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Testing Extended Rules of Thumb for the Dynamics of Volatility Surfaces

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Abstract

It is a common practise to quote option prices using their Black-Scholes implied volatility. A volatility surface describes an options implied volatility as a function of the strike price and time to maturity. It can be used as a tool for hedging but also valuation when prices are not directly observable. The short-term evolution of this surface has been described by a variety of apocryphal rules. Three of these rules are tested empirically for exchange traded S&P 500 index options for two distinguished periods. The square root of time rule, consistent with the no-arbitrage condition, has the highest explanatory and predictive power. Extended versions are also derived but with no significant improvement.

Keywords: Volatility Surface, Implied Volatility, Rules of Thumb, No-Arbitrage Condition, Index Options

Contents

1 Introduction

The variation of the volatility surface for options, has been described by a variety of apocryphal rules. Since the surface is used as a tool for market participants to hedge and value contracts, understanding changes are of great importance. This essay examines the practicality of these rules on index options, based on their explanatory power under two different volatility regimes. In comparison to earlier work, different extensions will be introduced to see if improvements may be possible. This will be done under a no-arbitrage framework. The rules will then be tested out of sample, to evaluate their predictive power.

1.1 Options and Terminology

An option is a type of derivative, therefore the value depends on (or derives from) the value of other underlying variables [6, p. 23]. There are numerous variations but the main types of options are calls and puts. A *call option* gives the holder the right to buy the underlying asset by a certain date for a certain price. A *put option* is similar but gives the holder the right to sell [6, p. 30]. Buyers of calls or puts are referred to as having *long positions*; while sellers are referred to as having *short positions* [6, p. 33].

The price set in the contract is called *exercise price* or *strike price*, denoted K. The *maturity* (T) is the date which the option expires. There are different subtypes of options that can be exercised differently. An American option can be exercised at any time while an European option can only be exercised at the expiration date [6, p. 33]. The price of the underlying asset at time t will be denoted by S_t and the price of buying an option is called the *premium* denoted by Π.

A call option will never be exercised if the underlying asset has a price below the strike price ($S_t < K$). Similarly for a put option but with reversed inequalities ($S_t > K$). This scenario is referred to as the option being *out-of-the-money.* If instead the asset price is equal to the strike price ($S_t = K$) the option is refereed to as *at-the-money.* Lastly, a call option where the asset price is above the strike price ($S_t > K$) and inequality reversed for put option ($S_t < K$), will be referred to as *in-the-money.* The *moneyness* of an option will be referring to the ratio of K/S.

1.2 Pricing of Options

In the early 1970s, Fischer Black, Myron Scholes and Robert Merton achieved a major breakthrough. This was due to the development of the pricing model of European stock options known as the Black-Scholes-Merton (or Black-Scholes) model [6, p. 343]. The importance of the model would later be recognized when Robert Merton and Myron Scholes were awarded the Nobel prize for economics in 1997. Fischer Black had passed away in 1995 but would undoubtedly been one of the recipients. Their contribution can not be overstated.

Since the early 70s, the field of pricing options have come a long way. Extensive analytic models have been developed to cope with some of the more rigid assumptions in the early model. Molecule models of a wide range of different types have been developed for different options [2, p. 147]. The rapid increase of computing power during these decades has also made different numerical models viable.

1.3 Volatility Surface

Volatility (σ) is a measure of the uncertainty of the return realized on an asset [6, p. 861]. The *implied volatility* is derived from an option price using Black-Scholes model. A plot of the implied volatility of an option with a certain life as a function of its strike price and time to maturity is known as the asset's *volatility surface* [10]. The moneyness of an option and the delta (sensitivity to changes in underlying $\Delta = \frac{\partial \Pi}{\partial S}$) may also be used as the strike dimension.

To derive a volatility surface, the implied volatilities of traded options are used. For European options and in the absence of arbitrage, the *put-call parity* implies that the implied volatility of a call and put is the same. These options may be traded on an exchange but can as well be *over-the-counter*. The mid price of the bid/ask is used to derive the implied volatility. If there are no contracts traded for a specific strike and maturity, one realizes that the surface will suffer from holes. This has been a heavily researched topic and there are countless of methods for interpolation and extrapolation. Note that the volatility surface need to satisfy the condition of static and dynamic no-arbitrage to be practically valid.

1.4 Rules of Thumb

The rules of thumb are trying to capture the dynamics of the volatility surface. This short-term evolution is described through dependence on plausible variables. The types of rules of thumb analyzed fall into two categories. The first one describes how the volatility surface changes through time. The second one is concerned with the relationship between implied volatility for different maturities at a point in time [10]. In physics or mathematics, parameters that do not change are called invariants. It is however customary to refer to what does not change as "sticky" in options trading [5].

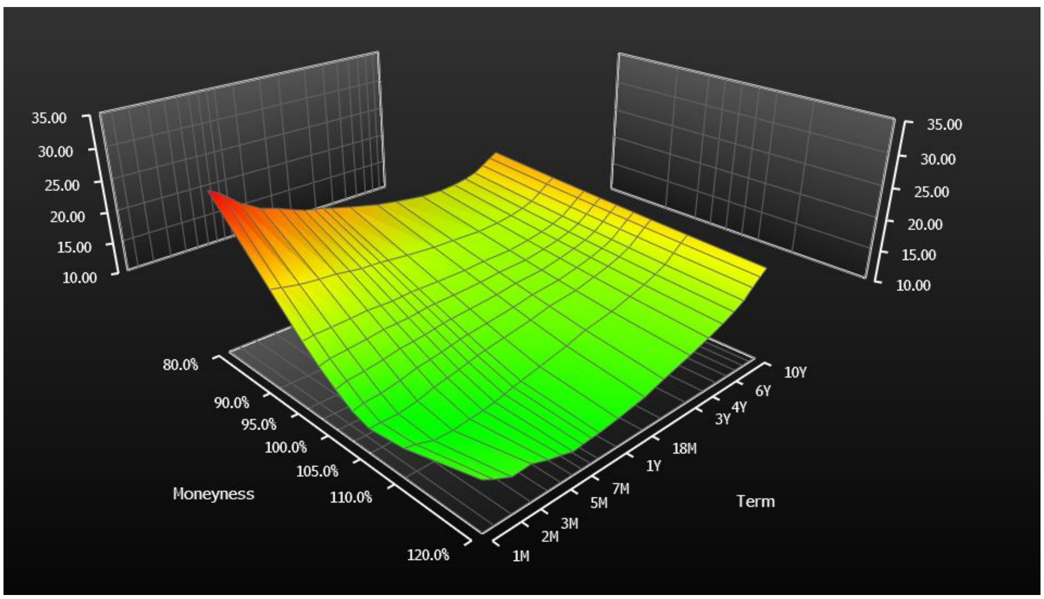

Figure 1: Volatility surface for the S&P 500 index the 3 December 2018. The vertical axis show the implied volatility, while the horizontal axes shows the maturity and the moneyness.

Three examples of different rules are the *sticky strike rule*, the *sticky delta rule* and the *square root of time rule*. The first two are concerned with changes in time and the last one fall into the second category. There are a variety of subrules of these rules. What follows are general ones stated in previous work like [10] and [5]. All of them assume a European call option with implied volatility σ_{TK} where T is the maturity and K the strike price.

1.4.1 The Sticky Strike Rule

The sticky strike rule is a poor man's attempt. The rule assumes that the σ_{TK} is independent of the asset price. In its most basic form, it varies only with K and T. A more generalized version describe σ_{TK} as independent of the underlying, but possibly dependent on other stochastic variable.

1.4.2 The Sticky Delta Rule

The sticky delta rule (also called sticky moneyness rule) is an alternative where the implied volatility is dependent on the asset price and strike price. This is through the moneyness variable. The intuition behind the rule is that an option that is X% out-the-money after the index moves should have the same implied volatility as the X% out-the-money option before the index move.

1.4.3 The Square Root of Time Rule

Another rule that is sometimes used is the square root of time rule. It is based on the time scaling of volatilities [7]. The time value of an option is approximately proportional to the square root of time. This is being captured by including dependence on the square root of time. The rule will then describe the relationship between implied volatilities for different maturities at a point in time.

1.5 Structure of the Essay

In the next section, already discussed concepts will be elaborated. Previous work of different rules of thumb and empirical observations will be discussed. Section 3 introduces theory and the no-arbitrage condition. This is trough a stochastic implied volatility model. The regression models will then be constructed for different rules of thumb in Section 4. In Section 5, the result is presented and in the last section conclusions will be made and discussed.

2 Review of the Existing Literature

This section provides an overview of previous work related to this essay.

2.1 Consistency with No-Arbitrage Condition

The paper 'Volatility Surfaces: Theory, Rules of Thumb, and Empirical Evidence' written by T. Daglish, J. Hull and W. Suo [10] relates to testing conventional rules. In the paper they focus on the consistency of different rules given a derived no-arbitrage condition. They show that a basic sticky strike rule that only depends on K and T is only internally consistent with a model where the volatility surface is flat. This is the original Black-Scholes model. A generalized version of the sticky strike rule is analyzed and showed only to be consistent with Mertons-model where the *instantaneous volatility* is a function of time. Therefore, any volatility surface that suffer from skewness will be inconsistent with any version of their sticky strike rule. For the basic sticky delta rule the same is concluded. A more general version of the sticky delta rule, where the implied volatility is stochastic and depends on the moneyness and time to maturity, can be consistent with the no arbitrage condition. The relative sticky delta rule, which models the excess volatility over the at-the-money volatility, and the square root of time rule are shown to be approximately consistent.

2.2 Empirical Evidence for different Rules of Thumb

The explanatory power of the rules seems to vary under different regimes. Emanuel Derman analyzes the variation of three-months implied volatilities for S&P 500 options [5]. With fourteen months of data from 1997-1998, Derman divides the period in seven different regimes, where the boundaries are somewhat subjective. Each regime has its own characteristics. Its concluded that each regime has a dominant rule that explains the observations. When the index and volatilities are fairly stable, they seem to follow the sticky strike rule. The sticky strike rule does also a good job of explaining the index when it exhibits an upward trend. A *sticky implied tree rule* is also analyzed. This rule seems to be good when the index enter a period of high volatility and appreciable downward jumps. It should be noted that the S&P index, realized an annualized volatility of 9% for the most tranquil period and 30% for the most turbulent.

Test of rules of thumb are also being conducted in [10]. The data used is monthly volatility surfaces from the over-the-counter market for 47 months (June 1998 to April 2002). In comparison to Derman, six maturities are considered ranging from six months to five years. The moneyness range from 0.8 to 1.2. The sticky strike rule tested, can be regarded as a Taylor expansion of second degree. The rule show poor explanatory power and sizable errors when tested out-of-sample. Using a similar Taylor expansion, the relative sticky delta rule and variants of square root of time rule show great promises. They also conclude that the latter can be marginally improved to explain volatility changes. This is through an exponent equal to 0.44 instead of the square root 0.5.

2.3 Time Effect Extensions

An extension of the sticky strike rule specification is made by Jacinto Romo in [8] and tested on monthly data for the Spanish index IBEX 35. A stochastic term is added to make the rule more flexible. The term is considered to aggregate the effect of random variables, such as news, that may affect the volatility surface. The one specified allows for a parallel shifts, together with preserving the overall term structure and skewness. The extended version outperforms the parsimonious version of the rule, but with the loss of economic interpretation.

3 Theory

3.1 Risk Neutral Valuation

The arbitrage free price, $\Pi(t, \Phi(S_T))$, of the claim $\Phi(S_T)$ at a time t is given by:

$$\Pi(t, \Phi(S_T)) = e^{-r(T-t)}\mathbb{E}_t^Q[\Phi(S_T)]$$

Where Q is the risk neutral measure and r the risk-free interest rate [3, p. 103].

3.2 The Black-Scholes Formula

The derivation will follow [3, p. 104-105]. The bank account B_t with risk-free interest rate r and the underlying asset S_t follows the processes:

$$dB_t = rB_t dt$$
$$dS_t = \alpha S_t dt + \sigma S_t dW_t$$

Where r and α are the drift terms for the processes. The diffusion component is the volatility σ and W is the standard Brownian Motion (BM). One can think of this component as adding the noise to the assets movements. A definition for the BM is included in the appendix A.1. It is important to note that the processes assumes absence of jumps.

The risk neutral Q-dynamics of S is given by:

$$dS_u = rS_u du + \sigma S_u dW_u$$
$$S_t = s$$

Finding S_T is done by integrating over $[t, T]$ [3, p. 67-69] to receive:

$$S_T = s \exp\left\{(r - \tfrac{1}{2}\sigma^2)(T - t) + \sigma(W_T - W_t)\right\} = se^z$$

Using the risk neutral valuation earlier, the pricing formula can be obtained:

$$\Pi(t, s) = e^{-r(T-t)} \int_{-\infty}^{\infty} \Phi(se^z) f(z) dz$$

Where f is the density of a random variable Z, where:

$$Z \in \mathcal{N}\left[(r - \tfrac{1}{2}\sigma^2)(T - t), \sigma\sqrt{T - t}\right]$$

3.3 Blacks-Scholes Pricing Formulas for European Options

The integral formula is general and must be evaluated numerically. However, analytic expressions for European options can be derived given the boundary conditions. What follows are the Black-Scholes price formulas for calls c and put p options.

$$c = S_t N(d_1^*) - Ke^{-r(T-t)} N(d_2^*)$$

$$p = Ke^{-r(T-t)} N(-d_2^*) - S_t N(-d_1^*)$$

Where

$$d_1^* = \frac{1}{\sigma\sqrt{T-t}} \Bigg[\ln(S_t/K) + (r + \sigma^2/2)(T-t) \Bigg]$$

$$d_2^* = d_1^* - \sigma\sqrt{T-t}$$

and $N(x)$ is the cumulative probability distribution function for a variable with a standard normal distribution.

3.4 Implied Volatility

The implied volatility σ_{TK} matches the value of an option observed in the market, Π_t, with the one obtained by Black-Scholes formula [6, p. 363]. It is not possible to derive it in closed form, so an iterative search procedure is used.

$$\sigma_{TK}(S_t, t) = BS^{-1}(\Pi_t, S_t, K, t, T)$$

3.5 The Implied Stochastic Volatility Model

In the Black-Scholes model, the underlying volatility is known and constant. This can be extended by assuming that the implied volatility follows a stochastic process. The process for the underlying asset (stock index) is assumed to have a drift that consists of the difference between the risk-free interest rate r and the the yield q. The diffusion component is the volatility σ and W is the BM. Note that r and q are assumed to be deterministic.

$$dS = (r-q)Sdt + \sigma SdW$$

In [9] the process for the implied volatility includes a term with the same BM as the underlying asset process. Here the process will instead follow the previous authors in [10], where all are specified at once. For algebraic convenience the *implied variance* ($V_{TK}(t,S) = (\sigma_{TK}(t,S))^2$) will be used. The process for the implied variance is then:

$$dV_{TK} = \alpha_{TK}dt + V_{TK}\sum_{i=1}^{N}\theta_{TKi}dW_i$$

Where θ_{TKi} measures the sensitivity of V_{TK} to the BM. Both of these processes assumes absence of jumps.

3.6 No-arbitrage condition

The process followed by Π the price for a call (note that given put-call parity, it could equally have been for a put) can be derived by Ito's lemma in two dimensions. For absence of arbitrage, the drift is equal to $r\Pi$. A comprehensive derivation can be found in the appendix $A.2$ with the help of [2, p. 118]. As derived in [10], an expression for the drift of the implied variance as a function of its volatility will provide a no-arbitrage condition.

$$\alpha_{TK} = \frac{1}{T-t}(V_{TK}-\sigma^2) - \frac{V_{TK}(d_1d_2-1)}{4}\left(\sum_{i=1}^{N}\theta_{TKi}^2 + \sum_{i\neq j}\theta_{TKi}\theta_{TKj}\rho_i\rho_j\right) + \sigma d_2\sqrt{\frac{V_{TK}}{T-t}}\sum_{i=1}^{N}\theta_{TKi}\rho_i$$

Where ρ_i is the correlation between the BM in the underlying asset process and the ith BM in the implied variance process ($\mathrm{corr}\{W,W_i\}$). The d_i are results of the Black-Scholes model:

$$d_1 = \frac{\ln(S/K)+\int_t^T[r(\tau)-q(\tau)]d\tau}{\sqrt{V_{TK}(T-t)}} + \frac{1}{2}\sqrt{V_{TK}(T-t)}$$

$$d_2 = \frac{\ln(S/K)+\int_t^T[r(\tau)-q(\tau)]d\tau}{\sqrt{V_{TK}(T-t)}} - \frac{1}{2}\sqrt{V_{TK}(T-t)}$$

3.6.1 The Sticky Strike Rule

For the most basic sticky strike rule, where σ_{TK} is a deterministic function only of T and K, the implied variance does no longer have a diffusion component. In other words, all $\theta_{TKi} = 0$. From the no arbitrage condition together with the implied variance process:

$$dV_{TK} = \alpha_{TK}dt = \frac{1}{T-t}(V_{TK} - \sigma^2)dt$$

such that:

$$\sigma^2 = -\frac{d[(T-t)V_{TK}]}{dt}$$

The volatilities are then the Black-Scholes constant volatility model. This type of sticky trike rule is inconsistent with any type of volatility skew. For a more generalized version where σ_{TK} is independent of the underlying asset, all ρ_i will equal zero and d_i depends on the underlying asset, so all $\theta_{TKi} = 0$. This is the same case as the basic sticky strike and the same conclusion can be drawn.

3.6.2 The Sticky Delta Rule

The most basic sticky delta rule, assumes σ_{TK} to be a deterministic function of moneyness K/S and T. This is the same case as for the basic sticky strike rule and all $\theta_{TKi} = 0$. The rule is then inconsistent with any type of skewness in the volatility surface.

For a generalized sticky delta model, where σ_{TK} is stochastic and dependent on the moneyness and T, may be consistent. To see this, note that if all the θ_{TKi} in the no-arbitrage condition have the same form of dependence on the moneyness and time, the same holds for α_{TK}. The d_1 and d_2 do already have a dependence on the moneyness. This type of rule can then be consistent with a volatility surface that has some type of skewness.

3.6.3 The Square Root of Time Rule

The rule is concerned with the relationship between implied volatilities for different maturities at a point in time. Anchoring the implied volatility will be the at-the-money one. Here it is defined to be an option where the strike price equals the underlying asset price. It is worth noting that sometimes the at-the-money volatility is defined as an option where the strike price equals the forward asset price. Here follows two general versions of the sticky strike rule:

$$\frac{\sigma_{TK}}{\sigma_{TS}} = \psi_1(\frac{ln(K/S)}{\sqrt{T}}) \qquad \sigma_{TK} - \sigma_{TS} = \psi_2(\frac{ln(K/S)}{\sqrt{T}})$$

For both expressions ψ is a function. The first one is concerned about describing the ratio while the second one is the excess of the volatility over the at-the-money volatility.

Since the time value of an option is approximately proportional to the square root of time [7], due to the assumption of the process driving the underlying asset is a BM, this rule is fairly intuitive. Although this is true when the underlying asset follows one, it is less clear when the volatility is also assumed to follow a stochastic process. Leaving the mathematical side, this rule is approximately consistent with the no arbitrage condition for α_{TK}. A mean-reverting volatility model might be an example, where the implied volatility approximately satisfies the square root of time rule, when the reversion coefficient is large [10]. In [1, p. 18] a more rigorous treatment of the topic can be found.

4 Models and Data Analysis

4.1 Data and Description of Variables

The data consists of 60 monthly volatility surfaces for the S&P 500 from the exchange traded market split into two distinguished periods. The first one is taken from 1 June 2007 to 1 June 2009 and the second one 1 February 2016 to 2 November 2018. The boundaries for each period have been chosen somewhat subjectively. A clear downward trend characterizes the first period, while the second a tranquil upward trend. In the appendix $A.3$, two graphs are included to show the stark difference in the periods.

The maturities range from one month up to two years. Higher maturities are available but a lot of the contracts are then not traded. This absence of trading will result in interpolations or extrapolations which are not as informative. Nine values of moneyness are considered, ranging from 80% to 120%. A total of 72 points on the volatility surface are provided each month with a total of 1872 and 2448 for each period. Around 18.5% of the implied volatilites in the first period have been interpolated or extrapolated while 7.7% for second. A large majority of these are in the higher range of the moneyness and lower maturity dimension together with high maturities. All of the data has been extracted from Bloomberg.

4.2 Regressions for Rules of Thumb

The α_i, β_i and γ_i are population parameters and ϵ is a normally distributed error term. A dummy variable δ representing a vector of $N-1$ parameters, where N represents the number of months, will be added to some of the models. This unspecific term will work as an aggregation for random variables affecting the volatility surface each month. This is to see if the models may be improved by for example macroeconomic indicators affecting it during a specific month. One can construe it as a parallel term shift for the respective months.

The functional specification of all models can be thought as a second order Taylor series expansion. A linear specification might be to restrictive, which is why quadratic terms will be favored. This will include squared terms and a cross-term.

A model's viability out of-sample will be tested by obtaining a root mean square error. It is based on a quadratic loss function where larger forecast error are punished more heavily [11]. This forecast evaluation will be tested for models excluding the dummy vector. This is due to the implausibility that the estimated coefficient for a certain month, will have the same impact a year later. The two periods will be split such that the first 19 and 25 months respectively will be used for the fitted model.

4.2.1 The Sticky Strike Rule

This model is inconsistent with the no-arbitrage condition setup earlier. However, for some volatility regimes, it has been shown to do a better job explaining the implied volatility then more structurally sound models [5]. The basic sticky strike rule will be tested where the implied volatility depends only on the strike K and the maturity T.

$$\sigma_{TK} = \alpha_0 + \alpha_1 K + \alpha_2 K^2 + \alpha_3 T + \alpha_4 T^2 + \alpha_5 KT + \epsilon$$

4.2.2 The Sticky Delta Rule

The basic sticky delta rule was earlier showed to be inconsistent with the no-arbitrage condition. For a generalized sticky delta rule, it was shown to be plausible given certain assumptions about the sensitivities of the variances. This version will be tested, where it is dependent on the moneyness K/S and T in a functional form of a second degree Taylor expansion together with the dummy vector δ.

A relative general version might also be plausible where the excess volatility over the at-the-money will be modeled. This will be through a dependence on the log of moneyness and T in a likewise functional form.

$$\sigma_{TK} = \beta_0 + \delta + \beta_1 (\tfrac{K}{S}) + \beta_2 (\tfrac{K}{S})^2 + \beta_3 T + \beta_4 T^2 + \beta_5 (\tfrac{K}{S}) T + \epsilon$$

$$\sigma_{TK} - \sigma_{TS} = \beta_0 + \beta_1 \ln(\tfrac{K}{S}) + \beta_2 \ln(\tfrac{K}{S})^2 + \beta_3 T + \beta_4 T^2 + \beta_5 \ln(\tfrac{K}{S}) T + \epsilon$$

4.2.3 The Square Root of Time Rule

Given the no arbitrage condition, this model will be approximately consistent. Two general versions were earlier introduced and it is the latter one that will be tested. The excess volatility over the at-the-money volatility will be dependent on a forth degree polynomial of the log of moneyness over the square root of time. This follows [10] but it is less clear what degree is most appropriate. A more parsimonious version might be tested depending on the result.

Extending the first version, a dummy vector δ will be added to see if it may capture shifts during each month.

$$\sigma_{TK} - \sigma_{TS} = \gamma_1 \frac{ln(K/S)}{\sqrt{T}} + \gamma_2 (\frac{ln(K/S)}{\sqrt{T}})^2 + \gamma_3 (\frac{ln(K/S)}{\sqrt{T}})^3 + \gamma_4 (\frac{ln(K/S)}{\sqrt{T}})^4 + \epsilon$$

$$\sigma_{TK} - \sigma_{TS} = \delta + \gamma_1 \frac{ln(K/S)}{\sqrt{T}} + \gamma_2 (\frac{ln(K/S)}{\sqrt{T}})^2 + \gamma_3 (\frac{ln(K/S)}{\sqrt{T}})^3 + \gamma_4 (\frac{ln(K/S)}{\sqrt{T}})^4 + \epsilon$$

5 Results

All of the referenced regression output can be found in the appendix $A.4$, sorted after each rule and period. Robust standard errors have been used for all regressions.

5.1 1 June 2007 - 1 December 2008

For the basic sticky strike rule, the T^2 is the only notably insignificant term at the 1% level but not 5%. The estimated coefficients K and T are clearly negative, which is in line with the general skewness observed (see the volatility surface in Section 1.4). However, the general sticky dela rule and relative sticky delta rule seem to both better explain the variation. They have as well clearly negative coefficients for the T and moneyness parameters. A comparison between the sticky strike rule and the sticky delta rules can be done through a F-test. If two models have equal explanatory power, the ratio of the squared errors should be F-distributed adjusted for their respective degree of freedom. A two-tailed test show a clear rejection at all levels. To compare the two sticky delta rules, all of the information criterion's unanimously point to the more parsimonious version. The criterion's incorporates a trade-off between goodness-of-fit and the number of regressors. The generalized sticky delta rule is heavily punished and the more parsimonious relative sticky delta rule is to be favoured.

For the square root of time rules, the second degree term is insignificant at all levels for both models. A more parsimonious version might somewhat marginally improve it. The first term is negative and all higher degrees are positive. In comparison to the relative sticky delta model, all information criterion's unanimously point to the square root of time rules. Comparing the two nested versions it is less clear cut. The Schwarz Bayesian Information Criterion tend to favour more parsimonious models because of the larger penalization of increased number of regressors. Although the Akaike's Information Criterion might be lower for the general square root of time rule, the more parsimonious version should here be preferred.

5.2 1 February 2016 - 2 February 2018

The null hypothesis for a significant T^2 term is clearly rejected at all levels for the basic sticky strike rule and the general sticky delta level. Some of the terms in the dummy vector are also insignificant but a joint test for all the regression coefficients to be zero is clearly rejected. As for the previous period, the two versions of the sticky delta rule are preferred. Once again, taking into account the trade-off of explanatory power and parsimony, the second trait will here be more desirable.

Both of the square root of time rules seems to better explain the variation observed in the second period. None of the degrees are insignificant but some of the terms in the dummy vector are. Testing for joint significance, it is clearly rejected that all coefficients be zero. Comparing the two versions, all information criterion's favour the extended or general version. It seems then that the extension might marginally improve the model and captures aggregate effects acting each month.

5.3 Forecast Evaluation

5.3.1 First Period

For the basic sticky strike rule, a root mean square error (RMSE) of 0.14 was obtained. The measure is dependent on the scale of the dependent variable but this represents a quite sizable error. There seems to be a tendency as well for the rule to systematically overestimate the implied volatility. For almost all predictions, the rule yield a higher implied volatility then observed in the market.

For the sticky delta rule and the square root of time rule, RMSE of 0.011 and 0.0074 were obtained. This represent a fairly close fit to the data observed. Even though the square root of time rule involves less parameters, it will still lead to improvements in the overall predictive power. No tendencies for over- and underestimating could be observed.

5.3.2 Second Period

A RMSE of 0.060 was obtained for the basic sticky strike rule. However, a reversed effect was observed for the predictions. The rule systematically underestimated the implied volatilities. Although true for a majority of the predicted values, the effect was not as strong as in the earlier period.

Out of sample performance for the sticky delta rule and the square root of time rule, resulted in a RMSE of 0.017 and 0.011 respectively. This is again a fairly close fit where the square root of time rule has a higher predictive power. There were no tendencies for over- and underestimating.

6 Conclusions

6.1 Rules of Thumb

The empirical tests of the rules of thumb, show a strong support for the square root of time rule. This is based on the information criterion. It is the most appropriate rule for a tumultuous downward trending index and a tranquil upward trending one. This rule will as well approximately satisfy the no-arbitrage condition. The simpler sticky strike rule and versions of the sticky delta will not, but show less explanatory and predictive power.

The square root of time rule is extremely useful in creating a complete volatility surface. If a relatively small number of options are available, one can compute the complete volatility surface. However, if one is interested in the sensitivity of underlying variable changes, the rule is not as useful. This is due to the modelling of excess volatility over the at-the-money. The "Greeks" as they are often referred to among option traders, are a set of values measuring the risk involved in options contract. Under this rule, they will not be easily calculated. Though the rule explains the dynamics of the volatility surface, a market participant with the intention of hedging, might not find it as useful when calculating overall risk exposure.

The extended versions seems to hardly improve the models, if anything marginally. Preserving the term structure and volatility skew, might not be the best way to capture the aggregate factors affecting the volatility surface. A more complex structural econometric modelling of the effect might improve. Although giving economic content, it is not always very useful for predictions. A time series extensions might then be a better alternative, but it is unclear how this will affect the no-arbitrage condition.

6.2 Interpolations and Extrapolations

When there is no available price data, the volatility surface will suffer from hole. Bloomberg uses a non-parametric numerical method for the moneyness dimension and a Hermite cube spline for the time to maturity [4]. It is in the company's interest of providing accurate data and one may conclude that the methods are sophisticate enough.

However, more turbulent periods will have less traded contracts which is the main reason why the first period has 18.5% interpolated or extrapolated implied volatilities while 7.7% for second one. This strong correlation will result in less informative data during more volatile periods. One should however note that not all trades are done on the exchange. It is not plausible to execute large volume trades on an exchange, so the data will not be able to capture transactions over-the-counter. Although the dynamics are slight different, empirical tests on the over-the-counter market in [10], seem to be in agreement with exchange traded in this paper.

6.3 Assumptions Made for the No-Arbitrage Condition

The no-arbitrage condition, underpinning the rules of thumb, is heavily dependent on the assumed stochastic process. In the stated form, the underlying asset and the implied variance will have continuous paths. Presence of jumps are not taken into account and will then be excluded. These type of jumps usually occur during turbulent market periods. Political and economical events but also extreme trade executions like a fat-finger error or a flash-crash may give rise to these movements.

Changing the no-arbitrage condition to include jumps, will increase the overall complexity. However, it would be of interest to see how theses rules of thumb will be affected.

6.4 Concluding Remarks

The square root of time rule outperforms the other rules of thumb for the two periods analyzed. However, there is room for improvement and as well finding a rule which appeals to a wider market audience. A more realistic no-arbitrage condition can also be derived, which may underpin a stronger theory for the rules of thumb.

A Appendices

A.1 Brownian Motion

Defintion: Let $W = \{W_t : t \in \mathbb{R}_+\}$ be a stochastic process. W is a BM if it satisfy the following properties:

$$W_0 = 0 \tag{1}$$

$$\text{The increments are independent and stationary.} \tag{2}$$

$$(W_{t+h} - W_t) \in \mathcal{N}(0, h) \tag{3}$$

$$W_t \;\text{ has continuous paths} \tag{4}$$

A.2

Recall the *Two-dimensional Itô Formula* [2, p. 118] applied to a function $\Pi(t, x, y) \in \mathcal{C}^{1,2,2}([0, T], \mathbb{R}^2)$ and the processes X_t and Y_t are given by:

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t^x$$
$$dY_t = \alpha(t, Y_t)dt + \beta(t, Y_t)dW_t^y$$

Where the two Brownian Motion has correlation ρ. The function $Z_t = \Pi(t, X_t, Y_t)$ will then follow the process (to ease the notation the arguments are skipped):

$$dZ_t = d\Pi(t, X_t, Y_t) = [\partial_t\Pi + \mu\partial_x\Pi + \alpha\partial_y\Pi + \tfrac{1}{2}(\sigma^2\partial_{xx}^2\Pi + \beta^2\partial_{yy}^2\Pi) + \rho\beta\sigma\partial_{xy}^2\Pi]dt + \sigma\partial_x\Pi dW_t^x + \beta\partial_y\Pi dW_t^y$$

Setting the drift term equal to $r\Pi$, or the interpretation that it must instantaneously earn the same rate of return as other short-term risk-free securities [6, p. 354], will result in no-arbitrage. Using instead $\Pi(t, S, V_{TK})$ and the processes defined in 3.2, where we have N different Brownian Motions for the variance, yield:

$$\partial_t\Pi + \mu\partial_x\Pi + \alpha\partial_y\Pi + \tfrac{1}{2}(\sigma^2\partial_{xx}^2\Pi + \beta^2\partial_{yy}^2\Pi) + \rho\beta\sigma\partial_{xy}^2\Pi = r\Pi$$

$$\frac{\partial\Pi}{\partial t} + (r-q)S\frac{\partial\Pi}{\partial S} + \alpha_{TK}\frac{\partial\Pi}{\partial V_{TK}} + \tfrac{1}{2}\sigma^2S^2\frac{\partial^2\Pi}{\partial S^2} + \tfrac{1}{2}V_{TK}^2\frac{\partial^2\Pi}{\partial V_{TK}^2}\Big[\sum_{i=1}^N \theta_{TKi}^2 + \sum_{i\neq j}\theta_{TKi}\theta_{TKj}\rho_i\rho_j\Big] + SV_{TK}\sigma\frac{\partial^2\Pi}{\partial S\partial V_{TK}}\sum_{i=1}^N \theta_{TKi}\rho_i = r\Pi$$

Using the *One-dimensional Itô Formula* [2, p. 123] on the underlying asset defined in 3.2 and holding V_{TK} constant:

$$\partial_t \Pi + \mu \partial_x \Pi + \tfrac{1}{2}\sigma^2 \partial^2_{xx} \Pi = r\Pi$$

$$\tfrac{\partial \Pi}{\partial t} + (r-q)S\tfrac{\partial \Pi}{\partial S} + \tfrac{1}{2}V_{TK}^2 S^2 \tfrac{\partial^2 \Pi}{\partial S^2} = r\Pi$$

Using the two results after Itô's Formula has been applied and solving for α_{TK} will yield:

$$\alpha_{TK} = -\tfrac{1}{2\partial\Pi/\partial V_{TK}} \Bigg[S^2 \tfrac{\partial^2 \Pi}{\partial S^2}(\sigma^2 - V_{TK}) + \tfrac{\partial^2 \Pi}{\partial V_{TK}^2} V_{TK}^2 \textstyle\sum_{i=1}^N (\theta_{TKi})^2 +$$

$$\tfrac{\partial^2 \Pi}{\partial V_{TK}^2} V_{TK}^2 \textstyle\sum_{i \neq j} \theta_{TKi}\theta_{TKj}\rho_i\rho_j + 2SV_{TK}\sigma \tfrac{\partial^2 \Pi}{\partial S \partial V_{TK}} \textstyle\sum_{i=1}^N \theta_{TKi}\rho_i \Bigg]$$

All of the above partial derivatives are the same as those for the Black-Scholes model, see [3] for a comprehensive list. Substituting and the relation for α_{TK} in section 3.2 will be the result.

A.3

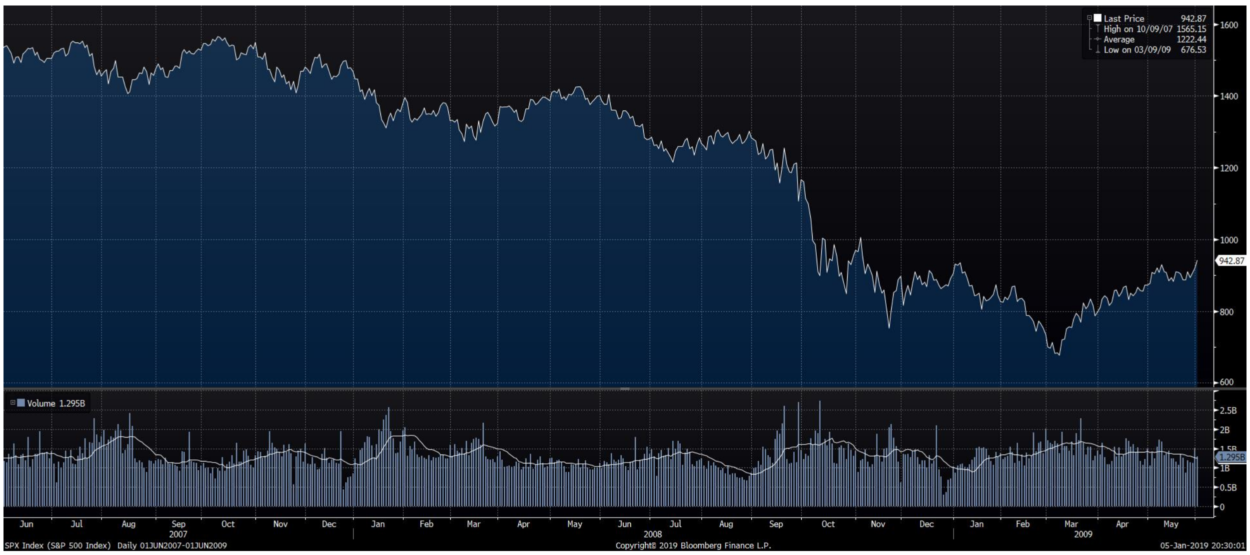

Figure 2: S&P 500 1 June 2007- 1 June 2009

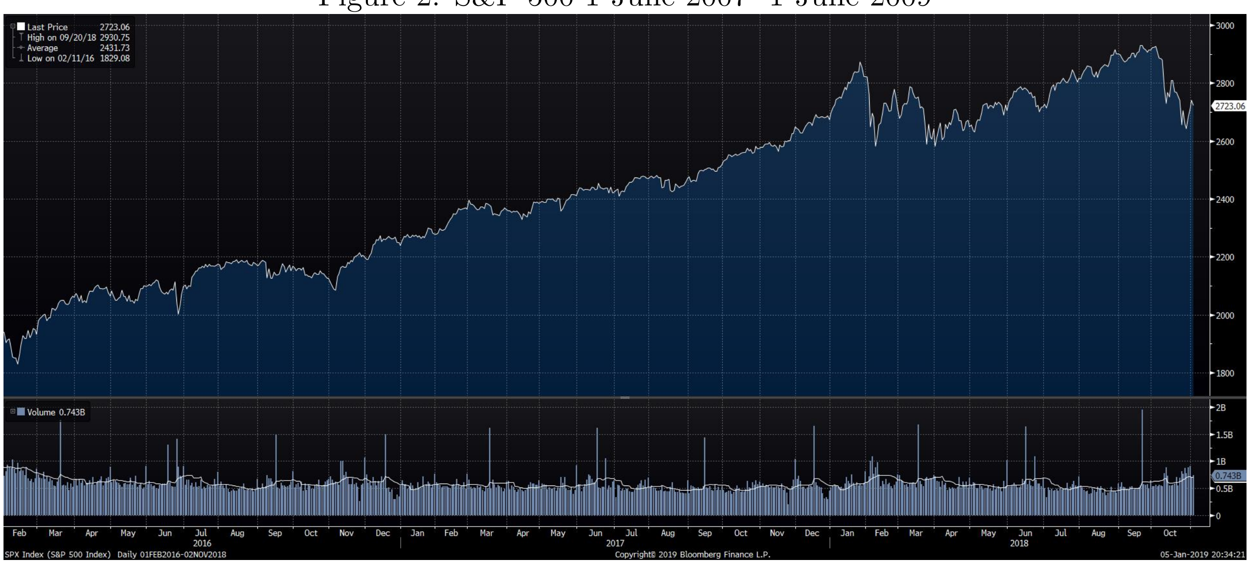

Figure 3: S&P 500 1 February 2016- 2 November 2018

A.4 Rules of Thumb - Regressions

A.4.1 The Sticky Strike Rule

Dependent Variable: Implied Volatility TK
Method: Least Squares
Sample: 1 1368
Included observations: 1368
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
A	1.434131	0.054149	26.48507	0.0000
K	-0.001356	6.81E-05	-19.91914	0.0000
K^2	3.47E-07	2.21E-08	15.71316	0.0000
T	-0.017976	0.002219	-8.101372	0.0000
T^2	8.61E-05	3.81E-05	2.259296	0.0240
KT	1.13E-05	1.18E-06	9.528704	0.0000

R-squared	0.890330	Mean dependent var	0.243926
Adjusted R-squared	0.889928	S.D. dependent var	0.097142
S.E. of regression	0.032229	Akaike info criterion	-4.027531
Sum squared resid	1.414711	Schwarz criterion	-4.004631
Log likelihood	2760.831	Hannan-Quinn criter.	-4.018960
F-statistic	2211.424	Durbin-Watson stat	0.488437
Prob(F-statistic)	0.000000	Wald F-statistic	386.7153
Prob(Wald F-statistic)	0.000000		

Figure 4: Basic Sticky Strike Rule: 1 June 2007 - 1 December 2008

Dependent Variable: Implied Volatility TK
Method: Least Squares
Sample: 1 1801
Included observations: 1801
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
A	1.026167	0.030797	33.32030	0.0000
K	-0.000630	2.62E-05	-24.08498	0.0000
K^2	1.05E-07	5.45E-09	19.17855	0.0000
T	-0.004964	0.000747	-6.641826	0.0000
T^2	5.47E-06	1.68E-05	0.325129	0.7451
KT	2.65E-06	2.76E-07	9.617439	0.0000

R-squared	0.732707	Mean dependent var	0.150992
Adjusted R-squared	0.731962	S.D. dependent var	0.049857
S.E. of regression	0.025812	Akaike info criterion	-4.472628
Sum squared resid	1.195937	Schwarz criterion	-4.454317
Log likelihood	4033.601	Hannan-Quinn criter.	-4.465869
F-statistic	984.0939	Durbin-Watson stat	0.807916
Prob(F-statistic)	0.000000	Wald F-statistic	686.5316
Prob(Wald F-statistic)	0.000000		

Figure 5: Basic Sticky Strike Rule: 1 February 2016 - 2 February 2018

A.4.2 The Sticky Delta Rule

Dependent Variable: Implied Volatility TK
Method: Least Squares
Sample: 1 1368
Included observations: 1368
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
B	1.298915	0.045358	28.63710	0.0000
K/S	-1.095988	0.075606	-14.49605	0.0000
(K/S)^2	0.316191	0.036315	8.707023	0.0000
T	-0.016653	0.001425	-11.68792	0.0000
T^2	8.61E-05	3.13E-05	2.753996	0.0060
(K/S)T	0.013842	0.000885	15.64236	0.0000
D1	-0.363888	0.021568	-16.87199	0.0000
D2	-0.349420	0.021344	-16.37054	0.0000
D3	-0.299806	0.021124	-14.19281	0.0000
D4	-0.298175	0.021153	-14.09644	0.0000
D5	-0.327385	0.021388	-15.30714	0.0000
D6	-0.287361	0.021279	-13.50443	0.0000
D7	-0.277321	0.021479	-12.91102	0.0000
D8	-0.278818	0.021282	-13.10143	0.0000
D9	-0.283113	0.021131	-13.39784	0.0000
D10	-0.264007	0.021063	-12.53409	0.0000
D11	-0.266561	0.021174	-12.58879	0.0000
D12	-0.312965	0.021583	-14.50062	0.0000
D13	-0.297779	0.021884	-13.60696	0.0000
D14	-0.282056	0.021221	-13.29156	0.0000
D15	-0.285260	0.021299	-13.39286	0.0000
D16	-0.284557	0.021531	-13.21634	0.0000
D17	-0.205179	0.023299	-8.806170	0.0000
D18	-0.075608	0.026431	-2.860624	0.0043

R-squared	0.934155	Mean dependent var	0.243926
Adjusted R-squared	0.933028	S.D. dependent var	0.097142
S.E. of regression	0.025139	Akaike info criterion	-4.511383
Sum squared resid	0.849385	Schwarz criterion	-4.419785
Log likelihood	3109.786	Hannan-Quinn criter.	-4.477102
F-statistic	829.0245	Durbin-Watson stat	0.387383
Prob(F-statistic)	0.000000	Wald F-statistic	179.3746
Prob(Wald F-statistic)	0.000000		

Figure 6: General Sticky Delta Rule: 1 February 2007 - 1 December 2008

Dependent Variable: Implied Volatility TK
Method: Least Squares
Sample: 1 1801
Included observations: 1801
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.563512	0.048989	31.91533	0.0000
K/S	-2.414929	0.098518	-24.51257	0.0000
(K/S)^2	0.977740	0.049941	19.57809	0.0000
T	-0.009664	0.000560	-17.27154	0.0000
T^2	4.23E-06	6.90E-06	0.612916	0.5400
(K/S)T	0.010874	0.000518	21.00643	0.0000
D1	0.042713	0.003596	11.87709	0.0000
D2	0.033797	0.002670	12.65852	0.0000
D3	0.007036	0.002739	2.568553	0.0103
D4	0.015151	0.002508	6.041772	0.0000
D5	0.009199	0.002719	3.383610	0.0007
D6	0.014557	0.002632	5.529891	0.0000
D7	0.004793	0.002905	1.649895	0.0991
D8	0.006596	0.002946	2.238536	0.0253
D9	0.006274	0.002702	2.322285	0.0203
D10	0.022329	0.003399	6.568558	0.0000
D11	0.011725	0.002657	4.412459	0.0000
D12	0.007143	0.002665	2.680169	0.0074
D13	-0.001959	0.002762	-0.709025	0.4784
D14	-0.001409	0.002503	-0.562814	0.5736
D15	-0.008924	0.002778	-3.212579	0.0013
D16	-0.015322	0.002572	-5.957135	0.0000
D17	-0.013414	0.003093	-4.337010	0.0000
D18	-0.012688	0.002807	-4.519859	0.0000
D19	-0.021941	0.002678	-8.191525	0.0000
D20	-0.014447	0.002540	-5.688565	0.0000
D21	-0.017778	0.002656	-6.693286	0.0000
D22	-0.017736	0.002623	-6.761347	0.0000
D23	-0.013038	0.002539	-5.135138	0.0000
D24	-0.015206	0.002717	-5.595878	0.0000

R-squared	0.883391	Mean dependent var	0.150992
Adjusted R-squared	0.881482	S.D. dependent var	0.049857
S.E. of regression	0.017164	Akaike info criterion	-5.275500
Sum squared resid	0.521736	Schwarz criterion	-5.183949
Log likelihood	4780.588	Hannan-Quinn criter.	-5.241706
F-statistic	462.6404	Durbin-Watson stat	1.245640
Prob(F-statistic)	0.000000	Wald F-statistic	438.8116
Prob(Wald F-statistic)	0.000000		

Figure 7: General Sticky Delta Rule: 1 February 2016 - 2 February 2018

Dependent Variable: Implied Volatility TK - Implied Volatility TS
Method: Least Squares
Sample: 1 1368
Included observations: 1368
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
B	0.005760	0.001086	5.302608	0.0000
ln(K/S)	-0.456840	0.011101	-41.15182	0.0000
ln(K/S)^2	0.124295	0.026365	4.714331	0.0000
T	-0.001279	0.000204	-6.265398	0.0000
T^2	4.23E-05	6.67E-06	6.343205	0.0000
ln(K/S)T	0.013934	0.000639	21.81935	0.0000

R-squared	0.911500	Mean dependent var	0.003447
Adjusted R-squared	0.911175	S.D. dependent var	0.040416
S.E. of regression	0.012045	Akaike info criterion	-5.995914
Sum squared resid	0.197610	Schwarz criterion	-5.973014
Log likelihood	4107.205	Hannan-Quinn criter.	-5.987344
F-statistic	2805.552	Durbin-Watson stat	1.398768
Prob(F-statistic)	0.000000	Wald F-statistic	856.1284
Prob(Wald F-statistic)	0.000000		

Figure 8: Relative Sticky Delta Rule: 1 June 2007 - 1 December 2008

Dependent Variable: Implied Volatility TK - Implied Volatility TS
Method: Least Squares
Sample: 1 1801
Included observations: 1801
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
B	0.024419	0.001425	17.14154	0.0000
ln(K/S)	-0.437324	0.010726	-40.77419	0.0000
ln(K/S)^2	0.742512	0.045116	16.45768	0.0000
T	-0.004604	0.000249	-18.46535	0.0000
T^2	0.000139	8.06E-06	17.18584	0.0000
ln(K/S)T	0.011570	0.000550	21.02269	0.0000

R-squared	0.860884	Mean dependent var	0.012694
Adjusted R-squared	0.860497	S.D. dependent var	0.047950
S.E. of regression	0.017909	Akaike info criterion	-5.203666
Sum squared resid	0.575734	Schwarz criterion	-5.185356
Log likelihood	4691.901	Hannan-Quinn criter.	-5.196907
F-statistic	2221.588	Durbin-Watson stat	1.219636
Prob(F-statistic)	0.000000	Wald F-statistic	1007.656
Prob(Wald F-statistic)	0.000000		

Figure 9: Relative Sticky Delta Rule: 1 February 2016 - 2 February 2018

A.4.3 The Square Root of Time Rule

Dependent Variable: Implied Volatility TK - Implied Volatility TS
Method: Least Squares
Sample: 1 1368
Included observations: 1368
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ln(S/K)/sqrt(T)	-0.841494	0.012594	-66.81804	0.0000
[ln(S/K)/sqrt(T)]^2	0.149293	0.134162	1.112785	0.2660
[ln(S/K)/sqrt(T)]^3	12.05286	0.907691	13.27859	0.0000
[ln(S/K)/sqrt(T)]^4	31.58730	5.161682	6.119576	0.0000

R-squared	0.953452	Mean dependent var	0.003447
Adjusted R-squared	0.953350	S.D. dependent var	0.040416
S.E. of regression	0.008729	Akaike info criterion	-6.641369
Sum squared resid	0.103935	Schwarz criterion	-6.626103
Log likelihood	4546.697	Hannan-Quinn criter.	-6.635656
Durbin-Watson stat	1.346974		

Figure 10: Square Root of Time Rule: 1 June 2007 - 1 December 2008

Dependent Variable: Implied Volatility TK - Implied Volatility TS
Method: Least Squares
Sample: 1 1801
Included observations: 1801
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ln(K/S)/sqrt(T)	-0.845864	0.015664	-54.00063	0.0000
[ln(K/S)/sqrt(T)]^2	3.918291	0.169615	23.10113	0.0000
[ln(K/S)/sqrt(T)]^3	12.70816	0.785017	16.18840	0.0000
[ln(K/S)/sqrt(T)]^4	-17.84485	4.653719	-3.834536	0.0001

R-squared	0.940799	Mean dependent var	0.012694
Adjusted R-squared	0.940700	S.D. dependent var	0.047950
S.E. of regression	0.011677	Akaike info criterion	-6.060251
Sum squared resid	0.245006	Schwarz criterion	-6.048045
Log likelihood	5461.256	Hannan-Quinn criter.	-6.055745
Durbin-Watson stat	0.976284		

Figure 11: Square Root of Time Rule: 1 February 2016 - 2 February 2018

Dependent Variable: Implied Volatility TK - Implied Volatility TS
Method: Least Squares
Sample: 1 1368
Included observations: 1368
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ln(K/S)/sqrt(T)	-0.840874	0.012475	-67.40619	0.0000
[ln(K/S)/sqrt(T)]^2	0.196795	0.134942	1.458366	0.1450
[ln(K/S)/sqrt(T)]^3	12.00694	0.893574	13.43698	0.0000
[ln(K/S)/sqrt(T)]^4	30.55444	5.281891	5.784754	0.0000
D1	0.005786	0.002118	2.732481	0.0064
D2	0.002417	0.001272	1.900155	0.0576
D3	0.000906	0.000732	1.237577	0.2161
D4	-1.36E-05	0.000663	-0.020546	0.9836
D5	0.002789	0.001462	1.908379	0.0566
D6	0.000113	0.000462	0.244001	0.8073
D7	-0.002384	0.000726	-3.285827	0.0010
D8	-0.002932	0.001090	-2.690100	0.0072
D9	-0.000639	0.000401	-1.593162	0.1114
D10	-0.002646	0.001029	-2.570768	0.0103
D11	-0.002725	0.000910	-2.992592	0.0028
D12	0.000834	0.000890	0.936293	0.3493
D13	0.000495	0.000777	0.636536	0.5245
D14	-0.001457	0.000655	-2.222781	0.0264
D15	-0.000911	0.000698	-1.304096	0.1924
D16	-0.000958	0.000735	-1.302757	0.1929
D17	-0.002455	0.000818	-3.000869	0.0027
D18	-0.001847	0.001583	-1.166706	0.2435

R-squared	0.956379	Mean dependent var	0.003447
Adjusted R-squared	0.955699	S.D. dependent var	0.040416
S.E. of regression	0.008507	Akaike info criterion	-6.680000
Sum squared resid	0.097399	Schwarz criterion	-6.596035
Log likelihood	4591.120	Hannan-Quinn criter.	-6.648576
Durbin-Watson stat	1.428378		

Figure 12: General Square Root of Time Rule: 1 June 2007 - 1 December 2008

Dependent Variable: Implied Volatility TK - Implied Volatility TS
Method: Least Squares
Sample: 1 1801
Included observations: 1801
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ln(K/S)/sqrt(T)	-0.844293	0.015116	-55.85548	0.0000
[ln(K/S)/sqrt(T)]^2	4.039312	0.141833	28.47941	0.0000
[ln(K/S)/sqrt(T)]^3	12.59192	0.729130	17.26979	0.0000
[ln(K/S)/sqrt(T)]^4	-20.48044	4.102688	-4.991956	0.0000
D1	-0.008593	0.001561	-5.505532	0.0000
D2	-0.008273	0.001432	-5.775621	0.0000
D3	-0.004546	0.000663	-6.861088	0.0000
D4	-0.003706	0.000756	-4.902763	0.0000
D5	-0.004632	0.000666	-6.958238	0.0000
D6	-0.004800	0.000674	-7.124022	0.0000
D7	-0.000464	0.001113	-0.417021	0.6767
D8	-0.003236	0.000714	-4.531733	0.0000
D9	-0.003795	0.000717	-5.292060	0.0000
D10	-0.007328	0.001407	-5.206946	0.0000
D11	-0.002257	0.000790	-2.856304	0.0043
D12	-0.001277	0.000975	-1.309693	0.1905
D13	0.001658	0.001490	1.113187	0.2658
D14	0.000122	0.001018	0.119600	0.9048
D15	0.001732	0.001437	1.205415	0.2282
D16	0.002883	0.001663	1.733650	0.0832
D17	0.004340	0.002648	1.638797	0.1014
D18	0.000268	0.000956	0.280099	0.7794
D19	0.004765	0.002128	2.238707	0.0253
D20	0.002421	0.001642	1.473973	0.1407
D21	0.005015	0.002300	2.180162	0.0294
D22	0.004382	0.001952	2.244661	0.0249
D23	0.002018	0.001687	1.196382	0.2317
D24	0.004462	0.002276	1.960512	0.0501

R-squared	0.948317	Mean dependent var	0.012694
Adjusted R-squared	0.947530	S.D. dependent var	0.047950
S.E. of regression	0.010984	Akaike info criterion	-6.169416
Sum squared resid	0.213891	Schwarz criterion	-6.083969
Log likelihood	5583.559	Hannan-Quinn criter.	-6.137875
Durbin-Watson stat	1.119841		

Figure 13: General Square Root of Time Rule: 1 February 2016 - 2 February 2018

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