Pricing fixed price electricity contracts in the Nordic region

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Abstract

In a competitive market like electricity retailing it is crucial for energy providers to estimate risk and cost for fixed price contracts in order to set competitive but not unprofitable prices. This report calculate three price components that covers the wholesale and retail costs. Profile price component is the expected electricity load to the expected spot prices in every hour in the delivery period. Correlation price component is added due to the fact that we do not know the realized outcome of load and spot price, but from historical data we see that they are highly correlated. The dependence structure between load and spot causes a higher expected cost. The third component is a risk premium that quantifies the risk that the retailer relieves from the customer.

One retailers portfolio load and electricity spot price in south Sweden, SE4, is modeled with autoregressive based models and simulated. Depending on a customer's risk exposure to the portfolio, this report also calculate the three price components for individual customers. The time series model approach was successful in capturing dependence in data. The results also shows that it is possible to set the three price components based on an individual customer's consumption behavior in an efficient way.

Keywords: Fixed price contract, Electricity load, Electricity spot price, vector autoregressive model, RAROC, Profile price, Correlation price, Volume risk

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Contents

1 Introduction

The purpose of this report is to calculate the underlying components in a fixed price electricity contract. Unlike most other commodities, electricity is not storable to a significant extent. This leads to complications that is reflected in electricity spot prices. Due to the extreme behavior of electricity spot price it is crucial for the retailer to understand the risks and costs of a fixed price contract. In a progressively more competitive business like electricity retailing, participants must have good estimates of future costs in order to offer fair prices to customers. When a customer wants to buy electricity at a fixed price, energy providers estimates the costs of having this customer in its portfolio. This cost depends on behavior of customer's load and the spot price. Energy providers will sell electricity to the customer at a fixed price agreed up on today. The electricity load consumed by the customer, which is unknown today, will be bought by retailer for an unknown price from the spot market at future time points. The fixed price will therefore depend on customers expected load profiles and the deviation from these as well as the expected spot prices and the deviation from these. A more detailed explanation is given in the method section. The report will also give suggestions of how to price contracts with volume limits, which is the case in real life, were the customer only pays fixed price if the consumed volume lies inside a certain interval from the prognosis. The project includes modeling of electricity spot price and the retailer's portfolio load at an hourly granularity in order to simulate scenarios and calculate price components. The modeling approach is based on additive models, which includes a deterministic part and a stochastic part. A similar approach was used in *Modeling electricity spot prices: combining reversion, spikes, and stochastic volatility* [Mayer, Schmid, Weber, 2015]. The modeling of stochastic spot price and stochastic portfolio load is done with three independent time series models, one autoregressive and two vector autoregressive processes. The dependence structure of data analyzed in this report is the key ingredient, which motivated the use of a time series approach which effectively uses lagged information between seasonal patterns and intra-day dependencies [Clements, Hurn, Li, 2015]. Based on the results of the portfolio's price components, this report will also provide a suggestion on how the cost can be allocated to individual customers. Contracts for three randomly selected customers and two fictional extreme customers will be priced and evaluated. The results indicates that a fixed price contract should not be priced only based on expected load and expected spot price. A risk premium and a correlation price component that arises from the stochastic movements are needed in order to set a fair price. For individual customers, these suggested price components can differ significantly.

1.1 Research questions

The two fundamental issues to solve is summarized in the sentences below.

- Is it possible to construct joint a model for electricity spot price and consumption load and use this model for pricing fixed price contracts and their underlying risk premium?
- Is there an efficient way of setting customer-specific risk premiums?

In order to answer the two main questions, the following questions must be clarified first.

- What characterizes spot prices for electricity?
- What characterizes consumption load for electricity?
- What is the relationship between consumption load and spot price ?
- How does the consumption load pattern impact the risk premium for fixed priced contracts?

1.2 Contribution statement

A large amount of research can be found on how to model electricity spot price and electricity load. However, research on how these models can be used in order to price fixed price contracts is sparse. In this thesis we use ideas from a wide range of different research reports and construct, to our knowledge, a new approach of pricing electricity fixed price contracts in the Swedish electricity market. We are not only modeling spot price and load, but also the dependence structure.

1.3 Disposition

This report follows the following disposition.

Section 2: This segment provides an introduction to the Nordic electricity market, both physical and financial. It discusses ways to hedge fixed price contracts. Also characteristics of spot price and load are examined.

Section 3: The approach of how to quantify the price components of a fixed price contract is explained in this section. It is derived from a profit and loss function, where the remaining risk after hedge is mathematically explained.

This chapter also explains how to allocate the portfolio price component into individual customer contracts.

Section 4: This section explains the models used in this report, how they are separated into different sub-models and what they aim to capture.

Section 5: In order to make this thesis easier to reconstruct, section 5 provides a more detailed explanation of how models were selected and parameters estimated.

Section 6: The purpose and approach of simulating outcomes for fixed price contracts are explained here. It also describes how to adjust the outcomes from P-measure to Q-measure.

Section 7: This section provides the results from simulating fixed price contracts. It shows the results from both portfolio load and individual customer load.

Section 8: Results are discussed in this section. Conclusions and recommendations are given. Also ways of improvements are suggested and discussed.

2 Nordic electricity market

In the year of 1996 the Swedish energy market was deregulated, which means that electricity prices went from being set by regulation to be set by market competition [Möller, Kahvedzic, 2008]. The following years, the Nordic market included Sweden and Norway. The Energy markets of Finland was deregulated in 1998 followed by Denmark in 1999/2000 [Kristiansen, 2004]. Nowadays the Nordic market is split into several different pricing areas for electricity. Sweden was divided into four different bidding areas in November 2011. The differences in price for the bidding areas is due to transmission capacity that may congest the electricity flow between areas [Nordpool, 2017]. Electricity is transmitted from areas with lower demand-to-supply ratio to areas with higher demand-to-supply ratio to make the spot prices less volatile. The financial energy market Nord Pool organizes two different markets for physical delivery of electricity. Elspot is a day-ahead auction where power is traded for delivery for each hour the next day. There is also an intraday market provided by Nord Pool called Elbas which makes it possible to balance the supply and demand up to one hour before delivery [Kristiansen, 2004]. The purely financial instruments are traded at NasdaqOMX. This report will further on focus on the day-ahead spot prices from Elspot in the bidding area of south Sweden, SE4. An electricity retailer's role in the market is to buy electricity from producers and sell to consumers. This can be done with floating price contracts or fixed price contracts.

Figure 1: Nord Pool bidding areas [Nordpool, 2017]

2.1 Production and consumption

Just like any other commodity in a non-regulated market, the price of electricity is determined by the equilibrium between supply and demand. In the electricity market this varies every given hour, which gives electricity spot price special characteristics. The demand curve is known to be inelastic compared to the supply curve, also known as the merit order curve. A small change in demand can result in a small change or an extreme change in spot price. This depends on the amount of available electricity at a particular time point. Because of its non-storability, the supply is very much impacted by factors such as water levels in the water reservoir and wind speed for wind power production [Lindström, Norén, Madsen, 2015]. The demand can vary from expected due to factors like unexpected temperature, which makes the energy production planning difficult for the producers. Both in Denmark and in Germany the spot price markets have experienced negative prices at several occasions the last few years. With increasing amount of renewable energy, one can expect it to be more difficult to plan energy production for upcoming years, which can result in extreme prices more frequently [Lindström, Norén, Madsen, 2015].

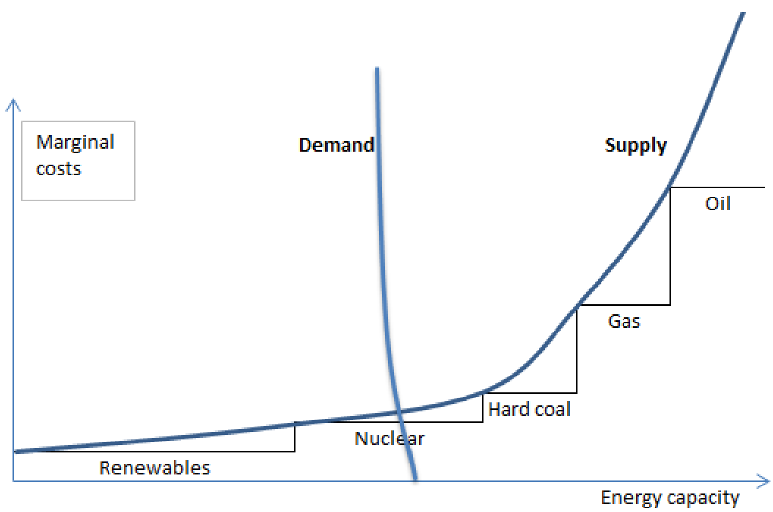

Figure 2: Merit order curve

An indication on dependence between spot price and load can be seen in Figure 3.

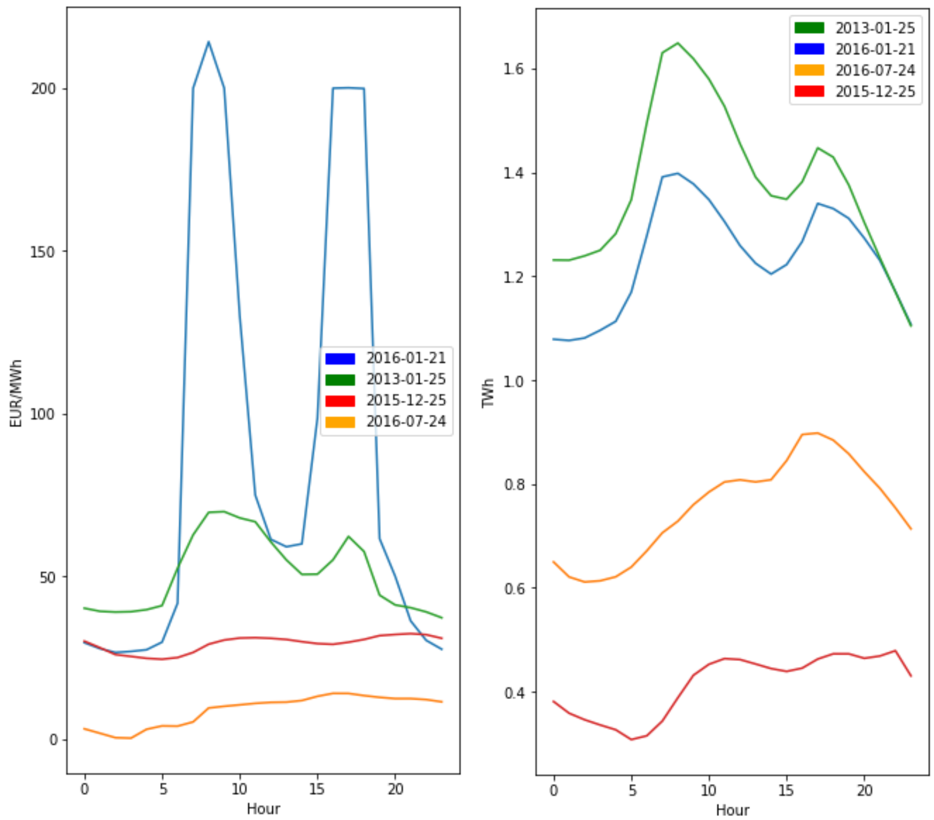

Figure 3: The plots show spot price (left) and portfolio load (right), for four different days. These days was chosen because they show maximum and minimum values for spot price and portfolio load.

2.1.1 Spot price characteristics

As mentioned before, Spot price for electricity has several special characteristics. From historical data one can see seasonal behavior in spot price, where the price in general is higher during winter than in the summer. One can also in general see higher prices when the demand is high, i.e in the middle of the day and lower prices when demand is low, i.e on weekends or during night time. One can also see that the spot price can be extremely volatile with time-varying volatility, were spikes and drops are known to be a difficult task to model. Another characteristic is the mean reverting behavior. Shortly after an extreme price jump, it tends to return back to a mean level. The mean reversion is generally much faster after a jump than after a small deviation [Mayer, Schmid, Weber, 2015]. Changes in spot price can be explained by a various number of factors. Important factors for the Nordic market is temperature, grid load and deviations from normal water levels in the water reservoir. Half of all electricity in Sweden are generated from hydro power, which stabilize the spot prices in this region[Bureger, Graeber, Schindlmayr, 2014]. In this report we consider the spot price in SE4 from 1 January 2013 to 31 December 2016.

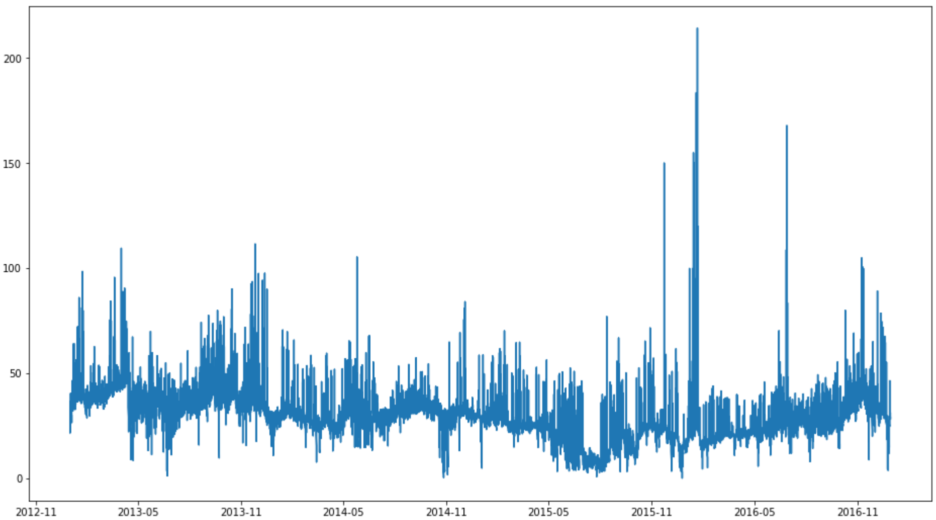

Figure 4: Four years of hourly spot price in EUR/MWh

During 2015, the spot price level was significantly lower than usual. If we look at the water reservoir level in Sweden for the same period one can see a clear elevated level in data for 2015, see Figure 5 and 6. This impacts not only spot price, but also as we will see later in this report, it impacts correlation between spot price and load.

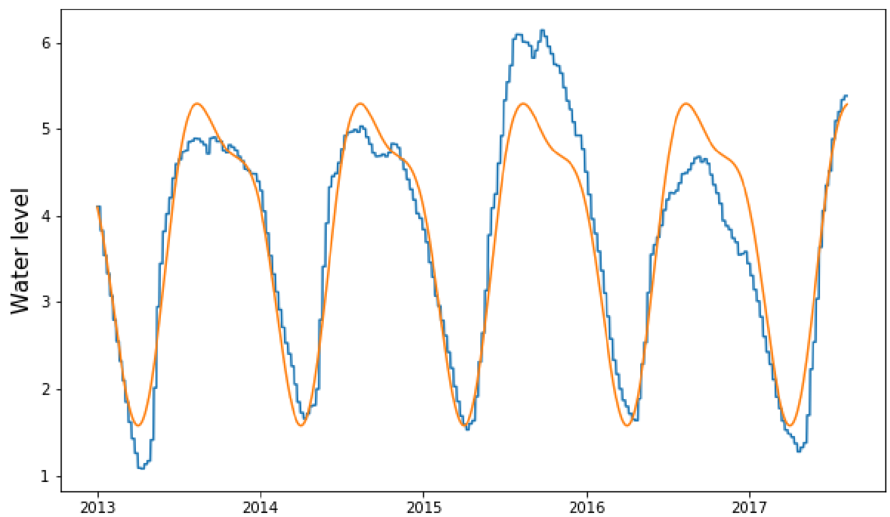

Figure 5: Water level in Swedish reservoirs in blue and a seasonal mean value in orange

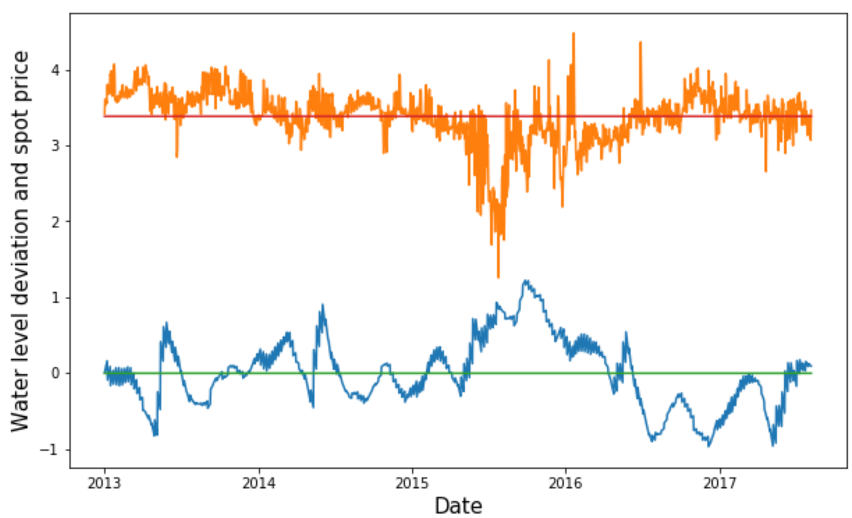

Figure 6: Water level deviation from expected in Swedish reservoirs in blue and a logarithmic spot price in orange

2.1.2 Load characteristics

Grid load often show strong seasonal behavior and repetitive patterns. In the Nordic market, the consumption is higher in the winter. Also consumption volume depends on the day of the week and hour of the day. Special pattern is often seen on holidays and bridge days. Load is also known to have structural changes and old load data could often be irrelevant. It is therefore important to identify structural changes in historical load in order to get a good model and forecast. When forecasting short time horizons it is also relevant to consider weather forecasts as an influencing factor[Bureger, Graeber, Schindlmayr, 2007]. The energy provider that is being analyzed in this report is a big market player in the Swedish electricity retail market. The portfolio load of fixed price customers shows similar characteristics as the grid load. In this report we consider the portfolio load in SE4 from 1 January 2013 to 1 December 2016.

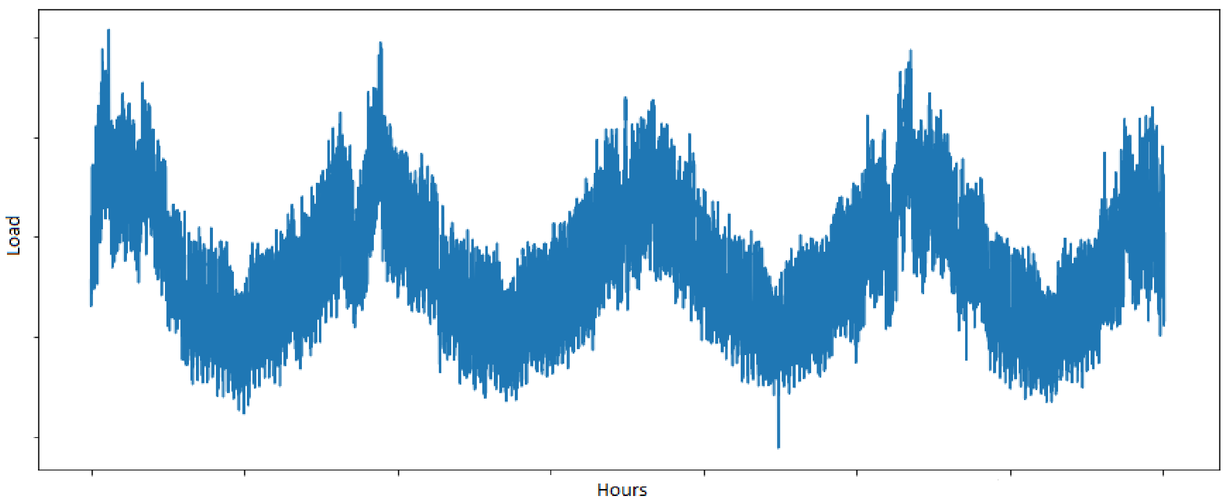

Figure 7: Hourly portfolio load from 1 Januari 2013 to 1 December 2016

2.2 Physical electricity retailing

The physical electricity trading in the Nordic region is done at Nord Pool. Market players that are interested in retailing electricity must send a purchase offer to Nord Pool before noon in order to deliver electricity for the hours the next day. The same deadline goes for producers who want to sell electricity for the following day. This creates a double auction, where the day-ahead prices are settled. It is called a double auction because not only buyers but also sellers have submitted orders [Houmoller, 2010]. As a secondary market, Nord Pool offers intra-day trading on Elbas. This market allows players to balance between supply and demand within the same day. One can expect the intra-day trading on Elbas to increase in the future with more variable renewable energy sources such as wind power [Scharff, Amelin, 2016]. In Sweden the customers of an energy provider can choose to buy electricity for a floating price or a fixed price. With floating price, the retailer will invoice the customer according to what the electricity price has been in the spot market every hour in the delivery period. With a fixed price contract, the customer knows what the electricity costs per MWh before consuming.

2.3 Financial electricity trading

Several factors distinguish electricity trading from other commodity trading. First of all, electricity is hard to store. One exception is hydro pumped storage power plants, whose capacity though is small in comparison to the total consumption. Also the trading of electricity is constrained by transmissions networks, which impede a global market. Because of the volatile behavior of spot prices, many producers, retailers and consumers want to secure their profit with futures or forward contracts. The delivery period for these kind of contracts could be a specific day, week, month or a year [Bureger, Graeber, Schindlmayr, 2014]. NasdaqOMX offers purely financial contracts on the Nordic system price. Futures and DS futures are flat baseload contracts that can be traded as options or futures. These contracts cover every hour in a delivery period and pay the difference between system spot price and forward price for every hour [Ernstsen, Broomsma, Tegnér, Skajaa, 2017]. The financial contracts that can be bought on NasdaqOMX are shown in Table 1.

	Granularity
Futures	
Nordic electricity	Day, Week, Month, Quarter, Year
EPAD*	Week, Month, Quarter, Year
DS Futures	
Nordic Electricity	Month, Quarter, Year
EPAD*	Month, Quarter, Year
Options	
Futures	Month, Quarter, Year
DS Futures	Quarter, Year
Minimum contract size	1 MW
*EPAD	Difference between Area price and Nordic System Price

Table 1: Financial contracts offered on NasdaqOMX

These financial instruments help participants in the physical market to manage their risks. Because of the fact that Futures and DS Futures takes the Nordic system spot price as the underlying asset, there are remaining risks concerning the area prices. The local pricing in the four areas can differ from system price, especially when consumption is high and the risk of congestion in the grid is high. In order to manage area risk, one can buy or sell Electricity Price Area Differentials (EPAD) [Energimarknadsinspektionen, 2016]. The buyer of an EPAD obligation will be payed the difference between system price and area price for a specific bidding area. If this difference is negative, the holder will be obligated to instead pay this difference. EPADs are also traded as options where the holder will not be obligated to pay if the difference is negative.

After hedging with these contracts there are still risks remaining. We can not hedge a customer's expected load profile within a day and we can not hedge against stochastic deviation from expected load. When a retailer makes a fixed price contract deal with a customer, they also buy a futures contract for the same delivery period as a hedge. As time passes by, the retailer buys contracts with finer granularity. At the time point when a fixed price is calculated, we know the cost of a futures contract for the same period. For this reason, in this report a flat futures contract will be set for the given period.

3 Method

As described in the previous section some risk remains after hedging. First of all a retailer must estimate the cost of a customer's load profile, which gives the profile price component. If the customer is expected to consume more electricity when the expected spot price is high, the profile price is higher. Apart from this, the unexpected movements of spot price and customer load must be considered. If load has a positive correlation with spot price, which is a common scenario, we expect a higher cost. A drop in load causes the retailer to sell the pre-bought electricity on the spot market for a lower price and an unexpected peak in load causes the retailer to buy extra electricity on the spot market for a higher price. The extra cost that arises from correlation between spot price and load will be covered by a *correlation price component*. The remaining uncertainties will be quantified and priced as a *risk premium*.

This report will take three pricing components, profile price, correlation price and risk premium in consideration. When the invoice reaches the customer, more components have been added to the price such as taxes and network grid fee, which will not be examined in this report. Here we will focus on the wholesale cost for the retailer and how this affects the price components just mentioned. In order to calculate these components we will set up autoregressive based models and simulate possible scenarios.

3.1 Price components derived from the P&L function

The following equation explains the profit and loss function for a fixed priced contract.

$$profit = P\sum_{t=1}^{n} L(t) - \sum_{t=1}^{n} F(0,t)\lambda - \sum_{t=1}^{n}(L(t) - \lambda)S(t) \tag{1}$$

For every hour in the contract, the retailer will sell a volume $L(t)$ at a fixed price P EUR/MWh. As a hedge, the retailer will buy a volume λ as close to $\hat{L(t)}$ as possible when the contract is signed. $\hat{L(t)}$ is the expected customer load and λ is bought at a forward price $F(0,t)$. There are several restrictions for λ in the market today. Eon can only hedge with base load contracts and at a quantity unit of 1 MWh. λ will therefore be a constant integer during the settlement period. The load that deviates from the hedged volumes must be bought from the spot market at a price of $S(t)$. $L(t)$ and $S(t)$ are unknown when the fixed price P is set.

In order to set a fixed price P, the approach is to first set a fair price, where the expected profit and loss is zero. The second step is to price the risk of loss with a risk premium. The profile price component describes the expected costs due to the fact that $\hat{L(t)}$ deviates from λ. The correlation price component describes the expected costs due to the fact that $L(t)$ deviates from $\hat{L(t)}$. This deviation is generally correlated with deviations in the spot price and the fixed price must take this in consideration. The price components are explained in the equations below, which are derived from the profit and loss function.

$$profit = P\sum_{t=1}^{n} L(t) - \sum_{t=1}^{n} F(0,t)\lambda - \sum_{t=1}^{n}(L(t) - \lambda + \hat{L(t)} - \hat{L(t)})S(t)$$

$$P\sum_{t=1}^{n} L(t) = \sum_{t=1}^{n} F(0,t)\lambda - \sum_{t=1}^{n} S(t)\lambda + \sum_{t=1}^{n} S(t)\hat{L(t)} + \sum_{t=1}^{n}(L(t) - \hat{L(t)})S(t) + profit$$

$$P = \frac{\sum_{t=1}^{n} F(0,t)\lambda + \sum_{t=1}^{n} S(t)(\hat{L(t)} - \lambda)}{\sum_{t=1}^{n} L(t)} + \frac{\sum_{t=1}^{n}(L(t) - \hat{L(t)})S(t)}{\sum_{t=1}^{n} L(t)} + profit \tag{2}$$

From equation 2 one can find the profile price component and the correlation price component.

$$P_{profile} + risk = \frac{\sum_{t=1}^{n} F(0,t)\lambda + \sum_{t=1}^{n} S(t)(\hat{L(t)} - \lambda)}{\sum_{t=1}^{n} L(t)} \tag{3}$$

$$P_{correlation} + risk = \frac{\sum_{t=1}^{n}(L(t) - \hat{L(t)})S(t)}{\sum_{t=1}^{n} L(t)} \tag{4}$$

As mentioned earlier the first step is to set a fair price by expect a zero return on the contract. The expected profit and loss function is then set to be equal to zero.

$$(P_{profile} + P_{correlation})\sum_{t=1}^{n} \hat{L(t)} = E[\sum_{t=1}^{n} F(0,t)\lambda + \sum_{t=1}^{n} S(t)(\hat{L(t)} - \lambda)] + E[\sum_{t=1}^{n}(L(t) - \hat{L(t)})S(t)]$$

Because of the assumption that $F(0,t) = \hat{S(t)}$, the two price components can be written as the following.

$$P_{profile} = \frac{\sum_{t=1}^{n} \hat{S(t)}\hat{L(t)}}{\sum_{t=1}^{n} \hat{L(t)}} \tag{5}$$

$$P_{correlation} = \frac{\sum_{t=1}^{n}(\hat{L(t)}\hat{S(t)} + cov[S(t), L(t)] - \hat{L(t)}\hat{S(t)})}{\sum_{t=1}^{n} \hat{L(t)}} = \frac{\sum_{t=1}^{n} cov[S(t), L(t)]}{\sum_{t=1}^{n} \hat{L(t)}} \tag{6}$$

If the fixed price is set to $P = P_{profile} + P_{correlation}$ we expect to have a zero return on the contract. However, the fixed price agreement puts the risks on the retailer and for this they want to be compensated with a risk premium, rp. The risk premium is calculated with a Risk adjusted Return on Capital (RAROC) based approach, with potential exposure as risk measure [Prokopczuk, Schindlmayr, Rachev, Trück, 2007].

$$RAROC = \frac{Expected\ return}{Economic\ capital} \tag{7}$$

In the case of an electricity retailer, RAROC can be seen as a hurdle rate μ. The rate that is the minimum yield at which an investment is considered profitable enough to make. Economic capital is often measured in Value at risk, which measures a worst case scenario at a certain confidence level. In our case economic capital is calculated as the 99% quantile of the profit and loss distribution. The hurdle rate will in this report be 10%.

$$rp = \left(q_{0.99}\left[\frac{\sum_{t=1}^{n} F(0,t)\lambda + \sum_{t=1}^{n}(L(t) - \lambda)S(t)}{\sum_{t=1}^{n} L(t)}\right] - (P_{profile} + P_{correlation})\right)\mu \tag{8}$$

Final expression of the fixed price is given by equation 9.

$$P = P_{profile} + P_{correlation} + rp \tag{9}$$

3.2 Portfolio and individual customer perspective

An electricity retailer with a large portfolio will not be able to model every individual customer's load. Therefore this report aims to simulate the portfolio load and from this set individual prices to each customer depending on their contribution to the portfolio risk. One way of doing this, showed by [Prokopczuk, Schindlmayr, Rachev, Trück, 2007], is to approximate the customer load as $l_i(t) = \hat{l_i(t)} + \beta_i(L(t) - \hat{L(t)})$, where $\hat{l_i(t)}$ is the expected load of customer i. $l_i(t)$ can be seen as customer i's load prognosis plus deviation that impacts the portfolio. $L(t) - \hat{L(t)}$ is the deviation from expected portfolio load. Beta is calculated from historical data and is an estimate for how risky a customer is to the portfolio.

$$\beta_i = \frac{covariance(l_i - \hat{l_i}, L_p - \hat{L_p})}{variance(L_p - \hat{L_p})} \tag{10}$$

However this requires several years of historical data from a customer to get this approximation of β staunch. The risk of doing this with only one year of data would be that we overestimate the deterministic load. Instead, in this report a similar method will be used inspired by component value at risk, where the sum of individual assets value at risk will be equal to the portfolio value at risk which can be written as the following [Hull, 2015].

$$\beta_i = \frac{covariance(l_i, L_p)}{variance(L_p)} \tag{11}$$

This β will then be used in order to estimate customer load paths.

3.3 Volume limits for fixed price contracts

In a fixed price contract one upper limit and one lower limit of total delivery period load is determined together with the customer. This report will focus on one year contracts and thus the accumulated load for one year. The retailer hedges the expected accumulated volume and the exposure to volume risk increases with the load deviation from these hedged volumes, because of larger exposure to the spot price market. The risk premium must therefore reflect how the limits are set. If the customer consume more or less than the limits they will pay the extra costs that arose, if it turned out to be an additional cost. From historical data it is hard to estimate a probability for unsystematic load deviation. For example if a high electricity consuming machine breaks down or is installed during the contract period.

The first suggested method was based on the historical volatility of customer load deviation. That is the deseasonalized daily load returns of the customer. Deseasonalized with the same type of season function as for portfolio load model which will be explained in the model description section. The aim is to calculate a probability of being inside the limits and multiply this with the risk premium calculated from the scenario without limits. Goodness of fit tests were done for the daily stochastic load returns for four customers in order to find a good distribution for the calculations. These returns are approximately independent identically distributed. Many distributions could be rejected just by looking at histogram of data. In order to get a distribution of yearly deviation from daily distribution, it is preferable for the distribution to be infinitely divisible. Data also shows heavy tails and skewness. Normal inverse Gaussian (NIG) distribution is a distribution that has these properties and was therefore chosen as the best fit to data. The property of infinitely divisibility means that the sum of NIG-variables is also NIG-distributed. If $X \sim NIG(\alpha, \beta, \mu, \delta)$, then the sum of independent X is also NIG distributed, $X_{sum} \sim NIG(\alpha, \beta, c\mu, c\delta)$ [Schlösser, 2014].

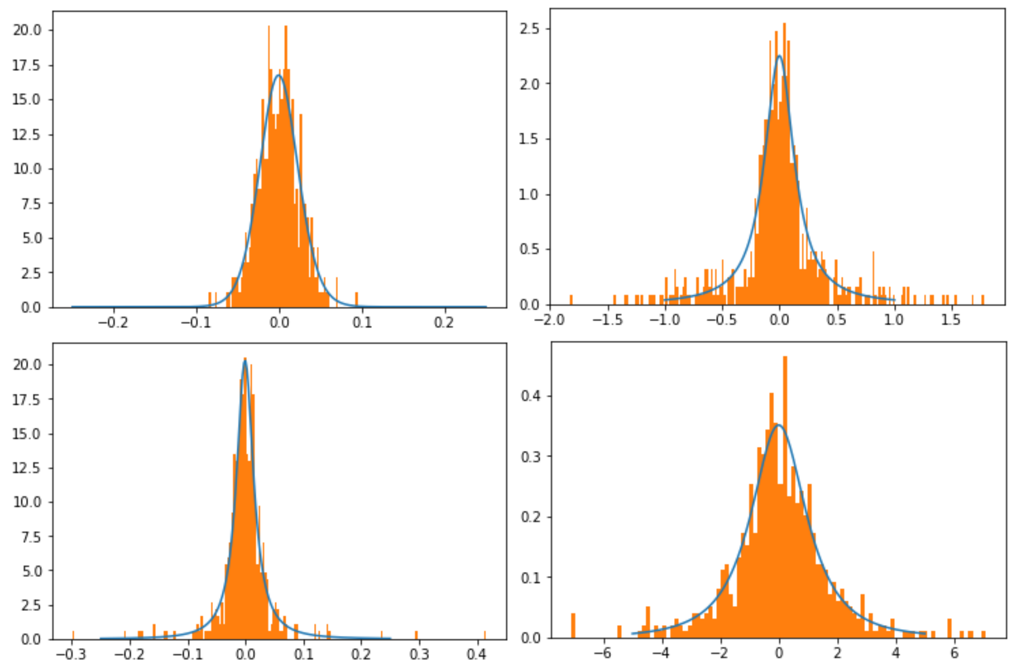

Figure 8: Normalized histogram of daily stochastic load returns from four different customers and fitted NIG distribution represented by the blue lines. In some cases the distribution shows clear skewness and heavy tails, which can be seen in these figures

The results from this method was however not sufficient to give a good estimation of the real world probability distribution for a customer's accumulated load. The results showed zero probability for total customer load to be outside 10% from expected for a whole year. The same results was found with a bootstrap method were 365 historical stochastic load returns was randomly chosen with replacement and summed up. One conclusion is that its not sufficient to look at historical load deviations in order to estimate the risk of large changes in total yearly load. As mentioned before this can be explained by unsystematic risks.

The third suggestion is to look at actual contract outcomes and see how much customers have under- or over consumed during several delivery periods. This will give an indication on how the accumulated load can differ from expected. It is hard to look at historical data and estimate historical prognosis, due to the fact that we do not know what information the retailer had at the time. If a customer double its consumption from one year to another, it is hard to tell today if this was included in the prognosis or not. With this approach we will not have to estimate historical prognosis. The data from actual contract outcomes will provide the information needed. Accumulated load deviation from expected for 3226 contracts was examined and fitted into a NIG distribution. With this method, a customer will only pay for the risk inside these volume limits, which is the risk the retailer takes.

4 Model description

The purpose of modeling spot price and load is to simulate spot price paths and load paths pairwise in order to capture their correlation. Because of this, the model must include a dependency between spot price and load. The model tested in this report is divided in three different parts. Deterministic parts of spot price and load are estimated separately. Daily stochastic movements are modeled by a vector autoregressive model, that include daily spot price deviation, load deviation and temperature deviation from expected. Temperature is included because it is a key driver for spot price and load. Generally customers consume more electricity than expected when the temperature is lower than expected and this results in a negative correlation between system load and temperature. The temperature deviation data used in the model comes originally from historical temperatures in the region of SE4, which are weighted according to the energy provider's portfolio. From this weighted temperature the seasonal trend is removed and remaining data is scaled by a factor. This remaining stochastic temperature data is the temperature deviations used for estimating the vector autoregressive model. For both spot and load, an additional hour is added in spring due to day-light saving, equal to the mean of the two hour surrounding it. The double hour in fall is replaced by one hour with the right mean value.

For the inner daily, hourly stochastic we tried to model the deviation in load with an ARMA model, which can also be referred to as a $SARIMA(1,0,1)\text{x}(1,0,1)_{24}$ model [Bureger, Graeber, Schindlmayr, 2007]. However, this model was not sufficient to capture the negative autocorrelations around lag 12. Instead, the hourly load deviations were modeled by an AR(p) process. A more thorough explanation is given in the next section. The corresponding spot price deviation will be modeled by a vector autoregressive process that includes 24 hour processes, that is updated once a day. This is also how the real hourly prices are set in the market. At around 12:42 CET prices for all 24 hours for the next day are determined [Nordpool, 2007]. The innovations for spot and load at hourly granularity will be correlated according to historical data.

The spot price model is built in three steps. As mentioned before one part of the function captures the deterministic prices, seasonal behaviors and is referred to as $f_{spot}(t)$. The second part captures stochastic prices at a daily granularity, $Y(d)$ and the third part captures stochastic prices at hourly granularity, $X_{spot}(t)$. here t denotes the time in hour and d denotes the time in days.

$$S(t,d) = f_{spot}(t)e^{Y(d)}e^{X_{spot}(t)} \tag{12}$$

Portfolio load is modeled similar to the spot model and can also be described in three sub-models. One deterministic function $f_{load}(t,d)$, one stochastic process for daily deviation $load(d)$ and a process for the hourly deviation $X_{load}(t)$.

$$L(t,d) = f_{load}(t,d) + load(d) + X_{load}(t) \tag{13}$$

4.1 Breaking down spot model in different processes

For a better understanding of what data the different processes aim to capture, we will have to break it down in detail. $S(t)$ denotes the spot price in hour t and $\bar{S(t)}$ is the price in daily granularity, i.e the mean of the 24 hourly prices for one day. Deterministic spot will be captured by $\alpha \exp(m+b+h)$, were m, b, and h are dummy variables for months, days and hours respectively. The spot price can be separated in the following way, derived from equation 12.

$$S(t) = \alpha \exp(m+b+h) \frac{\bar{S(t)}}{\alpha} \exp(-m-b) \frac{S(t)}{\bar{S(t)}} \exp(-h)$$

$$\ln(S(t)) = \ln(\alpha) + m + b + h + \ln(\bar{S(t)}) - \ln(\alpha) - m - b + \ln(S(t)) - \ln(\bar{S(t)}) - h$$

$$\ln(f_{spot}(t)) = \ln(\alpha) + m + b + h \tag{14}$$

$$Y(d) = \ln(\bar{S(t)}) - \ln(\alpha) - m - b \tag{15}$$

$$X_{spot}(t) = \ln(S(t)) - \ln(\bar{S(t)}) - h \tag{16}$$

No correlation or dependency between $X_{spot}(t)$ and $Y(d)$ was found in the data, which enabled the split to be made in this way. This is shown in Figure 9 and 10.

Figure 9: Orange graph shows daily spot price plus the hourly profiles. How the hourly profiles are determined is explained in the next section. Blue shows the hourly spot price and the green graph shows the hourly spot price deviations, $X_{spot}(t)$. The data includes four years of hourly spot price from area SE4, from 1 January 2013 to 31 December 2016

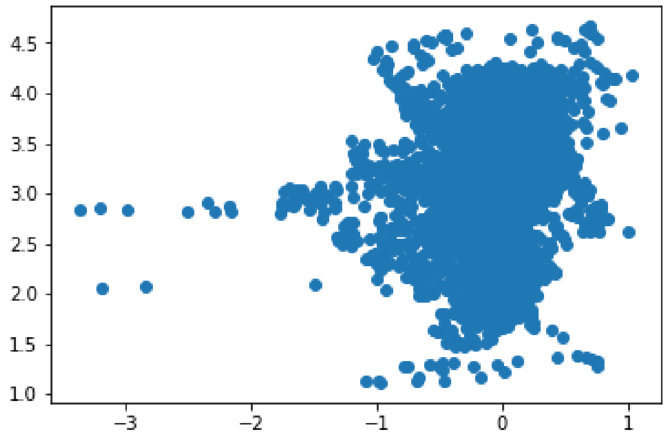

Figure 10: Scatter plot between hourly stochastic spot price and the remaining spot price. The plot shows no clear dependency.

4.2 Deterministic spot and load models

Deterministic functions are estimated for both spot prices and load in order to find the expected values for every hour. Around this, stochastic processes will later be applied. When simulating future outcomes, the spot forward curve will be used instead of the seasonal spot function. It will be multiplied with a stochastic factor with expectancy 1, for every simulated hour.

$$
\begin{aligned}
f_{spot}(t) \quad = \quad & (\alpha_0 + \alpha_1 t)\exp\Big(\sum_{i=1}^{12} \mathbf{1}(month)_i(t)m(i) + \sum_{k=1}^{7}(\mathbf{1}(day)_k(t)b_{spot}(k)) + \\
& \sum_{i=1}^{12}\sum_{k=1}^{7}\sum_{j=1}^{24}(\mathbf{1}(hour)_{i,k,j}(t)h_{spot}(i,k,j))\Big) \qquad (17)
\end{aligned}
$$

First of all the spikes and drops are removed from data and replaced by the 24 hour moving average. This is done by removing the prices which lies below the first quartile minus three times the inter-quartile range or above the third quartile plus three times the inter-quartile range [Mayer, Schmid, Weber, 2015]. The spikes and drops are replaced by the 24 hour moving average value. Besides from the linear trend estimations, the estimations are made for the logarithm of the spot price $\ln(S(t))$. α_0 and α_1 are estimated as a linear regression of four years of spot prices and captures a linear trend. Every month has a dummy variable denoted m(i). $b_{spot}(k)$ are dummy variables for every weekday. Besides $b_{spot}(k)$, every weekday in every month has dummies for every hour $h_{spot}(i,k,j)$, which will capture the inner daily expected profile. For the deterministic load, real data are fitted into the model in equation 18.

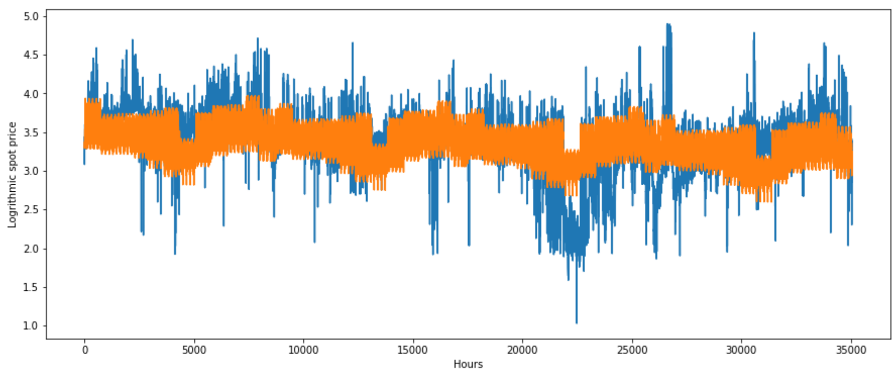

Figure 11: Four years of hourly $\ln(S(t))$ and the deterministic seasonal function

$$
\begin{aligned}
f_{load}(t,d) \quad = \quad & \lambda_1 + \lambda_2 \cos\big(\frac{2\pi}{365}(d-\lambda_3)\big) + \lambda_4 \cos\big(\frac{4\pi}{365}(d-\lambda_5)\big) + \\
& \sum_{k=1}^{7}(\boldsymbol{1}\,(day)_k(t) b_{load}(k)) + \sum_{i=1}^{12}\sum_{k=1}^{7}\sum_{j=1}^{24}\big(\boldsymbol{1}\,(hour)_{i,k,j}(t) h_{load}(i,k,j))\big)
\end{aligned} \tag{18}
$$

Every weekday is given a dummy variable $b_{load}(i)$ and every hour in every weekday in every month is given a dummy variable $h_{load}(i,k,j)$.

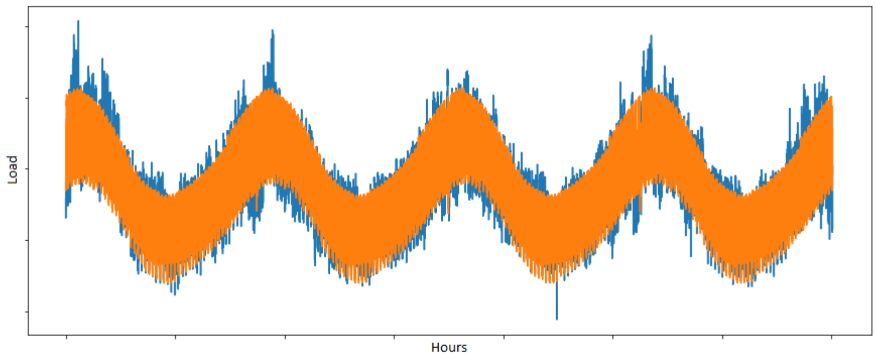

Figure 12: Four years of hourly $L(t)$ and the deterministic seasonal function

For both deterministic load and spot price 25 December and 1 January are estimated as Sundays. 24 December and 31 December are estimated as Saturdays. The deterministic models results in 2016 unique hourly dummy values of 8760 hours in a normal year.

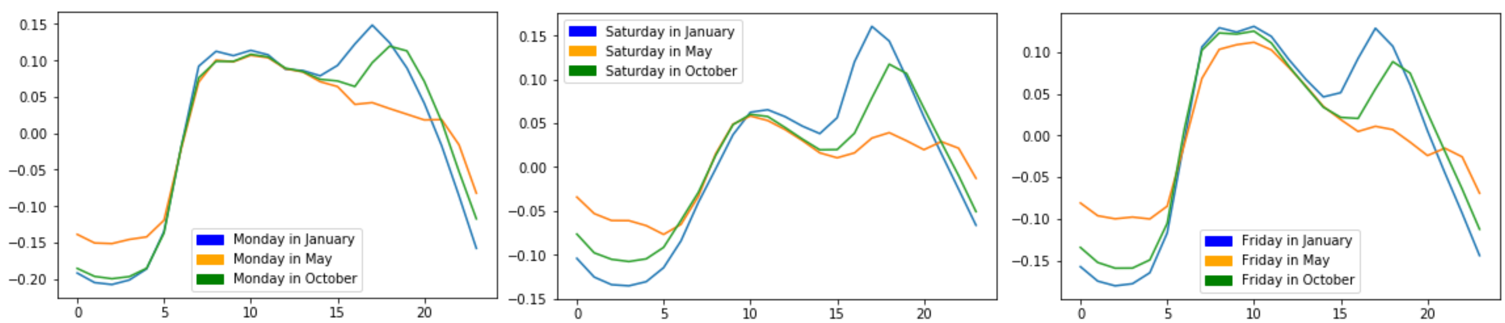

Figure 13: Daily portfolio load profiles for different months and weekdays

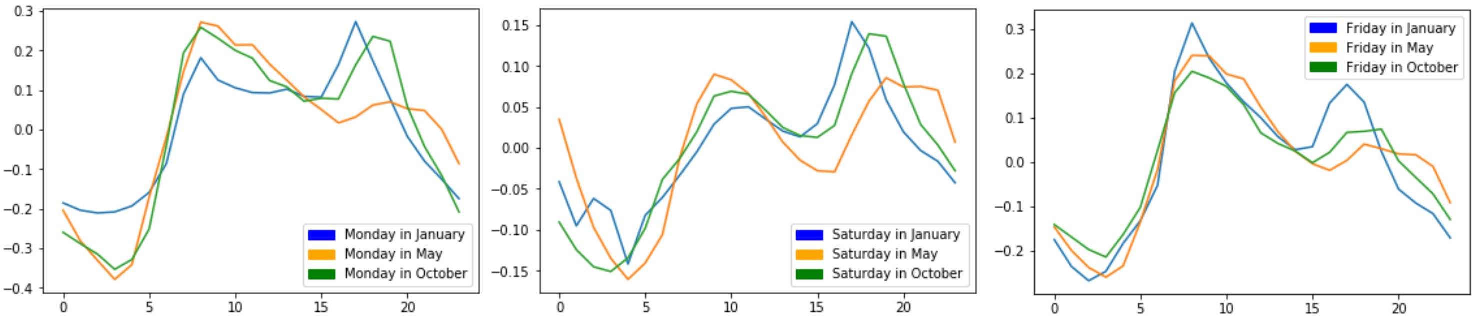

Figure 14: Daily spot price profiles for different months and weekdays

4.3 Daily stochastic models

When estimating daily stochastic $Y(d)$ and $load(d)$, daily $f_{spot}(t)$ and $f_{load}(t, d)$ are subtracted from real daily data. The daily data consist of one value per day. This value is the mean of all 24 hours within that day. This means that $h_{spot}(i, k, j)$ and $h_{load}(i, k, j)$ dummies are unused for this task. The remaining process will be covered by following vector autoregressive model.

$$Y(d) = \eta_1 Y(d-1) + \eta_2 temp(d-1) + \eta_3 Y(d-2) + \varepsilon_1 \quad (19)$$

$$\text{load}(d) = \eta_4 \text{load}(d-1) + \eta_5 temp(d-1) + \varepsilon_{2,k} \quad (20)$$

$$temp(d) = \eta_6 temp(d-1) + \eta_7 temp(d-2) + \varepsilon_{3,k} \quad (21)$$

Temperature residuals, $\varepsilon_{3,k}$ are estimated as normal distributions with different standard deviation for every month, $k = 1, 2, ..., 12$. Load residuals, $\varepsilon_{3,k}$, $k = 1, 2, ..., 12$ are estimated as non central Student's t distributions with different parameter values for every month. The reason for using non central Student's is the heavy tails and skewness shown in data. Figure 15 shows probability plots for two different month, comparing load residuals fitted as normal distribution with load residuals fitted with non central Student's t.

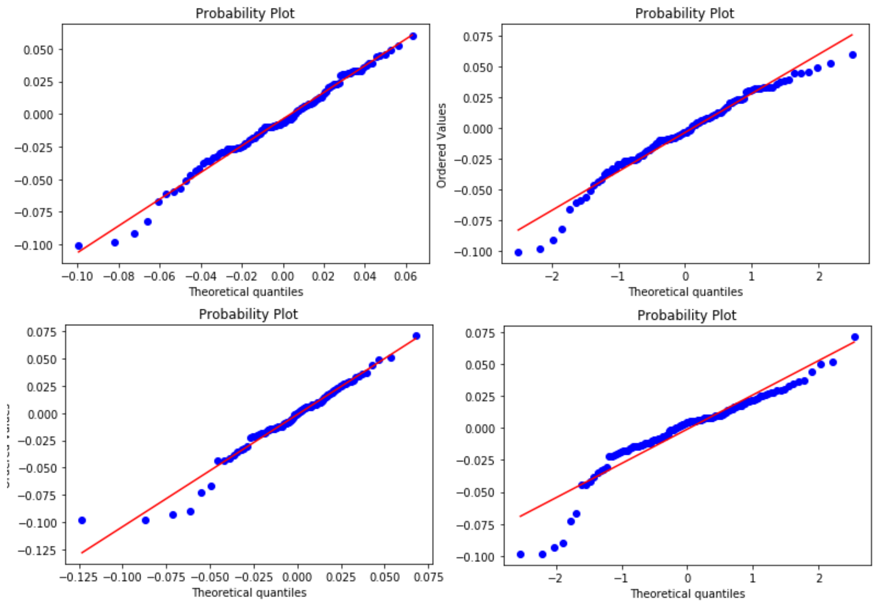

Figure 15: The plots on the left shows load residuals fitted with non central t. The plots on the right shows residuals fitted as normal distributions.

Spot price residuals, ε_1 are heavy tailed and shows no seasonal pattern. They are estimated as Laplace distributions. In order to capture the dependence structure from residuals, they will be generated from a multivariate normal distribution with different correlation for every month. Load residuals and spot price residuals are then quantile transformed to non central t and Laplace distribution respectively. The choice of lags of each vector process was determined with Likelihood ratio tests.

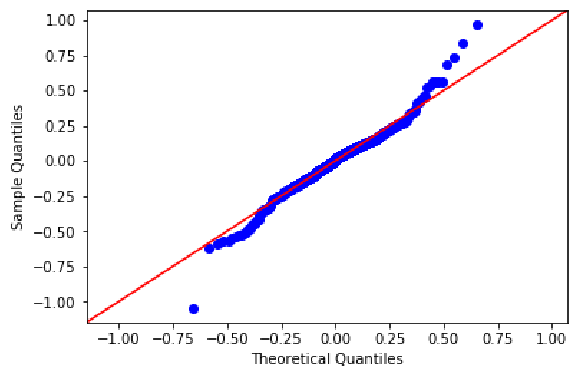

Figure 16: qq-plot of daily spot residuals against the fitted Laplace distribution

Another approach to fit a distribution to residuals from daily spot price process is to transform them to standard normal distribution with the following.

$$\widetilde{x} = \frac{x}{(1 + |x|^{\alpha})^{\gamma}}$$

The parameters α and γ are optimized with OLS, so that the transformed residuals $\widetilde{x}$ fits a standard normal distribution. x are the residuals before transformation scaled by the standard deviation.

$$x = \frac{\varepsilon_1}{\sigma}$$

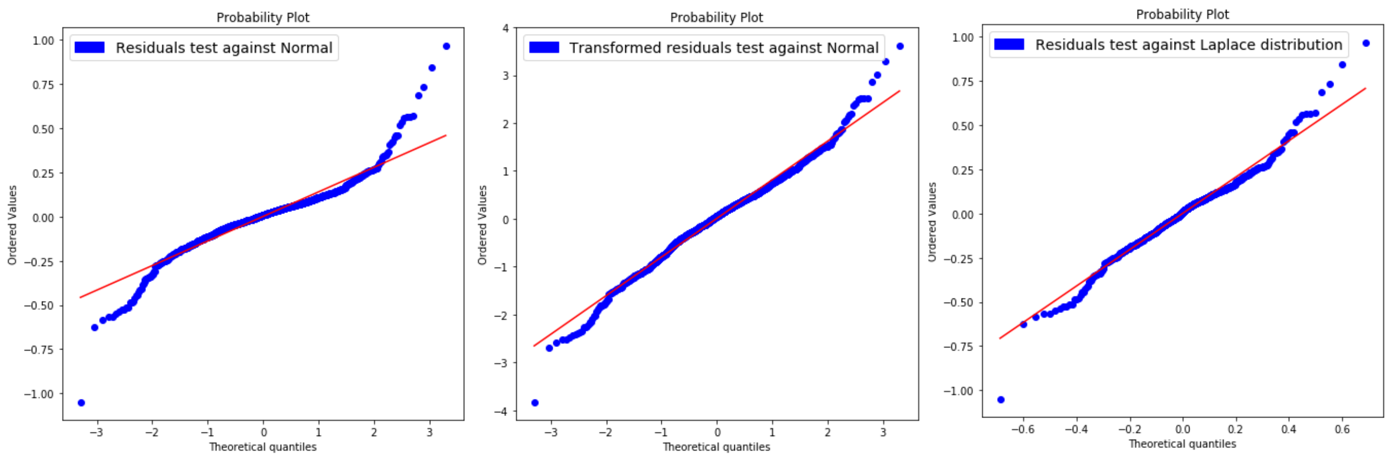

Figure 17: probability plots of daily spot residuals against Normal distribution to the left and the fitted Laplace distribution to the right. The plot in the middle shows transformed residuals against standard normal distribution

Laplace distribution was chosen as the best fit and will be used when simulating.

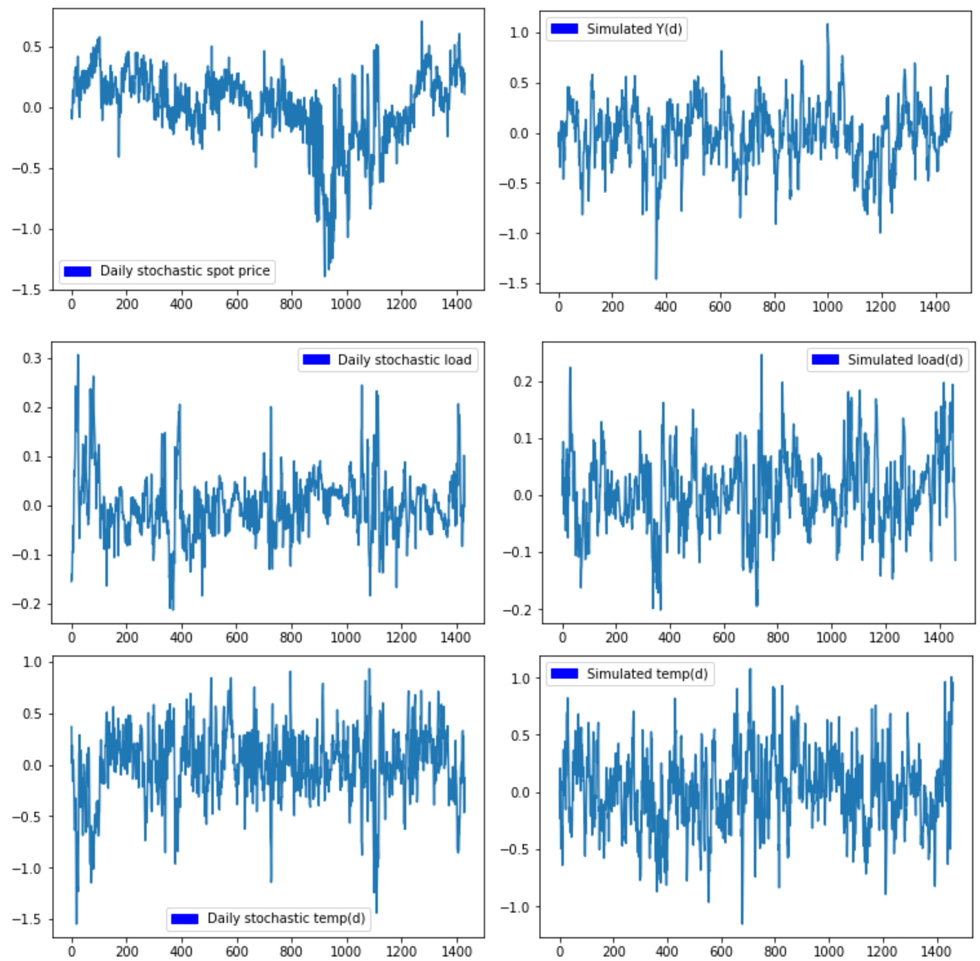

Figure 18: Real data and simulations from the daily vector autoregressive process

4.4 Hourly stochastic models

To model the remaining load and spot price movement, the hourly deviations, daily mean load from portfolio and spot prices from Nord Pool and the hourly deterministic dummies are subtracted from the hourly load from portfolio and spot prices from Nord Pool. The remaining data for load shows high dependency for time lags. For this reason, an AR(p) process will be estimated to the data.

$$X_{load}(t) = \gamma_{1,k} X_{load}(t-1) + \gamma_{2,k} X_{load}(t-2) + ... + \gamma_{p,k} X_{load}(t-p) + \sigma_k \xi_{load} \quad (22)$$

For all parameters in the load process, two different values will be given ($k = 1, 2$) depending on whether it is summer or winter. During the period from 1 May to 31 October the parameters has the notation $k = 1$ and for the other half of the year the notation is $k = 2$. This is because the stochastic hourly data behaves differently over a year. Also the standard deviation of innovations will be different for the two periods. The approach of parameter estimation and lag selection are explained in the next section of this report. As mentioned before, hourly spot deviation is modeled by a Vector-Auto Regressive model updated once a day according to equation 23.

$$X_{d+1} = AX_d + E_{d+1} \quad (23)$$

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,24} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,24} \\ \vdots & \vdots & \ddots & \vdots \\ a_{24,1} & a_{24,2} & \cdots & a_{24,24} \end{pmatrix}$$

$$X_d = \begin{pmatrix} x_{t-24} \\ x_{t-23} \\ \vdots \\ x_{t-1} \end{pmatrix}$$

$$E_{d+1} = \begin{pmatrix} e_t \\ e_{t+1} \\ \vdots \\ e_{t+23} \end{pmatrix}$$

Load process innovations, ξ_{load} are normally distributed. Spot innovations e_{spot} are Laplace distributed and will be correlated not only with other spot innovations in the same day, but also with the corresponding innovation in the load process. e_{spot} will be generated by a 24-multivariate normal distribution and quantile transformed into Laplace distribution. In order to get the right correlation between ξ_{load} and e_{spot}, we use the already generated e_{spot} and extract uniform distributed variables by putting the standard normal distributed e_{spot} in cumulative normal distribution functions. We denote the uniformly distributed variables from the spot process by V_1. In order to get correlation between spot and load innovations one can follow the method from *Generating Bivariate Uniform Data with a Full Range of Correlations and Connections to Bivariate Binary Data*, [Demitras, 2013].

1. Calculate a from $a = -\frac{5}{2} + \frac{1}{2}\sqrt{\frac{\rho+49}{\rho+1}}$, where ρ is Spearman's correlation.
2. Generate u from a uniform distribution $U(0,1)$
3. Generate W from a Beta distribution, $Beta(a,1)$
4. If $u < 0.5$, set $V_2 = |W - V_1|$, otherwise set $V_2 = 1 - |1 - W - V_1|$

V_2 is then put into a inverse cumulative normal distribution with the wanted standard deviation and we have generated load innovations with correlation with the spot innovations. Parameters in the hourly spot price process are estimated in R with the *vars* package. The unrestricted model, containing 576 parameters was tested against restricted models where insignificant parameters were set to zero, with likelihood ratio tests. These test resulted in a model with 370 parameters.

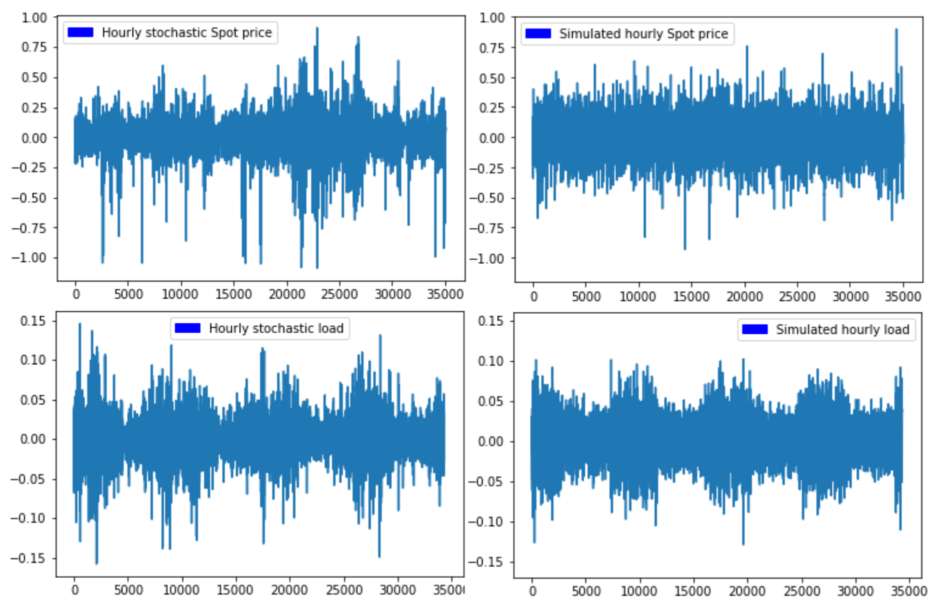

Figure 19: Real data and simulations from the two hourly processes

Values of estimated parameters from the stochastic models can be seen in Appendix.

5 Model estimation and selection process

This section aims to describe how the model parameters was estimated. It will also explain the selection of lags for each model.

5.1 Deterministic functions

All dummy variables in the deterministic spot price function and the deterministic portfolio load functions was determined through ordinary least squares regression. For both load and spot price, the first estimation was the yearly trends. When the yearly trend was removed from spot price data, dummies for months was estimated. In the corresponding load process, the seasonal cosinus trend was estimated. These trends was then removed from data and daily dummies was estimated for both spot price and load. The last step was to estimate the hourly dummies. Daily mean values for spot and load was removed from the data, in order to estimate the daily profiles with OLS.

5.2 Daily Vector autoregressive model

The first step of estimating the daily vector autoregressive model, was to find good start values. This was done with the Python package *statsmodels*, where three different maximum time lags was tested, one, two and three days. The model with two days of time lag was chosen as the best joint model according to BIC and AIC.

$$Y(d) = \eta_1 Y(d-1) + \eta_2 temp(d-1) + \eta_3 \text{load}(d-1) + \eta_4 Y(d-2) + \eta_5 temp(d-2) + \eta_6 \text{load}(d-2) + \varepsilon_1 \tag{24}$$

$$load(d) = \eta_7 Y(d-1) + \eta_8 temp(d-1) + \eta_9 \text{load}(d-1) + \eta_{10} Y(d-2) + \eta_{11} temp(d-2) + \eta_{12} \text{load}(d-2) + \varepsilon_{2,k} \tag{25}$$

$$temp(d) = \eta_{13} Y(d-1) + \eta_{14} temp(d-1) + \eta_{15} \text{load}(d-1) + \eta_{16} Y(d-2) + \eta_{17} temp(d-2) + \eta_{18} \text{load}(d-2) + \varepsilon_{3,k} \tag{26}$$

In order to get a more flexible model with certain parameters set to zero, the wanted parameters were optimized with maximum likelihood. The parameters that maximized a multivariate normal distribution with an initial estimation of the covariance matrix Σ was chosen. The initial value of Σ was set to the covariance matrix of the dataset containing spot price, load and temperature.

First of all, parameters in the temperature process, equation 26 concerning old values of load and spot price was set to zero. This was an intuitive assumption that load and spot price does not effect the temperature. At least not within a time frame of a couple of days.

The selection process after that was an iterative process where one parameter at a time was added or set to zero. The new model was tested against the old model with likelihood ratio test, selecting the new model if it could not be rejected at a 5% significance level.

$$Likelihood\ ratio = 2(L(\theta*) - L(\theta)) \sim \chi^2(1) \tag{27}$$

Given the chosen parameters, we could study the behavior of the residuals. Marginal distributions were fitted for load, temperature and spot price residuals. In order to capture dependence structure, correlation matrices for all twelve months were estimated in order to find a dependence structure for the joint distribution. Then marginal distribution for the three residual types was estimated. Load and temperature residuals was given different distribution parameters every month.

5.3 Gaussian copula approach for residuals

The theory behind the estimation of daily VAR-process can be explained by copula theory. When estimating the daily VAR-process, the optimal parameters maximized the log likelihood value of a multivariate normal distribution for the residuals. A gaussian copula model can be written as equation 28 [Kwak, 2016].

$$C(x_{Load}, x_{Temp}, x_{Spot}) = \Phi_{joint}(\Phi_1^{-1}(F_1(\varepsilon_{Load})), \Phi_2^{-1}(F_2(\varepsilon_{temp})), \Phi_3^{-1}(F_3(\varepsilon_{Spot}))) \tag{28}$$

Φ_{joint} is the joint distribution function, which in this case is a multivariate normal distribution with a 3x3 correlation matrix specified by the correlation between the inverse univariate normal distributions $\Phi_1^{-1}(u)$, $\Phi_2^{-1}(v)$ and $\Phi_3^{-1}(w)$. The variables u, v and w are uniform distributed and $F_{k=1,2,3}$ are the marginal distributions of the copula. The following properties can be shown.

$$If\ \varepsilon_{load} \sim Non\ central\ t\ , F_1,\ then\ u = F_1(\varepsilon_{Load})\ where\ u \sim uniform(0,1)$$

$$If\ \varepsilon_{Temp} \sim Normal, F_2,\ then\ v = F_2(\varepsilon_{Temp})\ where\ v \sim uniform(0,1)$$

$$If\ \varepsilon_{Spot} \sim Laplace, F_3,\ then\ w = F_3(\varepsilon_{Spot})\ where\ w \sim uniform(0,1)$$

5.4 Hourly spot price vector autoregressive model

The model for hourly spot price was estimated using *vars* package in R. Insignificant parameters were set to zero, with t statistics. The threshold value for the t statistics was chosen by likelihood ratio test of the joint model. The best model according to the likelihood ratio test, was found with threshold equal to 0.92. With this threshold value, 370 significant parameters were left. In the Figure 20, one can see the autocorrelation function for both real data and simulated data from the estimated model.

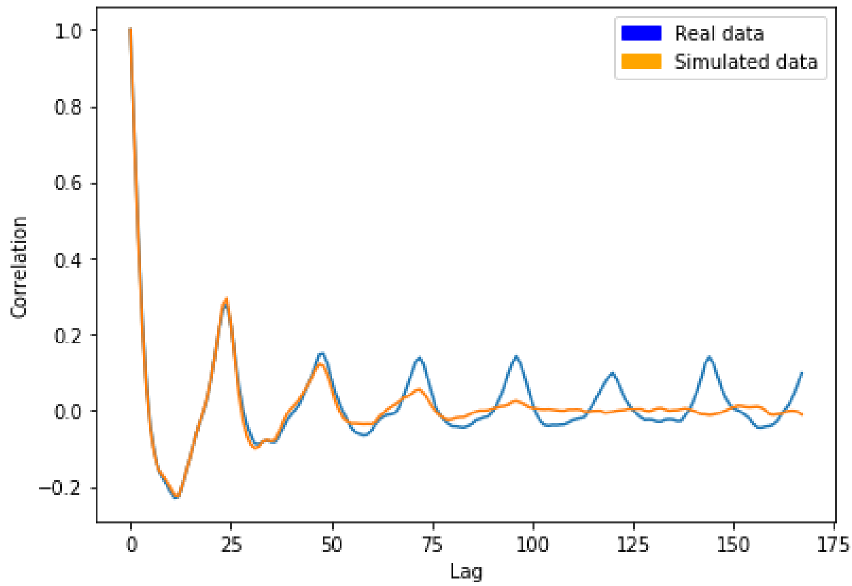

Figure 20: Autocorrelation functions for real data and simulated data

5.5 Hourly load AR(p) process

In the hourly load model, the parameters was estimated using *statsmodels.api.tsa* package in python. As explained earlier, the estimation includes two models. One AR(p) model for summer months and one AR(p) model for winter months. Which months to include in each process was determined by maximizing the sum of log likelihood for the two model estimations. The optimal was found by explaining the data from 1 May to 31 October as the summer process and the rest as the winter process. The number of lags in each model was automatically chosen with BIC-selection, as a built in method in the python package. This resulted in 27 parameters for the summer process and 26 parameters for the winter process, see Appendix. The Autocorrelation functions for both real data and simulated data can be seen in Figure 21.

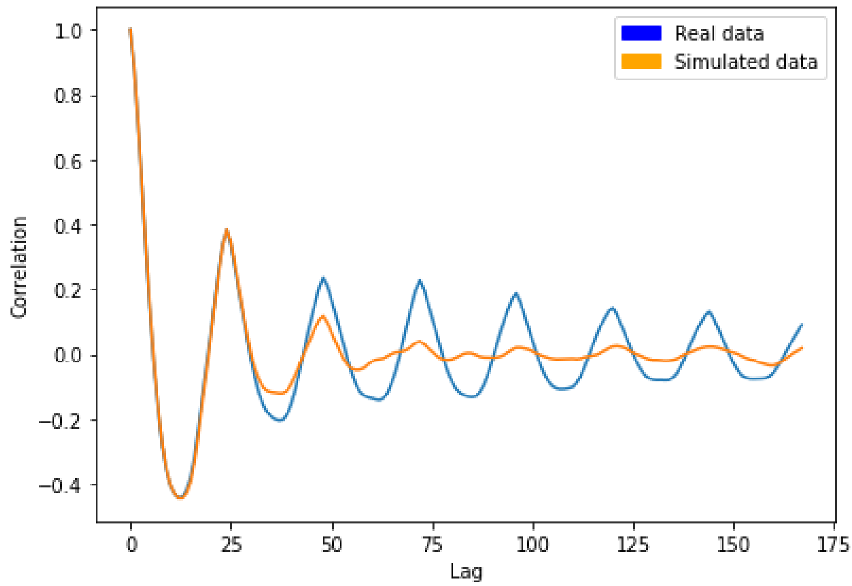

Figure 21: Autocorrelation functions for real data and simulated data

6 Simulation approach

The purpose of simulating spot prices and portfolio electricity load is to get a distribution of fixed price outcomes, i.e fixed prices that would level out the profit and loss function to zero. For every simulation we calculate a fair price P. In this report, 10 000 simulations are made for spot price deviations with corresponding portfolio load deviations. The simulated period will be the whole year of 2018 with hourly granularity. These deviations are the stochastic parts of 2018, and will be combined with prognosis in the following way.

$$S(t) = forward_{spot}(t)e^{X(t)-\mu_{spot}(t)} \tag{29}$$

$$L(t) = prognosis_{load}(t) + Y(t) - \mu_{load}(t) \tag{30}$$

$X(t)$ and $Y(t)$ are simulations. μ_{spot} and μ_{load} are adjustments to force the mean of every simulated hour to be equal to the forward curve and load prognosis respectively. They can be seen as empirical Girsanov kernals that transform the simulated values from P-dynamics to Q-dynamics. Both retailers and producers want to secure future incomes by eliminate the risk of high spot price and low spot price respectively. It results in a symbiosis between producers and retailers in a complete market. In fact both negative and positive risk premiums for forward contracts have been found empirically[Benth, Ortiz-Latorre, 2013]. Geman and Vasicek argued that the general case in power markets, the consumers hedge using forward contracts which are close to delivery, while producers hedge their power generation in the long end. This results in a tendency for positive risk premiums in the short term and negative in the long term [Geman, 2005]. When transforming P-dynamics to Q-dynamics in electricity pricing one does not demand Q to be a martingale measure and we can keep the mean reverting dynamics in the stochastic processes. The reason is the non-storability, which unable traders to invest by purchasing electricity in the spot market and sell at a future time. Electricity must be consumed instantly after bought on the spot market. This enable us to use μ_{spot} and μ_{load} as Esscher transforms for spot price and load dynamics from P to Q [Benth, Schmeck, 2014].

Every simulation gives a one year scenario of S(t) and L(t) which results in a fair price P. After 10 000 simulations we receive a distribution of 10 000 possible fair price outcomes and we can calculate the price components explained in the method section.

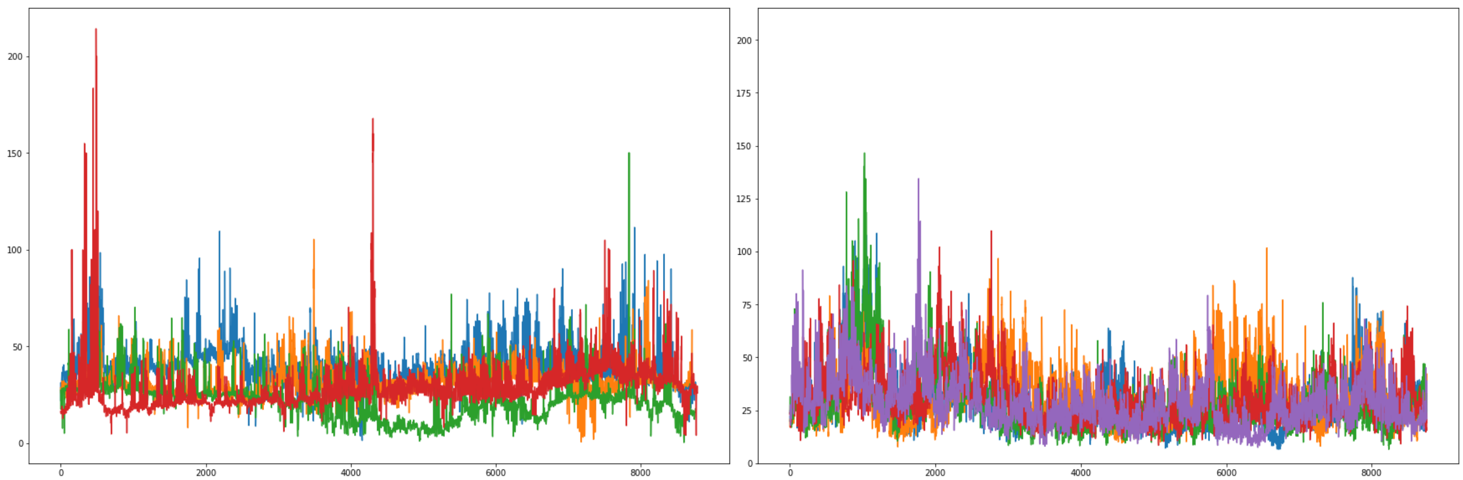

Figure 22: Four different years of real spot price in the plot to the left. The right plot shows five different simulations of 2018

7 Results

The performance of the model is crucial to the resulting price components. Because of this, the results are split in two sections. One section will show the model dynamics and performance, as well as the best estimation of parameters. The second result section will give the resulting price components for different customers in different scenarios.

7.1 Model results

The estimated parameters in the model is shown in Appendix. This section will provide some measures of the model's performance. Table 2 shows the relevant moments of the stochastic processes, without deterministic functions, compared to the real data. Skewness and Kurtosis are measured as Pearson's standardized moments. The results from simulated data are calculated from all 10 000 simulations.

	Variance	Skewness	Kurtosis
Real load	4641.6470	0.699115	5.650725
Simulated load	4235.3476	-0.0994	4.3535
5% and 95% simulated percentiles	(3027, 5756)	(-0.8236, 0.6413)	(3.1391, 6.1729)
Real spot	0.1065759	0.98716	15.53115
Simulated spot	0.1126	1.0006	5.0703
5% and 95% simulated percentiles	(0.0684, 0.1806)	(0.4901, 1.7721)	(3.2295, 8.7792)

Table 2: Moments from real data and simulated data

Table 3 shows different correlation measures between stochastic spot price and stochastic load, both real data and simulated. The results from simulated data are calculated from all 10 000 simulations.

	$Pearson's\ \rho$	$Kendall's\ \tau$	$Spearman's\ \rho$
Real data 2013	0.4077	0.2368	0.34818
Real data 2014	0.2827	0.1849	0.2773
Real data 2015	0.0652	-0.0063	-0.0115
Real data 2016	0.3991	0.2755	0.3882
Simulated data	0.2505	0.1618	0.2395
5% and 95% simulated percentiles	(0.0647, 0.4279)	(0.0391, 0.2807)	(0.0581, 0.4124)

Table 3: Dependence measures from real data and simulated data

Low dependence between spot price and load is found in data from 2015. This can partly be explained by high levels in the Swedish water reservoirs this year, which impacted spot price. This was explained in the subsection *Spot price characteristics*.

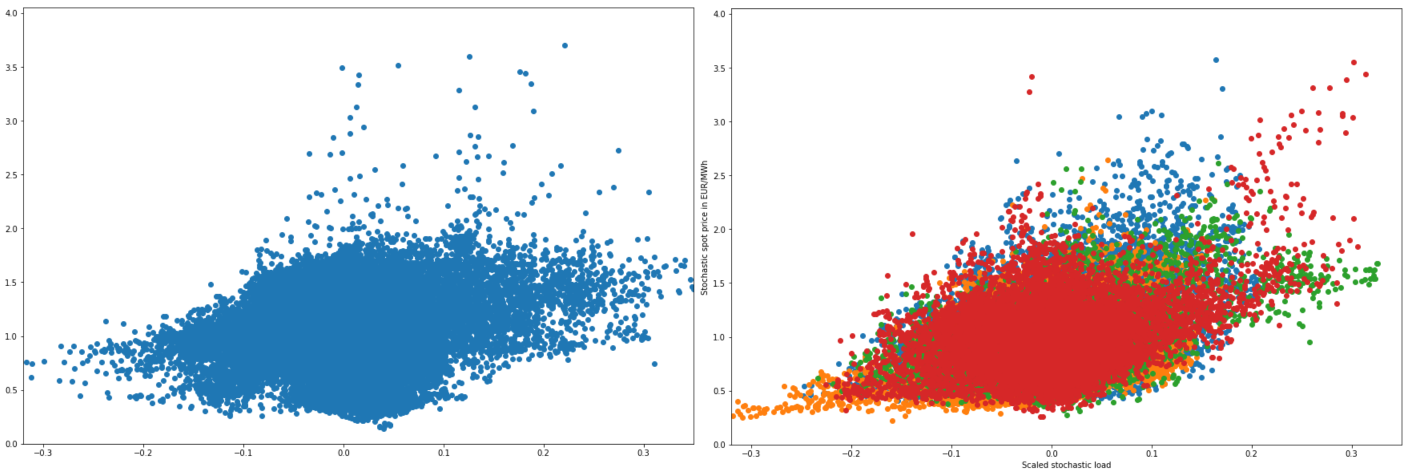

Figure 23: Scatterplot of four years of real stochastic load and spot price to the left. The right scatterplot shows stochastic load and spot price from four simulations of 2018

7.2 Price components results

7.2.1 Volume limits

This subsection will demonstrate what impact the volume limits have to the risk premium. During a delivery period, a customer is allowed to consume an accumulated load within the upper and lower limit. The most common limit is 10% deviation from expected accumulated load for yearly contracts. But in some cases this number can vary from 5% to 30% depending on what the customer wants. In the graphs below one can see how the risk increases with accumulated load deviating from expected. From the plots one can see that the total load volume does not impact the profile price nor the correlation price component. These price components only take into account *when* the extra electricity is consumed. However, if a customer consumes a total volume that is far off from the hedged volume, it will increase risk exposure. The plot below shows 5% and 95% quantiles, the profile price and fair price of a contract when risk is neglected. The x-axis represent the accumulated load in percentage of the expected accumulated load, i.e the load that is hedged when the contract is signed.

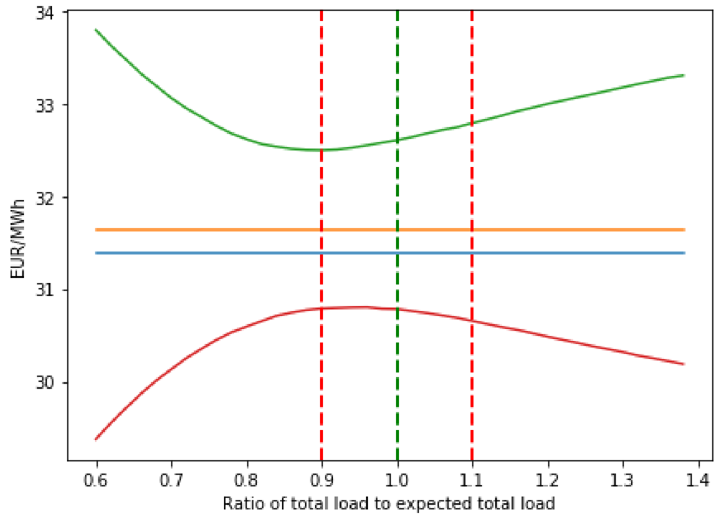

Figure 24: Price distribution on the y-axis and total deviation from expected total load on the x-axis. Horizontal green and red lines are the 95% and 5% quantile of the price distribution. The horizontal blue line is the profile price component and the orange line has correlation price component added. The vertical green line is the hedged volume and the other two red lines are 10% deviation from this.

It is clear that the price distribution changes when accumulated load deviates from hedged volume. A small difference between the 95% and 5% quantile means low risk. From the Figure above the minimum risk is found when accumulated load is lower than the hedged volume. A slight over-hedge of the portfolio load seems to be optimal in order to minimize risks.

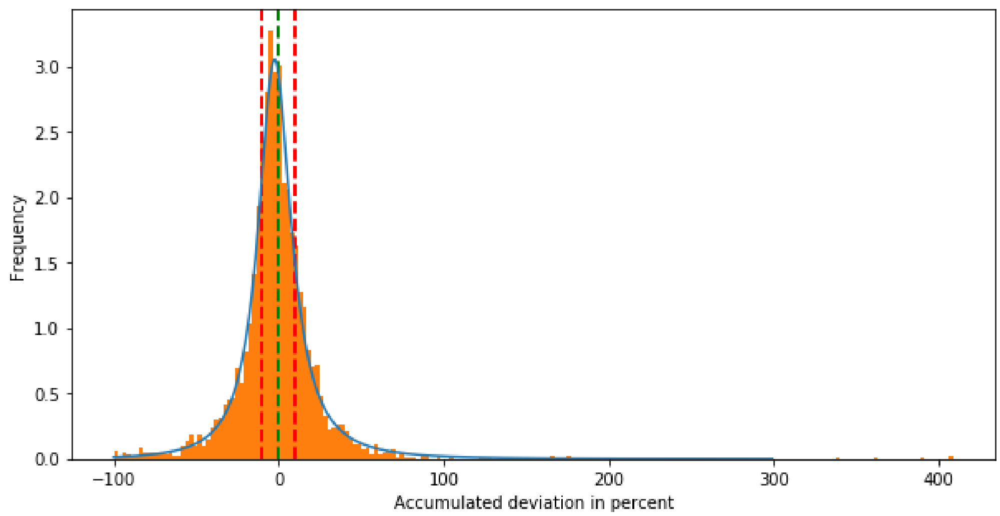

Figure 25: Probabilities of accumulated load in percentage. The data is outcomes from 3226 different contracts. The red vertical lines are the lower and upper limits of 10% from expected accumulated load

The histogram in Figure 25 shows the probability for customers accumulated load deviation from expected, i.e the probabilities on the x-axis from the plot in Figure 24. A fitted NIG distribution can be seen as the blue line from the plot in Figure 25. The risk premium for a customer calculated without limits will be multiplied with the probability of ending up inside these limits in order to get a risk premium for a certain volume limit interval. In the case of lower and upper limits of 10% from expected, risk premium included in the fixed price will be calculated according to equation 31. $NIGcdf$ refers to the fitted Normal inverse gaussian cumulative distribution function. Notice that the probability calculated is conditional to values larger than -1, due to the fact that accumulated load can not be less than -100% from expected.

$$rp_{with\ limits} = rp\frac{NIGcdf(0.1) - NIGcdf(-0.1)}{1 - NIGcdf(-1)} \tag{31}$$

In the United Kingdom, the market experienced a 12.6% decline in electricity demand for industrial consumers from the year of 2008 to 2009 [UK Government, 2017]. This could be explained by the financial crisis at the time. As we saw from spot price characteristics, a decline in demand can result in lower spot prices. Electricity retailers would in this case be forced to sell superfluous electricity for a low price on the spot market. The seemingly large dependence in grid consumption during extreme events is a risk factor that is not included in this paper. This could be analyzed in order to fully understand the risk of accumulated load deviation from expected.

7.2.2 Portfolio

For all prices calculated in the following subsections are calculated with a hurdle rate of 10%. Also the volume limits are set to -10% and +10%. The fixed prices are calculated with two different β values. Summer β, is used from 1 May to 30 September. For the other dates, winter β is used. Results from portfolio simulations is shown in this subsection. A common way of pricing contracts, as mentioned earlier in the report is to price customer profile components individually and add portfolio risk premium and portfolio correlation price. When viewing the whole SE4 portfolio as one customer, the price components in EUR/MWh can be seen in Table 4. These results are calculated for a yearly contract of 2018.

Summer β	1
Winter β	1
Profile price	31.3822
Correlation price	0.2496
Risk premium	0.0726
Total fixed price	31.7044

Table 4: Price components in EUR/MWh for portfolio.

The price distribution can be seen in Figure 26. The left red line is profile price component, the right red line is profile price plus correlation price component which is the mean of the price distribution. The right green line is the 99% quantile and the left green line is the mean price plus the risk premium.

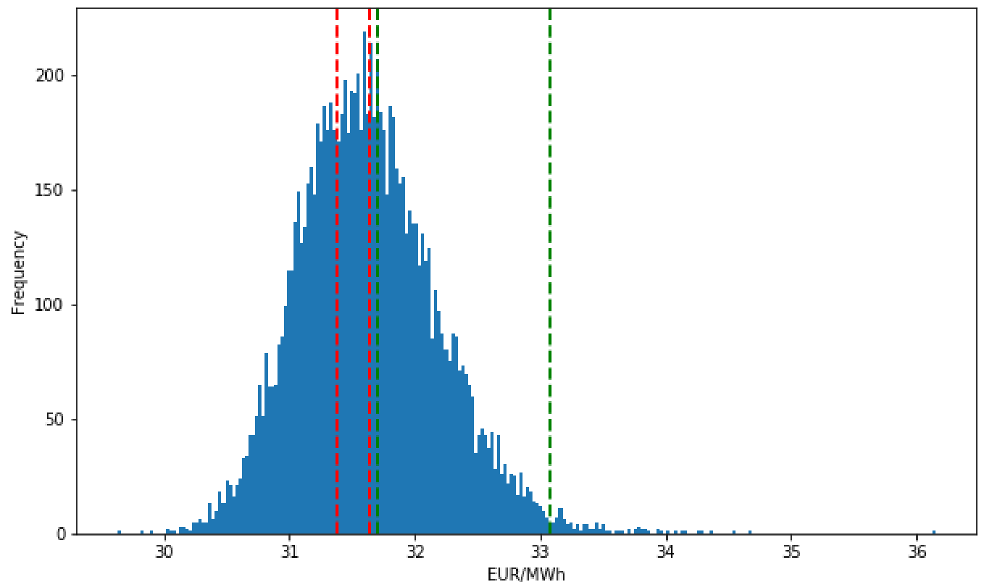

Figure 26: Price distribution for portfolio, with the three pricing components and the 99% quantile.

Contracts with shorter delivery periods was also calculated. Results from first and second quarter of 2018 is shown in Table 5.

	2nd Quarter	4th Quarter
Profile price	28.3194	32.1451
Correlation price	0.1619	0.4153
Risk premium	0.0767	0.0981
Total fixed price	28.5580	32.6585

Table 5: Price components in EUR/MWh for portfolio for quarterly contracts.

A significant difference can bee seen in the profile price and the correlation price component for the two different contract periods. Historical data shows that the dependence between spot price and load is higher in the 4th quarter than in the 2nd quarter, which explains the difference in correlation price component.

7.2.3 Customer A

From looking at customer A's historical load data and the prognosis for 2018, one can expect a lower fixed price. Costumer A consumes more electricity in summer time than in winter time.

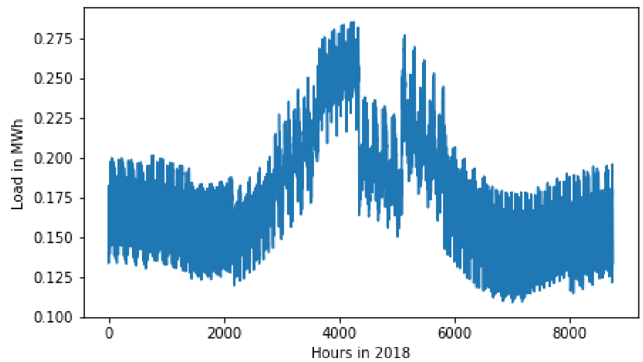

Figure 27: 2018 load prognosis for customer A

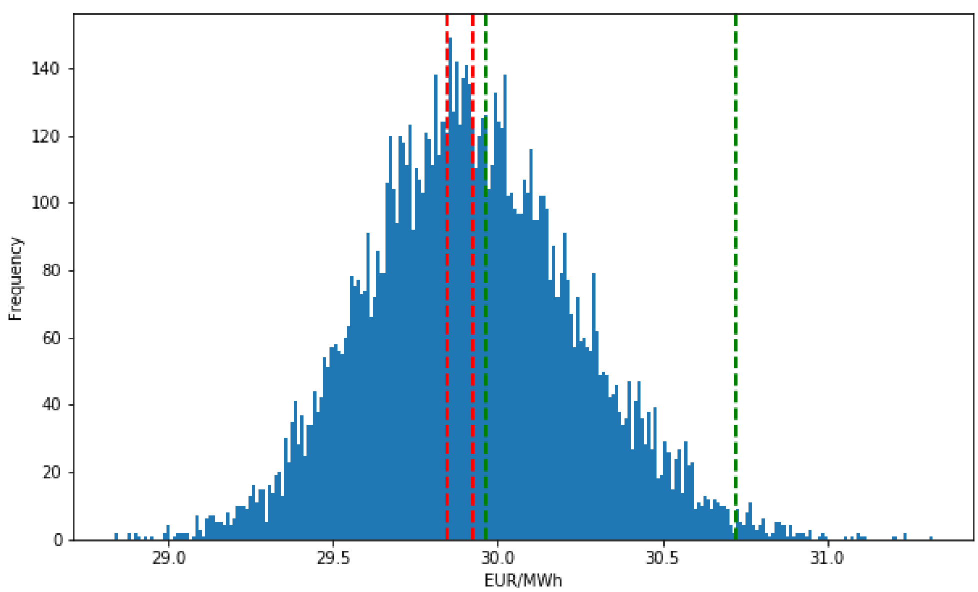

Figure 28: Price distribution for customer A, with the three pricing components and the 99% quantile.

Summer β	$1.5723\ 10^{-5}$
Winter β	$8.4100\ \ 10^{-5}$
Profile price	29.8439
Correlation price	0.0808
Risk premium	0.0400
Total fixed price	29.9647

Table 6: Price components in EUR/MWh for customer A.

The winter β is larger than the summer β. One can expect a large difference in fixed price depending on the delivery period. Table 7 shows fixed price components for quarter 2 and quarter 4.

	2nd Quarter	4th Quarter
Profile price	27.5870	31.8732
Correlation price	0.0316	0.1899
Risk premium	0.0777	0.0440
Total fixed price	27.6963	32.1071

Table 7: Price components in EUR/MWh for quarterly contracts for customer A

7.2.4 Customer B

By looking at Customer B's prognosis one can see that this customer consumes more electricity in the winter than in the summer.

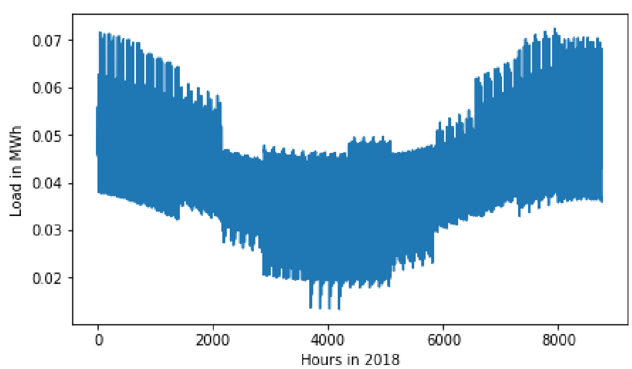

Figure 29: 2018 load prognosis for customer B

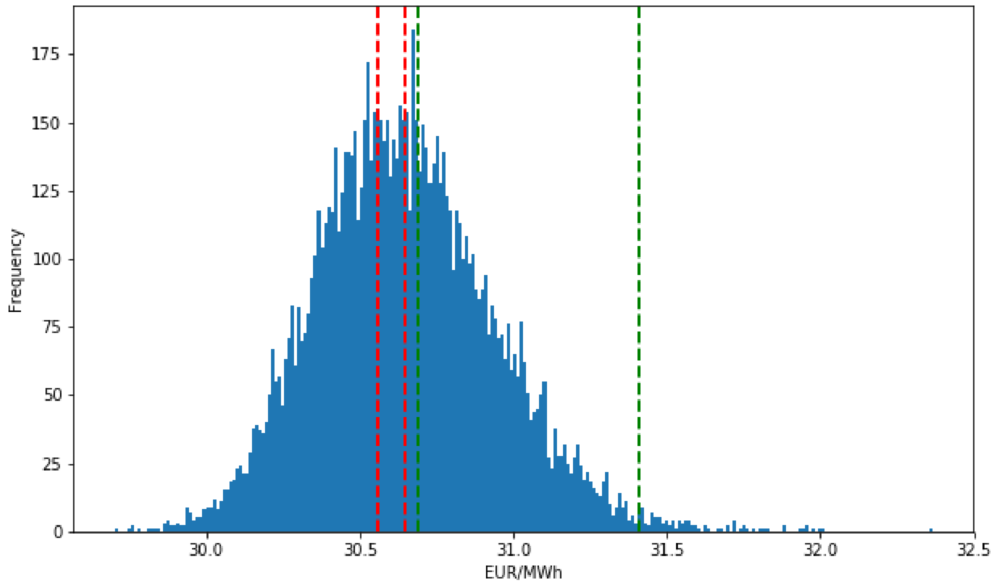

Figure 30: Price distribution for customer B, with the three pricing components and the 99% quantile.

Price components in EUR/MWh for a yearly contract and two quarterly contracts for customer B can be seen in the Table 8.

	Yearly contract	2nd quarter	4th quarter
Summer β	$2.9196\ 10^{-5}$		
Winter β	$2.0892\ 10^{-5}$		
Profile price	30.5558	27.5762	31.8136
Correlation price	0.0934	0.0655	0.1483
Risk premium	0.0382	0.0160	0.0510
Total fixed price	30.6896	27.6577	32.0129

Table 8: Price components in EUR/MWh customer B. Both for yearly and quarterly contracts.

7.2.5 Customer C

The third and last real customer examined in this report has the strongest correlation to the portfolio consumption behavior. This implies higher correlation price and risk premium.

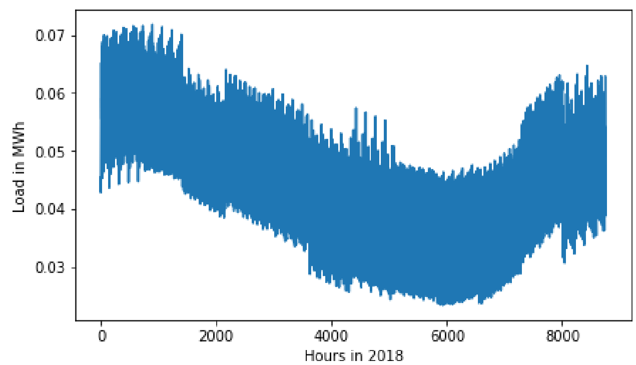

Figure 31: 2018 load prognosis for customer C

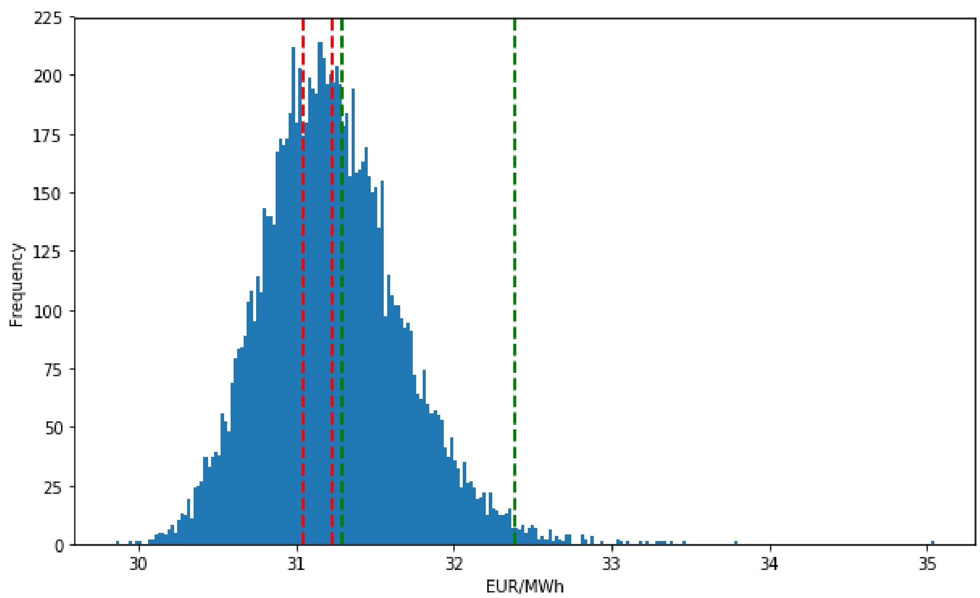

Figure 32: Price distribution for customer C, with the three pricing components and the 99% quantile.

	Yearly contract	2nd quarter	4th quarter
Summer β	$5.9396\ 10^{-5}$		
Winter β	$4.3349\ 10^{-5}$		
Profile price	31.0446	28.2195	32.1005
Correlation price	0.1863	0.1160	0.3378
Risk premium	0.0582	0.0588	0.0890
Total fixed price	31.2891	28.3943	32.5273

Table 9: Price components in EUR/MWh customer C. Both for yearly and quarterly contracts.

7.2.6 Fictional extreme customers

Two fictional customers has been examined. The first example of an extreme customer has $historical\ load = 2 - portfolio/1000$, which will have an opposite consumption pattern from the portfolio. The prognosis for 2018 for this customer can be seen in Figure 33.

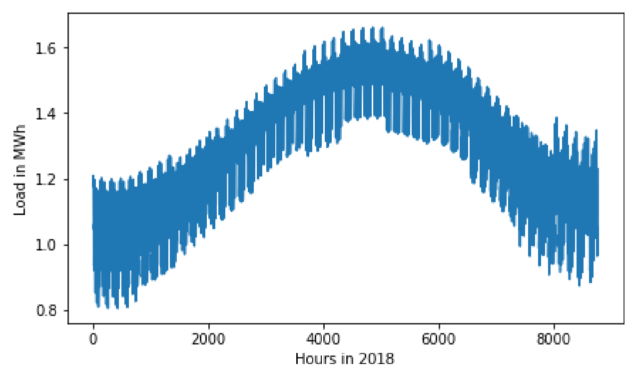

Figure 33: 2018 load prognosis for a low cost fictional customer

The price distribution from a yearly contract is shown in Figure 34. Notice that the right red line shows the profile price component and the left red line is profile plus correlation price component.

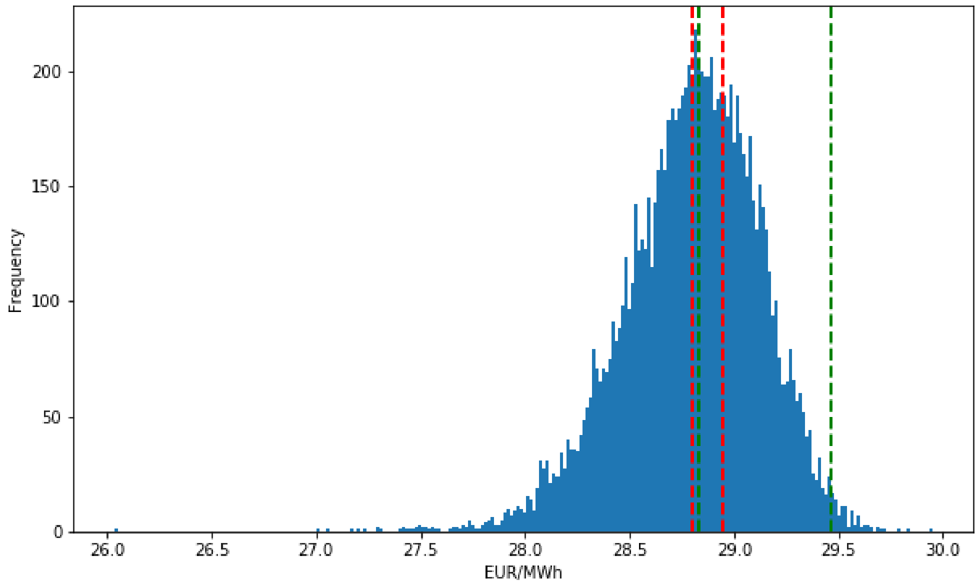

Figure 34: Price distribution for a low risk customer, with the three pricing components and the 99% quantile.

You can see from the result that this customer diversify the risk of the portfolio. For this reason the correlation price component will be negative. One can also notice that the final fixed price is lower than the profile price.

Summer β	-1.000 10^{-3}
Winter β	-1.000 10^{-3}
Profile price	28.9433
Correlation price	-0.1449
Risk premium	0.0334
Total fixed price	28.8318

Table 10: Price components in EUR/MWh for an example customer with negative β values

The second example of an extreme customer has $historical\ load = (portfolio/1000)^2$. This customer will have relatively high β values. The following Figure shows the prognosis for 2018.

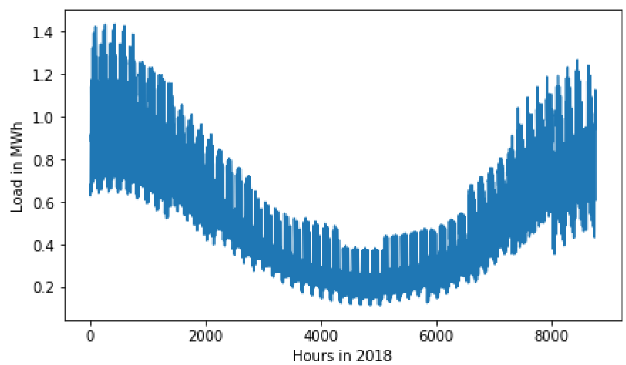

Figure 35: 2018 load prognosis for a high cost fictional customer

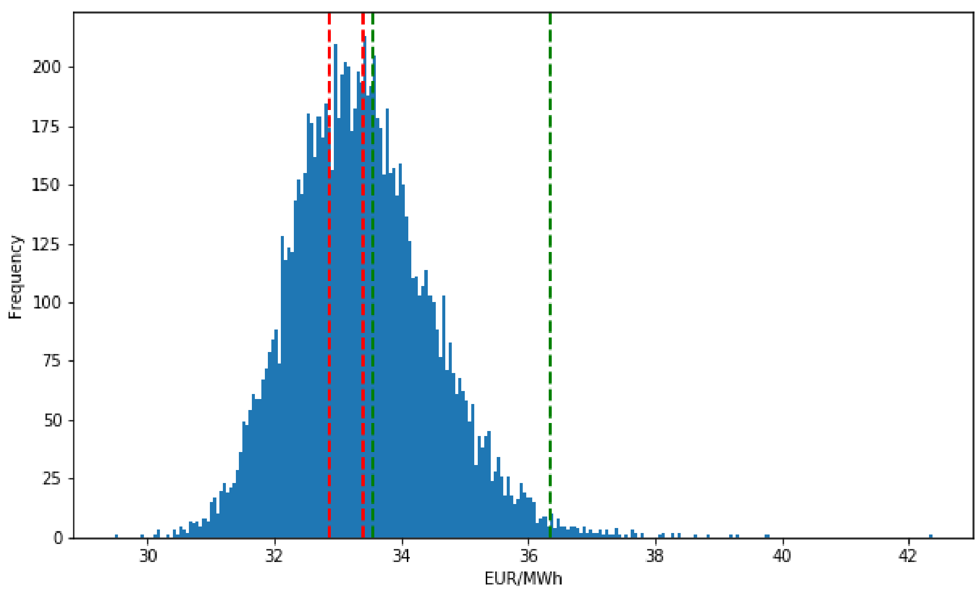

Figure 36: Price distribution for the high risk fictional customer, with the three pricing components and the 99% quantile.

Table 11 shows fixed price components in EUR/MWh for 2018.

Summer β	$1.1269\ 10^{-3}$
Winter β	$1.7342\ 10^{-3}$
Profile price	32.8601
Correlation price	0.5291
Risk premium	0.1483
Total fixed price	33.5375

Table 11: Price components in EUR/MWh for an example customer with high β values

8 Conclusions and discussion

In this paper we propose simulation based autoregressive models to estimate costs and risks of fixed price contracts for a retailer in the Nordic electricity market. The autoregressive model approach turned out successful in reflecting the real world dependence structure between load and spot price. The model was tested for several real world market situations and in all cases we found reasonable outputs. Results indicates that the model adequately captures the given risk factors. Results also shows that the costs of having a customer in a fixed price contract portfolio differs significantly depending on customers consumption behavior and delivery period. The common strategy of pricing a contract is to price the expected load of a customer individually, the profile price component and then add a portfolio based correlation price component and a risk premium. It would be unwise to set an equal correlation price component and risk premium for all customers and independent of delivery season. This could cause a scenario where the price is only considered profitable for the costly customers, while less costly customers chooses other retailers. With deeper knowledge of how costly a customer is, a retailer could potentially offer lower prices to low risk customers while avoiding unprofitable contracts with costly customers. The method used in this paper turned out to be an efficient way of pricing the contracts individually. One of the most time consuming activity is estimating model parameters, which in practice can be updated once a year. Also the simulation part was time consuming. It is not however necessary to simulate every time a contract needs to be priced. The 10 000 portfolio simulations are saved and used when calculating a new contract, together with the Forward curve for spot, customer load prognosis and the β values, which are easily estimated.

The correlation price component is significantly different for different contract periods. Deviation in load and spot price are more correlated in the fourth quarter of the year than in the second. Generally this results in higher correlation price component in the fourth quarter of the year. This shows the importance of pricing different correlation price components depending on the contract period.

A further development of the models could include multivariate non gaussian copula models to generate the residuals. One could possibly find a good fitted copula and capture the dependence structure of Spot price and load more accurately. Also the tail dependence could be examined more and captured by a non gaussian multivariate copula. It is common to use a spike process or a jump diffusion process in order to model electricity spot price. For this to be relevant in the case of this project we would need to know the dependency between load and spot price in extreme events. Another improvement for further development would be to estimate customer load prognosis more thoroughly. For a better approximation of customer prognosis one could adjust the prognosis after ”normal year conditions” and for example estimate how the load would behave with normal temperature. Often a prognosis can be adjusted with input from the customer itself. The profile price component will lack in accuracy with badly estimated prognosis. However, the important thing in the end is to find the fair price of the contract. Overestimated profiles will end up with underestimated correlation price components. If implemented in a pricing system for an electricity retailer and the retailer wants to use a better customer prognosis, one can see from the section *Simulation approach* that this is easily done.

Concerning the volume limits in a fixed price contract, a further research could include a deeper analysis of other risk aspects. One could estimate the probability for large deviations in yearly grid load and what impact this has on a yearly spot price. From historical data from the UK market, we found extreme events that caused the whole grid load to deviate from expected, which would cause problems for a retailer. The retailer is likely to sell the hedged volumes back to the spot market for a lower price.

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9 Appendix

9.1 Daily model parameters

Parameters in the daily stochastic VAR-process can be seen below.

$$
\begin{aligned}
Y(d) &= \eta_1 Y(d-1) + \eta_2 temp(d-1) + \eta_3 Y(d-2) + \varepsilon_1 \\
load(d) &= \eta_4 load(d-1) + \eta_5 temp(d-1) + \varepsilon_{2,k} \\
temp(d) &= \eta_6 temp(d-1) + \eta_7 temp(d-2) + \varepsilon_{3,k}
\end{aligned}
$$

η_1	η_2	η_3	η_4	η_5	η_6	η_7
0.7650	-0.0192	0.12340	0.7491	-0.0277	0.9201	-0.1009

Table 12: Parameters in daily VAR-model

Correlation matrices for every month of the year, for the multivariate normal distribution which generates residual dependence.

Table 13: Explanation

Load		
	Temp	
		Spot

Table 14: January

1	-0.53	0.26
-0.53	1	-0.36
0.26	-0.36	1

Table 15: February

1	-0.61	0.20
-0.61	1	-0.23
0.20	-0.23	1

Table 16: March

1	-0.60	0.09
-0.60	1	-0.19
0.09	-0.19	1

Table 17: April

1	-0.36	0.29
-0.36	1	-0.40
0.29	-0.40	1

Table 18: May

1	-0.36	0.13
-0.36	1	0.09
0.13	0.09	1

Table 19: June

1	-0.08	0.05
-0.08	1	-0.03
0.05	-0.03	1

Table 20: July

1	-0.01	0.12
-0.01	1	0.14
0.12	0.14	1

Table 21: August

1	-0.21	0.02
-0.21	1	-0.15
0.02	-0.15	1

Table 22: September

1	-0.58	0.07
-0.58	1	-0.09
0.07	-0.09	1

Table 23: October

1	-0.58	0.21
-0.58	1	-0.22
0.21	-0.22	1

Table 24: November

1	-0.70	0.50
-0.70	1	-0.52
0.50	-0.52	1

Table 25: December

1	-0.65	0.48
-0.65	1	-0.49
0.48	-0.49	1

Load ε_3			Temp ε_2	Spot ε_1
Non central t			Normal	Laplace
v	σ	nc	σ	s
11.57	0.04	0.33	0.05	0.10
23.67	0.02	-9.58	0.03	0.10
8.02	0.03	0.50	0.04	0.10
5.33	0.02	-0.64	0.03	0.10
3.24	0.02	-0.83	0.03	0.10
2.39	0.01	-0.74	0.03	0.10
3.24	0.02	-0.83	0.02	0.10
2.39	0.01	-0.74	0.02	0.10
9.83	0.01	-1.81	0.02	0.10
14.22	0.01	-5.33	0.03	0.10
6.84	0.01	0.31	0.04	0.10
8.86	0.02	-0.24	0.06	0.10

Table 26: Residual distribution parameters. The location parameters for all the distributions are set, so that the expected value is zero. The rows represent twelve different months starting from January

The probability density function of non central t can be written as the following.

$$f(x;v,nc,\mu,\sigma) = \frac{v^{v/2}\Gamma(v+1)}{\sigma 2^v exp(nc^2/2)(v+(\frac{x-\mu}{\sigma})^2)^{v/2}\Gamma(v/2)}\left(\sqrt{2}nc(\frac{x-\mu}{\sigma})\frac{F_1(\frac{v}{2}+1;\frac{3}{2};\frac{nc^2(\frac{x-\mu}{\sigma})^2}{2(v+(\frac{x-\mu}{\sigma})^2)})}{(v+(\frac{x-\mu}{\sigma})^2)\Gamma(\frac{v+1}{2})}+\frac{F_1(\frac{v+1}{2};\frac{1}{2};\frac{nc^2(\frac{x-\mu}{\sigma})^2}{2(v+(\frac{x-\mu}{\sigma})^2)})}{\sqrt{v+(\frac{x-\mu}{\sigma})^2}\Gamma(\frac{v}{2}+1)}\right)$$

F_1 is Kummer's confluent hypergeometric function. The probability density function of Laplace distribution is written as the following.

$$f(x;\mu,s) = \frac{1}{2s}exp(-\frac{|x-\mu|}{s})$$

9.2 Hourly spot price process parameters

Optimal parameters for the hourly spot process is shown on the next page.

0.25	−0.09	0	0.14	0	0	−0.07	0.18	0	−0.06	0	0	0.35	−0.30	0.26	0	−0.12	0.08	−0.09	0	0.08	−0.29	−0.22	0.61
0.22	−0.12	0.07	0.16	0	0	0	0.14	0	−0.08	0.06	0	0.27	−0.28	0.27	0	−0.08	0.07	−0.10	0.08	0	−0.26	−0.40	0.70
0.14	−0.22	0.12	0.19	0	0.06	−0.05	0.15	0	−0.10	0	0	0.29	−0.19	0	0.15	−0.13	0.06	−0.09	0	0	−0.41	0	0.47
0.10	−0.20	0.08	0.17	0.09	0.09	−0.11	0.17	0	−0.14	0	0	0.39	−0.40	0.21	0	−0.18	0.10	−0.10	0	0	−0.31	0	0.33
0.22	−0.23	0.11	0	0.10	0.10	0	0.18	0	−0.11	0	0	0.55	−0.67	0.51	0	−0.16	0.08	−0.15	0.09	−0.11	−0.15	0	0.27
0	−0.12	0.18	0	0	0.15	0	0.26	0.05	−0.10	0.09	−0.10	0.42	−0.27	0.18	0	0	0.12	−0.15	0.08	0	0.13	0.17	0
−0.08	0	0	0.06	0	0	−0.06	0.09	0.14	0	0	0	−0.20	0.20	−0.10	0.20	−0.06	0.05	0	0.19	0	0.28	−0.25	0.07
−0.11	0	0	0.09	−0.06	0.06	0	0	0.13	0.10	0	0.10	−0.22	0	−0.18	0.25	0	−0.06	0.10	0.16	0	0.14	−0.16	0
−0.09	0	0	0	−0.03	0	0	−0.06	0.11	0.09	0	0.18	−0.21	0.30	−0.57	0.41	−0.18	0.04	0.05	0	0.18	0	0	−0.08
−0.09	0	0	0	0	0	−0.06	−0.05	0	0.13	0	0.13	−0.08	0	−0.07	0.08	0	−0.05	0.09	0	0.10	0	0	−0.07
−0.06	0	0	0	0	0	−0.05	−0.04	−0.04	0.12	−0.07	0.23	−0.08	0	0	0.04	0	−0.04	0.08	0	0.04	0	0.12	−0.15
0.04	−0.04	0	0	0	−0.04	0	−0.03	−0.05	0.12	−0.12	0.19	0	0.05	−0.08	0.11	0	−0.06	0.10	−0.08	0.09	0	0.11	−0.16
0.06	0	−0.02	0	0	−0.03	0	−0.02	−0.04	0.06	−0.07	0.13	0	0.11	0	0	0.04	0	0.03	−0.04	0	0.08	0	−0.09
0.08	0	0.03	−0.04	0.02	0.04	0	0	−0.03	0.06	0	0.04	0	0.19	−0.06	0.11	0	0.03	0.02	0	0	0.11	0	−0.10
0.04	0	0.06	−0.07	0.03	0.04	0	−0.03	0	0.04	0	0	0	0.20	−0.09	0.10	0.06	0.06	0	−0.04	0	0.13	0	−0.10
0	0.09	0	−0.07	0.03	0	0	−0.05	0	0.06	0	−0.06	0	0.09	0	−0.14	0.42	0.03	0	0.05	−0.08	0.13	−0.07	−0.05
−0.10	0.16	0	0	0	0.05	0	0.05	0	0.10	0.14	−0.16	0	0.16	0	−0.16	0.23	0.14	0.09	0	0.13	0	0	0
−0.10	0.17	−0.09	0.08	−0.06	0.07	−0.04	0	0	0.17	0	−0.19	0	0.15	0	−0.24	0.32	0	0.15	0.09	0.07	−0.13	0.15	−0.07
0	0.08	0	0	0	0	0.09	0	0.06	0	0.06	−0.08	0.09	0	0.17	−0.16	0.11	0	0.11	0	0.08	0.15	0	−0.06
0	0.19	−0.08	0	0	0.03	0.10	0.08	0	0	0.11	−0.11	0.15	0	0.18	−0.10	0	0.05	−0.09	0.05	−0.11	0.28	0.17	−0.11
0.10	0.12	−0.06	0	0	0	0.10	0.09	0.04	0	0	−0.11	0.31	−0.17	0.19	0	−0.07	0.10	−0.14	0	0	0	0.33	−0.08
0.22	[illegible]	0	0	0.06	0	0.09	0.15	0	0	0	0	0.17	−0.22	0.30	0	−0.04	0.07	−0.15	0.07	0	0	0	0.20

The inovations are modeled as Laplace distributions with location 0 and the following scale parameters.

Hour	1	2	3	4	5	6	7	8	9	10	11	12
s	0.08	0.08	0.09	0.10	0.09	0.08	0.07	0.07	0.08	0.07	0.06	0.06
Hour	13	14	15	16	17	18	19	20	21	22	23	24
s	0.05	0.05	0.05	0.05	0.06	0.07	0.08	0.07	0.06	0.06	0.07	0.08

Table 27: Parameters in hourly VAR(1)-model. s parameters are rounded to 2 decimals

9.3 Hourly load process parameters

The hourly load AR(p) process can be seen below.

$$X_{load}(t) = \gamma_{1,k} X_{load}(t-1) + \gamma_{2,k} X_{load}(t-2) + ... + \gamma_{p,k} X_{load}(t-p) + \sigma_k \xi_{load}$$

	$\gamma_{1,k}$	$\gamma_{2,k}$	$\gamma_{3,k}$	$\gamma_{4,k}$	$\gamma_{5,k}$	$\gamma_{6,k}$	$\gamma_{7,k}$	$\gamma_{8,k}$	$\gamma_{9,k}$	$\gamma_{10,k}$	$\gamma_{11,k}$	$\gamma_{12,k}$	$\gamma_{13,k}$	$\gamma_{14,k}$
k=1	1.03	-0.22	-0.01	-0.02	-0.04	-0.08	0.02	-0.02	0.01	-0.09	0.06	-0.08	0.03	-0.07
k=2	1.09	-0.22	-0.03	-0.03	-0.02	-0.05	0.01	-0.03	0.00	-0.02	-0.01	-0.01	-0.04	0.00
	$\gamma_{15,k}$	$\gamma_{16,k}$	$\gamma_{17,k}$	$\gamma_{18,k}$	$\gamma_{19,k}$	$\gamma_{20,k}$	$\gamma_{21,k}$	$\gamma_{22,k}$	$\gamma_{23,k}$	$\gamma_{24,k}$	$\gamma_{25,k}$	$\gamma_{26,k}$	$\gamma_{27,k}$	σ_k
k=1	-0.02	-0.01	0.03	0.00	-0.05	-0.02	0.00	-0.05	0.08	0.34	-0.41	0.02	0.04	0.0072
k=2	-0.05	-0.01	0.02	0.01	-0.04	0.00	-0.01	-0.01	0.02	0.25	-0.32	0.05		0.0109

Table 28: Parameters in hourly AR(p)-model. γ parameters are rounded to 2 decimals