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# Modified second-price auctions in real-time bidding

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## Abstract

The surge in revenue from real-time auctions for online display advertisement has spurred great research interest into how to design optimal mechanisms for these auctions, with respect to both buyers and sellers. This thesis focuses on some recent proposals on how to modify the Vickrey auction, which has been dominating the world of real-time auctions for the past decade, in order to better accommodate for the dynamics of this specific auction environment while also increasing the revenue of the seller. Specifically, this thesis aims to evaluate the so-called *boosted second-price auction* on a dataset provided by Adform, which is a global advertisement technology company based in Copenhagen. In this specific case, the implementation of the boosted second-price auction does not seem to be justified. The changes in allocation as a result of implementing the mechanism are unstable and hard to constrain, while there doesn't seem to be a reason to assume that the Vickrey auction does not already achieve an efficient allocation. All of the code used is provided in the following GitHub repository: <https://github.com/Ostigland/econ-rtb>

**Key words:** *real-time bidding, Vickrey auction, second-price auction, modified second-price auction, boosted second-price auction, adverse selection*

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## Abbreviations

<b>RTB</b>	Real-time bidding
<b>DSP</b>	Demand-side platform
<b>AdX</b>	Ad exchange
<b>CTR</b>	Click-through rate
<b>FP</b>	First price
<b>SP</b>	Second price (i.e. Vickrey)
<b>MSP</b>	Modified second price
<b>BSP</b>	Boosted second price
<b>OMN</b>	Omniscient
<b>CDF</b>	Cumulative distribution function
<b>BSP-AM</b>	BSP alternating minimizer
<b>BSP-MC</b>	BSP Monte Carlo

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# Chapter 1

## Introduction

Auctions and competitive bidding account for economic activity across markets of all kinds, spanning both public and private transactions, including tobacco, treasury bills, fish, electromagnetic spectrum, and tracts for oil fields (Krishna, 2010; Milgrom and Weber, 1982). Since the early 2000s, auctions have also dominated the growing market of online display advertisement, with early pioneers such as Google and Yahoo! (Varian, 2009; Edelman, Ostrovsky and Schwarz, 2007; Yuan et al., 2014). The value of goods and services allocated through different kinds of auctions is immense. Hence, it has been of great interest to the field of economics to study auctions and competitive bidding during the last decades (Krishna, 2010). This area of study, broadly referred to as *auction theory*, was initiated in 1961 by the pioneering work of William Vickrey (Krishna, 2010; Vickrey, 1961).

Since then, the field of auction theory has been the subject of many interesting results and significant publications, such as the *revenue equivalence theorem*, initially discussed by Vickrey (1961) and subsequently developed by the seminal works of Myerson (1981) and Riley and Samuelson (1981). More broadly, important questions in auction theory concern strategic behavior on behalf of buyers and sellers given certain environmental conditions. For example, what is the optimal choice of strategy for a certain bidder given a specific auction

format? How can a seller achieve maximum revenue from selling an item through an auction? These kinds of questions open up a vast research landscape, a small part of which will be discussed in this thesis.

There are many kinds of auction formats, some of the most common being the *Dutch* auction, the *English* auction, the *sealed-bid first-price* auction and the *sealed-bid second-price* auction, which is also referred to as a *Vickrey* auction or a *Vickrey-Clarke-Groves* auction when generalized to auctions for multiple items (Krishna, 2010; Clarke, 1971; Groves, 1973). The Vickrey auction is one of the major auction formats and has been dominating the aforementioned case of online display advertisement (Krishna, 2010; Edelman, Ostrovsky and Schwarz, 2005). See Varian (2007, 2009), Edelman, Ostrovsky and Schwarz (2007), and Yuan et al. (2014) for great overviews on the process of auctioning out slots for online display advertisement, which is often referred to as *real-time bidding* (RTB).

RTB auctions are interesting for a number of reasons. For example, there is significant asymmetry amongst bidders, which means that the aforementioned revenue equivalence theorem is not applicable (Maskin and Riley, 2000). Rather, the Vickrey auction has become the dominating auction format due to its simplicity and nice behavioral properties (Edelman, Ostrovsky and Schwarz, 2005; Golrezaei et al., 2017). However, recent research by Arnosti, Beck and Milgrom (2016) suggests that the Vickrey auction can lead to suboptimal outcomes, with respect to both revenue and allocative efficiency. They propose a modification to the Vickrey auction which aims to reduce adverse selection for specific types of bidders, with the effect of improving overall allocative efficiency and revenue. Golrezaei et al. (2017) propose a similar modification with the explicit aim of increasing revenue. Both methods use data-driven approaches to allocate the item depending on the behaviors of different bidders. Other methods to increase revenues in auctions for online display advertisement include the use of dynamic floor prices in Vickrey auctions, discussed by e.g. Ostrovsky and Schwarz

(2016) and Rong et al. (2017).

Due to the increased prevalence of RTB auctions, the research interest in the RTB ecosystem has grown significantly (Yuan et al., 2014). The purpose of this thesis is to add to the auction theoretical analysis of the RTB ecosystem by considering the modifications to the Vickrey auction proposed by Arnosti, Beck and Milgrom (2016) and Golrezaei et al. (2017). Specifically, I will be implementing and testing the approach suggested by Golrezaei et al. (2017) on a dataset provided by Adform, which is a global advertisement technology company. In testing the approach, I will be comparing the resulting allocation and revenue to the corresponding outcome in a normal Vickrey auction.

## Real-time bidding

In an RTB auction, a bidder, or *advertiser*, receives a request to bid on display advertisement slots for specific users *as* they access or are browsing on a website, often referred to as a *publisher* (Yuan et al., 2014). Showing an ad to a user is often referred to as an *impression*. Hence, we talk about buying and selling impressions. The whole process of auctioning out the ad slots from the moment the user starts loading the website takes less than 100 milliseconds; hence the name "real-time bidding". First, the publisher hosts an auction through a so-called *advertisement exchange* (AdX), which sends out bid requests to a number of so-called *demand-side platforms* (DSP). The role of a DSP is to use algorithmic know-how to participate in RTB auctions on behalf of advertisers. After the DSPs have received the bid requests, they submit bids to the AdX, and whoever posts the highest bid wins the auction and gets to display their advertisement on the website. This somewhat simplified example of the RTB ecosystem is illustrated in figure 1.1.

In this thesis, 'bidder' and 'advertiser' will be used interchangeably when referring to par-



Participants in an auction, i.e. DSPs. There are two main reasons why an advertiser enlists a DSP in order to buy ad slots. First, the DSP has the statistical tools and data-management skills necessary to value different impressions. This is often done by different performance-related metrics, of which one of the most common is the so-called *click-through rate* (CTR), i.e. the probability that a certain user, given their demographic information and other characteristics, will click on a given ad (Zhang, Yuan and Wang, 2014). Secondly, a normal DSP can participate in *billions* of RTB auctions *per day* (Yuan et al., 2014). There are thousands of websites selling ad slots every second. Hence, simply participating in auctions and finding valuable impressions is a task in its own right. All of these procedures are, of course, strictly algorithmic. Adform is active both as a DSP and an AdX. The data used in this thesis is generated from auctions held in their AdX.

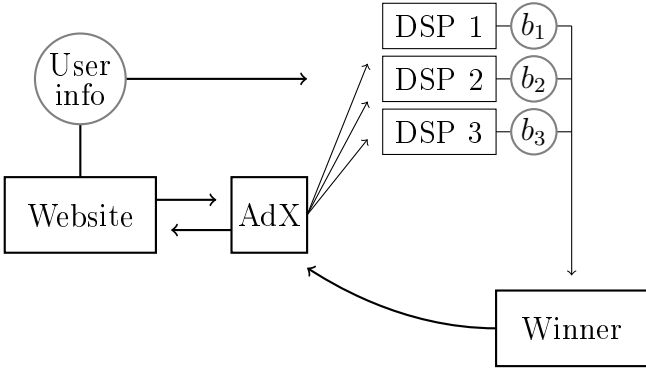


Figure 1.1: A simplified RTB ecosystem

## Chapter 2

# The second-price auction(s)

The Vickrey auction is the dominating auction mechanism in real-time auctions for online display advertising. As previously mentioned, the reason for this, as well as for the general popularity of the Vickrey auction mechanism, is its simplicity and desirable theoretical properties, which incentivizes bidders to always bid their true valuations for whatever item is up for auction. This thesis is mainly concerned with a variant of the Vickrey auction, a so-called *modified second-bid* auction. In this thesis, we will use "second price", rather than "second bid", and hence refer to this mechanism as MSP.

In this chapter, we will start by discussing the theoretical properties of the Vickrey auction. Then, we will consider a general framework for MSP auctions and to what extent the theoretical properties of the original Vickrey auction are retained in an MSP auction. This framework will rely on the recent work by Arnosti, Beck and Milgrom (2016). We will also consider a specific MSP-type mechanism, called the *boosted second-price* (BSP) auction and discuss how it relates to the framework developed by Arnosti, Beck and Milgrom (2016). The BSP auction was introduced in 2017 by Golrezaei et al. The main focus of this thesis is to implement the BSP auction on a dataset provided by Adform.

Finally, the chapter will also briefly consider the sealed-bid first-price (FP) auction and its historical role in online display advertisement, as well as why its revenue and allocative efficiency is not being compared along with the Vickrey auction and the BSP auction.

## 2.1 Vickrey auctions

Let's consider an auction where we have  $n$  bidders,  $i = 1, 2, \dots, n$ , who are all submitting sealed bids for one item. The bidders' valuations,  $v_i$ , are independent of each other and drawn from continuous distributions,  $v_i \sim V_i(\cdot)$ ,  $i = 1, 2, \dots, n$ . Each bidder has some bidding strategy, such that their bid is formulated as a function of their valuation, i.e.  $b_i = B_i(v_i)$ .

Assuming that all bids are ordered from largest to smallest,  $b_1$  will be the winning bid with the winner paying  $b_2$ . Then, it is a *weakly dominant strategy* for each bidder to always bid their valuation, i.e. such that  $B_i(v_i) = v_i$  for  $i = 1, 2, \dots, n$ . This means that the strategy earns the bidders a *payoff* at least as high as for any other strategy, regardless of what the other bidders do. The payoff is defined as the difference between the valuation,  $v_i$ , and the payment,  $p_i$ , i.e.  $v_i - p_i$ .

It is a weakly dominant strategy for any bidder,  $i$ , to bid their true valuation,  $v_i$  because they cannot affect their payoff positively by *not* bidding their true valuation. Let's consider bidder  $i$  employing some bidding strategy,  $B_i(\cdot)$ , such that  $B_i(v_i) > v_i$ . We denote the highest bid for all of the other bidders by  $b_*$ , i.e.  $\max_{j \neq i} b_j = b_*$ . Then, there are three possible cases:

(i)  $b_* > b_i$  and  $b_* > v_i$

(ii)  $b_i > b_* > v_i$

(iii)  $b_i > b_*$  and  $v_i > b_*$

In case (i), bidder  $i$  will not win the item regardless. In case (ii), bidder  $i$  will win the item,

but at the cost of a negative payoff, since  $b_* > v_i$ . Finally, in case (iii), bidder  $i$  will win the item with a positive payoff, since  $b_* < v_i$ . However, the payoff would be equally positive if  $b_i = v_i$ , since bidder  $i$  would still win the auction and pay  $b_*$ . Hence, in all three cases the bidder would have been at least equally well off, with the notable exception of (ii), where the bidder would actually have been better off not bidding such that  $b_i > v_i$ . We can show that the same result holds for a bidder employing a bidding strategy such that  $B_i(v_i) < v_i$  by following the same logic.

We say that the Vickrey auction is *truthful*, *efficient* and *individually rational*. The first is due to the incentive for bidders to always reveal their true valuations. The second, efficiency, means that the bidder with the highest valuation will always win the auction, which is clearly the case if all bidders bid their true valuations. Finally, individual rationality means that no bidder ever pays more than the bid they have posted. This will be the case for all auction mechanisms discussed in this thesis. Hence, it will not be discussed further.

It is important to bear in mind the truthfulness of the Vickrey auction, since the data we will be using is taken from Vickrey auctions, meaning that the empirical distribution of bids for each advertiser,  $i$ , reveals their actual value distribution,  $V_i(\cdot)$ . However, it should be noted that it's difficult to accurately assess the value of an impression in RTB. Hence, even if the advertisers have no reason to bid anything else than what they perceive to be the value of the impression, it doesn't mean that this perception is correct. On the other hand, we're working with a large dataset, often with millions, and in some cases tens of millions, of historical bids for different advertisers. Consequently, we will assume that the historical bid distributions are an accurate estimation of the bidders individual value distributions.

## 2.2 Modified second-price auctions

Arnosti, Beck and Milgrom (2016) discuss the problem of an AdX where there are two types of advertisers, *performance advertisers* and *brand advertisers*, who have positively correlated valuations for impressions, but where performance advertisers can estimate the value of individual impressions accurately while brand advertisers can't. A performance advertiser could be, e.g., an online store looking to sell items to specific customer segments, in which case the advertiser has some information on the CTR, which is often, in practice, used as a proxy for the true value. CTR estimations are not accessible to brand advertisers, who are concerned with, e.g., advertising an event or spreading a campaign message.

This environment might lead to brand advertisers being exposed to adverse selection, i.e. winning disproportionately many low-value impressions, and thus leading to an inefficient allocation in the AdX. The authors suggest that the value of any impression can be characterized by a *common value* component and a *match value* component. The common value represents generally desirable qualities in a user, such as high income, high susceptibility to online display advertisement, and so on. The match value, on the other hand, might represent information that is specific to a certain advertiser, such as if the user recently visited the advertiser's website. That is, due to the positive correlation of bids, the Vickrey auction may not be efficient with respect to the match value component when employed in an RTB setting.

The authors present the MSP auctions as the group of auction mechanisms that overcome the disadvantages of using Vickrey auctions in RTB, i.e. being free of adverse selection, while still maintaining full strategy-proofness. This means that there is no reason for bidders to shade their bids or employ strategies to increase their payoff. The MSP auction is formally defined by the authors as the mechanism parameterized by  $\alpha$ ,  $\mathbf{z}$ , and  $\mathbf{p}$ , where

- $\alpha$  is a constant,  $\alpha \geq 1$ ,

- $z_i(b)$  is the probability of advertiser  $i$  winning the auction given some bid,  $b$  and
- $p_i(b)$  is the expected payment of advertiser  $i$  given some bid,  $b$ ,

such that for performance advertisers,  $i = 1, 2, \dots, n$ , and a brand advertiser,  $i = 0$ , we have that

(i)

$$z_i(b) = P\left(b \geq \alpha \max_{j \neq i} b_j\right)$$

(ii)

$$p_i(b) = z_i(b) \times \alpha \mathbb{E}\left[\max_{j \neq i} b_j\right]$$

(iii)

$$z_0(b) = \prod_{i=1}^n P\left(b_i < \alpha \max_{j \neq i} b_j\right)$$

The first conditions say that the winner has to bid higher than and pay the second-highest bid multiplied by the constant  $\alpha$ . The third condition says that the brand advertiser only wins the auction when no performance advertiser wins the auction. The idea is that this happens if no performance advertiser matches a lower bid multiplied by  $\alpha$ . The authors describe this mechanism as being *deterministic*, *anonymous*, *fully strategy-proof*, and *adverse-selection free*. We're primarily interested in the two latter properties and how they relate to the Vickrey auction. Strategy-proofness is another word for truthfulness, meaning that the authors are saying that the MSP auction is truthful. In the rest of this thesis, 'truthful' and 'strategy-proof' will be used interchangeably. The BSP auction is, however, clearly not efficient.

The MSP auction mechanism is compared to the so-called *omniscient* (OMN) mechanism, which always achieves the optimal allocation by allocating the impression to the bidder with the highest match value, i.e. disregarding the common value component. In practice, it's obviously not feasible, or even possible, for an AdX - and even less so for an individual advertiser - to know the individual match value components for all advertisers. However, the

authors argue that the MSP auction, as opposed to the Vickrey auction, comes very close to the upper bound of allocative efficiency posed by the OMN auction, by choosing some optimal  $\alpha$ . Further, they argue that the MSP auction outperforms the Vickrey auction in an RTB setting in terms of revenue.

Next, we turn to the BSP auction, in which we will drop the anonymity of the general MSP auction and instead assign an individual multiplier to each advertiser. For discussion purposes and conceptual understanding, the distinction between brand and performance advertisers, as well as common value and match value, and the idea of an OMN auction as a benchmark will be retained throughout this thesis.

## 2.3 The boosted second-price auction

We consider the same auction environment as before, where we have  $n$  bidders who all submit sealed bids for one item. All of the bidders have valuations drawn from continuous distributions and each has some bidding strategy. However, each bidder is also assigned an individual *boost value*,  $\beta_i$ , corresponding to an individual  $\alpha$  in the general MSP auction. Similarly to the MSP auction, the winner of the auction is not the bidder with the highest bid, but the bidder with the highest bid multiplied by their individual boost value, i.e.

$$\text{winner} = \arg \max_i \beta_i b_i, \quad i = 1, 2, \dots, n$$

The payment of the winner is the second-highest boosted bid, scaled by the inverse of the boost value of the winner. If the bidders are ordered by the size of their boosted bids, the payment is

$$p = \max \left( b_2, \frac{\beta_2 b_2}{\beta_1} \right)$$

However, bidders can never pay more than their initial bid, such that if  $p > b_1$ , we get  $R_{\text{BSP}} = b_1$ . Hence, for any BSP auction, the revenue is

$$R_{\text{BSP}} = \min(b_1, p) = \min\left(b_1, \max\left(b_2, \frac{\beta_2 b_2}{\beta_1}\right)\right)$$

Golrezaei et al. (2017) also consider a set of reserve prices,  $\mathbf{r}$ , such that the auction is parametrized by  $\beta$  and  $\mathbf{r}$ , but we will set  $\mathbf{r} = \mathbf{0}$  and hence only be concerned with  $\beta$ . Let's consider the key distinctions with the general BSP presented in the previous section. We assume that we have  $n$  performance advertisers and one brand advertiser. Then, the BSP auction is parametrized by  $\beta$ ,  $\mathbf{z}$ , and  $\mathbf{p}$ , such that

(i)

$$z_i(b) = P\left(\beta_i b \geq \max_{j \neq i} \beta_j b_j\right)$$

(ii)

$$p_i(b) = z_i(b) \times \mathbb{E}\left[\max_{j \neq i} \frac{\beta_j b_j}{\beta_i}\right]$$

(iii)

$$z_0(b) = P\left(\beta_0 b \geq \max_{j \neq 0} \beta_j b_j\right)$$

The authors also distinguish between brand advertisers and performance advertisers, calling the latter *retargeting advertisers* rather than performance advertisers. However, the BSP auction only differentiates between them by their boost values. Rather than choosing an  $\alpha$  such that a proportionate amount of valuable impressions will be awarded to the brand advertiser by more or less making the performance advertisers forfeit the impression, the BSP auction can favor brand advertisers by assigning them higher boost values. Considering the framework by Arnosti, Beck and Milgrom (2016), we could think of this as that whenever a brand advertiser posts a relatively high bid, the AdX tries to capture the match value by assigning more weight to the brand advertiser's bid.



In contrast to Arnosti, Beck and Milgrom (2016), who, at best, give a very vague description of how they imagine their  $\alpha$  should be calculated, Golrezaei et al. (2017) give a more detailed account of how they calibrate  $\beta$ . They employ a data-driven algorithm they call *BSP alternating minimizer* (BSP-AM). First of all, their description of BSP-AM is somewhat lacking in practical details. Secondly, they admit themselves that BSP-AM aims to solve an NP-complete, non-convex optimization problem, where the algorithm does converge to a coordinate maximum but without guaranteeing that this is in fact an optimal solution. The approach is simple: iterate through a large number of randomized, historical auctions and compute the optimal boost value for each advertiser with respect to overall revenue in each auction, until the algorithm converges to a set of boost values. While the BSP-AM approach is intuitive and seemingly simple to implement, it can become complicated when working with a larger dataset. This will be discussed further in the method chapter.

In the previous section, we noted that Arnosti, Beck and Milgrom (2016) argue that MSP auctions are deterministic, anonymous, fully strategy-proof and adverse-selection free. The BSP auction is straightforward in the first two characteristics: it's certainly deterministic and definitely not anonymous. The interesting question is how we can characterize the BSP auction in terms of strategy-proofness and adverse selection. Golrezaei et al. (2017, p. 6) mention the paper by Arnosti, Beck and Milgrom (2016) briefly:

"Arnosti et al. (2016) study the adverse selection in online ad markets for the impressions that are sold via auctions vs. guaranteed-delivery contracts, where the valuations of the buyers are correlated via a common value component. They show that to address the adverse selection, the platform should sometimes allocate the impression to the guaranteed-delivery contracts, even when the bids from the auction are higher. This is similar to assigning higher boosts to those advertisers. In our private-value setting, we do not encounter the adverse selection problem. Nevertheless, we show that assigning boosts, based on the bidding patterns of the

advertisers, can increase revenue".

In addition to failing to mention that their own approach is very much similar to that of Arnosti, Beck and Milgrom (2016), the authors vaguely dismiss the notion of adverse selection with the argument that it "does not appear" in their private-value setting. The substance of this argument is unclear. Arnosti, Beck and Milgrom (2016), also consider a private-value setting. At least, Golrezaei et al. (2017) should show that there is no positive correlation between bids submitted by brand and performance advertisers in order to dismiss the existence of adverse selection. The point is not that the common value is individual and not disclosed in a private-value setting, but that if it exists, it represents a portion of any bidder's private valuation, such that it might be reasonable to assume that some bidders are exposed to adverse selection.

Intuitively, it seems reasonable that the BSP auction should also decrease adverse selection by assigning higher boost values to brand advertisers. Golrezaei et al. (2017) do not show or discuss what happens with the allocation of impressions in their data when the boost values are introduced. That is, while they report revenue increases, they do not show where this increased revenue actually comes from. In our dataset, we will see that unconstrained boosting leads to significant re-allocation of impressions, in which brand advertisers do seem to be heavily favored. However, it's hard to make conclusive statements on adverse selection as in Arnosti, Beck and Milgrom (2016), since they suggest to explicitly pick an  $\alpha$  such that the brand advertiser(s) does get a proportionate share of valuable impressions. The BSP auction, on the other hand, does not consider the allocative efficiency in such detail when calibrating boost values. Hence, while it certainly decreases adverse selection by the same logic as the general MSP decreases adverse selection, we can not say that it is adverse-selection free.

This leaves us with, perhaps, the most important characteristic: strategy-proofness. If the BSP auction is not strategy-proof and if truthfulness is not a weakly dominant strategy, our

results are less legitimate as we would have to consider changes in bidding strategies when testing. This is also why the lack of discussion from Golrezaei et al. (2017) is somewhat disappointing; the results and the discussion suffer from lack of dimensionality since potentially severe effects of introducing a boosting mechanism are not discussed properly. While Golrezaei et al. (2017) formally propose at the outset that the BSP auction mechanism is truthful, they never attempt to prove this proposition and more or less dismiss it at the end of the final discussion, where they suggest that the BSP auction mechanism may incentivize bidders to change their behavior in an attempt to increase their payoff. However, it's not clear that this is the case; again, there's a lack of scope and dimensionality.

Going forward, lacking the capacity to complete a more rigorous theoretical exposition, we will follow the proposition of Golrezaei et al. (2017) and assume that the BSP auction is, in fact, a truthful auction mechanism. The authors show that there's a correlation between boost values and certain parameters describing bidder behavior. Specifically, they show a significant relationship between volatile bidding behavior and low boost values. In other words, the BSP auction favors stable bidders, such as brand advertisers, while giving less favor to performance advertisers who are more prone to volatile bidding behavior because of, e.g., retargeting advertisement, more accurate estimation of CTR, and so on.

These results do seem to lend credence to the proposition of truthfulness. That is, we're not considering truthfulness with respect to a single bid, but with respect to entire bidding strategies. If the boost values are directly related to the behavior of bidders, broadly characterized by brand advertisers and performance advertisers, any bidder would have to change their long-term behavior in order to affect their boost value, which seems like an unlikely scenario. This will, of course, also depend on the length of the time period used to calibrate the boost values. However, it does seem likely that a bidder would change their short-term participation rates in response to sudden changes in the allocation of impressions,

e.g. because one advertiser might end up with more impressions than accounted for in the short-term budget. These are the types of issues that are not discussed by Golrezaei et al. (2017).

In conclusion, we consider the BSP to be a non-anonymous, deterministic, and strategy-proof form of MSP. It does seem, both from the results reported by Golrezaei et al. (2017) and from the results presented later in this paper, that the BSP mechanism results in an allocation of impressions that should decrease adverse selection in the AdX. However, we cannot state that it's adverse-selection free. In terms of the desirable properties of the Vickrey auction, we have assumed, and there seems to be some reason for this assumption, that the BSP auction is truthful. However, like the MSP auction, the BSP auction is clearly not efficient. Obviously, if the mechanism changes the allocation given in an Vickrey auction, it cannot be efficient.

## Example

Let's consider a simple example. We have three bidders, of which two, 1 and 2, are performance advertisers and one, 3, is a brand advertiser. They all post bids for an impression, such that

$$b_1 = 6, \quad b_2 = 4, \quad b_3 = 2.5$$

In a normal Vickrey auction, these would be truthful bids and advertiser 1 would win the auction, paying  $b_2$ , such that  $R_{\text{SP}} = 4$ . Now, let's consider a BSP setting with boost values  $\beta_1 = 1$ ,  $\beta_2 = 1$ , and  $\beta_3 = 2$ . The winner is again advertiser 1, since

$$\max_i \beta_i b_i = \max \{1 \times 6, 1 \times 4, 2 \times 2.5\} = 6$$

and the payment is

$$R_{\text{BSP}} = \min \left( 6, \frac{2 \times 2.5}{1} \right) = 5$$

which means that  $R_{\text{SP}} < R_{\text{BSP}}$ . This is, of course, a silly, albeit not entirely unrealistic, example. However, it can be useful for illustrating another important point. Let's instead assume that  $\beta_3 = 3$ . Then, the winner of the auction is bidder 3, since  $\beta_3 b_3 = 7.5$  and the payment is

$$R_{\text{BSP}} = \min\left(2.5, \frac{1 \times 6}{3}\right) = 2$$

which means that  $R_{\text{SP}} > R_{\text{BSP}}$ . Hence, the BSP generates less revenue than the Vickrey auction. However, while we might have less revenue, it might also mean a better overall allocation if the brand advertiser is exposed to adverse selection. At the same time, for a one-off auction, this is clearly not an efficient mechanism. Bidder 1 has the highest valuation, but bidder 3 wins the auction. More interestingly, it illustrates how a change in allocation is actually a spill-over effect from trying to increase revenues. As we're attempting to close the gap between the highest bids, we will necessarily assign higher boost values to lower bidders, e.g. brand advertisers, meaning that they will often win impressions for which they post relatively high bids. Essentially, this is exactly the function of the anonymous MSP, where an  $\alpha$  is chosen such that the gap between the highest bidders is narrowed, while simultaneously leading to the brand advertiser getting a larger portion of valuable impressions.

## 2.4 The shift from first price to second price

Edelman, Ostrovsky and Schwarz (2005) detail the institutional history of markets for online display advertisement. The FP auction had a brief stint of popularity from 1997 to 2002 due to its ease of use, low entry costs and transparency. However, both publishers and advertisers realized that it was prone to instabilities. The FP auction encouraged excessive gaming and bid shading since the bidder that was fast in reacting to other bidders' bidding strategies had an advantage. This also caused allocative inefficiencies. To remedy these problems, Google was the first publisher to turn to the Vickrey auction, which, as we know, is a truthful mechanism. Since then, the Vickrey auction has been the dominant auction format in RTB.

Golrezaei et al. (2016) also comment on the historical lack of popularity for FP auctions in RTB. They focus on the heterogeneity of bidders in RTB and how this leads to bid shading in FP auctions, as well as the fact that it's hard for the AdX to understand the bidders' behaviors and strategies. Thus, instabilities and insecurities have led to the Vickrey auction dominating RTB for almost two decades. These are also the reasons why we will not be focusing on the FP auction when considering revenue and allocation of the Vickrey auction and the BSP auction. In addition, the highest bid in a Vickrey auction doesn't really say anything about the revenue from an FP auction with the same bidders, since we have to consider the interplay of bidding strategies and bid shading that would've occurred in an FP auction.

While it would be interesting to consider how an FP auction would play out in terms of revenue and allocation given the data we have, it is simply too demanding and outside the scope of this thesis to attempt such simulations. Even if we know all of the bidders' true valuations, it's difficult, if not impossible, to accurately simulate how a number of intelligent bidders would've behaved to maximize their respective payoffs simultaneously.

## 2.5 Summary

We have considered how both revenue and allocative efficiency can be improved in an RTB setting where there are two types of advertisers, brand advertisers and performance advertisers, who have positively correlated valuations of impressions. We have looked at general MSP auctions, which can potentially increase both revenue and allocative efficiency while still retaining some of the nice theoretical properties of the Vickrey auction, especially with respect to bidder behavior. We have also considered a specific application of MSP, the BSP auction, for which the retainment of certain properties is not clear. If there exists adverse selection, both the general MSP and the BSP auction can increase revenue and allocative efficiency by, directly or indirectly, favoring brand advertisers.

The existence of adverse selection depends on whether bid from brand and performance advertisers are positively correlated, i.e. if there exists a significant common value component which primarily determines a bidder's valuation. That is, the degree of adverse selection should depend on the degree of dominance of the common value component. Conversely, if the common value component is insignificant and differences in bidder behavior are dependent on the so-called match value component, i.e. idiosyncratic valuations specific to each advertiser, then there is no reason to assume the existence of adverse selection. At the extreme end, if the common value component does not exist at all, the Vickrey auction should result in the same outcome as the OMN mechanism, since valuations are idiosyncratic and the Vickrey auction always awards the item to the bidder with the highest valuation.

Going forward, we want to investigate the correlation of bids from different advertisers and the performance of the BSP auction in relation to the Vickrey auction. In the latter investigation, we will, as noted above, assume that the BSP auction is indeed truthful and that an implementation of boost values on a dataset with historical RTB auctions would not have altered the bidders' value distributions. It is interesting to explore other auction mechanisms

than the Vickrey or FP auction, which seem to result in suboptimal outcomes in terms of revenue, allocation and stability, since RTB is a dynamic auction environment with heterogeneous bidders. MSP and BSP are attempts to amend the drawbacks of the traditional auction mechanisms by employing data-driven methods that aim to exploit the heterogeneity of bidders by understanding and adapting to the behavior and value distribution of each bidder.



# Chapter 3

## Method

As mentioned in the previous chapter, I haven't followed the approach by Golrezaei et al. (2017) when attempting to simulate the effects of introducing a BSP auction mechanism in an AdX. This is due to the fact that I have used a much larger dataset, spanning a longer time period and including more items (i.e. different ad slots). Not only does this mean that the algorithm they proposed is more cumbersome to use due to runtime, but it also means that the data preprocessing necessary for their algorithm becomes more complicated. This chapter is broadly dedicated to describing this problem and how I've attempted to deal with it.

The first section will describe the data used and how it differs from the data used by Golrezaei et al. (2017). Then, I will describe some of the bidders in the dataset in terms of their participation rate, bid average, and bid variance, as well as their estimated value distributions. Finally, the last sections will be devoted to describing the method I've used to calibrate the boost values, as well as how I've attempted to calibrate boost values for different advertisers when incorporating a constraint on their budget spending.

## 3.1 Data

The data consists of auctions for ad slots held by Adform on behalf of a single website in Denmark. The website is one of the largest in Denmark and has approximately 2.1 million unique visits per week, i.e. something like 300 thousand unique visits per day on average. There are a large number of ad slots on the website. These are characterized by height, width, and position. Summary statistics across the training dataset for some of the most popular ad slots are displayed in table 3.1. The dataset is split up into six different periods, of which the first five are used for calibrating the boost values while the last period is used for testing the revenue when using the boost values. There is a total of 116 million bids in the raw training data. When I have filtered out all auctions with only one bidder and with multiple ad slots, there are 8 million bids in 2.9 million auctions. When the testing data has been filtered for single-bid auctions and auctions with several ad slots, there are 634201 auctions left with 2 million bids. Summary statistics for the different periods are displayed in table 3.2.

Many of the auctions in the dataset only have one bidder. I have filtered out all of these auctions from the testing dataset, since applying boost values doesn't make any difference in terms of revenue when there's only one bidder. There are also many cases of the same bidder posting multiple bids. This is likely due to the fact that DSPs often run multiple advertisement campaigns at the same time, posting bids on behalf of several different advertisers depending on their specific budgets, targeting criteria, and so on. In the testing dataset, I've only considered the highest bid from an advertiser that has posted several bids.

The main reason why the approach suggested by Golrezaei et al. (2017), i.e. iterating through a number of historical auctions until the boost values converge, is problematic is that the large amounts of data make it difficult to randomize and shuffle the training data. Ideally, we want to shuffle the training data since RTB is a highly nonstationary environment, meaning that the "market price" of an ad slot can change drastically during a week, or even

Table 3.1: Ad slots

Slot ID	Width	Height	Position ID	# Bids
A	930	180	2	5033814
B	160	600	2	4896650
C	160	600	2	4839611
D	930	180	1	4498701
E	320	160	1	3260809
F	320	320	2	2984862
G	320	160	2	2566887
H	728	90	2	2485026

Table 3.2: Data periods

Period	Type	Start	End	# Bids	# Auctions
1	Training	2018-10-10	2018-10-17	25879377	10025222
2	Training	2018-10-17	2018-10-24	22878984	9117532
3	Training	2018-10-25	2018-11-01	27834936	10240755
4	Training	2018-11-02	2018-11-09	15843723	4533186
5	Training	2018-11-12	2018-11-19	23985352	6355267
6	Testing	2018-11-27	2018-12-04	24439817	7852427

during a day. Hence, if we calibrate boost values on a chronologically ordered set of auctions, the boost values may be fitted specifically to the last share of auctions, and may not be representative of the earlier auctions. This also highlights the major differences between the dataset used by Golrezaei et al. (2017) and the one I have used. First of all, I’m training and testing over much longer periods (several days, rather than just one day). Secondly, I’m considering bids for an number of different ad slots rather than for just one ad slot. As shown in table 3.1, the ad slots are in fact distinguishable. Hence, there is less granularity and more generality in my approach.

Obviously, I could focus on just one day and come closer to replicating Golrezaei et al. (2017). However, there is another important feature to consider which separates the datasets. They use data from Google’s AdX, which is the largest in the world. Hence, it has a large number of frequent bidders with high participation rates. Adform does not have as many auctions with as many bidders during one day for this particular website. This is not necessarily a

problem, but conducting a thorough analysis of the seasonality in the participation of different bidders is definitely a problem considering the scope of a Bachelor's thesis. Thus, I've chosen to use the entire dataset from Adform and create more general estimates of the boost values. I will show that this also leads to converging boost values and increased revenue, by the same evaluation method as Golrezaei et al. (2017).

## 3.2 Bidders

I have followed the approach of Golrezaei et al. (2017) in focusing on the largest bidders. This is sensible, since applying boost values to smaller bidders will have a negligible effect on the revenue. Some bidders participate in less than 0.1 % of the auctions. Essentially, among the 8 bidders with the highest spending, of which the two smallest have a participation rate of approximately 0.2 %, there is one brand advertiser and seven performance advertisers. This section will provide statistics on these bidders, including estimations of the distributions of their impression valuations. Summary statistics on the different bidders are presented in table 3.3.

There are some interesting points to be made from looking at the bidders. It's clear that bidder 2 is a typical brand advertiser, with a very high participation rate and relatively low bid mean and variance. The participation rate is defined as the number of auctions in which the bidder has posted at least one bid, divided by the total number of auctions. When considering all auctions (i.e. including all the auctions with only one bidder), bidder 2 has an even higher participation rate relative to the other bidders. At the other end of the spectrum, bidders 6 and 7 seem to be typical performance, or retargeting, advertisers, with extremely low participation rates but considerable spending due to a high bid mean and (especially for bidder 7) a very high bid variance. Obviously, this structure of bidders' characteristics will determine the possible revenue increase. For example, if we only would have had brand ad-

vertisers, with low bid variances and similar bid means, the possible revenue increase would be limited by the similarity of bidding strategies. In this sense, my results are not entirely comparable to those by Golrezaei et al. (2017). That is, even if I replicated their algorithm completely and limited the experiment to only one ad slot during one day, the results would still not be comparable since there is a fundamental difference in the bidders participating in the auctions. While Golrezaei et al. (2017) are not as transparent about their bidders, it seems like they have a more "complete" spectrum of bidders, including more brand advertisers.

Table 3.3: The eight largest bidders, ordered by number of bids

Bidder	Type	Participation rate	Number of bids	Bid mean	Bid var
1	Performance	0.71	4017328	1.89	2.10
2	Brand	0.90	2594512	0.60	1.06
3	Performance	0.22	631380	1.35	1.92
4	Performance	0.11	318965	1.66	0.91
5	Performance	0.08	229228	1.40	1.21
6	Performance	0.04	124821	2.22	4.61
7	Performance	0.02	63483	4.45	85.11
8	Performance	0.02	52920	2.07	0.57

The density estimates in figure 3.1-3.8 are purely for illustrative purposes. I have used a less granular bandwidth to give them a smoother appearance. The estimated cumulative distribution functions (CDFs) are, however, used in calibrating the boost values. However, when plotting the inverse CDFs I have used fewer bins in the underlying histograms. Hence, the figures should be understood primarily as a tool to illustrate the behavior and strategies of the different bidders.

Figure 3.1: Bidder 1

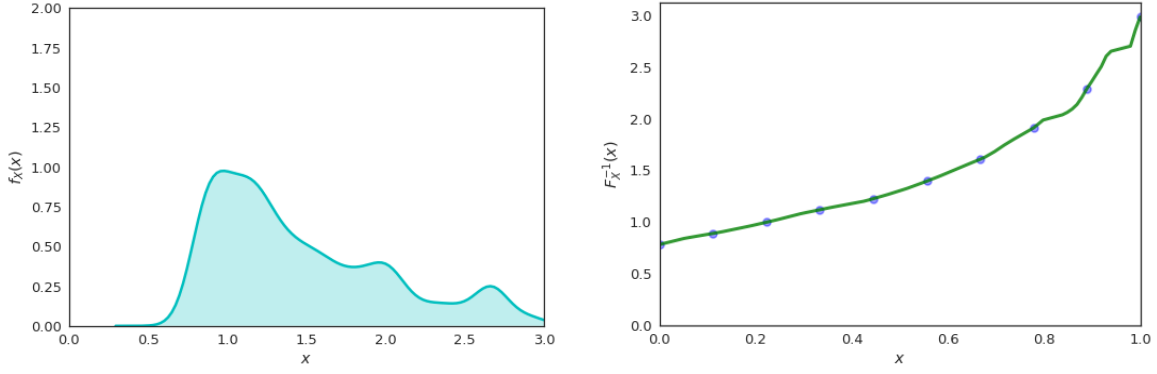


Figure 3.2: Bidder 2

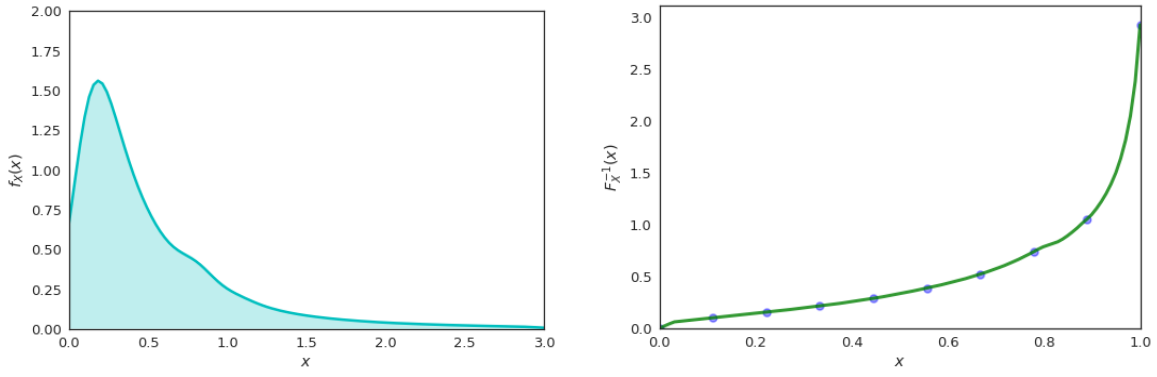


Figure 3.3: Bidder 3

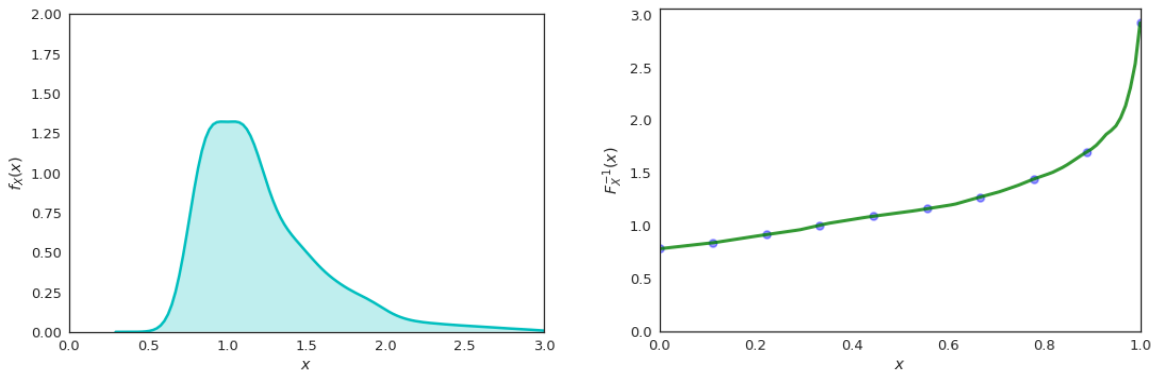


Figure 3.4: Bidder 4

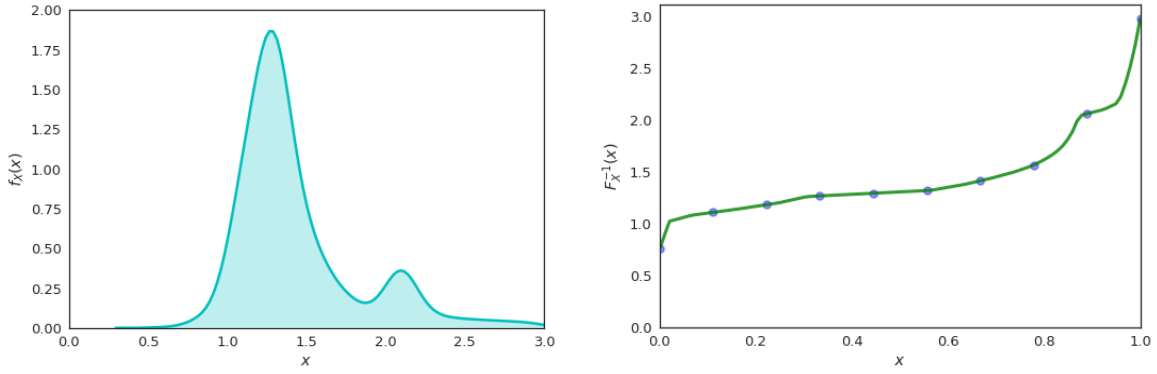


Figure 3.5: Bidder 5

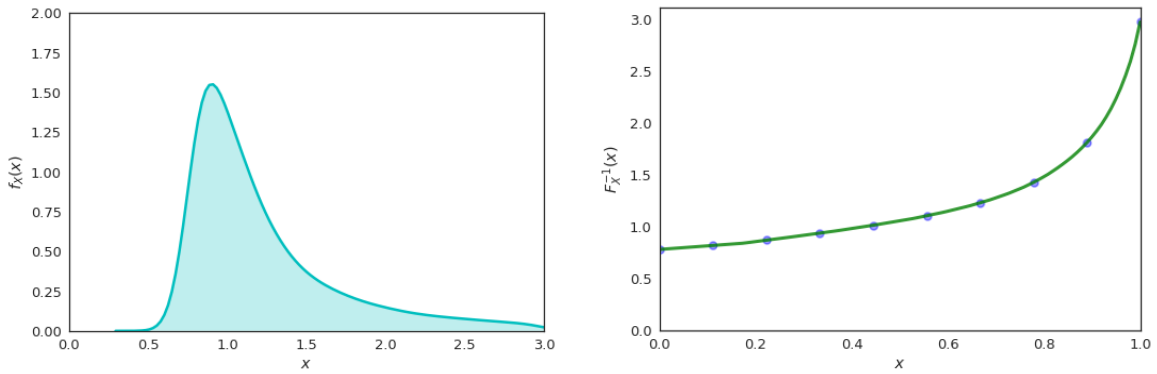
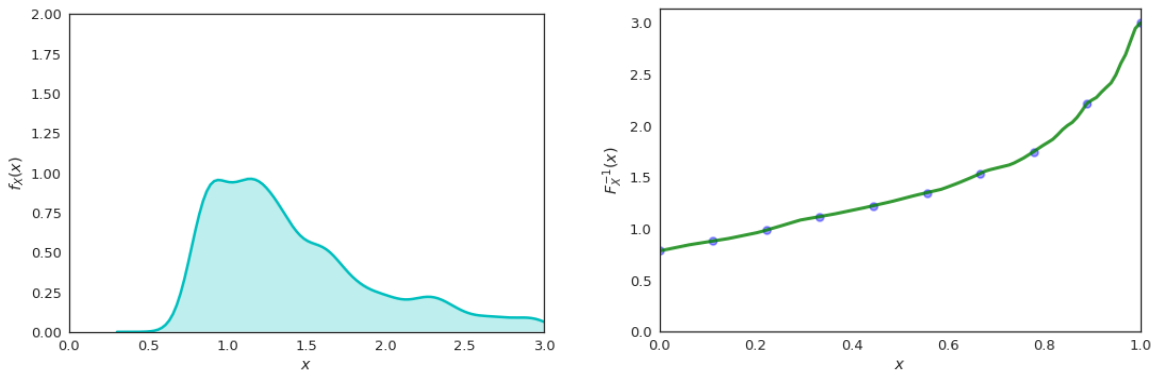


Figure 3.6: Bidder 6



### 3.3 Boosting by sampling

As described in the previous sections, I've been working with a dataset that is much larger than the one used by Golarezaei et al. (2017), spanning a longer time period and several different ad slots. Hence, a more general and dynamic approach is required to compute boost values which are representative of bidding behavior across all auctions. The large amounts of data also make it difficult to shuffle and iterate through the historical data. I've solved this by instead creating a stylized, randomized auction which is iterated a large number of times.

First, I extract all of the bidding data for each included advertiser. Then, I estimate all of the participation rates and bid distributions. This makes for a representation of any advertiser's "average" bidding behavior across all historical auctions. Hence, we can consider a simulated, average auction, more or less representative of all of the historical data. For each iteration, I sample from a uniform distribution for each advertiser to determine if they will participate in the auction. Then, if more than two advertisers are participating, I take a random sample from each of their bid distributions and hold a BSP auction. Finally, each boost value is updated incrementally with respect to maximizing the revenue from the auction. The whole procedure is described in more detail in the following subsections.

It is not entirely clear how Golrezaei et al. (2017) update the boost values in their algorithm, i.e. if they use incremental updates for every auction and, if so, how they scale the increments. The update rule I've suggested is heavily influenced by Monte Carlo methods and the idea that we want to move in the direction of an optimal set of boost values, which is assumed to exist. However, Golrezaei et al. (2017) do specify that they initialize the calibration of boost values by setting  $\beta_i = 1$  for all  $i = 1, 2, \dots, n$ . I will be following this way of initializing the simulation.



### 3.3.1 Estimating bid distributions

First, we need to create an *empirical distribution function* for each bidder. Let's consider bidder  $i$ . If we define  $\mathbf{b}_i = (b_i^{(1)}, b_i^{(2)}, \dots, b_i^{(n)})$  to be all bids by bidder  $i$ , we define the empirical CDF as

$$\bar{F}_i(x) = \frac{1}{n} \sum_{j=1}^n \mathbf{1}_{b_i^{(j)} \leq x}$$

such that  $\bar{F}_i : \mathbb{R} \rightarrow [0, 1]$ . We create a histogram and calculate the probability of some value  $x$  by considering the cumulative value of all the bins up to and including the one containing  $x$ . Specifically, I've used 500 bins in the range  $[0, 5]$ . This excludes some extreme bids for some of the performance advertisers. However, the algorithm converges quickly as a result, and it can be argued that there is no great loss with respect to revenue in using this restriction since the bidders with some extremely high bids also have very low participation rates.

### 3.3.2 Participation sampling

For each advertiser, we have a historical participation rate. Again, let's consider bidder  $i$ . We denote this participation rate as  $\rho_i$  and define it as

$$\rho_i = \frac{\text{numer of auctions in which bidder } i \text{ has posted a bid}}{\text{total number of auctions}}$$

Then, in each auction we sample from the uniform distribution between 0 and 1, such that we get  $u \sim \text{Uniform}(0, 1)$ . If  $u \leq \rho_i$ , bidder  $i$  will participate in the auction, and if  $u > \rho_i$ , bidder  $i$  will not participate. There are cases where is only one bidder or no bidders. In this case, there is no auction and hence no change to the boost values.

### 3.3.3 Bidding by inverse transform sampling

Whenever we have two or more bidders participating, we use so-called *inverse transform sampling* to get the bid from each bidder. This means that we take the *inverse* of the

aforementioned empirical CDF, i.e.  $\bar{F}_i^{-1}(x)$ . I've done this by interpolating a curve over the cumulative values and the bin edges, i.e. for  $\{(\bar{F}_i(x_j), x_j) \mid j = 1, 2, \dots, n\}$ . Thus, we get a continuous approximation of the inverse CDF for each bidder. Then, we sample another  $u$  from the same uniform distribution as before, i.e.  $u \sim \text{Uniform}(0, 1)$ . Since  $\bar{F}_i^{-1} : [0, 1] \rightarrow \mathbb{R}$ , this means that we can get random samples from bidder  $i$ ,  $b_i$ , by  $b_i = \bar{F}_i^{-1}(u)$ .

### 3.3.4 Boost calibration

For each advertiser, I've chosen to update the boost values using a rule inspired by the method of gradient descent. We consider the optimal boost value for bidder  $i$ ,  $\beta_i$ , with respect to maximizing the revenue in a given auction. We have two possibilities: either  $i$  is the winner of the auction or  $i$  is not the winner of the auction. In any case, bidder  $i$  will never pay more than the given bid, i.e.  $b_i$ . Let's consider another bidder,  $j$ . We have case (i), in which  $i$  is the winner of the auction and  $j$  is the second-highest (boosted) bidder, and case (ii), in which  $j$  is the winner of the auction.

- (i) If  $i$  is the winner of the auction, we want a low  $\beta_i$ , since the payment is scaled in inverse proportion to  $\beta_i$ . In any case,  $R_{\max} = b_i$ , such that the optimal beta,  $\beta_i^*$ , for that particular auction can be found by

$$R_{\max} = \frac{\beta_j b_j}{\beta_i^*} \iff b_i = \frac{\beta_j b_j}{\beta_i^*} \iff \beta_i^* = \frac{\beta_j b_j}{b_i}$$

- (ii) If  $j$  is the winner of the auction, we want a high  $\beta_i$ , since the payment is now scaled in proportion to  $\beta_i$ . Now, we have that  $R_{\max} = b_j$ , such that the optimal beta  $\beta_i^*$ , for this particular auction is given by

$$R_{\max} = \frac{\beta_i^* b_i}{\beta_j} \iff b_j = \frac{\beta_i^* b_i}{\beta_j} \iff \beta_i^* = \frac{\beta_j b_j}{b_i}$$

Then, we update  $\beta_i$  by

$$\beta_i^{(k+1)} = \beta_i^{(k)} - \alpha \times \left( \beta_i^{(k)} - \beta_i^* \right) = \beta_i^{(k)} - \alpha \times \left( \beta_i^{(k)} - \frac{\beta_j^{(k)} b_j^{(k)}}{b_i^{(k)}} \right)$$

where  $\alpha$  is the *learning rate* and where  $j$  is either the highest or second-highest bidder, depending on whether  $i$  won the auction. After each auction and subsequent calibration, all the boost values are divided by the smallest boost value, such that the smallest boost value is 1.0. In figure 3.7, I have plotted the tuning of the boost values using  $\alpha = 1 \cdot 10^{-4}$  for the largest advertisers by spending and iterating 500000 times. It is worth noting that the result is consistent with Golrezaei et al. (2017), since they report that typical brand advertisers are heavily favored by the BSP auction. This is clearly the case here as well.

Rather than letting the boost values converge to some exact value, I let the simulated auctions iterate for a fixed number of times. In order to get the final boost values, I take the average of the last 400000 iterations, i.e. after the boost values settle in some interval. The reason the boost values do not converge to exact values is probably due to the nonstationarity of the auction environment and the variations in bidding behavior, including the differences in participations rates. For example, a performance advertiser that has a typical retargeting behavior will not participate very often, but will likely have a large impact on the boost values when it does participate. As we're increasing the number of bidders, the calibration of the boost values gets increasingly unstable. One way of decreasing instability is to have a smaller  $\alpha$ , but this will also mean longer times for calibration. For example, in the case of 4 advertisers with  $\alpha = 1 \cdot 10^{-5}$ , it takes something like 1.5 million iterations before the boost values settles in some interval. Hence, we will be using  $\alpha = 1 \cdot 10^{-4}$ .

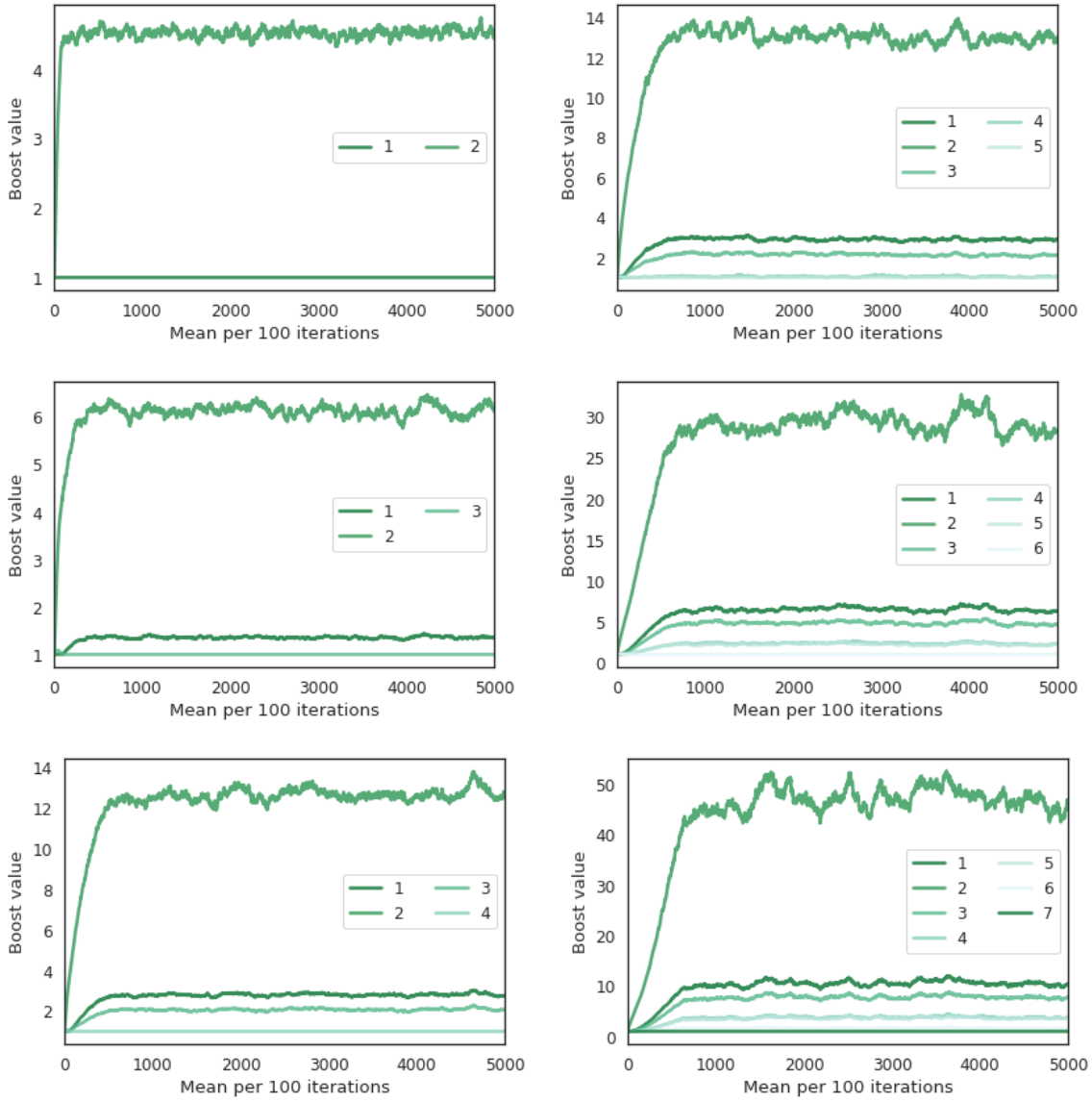


Figure 3.7: Calibration of boost values over 500000 iterations.

## 3.4 Summary

I have been working with a much larger and more extensive dataset than the one used by Golrezaei et al. (2017), even after filtering and preprocessing, which has required a different approach. Looking at the calibration of boost values in figure 3.7, the algorithm I've proposed does seem to produce a behavior similar to that reported by Golrezaei et al. (2017). That is, bidders with low variation in their bidding behavior, i.e. primarily brand advertisers, are favored and assigned high boost values. Additionally, the algorithm is fast and calculates a set of boost values in 1 – 2 minutes, which allows for large-scale simulations. Since the algorithm is inspired by Monte Carlo methods and is relying heavily on random sampling, I've chosen to call the procedure **BSP-MC**.

I've included the pseudocode in an appendix to give a better overview of the whole procedure. However, this procedure (and, seemingly, the method employed by Golrezaei et al. (2017)), is not entirely unproblematic. As we will see in the next chapter, it's important to consider exactly what happens when we start applying boost values and where the additional revenue actually comes from. We saw a brief example of this in the theory section. When some boost value is sufficiently high, specifically for the brand advertiser, we might see changes in the allocation of impressions. While this might be desirable to some extent, i.e. if some bidders are exposed to adverse selection, the algorithm does lack a naturally encoded constraint which prevents boost values from converging to levels where the resulting change in allocation might cause problems.

# Chapter 4

## Results

This chapter is divided into two sections. We will start by looking at the correlation between bids from the brand advertiser and two performance advertisers. Then, we will look at the changes in revenue and allocation from applying boost values calculated by the BSP-MC algorithm to the testing dataset. As mentioned previously, the results found by Golrezaei et al. (2017) and those presented here might be more or less incomparable. Hence, such comparisons will be held to a minimum. Rather, we will consider how our results relate more broadly to the discussions held in the theory section.

### 4.1 Bid correlation

In proposing the MSP, Arnosti, Beck and Milgrom (2016), discuss the positive correlation of bids from brand and performance advertisers as the main reason for the supposed existence of adverse selection in an AdX. Hence, this seems like an interesting aspect to consider when proposing to implement an auction mechanism which clearly changes the allocation of impressions. I've used the testing dataset and compared the brand advertiser, bidder 2, to two performance advertisers, bidder 1 and bidder 3. I looked at all auctions in which both bidders participate. If a bidder has posted multiple bids, I have taken the average of those bids. When comparing bidder 2 with bidder 1, there are 462 thousand auctions, and when

comparing bidder 2 with bidder 3 there is a thousand auctions.

In the first case, the correlation coefficient is  $-0.05$  and in the second case the correlation coefficient is  $-0.06$ . Hence, there's very little correlation, and no positive correlation, in the valuations of impressions in the auctions where the bidders participate. One could argue that it's also interesting to consider auctions in which one bidder participates and the other doesn't, in which case the bid, and valuation, of the bidder not participating could be considered to be zero. This would of course affect the correlation and possibly make it even smaller (due to the high participation rate of bidder 2 relative to the other bidders). However, as clarified by the discussion by Arnosti, Beck and Milgrom (2016), what we're interested in is the possible existence of a common value component, which should be captured by the current analysis. In contrast to their discussion, the small correlation coefficients suggest that there is little evidence of a common value component.

This result is of course not representative of RTB in general, only of auctions held by Adform for a specific website in Denmark. Nevertheless, the results are interesting and suggest that the proposed existence of a common value component is not entirely uncontroversial. One could also argue that due to the inability of brand advertisers to estimate the value of different impressions, they are less willing to post relatively high bids. However, looking at table 3.3, bidder 2 has significant variance, meaning there is also variation in the valuations of bidder 2. Hence, if a common value component did exist, any variations in the common value component should be captured by the bidding behavior and the correlation between the different bidders.

It's important to note that this result is underpinned by the truthfulness of the Vickrey auction. That is, attempting to capture the existence of a common value component only makes sense if the bids are wholly representative of the bidders' actual valuations. Given

the truthfulness of the Vickrey auction, there doesn't seem to exist a significant common value component or a positive correlation between bids submitted by brand and performance advertisers in our dataset.

## 4.2 Results from simulations

We want to consider the distribution of revenue increases for a number of simulated sets of boost values. Initially, I have deviated slightly from the method and run 100 simulations each when assigning boost values for 2, 3, 4, 5, and 6 bidders, which have been chosen randomly from the 8 top-spending bidders. In total, I ran 500 simulations. For each simulation, I've used 1000000 iterations and taken the average of the last 500000 iterations as the final boost values, with  $\alpha = 1 \cdot 10^{-4}$ . This is due to the fact that if we choose bidders randomly, we will sometimes end up only with bidders with low participation rates, meaning that we will have relatively few simulated auctions to calibrate boost values from. Each of the 500 sets of boost values are evaluated on the first 100000 auctions in the test data. The results are shown in figure 4.1 and table 4.1. The average revenue increase across all simulations is 84 %, while the maximum revenue increase is 220 %. As a comparison, Golrezaei et al. (2017) achieve revenue increases from 16.55 % to 29.28 %. However, it should be noted that even though the evaluation methods are comparable, the results are not necessarily comparable since the auction environments analyzed seem to be very different.

Table 4.1: Revenue increase

# Bidders	Avg. increase	Avg. $\beta$	$\beta$ var
2	86.4 %	1.42	1.22
3	68.4 %	1.36	1.03
4	119.5 %	3.57	85.59
5	126.9 %	4.21	106.10
6	20.5 %	4.97	125.56

We get the largest average revenue increase, 120 % and 127 %, when assigning boost values to 4 and 5 bidders, respectively. We look closer at the case of 4 bidders and run a simulation



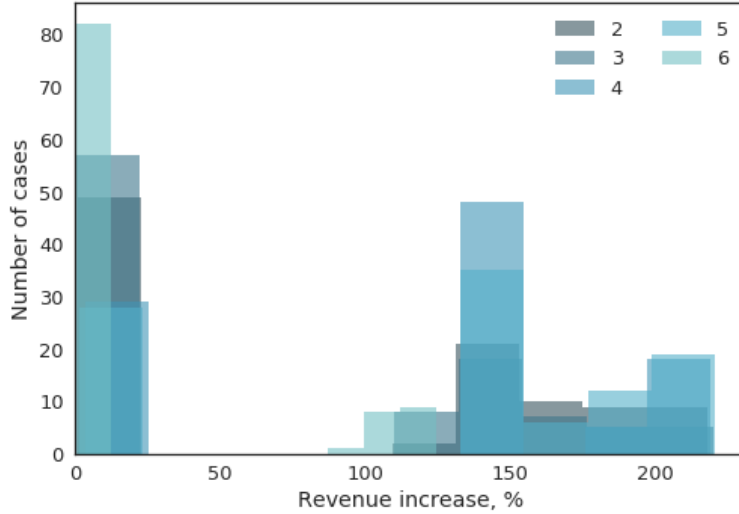


Figure 4.1: Histogram with revenue increases from unconstrained boosting

with the 4 top-spending bidders, using  $\alpha = 1 \cdot 10^{-4}$  for 500000 iterations, evaluating on the entire test dataset, in order to get a better understanding of where the revenue comes from. In figure 4.2, I've plotted the change in the distribution of payments (i.e. what the bidder with the winning bid actually pays) for the auctions in the test data, using the original Vickrey auction format and the BSP mechanism. There is a heavy shift to the right, which can be interpreted as the boost values "closing the gap" between the highest and second-highest bids. This is even clearer when looking at figure 4.3, in which I've plotted the difference in prices paid in the last 100 Vickrey and BSP auctions. The right figure illustrates the extra revenue in the BSP auction.

We also want to look at the changes in the allocation of impressions when implementing the boost values. As mentioned before, it's possible that these are significant since the boost values are calibrated without considering more or less realistic budget constraints on behalf of the bidders. That is, even if the auction is strategy proof, we should consider the fact that bidders can't be expected to be entirely flexible with respect to the amount of impressions they gain or lose as a result of using the boost values. In figure 4.4, I've plotted the changes in the allocation of impressions.

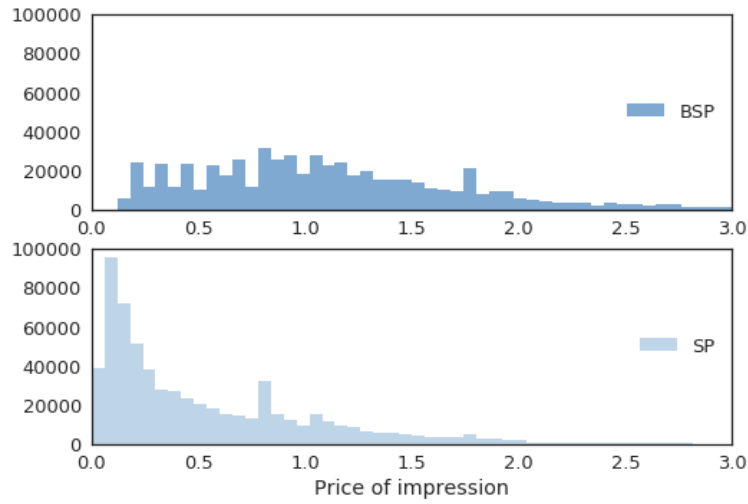


Figure 4.2: Change in price of impressions

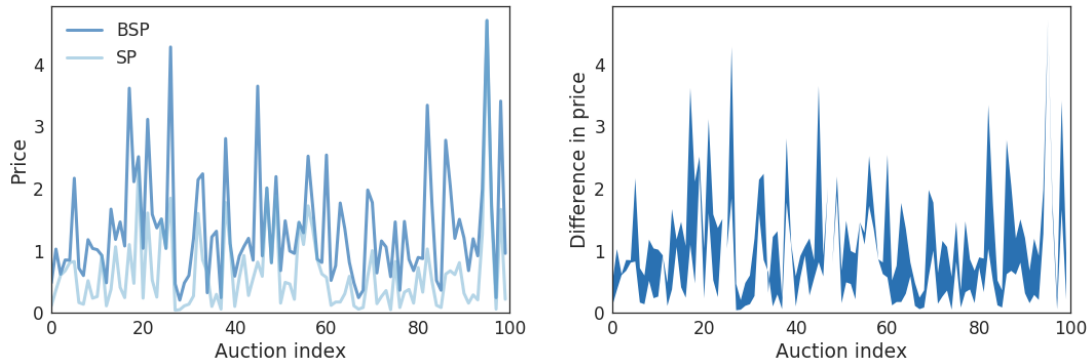


Figure 4.3: Difference in price of impressions

We're clearly dealing with an unrealistic scenario. The increase in revenue is 130 %, but the brand advertiser, 2, absorbs a lot more impressions, while bidders 1, 3 and 4 likely pays a lot more for fewer impressions. It seems unlikely that bidder 2 would maintain a high participation rate when implementing the boost values. The number of impressions won increases from 34939 to 297401, which clearly has serious budget implications. It's important to understand why this shift in allocation doesn't necessarily decrease the revenue. At first sight, it might seem like a bidder with a lower average bid winning more auctions would lead

to a lower average revenue. However, the point is that in the normal Vickrey auction, bidder 2 often has the second-highest bid such that the highest-bidder will pay  $b_2$ , while bidder 2 will often be the winner in the BSP auction and pay its own bid, i.e. such that the revenue is still  $b_2$ , due to its high boost value. The extra revenue then comes from the cases where bidder 2 does not win the impression despite its high boost value, meaning that the gaps between the highest bid and the other bids are narrowed.

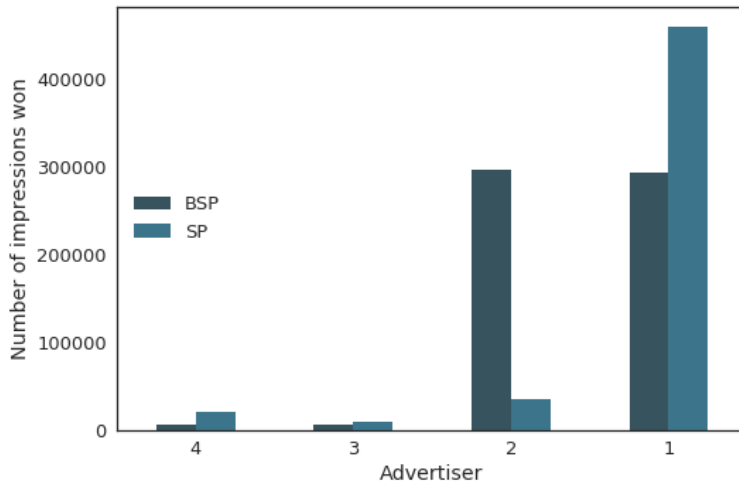


Figure 4.4: Change in allocation of impressions

Considering table 4.1, it's clear that while simulations with 4 and 5 bidders generate the largest average revenue increase, the calibration of boost values is very unstable. This is also clear from figure 3.7. Hence, while the average revenue increases from assigning boost values to 2 and 3 bidders are lower, we want to consider these cases as well since the calibration of boost values is more stable and might give a better representation of the results from implementing boost values in general and the BSP-MC algorithm in particular. I have chosen the 2 and 3 top-spending bidders, respectively, and have applied the boost values to the entire testing dataset. The revenue increases are 109 % and 116 % for assigning boost values to 2 and 3 bidders, respectively. The change in allocation is illustrated in figure 4.5.

Again, the change in allocation seems problematic. There is some change for bidders 3 and 4 in comparison to the previous case, but the allocation of impressions to bidder 2 is still extreme.

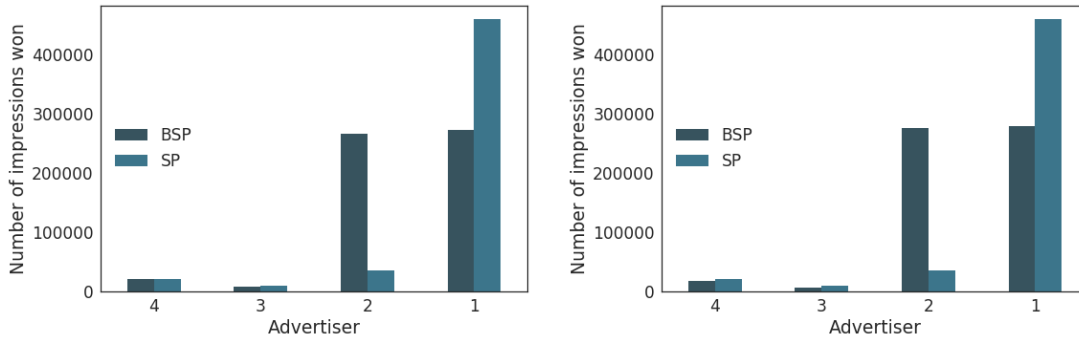


Figure 4.5: Change in allocation of impressions for 2 and 3 boost values

While the revenue increases are impressive, the change in the allocation of impressions shows a more problematic aspect of the boosting approach. It would be interesting too see the change in allocation for Golrezaei et al. (2017). Unfortunately, this information is not available. However, given that they also report significant increases in revenue, it’s likely that their simulations also resulted in significant changes in allocation. While some changes in allocation might be desirable if the brand advertiser is in fact exposed to adverse selection and is willing to pay for more impressions, the **BSP-MC** algorithm does not incorporate a constraint on how much the allocation is allowed to change as a result of applying the boost values.

This is likely true for Golrezaei et al. (2017) as well. However, it might be the case that their dataset is better suited for this type of implementation, i.e. if there is a better disposition of advertisers which prevents the resulting changes in allocation to become problematic. That is, it should be mentioned that even though the results presented here are consistent across the number of bidders that are assigned boost values, it might not be representative of the

performance of the boosting procedure in general. While it is obvious that implementing boost values will change the allocation of impressions, the extreme degree to which it happens in our results might be specific to this dataset and the disposition of bidders in the auctions hosted by Adform for a specific Danish website.

### 4.3 Summary

We found that there is no positive correlation between the bids posted by the brand advertiser and the two largest performance advertisers. This is an interesting result, since it should imply that there is in fact no significant adverse selection in the AdX, which in turn means that implementing a BSP is not entirely justified on behalf of the bidders. As mentioned in section 2.2, Arnosti, Beck and Milgrom (2016) discuss the positive correlation of valuations between brand and performance advertisers as the reason for brand advertisers being exposed to adverse selection. This does not mean that it is incorrect to assume a positive correlation in valuation for different types of advertisers; it means that their general discussion does not apply to the specific case we're dealing with in this thesis.

The lack of a positive correlation in bidding behaviors sheds another light on the changes in allocation from implementing boost values for the AdX represented in our dataset. That is, we can not justify the changes in the allocation of impressions by attempting to reduce adverse selection, since there doesn't seem to be any adverse selection to begin with. Thus, implementing boost values is only beneficial for the publisher's revenue. With that said, the BSP-MC clearly does a very good job in increasing revenue. It should, however, be noted that the large revenue increases reported may be a virtue of the dataset, i.e. because of the gap between the bids posted by the most frequent participants, rather than a virtue of the algorithm.

# Chapter 5

## Discussion and conclusion

Perhaps the most interesting result is the lack of positive correlation in bids from brand and performance advertisers. First of all, it undermines the justification for implementing a modified second-price auction on behalf of the advertisers. If there is no positive correlation, there is no adverse selection in the AdX. Hence, neither is there any justification for the changes in allocation with respect to fairness or efficiency. Secondly, this means that the Vickrey auction already achieves an efficient outcome in our RTB setting, with no adverse selection. More specifically, this should mean that the Vickrey auction actually achieves the same allocative efficiency as the aforementioned OMN mechanism. Then, the only reason for implementing boost values is to increase the publishers revenue.

If there is no adverse selection, such that the Vickrey auction achieves an efficient outcome on par with the OMN mechanism, any allocative changes imposed by the BSP auction should make the outcome less efficient. Hence, implementing boost values actually represents a trade-off between allocative efficiency and revenue; the greater the positive impact on revenue is, the less efficient the allocation will be. This is only true for the seller. In this case, it would seem that the seller is actually the only winner from implementing a BSP, or even an MSP, auction. The question is then how the seller can manage the trade-off between

revenue and allocative efficiency, i.e. how to increase revenue to a point where the changes in allocation are acceptable for the bidders.

Given that it's even possible to find such a balance, the boosting procedure would have to be improved to encode a natural constraint on the allocative changes. This is complicated and definitely outside the scope of a Bachelor's thesis. It doesn't seem like Golrezaei et al. (2017) incorporate such a constraint in their method either. At this point, it is reasonable to question the usefulness of a BSP auction in our case; it seems to only benefit the seller, and trying to reduce the adverse effects from implementing it is complicated. Considering these aspects of implementing the BSP, it seems like we could perhaps get a better outcome by considering a different approach.

Due to the extreme changes in allocation, the revenue increases from implementing the BSP-MC algorithm are obviously unrealistic. We assume that a more conservative boosting algorithm could achieve a more modest increase in revenue, along with a more modest change in the allocation of impressions. However, this still has the drawback of not being efficient in terms of not awarding the impression to the bidder with the highest valuation, and since there is no adverse selection, this is a less appealing outcome than the Vickrey auction with respect to overall allocation. Hence, the natural question is if there is a mechanism that can achieve a modest increase in revenue while still retaining the efficiency of the Vickrey auction.

We already have a lot of information on the bidders' value distributions and their expected willingness-to-pay. Thus, we could use this information to compute and implement optimal floor prices to extract surpluses from the bidders. This is, of course, also dependent on the disposition of bidders and their valuations. However, if the disposition of bidders is such that it is possible for the seller to capture some of the bidders' surpluses with a floor price, we can increase revenue while still retaining the efficiency and truthfulness of the Vickrey auction.

This is an example of a so-called *Bayesian-optimal mechanism*. As with the BSP auction, this is possible due to the information we have on the bidders' valuations.

In conclusion, implementing the BSP auction is problematic in our case. First of all, the disposition of bidders requires a more complete algorithm which incorporates constraints on the possible allocative changes resulting from the implementation. Secondly, even if such an algorithm was available, it's not clear what the justification for it would be, other than increasing the revenue of the seller. There doesn't seem to be reason to consider the existence of adverse selection. Third, if there is no adverse selection, it seems a Bayesian-optimal mechanism employing data-driven floor prices could achieve a better overall outcome than both the Vickrey auction and the BSP auction by possibly increasing revenue while still retaining the efficiency and truthfulness of the Vickrey auction. That is, we would achieve the same allocative outcome as the OMN mechanism, but with higher revenue. However, in order to get a better understanding of the nature of RTB auctions in terms of adverse selection and the possibility of implementing other auction mechanisms than the Vickrey auction, we would have to look at more cases and, e.g., verify the lack of adverse selection across auctions from a wide selection of different publishers.



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# Appendix A

## Pseudocode

### Algorithm - BSP-MC

Estimate  $\rho_i$  and  $\bar{F}_i^{-1}(\cdot)$  for  $i = 1, 2, \dots, n$   
Set  $\beta_i^{(0)} = 1$  for  $i = 1, 2, \dots, n$   
**for**  $k = 1$  to  $K$  **do**  
  **for**  $i = 1$  to  $n$  **do**  
    Sample  $u_i \sim \text{Uniform}(0, 1)$   
    **if**  $u_i \leq \rho_i$  **then**  
      Sample  $v_i \sim \text{Uniform}(0, 1)$  and compute  $b_i = \bar{F}_i^{-1}(v_i)$   
    **else**  
       $b_i = 0$   
    **end if**  
  **end for**  
  Set winner =  $\arg \max_i \beta_i^{(k-1)} b_i$   
  **for**  $b_i > 0$  **do**  
    **if**  $i = \text{winner}$  **then**  
       $j = \arg \max_{t \neq i} \beta_t^{(k-1)} b_t$   
       $\beta_i^* = \beta_j^{(k-1)} b_j / b_i$   
    **else**  
       $j = \text{winner}$   
       $\beta_i^* = \beta_j^{(k-1)} b_j / b_i$   
    **end if**  
     $\beta_i^{(k)} = \beta_i^{(k-1)} - \alpha \times (\beta_i^{(k-1)} - \beta_i^*)$   
  **end for**  
   $\beta_i^{(k)} = \beta_i^{(k)} / \beta_{\min}$  for  $i = 1, 2, \dots, n$   
**end for**