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Pairs Trading in Swedish Investment Companies:

Implementing a Market Neutral Trading Strategy

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Abstract

Title:	Pairs Trading in Swedish Investment Companies: Implementing a Market Neutral Trading Strategy
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Keywords:	Pairs trading, Investment Company, Traded at Discount, Bollinger Bands, Moving Average, Trading Strategy, Algorithmic Trading, Algo trading
Purpose:	The purpose of this paper is to examine if it is possible to profitably implement a market neutral trading strategy, so-called "pairs trading", on three different Swedish investment companies.
Theoretical Framework:	This paper is based on previous research who successfully implemented the pairs trading strategy on cointegrated prices series, combined with the empirical fact that Swedish investment companies many times are traded at a discount in relation to its underlying assets.
Sample:	The sample data consists of returns data of three Swedish investment companies and their underlying assets in the time period 2012.04.01 - 2017.03.31.
Methodology:	A pairs trading strategy is implemented on three Swedish investment companies using Bollinger Bands based on residual moving averages (MA) to create buy and short sell signals that execute trades.
Conclusions:	It can be concluded that applying a pairs trading strategy on [these three] Swedish investment companies and *their underlying listed assets* has the potential to beat the market Sharpe ratio. However, the profitability varies and depends on the parameter settings, and on which company the trading strategy is applied to.

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Table of Contents

List of Figures, Tables, and Graphs

1. Introduction

An investment company is a company whose prime objective is to invest and often take active ownership in other companies and thereby achieving the best possible returns for its investors. This is many times done by taking on substantial ownership and actively controlling its investments for long periods of time. An investment company may hold both private and public companies in its portfolio.

When valuing an investment company, it is of great importance to look at the value of its underlying assets. Unsurprisingly, the equity value of an investment company and the market capitalization of its underlying assets often move in tandem. However, the relationship is not perfect, and it's been empirically shown that Swedish investment companies often trade at a substantial discount in relation to its assets´ net asset value. This is known as the investment company is trading at a discount (Essen, 1997). If the opposite is true, the investment company is trading at a premium.

The rationale for this pricing divergence might appear irrational at first since the same cash flows are valued at different prices (Datar, 2001). Two possible theories are that the investment companies might acquire companies that are unwanted by the investors and that investment companies many times have unlisted assets, which are problematic to value at market price (Essen, 1997).

This paper acknowledges that there often exists a significant divergence in the pricing mechanism between the equity value of an investment company and the net asset value of its underlying assets. However, this paper does not aim to investigate further why this phenomenon is observed, but rather to examine if it is possible to implement a profitable trading strategy exploiting this empirical fact. The suggested trading strategy in this paper is called *pairs trading*.

Pairs trading is a mean-reverting trading strategy that exploits the observation that some pairs of traded assets are *cointegrated* and therefore move together over time. In this paper, the investment company will be one of these traded assets, and the other will be *a basket of the listed underlying holdings* of that investment company. The hypothesis behind this trading strategy is that cointegration between this traded pair is unlikely to break down since the value of the investment company is determined by its underlying assets. By logical reasoning, it’s clear that there is a limit to how much these can deviate from each other.

The fundamental idea in pairs trading is to initiate trades when two cointegrated assets have deviated far enough from each other and their long-term relationship, with the hypothesis that these two assets will mean-revert back towards their long-term equilibrium. To take advantage of this hypothesis in a trading strategy, an investor takes a long position in the asset that is relatively lower priced compared to its companion asset, while simultaneously taking a short position in the relatively high priced asset. By only speculating on the residual divergence between two assets, the trading strategy is not affected by whether or not the market is performing well. Consequently, the pairs trading strategy is considered market risk neutral (Figuerola-Ferretti, Paraskevopoulos, and Tang, 2017).

Since pairs trading always involves one long position and one offsetting short position, it is unlikely to outperform a stock that is doing well. However, because of the hedging aspects of the pairs trading strategy, it may be less volatile due to the market risk neutral aspects of the trades. Therefore, to evaluate the efficiency of the pairs trading strategy, the Sharpe ratio is used as a performance metric. The intuition behind using the Sharpe ratio as a performance metric is that it describes the risk-premium (expected excess return) per unit of risk, measured as standard deviation. The Sharpe ratio is discussed further in the theoretical framework section.

This leads to the purpose of this paper:

If Swedish investment companies are continuously valued at a different price than the value of their underlying assets, is this difference in value sufficiently stationary such that a mean-reverting pairs trading strategy can be profitably implemented?

This research is conducted in the following manner. First, the daily returns for three Swedish investment companies and their listed holdings are downloaded from a Bloomberg terminal. The unlisted assets are estimated at their reported book value, and these holdings are updated quarterly by looking at quarterly rapports. Next, the net asset value of the investment company's underlying assets is regressed on the equity value of that investment company. The residuals are then examined for stationarity, and if confirmed, it would imply a cointegration relationship between the investment companies and their assets. Next, the residuals are used to create the trade signals that are required when implementing a pairs trading strategy. This paper will divide the sample period into an in-sample and an out-of-sample period, to be able to build and evaluate the trading strategy successfully.

There are some limitations to this study. First, the sample consists of only three investments companies due to the time-consuming process of managing the comprehensive data that is needed to implement the pairs trading strategy. This is problematic in the way of not knowing if the results obtained are representative and therefore, would also apply to other investments companies. Secondly, in this paper, the holdings of the investment companies are only reported quarterly. In reality, the reweighting of the assets happen throughout the year, but for simplicity, this is only accounted for when quarterly and annual reports are released. Lastly, an investment company often hold both listed and unlisted assets, which problematizes the estimation of the daily difference between the two price series. Also, since it is not possible to buy or short sell unlisted assets, the listed assets will work as a proxy for the investment companies total asset value. However, it is worth pointing out that these limitations alone do not undermine the possibility to implement a pairs trading strategy successfully.

The results from this paper show that there seems to be at least some possibility for profitably applying a pairs trading strategy to investment companies and their underlying assets. An excellent example of this is Bure Equity, where the best strategy yielded a Sharpe ratio of above 2.0. This could be compared to the Swedish market's (OMX30) Sharpe ratio during the same period of –0.058. However, the profitability varies and depends on the parameter settings, and on which company the trading strategy is applied to. To increase the probability of successfully implementing this strategy, it would be recommendable to diversify the risk by applying the strategy on several investment companies at the same time.

The outline of the thesis is as follows. After the introduction, a review of the previous literature will take place, which will give guidance when structuring the trading strategy. Next, a description of the theoretical framework follows to provide the reader with the knowledge to fully grasp the content of this paper. In the following section, the methodology is outlined. Finally, the results and an ending discussion takes place.

2. Previous literature

Earlier studies on pairs trading differ from this paper in the manner that they mainly focus on pairs trading between different stocks that share similar movements over time. However, this paper will, in theory, trade the *same* asset (the investment company and a basket of its underlying assets). This does not fundamentally change how the pairs trading strategy is applied, but the rate of the mean reversion in the cointegration residual might affects the profitability of the trading strategy. Below are some of the main empirical findings related to pairs trading. These will be the foundation of this paper's pairs trading strategy.

Pairs trading can be based on different methodologies to determine which traded pairs that are suitable to use in a pairs trading strategy. For instance, the determining of the pairs can be done by looking at the correlation between two assets, or by looking at the distance in standardized prices between two traded assets, or by looking if there statistically exists a cointegration relationship between the traded pair. Carrasco et al. (2018) empirically investigated which method that generates the most robust results by looking at the S&P500 bank subgroup in a sample period over six years. Their findings concluded that cointegration is the most efficient method for structuring a pairs trading strategy.

Gatev et al. (2006) performed a study on pairs trading with daily data over 1962 - 2002 on stocks listed on the S&P500. This is often considered as the first comprehensive study of pairs trading and its profitability. They matched stocks into pairs by looking at the minimized distance between historical prices, and concluded that if the long and the short components fluctuate with some common nonstationary factors, then the two stocks would be assumed to be cointegrated and the pairs trading strategy could be implemented.

The authors formed stock pairs over a 12-month period (in-sample) and traded them in the next 6-month period (out-of-sample). The authors initiated trades when the divergence between a pair of stocks was more than two historical standard deviations away from each other, where the standard deviation is estimated during the in-sample period. They exited the positions when the next crossing of the prices occurs. Their empirical results suggest that the trading strategy generates more than 1.30% excess return on a monthly basis, which is considered significant in both an economical and a statistical sense. The authors suggest that pairs trading is profitable, with Sharpe ratios of between four to six times larger than the Sharpe ratio of the market.

Onwards, a key component when implementing a pairs trading strategy is determining when to initiate the buy and short shell signals. Huang and Martin (2018) investigated which strategies that work best for producing buy and short sell signals in a pairs trading strategy on a sample with 98 pairs of 152 stocks. Their in-sample period goes from 2012-01-01 to 2014-12-31, and their out-of-sample is from 2015-01-01 to 2016-06-01. They compared three main strategies for creating buy and short sell signals: the percentage strategy, the strategy of long-term standard deviation, and a Bollinger Bands strategy.

The percentage strategy sets the minimum of the residual as 0% and the maximum as 100%, and thereafter create buy and short sell signals depending on how the residuals between the cointegrated pairs are evolving over time. The strategy of long-term standard deviation is similar to the percentage strategy, but it uses multiples of long-term standard deviations to take the place of percentage levels. This means that the threshold for creating buy and sell signals are held constants throughout the period.

The Bollinger Bands strategy is a volatility indicator that creates an upper and a lower standard deviation band relative the moving average of the residual between the cointegrated stock prices. Their results showed that the Bollinger Bands strategy outperformed the other strategies and provided the highest annualized return rate per unit of risk. Moreover, 32% of their sample pairs ended up in a loss, in which 94% of the losses was explained by a break in the cointegration during the testing period.

Furthermore, when applying a pairs trading strategy, the length of the moving average used to create the buy and short shell signals have a substantial effect on the profitability of the trading strategy. Qvennerstedt and Svensson (2018) investigated the short– and long-run stability of cointegrated equity pairs on the Swedish equity market from 2005-04-01 to 2018-04-01. They applied a pairs trading strategy based on Bollinger Bands for creating buy and sell signals in the cointegrated stocks. The authors experimented with moving averages ranging between 30 and 200 days, and Bollinger Bands with standard deviations between 1 and 2.

The authors concluded that a shorter moving average and a higher standard deviation increased the returns, but in the end, they chose a higher moving average (MA200), because they wanted to investigate the long-run relationship of the selected pairs. They also applied two standard deviation Bollinger Bands. Their trading strategy generated the highest returns during volatile market conditions but underperformed the market during periods of lower volatility. For the whole examined period, the

Sharpe ratio from their trading strategy significantly beat the benchmark index during the same time period.

3. Theoretical Framework

Investment Companies & Investment Company Discount/Premium

An investment company is a company whose prime objective is to invest and often take active ownership in other companies and thereby achieving the best possible returns for its investors (Essen, 1997). In Sweden, investment companies enjoy tax reducing benefits if they attain the criteria of being an investment company. These criteria are somewhat approximate, and can be summarized to having a large and dispersed owner base, and that the holdings of the investment company are well diversified (Ne.se, 2019).

There are 14 investment companies on the primary Swedish stock market (Nasdaq OMX) that report their holdings quarterly (Ibindex.se, 2019). The largest company being Investor with an asset value of 448 BSEK (Quarterly report, 2019). The second largest is Kinnevik, with an asset value of 89 BSEK (Quarterly report, 2019).

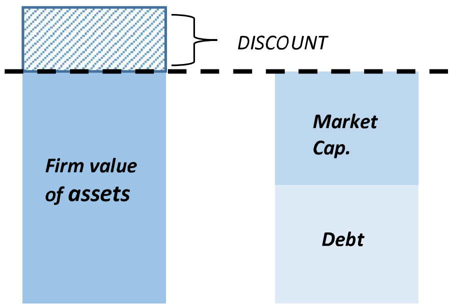


Figure 1. Concept of discount to net asset value

When valuing an investment company, it would be rational to assume that an investment company is fairly priced in relation to its assets and their cash flows (Datar, 2001). However, empirically this is not the fact, and many times Swedish investment companies are traded at a substantial discount in relation to its underlying assets. Historically, it is not uncommon for the discount to be more than 20 % (Essen, 1997). An illustration of the concept of an investment company’s discount to its net asset value is shown in figure 1. There have also been cases when the investment company's value exceeds that of its underlying assets; however, this is historically not as common. This is known as the investment company is traded at a premium in relation to its underlying assets.

There are many theories to why there exists such a divergence in fundamental pricing, but no clear answer to which is the real reason. The theories range from investment companies prioritizing the power of control rather than effective asset management, to lack of diversification possibilities, to the fact that investment companies often have unlisted assets which are hard to value at market price (ibid).

Pairs trading

Pairs trading is a mean-reverting trading strategy that exploits that some traded assets are cointegrated and therefore share a long-run relationship. The fundamental idea is to initiate trades when two cointegrated assets have deviated far enough from each other, with the hypothesis that these two assets will mean-revert back towards each other due to their cointegration. To take advantage of this hypothesis in a trading strategy, an investor takes a long position in the asset that is relatively lower priced compared to its companion asset, while simultaneously taking a short position in the relatively high priced asset. It is therefore only possible to make gains or losses when the traded assets move relative to each other. Pairs trading can be applied to any two pairs that are cointegrated, e.g., different commodities, indices, stocks, and currencies.

By only speculating on the residual divergence between two assets, the trading strategy is not affected by whether or not the market is performing well. Consequently, the pairs trading strategy is considered market risk neutral.

Consider the following cointegrated traded assets y_t and x_t as defined in Appendix 3:

(1.1) $$y_t = \beta_1 + \beta_2 x_t + u_t$$

The cointegration residuals can be written as:

(1.2) $$u_t = y_t - \beta_1 - \beta_2 x_t$$

When the divergence between the traded assets y_t and x_t reaches a pre-specified threshold value, a trade opportunity arises due to the cointegration between the variables being likely to close the divergence. If u_t is above (below) its threshold value, the pairs trading strategy is to take a long (short) position in x_t and a short (long) position in y_t.

The profit of the trade may be described as follows:

(1.3) $$\Pi_t = M(-\Delta y_t + \gamma_1 \Delta x_t) = -M\Delta u_t$$

where M is the amount invested, y_t and x_t are the traded stock prices, and γ_1 is the hedge ratio, offsetting the long and short positions in the trade. Equation (1.3) also applies if the cointegration residual is below its long-term level, but with opposite signs on the traded assets (Figuerola-Ferretti, Paraskevopoulos and Tang, 2017).

How to determine the pre-specified threshold values that initiate trades will be described next.

Moving Averages & Bollinger Bands

A moving average (MA) is commonly used in technical analysis, and is the average of a time series over a defined number of time units. It can formally be defined as follows:

(1.4) $$MA(n) = \frac{A_1 + A_2 + \cdots + A_n}{n}$$

where *A* is the average in period *n*, and *n* is the number of time units.

The Bollinger band was first invented by John Bollinger in the 1980s, and have evolved from the concept of trading bands used as volatility indicators. A Bollinger Band consists of an upper bound and a lower bound of a time series relative to the value of the time series (Huang and Martin, 2018). The following components are needed to construct Bollinger Bands:

1. An N-period moving average (MA) of time series data,
2. An upper band at K times an N-period standard deviation above the MA (MA + Kσ),
3. A lower band at K times an N-period standard deviation below the MA (MA – Kσ)

Short Selling

Short selling a stock can be defined as selling a security that the seller does not own, or that the seller owns but does not deliver to the purchaser. The seller borrows the security, usually from an institutional investor, on which the short sellers pay interest. The interest rate is less than 1% per annum for the majority of stocks that are possible to short. When the borrower pays back the loan, it will have profited if the stock price on the borrowed stock has declined (D'Avolio, 2002).

Sharpe ratio

The Sharpe ratio was introduced by Nobel laureate William F. Sharpe and is a performance metric that measures risk-adjusted returns (Rivin, 2018). The Sharpe ratio is defined as:

(1.5) $$SR_{\alpha} = \frac{E[R_{\alpha}] - R_f}{\sigma_{\alpha}}$$

where $E[R_{\alpha}]$ is the expected return of asset α, R_f is the risk-free return rate, and σ_{α} is the standard deviation of asset α's returns. The intuition behind using the Sharpe ratio as a performance metric is that it describes the risk-premium (expected excess return) per unit of risk, measured as standard deviation.

4. Methodology

Description of Data

There are 14 investment companies on the primary Swedish stock market (Nasdaq OMX) that report their holdings quarterly (Ibindex.se, 2019). The three investment companies with the highest percentage of listed assets are chosen to be part of this paper. The selected investment companies are Bure Equity, Industrivärden och Lundbergföretagen.

The reason for choosing these investment companies is that a higher percentage of listed assets will filter out unwanted "noise" in the cointegration regressions since it will not be possible to take long and short positions in unlisted assets. Additionally, the daily estimation of the asset value becomes more accurate since the value of the unlisted assets are relatively small.

The reason for only choosing three investment companies is due to the time-consuming process of downloading the needed data to apply a pairs trading strategy.

The sample period is from 2012-04-02 until 2019-03-29, which results in 1825 daily returns observations. The sample period will further be decomposed into an in-sample period for model building (2012-04-02 to 2017-03-29) and an out-of-sample for testing the model out-of-sample (2017-03-30 to 2019-03-29).

Research Approach

To successfully implement a pairs trading strategy based on cointegration, the assets must be cointegrated. Hence, the first step is to validate that the price series of the basket of underlying assets are cointegrated with the price series of the corresponding investment company. It is reasonable to believe that these price series are cointegrated since the net asset value of the investment company assets should be what drives the value of the investment company itself.

If a cointegration relationship is confirmed, the next step is to build an algorithm that initiates trade signals based on the discrepancy between the two prices series. The model is built over an in-sample period of five years (2012.04.01 - 2017.03.29).

During this period, the algorithm produces a long and short sell signals when the two price series have deviated significantly from their long-term equilibrium. The algorithm then produces an exit signal when the deviation has reverted back to its moving average. Although the algorithm for each investment company is based on the deviation between the price series, the sensitivity adjustments may differ from company to company as there their relationship with their underlying assets may differ.

The in-sample period is used to estimate the cointegration regression coefficients. These are applied to the two-year out-of-sample trading algorithm (2017-04-01 to 2019-03-27). Lastly, the performance of the trading strategy is evaluated by calculating the Sharpe ratio of the annualized returns and standard deviation of the portfolio.

Data Collection Method

For each investment company, two price series are needed to be able to apply the pairs trading strategy. The price series for the stock price of the investment company is retrieved from Bloomberg terminal. Daily prices are used, whereas each observation represents the last traded price of that day. The choice of daily prices is based on the nature of the trading strategy. The model is not supposed to perform high-frequency trading. This is based on the fact that the deviation between the two price series is believed to be a much slower moving process that develops over weeks or even months. Nonetheless, daily prices are chosen since it implies more observations and improved accuracy for the choice of initiating trading signals.

The second price series is not as straight-forward to calculate since it is a basket of the investment company's listed assets. The daily returns of each investment company's listed assets are downloaded from Bloomberg terminal. The value of the unlisted assets are estimated quarterly by the investment companies and are thus assumed to be constant until the next quarterly report is released. Since the unlisted assets are a minor part of the total value, it should not have any significant effect on the daily estimation. Moreover, since we are interested in the how the stock price of the investment company reacts to stock price changes in their listed assets, the relationship becomes more transparent and clearer to act upon when the unlisted assets are assumed to be constant.

To make the two price series comparable, the value of the total assets is divided by the number of shares outstanding of the investment firm such that it becomes a stock price of a sort. The calculation of how much each listed asset is contributing to the total stock price varies slightly depending on how the investment company structures their quarterly reports. While some firms specify precisely how many shares they hold of each listed asset, others only release the percentage of total shares they hold of each listed asset. If the investment firm announces the exact amount of shares they hold of each asset, the calculation is straight forward. The daily, last traded price of each listed asset is retrieved from Bloomberg terminal. The assets are then summed up to a total value and lastly divided by the number of shares outstanding in the investment company. For firms that only announce their percentage of ownership in each listed asset, the calculation is done differently. For each listed asset, the given percentage is multiplied by the total market capitalization of the company. Similarly, the calculated value of each listed asset is then summed up to total value and divided by the number of shares outstanding in the investment company.

Additionally, although not so frequently, investment companies reweight their holdings. The change in owned shares may increase or decrease depending on the investment companies new assessment of that asset. The investment company might also acquire new assets or sell existing assets. Though the changed position may occur at any point during the quarter, it is accounted for when the quarterly report is released. This may cause the value of the estimated asset price to jump significantly for the day when the report is released due to the new allocation of assets released in the report. To adjust for this jump, the trading algorithm is set up in such a way that an exit signal is produced the day before the report is released. A new enter signal is generated the day after the jump caused by the report. This is done to ignore any artificial potential gains or losses caused by an increase in asset value, which would not affect the portfolio if the strategy was implemented in actuality. In other words, all positions are terminated the day before the reports are released and re-entered once the holding allocation is updated.

Determining Cointegration

First, the prices series are tested for non-stationarity. According to financial theory, the prices series are expected to be non-stationary, which is also a condition for the price series to be cointegrated. This is tested with an Augmented Dickey-Fuller test, as shown in appendix 5, where the null hypothesis is not expected to be rejected.

A simple linear Ordinary-Least-Squares regression is used to estimate the residual time series. The regression coefficients are also saved and used for estimating the out-of-sample period residuals. The Augmented Dickey-Fuller test is then performed on the residual time-series to check for stationarity. If the null hypothesis is rejected for all investment companies, it implies that the residuals are stationary. Consequently, pairs trading strategies are justified. The statistical software used for this will be Eviews® 10.

For economic intuition, this paper uses the net asset value of the underlying assets as the regressor and the stock price of the investment company as the dependent variable since it is reasonable to believe that an investment company's value is determined by the value of its underlying assets.

Establishing Trade and Exit Signals

The basic idea of pairs trading says that if the cointegration residual is significantly less than zero, the strategy is to take a long position in the investment company and a short position in the basket of listed underlying assets. For example, if the investment company has on average been trading at a five percent discount in relation to the underlying assets, but is now trading at a 20 percent discount, which exceeds K standard deviations from the moving average, then the algorithm gives a buy signal for the investment company and hence, simultaneously, a short sell signal for the basket of underlying listed assets. The trades are then live in the market until an exit signal is given when the discount factor moves back to its moving average.

The algorithm works similarly for when the cointegration residual is significantly larger than zero. In this case, the algorithm is signaling to short sell the investment company and go long in the underlying assets, with the hypothesis that the divergence will return to its long-run relationship.

To determine the long-run relationship between the investment company and the underlying assets, a moving average is used on the residuals from the cointegration regression. An earlier study done by Qvennerstedt and Svensson (2018) suggested moving averages from 30 days up to 200 days. However, when examining the movement of the trading discount occurring within investment companies, it appears to move at a much slower rate. Thus, a short moving average such as 30 days will only capture noise in the cointegration relationship and not the real trend in their long-term equilibrium. Additionally, short moving averages become much more sensitive to estimation errors caused by the unlisted assets in the data set as one or two outliers affects the moving average a lot more than the longer and more robust moving averages. Therefore, the following five moving averages will be tested; MA50, MA100, MA200, MA300, and MA400.

Furthermore, the degree of deviation from the moving average is given by the Bollinger Bands. The width of the Bollinger Bands represents the sensitivity to initiating the buy and short sell trades. The wider the standard deviation in the Bollinger Bands, the further away the residuals, from the cointegration regression, must deviate from the moving average before the algorithm initiates the trades. Correspondingly, the narrower the width, the faster the algorithm acts on any deviation from the mean. Hence, a more narrow Bollinger Band results in a more sensitive and high-frequency trading model, since the deviation from the mean does not have to be particularly severe for the algorithm to produce long and short signals. The reason for using the Bollinger Bands to create trade signals is justified by being the method that empirically yielded the highest returns (Huang and Martin (2018).

When it comes to the exiting or the trade terminating signal, the most obvious choice is to exit when the two price series have reverted to their moving average, due to the mean-reverting process of the residual.

Evaluating the Trading Strategy

The portfolios Sharpe ratios will be compared to the Sharpe ratio of the market during the same time period, to provide a perspective of how good or bad the strategy performs. This will be done by taking the annualized return minus the risk-free rate, divided by the standard deviation. The Swedish risk-free rate is collected from Investing.com (2019).

The standard deviation will be calculated by subtracting the daily average return for each traded day and summing these up. Next, this is divided by the number of trading days to get the daily standard deviation. To arrive at the annual standard deviation, this will then be scaled up by the square root of 252 multiplied with the average number of traded days per annum.

In order to test the significance of the results, a p-value is estimated under the null hypothesis that the true value of each Sharpe ratio is zero. The p-values stem from the assumption that the Sharpe ratios are asymptotically normally distributed. The one-sided test statistic is then calculated by dividing the Sharpe ratio by its standard error. The standard error is estimated by using the following formula (Opdyke, 2007).

$$Standard\ Error = \sqrt{\frac{(1+0.5\times SR^2)}{N}} \tag{1.6}$$

where SR is the daily Sharpe ratio and N is the number of days in the period.

5. Results & Analysis

Stationarity Testing

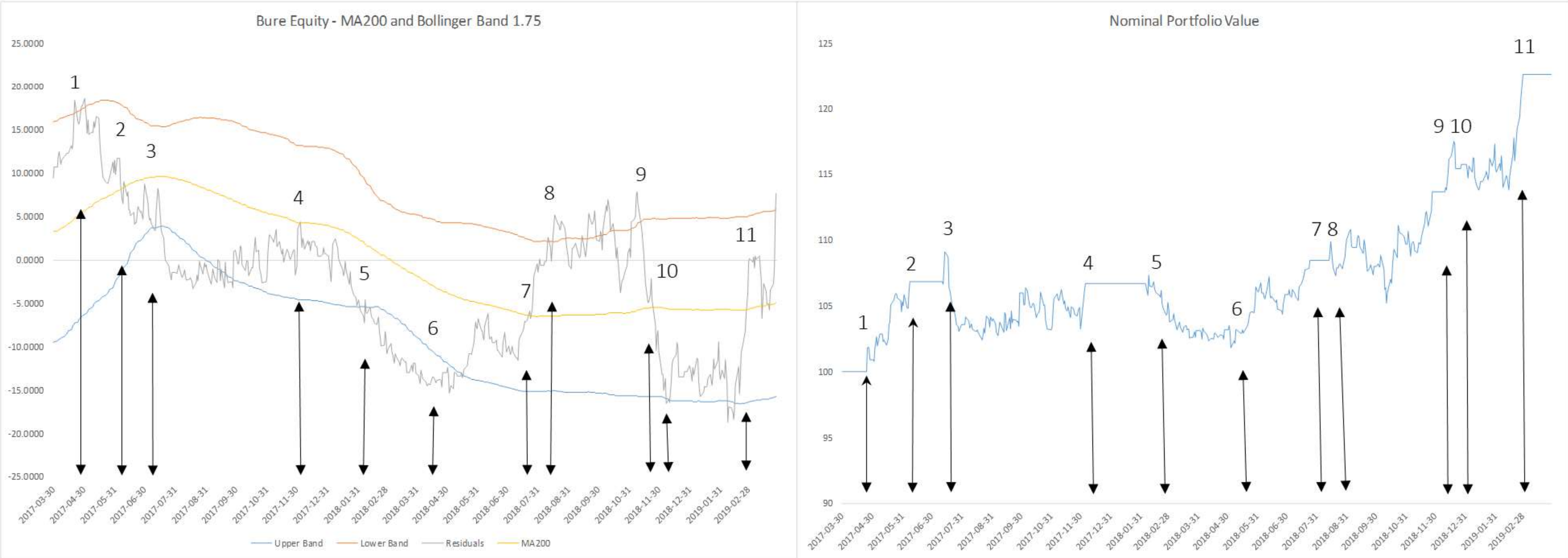
To verify that a pairs trading strategy is justified and have the potential to be profitable, the price series and the cointegration residuals are tested for stationarity for each of the three investment companies by using Augmented Dickey-Fuller tests. At a 90 percent confidence interval, the null hypothesis of non-stationarity in the cointegration residuals are rejected for all three investment companies. Conversely, at the same confidence interval, it is not possible to reject the null hypothesis of non-stationarity in the respective price series by themselves. Hence, the criteria for a successful pairs trading strategy might be prevailing in each of the three investment companies. The full results are presented in appendix 5.

Creating the Out-of-sample Coefficients

To estimate the out-of-sample regression coefficients, and thereby also the cointegration residuals, the in-sample data is used. In the sample period, the underlying assets of each of the investment companies are regressed on the market equity value of the investment companies. The coefficients obtained are presented in appendix 6.

Example of a Pairs Trading Strategy Using MA200 and 1.75 ST. DEV. Bollinger Bands

To give the reader a clearer picture of a pairs trading strategy, an example is illustrated below. The investment company in this example is Bure Equity, the moving average of the residual is set to 200 days, and the Bollinger Bands are based on 1.75 standard deviations from the moving average. These parameter settings resulted in a Sharpe ratio of 1.107. A detailed description of bullet points 1 – 11 follows on the next page.



Graph 1. Example of a Pairs Trading Strategy

At (1) the residual from the cointegration regression passes the upper Bollinger Band, which signals that the investment company is relatively highly valued in relation to its assets. This initiates a buy and short sell signal, based on the hypothesis that the prices series will mean-revert back to its long-run equilibrium. The gap between the price of the investment company and its listed assets is expected to increase back to its long-run average. The trading strategy is then to short sell the investment company, Bure Equity, and buy an offsetting position in the basket of the investment company's listed assets.

At (2) the residual has decreased and passes the 200-day moving average, which successfully closes the two positions. The profit is shown in the right-hand side graph, which illustrates the nominal portfolio value.

No new position is taken until the next trade signal is received. This is reflected in the right-hand side graph as the nominal portfolio value remains constant. The next trade signal happens at (3), where the residual crosses the lower band. At this time, the opposite scenario to (1) takes place. The residual is now relatively low, meaning that the investment company is now valued relatively low compared to its underlying assets, making the pairs trading strategy to take on a long position in the investment company, and short sell the basket of underlying listed assets. The positions are exited at (4), where the residuals and the MA200 briefly crosses.

At (5) the next trade signal appears, with the same strategy as at (3). This time, the gap between the price series initially becomes smaller, causing the portfolio to decrease in value. This is shown in (6) in the right-hand side graph. However, since the residuals haven't crossed the MA200, the positions are not un-winded, and are kept until (7), and thereby exiting with a profit. At (8) new positions are taken, which are closed at (9) when the residuals pass MA200. The last trade takes place between (10) and (11).

The strategy resulted in an absolute return of 16.16% during the out-of-sample period. The strategy held positions 59% of the time. This resulted in an annualized excess return of 8.13%, and an annual standard deviation of 7.35%, which in turn resulted in a Sharpe ratio of 1.107. The portfolios daily return had a -0.005 correlation with the Swedish market (OMX30).

In-sample and Out-of-sample Sharpe ratios

The Sharpe ratios obtained when applying a pairs trading strategy for the in-sample and out-of-sample period, for each of the investment companies, are presented below in table one through three. The tables show the Sharpe ratios for different Bollinger Bands, ranging from 1.00 – 2.25 standard deviations, combined with different moving averages, ranging from MA50 to MA400. The number in parenthesis seen below each Sharpe ratio is the calculated p-value with the null hypothesis that the true value of the strategy's Sharpe ratio is zero.

Bure Equity - In-Sample Sharpe Ratios

2012.04.02 - 2017.03.29

Bollinger Band	*MA50*	*MA100*	*MA200*	*MA300*	*MA400*
1.00	0.493 (0.18)	1.152 (0.01)	1.203 (0.01)	1.046 (0.02)	0.624 (0.12)
1.25	0.749 (0.08)	0.941 (0.04)	1.06 (0.02)	1.185 (0.01)	0.733 (0.08)
1.50	0.664 (0.1)	1.109 (0.02)	1.072 (0.02)	1.006 (0.03)	0.837 (0.06)
1.75	0.634 (0.12)	0.987 (0.03)	0.93 (0.04)	0.935 (0.04)	0.862 (0.05)
2.00	0.668 (0.1)	0.969 (0.03)	0.834 (0.06)	0.982 (0.03)	0.906 (0.04)
2.25	0.581 (0.14)	0.855 (0.05)	1.07 (0.02)	0.982 (0.03)	1.047 (0.02)

Market Sharpe ratio 0.371

Out-of-Sample Sharpe Ratios

2017.03.30 - 2019.03.29

Bollinger Band	*MA50*	*MA100*	*MA200*	*MA300*	*MA400*
1.00	-0.387 (0.71)	0.289 (0.34)	0.628 (0.18)	0.554 (0.21)	1.579 (0.01)
1.25	-0.282 (0.66)	0.465 (0.25)	1.001 (0.08)	0.854 (0.11)	1.362 (0.03)
1.50	-0.07 (0.54)	0.41 (0.28)	1.238 (0.04)	1.294 (0.03)	1.31 (0.03)
1.75	0.048 (0.47)	0.206 (0.38)	1.107 (0.06)	1.616 (0.01)	1.76 (0.01)
2.00	0.173 (0.4)	0.5 (0.24)	0.871 (0.11)	1.493 (0.02)	2.033 (0.00)
2.25	0.045 (0.47)	0.606 (0.19)	0.59 (0.2)	1.067 (0.06)	1.978 (0.00)

Market Sharpe ratio -0.058

Tabell 1 and 2. In and out-of-sample Sharpe ratios for Bure Equity.

Industrivärden - In Sample Sharpe Ratios

2012.04.02 - 2017.03.29

Bollinger Band	*MA50*	*MA100*	*MA200*	*MA300*	*MA400*
1.00	0.049 (0.46)	0.785 (0.07)	1.149 (0.01)	1.068 (0.02)	1.282 (0.01)
1.25	0.189 (0.36)	0.87 (0.05)	1.203 (0.01)	1.214 (0.01)	1.379 (0.00)
1.50	0.188 (0.36)	0.92 (0.04)	1.257 (0.01)	1.213 (0.01)	1.327 (0.01)
1.75	0.22 (0.34)	1.03 (0.03)	1.351 (0.01)	1.415 (0.00)	1.26 (0.01)
2.00	0.386 (0.23)	1.165 (0.01)	1.385 (0.00)	1.046 (0.02)	1.268 (0.01)
2.25	0.476 (0.18)	0.993 (0.03)	0.952 (0.04)	1.554 (0.00)	1.206 (0.01)

Market Sharpe ratio 0.371

Out-of-Sample Sharpe Ratios

2017.03.30 - 2019.03.29

Bollinger Band	*MA50*	*MA100*	*MA200*	*MA300*	*MA400*
1.00	-0.07 (0.55)	-0.183 (0.64)	-0.035 (0.53)	0.744 (0.08)	1.38 (0.00)
1.25	-0.273 (0.7)	-0.01 (0.51)	0.1 (0.42)	0.607 (0.13)	0.963 (0.03)
1.50	-0.339 (0.74)	-0.161 (0.62)	-0.186 (0.64)	0.371 (0.24)	0.811 (0.06)
1.75	-0.425 (0.79)	-0.41 (0.78)	-0.036 (0.53)	0.865 (0.05)	0.856 (0.05)
2.00	-0.204 (0.65)	-0.488 (0.82)	0.147 (0.39)	0.893 (0.05)	0.319 (0.27)
2.25	-0.512 (0.83)	-0.424 (0.79)	0.205 (0.35)	0.679 (0.1)	0.394 (0.23)

Market Sharpe ratio -0.058

Table 3 and 4. In and out-of-sample Sharpe ratios for Industrivärden.

Lundbergföretagen - In-Sample Sharpe Ratios

2012.04.02 - 2017.03.29

Bollinger Band	*MA50*	*MA100*	*MA200*	*MA300*	*MA400*
1.00	-0.092 (0.57)	0.223 (0.34)	0.768 (0.07)	0.645 (0.11)	0.46 (0.19)
1.25	0.041 (0.47)	0.272 (0.3)	0.759 (0.08)	0.654 (0.11)	0.415 (0.22)
1.50	-0.031 (0.52)	0.162 (0.38)	0.401 (0.22)	0.572 (0.14)	0.408 (0.22)
1.75	0.11 (0.42)	0.225 (0.33)	0.531 (0.16)	0.377 (0.24)	0.459 (0.19)
2.00	0.181 (0.37)	0.282 (0.3)	0.497 (0.17)	0.465 (0.19)	0.508 (0.17)
2.25	0.093 (0.43)	0.203 (0.35)	0.585 (0.13)	0.495 (0.17)	0.583 (0.14)

Market Sharpe ratio 0.371

Out-of-Sample Sharpe Ratios

2017.03.30 - 2019.03.29

Bollinger Band	*MA50*	*MA100*	*MA200*	*MA300*	*MA400*
1.00	0.873 (0.05)	0.095 (0.43)	-0.147 (0.61)	-0.363 (0.75)	-0.19 (0.64)
1.25	0.589 (0.13)	0.145 (0.39)	-0.029 (0.52)	-0.294 (0.71)	-0.131 (0.6)
1.50	0.823 (0.06)	0.318 (0.27)	-0.338 (0.74)	-0.236 (0.67)	-0.09 (0.57)
1.75	0.329 (0.27)	0.403 (0.22)	-0.308 (0.72)	0.042 (0.47)	-0.004 (0.5)
2.00	0.161 (0.38)	0.417 (0.22)	-0.241 (0.68)	-0.182 (0.63)	-0.249 (0.68)
2.25	-0.009 (0.51)	0.71 (0.09)	-0.182 (0.63)	-0.062 (0.55)	-0.182 (0.63)

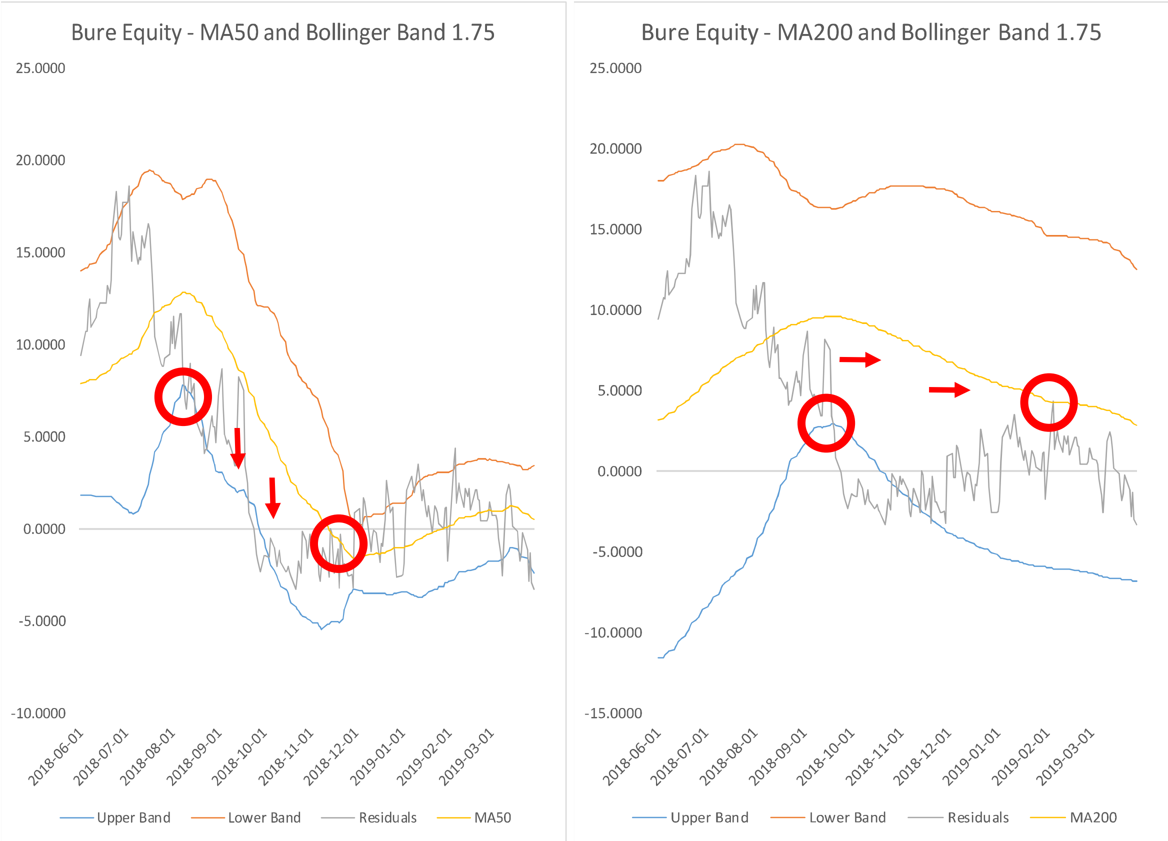
Market Sharpe ratio -0.058

Table 5 and 6. In and out-of-sample Sharpe ratios for Lundbergföretagen.

Interpretation of the In-Sample and Out-of-Sample Sharpe Ratios

When analyzing the results from the different pairs trading strategies, it is clear that the obtained Sharpe ratios for the three investment companies share some similarities. By comparing how the model performs for different moving averages, a clear pattern can be seen under the in-sample period. Here, the longer moving averages outperform the shorter moving averages, except for Bure Equity which is relatively stable for MA100 or higher. The relationship is best displayed in Industrivärden, where each step towards a longer moving average produces better results. During the in-sample period, the residuals show a stationary behavior with a clear mean-reverting property. The deviation away from its long-term equilibrium occurs under more prolonged periods of time, which suggest that the mean-reverting property is quite slow. Thus, explaining that the longer moving averages perform better as they capture the movement of the cointegration residual in a more accurate manner.

The shorter moving averages such as MA50 and MA100 may react too early on deviations away from the mean and hence predict the two time series to mean-revert back too soon when in reality the deviation may continue to grow for some time. This is illustrated in graph two below. On the left-hand side, MA50 is used, and on the right-hand side, the MA200 is used. Since the MA50 is too quick to adjust for the sudden drop of in the residual value, the unwinding of the positions results in a loss. This can be compared with the more slowly adjusting MA200, which stays at a higher level during the same time period, and thus lets the residuals bounce back, and the trades are closed with a profit.



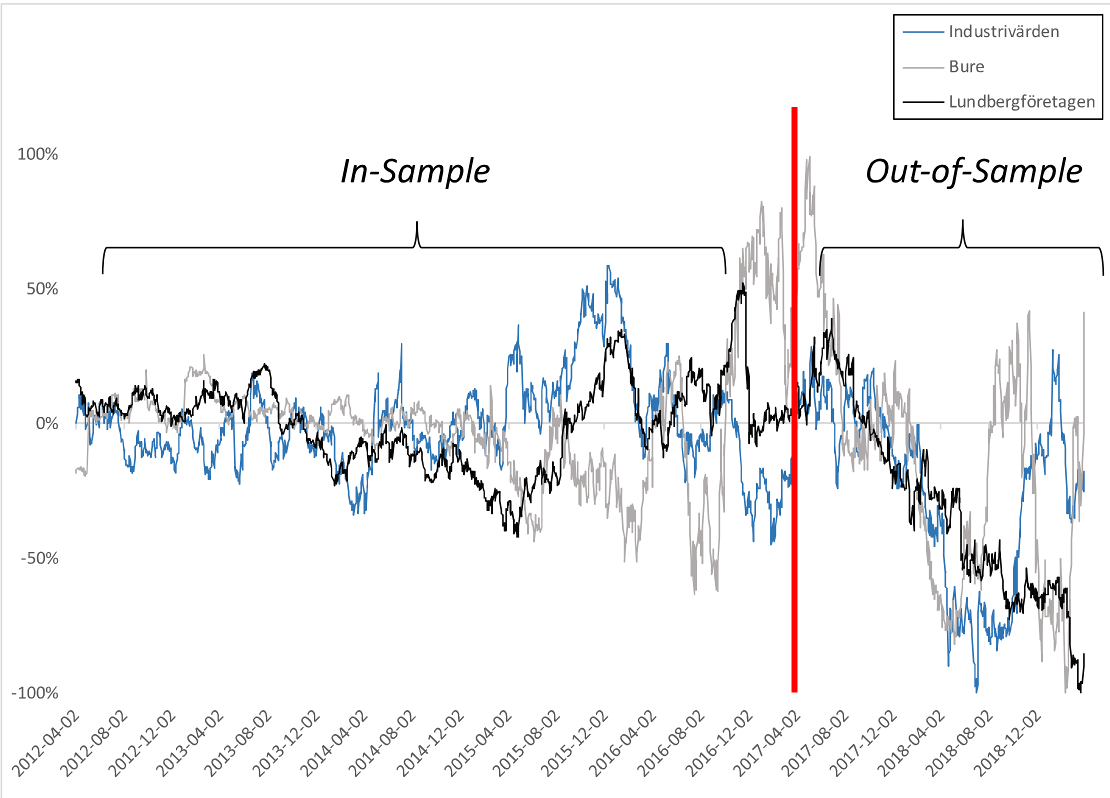
Graph 2. Trade comparison: MA50 and MA200

Moreover, the choice of Bollinger band appears to affect the results, although not as much as the choice of moving average. Wider Bollinger Bands such as 1.5 to 2 standard deviations away from the mean, tend to give more robust results. Intuitively, this makes sense since the model then only reacts to signals that are "strong," or perhaps more certain. However, too wide Bollinger bands display volatile results. This may be since as the Bollinger bands get wider, the number of trades become fewer. Consequently, the performance of the model may be evaluated on a minimal amount of trades, and thus, the proportionate importance of each trade becomes very high.

As can be seen in the graph below, the cointegration residual, and subsequently, the difference between each investment company and its underlying assets behave in a reasonably stationary manner during the in-sample period. There is a clear mean and a somewhat constant variance, although the variance appears to increase towards the end of the in-sample period. Accordingly, the pairs trading strategy performs rather well during this period and outperforms the market Sharpe ratio (0.371) in almost all cases.

The stationary properties observed during the in-sample period does not seem to be present in the out-of-sample period. A clear downward trend in the cointegration residual suggests a temporary structural break in the cointegration relationship for all three investment companies. This sudden change in the behavior of the residual may be what causes the poor results in the out-of-sample period for Lundbergföretagen. The moving averages react to the decreasing residual and produce a buy and short sell signals with the belief that it will revert to its long-term equilibrium; however, the trend continues downward, and thus the model performs poorly. Industrivärden and Bure Equity do seem to recover towards the end of the out-of-sample period while Lundbergföretagen continues to diverge from its former equilibrium. Moreover, the market observed negative returns over the same period (-0,058 Sharpe ratio). Thus, all positive returns from the pairs trading strategy during this period are outperforming the market.

The p-values indicate the statistical significance of the results. The Sharpe ratios produced from applying the pairs trading strategy to the in-sample period show that most of them are statistically larger than zero at a ten percent significance level. However, for the out-of-sample Sharpe ratios, only a few are able to reject the null hypothesis of the Sharpe ratio being zero. Since the out-of-sample period is shorter than the in-sample period, the standard error becomes larger, which makes it more difficult to reject the null hypothesis. Nonetheless, the out-of-sample Sharpe ratios from Bure Equity still produce returns that are statistically larger than zero, again using a 10 percent significance level.



Graph 3. In and out-of-sample cointegration residual time series for all firms

Bure Equity: Out-of-Sample Results and Analysis

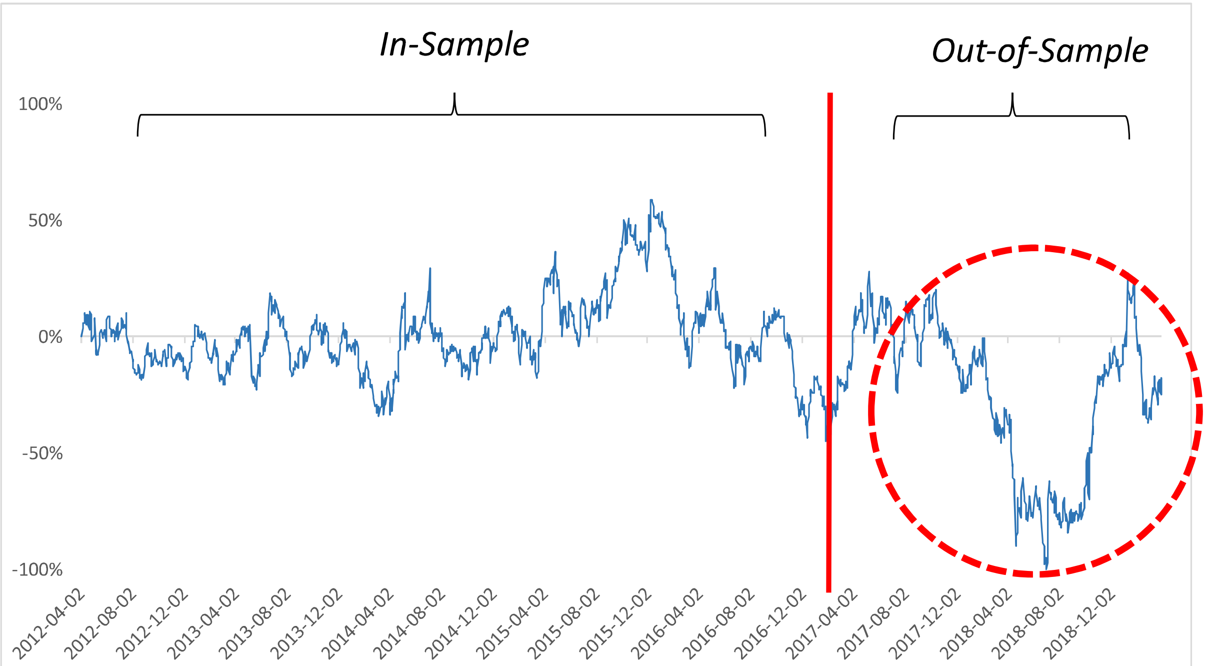
The out-of-sample results matrix for Bure Equity shows a clear progression of improved performance as it moves towards the longer moving average. The longest moving average, MA400 performs best with a peak of an impressive 2.033 in Sharpe ratio. The above mentioned structural break in the cointegration relationship is not as severe in Bure Equity´s case. Although the variance of the cointegration residual seems to increase during this period, it is deviating around the same mean as before. Correspondingly, the model still works as the long-term equilibrium relationship between the two time series does not seem to have changed. Furthermore, the increasing volatility happens gradually, so the Bollinger Bands can adjust for the increasing standard deviation and hence continue to give accurate buy and short signals under the continuously changing circumstances. This can be seen in the graph below.

Graph 4. Bure Equity cointegration residual time series

Since the deviations from the mean during this period are a lot more tenacious. The shorter moving averages do not work since they are too quick to adjust the mean as the deviation persists over a more extended period of time.

Industrivärden: Out-of-Sample Results and Analysis

Industrivärden somewhat follows the same pattern in the out-of-sample period as in the in-sample period. The shorter MA50 and MA100 yield negative excess returns, and even most of MA200 returns are negative. This is mostly due to one extreme deviation in the investment company's discount to its underlying assets. The deviation is depicted in graph five below. With the same reasoning as previously mentioned, this structural break in the cointegration explains why the short and medium long moving average performs poorly. Industrivärden has had a reasonably constant mean and variance up until this point in time, keeping the residuals stationary enough to profitable apply a pairs trading strategy. However, when the relationship between the investment company and its underlying assets fundamentally changes, the pairs trading strategy suffers.

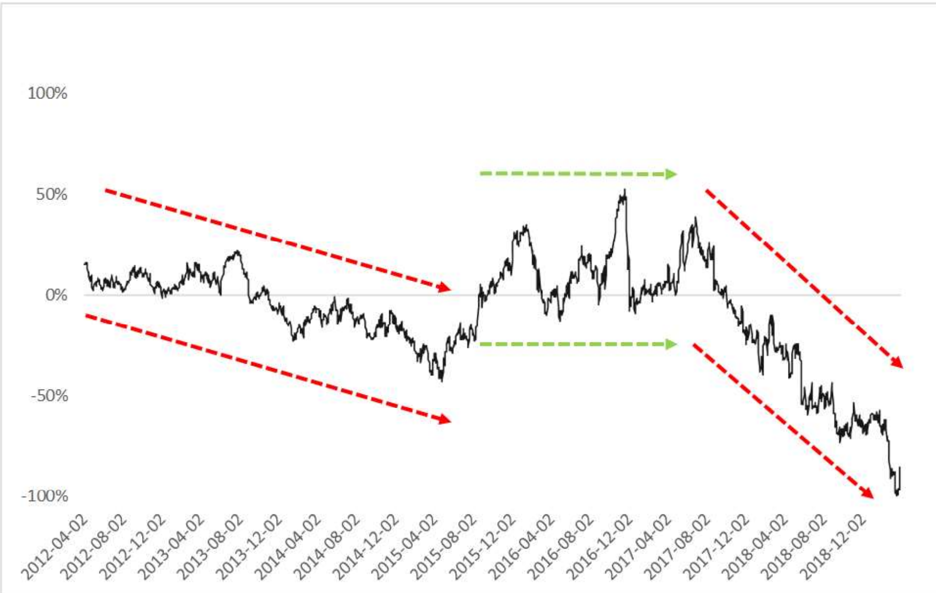


Graph 5. Industrivärden cointegration residual time series

If the chosen strategy had been using the MA300 or MA400, the trading strategy would have yielded positive excess returns. This is mostly because longer moving averages waits for these extreme deviations to mean-revert, and therefore not taking on significant losses in the meantime. This can be seen in the graph above since the sudden severe dip in the residual seems to last for about 1,5 years, which is approximately 375 trading days. Shorter moving averages would take on losses on the way down the dip, and thereafter take on losses on the way up. However, forecasting the sudden increase in residual volatility could be an impossible task.

Lundbergföretagen: Out-of-sample Results and Analysis

From a pairs trading point of view, Lundberg is behaving quite different from Bure Equity and Industrivärden. The overall Sharpe ratios from the in-sample period are generally lower than from the other investment companies. This is most likely due to that the residuals from the cointegration are behaving differently than would be preferred to apply a pairs trading strategy. In the first half of the in-sample period, the mean does not seem to be constant. This will cause the trading algorithm to take on positions in the hope of the residual bouncing back, which then never happens. In the second half of the in-sample period, this seems to change, and the residuals are frequently crossing the residual mean, resulting in a profitable pairs trading strategy. Here, the longer moving averages, such as MA300 and MA400, are performing relatively poorly, since the residuals come from low levels, the moving average needs some time to adjust back to the long-run mean. The next period of interest is the out-of-sample period. Here it can be seen that the residual, meaning the relationship in price level between Lundbergföretagen and its underlying assets, is continuously decreasing during the out-of-sample period. Thus, a pairs trading strategy is not applicable since the residual is not appearing to be stationary. This observation is in line with the test statistic observed in the Augmented Dickey-Fuller test, where Lundbergföretagen performed the worst in terms of the desired stationary properties of its cointegration residual.



Graph 6. Industrivärden cointegration residual time series

6. Discussion & Conclusion

Pairs trading is based on sophisticated statistical methods to develop a high-tech automated trading system, by taking out intuition and trading "skill" out of arbitrage and replacing it with disciplined, consistent filter rules. The trading strategy has proven to yield abnormal returns in several studies since it was first invented in the 1980s.

In this paper, the pairs trading strategy has been applied to the idea of a cointegration relationship between three investment companies and their underlying assets. The results from these trading strategies have been mixed, but some general trends are distinguishable. During both the in and out-of-sample period, the difference between the three investment companies and their underlying assets seem to be increasingly volatile. This volatility is necessarily not negative for applying a successful pairs trading strategy since higher volatility around the long-run mean would increase the profitability of a pairs trading strategy. However, it induces some issues when the algorithm creates buy and sell signals if the increase in volatility happens rapidly since the entering of some positions occurs too early and therefore the profitability of the strategy is not maximized.

The main issue with applying a pairs trading strategy to the investment companies seems to be that the overall long-run mean is changing over time. This explains why it is possible to obtain higher returns on Bure Equity since the mean-reverting process is more apparent than in, e.g., Lundbergföretagen. The main issue is that these structural breaks happen without notification, which brings a lot of uncertainty to the pairs trading strategy overall. Ideally, the optimal performing in-sample parameters would also be the optimal out-of-sample parameters to ensure the consistency of the pairs trading strategy. However, this is not what is being observed in this paper, since the optimal in-sample settings deviate from the optimal out-of-sample settings.

When putting the stationarity violations aside, there seems to be at least some possibility for successfully applying a pairs trading strategy to investment companies and their underlying assets. An excellent example of this is Bure Equity, where the highest excess returns yielded a Sharpe ratio of above 2.0. This could be compared to the Swedish market (OMX30) Sharpe ratio during the same period of –0,058.

Another major key takeaway is that the performance is more robust and steady in the in-sample period. This could be due to that the mean-reverting processes are stronger there, or it could only be because of a longer time period, which increases the probability of having a mean-reverting process. In a sense, the out-of-sample period in this paper might be deemed "unlucky" since both the mean and volatility seems to be changing at a much higher rate than during the in-sample period for all investment companies. However, it is not possible to say if this is because the fundamentals of the time series relationship have changed, or if these structural changes are only temporary. Whatever the reason, it stands clear that this has an apparent negative effect from a pairs trading strategy point of view. Relating to the study by Huang and Martin (2018), the majority of the failed pairs trading strategies was because there was a structural break in the cointegration relationship during the sample period. Similarly, the negative returns observed in Lundbergföretagen are caused by the same reason where the fundamental relationship suddenly changed during the sample period.

Furthermore, the issue of consistency in the performance of the different Bollinger bands and moving averages is problematic. There does seem to be some degree of randomness in what Bollinger bands perform best. Also, as the volatility of the regression residual increases in the out-of-sample period, the optimal choice of moving average changes. As a consequence, it is difficult to choose the optimal settings for the out-of-sample period by analyzing the results in the in-sample period. Thus, if this strategy was implemented, and based on the in-sample results, it is unlikely that one would choose the settings that would work best for the out-of-sample period. Nonetheless, everything is relative, although one may not have selected the ideal settings, the returns would almost certainly be positive (excluding Lundbergföretagen). It is the robustness of the performance that is desirable. If more data were available, for example, if tens or even hundreds of in-sample periods were available, the idea would be to choose the setting that maximizes the average Sharpe ratio across all periods. Hence, if one strategy from the Sharpe ratio matrix would have to be selected, it would not be the one that yields the highest Sharpe ratio, but the one that appears to be most robust in relation to its "neighbors" in the table.

It can be concluded that applying a pairs trading strategy on [these three] Swedish investment companies and their underlying listed assets has the potential to be profitable. Under certain conditions, it has the potential to beat the market Sharpe ratio. However, the profitability varies and depends on the parameter settings, and on which company the trading strategy is applied to. Thus, to increase the probability of successfully implementing this strategy, it would be recommendable to diversify the risk by applying the strategy on several investment companies at the same time. Relating this to the results obtained, both Bure Equity and Industrivärden generated lucrative Sharpe ratios during the out-of-sample period (using longer moving averages), while Lundbergföretagen yielded negative excess returns. Therefore, applying a trading strategy based on these three investment companies would have been likely to have yielded satisfactory excess returns compared to the Swedish market.

Some practical limitations have not been addressed in this paper. One would be the transaction cost of each trade, such as courtage and the issue of a bid-ask spread which could lower the overall profitability of the strategy. This would primarily affect the profitability of the shorter moving averages strategies, which tend to produce more trades than the longer ones. Another would be the issue of shorting. Not all listed assets are easily shorted and shorting comes at a cost in the form of paying a predetermined interest rate for borrowing the stock to be shorted. It could be the case that some of the assets included in the basket are not possible to short. It is worth mentioning that most of these large investment companies primarily own stocks in relatively large companies that are more likely to be able to short sell.

A listed asset that is not able to short sell would have to be treated as an unlisted asset. The number of unlisted assets may increase slightly, but the overall concept of the trading strategy would stay intact. The daily estimated difference between the two time series will not be as precise, but the longer trends modeled by the long moving average would not be implicated by the small estimation errors.

According to CAPM, the market should have the highest Sharpe ratio. However, when comparing the trading strategies´ Sharpe ratios with the market Sharpe ratio during the same period, both Bure Equity and Industrivärden outperform the market as the market index during the out-of-sample period saw a decline. Lundbergföretagen is the only company where the trading strategy performs worse than the market. Moreover, during the in-sample period, the market Sharpe ratio was 0.371, which is worse than almost all results observed in the Sharpe ratio tables.

Another trait that is easily observable from the Sharpe ratio tables is the difference in spread between the two periods. The out-of-sample Sharpe ratios vary significantly more than the in-sample period. This variation is most likely due to the difference in length between the periods. The in-sample period is five years, while the out-of-sample period is only two years long. As the period gets longer, Sharpe ratios above 1 are extremely difficult to sustain (Bolmeson, 2019). To put in contrast, Warren Buffett, who is considered by many to be the best investor of all time, had an estimated Sharpe ratio of 0.76 over 1976 to 2011 (ibid). So, the astronomical Sharpe ratio observed in Bure Equity of 2.033 would most certainly decrease if the trading strategy were to be implemented over a longer time period.

Since the problem of changing volatility in the cointegration residual is what causes problems in this attempt to create a profitable pairs trading strategy, one potential solution would be to build a more sophisticated algorithm that can interchange between different moving average during the trading period. The algorithm used in this paper can only handle one fixed choice of moving average that is applied for the entire trading period. The idea would be to build an algorithm that can change to longer or shorter moving averages based on the recent volatility in the residual. If the recent volatility has been low, a short moving average would be activated. Conversely, if the recent volatility has been high, a longer moving average would be activated. By implementing a more flexible algorithm that can interchange between different moving averages, the buy and short sell signals would be more in line with the prevailing market conditions and should produce more consistent returns.

In this paper, the exit signal has been initiated when the two price series have mean-reverted back to their moving average. However, it could very well be the case that the rate and certainty of the mean reversion are stronger the further away the deviation is from its mean. Consequently, the first part of the mean-reverting process may be more reliable than the last part of the mean reversion. The lower the certainty, the more noise the portfolio experiences and the higher the volatility of the trading strategy. Therefore, it could be suggested to experiment with different exit signals to examine if there are trading strategies that have the potential to generate even higher returns.

Another suggestion to improve this model might be to use a rolling regression window similar to the rolling moving average. The residuals produced for the out-of-sample period are currently based on the regression equation estimated in the in-sample period. In this way, the model may handle milder structural breaks in a better way as the rolling regression window would create continuously updated residuals that are in line with the current market conditions. As of now, the residuals for 2017-03-30 – 2019-03-29 are based on how the two time series behaved 2012-04-02 – 2017-03-29 which might be outdated. This may very well also be the reason why the residual starts behaving differently in the out-of-sample period.

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8. Appendix

Appendix 1. Stationary Processes

Stationarity is a statistical property of a time series variable. A stochastic process is considered weakly or covariance stationary if the statistical properties of the process, such as the mean, the variance and the auto-covariance structure remain constant over time. The three weak stationarity conditions are stated in the equations (1.7) – (1.9), and must hold for $t = 1, 2, \ldots, \infty$. (Brooks, 2017).

(1.7) $$E(y_t) = \mu$$

(1.8) $$E(y_t - \mu)(y_t - \mu) = \sigma^2 < \infty$$

(1.9) $$E\left(y_{t_1} - \mu\right)\left(y_{t_2} - \mu\right) = \gamma_{t_2 - t_1}, \quad \forall t_1, t_2$$

That is, a stationary time series is mean-reverting around its long-run mean, has a finite variance that is constant over time, and has theoretical correlogram which diminishes as the lag length increases (Asteriou and Hall, 2016).

In an informationally efficient market, the expected stock price of tomorrow is today's price. Consequently, the conditional mean is not constant. Stock prices are therefore considered non-stationary processes due to that the mean is changing over time. Moreover, the empirically observed phenomena that volatility of stocks often occurs in clusters, known as volatility clustering, further disproves the stationarity of stock prices (Brooks, 2017).

Appendix 2. Unit Root

Consider an AR(1) model:

$$Y_t = \theta Y_{t-1} + e_t \quad (1.10)$$

If theta in equation (1.10) is equal to one, it reflects a process where earlier shocks persist indefinitely in the model. This is also known as the process containing a unit root, or that the process is integrated of order 1 (I(1)).

As soon as the autocorrelation coefficient theta drops below the absolute value of one, the process becomes mean-reverting as previous shocks become less and less significant over time.

Appendix 3. Cointegration

Cointegration was first mentioned in an article written by Engle and Granger in 1987 (Engle and Granger, 1987), and is not to be confused with correlation, which reflects the short-term relationship between two time series. Cointegration, on the other hand, refers to two time series long-term relationship.

For two time series to be cointegrated, the time series must be at least integrated by order one, and there must exist a linear combination of them that is stationary. The order of integration refers to the number of differences needed to take before arriving at a stationary process, which synonymously with integrated by order zero. Engle and Granger (1987) formally define cointegration as follows:

Let w_t be a vector of $k \times 1$ variables, the components are integrated of order (d, b) if:

(1.11) All components of w_t are I(d)

(1.12) There exits at least one vector of coefficients α such that

$$\alpha' w_t \sim I(d-b)$$

Financial variables often contain a unit root and thus are I(1), which limits the order of integration to $d = b = 1$.

Appendix 4. Augmented Dickey-Fuller Tests

In order to test for a possible cointegration relationship between two or more times series, a cointegration regression is implemented. The regression can include more than one explanatory variable.

(1.13) $y_t = \beta_1 + \beta_2 x_t + u_t$

If y_t and x_t are cointegrated and they are both integrated of order one, I(1), the regression residual, u_t, will be stationary I(0). However, if there exists no such long-term relationship the residual will still be non-stationary (Brooks, 2017).

The Dickey-Fuller test is based on the null hypothesis that the cointegration residuals follow a first-order autoregressive process (AR(1)).

(1.14) $u_t = \theta u_{t-1} + \varepsilon_t$

The null hypothesis of the Dickey-Fuller test states that there exists a unit root in the residual AR-process, meaning that theta in equation (1.14) is equal to one. Therefore, under the null hypothesis, there exists no stationary linear combination between y_t and x_t, implying there is no cointegration between the variables (ibid).

The Augmented Dickey-Fuller test is an extension to the original Dickey-Fuller test to include multiple lags of the cointegration residual in equation (1.14).

Appendix 5. Augmented Dickey-Fuller Tests

Augmented Dickey Fuller tests			
Sample period:	2012/04/02 - 2017/03/29		
Samples:	1303		
Maximum Lag Length:	22		
Null Hypothesis:	Unit root is present		
	t-Statistic	P-value	Reject null?
Bure Equity			
Stock price:	0.28489	0.9775	No
Listed assets:	0.11509	0.9668	No
Cointegration residuals:	-3.07008	0.0291	Yes
Industrivärden			
Stock price:	-0.01302	0.9562	No
Listed assets:	-0.74517	0.8332	No
Cointegration residuals:	-3.31504	0.0144	Yes
Lundbergföretagen			
Stock price:	0.54566	0.9882	No
Listed assets:	0.09144	0.9651	No
Cointegration residuals:	-2.66943	0.0797	Yes

Appendix 6. Cointegration Coefficients

Cointegration Coefficients	
Sample period:	2012/04/02 - 2017/03/29
Samples:	1303
Bure Equity	
C	-5.5684
Coefficient	0.921462
Industrivärden	
C	-6411.62
Coefficient	0.962282
Lundbergföretagen	
C	-21245.9
Coefficient	1.348881

Bure Equity is denoted in equity value per share, while Industrivärden and Lundbergföretagen are denoted in equity market value.