Anticipated Events' Impact on FX Options' Implied Volatility

Lund Institute of Technology

LUND INSTITUTE OF TECHNOLOGY
Lund University

Frej Håkansson, Björn Nilsson

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Abstract

Understanding events' impact on financial instruments are crucial for the participants in the financial markets. Here we propose an approach to model an anticipated event's impact on the prices of FX options, represented in implied volatility. The model is implemented for anticipated event's with both Black and Scholes and SABR as the assumed underlying dynamic. The model generates an implied volatility frown, for both dynamics. Hence it contributes to the area regarding concave implied volatility functions, which at the time of writing has little published literature related to it.

Keywords: Volatility frown, implied volatility, jump model, anticipated event, SABR, FX Options

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1 Introduction

Events impact on financial markets has increased, and it is now essential to be able to understand the consequences an event may bring to the markets. This thesis model future events, taking place at a known time but with a stochastic outcome, and look further into how such events impact foreign exchange options with maturities after said event. A mixture model based on Black and Scholes dynamics is introduced, as well is a model with SABR dynamic. The behaviour of these models for different assumptions of the jump is then examined and discussed.

1.1 A Brief Introduction to the Foreign Exchange Market

1.1.1 Market Overview

The foreign exchange (FX) market started as an over-the-counter (OTC) market for banks, financial institutions and large international companies to hedge themselves against currency exposure [15]. The FX market is the most liquid and largest market which had an average daily volume in 2016 of about 5 trillion USD, where USD is the most traded currency with almost 90% of total turnover [1]. Similar to the FX spot market the FX options market has been considered as an interbank market, but at recent years this has changed and now individuals can participate in speculation or hedge positions in currencies [15].

1.1.2 FX Contracts and Terminology

Contracts There are several contracts that can be bought and sold in the FX market, a few examples are presented below.

A *spot* transaction is an agreement between two parties to exchange N_d units of domestic currency for N_f units of foreign currency, connected by the spot rate S_0. Often the transaction take place two business days after the spot exchange has been agreed [10, p. 5-7].

A *forward* is a transaction taking place in the future, the rate and volume are set when the parties enter the forward agreement at $t = 0$. At the maturity date the transaction will occur and both parties are obliged to follow through [6]. The relationship between the forward price, F_0, and the spot price, S_0, is given by the well-known interest parity relationship

$$F_0 = S_0 e^{(r^d - r^f)T}.$$

Here T denotes time to maturity and r^d and r^f the domestic and foreign risk-free rate respectively [18, p. 121].

An *option* gives the owner the right, but not the obligation to buy or sell the underlying asset, the writer of the option need to provide the underlying asset for the predetermined price. There are numerous variations of options, the simplest one is the European option where it has a definitive expiration structure, payout structure and pay amount. In the American option the expiration is set in beforehand but the buyer can exercise the option at any time up until the maturity date. There are also options that are called exotic i.e. they do not have an definitive structure where the contracts may have a change in one or all of the above features of a vanilla option. A few examples of exotic options are Asian-, Chooser-, Barrier- and Digital options [6].

FX Terminology The exchange rate is the ratio between a currency pair, for example GBPUSD (Great Britain Pound/US Dollar). The notation for currency pair is the following: the first three letters is the foreign currency and the three following letters is the domestic currency. Generally we say how many parts of the domestic currency one need to buy one part of the foreign currency. To clarify, if the GBPUSD spot is at 1.414 [14], that means that one need to pay 1.414 USD to buy one GBP.

Depreciation of a currency is a fall in value of the currency in a floating exchange system. If the GBP were to depreciate, the GBPUSD exchange rate would decrease, whereas if the USD were to depreciation the exchange rate would increase.

Currency appreciation is the opposite of depreciation, i.e. an increase in the value of one currency in terms of another. If the GBP would appreciate the GBPUSD exchange rate would increase, and vice versa if the USD would appreciate.

Option Terminology Below are some typical option terminologies that are frequently used.

The *premium* of an option is the price of buying the option.

The *maturity* is the date at which the option expires.

The *strike price* K is the predetermined price of the underlying asset, hence the option will not be exercised if the spot at maturity is below the strike price.

A call option is said to be *in-the-money (ITM)* at time t if $S_t > K$, and *out-the-money (OTM)* if $S_t < K$. For put options the inequalities are reversed. If $S_t = K$ the option is said to be *at-the-money (ATM)*. ITM, OTM and ATM is referred to as the *moneyness* of the option. [6]. When K is close to S_t an option is said to be *near-the-money* (NTM) [2].

1.2 Our Contribution

There are a limited amount of academic papers modelling anticipated events in the strike space. We contribute to the field by defining and implementing such models. Further the models generates a frown which is an interesting behaviour, and there is a scarce amount of research on models with said behaviour, as can be seen in Section 2.2.

1.3 Structure of the Thesis

The following Section goes through some of the previous work regarding volatility frown and event modelling. Section 3 gives a thorough introduction to the theory used in Section 4 where the methods and models are explained. Section 5 goes through the results obtained and the final conclusions are discussed in Section 6.

2 Literature Review

This section provides an overview of previous work and articles related to the thesis.

2.1 Event Modelling

The article 'Implied Distributions from GBPUSD Risk-Reversals and Implication for Brexit Scenarios' written by Ian J. Clark and Saeed Amen [11] relates to known events and looks at option prices and the implied volatility. In the paper they focus on understanding the market expectations for price action around the Brexit referendum date, i.e. an anticipated event with an unknown outcome. The constructed model has a specific probability P_L of a Brexit leave event. They model the exchange rate in the leave scenario with drift and variance μ_L and σ_L respectively, and similarly with μ_R and σ_R in the case of a remain. The price of a European option is then obtained by two, weighted, Black and Scholes prices, one for each scenario. Conclusions were drawn (from poll data and market data) that leaving EU would have a devaluation effect on the GBP currency rate and bring higher volatility than remaining in EU. The market implied volatility is used in order to predict the probability P_L and the possible exchange rate intervals for the different outcomes. A single point estimate for leave was made between 24 Feb 2016 - 22 June 2016 which had a mean of 1.3705 with a minimum of 1.2900 and a maximum of 1.4536. The post-referendum spot was 1.3622, well within the confidence interval and remarkably close to the estimate [11].

Poulsen, Hanke and Weissensteiner combines risk-neutral event probabilities implied from betting quotes with risk-neutral exchange rate densities extracted from currency option prices in [20]. The model's application is to predict exchange rates around events like referendums and elections. The authors use data around the Brexit referendum and the U.S. presidential elections to test the model. The betting quotes can be interpreted and translated to risk-neutral event probabilities, these are used as weights for risk-neutral densities implied by FX option prices, which are modelled as linear combinations of conditional densities; one for each possible outcome. The authors find that even though the probability of a Brexit fluctuates a lot, the exchange rate in the case of Brexit is relatively stable. The authors interpret this as that the markets were able to correctly separate the probability of Brexit from its potential consequences.

In [3] the authors propose a currency option model (E-model) that handle unscheduled and scheduled announcements. Their approach is to use a mixed jump-diffusion model and adding an indicator function that sets announcements dates in advance. The E-model is compared against MBS (Merton Black and Scholes) and BSAV (Black and Scholes Average event Variance). Where the E-model outperforms MBS significantly is in the pricing of short-term out-of-the-money options where MBS typically underprice. However this model is in time-space and not the strike-space. Hence they do not look further into the smile structure nor the implied volatility surface. In [4] the authors extends the above currency price model to hold for general options in other asset classes aswell.

2.2 Volatility Frowns

There are relatively little published on the topic of implied volatility frowns, i.e. implied volatility curves with a concave curvature. We have found three sources touching upon the subject.

In [16] the authors note that volatility frowns may arise in tranquil periods in the FX market. This is due to the fact that the return distribution has relatively thinner tails than the assumed normal distribution. Black and Scholes will then miss-price the option, giving deep in-the-money and out-of-the-money options relatively higher prices and options at-the-money relatively lower prices. This miss-pricing manifest itself in the implied volatility, causing a volatility frown. The authors use a model based on a generalised Student's t-distribution coupled with time-varying volatility to correct for the effects causing the volatility frown.

Hull mentions in [18, p. 440-441] that a volatility frown may arise when a single large jump is anticipated. For example in the extreme case when there are only two possible outcomes, this can be modelled as a mixture of two log-normal distributions. This will cause a bimodal distribution and the implied volatility will form a frown.

Matthias Thul looks further into binary events in [24] and [25]. The author's model adds a random jump at a specific time point to a stochastic process, to model for example quarterly earnings announcements, monetary policy or elections. He use a specific example, using a Brownian motion as the stochastic process and let the jump follow a normal mixture model, the resulting distribution and implied volatility becomes similar to the one in Hull [18, p. 440-441]. When the jump is symmetric, the implied volatility becomes concave near-the-money which indicates that the corresponding implied density is platykurtic i.e. negative excess kurtosis. The example used by the author illustrates that such a situation is not necessarily a violation of static no-arbitrage conditions.

2.3 Volatility Smiles and Skews

Brigo and Mercurio propose two asset-price models in [7], one log-normal mixture model and an extension of it with an affine transformation. The option prices indicated by these models are derived as linear combinations of Black and Scholes prices. The authors calibrate the model to the plain vanilla market and are then able to price exotic path-dependent claims through Monte Carlo simulation. The calibrated volatilities in the paper have the more common smile shape.

3 Theory

This section presents the theoretical background needed to understand the modelling approach taken in this thesis.

3.1 Present Value

When a future cost or benefit, Π, that occurs at time T is calculated as cash today (t), it is referred to as the present value of Π. This is done by discounting the cash-flow with a the relevant discount rate r,

$$PV(\Pi) = e^{-r(T-t)}\Pi$$

[5, p. 65].

3.2 Risk Neutral Valuation

The arbitrage free price, $\Pi\left(t, g\left(S_T\right)\right)$, of the claim $g\left(S_T\right)$ at time t is given by

$$\Pi\left(t, g\left(S_T\right)\right) = e^{-r(T-t)}\,\mathbb{E}_t^Q\left[g\left(S_T\right)\right], \tag{1}$$

where Q is the risk neutral measure. Hence it is given by taking the expectation of the final payment occurring at T and then discounting it to present value using the discount factor $e^{-r(T-t)}$ [6, p. 102-104].

3.3 Black and Scholes Formula

Black and Scholes model consist of two processes, the bank account B_t and the underlying price process S_t

$$\begin{aligned} dB_t &= rB_t dt \\ dS_t &= \alpha S_t dt + \sigma S_t d\bar{W}_t. \end{aligned}$$

The risk neutral Q-dynamics of S is given by

$$\begin{aligned} dS_u &= rS_u du + \sigma S_u dW_u \\ S_t &= s. \end{aligned} \tag{2}$$

As seen in (1), the solution to above differential equation (2) is needed in order to calculate the expected value of the payoff. This is done by integrating

(2) over $[t,T]$ to receive

$$S_T = se^{\left(r-\frac{\sigma^2}{2}\right)(T-t)+\sigma(W_T-W_t)}$$

where

$$\begin{aligned}
r &= \text{risk-free rate} \\
\sigma &= \text{volatility of returns} \\
W &= \text{Brownian motion} \\
s &= \text{spot price of underlying asset} \\
t &= \text{time of pricing} \\
T &= \text{time of maturity}
\end{aligned}$$

Using equation (1), the value at time t of a European call option is

$$V_t^c = S_t N(d_1) - Ke^{-r(T-t)} N(d_2)$$

and for a put option

$$V_t^p = Ke^{-r(T-t)} N(-d_2) - S_t N(-d_1)$$

where

$$\begin{aligned}
d_1 &= \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right] \\
d_2 &= d_1 - \sigma\sqrt{T-t} \\
K &= \text{strike price} \\
N &= \text{cumulative normal distribution function}
\end{aligned}$$

[6, p.103-105].

3.4 Garman and Kohlhagen's Extension

The Garman and Kohlhagen's formula is an extension of the Black and Scholes model to handle currency options where there are a domestic- and foreign interest rate, denoted r^d and r^f respectively. With this extension, the value of a European currency call option becomes

$$V_t^c = S_t e^{-r^f(T-t)} N(d_1) - Ke^{-r^d(T-t)} N(d_2)$$

and for a put option

$$V_t^p = Ke^{-r^d(T-t)}N(-d_2) - S_t e^{-r^f(T-t)}N(-d_1)$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r^d - r^f + \frac{\sigma^2}{2}\right)(T-t)\right]$$
$$d_2 = d_1 - \sigma\sqrt{T-t}$$

[10, p. 25].

3.5 Implied Volatility

The one parameter in Black and Scholes formula that cannot be observed is the volatility, σ, of the underlying asset, this can be estimated from the historical volatility. But in practice it is the implied volatility that is being used. By definition it is the value of σ that matches the value of an option observed in the market, Π_t, with the one obtained by the Black and Scholes formula

$$\sigma(S_t, K, t, T) = BS^{-1}(\Pi_t; S_t, K, t, T)$$

[18, p. 339].

3.6 Delta

The delta of an option denotes how much the value of the option change as an effect of a change in the spot price of the underlying asset. In other words, the delta is the instantaneous derivative of price with respect to changes in the asset price. Because there are many different ways to quote a price in the FX market, there are also different deltas, and it is important to know which one should be used in each scenario. Since the price used in this thesis is the so called domestic pips price, the delta that will be used is the pips spot delta, presented below.

$$\Delta_{S;pips} = \omega e^{-r^f T} N(\omega d_1)$$

Here ω takes the value 1 for a call option and -1 for a put option.

An interesting note is that a call option can be quoted in both domestic and foreign units. For example the V_t^c in Section 3.4 is the domestic/foreign price valued in domestic currency, this can be transformed into foreign/domestic price valued in foreign currency by the formula

$$\frac{V_t^c}{S_t K},$$

which will effect the delta calculations. Hence depending on how the investor manages and measures risk, the amount of currency needed to delta hedge can be in units of either foreign or domestic currency [10, p. 43-49].

3.7 Risk Reversal

The risk reversal is the difference between the volatilities of a call and put option with the same delta (with opposite signs), defined in (3). It can be interpret as a market parameter corresponding to the smiles skew

$$\sigma_{d-RR} = \sigma_{d-C} - \sigma_{d-P}. \tag{3}$$

One has to specify which delta should be used before the analysis. Then it is possible to derive strike-volatility pairs by solving the following equations, where d corresponds to the relevant delta level (e.g. 0.25)

$$\Delta\left(K_{d-C}, \sigma(K_{d-C}, 1)\right) = d$$

$$\Delta\left(K_{d-P}, \sigma(K_{d-P}, -1)\right) = -d.$$

A market consistent smile function, $\bar{\sigma}\left(K\right)$, has to match the information implied in the risk reversal

$$\bar{\sigma}\left(K_{d-C}\right) - \bar{\sigma}(K_{d-P}) = \sigma_{d-RR}.$$

The value of the instrument is the value of a portfolio consisting of being long a call and short a put option, denoted V_{RR}

$$V_{RR} = V^c\left(K_{d-C}, \sigma\left(K_{d-C}\right)\right) - V^p(K_{d-P}, \sigma\left(K_{d-P}\right))$$

[23].

3.8 Market Strangle

The strangle adds a third restriction to the function $\sigma(K)$, mapping the strike to the corresponding implied volatility. In the market, one can find the quoted strangle volatility, σ_{d-S-Q}. The market strangle volatility is defined as

$$\sigma_{d-S-M} = \sigma_{ATM} + \sigma_{d-S-Q}.$$

Given the volatility, one can extract a call strike $K_{d-C-S-M}$ and a put strike $K_{d-P-S-M}$, which using the market strangle volatility gives a delta of d and $-d$ respectively. With this information the value of a portfolio consisting of a long call option with strike $K_{d-C-S-M}$ and volatility σ_{d-S-M} and a long put option with strike $K_{d-K-S-M}$ and the same volatility can be calculated. The resulting price V_{d-S-M} is

$$V_{d-S-M} = V^c\left(K_{d-C-S-M}, \sigma_{d-S-M}\right) + V\left(K_{d-P-S-M}, \sigma_{d-S-M}\right)$$

which is the final variable of interest. This information must be regarded by a market consistent volatility function. The function can have different volatilities at the strikes, but the sum of the option prices must add up to V_{d-S-M}

$$\begin{aligned} V_{d-S-M} = & V^c\left(K_{d-C-S-M}, \sigma\left(K_{d-C-S-M}\right)\right) \\ & + V^p\left(K_{d-P-S-M}, \sigma\left(K_{d-P-S-M}\right)\right). \end{aligned}$$

Hence, the quoted strangle and the at-the-money volatility is used to convey information about a price of a strangle with certain strikes. A correct constructed smile must be built so that the volatilities at these strikes make the price of the corresponding strangle match the one calculated with a single volatility [23].

3.9 Butterfly

Instead of the market strangle, one can calculate the butterfly value, according to

$$\sigma_{d-BF} = \frac{\sigma_{d-C} + \sigma_{d-P}}{2} - \sigma_{ATM}$$

This is an option position corresponding to being long an out-of-the-money call and a out-of-the-money put with the same delta, d, and short two at-the-money options. It is, like the risk reversal and market strangle, denoted in volatility, hence σ_{ATM} denote the at-the-money volatility for put and call, σ_{d-C} the volatility for the out-of-the-money call and σ_{d-P} the volatility for the out-of-the-money put [27, p.42]. Figure 1 illustrates the risk reversal, market strangle and butterfly and is taken from [27, p.43].

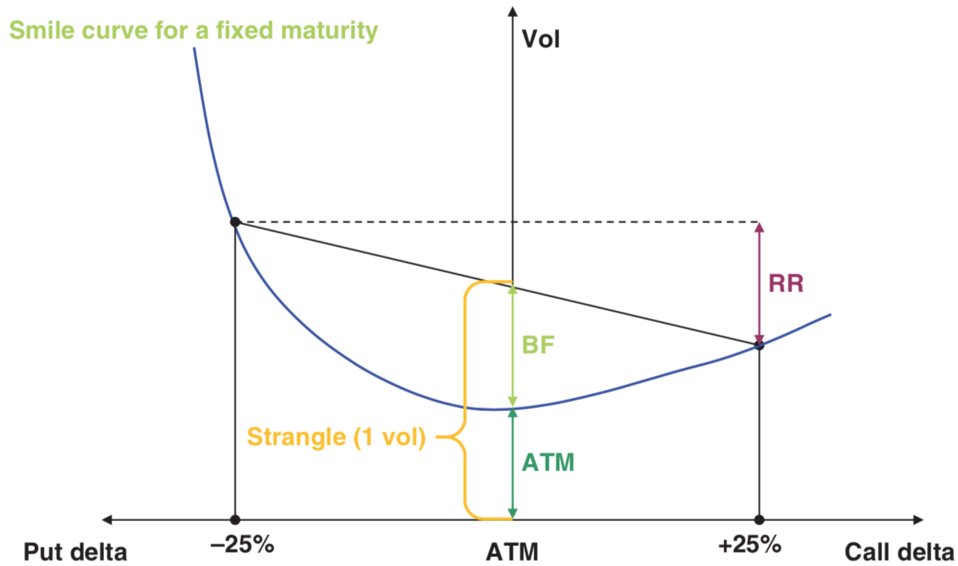


Figure 1: Schematic illustration of risk reversal, market strangle and butterfly. Figure taken from [27, p.43].

3.10 Implied Risk-Neutral Distributions from Volatility Smiles

Breeden and Litzenberger showed that risk-neutral probability distributions may be derived from volatility smiles and option prices. The value of a European call option on an asset with strike K and maturity T is given by

$$c = e^{-rT} \int_{S_T=K}^{\infty} (S_T - K)\, g(S_T)\, dS_T,$$

differentiating once with respect to K gives

$$\frac{\partial c}{\partial K} = -e^{-rT} \int_{S_T=K}^{\infty} g(S_T)\, dS_T.$$

If the expression is differentiated once again with respect to K this results in

$$\frac{\partial^2 c}{\partial K^2} = e^{-rT} g\left(K\right),$$

this expression gives the density function, with the strike price as variable, as

$$g(K) = e^{rT} \frac{\partial^2 c}{\partial^2 K}. \tag{4}$$

If c_1, c_2 and c_3 are prices of European call options with maturity T and strikes of $K - \delta, K$ and $K + \delta$ respectively [18, p. 446-448], and as δ goes to zero, the partial derivative in (4) can be approximated as

$$g(K) \approx e^{rT} \frac{c_1 + c_3 - 2c_2}{\delta^2}.$$

For very small values of δ round-off error will affect the calculation [26, p. 246-247]. Since the prices are assumed to be arbitrage free, c must be a monotonic and convex function in K. Also, since c_1, c_2 and c_3 are equidistant the implied risk-neutral probability density function will always be positive. However, since the second derivative is approximated numerically, the sum of the integrated density function may not be exactly one [13].

3.11 SABR Volatility Model

The SABR model is a stochastic volatility model which propose a stochastic evolution of the forward process, defined below

$$\begin{aligned} d\hat{F}_t &= \sigma_t \hat{F}_t^{\beta} \mathrm{d}W_t^{(1)}, \\ d\sigma_t &= \sigma_0 \sigma_t \mathrm{d}W_t^{(2)}, \\ \mathrm{d}W_t^{(1)} \mathrm{d}W_t^{(2)} &= \rho \mathrm{d}t \end{aligned}$$

[22]. The parameter β can be used to tune the model between normal, $\beta = 0$, and log-normal, $\beta = 1$. For FX, $\beta = 1$ is a common choice and in this case, one can obtain the following formula for Black's implied volatility

$$\sigma_B(K) = \sigma_0 \left[1 + \left(\frac{1}{4} \rho \sigma_0 \alpha + \frac{2 - 3\rho^2}{24} \sigma_0^2 \right) (T - t) \right] \frac{z}{\chi(z)},$$

where

$$z = \frac{\alpha}{\sigma_0} \log \left(\frac{F_{0,T}}{K} \right)$$
$$\chi(z) = \log \left(\frac{\sqrt{1 - 2\rho z + z^2} + z - \rho}{1 - \rho} \right)$$

[10, p. 60-61]. This implied volatility can be plugged into the Black and Scholes model to generate prices that are the result of the asset following SABR instead of a generalized Brownian motion [22]. The α ($\alpha > 0$) parameter is the volatility of volatility and ρ ($-1 \leq \rho \leq 1$) the correlation between spot and volatility [10, p. 60].

3.12 Shift in Implied Volatility

Anyone that is interested in an option's value would probably like to know how the implied volatility of the option is affected when the underlying asset price moves to another level S. There are a few ways to shift the implied volatility and the behaviour of the implied volatility depends on which method that is chosen. There are generally two ways of doing this, *sticky strike rule* and *sticky delta/moneyness rule* where the latter is the convention used. These rules make it convenient to quote and trade options written on different assets, and are not intended to model the evolution of the volatility smile. A drawback with shift in implied volatility is that it produced non-arbitrage free prices for models that are not the Black and Scholes model with a flat implied volatility [9, p. 93].

Sticky Strike This is the simplest rule and is a poor man's attempt to preserve Black and Scholes Model since it allows an independent existence of each option. In this rule the implied volatility curve is shifted sideways and each option keeps the exact same instantaneous volatility one could for example express it as

$$\sigma(S, K, t) = \sigma_0 - b(K - S_o) \tag{5}$$

where b is a constant. [12, p. 5-6]. A drawback of this rule is that it cannot be active in a very long time period. An example is explained to illustrate the problematic with this rule; when the index market rises, the implied volatility falls and leads to lower implied volatility for options traded at the money. Market makers lowers the implied volatility as the market rises which means that the higher the markets gets, the probability of a future catastrophe decrease [12, p. 5-6].

Sticky Moneyness This rule is usually used in over-the-counter markets (e.g., FX options) and is an approach where the option's volatility only depends on its moneyness, similar to (5) one can express it as

$$\sigma(S, K, t) = \sigma_0 - b(K/S - 1) S_0$$

where b is a constant. The effect of this is that the current level of at-the-money volatility, the volatility of the most liquid options, stays constant as the underlying moves. But at the same time the implied volatility (apart from at-the-money) will increase as the underlying asset increase. This rule assumes that the market mean reverts to definitive at-the-money volatility. In the Black and Scholes model, *sticky moneyness* equals *sticky Delta*, where *sticky Delta* is a rule where the delta moves according to the underlying asset movement. This rule should in general be used in a Stochastic Volatility, Jump Diffusion Model [12, p. 6-8].

4 Method

4.1 General Approach to Model an Expected Jump in Spot

At time T_J a large jump in the spot rate is expected. The spot rate at the time just after the event, T_J^+, is $S_{T_J^+} = S_{T_J^-}X$, where X is a binomial variable and $S_{T_J^-}$ the spot rate right before the event. The binomial variable X corresponds to the two possible outcomes, *up* and *down*, and is defined as

$$X = \begin{cases} e^{r_{up}} \text{ with probability } p_{up} \\ e^{r_{down}} \text{ with probability } (1 - p_{up}) = p_{down} \end{cases}$$

this implies that there are two possible spot rates at T_J^+,

$$S_{T_J^+} = \begin{cases} S_{T_J^-}e^{r_{up}} \\ S_{T_J^-}e^{r_{down}}. \end{cases}$$

T_J^+ is the right limit of T_J and T_J^- the left limit. Since the time is a continuous function these will be equal to each other ($T_J^- = T_J = T_J^+$). However, $S_{T_J^-}$ is not necessarily equal to $S_{T_J^+}$, and will not be in the model described above. The anticipated jump model introduced in this section is based on previous work from [17].

4.1.1 Risk Neutral Probabilities

Since the spot rate is a martingale and due to the fact that the market is assumed to be free of arbitrage, the expected jump of the underlying must net to zero. In other words, the expected future spot rate should be equal to the spot rate today, discounted with the risk-free rate. This leads to the following risk neutral probabilities

$$p_{up} = \frac{1 - e^{r_{down}}}{e^{r_{up}} - e^{r_{down}}}, \quad p_{down} = \frac{e^{r_{up}} - 1}{e^{r_{up}} - e^{r_{down}}} \tag{6}$$

where p_{down} and p_{up} are the probabilities of *down* and *up* respectively, and must sum to one. This impose the following relation between $e^{r_{up}}$ and $e^{r_{down}}$

$$e^{r_{down}} \leq 1 < e^{r_{up}} \text{ or } e^{r_{down}} < 1 \leq e^{r_{up}}.$$

Note that if $e^{r_{down}} = 1$, this implies that $p_{up} = 0$, and this means that there is no event occurring. Similarly for the case in which $e^{r_{up}} = 1$.

Fixation of $e^{r_{up}}$, $e^{r_{down}}$ and p_{up} From (6) it is clear that by fixating two of $e^{r_{up}}$, $e^{r_{down}}$ and p_{up}, the third will also be predetermined by the no-arbitrage condition. So the martingale argument reduce the number of parameters from three to two. Hence when the model is to be calibrated to market data, the fact that there are only two parameters to be calibrated makes it relatively cheap. While being cheap, it is still a flexible model since the two parameters may generate a wide range of implied volatility forms.

4.1.2 General Pricing Formula for Options with Maturity at T_J^+

From Section 3.2 we know that the arbitrage free call option price at time t before the event is

$$V_t^c = PV_t\left(S_{T_J^-}, K\right) = e^{-r^d(T_J-t)}\,\mathbb{E}_t^Q\left[\left(S_{T_J^-} - K\right)^+\right].$$

Assume that $S_{T_J^+} = S_{T_J^-}e^{r_{up}}$ with probability p_{up} and $S_{T_J^+} = S_{T_J^-}e^{r_{down}}$ with probability p_{down}, these are the key assumptions for pricing the event. They describe the behaviour at T_J and how $S_{T_J^-}$ and $S_{T_J^+}$ are connected. Also, it shows that the jump is linearly dependent on the start value S_t, which is important for the replication argument to hold. Given this, we can condition the expectation as

$$\begin{aligned}
V_t^c =& e^{-r^d(T_J-t)}\,\mathbb{E}_t^Q\left[\left(S_{T_J^+} - K\right)^+\right]\\
=& e^{-r^d(T_J-t)}\,\mathbb{E}_t^Q\left[\mathbb{E}_t^Q\left[\left(S_{T_J^-}X - K\right)^+ | S_{T_J^-}\right]\right]\\
=& e^{-r^d(T_J-t)}\left(p_{up}\,\mathbb{E}_t^Q\left[\left(S_{T_J^-}e^{r_{up}} - K\right)^+\right] + p_{down}\,\mathbb{E}_t^Q\left[\left(S_{T_J^-}e^{r_{down}} - K\right)^+\right]\right)\\
=& p_{up}e^{r_{up}}e^{-r^d(T_J-t)}\,\mathbb{E}_t^Q\left[\left(S_{T_J^-} - Ke^{-r_{up}}\right)^+\right]\\
&+ p_{down}e^{r_{down}}e^{-r^d(T_J-t)}\,\mathbb{E}_t^Q\left[\left(S_{T_J^-} - Ke^{-r_{down}}\right)^+\right]
\end{aligned}$$

As long as X is independent from the spot process and the jump is linear in S_t, this formula holds regardless of which process is driving the spot rate. This will also allow for pricing maturities longer than T_J^+.

4.1.3 Option Prices via Replication are Arbitrage Free

According to Section 3.2 $PV_t\left(T_J^-, K\right) = e^{r\left(T_J^- - t\right)}\mathbb{E}_t^Q\left[\left(S_{T_J^-} - K\right)^+\right]$ can be assumed to be arbitrage free, hence the prices are a monotonic and convex function in strike K. Via a replication argument the formula

$$PV\left(S_{T_J^+}, K\right) = p_{up}e^{r_{up}}PV\left(S_{T_J^-}, Ke^{-r_{up}}\right) + p_{down}e^{r_{down}}PV\left(S_{T_J^-}, Ke^{-r_{down}}\right)$$

is obtained in Section 4.1.2. Hence $PV\left(S_{T_J^+}, K\right)$ is a linear combination of two convex and monotonic functions, and as such, it is also a convex and monotonic function. According to this argument, the model will generate prices that are free of arbitrage.

4.2 Model the Jump with Black and Scholes Dynamic

If the exchange rate pre-event follows a standard Black and Scholes process extended according to Garman-Kohlhagen for FX options, the spot at time T, given spot at time t and a jump free model, is

$$S_T = S_t e^{\left(r^d - r^f - \frac{\sigma_1^2}{2}\right)(T-t) + \sigma_1(W_T - W_t)}.$$

To allow for options with maturities after the jump at T_J we impose a new variable $\sigma_{2,q}$

$$\sigma_{2,q} = \begin{cases} \sigma_{2,up} \\ \sigma_{2,down} \end{cases}$$

which is assumed to take the value $\sigma_{2,up}$ in the *up* scenario and $\sigma_{2,down}$ in case of a *down* event. Further, after the jump in the scenario of $X = e^{r_{up}}$ we expect the spot to continue with a Black and Scholes distribution with the volatility $\sigma_{2,up}$. Similarly, if $X = e^{r_{down}}$ the Black and Scholes volatility is assumed to be $\sigma_{2,down}$. Combining these assumptions the future spot rate process can be separated into different parts as in equation (7). It has the current spot rate, S_t, and the jump in the spot, X. Additionally it is separated into the return before the event, Y_1, and after , Y_2. Since the process is assumed to follow Black and Scholes dynamic and hence is a general Brownian motion, this yields (8)

$$S_T = S_t X \underbrace{\frac{S_{T_J^-}}{S_t}}_{Y_1} \underbrace{\frac{\widetilde{S}_T}{S_{T_J^+}}}_{Y_2} \tag{7}$$

$$= S_t X \underbrace{e^{\left(r^d - r^f - \frac{\sigma_1^2\theta_1 + \sigma_{2,q}^2\theta_2}{2}\right)(T-t) + \sigma_1\left(W_{T_J} - W_t\right) + \sigma_{2,q}\left(W_T - W_{T_J}\right)}}_{Y_q}, \tag{8}$$

where θ_1 denotes the proportion of the time to maturity spent in the state before the jump,

$$\theta_1 = \frac{T_J - t}{T - t}$$

and θ_2 the proportion spent after the jump

$$\theta_2 = \frac{T - T_J}{T - t}.$$

Y_q depends on the outcome of the event according to

$$Y_q = \begin{cases} e^{\left(r^d - r^f - \frac{\sigma_1^2\theta_1 + \sigma_{2,up}^2\theta_2}{2}\right)(T-t) + \sigma_1\left(W_{T_J} - W_t\right) + \sigma_{2,up}\left(W_T - W_{T_J}\right)} & \text{, if } up \\ e^{\left(r^d - r^f - \frac{\sigma_1^2\theta_1 + \sigma_{2,down}^2\theta_2}{2}\right)(T-t) + \sigma_1\left(W_{T_J} - W_t\right) + \sigma_{2,down}\left(W_T - W_{T_J}\right)} & \text{, if } down. \end{cases} \tag{9}$$

Important to note is that the process W_t is assumed to be unaffected by the event. So the Brownian motion is the same process in both scenarios, however the increments $W_{T_J} - W_t$ and $W_T - W_{T_J}$ are two independent normal variables.

Illustration with Monte Carlo Simulation Figure 2 illustrates one Monte Carlo simulated path of the spot process with a jump. The simulation assumes that the jump will occur at day 100 and we simulate a total of 365 days. The jump is symmetric with $e^{r_{up}} = 1.05$ and $e^{r_{down}} = 0.95$, and an initial spot rate $S_t = 1.0$. The initial volatility is set to 0.12 and jumps to 0.08 in the *up* scenario and 0.2 in the *down* scenario.

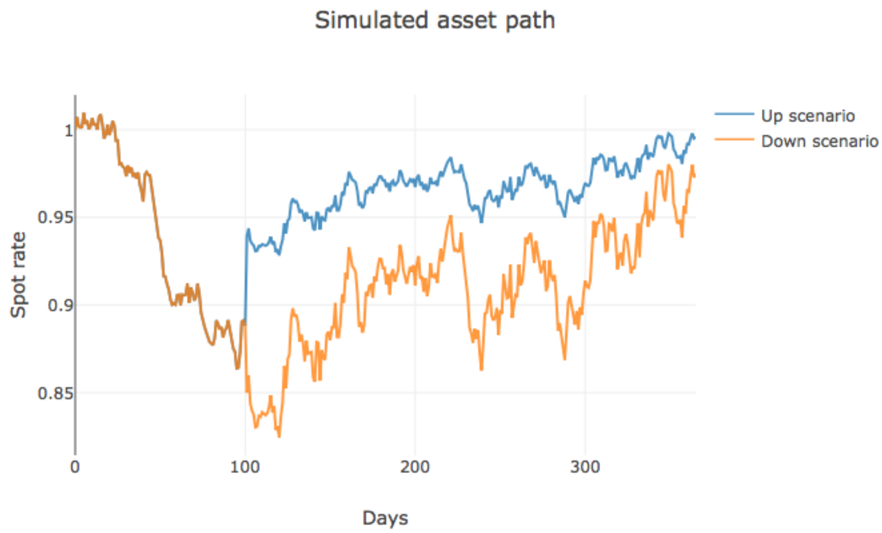


Figure 2: Monte Carlo simulation of the jump model with $S_t = 1.0$*,* $e^{r_{up}} = 1.05$*,* $e^{r_{down}} = 0.95$*,* $\sigma_1 = 0.12$*,* $\sigma_{2,down} = 0.2$*,* $\sigma_{2,up} = 0.08$*,* $t = 0$*,* $T_J = 100/365$ *and* $T = 365/365$*.*

Freedom of Parameters and Calibration $e^{r_{up}}$, $e^{r_{down}}$ and p_{up} are still connected and determined by the no arbitrage condition in (6). In addition we now also have a stochastic variable $\sigma_{2,q}$, which offers the model two additional parameters. The extended model with four parameters offers more flexibility, since with four parameters the range of possible implied volatility forms widens. However, the increased flexibility comes with a cost and the model is now more difficult to calibrate to market data. One need to keep this in mind since it is important from a practical point of view. Since if the model is to be used on a frequent basis by market practitioners it must be flexible, but even more important is the cost and complexity of the model.

4.2.1 Pricing the Expected Jump with Black and Scholes Assumptions

To obtain the option pricing formula, the expectation of the future pay-off is used as in Section 4.1.2. Furthermore we condition on the jump and the calculations is outlined below,

$$
\begin{aligned}
\mathbb{E}\left[(S_T-K)^+\right] =& \mathbb{E}\left[(S_tXY-K)^+\right] \\
=& \{\text{taking expectation over the up and down event,} \\
& \text{keeping all other random variables fixed}\} \\
=& \mathbb{E}\left[p_{up}\left(S_te^{r_{up}}Y_{up}-K\right)^+ + p_{down}\left(S_te^{r_{down}}Y_{down}-K\right)^+ |S_t\right] \\
=& p_{up}e^{r_{up}}\,\mathbb{E}\left[\left(\widetilde{S}_{T,up}-Ke^{-r_{up}}\right)^+ |S_t\right] \\
& + p_{down}e^{r_{down}}\,\mathbb{E}\left[\left(\widetilde{S}_{T,down}-Ke^{-r_{down}}\right)^+ |S_t\right].
\end{aligned}
$$

We will then use the definition of implied volatility, see Subsection 3.5 for details, to extend the pricing formula one step further. The definition gives us that

$$PV_t(S_T,K) = BS\left(S_t,K,\sigma\left(K,T\right)\right)$$

and in the *up* scenario

$$PV_t\left(S_T,Ke^{-r_{up}}\right) = BS\left(S_t,Ke^{-r_{up}},\sigma\left(Ke^{-r_{up}},T\right)\right) \tag{10}$$

and similarly for the *down* scenario.

To be able to use $BS\left(S_t,Ke^{-r_{up}},\sigma\left(Ke^{-r_{up}},T\right)\right)$, $\sigma\left(Ke^{-r_{up}}\right)$ must be derived, i.e. we have to find an effective volatility including the information of the assumed implied volatility both before and after the jump. Recall the use of the subscript q, denoting either *up* or *down*, which will be used here. According to (8) $\widetilde{S}_{T,q}$ in the calculations above may be written as S_tY_q, which in turn can be expressed as

$$S_tY_q = S_te^z$$

where

$$z = \left(r^d - r^f - \frac{\sigma_1^2\theta_1+\sigma_{2,q}^2\theta_2}{2}\right)(T-t) + \sigma_1\left(W_{T_J}-W_t\right) + \sigma_{2,q}\left(W_T-W_{T_J}\right).$$

Due to properties of the normal distribution and Brownian motion, it can be shown that (see Appendix A for details) z has the following distribution

$$z \sim \mathcal{N}\left(\left(r^d - r^f - \frac{\sigma_1^2\theta_1+\sigma_{2,q}^2\theta_2}{2}\right)(T-t)\,,\sqrt{\sigma_1^2\theta_1+\sigma_{2,q}^2\theta_2}\sqrt{T-t}\right).$$

Introducing a new variable

$$\widetilde{\sigma}_{eff,q} = \sqrt{\sigma_1^2\theta_1 + \sigma_{2,q}^2\theta_2}$$

this can be expressed as

$$z \sim \mathcal{N}\left(\left(r^d - r^f - \frac{\widetilde{\sigma}_{eff,q}^2}{2}\right)(T-t), \widetilde{\sigma}_{eff,q}\sqrt{T-t}\right).$$

Now according to [6, p.104-105] and using the notation in Section 3.3, $\widetilde{S}_{T,q}$ follows Black and Scholes dynamics with

$$\alpha = r^d - r^f$$

and

$$\sigma = \widetilde{\sigma}_{eff,q}.$$

Hence (10) can be priced as $BS\left(S_t, Ke^{-r_{up}}, \widetilde{\sigma}_{eff,up}\right)$, and the final pricing formula

$$\begin{aligned}\widetilde{BS}\left(S_t, K, \sigma\left(K,T\right)\right) =& p_{up}e^{r_{up}}BS\left(S_t, Ke^{-r_{up}}, \widetilde{\sigma}_{eff,up}\right)\\ &+ p_{down}e^{r_{down}}BS\left(S_t, Ke^{-r_{down}}, \widetilde{\sigma}_{eff,down}\right)\end{aligned}$$

is obtained.

4.2.2 Obtaining the Implied Volatility

Once the option prices are obtained, the implied volatility $\sigma(K,T)$ for the mixture prices is extracted with the bisection method. The bisection method is a root finding method that repeatedly bisects an interval and then selects a subinterval in which the root must lie. It is a very simple and robust method but a drawback is that it converges at a slow rate. The bisection method is also called *interval halving-*, *binary search-* and *dichomoty -method*. In the reference, *Dichotomous search* is used [8, p. 25].

4.3 Model the Jump with SABR Dynamic for Maturity at T_J^+

We now want to use SABR dynamic to model option prices for options with maturity at T_J^+ and extract the implied volatility curve. As mentioned in

Section 3.11, $\beta = 1$ is a reasonable choice when it comes to FX options. The general SABR process is simplified to

$$\begin{aligned} \mathrm{d}F_t &= \sigma_t F_t \mathrm{d}W_t^{(1)} \\ \mathrm{d}\sigma_t &= \sigma_0 \sigma_t \mathrm{d}W_t^{(2)} \\ \mathrm{d}W_t^{(1)} \mathrm{d}W_t^{(2)} &= \rho \mathrm{d}t. \end{aligned} \tag{11}$$

The solution to (11) is needed in order to derive the SABR prices further down, following the procedure in [21, p. 935] results in

$$\begin{aligned} F_T &= F_t e^{-\int_t^T \frac{\sigma_u^2}{2}\mathrm{d}u + \frac{\rho}{\sigma_t}(\sigma_T - \sigma_t) + \sqrt{1-\rho^2}\int_t^T \sigma_u \mathrm{d}W_u^{(1)}} \\ &= F_t e^z. \end{aligned}$$

where

$$z = -\int_t^T \frac{\sigma_u^2}{2}\mathrm{d}u + \frac{\rho}{\sigma_t}(\sigma_T - \sigma_t) + \sqrt{1-\rho^2}\int_t^T \sigma_u \mathrm{d}W_u^{(1)}.$$

A jump is added in accordance with Section 4.1, and results in the following formula

$$F_T = X F_t e^z.$$

4.3.1 Pricing the Expected Jump with SABR Dynamic

The jump is linear in the start value of the spot process also for the SABR model and by conditioning the procedure in Section 4.2.1 can be followed

$$\begin{aligned} \mathbb{E}^Q\left[(XF_te^z - K)^+\right] =& p_{up} e^{r_{up}}\, \mathbb{E}^Q\left[\left(F_T e^z - Ke^{-r_{up}}\right)^+\right] \\ &+ p_{down} e^{r_{down}}\, \mathbb{E}^Q\left[\left(F_T e^z - Ke^{-r_{down}}\right)^+\right]. \end{aligned}$$

This is then chosen to be expressed in terms of SABR prices

$$\begin{aligned} SABR(t, T, S_t, K) =& e^{r_{up}} p_{up} SABR\left(t, T, S_t, Ke^{-r_{up}}\right) \\ &+ e^{r_{down}} p_{down} SABR\left(t, T, S_t, Ke^{-r_{down}}\right). \end{aligned}$$

In order to calculate the SABR prices, volatilities are generated from the approximative formula in Section 3.11 and inserted in the Black and Scholes formula to price options in the *up* and *down* scenarios. These are then weighted to obtain the price of an option before the event with maturity at T_J^+, see (12),

$$\begin{aligned}\widetilde{BS}\left(S_t, K, \sigma\left(K, T_J^+\right)\right) = p_{up} e^{r_{up}} BS\left(S_t, K e^{-r_{up}}, \sigma_{SABR}\left(K e^{-r_{up}}, T_J^-\right)\right) \\ + p_{down} e^{r_{down}} BS\left(S_t, K e^{-r_{down}}, \sigma_{SABR}\left(K e^{-r_{down}}, T_J^-\right)\right).\end{aligned} \quad (12)$$

As proved in Section 4.1.2 the replication is agnostic to the dynamics of the underlying asset, as long as the jump is linear in the start value. Hence the replication argument holds whatever smile is generated or valid in the model. What is interesting to note, however, is that the shift in σ_{SABR} becomes the *sticky strike* method, see Section 3.12 for details. In order to extract the implied volatility from (12), the procedure in Subsection 3.5 is used.

5 Results

In the following section, the results from implementing the models explained in Section 4 are presented. In Section 5.1 the Black and Scholes dynamic is used to price options with maturity T_J^+, and is extended for longer maturities in Section 5.2. Finally the results from implementing the SABR dynamic is presented in Section 5.3.

The results includes the consequences different changes of the parameter values have on the implied volatility curve as well as the probability density function. Hence this section also shows the flexibility and behaviour of the models presented in Section 4 and one can study the difference between the different dynamics.

Options with maturity after T_J have different assumed implied volatility in the scenarios of *up* and *down*, as mentioned in Section 4.2. As a base case, the volatility in the *down* scenario is higher than before the jump, and the opposite for the volatility in the *up* scenario. To summarize, in the *up* scenario we have a higher spot rate and lower volatility, and in the *down* scenario we have a lower spot rate and a higher volatility.

$$S^+_{T_{J,down}} < S_{T_J^-} < S^+_{T_{J,up}} \text{ and } \sigma_{2,up} < \sigma_1 < \sigma_{2,down}$$

Recalling Brexit and the GBPUSD spot rate, this is a likely relation between spot and volatility. Once the market understood that the referendum was in favour of a Brexit, there was a depreciation of the GBP, hence the GBPUSD spot rate dropped, while the volatility in the market increased. If on the other hand the outcome of the referendum had been a remain, the authors in [11] suggest that the spot rate would move in the other direction, and since this outcome would had meant less uncertainty, it is likely that the volatility would had decreased. This was the case for Brexit, however, after big news releases the volatility usually increase, regardless of the outcome.

5.1 Black and Scholes Model with Jump in Spot Rate with Maturity at T_J^+

In Figure 3 the standard Black and Scholes is plotted together with the corresponding implied volatility. As can be seen the implied volatility is constant which is in line with the theory.

When observing Figure 4 the peaks of the probability density function are at different heights even though the jump in spot is symmetric, i.e. the same percentage change in both the *up* and *down* scenario. This is due to the asymmetry that occurs within the Black and Scholes model, and the scaling factor affecting the derivative when deriving the approximation of the density function. The implied volatility gets a concave curvature, which we refer to as a volatility frown.

In Figure 5 and 6 the impact of an asymmetric jump can be observed. When the jump is asymmetric the volatility get skewed towards the more extreme outcome, at which the probability density mass is smaller. It is also visually clear that the support of the probability density function now includes lower strikes. The support is also slightly reduced on the higher end.

As discussed in Section 4.1 the calibration capacity of this model is limited, since $T_J^+ = T$. But it is clear that one can skew the probability density function and the implied volatility curve by assuming different outcomes of the event. Different assumptions regarding the initial implied volatility will also affect the curves.

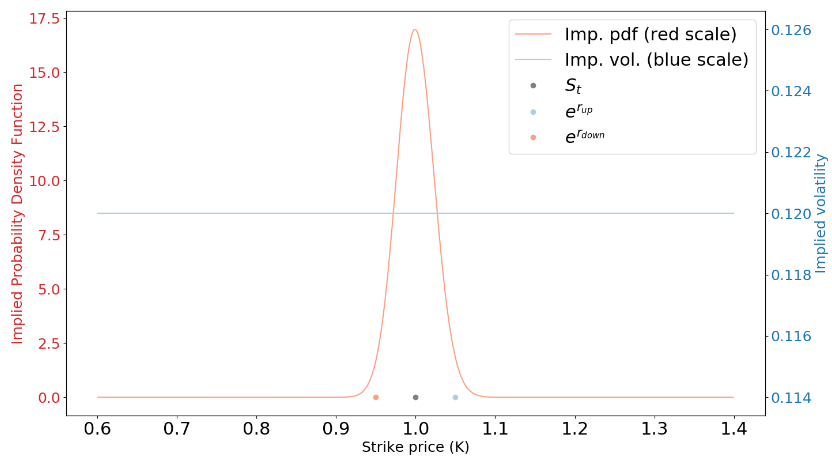


Figure 3: Graphic visualization of the effects of $e^{r_{up}}$ and $e^{r_{down}}$ on the implied probability density function and the implied volatility. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, blue and green dot respectively. The probability density function is the red curve and the implied volatility the blue curve where $S_t = 1$, $\sigma = 12\%$, $T = 14/365$.

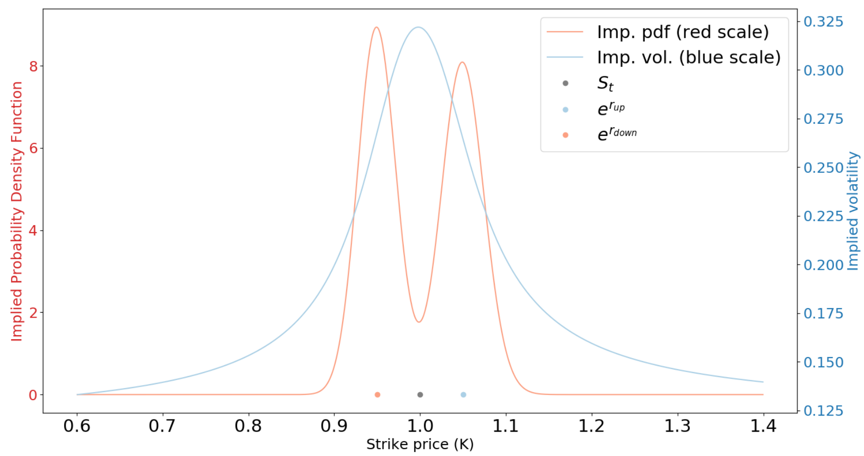


Figure 4: Graphic visualization of the effects of $e^{r_{up}}$ and $e^{r_{down}}$ on the implied probability density function and the implied volatility. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, blue and green dot respectively. The probability density function is the red curve and the implied volatility the blue curve where $S_t = 1$, $e^{r_{up}} = 1.05$, $e^{r_{down}} = 0.95$, $\sigma = 12\%$, $T = 14/365$.

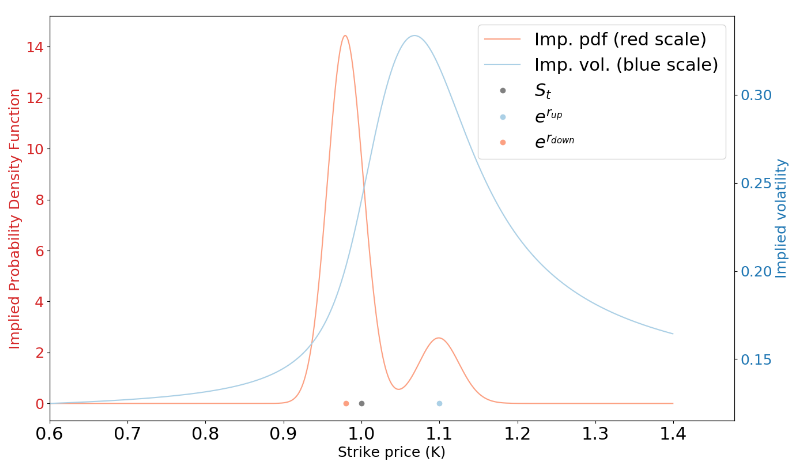


Figure 5: Graphic visualization of the effects of $e^{r_{up}}$ and $e^{r_{down}}$ on the implied probability density function and the implied volatility. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, blue and green dot respectively. The probability density function is the red curve and the implied volatility the blue curve where $S_t = 1$, $e^{r_{up}} = 1.10$, $e^{r_{down}} = 0.98$, $\sigma = 12\%$, $T = 14/365$.

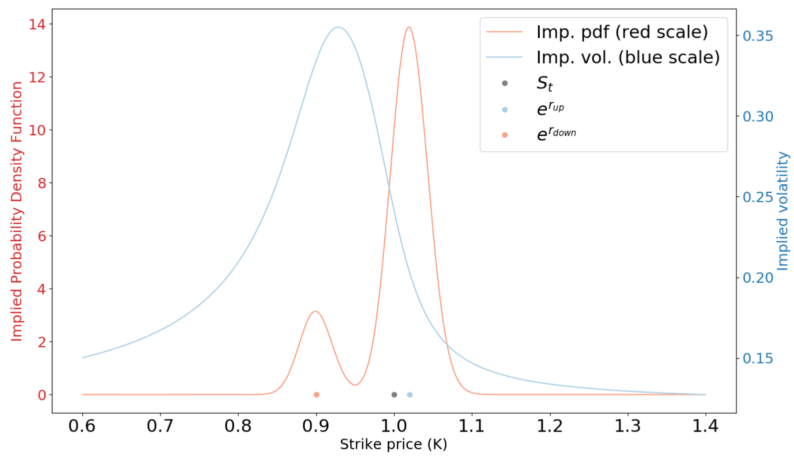


Figure 6: Graphic visualization of the effects of $e^{r_{up}}$ and $e^{r_{down}}$ on the implied probability density function and the implied volatility. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, blue and green dot respectively. The probability density function is the red curve and the implied volatility the blue curve where $S_t = 1$, $e^{r_{up}} = 1.02$, $e^{r_{down}} = 0.90$, $\sigma = 12\%$, $T = 14/365$.

5.2 Black and Scholes Model with Jump in Spot and Volatility, with Maturity at $T > T_J^+$

In this section we implement different scenarios for options with maturities longer than T_J^+, and observe the behaviour of the implied volatility and implied probability density function. To make this relevant we assume a jump in the volatility in all scenarios as well, but of different magnitude. We start by letting the time to maturity increase and then move on to review variations in the volatility jump. Lastly we let the jump in the spot rate vary.

Since $\sigma_{2,up}$ and $\sigma_{2,down}$ are assumed in this implementation, the flexibility of the model is increased compared to Section 5.1.

5.2.1 Increasing Time to Maturity

Here we present the result of letting the maturity stepwise increase. The result is presented with a symmetric jump in the spot rate and the result from a skewed jump can be found in Appendix B.1. What is interesting to see here is how time alter the option prices, and hence also the implied volatility curve. As can be seen in (8) this gives the volatility in terms of σ_1 and $\sigma_{2,q}$ as well as the Brownian motion W longer time to affect the spot process.

Symmetric Jump in Spot The parameter values for the symmetric jump in spot are presented in Table 1. The timing of the jump is kept constant ($T_J = 7/365$) while the time to maturity goes from 8 to 14 days. The result is presented in Figure 7. As the time to maturity increase the peak of the implied volatility curve decrease and the support of the implied probability distribution increase. The reason for this is that with a longer time to maturity the volatilities σ_1, $\sigma_{2,up}$ and $\sigma_{2,down}$ gets a longer time to impact the process, and the probability that the process hits values further out in the tails increase.

Property	Notation	Value
Initial spot-rate	S_t	1.0
Up jump	$e^{r_{up}}$	1.05
Down jump	$e^{r_{down}}$	0.95
Time for pricing	t	0
Time for jump	T_J	7 days (7/365 year)
Time to maturity	T	8 days $\leq T \leq$ 14 days
Domestic risk-free rate	r^d	0
Foreign risk-free rate	r^f	0
Volatility before jump	σ_1	12%
Volatility after jump in *up* scenario	$\sigma_{2,up}$	8%
Volatility after jump in *down* scenario	$\sigma_{2,down}$	20%

Table 1: Parameter values for increasing maturity.

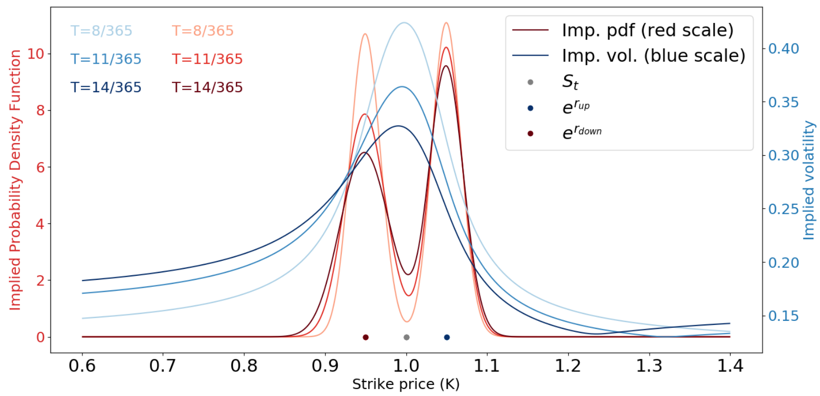


Figure 7: Implied volatility and implied probability density function when $S_t = 1.0$, $e^{r_{up}} = 1.05$, $e^{r_{down}} = 0.95$, $\sigma_{down} = 20\%$, $\sigma_{up} = 8\%$, $t = 0$, $T_J = 7/365$ and $T = \{8, 11, 14\}/365$. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, grey and a blue dot respectively. The shading of the implied volatility and the implied probability density function corresponds to different time to maturity T which is the only changing parameter. The darker shade corresponds to longer maturities.

Risk Reversal and Butterfly The risk reversal and butterfly is a way to quantify the structure of the implied volatility curve, and measures the skewness and kurtosis, respectively. The results are presented in Table 2.

As the time to maturity increase the 0.25 delta risk reversal decreases and becomes more and more negative. Hence the volatility curve gets more skewed with longer maturities. It implies that when the volatility is larger in the down scenario, as is the case here, the difference between the implied volatility for a 0.25 delta put and a 0.25 delta call increase, and that the implied volatility for the put is larger than the one for the call.

The 0.25 delta butterfly moves in the opposite direction, although also negative. Hence the peak decrease with longer maturities, which is visually clear from Figure 7.

Variable	Values		
T	8/365	11/365	14/365
Risk Reversal 0.25 delta	-2.08%	-3.27%	-3.88%
Butterfly 0.25 delta	-5.97%	-4.28%	-3.21%

Table 2: Risk Reversal and Butterfly values for increasing maturity and symmetric jump.

5.2.2 Changes in Volatility

We now look deeper into how changes in the jump of the volatility modifies the implied volatility and the implied probability density function. This may be interesting as part of a sensitivity analysis before a data release or election. The result from an increasing $\sigma_{2,up}$ can be found below, whereas the result from an increasing $\sigma_{2,down}$ is in Appendix B.2.

Increasing $\sigma_{2,up}$ Values of the parameters used in the implementation of the increasing $\sigma_{2,up}$ can be found in Table 3. $\sigma_{2,up}$ increase from 8% to 16% while the time aspect is constant with T_J being 7 days and T equal to 14 days.

Property	Notation	Value
Initial spot-rate	S_t	1.0
Up jump	$e^{r_{up}}$	1.02
Down jump	$e^{r_{down}}$	0.95
Time for pricing	t	0
Time for jump	T_J	7 days (7/365 year)
Time to maturity	T	14 days (14/365 year)
Domestic risk-free rate	r^d	0
Foreign risk-free rate	r^f	0
Volatility before jump	σ_1	12%
Volatility after jump in *up* scenario	$\sigma_{2,up}$	$8\% \leq \sigma_{2,up} \leq 16\%$
Volatility after jump in *down* scenario	$\sigma_{2,down}$	20%

Table 3: Parameter values for increasing $\sigma_{2,up}$.

Below the result of increasing the $\sigma_{2,up}$ is presented. As $\sigma_{2,up}$ increase the probability density function gets a heavier tail on the upside and the bi-modal attribute becomes less apparent. The peak gets lower as the volatility increase the support of the density function. The implied volatility increases as the probability density function gets wider on the up side, due to the increased volatility in the *up* scenario. The effects on the down side is not as obvious.

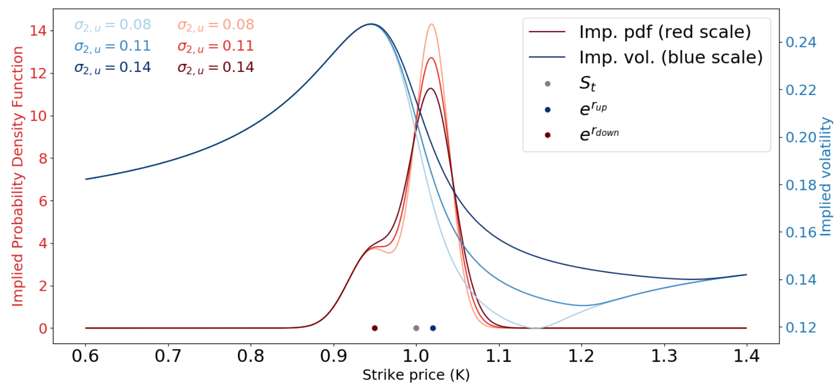


Figure 8: Implied volatility and implied probability density function when $S_t = 1.0$, $e^{r_{up}} = 1.02$, $e^{r_{down}} = 0.95$, $\sigma_{2,down} = 20\%$, $\sigma_{2,up} = \{8\%, 11\%, 14\%\}$, $t = 0$, $T_J = 7/365$ and $T = 14/365$. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, grey and a blue dot respectively. The shading of the implied volatility and the implied probability density function corresponds to different $\sigma_{2,up}$ which is the only changing parameter and can be seen in the top left corner. The darker shade corresponds to higher volatility in the up scenario.

Risk Reversal and Butterfly The risk reversal and butterfly quantifies the changes once more, and are presented in Table 4.

The risk reversal is negative, but increasing. Hence the implied volatility curve is skewed and the 0.25 delta put option is more expensive than the corresponding call. However, as $\sigma_{2,up}$ increase, the difference between the call and the put do so as well. Which is reasonable since the probability density function's support expand and includes higher strikes.

The butterfly is positive and decreasing, hence it moves in the opposite direction as the risk reversal. Again, this can be interpreted as the implied volatility being slightly convex in the interval of the butterfly, but decreasingly so.

Variable	Values		
$\sigma_{2,up}$	0.08	0.11	0.14
Risk Reversal 0.25 delta	-7.02%	-6.09%	-5.02%
Butterfly 0.25 delta	0.29%	0.21%	0.10%

Table 4: Risk Reversal and Butterfly values for increasing $\sigma_{2,up}$.

5.2.3 Changes in the Spot Jump

This section presents the result in the case with variation in the jump in the spot rate, keeping both the time aspect as well as $\sigma_{2,up}$ and $\sigma_{2,down}$ constant. This is interesting as a sensitivity analysis within risk management to see how the option prices change as the assumption for the jump is altered. The result from an increasing jump in the *up* scenario is found here, and for an increasing jump in the *down* scenario the reader is referred to Appendix B.3.

Increasing the Jump in *Up* Scenario As the jump size increase in the *up* scenario, the implied volatility gets skewed to the right. The upper peak of the probability density function decrease while the lower increase. Hence, option prices for higher strikes increase due to the fact that the density function gets more support for higher strikes, implicating that it becomes more likely that the option for these strikes actually will be exercised.

Property	Notation	Value
Initial spot-rate	S_t	1.0
Up jump	$e^{r_{up}}$	$1.02 \leq e^{r_{up}} \leq 1.08$
Down jump	$e^{r_{down}}$	0.95
Time for pricing	t	0
Time for jump	T_J	7 days (7/365 year)
Time to maturity	T	14 days (14/365 year)
Domestic risk-free rate	r^d	0
Foreign risk-free rate	r^f	0
Volatility before jump	σ_1	12%
Volatility after jump in *up* scenario	$\sigma_{2,up}$	8%
Volatility after jump in *down* scenario	$\sigma_{2,down}$	20%

Table 5: Parameter values for increasing $e^{r_{up}}$.

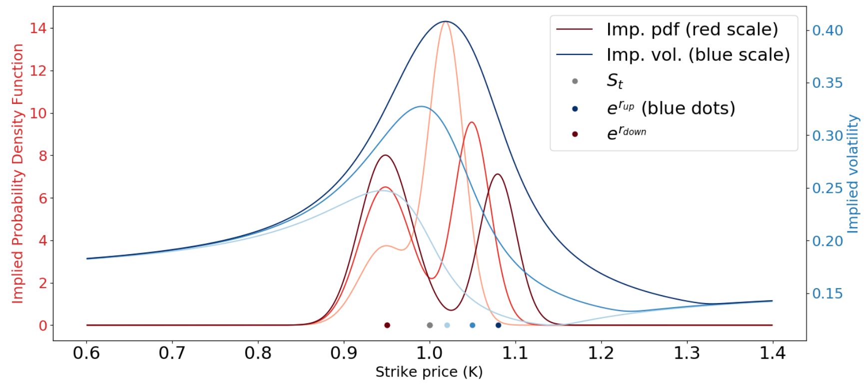


Figure 9: Implied volatility and implied probability density function when $S_t = 1.0$, $e^{r_{up}} = \{1.02, 1.05, 1.08\}$, $e^{r_{down}} = 0.95$, $\sigma_{2,down} = 20\%$, $\sigma_{2,up} = 8\%$, $t = 0$, $T_J = 7/365$ and $T = 14/365$. $e^{r_{down}}$, spot can be seen as a red and a gray dot. The only changing parameter is $e^{r_{up}}$ which is presented as shaded blue dots, the darker the colour the higher the jump This grading scale is used in the implied volatility and the implied probability density function as well.

Risk Reversal and Butterfly As $e^{r_{up}}$ increase, the risk reversal starts at a negative value and moves to a positive, as can be seen in Table 6. Thus, the order of magnitude between the 0.25 delta call and put shifts, and the $\sigma_{0.25-C}$ surpasses the $\sigma_{0.25-P}$.

Also the butterfly is switching sign, but from positive to negative. This is a consequence of that the peak of the implied volatility curve moves towards higher strikes. So the curvature in the interval for which the butterfly is calculated switches notably.

Variable	Values		
$e^{r_{up}}$	1.02	1.05	1.08
Risk Reversal 0.25 delta	-7.02%	-3.88%	3.49%
Butterfly 0.25 delta	0.29%	-3.22%	-4.18%

Table 6: Risk Reversal and Butterfly values for increasing $e^{r_{up}}$.

5.3 SABR Dynamic and Jump in Spot Rate for Maturity at T_J^+

In this section, different scenarios are implemented when the maturity is at T_J^+ and the jump sizes varies. The results below has a fixed time to maturity of 30 days, for results when time to maturity is 7 days, see Appendix C.1.

5.3.1 SABR Dynamic - 30 Days to Maturity

Here we present the result of letting time to maturity be 30 days and illustrating the outcome symmetric and skewed jumps have on the implied volatility. The SABR volatility is generated with the parameter values taken from [19] and presented in Table 7.

Property	Notation	Value
Initial spot-rate	S_0	1.0
Up jump	$e^{r_{up}}$	varies
Down jump	$e^{r_{down}}$	varies
Domestic risk-free rate	r^d	0
Foreign risk-free rate	r^f	0
Time to maturity	T	30 days (30/365 year)
Volatility at time $t = 0$	σ_0	10%
Alpha	α	0.4
Beta	β	1
Rho	ρ	-0.1

Table 7: Specification for jump, option and SABR data.

Symmetric Jump Figure 10 below illustrates the effect a symmetric jump has on the implied volatility. What is observed is a frown on the implied volatility when at the same time the SABR volatility has the typical smile shape. As seen in Figure 10 the jump affects the implied volatility. If the strike space were to widen, we would see that the implied volatility is not in fact a frown in itself, it experience a local frown and converges to the σ_{SABR} as strike price gets further away from S_t. Which is due to the smile curvature in SABR where prices are increasing quicker in the wings due to the smile. These result are a key difference between implied volatility with SABR dynamic and Black and Scholes dynamic.

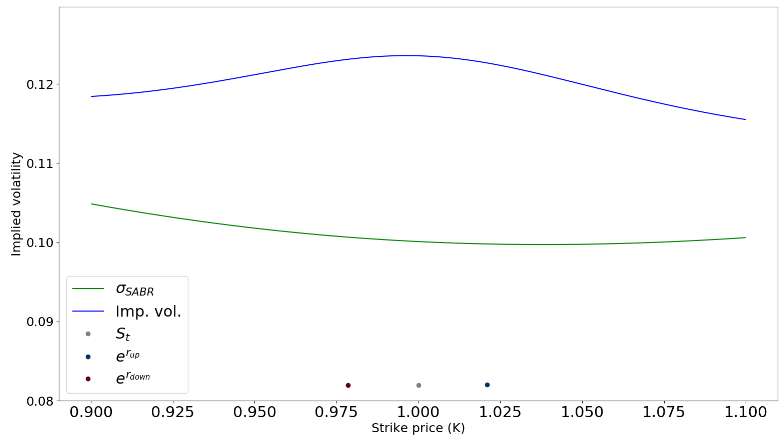
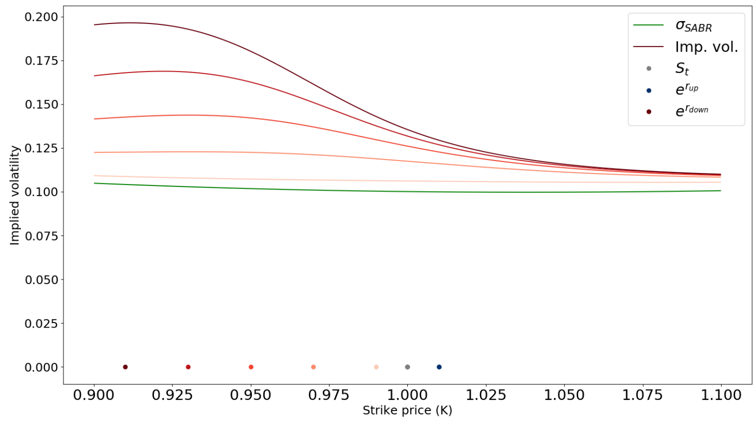
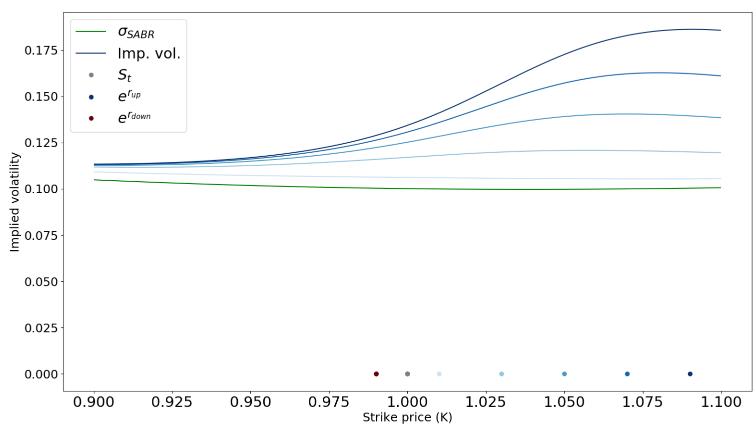


Figure 10: SABR Volatility generated by the SABR model with parameters in accordance with Table 7, and the implied volatility and implied probability density function after a jump with $S_t = 1.0$, $e^{r_{up}} = 1.02$ and $e^{r_{down}} = 0.98$. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, blue and a green dot respectively.

Skewed Jumps Instead of a symmetric jump we now look further into the behaviour of the implied volatility when the *up* and *down* jump varies in size. The effect of asymmetric jumps is illustrated in Figure 11a and 11b. The direction of the asymmetric jump has a direct impact on the implied volatility, we see that the skewness is drawn towards the side of the biggest jump size. These result follows the behaviour implied volatility with Black and Scholes dynamic has, even though, as explained in Section 5.3.1, it exhibits a local frown. The implied volatility forms a peak at the most extreme jump and at the same time the tail in the jumps direction gets fatter. This is due to the increased probability of reaching prices further away from at-the-money in that specific direction.



(a) $S_t = 1.0$, $e^{r_{up}} = 1.01$ *and* $e^{r_{down}} = \{0.91, 0.93, 0.95, 0.97, 0.99\}$.



(b) $S_t = 1.0$, $e^{r_{up}} = \{1.01, 1.03, 1.05, 1.07, 1.09\}$ *and* $e^{r_{down}} = 0.99$.

Figure 11: Several implied volatilities with different jump sizes generated by the SABR model with parameters in accordance with Table 7 and Figure 11a and 11b. The spot can be seen as a grey dot in the centre and then as $e^{r_{down}}$ *or* $e^{r_{up}}$ *increases in magnitude the shading changes, the same shading scale is used in the implied volatility curve. The lowest curve corresponds to the* σ_{SABR} *and not the implied volatility.*

An observations that has been made throughout Section 5.3 is that σ_{SABR} seems to be the lower limit that the implied volatility with SABR dynamic can exhibit. This seems natural because when an uncertainty is added to the process, the price should increase, and this follows from convexity. But also that implied volatilities far away from at-the-money seems to converge to σ_{SABR}, the effect on the implied volatility when a jump occurs near at-the-money does not effect implied volatility far away from that point.

6 Conclusions and Summary

Plain SABR Compared to a SABR process with Jump The SABR model can not fit a SABR process with jump, i.e. the parameters α, β and ρ can not be selected so that the SABR smile produce a local frown. A single SABR process might be a good model for option prices during normal market conditions, but as this thesis indicates, it might not be a good choice for options with maturities that extend over an anticipated event. In other words, SABR seems not to be enough when marking implied volatility smiles near important news announcements.

Frown and Option Prices The frown implies that options with strikes within an interval, starting at a point somewhat below the jump in the *down* scenario up to strikes somewhat above the jump in the *up* scenario, becomes more expensive relative options that does not include the event in its pricing. This is due to the fact that the probability of these options being in-the-money at maturity increase when the jump is considered. Since with no jump, the most likely spot at maturity is centred around the starting spot because there are no drift, and includes an area around it due to the variance of the process. With a jump, there are instead two areas, one for the *up* case and one for the *down* case, centred around the jump in each direction. They are also overlapping and therefore including also the area between them.

Even though the jump may be highly skewed, the interval in which the prices shows a significant, absolute, increase is relatively symmetric. The reason for this is the risk-neutral probabilities that weights the two outcomes, and gives the more extreme scenario less weight. This is in opposite of the frown, that may be highly skewed towards the more extreme scenario. The reason that the prices do not differ much in absolute values for extreme strikes even though the frown indicates a higher implied volatility is that if an option is deep in(out)-the-money, slight changes in the volatility does not have large impact on the price, since it is still very likely(unlikely) that the option will end up in(out)-the-money. In percentage terms the model increase the prices of options deep out-of-the-money a lot. A large jump on the downside increase the value of deep out-of-the-money put options more and a large jump on the upside has the same effect on deep out-of-the-money call options. This is since the likelihood that these will end up in the money increases drastically, and hence so does the tail risk for an option seller around these types of events. However one should remember that these options are very illiquid in the real market. Also, since the prices are very low, they may also be more sensitive to small numerical errors. Furthermore the larger the jump the higher the peak of the frown and hence also the absolute increase in option prices does increase with the jump.

An increased assumed implied volatility in one of the scenarios for options with maturity $T > T_J^+$ does increase the implied volatility in the same tail. This is due to the fact that an increased assumed implied volatility does increase option prices.

Also the SABR mixture model can generate a frown as the Black and Scholes mixture model. For a symmetric and quite large jump the frown is centred around the spot price observed at t. The frown generated by the SABR dynamics has similarities with the ones that has Black and Scholes dynamics, both skewness and the absolute value of the implied volatility increase in the same direction as the jump and also in the same way as Black and Scholes, see Figure 11a and 9. This behaviour is interesting because locally SABR and Black and Scholes have similar structure but globally, as mentioned, the implied volatility with SABR dynamics seems to converge toward the volatility smile generated by SABR.

Numerical Instability The implied volatility deep in the tails becomes numerically unstable in the Black and Scholes dynamic. The main reason for this is most likely that the implied volatility has no effect on the price of the option. This behaviour was seen both in the lower and upper tail of the implied volatility, but mainly in the lower side at put prices and just on a few places. However, this model is not intended to model the tails but rather the behaviour of the jump. A 40% decrease in a period of a week is highly unlikely and is therefore no big concern. For illustrative purposes and with above argument the implied volatility and the implied probability density function is only calculated with an interval of $S_t \pm 40\%$. Also these bounds are extreme, SABR is plotted within a tighter interval which is approximately $S_t \pm 10\%$, which still is very generous.

Numerical Approximation Since the implementation is made numerically some degree of error will arise. Problems with the implied probability density function was seen at a few places deep in(out)-the-money where they became negative with a factor $< 10^{-10}$, this is again not that important since it is of that magnitude and deep in(out)-the-money where we do not intend to model prices and implied volatilities.

The Model's Usefulness The model may be used as a risk management tool to understand the risks in options before an anticipated event. The research department may have a view of the outcome of such event, and then this model can price the positions given the input from the research team. Similarly it can be used in a sensitivity analysis to see how different outcomes impact the value of the firm's positions. Furthermore it may be used to seize investment opportunities if one can find options that does not price in the event.

Another area in which the model may be useful is to understand the markets view of an event, both the impact and the probability the market assign to different outcomes. Since if the model may be calibrated to market data with satisfying precision, one can estimate p_{up}, e^{up}, e^{down}, $\sigma_{2,up}$ and $\sigma_{2,down}$.

Suggested Further Research A suggested point for further research would be to calibrate the models to market data, to see if the model may capture the behaviour of FX options around events that fits the model in this thesis. One can also adjust the models slightly to match other asset classes and try to calibrate these to market data as well.

This thesis is focused solely on events with binary outcomes. There are events in which one may have a view that the market will react in a third way on a third option, a fourth on a fourth outcome etcetera. As an example one can think of a macroeconomic data release e.g. NonFarm Payrolls in the United States and have a view that if the number is in a certain interval that will have a certain effect on the markets.

Another extension of the model can be to make it compatible with more exotic options and not just European options. Further it could be interesting to implement it for other dynamics e.g. Heston and the Stein and Stein model.

Appendices

A Distribution of z in Section 4.2.1

Here the distribution of z is derived to be

$$\mathcal{N}\left(\left(r^d - r^f - \frac{\sigma_1^2\theta_1 + \sigma_{2,q}^2\theta_2}{2}\right)(T-t), \sqrt{\sigma_1^2\theta_1 + \sigma_{2,q}^2\theta_2}\sqrt{T-t}\right).$$

First we note that W has Gaussian increments and that all other parameters are constant, hence z is a sum of Gaussian variables and is therefore also Gaussian.

Expectation of z W_t is a standard Brownian motion, hence it has an expectation of 0 and variance of t, i.e.

$$\mathbb{E}[W_t] = 0$$
$$Var(W_t) = t.$$

Also, since W has Gaussian increments it follows that $W_{t_2} - W_{t_1} \sim \mathcal{N}(0, t_2 - t_1)$, where $t_2 > t_1$ and all other parameters are constants. Therefore the expectation of z can be calculated as

$$\begin{aligned}
\mathbb{E}[z] &= \mathbb{E}\left[\left(r^d - r^f - \frac{\sigma_1^2\theta_1}{2} - \frac{\sigma_{2,q}^2\theta_2}{2}\right)(T-t) + \sigma_1\left(W_{T_J} - W_t\right) + \sigma_{2,q}\left(W_T - W_{T_J}\right)\right] \\
&= \left(r^d - r^f - \frac{\sigma_1^2\theta_1}{2} - \frac{\sigma_{2,q}^2\theta_2}{2}\right)(T-t) + \sigma_1\,\mathbb{E}[W_{T_J} - W_t] + \sigma_{2,q}\,\mathbb{E}[W_T - W_{T_J}] \\
&= \left(r^d - r^f - \frac{\sigma_1^2\theta_1}{2} - \frac{\sigma_{2,q}^2\theta_2}{2}\right)(T-t) + \sigma_1\left(\mathbb{E}[W_{T_J}] - \mathbb{E}[W_t]\right) + \sigma_{2,q}\left(\mathbb{E}[W_T] - \mathbb{E}[W_{T_J}]\right) \\
&= \left(r^d - r^f - \frac{\sigma_1^2\theta_1}{2} - \frac{\sigma_{2,q}^2\theta_2}{2}\right)(T-t) + \sigma_1\left(0-0\right) + \sigma_{2,q}\left(0-0\right) \\
&= \left(r^d - r^f - \frac{\sigma_1^2\theta_1}{2} - \frac{\sigma_{2,q}^2\theta_2}{2}\right)(T-t).
\end{aligned}$$

Variance of z To be able to derive the variance we first have to state that the covariance of W at two different points in time is

$$Cov(t_1, t_2) = min(t_1, t_2).$$

Now we may calculate the variance

$$
\begin{aligned}
Var(z) =& Var\left(\left(r^d - r^f - \frac{\sigma_1^2\theta_1}{2} - \frac{\sigma_{2,q}^2\theta_2}{2}\right)(T-t) + \sigma_1\left(W_{T_J} - W_t\right) + \sigma_{2,q}\left(W_T - W_{T_J}\right)\right)\\
=& Var\left(\sigma_1\left(W_{T_J} - W_t\right) + \sigma_{2,q}\left(W_T - W_{T_J}\right)\right)\\
=& Var\left(-\sigma_1 W_t + (\sigma_1 - \sigma_{2,q})W_{T_J} + \sigma_{2,q}W_T\right)\\
=& \sigma_1^2 Var(W_t) + (\sigma_1 - \sigma_{2,q})^2 Var(W_{T_J}) + \sigma_{2,q}^2 Var(W_T)\\
& - 2\sigma_1(\sigma_1 - \sigma_{2,q})Cov(W_t, W_{T_J}) - 2\sigma_1\sigma_{2,q}Cov(W_t, W_T)\\
& + 2\sigma_{2,q}(\sigma_1 - \sigma_{2,q})Cov(W_T, W_{T_J})\\
=& \sigma_1^2 t + (\sigma_1 - \sigma_{2,q})^2 T_J + \sigma_{2,q}^2 T - 2\sigma_1(\sigma_1 - \sigma_{2,q})t - 2\sigma_1\sigma_{2,q}t\\
& + 2\sigma_{2,q}(\sigma_1 - \sigma_{2,q})T_J\\
=& \sigma_1^2(T_J - t) + \sigma_{2,q}^2(T - T_J)\\
=& \left(\sigma_1^2\theta_1 + \sigma_{2,q}^2\theta_2\right)(T-t).
\end{aligned}
$$

B Result Black and Scholes Dynamics

B.1 Increasing Time to Maturity with Skewed Jump in Spot

The jump in spot is now skewed towards the downside by decreasing $e^{r_{up}}$ to 1.01 (previously 1.05), all else being equal in accordance with Table 1. This is interesting since it is, for example, a possible scenario in case of an unexpected and unwelcome election outcome. A similar behavior as for the symmetric jump is observed. With a more skewed implied volatility with lower values in the center and higher in the outer regions. The implied probability density function gets a broader support and the bimodal behavior becomes less pronounced as T increase.

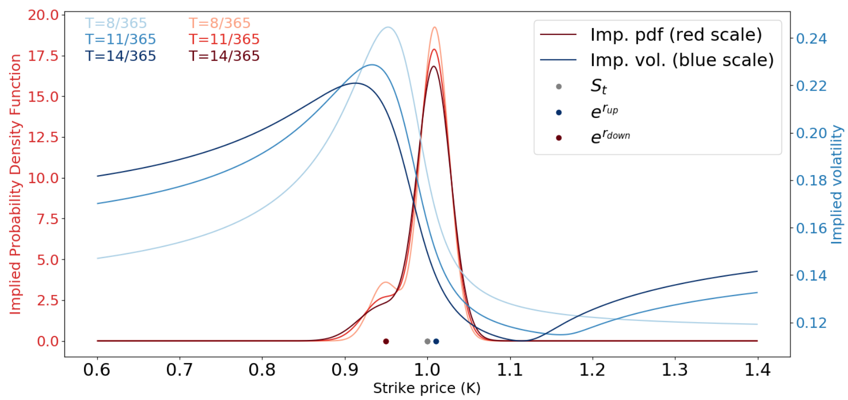


Figure 12: Implied volatility and implied probability density function when $S_t = 1.0$, $e^{r_{up}} = 1.01$, $e^{r_{down}} = 0.95$, $\sigma_{2,down} = 20\%$, $\sigma_{2,up} = 8\%$, $t = 0$, $T_J = 7/365$ and $T = \{8, 11, 14\}/365$. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, gray and a blue dot respectively. The shading of the implied volatility and the implied probability density function corresponds to different time to maturity T which is the only changing parameter. The darker shade corresponds to longer maturities.

Risk Reversal and Butterfly When the jump is skewed on the downside, the risk reversal increase as T increase. This is due to the fact that the implied volatility curve is skewed towards the downside, hence the curve is declining, but gets flatter and flatter in the area of the risk reversal calculations.

The butterfly is positive, which means that the curve is convex in the interval of the 25 delta call and put option. The butterfly is also decreasing with time, indicating that the curvature changes in the interval and flattens.

Variable	Values		
T	8/365	11/365	14/365
Risk Reversal 0.25 delta	-6.28%	-5.24%	-4.62%
Butterfly 0.25 delta	0.87%	0.79%	0.73%

Table 8: Risk Reversal and Butterfly values for increasing maturity and asymmetric jump.

B.2 Changes in Volatility - Increasing $\sigma_{2,down}$

Below the result of increasing the $\sigma_{2,down}$ is presented. The affect is in the opposite direction as for the case with an increasing $\sigma_{2,up}$. As $\sigma_{2,down}$ increases the lower peak gets wider and the density function gets support from lower strikes. The implied volatility increase for lower strikes as $\sigma_{2,down}$ increase, and it does make sense that the prices of options surge as the expected volatility in the market increase.

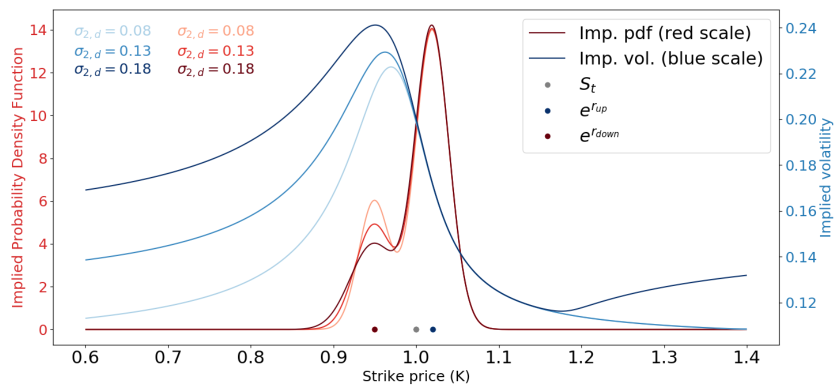


Figure 13: Implied volatility and implied probability density function when $S_t = 1.0$, $e^{r_{up}} = 1.02$, $e^{r_{down}} = 0.95$, $\sigma_{2,down} = \{8\%, 13\%, 18\%\}$, $\sigma_{2,up} = 8\%$, $t = 0$, $T_J = 7/365$ and $T = 14/365$. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, gray and a blue dot respectively. The shading of the implied volatility and the implied probability density function corresponds to different $\sigma_{2,down}$ which is the only changing parameter. The darker shade corresponds to higher volatility in the up scenario.

Risk Reversal and Butterfly How the risk reversal and butterfly is effected by an increase in $\sigma_{2,down}$ can be seen in Table 9. The direction in which they move is the opposite to the previous case, with an increase in $\sigma_{2,up}$. The butterfly switch sign in, from negative to positive as the volatility increase. Hence, the curve goes from first being concave to being convex as $\sigma_{2,down}$ changes.

Variable	Values		
$\sigma_{2,down}$	0.08	0.13	0.18
Risk Reversal 0.25 delta	-5.49%	-5.96%	-6.66%
Butterfly 0.25 delta	-0.27%	-0.08%	0.17%

Table 9: Risk Reversal and Butterfly values for increasing $\sigma_{2,down}$.

B.3 Changes in Spot Jump - Decreasing *down* Scenario

As the jump size increase in the *down* scenario, the downside risk increase for an option seller. Since it is now more likely that an option with a lower strike will be exercised. This is clear from examine the probability density function, that gets an increasing mass for lower strikes. The implied volatility gets skewed to the left and a heavier tail occurs in this direction, which indicates that an option seller wants a higher premium for an option with lower strike, since the risk has increased.

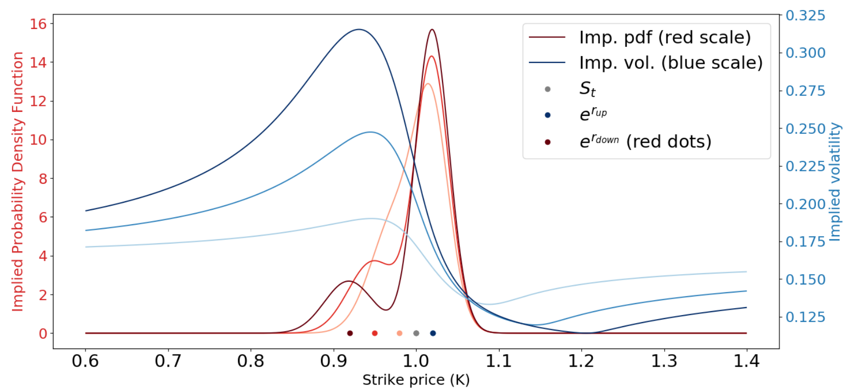


Figure 14: Implied volatility and implied probability density function when $S_t = 1.0$, $e^{r_{up}} = 1.02$, $e^{r_{down}} = \{0.92, 0.95, 0.98\}$, $\sigma_{2,down} = 20\%$, $\sigma_{2,up} = 8\%$, $t = 0$, $T_J = 7/365$ and $T = 14/365$. spot and $e^{r_{up}}$, can be seen as a gray and a blue dot. The only changing parameter is $e^{r_{down}}$ which is presented as shaded red dots, as the magnitude of the jump increases the colour gets darker. This grading scale is used in the implied volatility and the implied probability density function as well.

Risk Reversal and Butterfly The risk reversal is increasingly negative in this scenario. Indicating that the implied volatility gets more skewed, as the jump in the *down* scenario gets more drastic. In other words, the difference between implied volatility for the 0.25 delta put and call does increase.

The butterfly starts at a low, negative, value and becomes positive as the jump increase. One can see the steepening in the volatility curve, causing this, in Figure 14.

Variable	Values		
$e^{r_{down}}$	0.98	0.95	0.92
Risk Reversal 0.25 delta	-2.87%	-7.02%	-11.93%
Butterfly 0.25 delta	-0.08%	0.29%	1.40%

Table 10: Risk Reversal and Butterfly values for increasing $e^{r_{down}}$.

C Result SABR Dynamic

C.1 SABR Dynamic - 7 Days to Maturity

In this section time to maturity T is decreased from 30 to 7 days, all else being equal to Table 7. In comparison with section 5.3.1 the difference is that the local frown becomes more pronounced and because of that the tails gets thinner, otherwise the behavior is similar. This effect occurs since the probability of reaching a price deeper in-the-money or deeper out-the-money decrease as time to maturity decrease. This follow the same behaviour on the local implied volatility as the implied volatility with Black and Scholes dynamic has, compare Figure 10 and 15 with Figure 7, where these similarities are illustrated. The skewed jumps affect the implied volatility in the same way as in *Skewed Jumps*, Section 5.3.1, but with a more pronounced peak that follows from the shorter time to maturity as explained earlier.

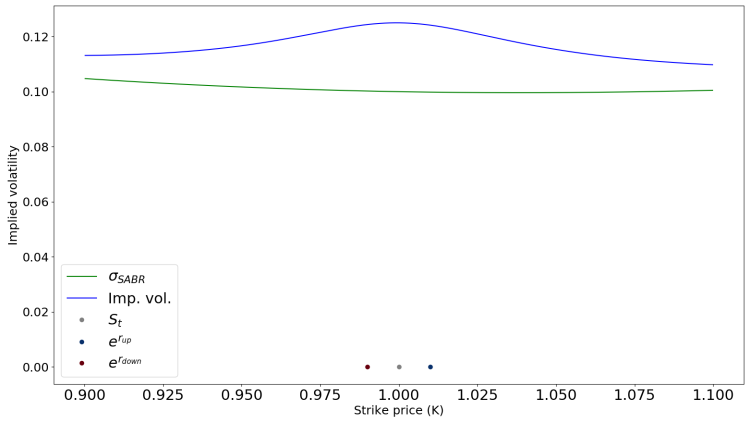


Figure 15: Volatility generated by the SABR model with parameters in accordance with Table 7 except for T which now is $7/365$*, and the implied volatility after a jump with* $S_t = 1.0$*,* $e^{r_{up}} = 1.01$ *and* $e^{r_{down}} = 0.99$*.*

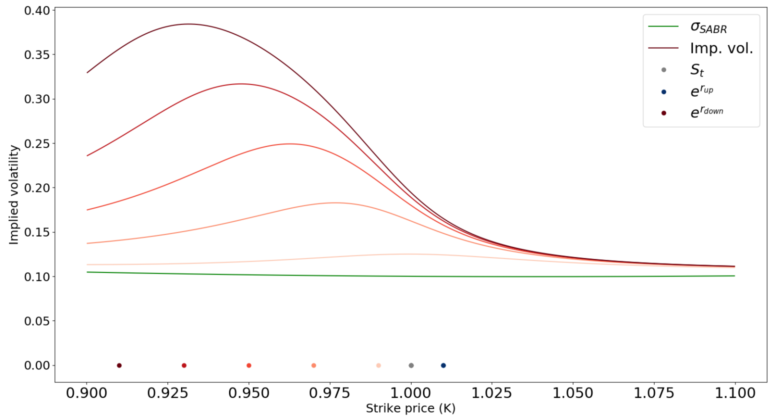


Figure 16: Several implied volatility curves generated by the SABR model with parameters in accordance with Table 7 except for T which now is $7/365$, and the implied volatility after a jump. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as red, grey and a blue dot respectively where $S_t = 1.0$, $e^{r_{up}} = 1.01$ and $e^{r_{down}} = \{0.91, 0.93, 0.95, 0.97, 0.99\}$. The lowest curve corresponds to the σ_{SABR} and not the implied volatility.

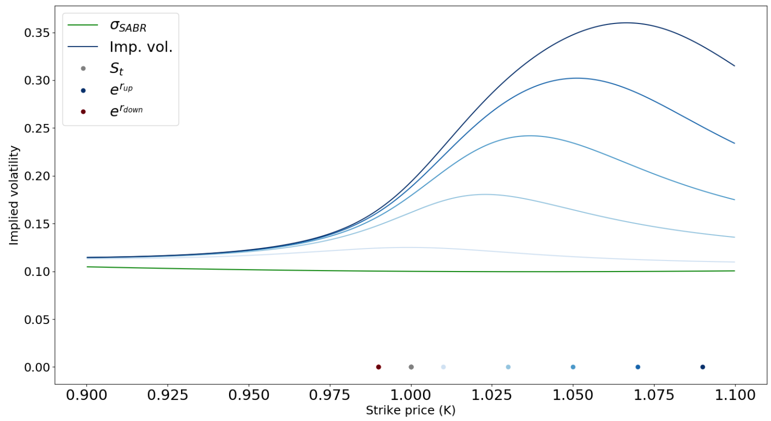


Figure 17: Several implied volatility curves generated by the SABR model with parameters in accordance with Table 7 except for T which now is $7/365$, and the implied volatility after a jump. $e^{r_{down}}$, spot and $e^{r_{up}}$ can be seen as a red, grey and blue dot respectively where $S_t = 1.0$, $e^{r_{up}} = \{1.01, 1.03, 1.05, 1.07, 1.09\}$ and $e^{r_{down}} = 0.99$. The lowest curve corresponds to the σ_{SABR} and not the implied volatility.

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