Lunds University - Department of economics

Bachelor thesis

Investigating the efficient market hypothesis using Fourier analysis

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Abstract

This study examines if the Swedish stock market adheres to the weak form efficient market hypothesis using Fourier analysis on past stock prices to identify possible cyclic returns. Fourier analysis is well suited for finding seasonalities which would violate the weak-form efficiency. 10 firms were randomly selected from stock market index OMX30 to represent the Swedish stock market.

All firms investigated showed signs of periodic behaviour in the long term but adhered to the weak form efficiency in the short term. Some of the cycles found supports some already known calendar effects, such as the U.S presidential election cycle, January effect and "Sell in May" strategy. However, the transforms were of poor resolution due to short data sets, making it difficult to differentiate potential cycles from white noise. In addition, this paper does not account for risk and assumes that all price mechanism are accurate.

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1 Introduction

1.1 Background

A *calendar effect* refers to any effect or anomaly that is related to the calendar or the season and are very prevalent in most time series data. It is present in unemployment rates, fatality rates at hospitals and even influences divorce rates among married couples (Bach 2016; Labor statistics 2001; Rogers 2012). While it may come as a surprise initially, with some reflection the observed behaviour is quite expected. Certain jobs are only available on specific seasons, most medical students graduate at a certain time of the year and certain seasons are often associated with different moods.

One would therefore expect to find seasonal behaviour on the stock market. People might be more optimistic during a certain time of the year or have more disposable income to invest with. However, the presence of calendar effects in stock markets would violate one of the very most central financial economic theorems, the *efficient market hypothesis*. The hypothesis in essence claims that all asset prices on the market fully reflect all available information and therefore impossible to consistently beat the market. If there was a way to exploit the market, all investors would join in until there no longer would be any potential gains. Eugene Francis Fama who developed the theorem eventually ended up receiving a Nobel prize in economics for his work.

Under recent times, the efficient market hypothesis has come under much scrutiny. It has failed to explain multiple market anomalies and some even claiming that blind faith in efficient markets caused the financial crisis of 2007 (Nocera 2009). Whether or not the efficient market hypothesis holds true or not would have big implications. If markets adhere to the efficient market hypothesis, hedge fund managers would only be an expense and not yield higher returns. The price the stock is traded at would always be its' fair value. But if the markets would not adhere to the efficient market hypothesis, investors could be making excess returns without any extra risk. Any calendar effect would imply that investors can simply earn money by buying and selling at the right time rather than buying the right stock. Information that could potentially save people from making bad investments or make really good ones.

1.2 Problem discussion

> ”Ships will sail around the world but the Flat Earth Society will flourish. There will continue to be wide discrepancies between price and value in the marketplace...”
>
> -Warren Buffet (Hagstrom 1999)

For a long time the efficient market hypothesis had been the dominating school of thought[1]. Based on the main principle of rational expectations where on average, every investor knows the model. However, the efficient market hypothesis has failed to describe multiple market anomalies around the world and has even been blamed for being the cause of some. Consequentially, behavioural economics has lately gained traction, contradicting the efficient market hypothesis' idea of rational investors. Fama has even conceded that misinformed investors could cause inefficient markets (Hilsenrath 2004).

A paper published by Grossman and Stiglitz (1980) argued that gathering information, processing and analyzing it is a time consuming and costly process for an investor. Therefore, prices can not reflect all available information. Since if it did, the investor who spent resources gathering information would receive no compensation. Concluding that an efficient market is effectively impossible.

Whether it is possible to obtain fully efficient markets or not is perhaps a question best left for philosophers. Whether markets are efficient or not is perhaps a more compelling question to answer, which has often resulted in mixed results. Basu (1977) found that stocks with low price to earnings ratio often outperform stocks with higher ratios, which would violate the efficient market hypothesis while others such as Chan, Gup, and Pan (1997) found that many markets adheres to the hypothesis. Many of these papers test for market efficiency using statistical tests of independence or time series data. Meaning investors look for some property or attribute a stock or firm has or how something changes over time. There is inherently nothing wrong with this approach by treating time linearly. After all, that is how time works. However, it neglects the repetitive attribute time usually has. Things tend to happen in regular time intervals. Earth rotates along its own axis once every 24^{th} hour and around the sun approximately once every 365 days. Therefore, another approach to determine whether a stock market is efficient or not is to look for calendar effects. If a stock price moves up and down in a predictable fashion, investors could exploit it to make excess returns without taking more risk. This is where Fourier analysis excels. Fourier analysis is a method to quickly identify any periodic behaviour within any signal. By using Fourier analysis any repetitive movement in stock prices can be identified.

Ten stocks are randomly chosen from the Stockholm stock market with the only criteria required being listed on the index OMX30 to determine whether or not Sweden's stock market is efficient. The Swedish market was chosen since it is the most interesting and relevant one from a Swedish perspective.

[1]Chicago school of economics to be more precise. The faculty has spawned many notable scholars such as Milton Friedman and Ronald Coase, all with the same basic principle of rational expectations.

1.3 Purpose

The purpose of this paper is to investigate if the Stockholm stock market adheres to the weak form efficient market hypothesis utilizing Fourier analysis on past stock prices.

1.4 Delimitations

Since this paper is geared towards undergraduates in economics, many mathematical aspects of the Fourier transform will be left out or only briefly mentioned. Instead, this paper will try to give the reader a more intuitive understanding of Fourier analysis[2]. In addition, due to time constraints the whole Swedish stock market can not be analyzed and therefore the number of firms will be restricted to 10 and no indices will be analyzed.

1.5 Disposition

The theoretical background will be presented in chapter 2 and is the foundation for the rest of the thesis. Chapter 3 will describe some of the calendar effects of interest that have some empirical evidence. Chapter 4 presents the empirical data obtained from the data source. The methodology is presented in chapter 5 with the results presented in chapter 6. At last the results will be analyzed in chapter 7 and concluded in chapter 8.

2 Theory

2.1 Efficient Market Hypothesis

The efficient market hypothesis (often abbreviated as EMH) is a theory in financial economics developed by E. F. Fama (1970) and implicates that it is impossible for any investor to consistently beat the market. Meaning that no amount of knowledge, computing power or *market timing* results in a consistently higher return than the market. Suggesting that the only possibility of obtaining higher returns is purely by chance or riskier investments. The model builds on certain assumptions[3], those being:

- Every investor is rational
- No information asymmetries and that all investors have homogeneous beliefs
- Markets are friction-less, i.e. no transaction costs, no short sale constraints, no distortions by the tax systems.

[2]For a more complete mathematical understanding of Fourier analysis, Firth (1992) is highly recommended.

[3]While these are the assumptions Fama makes, they are sufficient but not necessary. Meaning that the assumptions can be only partly fulfilled and there can still be an efficient market.

EMH is often stated in three different forms, *weak-form* efficiency, *semi-strong-form* efficiency and *strong-form* efficiency. Each of them has different ramifications for how markets work.

In **weak-form market efficiency** future prices cannot be predicted by analyzing past information. Asserting that stock prices already reflect all information that can be obtained by examining the history of the market. Excess returns are impossible to achieve using any algorithm or any special technique since share prices show no pattern or tendencies. If any tactic would reliably predict the future of the stock market, investors would exploit the tactic until it would become useless.

The **semi-strong-form market efficiency** incorporates the weak market efficiency and asserts that all *publicly* available knowledge regarding a firm and its prospects must also already be reflected in the stock price. Such information includes quality of management, balance sheet composition, patents held, fundamental data on the firm's product line, earning forecasts and accounting practices.

The **strong-form efficiency** asserts that, price and price changes reflect *all* information. Meaning that prices should follow a completely random walk and insiders are not able to beat the market since their knowledge is also public knowledge. In this thesis, Fourier analysis will be a test on the weak-form market efficiency since the analysis will be based on historic stock prices and no information about a firms balance sheet composition or similar will be regarded.

2.2 Stock prices and random walks

According to the EMH, stock prices should follow a random walk. However, it does not mean that prices move randomly, it only entails that new information should be unpredictable. Given that no new vital information regarding the firm is released, the stock prices tend to move in a certain direction. Stocks are expected to increase in value by a set percentage by the investors, independent of the stock's price (Hull 2012, p. 287). This means that the expected value of a stock is given by:

$$\mathrm{E}(S_T) = S_0 e^{\mu T} \tag{1}$$

where E is the expected value operator, S_T is the expected stock value at time T, where $\{T\} := t_1, t_2, ..., t_T$ and μ is the expected daily rate of return. Consequently, the stock price is expected to increase exponentially over time. While the relative expected return might stay constant over time, the absolute value of the changes in stock valuation will grow larger.

Note that these equations assumes non-dividend paying stock. For dividend paying stock, pricing is a bit trickier. In theory, on the pay out day, the price of the stock should fall with the same amount equal to the dividend pay out. However, step functions are difficult to model and is outside the scope of this thesis. In addition, it is often assumed that the dividends are spent on reinvesting in the same stock. Therefore, all stock will be treated as non dividend paying stock[4].

2.3 Risk adjusted returns and testing EMH

As stated by E. F. Fama (1970) it is impossible to earn more than the market unless taking on more risk. Therefore, if returns are adjusted for risk, it should not be possible to consistently beat the market. However, this poses a problem. From equation 1 the expected stock price is dependent on another stochastic variable, the expected return rate. The expected rate of return varies over time depending on another stochastic variable, the level of risk, hence

$$\mu_{\mathrm{t+1,t}} = \mathrm{E}(\mu_{\mathrm{t+1,t}}|\Phi_{\mathrm{t}}) + z_{\mathrm{t+1,t}} \tag{2}$$

where $\mathrm{E}(\mu_{t+1,t}|\Phi_t)$ is the expected rate of return for period t to t+1.Φ_{t} represents information available to the investor, which under the EMH should imply all possible information and $z_{\mathrm{t+1,t}}$ is any possible deviation from the observed return. For the EMH to hold, then:

$$\mathrm{E}(z_{\mathrm{t+1,t}}|\Phi_{\mathrm{t}}) = 0 \tag{3}$$

The deviations has to be *random*. However, risk is difficult to measure and predict. It is known to be time dependent even though it is often incorrectly assumed to be constant[5] and can often unpredictably surge upwards, as can be shown in figure 1. The problem occurs when testing for market efficiency that adheres both to equation 2 and equation 3 simultaneously. This is a *joint-hypothesis problem* which means it is impossible to test for market efficiency. Because if stocks deviate from what they are expected to be valued at, it could reflect that the market is inefficient or that the pricing models used are inaccurate due to flawed risk management, it is impossible to know which.

Therefore, to test for efficient markets, risk will be ignored. All asset pricing models will by assumption be accurate and correct. For the weak-form market efficiency to hold there should not be any reliable way of earning more than the expected return of the stock. Any deviations of the expected return must then be countered by the same offset in the opposite direction. If one month yields 2% more than the expected return, there should be one month that yields 2% less than the expected return since equation 3 must hold. In addition, there ought to be no patterns in the deviations either as this would lead to possible exploitation. For it to be impossible to exploit through market timings, the

[4]The step functions will result in a frequency spectrum. Giving rise to multiple frequency peaks equidistant from each other called ”harmonics” in the Fourier transform, further complicating the analysis.

[5]Most notably assumed in the Black-Scholes formula (Black and Scholes 1973).

deviations should not possess any periodical patterns. One of the most common means of testing for periodic patterns is using *Fourier analysis*.

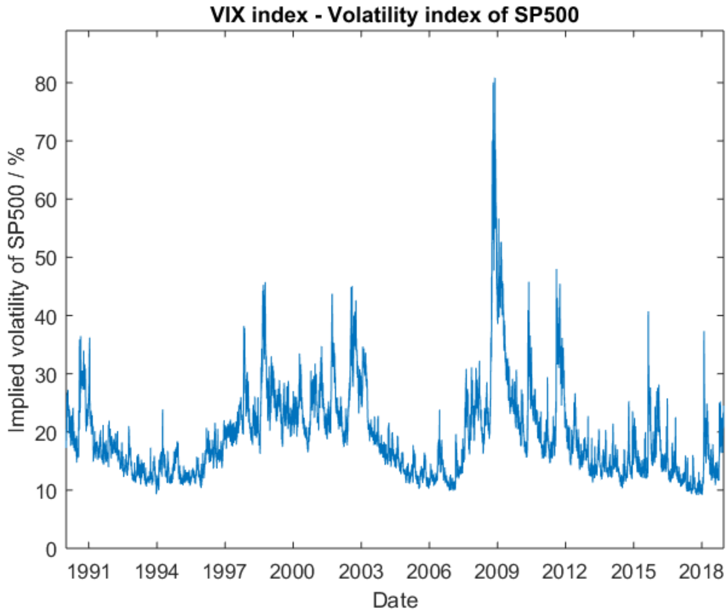

Figure 1: Shows the VIX index - A measure of implied volatility of the index SP500

2.4 Fourier analysis

2.4.1 Fourier transformations

Fourier analysis, named after the french mathematician Joseph Fourier who first investigated the phenomenon, is the study of how all general functions can be rewritten as a sum of trigonometric functions. This includes non-continuous wave forms such as a square wave or a saw tooth wave. Initially, Fourier analysis was used as a way to considerably simplify the study of heat transfer problems. However, the applications have now spread to many other scientific areas, including financial economics (Firth 1992).

The process of deconstructing a function to its' trigonometric functions is called a *Fourier transform*. The reverse process, by taking trigonometric functions and rebuilding the function, can also be done and is sometimes referred as *Fourier synthesis*. However, both operations are usually referred to as a Fourier transformation and Fourier analysis is often referred to the study of both operations.

2.4.2 Terminology

There are many technical terms that will be frequently used in this thesis, having a basic understanding of them will be necessary to understand the theory behind it.

- **Periodic function/sequence** (Sometimes referred to as cycle) - Any periodic function is a function that repeats its values in regular intervals or *periods*. Typical periodic functions are sine and cosine.

- **Oscillation** - An oscillation is a repetitive motion or pattern. If something oscillates at 10 times per second, it is said to repeat the same pattern 10 times per second.
- **Period** - The time required to complete a single cycle or oscillation.
- **Frequency** - Number of *oscillations* completed per unit time. Often given in the unit Hertz (Hz) which is number of repetitions per second.

More key concepts will be introduced in this chapter using the terminology above.

2.4.3 Advantages with Fourier analysis

A periodicity is a cyclic behaviour, a pattern that repeats itself in regular intervals. Usually, a function is represented how it changes over time. Instead, it can be represented as the presence of frequencies (or lack thereof) using Fourier transforms, as shown in the following figure. When Fourier transforming, the signal is transformed from the time domain to the frequency domain, the inverse of time domain.

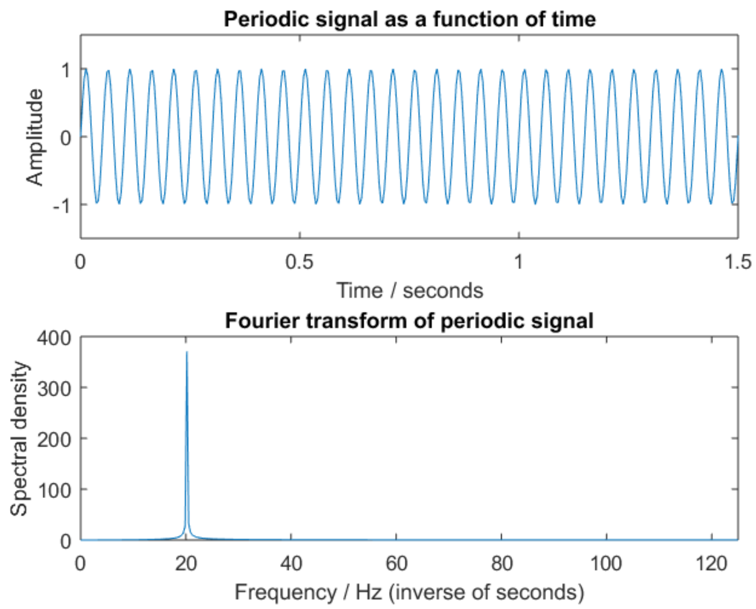

Figure 2: Fouriertransform of a simple sine signal

From figure 2, a simple oscillating signal is shown with 20 oscillations per second. A Fourier transform quickly picks out the frequency present. Fourier analysis is well suited to investigate if it is possible to earn returns by timing the market since the transformation will show any cyclic behaviour in stock returns. This is particularly advantageous because some cyclic behaviour can be difficult to spot when data is shown as a function of time. From figure 2 the oscillations are easy to spot and the frequency is also easy to calculate. However, signals often consist of multiple frequencies and disrupted by random noise.

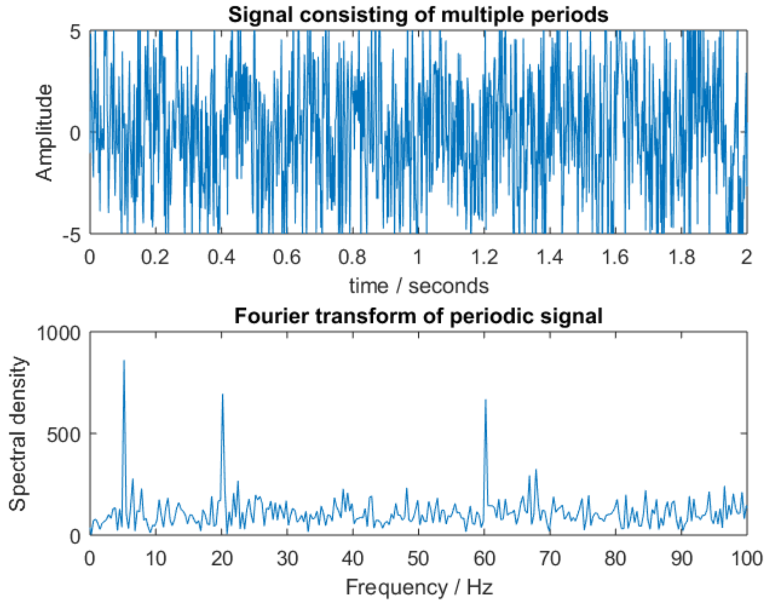

Figure 3: Fouriertransform of a noisy periodic signal

The signal in figure 3 consists of three different periodic functions oscillating at 5, 20 and 60 times per second, corresponding to 5, 20 and 60 Hertz. In addition, white noise is also added to further corrupt the signal. Despite this, the Fourier transform is still able to pick out the correct frequencies present. The presence of smaller peaks covering the entire frequency spectrum is the result of white noise added to the signal.

2.4.4 Discrete Fourier transform

While there are many different mathematical methods to Fourier transform a signal, the most popular and practical method of transforming is the Discrete Fourier transform (Spanne 1995, pp. 282–284) . A discrete Fourier transform converts a data set, $\{s_N\} := s_0, s_1, s_3, ..., s_{N-1}$, from the time domain to the frequency domain using the following equation:

$$\begin{aligned} S_k &= \sum_{n}^{N-1} s[n] e^{-2i\pi\frac{k}{N}n} \\ &= \sum_{n}^{N-1} s[n] \cdot [\cos(2\pi kn/N) - i \cdot \sin(2\pi kn/N)], \end{aligned} \tag{4}$$

where S_k is the transformed data set and k the sinusoids periodicity. The reverse operation, a *Fourier synethesis*, is also possible using the discrete fourier transform method:

$$s_N = \frac{1}{N} \sum_{k=0}^{N-1} S_k \cdot e^{2i\pi\frac{k}{N}n} \tag{5}$$

2.4.5 Sampling rate and period

Two important aspects to understanding discrete Fourier transforms are the *sampling period* and *sampling rate*. The sampling rate refers to the number of times a measurement is recorded for every unit of time. While the sampling frequency can be quite large, such as audio recordings (often around 100 000 times per second or more), most historical data of stock prices are sampled once per day. To resolve any signal, it has to be sampled *more than twice* per cycle, also known as the *Nyqvist sampling criterion*. In the case of stock prices, the Fourier transform would not be able to find cyclic periods that are shorter than two days, due to the sampling restriction of once per day. Figure 4 shows a signal with 120 repetitions per second (shown in red) but is only sampled 40 times per second (shown in black asterisk). Due to the low sample rate relative to the true signal frequency, the Fourier transform shows a frequency that does not occur in the true signal. This phenomena is called *aliasing*.

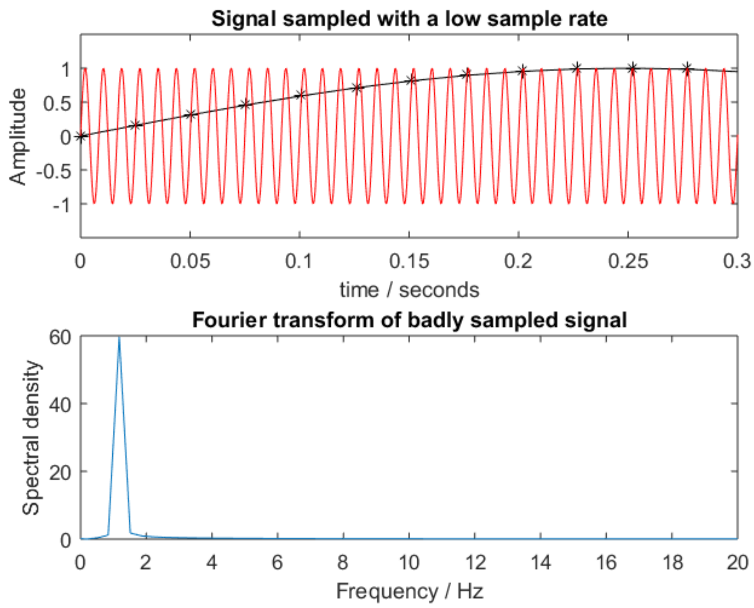

Figure 4: Fourier transform of a badly sampled (black) signal (red)

While the *Nyqvist sampling criterion* sets the upper bound for detectable frequencies, the sampling period sets the limit for how long a cycle can be for it to be detected. The sampling period refers to the time period a signal is being measured. If any cyclic behaviour lasts for a long period time, for example 4 years, the data set needs at least 4 years of stock prices for the transformation to be able to detect that cycle. More samples leads to better resolution of the transform and allows for longer periods to be detected.

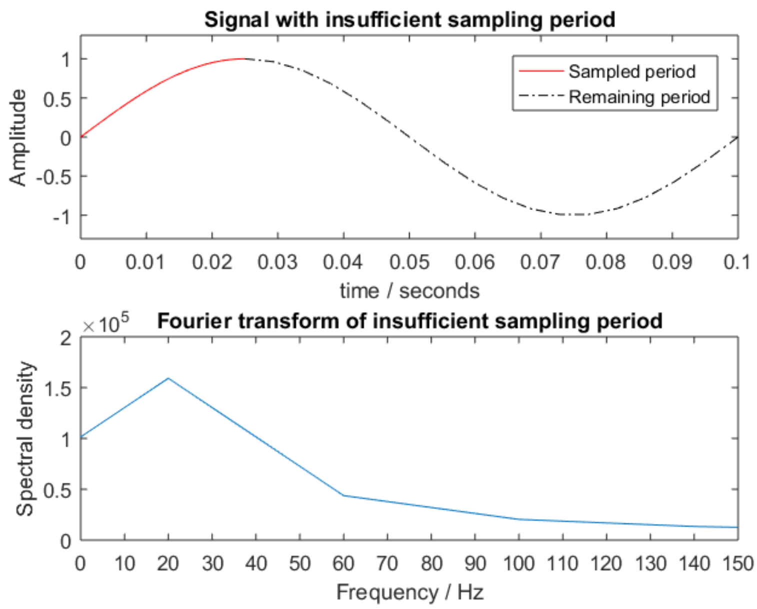

Figure 5: Fourier transform of a signal with insufficient sampling period

Figure 5 shows a signal oscillating at 10 times per second with a very high sample rate at 10 MHz. However, it is sampled for an insufficient period of time, red line representing the sampled part and the black dotted line showing the remaining period. Due to the insufficient sampling period, the Fourier transform shows *aliasing* (incorrect frequency being present). In addition the resolution of the transformation is very poor, with every distinct data point in the transform being 40 Hz apart from each other.

2.4.6 Weaknesses with the Fourier transform

Apart from the restrictions set by the *sampling rate* and *sampling period* there are some limitations due to the resolution of the transform. Resolution is the ability to distinctly tell two frequencies apart from each other, which is directly affected by the sampling period. It can be difficult to make distinctions between the frequency peaks if the oscillations are close to each other or if the observed peak is caused by white noise with short sampling periods. If the sampling period is short, two peaks close to each other will merge together due to the lower resolution of the transform, making it difficult to distinguish between two different peaks and their exact frequency. Aggravating the problem, there can be peaks hidden between two peaks but is drowned out due to the relatively larger amplitudes[6] of the other frequencies as seen in figure 6.

[6] Amplitude refers to the ”size” of the oscillation

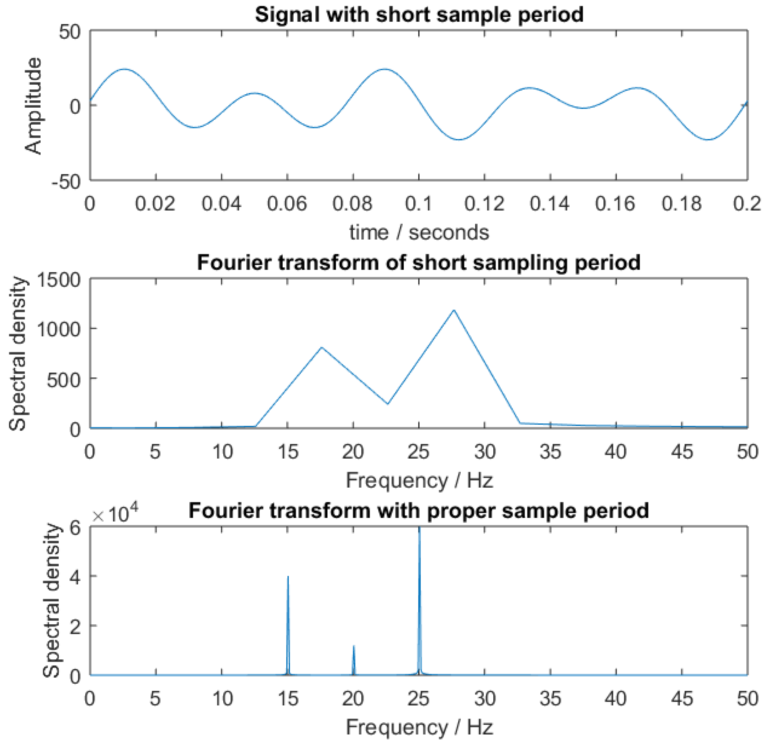

Figure 6: Fourier transform of a signal with 3 periodic functions

The signal transformed in figure 6 consists of three frequencies, 15, 20 and 25 Hz with a respective amplitude of 10, 3 and 15. However, from the transform it is easy to incorrectly draw the conclusion that there are only two frequencies present. The signal is sampled for a short time. Enough to let all three frequencies complete multiple oscillations so it satisfies the *sampling period criteria*, but short enough to leave the resolution unsatisfying. From the transform graph, the peaks are very wide and just slightly off (shifted to right) due to the short sampling period. With proper sample period, the transform should resemble the one at the bottom of figure 6.

When white noise is present in the signal it further complicates identifying relevant peaks. Figure 7 shows the exact same signal as before but with added white noise. From the transform, the smallest amplitude at 20 Hertz could easily be mistaken for being white noise. Vice versa, the peak occurring above 50 Hertz due to presence of white noise, could now be mistaken for being a frequency present in the signal.

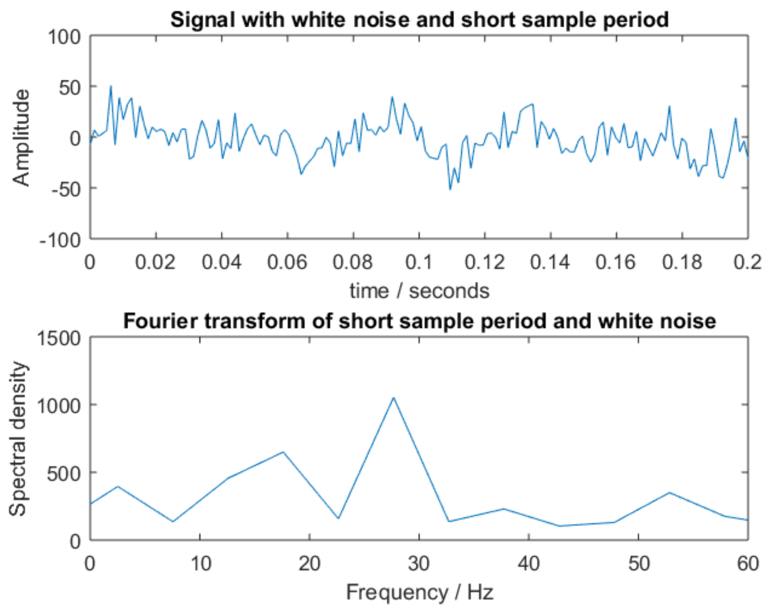

Figure 7: Fourier transform of a signal with 3 periodic functions

However, even if a clear frequency is presented by the transform there is still one fundamental problem with the transform. The Fourier transform can only pick out periodic behaviour and not much more. While the cycle can be identified and its period can be calculated, it is difficult to tell when the cycle starts. Since it is impossible to tell only from the transform when the cycle begins, it is also impossible to tell when to buy or sell. Information that is vital to investors if they wish to beat the market.

3 Previous research in calendar effects

There are plenty of strategies that tries to exploit allegedly present calendar effects on the stock market. While many strategies may lack conclusive evidence and is mostly speculation, there is empirical research that has found calendar effects in the stock market that could give some of the known tactics legitimacy, which would violate the weak-form efficiency.

3.1 January effect

There is empirical evidence suggesting that stock prices surge during the month of January, first documented by Wachtel (1942). In addition to Wachtels discovery, Banz (1981) documented that small firms portfolios consistently had higher returns than portfolios consisting of larger firms. Later studies on Banz paper found that the effect occurs virtually entirely in the month of January (Keim 1983; Reinganum 1983). This suggests that stock prices go up in the month of January, especially on small firm stock.

A plausible explanation for the January effect is that investors sell off stocks that have performed badly to offset potential capital gains for tax purposes. Then as the year ends, investors buy back the stocks. This would explain why small-firm stock prices fluctuate more since their prices are more prone to fluctuations caused by investors selling and buying (Ciccone 2013).

3.2 ”Sell in May”

The strategy ”Sell in May” is based on the belief that the stock market has significantly stronger growth during the months November to April compared to the rest of the year. When May rolls around an investor should sell their stock and invest in bonds until November (Twin 2008).

There has been research that support this strategy, which has found that several stock markets performs significantly stronger during seasons where temperature are lower, with Sweden being one of the countries examined (Cao and Wei 2015).

3.3 United States presidential election cycle

Data shows that for over four decades the stock market followed the presidential election closely. A paper published by Wong and McAleer (2009) found that the US stock market in general underperforms the initial two years of the presidential term before reaching a local minimum and then turning around the last two years. Showing that political uncertainty can affect the market and its' sentiment. While it is unclear whether the American election cycle affects the Swedish stock market, one would perhaps still expect a 4 year cycle since elections are also held every fourth year in Sweden.

4 Empirical data

All empirical data found and used were retrieved from Wharton Research Data Services. 10 stocks were selected at random from OMX30, these are presented in the table below along with their associated business sector.

Table 1: Firms selected to be analyzed with their respective business sector

Firm	Sector
Astra Zeneca	Pharmaceuticals
Atlas Copco	Industrial
Electrolux	Appliances
Hennes & Mauritz	Retail - Clothing
Investor	Investment banking
Kinnevik	Investment banking
Sandvik	Industrial
Swedbank	Banking
Tele2	Telecommunication Operator
Telia	Telecommunication Operator

Astra Zeneca's stock price as a function of time is presented in the following figure. The remaining firms' stock prices can be found in appendix A

Figure 8: Astra Zenecas stock price obtained from WRDS

The data obtained contains multiple duplicate prices for most dates and does not account for stock splits. The duplicates are due to the stock splitting up into A stock and B stock. In all cases, the stock that was chosen was the one that would give rise to the most continuous price growth, as step functions would complicate the analysis of the transform. Unfortunately, the time period where data is available also differs for each stock. Instead of confining the time period where data is available for all stocks selected, all data points will be utilized for each stock to increase the resolution of the transform. The stock price of Astra Zeneca adjusted for stock splits and removal of duplicates is presented in the following figure. Remaining firms' stock prices adjusted for stock splits and duplicates can be found in appendix B.

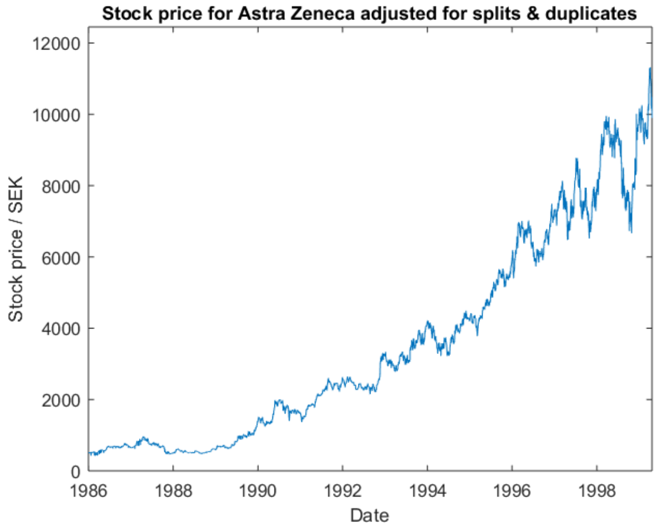

Figure 9: Astra Zenecas stock adjusted for stock splits and duplicates

5 Method

5.1 Linearizing data

The expected returns for each firm has to be calculated since the efficient market hypothesis concerns only excess returns and not in absolute prices. By taking the natural logarithm of both sides in equation 1 the following equality is obtained:

$$\ln S_T = \ln S_0 + \mu T \tag{6}$$

Since the logarithm of the initial stock price is a constant, equation 6 can be likened with the straight line equation.

$$\begin{cases} y = c + mx \\ \ln S_T = \ln S_0 + \mu T \end{cases} \tag{7}$$

The gradient, m is the equivalent of μ. By fitting a best fit linear line on the logarithmed stock prices, the gradient of said best fit line will be the expected return of the firms stock.

In addition, linearizing data is necessary for the Fourier transform to give any meaningful results. Had one not linearized the data, the Fourier transform would be dominated by the latest observations due to the exponential growth nature of stock prices.

5.2 Residuals

A residual is any deviation from the observed data and the expected value and is given by:

$$e_t = S_t - \widehat{S_t} \tag{8}$$

where e_t is the residual, $\widehat{S_t}$ is the expected stock price and S_t is the observed stock price. If $e_t > 0$ then the stock is overperforming since stock value is higher than expected. If $e_t < 0$ then the stock is underperforming due to stock value being lower than expected, might even be a loss in stock value. Keep in mind that these stock values are all logarithmed.

5.3 Fourier transform

Fourier transforming utilizing the DFT-method is done using the `fft`-command in Matlab. From the transform, relevant peaks are picked out.

$$T = \frac{1}{f} \tag{9}$$

The period length, T, for any cyclic peak is the inverse of the frequency, f, obtained.

6 Results

Linearizing Astra Zeneca's stock value and fitting a linnear trend is shown in the following figure.

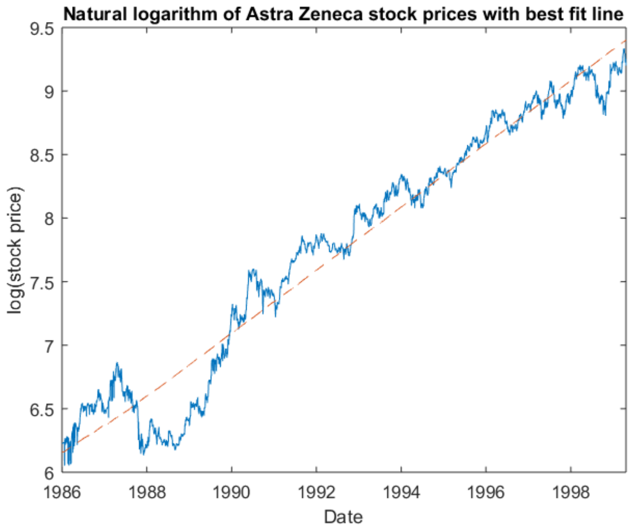

Figure 10: Astra Zenecas stock linearized with best fit line shown (dashed line)

Residuals calculated are represented in the following figure. Graphs for remaining firms can be found in appendix appendix C.

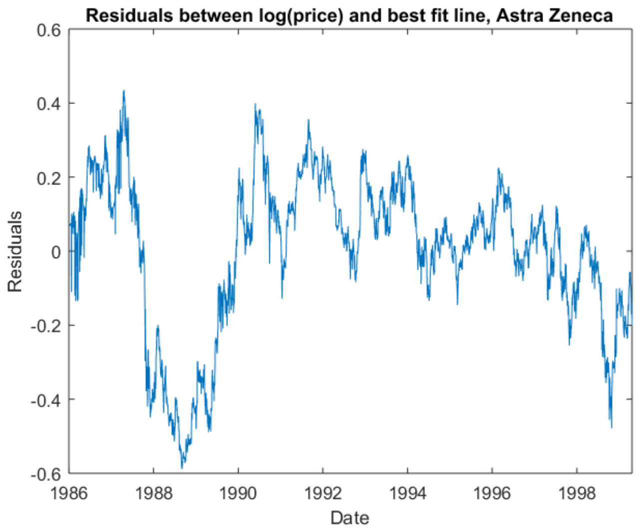

Figure 11: Astra Zenecas residuals between observed and expected stock prices.

The Fourier transform obtained from the residuals is shown in the following figure.

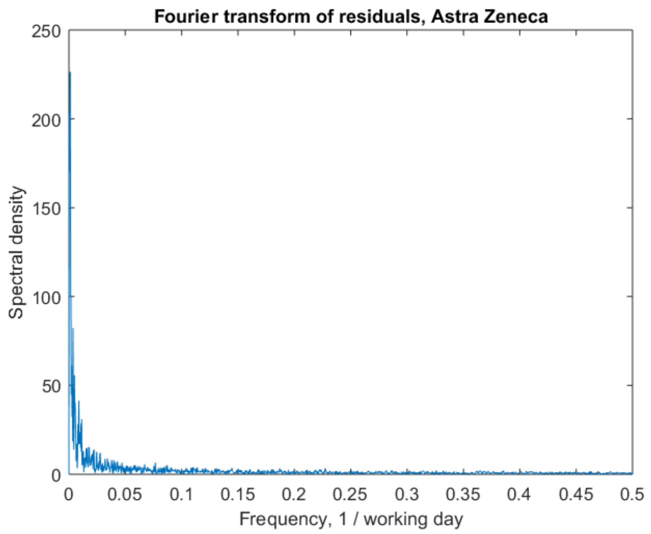

Figure 12: Fourier transform of Astra Zeneca residuals

Due to the relative differences in spectral density, it is easier to identify the first peaks by zooming in.

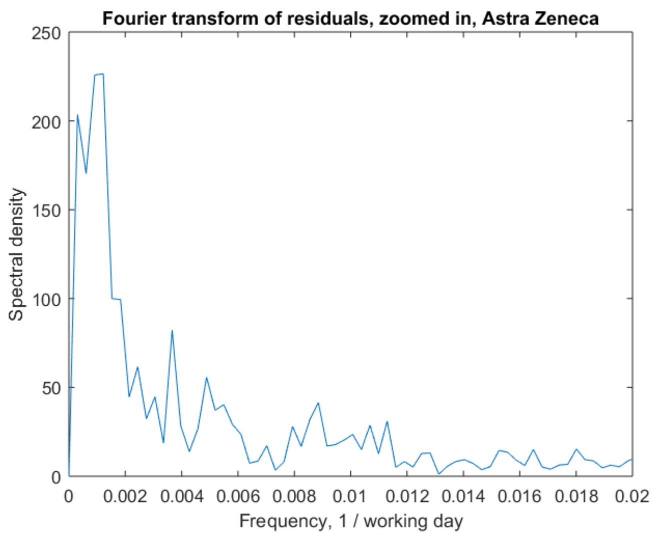

Figure 13: Fourier transform of Astra Zeneca residuals, zoomed in

Peaks with relative large amplitudes were picked out and the frequency noted. Table 2 shows a summary of all companies with their frequencies with relative large amplitudes with their corresponding periods. The period is also given in calendar years, calendar months and calendar weeks. Graphs for the remaining firms can be found in appendix D.

Table 2: Frequency presence for each firm and their corresponding period, given in number of working days, calendar years, calendar months and calendar weeks.

Firm:	**Peak / Freq**	**Period in working days**	**Calendar Years**	**Calendar Months**	**Calendar Weeks**	**Sampling period**
Astra Zeneca	0.0003053	3275.5	13.1	157.2	681.3	3277
	0.0009158	1091.9	4.4	52.4	227.1	
	0.001221	819.0	3.3	39.3	170.4	
	0.003663	273.0	1.1	13.1	56.8	
	0.004884	204.8	0.8	9.8	42.6	
	0.008852	113.0	0.5	5.4	23.5	
Atlas Copco	0.0003695	2706.4	10.8	129.9	562.9	8119
	0.0007391	1353.0	5.4	64.9	281.4	
	0.0009855	1014.7	4.1	48.7	211.1	
	0.001355	738.0	3.0	35.4	153.5	
	0.001971	507.4	2.0	24.4	105.5	
	0.003326	300.7	1.2	14.4	62.5	
	0.004435	225.5	0.9	10.8	46.9	
	0.01244	80.4	0.3	3.9	16.7	
	0.01478	67.7	0.3	3.2	14.1	
Electrolux	0.0002455	4073.3	16.3	195.5	847.3	8149
	0.0007364	1358.0	5.4	65.2	282.5	
	0.0009818	1018.5	4.1	48.9	211.9	
	0.001595	627.0	2.5	30.1	130.4	
	0.001964	509.2	2.0	24.4	105.9	
	0.00405	246.9	1.0	11.9	51.4	
	0.007609	131.4	0.5	6.3	27.3	
	0.01595	62.7	0.3	3.0	13.0	
HM	0.0001945	5141.4	20.6	246.8	1069.4	7712
	0.0005836	1713.5	6.9	82.2	356.4	
	0.0008430	1186.2	4.7	56.9	246.7	
	0.0009726	1028.2	4.1	49.4	213.9	
	0.001621	616.9	2.5	29.6	128.3	
	0.00214	467.3	1.9	22.4	97.2	
	0.004085	244.8	1.0	11.8	50.9	
	0.006549	152.7	0.6	7.3	31.8	
	0.01731	57.8	0.2	2.8	12.0	
Investor	0.0002209	4526.9	18.1	217.3	941.6	6790
	0.0005155	1939.9	7.8	93.1	403.5	
	0.0008101	1234.4	4.9	59.3	256.8	
	0.001252	798.7	3.2	38.3	166.1	
	0.002136	468.2	1.9	22.5	97.4	

Table 2 continued from previous page

Firm:	Peak / Freq	Period time / working days	Calendar Years	Calendar Months	Calendar Weeks	Sampling period
	0.004051	246.9	1.0	11.8	51.3	
	0.01083	92.3	0.4	4.4	19.2	
	0.0148	67.6	0.3	3.2	14.1	
	0.02644	37.8	0.2	1.8	7.9	
Kinnevik	0.0008026	1246.0	5.0	59.8	259.2	2493
	0.001204	830.6	3.3	39.9	172.8	
	0.002408	415.3	1.7	19.9	86.4	
	0.004013	249.2	1.0	12.0	51.8	
	0.008828	113.3	0.5	5.4	23.6	
	0.01605	62.3	0.2	3.0	13.0	
Sandvik	0.0001249	8006.4	32.0	384.3	1665.3	8009
	0.0007493	1334.6	5.3	64.1	277.6	
	0.001124	889.7	3.6	42.7	185.1	
	0.001499	667.1	2.7	32.0	138.8	
	0.001998	500.5	2.0	24.0	104.1	
	0.002747	364.0	1.5	17.5	75.7	
	0.003996	250.3	1.0	12.0	52.1	
	0.01061	94.3	0.4	4.5	19.6	
	0.01523	65.7	0.3	3.2	13.7	
Swedbank	0.0005968	1675.6	6.7	80.4	348.5	5866
	0.001279	781.9	3.1	37.5	162.6	
	0.00162	617.3	2.5	29.6	128.4	
	0.002813	355.5	1.4	17.1	73.9	
	0.003663	273.0	1.1	13.1	56.8	
	0.004518	221.3	0.9	10.6	46.0	
	0.001066	938.1	3.8	45.0	195.1	
	0.01731	57.8	0.2	2.8	12.0	
Tele2	0.0004429	2257.8	9.0	108.4	469.6	5646
	0.0007972	1254.4	5.0	60.2	260.9	
	0.000973	1027.7	4.1	49.3	213.8	
	0.002037	490.9	2.0	23.6	102.1	
	0.0132	75.8	0.3	3.6	15.8	
	0.02205	45.4	0.2	2.2	9.4	
Telia	0.0002162	4625.3	18.5	222.0	962.1	4627
	0.0006485	1542.0	6.2	74.0	320.7	
	0.0008647	1156.5	4.6	55.5	240.5	
	0.001081	925.1	3.7	44.4	192.4	
	0.001729	578.4	2.3	27.8	120.3	
	0.002594	385.5	1.5	18.5	80.2	
	0.003459	289.1	1.2	13.9	60.1	
	0.01362	73.4	0.3	3.5	15.3	

7 Analysis

The fact that the transform is able to pick out frequencies that are present from the residuals suggests that the Swedish stock market is not weak-form efficient. However, some of the frequencies obtained from table 2 are most likely incorrect due to low sampling period and poor resolution. Peaks that corresponds to a period that is in the same order of magnitude as their sampling period is most likely *aliasing* (See Astra Zeneca, Sandvik and Telia). None of the firms' Fourier transformation had a satisfying resolution due to the low sample period, meaning the cycle periods presented in table 2 should only be taken as an approximation of the period. Granger and Hatanaka (1964) maintains that data needs at least a length of seven times the length of the longest periodic cycle present to properly determine the cycles. Which is certainly not the case of any of our firms selected.

Whilst there is a presence of peaks, the methodology neglects risk and have not adjusted the returns accordingly. It could be very much so that the frequency peaks observed are caused by periodic behaviour in market risk. If risk is higher one would expect that in general the stock market would perform subpar usual levels. Many of the firms investigated share price cycles which could be an indicator that the whole market in general moves up and own in sync. Some of these cycles could still adhere to the weak-form EMH, which will be discussed.

7.1 Four and five year cycle

Nearly all 10 firms investigated share a price cycle of around 4 years with the exceptions being Investor and Kinnevik (note that these two companies are in the investment banking sector). This supports the idea that political uncertainty seem to have an affect on stock markets that was discussed in section 3.3. The national parliament elections are not the only political elections held in Sweden. Every fifth year the European Union holds elections to its own parliament. Many of these firms also show some sort of periodic behaviour occurring every fifth year. However, not as many firms seems to show this behaviour. Indicating that the European Union elections are perhaps not as consequential to the Swedish stock market as the national elections are, or simply not as unpredictable. While other factors could be the cause behind the four and five year cycles, the political variable seems to be the most plausible one since it supports earlier research.

Whether or not it is possible to exploit this pattern and if this cyclic pattern violates the EMH is difficult to determine from the transform alone. The outcome of an election could be seen as random new information being disclosed. However, one could argue that the outcome of an election may not be sufficiently random[7] and the fact that elections

[7]Elections can be seen as fairly predictable because of polls. With that comes a somewhat expected set of policies.

occur at a regular interval removes the aspect of randomness. While Wong and McAleer (2009) found that the market consistently underperformed the following two years after an election, the Fourier transform does not provide that information. It is unclear whether the markets go up or down after an election. It is reasonable to assume that market risk follows a pattern based on the time remaining until next election. Meaning the fluctuating price is not necessarily caused by a calendar effect, but a periodic volatility caused by the political uncertainty. If this was the case, then exploiting the market would prove to be difficult and would not necessarily violate EMH. Nevertheless, it does not explain why firms in the financial sector seem to be unfazed by the political landscape.

7.2 One year cycle

All firms except for Tele2 shows a cyclic pattern with a period of one calendar year. The January effect suggests a peak in price once per year during said month which the transform supports. However, as discussed in section 3.1, the strategy should mostly affect smaller firms. All firms analyzed are listed on OMX30 and should not really qualify as a small firm. However, the strategy does not exclude the effects on large firms, only said to be more pronounced on smaller firms. The second strategy discussed that should exploit a cycle of once per year is the ”Sell in May” tactic. With both support from the Fourier transform[8] and Cao and Wei (2015) findings, it is most likely that this periodicity is exploitable (if not already being exploited), violating the weak form efficiency.

Having said that, firms in Sweden have historically only given out dividends on an annual basis. While most firms have recently switched over to a semi-annual pay out scheme, most of the sampled data should show the dividend effects on an annual basis. It is unclear to determine the significance of these pay outs, as they can be very small relative to the stock price. A pay out of 1 SEK from a 100 SEK stock will likely not be detected by the transform since a 1% decrease in stock value can easily attributed to daily trading fluctuations. Without knowing the dividend yield it is difficult to say that the dividends are the cause of the periodic behaviour. However, E. Fama et al. (1969) concluded in their study that dividends adhered to the efficient market hypothesis. Assuming this is true, then the large periodic behaviour must be caused by a calendar effect[9].

[8]The presence of both January effect and ”Sell in May” strategy should exploit the same periodicity but with different phases. Meaning they share the same cycle length but the peaks occur at different times. The Fourier Transform can still tell that only one frequency is present and is unaffected by the phase shift.

[9]We know this to be true since a step function causes multiple frequency peaks and not one singular with harmonic frequencies. Since no harmonic frequencies are observed, no step function can be detected.

7.3 Quarterly cycle

Most of the investigated firms show a cycle with a period of around 3 months. This could be attributed to quarterly reports released by the firms. The quarterly reports can be seen as new information being presented to investors. However, the presence of peaks suggests that the information released may not be of random character, but somewhat predictable therefore not following a random walk. The peaks identified corresponding to a quarterly cycle do posses a very small amplitude in the Fourier transform compared to other cycles identified. From section 2.4.6 the weaknesses with the transform was discussed where the issue with identifying real frequency peaks from white noise was highlighted. The quarterly cycles in this case might just as well be white noise from the transform mistakenly identified as a calendar effect.

7.4 Cycles by sector

The investment banking firms not sharing the 4 year cycle like the other firms could be evidence of different industry sectors having their own cyclical behaviours. The lack of shared periodicity could be a potential method to earn excess returns. Investors could potentially hedge against risk caused by the political landscape which otherwise seems to affect everybody else on the stockmarket. Both Kinnevik and Investor show a period between 3.2 - 3.3 years, which is strikingly close considering the poor resolution of the Fourier transform.

Atlas Copco and Sandvik (industrial) share a peak almost identical to each other at 5.4 and 5.3 years. In addition, Telia and Tele2 (telecommunications operator) also more or less have identical peaks with a period of 73-75 days. It is plausible that these shared peaks between firms in the same sector is pure coincidence and that the rest of the sector does not show the same periodicity. These peaks have a seemingly arbitrary period which would suggests a plausible source of exploitation of the whole industry itself. Regardless, further research into firms within the same industry sector is required to validate any findings since two firms in each sector is an extremely small sample.

7.5 Long term and short term

There is a clear trend in the Fourier transforms for all firms investigated when considering the long term prices versus the short term prices. There are no clear frequency peaks for any stock at high frequencies[10] with the transform looking like white noise, which means that short term prices do seem to follow a random walk. Most cyclic behaviour are found at long term, often with a period time around a year or more. There are peaks present at

[10] While there is no definite definition of "High frequency", the highest frequency identified among any stock comes in at 0.02644 times per day. Anything higher than that could be considered high frequency in this study.

higher frequencies with periods of a couple of months, but far fewer and less pronounced. This would suggest that prices in the short term follow a random walk, adhering to the weak form of the efficient market hypothesis. Granger and Morgenstern (1963) found that the New York stock market also displayed the same characteristics of being efficient in the short run but having cyclic behaviour at longer time periods.

With the market being effective only in the short run could be due to investors investment horizon. Investors may just lack the required investment horizon to be able to exploit the longer calendar effects and therefore only exploit the shorter calendar effects until no anomaly can be capitalized on, resulting in an effective market in the short run but not the long run. Granger and Morgenstern (1963) argued that exploiting the long term anomalies would be difficult for an investor since it is unclear when any oscillation starts and the amount of work required to identify the phase of the oscillation increases the longer the period is.

8 Conclusion

Multiple peaks can be identified for all firms investigated. Assuming all stock price mechanism are accurate, the presence of peaks suggests that the Swedish stock market is inefficient due to possible exploitation through market timings. While the Fourier transform shows the presence of cyclic behaviour, it does not provide information about when they occur. To exploit the market further analysis has to be performed, but the Fourier transform does provide a good indicator what patterns to search for. The two strategies, *the January effect* and *Sell in May* are impossible to validate without further research. However, the presence of a 1-year peak is an indicator that is prerequisite for the strategy to work, which is present.

Some oscillations might be explained by reoccurring events that could alter the market risk such as upcoming elections or dividend payouts. As such, the efficient market hypothesis would still hold. However, it is impossible to confirm the underlying factors behind the periodic price movements since the Fourier transform does not provide information about when prices go up or down. To validate these findings, further analysis would have to be performed. The shared cycles between firms in the same industry sector would be another sign of some sort of inefficiency as well. The accuracy of the frequencies identified is left undesired. Increased sampling period would clarify each frequency present.

In the end, it is not too surprising that the findings suggests that the Swedish stock market is inefficient. The three assumptions that the EMH builds on are somewhat questionable. Sweden's market is most likely not friction-less due to taxes, regulations and transaction costs. That all investors have homogeneous beliefs is also a substantial assumption to make. The number of people being skeptical of the EMH shows that investors rarely share the same opinion. However, one should be cautious to draw the conclusion that the Swedish stock market is inefficient. Only 10 firms were investigated which is only a third of the firms listed on the OMX30 and even a smaller fraction of the Swedish stock market. With the small sample size it would be precipitously to assume the whole stock market behaves in a similar fashion. In addition, the transform does suggest that prices are efficient in the short term, while the peaks for long term are poorly resolved and inaccurate due to inadequate sampling period. Since this is a *joint hypothesis problem* it is impossible to know if the price mechanism are at fault or if the market is being inefficient.

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A Figures - Raw data

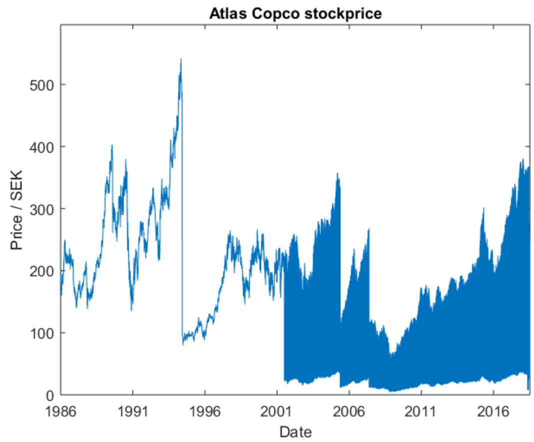

Figure 14: Atlas Copco stock price obtained from WRDS

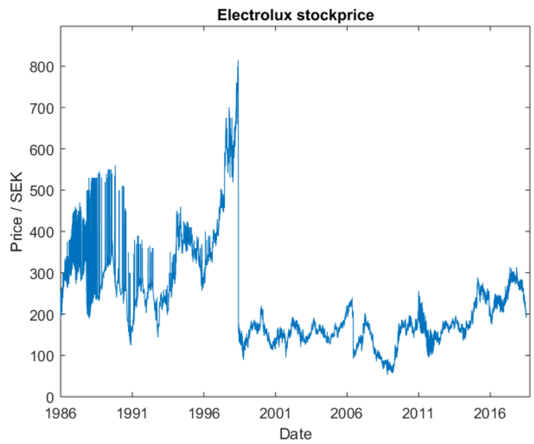

Figure 15: Electrolux stock price obtained from WRDS

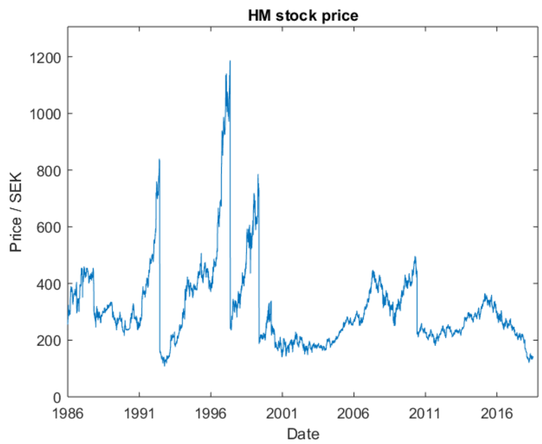

Figure 16: H&M stock price obtained from WRDS

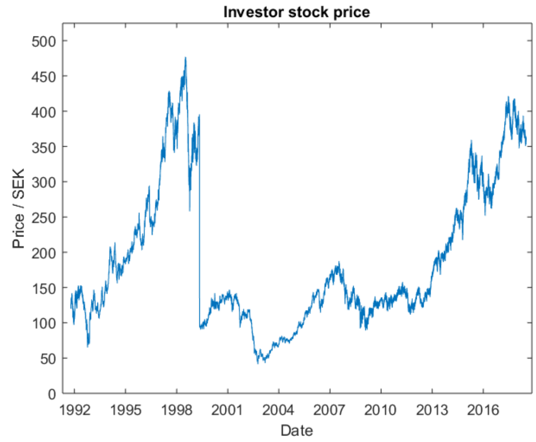

Figure 17: Investor stock price obtained from WRDS

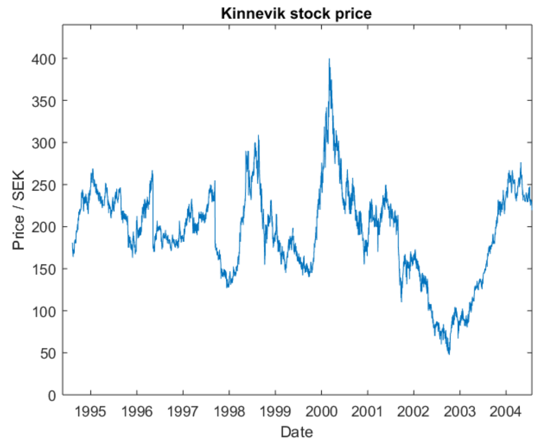

Figure 18: Kinnevik stock price obtained from WRDS

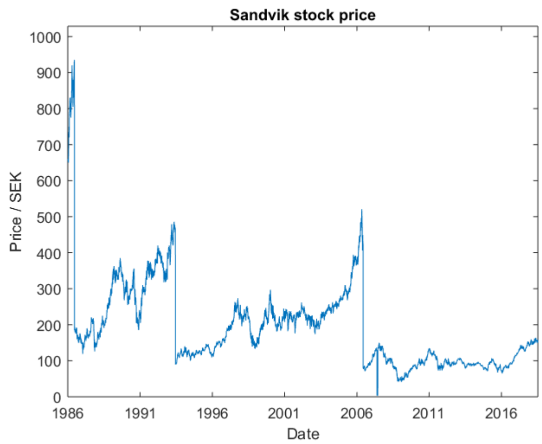

Figure 19: Sandvik stock price obtained from WRDS

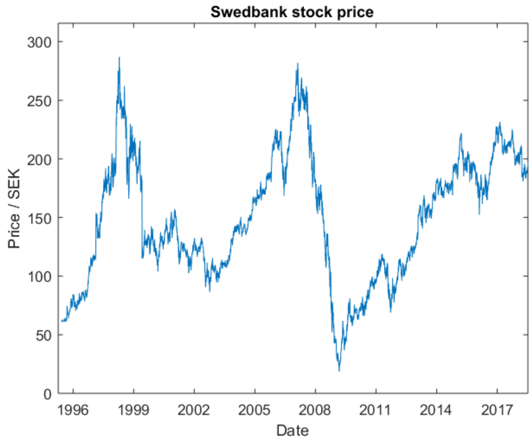

Figure 20: Swedbank stock price obtained from WRDS

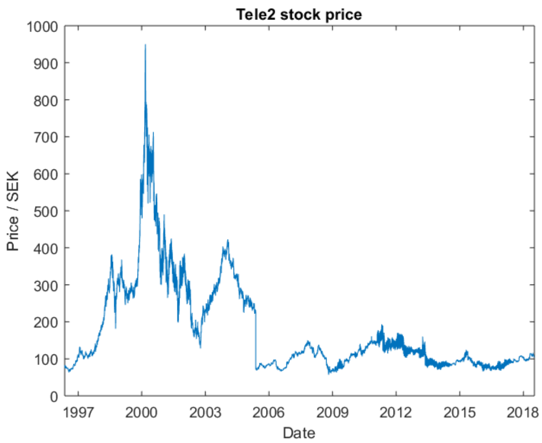

Figure 21: Tele2 stock price obtained from WRDS

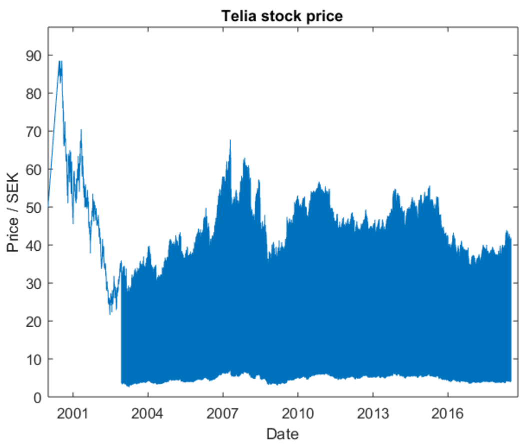

Figure 22: Telia stock price obtained from WRDS

B Figures - Removed duplicates and adjusted for stocksplits

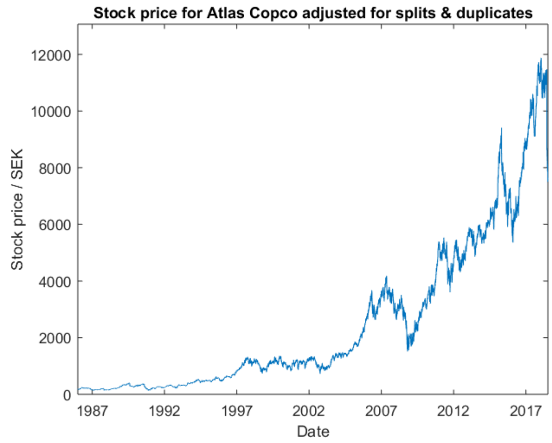

Figure 23: Atlas Copco stock adjusted for stock splits and duplicates

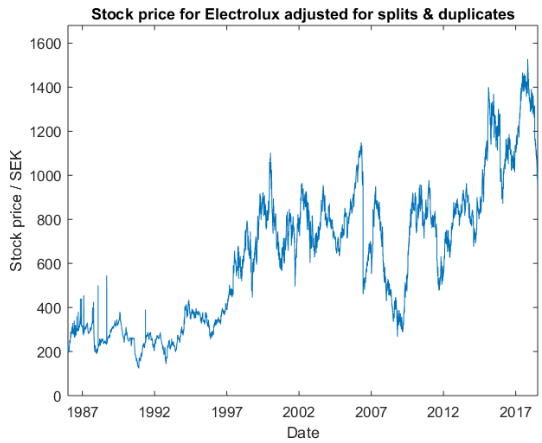

Figure 24: Electrolux stock adjusted for stock splits and duplicates

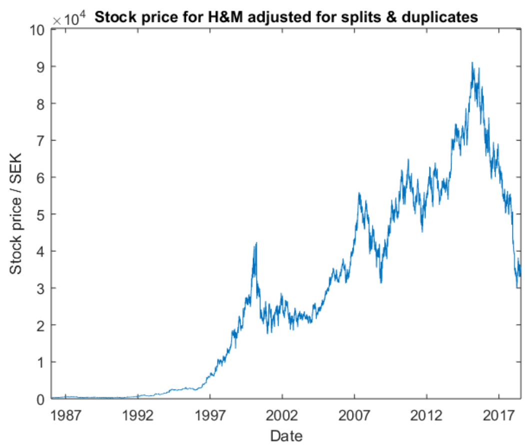

Figure 25: H&M stock adjusted for stock splits and duplicates

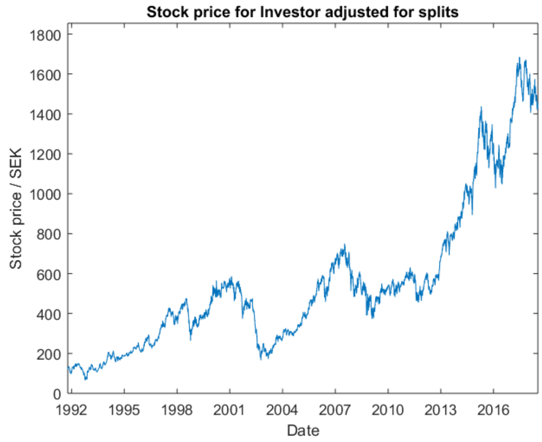

Figure 26: Investor stock adjusted for stock splits and duplicates

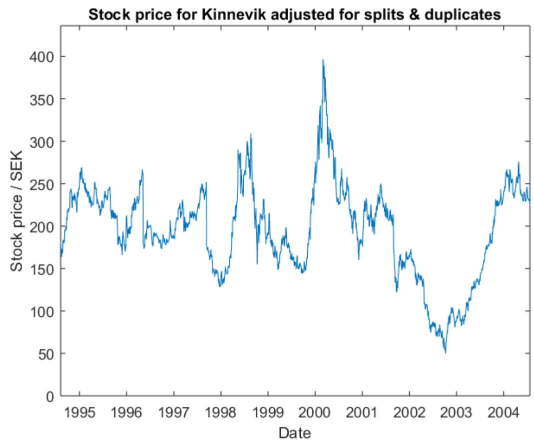

Figure 27: Kinnevik stock adjusted for stock splits and duplicates

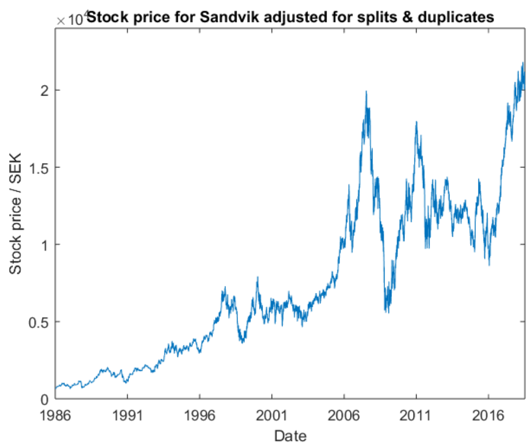

Figure 28: Sandvik stock adjusted for stock splits and duplicates

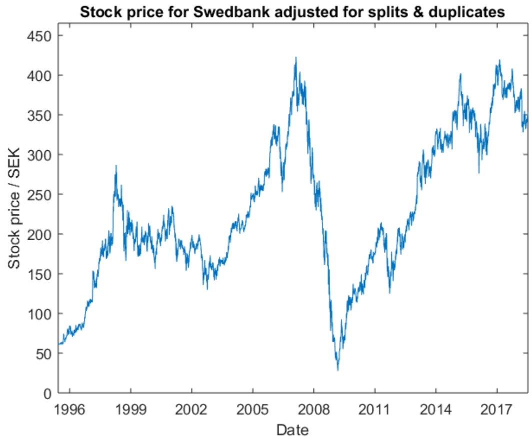

Figure 29: Swedbank stock adjusted for stock splits and duplicates

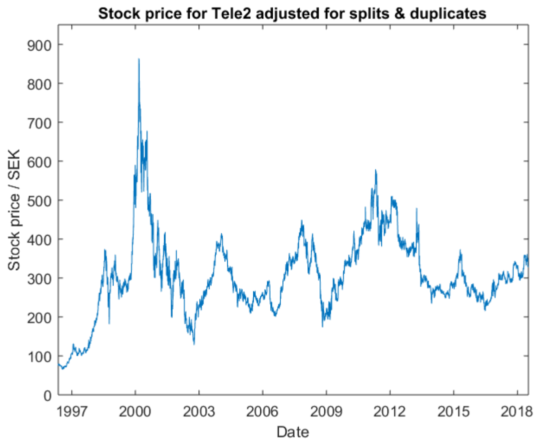

Figure 30: Tele2 stock adjusted for stock splits and duplicates

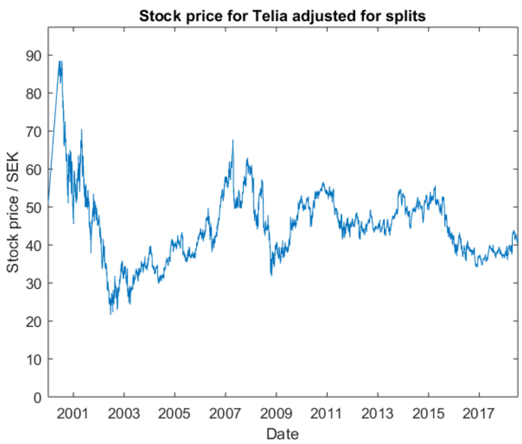

Figure 31: Telia stock adjusted for stock splits and duplicates

C Figures - Processed data; Linearized and residuals

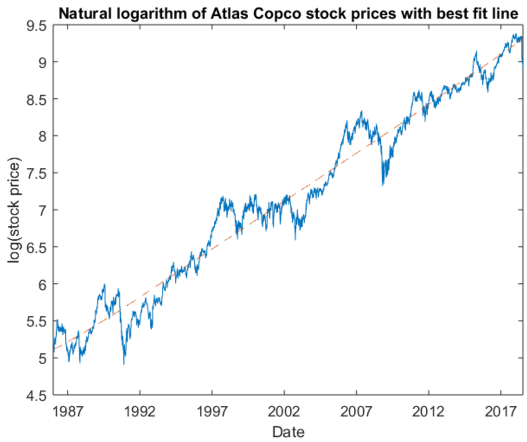

Figure 32: Atlas Copco stock linearized with best fit line shown (dashed line)

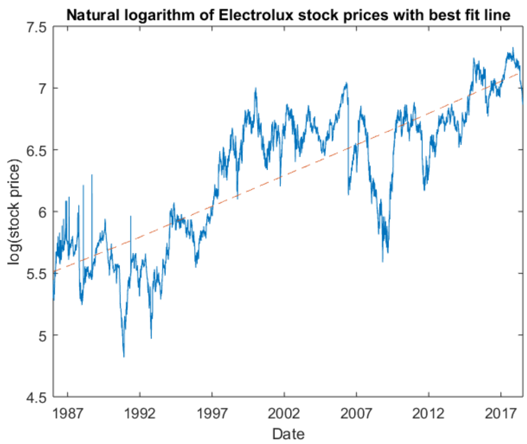

Figure 33: Electrolux stock linearized with best fit line shown (dashed line)

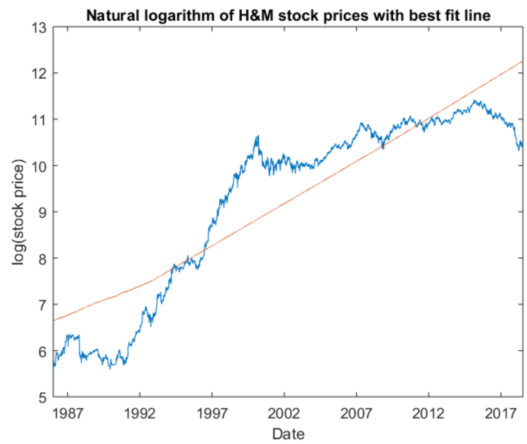

Figure 34: H&M stock linearized with best fit line shown (dashed line)

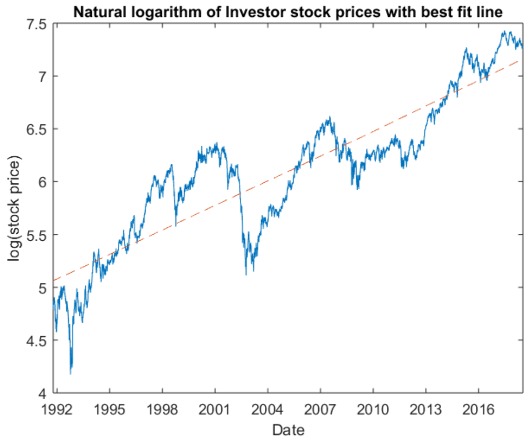

Figure 35: Investor stock linearized with best fit line shown (dashed line)

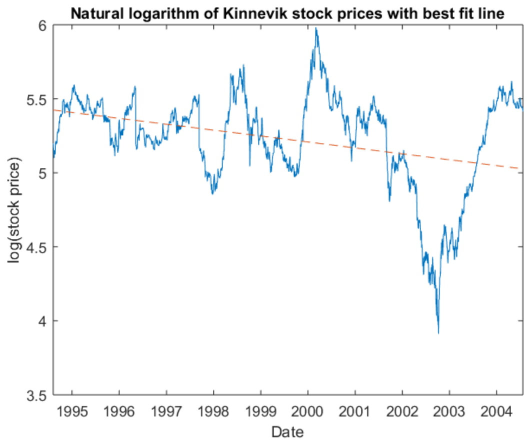

Figure 36: Kinnevik stock linearized with best fit line shown (dashed line)

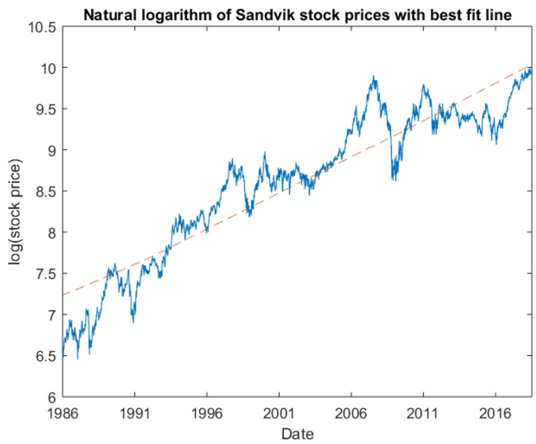

Figure 37: Sandvik stock linearized with best fit line shown (dashed line)

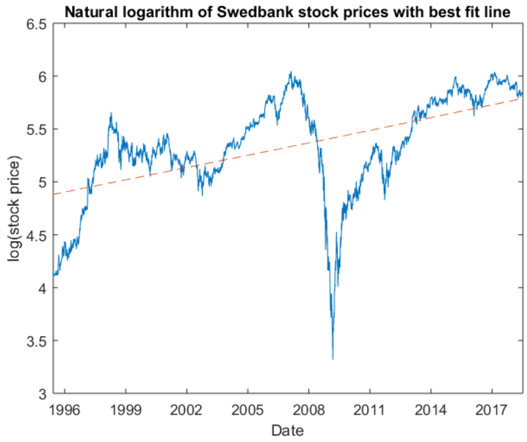

Figure 38: Swedbank stock linearized with best fit line shown (dashed line)

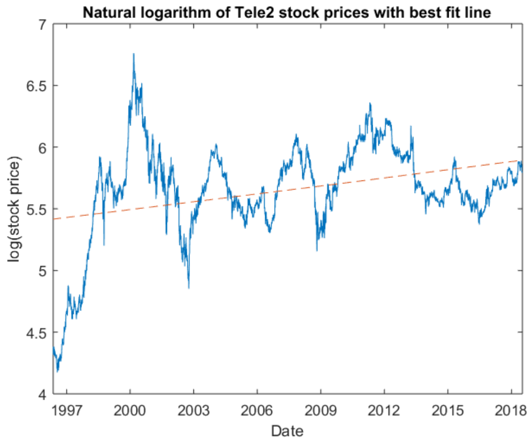

Figure 39: Tele2 stock linearized with best fit line shown (dashed line)

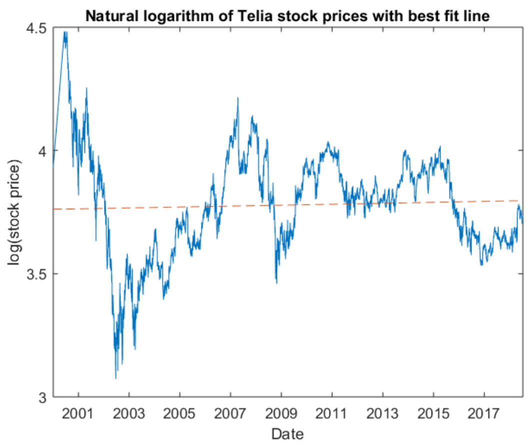

Figure 40: Telia stock linearized with best fit line shown (dashed line)

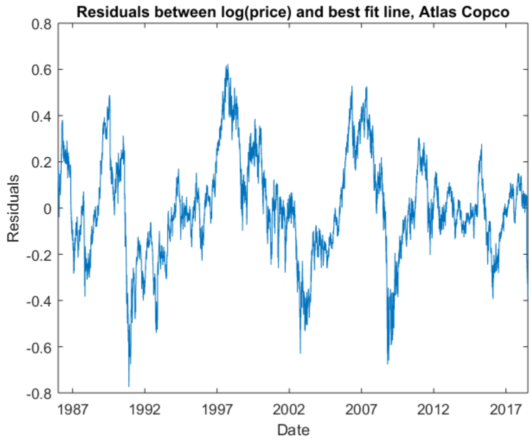

Figure 41: Atlas Copco residuals between observed and expected stock prices

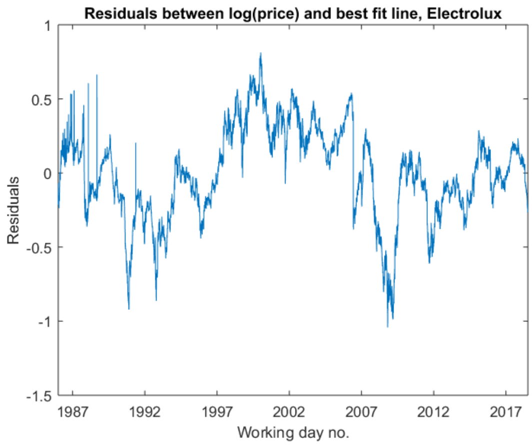

Figure 42: Electrolux residuals between observed and expected stock prices

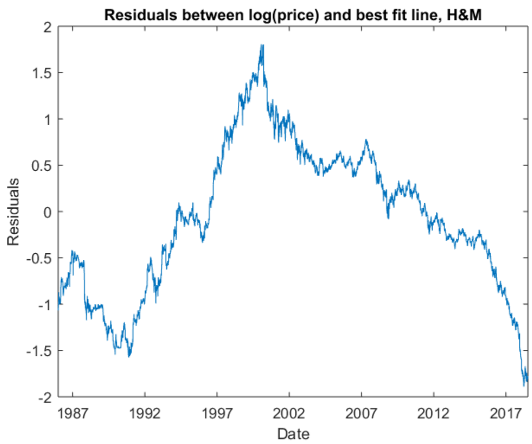

Figure 43: H&M residuals between observed and expected stock prices

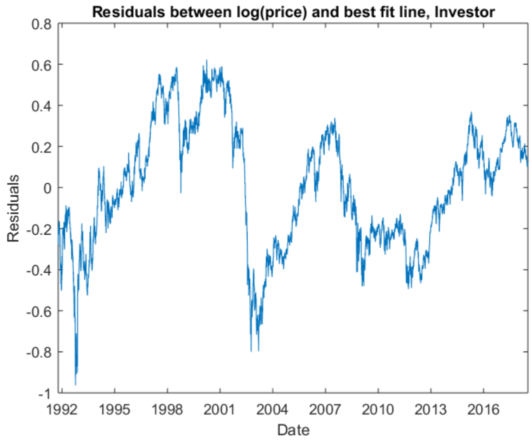

Figure 44: Investor residuals between observed and expected stock prices

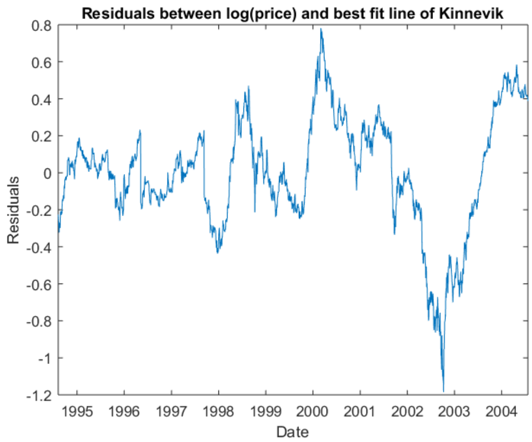

Figure 45: Kinnevik residuals between observed and expected stock prices

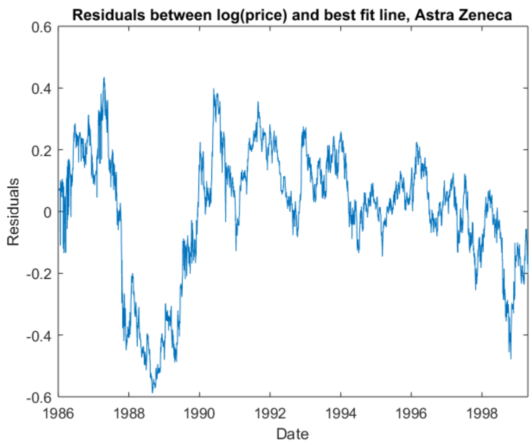

Figure 46: Sandvik residuals between observed and expected stock prices

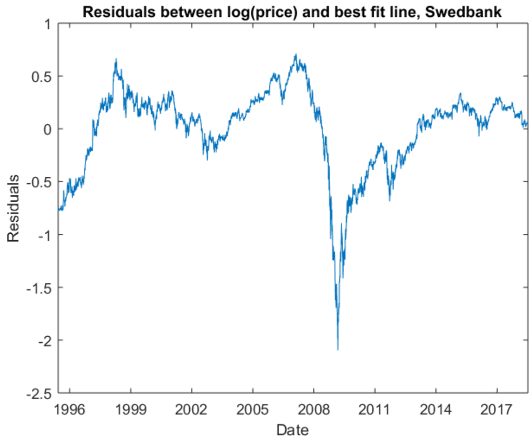

Figure 47: Swedbank residuals between observed and expected stock prices

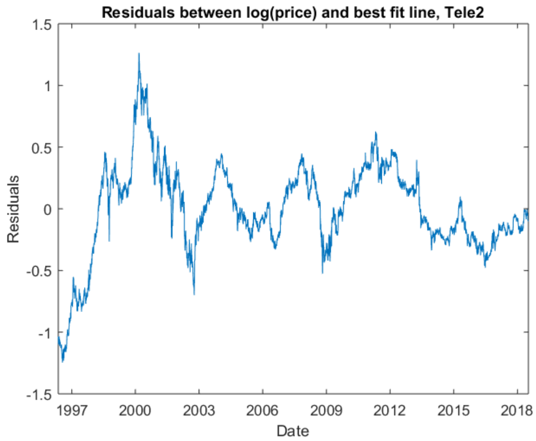

Figure 48: Tele2 residuals between observed and expected stock prices

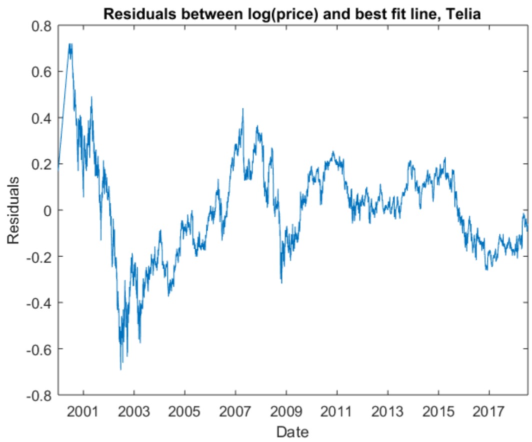

Figure 49: Telia residuals between observed and expected stock prices

D Figures - Processed data; Fourier transform

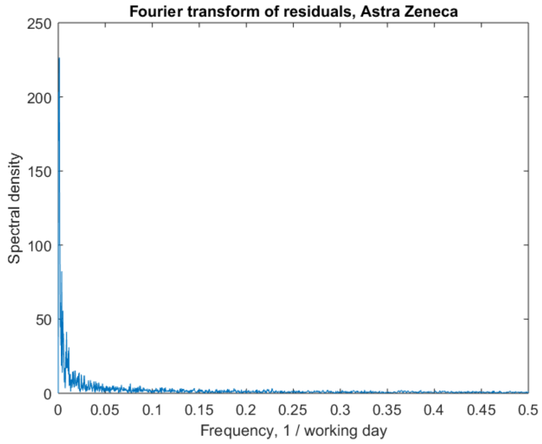

Figure 50: Fourier transform of Atlas Copco residuals

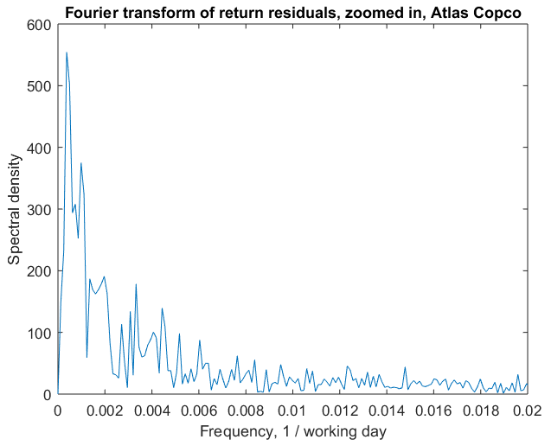

Figure 51: Fourier transform of Atlas Copco residuals, zoomed in

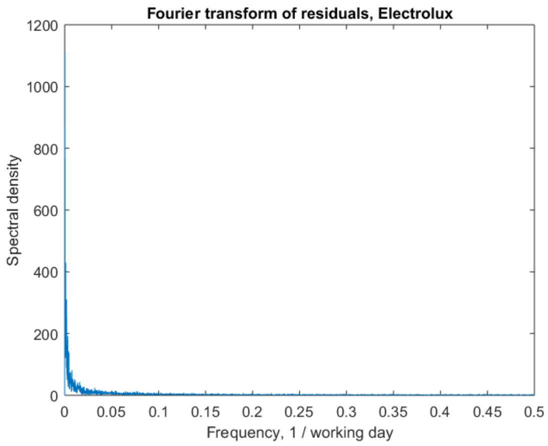

Figure 52: Fourier transform of Electrolux residuals

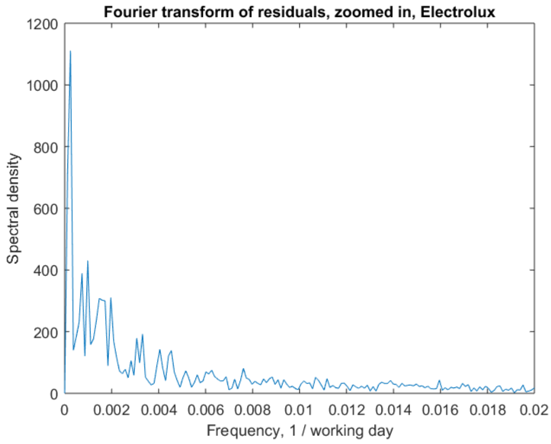

Figure 53: Fourier transform of Electrolux residuals, zoomed in

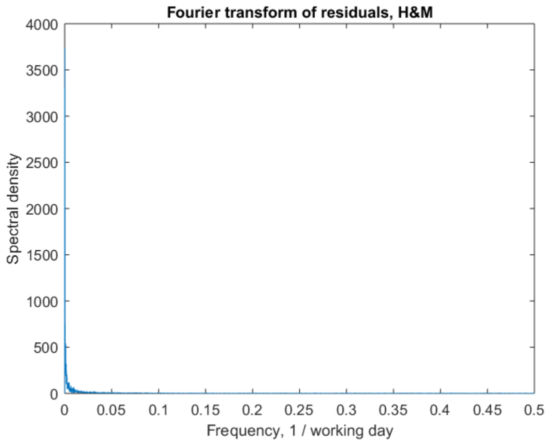

Figure 54: Fourier transform of H&M residuals

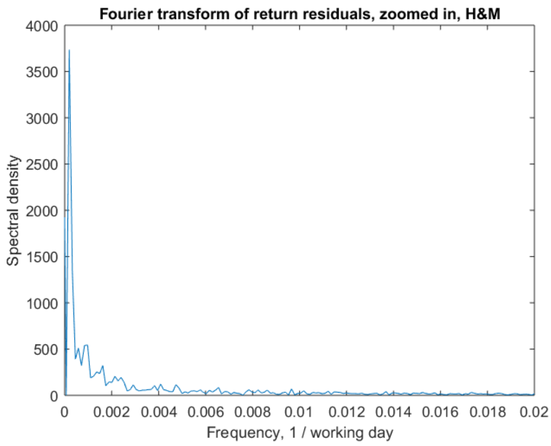

Figure 55: Fourier transform of H&M residuals, zoomed in

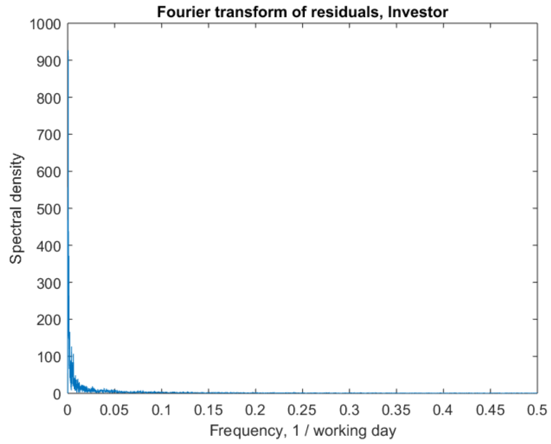

Figure 56: Fourier transform of Investor residuals

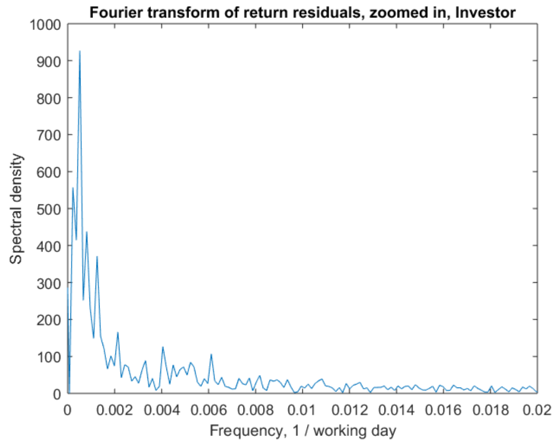

Figure 57: Fourier transform of Investor residuals, zoomed in

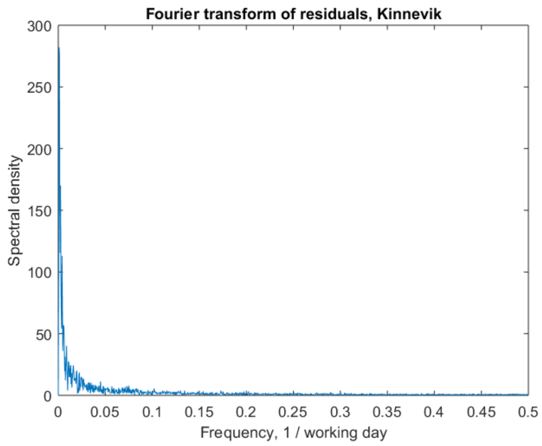

Figure 58: Fourier transform of Kinnevik residuals

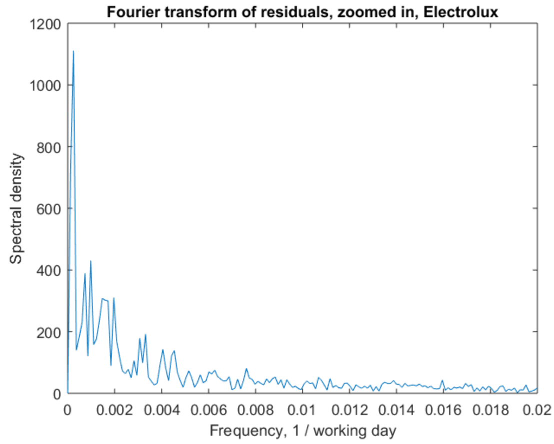

Figure 59: Fourier transform of Kinnevik residuals, zoomed in

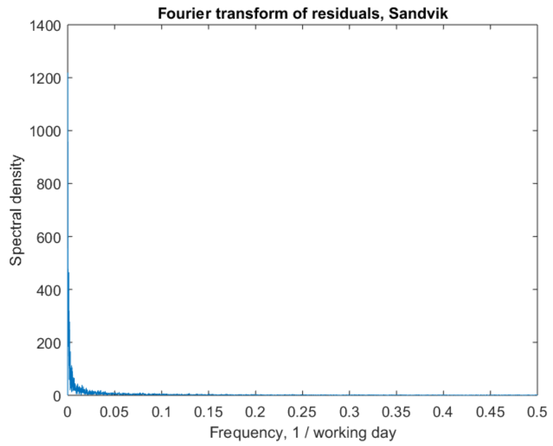

Figure 60: Fourier transform of Sandvik residuals

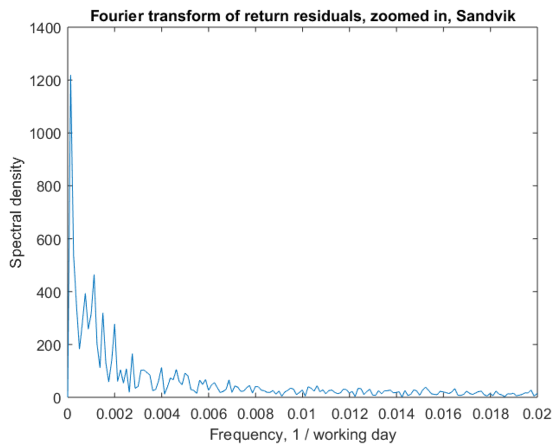

Figure 61: Fourier transform of Sandvik residuals, zoomed in

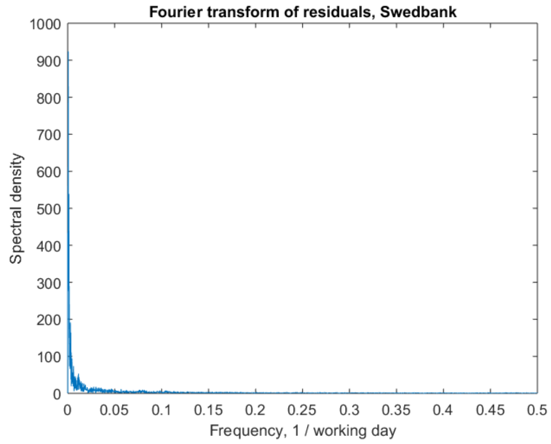

Figure 62: Fourier transform of Swedbank residuals

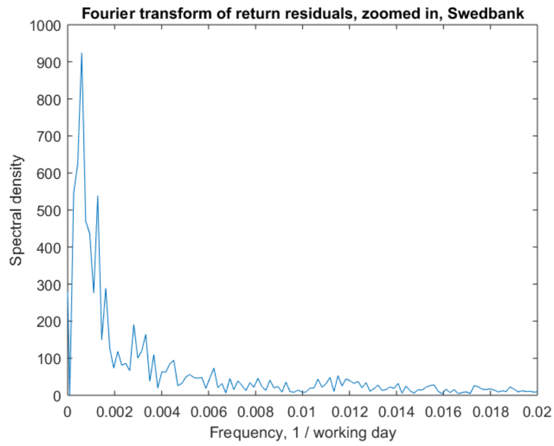

Figure 63: Fourier transform of Swedbank residuals, zoomed in

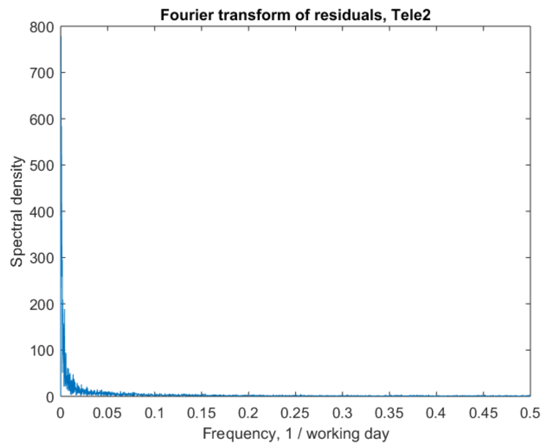

Figure 64: Fourier transform of Tele2 residuals

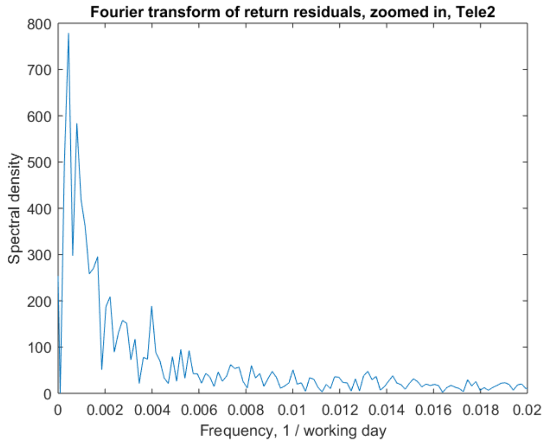

Figure 65: Fourier transform of Tele2 residuals, zoomed in

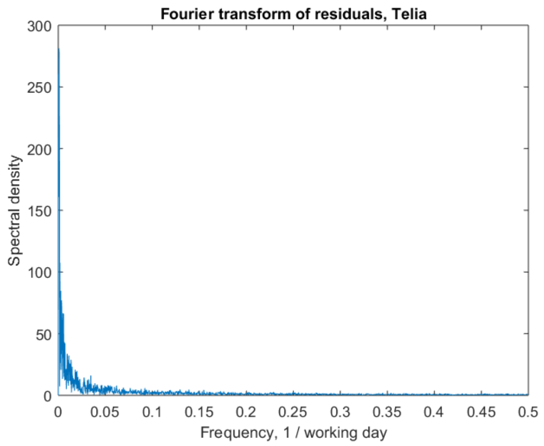

Figure 66: Fourier transform of Telia residuals

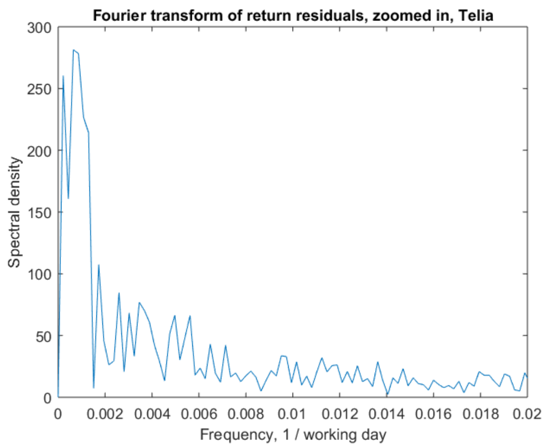

Figure 67: Fourier transform of Telia residuals, zoomed in

E MatLab code

The Matlab code used for the data processing and analysis can be provided by request.