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Bitcoin: The New Digital Gold?

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Abstract

Bitcoin has become a mainstream in the financial world. It has several similarities with Gold as low correlation with stocks, inflation hedge, government decentralization and the no currency attachment. Therefore, the name of the new “Digital Gold”. This study compares Gold and Bitcoin, offered as an alternatives financial assets. Thus, there are two research questions. First, if Bitcoin helps to reduce risk within a well-diversified equity portfolio, represented by the S&P 500. Second, if Bitcoin improves risk-return relation. Both questions are compared with Gold. Hence, two the financial approaches are the Minimum Portfolio Variance and the Tangency Portfolio. In order to have a solid base, this study evaluates in-sample and out-sample portfolios of rolling windows for 3, 6 and 12 months horizons. The results of the first question suggest that even the correlation coefficients are extremely low, they are not enough to mitigate risk because of Bitcoin high volatility. The in-sample and out-sample analysis demonstrate Bitcoin does not help to reduce risk in an equity portfolio as Gold does. For the second question, Bitcoin offers a better risk premium than Gold for in-sample and out-sample and considering both frameworks: long and long-short. However, Gold still gives a high premium per unit of risk. Therefore, Gold is also a good investment asset. Hence, even the similarities between both assets, Bitcoin cannot be considered as the new “Digital Gold” because it is still in the infancy stage to be treated as Gold.

Keywords: Bitcoin, Gold, Digital Gold, Minimum Variance Portfolio, Tangency Portfolio, In-sample and out-sample portfolios.

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1. Introduction

In recent years, Bitcoin has become a mainstream in the financial world. Due to several similarities with Gold, it has gained the name of the new “Digital Gold”. This study is interested in treat Bitcoin as an alternative financial asset and evaluates two aspects: first if Bitcoin help to reduce risk in an equity portfolio, as gold does. Second, if it has better risk-return relation compared with Gold.

1.1. Background

Since 2016, the largest investment banks start showing interest in cryptocurrencies and the technology behind them, for instance, blockchain. Especially Bitcoin became a mainstream in Wall Street discussions and news. The following are just a few events that confirm Bitcoin is here to stay.

In July 2017, Bank of America Merrill Lynch issued a report about Bitcoin. This paper analyzed the historical development of currencies thought time; with the conclusion that Bitcoin can be soon considered legitimate. Three characteristics were discussed: safety, liquidity and returns. Special emphasis was made in its exponential trading volume and similar patterns with gold, calling Bitcoin as the “digital gold”. The report suggested that in an early future Bitcoin can be considered a collateral or even a potential investment product. (CNBC, 2017)

In November 2017, Fidelity Investments announced his partnership with Coinbase, a cryptocurrency exchange. Allowing clients to see the digital cash they hold on this currencies. This is an example of how traditional financial institutions starting to integrate and adopt digital assets in their regular operations. (Investor’s Business Daily, 2017)

In June 2017, IBM announced is building a blockchain technology to be used for seven of the largest European Banks as Deutsche Bank, HSBC and Unicredit. This technology is basically a distributed digital ledger, its main function is to record transactions. In other

words, this is the technology required to trade cryptocurrencies. For now, this technology will exclusively be used to facilitate international trading. However, if in the future banks decides to adopt cryptos in their operational business they will already have the necessary technology. (CNCB, 2017)

In February 2018, Commercial banks as Barclays and HSBC allowed their clients to use their regular debit or credit card to buy cryptocurrencies legitimately due to the increasing demand.

In May 2018, Goldman Sachs became the first Wall Street bank that opens a trading desk for Bitcoin. Since January 2018, Lloyd Blankfein, Goldman Sachs' CEO, announced his interest to trade futures in Bitcoin due to the special demand of their customers. Therefore, now Goldman is helping clients who want to buy and sell Bitcoin futures on the Chicago Mercantile Exchange and the Chicago Board Options Exchange. (The New York Times, 2018)

From the mentioned events, it is clear the new banking era just begin. Goldman Sachs is the first mover to explicit incorporates Bitcoin in their operational business and it will not be a surprise if competitors follow the same path.

1.2. Research Purpose

Bitcoin has become a mainstream within investments banks calling it as “Digital Gold” due to similarities between them as low correlation with stocks, inflation hedge, government decentralization and the no currency attachment. Therefore, the purpose of this study is to compare Bitcoin against Gold and its effects generated in a portfolio. This investigation has two main research questions.

- 1) *Does Bitcoin actually help to reduce risk in a well-diversified equity portfolio compared with Gold?*
- 2) *Does Bitcoin improve the risk-return relation within a well-diversified equity portfolio, compared with Gold?*

Hence, this study is divided into two parts; question one will be answered through the Minimum Variance Portfolio and question two through the Tangency Portfolio. Moreover, within this study Bitcoin is treated as an alternative financial investment asset, so, it's trading as a currency is out of the scope of this investigation.

This analysis may be in the interest of asset portfolio managers, traders, investment bankers and academics. Moreover, this topic can be relevant to different financial institutions, central banks and regulators due to it represents a revolutionary investment product.

1.3. Objective

The objective is based on the construction of well-diversified equity portfolio which may cover three characteristics: increase returns by keeping the lowest possible risk; provide liquidity and incorporate alternative asset that has negative correlation against the rest of the portfolio. (Gerstein Fisher Invest with Reason, 2017). The analysis will be thought three assets: stocks, Gold and Bitcoin. Thus, two portfolios are created: S&P+Bitcoin and S&P+Gold. The Modern Portfolio Theory will be implemented to find the Minimum Portfolio Variance and Tangency Portfolio. The period of analysis is divided in 3, 6 and 12 months with overlapping rolling windows to compare the volatility in-sample and out-sample portfolios.

The main limitation of the study is the small period of analysis, due to Bitcoin is a recent trading asset compared with Gold. The study is a relationship between them, so, it covers the same trading days for both assets; which gives a study period of 5 years.

1.4. Research Structure

The structure of this study is the following. Chapter 2 describes the assets involved in this study. Chapter 3 explains previous academic research regarding gold and Bitcoin and their performance within a portfolio. Chapter 4 indicates the financial models used for the analysis. Chapter 5 presents the utilized data. Chapter 6 exposes all the result and analysis of this study. Chapter 7 indicates the main conclusions.

2. The Assets

This study is based on the interaction of three assets: Bitcoin, Gold and S&P. Therefore it is convenient to discuss and provide a framework for them.

2.1. Cryptocurrencies

In the recent years, the word “cryptocurrency” or “crypto” has become a financial mainstream; suddenly everyone is interested in its financial implications. In order to comprehend their role, it is necessary to understand the nature of any currency. The money is just an item to exchange goods and services. Its value is inherent to the piece of paper; thus, a currency has a value because the worldwide economic agents, usually a country, assigned it and agree with it. In this sense, cryptocurrencies can be used as an item of trade if both seller and buyer agree.

The cryptocurrency name has its origins in the combination of digital currencies and the use of cryptography that secure transactions. Its principal characteristic is due to they are not issued by any government, they are decentralized. This means no single institution controls the network. Instead, they are created and controlled by computer algorithms which record all the transactions. The cryptos use the blockchain technology, which means every user, known as a miner, keeps the record of every transaction and by solving complex computer problems can be reward with new coins. So, basically, the miners and their massive computer power to record all the transactions are the ones who control the network. (Nasdaq, 2018)

The second characteristic is the restricted supply which means the total amount in circulation is limited. In the case of conventional currencies issued by governments, they work under an inflationary system, where the money loses value through the time because central banks issue more money in terms of the economic growth. In contrast, for cryptocurrencies due to the limited amount, they work under a deflationary model meaning that through time if demand increases and supply stays (which is the case) the crypto increases in value but not in quantity.

The third characteristic is the anonymity. Traditional currencies are controlled by banks so the user identification is mandatory to obey legislation, for instance against money laundering. In the case of cryptos there is no a central validator, so users do not need to identify. The system works by only checking previous transactions to confirm if the sender has the number of cryptos required for the transaction, it is just a wallet revision. Since the miners are the ones recording all the transactions, they are visible to everyone. (Nasdaq, 2018)

Nowadays, there are around of 1,300 different cryptocurrencies. Bitcoin, Ethereum, Ripple and Litecoin are the ones with the largest market capitalization. (Coindesk, 2018)

2.1.1. Bitcoin

Bitcoin is the most popular and trade cryptocurrency. It was originally created by a software developer under the pseudonym of Satoshi Nakamoto in 2008. It has a market capitalization of about \$164,143,168,264 USD according to the Coindesk (2018) record. As all cryptocurrencies, Bitcoin has a limited supply which is a maximum of 21 million coins. Every hour a limited number of new Bitcoins are generated, using a decreasing rate until they reach the maximum, making this asset scarce and therefore attractive.

This study is interested in threat Bitcoin as an alternative financial investment asset that may help portfolio diversification because it is low correlated with stocks, it is out of the inflationary economic model and due to the exponential trading volume it has liquidity. Thus the properties of Bitcoin as a currency or as a medium of trading are out of scope for this analysis.

2.2. Commodities

Commodities are basic goods commonly used as raw materials for goods or services. Some examples are grains, gold, beef, oil, natural gas and also foreign currencies. Commodities are traded by future contracts within an established market, for instance, Chicago Board of Trade. These contracts are a legal obligation to buy or sell a commodity at a specific day in the future; also the commodity's quantity and quality are determined. There are two types of commodity's traders: the hedgers and the speculators. The first ones are buying and selling the actual commodity, they hedge against the change in price. The second ones are just interested in making profits by the volatile price movements. (Natale, 2016)

Commodities are commonly used by portfolio managers to diversify investment opportunities. The main reason is due to commodities have a negative correlation with other types of financial assets, specifically to stocks and bonds. This low correlation trend has been observed since around 30 years ago. (Morningstar, 2011) The second reason is that commodities are considered as an inflationary hedge because they have a positive correlation to changes in inflation. Furthermore, commodities overreacts to unexpected inflation changes. Thus, when there is an increment on inflation, commodities returns increase even more. (Whitton and Thuerbach, 2015)

2.2.1. Gold

Gold is the most used commodity to diversify a portfolio. According to the World Gold Council (2018) it is among other commodities, the less correlated with stocks and bonds. The reason is that it is not commonly used as a raw material in the industry, therefore it is the less exposed to the business cycle. Also, because the gold is the easiest commodity to store.

This study is interested in use exclusively gold instead of another commodity since it has all the properties of a commodity (low correlation with other assets and inflation hedge) and it is the most similar to Bitcoin. For instance, gold is not centralized so the price is not influenced by political decisions. Also, the gold price is not attached to a specific currency, it is determined by the demand and not by the supply of gold mines. Both Bitcoin and gold

are easy to store. (Radomski, 2015) Finally, the decision is also based on previous studies that will be discussed in the next section.

2.3. S&P 500

Standard and Poor's 500 Index is formed by 505 stocks issued by the 500 largest market capitalized companies in the US. The companies are selected based on their liquidity, size and industry. To be part of this index, companies must be in the US, with a market cap of at least \$6.1 billion and at least 50% of the stocks must be available for the public. S&P 500 rebalances quarterly. Some of the represented industries are information technology (24.9%), financials (14.7%), healthcare (13.7%), consumer discretionary (12.7%), and industrials (10.2), among others. For this reason, this index is the benchmark for the US stock market and foreign stock markets as well. (Bloomberg, 2018)

For the purpose of this study, the S&P 500 index will represent a well-diversified stock portfolio which will be the common base between the comparison with Bitcoin and Gold, with an especial interest to compare the change in volatility.

3. Previous Research

Due to the intrinsic gold properties, it has been a factor of study as a diversifier within a portfolio and its function to reduce risk. Moreover, recent academic investigations have been done to examine the correlation between Bitcoin and traditional financial assets, its role within a portfolio and its similarities with gold. The following are some previous research regarding gold and Bitcoin.

According to Lawrence (2003), the two principal differences between gold and the rest of commodities are the liquidity factor and the price changing. His study based on dynamic VaR demonstrated that commodities returns (oil, zinc, lead, aluminum and copper), except gold, are correlated with macroeconomic variables and with financial indexes the reason is that the risk is driven by the business cycle. In contrast, gold returns are uncorrelated to financial index performance. Therefore, this study will make an analysis using exclusively gold instead of using several commodities, for instance, a well-diversified commodity index.

Gold has been used as an alternative investment asset with the objective of helping diversification and therefore reduce the risk in a portfolio. Baur and Lucey (2010) investigated if gold could be used as a hedge against the stock and bond markets. They defined a hedge as an asset that is uncorrelated, on average, with another asset. By using an econometric approach and GARCH process in the error term, they conclude that gold is a hedge against stocks. Furthermore studies, Baur and McDermott (2010) also conclude that gold is a hedge especially in the US and European stock markets. Hood and Malik (2013) evaluated if gold and other precious metals, as silver and platinum, behaves as hedgers; the main conclusion was that gold has a statistically significant negative correlation with the S&P 500.

Based on the mentioned studies, this study will construct a portfolio of S&P+Gold and use it as a benchmark to compare what is the effect generated by Bitcoin. Additionally, this paper will use the gold hedge property against the stock market and evaluate if Bitcoin has the same property.

Recently studies as Dyhrberg (2015) establish that Bitcoin and gold present very similarities: both are scarce, costly to extract, decentralized, unregulated and are mined by independent users. Also, their intrinsic value does not justify the current market value. This study used GARCH model to show that neither good nor bad shocks affect Bitcoin and gold returns; so, they can be used as market risk hedgers. The main suggestion was that Bitcoin can be a tool for risk-averse investors because it has the advantages of commodities and currencies.

Moreover, Bouri, et al. (2017) used a bivariate DCC model proposed by Engle to compare correlations between Bitcoin, stocks, bonds, currencies and commodities. They concluded that it exist statistical evidence to consider Bitcoin as a hedge against the commodity index and the movements in the Chinese stocks, suggesting Bitcoin can reduce risk.

A similar study, Briere et al. (2015), the objective was to analyze the correlations between Bitcoin, traditional assets (bonds, stocks, currencies) and alternative investments (commodities, hedge funds, real state) by using mean-variance spanning test. The conclusion revealed that even though there is a low correlation between the mentioned assets, it does not imply that during crisis the correlations will maintain in the lower levels. Also, this study showed that using a well-diversified portfolio, Bitcoin can improve the relation risk-return; suggesting that it deserve to be considered as a diversifier.

Based on mentioned investigations, this study will use Bitcoin as potential diversifier because of the demonstrated low correlation with traditional assets. Also, the analysis will focus on investigating the effect that Bitcoin can potentially generate in the portfolio volatility.

Additionally, Eisl, et al. (2015) investigated if Bitcoin improves portfolio diversification by using the Conditional Value at Risk framework, which adapts better for returns that do not follow a normal distribution. One relevant aspect of this study was the calculation of monthly out-sample portfolios based on the optimal weights of 12 months, generating rolling horizons. The result was that an optimal well-diversify portfolio includes Bitcoin because it contributes the risk-return ratio. However, Bitcoin weights were very low; on the other hand, this was considering beneficial to investors in terms of liquidity. A very similar study was realized by Brauneis and Mestel (2018) by using the Markowitz mean-variance framework.

Based on Eisl et al. (2015), this study will also use a rolling window approach. However, this study divides the estimate horizon in 3, 6 and 12 months to also analyze the in-sample and out-sample portfolios. Additionally, the analysis will use Markowitz framework as the same approach implemented by Brauneis and Mestel (2018).

Wu and Pandey (2014) were interested in analyzing if Bitcoin enhances efficient portfolio, composed of traditional assets. The study was by generating random weights from a simulation of 1000 trials and calculating the Sharpe ratio. One important assumption was the valuation of only long portfolios. The conclusion was that Bitcoin does not have the essential characteristic of a currency but it can be considered as a financial asset. However, the limited acceptance of Bitcoin through investors makes it a poor medium of exchange; also, its very high volatility makes it a poor value asset. Besides the results, this study still recommends holding Bitcoin within a portfolio because it does improve the portfolio performance.

Based on Wu and Pandey (2014), this study will also calculate the tangency portfolio and use the Sharpe ratio as a performance indicator.

Likewise previous studies, Carpenter (2016) considered Bitcoin as an attractive investment because it improves risk-return. It mentioned that the return distribution is skewness and has an excess of kurtosis. Yet, the conclusion was that Bitcoin is at a young age to generalize the behavior of returns within a portfolio.

All the mentioned investigations give the framework and bases which drive this study.

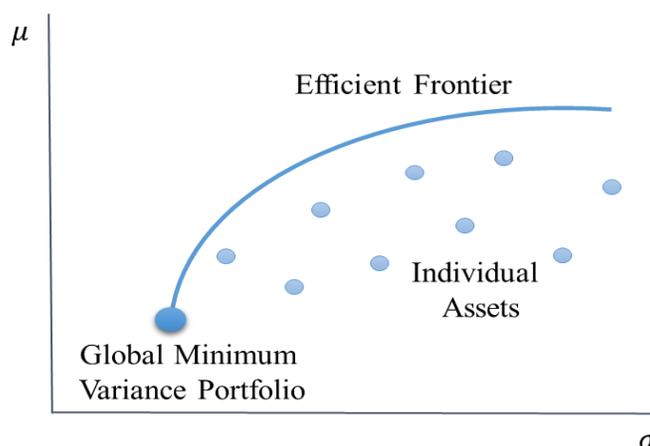
4. Methodology

The objective of this study is to investigate if Bitcoin, assumed to be treated as a financial asset rather than a digital currency, can help to reduce the risk in a well-diversified portfolio. To achieve it, different methodology has been used.

4.1. Minimum Variance Portfolio

Modern Portfolio Theory (MPT) is based on the assumption that investors have an increase and concave utility function, in other words, they are risk-averse. They will prefer higher returns and lower risk. In presence of two investment opportunities, they will choose the less risky rather than the riskier alternative. This theory, proposed by Markowitz in 1952, consists of portfolio construction that maximizes the expected return given a certain level of market risk. Therefore, it is possible to form an efficient frontier which can be generated by the linear combination of two distinct optimal portfolios. This set of portfolios represent the best return an investor can expect from the portfolio given the risk; or expressed in a different way, they are the minimum variance portfolios given an expected return. The vertex of this frontier represents the global minimum variance, which is the lowest standard deviation from the set of securities. (Bodie, Kane and Markus, 2014)

Figure 1. Efficient Frontier.



The optimization problem is very straightforward considering the two principal components: the expected return and the standard deviation. In this way, minimum variance portfolios are given by minimizing the standard deviation given the expected return. The expected return is calculated as the weighted sum of individual historical returns.

$$\text{Min } \sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \sigma_{ij} \quad \text{St: } \mu_p = \sum_{i=1}^n \omega_i \mu_i$$

Where:

μ_p : Portfolio expected return

σ_p^2 : Variance Portfolio

ω_i : Weight of the asset i

σ_{ij} : Covariance between assets i and j

This study analyzes portfolios formed by the combination between S&P 500 and one of this two assets: Gold or Bitcoin. Thus, the expected return and the standard deviation are given by the following.

$$\mu_p = w_1 \mu_1 + w_2 \mu_2$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2$$

The optimized asset weights are constrained to sum 1, meaning the portfolio is fully invested. In this particular study, two scenarios will be analyzed: (i) only long position and (ii) long and short investments are allowed.

$$w_1 + w_2 = 1$$

$$i) w_1, w_2 \geq 0$$

ii) *no constraint*

The optimization process, in this particular case the minimization of the portfolio variance is found by equal to zero the derivative of the portfolio variance respect the weight of asset one.

$$\frac{\partial \sigma_p^2}{\partial w_1} = 0$$

Thus, the minimum variance portfolio weights for asset one can be obtained by the subsequent formula. Due to the portfolio is based on two assets, the optimized weight of asset two is by subtracting the optimized weight of asset one to the full investment.

$$w_1^* = \frac{\sigma_2^2 - \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

$$w_2^* = 1 - w_1^*$$

This study uses the portfolio optimization theory to get the optimal weights of minimum variance portfolios: the S&P+Bitcoin and S&P+Gold, with the objective to analyze and compare if the risk is reduced.

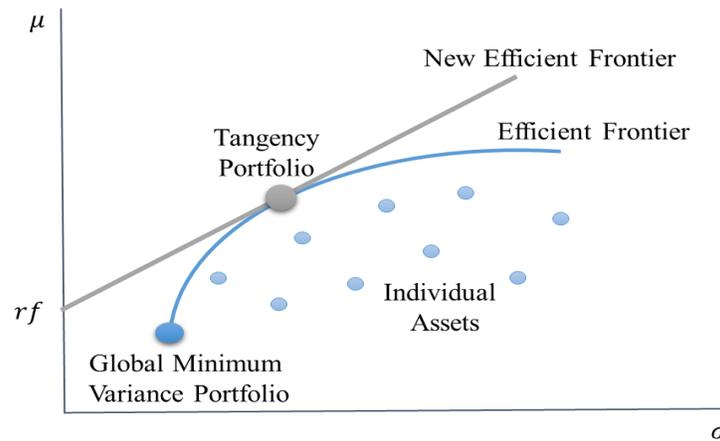
4.2. Tangency Portfolio

The Tangency Portfolio is based on the Markowitz Model Portfolio Theory. It assumes the possibility that investors can borrow or lend at a current risk-free rate. Also, it includes a tangency portfolio which is well-diversified and formed by risky assets.

$$\mu_p = rf + \left(\frac{\mu_T - rf}{\sigma_T}\right)\sigma_p$$

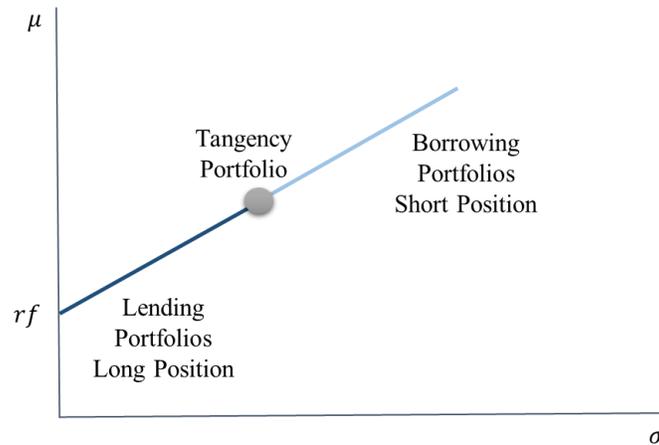
Thus a positive slope line is originated from the risk-free rate through making a tangency with the efficient frontier. Therefore, this new frontier reflects all the possible combinations between the tangency portfolio and the risk-free rate. Investors are better situated in this frontier because of the inclusion of the risk-free asset. (Bodie, Kane and Markus, 2014)

Figure 2. The efficient frontier with risk-free and tangency portfolio.



As mentioned, this study analyzes the short and long portfolio positions so the following graph illustrates where the investor may be situated in the new frontier with this two possible scenarios.

Figure 3. Long and Short Portfolios.



From the graph, it can be observed that if the investor chooses the tangency portfolio all the wealth will be invested in risky assets. The portfolios in the LHS of this portfolio indicates long positions with a combination of risk-free and risky assets. Thus, in the RHS of the tangency portfolio, the investor invest more than 100% of his wealth so a short position is held.

4.3. Sharpe Ratio

This is the most popular performance measurement used for portfolio managers because it measures the efficiency of the investment. Financially, it represents the excess return per unit of total risk, in other words, it tells investors how much reward they get per volatility. Mathematically, it is the slope of the frontier formed by the risk-free asset and the tangency portfolio. The Sharpe can be calculated by the following. (Reilly and Brown, 2012)

$$sr = \frac{\mu_p - rf}{\sigma_p}$$

When the Sharpe is based in q period returns, it is important to rescale in order to make it comparable. This study has a daily data returns, so in order to make it yearly, the following formula is needed.

$$sr(q) = \sqrt{q} sr_i$$

The second part of this study is based on the tangency portfolio, in other words, when the Sharpe ratio is maximized. Managers should make sure the investor gains the highest Sharpe ratio. The maximization process is found by equal to zero the derivative of the Sharpe ratio respect the weight of asset one.

$$\frac{\partial sr}{\partial w_1} = 0$$

Thus, the optimal weight for the asset one of the tangency portfolio is obtained by the following formula. Because this study is interested in only two assets, the optimal weight of asset two is by subtracting the optimized weight of asset one to the full investment.

$$w_1^* = \frac{(\mu_1 - rf)\sigma_2^2 - (\mu_2 - rf)\rho_{12}\sigma_1\sigma_2}{(\mu_1 - rf)\sigma_2^2 + (\mu_2 - rf)\sigma_1^2 - (\mu_1 - rf + \mu_2 - rf)\rho_{12}\sigma_1\sigma_2}$$

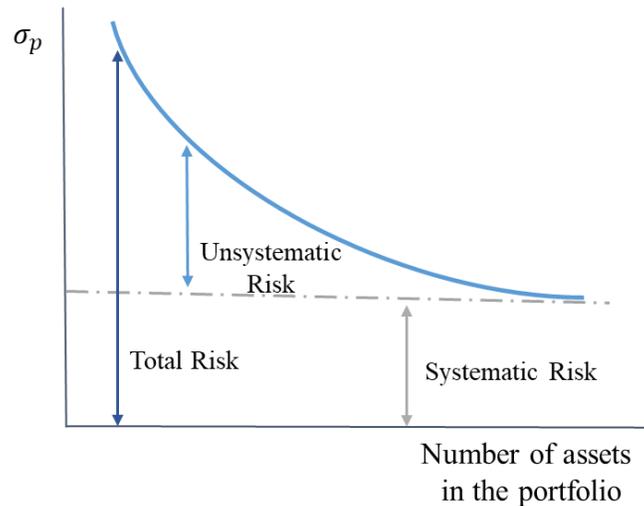
$$w_2^* = 1 - w_1^*$$

Similar to part one of the study (the MVP analysis), this second section calculates the tangency portfolio of S&P+Bitcoin and S&P+Gold. This is to give investors the alternative to include the risk-free asset.

4.4. Risk and Correlations

Risk management is important for portfolio analysis. There are two types: the systematic and unsystematic risk. The systematic, called “un-diversifiable” is the volatile attached to the entire market. The unsystematic, called “diversifiable” is the uncertainty involved with the firm and can be reduced by a mix of different investment alternatives within a portfolio, in other words, through diversification. (Travers, 2004).

Figure 4. Risk involve in a portfolio.

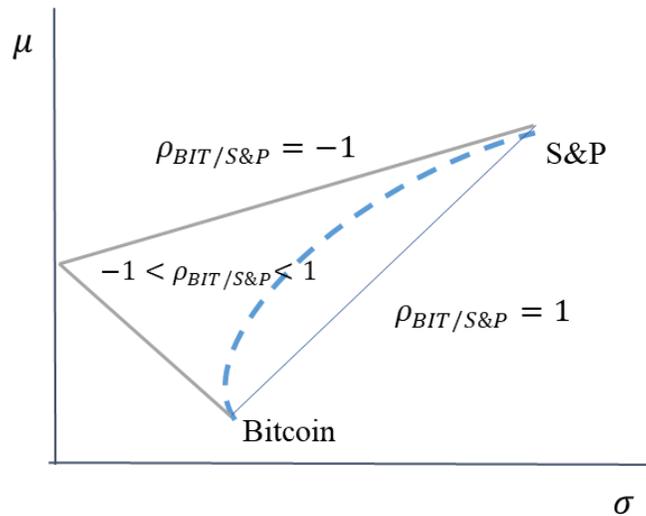


The MTP is based on the diversification which implies to look up for uncorrelated assets. The correlation coefficient is a statistical measure that determines the degree in which two variables are associated. This coefficient only considers the linear relationship and can vary in a range between -1 and 1.

$$\rho_{12} = \frac{cov(r_1, r_2)}{\sigma_1 \sigma_2}$$

In theory, it is possible to construct a zero risk portfolio within two assets that have a perfect negative correlation. One of the objectives of this study is to investigate the degree of correlation between S&P-Bitcoin and S&P-Gold. Figure 5 shows how the correlation coefficient between S&P and Bitcoin possible behaves.

Figure 5. Correlation between S&P and Bitcoin.



By determining the degree of correlation between assets it can be used within a portfolio to investigate if Bitcoin helps to reduce the unsystematic risk more than Gold.

4.5. In sample and out sample portfolios (rebalancing)

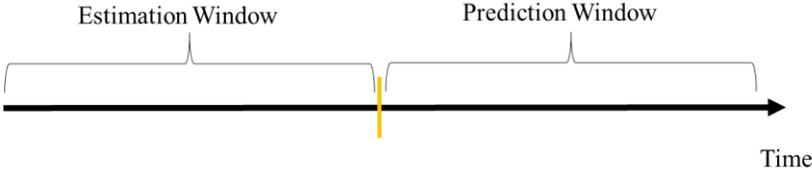
The rolling analysis of time series is commonly used to backtest a statistical model using the historical data to evaluate its future accuracy. The backtesting method splits the data into estimation horizon (in-sample) and prediction horizon (out-sample). Thus, the data within the estimation window is used to predict h-periods ahead. This method is used until it is not possible to make more h-step predictions. (Zivot & Wang, 2006)

In finance, rolling portfolio optimization is very common to evaluate investment performance. It helps managers to estimate future returns and portfolio volatility; especially in the rebalancing process which is a reallocation of asset weights within the portfolio. In practice, it is recommended to make it once per year.

This study analyzes three different periods: 3, 6 and 12 months. The aim is to evaluate the in-sample and out-sample returns and volatility of the S&P+Bitcoin and S&P+Gold portfolios. For the in-sample portfolio analysis, the optimized weights obtained within a certain period are used to estimate risk and return of the portfolio within the same period of time.

For the out-sample portfolios, first, it is necessary to divide the data and determine the estimation and prediction period. Then, optimize the asset weights by using historical returns within the estimation window. Finally, these optimized weights are used to predict returns and volatility in the prediction window.

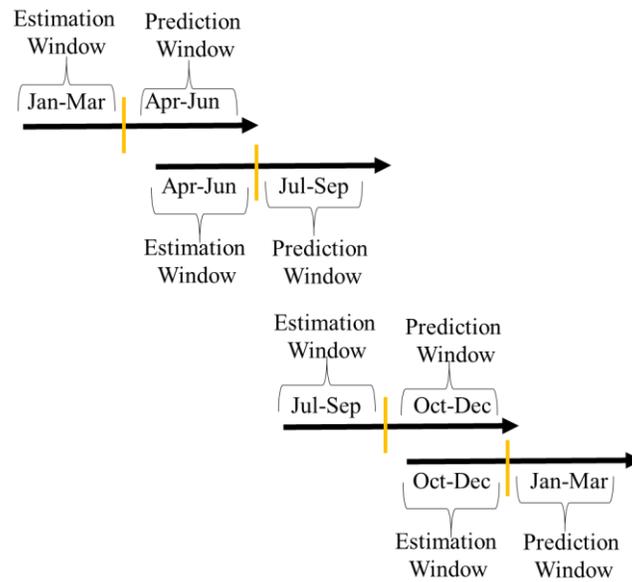
Figure 6. Estimation and Prediction window.



Therefore, both the estimation and prediction windows are divided into periods of 3, 6 and 12 months. The objective is by using the optimized weights in the estimate window to predict the out-sample risk (prediction window) and investigate if Bitcoin helps to reduce it more than Gold. It is important to mention the rolling windows overlap in all periods.

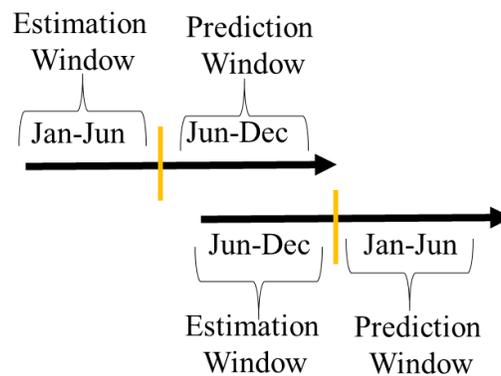
As mentioned, the first estimation and prediction window analysis covers three months. In other words, the optimized weights obtain within period January-March are used to estimate the risk and return of the S&P+Bitcoin and S&P+Gold portfolios of the period April-June. In the same way, this period (Apr-Jun) is the estimation window for the period July-September and the process continues over the 5 years of analysis. Figure 7 is a timeline that illustrates how 3 rolling windows move through time.

Figure 7. 3-months rolling windows.



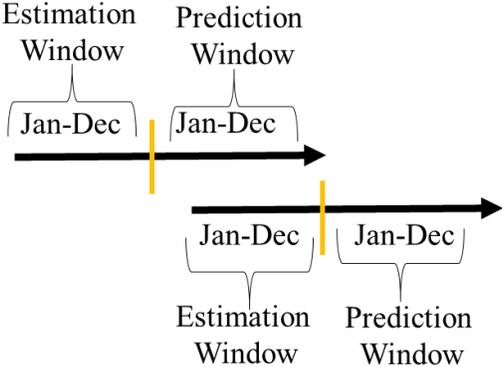
The second period of analysis covers 6 months rolling windows. Thus, the optimized weights of the period January-June are used to estimate the returns and risk of the mentioned portfolios of the period July-December. In this way, the period (Jul-Dec) is also the estimate window for the first semester of the next year, as the Figure 8 illustrates.

Figure 8. 6-months rolling windows.



The last rolling window is yearly, the approach is the same but now the optimized weights of year one are used to estimate the risk and returns of the portfolios in year two, until the 5 years of analysis of this study concludes.

Figure 9. 12-months rolling windows.



The objective of analyzing in-sample and out-sample methods is to have enough portfolios in different time horizons and have a solid base to evaluate the effect of Bitcoin and Gold in a well-diversified equity portfolio.

5. Data

This study uses a time series based on historical daily closing prices of S&P 500 Index and Gold. The Gold price is the well-known XAU/USD per ounce of gold. The Bloomberg database was used to obtain the mentioned historical prices. Regarding Bitcoin, the daily closing prices were obtained from CoinDesk.com database. The price is Bitcoin/USD. Therefore, both Gold and Bitcoin are in terms of the same currency, USD, in order to be consistent with the S&P 500 index which is the base of the analysis.

The data covers the period from January 2013 to March 2018. Furthermore, it is subdivided in 3, 6 and 12 months in order to generate rolling estimate windows. Despite the fact Bitcoin started its operations in 2008, it was until 2013 the trading volume became significant and started to generate returns different from zero. Additionally, one of the objectives is to generate portfolios that are liquid for investors, so, the first years of Bitcoin with low trading volume can affect the objective. Therefore, the analysis of the three assets starts in 2013.

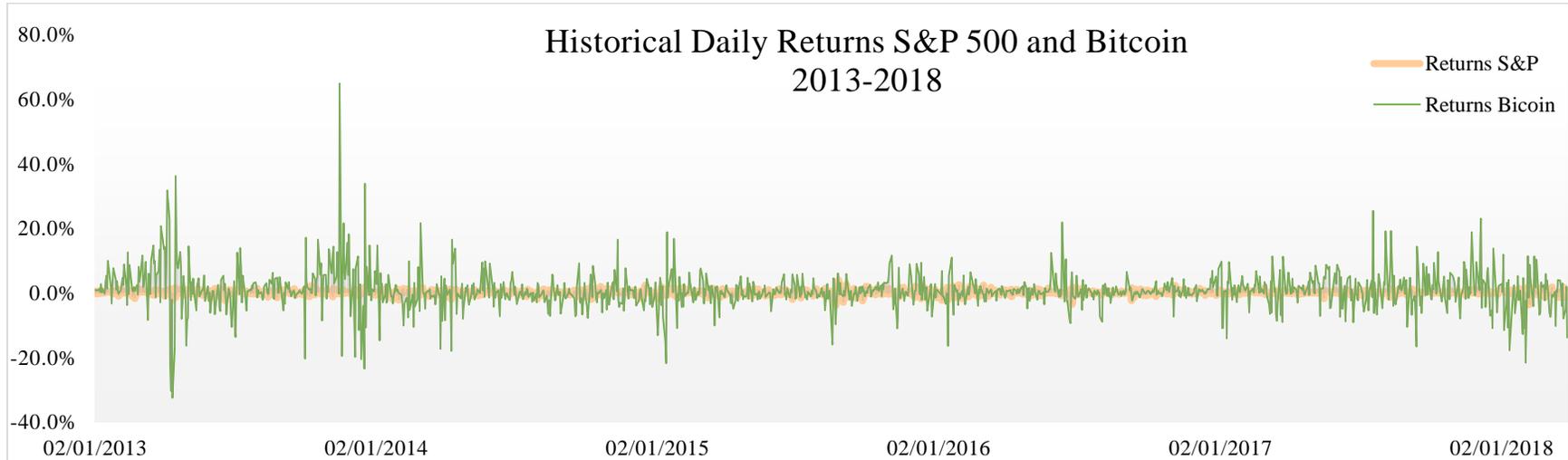
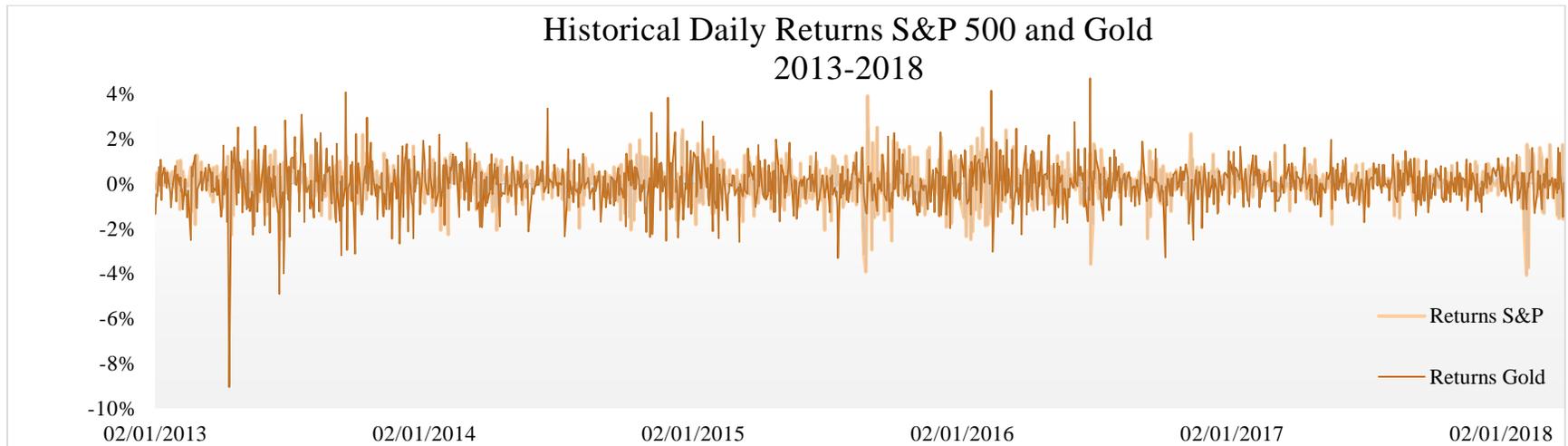
Based on prices, the daily returns were calculated by using the following formula.

$$\mu_i = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where $i = S\&P, Bitcoin \text{ and } Gold$

Figure 10 shows daily returns of Bitcoin and Gold against the S&P. It indicates Bitcoin and Gold, as other financial assets are sensitive to certain shocks and shows they are non-stationary. Moreover, both graphs suggest a random walk and volatility clustering because there are periods of high volatility and periods of relative calmness.

Figure 10. Historical daily returns of Gold and Bitcoin against S&P 500.



The descriptive statistical analysis of the returns is expressed in Table 1, from where Bitcoin has the maximum variance of 0.0035, among the others. Thus, the minimum return was -33% and the maximum achieve 65%. Clearly noticing, Bitcoin is the riskiest asset among Gold and S&P.

Table 1. Descriptive statistics daily returns of the historical data (Jan 2013- March 2018).

Descriptive statistics daily returns			
	S&P	Bitcoin	Gold
Mean	0.0005	0.0064	-0.0001
Median	0.0005	0.0032	0.0000
Minimum	-0.0410	-0.3257	-0.0907
Maximum	0.0390	0.6482	0.0469
Standard Deviation	0.0077	0.0591	0.0098
Variance	0.0001	0.0035	0.0001
Kurtosis	3.2302	15.8120	7.8286
Skewness	-0.5321	1.1588	-0.6560
# Observations	1319	1319	1319

Additionally, it is important to mention S&P follows a normal distribution because it has a kurtosis of 3.23. In contrast, Bitcoin has heavy tails because its kurtosis is 15.81, meaning it has many outliers. Concerning S&P and Gold, they are negatively skewed, -0.53 and -0.65 respectively; which implies returns are more concentrated in the left tail. To visualize them graphically, appendix A illustrates the histograms of the three assets.

As mentioned in chapter 4, correlation is crucial to achieving diversification. Table 2 shows the correlation matrix between the three assets.

Table 2 Correlation matrix between S&P, Bitcoin and Gold of the historical data (Jan 2013- March 2018).

Correlation Matrix			
	S&P	Bitcoin	Gold
S&P	1	0.01	-0.08
Bitcoin		1	0.03
Gold			1

The results are according to the previous literature (Lawrence (2003), Baur and Lucey (2010) Dyhrberg (2015) and Briere et al. (2015)) because Gold and Bitcoin present a very low correlation with the stock market. The correlation scenario between this two assets is interesting. On one hand, in absolute terms, Gold is higher correlated than Bitcoin. So, Bitcoin has the lowest correlation, potentially useful to reduce the risk portfolio. On the other hand, Gold presents a negative correlation which is essential to reduce risk within a portfolio. Likewise, it is important to notice that Gold and Bitcoin are also low correlated between them.

The negative correlation of Gold suggests it can be treated as a hedge. Therefore, the econometric approach suggested by the Baur and Lucey (2010) was used as a confirmation. The same procedure was used by just adapting the regression with the assets of this study.

$$\mu_{S\&P} = \alpha + \beta_1\mu_{Gold} + \beta_2\mu_{Bitcoin}$$

The results confirm Gold can be used as a hedge against the stock market because $\beta_1 = -0.0666$. The negative sign implies that investors with Gold in their portfolio will receive a compensation for the losses caused by negative stock returns. Regarding Bitcoin, $\beta_2 = 0.0016$ even it has a positive sign it is the value is very low. This implies that any change in Bitcoin returns affect just by 0.0016 the stock returns and this low relation may be an advantage to reduce risk within the portfolio.

6. Results and Analysis

The first objective is to determine if Bitcoin helps to reduce risk within an equity portfolio compared with Gold when they are offered as alternative investment assets. The second aim is to evaluate if Bitcoin improves the risk-return relation, also compared with Gold. Due to several similarities between both assets, Bitcoin has gained the name of “Digital Gold”, so the comparison will be exclusively with Gold, Hence, the analysis is based on the construction of two portfolios: S&P+Bitcoin and S&P+Gold within different sub-periods.

Two main parts will be discussed. Firstly, the Minimum Variance Portfolio for investors who are only interested in reducing risk by using a diversified portfolio. This analysis is limited to only long positions.

Secondly, the Tangency Portfolio for investors interested to include a risk-free asset. This analysis is over two frameworks: one only with long position and the other allowing short positions as well.

6.1. Minimum Variance Portfolio

This study calculates, from each sub-period, the returns, standard deviation, variance and correlations. Therefore, based on the Modern Portfolio Theory and by implementing the optimization process, the optimal weights for the Minimum Variance Portfolios (MVPs) are obtained. The formula used is presented in Chapter 4.

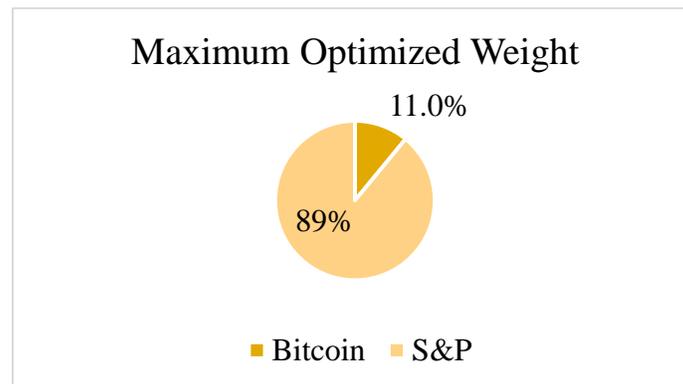
Thus, the returns and risk of the S&P+Gold and S&P+Bitcoin portfolios are calculated for each period. Even though the optimization problem is about the variance, this study considers standard deviation as the representation of risk. Furthermore, the standard deviation is rescaled by 252 trading days (one year) in order to facilitate the interpretation.

6.1.1. 3 months

The daily returns are divided by sub-periods of 3 months which is around 60 trading days. So, every year had 4 periods: January-March, April-June, July-September and October-December. Since the data is from January 2013 to March 2018, this analysis shows 21 estimate rolling windows which overlap with the prediction periods. Both periods cover the same trading days.

This study has a similar output to Eisl, et al. (2015) in terms of the optimized weights for the portfolio S&P+Bitcoin. In both cases, the optimization process gives to Bitcoin a very low weight, however, it does include as part of the diversified portfolio. Within this 3 months analysis, the maximum suggested weight for Bitcoin is 11%.

Figure 11. Maximum optimized weight of Bitcoin and S&P within the S&P+Bitcoin portfolio of the 21 rolling windows (3 months horizon analysis).



As explained in Chapter 4, the correlation between assets is important for diversification. Table 3 shows the maximum and minimum correlations' results between S&P/Bitcoin and S&P/Gold of the 21 rolling windows generated in the 3 months analysis. During this horizon, both assets have negative correlations against S&P. Before, when the analysis considers all the historical data from January 2013 to March 2018 (showed previously in Table 2) only Gold presents negative correlation to S&P. Another difference is that Bitcoin and Gold correlations increase, however they stay very low. Thus, they still can be considered as independent assets from S&P. The details of correlations for each sub-period are presented in appendix B.

Table 3. Maximum and minimum correlation coefficients between S&P/Bitcoin and S&P/Gold of the 21 rolling windows (3 months horizon analysis).

Correlations 3 Months Analysis		
	S&P/Bitcoin	S&P/Gold
Max	0.26	0.28
Min	-0.22	-0.58

6.1.1.1. In-sample Portfolios

The standard deviation of the S&P+Gold is compared with the individual risk of S&P. Separately, the risk of S&P+Bitcoin is also compared with S&P. Thus, the impact of Gold and Bitcoin within a portfolio can be quantified by measuring the effect of risk-reduction against the individual risk of S&P. As well, due to the analysis is in-sample, all the weights and comparisons are within the same period.

Table 4 shows the results of all the standard deviations. As expected, both portfolios, S&P+Gold and S&P+Bitcoin, reduce risk due to the incorporation of one asset and by taking the advantage of the low correlations. From the results it clear Gold generates the major risk reduction, being in line with the -0.58 negative correlation against S&P. Taking the most volatile period, Jul-Sep 2015, S&P registered a standard deviation of 20.97%. So, the incorporation of Gold helped to reduce risk to 10.57% whereas Bitcoin had a poor impact because the risk maintained almost the same with 20.08%. The results demonstrate even if Bitcoin has a negative and low correlation with S&P, it is not enough to generate an important risk-reduction. The principal reason is that Bitcoin presents a high volatility. Form the descriptive statistics daily returns (Table 1) it can be observed that Bitcoin has a standard deviation of 0.0591; In contrast, Gold has a lower standard deviation of just 0.0098. Also, the low correlation between S&P/Gold makes it a good hedge against an equity portfolio.

Table 4. Optimized standard deviations of the 21 rolling windows in-sample Minimum Variance Portfolios and the individual S&P standard deviations.

Optimized Standard Deviation of in-sample MVP				
Year	Months	S&P	S&P+Bitcoin	S&P+Gold
2013	Jan-Mar	9.09%	9.07%	7.05%
	Apr-Jun	14.05%	13.93%	13.81%
	Jul-Sep	9.03%	8.66%	8.57%
	Oct-Dec	10.36%	10.16%	9.18%
2014	Jan-Mar	11.99%	11.38%	8.24%
	Apr-Jun	9.38%	9.27%	6.76%
	Jul-Sep	9.41%	8.67%	6.40%
	Oct-Dec	14.22%	13.63%	10.99%
2015	Jan-Mar	14.14%	14.13%	9.50%
	Apr-Jun	10.11%	9.63%	8.13%
	Jul-Sep	20.97%	20.08%	10.57%
	Oct-Dec	14.85%	13.88%	10.79%
2016	Jan-Mar	18.53%	17.81%	10.15%
	Apr-Jun	13.67%	12.80%	7.02%
	Jul-Sep	9.82%	9.65%	7.50%
	Oct-Dec	8.59%	7.78%	6.97%
2017	Jan-Mar	6.70%	6.66%	5.51%
	Apr-Jun	7.35%	7.34%	4.91%
	Jul-Sep	7.11%	7.07%	5.00%
	Oct-Dec	5.56%	5.56%	4.34%
2018	Jan-Mar	18.44%	18.36%	9.76%

Furthermore, this study is interested in quantifying the risk reduction provided by Bitcoin. So, a scenario analysis is provided, by considering the best, the average and the worst risk-reduction. Table 5 shows the results of each scenario. Thus, the best situation represents the maximum risk-reduction of the portfolio against S&P, the comparison is between the same trading years to reflect potential differences between years.

Table 5. Scenario analysis of the risk reduction in-sample S&P+Bitcoin and S&P+Gold against individual S&P standard deviation (3 months horizon analysis).

Year	Months	In- Sample Scenarios S&P+Bitcoin			In- Sample Scenarios S&P+Gold		
		Worst	Average	Best	Worst	Average	Best
2013	Jan-Mar	0.12%	0.23%	0.38%	0.24%	0.63%	1.19%
	Apr-Jun						
	Jul-Sep						
	Oct-Dec						
2014	Jan-Mar	0.11%	0.48%	0.74%	2.62%	2.95%	3.23%
	Apr-Jun						
	Jul-Sep						
	Oct-Dec						
2015	Jan-Mar	0.48%	0.78%	0.96%	1.98%	5.48%	10.40%
	Apr-Jun						
	Jul-Sep						
	Oct-Dec						
2016	Jan-Mar	0.17%	0.62%	0.87%	1.62%	3.53%	6.64%
	Apr-Jun						
	Jul-Sep						
	Oct-Dec						
2017	Jan-Mar	0.002%	0.02%	0.04%	1.22%	1.22%	2.44%
	Apr-Jun						
	Jul-Sep						
	Oct-Dec						
Total		0.002%	0.43%	0.96%	0.24%	2.76%	10.40%

As already mentioned, 2015 was the most volatile year so it is not a surprise the biggest reduction happened within this year. The worst year to include Gold within a portfolio was 2013 and for Bitcoin was 2017. Bitcoin in the worst case helps to reduce risk only by .002% and in the best scenario contributes the risk-reduction by 0.96%, impressively the reduction is less than 1%. In the other hand, the Gold contribution is much higher. Gold provides by far the biggest risk-reduction with 10.4%. Therefore, despite the fact that both assets present a low and negative correlation with S&P, Gold is the best hedger within the 3 months analysis.

6.1.1.2. Out-sample Portfolios

The objective of the out-sample portfolio is to investigate if the hedge works for a future period, resembling a more realistic hedging situation. The optimal weights calculated in one period are used to estimate the portfolio standard deviation for the next period, generating rolling windows with the same number of trading days. In other words, the optimal weights of the estimation window are the base to forecast the returns and standard deviation of the portfolio in the prediction window. In finance, it is commonly used to forecast risks, and evaluate portfolio performance, thus managers use it to rebalance the weight if necessary.

This approach is another way to quantify the effect Bitcoin generates to reduce risk. Likewise, the comparison is through portfolios' standard deviation against the individual S&P risk. Table 6 indicates the results for the out-sample portfolios, the highlighted numbers are portfolios' standard deviations higher than individual S&P risk. In other words, these are portfolios with higher risk than S&P. These results behave opposite to diversification because when a portfolio presents higher risk than the individual asset it shows a non-subadditive property.

Table 6. Optimized standard deviations of the 21 rolling windows out-sample Minimum Variance Portfolios and the individual S&P standard deviations (3 months horizon analysis).

Out-sample portfolios Standard Deviation				
Year	Months	S&P	S&P+Bitcoin	S&P+Gold
2013	Jan-Mar	9.09%		
	Apr-Jun	14.05%	13.93%	16.69%
	Jul-Sep	9.03%	8.86%	8.62%
	Oct-Dec	10.36%	11.53%	9.41%
2014	Jan-Mar	11.99%	11.72%	9.09%
	Apr-Jun	9.38%	10.34%	6.77%
	Jul-Sep	9.41%	9.79%	6.46%
	Oct-Dec	14.22%	13.65%	11.26%
2015	Jan-Mar	14.14%	15.54%	9.81%
	Apr-Jun	10.11%	10.15%	8.20%
	Jul-Sep	20.97%	20.14%	13.05%
	Oct-Dec	14.85%	14.05%	11.08%
2016	Jan-Mar	18.53%	17.82%	10.30%
	Apr-Jun	13.67%	12.88%	7.27%
	Jul-Sep	9.82%	9.67%	7.51%
	Oct-Dec	8.59%	8.00%	7.19%
2017	Jan-Mar	6.70%	10.29%	5.51%
	Apr-Jun	7.35%	7.34%	5.07%
	Jul-Sep	7.11%	7.24%	5.04%
	Oct-Dec	5.56%	5.63%	4.35%
2018	Jan-Mar	18.44%	18.45%	13.46%

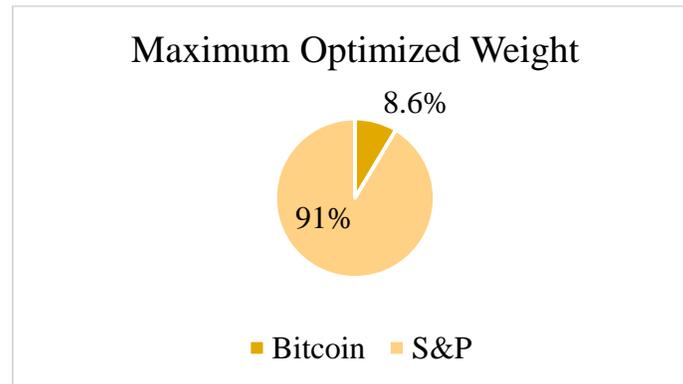
Analyzing the out-sample results, it is clear Bitcoin has a poor performance because from the 20 rolling windows, 9 of them present a higher risk than S&P. In percentage means that 45% of the cases Bitcoin does not help risk-reduction. The portfolios that have a higher risk than S&P, on average increment the risk by 0.86%. Furthermore, by considering all the sub-periods portfolios (when the risk decreases/increases) Bitcoin on average just help the risk-reduction by 0.14%. On the other hand, Gold maintains its position of a good hedge because only 1 out of 20 cases, the result is not favorable. On average, Gold reduces the risk by 2.91%, even out-sample.

6.1.2. 6 months

Now, the daily returns are divided into periods of 6 months which is approximately 120 trading days. Every year has two periods: January- June and July-September. Considering the 5 years of analysis, 10 sub-periods are analyzed.

Comparing the 6 months against the 3 months, the optimized weights of S&P+Bitcoin decrease, now, the maximum amount of wealth that may be invested in Bitcoin is 8.6% against the 11% before.

Figure 12. Maximum optimized weight of Bitcoin and S&P within the S&P+Bitcoin portfolio of the 10 rolling windows (6 months horizon analysis).



The correlation of Gold and Bitcoin against S&P are still very low and both continue to be negative. The difference against the 3 months analysis is that now Bitcoin/S&P correlation improves because it decreases almost by half, showing these assets become more independent of each other. For Gold, it still is more correlated with S&P. However, this correlation also falls. The details of correlations of each sub-period are presented in appendix B.

Table 7. Maximum and minimum correlation coefficients between S&P/Bitcoin and S&P/Gold of the 10 rolling windows (6 months horizon analysis).

Correlations 6 Months Horizon		
	S&P/Bitcoin	S&P/Gold
Max	0.12	0.23
Min	-0.13	-0.48

6.1.2.1. In-sample Portfolios

As expected, portfolios S&P+Bitcoin and S&P+Gold show less risk than the individual S&P in most of the sub-periods. Similar to the 3 months analysis, Gold continues as the best hedger for S&P, due to it reduces risk more than Bitcoin. By analyzing the less volatile period, Jul-Dec 2017, where S&P had a standard deviation of 6.36%. Gold helps to reduce risk to 4.67% in contrast Bitcoin does not help at all the risk reduction, maintaining the same risk as S&P.

Table 8. Optimized standard deviations of the 10 rolling windows in-sample Minimum Variance Portfolios and the individual S&P standard deviations.

In-sample portfolios Standard Deviation				
Year	Months	S&P	S&P+Bitcoin	S&P+Gold
2013	Jan-Jun	11.90%	11.83%	11.44%
	Jul-Dec	9.70%	9.53%	8.91%
2014	Jan-Jun	10.71%	10.63%	7.50%
	Jul-Dec	12.02%	11.41%	9.02%
2015	Jan-Jun	12.21%	12.19%	8.81%
	Jul-Dec	18.17%	17.37%	10.75%
2016	Jan-Jun	16.16%	15.41%	8.72%
	Jul-Dec	9.20%	8.81%	7.29%
2017	Jan-Jun	7.02%	6.99%	5.27%
	Jul-Dec	6.36%	6.36%	4.67%

Moreover, the scenario analysis of the 6 months periods shows that Bitcoin has a worse performance than in the 3 months analysis because all the scenarios decrease the risk-reduction. In other words, Bitcoin helps less to mitigate the risk. On average now, Bitcoin helps 0.34% and before it was 0.43%. On the other hand, Gold has a better overall performance compared to the 3 months analysis and the average risk reduction increases from 2.76% to 3.73%. In addition, the worst scenario has greatly improves from 0.24% to 1.70%. Therefore, the 6 months analysis also confirms that Gold is the best hedger because even the worst situation (1.70%), is still better than the best Bitcoin case (0.75%).

Additionally, the 6 months analysis helps to determine that the best year to include Gold and Bitcoin within a portfolio was 2016, because both assets help the most to reduce risk.

Table 9. Scenario analysis of the risk reduction in-sample S&P+Bitcoin and S&P+Gold against individual S&P standard deviation. (6 months horizon analysis).

Year	Months	In-Sample Scenarios S&P+Bitcoin			In-Sample Scenarios S&P+Gold		
		Worst	Average	Best	Worst	Average	Best
2014	Jan-Jun	0.08%	0.35%	0.62%	3.00%	3.10%	3.21%
	Jul-Dec						
2015	Jan-Jun	0.01%	0.42%	0.70%	3.39%	5.40%	7.42%
	Jul-Dec						
2016	Jan-Jun	0.39%	0.57%	0.75%	1.91%	4.67%	7.43%
	Jul-Dec						
2017	Jan-Jun	0.01%	0.02%	0.03%	1.70%	1.72%	1.75%
	Jul-Dec						
Total		0.01%	0.34%	0.75%	1.70%	3.73%	7.43%

6.1.2.2. Out-sample Portfolios

To obtain the out-sample portfolios same approach is used by using the optimized weights of the 6 months estimate window with the objective to minimize risk for the next 120 days standard deviation. In Table 10, the highlighted numbers are the portfolios' standard deviations higher than the individual S&P risk.

Table 10. Optimized standard deviations of the 10 rolling windows out-sample Minimum Variance Portfolios and the individual S&P standard deviations. (6 months horizon analysis).

Out-sample portfolios Standard Deviation				
Year	Months	S&P	S&P+Bitcoin	S&P+Gold
2013	Jan-Jun	11.90%		
	Jul-Dec	9.70%	9.54%	8.94%
2014	Jan-Jun	10.71%	10.63%	8.63%
	Jul-Dec	12.02%	11.78%	9.04%
2015	Jan-Jun	12.21%	12.90%	8.89%
	Jul-Dec	18.17%	18.03%	11.67%
2016	Jan-Jun	16.16%	15.43%	10.20%
	Jul-Dec	9.20%	8.81%	7.47%
2017	Jan-Jun	7.02%	8.51%	5.27%
	Jul-Dec	6.36%	6.48%	4.67%

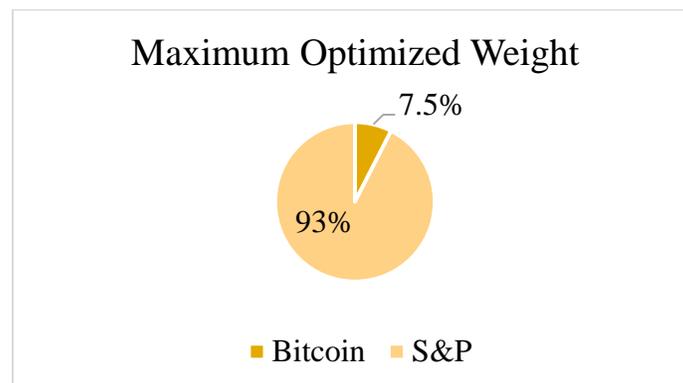
The 6 months out-sample results show that Bitcoin improves the performance compared to the 3 months, however, the outputs are still against the expectations “Bitcoin reduce risk within a portfolio”. Now only 3 rolling windows out of 9 have a higher risk than S&P. Meaning that 33.33% of the cases Bitcoin does not help the risk-reduction, before it was 45% of the cases. By considering only the situations of risk-increment, the risk grows by 0.77% on average, before was 0.86%. However, by analyzing all the cases (when the risk decrease/increase) Bitcoin gets worse by reducing risk in 0.06% on average and before it helped reducing risk by 0.14%. On the other hand, Gold improves its performance because in all the sub-periods the risk is reduced and on average it improves from 2.91% to 2.97%.

6.1.3. 12 months

In this last part of this analysis, the daily returns are divided yearly from January to December considering approximate 240 trading days. Since this study covers historical returns from 2013 to 2017 there are 5 rolling windows to evaluate.

By making a comparison with the previous time horizon analysis, the maximum optimal weight of S&P+Bitcoin portfolio decrease from 11% (3 months) and 8.6% (6 months) to 7.5% within the year horizon. The results of this study coincide with Eisl, et al. (2015), where the analysis was also divided by 12 months rolling windows and the optimization process gave very low proportions to Bitcoin against the rest of the assets.

Figure 13. Maximum optimized weight of Bitcoin and S&P within the S&P+Bitcoin portfolio of the 5 rolling windows (12 months horizon analysis).



Regarding Bitcoin/S&P correlation, within this 12 months analysis, it becomes more interesting for diversification because it decreases even more and now it is exceptionally low. Thus, the year Bitcoin's returns are completely independent of S&P's returns. Concerning Gold/S&P correlation also improves because it presents a significant reduction. As well, most of them are negatively related with S&P, appendix B presents the detail correlations of every year.

Table 11. Maximum and minimum correlation coefficients between S&P/Bitcoin and S&P/Gold of the 5 rolling windows (12 months horizon analysis).

Correlations 12 Months Horizon		
	S&P/Bitcoin	S&P/Gold
Max	0.06	0.16
Min	-0.07	-0.35

6.1.3.1. In-sample Portfolios

The results of this 12 months horizon are very similar to the previous 3 and 6 months, in the sense that all the in-sample S&P+Bitcoin and S&P+Gold portfolios reduce the individual S&P risk. However, this reduction is very low when Bitcoin is used, even if the correlation is very low it does not help. By taking the most and less volatile year, 2015 and 2017 respectively, it is clear that Bitcoin does not help to reduce risk as the impact on the risk is very small. In contrast, the Gold impact is more significant as it actually shows a non-trivial risk-reduction.

Table 12. Optimized standard deviations of the 5 rolling windows in-sample Minimum Variance Portfolios and the individual S&P standard deviations.

In-sample portfolios Standard Deviation			
Year	S&P	S&P+Bitcoin	S&P+Gold
2013	10.82%	10.70%	10.23%
2014	11.37%	11.15%	8.31%
2015	15.49%	15.25%	10.04%
2016	13.09%	12.50%	8.13%
2017	6.69%	6.69%	4.97%

The scenario analysis is through all the 5 years performance. It shows that Bitcoin fares worse compared with the 3 and 6 months horizons because in the best case it only helps to reduce risk by 0.59% and before the best case was 0.96% (3 months). Also, the average performance is reduced from the previous time horizons. So in the case of Bitcoin, by considering all the time horizons and all the scenarios, the results show that it helps to reduce risk by less than 1%. In contrast, Gold improves in the average state because the risk-reduction increase from 3.73% (6 months) to 3.80%.

Table 13. Scenario analysis of the risk reduction in-sample S&P+Bitcoin and S&P+Gold against individual S&P standard deviation. (12 months horizon analysis).

In-Sample Scenarios S&P+Bitcoin			In-Sample Scenarios S&P+Gold		
Worst	Average	Best	Worst	Average	Best
0.00%	0.27%	0.59%	1.72%	3.80%	5.45%

6.1.3.2. Out-Sample Portfolios

The out-sample portfolios are calculated by using the optimized weights of the year before with the returns and standard deviation of the current year. From the out-sample portfolio results, it is clear Bitcoin performs better than the in-sample situation. From Table 14, the highlighted number is the portfolios' standard deviation higher than the individual S&P risk.

Table 14. Optimized standard deviations of the 5 rolling windows out-sample Minimum Variance Portfolios and the individual S&P standard deviations. (12 months horizon analysis).

Out-sample portfolios Standard Deviation			
Year	S&P	S&P+Bitcoin	S&P+Gold
2013	10.82%		
2014	11.37%	11.24%	9.51%
2015	15.49%	15.26%	10.66%
2016	13.09%	12.63%	8.66%
2017	6.69%	9.48%	5.13%

The 12 months out-sample portfolios perform much better than the 3 and 6 months horizon because now only 1 portfolio out of 4 presents a higher risk. In percentage, it means that only 25% of the cases Bitcoin does not help for the risk-reduction. The improvement is significant considering the 3 months horizon was 45%. By considering all the years, the average of Bitcoin risk-reduction improves by increasing 0.49%, before it was 0.14% (3 months). Even the probabilities of having a higher risk than S&P decreases among the time horizons (3, 6 and 12 months) it is still very high but it depends on the risk-adverse investor profile. Regarding Gold performance, it also has an improvement because on average it helps to reduce risk by 3.17%, on the 6 months horizon was 2.97%. Based on the results Gold is the secure alternative to the risk reduction of S&P because all the portfolios present lower risk and the risk-reduction is in all time horizons around 3%.

6.1.4. Analysis

The first part of the analysis investigates if Bitcoin within an equity portfolio reduces risk compared to Gold. Regarding the optimized weights of Bitcoin within a portfolio, the results are similar to Eisl, et al. (2015) because the weights are low. In other words, due to the division of the 5 years analysis into periods of 3, 6, and 12 months, it is possible to conclude that optimized weights of Bitcoin decrease from 3 to 12 months. Figure 14 resumes the reduction of Bitcoin’s weight through time.

Figure 14. Comparison between the maximum optimized weight of Bitcoin and S&P within S&P+Bitcoin portfolio of all the analysis horizons (MVP analysis)



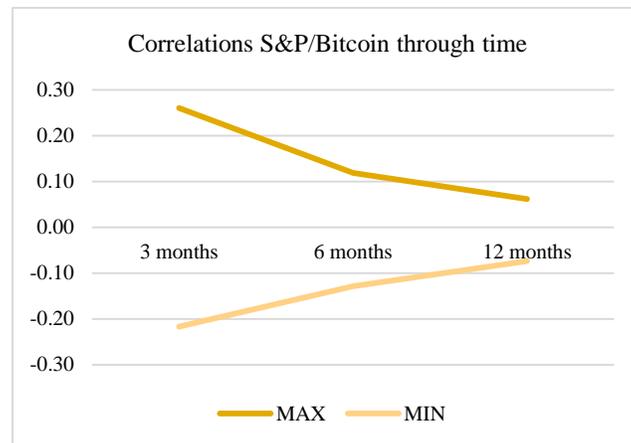
Regarding the correlations between Bitcoin and S&P, the results are similar to Briere et al. (2015) because the correlation coefficients are extremely small, very close to zero. This study concludes that the correlations are also proportional inverse to time; because when the analysis covers a larger period the correlation decrease.

Table 15. Comparison between the maximum and minimum correlation coefficient between S&P/Bitcoin and S&P/Gold of all the analysis horizons.

Correlations S&P/Bitcoin			
	3 months	6 months	12 months
Max	0.26	0.12	0.06
Min	-0.22	-0.13	-0.07

To see the relation graphically, the following Figure illustrates how when the horizon time is bigger the correlations are close to zero and even the gap between the max and min correlation coefficients decrease. However, these small correlations are not enough to reduce risk as the in-sample and out-sample portfolios conclude.

Figure 15. Comparison between the maximum and minimum correlation coefficient between S&P/Bitcoin and S&P/Gold through time.



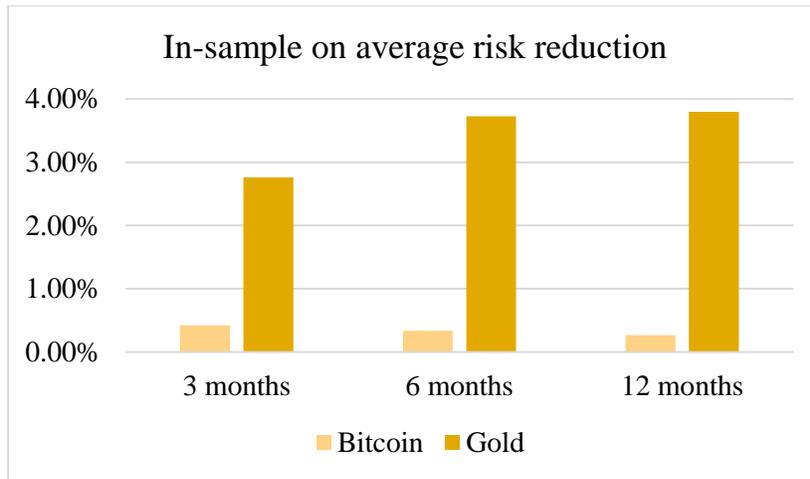
This first part has the aim to answer the following research question:

Does Bitcoin actually help to reduce risk in a well-diversified equity portfolio compared with Gold?

To answer, this study evaluates in-sample and out-sample approach within different horizons to have enough portfolios as a base.

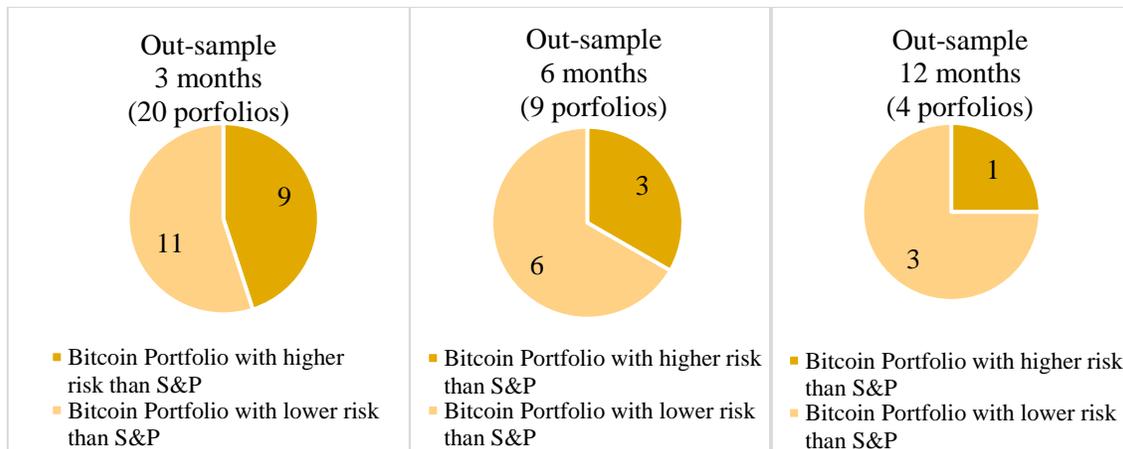
From the in-sample scenario analysis, it can be determined that Gold helps to reduce the individual risk of S&P on average 3.4%. Nevertheless, Bitcoin does not help to reduce the mentioned risk because its contribution reduces risk just by 0.3%. Therefore, Bitcoin is not a good diversification asset for S&P. In contrast, Gold can be considered as a risk-reduction asset against S&P.

Figure 16. Comparison between in-sample average risk reduction generated by Bitcoin and Gold of all the analysis horizons.



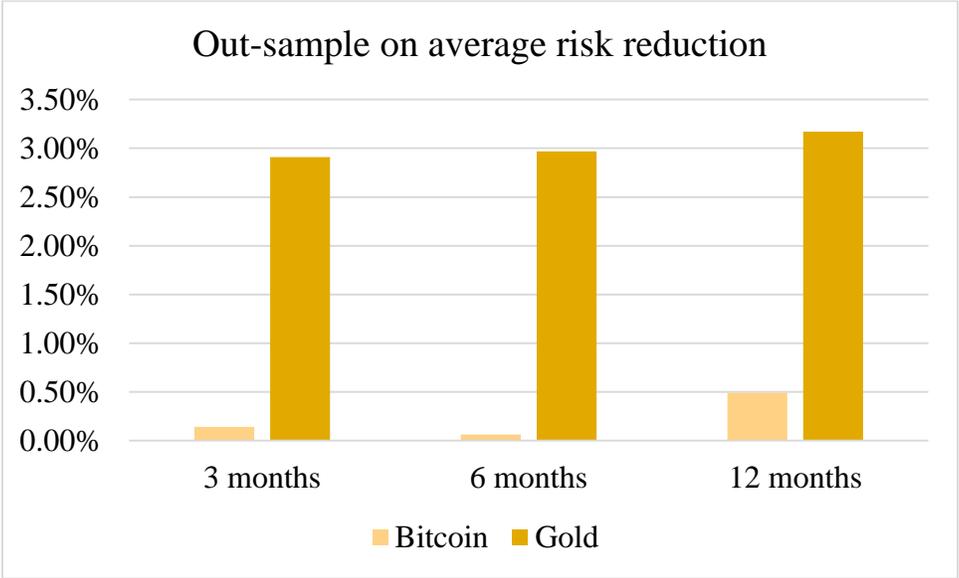
The out-sample analysis shows that Bitcoin portfolios with higher risk than S&P decrease over the time horizons of 3, 6 and 12 months. It is important to notice that the number of base portfolios changes due to the rolling windows. Thus, the 3 months horizon has 20 out-sample portfolios; the 6 months has 9 portfolios and the 12 months has 4 portfolios. So, for the 3 months analysis the number of portfolios with a higher risk than S&P is 45%, for 6 months is 33.33% and for 12 months 25%. Figure 17 shows the decrease of the number of portfolios with higher risk than S&P through the time horizon analysis increase.

Figure 17. Number of Bitcoin out-sample portfolios that present higher and lower risk than S&P in all the analysis horizons.



Despite the decrease of the number of portfolios with higher risk than S&P, the amount of contribution that actually Bitcoin helps to reduce risk is very low compared with Gold. Bitcoin on average helps risk-reduction by 0.23% and Gold decrease risk by 3.02%. From the Figure 18 it is clear Bitcoin is not even close to being consider a good diversifier for S&P. On the other hand, Gold maintains its position as a good hedger even in the out-sample analysis.

Figure 18. Comparison between out-sample average risk reduction generated by Bitcoin and Gold of all the analysis horizons.



Therefore, based on the low weights of Bitcoin within a well-diversified equity portfolio, the correlations that are not enough to reduce risk and the in-sample and out-sample portfolios from the different horizon periods, the answer is to the main research question is that Bitcoin does not help to reduce risk as Gold so it cannot be considered as the new “Digital Gold”.

6.2. Tangency Portfolio

The second part of this study aims at analyzing the performance of the tangency portfolio to evaluate if Bitcoin improves the risk-return ratio compared to Gold. This will give the investor an alternative to incorporate the risk-free asset. The objective is to evaluate the maximum risk premium per unit of risk when Bitcoin is included in a portfolio. The study is through the maximization of the Sharpe ratio. Now, the interest is to obtain the highest excess return of the portfolio S&P+Bitcoin over the total risk, this is a different approach to investigate if Bitcoin, offered as an alternative asset, compensates for a unit of total risk.

To be consistent with the previous MVP analysis, this second part of the study also uses in-sample and out-sample approach. The same time horizons are analyzed: 3, 6 and 12 months. For each sub-period the standard deviation, variance and correlations are calculated and based on the optimization process, now by maximizing the Sharpe ratio, the optimal weights are obtained. Thus, the risk-adjustment return is calculated. In order to compare the Sharpe ratios, they are rescaled yearly by using the formula discussed in Chapter 4. Two frameworks are considered: one where short and long positions are allowed and the other with the constraint of only long positions.

6.2.1. In-sample Sharpe Ratio with Long and Short positions

After the maximization process, the optimized weights gives in-sample Sharpe ratios showing that the limitation of short and long position only affects the S&P+Gold portfolio because it presents different Sharpe values. On the other hand, the position constraint does not affect the S&P+Bitcoin and therefore the results are the same.

Table 16 presents the maximum Sharpe ratios of both constraints considering all in-sample sub-periods. It is interesting that independently the constraints, within the same time horizon, the reward per unit of risk is much better by using Bitcoin than Gold in a portfolio. This can be explained because it is a more volatile asset than Gold, so the premium is higher. However, Gold still gives good risk-compensation because in the worst case it is 1.86.

Table 16. Maximum Sharpe ratio considering both long and long-short constraints of all the in-sample analysis horizons.

Maximum Sharpe Ratio Constraint to Long Position of in-sample portfolios					
3 Months Horizon		6 Months Horizon		12 Months Horizon	
S&P+Bitcoin	S&P+Gold	S&P+Bitcoin	S&P+Gold	S&P+Bitcoin	S&P+Gold
11.68; 5.93	3.24	4.51	2.56	4.04	1.86

Maximum Sharpe Ratio Long and Short Position of in-sample portfolios					
3 Months Horizon		6 Months Horizon		12 Months Horizon	
S&P+Bitcoin	S&P+Gold	S&P+Bitcoin	S&P+Gold	S&P+Bitcoin	S&P+Gold
11.68; 5.93	4.02	4.51	3.48	4.04	2.72

Bitcoin gives a maximum Sharpe ratio of 11.68, this occurred on the period January-March 2013, this result is as an extreme value and the reason can be due to this year was when Bitcoin started to generate positive returns since its origins in 2008. In other words, the first-mover investors received more risk-premium due to taking risk of a completely unknown asset. By excluding this extreme value, the maximum Sharpe ratio given by Bitcoin within the 3 months period is 5.93 in both long and short positions. This value is more comparable with the other results.

6.2.2. Out-sample Sharpe Ratio with Long and Short positions

In this second part of the study, the out-sample Sharpe ratios are calculated to evaluate the risk-return relationship for future periods, simulating a real situation. In the out-sample framework, the optimized weights obtained from the Sharpe maximization in the estimation window are used to calculate the premium for the following period.

The results are similar to the in-sample Sharpe ratios in the sense that S&P+Bitcoin have the same values in the long and long-short position. On the other hand, S&P+Gold have different results for the two discussed frameworks.

Table 17 presents the maximum Sharpe ratios of both constraints and considering all out-sample sub-periods. Similar to the in-sample ratios, the reward per unit of total risk is much better by using Bitcoin than Gold in a portfolio. This is due to the high returns of Bitcoin, so

investors get a better compensation per acquired risk. Regarding the out-sample Gold's Sharpe ratios, they still give a good premium. Moreover, by comparing the worst ratio of the in-sample, 1.86, now the Sharpe decreases to 0.73.

Table 17. Maximum Sharpe ratio considering both long and long-short constraints of all the out-sample analysis horizons.

Maximum Sharpe Ratio Constraint to Long Position of out-sample portfolios					
3 Months Horizon		6 Months Horizon		12 Months Horizon	
S&P+Bitcoin	S&P+Gold	S&P+Bitcoin	S&P+Gold	S&P+Bitcoin	S&P+Gold
5.58	2.28	4.06	2.56	0.11	0.81

Maximum Sharpe Ratio Long and Short Position of out-sample portfolios					
3 Months Horizon		6 Months Horizon		12 Months Horizon	
S&P+Bitcoin	S&P+Gold	S&P+Bitcoin	S&P+Gold	S&P+Bitcoin	S&P+Gold
5.58	3.57	4.06	1.63	0.11	0.73

It is interesting that all the out-sample ratios are lower than the in-sample ratios. Moreover for Gold, the largest ratio decrement occurs in the long-short position for 6 months period from 3.48 to 1.63. For Bitcoin, the largest decrement is from 4.04 to 0.11 in the 12 months analysis. Furthermore, the maximum Bitcoin Sharpe ratio for both in-sample and out-sample occurs in the 3 months period and they are very close each other: 5.93 (in-sample) and 5.58 (out-sample).

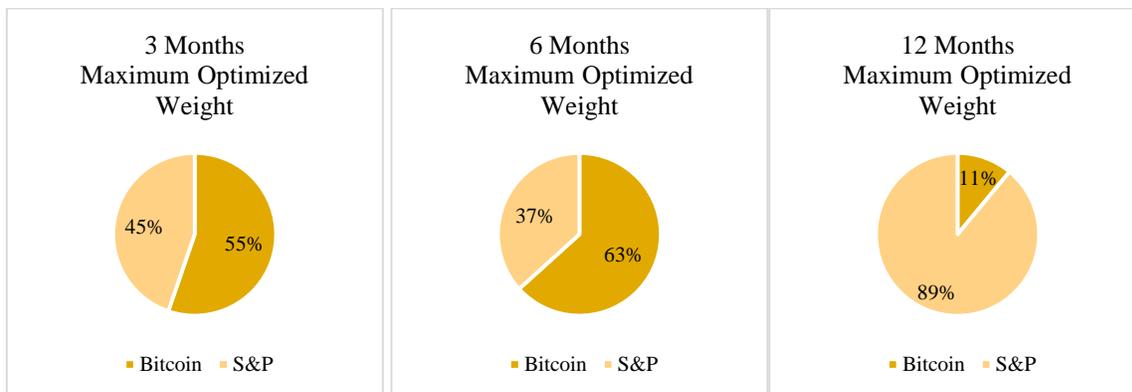
6.2.3. Analysis

For the second part of the study, the objective is to answer the following research question:

Does Bitcoin improve the risk-return relation within a well-diversified equity portfolio, compared with Gold?

Regarding, the maximum optimized weights of Bitcoin within the portfolio S&P+Bitcoin in the tangency analysis shows that under the 3 and 6 months' time horizons it has more than half of the invested wealth, with 55% and 63% respectively. This is a difference with the MVP analysis, where Bitcoin optimized weights are small proportion of the total portfolio. One reason may be because in the tangency analysis investor is interested in maximized the relation risk-return and because Bitcoin has high returns it is consistent to invest more in this asset.

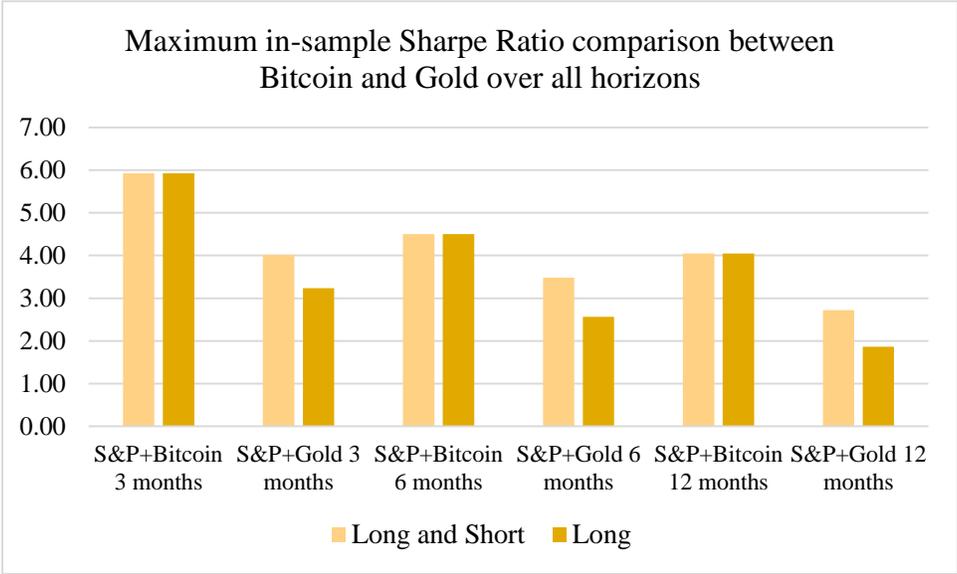
Figure 19. Comparison between the maximum optimized weight of Bitcoin and S&P within the S&P+Bitcoin portfolio of all the analysis horizons (Tangency analysis)



The results of the in-sample and out-sample Sharpe ratios based on two frameworks long and long-short constraints show that the portfolio S&P+Bitcoin is not affected by the constraints because the Sharpe's are the same in both cases.

Figure 20 makes a comparison of the maximum in-sample Sharpe ratio against all time horizons. It is clear Bitcoin improves risk-return ratio better than Gold in all periods. Moreover, it is interesting that Gold’s Sharpe decreases when the time horizons increase independently of the long-short constraint. Additionally, the best Sharpe ratios for both Gold and Bitcoin is within the 3 months horizon.

Figure 20. Comparison between the maximum in-sample Sharpe ratio generated by Bitcoin and Gold portfolios of all the analysis horizons and considering the long and long-short constraints.



Regarding the out-sample results, Figure 21 makes a comparison of the maximum out-sample Sharpe ratios for all horizons. Almost all the time periods, Bitcoin’s Sharpe ratios give the investor a better compensation per unit of total risk, except for the 12 months horizon, where the Gold gives a better premium. This is a difference with the in-sample analysis because before Bitcoin had a higher Sharpe for all periods. Another difference from the in-sample is that the out-sample Gold’s Sharpe does not decrease when the time horizons increases, now Gold does not follow any pattern. Moreover, the period for the best Sharpe ratio is 3 months horizon analysis for Gold and Bitcoin.

Figure 21. Comparison between the maximum out-sample Sharpe ratio generated by Bitcoin and Gold portfolios of all the analysis horizons and considering the long and long-short constraints.

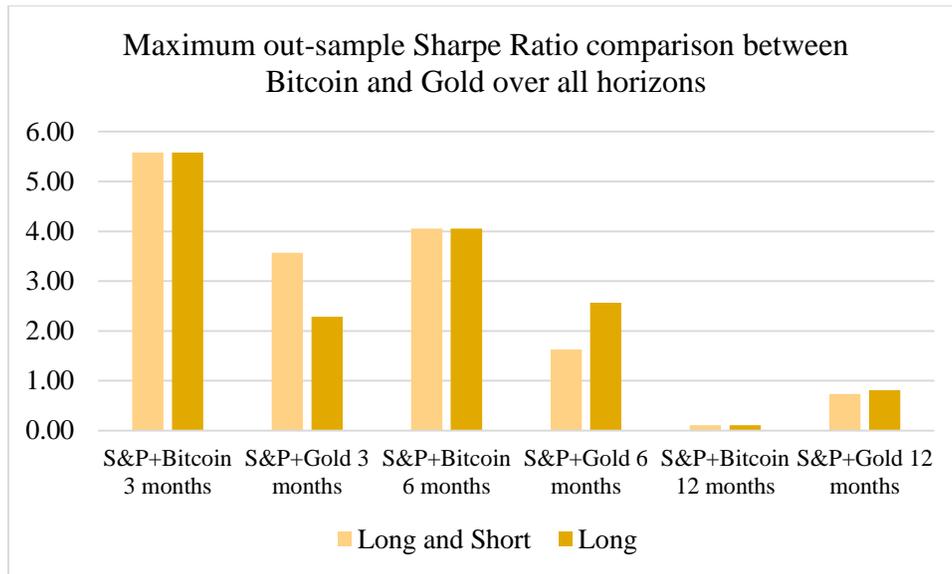
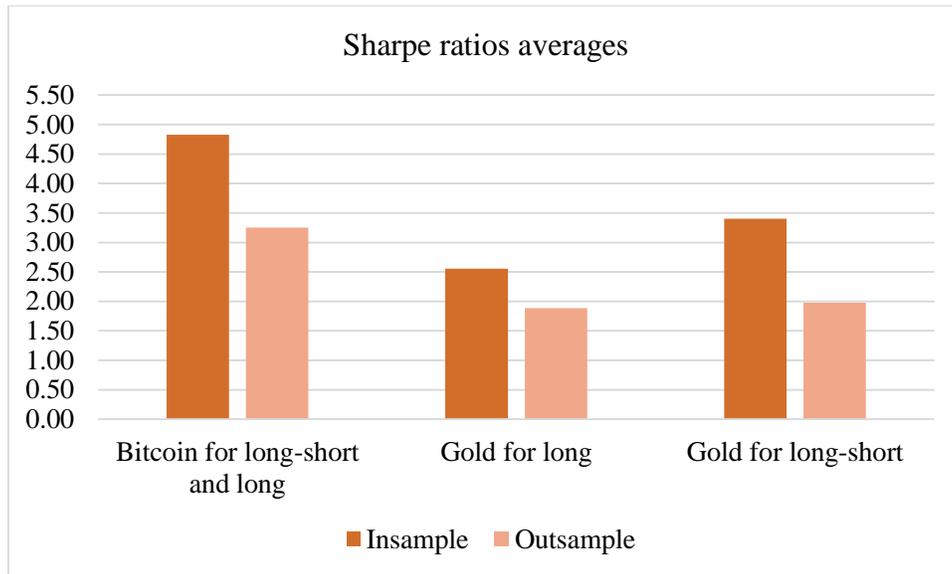


Figure 22 presents the comparison between the averages of in-sample and out-sample Sharpe ratios, considering all the periods and both frameworks: long and long-short. For the in-sample, on average the risk premium that Bitcoin offers per unit of extra volatility is 4.83. Regarding Gold, in the case of long position the Sharpe is 2.55 and when long-short positions are allowed is 3.41. For the out-sample, the risk premium offered by Bitcoin is on average 3.25. In the case of Gold, the Sharpe in the long position is 1.89 and in the long-short framework is 1.98. Now, it is visually clear that the out-sample ratios for both Gold and Bitcoin are lower than the in-sample.

Figure 22. Averages of in-sample and out-sample Sharpe ratios for all the periods and both frameworks: long and long-short.



Based on the in-sample and out-sample approaches and considering the long and long-short constraints, the answer to the research question is that Bitcoin improves the risk-return ratio more than Gold. This is due to Bitcoin gives higher returns. However, it is important to mention that Gold also offers an excellent premium. Hence Gold is also a good investment option.

7. Conclusion

Bitcoin and Gold have several similarities as low correlation with stocks, inflation hedge, government decentralization and the no currency attachment, these are the reasons why investment banks called Bitcoin as the new “Digital Gold”. Based on this similarities this study compares both assets.

The conclusion of the first question is that Bitcoin does not help to reduce risk in a well-diversified equity portfolio compared with Gold, so it cannot be considered as the new “Digital Gold”. Even if correlations between Bitcoin and S&P considering different time horizons are extremely low, almost zero, they are not enough to mitigate the risk due to its high standard deviation. The in-sample approach of all the horizon periods demonstrates Gold reduces risk on average 3.4% against Bitcoin only reduces risk by 0.3%. The out-sample approach shows that Gold decrease risk by 3.02% against Bitcoin only helps risk-reduction by 0.23%.

The conclusion of the second question is that Bitcoin improves risk-return relation in a well-diversified equity portfolio more than Gold. The reason is because of the higher returns of Bitcoin. Based on the long and long-short approach, Bitcoin on average offers a risk premium of 4.83. On the other hand, Gold Sharpe ratio is 2.55 in the long framework and 3.41 for the long-short scenario. Even though Bitcoin has a better premium per unit of total risk than Gold, this asset also offers a high Sharpe. Therefore, Gold is also a good investment asset because it has an excellent premium too.

Finally, Bitcoin as an investment asset cannot be considered a good hedge against S&P, as Gold does. Despite the similarities between both assets, Bitcoin cannot be considered as the new “Digital Gold” because even Bitcoin represents a new and revolutionary financial asset it is still in the infancy stage to be treated as Gold. Further investigations can be interested in using the same approach of this study but comparing Bitcoin against more financial assets or by modifying the rolling windows horizon. Additionally, this study recommends investment banks to use Bitcoin with moderation because it is too early to incorporate it as an alternative investment asset within an equity portfolio.

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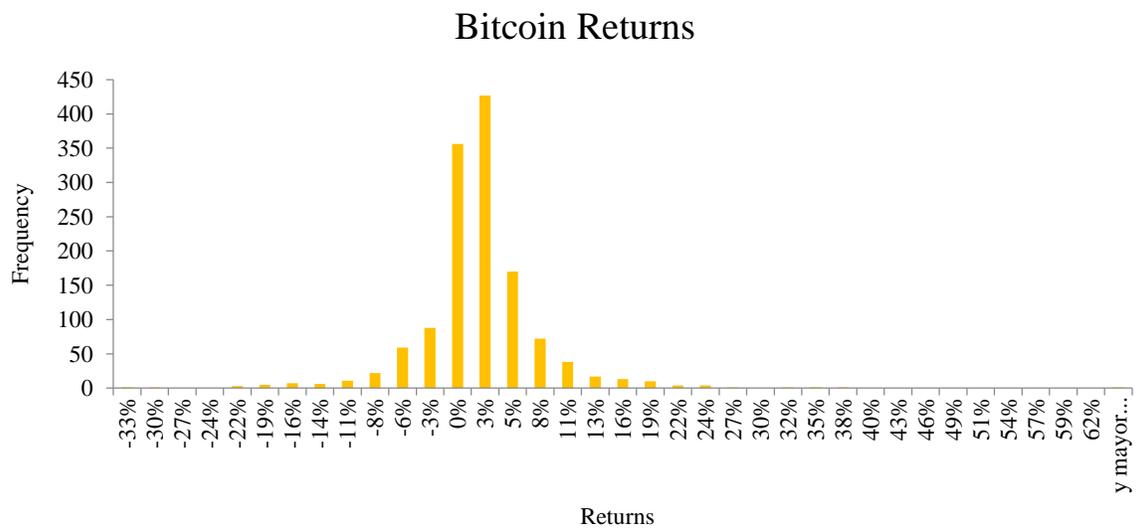
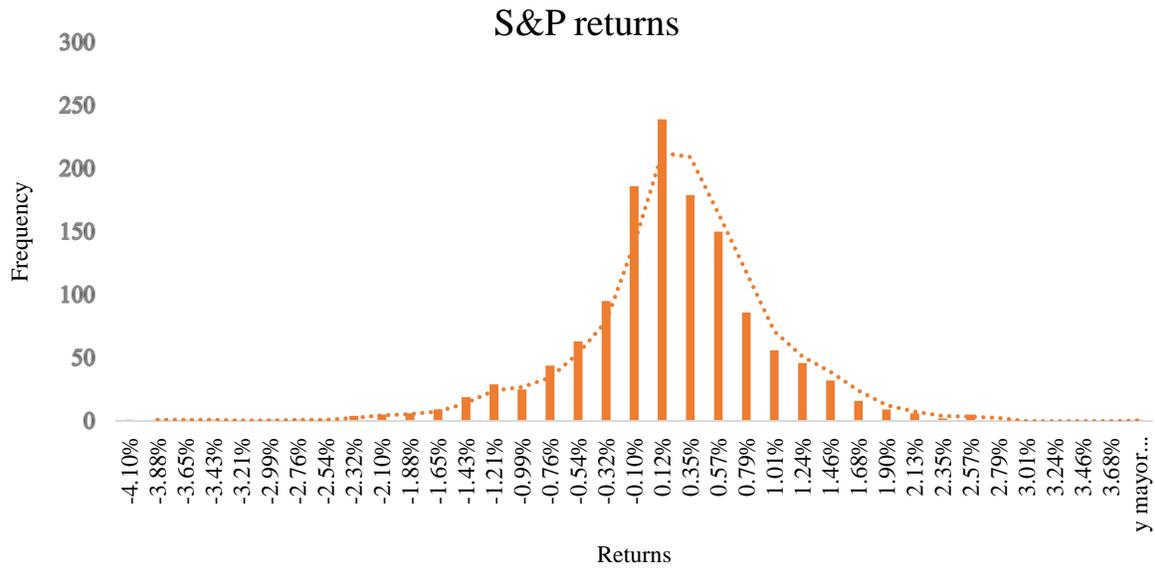
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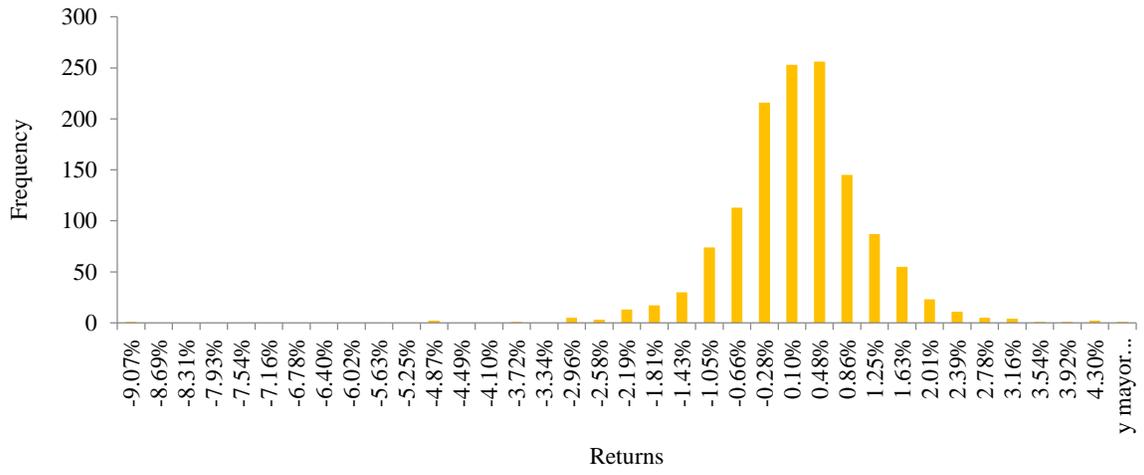
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Appendix A

Histogram of daily returns



Gold Returns



Appendix B

Correlation between S&P, Gold and Bitcoin for the different analysis periods.

Correlations 3 Months Horizon			
Year	Months	S&P/Bitcoin	S&P/Gold
2013	Jan-Mar	0.053	0.013
	Apr-Jun	-0.053	0.285
	Jul-Sep	-0.164	0.099
	Oct-Dec	-0.149	0.033
2014	Jan-Mar	-0.198	-0.229
	Apr-Jun	0.259	-0.176
	Jul-Sep	-0.217	-0.175
	Oct-Dec	-0.067	-0.100
2015	Jan-Mar	0.167	-0.173
	Apr-Jun	-0.038	0.080
	Jul-Sep	0.098	-0.154
	Oct-Dec	-0.143	0.141
2016	Jan-Mar	0.047	-0.420
	Apr-Jun	-0.166	-0.581
	Jul-Sep	0.093	0.000
	Oct-Dec	-0.193	-0.071
2017	Jan-Mar	-0.032	-0.071
	Apr-Jun	0.045	-0.329
	Jul-Sep	0.171	-0.316
	Oct-Dec	0.026	-0.161
2018	Jan-Mar	0.261	0.098

Correlations 6 Months Horizon			
Year	Months	S&P/Bitcoin	S&P/Gold
2013	Jan-Jun	-0.024	0.235
	Jul-Dec	-0.128	0.058
2014	Jan-Jun	-0.003	-0.209
	Jul-Dec	-0.114	-0.116
2015	Jan-Jun	0.119	-0.079
	Jul-Dec	0.001	-0.030
2016	Jan-Jun	-0.053	-0.483
	Jul-Dec	-0.032	-0.034
2017	Jan-Jun	-0.002	-0.195
	Jul-Dec	0.112	-0.257

Correlations 12 Months Horizon		
Year	S&P/Bitcoin	S&P/Gold
2013	-0.073	0.161
2014	-0.047	-0.154
2015	0.051	-0.048
2016	-0.048	-0.353
2017	0.062	-0.224