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The natural effect of multi-currency cross-hedging

An alternative hedging strategy for small- and medium-sized enterprises?

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ABSTRACT

This thesis investigates the power and accuracy of the natural effect of multi-currency cross-hedging, based on the non-zero correlation between currency pairs, with the purpose of determining if this could be a suitable hedging strategy for small- and medium-sized enterprises having transaction exposure to currency risk. The investigation, performed through an out-of-sample approach, is based on the minimization of the risk measure Conditional Value at Risk. By using a sample of exchange spot rates from 2010 to 2017, divided into two periods, we test a set of hedged portfolios (computed in the setting of a multi-currency cross-hedging strategy) from the estimation period on the data of the test period. The results show that the investigating hedging strategy performs poorly already in the estimation period, and even worse when applied to the test period. The conclusion of this thesis is that the natural effect of multi-currency cross-hedging lacks power and accuracy over time, probably due to the instability of correlations between periods, and is therefore not considered as a suitable hedging strategy for small- and medium-sized enterprises.

Key words: *Multi-currency cross-hedging, Small- and medium-sized enterprises, Conditional Value at Risk, Exchange rate risk*

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TABLE OF CONTENTS

LIST OF TABLES

1. INTRODUCTION

With the increasing interdependence of the world economy and ever fastening development of globalization, exchange rates and their movements have become of major interest for companies in their daily operations and risk management programmes. For companies involved in multinational trade, as soon as cash flows depend on a foreign currency whose exchange rate is not fixed to the home currency, there is an exchange rate risk. Considering the Swedish krona, which is not fixed against any other currency but has a floating exchange rate, exchange rate risk arises when a Swedish company deals with any other foreign currency in its operations. When this is the case, a company might choose to hedge its currency exposure, which can be done through the use of derivative instruments. However, given the complexity of derivative instruments, such a hedging strategy might not be optimal for small- and medium-sized enterprises that do not have the capacity or resources to engage in such activities. Small- and medium-sized enterprises usually do not have separate departments or special organizational structures for administrative functions, including their financial risk management, and these functions are often covered by one manager alone (Pennings and Garcia, 2004). In addition, ownership is often quite concentrated in such companies, meaning that the responsible manager's risk aversion or speculative interest can be a further motivation not to use derivative instruments. An alternative approach, more suitable for smaller companies with foreign exposure to two or more currencies at the same time, might be to use multi-currency cross-hedging. Applying this approach, one usually first makes use of the natural hedging effect (arising from the non-zero correlation between currencies) from operating with more than one currency, and then hedge the residual risk by the use of derivative instruments (Álvarez-Díez and al., 2015).

The purpose of this thesis is to investigate to what extent the natural hedging effect of multi-currency cross-hedging covers hedging needs without the use of any derivative instruments. This will be done by answering the following research question; *Is multi-currency cross-hedging (excluding derivative instruments) a suitable hedging strategy for small- and medium-sized enterprises?* Following the analysis of the results obtained, conclusion will be drawn on whether the investigated strategy can be an alternative for the hedging needs of Swedish small- and medium-sized import companies.

This given hedging strategy will be examined from the perspective of a Swedish small- and medium-sized enterprise with accounts payables in other currencies, i.e., involved in import trades. Since Sweden is a member of the European Union, its definition of small- and medium-sized enterprises follows that of the European Commission (2018). Small-sized enterprises are those having less than 50 employees and a turnover or balance sheet total less than or equal to 10 million euros, while medium-sized enterprises have less than 250 employees and either a turnover less than or equal to 50 million euros or a balance sheet total less than or equal to 43 million euros. In this thesis, imports refer to all trades where a Swedish company buys goods and services from outside of Sweden. In addition, the analysis only concentrates on the transaction risk of currency exposure, being the risk related to specific cash flows from foreign transactions and not the entire balance sheet (Papaioannou, 2006).

The investigation of the natural hedging effect of multi-currency cross-hedging in this given setting is done through the calculation of optimal hedge ratios. These optimal hedge ratios, which show what positions to take in other currencies for the purpose of hedging, are obtained through the minimization of the risk measure Conditional Value at Risk in MatLab. The currencies we aim to hedge are the US dollar, the Danish krone, the Euro, the British pound sterling and Norwegian krone, all against the Swedish krona. For the purpose of hedging, we also add the Japanese Yen, Australian Dollar, Canadian Dollar, Swiss Franc, Chinese Renminbi and Mexican Peso to the analysis. The currencies' spot rates are collected over the sample period 2010-01-01 to 2017-12-29 and divided into one estimation period, on which we calculate the optimal hedge ratios, and one test period, where we test these optimal hedge ratios.

It needs to be pointed out that in this thesis, when talking about the natural hedging effect, we refer to the hedging arising from trading with currencies, and not for example through having plants and subsidiaries abroad or cash inflows and outflows in the same foreign currency. One further issue that should be kept in mind is that the concept of cross-hedging generally refers to a method that can be applied when no market for derivative instruments, usually forwards and futures, is available (Eaker and Grant, 1987). In this thesis, we do not investigate cross-hedging from the perspective of inexistent derivatives markets, but from the perspective of an unwillingness or inability to use such markets.

Reviewing the literature, we find that there are many ways in which this thesis can develop certain aspects of previous research, as well as bring new aspects to light. The strategy investigated in this thesis builds on the article *“Hedging foreign exchange rate risk: Multi-currency diversification”* by Álvarez-Díez and al. (2015), where the authors investigate the best way to minimize exchange rate risk via a multi-currency cross-hedging strategy through the use of Value at Risk and Conditional Value at Risk. This thesis takes much inspiration from their work in terms of the matter of investigation and some methodological aspects. However, we only choose to focus on the minimization of Conditional Value at Risk, excluding that of Value at Risk, for the purpose of obtaining optimal hedge ratios. This because of the shortcomings of Value at Risk, especially with regards to the measure’s inadequacy in terms of optimization due to its undesirable mathematical properties. Moreover, by using CVaR as our measure of risk for optimization purposes, we make the analysis more up to date since the Basel Committee on Banking Supervision (BCBS) recently decided to change its recommended risk measure from Value at Risk to Conditional Value at Risk (BCBS, 2013). A further improvement of their work is that this thesis will apply an out-of-sample method, where we will test the accuracy of the obtained optimal hedge ratios.

Comparing the aim of this thesis with that of other similar research in the field, several components of uniqueness can be observed. This thesis focuses on an alternative way for small- and medium-sized enterprises to handle their currency risk, more adapted to their resources and capabilities, excluding the use of derivative instruments. Regardless of the context, research about multi-currency cross-hedging, its application and accuracy is scarce. In addition, no study performs this kind of analysis from the perspective of a Swedish firm with a special focus on its most relevant currencies for trade.

The thesis is structured as follows. Chapter 2 covers a review of the existing literature and previous research regarding the subject, including a theoretical background on risk measures. Chapter 3 provides the theoretical framework for the thesis, including definitions of exchange rate risk, the risk measures used, how they are computed and how the minimization of Conditional Value at Risk is performed in theory. Chapter 4 presents choice of data and its collection, as well as the methodology of the thesis and its limitations. Chapter 5 presents the results obtained through the application of the methodology on the collected data. Chapter 6 discusses and analyzes the results obtained in the previous section, including some suggestions for future research within this topic. Chapter 7 ends the thesis with conclusions related to the research question.

2. LITERATURE REVIEW

This chapter covers a review of the existing literature, including a section on the theoretical background of risk measures and a section on the previous research related to our analysis and research question.

2.1. THEORETICAL BACKGROUND

The discussion about risk measures within financial risk management has been widely covered in the existing literature. So has its relation to currency risk and hedging strategies within varying fields of finance. A central question in risk management, which underlies the whole discussion about a company's exposure to exchange rate risk, is of course how to measure this risk.

In the specific framework of portfolio optimization, there exists a variety of measures to quantify risk, which also apply to exchange rate risk in a portfolio setting. The most classical approach, at first introduced by Markowitz (1952) through his work on return/variance risk management, is to use variance as the measure of risk (Krokhmal and al., 2001). With this view, an investment alternative having a large variance can be considered as risky.

Markowitz' portfolio theory has been widely criticized in the literature for a variety of reasons, most of which are outside the scope of this thesis; see for example Michaud (1989) and Schulmerich and al. (2015) for further reading. However, one important drawback of using variance as the measure of risk, discussed by both Grootveld and Hallerbach (1999) and King (1993), relates to its assumption of elliptically distributed returns, which excludes the possibility of asymmetries in the return distribution. The presence of asymmetry in financial returns has been widely discussed in the literature, including but not limited to the works of Silvapulle and Granger (2001), Ang and Chen (2002) and Garcia and Tsafack (2011). Using variance as risk measure in the presence of asymmetrical return distributions punishes advantageous upside movements to the same extent that disadvantageous downside movements, meaning that variance is under such conditions an unsatisfactory measure of investment risk (Grootveld and Hallerbach, 1999). Thus, thinking of risk from the perspective of an investor, any risk measure that separates advantageous upside movements from disadvantageous downside movements serves its purpose better than variance. For the sake of analyzing financial returns, it is therefore important to distinguish between symmetric and asymmetric risk measures, as has been done for example by Harris and Shen (2006). Their definition is straightforward; asymmetric risk measures take asymmetries in the return distribution into account, especially in terms of their skewness and kurtosis, while symmetric risk measure do not.

Two asymmetric risk measures that have been developed in more recent times are Value at Risk (henceforth referred to as VaR) and Conditional Value at Risk (henceforth referred to as CVaR). According to Holton (2002), VaR was first introduced to the wide audience in 1994 by J.P. Morgan, and became very popular when the Basel Committee on Banking Supervision adopted VaR as its standard for risk measurement.

However, as discussed by Sarykalin and al. (2008), VaR has both advantages and disadvantages. The intuition behind VaR is easily understood and interpreted. It has the advantage of measuring risk through a single number based on the given confidence level and of having stable estimation procedures, in the sense that it does not take losses above VaR into account. The latter can be both an advantage and disadvantage. It is an advantage in the sense that extreme tail losses do not affect VaR, and that these very high losses can often be difficult to measure. However, this property is also of great disadvantage, since it entails the possibility to take on very large risks and still satisfy a certain VaR level. As soon as a loss is above VaR, the magnitude of the losses is not accounted for, meaning that one could take on a very high risk and still satisfy the VaR level. Using VaR as a risk measure might therefore lead to an unforeseen carrying of high risks. Another problem with VaR, also discussed by Acerbi and Tasche (2002), is that when such a tail risk measure is applied to a discontinuous distribution, it is very sensitive to changes in the confidence level. In case of a discontinuous distribution, even small changes of the confidence level will have an impact on the risk estimated by VaR, meaning that VaR is in general not continuous relative to the confidence level. When returns have a discrete distribution, VaR is also a non-convex function with respect to portfolio positions, which complicates VaR optimization (Sarykalin and al., 2008). In addition to its problems under discrete distributions, VaR is also unstable when applied to losses that are not normally distributed (Rockafellar and Uryasev, 2002). This is often a relevant problem as losses tend to have fat tails (Nath, 2015), thereby not being normally distributed.

As a consequence of the shortcomings of VaR as a risk measure, CVaR[1] was later introduced by Rockafellar and Uryasev (2000). As opposed to VaR, CVaR is continuous relative to the confidence level (Acerbi and Tasche, 2002). This implies that there will be no extensive change in the risk estimated by CVaR when we change the confidence level by some base points, regardless of the underlying distribution we use to compute returns and losses. The choice between VaR and CVaR has been a widely discussed topic, especially in financial risk management (Sarykalin and al., 2008). Most scholars argue in favor of CVaR because of the undesirable properties of VaR discussed above; see for example Sarykalin and al. (2008) and Rockafellar and Uryasev (2000, 2002).

2.2. Previous research

As has been outlined by many scholars in the field, see for example Allayannis and al. (2001) and Papaioannou (2006), the exposure to exchange rate fluctuations is a major source of risk for companies operating outside of their home market or having foreign trades.

Allayannis and Ofek (2001) highlight the effect of exchange rate movements on expected future cash flows of large multinationals, small importers (exporters) and import competitors. They notice that previous research in the area has determined that exchange rate movements do not have a significant effect on this type of enterprises, and argue that the use of foreign currency derivatives and other hedging instruments can explain this result. The aim with their paper is to investigate whether firms' usage of foreign currency derivatives is for hedging or for speculative purposes. By estimating a multivariate regression, where a firm's exchange rate exposure is measured both by its foreign sales and its financial hedging activities, they obtain the result that firms use foreign currency derivatives for hedging purposes and not to speculate. By including foreign sales as a part of a firm's exchange rate exposure, they also conclude that firms with larger size and more exposure to foreign sales are more likely to use currency derivatives. This latter result is consistent with Froot et al.'s (1993) theory of optimal hedging, also focusing on the fact that hedging is related to high fixed start-up costs.

The first application of CVaR on portfolio optimization and hedging was made by Rockafellar and Uryasev (2000). Their paper focuses on minimizing CVaR instead of VaR for a portfolio

[1] The properties of CVaR and further discussions about its advantages over VaR are presented in Section 3.2 in "Theoretical Framework".

of financial instruments, in order to reduce risk. The uniqueness of this new approach is the technique used, namely that VaR is calculated at the same time as CVaR is optimized. They argue that CVaR can be a suitable risk measure to enterprises that evaluate risk, investment companies, mutual funds and brokerage firms. In addition to being a risk measure with better properties than VaR, they come to the conclusion that CVaR minimization works beyond the one-instrument setting. Thus, the technique can be applicable on several instruments and therefore yields a broader approach regarding hedging.

As mentioned above, Rockafellar and Uryasev (2000) argue that CVaR is a risk measure that can be applied to a broad range of risk-related problems, not only exchange rate risk. To mention some examples, Capiński (2015) uses CVaR to hedge stock positions with put options, Andersson and al. (2001) use CVaR as a risk measure for evaluating credit risk and Sheena and al. (2011) use CVaR to analyze optimal strategies in the context of contract obligations in the energy supply sector under the uncertainty of spot prices. However, since this thesis only focuses on CVaR and exchange rate risk, the application of CVaR and hedging on other fields will not be discussed in more depth. Krokhmal and al. (2001) further develop the field of usage of the risk measure, showing that CVaR not only can be used to reduce risk, but also to maximize expected returns under CVaR constraints. For example, Gototh and Takano (2007) minimize CVaR in the context of the newsvendor problem, i.e., the maximization of expected profits or minimization of expected costs. Yet again, the maximization of expected returns and profits is not relevant for this thesis and will not be further discussed.

As mentioned previously, the article of Álvarez-Díez and al. (2015) provides the basis for this thesis. They investigate, through VaR and CVaR minimization, the best way to minimize the exchange risk via a multi-currency cross-hedging strategy. Their usage of VaR is motivated by the fact that VaR is the risk measurement assumed by the Basel Committee on Banking Supervision. Since the acceptance of CVaR has increased over time and is a risk measure that makes optimization easier because of its convexity, they also use CVaR in addition to VaR. The aim of their article is to find optimal hedge ratios, namely how large and which type of position an investor should take in another currency in order to minimize VaR and CVaR. To calculate the optimal hedge ratios using VaR, they employ a multi-objective genetic algorithm. In the case where CVaR is considered, they instead linearize CVaR and obtain a linear function that generates hedge ratios when solving for the linear problem. This approach is in great part based on the work of Rockafellar and Uryasev (2000, 2002), discussed previously. The currencies they want to hedge are 10 developed market currencies, all measured against the EUR. They minimize VaR and CVaR for a two-currency hedge portfolio (where only two currencies are used; one representing the position held, and the other used for hedging this position) as well as a ten-currency hedge portfolio (where all remaining nine currencies are used to hedge the specific position in a currency). Their main conclusion is that the multi-currency cross-hedging strategy reduces both VaR and CVaR for both type of portfolios. In addition, when increasing the number of hedging currencies from one (two-currency hedged portfolio) to nine (ten-currency hedged portfolio), this decreases VaR and CVaR on average around 9%.

Another related subject of interest that Álvarez-Díez and al. (2015) only briefly mention is that derivatives may not be the perfect way for small- and medium-sized enterprises (hereafter abbreviated SME) to hedge their currency risk, but without analyzing the issue any further. Nevertheless, this issue is studied in Kantox's (2013) research paper, where the authors analyze over 100 SMEs and mid-caps dealing with foreign currencies. The challenges with hedging currency risk were many; to mention some, the companies found it difficult to quantify the foreign exchange (henceforth referred to as FX) exposure, they had a lack of FX knowledge/skills and outlined the complexity of the matter. The majority (77%) state to have a formal FX risk management policy, but only 38% monitor their FX risk daily. In periods with high volatility, it has been showed that enterprises that do not monitor their exposure at least on a weekly or daily basis are running a high risk of losing money on their FX exposures. Pennings and Garcia (2004) further argue that use of derivative instruments in SMEs might not be common. According to their research, ownership in this type of enterprises are often concentrated, meaning that the responsible manager's risk aversion or speculative interest can harm the company's actual hedging need.

3. THEORETICAL FRAMEWORK

This chapter covers the most important theories and definitions related to our analysis and research question, in order to enable the reader to understand the concepts used and investigated in the thesis. The chapter ends with an explanatory example that ties together these concepts.

3.1. Exchange rate risk and exposure

Exchange rate risk, sometimes also called currency risk, refers to the uncertainty that is automatically present when dealing with two or more currencies that are not fixed against each other (Business Dictionary, n.d.a). In other words, it reflects the uncertainty about a currency's future value. Adler and Dumas (1984) state that one should not consider a currency as risky simply because we believe it to devalue or appreciate in the future, but that the risk is related to uncertainty in the anticipation of these movements. As a consequence of this exchange rate risk, exchange rate exposure can then be defined as what one has to risk in monetary terms.

Exchange rate risk can be divided into three components; transaction risk, translation risk and economic risk (Papaioannou, 2006). However, as this thesis only covers transaction risk, translation and economic risk will not be further discussed. Transaction risk consists of the risk of having cash flows in a foreign currency, i.e., the risk that exchange rates imply on accounts receivables, accounts payables and dividends.

The focus of this thesis lies on import firms and the uncertainty of their future payments in terms of the amount that one has at risk, or rather the possibility of having to pay greater sums for an invoice because of exchange rate fluctuations. We are thus working with an exchange risk exposure, and more specifically the transaction exposure of accounts payables.

3.2. Risk measures

Within the field of finance, individuals as well as corporations are often exposed to risk. It is therefore convenient to have some way in which to quantify the riskiness of a specific position, allowing us to decide whether it is acceptable or not (Frittelli and Gianin, 2002). According to Roccioletti (2006), risk measures are a way to summarize the riskiness of a position into one single number. Quite logically, riskier positions will yield higher risk measure outcomes.

As mentioned previously, classical risk measures focusing on a specific portfolio's variance have in recent times been increasingly replaced (in practice) by more modern risk measures, such as VaR and CVaR. Following the notation of Sarykalin and al. (2008), where X represents a random variable having the cumulative distribution function $F_X(z) = P\{X \leq z\}$ and α represents the confidence level, set between 0 and 1, VaR and CVaR can be defined as follows:

$$VaR_\alpha(X) = min\{\, z \mid F_X(z) \,\geq\, \alpha \,\}. \qquad (3.1)$$

$$CVaR_\alpha(X) = \int_{-\infty}^{\infty} z\, F_X^\alpha(z), \qquad (3.2)$$

where $F_X^\alpha(z) = \begin{cases} 0, & when\ z < VaR_\alpha(X), \\ \frac{F_X(z)-\alpha}{1-\alpha}, & when\ z \geq VaR_\alpha(X). \end{cases}$

In practice, VaR represents the largest loss which we can expect to suffer with some probability (confidence level) over a given holding period (Business Dictionary, n.d.b). Thus, $VaR_\alpha(X)$ can be interpreted as that we are α% certain that we will not lose more (in monetary terms) than $VaR_\alpha(X)$ over the investigated holding period, based on the given loss distribution. In turn, $CVaR_\alpha(X)$ can be interpreted as the average of losses above $VaR_\alpha(X)$ (Acerbi, 2002). VaR can therefore be understood in practice as how bad losses can get, while CVaR grasps the extent of the expected losses if losses do get this bad (Hull, 2015).

Alongside the development of VaR and CVaR, Artzner and al. (1999) also established the definition of coherent risk measures. Following their terminology, a risk measure is said to be coherent if it satisfies four properties; monotonicity, translation invariance, homogeneity and sub-additivity. According to Hull (2015), the property of monotonicity concerns the fact that if a certain portfolio yields a worse result than another portfolio in all possible states, the risk measure of this portfolio should also be greater than that of the other portfolio. The property of translation invariance implies that if we add some amount of cash to the portfolio, the risk measure of the portfolio should decrease by this amount. The property of homogeneity means that if we change the size of a portfolio by some factor, while keeping all relative amounts inside the portfolio the same, the risk measure of the portfolio should be multiplied by this same factor. The property of sub-additivity states that when we combine two portfolios into one, the risk measure of the combined portfolio should not be greater than the sum of the individual portfolio's risk measures before their combination. Acerbi and Tasche (2002) argue that sub-additivity may be the feature of a risk measure that characterizes it the most, and that it captures the core aspect of the behavior of a risk measure under the composition of a portfolio.

Acerbi and Tasche (2002) go so far as to say that the four properties presented by Artzner and al. (1999) entirely define the concept of a risk measure, meaning that they do not define a non-coherent measure as a risk measure. From their point of view, VaR should therefore not even be considered as a proper risk measure. They thereby also argue that the most important property of CVaR is its coherence.

The most relevant difference between VaR and CVaR for the sake of this essay's purpose is related to the disadvantageous mathematical properties of VaR, since convexity and the property of sub-additivity are essential in the optimization of risk measures. As was briefly mentioned previously[2], VaR optimization is complicated due to its non-convexity. This non-convexity is in turn due to its lack of sub-additivity, since convexity follows from the properties of sub-additivity and positive homogeneity (Acerbi and Tasche, 2002). In the case of a normal or elliptical distribution, CVaR and VaR minimization leads to the same results; one can equivalently work with CVaR, VaR or even Markowitz's minimum variance under such circumstances. The problem of VaR being complicated to optimize numerically arises when losses are not normally distributed (Sarykalin and al. 2008).

3.3. CVaR optimization

Taking the field of portfolio management in general, optimization models can help us to satisfy constraints with some specific probability level, since we usually want to make sure that a portfolio's loss at a certain future date does not exceed a certain value (Sarykalin and al., 2008).

CVaR's most important feature in the context of its minimization (which is a key component of this thesis) is that it can be expressed by a minimization formula, as suggested by Rockafellar and Uryasev (2000). This minimization formula can in turn be incorporated into the given

[2] See Section 2.1 in "Literature Review".

optimization problem together with some specific decision variables related to target returns and portfolio positions. The decision variables allow us to shape the risk within some bound or minimize it as a whole. Thus, by applying the methodology of a minimization formula, we can conserve the convexity feature of CVaR while making its minimization less cumbersome and complex (Sarykalin and al., 2008). Since CVaR is a risk measure, one can interpret the optimization of CVaR as a maximization or minimization of risk, given the predetermined settings.

3.4. HEDGING

Hedging is defined as a strategy that risk managers can apply to limit or totally offset the probability of a loss, resulting from price fluctuations. In practice, hedging can be interpreted as a transfer of risk without the involvement of insurance policies (Business Dictionary, n.d.c). In the field of foreign exchange, hedging strategies concern the reduction or total elimination of the currency risk (Papaioannou, 2006).

Hedging is often performed with derivatives instruments, such as forwards, futures and options. This kind of hedging usually consists of locking in a certain price today, allowing the hedger to better plan for the future, knowing that the exposure to a given risk is minimized (Sucden Financial, n.d.). However, all hedging strategies do not aim at locking in a future price at the current date for the entire exposure. First of all, one can choose only to hedge a part of the future obligation, referred to as partial hedging. Thus, partial hedging reduces the effect of undesired or unanticipated movements, without eliminating it entirely (Oxford Reference, n.d.). In addition, all hedging strategies do not include the use of derivative instruments, as will be the case in this thesis with the natural effect of multi-currency cross-hedging.

Multi-currency cross-hedging can be applied when having a non-zero correlation between two or more currencies. From this non-zero correlation, a part of the currency risk can then be hedged by natural hedging, which arises when the currencies move in such a way that investing in two or more currencies will naturally make them hedge one another (Álvarez-Díez and al., 2015).

It also needs to be pointed out that hedging is not a miracle remedy for risky positions. Hedging gives the risk manager the benefit of being able to manage the given risk to a certain degree, either through its reduction or total elimination, which can allow for e.g., cash flow stability and better planning. However, at the same time, the risk manager might miss out on potential profits if the given market fluctuation were to move in the opposite direction of what was initially anticipated, or in the opposite direction of what one feared it would do (Sucden Financial, n.d.).

3.5. Long and short positions

Within the field of currencies, going long means buying the base currency while going short means selling the base currency (MahiFX, n.d.). Taking a long position in SEK/USD thus implies buying SEK for USD, while taking a short position in SEK/USD implies selling SEK for USD. From this definition, it is also easily seen that whenever we are taking either a short or long position, we are short one currency and long the other (DailyForex, n.d.).

Taking a long position, we expect the market price of the currency to rise, enabling us to sell it back in the future and make a profit. Taking a short position, we expect the market price of the currency to decline (MahiFX, n.d.).

3.6. Optimal hedge ratio

Optimal hedge ratios are often encountered in the field of derivative instruments, where they represent the size of a short position in the futures markets as a proportion of a long position in the spot market. The short position is estimated by maximizing one's expected utility, which in turn depends on the risk and expected return of this hedged portfolio (Harris and Shen, 2003). In this setting, optimal hedge ratios are used due to the fact that futures contracts are not in themselves always effective for the purpose of hedging. By computing optimal hedge ratios, one can select the most appropriate types of futures contracts and how many of them are needed to hedge the given exposure (Kantox, n.d.). Nevertheless, this same methodology can also be applied to the case where we hedge without the use of derivative instruments. For example, in the setting of this thesis, we apply the same approach but somewhat reversed.

In order to compute optimal hedge ratios, the setting of the given hedged portfolio must be defined in terms of its composition and size. When talking about currencies, the hedged portfolio is a portfolio consisting of a set of two or more currencies, where the first currency is the one that needs to be hedged, and the remaining currencies are the ones used for hedging the exposure in this first currency. The multi-currency cross-hedging approach implies that, given that the currencies in the hedged portfolio have a non-zero correlation with one another, a long (short) position in one currency can be used to hedge a short (long) position in another currency. For a set of two or more currencies, the return of the hedged portfolio can be defined as follows:

$$r_p = r_0 + \sum_{i=1}^{n} h_i r_i. \qquad (3.3)$$

In this formula, r_p represent the return of the hedged portfolio, $r_0$represent the return of a short (long) position in the currency that needs to be hedged, r_i represent the return of a long (short) position in the currencies used for hedging and h_i represents the optimal hedge ratios related to the currencies used for hedging. The optimal hedge ratio is the proportion one should long or short in the currency used for hedging in order to minimize the risk and/or maximize the returns of the hedge portfolio. The procedure to compute the optimal hedge ratio can be single-objective or multi-objective. If single-objective, optimal hedge ratios are calculated to only minimize risk. If multi-objective, optimal hedge ratios are calculated both with regards to risk and returns (Álvarez-Díez and al., 2015).

3.7. Explanatory example

In order to interlink the concepts explained above and give an intuitive sense to the hedging strategy that we analyze, we hereunder present an explanatory example of a very generalized way in which multi-currency cross-hedging can be performed.

Assume we are a Swedish company, having a 1'000 USD invoice due in one month. Further assume that the current SEK/USD exchange rate is of 8.0, but that we expect a devaluation of the Swedish krona against the US dollar, so as to be at a rate of 8.5 in one month. This would imply that instead of having to pay 8'000 SEK (being the value of our invoice today) we will, in one month, have to pay 8'500 SEK. Thus, we will have to pay more for our 1'000 USD invoice in one month than if we would have paid it today, because of the devaluation of the SEK against the USD during this time.

Now assume that, through CVaR optimization, we find that the best currency (among our sample of currencies) for hedging this devaluation is the euro, with an optimal hedge ratio of 0.5. This means that adding an exposure in EUR to the exposure in USD is the currency combination that results in the highest CVaR reduction (when comparing to the CVaR of the unhedged USD exposure). A hedge ratio of 0.5 means that we should have an EUR exposure that corresponds to 50% of the USD exposure, in terms of SEK. Note that we will ignore transaction costs in this thesis, and therefore in this example as well. Using the invoice amount above, we thus get that 4'000 SEK (0.5·1'000·8.0 = 4000) should be used for taking a short position in SEK/EUR. Assuming that the SEK/EUR exchange rate is of 9.0 today (the day we receive the USD invoice and enter the SEK/EUR short position) and of 9.5 in one month (the day we pay the USD invoice), entering a short position today means that we sell our 4'000 SEK and receive 444 EUR (4'000÷9.0≈444).

In one month, when the USD invoice is to be paid, we take a long position in SEK/EUR (corresponding to the amount we shorted on the day we received the invoice) and a short position in SEK/USD (corresponding to the amount of the invoice). Since the SEK/EUR rate is now of 9.5, we get 4'218 SEK for our 444 EUR through the long position (444·9.5 = 4'218). The SEK/USD rate being of 8.5, we today sell 8'500 SEK in order to get 1'000 USD (covering our invoice) through the short position (8.5·1'000 = 8'500). Our cash inflows and outflows for the whole hedging procedure are thus -4'000 SEK (short SEK/EUR), +4'218 SEK (long SEK/EUR) and -8'500 SEK (short SEK/USD), summing up to a cash outflow of -8'282 SEK.

Short position SEK/USD (invoice)	-8'500 SEK
Cash inflow/outflow without hedging	**-8'500 SEK**

Short position SEK/EUR	-4'000 SEK
Long position SEK/EUR	+4'218 SEK
Short position SEK/USD (invoice)	-8'500 SEK
Cash inflow/outflow with hedging	**-8'282 SEK**

By applying this natural hedging strategy, we thus only have to pay 8'282 SEK in total instead of the 8'500 SEK we would have had to pay on the day of the invoice without any hedging at all.

4. METHODOLOGY

This chapter describes the methodology used to obtain the results of this thesis, including a description of the data, its collection and sampling.

4.1. Choice of currencies

The selection of currencies was based on Swedish SMEs that deal with other foreign currencies in their operations, through imports from outside of Sweden. To stay within the scope of this thesis, we assumed that the general Swedish small- and medium-sized import company follows the same import pattern as the country as a whole regarding the main currencies of trade. According to Statistics Sweden's (n.d.) numbers for 2017, the most important countries for Swedish imports are Germany (18.9% of Swedish imports), the Netherlands (8.9%), Norway (8.1%), Denmark (7.2%) and the United Kingdom (5.2%). Apart from the fact that they are Sweden's main import countries, they are also interesting countries in this specific analysis as they represent four different currencies; euro (EUR), Norwegian krone (NOK), Danish krone (DKK) and British pound sterling (GBP). In addition, even though the United States only represents 2.4% of Sweden's imports, the U.S. dollar (USD) was added as a currency of interest. This because international trade within the European Union is often denominated in USD if not in the local currency or in EUR (Eurostat, n.d.). These five currencies, which hereafter will be referred to as *central currencies* in this thesis, are the main currencies of interest, being the ones we aim to hedge exposures in.

For the purpose of hedging the central currencies, what we refer to as *additional currencies* were also introduced, since it is possible that the optimal hedge may be outside the combination of central currencies. The additional currencies were taken from the list of the 10 most traded currencies in the first half of 2017 (Bullmarketz, 2017), being the most recent statistics we could find. However, four of these most traded currencies were already a part of the central currencies group, leaving six currencies as additional currencies. These additional currencies are Japanese yen (JPY), Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), Chinese renminbi (CNY) and Mexican peso (MXN).

We limited ourselves to these currencies, including the central currencies, for computational reasons. The use of the most traded currencies for hedging purposes was motivated by the small-and medium-size enterprise perspective of this thesis. It might be the case that some other currency actually hedges a central currency better than another central currency or one of the additional currencies. However, first, this would have gone outside of the scope of this thesis (mainly for computational reasons) and second, we wanted to keep the analysis on a level suitable for SMEs operating in Sweden. For such companies, sometimes lacking both the resources and competences for extensive foreign exchange activities, a focus on the most traded currencies worldwide seemed more appropriate. One alternative way could have been to send out surveys to a sample of Swedish SMEs in order to more thoroughly analyze what currencies they most often have exposures to. This could have given us more accuracy in the choice of currencies, truly reflecting the hedging needs of Swedish SMEs. However, this thesis focuses on the power and accuracy of multi-currency cross hedging, and not directly on finding the optimal hedge ratio for a given currency in a limited (but more accurate) or worldwide sample of currencies. For the purpose of testing the validity of the approach, a sample of 11 currencies based on recent statistics was therefore regarded as sufficient.

4.2. Collection of data

The sample of currencies was collected from Thomson Reuters Datastream. The data, consisting of exchange rates (closing spot rates), was retrieved on a daily basis from 2010-01-01 to 2017-12-29, summing up to 2086 observations for each currency, yielding a total of 22’946 observations. A further comment on the sample of currencies is that data was retrieved for every trading day for every currency, meaning that there was no reduction of the data. The decision to start in 2010 was based on the fact that Álvarez-Díez and al. (2015) end their analysis in 2009 (included) and that we, as one part of this thesis, want to investigate their approach on a new data set. In addition, when data was retrieved from 2010 and onwards, the financial crisis (starting in 2007-2008) was not included, which could have led to somewhat biased results. No adjustments of the data were required to complete the final data set.

4.3. Partitioning of data

As we, in contrast to Álvarez-Díez and al. (2015), apply an out-of-sample approach in this thesis, the data set was divided into two periods. The first period, ranging from 2010 to 2013 (included), is used to estimate the optimal hedge ratios. The second period, ranging from 2014 to 2017 (included), is used to test the power and accuracy of the optimal hedge ratios obtained in the first period on data from the second period. For this purpose, the first period is defined as an estimation period and the second period as a test period.

4.4. COMPUTATION OF VARIABLES

For each currency, daily returns (r_t) were calculated by dividing the difference between yesterday's and today's spot rates ($s_{t+1} - s_t$) with the spot rate (s_t) of the current day, as below:

$$r_t = \frac{s_{t+1} - s_t}{s_t}. \qquad (4.1)$$

In our setting, a positive return implies a loss to the company dealing with imports (where the invoice is denominated in the foreign currency), since a higher spot rate at the payment date makes the transaction more expensive[3].

However, as we want to investigate the multi-currency cross-hedging strategy on a short position (as the company will have to short the amount of their invoice at the date of the payment), the daily returns were converted into those of a short position by multiplying them by minus one[4]. Taking this into account, the returns on which all following computations have been performed are actually the negative of the return formula presented above (Formula 4.1). Returns that were initially positive (corresponding to a current spot rate higher than yesterday's spot rate) thereby become negative, and vice versa. Thus, after such a conversion, negative returns are those that do not benefit the company, since it will have to pay more SEK for the invoice denominated in the foreign currency. This needs to be noted in order to understand how returns and losses are related to each other. For the ease of future referencing, we can define these returns as follows:

$$r_{t;SHORT} = (-1)\left(\frac{s_{t+1} - s_t}{s_t}\right). \qquad (4.2)$$

Daily losses were calculated by multiplying the daily returns (after their conversion into a short position) with a negative amount of 100, meaning that the daily losses can be interpreted both in percentage terms or in currency units. Multiplying returns by a negative amount means that

[3] Review the explanatory example (Section 3.7 in "Theoretical Framework") if this way of reasoning is still not clear.
[4] This formulation will be further explained in Section 4.6.

positive returns become negative losses, i.e., gains, while negative returns positive losses, i.e., losses. The conversion of returns into losses was made through the following formula:

$$l_t = (-100) * r_{t;SHORT}. \quad (4.3)$$

After computing the daily losses for each of the currencies, their respective distributions were investigated. The results of this investigation, performed using EViews, can be found in Appendix 1. The results concluded on a non-normal distribution for each respective currency, through the rejection of the Jarque-Bera test. As can be seen from these results, the skewness and kurtosis of the respective loss distributions vary widely between currencies, exhibiting features of asymmetrical distributions. This strengthened our choice of an asymmetric risk measure based on a historical simulation approach[5]. Similarly, the conclusion of non-normal loss distributions strengthened our choice of CVaR as the appropriate asymmetrical risk measure to be used in the optimization procedure[6].

TABLE 4.1. SUMMARY STATISTICS OF THE LOSS DISTRIBUTIONS.

Short position	Skewness	Kurtosis	Jarque-Bera	P-value
Central currencies				
USD	0,3199	5,4581	560,4731	0,0000
DKK	0,4066	5,1680	465,7717	0,0000
EUR	0,3958	5,1283	447,9265	0,0000
GBP	-0,2932	8,3993	2 562,5040	0,0000
NOK	0,0792	6,1719	876,2104	0,0000
Additional currencies				
AUD	-0,0729	4,5677	215,3599	0,0000
CAD	0,0458	4,3012	147,8344	0,0000
JPY	0,5364	7,1171	1 572,5420	0,0000
CHF	5,1976	150,2079	1 891 983,0000	0,0000
CNY	2,7904	457,3697	17 938 208,0000	0,0000
MXN	-0,3583	7,5229	1’821,7980	0,0000

Table 4.1 presents the skewness, kurtosis (excess kurtosis coefficient), Jarque-Bera statistic and P-value for the loss distributions over the whole sample period (2010-01-01 to 2017-12-29).

Having the losses for each currency, their respective VaR and CVaR were calculated over a one-day holding period with a confidence level of 99%. The choice of confidence level was based on two factors. First, 99% is the highest standard value[7] of confidence levels. Since we deal with the minimization of risk of real monetary exposures, we wanted to get as accurate risk measures as possible. Second, 99% is the confidence level used by Álvarez-Díez and al. (2015).

[5] Review Section 2.1 in ”Literature Review” for a discussion of asymmetric risk measures.
[6] Review Section 3.2 in ”Theoretical Framework” for a discussion of VaR not being well-suited for optimization procedures when losses are not normally distributed.
[7] Usually used in research and academic writing.

As we take inspiration from their research paper and aim to compare some of our results to theirs, it seemed logical to use the same confidence level.

VaR was calculated as the 99th percentile of losses for each currency. CVaR was subsequently calculated as the mean loss of those losses exceeding VaR for that specific currency[8]. These are referred to as initial VaR and initial CVaR as they correspond to the unhedged initial short position, i.e., the one we want to compare CVaR changes with. As we apply an out-of-sample approach, one initial VaR and one initial CVaR were computed for the estimation period, and one initial VaR and one initial CVaR were computed for the test period. The initial VaR is used in the optimization process as a target return (lower level for losses) and, as mentioned before, the initial CVaR is used to compare how CVaR changes from the unhedged position to the hedged portfolio. Table 4.2 presents the initial VaR and initial CVaR as well as the annual average return of the two periods.

TABLE 4.2. SUMMARY STATISTICS OF SPOT RATES AND LOSSES.

Short position	Average return	$VaR_{99\%}$	$CVaR_{99\%}$
Estimation period			
USD	-2,09%	2,4121	2,7715
DKK	-3,23%	1,2377	1,6294
EUR	-3,15%	1,2471	1,6274
GBP	-1,32%	1,6744	2,0430
NOK	-3,78%	0,8921	1,1445
Test period			
USD	+6,59%	1,3794	1,7770
DKK	+2,60%	0,8767	1,0634
EUR	+2,54%	0,8831	1,0598
GBP	+1,09%	1,4705	1,6909
NOK	-1,12%	1,3186	1,7180

Table 4.2 presents the returns (average annual returns in percentage, calculated on spot rates), $VaR_{99\%}$ and $CVaR_{99\%}$ are the 1-day VaR and CVaR computed on daily losses. All three measures are calculated for the estimation period (2010-01-01 to 2013-12-31) and the test period (2014-01-01 to 2017-12-29).

4.5. COMPOSITION OF HEDGED PORTFOLIOS

The focus of this thesis lies on hedging an exposure in one of the central currencies in order to lose as little money as possible if the SEK were to devaluate against the currency in which we have the given exposure. The composition of the different hedged portfolios was therefore based on the aim of hedging these currencies. As mentioned previously, the currencies we want to hedge are EUR, NOK, DKK, GBP and USD, all against the SEK. To compose the hedged portfolios, we also used the currencies that we refer to as additional currencies, being JPY,

[8] See Section 3.2 in ”Theoretical Framework”.

AUD, CAD, CHF, CNY and MXN. Note that the central currencies were also used as hedging currencies. As mentioned previously[9], no transaction costs are accounted for in this thesis. This because many Swedish banks offer foreign currency accounts where companies do not have to pay any exchange fee, therefore no costs are incurred by such a transaction; see for example SEB (n.d.) and Swedbank (n.d.).

The composition was done either in a two-, five- or 11-currency hedged portfolio. Independently of the size of the hedged portfolio, the first currency in the portfolio was always the one we aimed to hedge, whereas the remaining currencies were used for hedging. In a two-currency hedged portfolio, the hedged portfolio consists of two currencies. This means that only one currency was used for hedging. In this setting, each central currency was hedged with all other currencies (additional currencies as well as central currencies, excluding the one under investigation), one by one. In the five- and 11-currency hedged portfolios, more than one currency was used for hedging. The five-currency hedged portfolios only consist of central currencies, where one needs to be hedged and the remaining ones are used for hedging. In contrast, both central and additional currencies were included in the 11-currency hedged portfolios. Since previous research (Álvarez-Díez and al., 2015) claims that adding more currencies reduces CVaR, the computation of five- and 11-currency hedged portfolios in addition to that of a two-currency hedged portfolio was considered relevant to test.

Since the natural hedge part of multi-currency cross-hedging is based on the non-zero correlation between currencies we first, before any portfolio composition, needed to compute these correlations to check if all currencies can be included. If for example SEK/USD would have zero correlation with SEK/AUD, AUD could not be used to hedge the USD position through multi-currency cross-hedging and would therefore need to be removed from any hedged portfolio related to an exposure in USD. The correlation matrices, for the estimation and test period data, are presented in Appendix 2. We found that all currencies have a non-zero correlation with one another for both data samples and could therefore conclude that all currencies could be included.

However, there is a significant limitation to our choice of currencies that we had to take into account in the composition of the hedged portfolios, namely that Denmark has a fixed exchange

[9] See Section 3.7 in ”Theoretical Framework”.

rate policy against the euro. Since the introduction of the euro in 1999, the Danish krone has been fixed at a central rate of 746.038 DKK per 100 EUR, as a part of the European Exchange Rate Mechanism (ERM 2). Within ERM 2, 15% is set as the standard fluctuation band for fixed exchange rates, meaning that the exchange rate of a given fixed currency can fluctuate plus or minus 15% from its central rate. However, Denmark has a lower fluctuation band of 2.25% following an agreement with European Central Bank, meaning that the Danish krone can only move between 729.252 DKK and 762.824 DKK per 100 EUR. Despite an already narrower fluctuation band, the Danish central bank has in addition managed to stabilize their currency even closer to the central rate (Nationalbanken, 2015).

When hedging DKK and EUR, we could therefore not use them against each other as hedging currencies. This implies that when hedging DKK in the two-currency approach, EUR was not used as a hedging currency, and vice versa. The same applies to the five- and 11-currency hedged portfolios, where EUR had to be removed from the sample of hedging currencies used when hedging DKK, and vice versa. Thus, instead of five- and 11-currency hedged portfolios, we have four- and 10-currency hedged portfolios when hedging DKK and EUR.

Staying within the field of fixed exchange rates, a further issue that has to be clarified is the pegging of the Swiss franc to the euro. The Swiss National Bank (SNB) decided to peg the Swiss franc against the euro in September 2011. This decision implied that EUR/CHF exchange rate would be kept above 1.20 CHF per unit of EUR (SNB, 2011). This lower floor was then removed in January 2015, meaning that the Swiss franc was pegged to the euro for approximately three years (Bishop, 2015). Since our data set stretches over the years 2010-2017 (included), the EUR/CHF peg affects approximately two years in the estimation period data and approximately one year in the test period data. Thus, we decided to still include CHF in all hedged portfolios since the pegging only affects three out of eight years in total, but keep this potential source of bias in mind when analyzing results. This decision was further strengthened by the fact that the correlation between CHF and EUR spot rates did not stand out as compared to other non-pegged currencies, while the correlation between DKK and EUR was very close to one (see Appendix 2).

Taking the DKK/EUR pegging into account, different data set were then formed for each hedged portfolio that we wanted to test. Since we had five central currencies that we wanted to apply the two-currency portfolio composition on with ten (nine for initial short positions in

DKK and EUR) hedging currencies, this first composition alone yielded 48 data sets to perform CVaR optimization on. The five- and 11-currency portfolio compositions yielded five data sets each (one for each initial central currency short position) to perform the optimization on.

4.6. Optimization and related computations

Optimal hedge ratios were derived by minimizing CVaR of each hedged portfolio. The minimization was done in MatLab, using a code developed by Vogiatzoglou (2008) that we made some minor adjustments to. This code estimates the optimal hedge ratios (portfolio weights) that minimize CVaR under a given target return R_0. The target return was set to a loss of the initial VaR, implying a target return of $R_0 = (-VaR_{0.99})$. In practice, the company will at some point in time experience a loss; putting a target return equal to zero would therefore be over-optimistic. It also needs to be pointed out that the target return is not a return requirement, but a lower bound for the acceptable loss. When setting $R_0 = (-VaR_{0.99})$ we simply told the optimization model that we do not accept a loss greater than $VaR_{0.99}$ for the calculation of the optimal hedge ratio. Since we computed the optimal hedge ratios only with the aim of minimizing risk, and not maximizing returns, our optimization procedure was therefore single-objective.

To find the optimal hedge ratios, we forced r_0 in the function of the return of the hedged portfolio (Formula 3.4[10]) to be a short position, multiplying r_0 with minus one. This can be interpreted as if the currency we aim to hedge always has a weight of minus one in the hedged portfolio. The returns of the hedged portfolio were therefore defined as follows:

$$r_p = (-r_0) + \sum_{i=1}^{n} h_i r_i. \quad (4.4)$$

Usually, a short position is hedged with a long position. We therefore expect all optimal hedge ratios (h_i) using the two-currency approach to be positive. However, when having the setting of a five- or 11-currency hedged portfolio, the initial short position can be hedged with both long and short positions. What should be mentioned is that in the optimization, we did not put any bounds on h_i in terms of long and short positions; in any setting, they could become both positive (long positions) and negative (short positions). The only bound that was set on h_i was that they cannot be lower than minus one or greater than one, meaning that one single optimal hedge ratio could not hedge more than the entire initial short position. In addition, no bounds

[10] Section 3.7 in "Theoretical Framework".

were set on the sum of all optimal hedge ratios in a hedged portfolio, meaning that the sum of portfolio weights could exceed one.

Having obtained the optimal hedge ratios, we could then compute the returns of each hedged portfolio using the formula above (Formula 4.4). These returns were then converted into losses in the same way as before[11], from which CVaR was subsequently computed[12]. This in turn allowed us to calculate the CVaR change of each hedged portfolio (with respect to the CVaR of the unhedged initial short position), which was done in the following way:

$$\Delta CVaR = \frac{Hedged\ portfolio\ CVaR - Initial\ CVaR}{Initial\ CVaR}. \qquad (4.5)$$

We then also measured the CVaR change relative to the other portfolio compositions. This means that for a two-currency hedged portfolio, we first compared its CVaR to the initial CVaR, but then also with the CVaR of its corresponding five- and 11-currency hedged portfolios, and vice versa. These additional CVaR changes were calculated as follows (for the case where we compared the two-currency hedged portfolio CVaR with the five-currency hedged portfolio CVaR, but the same principle applies to the 11-currency portfolio):

$$\Delta CVaR = \frac{Two\ currency\ hedged\ portfolio\ CVaR - Five\ currency\ hedged\ portfolio\ CVaR}{Five\ currency\ hedged\ portfolio\ CVaR}. \qquad (4.6)$$

What is worth noting at this point is that in the estimation period, the CVaR of a hedged portfolio containing more hedging currencies than another hedged portfolio should always be lower than that of this other portfolio as long as it does not contain any optimal hedge ratios of zero[13]. For example, for the same initial short position, an 11-currency hedged portfolio should always yield a lower CVaR than that of a five- or two-currency hedged portfolio. This in turn means that the CVaR change (Formula 4.6) should always be negative when comparing the 11-currency hedged portfolio to the two other smaller hedged portfolios, since the 11-currency hedged portfolio will always yield a lower CVaR. Similarly, the CVaR change should always be

[11] Using Formula 4.3 but with the returns of the hedged portfolio.
[12] In the same way as in Section 4.4.
[13] Since adding more currency exposures to an already existing hedged portfolio should theoretically never lead to an increase in CVaR in the setting of its minimization. If we add a currency to an already existing hedged portfolio and that this additional currency would increase the CVaR of the hedged portfolio, the optimization algorithm would put a weight (optimal hedge ratio) of zero on this additional currency, as we know that a lower CVaR can be achieved with the combination of optimal hedge ratios of the existing (excluding the additional currency) hedged portfolio.

negative when comparing the five-currency hedged portfolio with the two-currency hedged portfolio.

4.7. Test of optimal hedge ratios

To evaluate the power and accuracy of multi-currency cross-hedging, we tested the optimal hedge ratios obtained from the CVaR optimization on estimation period data on the test period data. This testing procedure was initiated by calculating the return of the hedged portfolio[14] using the test period data, where the optimal hedge ratios (h_i) from the estimation period were used. Thus, taking the example of an initial short position in SEK/USD and applying the two-, five- and 11-currency approaches resulted in the following calculations:

$$r_{p;USD2} = (-r_{USD}) + h_{AUD}r_{AUD}. \qquad (4.7)$$

$$r_{p;USD5} = (-r_{USD}) + h_{NOK}r_{NOK} + h_{EUR}r_{EUR} + h_{DKK}r_{DKK} + h_{GBP}r_{GBP}. \qquad (4.8)$$

$$r_{p;USD11} = (-r_{USD}) + h_{NOK}r_{NOK} + h_{EUR}r_{EUR} + h_{DKK}r_{DKK} + h_{GBP}r_{GBP} + \\ + h_{AUD}r_{AUD} + h_{CHF}r_{CHF} + h_{CNY}r_{CNY} + h_{JPY}r_{JPY} + h_{MXN}r_{MXN} + h_{CAD}r_{CAD}. \qquad (4.9)$$

Note that for the $r_{p;USD2}$ hedged portfolio, the example above is with AUD, chosen randomly among the sample of currencies. In practice, we computed 10 different $r_{p;USD2}$ (nine when the initial short position is in DKK or EUR), i.e., one for each hedging currency. Also note that we calculated a new $r_{p;USDX}$ for each day in the test period.

The hedged portfolio returns were then converted into losses, which allowed us to calculate VaR and CVaR in the same way as before[15]. From these values, we were then able to investigate if the CVaR of the hedged portfolio (using the optimal hedge ratios from the estimation period) decreased as compared to the CVaR of the unhedged initial short position. To see if the pattern of CVaR changes is still consistent, we here again also computed the CVaR change of a given hedged portfolio relative to its corresponding hedged portfolios using the other approaches.

[14] Using Formula 4.4.
[15] See Section 4.4.

5. RESULTS

This chapter presents the results obtained when applying the methodology on the data. Since the data is partitioned into an estimation period and a test period, the results obtained in these two periods are presented separately. The chapter starts with the presentation of estimation period results, since some of these results have been used to obtain the results of the test period.

5.1. ESTIMATION PERIOD RESULTS

This first set of results relates to the case where we minimize CVaR on the estimation period data in order to obtain the optimal hedge ratios (and thereby composition) of each respective hedged portfolio.

5.1.1. TWO-CURRENCY HEDGED PORTFOLIO

Table 5.1.1 summarizes the results from the two-currency approach on the data from the estimation period. For each short position, it presents the hedging currency, among all investigated hedging currencies, that yields the lowest CVaR in the sample. The entire result tables, presenting the results for all two-currency hedged portfolios relative to each initial short position, can be found in Appendix 3.

As can be seen from these results, short positions in USD and DKK are best hedged with GBP, while short positions in EUR, GBP and NOK are best hedged with USD. Since all optimal hedge ratios are positive, all short positions are best hedged by long positions. In addition, all hedged portfolios yield a reduction of CVaR as compared to their respective unhedged initial short position.

When comparing the CVaR of each hedged portfolio with that of its corresponding hedged portfolio under the five- and 11-currency approach[16], all hedged portfolios of the two-currency approach yield greater values of CVaR than those of the five- and 11-currency approaches. The positive hedge ratios and patterns of CVaR changes are also true for all other two-currency hedged portfolios (see Appendix 3). All these results are in line with expectations.

[16] Which will both be presented later on, in Section 5.1.2 and 5.1.3.

TABLE 5.1.1 TWO-CURRENCY HEDGED PORTFOLIOS.

			CVaR change		
Short position	**Hedging currency**	**Optimal hedge ratio**	**Total**	**5 currencies**	**11 currencies**
USD	GBP	0,6796	-36,79%	+14,83%	+56,23%
DKK	GBP	0,4931	-27,52%	+7,92%	+20,38%
EUR	USD	0,2163	-19,24%	+9,87%	+23,28%
GBP	USD	0,3067	-32,86%	+15,74%	+26,62%
NOK	USD	0,2069	-6,27%	+0,40%	+16,72%

For each initial short position, Table 5.1.1 reports the optimal hedge ratios that yield the lowest CVaR in the given sample and the currency to which they correspond. Note that only the hedging currencies (and their respective positions) yielding the lowest CVaR are presented in the table. The CVaR change is both computed as compared with the unhedged initial short position ("Total"), as compared with the corresponding five-currency hedged portfolio and as compared with the corresponding 11-currency hedged portfolio.

5.1.2. FIVE-CURRENCY HEDGED PORTFOLIO

Table 5.1.2 presents the results of the five-currency approach on the estimation period. All additional information (e.g., the exact value of optimal hedge ratios) can be found in Appendix 4. Applying the five-currency approach, we obtain both long and short positions in the hedged portfolio for three out of five initial short positions. Only initial short positions in GBP and NOK yield positive optimal hedge ratios, implying long positions, through the entire hedged portfolio. The hedged portfolio corresponding to an initial short position in USD contains three long positions and one short position. Both for an initial short position in DKK and an initial short position in EUR, we obtain two long positions and one short position.

All hedged portfolios reduce CVaR as compared to their respective unhedged short position. Comparing the CVaR of each hedged portfolio with that of its corresponding hedged portfolio under the two- and 11-currency approach[17], all hedged portfolios of the five-currency approach yield lower values of CVaR than those of the two-currency approach, while they yield higher values of CVaR than those of the 11-currency approach. All these results are in line with expectations.

TABLE 5.1.2. FIVE-CURRENCY HEDGED PORTFOLIOS.

			CVaR change		
Short position	**# h>0**	**# h<0**	**Total**	**2 currencies**	**11 currencies**
USD	3	1	-44,96%	-12,92%	+36,05%
DKK	2	1	-32,84%	-7,34%	+11,54%
EUR	2	1	-26,50%	-8,99%	+12,20%
GBP	4	0	-41,99%	-13,60%	+9,41%
NOK	4	0	-6,64%	-0,40%	+16,26%

For each initial short position, Table 5.1.2 reports the number of long positions (# h>0) and the number of short positions (# h<0) in its respective hedged portfolio. The CVaR change is both computed as compared with the unhedged initial short position ("Total"), as compared with the corresponding two-currency hedged portfolio and as compared with the corresponding 11-currency hedged portfolio.

[17] Which will be presented later on, in Section 5.1.3.

5.1.3. 11-CURRENCY HEDGED PORTFOLIO

Table 5.1.3 presents the results of the 11-currency approach on the estimation period. Additional information, such as the exact value of optimal hedge ratios, can be found in Appendix 5. Applying the 11-currency approach, we obtain both long and short positions in the hedged portfolios for four out of five initial short positions. Only an initial short position in USD yields positive optimal hedge ratios, implying long positions, all through its hedged portfolio. The hedged portfolio corresponding to an initial short position in DKK contains six long positions and three short positions, that of an initial short position in EUR contains seven long positions and two short positions, that of an initial short position in GBP contains eight long positions and two short positions and that of an initial short position in NOK contains nine long positions and one short position.

All hedged portfolios reduce CVaR as compared to their respective unhedged short position. Comparing the CVaR of each hedged portfolio with that of its corresponding hedged portfolio under the two- and five-currency approach, all hedged portfolios of the 11-currency approach yield lower values of CVaR than those of the two-currency and five-currency approaches. These results are in line with expectations.

TABLE 5.1.3. 11-CURRENCY HEDGED PORTFOLIOS.

			CVaR change		
Short position	**# h>0**	**# h<0**	**Total**	**2 currencies**	**5 currencies**
USD	10	0	-59,54%	-35,99%	-26,50%
DKK	6	3	-39,79%	-16,93%	-10,43%
EUR	7	2	-34,49%	-18,89%	-10,88%
GBP	8	2	-46,98%	-21,03%	-8,60%
NOK	9	1	-19,70%	-14,33%	-13,98%

For each short position, Table 5.1.3 reports the number of long positions (# h>0) and the number of short positions (# h<0) in its respective hedged portfolio. The CVaR change is both computed as compared with the unhedged initial short position ("Total"), as compared with the corresponding two-currency hedged portfolio and as compared with the corresponding five-currency hedged portfolio.

5.2. TEST OF OPTIMAL HEDGE RATIOS

This set of results relates to the case where we test the optimal hedge ratios obtained from the estimation period on the test period. Note that all CVaR changes are computed from the initial CVaR of the test period[18], and not the estimation period.

[18] Which can be found in Table 4.2 in "Methodology".

5.2.1. Two-currency hedged portfolio

Table 5.2.1 summarizes the results of the two-currency approach when the optimal hedge ratios from the estimation period are applied on the test period. All results presented below can be found in more detail in Appendix 6.

As can be seen from Table 5.2.1, when applying the optimal hedge ratios from the estimation period, an initial short position in USD is best hedged with JPY, initial short positions in DKK and EUR are best hedged with CHF, an initial short position in GBP is best hedged with USD and an initial short position in NOK is best hedged with MXN. All hedged portfolios yield a reduction of CVaR as compared to their respective unhedged initial short position. It can however be noted (see Appendix 6) that for each of the initial short positions, when analyzing all of the two-currency hedged portfolios, there is always one hedged portfolio that yields a CVaR increase.

Comparing the two-currency approach to the five-currency approach[19], four out of five hedged portfolios of the two-currency approach yield a lower CVaR (implying a negative CVaR change) than those of the five-currency approach. The hedged portfolio of an initial short position in GBP is the only one with a positive CVaR change, meaning that it yields a higher CVaR using the two-currency approach than the five-currency approach. Compared to the 11-currency approach[20], CVaR is higher in the two-currency approach for three out of five hedged portfolios (corresponding to initial short positions in USD, EUR and GBP) and lower for the remaining two (corresponding to initial short positions in DKK and NOK).

Table 5.2.1. Two-currency hedged portfolios.

			CVaR change		
Short position	**Hedging currency**	**Optimal hedge ratio**	**Total**	**5 currencies**	**11 currencies**
USD	JPY	0,3632	-25,92%	-26,31%	+12,53%
DKK	CHF	0,2079	-18,14%	-19,23%	-8,70%
EUR	CHF	0,1977	-17,09%	-6,60%	+9,02%
GBP	USD	0,3067	-6,99%	+2,72%	+1,16%
NOK	MXN	0,1585	-5,29%	-2,49%	-3,98%

Table 5.3.1 reports the results for when the optimal hedge ratios from the estimation period are applied on the test period. Note that only the hedged portfolios yielding the lowest CVaR are presented in the table. The CVaR change is both computed as compared with the unhedged initial short position, as compared with the five-currency hedging portfolio and as compared with the 11-currency hedged portfolio of the given initial short position.

[19] Which will be presented later on, in Section 5.2.2.
[20] Which will be presented later on, in Section 5.2.3.

5.2.2. Five-currency hedged portfolio

Table 5.2.2 presents the results of the five-currency approach when the optimal hedge ratios from the estimation period are applied on the test period. All results presented below can be found in more detail in Appendix 7. When testing the optimal hedge ratios of the five-currency approach on the test period, three hedged portfolios (corresponding to initial short positions in EUR, GBP and NOK) reduce CVaR and two portfolios (corresponding to initial short positions in USD and DKK) increase CVaR as compared to their respective unhedged initial short position. All hedged portfolios of the five-currency approach yield higher CVaR values than those of the two-currency approach (implying a positive CVaR change), except for that of an initial short position in GBP. Comparing the five-currency approach with the 11-currency approach[21], CVaR is higher in the five-currency setting for three hedged portfolios (corresponding to initial short positions in USD, DKK and EUR) and lower for two (those of initial short positions in GBP and NOK).

Table 5.2.2. Five-currency hedged portfolios.

			CVaR change		
Short position	**# h>0**	**# h<0**	**Total**	**2 currencies**	**11 currencies**
USD	3	1	+4,94%	+35,07%	+52,70%
DKK	2	1	+1,35%	+23,80%	+13,04%
EUR	2	1	-11,23%	+7,07%	+16,72%
GBP	4	0	-9,46%	-2,65%	-1,52%
NOK	4	0	-2,87%	+2,55%	-1,53%

Table 5.3.2 reports the results for when the optimal hedge ratios from the estimation period are applied on the test period. The number of long positions (# h>0) and the number of short positions (# h<0) in each respective hedged portfolio is computed from the estimation period. The CVaR change is both computed as compared with the unhedged initial short position, as compared with the two-currency hedging portfolio and as compared with the 11-currency hedged portfolio of the given initial short position.

5.2.3. 11-currency hedged portfolio

Table 5.2.3 presents the results of the 11-currency approach when the optimal hedge ratios from the estimation period are applied on the test period. All results presented below can be found in more detail in Appendix 8. When testing the optimal hedge ratios of the 11-currency approach on the test period, all hedged portfolios reduce CVaR as compared to their respective unhedged short position. Compared to the two-currency approach, the hedged portfolios of three initial short positions (USD, EUR and GBP) have a negative CVaR change, whereas those of initial short positions in DKK and NOK have a positive CVaR change. Comparing the 11-currency approach with the five-currency approach, CVaR is lower for three hedged portfolios (corresponding to initial short positions in USD, DKK and EUR) and higher for those of initial short positions in GBP and NOK.

[21] Which will be presented later on, in Section 5.2.3.

TABLE 5.2.3. 11-CURRENCY HEDGED PORTFOLIOS.

			CVaR change		
Short position	**# h>0**	**# h<0**	**Total**	**2 currencies**	**5 currencies**
USD	10	0	-31,28%	-11,13%	-34,51%
DKK	6	3	-10,34%	+9,52%	-11,53%
EUR	7	2	-23,95%	-8,27%	-14,33%
GBP	8	2	-8,06%	-1,14%	+1,55%
NOK	9	1	-1,37%	+4,14%	+1,55%

Table 5.3.3 reports the results for when the optimal hedge ratios from the estimation period are applied on the test period. The number of long positions (# h>0) and the number of short positions (# h<0) in each respective hedged portfolio is computed from the estimation period. The CVaR change is both computed as compared with the unhedged initial short position, as compared with the two-currency hedging portfolio and as compared with the five-currency hedged portfolio of the given initial short position.

6. ANALYSIS AND DISCUSSION

This chapter consists of the analysis and discussion of our results, which will later allow us to answer our research question; *Is multi-currency cross-hedging (excluding derivative instruments) a suitable hedging strategy for small- and medium-sized enterprises?* The analysis of results will be done in two steps, first analyzing the estimation period results and then analyzing how well or bad the optimal hedge ratios obtained in the estimation period perform on the test period. After this, we will provide a brief comparison of our results to those of Álvarez-Díez and al. (2015). As this thesis concentrates on a quite specific topic on which the previous literature is scarce, it is hard to analyze our results in line with other existing research. In addition, as there is little academic theory about the investigated hedging strategy, it is also hard to analyze the results in line with theoretical expectations. As a last point of discussion, we also highlight a limitation of the given optimization procedure and present our suggestions for future research within this topic.

For each central currency, the results show that the biggest reduction in CVaR is obtained through the 11-currency hedging approach. However, since we analyze multi-currency cross-hedging from the perspective of a SME, the best result would be if the two-currency approach yields a sufficient CVaR reduction, so that such an enterprise does not have to invest in many currencies and still can obtain a good hedge. However, since only the natural hedging effect of the multi-currency cross-hedging strategy is considered, we will never be able to obtain a full elimination of the risk. This leads us into the probably most important discussion of this analysis; when is a hedge considered as good?

There is no straight answer to when our investigated hedged portfolios can be considered as yielding a sufficiently good hedge to the given initial short position. This depends on many factors, but the most important is probably how much risk reduction can be obtained in relation to its costs. By costs, we mainly mean in terms of time consumption, complexity of the process and other efforts related to the implementation of the given hedging strategy. No transaction costs are taken into account in the computations and analysis of this thesis, but one must of course keep in mind that such (and other related) costs can arise in practice. This trade-off makes it hard to define any specific risk reduction (e.g., in terms of percentage) that makes the given hedged portfolio a good hedge. As will be discussed in more detail later on, the definition of a good hedge can also be specific to a given setting. Based on the amount of work that has been required to obtain the results presented in this thesis and the complexity of their computations, our own view is that a rather great CVaR reduction should have to be achieved in order for the investigated hedging strategy to be considered as satisfactory.

6.1. Analysis of estimation period results

In this section, we analyze the results from the point of view of risk reduction. Taking the average CVaR reduction of the hedged portfolios of each approach[22], the two-currency approach reduces CVaR by on average 11.17%, the five-currency approach by 30.59% and the 11-currency approach by 40.10%. At first sight, these averages might look quite satisfying. However, as will be discussed hereunder, a more thorough analysis, breaking down the results and taking surrounding factors into account, shows on a rather poor performance of the investigated hedging strategy.

For the ease of comparison and in order to facilitate the analysis, we have summarized the results of the CVaR reductions (compared to the unhedged initial short positions) under each respective approach in Table 6.1 below. Looking at these results, we first of all note that the CVaR reductions vary widely between different hedged portfolios under the same approach; between 6.27% and 36.70% for the two-currency hedged portfolios, between 6.64% and 44.96% for the five-currency hedged portfolios and between 19.70% and 59.54% for the 11-currency hedged portfolios. An interesting observation is that all the best CVaR reductions discussed in the previous sentence correspond to an initial short position in USD, while all the worst CVaR reductions correspond to an initial short position in NOK. This is a first indication of that the investigated hedging strategy works differently well on different initial short positions.

Starting with the two-currency hedged portfolios we would, based on our experiences of the practical implementation of the hedging strategy, not say that the hedged portfolios should be considered as particularly good. The only hedged portfolios that could potentially be regarded as such are those corresponding to initial short positions in USD and GBP and, at the utmost, DKK. For the five-currency hedged portfolios we would, based on our experiences, here again

[22] From Appendix 3, Table 5.1.2 and Table 5.1.3.

say that no hedged portfolio really provides what one could consider a good hedge. The hedged portfolios corresponding to initial short positions in USD and GBP could possibly be regarded as such, while all remaining hedged portfolios perform quite poorly. Note that the CVaR reductions here are indeed greater than those of the two-currency approach, but that we need to take the supplementary work implied by the addition of currencies into account in the decision of the performance of the hedged portfolio. This issue will be further discussed later on in the analysis. For the 11-currency hedged portfolios we would, based on our experiences, say that the results are actually quite poor with regards to the fact that we have increased the number of hedging currencies that much. The only hedged portfolio that we consider as sufficiently good in this setting is the one corresponding to a short position in USD, and at the utmost that of an initial short position in GBP. Simply by considering these results, we can already notice that for what we define as a good hedge, the investigated hedging strategy does not seem to perform very well.

TABLE 6.1. CVAR REDUCTIONS OF HEDGED PORTFOLIOS.

Hedged portfolio	**CVaR reduction**
Initial short position in USD	
Two-currency hedged portfolio	36,70%
Five-currency hedged portfolio	44,96%
11-currency hedged portfolio	59,54%
Initial short position in DKK	
Two-currency hedged portfolio	27,52%
Five-currency hedged portfolio	32,84%
11-currency hedged portfolio	39,79%
Initial short position in EUR	
Two-currency hedged portfolio	19,24%
Five-currency hedged portfolio	26,50%
11-currency hedged portfolio	34,49%
Initial short position in GBP	
Two-currency hedged portfolio	32,86%
Five-currency hedged portfolio	41,99%
11-currency hedged portfolio	46,98%
Initial short position in NOK	
Two-currency hedged portfolio	6,27%
Five-currency hedged portfolio	6,64%
11-currency hedged portfolio	19,70%

Table 6.1 presents each hedged portfolio's CVaR reduction, i.e., how much the given hedged portfolio reduces CVaR from its initial value.

As a further matter of analysis, briefly discussed above, we have computed[23] the additional increase in CVaR reduction when we increase the number of hedging currencies. From this, we see that an increase in the number of hedging currencies from two to five yields an average

[23] From Table 5.1.2 and 5.1.3 in "Results".

increase in CVaR reduction of 8.65%. An increase in the number of hedging currencies from five to 11 in turn yields an average increase in CVaR reduction of 14.08%. Comparing the two-currency approach to the 11-currency approach, an increase in the number of hedging currencies from two to 11 yields an average increase in CVaR reduction of 21.43%.

However, these averages are not to base any reliable conclusions on directly. They show us that on average, a quite good increase in risk reduction can be obtained by adding more currencies to the hedged portfolio. On the other hand, adding more currencies complicates the hedging process, both in terms of computations, time and all issues related to the purchase of these currencies. Thus, a trade-off needs to be made between how much the company can increase its risk reduction by adding more hedging currencies and how much more complex the process becomes when adding these currencies (including eventual costs, which are not accounted for in this thesis). Furthermore, the addition of hedging currencies seems to be more relevant in certain hedge portfolios than others[24]. For example, for an initial short position in USD, the CVaR of its 11-currency hedged portfolio is 35.99% lower than that of its two-currency hedged portfolio. For such a position, it could therefore be worthwhile to consider the construction of an 11-currency hedged portfolio. This can be compared to an initial short position in DKK, where the CVaR of its 10-currency hedged portfolio is only 16.93% lower than that of its two-currency hedged portfolio. For an initial short position in DKK, it might therefore not be worth the additional work to extend the hedged portfolio from two currencies to 10 currencies.

Taken the amount of work an addition of hedging currencies implies, we consider that going from a two- to a five-currency hedged portfolio is only worthwhile for the hedged portfolios corresponding to initial short positions in USD and GBP (based on the increase in CVaR reduction that such an extension can contribute with). The same applies to an extension of the number of currencies from five to 11. Thus, only initial short positions in USD and GBP should be considered to be hedged with more than one currency. This conclusion is actually quite satisfactory, keeping the SME perspective of this thesis in mind[25].

An observation that requires some further comments is the poor performance of the investigated hedging strategy on an initial short position in NOK. As can be seen from Table 6.1, the CVaR reductions of this position's hedged portfolios are of 6.27% (two-currency), 6.64% (five-

[24] See Table 5.1.3 in "Results".
[25] Remember the discussion in the beginning of Chapter 6.

currency) and 19.70% (11-currency). Thus, no approach yields any satisfactory CVaR reduction. Based on these results, it would probably not be worth to hedge a short position in NOK at all. However, the CVaR reduction is (even though quite bad) much better in the 11-currency hedged portfolio than in the two other hedged portfolios. Hence, if a company would still want to hedge such an exposure in NOK using this hedging strategy, it would probably be more valuable to use the 11-currency hedged approach. This once again shows that the analysis of the performance of the hedging strategy is rather dependent on each initial short position, i.e., exposure we want to hedge.

6.2. Hedge portfolio and optimal hedge ratios

Even though the main analysis of this thesis focuses on the power and accuracy of the investigated hedging strategy, the composition of the hedged portfolios and their respective optimal hedge ratios are also interesting to analyze to some extent.

Table 6.2 presents the average CVaR reduction under the two-currency approach[26]. We can see that the average CVaR reduction is almost always smaller (and otherwise just slightly larger) than half of the reduction obtained with the best hedging currency for each respective initial short position. This shows on the importance of finding the best hedging currency in the two-currency approach, and not just use any currency for hedging.

Table 6.2. Average CVaR reduction of two-currency hedged portfolios.

Initial short position	Average CVaR reduction
USD	14,37%
DKK	11,88%
EUR	10,01%
GBP	15,71%
NOK	3,81%

Table 6.2 presents the average CVaR reduction of the two-currency hedged portfolio corresponding to each initial short position.

Staying within the two-currency approach, we can also note that the best hedging currency is not always the one having the highest (in absolute value) correlation with that of the initial short position (see Appendix 2 and Appendix 3). This observation, together with the importance of finding the best hedging currency, shows how even the two-currency approach is actually quite complex for the unexperienced practitioner. This hedging strategy does not simply consist of hedging one currency exposure with another, relying on a high correlation between these

[26] Taken from Appendix 3.

currencies. Quite cumbersome computations need to be performed simply to find the best hedging currency, which then in itself does not guarantee a satisfactory hedge.

In addition, there is no clear consistency between the best hedging currency in the two-currency approach and the optimal hedge ratios in the five- and 11-currency approaches. The best hedging currency does not always correspond to the highest weight in the five- and 11-currency hedged portfolios. Similarly, the highest weight is not always that of the currency with the highest correlation. Here again, laborious computations must be performed to find the optimal hedge ratios and thereby the composition of each hedged portfolio.

Another observation is that in the five- and 11-currency hedge portfolios, some currencies have very small hedge ratios. For example, the USD short position hedged with the 11-currency approach yields a very small hedge ratio for AUD (0.0360). If we have an invoice of 1’000 USD, this means that we should take a position in AUD representing 3.60% of the SEK amount corresponding to the given invoice. The question thus arises if the work and costs related to taking such a small position are worth the risk reduction that the given hedged portfolio yields. In most cases, investing such a small amount would seem unnecessarily cumbersome. However, if this position is not taken, the optimal hedged portfolio is no longer satisfied and the same risk reduction will no longer be attained. This brings yet another criticism of the investigate hedging strategy to light; a hedged portfolio might yield a satisfactory hedge, but the nature of its composition does not make it worthwhile to implement. A good example of a hedged portfolio that would probably not be worth to implement is that of an initial short position in EUR, where five out of nine currencies in the 11-currency hedged portfolio have smaller hedge ratios than five percent (see Appendix 5).

6.3. Analysis of out-of-sample approach

Testing the optimal hedge ratios obtained from the estimation period on the test period, we overall get worse results (in terms of risk reduction) than in the estimation period. However, some impairment of the results can be expected, as the applied hedge ratios are optimal for the estimation period and not the test period, and that the two periods naturally differ from one another to some extent. Looking at our data, we indeed see that the two data sets are quite different[27]. In the estimation period, all average yearly returns are negative, while in the test period, all average yearly returns except for one are positive. In addition, the VaR and CVaR values of the unhedged initial short position are in general higher in the estimation period than in the test period.

Looking more deeply into these differences, we see that when applying the optimal hedge ratios on the test period, it no longer holds that an 11-currency portfolio always yields a higher CVaR reduction than its respective five-currency hedged portfolio, which in turn does not always yields a higher CVaR reduction than its respective two-currency portfolio. We even obtain some hedged portfolios that increase CVaR as compared to the unhedged initial short position. In addition, four out of five best hedging currencies in the two-currency approach change. It thus seems as if the optimal hedge ratios are not very sustainable over time, i.e., that they are more related to a specific data set or time period than being some globally applicable standard weights.

This finding can be due to different reasons. First of all, the calculation of optimal hedge ratios is done on the CVaR values of the estimation period, while we compute the CVaR changes of the testing procedure on the initial CVaR values of the test period. Since the CVaR values of the unhedged initial short position differ rather much between the two periods, it seems natural that the results of the testing procedure in terms of risk reduction are not so good. If the unhedged initial short position is much riskier in one data set, its optimal hedge ratios and thereby hedged portfolios will not work well on another data set where the risk of the unhedged initial position is lower. However, the lack of power and accuracy of the multi-currency cross-hedging strategy (i.e., that the optimal hedge ratios do not function well in terms of their hedging capacity when applied to the test period data) is most probably due to an instability of the correlation between currencies over time. If the correlations remain stable over the investigated periods (data sets) but that the currencies in the hedged portfolio become more or less risky, the hedge observed in the first period would probably still work decently in the second period. When we observe a lack of fit of the hedged portfolios on the second period, this is therefore most probably due to changes in the correlation between the currencies included in these portfolios. In the extreme case of a change of sign in some of the correlations (going from positive to negative or vice versa) between the two periods, the hedge would most certainly not work at all.

[27] See Table 4.2 in ”Methodology”.

Analyzing this last cause, we can see that the correlations between our currencies of investigation vary very much between the estimation period and the test period (see Appendix 2). In addition to sometimes great variations in the correlations, 23 out of the total 55 correlations also change sign between the two periods. It is thus no surprise that we obtain such a lack of fit when testing the optimal hedge ratios from the estimation period on the test period, since there has been a radical change in correlations between these two periods. Taking for example the case of an initial short position in DKK under the two-currency approach[28], the best hedging currency changes from GBP to CHF when applying the optimal hedge ratios from the estimation period on the test period data. With its optimal hedge ratio from the estimation period, the GBP hedge yields a CVaR increase of 19.82% in the test period. Between these two periods, the correlation between DKK and GBP changes from 0.72 to -0.10, while the correlation between DKK and CHF changes from -0.03 to 0.71. This clearly shows one of the cases where the given hedge is completely worthless when applied on a new period, due to an extensive change in correlations including a change of sign.

One further interesting observation that is worth commenting is the poor performance of the five- and 11-currency hedged portfolios in the test period, when comparing them to the two-currency hedged portfolios. In terms of risk reduction, the two-currency hedged portfolios always perform better, or just slightly worse, than their respective five- and 11-currency hedged portfolios[29]. In the few cases where a two-currency hedged portfolio performs worse than its respective five- or 11-currency hedged portfolio, the difference in terms of CVaR change is always much lower than it was in the estimation period[30]. Thus, it seems as if the decision to add more currencies to the hedged portfolio in the estimation period punishes itself when actually implementing the given hedging strategy, perhaps showing signs of some over-fitting issue. If a risk manager would still want to implement this hedging strategy (despite its lack of power and accuracy and general poor performance), it would therefore probably be more beneficial to stay within the simplest setting of the strategy, only applying a two-currency approach.

The fact that we get poor results when applying the optimal hedge ratios on the test period strengthens our choice of an out-of-sample approach, as this has allowed us to observe the lack

[28] See Table 3.2 in Appendix 3 and Table 6.2 in Appendix 6.
[29] See Table 5.2.1, Table 5.2.2 and Table 5.2.3 in "Results".
[30] See Table 5.1.1 and Table 5.2.1 in "Results".

of sustainability over time of the investigated hedging strategy and discuss on its plausible causes.

6.4. Comparison with previous research

As this thesis is mainly based on the work of Álvarez-Díez and al. (2015), a remark on the differences between our results and conclusions seems appropriate. Comparing our results with theirs, we get similar CVaR reductions for the two-currency hedged portfolios and the 11-currency hedged portfolios. The authors only covered these two types of hedged portfolios, so no comparison can be made with our five-currency hedged portfolio. In addition, they do not apply an out-of-sample approach in their analysis, making it hard to compare anything else than the CVaR reductions in the estimation period in terms of results.

Overall, Álvarez-Díez and al. (2015) seem to have a rather positive view on the multi-currency cross-hedging strategy, highlighting their results in terms of CVaR reduction without providing any critique of the method. This is the main difference between their analysis and ours, as we have a more skeptical view on the results obtained. However, this is also in great part due to the fact that they do not apply an out-of-sample approach. Even though we are not entirely satisfied with the results of the investigation on the estimation period in terms of risk reduction, our greatest criticism of the multi-currency cross-hedging strategy lies in the fact that it lacks power and accuracy when applied to future exposures. Our application of an out-of-sample approach also allowed us to draw the conclusion that the natural effect of multi-currency cross-hedging is actually very dependent on a stability in correlations over time, which Álvarez-Díez and al. (2015) were not able to discern through their in-sample analysis.

6.5. Limitations

In the procedure of obtaining our results, we have encountered some problems in the minimization of CVaR, probably related to the difficulty of the optimization algorithm to find a global minimum. By investigating the results obtained when minimizing CVaR in MatLab, we have on some occasions discovered that by changing the optimal hedge ratios provided by the program, CVaR could actually be reduced further. This finding suggests the existence of local minima that would complicate the given procedure for any optimization algorithm. Note that we here only talk about very minor changes to the CVaR reduction and nothing that invalidates the results obtained and presented in this thesis. Through the discussion of this limitation, we simply want to highlight the complexity of the given optimization procedure.

6.6. Suggestions for future research

In the following part, we present some suggestions for future research within the topic of multi-currency cross-hedging. Since this thesis focuses on the scenario where we do not want to use derivative instrument, it would for example be interesting to analyze the power of multi-currency cross-hedging on a scenario where derivative instruments (especially forward markets) are not easily available, e.g., on emerging market currencies. Another interesting subject, staying within the topic of multi-currency cross-hedging, would be to investigate it through the use of another risk measure than CVaR. As mentioned in the previous section, we have experienced that CVaR optimization is complex and that it takes much time and many computations to find optimal hedge ratios. The optimization problem in itself, and what risk measure to base it on, could therefore be a relevant topic for future research.

A further possible topic of interest could be to, instead of testing the multi-currency cross-hedging method as we do, try to find the optimal hedging currency of another given currency. In such a research, a worldwide sample of currencies could be investigated, leading to a conclusion of the type that for a short position in USD the best hedging currency is for example always the New Zealand dollar, regardless of settings and time horizons. As mentioned previously, we believe that the lack of power and accuracy of multi-currency cross-hedging is due to an instability of correlations over time. Such a study on the optimal hedging currency of a given currency would therefore most probably be related to the problem of unstable correlations in some way, and seek to investigate the stability of correlations between currencies for the purpose of hedging that is sustainable over time.

7. CONCLUSION

This thesis has sought to provide insights on whether the natural effect of multi-currency cross-hedging can be considered an appropriate hedging alternative when one wants to hedge a currency exposure without the use of derivative instruments. This could be a suitable strategy for small- and medium-sized enterprises that may not have the resources or competences necessary for hedging through derivatives. The analysis has been performed through the minimization of CVaR, yielding optimal hedge ratios that define the composition of the hedged portfolio that should be implemented to obtain the greatest risk reduction for a given initial short position (corresponding to the payment of an invoice denominated in a foreign currency). The analysis has been performed using an out-of-sample approach, where we test the power and accuracy of the investigated hedging strategy when applied to new currency exposures.

The results of the estimation period show that a two-currency hedged portfolio approach can reduce the CVaR of the unhedged initial short position by on average 11.17%. When increasing the size of the hedged portfolio to five currencies, CVaR is on average reduced by 30.59%. Increasing it to 11 currencies, CVaR is on average reduced by 40.10%. Even though all these approaches lead to a reduction of the initial CVaR (as should be expected), we are of the opinion that the CVaR reductions are in general not sufficiently large to make this hedging strategy worthwhile, especially when considering the complexity of related computations and the costs that will be incurred in practice but that we have not taken into account in our analysis.

Furthermore, when we apply the out-of-sample approach, testing the optimal hedge ratios obtained in the estimation period on the test period, the results get even worse. Most hedged portfolios still yield a reduction of CVaR, however generally being smaller than those obtained in the estimation period, while some even increase CVaR as compared to its initial value. In addition, in the two-currency hedged portfolio approach, all but one best hedging currency change between the samples. The poor results in terms of risk reduction and the changes in best hedging currencies lead us to conclude that the optimal hedge ratios are not sustainable over time. This in turn seems to be due to the instability of the correlation between currencies over time, meaning that the investigated hedging strategy will most certainly always perform poorly as long as the correlations are not stable over the periods that one seeks to apply this strategy on.

Overall, we therefore conclude that the natural effect of multi-currency cross-hedging is not a suitable alternative to traditional hedging strategies. The strategy performs poorly in terms of risk reduction even when simply estimating the optimal hedge ratios, which makes it no surprise that the results get even worse when applying these hedge ratios on future exposures, especially in the case where correlations are not stable over time. Thus, the strategy seems to lack power and accuracy, which makes it quite worthless for the purpose of hedging future exposures. Even if one could discern a pattern of stable correlations between some currencies in the past, there is no guarantee that they will also be stable in the future. The whole purpose of a hedging strategy is that it can be applied on future exposures. Since the power and accuracy of multi-currency cross-hedging seems to rely heavily on the stability of correlations over the periods on which we estimate hedged portfolios and apply them on (for actual hedging), we find it difficult to see how such a strategy could ever be considered as an alternative to traditional, much safer and more reliable, hedging strategies.

Our final conclusion, providing an answer to the research question of this thesis, is that multi-currency cross-hedging excluding the use of derivative instruments is not a suitable hedging strategy for small- and medium-sized enterprises. First of all, this strategy performs poorly as a whole, which does not make it beneficial for any type of company, independent of size. Secondly, it must be kept in mind that this thesis does not account for the costs arising from the implementation of such a hedging strategy. A well-functioning strategy could have been worth the investment, but with the results obtained, this is doubtful. Lastly, the purpose of focusing on small- and medium-sized enterprises was to try to find a suitable hedging strategy for such companies, given their often disadvantaged conditions in terms of risk management. In addition to the hedging strategy performing poorly, the optimization procedure needed for its implementation is complex and time-consuming, which is exactly the opposite of what would be optimal for small- and medium-sized enterprises. The only somewhat positive feature of multi-currency cross-hedging from this perspective is that its application on future exposures seems to perform best when kept in its most simple setting, hedging a given exposure with only one other foreign currency exposure.

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APPENDICES

APPENDIX 1

Figure 1.1. Histogram and summary statistics for losses corresponding to an initial short position in USD.

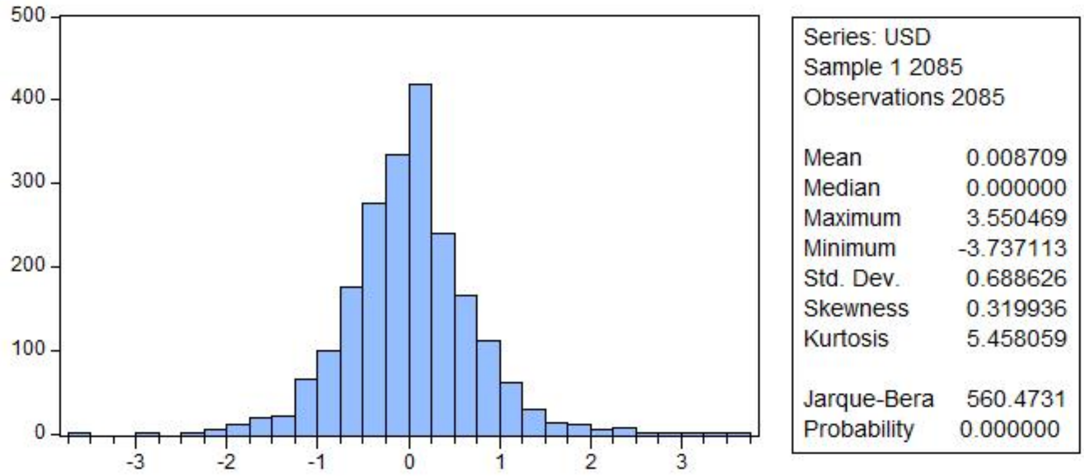


Figure 1.2. Histogram and summary statistics for losses corresponding to an initial short position in DKK.

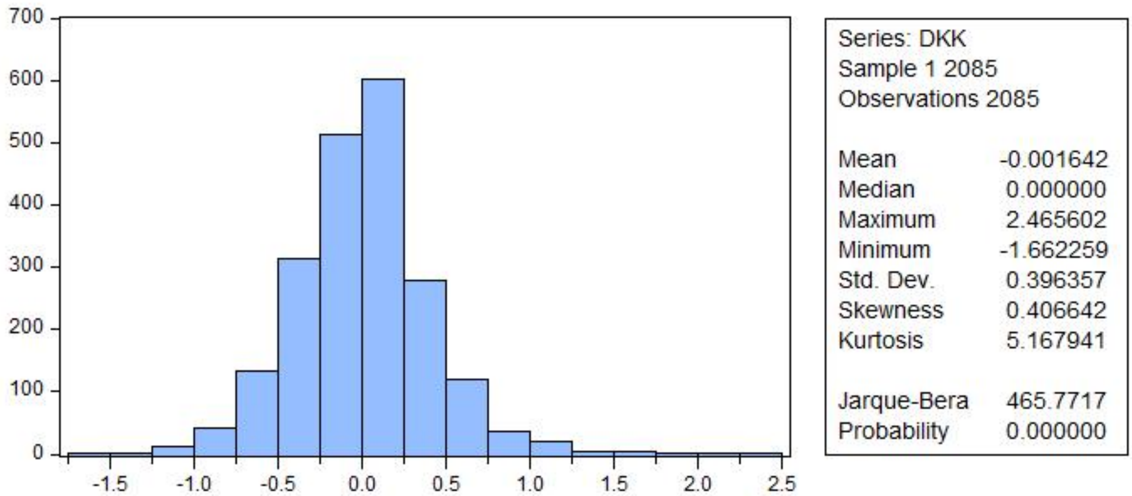


Figure 1.3. Histogram and summary statistics for losses corresponding to an initial short position in EUR.

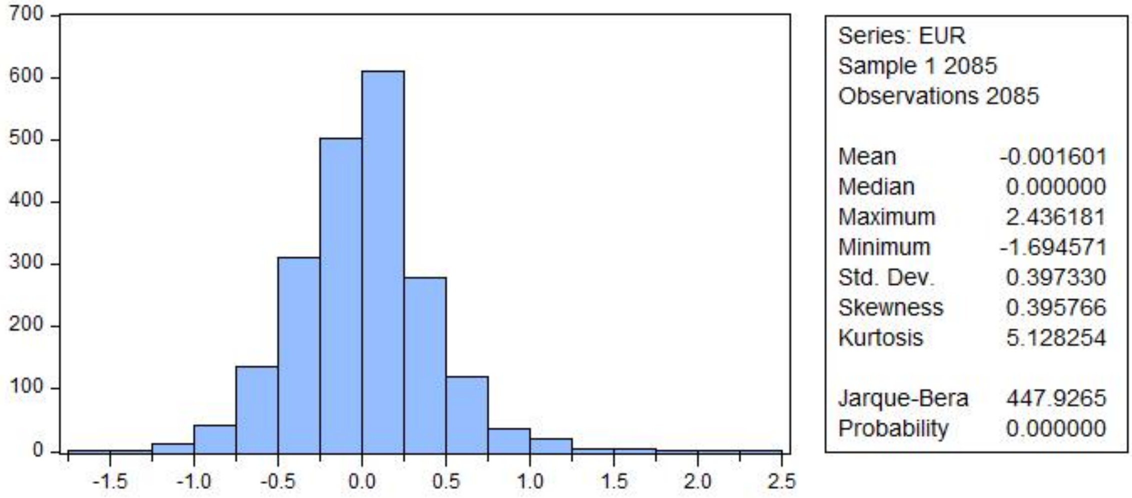


Figure 1.4. Histogram and summary statistics for losses corresponding to an initial short position in GBP.

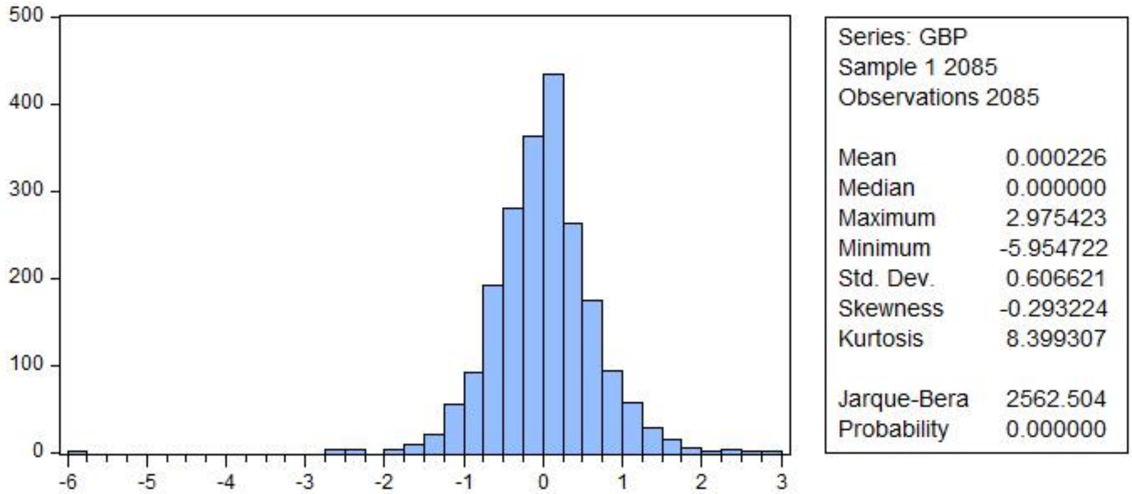


Figure 1.5. Histogram and summary statistics for losses corresponding to an initial short position in NOK.

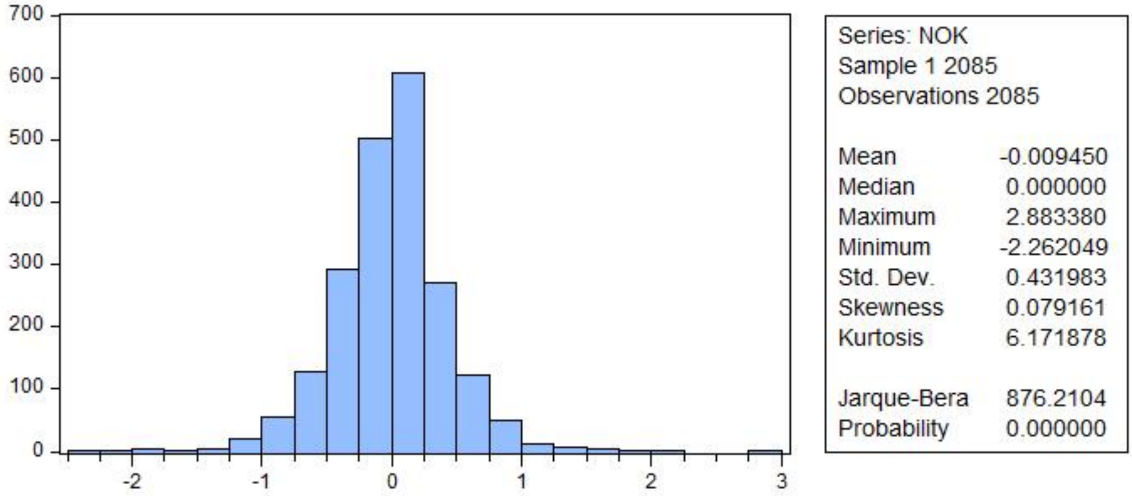


Figure 1.6. Histogram and summary statistics for losses corresponding to an initial short position in AUD.

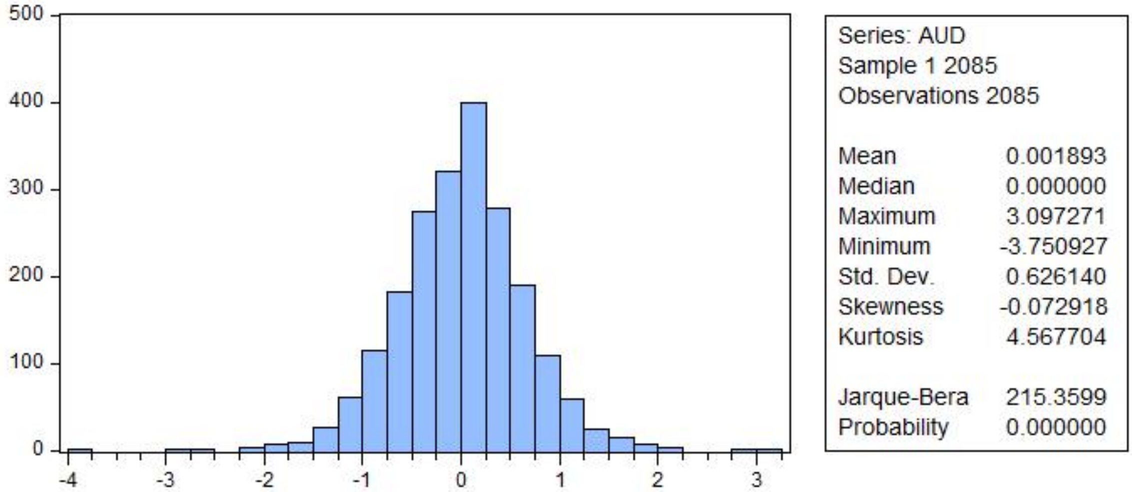


Figure 1.7. Histogram and summary statistics for losses corresponding to an initial short position in JPY.

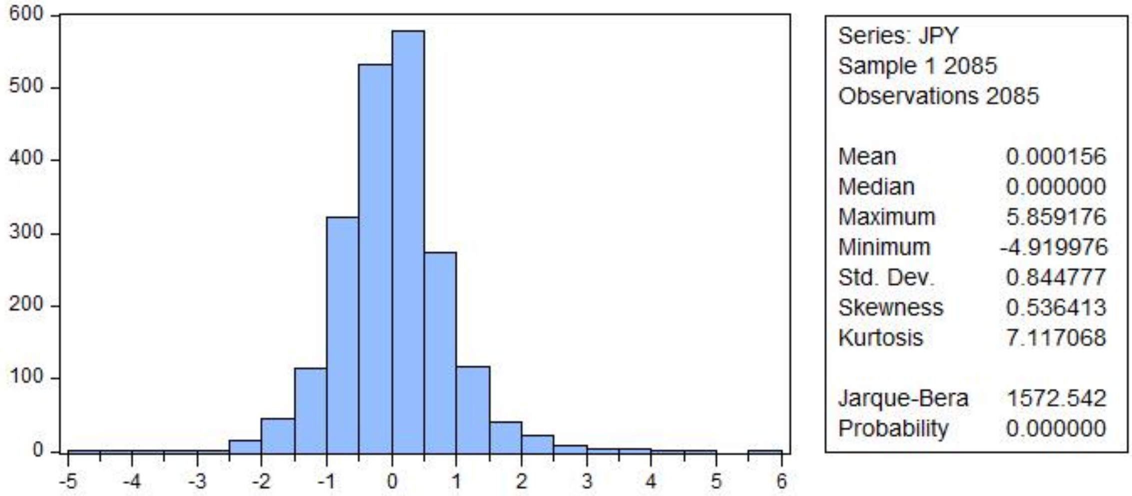


Figure 1.8. Histogram and summary statistics for losses corresponding to an initial short position in CAD.

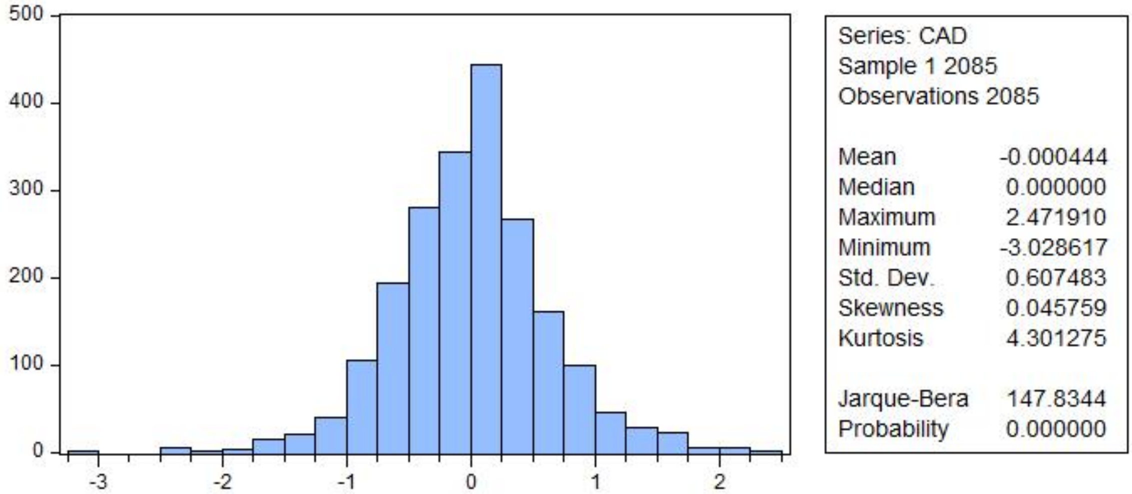


Figure 1.9. Histogram and summary statistics for losses corresponding to an initial short position in CHF.

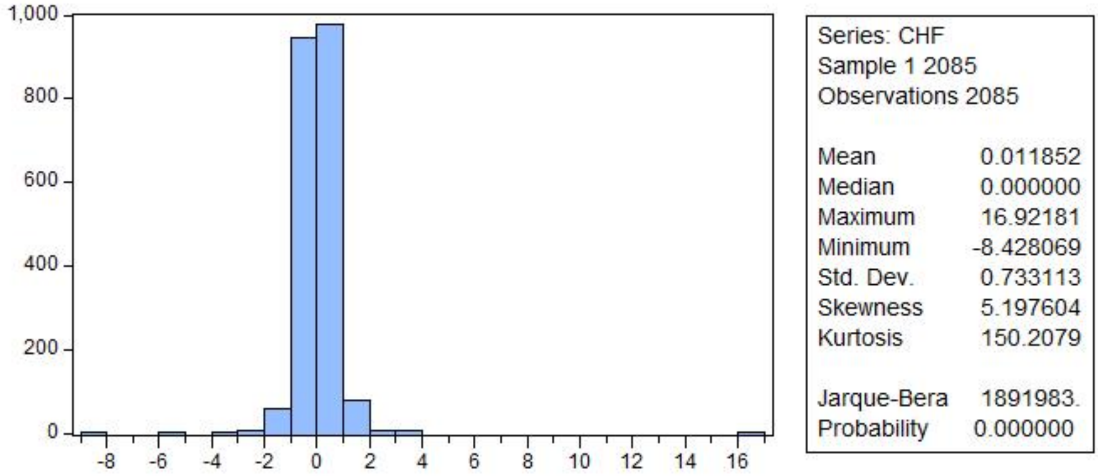


Figure 1.10. Histogram and summary statistics for losses corresponding to an initial short position in CNY.

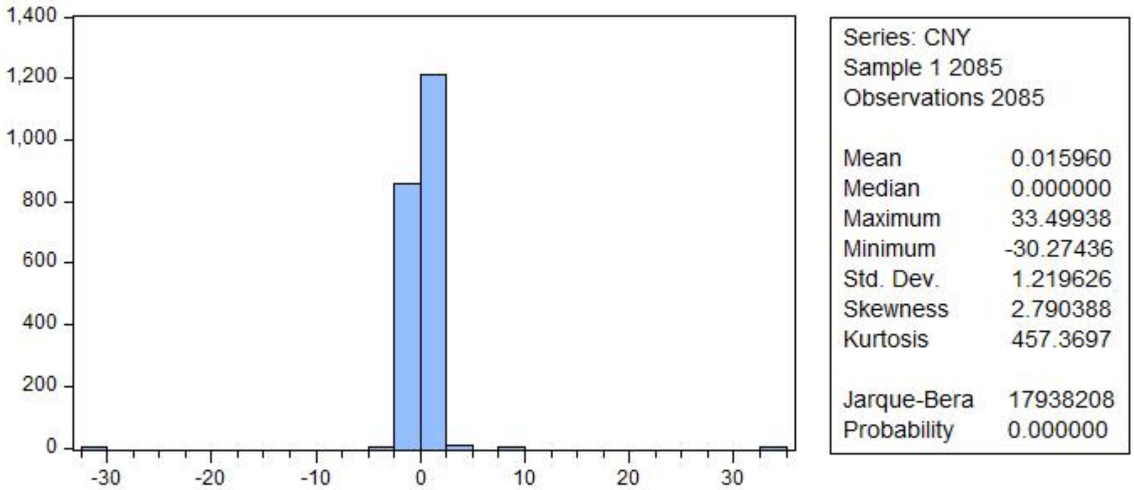
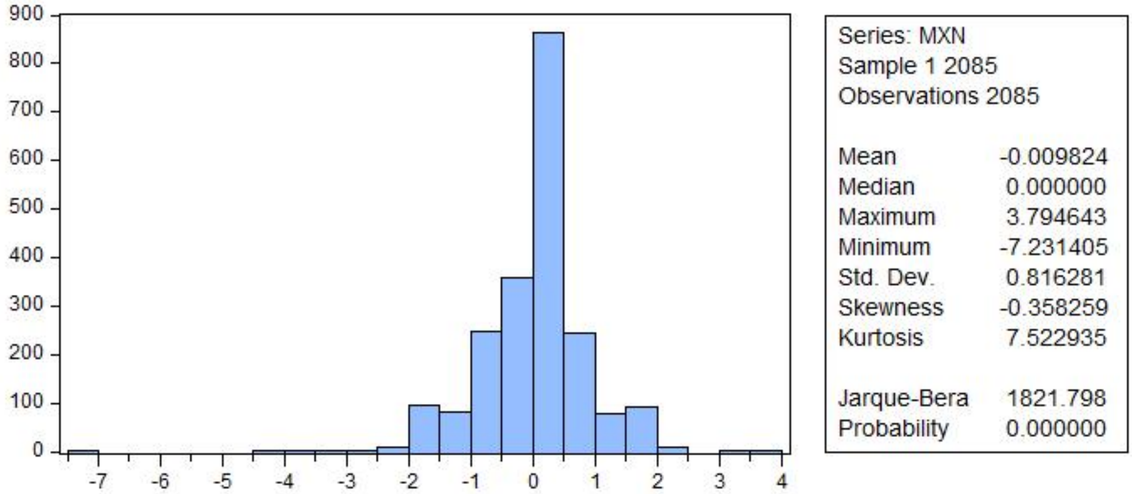


Figure 1.11. Histogram and summary statistics for losses corresponding to an initial short position in MXN.



APPENDIX 2

Table 2.1. Correlation matrix for spot rates over the estimation period 2010/01/01 to 2013/12/31.

	USD	AUD	CHF	CNY	DKK	EUR	GBP	MXN	JPY	CAD	NOK
USD	**1**	0,18	-0,19	0,70	0,65	0,64	0,86	0,69	0,49	0,87	0,62
AUD	0,18	**1**	0,29	0,09	-0,06	-0,07	0,22	0,06	0,75	0,49	0,52
CHF	-0,19	0,29	**1**	0,01	-0,03	-0,04	-0,01	-0,32	0,31	-0,09	0,06
CNY	0,70	0,09	0,01	**1**	0,13	0,12	0,53	0,14	0,21	0,47	0,19
DKK	0,65	-0,06	-0,03	0,13	**1**	1,00	0,72	0,68	0,34	0,59	0,69
EUR	0,64	-0,07	-0,04	0,12	1,00	**1**	0,71	0,68	0,32	0,58	0,68
GBP	0,86	0,22	-0,01	0,53	0,72	0,71	**1**	0,58	0,59	0,81	0,65
MXN	0,69	0,06	-0,32	0,14	0,68	0,68	0,58	**1**	0,23	0,75	0,57
JPY	0,49	0,75	0,31	0,21	0,34	0,32	0,59	0,23	**1**	0,69	0,64
CAD	0,87	0,49	-0,09	0,47	0,59	0,58	0,81	0,75	0,69	**1**	0,78
NOK	0,62	0,52	0,06	0,19	0,69	0,68	0,65	0,57	0,64	0,78	**1**

Table 2.2. Correlation matrix for spot rates over the test period 2014/01/01 to 2017/12/29.

	USD	AUD	CHF	CNY	DKK	EUR	GBP	MXN	JPY	CAD	NOK
USD	**1**	0,57	0,95	0,92	0,75	0,76	0,29	-0,32	0,77	0,69	-0,43
AUD	0,57	**1**	0,58	0,47	0,64	0,63	-0,07	-0,04	0,58	0,84	0,34
CHF	0,95	0,58	**1**	0,90	0,71	0,71	0,35	-0,21	0,73	0,70	-0,32
CNY	0,92	0,47	0,90	**1**	0,58	0,59	0,60	0,02	0,53	0,73	-0,34
DKK	0,75	0,64	0,71	0,58	**1**	1,00	-0,10	-0,45	0,73	0,63	-0,23
EUR	0,76	0,63	0,71	0,59	1,00	**1**	-0,07	-0,44	0,72	0,63	-0,23
GBP	0,29	-0,07	0,35	0,60	-0,10	-0,07	**1**	0,61	-0,21	0,27	-0,09
MXN	-0,32	-0,04	-0,21	0,02	-0,45	-0,44	0,61	**1**	-0,61	0,10	0,51
JPY	0,77	0,58	0,73	0,53	0,73	0,72	-0,21	-0,61	**1**	0,48	-0,31
CAD	0,69	0,84	0,70	0,73	0,63	0,63	0,27	0,10	0,48	**1**	0,19
NOK	-0,43	0,34	-0,32	-0,34	-0,23	-0,23	-0,09	0,51	-0,31	0,19	**1**

APPENDIX 3

Estimation period results.

Table 3.1. Two-currency portfolio for USD hedging.

Short	Hedge	Weight	VaR	CVaR	CVaR change
USD	DKK	0,2275	2,2405	2,5393	-8,38%
USD	NOK	0,0363	2,4146	2,7625	-0,33%
USD	EUR	0,5620	2,0097	2,2641	-18,31%
USD	GBP	0,6796	1,5262	1,7518	-36,79%
USD	AUD	0,1630	2,3078	2,6458	-4,54%
USD	CAD	0,2861	2,0713	2,4084	-13,10%
USD	JPY	0,3632	1,5687	1,8354	-33,78%
USD	MXN	0,1910	2,1430	2,5854	-6,72%
USD	CHF	0,3270	2,0279	2,2824	-17,65%
USD	CNY	0,0413	2,3072	2,6584	-4,08%

Table 3.2. Two-currency portfolio for DKK hedging.

Short	Hedge	Weight	VaR	CVaR	CVaR change
DKK	USD	0,2188	1,0278	1,3096	-19,63%
DKK	NOK	0,1036	1,2397	1,6083	-1,30%
DKK	GBP	0,4931	1,0026	1,1810	-27,52%
DKK	AUD	0,1133	1,2754	1,5497	-4,89%
DKK	CAD	0,2849	1,1496	1,4709	-9,73%
DKK	JPY	0,1878	1,0494	1,3339	-18,14%
DKK	MXN	0,0975	1,2122	1,5831	-2,84%
DKK	CHF	0,2079	0,9914	1,3180	-19,11%
DKK	CNY	0,0334	1,2391	1,5674	-3,80%

Table 3.3. Two-currency portfolio for EUR hedging.

Short	Hedge	Weight	VaR	CVaR	CVaR change
EUR	USD	0,2163	1,0358	1,3142	-19,24%
EUR	NOK	0,1024	1,2257	1,6125	-0,92%
EUR	GBP	0,1939	1,1251	1,3996	-14,00%
EUR	AUD	0,1166	1,2455	1,5436	-5,15%
EUR	CAD	0,1316	1,2380	1,5182	-6,71%
EUR	JPY	0,1878	1,0462	1,3336	-18,05%
EUR	MXN	0,1745	1,2471	1,5688	-3,60%
EUR	CHF	0,1977	0,9556	1,3225	-18,73%
EUR	CNY	0,0349	1,2290	1,5671	-3,70%

Table 3.4. Two-currency portfolio for GBP hedging.

Short	Hedge	Weight	VaR	CVaR	CVaR change
GBP	USD	0,3067	1,1731	1,3716	-32,86%
GBP	DKK	0,2081	1,5048	1,8292	-10,46%
GBP	NOK	0,0562	1,6419	2,0269	-0,79%
GBP	EUR	0,2046	1,5058	1,8349	-10,18%
GBP	AUD	0,1858	1,4638	1,8815	-7,90%
GBP	CAD	0,6872	1,2312	1,4350	-29,76%
GBP	JPY	0,3695	1,1357	1,4212	-30,43%
GBP	MXN	0,1973	1,4965	1,8557	-9,17%
GBP	CHF	0,3246	1,3597	1,6400	-19,72%
GBP	CNY	0,0488	1,6082	1,9232	-5,86%

Table 3.5. Two-currency portfolio for NOK hedging.

Short	Hedge	Weight	VaR	CVaR	CVaR change
NOK	USD	0,2069	0,8576	1,0727	-6,27%
NOK	DKK	0,2016	0,8001	1,0950	-4,32%
NOK	EUR	0,2510	0,7907	1,0932	-4,48%
NOK	GBP	0,0672	0,8830	1,1227	-1,91%
NOK	AUD	0,1063	0,8571	1,0956	-4,27%
NOK	CAD	0,1146	0,8561	1,0797	-5,66%
NOK	JPY	0,0670	0,8907	1,1259	-1,62%
NOK	MXN	0,1585	0,8941	1,0922	-4,57%
NOK	CHF	0,0039	0,8981	1,1426	-0,16%
NOK	CNY	0,0324	0,8862	1,0897	-4,79%

APPENDIX 4

Estimation period results.

Table 4.1. Minimum CVaR 5-currency (central currencies) portfolio for USD hedging (short position in USD).

Hedging currency	Weight
DKK	0,3449
NOK	−0,0027
EUR	0,3115
GBP	0,5692

VaR	1,3678
CVaR	1,5255
CVaR change	-44,96%

Table 4.2. Minimum CVaR 4-currency (central currencies) portfolio for DKK hedging (short position in DKK).

Hedging currency	Weight
USD	0,2319
GBP	0,3311
NOK	-0,0329

VaR	0,8882
CVaR	1,0943
CVaR change	-32,84%

Table 4.3. Minimum CVaR 4-currency (central currencies) portfolio for EUR hedging (short position in EUR).

Hedging currency	Weight
USD	0,1907
GBP	0,1814
NOK	-0,0216

VaR	0,9623
CVaR	1,1961
CVaR change	-26,50%

Table 4.4. Minimum CVaR 5-currency (central currencies) portfolio for GBP hedging (short position in GBP).

Hedging currency	Weight
USD	0,3065
DKK	0,1461
EUR	0,1453
NOK	0,0326

VaR	1,0561
CVaR	1,1851
CVaR change	-41,99%

Table 4.5. Minimum CVaR 5-currency (central currencies) portfolio for NOK hedging (short position in NOK).

Hedging currency	Weight
USD	0,1487
DKK	0,0594
EUR	0,0427
GBP	0,0102

VaR	0,8000
CVaR	1,0684
CVaR change	-6,64%

APPENDIX 5

Estimation period results.

Table 5.1. Minimum CVaR 11-currency portfolio for USD hedging (short position in USD).

Hedging currency	Weight
AUD	0,0360
CHF	0,0912
CNY	0,0260
DKK	0,0962
EUR	0,0921
GBP	0,2042
MXN	0,0806
JPY	0,2358
CAD	0,1913
NOK	0,0353

VaR	0,9498
CVaR	1,1213
CVaR reduction	-59,54%

Table 5.2. Minimum CVaR 10-currency portfolio for DKK hedging (short position in DKK).

Hedging currency	Weight
USD	0,1457
AUD	-0,0748
CHF	0,1205
CNY	0,022
GBP	0,3293
MXN	0,0808
JPY	0,1368
CAD	-0,1934
NOK	-0,0023

VaR	0,8702
CVaR	0,9811
CVaR reduction	-39,79%

Table 5.3. Minimum CVaR 10-currency portfolio for EUR hedging (short position in EUR).

Hedging currency	Weight
USD	0,0903
AUD	-0,0075
CHF	0,1325
CNY	0,0279
GBP	0,1247
MXN	0,0103
JPY	0,0953
CAD	0,0047
NOK	-0,0439

VaR	0,9001
CVaR	1,0660
CVaR reduction	-34,49%

Table 5.4. Minimum CVaR 11-currency portfolio for GBP hedging (short position in GBP).

Hedging currency	Weight
USD	0,3306
AUD	-0,0637
CHF	0,0629
CNY	0,0127
DKK	0,0615
EUR	0,0652
MXN	0,0002
JPY	0,0357
CAD	0,1969
NOK	-0,0716

VaR	0,9510
CVaR	1,0832
CVaR reduction	-46,98%

Table 5.5 Minimum CVaR 11-currency portfolio for NOK hedging (short position in NOK).

Hedging currency	Weight
USD	0,0701
AUD	0,1625
CHF	-0,0832
CNY	0,0081
DKK	0,1007
EUR	0,0829
GBP	0,0518
MXN	0,0778
JPY	0,0124
CAD	0,0484

VaR	0,7637
CVaR	0,9190
CVaR reduction	-19,70%

APPENDIX 6

Test period results.

Table 6.1. Optimal hedge ratios applied on 2014-2017 data for two-currency portfolio USD hedging (short position in USD).

Hedging currency	VaR	CVaR	CVaR change
DKK	1,2819	1,7073	-7,96%
NOK	1,3897	1,7701	-4,58%
EUR	1,2564	1,7210	-7,22%
GBP	1,2484	2,0147	+8,61%
AUD	1,2900	1,6237	-12,47%
CAD	1,1694	1,5170	-18,22%
JPY	1,2177	1,3742	-25,92%
MXN	1,2550	1,6292	-12,17%
CHF	1,1955	1,5590	-15,96%
CNY	1,3259	1,7124	-7,69%

Table 6.2. Optimal hedge ratios applied on 2014-2017 data for two-currency portfolio DKK hedging (short position in DKK).

Hedging currency	VaR	CVaR	CVaR change
USD	0,7497	0,9338	-12,19%
NOK	0,8344	1,0419	-2,02%
GBP	0,8335	1,2742	+19,82%
AUD	0,8360	0,9998	-5,98%
CAD	0,7864	0,9841	-7,46%
JPY	0,7434	0,9190	-13,58%
MXN	0,8586	1,0207	-4,02%
CHF	0,7541	0,8705	-18,14%
CNY	0,8566	1,0324	-2,92%

Table 6.3. Optimal hedge ratios applied on 2014-2017 data for two-currency portfolio EUR hedging (short position in EUR).

Hedging currency	VaR	CVaR	CVaR change
USD	0,7623	0,9386	-11,44%
NOK	0,8179	1,0422	-1,67%
GBP	0,8128	1,0505	-0,88%
AUD	0,8407	1,0005	-5,60%
CAD	0,8110	0,9785	-7,67%
JPY	0,7410	0,9192	-13,27%
MXN	0,8888	1,0909	+2,93%
CHF	0,7276	0,8787	-17,09%
CNY	0,8510	1,0300	-2,82%

Table 6.4. Optimal hedge ratios applied on 2014-2017 data for two-currency portfolio GBP hedging (short position in GBP).

Hedging currency	VaR	CVaR	CVaR change
USD	1,3259	1,5727	-6,99%
DKK	1,3569	1,6447	-2,73%
NOK	1,4645	1,6867	-0,25%
EUR	1,3557	1,6479	-2,54%
AUD	1,3099	1,6637	-1,61%
CAD	1,3814	1,6776	-0,79%
JPY	1,4365	1,7043	+0,79%
MXN	1,2810	1,6673	-1,40%
CHF	1,3000	1,6236	-3,98%
CNY	1,4172	1,6606	-1,79%

Table 6.5. Optimal hedge ratios applied on 2014-2017 data for two-currency portfolio NOK hedging (short position in NOK).

Hedging currency	VaR	CVaR	CVaR change
USD	1,2996	1,6723	-2,66%
DKK	1,3083	1,6873	-1,79%
EUR	1,2837	1,6852	-1,91%
GBP	1,2762	1,7047	-0,77%
AUD	1,2734	1,7045	-0,79%
CAD	1,2034	1,6558	-3,62%
JPY	1,3268	1,7231	+0,30%
MXN	1,2175	1,6271	-5,29%
CHF	1,3172	1,7173	-0,04%
CNY	1,3086	1,7129	-0,30%

APPENDIX 7

Test period results.

Table 7.1. Optimal hedge ratios applied on 2014-2017 data for 5-currency portfolio (central currencies) USD hedging (short position in USD).

VaR	1,2389
CVaR	1,8648
CVaR change	+4,94%

Table 7.2. Optimal hedge ratios applied on 2014-2017 data for 4-currency portfolio (central currencies) DKK hedging (short position in DKK).

VaR	0,7817
CVaR	1,0777
CVaR change	+1,35%

Table 7.3. Optimal hedge ratios applied on 2014-2017 data for 4-currency portfolio (central currencies) EUR hedging (short position in EUR).

VaR	0,7102
CVaR	0,9408
CVaR change	-11,23%

Table 7.4. Optimal hedge ratios applied on 2014-2017 data for 5-currency portfolio (central currencies) GBP hedging (short position in GBP).

VaR	1,2596
CVaR	1,5310
CVaR change	-9,46%

Table 7.5. Optimal hedge ratios applied on 2014-2017 data for 5-currency portfolio (central currencies) NOK hedging (short position in NOK).

VaR	1,2758
CVaR	1,6686
CVaR change	-2,87%

APPENDIX 8

Test period results.

Table 8.1. Optimal hedge ratios applied on 2014-2017 data for 11-currency portfolio USD hedging (short position in USD).

VaR	0,9004
CVaR	1,2212
CVaR change	-31,28%

Table 8.2. Optimal hedge ratios applied on 2014-2017 data for 10-currency portfolio DKK hedging (short position in DKK).

VaR	0,7505
CVaR	0,9534
CVaR change	-10,34%

Table 8.3. Optimal hedge ratios applied on 2014-2017 data for 10-currency portfolio EUR hedging (short position in EUR).

VaR	0,6384
CVaR	0,8060
CVaR change	-23,95%

Table 8.4. Optimal hedge ratios applied on 2014-2017 data for 11-currency portfolio GBP hedging (short position in GBP).

VaR	1,2759
CVaR	1,5547
CVaR change	-8,06%

Table 8.5. Optimal hedge ratios applied on 2014-2017 data for 11-currency portfolio NOK hedging (short position in NOK).

VaR	1,1985
CVaR	1,6945
CVaR change	-1,37%