On Robustness of Equilibria in Transportation Networks

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As infrastructural networks become ever more complex and interconnected, the risk of local disturbances causing system-wide problems become increasingly present. In light of this issue, the concept of resilience—defined as a system's ability to withstand and recover from disturbances and/or changing conditions—has become increasingly important in their design and maintenance.

1 Theory

One prominent model used to describe transportation networks is Daganzo's Cell Transmission Model. It imagines a transportation network as a set of cells of comparable lengths. Each cell then has a corresponding traffic volume (or mass) and a total outflow and the change in mass for each cell is equal to:

$$\text{Change of mass} = \text{total inflow}^{*} - \text{total outflow}. \quad (1)$$

The freeflow equilibrium—corresponding to having unhindered flow throughout the network and no changes in mass—can be shown to have very advantageous stability properties, whereby it is of great use to know which perturbations remove this property of freeflow.

2 Results

The results of this report can be divided into two parts: analytical and simulation results.

2.1 Analytical results

For deterministic inflow perturbations, the smallest increase to one or more exogenous inflows necessary for at least one equilibrium flow to reach its capacity was found. It proved insufficient to merely consider the cells that were closest to their capacity; it was also necessary to consider to what degree they react to any added inflow.

For stochastic inflow perturbations, bounds were found for the probability of the resulting equilibrium to not have freeflow. This was done both for normally and exponentially distributed inflows. These bounds may serve as qualitative measures of how likely the network is to be in freeflow when system-wide data is lacking.

For single-row routing matrix perturbations, a condition was found which must be fulfilled for the equilibrium to retain freeflow. It showed (somewhat intuitively) that rerouting large outflows give less room for errors than redirecting small outflows, and keeping track of changing behaviour at such cells is thus of greater importance.

2.2 Simulation results

For periodic inflows, it was observed that inflows which average an inflow for which the system has freeflow may still cause the system to deadlock. It also implied that—at the extreme cases—the amplitude and period for the variation of these inflows are near-inversely proportional, where e.g. doubling the amplitude requires one to roughly halve the period.

For cell mass increments, it was observed that cells further away from any out-connected cells (i.e. cells which route some of their outflow outside the system) are more vulnerable. This worked in conjunction with an effect where other cells worked as buffers for the increments, spreading the load and thereby lessening the adverse effect on congestion. It was also found that cells may be able to accept (and retain) increased mass even in cases where the system does not allow any additional (constant) inflow.

[*] Whether exogenous or from other cells.