

# Volatility of Bitcoin in a European Context

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# Abstract

In 2009, Bitcoin was introduced to the world. Today, ten years later, there are still gaps in the research of how to model the cryptocurrency. In this thesis, the capacities of different volatility models to capture the high volatility of Bitcoin returns are investigated. The models used are GARCH-type models: GARCH(1,1), IGARCH(1,1) and GJR-GARCH(1,1). Jumps, or outliers, are also detected and filtered using two different methods to see how these observations affect the suitability of the models. The volatility of Bitcoin is modelled with other explanatory variables, namely the biggest stock market indices in Europe. The reason for this is to investigate if the volatility of the chosen indices can explain the volatility of Bitcoin. If so, the stock market indices may be used as a tool for forecasting the development of Bitcoin. The volatility of the stock market indices is also modelled with Bitcoin as an explanatory variable. The aim of this analysis is to examine if Bitcoin can be used as a tool for forecasting stock market events. After investigation, there are however no strong evidence that the volatility of Bitcoin can be explained by the volatility of the indices or vice versa.

Another conclusion is that the model assumptions are not completely fulfilled regardless of the chosen GARCH-type model when Bitcoin return is used as response variable. The innovations are autocorrelated and seem to follow a Laplace distribution. The modelling of Bitcoin volatility using GARCH-type models must therefore be supplemented, and one possible way of doing this is to filter the jumps. However, neither of the two methods used to detect and filter the jumps are fully satisfying, since, for example, the innovations still are autocorrelated after fitting GARCH-type models to the filtered Bitcoin returns.

**Keywords:** GARCH, IGARCH, GRJ-GARCH, Jumps, Bitcoin, cryptocurrency, European market, Laplace distribution

# 1. Introduction and background

Bitcoin was introduced to the world in 2009 – created by an unknown person with the pseudonym Satoshi Nakamoto. Since then, several hundred cryptocurrencies have been developed, although Bitcoin is the most widely used of them all. Money is still continuously flowing in to Bitcoin and more uses have emerged. It seems like Bitcoin is here to stay (Marr, 2017).

As for the market of Bitcoin, it is rather dispersed as top 1000 Bitcoin holders only account for 40 percent of the market. The world market of Bitcoin is led by Japan and the US, followed by China and Europe (Jones, 2017; Khariv, 2017).

Since 2009, Bitcoin returns have grown in an incredible rate. From March 2010 to December 2017, Bitcoin rallied from less than \$0.01 to more than \$20,000 per token. This is a development which has never been seen for any stock return (Williams, 2018). The returns of Bitcoin have, however, not grown continuously. Even though Bitcoin returns are more stable than the returns of other cryptocurrencies, Bitcoin's weekly volatility reached up to 60 % per annum during the previous years (CryptoCompare, 2018). In 2017, Bitcoin dipped by at least 30 % in five separate occasions and in 2013, Bitcoin went through a price fall of 70 %. In comparison to traditional assets, Bitcoin is much more volatile (Williams, 2018). The largest price falls of Bitcoin have coincided with for example rumours about the cryptocurrency being hacked (in 2014) and rumours about the Chinese government planning to ban trading with cryptocurrencies (in 2017) (Roberts, 2017).

Recently, discussions have been going on among some investors on the web. They are talking about the possibility of using Bitcoin as a tool for forecasting what is going to happen on the stock market. But is there actually a relationship between returns and volatility of Bitcoin and the returns and volatility on the stock market? If so, it may be possible to predict the movements on the stock market by looking at Bitcoin returns. Or is the relationship actually the opposite?

For a statistician, the volatility of Bitcoin seems like an interesting, and maybe challenging, concept to model. Are the widely used ARCH and GARCH models appropriate when modelling Bitcoin? Or does the high volatility require other approaches? Another interesting question is if there exists a relationship between the Bitcoin volatility and the volatility on the stock market. This thesis is an attempt to answer these questions.

## 1.1 Purpose and problems

The purpose of the study is to investigate how models from the field of financial statistics work when analysing the extreme volatility of Bitcoin returns. To explore this, the volatility of Bitcoin is modelled with some chosen European stock market indices as explanatory variables. The following question works as an instrument when modelling Bitcoin volatility:

*Can volatility of the chosen European stock indices explain the volatility of Bitcoin?*

Due to the recent discussion of Bitcoin as a tool for forecasting the movements on the stock market, the opposite relation is also further examined. In this context:

*Can volatility of Bitcoin explain the volatility of the chosen European indices?*

Our aim of the thesis is thus to investigate how to model Bitcoin volatility and simultaneously give reasonable answers to the empirical questions stated above. As a result, the knowledge about both how to explain and how to model the high volatility of Bitcoin will hopefully increase. Additionally, our conclusions from the thesis may perhaps be useful also when modelling other highly volatile time series.

## 1.2 Literature review

This literature review is only considering the discussion of a possible relationship between Bitcoin returns and the returns on the stock market. As for the concept of volatility and possible models in previous research, this subject is further discussed with a literature review in the model building section of the thesis.

To discuss the relationship between stock market returns and Bitcoin returns seems to have become more and more popular recently, as mentioned above. Although there have not been carried out so many academic works, many newspapers and websites are discussing the subject frequently. In these articles, there seem to be different views on how big the impact and predictive power of Bitcoin returns is in forecasting the movements on the stock market. Some of these articles are discussed below, along with information about the writers.

Investopedia, a website that focuses on investing and finance education and analysis, published an article on the subject in March 2018. The writer, Mark Kolakowski, has a master's in business administration and a long education and work experience. Kolakowski presents a hypothesis in the article that Bitcoin and the stock market seem to follow each other's movements when sentiments and emotions have a large impact on the financial market. When investors for some reason abandon Bitcoin, there will be a downturn also in asset returns. But the relationship is also challenged in the same article. Stock returns are said to depend mostly upon, for example, the economy and inflation, and these are aspects that Bitcoin investors pay no attention to (Kolakowski, 2018).

Cointelegraph, a website covering cryptocurrency and blockchain, published an article in February 2018, written by the journalist Darryn Pollock. He highlights that Bitcoin returns seem to follow the returns of the great American indices S&P 500 and Dow Jones too good to be just a coincidence. It is discussed that fear among the investors is not isolated to the stock market. But this article does also challenge the relationship later on as it is quoted "the connection between the two is really, really limited" (Pollock, 2018).

There are a lot of speculative articles of this kind<sup>1</sup>. Many of them are comparing Bitcoin with the two big American indices S&P 500 and Dow Jones. The biggest European indices are not considered though, and surprisingly little is heard from the academic world regarding

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<sup>1</sup> See for example Manalo (2018) and Landsman (2018).

comparisons of this kind. It seems to exist only one exception. A paper from 2016, written by Dyhrberg, examines the relationship between Bitcoin volatility and volatility of the FTSE index, which is an index of 100 companies listed on the London Stock Exchange. The model used is a GARCH(1,1), among others. Notable is that this comparison is not the main objective of the paper, as the author's goal is to examine the relationship between Bitcoin, gold and the dollar. Her approach is however to include explanatory variables when modelling Bitcoin volatility (see Dyhrberg, 2016, p. 90), and one of the conclusions is that a positive volatility shock to the FTSE index decreases the volatility of Bitcoin returns. Thus, it seems like Bitcoin has some hedging capabilities in times when the stock market is volatile, at least in the UK (Dyhrberg, 2016, p. 91). The opposite relation, that is if Bitcoin volatility can explain the FTSE index' volatility, is not examined. The paper of Dyhrberg (2016) has however been an inspiration in how to approach the empirical questions stated above and is further discussed in the model building part of the thesis (Section 2.4.1 and 2.4.3).

## 2. Methodology and calculations

### 2.1 Data sources

The dataset contains two types of information. First, data about stock market prices for the chosen European market indices, namely UK, France, Germany, Italy, Belgium, Spain, Norway and Ireland, have been collected. The mentioned countries have great economic power in Europe and are therefore of special interest. The dataset consists of each country's greatest stock market index (for example FTSE 100 in UK and CAC 40 in France) and is available from Investing.com's on-line database. Second, the price of Bitcoin is necessary for the aims of the thesis. This dataset can be found on coindesk.com.

A timespan from January 2011 to May 2018 is used. Daily observations are available for both the stock market indices and the price of Bitcoin.

### 2.2 Variable transformations

First, it is important to note that this thesis is based upon the returns on stock market indices and Bitcoin rather than the prices of them. This is a common approach to many problems of interest in finance, for example because it can redress problems related to stationarity (see Section 2.3). The simple return is defined as

$$R_t = \frac{p_t - p_{t-1}}{p_{t-1}} \quad (1)$$

where  $R_t$  denotes the simple return at time  $t$  and  $p_t$  denotes, for example, the price of the stock market indices at time  $t$ . The advantages of using returns prior to prices are for example that returns are unit-free, i.e. one can compare changes in percentages of the stock market indices rather than just look at changes that are dependent of the different scaling for various indices (Brooks, 2008, p. 7; Tsay, 2010, p. 3).

A related approach is to use a log-transformation of the prices. This log-return formulation, that is used in this thesis, is defined as

$$r_t = \ln(p_t) - \ln(p_{t-1}) = \ln\left(\frac{p_t}{p_{t-1}}\right) \quad (2)$$

where  $r_t$  denotes the log-return at time  $t$  and  $p_t$  again, for example, denotes the price of the stock market indices at time  $t$ . The log-returns are often interpreted as continuous compounded returns. There are two main reasons for using continuous compounded returns instead of simple returns. First, the frequency of compounding of the return does not matter when calculating the log-returns. This makes the different indices more comparable. Second, continuous compounded returns are time-additive (Brooks, 2008, p. 8).

From coindesk.com, the Bitcoin prices are available in US dollars. To be able to compare Bitcoin log-returns with the log-returns of the stock market indices from different European countries, a transformation of Bitcoin prices to pound and euro is needed. This is done by dividing the Bitcoin price in USD by the exchange rate of pound and euro.

## 2.3 Unit root test

The concept of stationarity is important in the field of time series analysis. In this thesis, the definition of weak stationarity is used, i.e. a stationary series is a series with constant mean, constant variance and constant autocovariance for each given lag (Brooks, 2008, p. 318). There are several reasons for the importance of the concept. Whether a series is stationary or not influences for example its behaviour and properties and therefore the possibilities of forecasting. For a non-stationary series, the use of for example t-statistics or F-statistics lead to non-valid results (Brooks, 2008, p. 318-320). Another problem with non-stationary series is spurious regression. This can occur when one non-stationary variable is regressed on another non-stationary variable. If the variables are trending over time, a regression of one on the other can have a high  $R^2$  value and significant slope coefficient even though the two are totally unrelated (Brooks, 2008, p. 319).

Thus, it is necessary to test whether a series is stationary or not. For this purpose, one can use the Dickey-Fuller test. The hypotheses are verbally formulated as:

$H_0$ :The series contains a unit root

$H_1$ :The series is stationary

Mathematically, one can express the null hypothesis as  $\phi = 1$  in the simple case

$$r_t = \phi r_{t-1} + u_t \quad (3)$$

i.e. an AR(1)-process where  $u_t$  is the innovation at time  $t$  and  $r_t$  is the log-return at time  $t$ . The one-sided alternative hypothesis is in this case  $\phi < 1$  (Brooks, 2008, p. 327; Cryer and Chan, 2008, p. 128-129).

Another possibility is to use an Augmented Dickey-Fuller test. This form of test is used when the error terms might be autocorrelated. The solution is to consider  $p$  lags of the dependent variable  $\Delta r_t$  which leads to the model

$$\Delta r_t = \psi r_{t-1} + \sum_{i=1}^p \alpha_i \Delta r_{t-1} + u_t. \quad (4)$$

If there is a dynamic structure in the dependent variable, this is soaked up by the lags of  $\Delta r_t$  and, as a result,  $u_t$  is not autocorrelated. The ADF-test statistics is defined as

$$ADF = \frac{\hat{\psi}}{SE(\hat{\psi})}. \quad (5)$$

The test statistics follow a specific distribution known as the Dickey-Fuller table. When the null hypothesis is rejected, the series is assumed to be stationary (Brooks, 2008, p. 329; Tsay, 2010, p. 77).

Usually, non-stationary series can be transformed into stationary ones by taking first differences, or by different types of transformations (Cryer and Chan, 2008, p. 87 ff.). Stock market prices and Bitcoin prices are all tested for stationarity and so are the log-returns of both the stock market and Bitcoin.

## 2.4 Model building

The concept of cryptocurrencies is fairly new and the research about Bitcoin volatility is therefore limited. In this section of the thesis, there is a presentation of the limited number of models proposed in previous research. These models are described, as well as two different jump detection methods. After that, a section about how the models can be utilized to answer the empirical questions follows. A discussion about model validation ends this section.

### 2.4.1 Possible models

Two of the most widely used models when analysing volatility is ARCH and GARCH. These models are often used for modelling financial data containing volatility clusters (Brooks, 2008, p. 386-387; Tsay, 2010, p. 111).

The autoregressive conditional heteroskedasticity (ARCH) model is commonly used in the field of financial statistics. The important issue is to be able to model the above-mentioned volatility clusters, and the ARCH model does this by using conditional variance of the error term. This conditional variance can be defined as

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 \tag{6}$$

where  $\sigma_t^2$  is the conditional variance,  $u_t$  is the innovation,  $\omega$  is a constant and  $\alpha$  is a parameter that needs to be estimated. Equation (6) above is an example of an ARCH(1) model since the conditional variance depends on only one lagged square error. However, the conditional variance can easily be extended to a general case:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \dots + \alpha_q u_{t-q}^2 \tag{7}$$

i.e. an ARCH(q) model. When testing for ARCH-effects, the joint null hypothesis is that all q lags of the squared innovations have parameter  $\alpha$ :s that are not significant different from zero (Brooks, 2008, p. 389-390).

A generalised ARCH (GARCH) model is even more popular than the ARCH model, mainly because it allows the conditional variance to be dependent upon previous own lags. This model is therefore one of the models chosen to analyse the volatility in the dataset. The conditional variance equation can be defined as

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2. \tag{8}$$

The difference from (6) is the last term, which makes the conditional variance dependent upon previous own lags. Equation (8) is the conditional variance equation for a GARCH(1,1) model. In the general case, the notation is GARCH(p,q) (Brooks, 2008, p. 392-394).

Note that the conditional variance equations above only are one part of the ARCH and GARCH models. It is necessary to formulate a conditional mean equation as well. This equation describes how the dependent variable  $r_t$  varies over time. An example of a conditional mean equation is

$$r_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t \quad (9)$$

where  $u_t \sim N(0, \sigma_t^2)$ . Here,  $u_t$  is the innovation (Brooks, 2008, p. 388).

In the context of Bitcoin volatility, the GARCH(1,1) model has been used in some scientific papers. First of all, Dyhrberg, discussed in the beginning of this thesis, uses a GARCH(1,1) to investigate the similarities between Bitcoin, gold and dollar. In her paper, Bitcoin return is used as the response variable and the FTSE index is one of the explanatory variables used. Other explanatory variables are for example the dollar-euro exchange rate and the price of gold. Her conclusion is that Bitcoin has similarities both to dollar and gold. Bitcoin seems, for example, to possess hedging capabilities similar to that found for gold. Also, a positive volatility shock to the FTSE index seems to decrease the volatility of Bitcoin returns (Dyhrberg, 2016, p. 82, 91).

Other papers expand this approach. Chu *et al.* (2017) tries several GARCH-type models where the simplest one is a standard GARCH(1,1) model. Their aim is to investigate how to model volatility for the seven most popular cryptocurrencies, where Bitcoin is included. The conclusion is that an IGARCH(1,1) model is the best when modelling Bitcoin volatility. The authors reach this conclusion after comparing information criteria for different models (Chu *et al.*, 2017, p. 9).

An IGARCH is an integrated GARCH model. Recall the conditional variance equation for a GARCH(1,1) model (8). In an IGARCH,

$$\alpha + \beta = 1 \quad (10)$$

which results in a nonstationary GARCH(1,1), or a nonstationary variance process (Cryer and Chan, 2008, p. 297). Thus, the variance forecast from an IGARCH model does not converge to the long-term mean value of the variance, as is the case for a standard GARCH model (Engle and Bollerslev, 1986, p. 26). The conditional variance can in this case be expressed as follows:

$$\sigma_t^2 = \omega + (1 - \beta)u_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (11)$$

which is a standard IGARCH(1,1) model. This model resembles a model with a random walk with a drift in the mean, but notable is that the drift is in this case in the conditional variance equation (Engle and Bollerslev, 1986, p. 28).

In summary, an IGARCH model is suitable when modelling Bitcoin volatility according to Chu *et al.* (2017). However, another paper, written by Azzi *et al.* (2017), uses another type of GARCH model, which is an asymmetric GARCH. The authors investigate whether Bitcoin can be considered as a valuable asset in downturn periods. With an asymmetric GARCH

model, the authors examine the volatility of Bitcoin returns before and after the Bitcoin price crash of 2013. The findings are that positive shocks to Bitcoin returns were positively correlated with shocks to Bitcoin volatility prior to the price crash of 2013, which is contrary to that found in equities. It seems like if Bitcoin prices increase in periods of economic turmoil, during which stock market prices fall, investors purchase Bitcoin and transmit the increased uncertainty and volatility of the stock market to the Bitcoin market. The conclusion is that Bitcoin was an effective investment tool prior to its crash in 2013 (Azzi *et al.*, 2017, p. 9).

The model used in this study is asymmetric GARCH, also known as GJR-GARCH (named after Glosten, Jagannathan and Runkle (1993)). This model is also used by Chu *et al.* (2017). As mentioned above, the authors conclude that an IGARCH-model is best when modelling Bitcoin volatility, although other cryptocurrencies are best modelled with GJR-GARCH models (Chu *et al.*, 2017, p. 9). The conditional variance of a GJR-GARCH(1,1) is

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}. \quad (12)$$

The GJR-GARCH model allows positive and negative shocks to have different influence on the volatility. If the parameter  $\gamma$  is positive and significant, negative shocks generate more volatility than positive shocks and vice versa. When  $\gamma=0$ , the model reduces to a GARCH(1,1) model (Azzi *et al.*, 2017, p. 6). Note that  $I_{t-1}$  in (12) is an indicator function defined as

$$I_{t-1} = I(u_{t-1} < 0). \quad (13)$$

The rule in (13) implies that  $I_{t-1}=1$  when  $u_{t-1} < 0$  and 0 otherwise (Amado and Teräsvirta, 2014, p. 16-17).

To conclude this section, there are many GARCH-type models that could have been used. The ones described above have however been shown to work best when modelling Bitcoin volatility in previous research when considering different information criteria. This is the reason why, for example, the AP-ARCH model and E-GARCH model are not considered in this thesis. Higher model orders are not deeper investigated for the same reason.

## 2.4.2 Jump detection

The final issue to discuss is jump detection and filtration. This subject is discussed in a paper of Charles and Darné (2019). The authors fit several GARCH-type models to Bitcoin returns, where the simplest one is a GARCH(1,1) (see Charles and Darné, 2019, p. 26). The aim of their study is to model volatility of Bitcoin returns. Additionally, the authors propose that in order to correctly model Bitcoin volatility, one must take into account the volatility jumps, or drastic shocks, that occurs. Neglecting the jumps can lead to overestimation of the volatility during several days after their occurrence (Laurent *et al.*, 2016, p. 383).

Their approach is as follows: first, detect the jumps and filter them, and after that the models can be fitted to the adjusted dataset. The detection and filtration of jumps is important in order to get correct estimates of the parameters in the GARCH-type models (Charles and Darné, 2019, p. 31). Different approaches can be used to detect and filter the jumps, and two of these are discussed in this section of the thesis.

*Method one: constant estimates of mean and standard deviation*

In the first approach, the daily changes in Bitcoin log-return are considered. This method was created by us as an initial step in the investigation of how to find the jumps and filtrate them.

To decide how big the changes must be to be classified as a jump, a standardization is done, that is:

$$\tilde{J}_t = \frac{d_t^* - \bar{d}^*}{s}. \quad (14)$$

Note that  $\tilde{J}_t$  is the standardized daily changes in log-returns and  $d_t^*$  is the daily changes in observed log-returns with jumps included,

$$d_t^* = d_t + a_t I_t. \quad (15)$$

Here,  $d_t$  is the daily changes in log-returns without jumps and  $a_t I_t$  is a jump component that needs to be filtered.

If  $\tilde{J}_t$  is too large, compared to a critical value,  $k$ , it must reflect a jump (Laurent *et al.*, 2016, p. 386). When deciding a critical value, one must for example take into account the number of observations in the dataset. This subject is further discussed in Section 3.3.2 of the thesis.

This first approach is to use a constant mean and a constant standard deviation in the standardization procedure (14). That is,

$$\bar{d}^* = \frac{\sum_{t=1}^n d_t^*}{n} \quad (16)$$

$$s = \sqrt{\frac{\sum_{t=1}^n (d_t^* - \bar{d}^*)^2}{n}}. \quad (17)$$

The detected jumps are then filtered using the following formula:

$$\tilde{r}_t = r_t^* - (r_t^* - \bar{r}^*) \tilde{I}_t \quad (18)$$

where  $r_t^*$  is the observed log-returns,  $\tilde{r}_t$  is the filtered log-returns and  $\tilde{I}_t$  is an indicator function defined as

$$\tilde{I}_t = I(|\tilde{J}_t| > k). \quad (19)$$

The rule in (19) implies that  $\tilde{I}_t = 1$  when a jump is detected at observation  $t$  and  $\tilde{I}_t = 0$  otherwise (Laurent *et al.*, 2016, p. 385). Note that because the standardization of daily changes is considered when identifying the jumps, the filtration is performed on the observation following the jump. An alternative would be to filtrate the daily changes in log-returns in order to get  $d_t$ , i.e. the daily changes in log-returns without jumps.

The algorithm for finding potential jumps using this first method is presented below.

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**Algorithm 1. Detection and filtration of jumps using the first method**

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1. The daily changes in Bitcoin log-returns are calculated by taking the difference between the observation at time  $t$  and the observation at time  $t-1$ .
  2. The mean of the daily changes in Bitcoin log-returns is calculated using (16).
  3. The standard deviation of the daily changes in Bitcoin log-returns is calculated using (17).
  4. The standardized value,  $\tilde{J}_t$ , is calculated for every time point using (14).
  5. For the standardized values exceeding the absolute critical value  $k$ , the observation following the jump is set to the mean of Bitcoin log-returns using (18).
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One possible drawback of this approach is that the jumps actually have an influence on the estimates of  $\bar{d}^*$  and  $s$ . That is, distant jumps influence the estimations of the mean and the standard deviation in the entire series. One can therefore expect that the jumps cause the estimates of the mean and standard deviation to be biased. Robust estimates, that are not affected by potential jumps, are therefore required. The second approach takes this problem into account.

*Method two: time varying estimates of mean and standard deviation*

The method was presented in 2016 by Laurent *et al.* and is also used by Charles and Darné in the paper discussed above (see Charles and Darné, 2019, p. 27, 29, 31).

The following equations have some similarities with (14) and (15) above. Note however that the log-returns are used instead of the daily changes in log-returns and that the mean and standard deviation are time varying.

The returns, with jumps included, can be defined as:

$$r_t^* = r_t + a_t I_t \tag{20}$$

where  $r_t^*$  is the observed log-returns and  $a_t I_t$  again is a jump component that needs to be filtered.

To detect the jumps one can use their standardized values, that is

$$\tilde{J}_t = \frac{r_t^* - \tilde{\mu}_t}{\tilde{\sigma}_t} \tag{21}$$

When a suitable critical value,  $k$ , has been selected, one can then filter the detected jumps from  $r_t^*$  as follows:

$$\tilde{r}_t = r_t^* - (r_t^* - \tilde{\mu}_t) \tilde{I}_t \tag{22}$$

Here,  $\tilde{r}_t$  is thus the filtered log-returns. What is left to estimate is  $\tilde{\mu}_t$  and  $\tilde{\sigma}_t$ . As for  $\tilde{\mu}_t$ , it can, according to Laurent *et al.* (2016, p. 386), be estimated from the following formula:

$$\tilde{\mu}_t = \mu + \sum_{i=1}^{\infty} \lambda_i \tilde{\sigma}_{t-i} w_{z_\delta}(\tilde{J}_{t-i}), \quad (23)$$

where  $w_{z_\delta}(\tilde{J}_{t-i})$  is a weight function needed to down weight the effect of past values of  $r_t^*$ ,  $\tilde{\sigma}_{t-i}$  is defined below and  $\lambda_i$  are the coefficients from a MA( $\infty$ ) representation of the ARMA process<sup>2</sup> (Muler *et al.*, 2009, p. 818). The weight function can more specifically be expressed as:

$$w_{z_\delta}(u) = \text{sign}(u) \min(|u|, z_\delta). \quad (24)$$

Here,  $\delta$  is a quantile of a standard normal distribution, commonly chosen to be 0.975. Thus, if the absolute values of the standardized Bitcoin returns exceed 1.96, the value of 1.96 is used instead of the true value of the standardized return in the summation in (23). In this way,  $\tilde{\mu}_t$  is robust to potential presence of jumps (Laurent *et al.*, 2016, p. 386), i.e. distant jumps do not influence the estimation of the mean at time  $t$ . Note that (24) plays a key role in the robustification of the ARMA model.

Also, the estimation of the standard deviation must be robust to potential presence of jumps (Charles and Darné, 2019, p. 29). For this purpose, the following function can be used:

$$\tilde{\sigma}_t^2 = \omega + \alpha \tilde{\sigma}_{t-1}^2 c_\delta w_{z_\delta}(\tilde{J}_{t-1})^2 + \beta \tilde{\sigma}_{t-1}^2. \quad (25)$$

Again, if the standardized returns are expected to be jumps, their effect on the variance equation are limited by the weight function. Note that  $c_\delta$  in (25) is a correction factor recommended to ensure that returns that are not affected by jumps are not down weighted (Laurent *et al.*, 2016, p. 386; Boudt *et al.*, 2013, p. 246). A higher value of  $c_\delta$  results in a smaller number of detected jumps. Following the recommendations of Laurent *et al.*, this value is set to 1.0953. Note that if the weight function is ignored, a jump has a large and slowly decaying effect on future volatility predictions.

As the theory behind the estimations of  $\tilde{\mu}_t$  and  $\tilde{\sigma}_t^2$  is rather complicated, the interested reader can learn more about these robust estimators in the papers of Laurent *et al.* (2016), Muler and Yohai (2008), and Muler *et al.* (2009).

The algorithm for finding potential jumps using this second method is presented below.

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<sup>2</sup> Recall that the GARCH-type models must contain a mean equation. In this thesis (following for example Laurent *et al.* (2016)), this mean equation is modelled as an ARMA(1,1) (see also Section 2.4.3).

## Algorithm 2. Detection and filtration of jumps using the second method

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1. An ARMA(1,1) model of the logarithmic Bitcoin returns is estimated. The model is represented as an MA( $\infty$ ) model and used to estimate  $\tilde{\mu}_t$ , see (23).
  2. A GARCH(1,1) model of the logarithmic Bitcoin returns is estimated. The parameters  $\omega$ ,  $\alpha$ , and  $\beta$  is used to estimate  $\tilde{\sigma}_t^2$  as in (25) (Boudt *et al.*, 2013, p. 246).
  3. The standardized value,  $\tilde{J}_t$ , is calculated using (21).
  4. If the absolute value of  $\tilde{J}_{t-1}$  exceeds 1.96, the value of 1.96 is used instead of  $\tilde{J}_{t-1}$  in the summation to estimate  $\tilde{\mu}_t$  in (23). If the absolute value of  $\tilde{J}_{t-1}$  does not exceed 1.96, the value of  $\tilde{J}_{t-1}$  is used in the summation to estimate  $\tilde{\mu}_t$  in (23).
  5. If the absolute value of  $\tilde{J}_{t-1}$  exceeds 1.96, the value of 1.96 is used instead of  $\tilde{J}_{t-1}$  to estimate  $\tilde{\sigma}_t^2$  in (25). If the absolute value of  $\tilde{J}_{t-1}$  does not exceed 1.96, the value of  $\tilde{J}_{t-1}$  is used to estimate  $\tilde{\sigma}_t^2$  in (25).
  6. Step 3-5 is repeated for every time point.
  7. The values of Bitcoin returns with standardized values exceeding the critical value  $k$  is filtered using (22).
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### 2.4.3 Utilisation of the models

The next step is to present how the GARCH-type models are utilized in order to be able to answer the empirical questions stated in Section 1.1. Here, the paper of Dyrberg written in 2016 is an inspiration. Recall that Dyrberg includes explanatory variables to the GARCH-type models. As an example, the FTSE index is used as an explanatory variable when Bitcoin volatility is modelled (Dyrberg, 2016, p. 90), but the opposite relation is not considered.

In this thesis however, every model is used twice. First, the influence of Bitcoin volatility on the stock market volatility is investigated (i.e. Bitcoin returns are used as an explanatory variable) and second, the influence of stock market volatility on Bitcoin volatility is considered (i.e. Bitcoin returns are used as a response variable). Note the conditional variance equation for every model ((27), (29), (31), (33), (35), and (37)) can be used to explore Bitcoin volatility.

For the GARCH(1,1) model, the following conditional mean and variance equations for stock market returns are utilized:

$$r_t = \mu + \phi r_{t-1} + \theta u_{t-1} + B_1 R_{Bt} \quad (26)$$

$$\sigma_t^2 = (\omega + \alpha u_{t-1}^2 + \beta_0 \sigma_{t-1}^2) + B_2 R_{Bt}. \quad (27)$$

The conditional mean equation is, following for example Laurent *et al.* (2016), modelled as an ARMA(1,1) where  $B_1$  is the effect of Bitcoin returns on mean stock market returns. The significance of the parameter  $B_1$  for every model is discussed in Section 3. As for the

conditional variance part (26), the parenthesis includes standard GARCH variables and  $B_2$  is the effect of Bitcoin volatility on stock market volatility. Thus, the parameter  $B_2$  shows how the stock market reacts to a positive shock in Bitcoin volatility. This parameter is also considered in Section 3.

As mentioned above, the model is used twice. In the second case, the conditional mean and variance equations for Bitcoin returns are modelled as:

$$r_t = \mu + \phi r_{t-1} + \theta u_{t-1} + B_1 R_{It} \quad (28)$$

$$\sigma_t^2 = (\omega + \alpha u_{t-1}^2 + \beta_0 \sigma_{t-1}^2) + B_2 R_{It}. \quad (29)$$

Now,  $B_1$  is the effect of stock market returns on mean Bitcoin returns and  $B_2$  is the effect of stock market volatility on Bitcoin volatility.

For the IGARCH(1,1) model, the following conditional mean and variance equations for stock market returns are utilized:

$$r_t = \mu + \phi r_{t-1} + \theta u_{t-1} + B_1 R_{Bt} \quad (30)$$

$$\sigma_t^2 = \omega + (1-\beta)u_{t-1}^2 + \beta\sigma_{t-1}^2 + B_2 R_{Bt}. \quad (31)$$

For the Bitcoin returns, the equations look as follows:

$$r_t = \mu + \phi r_{t-1} + \theta u_{t-1} + B_1 R_{It} \quad (32)$$

$$\sigma_t^2 = \omega + (1-\beta)u_{t-1}^2 + \beta_0 \sigma_{t-1}^2 + B_2 R_{It}. \quad (33)$$

For the GJR-GARCH(1,1) model, the conditional mean and conditional variance equations for stock market returns are:

$$r_t = \mu + \phi r_{t-1} + \theta u_{t-1} + B_1 R_{Bt} \quad (34)$$

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} + B_2 R_{Bt}. \quad (35)$$

Finally, the conditional mean and variance equations for the Bitcoin returns are:

$$r_t = \mu + \phi r_{t-1} + \theta u_{t-1} + B_1 R_{It} \quad (36)$$

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} + B_2 R_{It}. \quad (37)$$

In all the above-mentioned equations, the main focus is to investigate which parameters are significantly different from zero. The parameters  $B_1$  and  $B_2$  are of special interest as this is where an answer to the empirical questions can be found. If, for example,  $B_2$  is significant in (27), we can assume that Bitcoin volatility actually explain the stock market volatility in the GARCH(1,1) model.

As for the jump detection test, the two mentioned methods are used to filter the jumps. All the above-mentioned models can then be reused with the filtered Bitcoin returns as an explanatory variable when modelling the indices and as a response variable when modelling Bitcoin returns. After this, one can conclude if the jumps actually affect the parameter estimates and conclusions from the previous model fitting procedure.

#### 2.4.4 Model validation and comparisons

This section of the thesis contains a discussion about both how one must validate each of the models and how to compare different models.

As for the model validation, two aspects are considered. First, one must check if the autocorrelation in the standardized innovations has successfully been removed. If not, the model fails to describe the structure of the innovations and the proposed model is not adequate (Brooks, 2008, p. 382). The presence of autocorrelation in the innovations can be checked using Ljung-Box test. The null hypothesis in this test is that no serial correlation exists. The test value is attained from

$$Q_k = T(T+2) \sum_{i=1}^K \frac{r_i^2}{T-i}, \quad (38)$$

where  $T$  is the sample size,  $K$  is the number of lags being tested and  $r_i$  is the  $i$ :th autocorrelation. Under the null hypothesis,  $Q_k$  is  $\chi^2$ -distributed (Brooks, 2008, p. 210).

For all the above discussed models, it is assumed that the innovations are normally distributed. This must of course be further examined. The Jarque-Bera test is suitable for this purpose, mainly because the dataset is big and other normality tests therefore tend to fail. The null hypothesis in the test is that the innovations are normally distributed, and the test value is then  $\chi^2$ -distributed. In simple terms, the test investigates if the skewness and the kurtosis of the observed distribution matches a normal distribution using the following formula:

$$JB = \frac{n}{6} \left( S^2 + \frac{(K-3)^2}{4} \right). \quad (39)$$

In the above formula,  $S$  is the sample skewness and  $K$  is the sample kurtosis. The basic idea behind the test is that a normal distribution with any mean and variance has a skewness of zero and a kurtosis of three (that is zero excess kurtosis) (Thadewald and Büning, 2007, p. 88). The normal assumption of the innovations can also be checked graphically from histograms of the innovations. If it turns out that the innovations are not normally distributed, the confidence of the parameter estimates is affected (Brooks, 2008, p. 399).

To compare models, it is common to use different versions of information criteria. In this thesis, Akaike's information criterion, Hannan-Quinn's information criterion and the Bayesian information criterion are considered. These measures all strive to find balance between goodness of fit and model complexity. For the three measures, the model with the lowest values is preferable (Brooks, 2008, p. 232-233).

## 2.5 Software

Throughout the analysis, R (R Core Team, 2016) is used. There are quite a few packages that are useful; especially 'rugarch' (Ghalanos, 2018). This package is used to evaluate different types of GARCH models. It should be noted that the parameter estimates in this package are based upon maximum likelihood.

### 3. Results

In this section, the empirical results of the analysis are presented. Daily frequencies for the period January 2011 to May 2018 are used for all the chosen stock market indices and also for Bitcoin. First, some descriptive statistics regarding both Bitcoin and the chosen stock market indices are presented to give the reader some background information. Both returns and log-returns are considered. The results from the Augmented Dickey-Fuller test follows, and after that the discussed models are implemented to the data. Parameter estimates are presented as well as the model validations. The analysis is then be repeated for the filtered data. This procedure hopefully ends up with new knowledge about, for example, how to model the extreme volatility of Bitcoin and its relationship with the volatility on the chosen stock market indices.

The results are finally summarized and a brief discussion about the findings and possible alternative approaches end the thesis.

#### 3.1 Descriptive statistics

The table below shows descriptive statistics in percent for log-returns. Note that Bitcoin had the highest mean annualised log-return and also the highest volatility based on the annualised standard deviation. To display this volatility differences graphically, time series of Bitcoin log-returns and UK index log-returns have been inserted in the Appendix (Figure 3).

**Table 1. Summary statistics of log-returns using daily data (in percent)**

|           | BTC   | UK   | FRA  | GER  | ITA   | BEL  | SPA   | NOR  | IRE   |
|-----------|-------|------|------|------|-------|------|-------|------|-------|
| Mean*     | 101.7 | 5.5  | 8.2  | 11.6 | 5.3   | 8.2  | 1.9   | 12.7 | 15.9  |
| Median*   | 73.1  | 9.7  | 13.2 | 22.6 | 16.8  | 8.8  | 13.5  | 15.3 | 15.6  |
| St dev*   | 119.1 | 14.9 | 19.9 | 19.8 | 25.5  | 16.9 | 22.6  | 18.3 | 17.7  |
| Min daily | -84.7 | -4.8 | -8.4 | -7.1 | -12.9 | -6.6 | -13.2 | -6.1 | -10.6 |
| Max daily | 147.5 | 3.9  | 6.1  | 5.2  | 6.4   | 4.5  | 5.9   | 4.5  | 5.0   |

\* Mean, median and standard deviation are annualised for interpretability reasons.

Figure 1, on the next page, shows dynamics of £1.0 invested into each stock market and also £1.0 invested in Bitcoin.

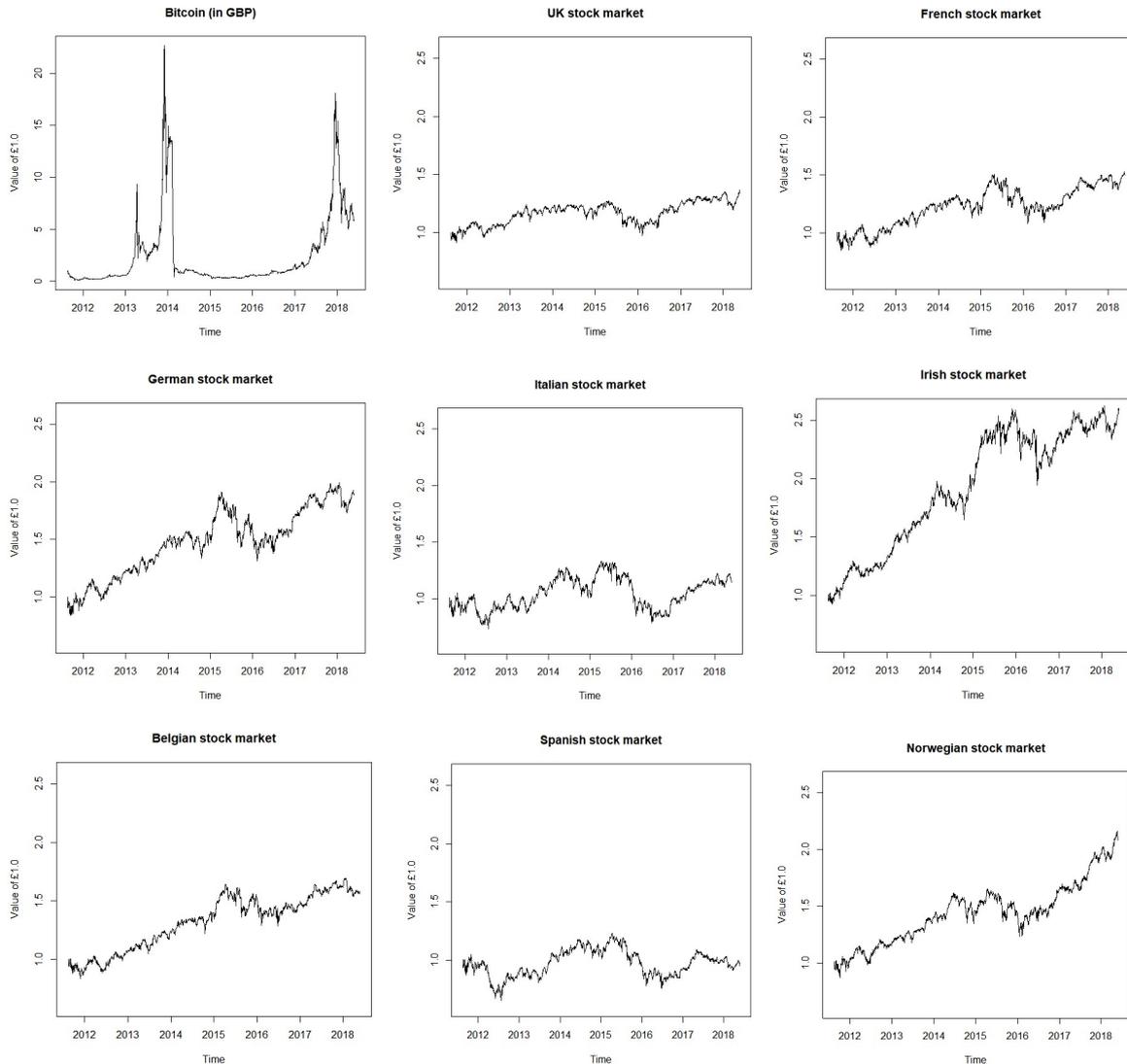


Figure 1. Daily time series plots of the considered stock market indices (value of £1.00)

It is important to note the different scaling on the y-axis for different plots. It is obvious that Bitcoin have gone through more dramatic changes than any of the stock market indices. When Bitcoin increases in value, it can increase more than 20 times the stock market indices. This has happened twice – one time in the end of 2013 and one time in the end of 2017. When Bitcoin decreases in value however, the drop can be just as deep as the previous increase. The volatility of Bitcoin is much higher than the volatility of any of the stock market index.

Among the considered stock market indices, the highest increase in values are those for Ireland, Norway and Germany. This matches with the highest annualised mean return in Table 1 above. In these countries, the value of £1.00 increased the highest among all the considered stock market indices. During the considered timespan, £1.00 increased up to £2.59 for Ireland, £2.08 for Norway, and £1.89 for Germany. The corresponding value for Bitcoin is £5.83. Among the countries with smallest volatility is UK (with the lowest standard deviation in Table 1) but also the German and French stock market indices show relatively stable growth. The Irish stock market also appears to be steadily growing.

### 3.2 Unit root tests

As stated in methodology part of the thesis, an augmented Dickey-Fuller test is used to investigate whether a time series is stationary or not. Both the prices and the log-returns are considered in the testing which yields the following table:

**Table 2. Augmented Dickey-Fuller test**

|     | Prices   | Log-returns |
|-----|----------|-------------|
| BTC | -1.743   | -40.908***  |
| UK  | -3.278*  | -39.959***  |
| FRA | -3.606** | -40.670***  |
| GER | -3.195*  | -39.133***  |
| ITA | -2.715   | -42.306***  |
| BEL | -3.343*  | -39.315***  |
| SPA | -2.496   | -39.331***  |
| NOR | -2.496   | -41.831***  |
| IRE | -2.778   | -37.763***  |

Note: Table shows Dickey-Fuller-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

Recall that the null hypothesis is that the series contains a unit root, i.e. that the series is non-stationary. Considering the log-returns, this null hypothesis is rejected for Bitcoin as well as for every stock market index. All the log-return series can therefore be assumed to be stationary. As for the prices, the daily frequencies of UK, France, Germany, and Belgium stock market indices seem stationary. In these cases, the null hypothesis of the series containing a unit root is rejected.

In summary, the transformation from prices to log-returns seems like a good idea. As the series are shown to be stationary, one can for instance avoid problems with spurious regression.

### 3.3 Volatility modelling

It is now time to implement the models built in Section 2.4.3 to the dataset. The goal of this analysis is both to evaluate the performances of the GARCH-type models and to investigate the relationship between returns and volatility of Bitcoin and the returns and volatility of the stock market indices. The section is divided into two parts. The first part displays models implemented on Bitcoin log-returns and the second part displays models implemented on filtered Bitcoin log-returns, following the two methods described earlier. The second part of the analysis thus accounts for the effect of jumps.

### 3.3.1 Modelling volatility of Bitcoin returns

ARCH- and GARCH-type models contain both a conditional mean part and a conditional variance part. In Table 3 below, the numbers in column B1 is the effect of stock market returns on mean Bitcoin returns for all the chosen indices. In column B2, the effect of stock market volatility in the variance equation is displayed. For the GARCH(1,1) model used in this thesis, the number of coefficients to be estimated is four in the mean equation and four in the variance equation (recall (26), (27), (28) and (29)). Only two of a total number of eight coefficients are thus displayed in the table below. This is because the parameters B1 and B2 from the equations in Section 2.4.3 are of special interest as this is where the relationship between the indices and Bitcoin can be investigated. Some of the tables below also contain the information criteria discussed in Section 2.4.4. The bolded numbers are the lowest and thus shows the models which seem to perform best. The values are further discussed later in the section.

Note that this thesis focuses on modelling the volatility of Bitcoin returns. The most interesting models for this purpose are thus the models with the indices as explanatory variables and Bitcoin returns as the response variable. The parameter estimates and information criteria from these models follow in Table 3, Table 4, and Table 5. Table 6, Table 7, and Table 8 display the effect of Bitcoin returns on the stock market indices. These three tables are attached to the thesis to serve as an interesting comparison to tables 3, 4 and 5. Additionally, the content in these tables is interesting due to the previous discussion of Bitcoin as a tool for forecasting the movements on the stock market. However, as the aim of the thesis is not to discuss how to model the volatility of the stock market indices, the values of the information criteria are not attached to Table 6, Table 7, and Table 8.

**Table 3. GARCH(1,1) – Index as an explanatory variable**

|     | <i>B1</i>   | <i>B2</i> | <i>AIC</i>     | <i>BIC</i>     | <i>HQIC</i>    |
|-----|-------------|-----------|----------------|----------------|----------------|
| UK  | 0.176194*   | 0.000000  | <b>-3.1285</b> | -3.1024        | <b>-3.1188</b> |
| FRA | 0.174363**  | 0.000000  | <b>-3.1391</b> | <b>-3.1131</b> | <b>-3.1295</b> |
| GER | 0.186610**  | 0.000000  | <b>-3.1395</b> | <b>-3.1134</b> | <b>-3.1298</b> |
| ITA | 0.149413*** | 0.000000  | <b>-3.1400</b> | <b>-3.1139</b> | <b>-3.1303</b> |
| BEL | 0.192404**  | 0.000000  | <b>-3.1386</b> | <b>-3.1125</b> | <b>-3.1289</b> |
| SPA | 0.081612    | 0.000000  | <b>-3.1368</b> | <b>-3.1107</b> | <b>-3.1271</b> |
| NOR | 0.145267*   | 0.000000  | <b>-3.1378</b> | <b>-3.1118</b> | <b>-3.1282</b> |
| IRE | 0.209426**  | 0.000000  | <b>-3.1393</b> | <b>-3.1132</b> | <b>-3.1296</b> |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

In column B1 in Table 3, the parameter estimates show if a positive shock to the returns of the stock market indices has a significant influence on Bitcoin returns. In the above table, all the B1 parameters (except Spain) in the mean equations are positive and significant. The null hypotheses of B1 being zero are therefore rejected in almost all cases. It seems like a

positive shock to the stock market indices may make investors more risk seeking and inclined to invest in alternative assets like Bitcoin. The conclusion follows the result from previous research (Dyhrberg, 2016, p. 90). Additionally, the values of the parameters in the variance equations (column B2) in Table 3 shows that a positive shock to the volatility of the stock market indices does not seem to affect the volatility of Bitcoin returns as all the parameters are insignificant. The null hypotheses of B2 being zero cannot be rejected. It seems like the volatility of Bitcoin returns cannot be explained by the volatility of the chosen stock market indices. This is not the same conclusion as Dyhrberg (2016, p. 91)<sup>3</sup> reaches in her paper regarding the FTSE index.

The following table displays the IGARCH(1,1) model, also with the indices as explanatory variables.

**Table 4. IGARCH(1,1) – Index as an explanatory variable**

|     | <i>B1</i>   | <i>B2</i> | <i>AIC</i> | <i>BIC</i>     | <i>HQIC</i> |
|-----|-------------|-----------|------------|----------------|-------------|
| UK  | 0.186088*   | 0.000000  | -3.1254    | <b>-3.1026</b> | -3.1169     |
| FRA | 0.178064**  | 0.000000  | -3.1351    | -3.1123        | -3.1267     |
| GER | 0.205704*** | 0.000000  | -3.1353    | -3.1125        | -3.1268     |
| ITA | 0.149116**  | 0.000000  | -3.1349    | -3.1121        | -3.1265     |
| BEL | 0.194275**  | 0.000000  | -3.1345    | -3.1117        | -3.1261     |
| SPA | 0.081404    | 0.000000  | -3.1328    | -3.1100        | -3.1243     |
| NOR | 0.148059*   | 0.000000  | -3.1338    | -3.1110        | -3.1254     |
| IRE | 0.197796**  | 0.000000  | -3.1348    | -3.1120        | -3.1264     |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

As in the GARCH(1,1) model (Table 3), the majority of the B1 parameters in Table 4 are significant. One can again suspect that a positive shock to the indices makes the investors more risk seeking which results in a positive shock also in Bitcoin returns. As for the variance equations, all the considered parameters are again estimated to be zero.

The final step is to fit the GJR-GARCH(1,1) model. In the following table, a column for  $\gamma$  is also included. Recall that if the parameter  $\gamma$  is positive and significant, negative shocks generate more volatility than positive shocks and vice versa.

<sup>3</sup> Dyhrberg instead states that a positive volatility shock to the FTSE index results in less volatility in Bitcoin returns (2016, p. 91).

**Table 5. GJR-GARCH(1,1) – Index as an explanatory variable**

|     | <i>B1</i>  | <i>B2</i> | $\gamma$  | <i>AIC</i> | <i>BIC</i> | <i>HQIC</i> |
|-----|------------|-----------|-----------|------------|------------|-------------|
| UK  | 0.171146*  | 0.000000  | -0.032129 | -3.1279    | -3.0986    | -3.1171     |
| FRA | 0.170682** | 0.000000  | -0.028034 | -3.1385    | -3.1091    | -3.1276     |
| GER | 0.184962** | 0.000000  | -0.029891 | -3.1389    | -3.1096    | -3.1280     |
| ITA | 0.145447** | 0.000000  | -0.025814 | -3.1392    | -3.1099    | -3.1283     |
| BEL | 0.189844** | 0.000000  | -0.029820 | -3.1380    | -3.1086    | -3.1271     |
| SPA | 0.078346   | 0.000000  | -0.029345 | -3.1362    | -3.1068    | -3.1253     |
| NOR | 0.141952*  | 0.000000  | -0.029156 | -3.1372    | -3.1079    | -3.1263     |
| IRE | 0.208528** | 0.000000  | -0.031094 | -3.1388    | -3.1094    | -3.1279     |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

Table 5 has many similarities with Table 3 and 4. However, a new, interesting finding is the insignificance of  $\gamma$ . Negative shocks to Bitcoin returns do not seem to generate more volatility than positive shocks as the null hypotheses of  $\gamma$  being zero cannot be rejected.

Below follows a short section with Bitcoin as an explanatory variable when modelling the volatility of the stock market indices.

**Table 6. GARCH(1,1) – Bitcoin as an explanatory variable**

|     | <i>B1</i>   | <i>B2</i>   |
|-----|-------------|-------------|
| UK  | 0.000430    | 0.000000    |
| FRA | 0.000974    | 0.000022    |
| GER | 0.001787    | 0.000000    |
| ITA | 0.000619*** | 0.000076*** |
| BEL | 0.000903    | 0.000000    |
| SPA | 0.003329    | 0.000000    |
| NOR | 0.001987    | 0.000000    |
| IRE | 0.000660    | 0.000007    |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

In the mean equations in Table 6, the parameter estimates of B1 show if a positive shock to Bitcoin return has a significant influence on the stock market indices. In the case of Italy, positive shocks to Bitcoin returns seems to explain positive shocks to the stock market index. Additionally, the variance equations in Table 6 shows that a positive shock to Bitcoin volatility cannot explain the volatility of the chosen stock market indices. The only exception is again the stock market index of Italy, where the volatility seems to be explained by the volatility of Bitcoin returns.

**Table 7. IGARCH(1,1) – Bitcoin as an explanatory variable**

|     | <i>B1</i> | <i>B2</i>   |
|-----|-----------|-------------|
| UK  | 0.000669  | 0.000003    |
| FRA | 0.000873  | 0.000027*** |
| GER | 0.002253  | 0.000021*   |
| ITA | 0.000967  | 0.000072*** |
| BEL | 0.000865  | 0.000006    |
| SPA | 0.003492  | 0.000000    |
| NOR | 0.002000  | 0.000000    |
| IRE | 0.000644  | 0.000009    |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

All the B1 parameters in the mean equations in Table 7 are insignificant. In the variance equations however, the parameter B2 is significant in three equations. A positive shock in Bitcoin volatility seems to affect the volatility of the indices in France, Germany and Italy. The explanation to this rather surprising result might be confounding variables, i.e. lack of causality between the studied variables. One cannot exclude the possibility that an underlying cause is affecting both the indices and Bitcoin. This is further discussed in Section 4 of the thesis.

**Table 8. GJR-GARCH(1,1) – Bitcoin as an explanatory variable**

|     | <i>B1</i>   | <i>B2</i>   | $\gamma$    |
|-----|-------------|-------------|-------------|
| UK  | 0.000024    | 0.000004    | 0.204657*** |
| FRA | -0.000127   | 0.000029**  | 0.188736*** |
| GER | 0.004256**  | 0.000037*** | 0.135477*** |
| ITA | 0.002106*** | 0.000087*** | 0.138604*** |
| BEL | 0.000626    | 0.000029*** | 0.156761*** |
| SPA | 0.002915    | 0.000000    | 0.123793*** |
| NOR | 0.001647    | 0.000000    | 0.116076*** |
| IRE | 0.000783    | 0.000017    | 0.091744*** |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

In Table 8, the conclusions are that a positive shock to Bitcoin returns seems to result in a positive shock in the stock market indices of Germany and Italy. Additionally, a positive volatility shock in Bitcoin return seems to make the stock market indices of France, Germany, Italy and Belgium more volatile. Again, confounding variables might be an explanation to these results. As for the parameter  $\gamma$ , it is positive and significant for all the countries and the null hypotheses of  $\gamma$  being zero are therefore rejected. Negative shocks to the indices seem to generate more volatility than positive shocks. In this aspect, the stock

market indices behave differently than Bitcoin. Recall that negative shocks to Bitcoin returns did not seem to generate more volatility.

Now, when all the models have been fitted to the data, it is appropriate to discuss which of them seems best when modelling Bitcoin volatility (i.e. when Bitcoin return is modelled as the response variable). When comparing the information criteria, it is clear that the values do not differ a lot between the three models (see Table 3, Table 4, and Table 5). Regarding these values, the GARCH(1,1) model however seems preferable. As for the parameter estimates, these are also nearly the same for all the three models with the same conclusions. Note that when the indices are used as an explanatory variable, the parameter  $\gamma$  is insignificant in the GJR-GARCH(1,1) model. This means that the model reduces to a GARCH(1,1) model.

As for the model assumptions, it is notable that the assumptions are met when the indices are modelled as response variables (Table 6, Table 7, and Table 8). Unfortunately, this is not the case when Bitcoin is modelled as the response variable. The first problem is that the Ljung-Box test shows significant autocorrelation in the standardized innovations for all the models, independently of which index is used as explanatory variable. Eventually, the models cannot capture the dependence in the innovations and do not work properly when modelling the high volatility of Bitcoin returns. For the interested reader, a plot of the autocorrelation in the innovations can be found in Figure 4 in the Appendix.

One way to deal with the autocorrelation in the innovations is to model the innovations with an ARIMA model of an unknown order. By studying the autocorrelation function (see again Figure 4 in the Appendix) and the extended autocorrelation function (EACF), an appropriate model for the innovations seems to be a MA(3) model with an additional MA(1) component every forth lag. When fitting this model to the innovations, the Ljung-Box test yields a p-value of around 0.75. The autocorrelation function is attached in the Appendix and shows what appears to be white noise (Figure 5). Notable is that this modelling of the innovations is appropriate for all the three models, i.e. the GARCH(1,1) model, the IGARCH(1,1) model and the GJR-GARCH(1,1) model.

The second problem is that the innovations do not seem to be normally distributed as the Jarque-Bera tests give low p-values. A histogram of the innovation is displayed below.

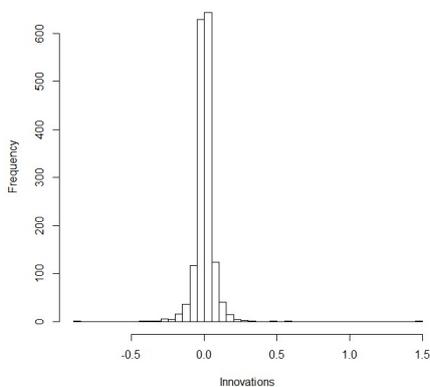


Figure 2. Histogram of innovations from the GARCH(1,1) model

The histogram shows that the innovations seem to have a too high peak around the mean to be normally distributed. Considering the high peak, it seems reasonable to suspect that the innovations follow a Laplace distribution. It is notable that the histogram looks practically the same regardless of which of the GARCH-type model that have been fitted to the data.

In summary, it is not sufficient to model Bitcoin volatility with GARCH-type models as the model assumptions are not fulfilled. The analysis must therefore be supplemented in order to better meet these assumptions. The next step in the analysis is to apply jump detection and filtration of the Bitcoin returns. Maybe this approach can solve the problems with the unfulfilled model assumptions. It is also interesting to see how the filtration effects the parameter estimates and conclusions.

### 3.3.2 Modelling volatility of filtered Bitcoin returns

The first thing to do is to determine a critical value,  $k$ . This value serves as a limit; if the absolute standardized values exceed  $k$ , they are filtered. This holds for both the methods discussed in Section 2.4.2. Earlier research suggests that for larger datasets, with more than 200 observations, a suitable critical value is probably somewhere above 3 (Chen and Liu, 1993, p. 290). However, note that the choice of critical value is not trivial as both the number of observations and the distribution of the observations may affect which value is the most suitable to serve as a limit for filtration. In the case of Bitcoin returns, both the standardized log-returns and the standardized changes in log-returns show a possible leptokurtic behaviour with heavy tails and a high peak around the mean (see Figure 6 in the Appendix for a histogram of the standardized log-returns). This ought to be accounted for when choosing a critical value.

A possible approach is to analyse the sensitivity of the results to different choices of critical values (Chen and Liu, 1993, p. 290). This is done in the tables below. In Table 9, a GARCH(1,1) model has been fitted to the log-returns before and after filtration. Different critical values,  $k$ , are used for detecting the jumps. Note that the method used is the one proposed by Laurent *et al.* (2016), with robust estimates of  $\tilde{\mu}_t$  and  $\tilde{\sigma}_t^2$  (recall (23) and (25)).

**Table 9. Sensitivity analysis for different choices of k**

|                                 | $\omega$        | $\alpha$        | $\beta$         | Q(10)         | Number of filtered obs. |
|---------------------------------|-----------------|-----------------|-----------------|---------------|-------------------------|
| GARCH(1,1) without filtration   | 0.000087<br>*** | 0.201983<br>*** | 0.797017<br>*** | 23.728<br>*** | -                       |
| GARCH(1,1) filtration $\pm 7$   | 0.000082<br>*** | 0.199176<br>*** | 0.799824<br>*** | 11.456<br>*** | 4                       |
| GARCH(1,1) filtration $\pm 6$   | 0.000084<br>*** | 0.204729<br>*** | 0.794271<br>*** | 10.978<br>*** | 5                       |
| GARCH(1,1) filtration $\pm 5$   | 0.000054<br>*** | 0.165954<br>*** | 0.833046<br>*** | 12.026<br>*** | 9                       |
| GARCH(1,1) filtration $\pm 4$   | 0.000070<br>*** | 0.208327<br>*** | 0.790673<br>*** | 10.309<br>*** | 15                      |
| GARCH(1,1) filtration $\pm 3$   | 0.000059<br>*** | 0.179476<br>*** | 0.811448<br>*** | 8.989<br>**   | 34                      |
| GARCH(1,1) filtration $\pm 2.5$ | 0.000061<br>*** | 0.182686<br>*** | 0.801049<br>*** | 5.940         | 65                      |
| GARCH(1,1) filtration $\pm 2$   | 0.000030<br>**  | 0.170799<br>*** | 0.825745<br>*** | 2.6041        | 119                     |

Here,  $\omega$ ,  $\alpha$  and  $\beta$  are the estimated parameters from the GARCH(1,1) model and Q(10) is the Ljung-Box statistics with lag 10 of the standardized innovations. Note that the value of  $\alpha$  is decreasing after filtration and the value of  $\beta$  is increasing. These results are in line with those of Carnero *et al.* (2012, p. 87), which implies that the behaviour of the volatility parameters might be due to jumps. The results also agree with the findings of Charles and Darné (2019, p. 31). It is notable that the Ljung-Box statistics get smaller after the filtration of jumps. The autocorrelated innovations are not a problem when a critical value below 2.5 is chosen. As for the innovations, they still seem to follow a leptokurtic distribution.

Table 10 shows a similar sensitivity analysis as Table 9, but in this case the jump detection method using a constant mean and a constant variance is used (recall (16) and (17)). The standardization is thus based upon the daily changes in Bitcoin log-returns.

**Table 10. Sensitivity analysis for different choices of k**

|                                 | $\omega$        | $\alpha$        | $\beta$         | Q(10)         | Number of filtered obs. |
|---------------------------------|-----------------|-----------------|-----------------|---------------|-------------------------|
| GARCH(1,1) without filtration   | 0.000087<br>*** | 0.201983<br>*** | 0.797017<br>*** | 23.728<br>*** | -                       |
| GARCH(1,1) filtration $\pm 7$   | 0.000065<br>*** | 0.169741<br>*** | 0.829259<br>*** | 38.64<br>***  | 4                       |
| GARCH(1,1) filtration $\pm 6$   | 0.000051<br>*** | 0.145435<br>*** | 0.853565<br>*** | 22.023<br>*** | 6                       |
| GARCH(1,1) filtration $\pm 5$   | 0.000048<br>*** | 0.142681<br>*** | 0.856319<br>*** | 10.513<br>*** | 9                       |
| GARCH(1,1) filtration $\pm 4$   | 0.000042<br>*** | 0.129485<br>*** | 0.869515<br>*** | 8.312<br>**   | 14                      |
| GARCH(1,1) filtration $\pm 3$   | 0.000065<br>*** | 0.160670<br>*** | 0.834105<br>*** | 6.711         | 22                      |
| GARCH(1,1) filtration $\pm 2.5$ | 0.000056<br>*** | 0.147877<br>*** | 0.847608<br>*** | 7.735<br>*    | 29                      |
| GARCH(1,1) filtration $\pm 2$   | 0.000049<br>*** | 0.128915<br>*** | 0.863030<br>*** | 9.209<br>**   | 52                      |

The problem of autocorrelated innovations is less apparent than before filtration, at least when the critical value gets closer to zero. The autocorrelation is however not decreasing as steadily as when the filtration method proposed by Laurent *et al.* (2016) is used. The estimated parameters follow the pattern discussed above as the value of  $\alpha$  is decreasing after the filtration and the value of  $\beta$  is increasing. Note however that the number of filtered observations is less than in Table 9. When investigating the filtered observations, it appears that this method detects the biggest jumps in absolute terms. It is notable that these observations seem to cluster. While the method with time varying estimations of the mean and standard deviation correct for the high volatility and only filtrate some of the observations in the clusters, the method with constant estimations of the mean and standard deviation filtrates a greater number of observations in the volatile periods. However, in periods with small scale jumps, the method proposed by Laurent *et al.* (2016) filters some of these while the second method does not filter at all. This can be the reason for the fewer number of filtered observations when using constant estimators of the mean and standard deviation. The time series before and after filtration using the two different methods can be found in Figure 7 and 8 in the Appendix.

One can assume that the bandwidth influences the result in Table 10. If the mean and standard deviation had been estimated using only the closest observation in the series, the estimates would have been time-dependent. Consequently, distant jumps would not affect the estimations of the mean and standard deviation and the results would probably be more similar to the results in Table 9. Additionally, it can be problematic that the filtration is performed on the observation following the big change in log-return. The second method thus ought to be improved.

The conclusions from the above tables are that the second method presented in Section 2.4.2, i.e. the method proposed by Laurent *et al.* (2016), seems preferable. This method is therefore used to filtrate Bitcoin log-returns when the analysis from Section 3.3.1 is repeated for the adjusted dataset. Recall that the purpose is, for example, to investigate how the estimates of  $B1$  and  $B2$  are affected by the filtration procedure. Additionally, it is interesting to see whether the filtration procedure makes the GARCH-type models more suitable for the dataset. The choice of critical value is however still not trivial. The critical value in the following tables is, following the example of Chen and Liu (1993, p. 290), set to  $\pm 3.5$ . Also, a sensitivity analysis is again performed for different choices of critical values. The result from this analysis is presented in the text following the tables.

As before, the first three tables display the result for the three models with the indices as explanatory variables and Bitcoin log-returns as response variable. After that, the result from the models with Bitcoin as an explanatory variable follows.

**Table 11. GARCH(1,1) – Index as an explanatory variable**

|     | $B1$       | $B2$     | $AIC$   | $BIC$   | $HQIC$  |
|-----|------------|----------|---------|---------|---------|
| UK  | 0.079728   | 0.000000 | -3.4183 | -3.3922 | -3.4086 |
| FRA | 0.090856   | 0.000000 | -3.4197 | -3.3936 | -3.4100 |
| GER | 0.098379   | 0.000000 | -3.4199 | -3.3938 | -3.4102 |
| ITA | 0.080783   | 0.000000 | -3.4201 | -3.3940 | -3.4104 |
| BEL | 0.103626   | 0.000000 | -3.4196 | -3.3935 | -3.4099 |
| SPA | 0.061265   | 0.000000 | -3.4193 | -3.3932 | -3.4096 |
| NOR | 0.033321   | 0.000000 | -3.4188 | -3.3927 | -3.4091 |
| IRE | 0.175045** | 0.000000 | -3.4216 | -3.3955 | -3.4119 |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

Even though only 24 observations have been filtered, there are some evident changes in comparison to Table 3. Nearly all the considered parameters in the mean equations are now insignificant (Ireland is the only exception) and the null hypotheses of the  $B1$ 's being zero can therefore not be rejected. The conclusion from before, that a positive shock to the stock market indices may make investors more risk seeking and inclined to invest in alternative assets like Bitcoin, does not seem to be true for the filtered data.

**Table 12. IGARCH(1,1) – Index as an explanatory variable**

|     | <i>B1</i> | <i>B2</i> | <i>AIC</i>     | <i>BIC</i>     | <i>HQIC</i>    |
|-----|-----------|-----------|----------------|----------------|----------------|
| UK  | 0.078769  | 0.000000  | <b>-3.4195</b> | <b>-3.3966</b> | <b>-3.4110</b> |
| FRA | 0.092637  | 0.000000  | <b>-3.4206</b> | <b>-3.3978</b> | <b>-3.4122</b> |
| GER | 0.101004  | 0.000000  | <b>-3.4208</b> | <b>-3.3980</b> | <b>-3.4124</b> |
| ITA | 0.080784  | 0.000000  | <b>-3.4210</b> | <b>-3.3982</b> | <b>-3.4125</b> |
| BEL | 0.105093  | 0.000000  | <b>-3.4205</b> | <b>-3.3977</b> | <b>-3.4120</b> |
| SPA | 0.061321  | 0.000000  | <b>-3.4202</b> | <b>-3.3974</b> | <b>-3.4117</b> |
| NOR | 0.034784  | 0.000000  | <b>-3.4197</b> | <b>-3.3969</b> | <b>-3.4112</b> |
| IRE | 0.174993* | 0.000000  | <b>-3.4225</b> | <b>-3.3997</b> | <b>-3.4140</b> |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

The conclusions from the IGARCH(1,1) model is similar to the ones from the GARCH(1,1) model. The explanatory variables in the mean equations are now mostly insignificant.

**Table 13. GJR-GARCH(1,1) – Index as an explanatory variable**

|     | <i>B1</i>  | <i>B2</i> | <i>γ</i>  | <i>AIC</i> | <i>BIC</i> | <i>HQIC</i> |
|-----|------------|-----------|-----------|------------|------------|-------------|
| UK  | 0.078657   | 0.000000  | -0.020943 | -3.4175    | -3.3881    | -3.4066     |
| FRA | 0.090872   | 0.000000  | -0.011107 | -3.4186    | -3.3892    | -3.4077     |
| GER | 0.099009   | 0.000000  | -0.011877 | -3.4188    | -3.3894    | -3.4079     |
| ITA | 0.080473   | 0.000000  | -0.010449 | -3.4190    | -3.3896    | -3.4081     |
| BEL | 0.108128   | 0.000000  | -0.013552 | -3.4152    | -3.3859    | -3.4043     |
| SPA | 0.061219   | 0.000000  | -0.010937 | -3.4182    | -3.3888    | -3.4073     |
| NOR | 0.033429   | 0.000000  | -0.010913 | -3.4177    | -3.3883    | -3.4068     |
| IRE | 0.175673** | 0.000000  | -0.012176 | -3.4205    | -3.3912    | -3.4096     |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

Also when applying a GJR-GARCH(1,1) model to the data nearly all of the parameters in the mean equations are insignificant. Additionally, negative shocks to Bitcoin return do not seem to generate more volatility than positive shocks. This is the same conclusion as before (see Table 5).

The three following tables show the parameter estimates when the indices are used as response variables. Comments follow the tables.

**Table 14. GARCH(1,1) – Bitcoin as an explanatory variable**

|     | <i>B1</i> | <i>B2</i> |
|-----|-----------|-----------|
| UK  | -0.000033 | 0.000000  |
| FRA | 0.000588  | 0.000000  |
| GER | -0.000905 | 0.000000  |
| ITA | 0.001494  | 0.000000  |
| BEL | 0.000243  | 0.000000  |
| SPA | 0.001904  | 0.000000  |
| NOR | 0.004932  | 0.000000  |
| IRE | 0.003843  | 0.000000  |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

**Table 15. IGARCH(1,1) – Bitcoin as an explanatory variable**

|     | <i>B1</i> | <i>B2</i> |
|-----|-----------|-----------|
| UK  | -         | -         |
| FRA | -0.000230 | 0.000000  |
| GER | -0.000937 | 0.000000  |
| ITA | 0.001649  | 0.000000  |
| BEL | 0.000263  | 0.000000  |
| SPA | 0.002066  | 0.000000  |
| NOR | 0.004961  | 0.000000  |
| IRE | 0.003438  | 0.000000  |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

In case of UK, the model could not be estimated. The software reported non-convergence of the optimisation algorithm so that the maximum likelihood function could not be optimised.

**Table 16. GJR-GARCH(1,1) – Bitcoin as an explanatory variable**

|     | $B1$      | $B2$     | $\gamma$    |
|-----|-----------|----------|-------------|
| UK  | -0.000710 | 0.000003 | 0.205747*** |
| FRA | -0.001283 | 0.000000 | 0.180390*** |
| GER | -0.001953 | 0.000000 | 0.119285*** |
| ITA | -0.000808 | 0.000042 | 0.130036*** |
| BEL | -0.001399 | 0.000015 | 0.146417*** |
| SPA | -0.000126 | 0.000000 | 0.122247*** |
| NOR | 0.003711  | 0.000000 | 0.114984*** |
| IRE | 0.003835  | 0.000009 | 0.089369*** |

Note: Table shows t-statistics of the tests. Significance: \* 10%, \*\* 5%, \*\*\* 1%

The filtered data leads to different conclusions also when it comes to modelling the indices with Bitcoin as an explanatory variable (see Table 14, Table 15, and Table 16). Before, significance was found for a few countries in both the mean and the variance equations (see for example the significant parameters B2 in the variance equations in Table 8). These relationships have apparently disappeared with the filtering of the data as the null hypotheses of B1 and B2 being zero now cannot be rejected. Note that negative shocks to the indices still seem to generate more volatility than positive shocks as the parameter  $\gamma$  is positive and significant in Table 16.

Moving on to the evaluation of the models. The IGARCH(1,1) model yields the lowest values of the information criteria. When comparing the information criteria however, it is clear that the values do not differ a lot between the three models.

What is more interesting is to discuss the model assumptions and parameter estimates<sup>4</sup>. As concluded above, the value of  $\alpha$  is decreasing and the value of  $\beta$  is increasing after filtration and this is true also when including the indices as explanatory variables (these parameters are however not included in the above tables). The magnitudes of the changes in the parameter estimates depend on the choice of critical value. This pattern is also apparent in Table 9. Additionally, the estimates of parameters B1 from (28), (32) and (36) are decreasing in every model after the filtration. With an absolute critical value of 3.5 (i.e. the critical value chosen for filtration in the above analysis), the estimates of B1 are now insignificant, indicating that a positive shock in the stock market indices does not affect Bitcoin returns. These changes in the estimations of B1 are also dependent of the choice of critical value. When a value below the absolute value of 3.5 is chosen, the estimates of B1 become even more insignificant. When a critical value above the absolute value of 3.5 is chosen, the

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<sup>4</sup> Note that this discussion only considers the models with Bitcoin modelled as response variable. The jump detection and filtration are performed on the log-returns of Bitcoin and the changes in the parameter estimates of  $\omega$ ,  $\beta$  and  $\alpha$  when using the indices as response variables (i.e. filtered Bitcoin returns as an explanatory variable) are limited after filtration, independently of choice of critical value in the filtration procedure. The conclusion from before, that the model assumptions are met when the indices are used as response variables, is still true.

estimations of B1 become more significant. The parameter B2 is insignificant in all models both before and after the filtration.

The choice of critical value also affects the autocorrelation in the innovations of the GARCH-type models. It was previously concluded that a MA(3) model with an additional MA(1) component every fourth lag was suitable for the autocorrelated innovations of the unfiltered data. After the filtration procedure, the innovations are less autocorrelated (see Figure 9 in the Appendix) and the previous proposed model for the innovations is no longer suitable. If the chosen critical value is closer to zero, the autocorrelation in the innovations becomes smaller (the p-value from Ljung-Box test gets higher and the null hypothesis of no serial correlation cannot be rejected). However, if the absolute critical value is greater than 3.5, the autocorrelation in the innovations does not get more prominent. This result is apparent also in the column Q(10) in Table 9, where the innovations seem to be less autocorrelated after filtering only four observations. A small number of outliers in the series seem to have a big influence on the autocorrelation function of the innovations.

As for the distribution of the innovations, a Laplace distribution still seems like a possible solution to the problem of non-normality (see Figure 10 in the Appendix). This is true independently of choice of critical value or choice of model.

## 4. Summary and discussion

The purpose of the study was to investigate how models from the field of financial statistics, such as GARCH(1,1), work when analysing the extreme volatility of Bitcoin returns. The findings regarding this purpose is discussed in Section 4.1 below. As an instrument in the analysis, two empirical questions were formulated:

*Can volatility of the chosen European stock indices explain the volatility of Bitcoin?*

The opposite relationship was also considered:

*Can volatility of Bitcoin explain the volatility of the chosen European stock indices?*

Possible answers to these questions are discussed in Section 4.2. Section 4.3 contains a short discussion of how the result from the thesis can be generalized to other highly volatile time series.

### 4.1 The performance of GARCH-type models

The investigation in previous sections has shown that the three chosen models (GARCH(1,1), IGARCH(1,1) and GJR-GARCH(1,1)) are not sufficient when modelling Bitcoin returns. Neither of the models can capture the structure of Bitcoin returns as the autocorrelation in the innovations are significant. Additionally, the innovations follow a Laplace distribution rather than a normal distribution. Since the model assumptions are not fulfilled, it is necessary to consider additional analyses. This conclusion is important and adds new knowledge about how to approach highly volatile time series. Previous research about Bitcoin volatility have focused only on comparing information criteria of different GARCH-type models (see for example Chu *et al.*, (2017)), but the findings in this thesis suggest that these analyses should be supplemented.

The unsatisfactory performance of the GARCH-type models might be due to the jumps. As stated previously, neglecting the jumps can lead to an overestimation of the volatility during several days after their occurrence (Laurent *et al.*, 2016, p. 383). These conclusions have been reached also in other papers, showing that the volatility actually can be overestimated through the whole sample period (see for example Carnero *et al.*, 2012, p. 87). The reason for this overestimation of volatility in presence of outliers is that the GARCH-type models contain, by construction, assumptions about continuity. That is, a jump is assumed to be followed by another by the construction of the conditional variance equation (Cheng, 2008, p. 2). When a jump is not followed by another, the model overestimates the volatility of the following observations. Following the conclusions from these papers, it is reasonable to assume that the volatility of the GARCH-type models is overestimated when modelling Bitcoin returns. This overestimation of the volatility might cause the autocorrelated innovations. According to Chan (1995, p. 186), the autocorrelation function can be highly affected by outliers. The analyses from previous sections in this thesis have indeed shown

that a small number of jumps seem to have a large influence on the autocorrelation function of the innovations.

To solve the problem of overestimation of volatility, earlier researchers have proposed methods to detect and filter the jumps in the series. This is one way to supplement the analysis and can be done in different ways. When implementing the jump detection test proposed by Laurent *et al.* in 2016, the choice of critical value seems to be crucial for the results. Sensitivity analyses have shown that both the parameter estimates and the autocorrelation in the innovations in the GARCH-type models are dependent of the choice of critical value for filtration. Earlier researchers, such as Chen and Liu (1993) have suggested that with a dataset of this size, a suitable absolute critical value is probably somewhere above 3. When using a critical value of  $\pm 3.5$  for filtration however, autocorrelation in the innovations after fitting GARCH-type models is still present after filtration. This is an indication that the proposed models are not adequate. The reason for the remaining autocorrelation might be that the normality assumption of the filtered returns seems to be violated (see a normal probability plot of the filtered Bitcoin returns in Figure 11 in the Appendix). This is an important assumption of the method proposed by Laurent *et al.* (2016). If the assumption is violated, spurious jumps can be detected and filtered while some of the true jumps might remain undiscovered (Laurent *et al.*, 2016, p. 399). Another possible explanation to the remaining autocorrelation might be a wrongly specified mean equation (Matteson and Ruppert, 2011, p. 75).

With current state of knowledge, a good first step when modelling Bitcoin volatility is to filter the jumps. The method used for this purpose ought to be carefully evaluated to ensure that it can account for the leptokurtic distribution of Bitcoin log-returns. The method proposed by Laurent *et al.* in 2016 seems to fail in this respect, along with many other jump detection tests that are also built upon the normality assumption of the filtered returns (see Laurent *et al.*, 2016, p. 384). As for the other tried method, the wide bandwidth is a problem. This allows the jumps to have a large influence on the parameter estimates. By elaborating with the bandwidth, this jump detection method can probably be improved.

After the filtration, a GARCH-type model can be fitted to the filtered returns in order to find parameter estimates that are not affected by jumps. The type of model does not seem to be of great importance as the values of the information criteria do not differ a lot. The innovations might be modelled with a Laplace distribution. Jumps occur with a small probability and these can perhaps be modelled with a Poisson process.

## 4.2 The relationship between the stock market and Bitcoin

Moving on to the empirical questions. It is notable that different results are achieved depending on if the jumps are filtered or not. Without filtration, it seems like a positive shock to the indices is related to a positive shock in Bitcoin returns. The null hypothesis of  $B1$  being zero is rejected in all three GARCH-type models for all countries except Spain. This relationship disappears with the filtration of jumps as the null hypothesis of  $B1$  being zero cannot be rejected anymore. Additionally, the volatility of the indices cannot explain the

volatility of Bitcoin neither before nor after filtration, as the parameter B2 is insignificant in the variance equations in all GARCH-type models.

Additionally, a positive shock to Bitcoin return is related to a positive shock to the index in some of the countries and for some of the models (it seems for example like positive shocks to Bitcoin returns can explain positive shocks to the stock market index of Italy). In these cases, the parameter B1 is positive and significant. The volatility of Bitcoin return seems to explain the volatility of the indices in some of the countries as the parameter B2 is positive and significant. However, these relationships disappear after filtration of jumps.

In summary, there is almost no evidence that volatility of the chosen indices can explain volatility of Bitcoin or vice versa. To use Bitcoin as a tool to forecast what is going to happen on the stock market seems like a bad idea. Recall that the method of including different explanatory variables, such as stock market indices, to explain the volatility of Bitcoin was proposed by Dyhrberg in 2016. After using the method, it is clear that it has some shortcomings. A third variable might, for example, influence both Bitcoin and the indices and even if there is a relationship it is impossible to distinguish between cause and effect using this method. An alternative would be to use some sort of causality test (see for example Cheung and Ng, 1996).

Finally, it appears that the jumps of Bitcoin returns in some sense are related to the returns of the indices as many of the significant relationships in the mean equations disappear after filtration (the parameter B1 becomes insignificant in almost all cases). The jumps therefore seem to connect Bitcoin and the stock market indices in the mean equations when the indices are used as explanatory variables. One might suspect that the indices also are affected by jumps and that an unknown underlying process is generating jumps in a certain manner. Bitcoin returns are however reacting more intense to these events than the indices. The characteristics of this possible underlying process must be further investigated in order to correctly model and predict the jumps, both for the stock market and for Bitcoin. This subject is further discussed in Section 5. The conclusion from this discussion is however again the inaccuracy of suggesting that it is possible to predict the movements on the stock market by looking at Bitcoin returns or vice versa. There is evidence that both Bitcoin and the indices are affected by another process that needs further investigation.

### 4.3 Generalizations

The results from this thesis might be generalized to other highly volatile time series. If the series contains jumps, jump filtration ought to be considered before GARCH-type models can be used. It is however important to note that the filtration procedure might cause problems if explanatory variables have been added to the models. Additionally, it is important to be aware of the complexity of filtering the jumps, for example the possible crucial decision of how big an outlier shall be to be considered as a jump (i.e. the decision of critical value). If the filtration method used has assumptions about normality of filtered returns, this must be investigated.

## 5. Other approaches and future directions

A short discussion about alternative approaches and future directions ends the thesis. First, one could have used other jump-detection tests. In a paper from 2015, Caporin *et al.* propose, for example, a test to detect multi-jumps occurring simultaneously in stock prices. There is also plenty of univariate jump detection tests (Caporin *et al.*, 2015). Recall that the primary test used in this thesis, proposed by Laurent *et al.* (2016), had been used previously when filtering time series of Bitcoin returns (see Charles and Darné, 2019). Drawbacks of this test in the context of Bitcoin returns are however the unfulfilled model assumptions and also the crucial choice of the critical value used in the filtration procedure. The leptokurtic distribution of the innovations seems to require another approach, for example a generalized autoregressive score (GAS) model (see Laurent *et al.*, 2016, p. 390).

Second, other conclusions could maybe have been reached with weekly or monthly frequencies of data. The choice to use daily frequencies is however based on the small value of Bitcoin in comparison to the values of the stock market indices. If Bitcoin would have had any connection to the indices, the effect is not assumed to last for a longer period of time.

Third, the world market of Bitcoin is led by Japan and the US. Maybe the Bitcoin behavior can be better explained by these countries' indices? As mentioned in the beginning of the thesis, some discussions have been going on on the web about this matter. There is, however, a lack of scientific research about questions of this type.

Forth, it would be interesting to further investigate possible explanatory variables when modelling Bitcoin volatility. In this thesis, the only ones considered are the chosen stock market indices. The amount of such studies for the stock market is huge. Popular explanatory variables in these studies are different type of macroeconomic variables such as oil price, inflation and real exchange rate (see for example Hussain *et al.*, 2015 and Nikmanesh *et al.*, 2014). It would be interesting to investigate if these variables can explain Bitcoin volatility. It is however again worth mentioning that confounding variables might exist, i.e. that other variables might cause the detected relationships. This is a phenomenon that must not be ignored in these types of analyses.

Finally, the leptokurtic distribution of the innovations from the GARCH-type models, both before and after filtration, need to be further investigated and in order to correctly model the jumps, one might examine the use of a Poisson process. One possible approach could be to apply a hidden Markov model and letting the Poisson intensity vary for different parts of the series. As concluded above, there is some evidence that jumps exist both in the time series of the indices and in the time series of Bitcoin, but the jump generating process might even though have different characteristics for the different series. This is an interesting subject for future research.

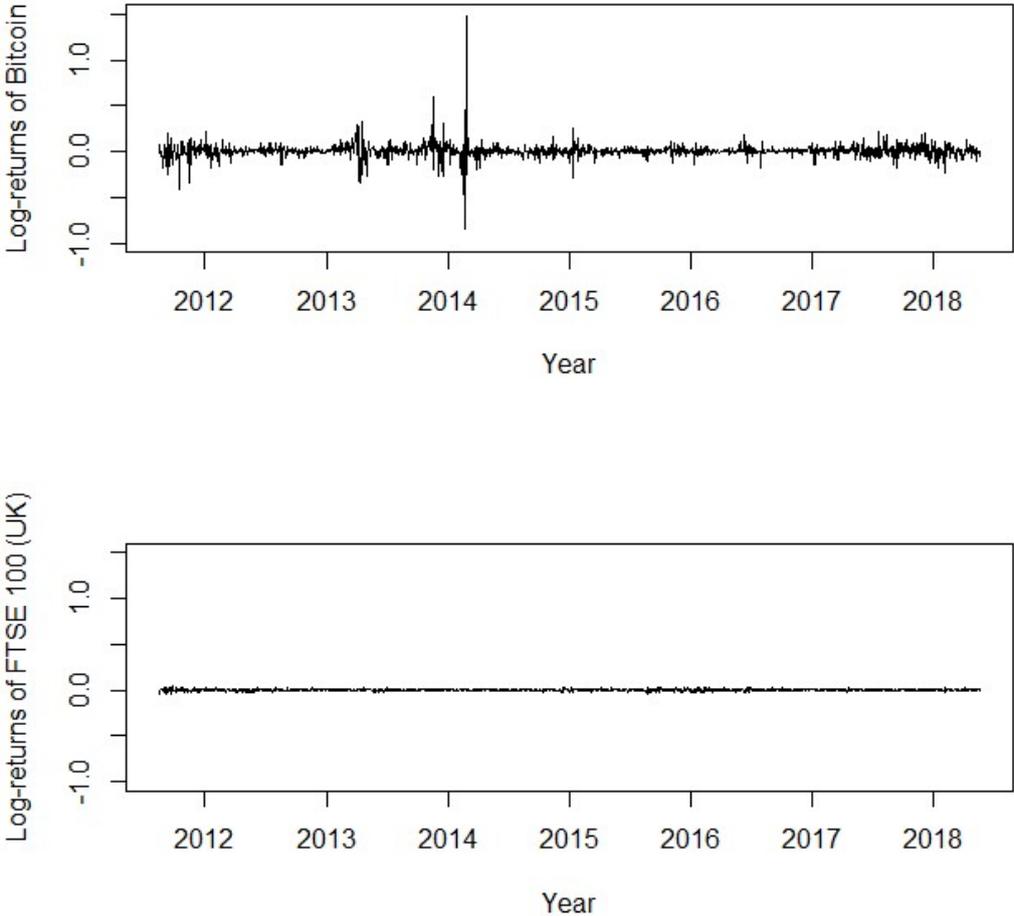
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# 7. Appendix



*Figure 3. Comparison of log-returns for Bitcoin and UK stock market. The figure serves as a graphical comparison of the volatility of the log-returns. At the top is the Bitcoin log-returns and at the bottom is the UK stock market log-returns. One can clearly see that the Bitcoin log-returns are more volatile.*

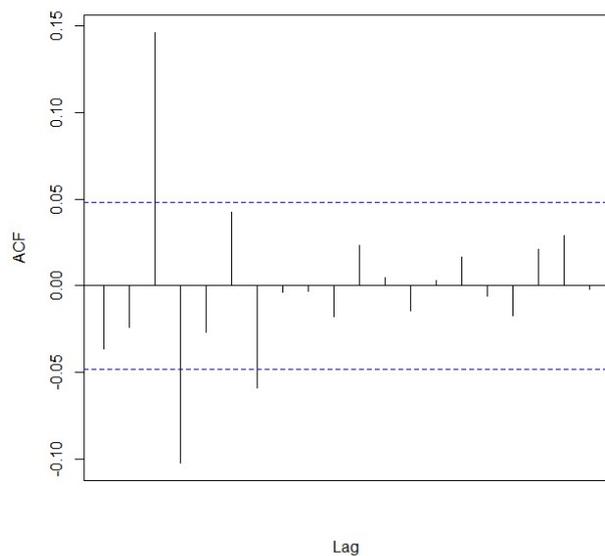


Figure 4. ACF of the innovations from a GARCH(1,1) model. Apparently, the innovations are autocorrelated after the model has been fitted to the unfiltered data. Here, the index FTSE 100 in UK is used as an explanatory variable, but the pattern is the same independently of which index is chosen in the model. To exclude the index as an explanatory variable does not affect the autocorrelation.

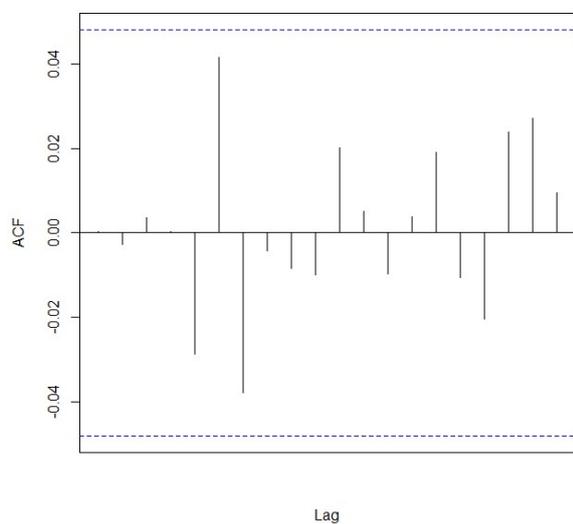
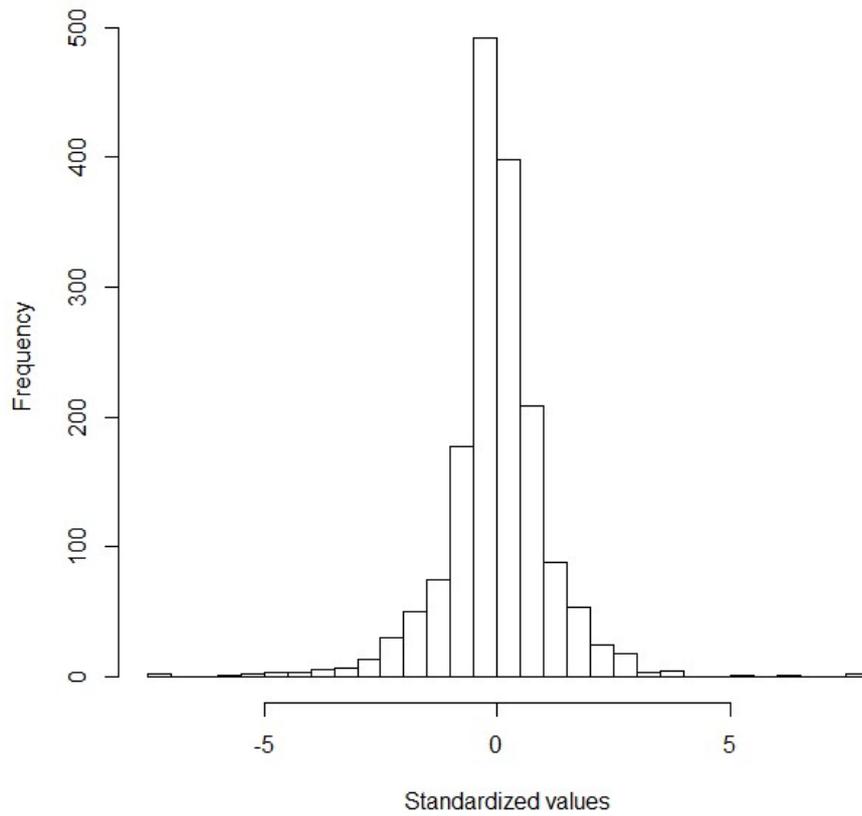


Figure 5. ACF of the innovations. After modelling the innovations with a MA(3) model with an additional MA(1) component every fourth lag, the autocorrelation seems to disappear.



*Figure 6. Histogram of standardized Bitcoin log-returns. A possible leptokurtic distribution of the standardized Bitcoin log-returns must be accounted for when deciding a critical value for filtration. The method proposed by Laurent et al. (2016) is used in the standardization procedure.*

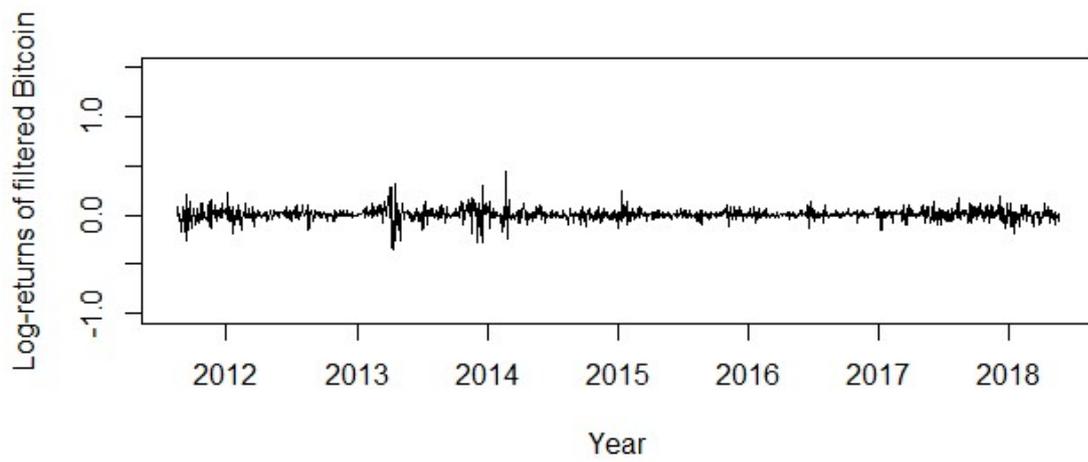
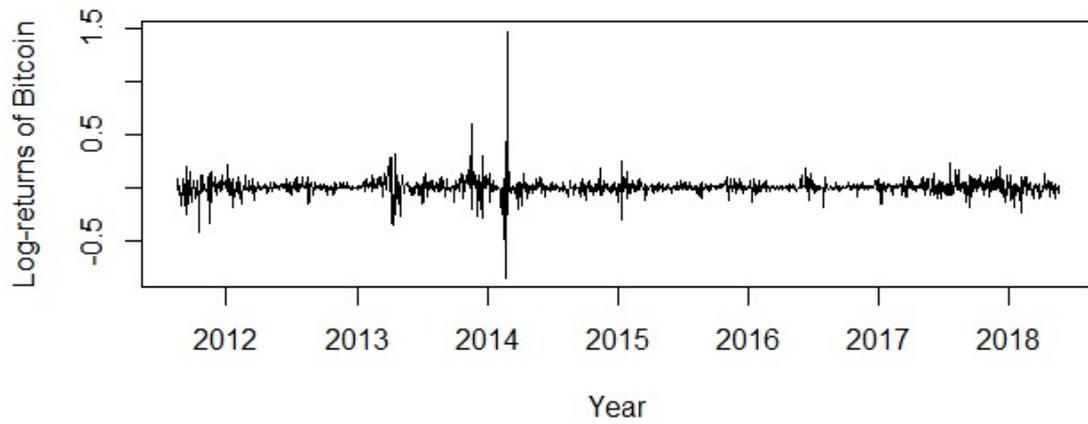
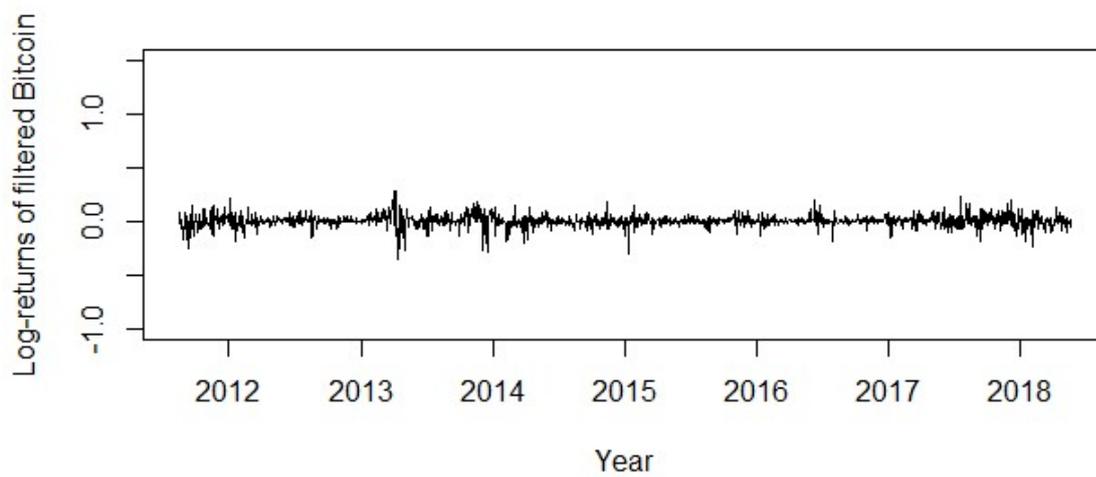
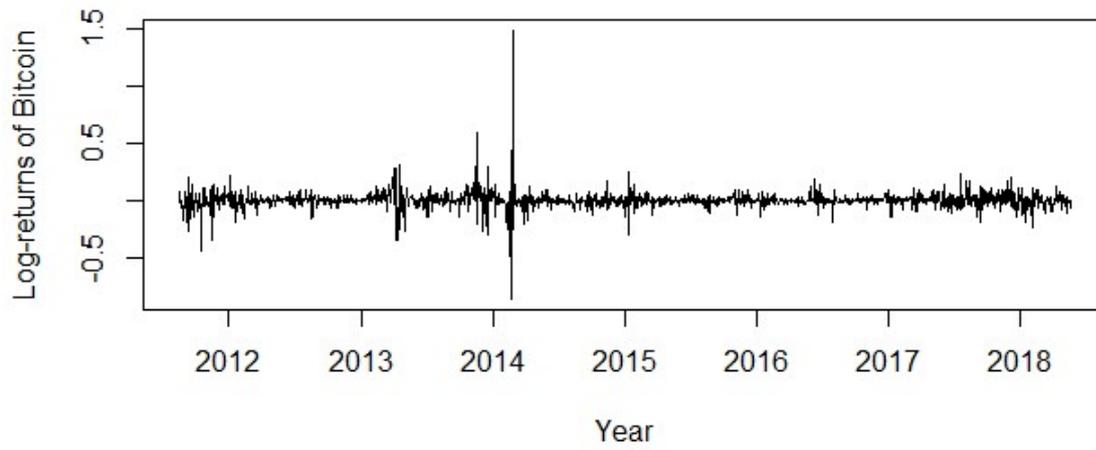


Figure 7. Bitcoin log-returns before and after filtration. The method with time varying mean and standard deviation proposed by Laurent et al. (2016) has been used to find and filtrate the jumps. The critical value used is  $\pm 3.5$ , resulting in 24 filtered observations.



*Figure 8. Bitcoin log-returns before and after filtration. The method with constant mean and standard deviation has been used to find and filtrate the jumps. The critical value used is  $\pm 3.5$ , resulting in 19 filtered observations.*

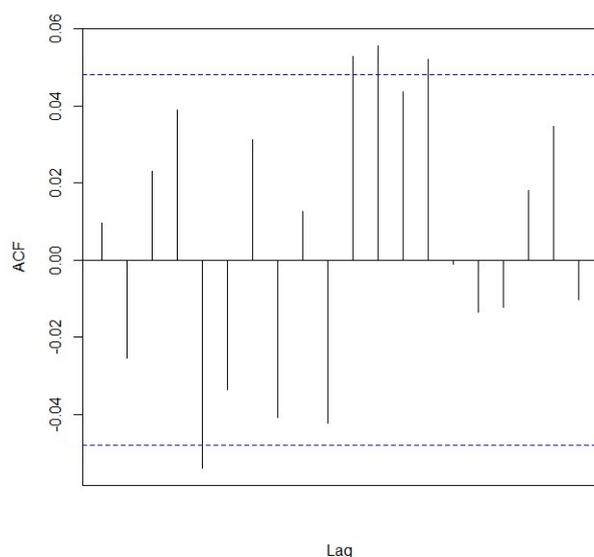


Figure 9. ACF of the innovations from a  $GARCH(1,1)$  model after filtration when an absolute critical value of 3.5 is used. This figure shall be compared to the ACF of the innovations before filtration (Figure 4). It is clear that the innovations are less autocorrelated after filtration.

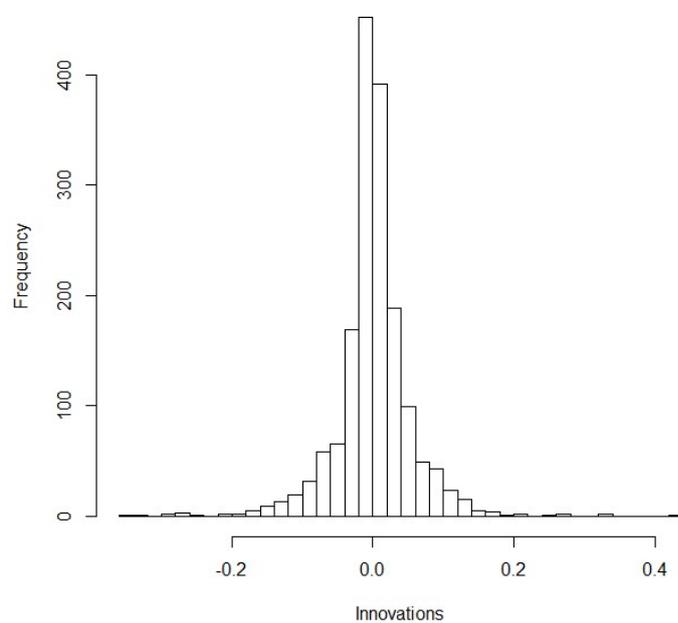
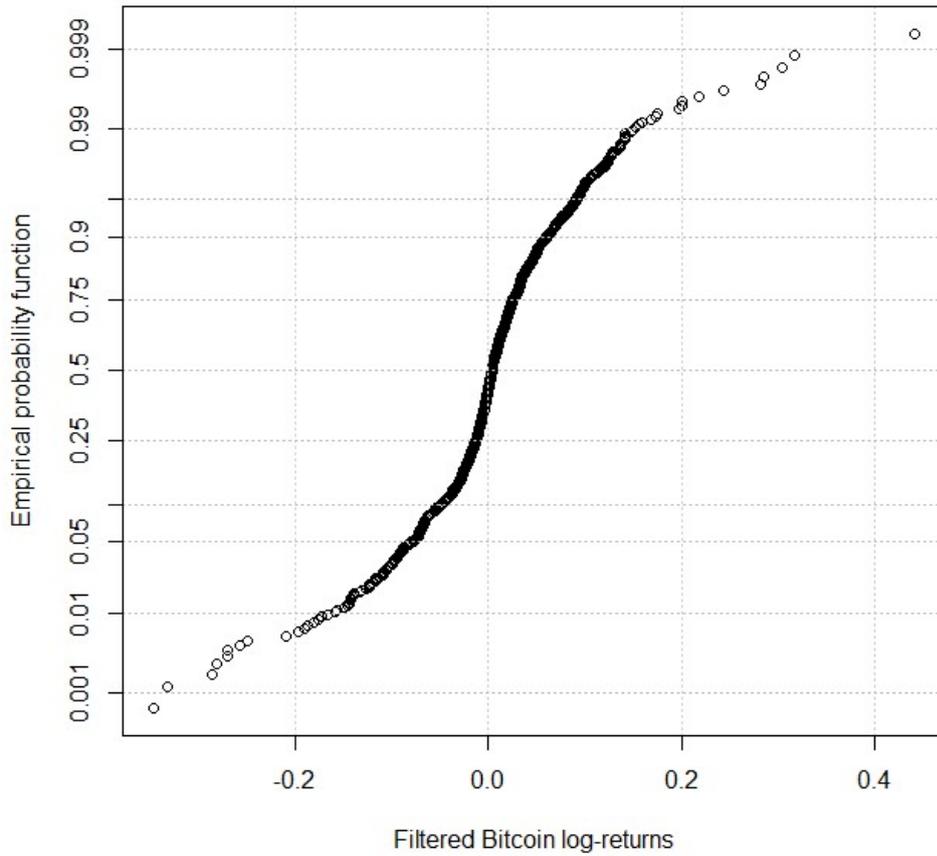


Figure 10. Innovations of  $GARCH(1,1)$  after filtration. The histogram shows innovations of a  $GARCH(1,1)$  model when using a critical value of 3.5 for filtration. Number of filtered observations is 24. The innovations still seem to be Laplace distributed, just as in Figure 2.



*Figure 11. Normal probability plot of the filtered Bitcoin log-returns. In the plot, a critical value of 3.5 is chosen to filtrate the jumps. The method proposed by Laurent et al. (2016) is used to detect and filtrate the outliers in the time series. One important assumption of this method is that the filtered observations shall be normally distributed (Laurent et al., 2016, p. 384). When looking at Figure 11 above, this assumption seems to be violated.*