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Volatility Spillovers between Stock and Bond Returns

Evidence from Nordic Countries

by

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Abstract

This paper uses a bivariate BEKK model to estimate volatility spillover effects between stock and bond markets for the Nordic countries Sweden, Denmark, Finland and Norway. Daily log returns between the years 2001 and 2018 are analyzed. No spillover effect from the bond to the stock market is found in any of the countries which is a result that is unusual compared to a large part of the preexisting literature. Regarding spillover effects from the stock market to the bond market, significant effects are found in Sweden, Denmark and Finland but not in Norway. Volatility Impulse Response Functions (VIRFs) are computed to present the results more intuitively. The VIRFs describe the effects on stock and bond markets after being hit by a shock which originates from either of the markets. The main conclusions drawn from the VIRFs are that a shock originating from the stock market has a persistent effect on bond market volatilities and that stock market to bond market volatility spillovers are higher when the financial markets are more unstable.

Keywords: Volatility spillover, stocks, bonds, BEKK, Volatility Impulse Response Function

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1 Introduction

With stocks and bonds being the two main asset classes traded and held by portfolio managers around the world, the relevance of studying the interdependence of stock and bond markets is high. Since the degree of risk in the two asset classes differs, asset managers often use bonds to counter the risk that stocks bring to the portfolio. This makes the relation between volatilities for stock and bond markets an interesting topic to study.

Research on the interdependence between stock and bond markets has a long history. Shiller (1982), Barsky (1989) and Shiller and Beltratti (1992) all provide and discuss models on the relationship between stock and bond markets. In more recent years, modelling the conditional covariance of financial data has been given more attention from researchers after the advancement of the ARCH framework started by Engle (1982) and improved in the multivariate setting by Bollerslev et al. (1988), Bollerslev (1990) and Engle and Kroner (1995) among others. Naturally, most of the research is focusing on markets in larger economies. Glabadanidis and Scruggs (2003) estimate the dynamic variances and covariance of the returns from the NYSE-AMEX index and the US one-month Treasury bill using monthly data in an interval spanning from January 1953 to December 1997. They find that bond return shocks spill over to stock market variance. However, their results indicate that there are no spillover effects from the American stock market to bond market variance. Another paper from the US context that focuses on the covariance is written by De Goeij and Marquering (2004). The authors use a diagonal VECH-model on returns on the 1-year and 10-year Treasury bond as well as the S&P 500 index and the NASDAQ index between January 1982 and August 2001. The main finding of this paper is that both variances and covariances react differently to positive and negative shocks to the stock and bond market. Covariance between the stock and bond market is higher after a negative shock to the stock market than after a positive shock. Moreover, it is found that cross effects in asymmetries also influence the covariances. Dean et al. (2010) also look at asymmetric effects in volatility spillovers between the two Australian indices ASX All Ordinaries Total Return Index and the All Lives Government Bond Total Index during 1992-2006. Similarly to Glabadanidis and Scruggs (2003), the authors find that there was a spillover effect in volatility from the bond market to the equity market, but no effects in the other direction are found. They also discover that the size of the spillover is different for different combinations of signs of the shocks for the two markets.

Wang et al. (2013) take a broader approach than previous articles that have been mentioned by not only focusing on single country markets but rather all of the G7 countries together with the BRICS countries for the years 1989-2012. Using what they refer to as an LM-GARCH model, they find volatility spillovers between the stock and bond market in both directions for France, Brazil and South Africa; and one-directional spillovers from the bond to the stock market in the US, the UK and Germany. Also looking at the G7 countries, Liow (2015) calculates the so-called volatility spillover index (developed by Diebold and Yilmaz, 2012) for the asset classes public real estate, stocks, bonds, money and currency between January 1997 and December 2013. A BEKK model is the underlying GARCH model used to calculate these indices and the general finding is that the stock market is the primary source of spillovers for these asset classes. Regarding volatility spillover effects between stock and bond markets, his results are mixed. In the US and Canada, evidence of unidirectional spillover effects from the stock to the bond market was found. Meanwhile, in the UK, Germany and Italy spillovers going in the other direction were found. For the rest of the G7 countries, volatility spillover effects in neither direction were found. Boujelbene and Saadaoui (2014) also study volatility spillovers for several countries at the same time, namely some emerging markets (Argentina, Australia, Greece, Hong Kong, Hungary, Mexico, Peru, Spain, Turkey and Poland) between August 2009 and January 2011 using a BEKK-model. They conclude that there are spillovers in both directions in the studied markets.

Looking at some of the few papers that look at Nordic countries we have Byström (2004) who studies blue-chip stock indices for Sweden, Denmark, Finland and Norway. He constructs an orthogonal GARCH model to forecast stock market volatilities for these countries during the Asian financial crisis of 1997-1998. Booth et al. (1997) who also only looks at Nordic blue-chip stock indices between May 1988 and June 1994, use a multivariate EGARCH model to find that volatility spillover effects between the countries are weak if anything. Christiansen (2010) investigates whether there is volatility spillover from the US as well as European aggregate stock and bond markets into some European markets including Denmark and Sweden for the period from January 1988 to December 2003. Her results indicate that volatility from US and European bond markets spills over to European national bond markets after the year 1999. In the European national stock market case, there is volatility spillover from US and European stock markets after 1999.

Hafner and Herwartz (2006) alter the impulse response function methodology to fit for volatility models into what they call a Volatility Impulse Response Function (VIRF) to get an impression of how exchange rate volatilities are affected by specific shocks. The VIRF methodology has since then been used as a useful tool for research on volatility spillover effects. Allen et al. (2017) combine a BEKK model with a VIRF to analyze volatility spillover effects between returns from the New York Stock Exchange Index and the FTSE 100 index from January 2005 to January 2015. They find that negative shocks from an asymmetric BEKK specification have a more considerable initial effect on volatilities than shocks in a symmetric setting. However, shocks in a symmetric specification seem to last longer. Jin (2015) uses a VIRF together with a BEKK model to evaluate spillover effects between the three stock market indices MSCI China index, MSCI, Hong Kong index and MSCI Taiwan index between July 1993 and June 2013. He finds spillover effects between the markets and that a shock the same size as the 1997-1998 Asian financial crisis or the 2008 global financial crisis would have a lower impact on conditional variances today compared to when these financial crises occurred. Similarly, Jin and An (2016) studies volatility spillover between stock markets in the United States and the BRICS-countries mainly during the global financial crisis using a bivariate BEKK model together with VIRFs for the sample period July 1997 to December 2013. They find that the BRICS countries that were more connected to the US had more strong reactions in volatility to the crisis. Further, they find that if a shock similar to that of the global financial crisis happened today, reactions in volatilities on stock markets in the BRICS countries would be of more vast proportions.

The purpose of this paper is to analyze whether or not there exist volatility spillover effects between stock and bond markets for the Nordic countries except Iceland[1]. Further, this paper aims to analyze the magnitude of these potential volatility spillover effects and how they change over time. The data consists of stock and bond returns from Sweden, Denmark, Finland and Norway between September 2001 and April 2018. Results could be of interest to portfolio managers when calculating risk measures and setting their strategies as well as regulators who should be aware of any potential effect that regulation on a particular market could have on another market. Moreover, determining the sizes of any possible volatility spillover effects between markets and how these effects change over time could be crucial when figuring out the potential effects of a financial crisis. No previous literature that analyzes Nordic financial markets or stock-bond interdependencies using a VIRF has been

[1] The reason for omitting Iceland from the analysis is that Iceland is a much smaller economy compared to the other Nordic countries. Comparing the Nordic countries excluding Iceland thus is more compatible. Correspondingly, previous studies focusing on the Nordic countries also tend to omit Iceland (e.g. Booth et al., 1997 and Zhang, 2015).

found. Therefore the main contribution of this paper is the application of a BEKK model together with a VIRF on stock and bond returns which to the author's knowledge has been missing from the literature. Also, this paper is the first to model volatility spillovers between stock and bond markets in a Nordic context.

The method for doing this is based on a multivariate extension of the GARCH methodology, called BEKK, which was developed by Engle and Kroner (1995). The BEKK-model has an advantage towards other multivariate GARCH models in that the presence of volatility spillover effects can be attained directly from the coefficients of the model. However, the BEKK model does not provide an intuitive interpretation concerning the size of the spillover effects. A way to get a more interpretive view of the BEKK model estimates is to combine the model with the use of a Volatility Impulse Response Function which will be done in this paper.

The remainder of this thesis is structured as follows. The next section presents and describes the data used for the analysis as well as the underlying theory of the BEKK model and the Volatility Impulse Response Function. In section 3, results and estimations from the different models are presented and discussed. Finally, section 4 concludes.

2 Methodology

2.1 Data

All data were collected from Thomson Reuters Datastream. Following the methodology used in similar research (Zhang, 2015 and Ajayi et al., 2018) daily stock indices for blue-chip stocks were used for the analysis. This means that data from OMXS30 (Sweden), OMXC20 (Denmark), OMXH25 (Finland) and OBX (Norway) was gathered. For the bond markets, the daily Benchmark 5 Year Datastream Government Indices for each country were used. A choice was made to set the starting point of the time series evaluated to 3rd September 2001. The argument for doing this was that in September 2001 the Finnish index FOX was replaced by the new index and alterations to how it was calculated was made (Zhang, 2015). The endpoint for the analysis was set to be 3rd April 2018. Observations during holidays when no trading was done were deleted from the sample. This leads to 4118 observations for each of the eight variables, i.e., 32 944 observations in total.

Daily log-returns were generated for all stock- and bond indices. Figure 1 presents these daily log-returns below. The figure indicates that there is volatility clustering for all the countries and both asset classes. It is not surprising to see that the most volatile periods for all the countries come around the financial crisis of 2008. Furthermore, the return series for the stock indices seem to start with a cluster of rather large volatility, likely stemming from the dot-com bubble. We see that the periods with the lowest volatility were 2004-2005, around 2014 and the most recent period in 2018. Graphs comparing the indices in levels of the stock and bond markets can be found in Figure A1 in the appendix.

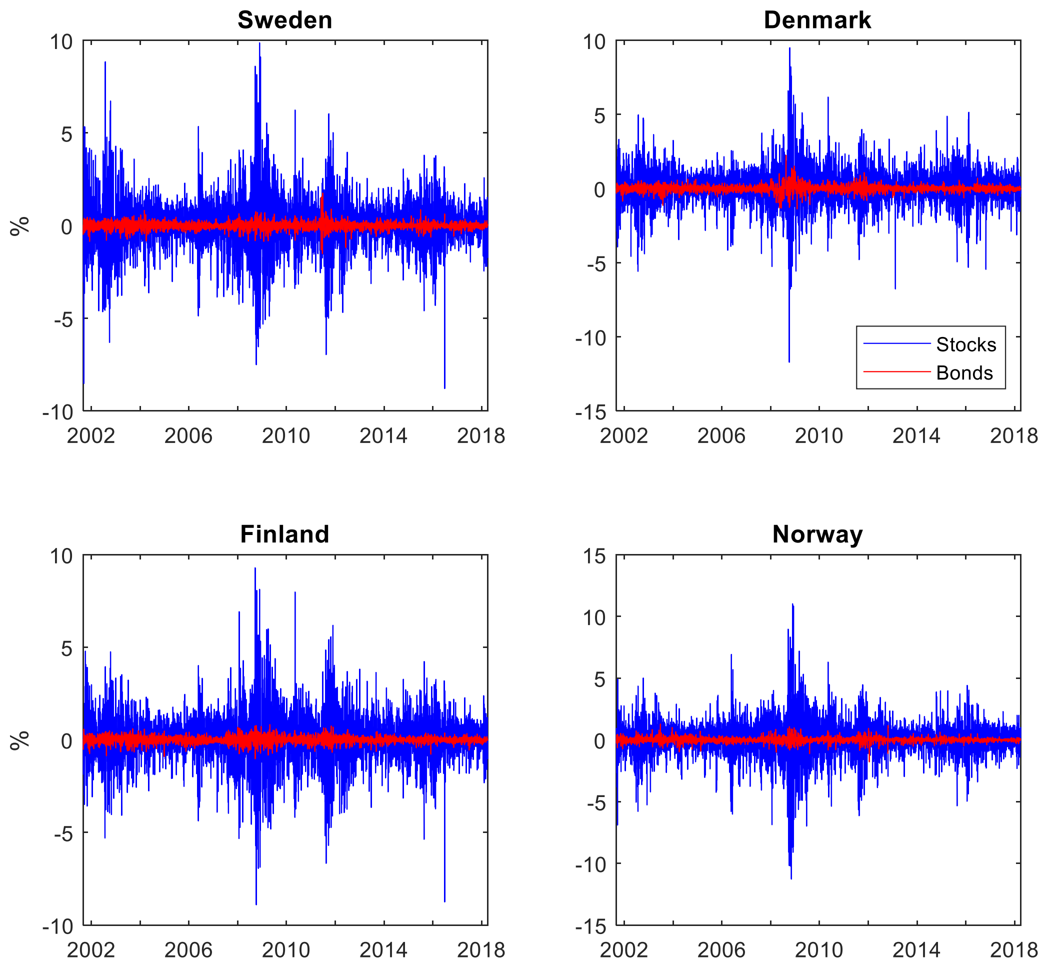

Figure 1: Time series of daily log returns

Table 1 displays descriptive statistics for the different indices. As expected by the efficient market hypothesis, both the mean and the median are very close to or equal to zero in all cases. What is also expected is the fact that bond returns move in a shorter interval than stock returns and that their standard deviations are lower. Moreover, Table 1 indicates that all time series are leptokurtic. This motivates the use of distributional assumptions that account for heavy tails such as a Student t-distribution. The correlations presented in Table 1 are correlations measured over the whole sample period between the stock and bond returns for each country. In all four countries, stock and bond returns tend to move in opposite directions. Sweden and Finland are the two countries that have the strongest negative correlation between their respective stock and bond markets. Table 1 also shows correlations between squared returns of each country's respective stock and bond market over the whole sample period. These correlations measure how volatilities move together and are positive for all countries. Finland has by far the highest correlation while Denmark has the lowest. Sweden and Norway have correlations closer to Denmark than to Finland. Overall, the fact that there is correlation between stock and bond market variances motivates a further analysis of volatility spillovers.

Table 1: Descriptive statistics of daily log-returns

	Sweden	Denmark	Finland	Norway
	STOCKS			
Mean (%)	0,01	0,03	0,03	0,02
Median (%)	0,01	0,08	0,06	0,09
Maximum (%)	9,87	9,5	9,29	11,02
Minimum (%)	-8,8	-11,72	-8,91	-11,28
Std. Dev. (%)	1,44	1,29	1,39	1,55
Skewness	0,02	-0,28	-0,09	-0,55
Kurtosis	7,5	8,69	6,91	9,93
	BONDS			
Mean (%)	0	0	0,01	0
Median (%)	0	0	0	0
Maximum (%)	1,75	2,24	0,82	1,28
Minimum (%)	-1,52	-1,38	-1,02	-1,79
Std. Dev. (%)	0,19	0,2	0,18	0,19
Skewness	0,02	0,18	-0,21	-0,04
Kurtosis	9,04	13,3	5,3	9,59
Correlation	-0,34	-0,2	-0,37	-0,23
Correlation between squared returns	0,2	0,12	0,38	0,16
Obs	4118	4118	4118	4118

2.2 Model framework

2.2.1 Mean equation

Assuming that the conditional mean follows a VAR (p) structure, the mean equation to model stock and bond returns can be written as follows:

$$\boldsymbol{r}_t = \boldsymbol{\alpha_0} + \sum_{n=1}^{p} \boldsymbol{\alpha_n} \boldsymbol{r_{t-n}} + \boldsymbol{\epsilon_t}, \tag{1}$$

where $\boldsymbol{r_t}$ is a 2 × 1 vector of returns at time t. The first element in this vector is the element for the stock market in one of the countries Sweden, Denmark, Finland or Norway. The second cell in the $\boldsymbol{r_t}$ vector represents the returns at time t on the bond market for the same country. $\boldsymbol{\alpha_0}$ is a 2 × 1 vector and $\boldsymbol{\alpha_n}$ are 2 × 2 vectors of constants. $\boldsymbol{\epsilon_t}$ is a 2 × 1 vector of error terms with zero mean and conditional covariance matrix $\boldsymbol{H_t}$.

2.2.2 Covariance equation

In the multivariate GARCH framework, the covariance matrix $\boldsymbol{H_t}$ can be modelled in many ways. When the focus as is in this case lies on estimating volatility spillover effects between variables, the BEKK model is preferable since the spillover effects can be obtained directly from the estimated coefficients. Additionally, the BEKK model has the advantage that it guarantees positive definitiveness of $\boldsymbol{H_t}$. In this version of the BEKK model, the covariance matrix $\boldsymbol{H_t}$ is modelled as:

$$\boldsymbol{H_t} = \boldsymbol{\Omega\Omega'} + \mathbf{A}'\boldsymbol{\epsilon_{t-1}\epsilon'_{t-1}}\mathbf{A} + \mathbf{B}'\boldsymbol{H_{t-1}}\mathbf{B}, \tag{2}$$

where $\boldsymbol{\Omega}$ is a 2 × 2 lower triangular matrix and both **A** and **B** are 2 × 2 matrices. The residual vector $\boldsymbol{\epsilon_{t-1}}$ is obtained from Equation 1. Note that Equation 2 is a BEKK(1, 1). This specification that only looks back one period is the main one used in the literature (e.g. Boujelbene and Saadaoui, 2014 and Chen et al., 2012) mainly because of the large increase in the number of parameters that has to be estimated when including a higher number of lags.

Equation 2 implies that the variance of the stock market at time t is modeled as:

$$h_{11t} = \omega_{11}^2 + a_{11}^2\epsilon_{1t-1}^2 + 2a_{11}a_{21}\epsilon_{2t-1}\epsilon_{1t-1} + a_{21}^2\epsilon_{2t-1}^2 + b_{11}^2h_{11t-1} + 2b_{11}b_{21}h_{21t-1} + b_{21}^2h_{22t-1}, \tag{3}$$

while the variance of the bond market at time t is modeled as:

$$h_{22t} = \omega_{21}^2 + \omega_{22}^2 + a_{12}^2\epsilon_{1t-1}^2 + 2a_{12}a_{22}\epsilon_{2t-1}\epsilon_{1t-1} + a_{22}^2\epsilon_{2t-1}^2 + b_{12}^2h_{11t-1} + 2b_{12}b_{22}h_{21t-1} + b_{22}^2h_{22t-1}. \quad (4)$$

These two equations tell us that the parameters that capture any eventual spillover effects are the off-diagonal elements of **A** and **B**, that is, a_{21} and $b_{21}$for spillover effects from the bond market to the stock market. Similarly, a_{12} and b_{12} captures spillover effects from the stock market to the bond market. Because the terms are quadratic, the sizes of these parameters are difficult to interpret. The volatility impulse response function discussed in the next section is therefore useful to use together with a BEKK model to get an even more fruitful analysis of volatility spillovers. Note also that in Equations 3 and 4, $2b_{11}b_{21}$and $2b_{12}b_{22}$ capture the effect that a change in covariance between two variables has on the proceeding period's volatility and that the parameters $2a_{11}a_{21}$ and $2a_{12}a_{22}$ lacks an intuitive interpretation. In some cases, **A** and **B** can be defined to be diagonal. This makes an estimation of the model easier because of the lesser amount of parameters that need to be estimated but with a_{12}, a_{21}, $b_{12}$and b_{21}all being restricted to be zero, no volatility spillover effects can of course be found. Anyhow, in this paper a diagonal BEKK is used as a robustness check to see if estimates are sensitive to the exact specification used or not.

Parameters from both the mean and covariance equation are then estimated using the maximum likelihood estimation method where the residuals are assumed to be t-distributed to account for the fat tails displayed in Table 1.

2.2.3 Volatility Impulse Response Function

A drawback with the BEKK-model is, as noted earlier that an intuitive interpretation of the size of the estimated spillover effects is difficult to make. This difficulty motivates the use of a so-called Volatility Impulse Response Function (VIRF) first proposed by Hafner and Herwartz (2006), which define the volatility impulse response at time $t, \boldsymbol{V_t}(\boldsymbol{\epsilon_0})$, as the following:

$$\boldsymbol{V_t}(\boldsymbol{\epsilon_0}) = \mathrm{E}[\mathrm{vech}(\boldsymbol{H_t})|\boldsymbol{\epsilon_0}, \boldsymbol{I_{-1}}] - \mathrm{E}[\mathrm{vech}(\boldsymbol{H_t})|\boldsymbol{I_{-1}}], \quad (5)$$

where $\boldsymbol{\epsilon_0}$ is the initial shock and $\boldsymbol{I_{-1}}$ is the information set at the period prior to the one when this shock occurs. The vech-operator is an operator that stacks the lower triangular of a matrix into a vector i.e., in this case, it stacks the two variances and the covariance in $\boldsymbol{H_t}$ into a 3×1 vector. To incorporate the estimated BEKK model with the VIRF framework, Equation 2 is rewritten as a VEC(1, 1) model (Bollerslev et al., 1988) in the following way:

$$\text{vech}(\boldsymbol{H_t}) = \text{vech}(\boldsymbol{\Omega}) + \mathbf{A}^*\text{vech}(\boldsymbol{\epsilon_{t-1}\epsilon'_{t-1}}) + \mathbf{B}^*\text{vech}(\boldsymbol{H_{t-1}}), \tag{6}$$

where $\mathbf{A^*}$ and $\mathbf{B^*}$ are 3 × 3 matrices. The relation between these two matrices and the parameter matrices **A** and **B** from Equation 2 is:

$$\mathbf{A}^* = \boldsymbol{L_2}(\mathbf{A}' \otimes \mathbf{A}')\boldsymbol{D_2} \tag{7}$$

$$\mathbf{B}^* = \boldsymbol{L_2}(\mathbf{B}' \otimes \mathbf{B}')\boldsymbol{D_2}, \tag{8}$$

where $\boldsymbol{D_2}$ is the duplication matrix such that $\text{vec}(\mathbf{Z}) = \boldsymbol{D_2}\text{vech}(\boldsymbol{Z})$ for any 2 × 2 matrix $\mathbf{Z}$ and similarly $\boldsymbol{L_2}$ is the elimination matrix such that $\text{vech}(\mathbf{Z}) = \boldsymbol{L_2}\text{vec}(\boldsymbol{Z})$.[2] The $\otimes$ operator denotes the Kronecker product. When applying Equation 6 to the VIRF, the following expression for the initial volatility impulse response is obtained by Hafner and Herwartz (2006):

$$\begin{aligned} \boldsymbol{V_1}(\boldsymbol{\epsilon_0}) &= \mathbf{A}^*\left\{\text{vech}\left(\boldsymbol{H_0^{1/2}\xi_0\xi'_0H_0^{1/2}}\right) - \text{vech}(\boldsymbol{H_0})\right\} \\ &= \mathbf{A}^*\boldsymbol{D_2^+}\left(\boldsymbol{H_0^{1/2}} \otimes \boldsymbol{H_0^{1/2}}\right)\boldsymbol{D_2}\text{vech}(\boldsymbol{\xi_0\xi'_0} - \boldsymbol{I_2}), \end{aligned} \tag{9}$$

where $\boldsymbol{D_2^+}$ is the Moore-Penrose inverse of $\boldsymbol{D_2}$ and $\boldsymbol{I_2}$ is the 2 × 2 identity matrix.[3] Further, for $t \geq 2$ the volatility impulse response can be written as:

$$\begin{aligned} \boldsymbol{V_t}(\boldsymbol{\epsilon_0}) &= (\mathbf{A}^* + \mathbf{B}^*)^{t-1}\mathbf{A}^*\boldsymbol{D_2^+}\left(\boldsymbol{H_0^{1/2}} \otimes \boldsymbol{H_0^{1/2}}\right)\boldsymbol{D_2}\text{vech}(\boldsymbol{\xi_0\xi'_0} - \boldsymbol{I_2}) \\ &= (\mathbf{A}^* + \mathbf{B}^*)\boldsymbol{V_{t-1}}(\boldsymbol{\xi_0}). \end{aligned} \tag{10}$$

Looking back at Equation 5, volatility impulse response can be interpreted as the difference between two expectations of $\boldsymbol{H}_t$, one conditioning on the initial shock $\boldsymbol{\epsilon_0}$ as well as history, and one conditioning only on history. The expectation conditioning only on history is often referred to as the *baseline expectation* of volatility. Consequently, when computing a VIRF, one has to decide on the size of the initial shock $\boldsymbol{\epsilon_0}$. It is common to set $\boldsymbol{\epsilon_0}$ as one of its estimated past values. This methodology can be good for analysing the effects of shocks that occurred on some particularly interesting dates (see for example Jin, 2015 and Jin and An, 2016) but has a disadvantage in that it does not give a general view. Furthermore, this methodology does not distinguish between shocks originating from either of the markets. Instead, this paper analyses a shock drawn from the underlying distribution of the residuals.

[2] The vec operator stacks an m × n matrix into a mn × 1 column vector.
[3] The VIRF computation has been made in RATS using code originally intended to replicate the study made by Hafner and Herwartz (2006) but modified to fit the purpose of this paper.

Because the two markets in this case are correlated, shocks that occur in them are not independent. Thus, constructing a shock where one of the residuals is set equal to zero would be unrealistic. Therefore, to construct a shock that originates from only one market, one has to implement the following procedure also presented by Hafner and Herwartz (2006). First, they define the residual vector $\boldsymbol{\epsilon_t}$ as:

$$\boldsymbol{\epsilon_t} = \boldsymbol{H_t^{1/2}}\, \boldsymbol{\xi_t}, \tag{11}$$

where $\boldsymbol{\xi_t}$ is an i.i.d. random vector whose elements will be referred to as *news* and $\boldsymbol{H_t^{1/2}}$ is calculated using a Jordan decomposition of $\boldsymbol{H_t}$. That is,

$$\boldsymbol{H_t^{1/2}} = \boldsymbol{\Gamma_t \Lambda_t^{1/2} \Gamma'_t}, \tag{12}$$

where $\boldsymbol{\Gamma_t}$ is a matrix consisting of all eigenvectors of $\boldsymbol{H_t}$ and $\boldsymbol{\Lambda_t}$ is a diagonal matrix with the corresponding eigenvalues along the diagonal. In this setup, one can set the news vector $\boldsymbol{\xi_t}$ to have a zero in one of its elements and any other value in the other. As a consequence, a realistic shock $\boldsymbol{\epsilon_t}$ corresponding to news from only one market can be constructed via Eq.11. For this paper, while keeping the news of one market at zero, the news hitting the other market was set to be the 99^{th} percentile of the t-distribution (following the assumption that the residuals are t-distributed) which is equal to 2,3273. Note that this corresponds to a positive shock occurring on average twice or three times every year. The BEKK model described in the previous section is symmetric. Whether the shock that hits the system is positive or negative therefore has no effect on the reaction. Constructing news as the top percentile is as a result sufficient; there is no need to construct any negative news. To obtain responses that are comparable over time as well as over stock and bond markets, each response is normalized with the estimated conditional volatility the day after the shocks occurs. These normalized responses are hence expressed in percentage terms.

Since correlations are time-dependent, VIRF calculations will be different depending on what date the initial shock occurs. To get a more general VIRF, a shock that takes place in the middle of each of the 66 quarters of the sample period is constructed. For each response, the average of the normalized responses from all quarters is then calculated.

3 Results and analysis

The mean equation was after calculating the Bayesian information criterion (BIC) for different VAR (*p*) models and an equation containing only constants selected to be a VAR (1) model for all countries. Results from the VAR (1) estimation can be found in Table A1 in the appendix. To determine whether or not there are any ARCH effects in the residuals, Engle's ARCH test was used. The null of homoscedastic residuals was rejected for all variables. This is a result that motivates the modeling of conditional covariance with a GARCH-model. Jarque-Bera tests on the residuals from the VAR (1) estimation strongly rejected the null hypothesis of Gaussian distribution in the residuals for all variables which further motivates the distributional assumption of a t-distribution.

3.1 BEKK estimation

Table 2: Parameter estimates from the BEKK estimation

	Sweden	Denmark	Finland	Norway
a_{11}	0,2753***	0,3442***	0,3110***	0,3180***
	(0,0176)	(0,0206)	(0,0159)	(0,0150)
a_{12}	-0,0084***	-0,0002	-0,0045***	-0,0022
	(0,0019)	(0,0019)	(0,0015)	(0,0015)
a_{21}	0,0376	-0,1233	-0,0079	0,0133
	(0,1069)	(0,1412)	(0,1033)	(0,0812)
a_{22}	0,1711***	0,1521***	0,1776***	0,1666***
	(0,0145)	(0,0121)	(0,0123)	(0,0090)
b_{11}	0,9581***	0,9234***	0,9418***	0,9393***
	(0,0052)	(0,0093)	(0,0060)	(0,0057)
b_{12}	0,0021***	-0,0012*	0,0007	-0,0002
	(0,0005)	(0,0006)	(0,0005)	(0,0005)
b_{21}	-0,0347	-0,0194	-0,0378	0,0018
	(0,0347)	(0,0400)	(0,0302)	(0,0210)
b_{22}	0,9821***	0,9827***	0,9815***	0,9843***
	(0,0039)	(0,0024)	(0,0027)	(0,0017)
ω_{11}	0,0012***	0,0022***	0,0017***	0,0019***
	(0,0002)	(0,0002)	(0,0002)	(0,0002)
ω_{21}	0,0000	0,0000	0,0000	0,0000
	(0,0000)	(0,0000)	(0,0000)	(0,0000)
ω_{22}	0,0001***	0,0001***	0,0001***	0,0001***
	(0,0000)	(0,0000)	(0,0000)	(0,0000)

Note: Parameters correspond to elements in the **A**, **B** and **Ω** matrices of Equation 2. For the elements a_{ij}, b_{ij} *and* ω_{ij}, i,j = 1 (stocks) and 2 (bonds). Since **Ω** is lower triangular, only three estimates are presented. Statistical significance is denoted by * (10%), ** (5%) and *** (1%). Standard errors are in parenthesis.

Table 2 presents the estimation results from the BEKK model discussed in section 2. The results for the diagonal elements of **A**, **B** and **Ω** all display a high degree of statistical significance for all four countries. The estimates of the diagonal elements of the **B** matrix are all close to one indicating a high degree of volatility clustering both for stock and bond markets in all the countries analyzed. Looking at the estimates for the off-diagonal elements of the **A** and **B** matrices, fewer of them are statistically significant. Spillover effects from the bond market to the stock market (estimates a_{21} and b_{21}) are not found to be statistically significant in any country. Regarding volatility spillover effects from the stock market to the bond market, results are mixed. In Sweden, both the estimates a_{12} and b_{12} are significant at the 1% level, strongly implying the presence of a spillover effect from the stock market to the bond market. Results for Denmark and Finland also suggest unidirectional volatility spillover effects from the respective stock markets to bond markets. For both countries, one of the two parameters indicating spillover effects from the stock market to the bond market is significant while the other is not. Meanwhile, results for Norway do not suggest any spillover effects in either direction. These results are partly in line with previous research in the sense that the existence of any spillover effects from the stock market to the bond market seem to differ from country to country (see Scruggs and Glabadanidis (2003), Dean et al. (2010) and Wang et al. (2013) among others). In any case, results from all countries differ from previous literature in the sense that no spillovers from the bond market to the stock market have been found at all.

As robustness checks, simpler GARCH models were also estimated. In Table A2 and A3 in the appendix, results from both univariate GARCH(1, 1) and a diagonal BEKK(1, 1) are presented. The diagonal BEKK estimates do not differ notably compared to the full BEKK estimates. Comparing the full BEKK to the univariate GARCH estimates, a difference can be noted in the ARCH effects while GARCH effects stay approximately the same.

3.2 Volatility Impulse Response Functions

In this section, normalized VIRFs are presented in two different ways. First, the average VIRFs calculated from the 66 quarters over the whole period are presented. As noted earlier this gives a general view of how each market reacts to different kinds of shocks. Following that, time series of the initial responses in volatilities after news in either the stock or bond markets are presented. The reason for doing this is to get an idea of how volatility spillovers change over time and potentially how they are affected by specific historical events, e.g., the global financial crisis.

3.2.1 General VIRF

Volatility impulse responses coming from news from the stock and bond market respectively are presented in Figures 2 and 3. Looking at the top graph in Figure 2, we see that Denmark is the country that has the most strong reaction in stock market volatility to the shock. The initial shock leads to a 55% higher volatility than what was expected otherwise which is the highest initial reaction. However, compared to the other countries this shock is not persistent; after about 100 days the VIRF has reduced to 5%. Then we have Sweden and Norway which show similar reactions to their respective shocks. The initial rise in stock market volatility is more substantial for Norway, but it also reverts to normal at a higher rate initially. Over the 400 days, the responses almost follow identical paths. In both of these countries, the shocks are more persistent than the other two. Moving over to the Finnish curve, it shares more traits with the Danish curve than with the Swedish and Norwegian initially as we see a VIRF that decreases more quickly in the beginning. However, the rate at which the VIRF decreases eventually becomes higher in Sweden and Denmark. After approximately 150 days the gap between the Finnish curve and the Swedish and Norwegian curves start to decrease.

Shifting focus to the lower graph of Figure 2, the VIRF of the bond volatility from news in the stock market, the VIRFs are smaller, but the initial shocks are also more persistent. The curve that stands out the most in the bond market volatility graph is the one that represents Finland. Initially, Finnish bond market volatility does not seem to react at all to the stock market news. Over the next 180 days or so the VIRF is continuously increasing to the point where it reaches 7%. After that, the VIRF starts to revert to zero but at a slow rate. As a matter of fact, after 400 days the effect of the shock is still more extensive than what it was initially. In other words, although the initial impact on the bond market volatility of the shock is minimal, the persistence seems to be extremely high compared to at least Denmark and Sweden. In Norway, the responses follow a similar path to the Finnish ones but not as pronounced. This is a contradiction to the BEKK estimation that did not find any evidence of volatility spillover from the Norwegian stock market to the bond market. Note that this is because the VIRF only uses the point estimates of the BEKK and does not take statistical significance into account. In this sense, the curves for the other countries are more “reliable”, but the Norwegian curve should not be dismissed because of this. The Danish market is again the one where the effects of the initial shock die out the quickest. At the start, we see a rather large positive reaction in bond market volatility to the shock but after a little under 100 days, it starts to decline again. Further, the Swedish responses differ from the others in that no notable increase in the VIRF is in the months after the shock. Instead, the VIRF is constant at around 4% for about 100 days. After that, the VIRF starts to revert to zero but at a small rate. The Swedish VIRF also has the initial response of the highest magnitude which is consistent with the finding of the BEKK model where evidence for stock market to bond market volatility spillovers was most pronounced in Sweden.

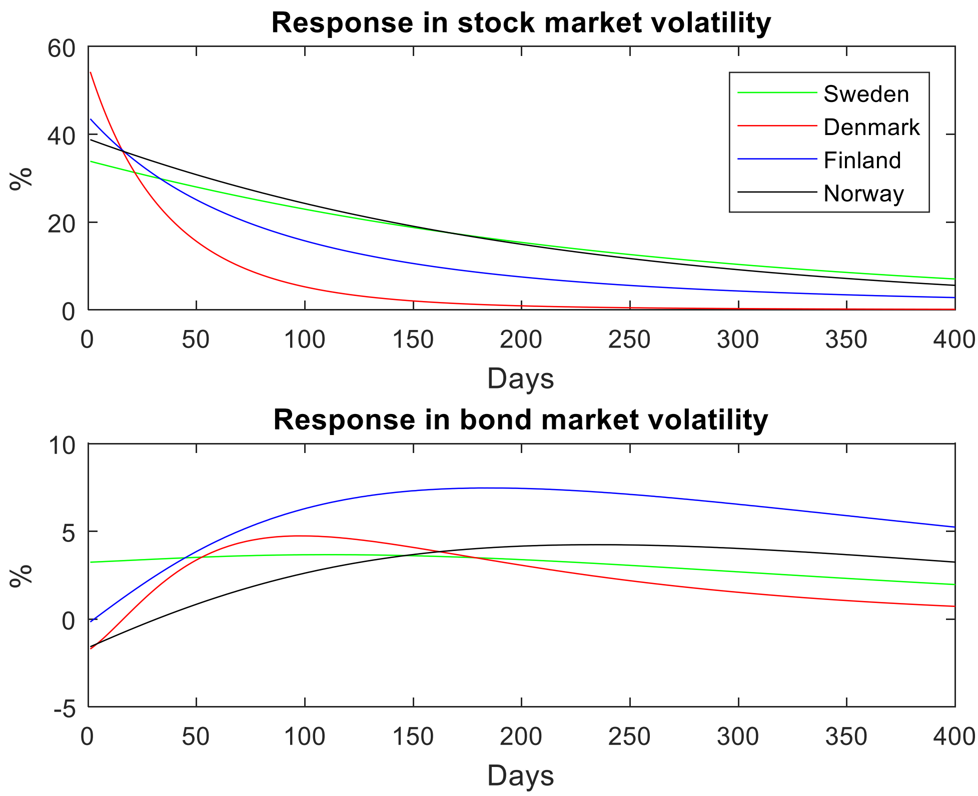

Figure 2: Volatility Impulse Response Function for news happening in the stock market

Moving over to Figure 3 which shows the volatility impulse responses for the two markets after news in the bond market, the reactions to the initial shocks are more similar to each other than what was the case in Figure 2. In the top graph, we see that VIRFs are negative for all countries. This suggests that volatility is expected to be lower when conditioning on the news in bond market than otherwise. These results are not consistent with the BEKK results which could not determine any spillover effects in this direction. Again, it should be noted that the VIRF does not take statistical significance from the BEKK estimates into account which is why we see contradicting results. Once again, the country where the shock is the least persistent is Denmark. After 100 days we can see that the trace of the shock is almost gone. The Finnish curve is relatively similar to the Danish although the reaction is quite a bit slower; it takes in approximately 160 days for the shock to be canceled out. The Swedish and Norwegian stock market volatilities show very similar reactions to the bond market news; the initial response is seven or eight percent smaller than the baseline expectation, and the shocks are more consistent compared to Denmark and Finland. Comparing the stock market volatility graphs from Figures 2 and 3 the most notable difference is the sign and size of the shocks. Apart from this, the rate of convergence towards zero is similar when comparing the same country over the two graphs. The main graphs concerning volatility spillover effects are the bottom graph of Figure 2 and the top graph of Figure 3. When comparing these two graphs, there is a pattern in that the initial responses tend to be negative with the only exception being the Swedish bond market's volatility's response to news in the stock market. Additionally, there are two other aspects of these graphs that are worth pointing out. First, the initial responses are of greater magnitude for the stock market than the bond market after news in the other market. Second, shock persistency from news in the other market is higher for the bond market than for the stock market.

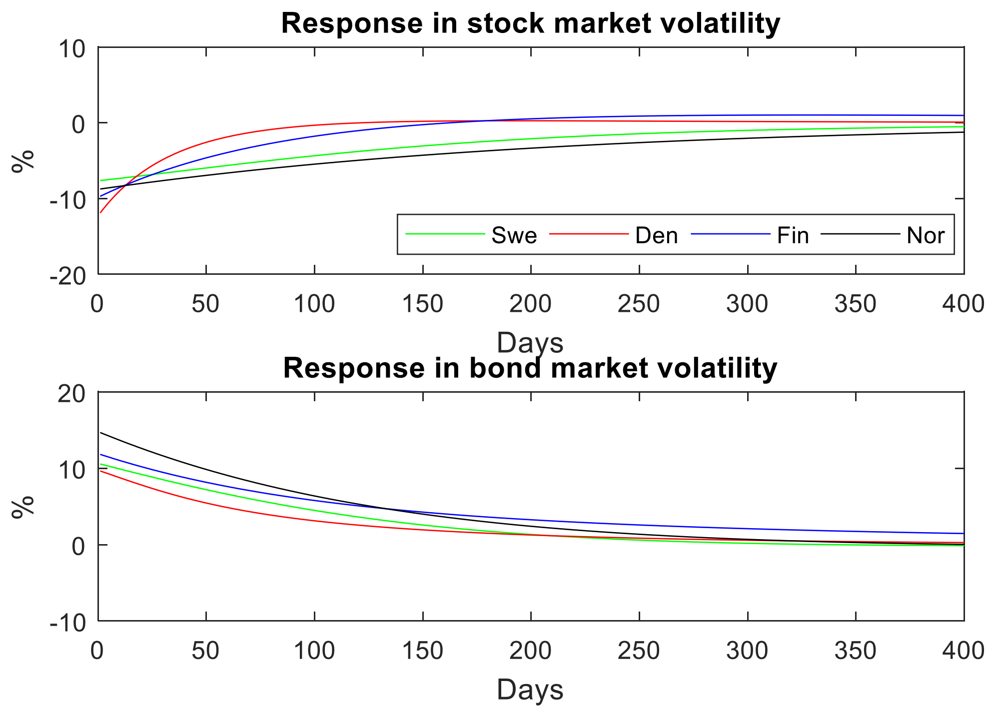

Figure 3: Volatility Impulse Response Function for news happening in the bond market

Lastly, for the response in bond market volatilities from news happening in the same market, all the countries display similar responses. The shocks are the least persistent in the Swedish and Danish markets. Norway has the most substantial initial response with bond market volatility at 15% above the baseline expectation, but “catches up” with Finland after about 140 days. Comparing the lower graphs of Figures 2 and 3, i.e. the two graphs displaying responses in bond market volatilities there are more noticeable differences in the shape of the curves than when comparing the stock market equivalents. Shocks that originates from the stock market are more persistent on bond market volatilities than shocks originating from the own market. Once again this is consistent with the findings of the BEKK estimation that volatility does spill over from the stock market to the bond market.

The results of the BEKK estimation showed that spillover effects from the stock market to the bond market were the most prominent. These results are presented more explicitly in Figures 2 and 3. Comparing the response in stock market volatility from news in the bond market with the response in bond market volatility from news in the stock market clearly shows that news in the stock market has a longer lasting effect on the bond market volatility than vice versa while the initial responses tend to be negative in both cases.

3.2.2 Time variation in initial responses

Day-ahead responses in stock and bond market volatilities to news happening in each quarter in the sample are presented in Figures 4 and 5. Similarly to the previous section, there is one figure for each type of shock. Figure 4 depicts the initial volatility responses over time to news in the stock market. In the upper graph illustrating the initial responses in the stock market, we see that the initial Danish responses have the most considerable variation over time generally moving between 40% and 60% of the baseline expectations. The biggest movement, not only for Denmark but for all countries can be seen in 2006 when responses drop to below 20% for both Denmark and Norway. Swedish and Finnish responses drop as well but not as substantially. Overall, no patterns in the time series can be seen. The global financial crisis, for example, does not seem to affect these responses. In the bond market equivalent, the lower graph of Figure 4, Denmark instead has the smallest variation over time in initial responses. In this case, it is Sweden and Finland that has the largest variation. An essential aspect of this graph is that the initial responses seem to be higher during periods of high volatility (early 2000’s, around the global financial crisis and around 2016) in all countries but least prominently in Denmark. This implies that stock market to bond market volatility spillovers are higher during times of financial instability. Another interesting result is that the signs of the initial responses are not constant over time. For example between the years 2004-2007 there is an extended period where news in the stock market results in volatilities around two or three percent smaller than what would otherwise have been expected in all countries except Sweden.

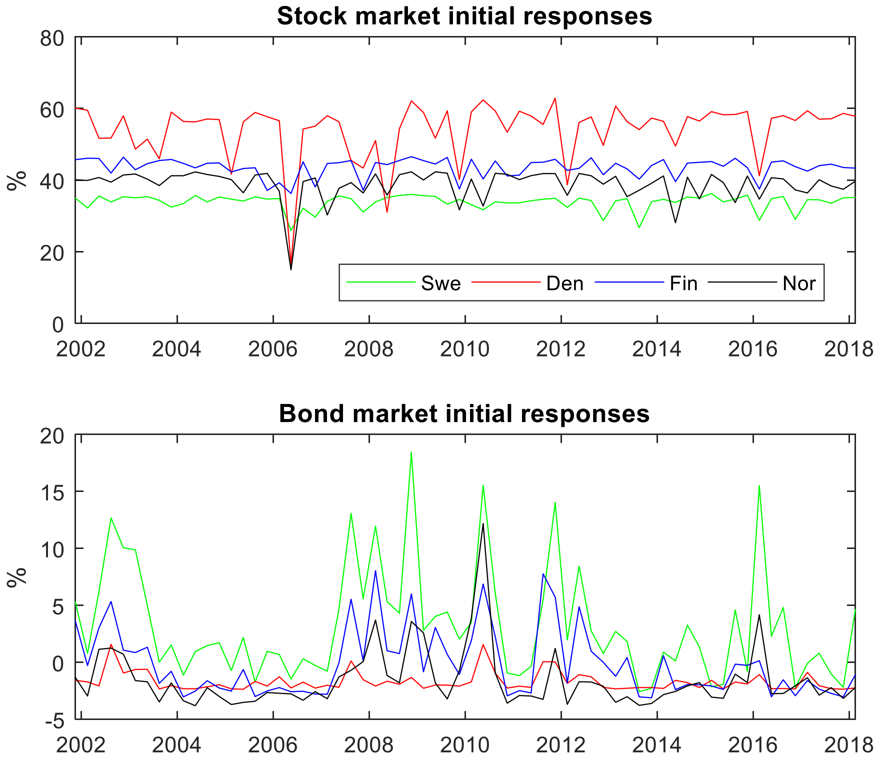

Figure 4: Day-ahead responses to news in the stock market

Figure 5 depicts day-ahead responses of bond and stock market volatilities to news in the bond market. Initial responses in stock market volatilities seen in the upper graph are negative over the whole sample and for all countries. It is once again Denmark who has the responses of the highest magnitude and the greatest variation over time moving between around the 13 to 10 percent marks. An anomaly is once more seen at the beginning of 2006 when there is an upward spike in Danish and Norwegian responses. The graph does not indicate that volatility spillovers from the bond market to the stock market varies noticeably over time as was the case with the spillovers in the opposite direction. The variation in bond market volatility responses to news in the own market seen in the bottom graph resembles the one observed for bond market volatilities in Figure 4 in that we can see patterns with the overall volatility in stock and bond markets. In this case, however, the effect is reversed compared to the responses to stock market news; in times of higher volatility in the stock and bond markets, the initial responses are lower than at other times. The Danish bond market volatility displays the most moderate degree of variation while the other three countries seem to vary equally much over the sample period.

To summarize this section, the initial responses of the VIRFs for the stock markets does not seem to vary particularly much with time irrespective of whether the response is to news in the stock or bond market. Nonetheless, for the bond markets, the story is different. Responses in bond market volatilities increase in size during times of financial instability regarding news from the stock market. For news in the own market, responses are lower during times of uncertainty.

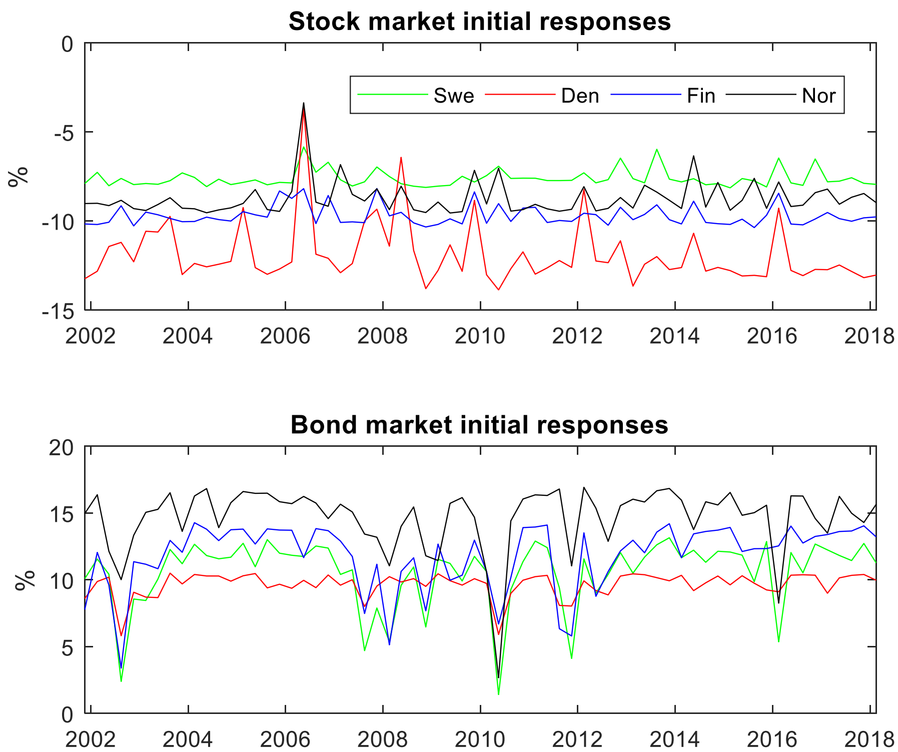

Figure 5: Day-ahead responses to news in the bond market

4 Conclusions

The goal of this paper has been to analyze the interdependence between stock and bond markets for the Nordic countries Sweden, Denmark, Finland and Norway by estimating volatility spillover effects with a BEKK model. Contrary to a lot of the existing literature but in line with for example Liow (2015), no volatility spillover effects from bond markets to stock markets have been found. Regarding effects in the opposite direction, results imply that there indeed is an effect in Sweden, Denmark and Finland but not in Norway. To get a more intuitive comprehension of these results, Volatility Impulse Response Functions have been computed as a way of quantifying the sizes of the spillover effects. The VIRFs have shown that news in the stock markets lead to long-lasting effects on bond market volatilities. The Finnish bond market displays the most sizable reaction to news in the stock market as 400 days after the news happen the VIRF is still much larger than the initial response. At the same time, the VIRFs for the stock markets have shown that although the initial responses in volatilities are positive after news in the own market and negative after bond market news, the way in which they revert to more normal states are similar. The VIRFs have also been used to see how initial volatility responses change over time to get a more dynamic view of the spillover effects. This analysis has led to the conclusion that volatility spillovers from the stock market to the bond market are more sizable during periods when the financial markets are more volatile. In general, news in one market tends to have a negative effect on the initial response in the cross-market volatility. Any potential variations in volatility spillovers from the bond to the stock market have not been found.

With the existence of volatility spillover effects from the stock market to the bond market in Sweden, Denmark and Finland, asset managers could consider including these countries stock markets in their models when calculating risk measure of bonds or when pricing them. This result also means that a financial crisis in any of these three countries' stock markets is likely to spread over to the respective bond markets especially with volatility spillover effects being more prominent during turbulent times. Equivalently, this paper can conclude that asset managers need not necessarily take into account cross-effects between Norwegian stock and bond markets and financial crises in any of these markets does not spread as easily. Also, this BEKK model can of course be useful when making forecasts of future volatilities and covariances.

A large part of the preexisting literature where multivariate GARCH models are used to model conditional covariances have more complex model specifications which include asymmetric effects. The data analysis in this paper indicates that log-returns for the markets in question can be skewed. This motivates the use of models that take leverage effects into account like De Goeij & Marquering (2004) and Dean et al. (2010). Estimating simpler models as in this case has its advantages but to get a more widespread view of the interdependence of Nordic stock and bond markets, future researchers can extend the model used here with terms that account for asymmetric effects. Moreover, this paper has focused on the two main asset classes: stocks and bonds. A natural extension to this could be to estimate a multivariate BEKK model including more than two asset classes rather than a bivariate BEKK as in this paper. Proposedly, one could follow the footsteps of e.g. Liow (2015) and estimate volatility spillovers between stocks and bonds but also public real estate prices and exchange rates.

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Appendix

Table A1: Estimates of the mean equation using a VAR (1) specification

	Sweden	Denmark	Finland	Norway
α_{01}	0,0007***	0,0010***	0,0009***	0,0010***
	(0,0002)	(0,0002)	(0,0002)	(0,0002)
α_{02}	0,0000	0,0000*	0,0000	0,0000**
	(0,0000)	(0,0000)	(0,0000)	(0,0000)
α_{11}	-0,0452***	-0,00043	0,0287**	-0,0241*
	(0,0139)	(0,0137)	(0,0144)	(0,0140)
α_{12}	-0,0033**	0,0013	0,0050***	-0,0019
	(0,0017)	(0,0022)	(0,0017)	(0,0019)
α_{21}	0,0877	0,0351	-0,0897	0,2594**
	(0,0961)	(0,0812)	(0,0991)	(0,1052)
α_{22}	0,0857***	-0,0172	0,0366***	0,1028***
	(0,0161)	(0,0148)	(0,0136)	(0,0140)

Note: Parameters from Eq.1. Statistical significance is denoted by * (10%), ** (5%) and *** (1%). Statistically significant results containing zeros are not exactly equal to zero but rounded. Standard errors are in parenthesis.

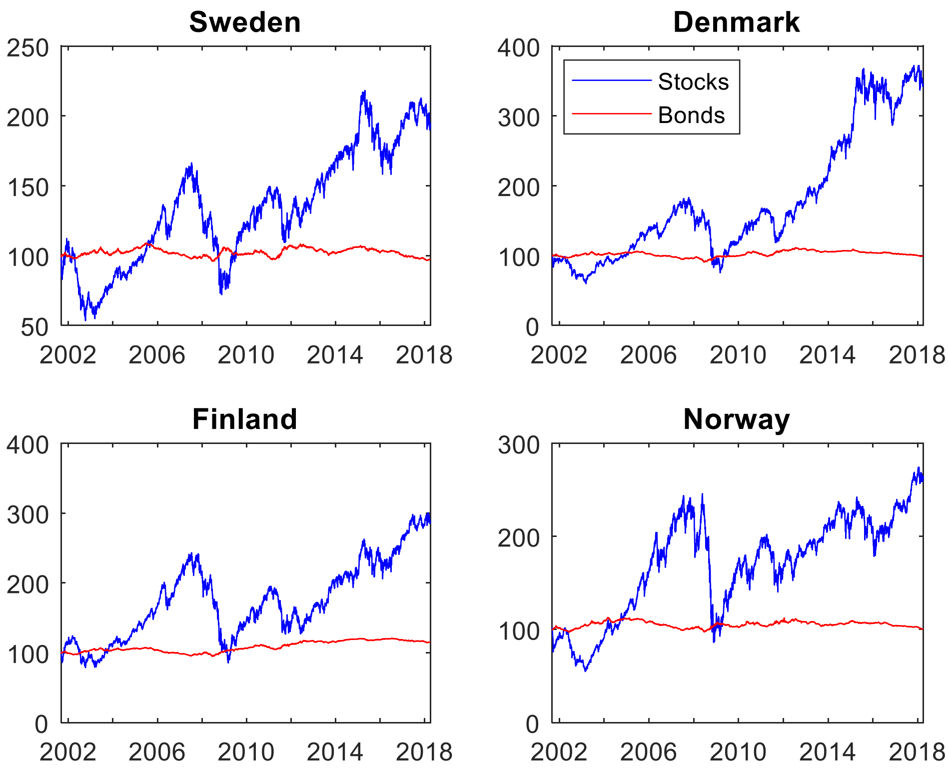

Figure A1: Graphs of Nordic stock and bond markets. Note that all indices have been rebased so that the first observation takes the value 100.

Table A2: Output from univariate GARCH estimations

	STOCKS				BONDS			
	Sweden	Denmark	Finland	Norway	Sweden	Denmark	Finland	Norway
a	0,0773***	0,1065***	0,0815***	0,0915***	0,0404***	0,0382***	0,0519***	0,0665***
	(0,0083)	(0,0116)	(0,0086)	(0,0093)	(0,0054)	(0,0060)	(0,0065)	(0,0083)
b	0,9164***	0,8670***	0,9113***	0,8980***	0,9539***	0,9548***	0,9452***	0,9331***
	(0,0084)	(0,0132)	(0,0090)	(0,0010)	(0,0061)	(0,0065)	(0,0065)	(0,0071)
ω	0,0000***	0,0000***	0,0000***	0,0000***	0,0000***	0,0000***	0,0000**	0,0000***
	(0,0000)	(0,0000)	(0,0000)	(0,0000)	(0,0000)	(0,0000)	(0,0000)	(0,0000)

Note: Correspondingly to Table 1, *a* denotes the ARCH-effect, *b* is the GARCH-effect and ω is the constant. Statistical significance is denoted by ** (5%) and *** (1%). Statistically significant results containing zeros are not exactly equal to zero but rounded. Standard errors are in parenthesis.

Table A3: Output from diagonal BEKK estimations

	Sweden	Denmark	Finland	Norway
a_{11}	0,2471***	0,3313***	0,2796***	0,2876***
	(0,0126)	(0,0171)	(0,0136)	(0,0154)
a_{22}	0,1914***	0,1658***	0,2009***	0,1997***
	(0,0102)	(0,0100)	(0,0104)	(0,0095)
b_{11}	0,9662***	0,9307***	0,9550***	0,9549***
	(0,0032)	(0,0065)	(0,0041)	(0,0046)
b_{22}	0,9771***	0,9823***	0,9769***	0,9765***
	(0,0024)	(0,0021)	(0,0023)	(0,0020)
ω_{11}	0,0000***	0,0000***	0,0000***	0,0000***
	(0,0000)	(0,0000)	(0,0000)	(0,0000)
ω_{22}	0,0000***	0,0000***	0,0000***	0,0000***
	(0,0000)	(0,0000)	(0,0000)	(0,0000)

Note: Parameters correspond to elements in the **A**, **B** and **Ω** matrices of Eq.2. Statistical significance is denoted by *** (1%). Statistically significant results containing zeros are not precisely equal to zero but rounded. Standard errors are in parenthesis.