DATA ENVELOPMENT ANALYS – EFFICIENCY ANALYSIS ON 17 MIDDLE-SIZED HOSPITALS IN SWEDEN

Data Envelopment Analysis as method of comparison for specialized care production at 17 hospitals between 2012 and 2016

Master Thesis

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Abstract

Rising costs of healthcare in most OECD-countries have contributed to a quest for research into the field of healthcare costs and efficiency, something that has not been free of controversies. As a result of that, healthcare financers as well as providers have become more inclined to measure performance and compare themselves to others with the same responsibilities. In Sweden, the Association of Local Authorities and Regions, SKL, has since 2006 measured and compared the costs and production of entities in the healthcare sector in what is called “Öppna Jämförelser”, Open comparisons (SKL.se/oppnajamforelser). However, most of the comparisons have been between county councils.

In this thesis, the entities compared are hospitals. The technique used is Data Envelopment Analysis (DEA), in its different revised forms. The entities, i.e. hospitals are seen as Decision-making units (DMUs) that are compared in order to find out which hospitals that are efficient in relation to other hospitals and which ones that have potential to increase their efficiency levels. One of the aims of the thesis is to find whether the technique is robust and reliable. A revised version of DEA, namely Multiple-criteria DEA (MCDEA), is also used and compared with the classical one.

As hospitals in Sweden all are financed by county councils, their sizes and patient bases differ depending on how many people live in the region or sub-regional area and how many hospitals are active there. Some county councils that had not reported complete data on hospital level to Swedish Association of Local Authorities and Regions (SKL) have no hospitals represented in this study.

Of the 17 hospitals included, results show that the smallest ones face increasing returns to scale while the bigger ones face either constant or decreasing returns to scale. Spearman’s rank correlation tests show correlations between the efficiency ranks of the hospitals and their ranking orders in some other usually used indicators used in healthcare, such as length of stay, patient satisfaction rate, overcrowding and mean DRG-point.

Compared to the classical DEA, MCDEA performs much better and easing the efficiency score limit of 1.00 shows that the efficient hospitals get different scores higher than 1.00 and are thus discernible and rankable. It is concluded that DEA is a reliable and robust technique and the revised version, MCDEA, is better than the classical one.

Key words: Data Envelopment Analysis, efficiency in healthcare, specialized care, Swedish healthcare system, hospital efficiency, MCDEA

Table of contents

1. Background and earlier research 3
1.1 Purpose and questions 7
1.2 Delimitation 7
2. Method and theories 8
2.1 Production theories 8
2.2 Productivity, efficiency and effectiveness 10
2.2.1 Different types of efficiency 11
2.3 Choice of variables 12
2.4 Data Envelopment Analysis (DEA) 13
2.4.1 Sources of Variation 13
2.4.2 The Mathematical Model 15
2.4.3 Multiple-Criteria Data Envelopment Analysis (MCDEA) 18
2.5 Correlation tests – Spearman's rho 20
2.6 Data collection 20
3. Results 22
3.1 Model A: *2 output and 4 input variables with Classical DEA* 22
3.2 Model B: 2 output and 2 input variables with Classical DEA 27
3.3 Model C: *1 output and 2 input variables with Classical DEA* 28
3.4 Model D: *2 output and 4 input variables with BiO-MCDEA-Supper-efficiency* 29
3.5 Correlation between efficiency scores and environmental factors 33
4. Discussion and conclusion 35
4.1 Analysis and discussion 35
4.2 Conclusion 37
References 40
Appendices 45

1. Background and earlier research

The healthcare sector in Sweden has been one of the areas with steady increase in expenditures and was the biggest expenditure area of the public sector in 2015, standing for 25% (Statskontoret, Den offentliga sektorn i korthet, 2017, p. 29). Between years 2000 and 2016, especially after 2008 when the geriatric care was included in order to make it comparable with other OECD countries, Sweden's increase in healthcare expenditure as share of GDP was one of the highest among OECD countries. A comparison between Sweden and some other OECD countries in terms of healthcare expenditure as share of GDP is in table 1.

Countries	*2000*	*2008*	*2016*	*Change 2000-2008*	*Change 2008-2016*	*Change 2000-2016*
Sweden	7,41	8,31	11,01	12%	32%	48,5%
Denmark	8,10	9,51	10,37	17%	9%	27,9%
Norway	7,71	7,97	10,46	3%	31%	35,7%
Finland	6,84	8,08	9,41	18%	16%	37,6%
Germany	9,84	10,18	11,27	3%	11%	14,5%
United Kingdom	6,01	7,74	9,75	29%	26%	62,1%
France	9,54	10,10	10,98	6%	9%	15,1%
Italy	7,58	8,56	8,94	13%	4%	17,9%
Australia	7,60	8,26	9,59	9%	16%	26,1%
The Netherlands	7,06	9,51	10,50	35%	10%	48,8%
United States	12,51	15,29	17,21	22%	13%	37,6%
Japan	7,15	8,20	10,85	15%	32%	51,7%
Korea	4,00	5,82	7,67	45%	32%	91,9%
Canada	8,28	9,47	10,34	14%	9%	24,9%
Austria	9,22	9,64	10,37	4%	8%	12,4%
Switzerland	9,34	9,78	12,38	5%	27%	32,6%
Chile	6,29	6,69	8,45	6%	26%	34,3%

Table 1: Healthcare expenditure as share of GDP, 2000-2016, source: (OECD, 2017)

Sweden has a decentralized healthcare system, with 20 county councils having the statutory responsibility to provide the residents within their geographical areas with healthcare. The residents of Sweden are a bit over 10 million. The population increased by 7.4% between 2009 and 2016 (SCB, Preliminary population statistics, by month, 2017). During the same period, people of the age 65 years and older, who are the main consumers of specialized healthcare, as share of the total population grew from 18.1% to 19.8% (SCB, Population statistics, 2018). Beside the county councils, there are 290 municipalities that have a wide range of responsibilities, amongst them care for elderly and home-based healthcare. One municipality, Gotland, functions both as municipality and county council (Levin, Paul T., 2013, pp. 15-20 & 26).

Lately, there are omnipresent discussions about efficiency, in reality lack of it, in the healthcare sector and one of the most debated national enquiries in latest years; SOU 2016:2 *Effektiv Vård* led by Göran Stiernstedt, claimed that Swedish healthcare is inefficient and too hospital-centered. The enquiry proposed a transfer of some tasks from hospitals to the primary care. Efficiency is an economic term which denotes quality of doing something well and effectively, without wasting time, money, or energy (Longman English Dictionary, 2018). However, in comparison to some comparable countries, Swedish healthcare system was not as hospital-centered as often claimed. The number of hospital beds per 100000 inhabitants in Sweden which already was one of the lowest in the EU, decreased remarkably by 7.6% between 2008 and 2015 (Eurostat, Hospital bed per 100000 inhabitants). Probably as a consequence of that, bed occupancy rate between 2010 and 2016 increased from 89% to 92% (Väntetider.se, 2018). The number of discharges from inpatient care was decreasing and lower than both the Nordic and EU averages between 1990 and 2014. That can be seen in Diagram 1.

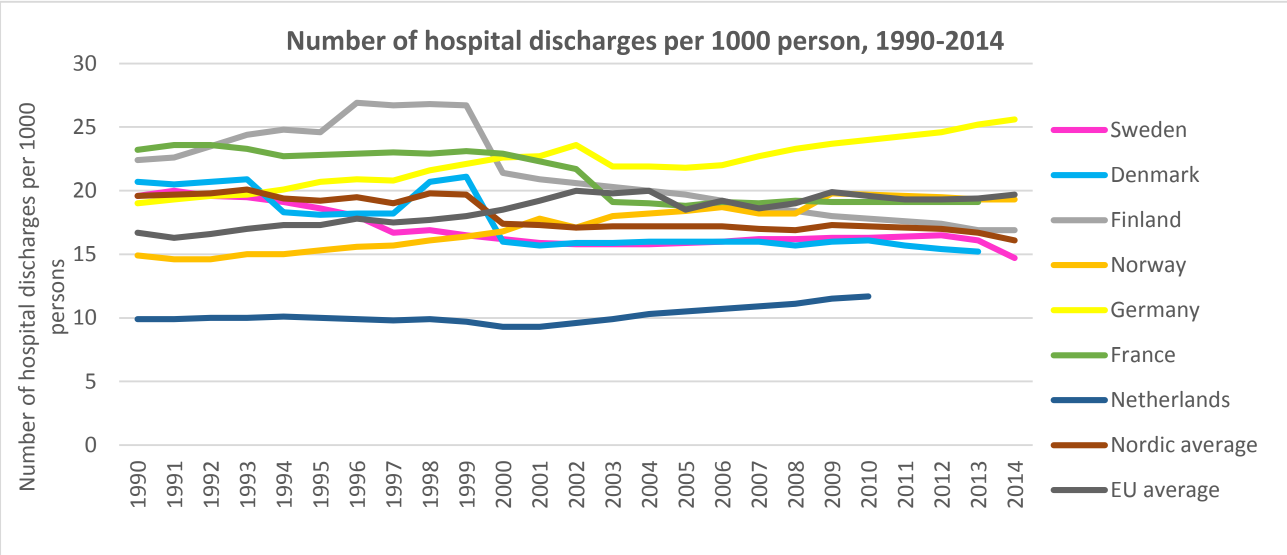

Diagram 1: Number of discharges from inpatient care per 1000 persons, 1990-2014, source: WHO

Anell et al. (2012) discussed the extent of primary care in Sweden in their Health System Review and concluded that the primary care was on rise since the beginning of 1990s but that the past investments have affected the priorities to be hospital-based (Anell et al., 2012: 135). Diagram 2 shows that the rise has not been significant as Sweden continues to be the country with fewest number of outpatient contacts per person among both Nordic and EU-member states.

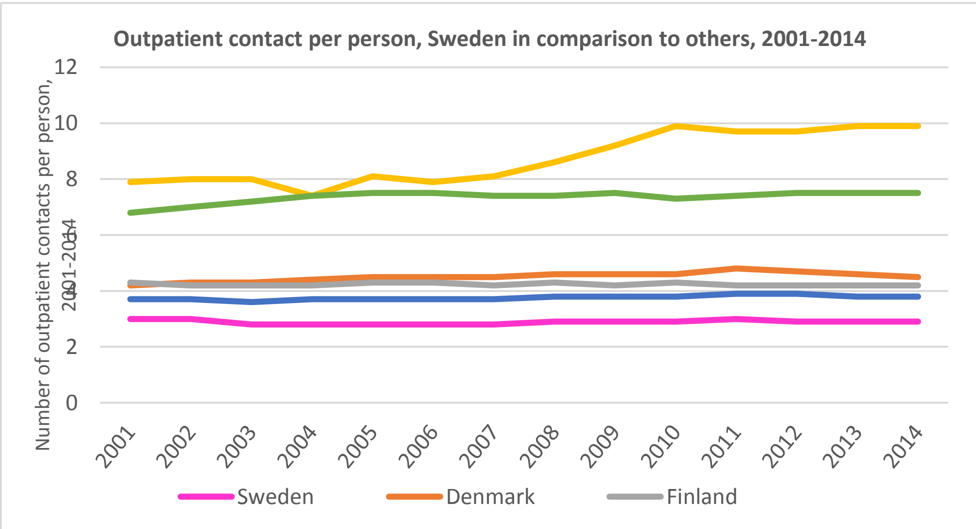

Diagram 1: Number of outpatient contacts per person, 2001-2014, source: WHO

Nevertheless, there has been a shift in shares in the portion of the costs incurred by the respective care forms. Between 2001 and 2007, per-capita-costs of inpatient curative and rehabilitative care increased by 31% while the increase for outpatient care form was 25%. Between 2007 and 2015, the increase for inpatient per-capita-costs was 20% but the increase for outpatient per-capita-costs was 32%. That can be a sign of cost-consciousness among the financing county councils as inpatient care is more expensive than outpatient care. Diagram 3 shows annual costs per capita in Euro.

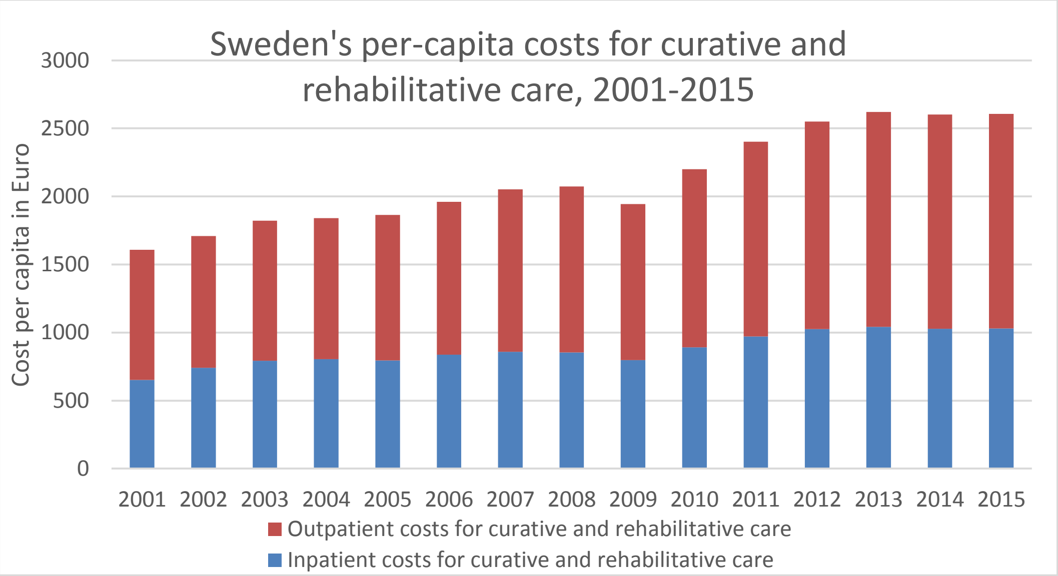

Diagram 3: Sweden's per-capita-costs for curative and rehabilitative care 2001-2015, sources: Eurostat for healthcare cost data and Statistics Sweden for population. The per-capita-costs have been calculated by the author of this paper. Eurostat has total costs and inpatient costs for curative and rehabilitative care but not outpatient cost.

The data on outpatient care in diagram 2 and inpatient care based on discharges from hospitals in diagram 1 show that Sweden has very low consumption of outpatient care and lower than EU- and Nordic averages of inpatient care. This can be interpreted as comparably low production of healthcare, even though Sweden is among the countries with highest numbers of doctors and nurses per 1000 inhabitants. That undesired development has led to numerous reports on healthcare efficiency in Sweden, among them the above-mentioned enquiry "SOU 2016:2 – Effective care".

The decreased level of productivity per specialist staff at Swedish hospitals has become a research area within the field of economics. Comparison between county councils and hospitals has gained in both importance and extent and better access to data during the latest years has made it easier. Anell et al (2012) looked into the efficiency levels of healthcare in Sweden and referred to Gerdtham et al. (1999) who had found that the technical efficiency was significantly low in both inpatient and outpatient care before 1990s but started to bounce upwards from the beginning of 1990s. However, productivity has not increased during the last decade and one the reasons, as Gerdtham (1993) points out, can be that elderly patients as share of total patient population has increased and it is harder to achieve productivity gains. Both Anell et al. (2012) and Gerdtham et al. (1993 & 1999) had county councils as units of analysis, not individual hospitals.

Kittelsen et al. (2015) compared hospitals in Nordic countries in terms of efficiency and found that there was differences in efficiency between the countries and that it was mostly the country specific possibility sets that distinguished them from each other. In their study, Swedish hospitals was said to have the lowest efficiency rates while Finnish hospitals were deemed as the most efficient ones. However, Kittelsen et al. (2015) had hospitals as units from Finland, Denmark and Norway but county councils from Sweden. Hospitals are providers while county councils are purchasers and financers of healthcare. Their acknowledgment was “As so often, the strongest results are not what we can explain, but what we cannot explain. There is strong evidence, independent of method, that there are large country-specific differences that are not correlated with any of our other variables”. They theorized the optimal hospital size to be small while optimal county council size to be large. That meant that the country-specific productivity frontier that Sweden was compared to was higher than the other countries’ and therefore it was deemed as inefficient relative to what was achievable for county councils. That was also reflected in the difference in scale efficiency which was <0.1 for Denmark, Norway and Finland but >1.0 for Sweden. In light of that, comparing Swedish hospitals with each other is a much desired research theme and this paper is an attempt to fulfill that goal.

Sweden’s healthcare system concerning quality of care is often ranked high in international comparisons, not least among OECD countries. One of the most prestigious reports, GBD 2015 Healthcare Access and Collaborators, published in the Lancet 2017 ranked Sweden as 4th best in the world in terms of health outcomes, behind only Andorra, Iceland and Switzerland (Lancet 2017; 390: 231–66, p. 237).

This master thesis in economics aims to focus on relative efficiency of Swedish hospitals by using Data Envelopment Analysis (DEA) as comparison technique. Here, the production data are case-mix adjusted by the classification method of Diagnosis-Related Groups (DRGs). Procedures and healthcare episodes are grouped in terms of resource use. Derived from Reyes and Högberg (2015) the formula for calculation of casemix-adjusted DRG weights for inpatient or outpatient at hospital level can be written as:

$$\mathrm{Ij} = \frac{\sum_{\mathrm{i}=1}^{n} \left(\mathrm{w_i}\ \mathrm{N_{ij}} \right)}{\mathrm{N_j}}$$

Where j= hospital j=1,2,….17

I_j = casemix for hospital j

i = NordDRG i where i=1,2,…n

N_j = number of inpatient episodes of care or outpatient care contacts

1.1 Purpose and questions

In my master's thesis of year 1 I focused on comparing county councils in terms of efficiency in provision of somatic and psychiatric specialized care. This thesis follows on the same path with the focus on the entities that produce specialized care, i.e. hospitals, rather than the county councils they are financed by. As there have been research on comparisons between university hospitals, in this paper the entities compared are middle-sized hospitals, based on categorization of the Medical newspaper, Dagens Medicin, which annually ranks the hospitals in three size categories based on their performances in some indicators (Dagens Medicin, Bästa sjukhusets databas).

The research questions to be answered are as follows:

- ✓ Are there differences in efficiency between Swedish middle-sized hospitals?
- ✓ Is DEA robust as a technique for efficiency comparison?
- ✓ Are the relative efficiency scores of the hospitals robust and consistent over years?
- ✓ Does hospital size play role in relative efficiency of the hospitals?
- ✓ Does a change in the number of input and output variables cause efficiency scores to change?
- ✓ If the classical DEA method does not discriminate enough, do revised versions perform better?

In many earlier studies, productivity has been the main theme. Productivity and efficiency are not interchangeable as productivity is a simpler indicator than efficiency which takes into consideration optimal mix of inputs and outputs. For the purpose of measuring compatibility between productivity and efficiency, potential correlation between efficiency scores and productivity beside some other factors such as length of stay, ratio between outpatient and inpatient care and hospital size will be tested for.

1.2 Delimitation

It would be optimal to compare all hospitals in Sweden, even university and small hospitals. There are inherent structural differences due to different responsibilities, such as highly specialized care being the task of university hospitals only, and lack of emergency sections in small hospitals. With that in mind, I have chosen to have as comparable hospitals as possible and middle-sized hospitals with emergency sections live up to that criteria. However, in Sweden, most of the nationally available cost data have been on county council level, only in recent years data on hospital levels have been made available by SKL. Due to lack of data for many middle-sized hospitals for most of the 5 years of interest, I've chosen 17 hospitals with complete data for all years to be used as decision-making units (DMUs) in the thesis.

2. Method and theories

This thesis is based on data accumulated by the Swedish Association of Local Authorities and Regions, SKL, and the Swedish National Board of Health and Welfare, Socialstyrelsen, at hospital level. The thesis is delimited to inpatient and outpatient specialized somatic care, and inpatient specialized psychiatric care. As different hospitals can have different patient compositions, the production data are casemix-adjusted with DRG. Complete data on total DRG-points for inpatient care have been retrieved from Socialstyrelsen's statistical database for each of the 17 hospitals used in this thesis, for the opted period of 2012-2016. Outpatient somatic care data have been collected with the help of a statistician at Socialstyrelsen.

Collection of cost data had rather a longer path. A statistician at SKL helped by asking each of the county councils who administer the hospitals for permission to make the data available to me. After a waiting period of some weeks I got permission for the 17 hospitals included in this thesis. Some of the county councils have no hospital in this study as their cost data on hospital level, reported to SKL, were complete only for 2016. Lack of data at hospital level has been one of the main problems behind some earlier studies on efficiency measurement using county councils as proxy-DMUs for hospitals, such as Kittelsen et al. (2015). Nevertheless, I hope that in the future, with more detailed data at hospital level, comparison across all hospitals in Sweden will be easier.

2.1 Production theories

Healthcare production is not entirely similar to industrial production of goods. Healthcare is not a "good" in terms of people demanding it as much as possible. Healthcare, rather, is something that people are obliged to seek when illness and diseases affect them. Thus, the maximum production of healthcare is not the goal, because the goal is maintaining maximum healthiness. If it is possible to achieve maximum healthiness without needing to be admitted to a hospital, then that should be strived for. The goal of remedy from illness is achieved in two stages, where inpatient and outpatient specialized care are output variables in the first stage and input variables in the second stage. Healthiness which is an *outcome* of a combination of healthcare-related factors, such as inpatient care, and non-healthcare-related factors, such as life-style, is the output variable in second stage. The first stage, in which sheer quantitative production data is calculated and analyzed, measures *efficiency* while the second stage which takes into account even qualitative data measures *effectiveness* (Javid, 2017). In this thesis, measuring *relative efficiency* is the main target so potential differences across hospitals in effects of treatments on patients' health are not captured.

In producing healthcare, similar to goods production, there are inputs that together produce the output(s). Cobb-Douglas classical production function is the most famous and built on two inputs;

capital and labour. The mathematical equation is: $Q = AK^{\alpha}L^{1-\alpha}$ where K stands for capital, L for labour and A is a constant representing total factor productivity - i.e. the part of the output that is not explained by either capital or labour – α is the parameter that shows the rate of capital's contribution to production while 1-α shows the labour's (Borjas, 2010, p. 170).

The unit cost of capital is $r = \alpha A \left(\frac{K}{L}\right)^{1-\alpha}$ while the unit cost of labour is $w = (1-\alpha)A\left(\frac{K}{L}\right)^{\alpha}$. If this basic formula is used for healthcare, all forms of staff- and service-related contributions are summed to one variable, *labour*, while all forms of material inputs are assumed to be represented by *capital*. As in hospitals there are different types of cost sources, both within labour part – such as different wages for different professional groups - and within capital part, the simple Cobb-Douglas function would be too simplistic. In this paper, as proxies for production factors, I have 4 monetary variables so the hospitals will have a production function:

$$Q = Q_1 + Q_2 = AX_1^{\alpha}X_2^{\beta}X_3^{\gamma}X_4^{\delta}$$

Where Q_1 = DRG-points in inpatient care, Q_2=DRG-points in outpatient somatic care, X_1=ward costs for inpatient care, X_2=indirect costs for inpatient care, X_3=reception costs for outpatient care and X_4=indirect costs for outpatient care. A, β, γ and δ are coefficients of X_1-X_4 and sum up to 1.0.

As the statutory responsibility of healthcare provision is on county councils rather than individual hospitals, the county councils may follow different strategies in distributing specialties at different hospitals depending on their catchment area, population size, perceived economies of scale and patient safety. Bigger regions/county councils with university hospitals might be more inclined to concentrate highly specialized care at their university hospitals so that other hospitals, i.e. non-university middle-sized or small hospitals, in the county council probably will have lower average DRG-weights compared to hospitals of the same size in county councils without university hospitals. That can affect production patterns and efficiency levels of the individual hospitals.

The basic Cobb-Douglas function with α+β=1 assumes *constant returns to scale (CRS)* from the inputs. Many studies, such as: Kristensen (2008) concerning Danish hospitals; Wang et al. (2006) concerning Australian hospitals; and Marini & Miraldo (2009) concerning British hospital, have all concluded that healthcare production at hospitals doesn't follow CRS. Like many other sectors, the hospitals' production follows variable returns to scale (VRS), with increasing returns to scale (IRS) in the beginning, before reached optimum, and decreasing returns to scale (DRS) after the optimal production solution where the ratio between output and input is as largest.

2.2 Productivity, efficiency and effectiveness

The relationship between effectiveness and efficiency is not linear but efficiency (low waste) mostly is associated with high effectiveness (high attainment). The ratio between effectiveness and efficiency is called *productivity* (OEECoach.com). In medical field, there is also another term, efficacy, which is widespread and is defined as *the ability, especially of a medicine or a method of achieving something, to produce the intended result* (Cambridge Dictionary of English).

Efficient resource allocation in healthcare requires both productivity enhancement, with the help of innovations, and optimal mixture of input variables. Output variables often are uncontrollable, especially for inpatient care, due to most of the patients being admitted acutely. At hospital level, differences in share of acute admissions can be a source of efficiency variation. Acute/unplanned admissions have dominated over planned admissions for most of the modern healthcare history. The share of acute admissions have even increased during the latest years, which has resulted in longer waiting times at emergency receptions and overcrowding at inpatient wards (Socialstyrelsen, 2017, p. 16). As output is mostly uncontrollable in the context of Sweden's tax-funded healthcare system with universal coverage, inputs are used as tools for enhancing efficiency.

Cost minimization, like profit maximization, can be calculated by the help of *marginal rate of technical substitution* (MRTS) which denotes increase of capital needed in order to keep output unchanged given a unit decrease in labour. Cost is minimized when the increase in capital in order to offset a unit decrease of labour is equal to the ratio of unit cost of labour over unit cost of capital; $MRTS_{KL}x = \frac{w}{r}$. That state is called Pareto Optimum. (Connolly & Munro, 1999, p. 25).

If we take specialized somatic care with two levels, i.e. *outpatient* care where patients visit the hospitals and are consulted or treated during the day so that they return home, and *inpatient care*

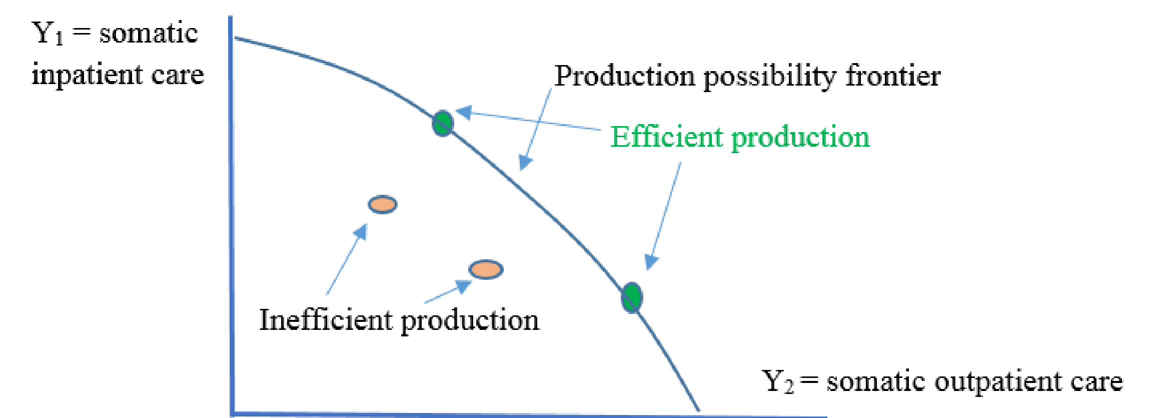

Figure 1: Production possibility curve

where patients are admitted to the wards and stay at the hospital at least one night, the optimal resource allocation can be achieved in a way that can be depicted in a production possibility curve, as in figure 1. Y_1 denotes *DRG points produced in inpatient specialized somatic care* while Y_2 stands for *DRG-points produced in outpatient specialized somatic care.* Then optimum is reached when $MRTS_{KL}Y_1 = MRTS_{KL}Y_2$.

2.2.1 Different types of efficiency

Efficiency, as described in earlier section, is focusing on inputs in order to minimize waste, subject to given amount of outputs. For finding inefficiencies in the hospitals compared, it is important to be sure that the reason behind inefficiencies are suboptimal use of available resources. If the reason is inherent differences in the input variables across compared hospitals, then any attempt to make the seemingly inefficient hospitals efficient may lead to failure and despair. In that sense, the technique used in this thesis, Data Envelopment Analysis (DEA), assumes that the variables are homogenous across Decision-Making Units (Zhu, 2015).

If the compared hospitals use exactly the same variables with the same characteristics and produce differently efficiently, then the relatively inefficient units face *technical inefficiency*. It means that the inefficient hospitals have a gap between their actual production level and what is possible to produce, given their current level of inputs. The maximum production possibility cannot be read from the comparison and can be higher than the most efficient units in the model have managed to achieve. If the units compared face known maximum output levels that differ between them, then a difference in production can be due to the difference in their sizes, i.e. *scale inefficiency* (Sherman & Zhu, 2006, p. 52). This was one of the conclusions that Kittelsen et al. (2015) came to, when they found that for Denmark, Norway and Finland the optimal efficiency levels were lover as the units in the study were hospitals while for Sweden the optimal efficiency level was higher as the county councils were the used units.

If instead of physical units, such as number of physicians, number of nurses, number of beds, etc. the input variables are in monetary terms, one of the reasons behind the differences in production costs may be due to differences in price of the same input variables across the compared units. Demand and supply of the professional groups in different parts of Sweden can play a central role in potential wage disparities within each professional group across hospitals. If, for example, the average monthly wage of employees at hospital A is 40000 SEK while it is 36000 SEK at hospital B, and the same number of employees in both hospitals produce the same quantity of healthcare, hospital A will still be more expensive. The difference in efficiency in this case is *price inefficiency* (Sherman & Zhu, 2006, p. 53) and if not correctly understood, the financers and managers might assume that hospital A is technically inefficient because its cost per produced unit of care is higher than that of hospital B. The input variables in this thesis, as outlined in the next section, are based on monetary values so price inefficiency may arise.

When multiple inputs and outputs are involved, the mixture of the input variables becomes an important issue. If in the example above, the two hospitals A and B used several inputs with the same price and producing potentials but still got different output levels, then they might have had different mixtures of inputs. The compared hospitals that are relatively inefficient in comparison to some other ones are then affected by *allocative inefficiency* and must change their mixture of inputs in order to reach production levels of the efficient units (Sherman & Zhu, 2006, p. 54).

2.3 Choice of variables

The complete version of the DEA models will have the variables depicted in figure 2.

As output variables I have chosen:

1. Total DRG-points in inpatient somatic and psychiatric specialized care
2. Total DRG-points in outpatient somatic care

As input-variables I have chosen the following cost variables, depicted as well in figure 4:

1. *Direct costs for inpatient somatic and psychiatric care ("avdelningskostnader")*
2. *Indirect costs for inpatient somatic and psychiatric care*
3. *Directs costs for outpatient somatic and psychiatric care ("mottagningskostnader")*
4. *Indirect costs for outpatient somatic and psychiatric care*

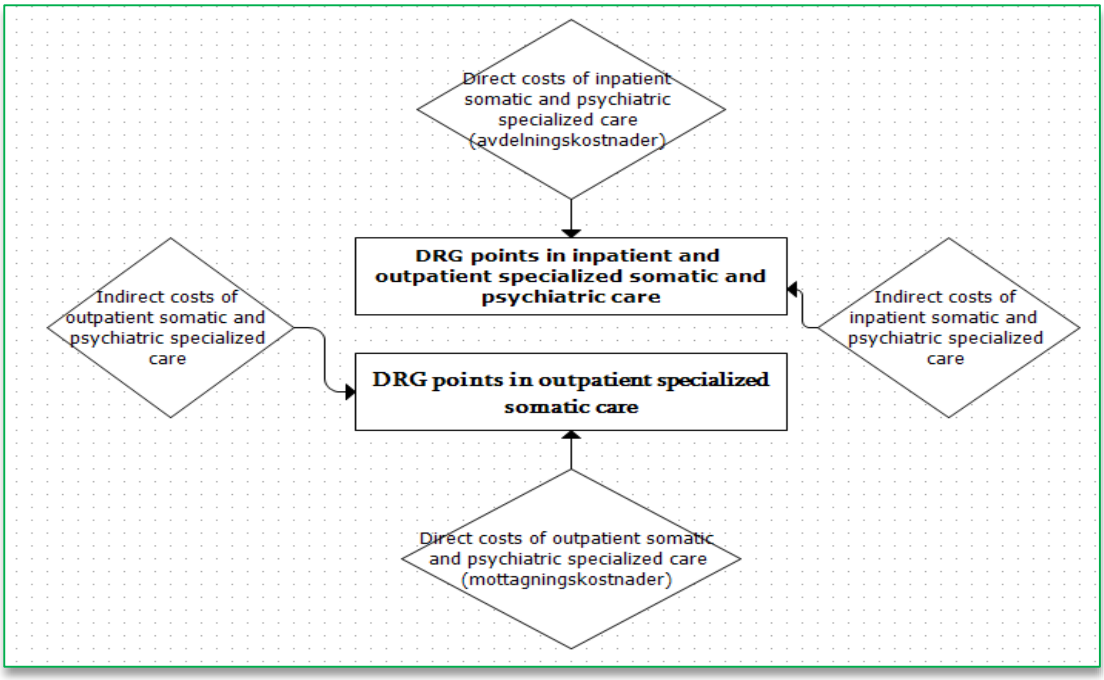

Figure 2: Chosen input and output variables in the final model

2.4 Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a non-parametric statistical comparison technique that first was developed by A. Charnes, W.W. Cooper and E. Rhodes in 1978 in an article by the name of "*Measuring the Efficiency of Decision Making Units*" with the purpose of measuring efficiency in not-for-profit firms in the public sector (Charnes et al., 1978*)*. Since then, all units that are compared with the help of DEA are called Decision Making Units (DMUs), which either maximize their production given a fixed amount of inputs or minimize their costs given a fixed level of production. An advantage of non-parametric tests is that they do not rely on assumptions about characteristics of unknown parameters, e.g. normal distribution in mean or stationarity in variance.

In the beginning the monetary costs of compared units were not the main variables to compare efficiency with. That was one of the reasons why Charnes et al. (1978) first applied the model to nonprofit organizations. Nevertheless, DEA has caught the eyes of the management in the for-profit sectors of the economies and been applied to DMUs for goods production. Though, in the case of hospitals, the main purpose is to maximize the outcomes in terms of the patients' health, doing it at as low cost as possible is not only desirable but also demanded by the healthcare law *Hälso- och sjukvårdslagen 1982:763*.

If we suppose that there are *n* DMUs the classical DEA model (Wen et al., 2017) is consisting of properties below:

DMU_k: the *k*th DMU, where k=1,2,…n
DMU_0: the target DMU
$x_k \in R^{p*1}$: the input vector of DMU_j, where k=1,2,…,n
$x_0 \in R^{p*1}$: the input vector of the target DMU_0.
$y_k \in R^{q*1}$: the output vector of DMU_j, where k=1,2,…,n
$y_0 \in R^{q*1}$: the output vector of the target DMU_0
$u \in R^{p*1}$: the vector of input weights, and
$v \in R^{q*1}$: the vector of output weights

2.4.1 Sources of Variation

There are many sources of variation in a DEA formulation so it is important to define the model in use as precisely as possible so that it fits the data being analyzed (Hayes, 2005). The first of the sources of variation is ***formulation*** such that DEA can has either;

- *Primal form,* with rows as representing the model, or
- *Dual form* in which columns are representing the model.

When there are many DMUs to be compared, the dual form is the most common approach. In this paper 17 hospital are compared in respect to relative efficiency so the formulation of the DEA here is the *dual form.* The second source of variation that it important to define is the ***orientation*** with either;

- *Output-oriented DEA*, where the main focus is maximizing production, or
- *Input-oriented DEA*, where the main target is minimizing usage of resources given certain levels of output.

The main purpose of this thesis is to compare a number of middle-sized hospitals in Sweden with respect to their healthcare production subject to the output levels treated as non-discretionary and thus minimizing inputs as the main target. Though, both input- and output-oriented versions will be tested and compared. Different specialties have different distributions of care on acute (unplanned) and elective (planned) parts. Those with higher shares of elective care have more opportunities to control their costs than hospital with lower shares of elective care. In that sense, differences in allotment of specialties can be a reason behind eventual differences in efficiency scores. Nevertheless, in a comprehensive study, Rajasekar and Deo (2014) used both input- and output-orientation in many different DEA models for comparing Indian ports in terms of efficiency and came to the conclusion that there was no major difference between them in efficiency scores.

The third important source of variation is ***returns to scale***, divided and described by Burda & Wyplosz (2005) as:

- *Constant returns*, so that the output/input ratio is constant over time, irrespective of the amount of inputs, or
- *Variable returns*, so that the contribution of the inputs to outputs changes over time, increasing sometimes and decreasing other times.

Ample studies from hospitals in different countries, such as: Wang et al. (2006) from Australian hospitals; Kristen (2008) from Danish hospitals; and Marini & Miraldo (2009) concerning British NHS hospitals, have all concluded that there are economies of scale at small hospitals and diseconomies of scale at large size hospitals. Economies of scale means increasing returns to scale while diseconomies of scale denotes decreasing returns to scale. There are many reasons, such as limited space, operation rooms, shared medical equipment, and etcetera. In this paper, both Charnes-Cooper-Rhodes (CCR) model assuming constant returns to scale and Banker-Charnes-Cooper (BCC) model assuming variable returns to scale will be used to find out whether hospitals face constant or variable returns to scale in their healthcare production.

The fourth source of variation in a DEA is the ***manager's ability to control*** the variables as they wish. Thus, variables can be either;

- *Discretionary*, so that they are fully controllable and changeable, or
- *Non-discretionary*, so that managers at the counties cannot fully control them.

In this thesis, the outputs, which depend on the care-seeking patterns of patients and the circumstances of diseases in the country, are treated to be uncontrollable and thus non-discretionary. Though county councils with more than one hospitals can choose to steer the patients with specific needs to specific hospitals, having some control on the seeking patterns. Input variables are controllable, thus discretionary. In assessing efficiency with DEA, both discretionary and non-discretionary variables are considered, but in determining whether to maximize or minimize the function, only the discretionary variables are included (Hayes, 2005, p 71).

The fifth and last source of variation is the **interaction** between the input variables and also between the output variables. Thus, the model can be either;

- *Additive*, in which input or output variables add up when changed, or
- *Multiplicative*, in which input or output variables are multiplied to each other once changed.

The weights of the variables are chosen by the software itself and the operator can put some constraints such as requiring the weights to be non-negative or strictly positive. In some cases, the *a priori* weights for a variable chosen by the Excel Solver program itself might be unrealistic so more constraints may be needed in order to limit its over- or underestimation (Hayes, 2005, p. 49).

The mathematical formula used in DEA handles the weighted ratio between sum of outputs and sum of inputs ranging from 0 to 1.00 with the restriction that both inputs and outputs are non-negative. The DMUs that have optimal efficiency in comparison to other DMUs, and thus are not able to further increase their efficiency levels, will have efficiency value 1.00 and will be on the *efficiency frontier*. For ranking purpose, the efficiency scores of the efficient hospitals in this paper will be allowed to exceed 1.00, letting the hospitals to be super-efficient. Then, the most efficient hospitals can be discerned and output excesses, i.e. slacks, will be taken into account. The DEA model here uses optimization technique of Excel Solver where the output and input variables can be conditioned to be non-negative.

2.4.2 The Mathematical Model

The first ones who developed the DEA were Charnes, Cooper and Rhodes so the model was dubbed CCR and focused on output maximization and assumes constant returns to scale. That model (Charnes, et al. 1978, p. 430) has the formula:

$$\max h_0 = \frac{\Sigma_{r=1}^{s} u_r y_{r0}}{\Sigma_{i=1}^{m} v_i x_{i0}}$$

Subject to:

$$\max h_0 = \frac{\Sigma_{r=1}^{s} u_r y_{rj}}{\Sigma_{i=1}^{m} v_i x_{ij}} \leq 1,$$

$$j = 1, 2, \dots . n; \qquad u_r, v_i \geq 0 \qquad r = 1, 2, \dots s \qquad \& \quad i = 1, 2, \dots m$$

j=1,2…n stands for the DMUs, y for outputs and x for inputs. It is assumed that there are r=1,2…s different outputs and i=1,2…m different inputs. Each output and input has its own vector of weights represented by u and v respectively. A revised model based on variable returns to scale and first developed by Banker, Charnes and Cooper, in 1984, called BCC model. Here is the input-oriented BCC model formulation:

$$\theta_B = \min \theta,$$

Subject to:

$$\sum_{j=1}^{n} x_{ij}\lambda_j \leq \theta x_{ij_0}, i = 1,2, \dots p$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{rj_0}, where\ r = 1,2, \dots q$$

$$\sum_{k=1}^{n} \lambda_k = 1$$

$\lambda_k \geq 0$, such that k = 1,2,…,n

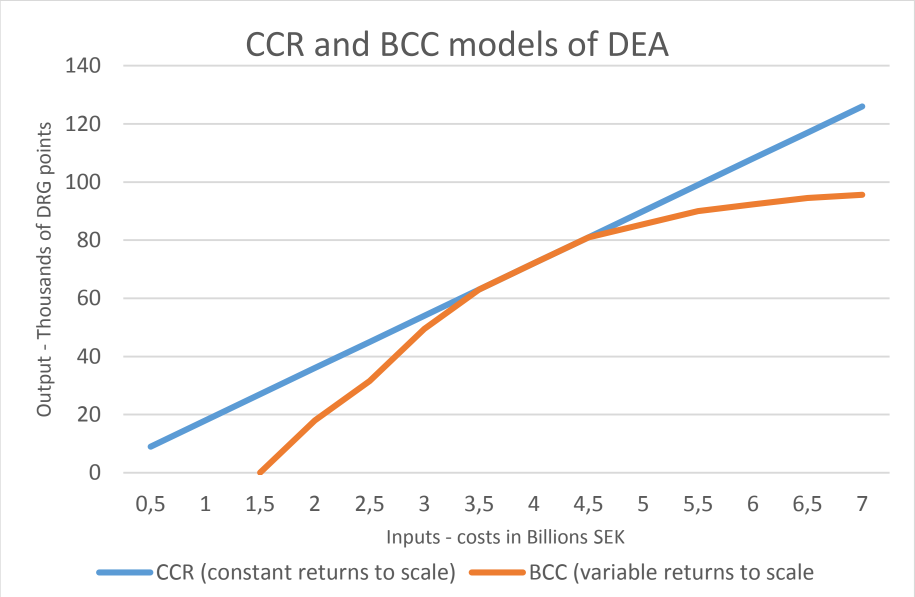

Figure 3: Classical DEA models, CCR and BBC

Hypothetically, the relationship between costs and produced DRG points per hospital, according to the two DEA models would be as in figure 3. It would mean that in BCC model, there is a threshold of optimal size for hospitals. In the hypothetical model of figure 3, a hospital with costs less than 1.5 billion SEK is best to stop its activitities. The reason is that there are some fixed costs, sunk costs such as investment in building, equiment, recruitment of staff etcetera. Left of the 1.5 Billion SEK input-threshold, the production may not take off and the hospital might not be able to operate efficiently.

On the upper end, a hospital with total costs exceeding 5 Billion SEK would benefit from moving some of its activities to other hospitals and thus reducing its size because it produces fewer and fewer DRG points per cost unit. The reason can be that it has reached its limits with current fixed cost factors such as rooms and equipments so for further increase of healthcare production it has to purchase more expensive equipments, invest in new buildings. New investments would make marginal cost of new DRG points much more expensive.

Not all DMUs deemed as efficient will have 1.00 as efficiency score, some will be efficient but still have a very small distance to the frontier, i.e. non-zero slacks; input excesses $s_i^- \neq 0$ and/or output shortfalls $s_r^+ \neq 0$. The DMUs with $\theta^* = 1$ and non-zero slacks are said to be weakly efficient while DMUs with $\theta^* = 1$ and $s_r^+ = s_i^- = 0$ are said to be strictly efficient. In order to get rid of the problem with slacks, the BCC model proposes to take the slacks to their maximal values, as formulated to in Appendix 1.

The optimal number of DMUs in a model, with the aim of having good discriminating power, i.e. being able to find the relatively efficient and non-efficient ones and ranking them based on efficiency scores, is an issue that not all operational scientists are agreeing upon. Golany and Roll (1989) name two conflicting consequences of choosing large number of DMUs; 1. It helps the model in its discriminating power to find the most efficient DMUs that determine the frontier and 2. It might actually decrease the level of homogeneity across DMUs, something that is a precondition for DEA to function properly.

Boussofiane et al. (1991) assume that when many input and output variables are in place, many DMUs will be deemed efficient and the minimum number of efficient DMUs is the product of input and output variables as each of the DMUs try to find at least one output-input ratio that they are most efficient at. If there are X inputs and Y inputs, then the minimum number of efficient DMUs is expected to be Z = X*Y DMUs. Therefore, they suggest that the minimum number of DMUs included in the model should be the product of number of inputs and number of outputs for the model to have any discrimination power. In this study, there are 4 input variables and 2 output variables so the minimum number of DMUs would be 4*2=8 in order for the model to have enough discriminating power to discern the efficient DMUs from the inefficient ones. This study has 17 hospitals as DMUs so that threshold is well fulfilled.

Excel Solver has three methods for DEA, Simplex Linear Programming (LP), Non-linear Generalized Reduced Gradients (GRG) and Evolutionary. Because the Evolutionary method does not rely on derivative or gradient information, it cannot determine whether a given solution is optimal – so it never really knows when to stop (Solver.com, Excel Solver - Evolutionary Solving

Method Stopping Conditions). That method is not used in this thesis. A non-linear problem can have more than one possible region, or a collection of similar values for decision making, where all requirements are fulfilled. More than one top concerning maximization problem or more than one bottoms in relation to minimization problem are possible to prevail and some of them can be false tops or bottoms, which are known as saddle points. Taking these properties into account, non-linear optimal solutions give few guarantees (solver.com, The Standard Excel Solver Limitation). Therefore, in this thesis the main method used is Linear Programming (LP).

2.4.3 Multiple-Criteria Data Envelopment Analysis (MCDEA)

As mentioned earlier, operational scientists have, since the launch of the basic classical DEA model, faced both impediments and proposed solutions in order to mitigate those impediments. The first revision of DEA was made by the founding scientists themselves as the CCR model developed by Charnes, Cooper and Rhodes assumed constant returns to scale so Charnes and Cooper together with Banker revised and came with the BCC model with variable returns to scale.

During the latest years, as DEA has become a popular technique for comparing and evaluating units with each other, especially in the public sector, more and more of the shortcomings have been unmasked. One of the shortcomings has been the problem with low discriminating power as when the number of variables increases to higher than 2, more and more of the DMUs get the efficiency score 1.00 or very close to it, because the increased number of output-input combinations complicate the model so the DMUs can find a niche for themselves and seem efficient. That low-differentiating problem arises even when the number of DMUs is small in relation to the number of variables (Hatami-Morbini & Toloo, 2017). When some of the variables are aggregated so that the number of variables decreases, some of the previously efficient DMUs become inefficient as the combination they might have had an advantage in disappears.

One of the reasons behind low discriminating power in classical DEA is that it has a single criterion of maximizing weighted outputs given the input levels or minimizing the weighted inputs given the output levels. A frequently used alternative version is Multiple Criteria DEA (MCDEA) which puts some new restrictions on the weights of the inputs and outputs so that at least one of them has to be strictly positive for each of the DMUs when they maximize their efficiency scores relative to others.

Multiple Criteria DEA was first used in 1990s. Xiao-Bai Li and Gary R. Reeves article from 1999 is among the most referenced ones in this field. In that article, they go through the two main problems facing the classical DEA, low discriminating power for the DEAs to be ranked and unrealistic weight dispersion among the variables. In their article, they used some examples and calculate with both classical DEA and two forms of MCDEA;

- *minimax*, whose objective is to minimize the maximum deviation between weighted sum of the outputs and weighted sum of the inputs for each DMU, and
- *minsum*, which has the objective to minimize the sum of all deviations between weighted sum of outputs and weighted sum of inputs for the DMUs.

Li and Reeves (1999) concluded that the MCDEA method is much more robust, discriminate better and use more reasonable weights for the variables. The mathematical formulation of MCDEA which is based on the classical DEA, with supplementary criteria, is;

$$\min d_o (or \max h_0 = \sum_{r=1}^{s} u_r y_{rj0}$$

$$\min M,$$

$$\min \sum_{j=1}^{n} d_j$$

$$S.t. \quad \sum_{i=1}^{m} v_i x_{ij0} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{xij} + d_j = 0\,,$$

j=1,…,n, $M - d_j \geq 0$, j=1,…, n, u_r, v_i, $d_j \geq 0$, for all r, i and j

The d_0 is bounded on the interval [0, 1] and represents the deviation variable for DMU_0, i.e. the DMU for which one wants to maximize the efficiency score, while d_j is the deviation variable for the *j*th DMU. M variable in this model represents the maximum quantity among all deviation variables d_j (j=1,…,n). The objective is to minimize the sum of the deviations between weighted outputs and weighted inputs. When there is no deviation between weighted sum of outputs and weighted sum of inputs, we get a ratio of 1, which indicates that the DMU in question is efficient. Ghasemi et al. (2014) came with a revised version of MCDEA by reducing the three objectives from Li & Reeves (1999) to two objectives by arguing that *min d_0* was already captured in $\min \sum_{j=1}^{n} d_j$, and thus not necessary to have both of the objectives. They omitted the first objective, i.e. $\min d_o (or \max h_0 = \sum_{r=1}^{s} u_r y_{rj0})$. They called their model Bi-objective Multiple Criteria Data Envelopment Analysis (BiO-MCDEA) model. By using their model on data for European countries' CO_2 emission, they got satisfactory results in terms of discriminating power for the DMUs, so that they were more easily ranked, and higher weight dispersion, so that all variables were given reasonable weights. They also let the efficiency scores to exceed 1.00, which further increased the possibility to rank as the efficient DMUs, that otherwise would all have 1.00.

2.5 Correlation tests – Spearman's rho

Oftentimes, when there are many data points and the variables are continuous and normally distributed, the parametric statistical test, Pearson's correlations coefficient, is used to test whether a change in a target variable, y, over time can be explained by a change in another variable, x, over that period (Björk, 2015, p. 204-207). In this thesis, in an effort to find some explanations and test reasonability of the efficiency scores, the correlation of efficiency scores of the hospitals will be tested in relation to some factors perceived as impacting the output and input variables. However, as the number of data points is small and the parameters' means or variances are not of interest, another non-parametric test which corresponds to Pearson's correlation coefficient test but which is for non-parametric rank-based data, is used. That test is Spearman's rank-correlation test which has the following formula (Gujarati & Porter, 2009, p. 86);

$$\rho = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

where ρ is the Spearman's correlation coefficient, $\sum d^2$ is the sum of squared differences in rank between the two variables, in this case the hospitals' rank in efficiency score and another variable of interest, while n is the number of units, i.e. 17 for this study.

The coefficient values will be between -1 and +1, in which -1 means perfect negative correlation and +1 perfect positive correlation. A ρ-value of 0 implies no correlation at all. If ρ-value is between +0.5 and +1, then it is said to be strong positive correlation between the variables and conversely, a ρ-value between -0.5 and -1 means strong negative correlation.

2.6 Data collection

The main sources of input data, i.e. cost data, is the Swedish Association of Local Authorities and Regions (SKL). Earlier years, data on hospital level were difficult to access. That was one of the reasons for Kittelsen et al. (2015) to use county councils as DMUs for Sweden though their goal was to compare individual hospital in the four major Nordic countries. However, Recently, SKL has started collecting data on hospital level, but it is an ongoing project that has not included all the hospitals for earlier than 2016. Due to lack of data for some middle-sized hospitals that I wanted to include in the model during the entire period, the study is delimited to 17 middle-sized hospital for which SKL had cost data for the whole period and which also gave permission to use their data in this thesis. I have sorted the hospitals based on the order their county council's order in Swedish official documents. The hospitals chosen for the study are in table 2.

For the output data, in terms of DRG points, the source is Swedish Board of Health and Welfare, Socialstyrelsen. Data on DRG points in inpatient care per hospital are available at their statistical database but data on DRG points in outpatient care is rather scarcer. Therefore the data unit was contacted for data on DRG points in outpatient care for the named hospitals. Unfortunately, the data for outpatient care provided contains only somatic care as psychiatric care is not accessible. That can raise validity questions as costs include both somatic and psychiatric care. It is assumed that the share of outpatient psychiatric care is the same at all hospitals.

Socialstyrelsen has also data on many other aspects of healthcare in Sweden, not least diagnosis-based quality data, data on average length of stay, number of inpatient episodes of care etc. and some of these data are used for performing Spearman's correlation tests.

Order	Hospital name in English	Hospital name in Swedish
1	S:t Göran's Hospital	S:t Görans sjukhus
2	Stockholm South General Hospital	Södersjukhuset
3	Danderyd's Hospital	Danderyds sjukhus
4	Södertälje Hospital	Södertälje sjukhus
5	Vrinnevi Hospital	Vrinnevisjukhuset
6	Västervik's Hospital	Västerviks sjukhus
7	Kalmar County Hospital	Länssjukhuset Kalmar
8	Halland's Hospital	Hallands sjukhus
9	NU-Hospital group	NU-sjukvården
10	Southern Älvsborg's Hospital	Södra Älvsborgs sjukhus
11	Skaraborg's Hospital	Skaraborgs sjukhus
12	Västmanland's Hospital	Västmanlands sjukhus
13	Gävle-Sandviken hospital	Länssjukhuset Gävle-Sandviken
14	Sundsvall's Hospital	Sundsvalls sjukhus
15	Östersund's Hospital	Östersunds sjukhus
16	Skellefteå'l Hospital	Skellefteå lasarett
17	Sunderbyn's Hospital	Sunderbyns sjukhus

Table 2: Included hospitals in the thesis

3. Results

3.1 Model A: *2 output and 4 input variables with Classical DEA*

In this first model, the Classical DEA CCR with constant returns to scale is used to compare the 17 middle-sized hospitals included in the study. The objective here is to maximize the ratio of the weighted outputs, i.e. *DRG points*, over the weighted inputs, i.e. *inpatient and outpatient care costs*, for each hospital relative to other 16 hospitals by giving weights to variables in a manner that favors the production pattern of the hospital in question. If hospital A has huge advantage in one or more of the inputs or outputs, then DEA gives higher weights to that or those variables so that hospital A maximizes its efficiency relative to others. On the contrary if hospital A has a clear disadvantage in one or more variables, DEA gives either zero or very small weight to that or those variables. This means that if hospital A has an absolute advantage in any combination of weighted inputs and outputs over other hospitals, it will get 1.00 in efficiency score and be deemed as relatively efficient.

The results of the first, complete input-oriented DEA model with 2 output variables; *DRG points for inpatient and outpatient care*, and 4 input variables; *ward costs*, *indirect inpatient care costs*, *reception costs* and *indirect outpatient costs* shows efficiency levels in table 3.

DMU no.	Hospital	2012	2013	2014	2015	2016
1	S:t Göran's Hospital	100%	100%	100%	100%	100%
2	Stockholm South General Hospital	100%	100%	100%	91%	100%
3	Danderyd's Hospital	100%	100%	100%	100%	99%
4	Södertälje Hospital	100%	100%	100%	100%	100%
5	Vrinnevi Hospital	95%	94%	92%	88%	89%
6	Västervik's Hospital	95%	100%	100%	100%	100%
7	Kalmar County Hospital	100%	95%	99%	100%	100%
8	Halland's Hospital	100%	100%	90%	83%	93%
9	NU-Hospital group	77%	85%	93%	92%	86%
10	Southern Älvsborg's Hospital	80%	91%	98%	94%	90%
11	Skaraborg's Hospital	89%	85%	85%	85%	90%
12	Västmanland's Hospital	71%	76%	76%	74%	80%
13	Gävle-Sandviken Hospital	56%	56%	61%	60%	72%
14	Sundsvall's Hospital	100%	100%	100%	100%	100%
15	Östersund's Hospital	100%	90%	93%	100%	88%
16	Skellefteå's Hospital	92%	99%	93%	92%	90%
17	Sunderbyn's Hospital	75%	74%	69%	71%	80%

Table 3: Relative efficiency levels of 17 middle-sized Swedish hospital with 2 output and 4 input variables, through classical DEA CCR model

As can be seen in table 3, many hospitals, between 6 and 8, have efficiency scores 1.00, i.e. 100% relative efficiency. According to Boussofiane et al. (1991), the number of efficient DMUs would be product of the number of output and input variables, which for this model would be 2*4 =8 for each year. Considering that some are very close to 100% efficiency, that hypothesis is almost met for all the years. This poses, a problem of differentiation as many hospitals simultaneously deemed as efficient makes ranking of the hospitals a tedious, if not impossible, task.

According to the theories in chapter2, the ratio between the number of DMUs and the number of variables is of great importance here. Though the threshold of 8 DMUs is well met, the problem of low discriminating power prevails. One problem that lurks and that is not visible here is that some hospitals get efficiency score 1.00 only for having the highest value in an output variable or the lowest value in an input variable. Based on the assumption by Ghasemi et al. (2014), the single criteria in the classical DEA model is not strict enough and therefore the weights might be unrealistic and give results with low discriminating power when the number of DMUs is low at the same time as there are many variables.

In order to get a picture of whether the hospitals get different efficiency scores based on CCR model (constant returns to scale) and BCC (variable returns to scale) and thus have economies or diseconomies of scale, both types are compared in an output-oriented DEA in table 4.

		2012		**2013**		**2014**		**2015**		**2016**	
DMU	**Hospital name**	CCR	BCC	CCR	BCC	CCR	BCC	CCR	BCC	CCR	BCC
16	Skellefteå's Hospital	92%	148%	99%	160%	93%	139%	92%	134%	90%	150%
4	Södertälje Hospital	118%	130%	123%	144%	150%	153%	153%	158%	140%	148%
1	S:t Göran's Hospital	120%	135%	150%	153%	134%	144%	138%	143%	134%	137%
6	Västervik's Hospital	95%	115%	108%	124%	100%	115%	110%	121%	111%	121%
7	Kalmar Regional Hospital	101%	109%	95%	96%	99%	99%	110%	110%	112%	112%
14	Sundsvall's Hospital	114%	116%	120%	120%	125%	125%	105%	105%	111%	111%
3	Danderyd's Hospital	115%	115%	107%	107%	105%	105%	101%	101%	99%	99%
2	Stockholm South General Hospital	100%	100%	106%	106%	111%	111%	91%	91%	98%	98%
8	Halland's Hospital	109%	109%	102%	102%	90%	90%	83%	83%	93%	93%
5	Vrinnevi Hospital	95%	98%	94%	94%	92%	93%	88%	88%	89%	91%
10	Southern Älvsborg's Hospital	80%	83%	91%	91%	98%	98%	94%	94%	90%	90%
11	Skaraborg's Hospital	89%	89%	85%	85%	85%	85%	85%	85%	90%	90%
15	Östersund's Hospital	90%	93%	90%	91%	93%	95%	137%	150%	88%	89%
9	NU-Hospital group	77%	77%	85%	85%	93%	94%	92%	92%	86%	86%
12	Västmanland's Hospital	71%	71%	76%	76%	76%	76%	74%	74%	80%	80%
17	Sunderbyn's Hospital	75%	82%	74%	76%	69%	73%	71%	71%	80%	80%
13	Gävle-Sandviken hospital	56%	57%	56%	56%	61%	61%	60%	60%	72%	72%

Table 4: Efficiency scores of hospitals based on CCR and BCC models, with 4 input and 2 output variables

Because there are many relatively efficient hospitals, a comparison for those hospitals would not be possible if maximum efficiency scores were restricted to 1.00. Therefore that limit restriction was abandoned in order to get super-efficiency scores. A comparison between efficiency scores and ranking of input-oriented CCR DEA efficiency scores in table 3 and output-oriented CCR DEA super-efficiency model of table 4 shows almost perfect correlation, which means that it does not play any important role if input-oriented or output-oriented model is used, the optimal solutions are almost the same. This confirms robustness of DEA as comparison technique. The comparison between the two types of the output-oriented classical models give the results in table 4. The hospitals are sorted by efficiency score of BCC model for 2016.

The results in table 4 show that that the smallest hospitals in terms of total aggregated output – which are displayed in appendix 3 - benefit from using variable returns to scale instead of constant returns to scale. That is obvious for the three smallest ones, Skellefteå's hospital, Södertälje's hospital and Västervik's hospital. It is an indication that these hospitals have sizes smaller than the optimal hospital size. The model difference between CCR and BCC models is a new variable C_0 which represents scale efficiency. In pursuit of explanations, the weights of BCC-DEA-super efficiency model for the variables and for C_0 for each of the hospitals are displayed in table 5. It's important to note that the model assumes that all input variables are combined for production of each of the output variables.

DMU	Hospital	u_1	u_2	v_1	v_2	v_3	v_4	C_0
1	S:t Göran's Hospital	0,0000339	0,000008	-	-	0,0011799	0,0030840	0,1481973
2	Stockholm South General Hospital	0,0000188	-	0,0006600	-	-	-	-
3	Danderyd's Hospital	0,0000223	-	0,0003054	0,0009343	-	-	-
4	Södertälje Hospital	-	0,000091	-	-	0,0042463	-	0,6743860
5	Vrinnevi Hospital	0,0000225	0,000040	0,0015802	-	-	-	0,0755981
6	Västervik's Hospital	0,0000525	0,000068	0,0006908	0,0062422	-	-	0,2729230
7	Kalmar County Hospital	0,0000518	-	-	0,0031476	-	0,0014568	-
8	Halland's Hospital	0,0000109	0,000023	0,0005886	0,0003976	-	-	-
9	NU-Hospital group	0,0000225	0,000001	0,0003142	0,0009584	-	-	-
10	Southern Älvsborg's hospital	-	0,000053	-	0,0005375	-	0,0046319	-
11	Skaraborg's Hospital	0,0000102	0,000029	0,0008736	-	-	-	-
12	Västmanland's Hospital	0,0000111	0,000024	0,0006016	0,0004063	-	-	-
13	Gävle-Sandviken hospital	0,0000107	0,000031	0,0009166	-	-	-	-
14	Sundsvall's Hospital	0,0000002	0,000089	-	0,0045618	-	-	-
15	Östersund's Hospital	0,0000224	0,000045	-	0,0022159	-	0,0037995	0,0860623
16	Skellefteå's Hospital	-	-	-	-	-	0,0184393	1,5026198
17	Sunderbyn's Hospital	0,0000240	0,000029	0,0001101	0,0021265	0,0002718	-	-

Table 5: Weights of variables in Classical BCC-model.

From table 5, it can be read that six hospitals benefit from using variable returns to scale, instead of constant returns to scale, as they have C_0 >0. It is clear that the absolutely smallest ones benefit the most. With BCC model, the only advantage Skellefteå's hospital has over the other DMUs is in v_4, which represents indirect costs associated to outpatient care, as it has lowest cost of all DMUs in this variable. Almost all of its weight in efficiency score is associated to C_0. Just because of that, it is shown as efficient, which shows the weakness of a classical DEA model with one restriction. As the weights given to both output variables is zero for Skellefteå's hospital, the optimal solution – if economic efficiency was the only criteria - for it would be to close down. If the weights were not displayed, one would get wrong perception of the efficiency scores. The reason behind Skellefteå's hospital being efficient is that it's expected that other hospitals with Skellefteå's hospitals size would probably close down, so the mere existence of its production is enough for being seen as efficient.

Södertälje's hospital, which is the next smallest is deemed as efficient thanks to having lowest costs in input variables 3 and 4, which are outpatient costs, in relation to its total DRG-production. Its inputs 1 and 2, i.e. costs associated to inpatient care, are among the highest in comparison to its total DRG points. Therefore, the DEA model with variable returns to scale proposes that the best option for Södetälje's hospital and Southern Älvsborg's hospitals is to stop their inpatient care production and move their resources to only outpatient care production. One reason is that these two already have higher shares of outpatient care, compared to other hospitals. For Stockholm South General Hospital, Danderyd's hospital and Kalmar county hospital, the optimal choice is the opposite way, i.e. closing their outpatient care and shifting all their resources to inpatient care as their inpatient-care costs per DRG-points are amongst the lowest while their outpatient-care costs per DRG-point are among the highest. The data on costs and DRG-points for 2016 are displayed in appendix 4.

What is interesting from the weight distribution in table 5 is that in all regions where only one of either inpatient or outpatient care is proposed for the hospitals, there are more than one major hospital, in three of the four county councils there are university hospitals present. It can be interpreted as the county councils having distributed their care productions over many hospitals and probably concentrated more of the inpatient care to the biggest hospital within the county council so that the middle-sized hospitals from those counties are affected more by external factors.

The Efficiency frontier of the BCC model tends to follow a logarithmic curve, as displayed in figure 4. Except Vrinnevi hospital and Östersund's hospital who are deemed as inefficient but lie over the efficiency frontier, all other hospitals lie on the right side of the frontier with the right distance to the frontier. The efficiency scores follow a clear pattern of variable returns to scale. It's noteworthy that Sundsvall's hospital, in unweighted calculation, has the second highest cost per DRG point, after only Gävle-Sandviken hospital, but as it has very small indirect costs for both inpatient and outpatient care, it is deemed as having efficient combinations of inputs and therefore is efficient as a DMU.

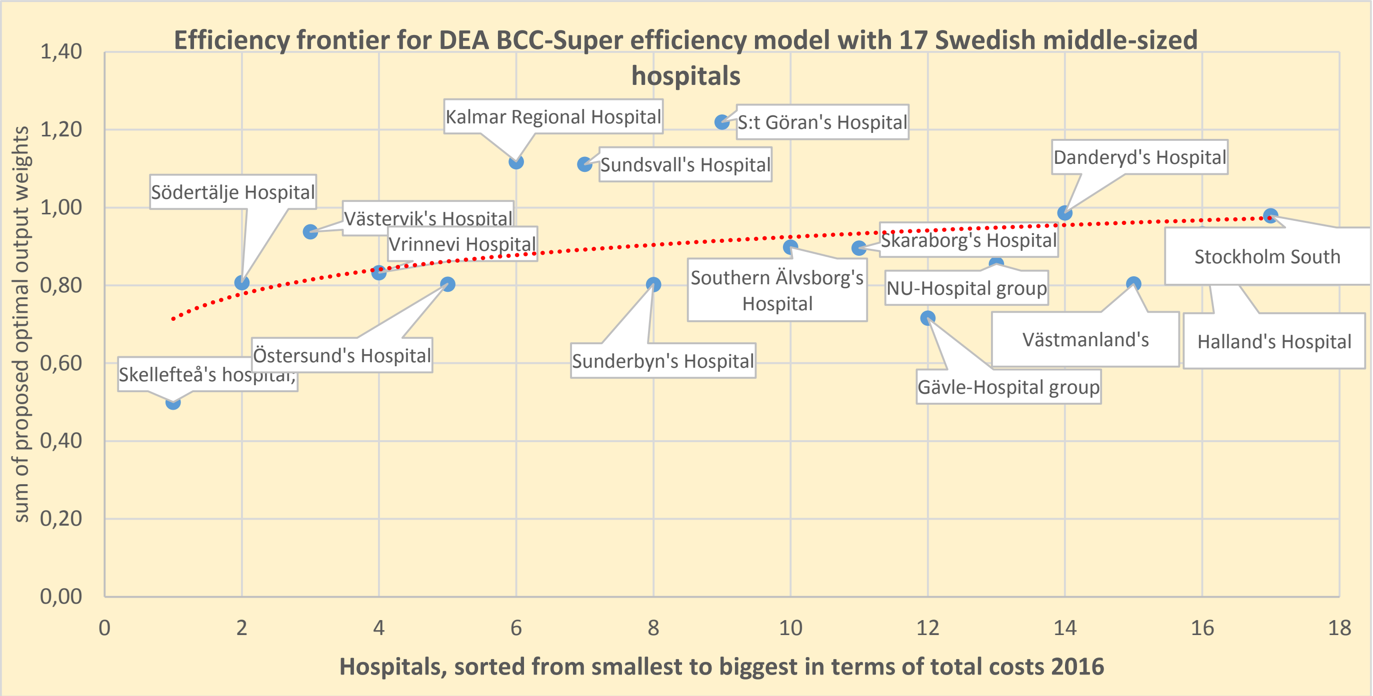

Figure 4: Efficiency score with DEA BCC and smoothing with logarithmic line

DEA assumes all of the input variables as different from each other and combined for production of each of the outputs. When we have monetary variables that are aggregated costs of many factors, instead of the real factors behind; such as staff, equipment and rooms, it can give wrong indications, which operational scientist have to take into consideration.

Table 6 shows the reference sets of inefficient hospitals so that they follow production patterns of nearest efficient hospitals on the frontier. According to Banker et al. (2004), the Input-oriented dual CCR DEA model gives indication on whether the DMUs follow increasing, constant or decreasing returns to scale. If $\sum\lambda j$ =1, then the DMU_j follows constant returns to scale, if $\sum\lambda j$ <1, they follow increasing returns to scale and if $\sum\lambda j$ >1, they follow decreasing returns to scale. Even that model, alike the BCC Model behind Appendix 5, attests that most hospitals in year 2016 had decreasing returns to scale while some had increasing returns to scale. In 2012, Södertälje's, Danderyd's and Halland's hospitals were the most referenced efficient hospitals but in 2016, only Södertälje's hospital had maintained its position while S:t Göran and Kalmar had replaced Halland's and Danderyd's as the other most referenced efficient hospitals.

The results in table 6 indicate that most DMUs in 2016 were following decreasing returns to scale. Appendix 5 shows that in 2012, only two had decreasing returns to scale while 7 had increasing returns to scale. It can be interpreted as increase in the mean size of the hospitals in this study in terms of case-mix adjusted inpatient and outpatient specialized healthcare production. However, as mean DRG points for all of these hospitals are below national average, a conclusion cannot be drawn for all Swedish hospitals.

Reference sets for 17 hospitals, 2016	λ S:t Göran's Hospital	λ Stockholm South General	λ Danderyd's Hospital	λ Södertälje Hospital	λ Vrinnevi Hospital	λ Västervik's Hospital	λ Kalmar County Hospital	λ Halland's Hospital	λ NU-Hospital group	λ Southern Älvsborg's	λ Skaraborg's Hospital	λ Västmanland's Hospital	λ Gävle-Sandviken	λ Sundsvall's Hospital	λ Östersund's Hospital	λ Skellefteå's Hospital	λ Sunderbyn's Hospital
S:t Göran's Hospital	1,00		1,13		0,50			0,80	1,05	0,55	0,71	0,42	0,29		0,09	0,09	0,20
Stockholm South General Hospital	0,00	1,00															
Danderyd's Hospital																	
Södertälje Hospital				1,00	0,26	0,00		0,72		0,91	0,92	1,25	1,27		0,41	0,09	0,30
Vrinnevi Hospital																	
Västervik's Hospital						1,00		0,53	0,18					0,00			0,53
Kalmar Regional Hospital			0,37			0,00	1,00		0,09			0,39			0,44	0,23	0,23
Halland's Hospital																	
NU-Hospital group																	
Southern Älvsborg's Hospital																	
Skaraborg's Hospital																	
Västmanland's Hospital																	
Gävle-Sandviken hospital																	
Sundsvall's Hospital														1,00			
Östersund's Hospital																	
Skellefteå's Hospital																	
Sunderbyn's Hospital																	
$\sum\lambda_j$	**1,00**	**1,00**	**1,49**	**1,00**	**0,76**	**1,00**	**1,00**	**2,04**	**1,31**	**1,46**	**1,63**	**2,06**	**1,56**	**1,00**	**0,93**	**0,40**	**1,26**

Table 6: Reference sets of the hospitals, for inefficient ones to follow efficient ones, 2016

3.2 Model B: 2 output and 2 input variables with Classical DEA

In this second model with classical DEA as technique, the two cost variables related to inpatient care are aggregated into one variable and the two cost variables concerning outpatient care into another one. Thus the model has 2 input variables and 2 output variables. Based on Boussofiane et al.'s (1991) theory, the minimum number of efficient DMUs should decrease from 8 to 4 as the product of 2 input variables multiplied by 2 output variables is 4.

From table 7, it can be read that the *scale efficiency* problems for the smallest hospitals persist. Now, the number of efficient hospitals have, as expected according to the assumption by Boussofiane et al. (1991), decreased and are 3-5 per year. That means that even with restriction of efficiency score ≤1.00, the discriminating power is bigger so the hospitals can be ranked more easily. The main reason, based on theories in chapter 2, is that the combinations of inputs and outputs have decreased and there are fewer possibilities for some of the hospitals, which were efficient in model A, to maintain their positions.

Hospitals with similar efficiency scores in CCR and BCC have their scores related to *technical efficiency* without disturbance from *scale efficiency* as they face constant returns to scale. The weights given to the variables and C_0 in the reformulated BCC model for 2016 is displayed in Appendix 6.

DMU no.	Hospital	2012 CCR	2012 BCC	2013 CCR	2013 BCC	2014 CCR	2014 BCC	2015 CCR	2015 BCC	2016 CCR	2016 BCC
4	Södertälje Hospital	116%	127%	118%	129%	144%	146%	138%	146%	129%	139%
1	S:t Göran's Hospital	114%	127%	149%	152%	127%	138%	133%	138%	128%	132%
6	Västervik's Hospital	94%	107%	99%	106%	99%	112%	98%	116%	102%	116%
3	Danderyd's Hospital	114%	114%	101%	101%	101%	101%	95%	95%	96%	96%
8	Halland's Hospital	82%	83%	86%	86%	90%	90%	82%	82%	93%	93%
7	Kalmar Regional Hospital	88%	93%	94%	94%	96%	96%	87%	90%	89%	91%
11	Skaraborg's Hospital	80%	81%	80%	80%	85%	85%	84%	84%	88%	88%
2	Stockholm South General Hospital	90%	90%	96%	96%	100%	100%	88%	88%	88%	88%
5	Vrinnevi Hospital	92%	97%	92%	92%	92%	92%	86%	86%	88%	88%
10	Southern Älvsborg's Hospital	77%	80%	88%	88%	95%	95%	80%	80%	86%	86%
9	NU-Hospital group	76%	77%	85%	85%	92%	92%	89%	89%	85%	85%
14	Sundsvall's Hospital	77%	80%	83%	83%	86%	86%	80%	80%	84%	84%
16	Skellefteå's Hospital	80%	118%	87%	114%	85%	110%	76%	108%	80%	116%
12	Västmanland's Hospital	67%	67%	72%	72%	75%	75%	74%	74%	80%	80%
17	Sunderbyn's Hospital	73%	78%	73%	76%	69%	72%	71%	71%	78%	78%
15	Östersund's Hospital	77%	84%	78%	81%	83%	84%	73%	75%	78%	78%
13	Gävle-Sandviken hospital	55%	57%	54%	54%	57%	57%	60%	60%	66%	66%

Table 7: Efficiency scores from CCR and BCC models with 2 input and 2 output variables

The weights for the individual hospitals in Appendix 6 show that the suggestions by the DEA for most of the hospitals persist. Skellefteå's hospital is still recommended to close down as its size is far from optimal size of a hospital with specialized care. Södertälje's hospital still would benefit from transferring all its resources to outpatient care, but Southern Älvsborg's hospital in this model should keep on producing both inpatient and outpatient care. In this model, beside Stockholm Southern General Hospital, Danderyd's hospital and Kalmar County hospital, even S:t Göran's hospital could be better-off abandoning its outpatient care production and focusing entirely on inpatient care production. Nevertheless, S:t Göran's hospital's size is smaller than optimal.

3.3 Model C: *1 output and 2 input variables with Classical DEA*

In this model, the two output variables are aggregated to one, representing all DRG points produced by the hospitals. The number of input variables are, as in model B, two, standing for costs of inpatient and outpatient care respectively. The results of input-oriented DEA model C, are as in table 8. The results show that the number of efficient hospitals according to CCR model with constant returns to scale has diminished and, for 3 of the 5 years, only 1 hospital is deemed as efficient. The BCC model, however, finds more efficient combinations of weighted outputs and inputs so there are 4-5 efficient hospitals per year. Once again, the theory of increasing discriminating power, thanks to decreased complexity as a result of fewer variables, is confirmed.

Aggregating similar variables, such as cost data in which a monetary unit is valued homogenously across variables, helps in minimizing the risk for inefficient DMUs to be deemed as efficient only due to having smallest value in a certain input variable or biggest value in a certain output variable.

		2012		2013		2014		2015		2016	
DMU	**Hospital**	*CCR*	*BCC*	*CCR*	*BCC*	*CCR*	*BCC*	*CCR*	*BCC*	*CCR*	*BCC*
1	S:t Göran's Hospital	113%	124%	144%	151%	126%	132%	131%	135%	127%	128%
2	Stockholm South General Hospital	86%	86%	93%	93%	99%	99%	86%	86%	87%	87%
3	Danderyd's Hospital	114%	114%	96%	96%	100%	100%	94%	94%	94%	94%
4	Södertälje Hospital	89%	105%	91%	124%	100%	122%	92%	112%	90%	113%
5	Vrinnevi Hospital	91%	96%	90%	92%	92%	92%	85%	86%	87%	87%
6	Västervik's Hospital	90%	105%	94%	105%	99%	105%	96%	110%	98%	114%
7	Kalmar Regional Hospital	85%	88%	93%	94%	96%	96%	87%	88%	89%	90%
8	Halland's Hospital	82%	83%	85%	85%	90%	90%	80%	80%	91%	91%
9	NU-Hospital group	73%	73%	83%	83%	90%	90%	89%	89%	85%	85%
10	Southern Älvsborg's Hospital	75%	77%	88%	88%	95%	95%	78%	78%	83%	83%
11	Skaraborg's Hospital	80%	81%	79%	79%	85%	85%	83%	83%	86%	86%
12	Västmanland's Hospital	65%	66%	69%	69%	74%	74%	71%	71%	76%	76%
13	Gävle-Sandviken hospital	54%	56%	53%	53%	57%	57%	57%	58%	62%	62%
14	Sundsvall's Hospital	76%	79%	81%	82%	86%	86%	77%	78%	78%	78%
15	Östersund's Hospital	77%	82%	77%	80%	83%	84%	72%	74%	76%	78%
16	Skellefteå's Hospital	79%	118%	85%	114%	85%	110%	75%	108%	79%	116%
17	Sunderbyn's Hospital	70%	74%	73%	75%	67%	67%	70%	71%	76%	76%

Table 8: Output-oriented DEA models, CCR and BCC compared, with 1 output and 2 input variables.

As table 8 shows, the number of efficient hospitals has diminished from 6-8 per year, in model A with 4 input and 2 variables, to only 1-2 per year in model C where the 4 input variables are aggregated to two and the 2 output variables aggregated to one. This has also increased the possibility to rank all the hospitals. What is pleasing is also that the relative standings have not changed so even though the efficiency scores have diminished for most hospitals, they still have the same standing in the ranks, as in models A and B. This proves the robustness of the DEA as an efficiency measurement technique.

It is noteworthy that there is a risk for oversimplification as healthcare production in reality needs both direct costs incurred in the wards and indirect costs such as administration, transportation etc. The goal is finding the optimal combinations of those cost types, not aggregating them for getting higher discriminating power. Therefore, a new version of DEA with all input and output variables, without aggregation, is used to test whether that goal can be achieved.

3.4 Model D: *2 output and 4 input variables with BiO-MCDEA-Supper-efficiency*

As model A and B showed, a single objective of maximizing the weighted outputs given inputs, or inversely stated, minimizing weighted inputs given output levels, has high risk of false alarms in terms of showing too many DMUs as efficient. For some DMUs it suffices to have the "best" values in one combination in order to be shown as efficient. In model A, it was enough for Södertälje's hospital to have the lowest outpatient costs as share of total costs in order to be deemed efficient in both CCR and BCC models in all 5 years. For Skellefteå's hospital, having the smallest cost value in outpatient indirect costs was enough for being deemed as efficient in BCC model, despite the fact that proposed weighted outputs recommended it to discontinue its production.

As all input variables are in monetary terms, aggregation of variables, in order to get fewer variables resulting into more realistic weights and higher discriminating power, was possible. In most real life cases the input variables are of different nature and cannot be aggregated so the model has to give realistic weights and have high discriminating power even with complete number of variables. Here, BiO-MCDEA is used, in order to test if it can fulfill the objective of higher discriminating power and better weight dispersion. The efficiency scores achieved from BiO-MCDEA are in table 9. In order for the full spectra of the efficiency scores to be visible and thus the hospitals to be rankable, the upper score restriction of 1.00 is abandoned. It means that the efficiency scores are so-called super-efficiency scores, i.e. allowed to exceed 1.00. A computational model of BiO-MCDEA-Super efficiency on Excel can is displayed in Appendix 7.

DMU no.	Hospital	2012	2013	2014	2015	2016
1	S:t Göran's Hospital	83,1%	101,9%	99,7%	109,3%	108,8%
2	Stockholm South General Hospital	80,8%	87,2%	88,1%	83,8%	88,7%
3	Danderyd's Hospital	114,4%	97,8%	103,1%	99,6%	92,6%
4	Södertälje Hospital	75,3%	94,7%	135,0%	148,2%	124,4%
5	Vrinnevi Hospital	92,8%	94,1%	92,3%	87,5%	88,6%
6	Västervik's Hospital	83,9%	90,3%	98,8%	109,1%	93,5%
7	Kalmar Regional Hospital	92,9%	95,0%	95,4%	88,8%	78,6%
8	Halland's Hospital	70,3%	98,4%	89,8%	80,0%	93,3%
9	NU-Hospital group	76,6%	85,0%	81,8%	89,6%	84,9%
10	Southern Älvsborg's Hospital	74,9%	85,6%	87,0%	82,1%	86,5%
11	Skaraborg's Hospital	88,7%	84,9%	84,6%	83,2%	89,2%
12	Västmanland's Hospital	66,7%	76,3%	74,4%	73,4%	80,4%
13	Gävle-Sandviken hospital	53,5%	55,9%	56,6%	56,9%	67,6%
14	Sundsvall's Hospital	62,9%	102,2%	84,2%	82,0%	74,0%
15	Östersund's Hospital	73,6%	89,8%	82,2%	74,8%	71,0%
16	Skellefteå's Hospital	73,9%	98,4%	83,7%	77,3%	73,1%
17	Sunderbyn's Hospital	70,5%	73,8%	67,3%	70,7%	74,1%

Table 9: Efficiency scores of hospitals based on output-oriented BiO-MCDEA super-efficiency model with 4 input and 2 output variables

As can be seen in table 9, the small hospitals have got much lower efficiency scores while the big hospitals - in terms of total DRG points which can be found in Appendix 3 - maintain their standings. This means that big hospitals are not as dependent on one or few variable combinations as small ones are. Even the ones that previously had been deemed as efficient only thanks to having the highest value in an output variable loses their advantage and become inefficient if they have not an optimal combination of weighted output and weighted input variables. That is the case for Stockholm South General Hospital which in model A was deemed as efficient in 4 out of 5 years because it had biggest number of inpatient DRG points.

A Spearman's rank correlation test between CRR model of table 4 and CCR BiO-MCDEA model of table 9 for year 2016, outlined in Appendix 8, shows a ρ-value of 0.61. It means that despite inclusion of new constraints, the standings of the hospitals are quite similar in both models. Though efficiency scores of some hospitals in the BiO-MCDEA have decreased in comparison to the CCR model in table 4, the rank orders of the hospitals have not changed much. That can be interpreted as a sign of robustness of the DEA. Results in table 10, which compares CCR model of model C and CCR BiO-MCDEA shows a strong positive correlation with Spearman's $\rho = 0.88$. As the BiO-MCDEA model seems to calculate the relative efficiency scores of the 17 hospitals in the paper with all variables included, almost as precisely as classical model in which variables are aggregated, it can be said that BiO-MCDEA model is superior to the classical DEA models.

DMU no.	Hospital	BiO-MCDEA, 2 output and 4 input variables	CCR Model, 1 output and 2 input variables	Rank, BiO-MCDEA model	Rank, CCR model	d	d^2
1	S:t Göran's Hospital	108,8%	100%	2	1	-1	1
6	Västervik's Hospital	93,5%	98%	3	2	-1	1
3	Danderyd's Hospital	92,6%	94%	5	3	-2	4
8	Halland's Hospital	93,3%	91%	4	4	0	0
4	Södertälje's Hospital	124,4%	90%	1	5	4	16
7	Kalmar county hospital	78,6%	89%	12	6	-6	36
2	Stockholm South General hospital	88,7%	87%	7	7	0	0
5	Vrinnevi hospital	88,6%	87%	8	8	0	0
11	Skaraborg's Hospital	89,2%	86%	6	9	3	9
9	NU hospital group	84,9%	85%	10	10	0	0
10	Southern Älvsborg's Hospital	86,5%	83%	9	11	2	4
16	Skellefteå's Hospital	73,1%	79%	15	12	-3	9
14	Sundsvall's Hospital	74,0%	78%	14	13	-1	1
12	Västmanland's Hospital	80,4%	76%	11	14	3	9
17	Sunderby Hospital	74,1%	76%	13	15	2	4
15	Östersund's Hospital	71,0%	76%	16	16	0	0
13	Gävle Hospital-group	67,6%	62%	17	17	0	0
Sum of differences in ranks of the hospitals in terms of efficiency between the two models							94
Spearman's rank ρ for 2016: $1 - (6 \cdot \sum d^2)/n(n^2-1) = 0.88$							0,88

Table 10: Comparison between decimated CCR model with 1 aggregated output and 2 aggregated input variables and complete BiO-MCDEA model with 2 output and 4 input variables.

In order to be able to talk about whether the weights are more realistic in BiO-MCDEA than in the classical DEA models, it's important to display them. In table 11, the weights given to the included output and input variables and the maximum quantity of all variations between weighted sum of outputs and weighted sum of inputs, i.e. variable M, for each hospital are demonstrated.

DMU no.	Hospital	u_1	u_2	v_1	v_2	v_3	v_4	M
1	S:t Göran's Hospital	0,000020	0,000028	0,000912	0,000309	0,000035	-	0,361960
2	Stockholm South General Hospital	0,000008	0,000018	0,000452	0,000305	-	-	0,221337
3	Danderyd's Hospital	0,000010	0,000022	0,000565	0,000382	-	-	0,277010
4	Södertälje Hospital	-	0,000141	0,002423	-	0,000092	-	0,518142
5	Vrinnevi Hospital	0,000022	0,000047	0,001178	0,000796	-	-	0,577233
6	Västervik's Hospital	0,000034	0,000097	0,002881	-	-	-	0,891003
7	Kalmar Regional Hospital	0,000015	0,000042	0,001263	-	-	-	0,390511
8	Halland's Hospital	0,000011	0,000023	0,000589	0,000398	-	-	0,288483
9	NU-Hospital group	0,000011	0,000023	0,000589	0,000398	-	-	0,288713
10	Southern Älvsborg's Hospital	0,000014	0,000030	0,000757	0,000511	-	-	0,370785
11	Skaraborg's Hospital	0,000012	0,000026	0,000657	0,000444	-	-	0,322003
12	Västmanland's Hospital	0,000011	0,000024	0,000602	0,000406	-	-	0,294818
13	Gävle-Sandviken hospital	0,000012	0,000026	0,000661	0,000447	-	-	0,215275
14	Sundsvall's Hospital	0,000014	0,000041	0,001233	-	-	-	0,381382
15	Östersund's Hospital	0,000016	0,000046	0,001373	-	-	-	0,424503
16	Skellefteå's Hospital	0,000033	0,000096	0,002853	-	-	-	0,882326
17	Sunderbyn's Hospital	0,000014	0,000040	0,001193	-	-	-	0,369089

Table 11: Weights given to variables in BiO-MCDEA for 2016

As it can be witnessed in table 11, unfortunately, the problem of unrealistic weight dispersion prevails. In fact, the problem has exacerbated, compared to the classical model with 2 output and 4 input variables, whose weights are displayed in table 5. In Cobb-Douglas sense, the DEA model here treats the input variables as substitutes such that when the utility of one unit increase in one factor, e.g. direct ward costs, exceeds the disutility of one unit decrease in another factor, e.g. indirect outpatient costs, it's optimal to abandon the latter one and increase the first one. It's obvious from Appendix 4 that costs per DRG points are much higher for outpatient care than for inpatient care, for almost all of the hospitals. As DEA perceives that both of the outputs; inpatient somatic and psychiatric specialized care and outpatient specialized somatic care, are achievable by any combination of the included input variables, it proposes the combination with highest efficiency for each DMU, given other DMUs' input-output combinations. Table 14 shows that, except for S:t Göran's hospital and Södertälje's hospital who have low outpatient costs per DRG point, optimal solution for all other hospitals is to abandon outpatient costs and produce both inpatient and outpatient care using only inpatient care costs as inputs.

As DEA compares the same variables across all DMUs, the variables with lowest deviations will have strongest prediction powers. With low standard deviation, the manner one particular DMU benefits from that variable can be generalized to all DMUs. When there are large deviations across DMUs in benefit of a particular input variable for production of the same output, the predictability will be a victim. Therefore, value of each input variable per unit of output variable, for each hospital in 2016, is displayed in Appendix 9. The standard deviations and coefficients of variation (CV), i.e.

standard deviation of mean, are calculated to give a picture of which variables that have lowest CV-values and, thus, highest predictability. The reason behind using CV is that the input variables are of obviously different sizes, CV normalizes their standard deviations with respect to their sizes.

The input variable *ward costs per DRG point* has the smallest standard deviation and subsequently smallest coefficient of variation. It means that it has the highest prediction power. The second best variable is *indirect inpatient care costs*. The standard deviations (SD) and coefficients of variation (CV) are displayed in Appendix 9. As a result of the SD and CV, the weight dispersion for each of the DMUs in table 11 is expected. The reason behind S:t Göran's and Södertälje's hospitals having weights for the variable *direct reception costs* is that they have comparative advantages as their costs per DRG point is much lower than the average cost. Therefore, using variable 3 may give them advantage over the other hospitals.

3.5 Correlation between efficiency scores and environmental factors

In this section, the efficiency scores obtained by different DEA models are analyzed by relating the efficiency scores to some environmental factors that are important for hospitals' specialized healthcare provision. The analysis is done mainly by Spearman's rank-order tests. The included hospitals are ranked by efficiency scores and each of the other indicators. If the Spearman's rank coefficient gives a value between -0.5 and -1 or +0.5 and +1, then it can be concluded that there is strong correlation between efficiency score and the other indicator/variable. That can then be interpreted as the indicator playing vital role in the hospitals' performance in terms of efficiency. In table 12, Spearman's correlation coefficients for correlation between efficiency scores of BiO-MCDEA-Super-efficiency model and some chosen indicators, are displayed.

Indicator no.	***Indicators***	**2012**	**2013**	**2014**	**2015**	**2016**
1	*Mean length of stay (LOS)*	-0,43	-0,43	-0,74	-0,61	-0,54
2	*Mean DRG-weight*	-0,24	0,12	-0,20	-0,11	-0,03
3	*Inpatient care as share of total healthcare production*	0,30	-0,01	-0,05	0,31	0,18
4	*Share of indirect/administrative costs*	0,32	-0,28	0,25	0,14	0,37
5	*Mean number of diagnoses per care episode in inpatient care*	0,17	0,31	0,03	0,22	0,18
6	*Total DRG-points*	0,13	0,56	0,03	0,07	0,27
7	*Total costs*	-0,24	-0,34	-0,34	-0,31	0,05
8	*DRG points per million SEK*	0,69	0,56	0,85	0,82	0,78
9	*Overcrowding and relocating rate*		-0.36	0.15	-0.40	-0.51
10	*National Patient Survey – inpatient care – total impression*	-0.04		0.28		0.14
11	*National Patient Survey – outpatient care – total impression*	0.57		0.47		0.60

Table 12: Spearman's ρ for correlation between efficiency scores of Model D and some factors

The coefficients in table 12 can be interpreted for the purpose of gauging whether efficiency scores are reasonable and logic. Red color indicates negative correlation between efficiency score and the indicator in question while green color shows positive correlation. As stated in chapter 2.5, ρ-values in the interval $-1 \leq \rho \leq -0.5$ shows strong negative correlation while ρ-values in the interval $0.5 \leq \rho \leq 1$ indicates a strong positive correlation between the efficiency score and the indicator in hand. In that sense, *mean length of stay* has a strong negative correlation with efficiency scores of the hospitals. At the same time, it is pleasing that the crude productivity measure of number of *DRG points per million SEK* has a strong positive correlation with the efficiency scores. If that had not been the case, the efficiency scores and DEA's ability to find optimal combinations of weighted outputs and inputs could be starkly questioned.

The results in table 12 also attests an aversion towards costs in terms of absolute values, at the same time as total DRG points has almost no correlation with efficiency score. Nevertheless, the coefficient attests that hospitals with higher shares of inpatient care get higher efficiency scores. That is logical as the data on costs per DRG points divided over inpatient and outpatient care respectively shows that inpatient care has had far more DRG points per million SEK expenditures. The efficiency scores of 2013 seems to deviate from those of other 4 years in its correlation to the included indicators.

Share of indirect costs, which entails administrative costs, are higher at bigger hospitals. As bigger hospitals in terms of total DRG points are more likely to be efficient than smaller hospitals, share of indirect costs also tend to have positive correlation with efficiency score.

The last three indicators, 9-11, can be seen as proxies for quality. Correlations analysis between efficiency scores of the hospitals and the indicators attest that the hospitals deemed as efficient also have higher quality as they have lower overcrowding and relocating rates, and have more satisfied patients according to National Patient Survey (NPS). Nevertheless, the NPS is a biannual survey so all the five years cannot be caught.

4. Discussion and conclusion

4.1 Analysis and discussion

The classical version of DEA, without adjustments, shows many hospitals as relatively efficient by giving them score of 1.00. Due to that, the purpose of the thesis could not be optimally achieved by using only that version. If all of the intended hospitals, i.e. all middle-sized from all county councils in Sweden, had been included in the model, there could have been higher discrimination power and thus better rankability between the hospitals. As data for many hospital were not present, I was compelled to abandon that aim and focus on studying relative efficiencies of the hospitals that had reported cost data and also had given permission to use their data.

Size of the hospitals and the distribution of costs on different cost areas are often debated in connection to efficiency in healthcare. The results imply that share of indirect costs, which entail administrative costs, are higher at bigger hospitals. As bigger hospitals, in terms of total DRG points, are more likely to be efficient than smaller hospitals, share of indirect costs also tend to have positive correlation with efficiency score. This is interesting because, as outlined in chapter 1, some of the critiques against inefficiencies in the healthcare sector point out the increased administration as one of the main problems. Though cautiously, due to not having all the hospitals in the model, it points out that indirect costs *per se* may not be a reason for lower efficiency. Indirect/administrative costs may increase quality of the healthcare and decrease length of stay, which is strongly negatively correlated with efficiency scores. Indirect costs in terms of better coordination, leadership and registrations may also prevent unplanned readmissions for discharged patients. That can be the case if administrations help the processes to be seamless and patients who are often in contact with the hospitals to go through without much delay and get proper treatment.

Though the different models of DEA differ in efficiency scores given to the hospitals, the rank of the hospitals do not change, more than marginally, between the models. When the efficiency frontier is allowed to get a super-efficiency score, obviously, the ones on the frontier get changed scores and become rankable, as a comparison between table 3 with efficiency limit of 1.00 and table 4 without that limit shows. The relatively inefficient hospitals maintain their scores. It is important to mention that lack of data on outpatient somatic care can have had its impact on efficiency scores. Unfortunately, that problem couldn’t be solved in this paper.

The most obvious changes in efficiency scores occur when the condition of “constant returns to scale” is eased and the model is allowed to follow variable returns to scale along its production path. That can be seen in table 4. The hospitals that get the most benefits from variable returns to scale are the smallest hospitals, especially Skellefteå’s hospital and Södertälje’s hospital. They perform better than they are expected to do, given their size. Smaller hospitals are expected to produce less per cost unit as their fixed costs make up a large share of their total costs and they have not yet reached a state of optimal marginal cost per DRG-point produced. Without taking fixed costs into consideration, they are expected to produce more per variable cost units and follow increasing returns to scale – as shown in the BCC model of figure 3.

Large hospitals have surpassed that state and face decreasing returns to scale. They need new investments in fixed cost post, such as buildings, machinery, new recruitments, etc. if they do not want the decreasing returns to scale to prevail. From table 6 and Appendix 5, it could be read that in 2012 few hospitals were big enough to have reached the state of decreasing returns to scale but in 2016 their number was 7. Some of the university hospitals, which are omitted in this thesis, are big. They could as well have shown decreasing returns to scale if they had been included in the model as they probably have passed the optimal size state.

In this thesis, only middle-sized hospitals are added, if the smallest ones had been added, many more could have been deemed to have too small sizes and face increasing returns to scale. The results give rise to the question of whether merging some small hospitals could increase efficiency within the Swedish healthcare system as they are too small and cannot achieve cost-efficient production. The question of whether there are some too big hospitals can also be posed as many hospitals showed decreasing returns to scale.

The three years included still shows clear tendency towards positive correlation between efficiency rank of the hospitals and the satisfaction rate among patients who have been treated there. The Spearman's rank-correlation coefficients for the 17 hospitals – based on indicators 9-11 in table 12 - indicate that there is no trade-off between efficiency and quality of care, in fact the relationship is positive. This gives support to the idea of efficiency being included in the concept of quality. Even indicator 1 in the mentioned table can be seen as a proxy for quality as it can be risky for the patients to be at hospitals after having been treated and deemed as ready for discharge. The risks include hospital-related infections. Many elderly patients with multiple simultaneous diseases need care planning for after-care by their respective municipalities so in this sense, an unnecessarily long length of stay can also be due to inefficiency at municipality level, not only the hospitals themselves. However, on the other side, too short length of stay can have the risk for under-treatment and therefore higher probability of readmission soon after discharge.

A comparison between results of the classical DEA model, with one objective of maximizing output, given the inputs, or minimizing the inputs, given the output levels, on the one hand and multiple-criteria DEA (MCDEA) model on the other hand shows that the latter is better in having high discriminating power. In the classical model, many hospitals are deemed as efficient, some only for having the highest value for an output variable or lowest value for an input variable. The MCDEA version, nevertheless, omits that problem. MCDEA models also have better weight dispersion between the input and output variables, though the variables with highest weights in the classical model maintain that position for most of the hospitals even in the MCDEA models. That can be interpreted as a strength of the DEA technique as the variables most favorable for respective hospitals retain their position irrespective of the DEA version.

Easing the limit of 1.00 for the efficiency scores makes it much easier to discern the efficient hospitals and rank them. That is especially advantageous when we have few DMUs and more than 2 variables because then many DMUs are expected to get score of 1.00 and be deemed as efficient.

4.2 Conclusion

This thesis has had two concurrent targets; A: to compare middle-sized hospitals in Sweden in order to find efficient and inefficient ones so that the latter ones can have the former ones as benchmarks for efficiency enhancement of their own, and B: to dissect and analyze the Data Envelopment Analysis (DEA) technique in order to find strengths and weaknesses that might have desirable or undesirable impacts on decisions based on it. In fact, the latter objective has been the stronger one so that other researchers in the future think upon the potential deficiencies of DEA in their studies of efficiency. The motivating factor has been that in recent years many researchers and also policy-makers have given studies through DEA great importance and attention. In light of the possibility of studies done with DEA as technique leading to decisions, it is of vital importance that the model is used cautiously and with full spectra of its strengths and weaknesses in mind.

The Data Envelopment Analysis as a technique is very interesting and as it has showed both robustness and efficiency in terms of relatively quick calculation with many dimensions included in the model. However, the technique has its weaknesses which many researchers have pointed out, some of whom are included and discussed in this paper. One of the main weaknesses has been the low discriminating power, especially when there are many variables and few DMUs. The results of this paper has also testified that problem as in the classical unedited version many hospitals get efficiency scores of 1.00 and are reckoned as efficient. In the beginning I wanted to break down the costs into detailed cost posts, which would lead into many variables. With the results in hand, the hospitals would be even less rankable as more hospitals would get score of 1.00. In that sense, the number of underlying variables used is satisfactory. Nevertheless, the number of underlying DMUs is not satisfactory as some middle-sized hospital had not reported their cost data to SKL for much of the time span and therefore were excluded. In future research, when data for all hospitals are available, it can be very interesting to include all and do the analysis.

The field is very dynamic and a lot of new insights and dimensions have been discussed and included through years by researchers interested in the field, one of which is MCDEA that I have opted to include here in order to analyze whether problems faced in my classical model could be solved. With all dimensions in this study considered, MCDEA has performed better and is recommended for future studies. It is also recommended to use super-efficiency models if one is interested in being able to rank all DMUs.

By tracking back to my research questions the answers are as follows.

- ✓ *Are there differences in efficiency between Swedish middle-sized hospitals?*

  Answer: Yes, there is obvious variation in relative efficiency among the 17 hospitals included in this study
- ✓ *Is DEA robust as a model for efficiency comparison?*

  Answer: Mostly, yes. Though the efficiency scores change between different versions of DEA, the relative rank order of the DMUs seldom changes.
- ✓ *Are the relative efficiency scores of the hospitals robust and consistent over years?*

  Answer: For most part but not always. Overall, the most efficient hospitals maintain their positions, while the less efficient ones get changes in their efficiency scores. However, the differences between hospitals diminish from 2012 to 2016 so the less efficient ones improve their performances over the period.
- ✓ *Does hospital size play role in relative efficiency of the hospitals?*

  Answer: Yes. Depending on size, hospitals are compared to different target levels. Therefore, their size is vital in their performance measure in terms of relative efficiency. Smallest hospitals showed increasing and biggest ones increasing returns to scale.
- ✓ *Does a change in the number of input and output variables cause efficiency scores to change?*

  Answer: Yes. The ratio between number of variables and number of DMUs is important in efficiency scores through DEA. The larger the number of DMUs in relation to the number of variables the better.
- ✓ *If the classical DEA method does not discriminate enough, do revised versions perform better?*

  Answer: Yes. The revised version in this paper, multiple-criteria DEA (MCDEA) performs much better than the classical DEA with one objective.

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Appendices

$$\max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$$

Subject to:

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- \leq \theta x_{ij_0}\,, i = 1,2, \ldots, p$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j + s_r^+ \geq y_{rj_0}\,, where\ r = 1,2, \ldots q$$

$$\sum_{k=1}^{n} \lambda_k = 1$$

$$\begin{array}{ll} \lambda_k \geq 0, & \mathrm{k} = 1,2,\ldots,\mathrm{n} \\ s_i^- \geq 0, & \mathrm{i} = 1,2,\ldots,\mathrm{p} \\ s_r^+ \geq 0, & \mathrm{r} = 1,2,\ldots,\mathrm{q} \end{array}$$

The resulting problem that should be solved, based on the two steps above, will therefore be:

$$\text{mix}\ \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

Subject to:

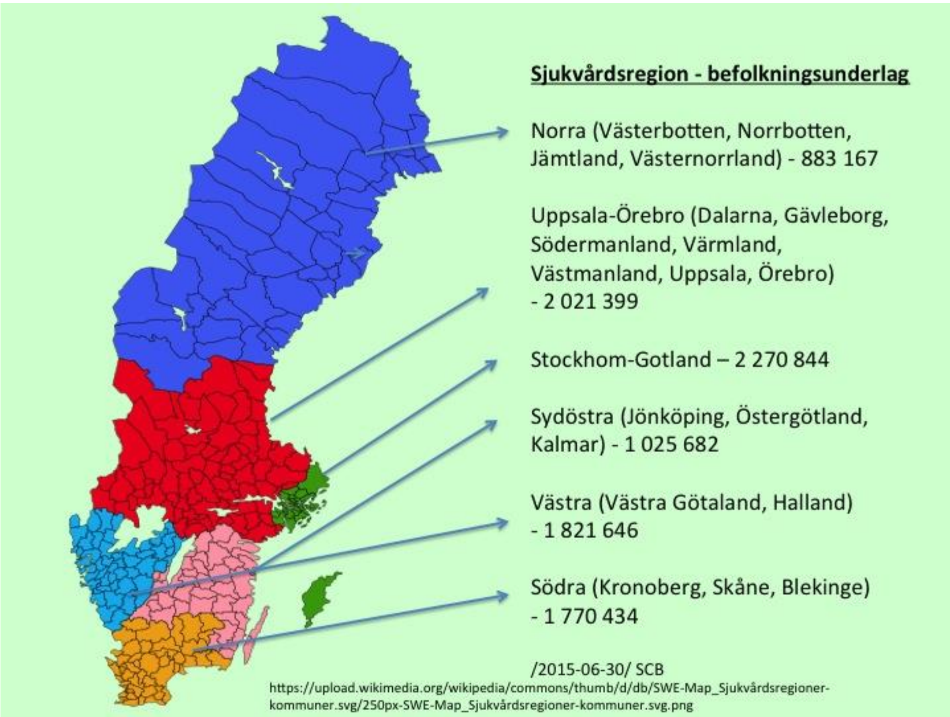
$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- \leq \theta x_{ij_0}\,, i = 1,2, \ldots, p$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j + s_r^+ \geq y_{rj_0}\,, där\ r = 1,2, \ldots q$$

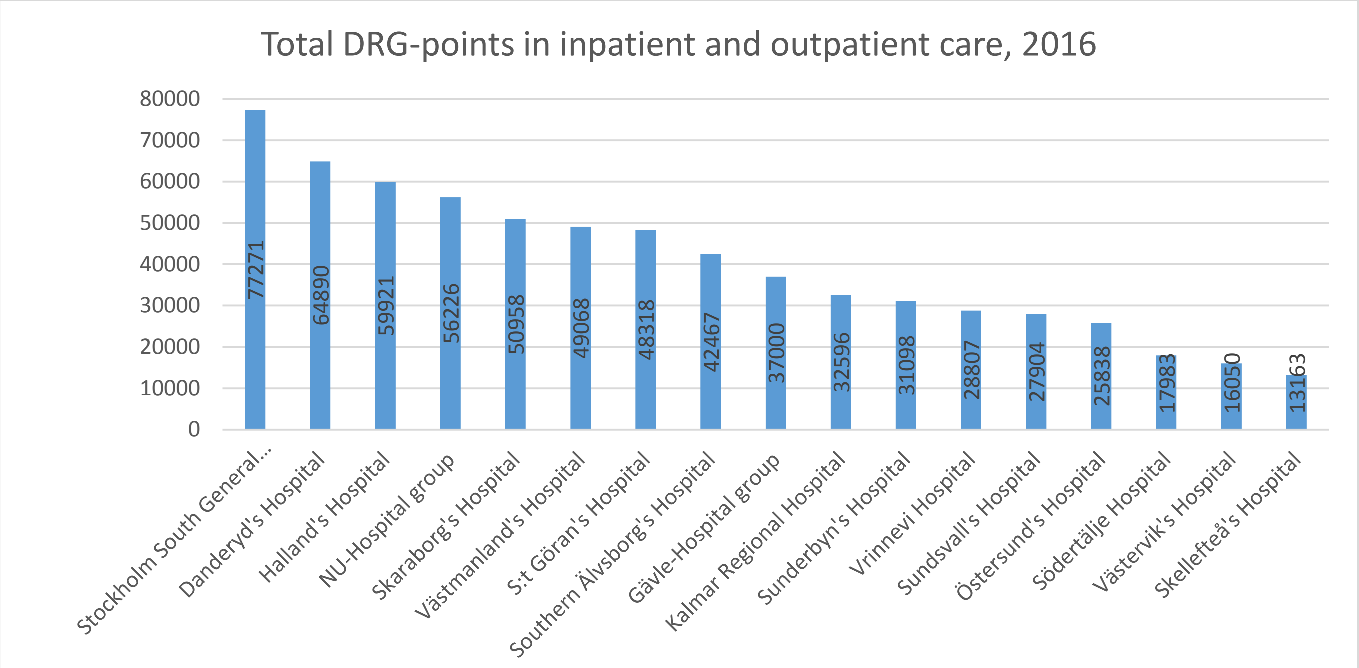
$$\sum_{k=1}^{n} \lambda_k = 1$$

$$\begin{array}{ll} \lambda_k \geq 0, & \mathrm{k} = 1,2,\ldots,\mathrm{n} \\ s_i^- \geq 0, & \mathrm{i} = 1,2,\ldots,\mathrm{p} \\ s_r^+ \geq 0, & \mathrm{r} = 1,2,\ldots,\mathrm{q} \end{array}$$

Appendix 1: Mathematical formula for input-oriented CCR DEA, with slacks



Appendix 2: Healthcare regions of Sweden, collaborations between county councils for highly specialized care



Appendix 3: Aggregated sum of DRG points in inpatient and outpatient specialized care, 2016

DMU	Hospital	*Total DRG points in inpatient care*	*Total DRG points in outpatient care*	*Direct inpatient costs in million SEK*	*Indirect inpatient costs in million SEK*	*Direct outpatient costs in million SEK*	*Indirect outpatient costs in million SEK*	*Costs per aggregated DRG points*	*Cost per inpatient DRG-point*	*Cost per outpatient DRG-point*
1	S:t Göran's Hospital	32201	16116	918,3	469,3	506,0	130,7	41893	43090	39502
2	Stockholm South General Hospital	52030	25241	1515,1	1035,0	988,6	307,0	49768	49012	51327
3	Danderyd's Hospital	44198	20692	1342,7	631,4	969,0	216,6	48694	44665	57300
4	Södertälje Hospital	9153	8830	403,8	171,8	235,5	81,5	49637	62889	35900
5	Vrinnevi Hospital	18458	10349	632,8	320,1	550,2	147,0	57283	51627	67371
6	Västervik's Hospital	9799	6251	347,1	121,8	433,8	85,2	61547	47853	83013
7	Kalmar County Hospital	21593	11003	792,0	257,1	739,4	131,0	58887	48584	79108
8	Halland's Hospital	37433	22488	1292,0	602,4	1199,2	266,8	56080	50606	65191
9	NU-Hospital group	37304	18923	1275,1	625,3	1039,4	272,3	57129	50945	69319
10	Southern Älvsborg's Hospital	25581	16886	1010,6	460,6	885,3	162,4	59316	57514	62047
11	Skaraborg's Hospital	31354	19604	1144,8	558,5	991,6	251,8	57825	54323	63425
12	Västmanland's Hospital	28821	20246	1276,1	571,8	1213,0	280,7	68102	64115	73778
13	Gävle-Sandviken hospital	21077	15924	1091,0	623,5	1118,4	318,1	85158	81341	90210
14	Sundsvall's Hospital	15394	12510	811,0	219,2	845,1	169,3	73273	66922	81087
15	Östersund's Hospital	16008	9831	728,6	253,0	528,5	115,6	62921	61321	65526
16	Skellefteå'l Hospital	8492	4671	350,5	127,0	259,4	54,2	60111	56237	67154
17	Sunderbyn's Hospital	19370	11728	838,0	335,4	716,2	164,3	66043	60576	75073
	Mean of all hospitals	**428264**	**251293**	**15769**	**7383**	**13219**	**3154**	**58164**	**54061**	**65156**

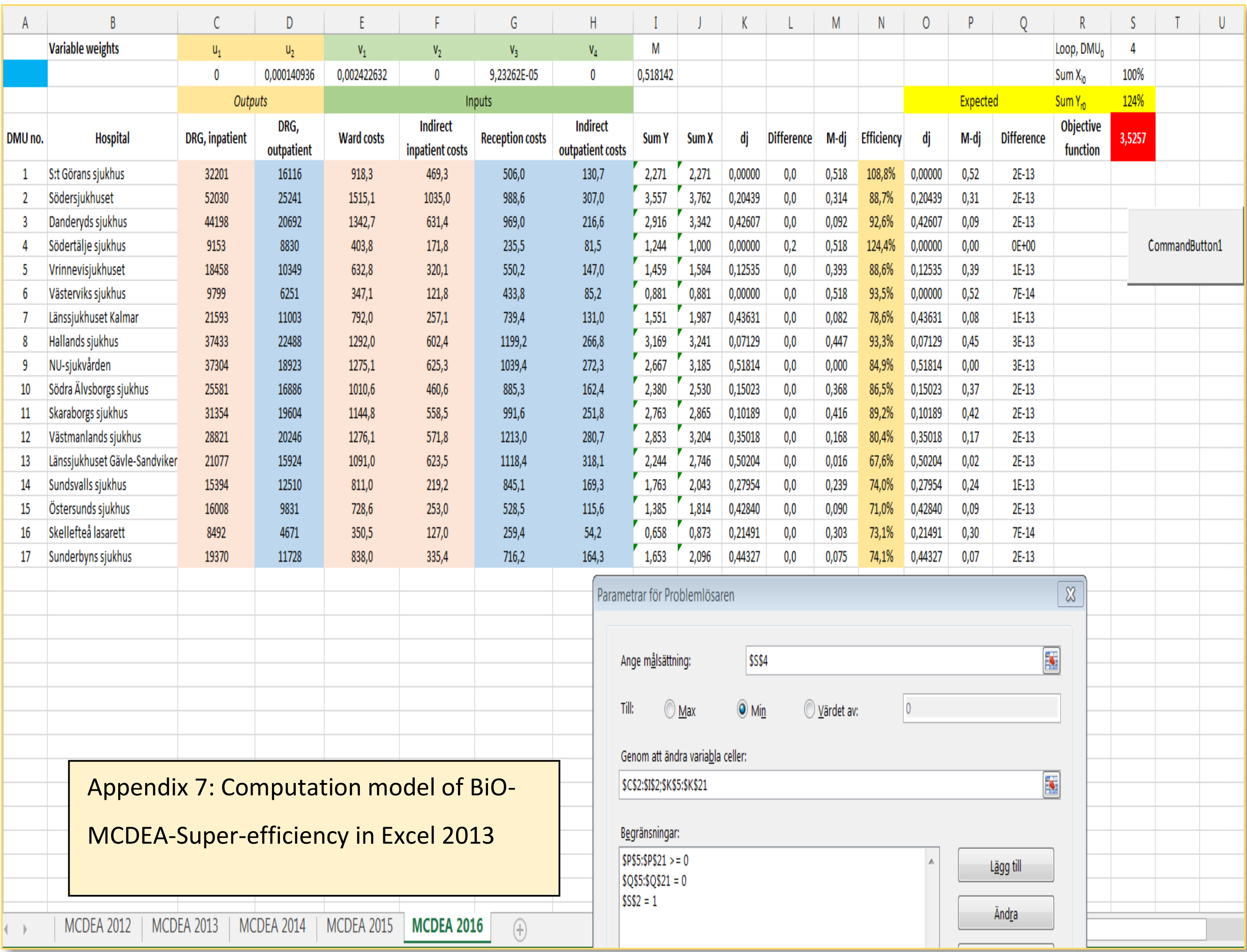
Appendix 4: Costs and DRG production for each of the hospitals, 2016

Reference sets for 17 hospitals, 2012	*λ S:t Göran's Hospital*	*λ Stockholm South General Hospital*	*λ Danderyd's Hospital*	*λ Södertälje Hospital*	*λ Vrinnevi Hospital*	*λ Västervik's Hospital*	*λ Kalmar Regional Hospital*	*λ Halland's Hospital*	*λ NU-Hospital group*	*λ Southern Älvsborg's Hospital*	*λ Skaraborg's Hospital*	*λ Västmanland's Hospital*	*λ Gävle-Sandviken hospital*	*λ Sundsvall's Hospital*	*λ Östersund's Hospital*	*λ Skellefteå's Hospital*	*λ Sunderbyn's Hospital*
S:t Göran's Hospital	1,00																
Stockholm South General Hospital		1,00								0,15							0,10
Danderyd's Hospital		0,00	1,00	0,00	0,34	0,14	0,00	0,00	0,84	0,45	0,44	0,28	0,45		0,00	0,06	0,34
Södertälje Hospital			0,00	1,00	0,14	0,38						0,64	0,20			0,05	
Vrinnevi Hospital																	
Västervik's Hospital																	
Kalmar Regional Hospital							1,00										
Halland's Hospital					0,06			1,00	0,03		0,16	0,26		0,00		0,09	
NU-Hospital group																	
Southern Älvsborg's Hospital																	
Skaraborg's Hospital																	
Västmanland's Hospital																	
Gävle-Sandviken hospital																	
Sundsvall's Hospital			0,00					0,00			0,42			1,00	0,00	0,11	
Östersund's Hospital														0,00	1,00		
Skellefteå's Hospital																	
Sunderbyn's Hospital																	
$\sum\lambda_j$	**1,00**	**1,00**	**1,00**	**1,00**	**0,54**	**0,52**	**1,00**	**1,00**	**0,87**	**0,61**	**1,01**	**1,18**	**0,65**	**1,00**	**1,00**	**0,30**	**0,44**

Appendix 5: Reference sets of hospitals, for inefficient ones to follow efficient ones, 2012

DMU	Hospital	u_1	u_2	v_1	v_2	c_0
1	S:t Göran's Hospital	0,000036	-	-	0,001571	0,169771
2	Stockholm South General Hospital	0,000017	-	0,000392	-	-
3	Danderyd's Hospital	0,000022	-	0,000507	-	-
4	Södertälje Hospital	-	0,000123	0,001190	0,000994	0,307209
5	Vrinnevi Hospital	0,000022	0,000045	0,001021	0,000039	-
6	Västervik's Hospital	0,000062	0,000040	0,002133	-	0,300321
7	Kalmar Regional Hospital	0,000039	-	0,000953	-	0,063949
8	Halland's Hospital	0,000011	0,000022	0,000513	0,000020	-
9	NU-Hospital group	0,000014	0,000018	0,000526	-	-
10	Southern Älvsborg's Hospital	0,000014	0,000029	0,000662	0,000025	-
11	Skaraborg's Hospital	0,000012	0,000025	0,000571	0,000022	-
12	Västmanland's Hospital	0,000011	0,000023	0,000525	0,000020	-
13	Gävle-Sandviken hospital	0,000012	0,000025	0,000565	0,000022	-
14	Sundsvall's Hospital	0,000018	0,000045	0,000971	-	-
15	Östersund's Hospital	0,000022	0,000044	0,000994	0,000038	-
16	Skellefteå's Hospital	-	-	0,001554	0,000822	1,155188
17	Sunderbyn's Hospital	0,000018	0,000036	0,000829	0,000032	-

Appendix 6: Weights given to input and output variables and C_0



A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
	Variable weights	u_1	u_2	v_1	v_2	v_3	v_4	M									Loop, DMU_0	4
		0	0,000140936	0,002422632	0	9,23262E-05	0	0,518142									Sum X_{i0}	100%
		Outputs		Inputs										Expected			Sum Y_{r0}	124%
DMU no.	Hospital	DRG, inpatient	DRG, outpatient	Ward costs	Indirect inpatient costs	Reception costs	Indirect outpatient costs	Sum Y	Sum X	dj	Difference	M-dj	Efficiency	dj	M-dj	Difference	Objective function	3,5257
1	S:t Görans sjukhus	32201	16116	918,3	469,3	506,0	130,7	2,271	2,271	0,00000	0,0	0,518	108,8%	0,00000	0,52	2E-13		
2	Södersjukhuset	52030	25241	1515,1	1035,0	988,6	307,0	3,557	3,762	0,20439	0,0	0,314	88,7%	0,20439	0,31	2E-13		
3	Danderyds sjukhus	44198	20692	1342,7	631,4	969,0	216,6	2,916	3,342	0,42607	0,0	0,092	92,6%	0,42607	0,09	2E-13		
4	Södertälje sjukhus	9153	8830	403,8	171,8	235,5	81,5	1,244	1,000	0,00000	0,2	0,518	124,4%	0,00000	0,00	0E+00		
5	Vrinnevisjukhuset	18458	10349	632,8	320,1	550,2	147,0	1,459	1,584	0,12535	0,0	0,393	88,6%	0,12535	0,39	1E-13		
6	Västerviks sjukhus	9799	6251	347,1	121,8	433,8	85,2	0,881	0,881	0,00000	0,0	0,518	93,5%	0,00000	0,52	7E-14		
7	Länssjukhuset Kalmar	21593	11003	792,0	257,1	739,4	131,0	1,551	1,987	0,43631	0,0	0,082	78,6%	0,43631	0,08	1E-13		
8	Hallands sjukhus	37433	22488	1292,0	602,4	1199,2	266,8	3,169	3,241	0,07129	0,0	0,447	93,3%	0,07129	0,45	3E-13		
9	NU-sjukvården	37304	18923	1275,1	625,3	1039,4	272,3	2,667	3,185	0,51814	0,0	0,000	84,9%	0,51814	0,00	3E-13		
10	Södra Älvsborgs sjukhus	25581	16886	1010,6	460,6	885,3	162,4	2,380	2,530	0,15023	0,0	0,368	86,5%	0,15023	0,37	2E-13		
11	Skaraborgs sjukhus	31354	19604	1144,8	558,5	991,6	251,8	2,763	2,865	0,10189	0,0	0,416	89,2%	0,10189	0,42	2E-13		
12	Västmanlands sjukhus	28821	20246	1276,1	571,8	1213,0	280,7	2,853	3,204	0,35018	0,0	0,168	80,4%	0,35018	0,17	2E-13		
13	Länssjukhuset Gävle-Sandviker	21077	15924	1091,0	623,5	1118,4	318,1	2,244	2,746	0,50204	0,0	0,016	67,6%	0,50204	0,02	2E-13		
14	Sundsvalls sjukhus	15394	12510	811,0	219,2	845,1	169,3	1,763	2,043	0,27954	0,0	0,239	74,0%	0,27954	0,24	1E-13		
15	Östersunds sjukhus	16008	9831	728,6	253,0	528,5	115,6	1,385	1,814	0,42840	0,0	0,090	71,0%	0,42840	0,09	2E-13		
16	Skellefteå lasarett	8492	4671	350,5	127,0	259,4	54,2	0,658	0,873	0,21491	0,0	0,303	73,1%	0,21491	0,30	7E-14		
17	Sunderbyns sjukhus	19370	11728	838,0	335,4	716,2	164,3	1,653	2,096	0,44327	0,0	0,075	74,1%	0,44327	0,07	2E-13		

Appendix 7: Computation model of BiO-MCDEA-Super-efficiency in Excel 2013

DMU no.	Hospital	Efficiency, BiO-MCDEA, 2 output and 4 input variables	Efficiency CCR 2 output and 4 input variables	Rank of CCR BiO-MCDEA	Rank of CCR DEA	d	d^2
1	S:t Göran's Hospital	108,8%	134%	2	2	0	0
2	Stockholm South General Hospital	88,7%	98%	7	7	0	0
3	Danderyd's Hospital	92,6%	99%	5	6	1	1
4	Södertälje Hospital	124,4%	140%	1	1	0	0
5	Vrinnevi Hospital	88,6%	89%	8	12	4	16
6	Västervik's Hospital	93,5%	111%	3	5	2	4
7	Kalmar Regional Hospital	78,6%	112%	12	3	-9	81
8	Halland's Hospital	93,3%	93%	4	8	4	16
9	NU-Hospital group	84,9%	86%	10	14	4	16
10	Southern Älvsborg's Hospital	86,5%	90%	9	9	0	0
11	Skaraborg's Hospital	89,2%	90%	6	11	5	25
12	Västmanland's Hospital	80,4%	80%	11	15	4	16
13	Gävle Hospital group	67,6%	72%	17	17	0	0
14	Sundsvall's Hospital	74,0%	111%	14	4	-10	100
15	Östersund's Hospital	71,0%	88%	16	13	-3	9
16	Skellefteå's Hospital	73,1%	90%	15	10	-5	25
17	Sunderby Hospital	74,1%	80%	13	16	3	9
		Sum of differences in ranks of the hospitals in terms of efficiency between the two models					318
Spearman's rank ρ for 2016		$1 - (6*\sum d^2)/n(n^2-1) = 0.61$					0,61

Appendix 8: Comparison between efficiency scores of BiO-MCDEA Super-efficiency with 2 output and 4 input variables & BCC DEA Super-efficiency with 2 output and 4 input variables

DMU no.	Hospital	Total DRG points	Direct ward costs per DRG point	Indirect inpatient costs per DRG points	Reception costs per DRG point	Indirect outpatient cost per DRG point
1	S:t Göran's Hospital	48318	19005	9712	10471	2705
2	Stockholm South General Hospital	77271	19608	13395	12793	3973
3	Danderyd's Hospital	64890	20692	9731	14934	3338
4	Södertälje Hospital	17983	22454	9556	13096	4531
5	Vrinnevi Hospital	28807	21968	11112	19101	5102
6	Västervik's Hospital	16050	21628	7588	27025	5306
7	Kalmar Regional Hospital	32596	24298	7886	22684	4019
8	Halland's Hospital	59921	21561	10053	20013	4452
9	NU-Hospital group	56226	22678	11122	18486	4843
10	Southern Älvsborg's Hospital	42467	23798	10846	20847	3825
11	Skaraborg's Hospital	50958	22465	10960	19460	4941
12	Västmanland's Hospital	49068	26008	11652	24722	5721
13	Gävle-Sandviken hospital	37000	29485	16850	30227	8596
14	Sundsvall's Hospital	27904	29063	7856	30288	6066
15	Östersund's Hospital	25838	28198	9792	20456	4475
16	Skellefteå's Hospital	13163	26631	9651	19709	4120
17	Sunderbyn's Hospital	31098	26946	10784	23030	5283
	Mean	**679557**	**23205**	**10865**	**19452**	**4642**
	Standard deviation (SD)		3263	2194	5678	1299
	Coefficient of variation (CV)		**14,1%**	**20,2%**	**29,2%**	**28,0%**

Appendix 9: Each of the input variables contribution to the sum of the outputs, standard deviation and coefficient of variation for the input variables, for year 2016