{\delta+w} * A\left(\frac{s}{\delta+w} * A * k^{\alpha} N^{\beta}\right)^{\frac{1}{1-\beta}\beta} N^{\beta}\right)^{\frac{1}{1-\alpha}} \quad \text{(A1: 13)}$$

By simplifying and rewriting the expression, we get the steady state levels for both physical capital and human capital:

$$k^{*} = \left(\frac{s}{\delta+w} * A * N^{\beta}\right)^{\frac{1}{1-\alpha-\beta}} \quad \text{(A1: 14)}$$

$$h^{*} = \left(\frac{s}{\delta+w} * A * N^{\beta}\right)^{\frac{1}{1-\alpha-\beta}} \quad \text{(A1: 15)}$$

The steady state growth for physical capital (and human capital) depends on the savings rate divided by the depreciation rate and population growth, multiplied by the technology level of the country and the age groups share of the population.

Rewrite the steady state for the growth of GDP per worker minus growth for GDP per worker, $ln\ y^{*} - ln\ y$ by using $k^{*}$ (A1: 14) and $h^{*}$ (A1: 15):

$$ln\ y^{*} - ln\ y = \frac{1}{1-\alpha-\beta} lnA * \frac{\alpha+\beta}{1+\alpha+\beta} ln\frac{s}{\delta+w} + \frac{\beta}{1-\alpha-\beta} lnN - ln\ y \quad \text{(A1: 16)}$$

By using the equations for technology (A1:7) and the equations for physical capital accumulation and human capital accumulation (A1:8, A1:9) and (A1:10), we compute and simplify, and get:

$$ln\ y^{*} - ln\ y = \frac{1}{1-\alpha-\beta} * \left[\frac{ln\left(\left(A+\frac{\dot{A}}{\gamma}\right)\right)}{\frac{\dot{A}}{\gamma}} * \frac{\dot{A}}{\gamma}(\alpha-\beta)\frac{ln\left(k+\frac{\dot{k}}{\delta+w}\right)}{\frac{\dot{k}}{\delta+w}} * \frac{\dot{k}}{\delta+w}\right] \quad \text{(A1: 17)}$$

Simplify with the definition of logarithmic derivation:

$$ln\ y^{*} - ln\ y = \frac{1}{(1-\alpha-\beta)\gamma(\delta+w)} * \left[(\delta+w)\frac{dlnA}{dt} + \gamma(\alpha-\beta)\frac{dlnk}{dt}\right] \quad \text{(A1: 18)}$$

The expression within the square brackets is the directional time derivative of *y*. Lindh and Malmberg mean that a dimension of *h* could be added and then decompose physical and human capital. The reason for not doing so is that it only adds algebra and no content since they assumes that *N* is constant to the value of $N = \gamma\frac{dlny}{dt}$ when $(\delta+w)$ and $\gamma$ have the same value, which might seem strange and drastic. However, consider $\lambda = (1-\alpha-\beta)\ \gamma\ (\delta+w)$ and you get:

$$\frac{d\ln y}{dt} \approx \lambda(\ln y^{*} - \ln y) * D \qquad \text{(A1: 19)}$$

$\frac{d\ln y}{dt}$ is the growth rate of GDP per capita when the economy is transitioning into its steady state. $D$ is a factor of the same quantity as $(\delta+w)$ and $\gamma$. $D$ will however be ignored since $\gamma$ is measured in terms of the gap in GDP per capita, which is assumed to be proportional to $\gamma$.

To conclude, this leads to the growth rate of the economy when transitioning to its steady state:

$$g_y = \frac{d\ln y}{dt} = \lambda(\ln y^{*} - \ln y) + u \qquad \text{(A1: 20)}$$

where:
- u is an error term
- $\tilde{\gamma}$ is assumed to proportional to $\gamma$

$$\lambda = \tilde{\gamma}(\delta+w)(1-\alpha-\beta) \qquad \text{(A1: 21)}$$

Equation (A1: 20) states that the transitional growth rate for an economy depends on the technological adjustment rate multiplied by the depreciation rate and the population growth.

Insert values for steady state stocks and divide by $\Gamma = \tilde{\gamma}(\delta+w)$, and you get the basic equation for economic growth:

$$\frac{g}{\Gamma} = \ln A^{*}(\alpha+\beta)\ ((\ln s - \ln(\delta+w)) - (1-\alpha-\beta)\ \ln y + \beta \ln N + \frac{u}{\Gamma} \qquad \text{(A1: 22)}$$

Since the GDP of the United States is used as a proxy for the world technology, the error term $u$ in equation (A1: 22), really is written as $u = \tilde{u} + g_{US}$, where $\tilde{u}$ is an approximation of the error term, that is assumed to be distributed with an average of 0 over the observations: $E(u) = E(g_{US})$

This means that the estimation of $\lambda$ is:

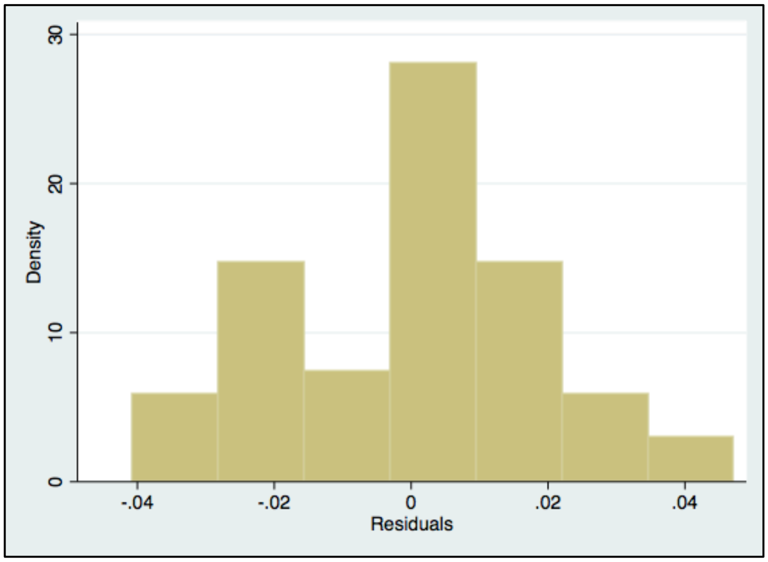
$$\lambda = (1-\alpha-\beta)[E(gUS) + E(\Gamma\pi)] = (1-\alpha-\beta)\ \beta_5 - \beta_3\Gamma \qquad \text{(A1: 23)}$$

The equation (A1: 22) is the economic growth equation that Lindh and Malmberg rewrite in order to test empirically.

## 8.2 Test results

### *8.2.1 Normality test: histogram*

**Figure 7: Normality test**




**Result:** Normally distributed residuals.

### *8.2.2 Test for multicollinearity: correlation matrix*

**Table 3: Correlation matrix**

| | $y_{g,t}$ | $\Delta\ inv_t$ | $\Delta\ govex_t$ | $\Delta\ nfb_t$ | $\pi_t$ | $ln\ y_{t-1}$ | $(\delta+w_t)$ | $ln\ youth_t$ | $ln\ prime_t$ | $ln\ middle_t$ | $ln\ old_t$ | $\left(\frac{1}{\Gamma}\right)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y_{g,t}$ | 1.00 | | | | | | | | | | | |
| $\Delta\ inv_t$ | 0.19 | 1.00 | | | | | | | | | | |
| $\Delta\ govex_t$ | -0.26 | 0.03 | 1.00 | | | | | | | | | |
| $\Delta\ nfb_t$ | 0.09 | -0.23 | -0.16 | 1.00 | | | | | | | | |
| $\pi_t$ | 0.28 | -0.09 | 0.15 | -0.13 | 1.00 | | | | | | | |
| $ln\ y_{t-1}$ | 0.15 | -0.06 | 0.13 | -0.18 | 0.76 | 1.00 | | | | | | |
| $(\delta+w)$ | -0.09 | 0.16 | -0.15 | 0.15 | -0.70 | -0.97 | 1.00 | | | | | |
| $ln\ youth_t$ | -0.33 | 0.07 | 0.02 | 0.05 | -0.48 | -0.36 | 0.24 | 1.00 | | | | |
| $ln\ prime_t$ | 0.15 | -0.26 | 0.15 | -0.17 | 0.75 | 0.95 | -0.98 | -0.29 | 1.00 | | | |
| $ln\ middle_t$ | 0.22 | -0.06 | 0.11 | -0.19 | 0.72 | 0.96 | -0.90 | -0.54 | 0.90 | 1.00 | | |
| $ln\ old_t$ | 0.18 | -0.14 | 0.13 | -0.16 | 0.75 | 0.98 | -0.97 | -0.40 | 0.97 | 0.94 | 1.00 | |
| $\left(\frac{1}{\Gamma}\right)$ | 0.28 | -0.02 | 0.05 | -0.15 | 0.71 | 0.88 | -0.79 | -0.71 | 0.78 | 0.93 | 0.87 | 1.00 |

Number of observations = 53

**Result:** the regression suffers from collinearity.

### *8.2.3 Test for heteroscedasticity: regression on residuals*

- H0: heteroscedasticity
- H1: homoscedasticity

**Table 4: Regression result, test for heteroscedasticity, $\hat{e}^2$ as dependent variable**

| **Independent variables** | **Coefficients** |
|---|---|
| Growth in investment rate | -0.00<br>(0.00) |
| Growth in government expenditures | 0.00<br>(0.00) |
| Growth in net foreign balance | -0.00<br>(0.00) |
| Inflation rate | -0.00<br>(0.00) |
| Initial GDP per worker | 2.40<br>(0.00) |
| Population growth | -0.01**<br>(9.72) |
| Change in age share young (0-14 year olds) | N.I |
| Change in age share youth (15-24 year olds) | -0.00<br>(0.00) |
| Change in age share prime (25−49 year olds) | -0.00<br>(0.00) |
| Change in age share middle aged (50−64 year olds) | 0.00<br>(0.00) |
| Change in age share old (65+ year olds) | -0.00<br>(0.00) |
| Inverse technology convergence term $\left(\frac{1}{r}\right)$ | -0.02<br>(0.10) |
| Constant | -0.02<br>(0.04) |
| Number of observations | 54 |

*Note: Standard errors in parenthesis, N.I means not included, * p<0,01 %; ** p<0,05 %; *** p<0,1 %*

**Result:** H0 cannot be rejected i.e. the residuals are heteroscedastic.

Treatment: correct the model with White's robust standard errors.

### *8.2.4 Test for autocorrelation*

- H0: no autocorrelation
- H1: autocorrelation

**Table 5: Regression result, test for autocorrelation, $\hat{e}$ as dependent variable**

| Independent variables | Coefficients |
|---|---|
| $\hat{e}$ lag (1) | 0.01<br>(0.17) |
| Growth in investments | 0.00<br>(0.03) |
| Growth in government expenditures | 0.00<br>(0.05) |
| Growth in net foreign balance | -0.00<br>(0.02) |
| Inflation rate | 0.00<br>(0.00) |
| Initial GDP per worker | -0.02<br>(0.09) |
| Population growth | -0.01<br>(0.14) |
| Change in age share young (0-14 year olds) | N.I |
| Change in age share youth (15-24 year olds) | 0.00<br>(0.22) |
| Change in age share prime (25−49 year olds) | -0.01<br>(0.19) |
| Change in age share middle aged (50−64 year olds) | 0.01<br>(0.10) |
| Change in age share old (65+ year olds) | 0.03<br>(0.11) |
| Inverse technology convergence term $\left(\frac{1}{r}\right)$ | 0.00<br>(2.49) |
| Constant | 0.33<br>(1.89) |
| Number of observations | 53 |

*Note: Standard errors in parenthesis, N.I means not included, * p<0,01 %; ** p<0,05 %; *** p<0,1 %*

**Result:** H0 cannot be rejected i.e. the residuals are not auto-correlated.

#### *8.2.5 Test for joint significance*

**Table 6: Wald test**

| | **Variable** |
|---|---|
| 1 | ln youth = 0 |
| 2 | ln prime = 0 |
| 3 | ln middle = 0 |
| 4 | ln old = 0 |
| F(4,42) | 4.77 |
| Prob > F | 0.0029 |

## 8.3 Variable description table

**Table 7: Variable description table, sources**

| **Variable name** | **Source codes** | **Source** |
|---|---|---|
| $y_{g,t}$ | empl | PWT, 9.0, NA |
| $\Delta inv_t$ | q_i | PWT, 9.0, NA |
| $\Delta govex_t$ | q_g | PWT, 9.0, NA |
| $\Delta nfb_t$ | q_x and q_m | PWT, 9.0, NA |
| $\pi_t$ | CPI SL | WDI |
| $ln\ y_{t-1}$ | q_gdp SL | PWT, 9.0, NA |
| $(\delta+w_t)$ | popSL | UN |
| $ln\ young_t$ | pop ages 0-14 | UN |
| $ln\ youth_t$ | pop ages 15-24 | UN |
| $ln\ prime_t$ | pop ages 25-49 | UN |
| $ln\ middle_t$ | pop ages 50-64 | UN |
| $ln\ old_t$ | pop ages 65+ | UN |
| $ln\left(\frac{1}{r_t}\right)$ | q_gdp US and q_gdp SL | PWT, 9.0, NA |

*Note: SL means Sri Lanka, CPI means Consumer price Index, PWT means Penn World Tables, NA means National Accounts, UN means United Nations, WDI means World Development Indicators*