How Stellar Tides Affect Planet Evolution

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Abstract

Planets that orbit their host star closely experience tidal forces due to the strength of gravity not being uniform all over the planet. This leads to effects such as tidal spin synchronization, tidal eccentricity damping and tidal semi-major axis damping. This project aims to study how the time scales for these phenomena are affected by the planetary parameters such as mass, radius and the tidal dissipation quality factor Q as well as the initial value of the semi-major axis.

Simulations were made using an averaging code which was based on the fact that the Runge-Lenz vector $\boldsymbol{e}$ and the orbital angular momentum vector $\boldsymbol{h}$ vary slowly under external perturbations such as tidal forces. Because of this the average of these vectors can be used to calculate orbital elements instead of using N-body simulations which decreases the calculation time.

We found that the time scales were of different orders of magnitude, with the spin time scale of the order of 10^4 years, the eccentricity time scale of the order of 10^7 years and the semi-major axis time scale of the order of 10^8 years. An increase of mass and the Q-value both increased the time scale while an increase of radius decreased the time scale. For the initial value of the semi-major axis a lower value gives a shorter time scale compared to larger values.

Populärvetenskaplig beskrivning

Som de flesta av oss vet så kretsar planeter i elliptiska banor runt stjärnor. Vissa planeter som ligger väldigt nära de stjärnor som de kretsar får uppleva visa effekter som inte märkbart påverkar de planeterna i omloppsbana längre bort. Dessa effekter uppkommer ur så kallade tidvattenskrafter som härstammar från det faktum att gravitationskrafter beror väldigt mycket på avstånd mellan objekt. Det innebär t.ex att olika delar av en planet kan uppleva gravitationen från en stjärna den kretsar runt olika starkt då punkten på planeten närmast stjärnan och längst bort från stjärnan kan vara tusentals kilometer ifrån varandra.

Detta leder till effekter som spinsynkronisering, excentricitetsdämpning och dämpning av halva storaxeln. Spinsynkronisering handlar om att rotationen av planeten synkroniserar sig med rotationen av stjärnan på ett sånt sätt att det ser ut som att samma sida av planeten alltid är vänd mot stjärnan, precis som hur vår måne alltid har samma sida mot jorden. Excentricitetsdämpningen leder till att planetens elliptiska omkrets istället blir cirkulär, och dämpningen av halva storaxeln innebär att planeten rör sig närmare stjärnan och får en mindre omkrets på omloppsbanan.

Dessa processer påverkar klimatet på planeten på flera olika sätt. Genom spin synchronizationen så blir det evig dag på ena sidan planeten och evig natt på andra, och när omloppsbanan blir cirkulär så försvinner variationer av temperatur som tidigare har påverkts av hur nära planeten ligger stjärnan under sin omloppsbana. Till slut så blir planeten varmare när den rör sig närmare stjärnan, vilket kan få en planet att gå från frusen till en öken. Dessa effekter kan även påverka eventuella planeter som kretsar stjärnan längre ut genom gravitation. Ändras planetens omloppsbana så ändras hela solsystemets dynamik, så även om tidvattenskrafterna inte påverkar alla planeter direkt så påverkas hela systemet indirekt i det långa loppet.

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Chapter 1

Introduction

When a planet orbits a star it will feel a force raised by the gravity between them. Usually this just keeps the planet in orbit around the star, but when the planet is very close to the star other phenomena will occur. These phenomena are tidal locking, tidal circularization and tidal semi-major axis damping. The time scale for these phenomena can vary with parameters such as mass, radius, semi-major axis and so forth. The aim of this project is to see how the time scales are affected by changes of different planetary parameters. What these results gives us is a way to roughly determine the age of planets so close to their host stars that they would experience the tidal phenomena.

The reason why these phenomena occur has to do with gravity and distance. In equation (1.1) the gravitational force between objects is defined. The strength of gravity depends on the inverse of the square of the distance r between the objects with mass m_1 and m_2 exerting the force multiplied by the gravitational constant G.

$$F = G\frac{m_1 m_2}{r^2} \tag{1.1}$$

By differentiating equation (1.1) we get the expression for the tidal forces, which is

$$F = 2GR\frac{m_1 m_2}{r^3} \tag{1.2}$$

where R is the radius of the object in question.

It works in such a way that the closer the objects are to each other the stronger the force is. This means that some parts of an object will experience different strengths since not all parts of the object are equally close to the other object. Usually when a planet orbits a host star that is so far away that its own diameter is very small compared to the total distance. The difference between the gravity at the point of the planet closest to the star and the point furthest away from the star would therefore be so small that it would be negliable. However, when a planet is very close to its host star this changes. When the diameter of the planet compared to the distance between the planet and the star isn't negliable the force difference between the different parts of the planet is large enough to affect its evolution. This is the origin of the tidal forces and the phenomena mentioned earlier.

Another important equation describing the planet that needs to be introduced before the phenomena due to tidal forces are explained more in detail is the equation for the period of the planet. The period of the planet, which is the amount of time needed to orbit its host star once, is defined as (Roy [2005])

$$P = \frac{2\pi}{n} = 2\pi\sqrt{\frac{a^3}{GM_*}} \tag{1.3}$$

where a is the semi-major axis, M_* is the mass of the star and n is the so-called mean motion (Roy [2005]) given in radians per unit time.

$$n = \sqrt{\frac{GM_*}{a^3}}. \tag{1.4}$$

The mean motion is the angular frequency needed for a planet to complete an orbit around a star and helps describe the amount of time needed for a planet to orbit its host star and is usually given in revolutions per day.

1.1 Tidal locking

One of the effects that a planet orbiting its host star closely will experience is tidal locking, also known as spin synchronization. Tidal locking arises from the fact that the planet and the star are tugging at each other due to gravity when they are rotating: the side of the planet moving away from the star as its rotating will feel a force that is slowing it down and pulling it back.

The spin of the planet will be adjusted by the tidal forces from the star. If the planet spins quickly the side of the planet moving away from the star will be slowed down, while the side moving towards the planet will be accelerated. If it on the other hand spins slowly the opposite will happen. This can be seen in equation (1.5) which depends on the difference between the spin Ω and the mean motion n. If Ω is larger than n the whole term will become negative and thus the rate of change will also become negative and slow the spin down. If Ω on the other hand is smaller than n then the equation will remain positive and the spin will increase. The rate at which the spin of the planet Ω_p changes is defined in equation (1.5) (Gu et al. [2003]).

$$\dot{\Omega}_p = \frac{9n}{2\alpha_p Q'_p}\left(\frac{M_*}{M_p}\right)\left(\frac{R_p}{a}\right)^3 (n - \Omega_p) \tag{1.5}$$

Equation (1.5) depends on several parameters such as the mass of the star and the planet M_* and M_p as well as the radius of the planet R_p and the semi-major axis. There are also two other parameters included which are less intuitive in their description, namely α_p which is determined by the internal structures of the planet, and the modified Q-value *Q'*.

The modified Q-value for the planet is defined by $Q'_p = 3Q_p/2k_p$ (Gu et al. [2003]) which in turn depends on the Q-value and the k-value for the planet (Murray and Dermott [1999]). The Q-value is called the tidal dissipation quality factor and describes how easily energy is dissipated throughout the object (Alvardo-Montes et al. [July 2017]). This means that the higher the Q-value is the easier the planet in question dissipates energy. Q is defined as (Goldreich and Soter [1966])

$$Q^{-1} = \frac{1}{2\pi E_{max}} \oint \left(-\frac{dE}{de} \right) \tag{1.6}$$

where E_{max} is the maximum energy stored in the distortion.

k is one of 3 dimensionless parameters called the Love numbers which describe the rigidity and susceptibility to change due to tidal forces of a planet (Souchay et al. [2012]). The k Love number describes the ratio between the deforming potential from the tidal forces and the potential produced by the redistribution of mass.

1.2 Tidal circularization

The majority of planetary orbits are elliptical to some degree. Eccentricity e is the parameter which defines how elliptical an orbit is and takes values between 0 and 1 with 0 being completely circular and 1 being a parabola. In our solar system the values for the eccentricity are between 0.205 and 0.07 which means that all orbits are slightly elliptical (Williams). Tidal circularization, also known as eccentricity damping, is the phenomena of the eccentricity decreasing until it reaches zero and thus have the orbit becomes completely circular.

The cause of eccentricity damping come from tidal friction forces from the star. When a planet is moving away from its host star in its elliptical orbit it will feel a tug towards the star which will slightly affect its orbit until the orbit is adjusted. The planet will move closer and closer to the star each time it completes an orbit so that the part furthest away from the star, the aphelion, will be located more closely to the star. It will continue to do so until all points of the orbit are equally distant from the star.

The rate of change for the eccentricity is defined in equation (1.7). This is an exponential equation which shows that the speed at which the eccentricity changes will decrease over time (Gu et al. [2003]).

$$\frac{de}{dt} = -\frac{63ne}{4Q'_p} \left(\frac{M_*}{M_p} \right) \left(\frac{R_p}{a} \right)^5 \tag{1.7}$$

1.3 Tidal semi-major axis damping

The last effect from the tidal forces is the tidal semi-major axis damping. The semi-major axis is half of the longest diameter in an ellipse and defined to be the average distance of a planet to its host star. It's usually described in terms of astronomical units (AU) with 1 AU being defined as the average distance between the Sun and Earth which is approximately 150 million kilometers (IAU). Mercury for example, which is the planet closest to the Sun in our solar system, is approximately 0.387 AU away from the Sun. For tidal semi-major axis damping to occur the planet needs to be much closer, so in this report the semi-major axis will be 10 times smaller than that of Mercury's.

The rate of change for the semi-major axis depends on the eccentricity and its change as well as the angular momentum of the system which can be seen in equation (1.8) (Gu et al. [2003]). When an orbit becomes more circular due to tidal forces it's the outer points that move inwards rather than the inner moving outward. Therefore the semi-major axis will decrease too since the average distance now decreases.

$$\dot{a} = a\left(\frac{2e\dot{e}}{1-e^2} + \frac{2(\dot{J}_* + \dot{J}_p)}{J_o}\right) \tag{1.8}$$

where J_* and J_p are the angular momenta from the spin of the star and the planet on the planet's orbit and J_o is the total angular momentum from the planet's orbit.

1.4 Time scales

The forces which play a part in the evolution of the semi-major axis, eccentricity, and spin of the planet with different magnitudes and contributions. They all depend on the same type of parameters such as mass, semi-major axis, radius and Q-value. Those parameters therefore make good variables to make different simulations for since it would affect all the time scales.

The spin synchronization time scale defined in equation (1.9) which can be obtained from the time-averaged equations of motion depends not only on the variables already mentioned but also on the initial value of the spin. It is also the only time scale that depends on α_p and ϵ_p. The factors α_p and ϵ_p are determined by the internal structures of the planet as well as the fraction of the planet which participates in the angular momentum exchange induced by the tidal forces (Gu et al. [2003]). We use a value of 0.4 for their total contribution in this paper.

$$\begin{aligned}\tau_{\Omega p} &= 0.187\alpha_p\epsilon_p\left|\frac{1\text{day}/(2\pi/\Omega_p)}{0.34(M_*/M_\odot)^{1/2}(0.04\text{AU}/a)^{3/2} - 1\text{day}/(2\pi/\Omega_p)}\right| \\ &\times\left(\frac{Q'_p}{10^6}\right)\left(\frac{M_p}{M_J}\right)\left(\frac{M_\odot}{M_*}\right)^{3/2}\left(\frac{a}{0.04\text{AU}}\right)^{9/2}\left(\frac{R_J}{R_p}\right)^3\text{Myr}\end{aligned} \tag{1.9}$$

The time scale for the eccentricity and the semi-major axis are defined in equation (1.10) and (1.11) respectively. Whereas the semi-major axis depend directly on the mean motion defined in equation (1.4) the eccentricity time scale does so indirectly via its rate of change. The semi-major axis also depend on the spin of the star Ω_*.

$$\tau_e = 0.33 \left(\frac{Q'_p}{10^6}\right) \left(\frac{M_p}{M_J}\right) \left(\frac{M_\odot}{M_*}\right)^{3/2} \left(\frac{a}{0.04\text{AU}}\right)^{13/2} \left(\frac{R_J}{R_p}\right)^3 \text{Gyrs} \tag{1.10}$$

$$\tau_a \equiv \frac{a}{\dot{a}} \simeq \left(\frac{7n}{\Omega_* - n}\right) \left(\frac{Q'_*}{10^6}\right) \left(\frac{M_*}{M_\odot}\right)^{1/2} \left(\frac{M_J}{M_p}\right) \left(\frac{a}{0.04\text{AU}}\right)^{13/2} \left(\frac{R_\odot}{R_*}\right)^5 \text{Gyr} \tag{1.11}$$

Here we also have Ω_* which is the spin of the star. This has a value of 0.05 years in the code, or about 18.26 days.

Chapter 2

Method

To be able to examine how different parameters affect the time scale a number of simulations were made. The code is based on the equations found in Rosemary A. Mardlings and D.N.C Lins paper *Calculating the tidal, spin, and dynamical evolution of extrasolar planetary systems* (Mardling and Lin [2002]) and the orbit averaging method used there. It was written by Thijs Kouwenhoven at Xi'an Jiaotong-Liverpool University in Suzhou, China.

2.1 The averaging code

For an unperturbed orbit the Runge-Lenz vector $\boldsymbol{e}$ and the orbital angular momentum vector for the inner orbit $\boldsymbol{h}$ are constant and their components will vary slowly compared to the orbital period under external perturbations such as tidal forces. The Runge-Lenz vector $\mathbf{e}$ is a vector with magnitude equal to the eccentricity and the direction of periastron. This vector, together with the angular momentum vector of the inner orbit $\mathbf{h} = r \times \dot{r}$ where r is the distance between the bodies, are used to calculate the secular evolution of the orbital elements. These elements include the eccentricity e, semi-major axis a, inclination i, argument of periastron ω, and longitude of the ascending node Ω. Therefore, by having the vectors vary very slowly, the average of the vectors can be used to calculate the orbital components instead of using a N-body simulation and the calculation time will be reduced.

By assuming that the inner orbit is Keplerian and that its orbital elements remain unchanged for one orbit so-called orbit averaging can be performed. Possible outer bodies affect inner one and causes perturbations which depend on the ratio between distances between the bodies and the host star r$/R$, but these perturbations will not be included in this paper since we will be looking at a 2-body system. Spin on the other hand depends on the ratio between the radius of the star S_1 and the distance between it and bodies which is S_1/R. By taking the average of both of these perturbations we obtain averaged equations for the Runge-Lenz vector and the angular momentum vector which are

$$\langle\frac{dh}{dt}\rangle = \langle d_{quad}\rangle + \langle d_{oct}\rangle + \langle d_{QD}\rangle + \langle d_{TF}\rangle \tag{2.1}$$

$$\langle\frac{de}{dt}\rangle = \langle g_{quad}\rangle + \langle g_{oct}\rangle + \langle g_{QD}\rangle + \langle g_{TF}\rangle \tag{2.2}$$

where "quad" is the quadrupole contributions, "oct" is the octupole contributions, "QD" is the quadrupole distortion, and "TF" is the tidal friction. These contributions are obtained by averaging each one of them. The quadrupole and octupole contributions come from possible outer bodies and is therefore unimportant for this report. The most important contributions come for the tidal friction forces.

Chapter 3

Results

All the simulations depend on some variable parameters which were changed to see how they affected the time scale. To be able to see how they change we use a nominal model which we will compare to other simulations. Table 3.1 displays the values used in the nominal models. The planet has a radius which is roughly twice the size of Jupiter's, and the star has a radius which is roughly the same as the Sun's radius. Unless otherwise stated in the coming sections the values displayed in table 3.1 were the parameters used in the simulations. The period P was set to have a value of 2.8 days.

Table 3.1: The values of the parameters used in the simulations.

R_p	R_*	M_p	M_*	Q-value	k-value	Ω_{init}	a
$1.50 \cdot 10^5$ km	$7.48 \cdot 10^5$ km	2 M_J	1 $M_\odot$	10^5	0.1	18 days	0.04 AU

3.1 Nominal model

In the code used for the simulations we have had the ability to choose if we wished to include the contributions from the tidal friction, which arises from stretching of bodies due to gravity, and the quadrupole effects, which is due to rotation of the bodies and them being flattened at the pole (Marchenko [1979]). This can be seen in equation (2.1) and (2.2), and it could be included to the tidal forces for both the planet and the star. In this section and the coming ones we will only use the tidal friction force, but plots for other configurations can be found in appendix A. Those include one simulation for when both the tidal friction and quadrupole contributions were included, one when only the quadrupole contributions were included, and one where only the tidal friction from the star felt by the planet was included and not from the star. We will look at the different time scales for the spin evolution, the eccentricity evolution and the semi-major axis evolution.

Spin evolution for the nominal model

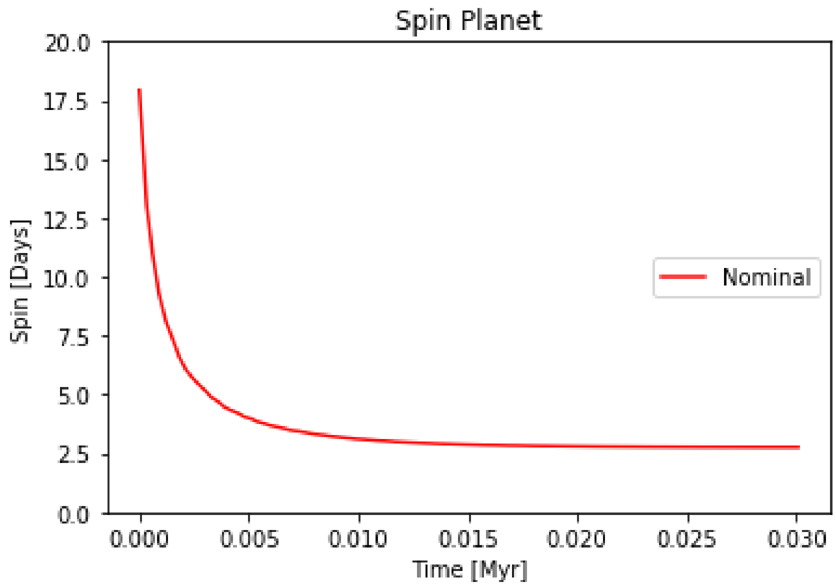

Figure 3.1: The spin evolution.

We have the spin evolution when the quadrupole contributions are excluded and only the tidal friction forces are enabled in figure 3.1. The initial value of the spin of the planet is 18 days, and it will decay to 3 days. The reason why we get down to 3 days is related to equation (1.5) which describes what the change in the spin is at any given moment. The spin of the planet will continue to change as long as $\dot{\Omega}_p$ has a non-zero value, and this is the case as long as the spin is different from the mean motion. The value for the mean motion in our case is 3 days, which explains why the spin goes to a value of 3 days in our plot.

Of all the effects the time synchronization have the shortest time scale of a few thousand years. In the cases simulated in figure 3.1 the time scales have a value of roughly 15 000 years, before settling on a new spin value of 3 days. Putting the values from table 3.1 into equation (1.9) we get an estimate for the time scale which agrees with the simulation.

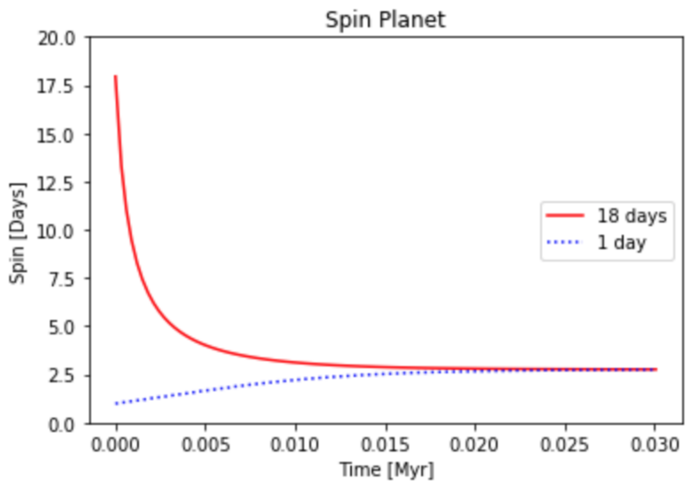

Figure 3.2: The spin evolution with initial value 1 day and 18 days.

The spin will also change even if its initial value is lower than that of the mean motion. This is shown in figure 3.2 where we tried a simulation for an initial spin value of 1 day. Instead of slowing down the parts of the planet moving away from the star when it's rotating, the parts moving towards the star will be accelerated due to being pulled towards the star. For the case simulated the time scale for a spin of 1 day is about 0.020 Myr, which is slightly longer than for an initial value of 18 days.

Eccentricity evolution for nominal model

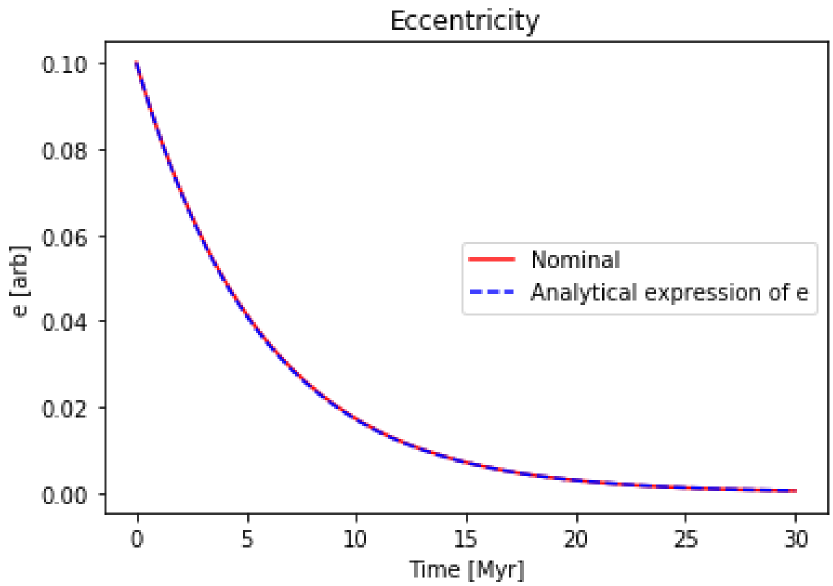

Figure 3.3: The eccentricity evolution.

The time scale for the eccentricity is about 100 times longer than the time scale for the spin synchronization which is seen in figure 3.3. Inserting the values in table 3.1 into equation (1.10) gives an estimate of the time scale which agrees with the values in the plot. Also included in the plot is an analytical result for how the eccentricity should behave. This comes from the differential equation (1.7) which was solved by hand which gave

$$e = e^{-c} \tag{3.1}$$

with

$$c = \left(\frac{63n}{4Q_p'}\right)\left(\frac{M_*}{M_p}\right)\left(\frac{R_p}{a}\right)^5 . \tag{3.2}$$

An array of different values for the time between 0 and 30 Myr was put into the equation which then gave the analytical result represented by the blue dotted line in figure 3.3. As can be seen, the analytical expression of e and the simulation agrees well with each other and gives the same time scale for the nominal model.

Semi major axis evolution for nominal model

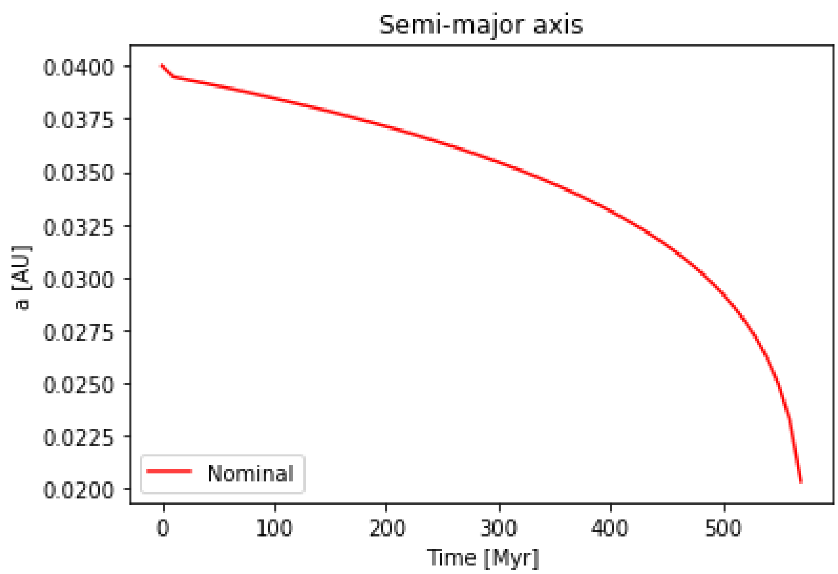

Figure 3.4: The semi-major axis evolution.

The semi-major axis evolution for the nominal model can be seen in figure 3.4. The result can be related to equation (1.11). Inserting the values gives us an estimated timescale of 0.71 Gyrs. The reason why the plot does not show the evolution until 0.71 Gyrs was that the simulations stopped working after roughly 500-600 Myrs. We do however see that the semi-major axis seem to approach the values estimated by equation (1.11), but we cannot say with certainty that they agree. From what is seen in the figure 3.4 the rate of change seems to increase over time and make the line steeper the further along the x-axis it is. It is therefore not unreasonable to believe that the time scale could be 0.71 Gyrs.

In the beginning of the plot we see that the semi-major axis decreases more rapidly than it does later on. This occurs over a time period of about 30 Myrs which corresponds to the eccentricity evolution time scale. In fact, what is visible in the plot for the first 30 Myrs is the circularization of the eccentricity. This can be explained by equation (1.8). There are two terms that affect the change in a: one that depends on the eccentricity and the change of eccentricity over time, and one that depends on the angular momentum. Once the eccentricity is zero, that term will stop contributing, which explains why there's a change in the rate of change.

One can also think about how the circularization will affect the semi-major axis: the distance to the aphelion will become shorter when the elliptical orbit becomes circular. After about 30 Myr the orbit of the planet will have become circular, and therefore the first term in equation (1.8) will become 0 and not contribute anymore. The second term, which includes J_p and J_*, will still be there with $J_p = 0$ and be nonzero, but this term will be very small because the force on the star exerted by the planet is small.

3.2 Variation of mass and radius of the planet

There are 3 simulations in each plot in this section. One is the nominal graph for comparison with the simulations with new values for the parameters. The other two are when the mass have been doubled, and when the radius have been increased by 50%.

Spin evolution for the nominal, increased mass, and increased radius.

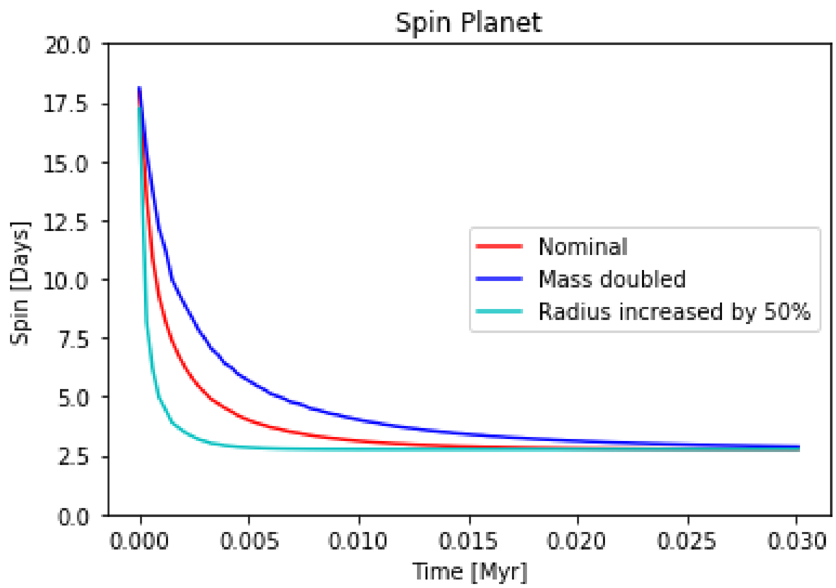

Figure 3.5: The spin evolution for different mass and radius.

In figure 3.5 the three different simulations for the nominal model, increased mass and increased radius can be seen. The nominal model is visible in red and have the same time scale as before in section 2.1 where it reaches a spin of 3 days after about 0.015 Myr. The dark blue line represents the spin evolution for a planet that's twice as massive as the one used in the nominal model. The cyan line represents the evolution for a planet with a radius that's 50% larger than that of the planet in the nominal model.

For the spin evolution increasing the mass by 100% so that it reaches a value of 2 Jupiter mass makes the time scale twice as long. Instead of reaching a spin value of 3 after roughly 0.015 Myr it converges to it after 0.030 Myr. This we see in equation (1.9) where we have the factor (M_J/M_p), and we can there see that the timescale is proportional to the mass of the planet.

The radius on the other hand doesn't change the time scale in the same way like the mass does. Instead the time scale depends on the cube of the radius of the planet which can be seen at the end of equation (1.9). An increase in radius will actually decrease the time scale since the radius of the planet is in the denominator. For our case in figure 3.5 the simulation where the radius was increased by 50% the time scale was decreased to about 0.005 Myr. This means that our time scale decreased by 33% from 0.015 Myr to 0.005 Myr.

Eccentricity evolution for the nominal, increased mass, and increased radius.

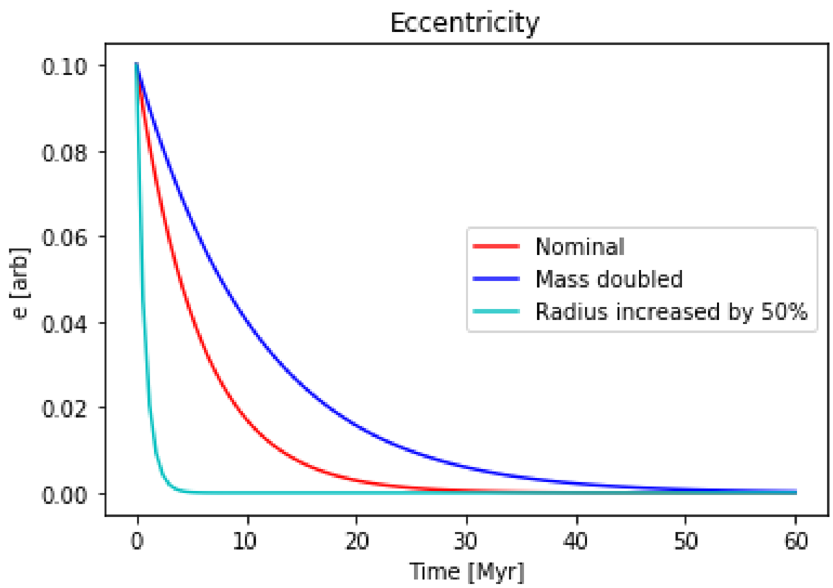

Figure 3.6: The eccentricity evolution for different mass and radius.

The three simulations described earlier can be seen in figure 3.6. The red line is from the nominal model which can also be seen in figure 3.3 in section 2.1. The blue line represents the eccentricity evolution when the mass of the planet in table 3.1 have been doubled, and the cyan line represents the eccentricity evolution for a 50% increase of the radius in the same table.

For the increased mass simulation we expect that the time scale depends directly on M_p by comparing with equation (1.10), and by doubling M_p we expect the timescale to also be doubled just like in the case for the spin evolution. From the plot we can see that the estimation agrees with the plot. The eccentricity reaches 0 at 60 Myr which is twice as long than for the nominal model where the circularization is complete at 30 Myr.

We also checked what happens if the radius is increased by 50%. Once again checking with equation (1.10) we can estimate the time scales. This time we also see that the time scale depends on the cube of the radius, just like for the spin evolution. Therefore, by increasing R_p by 50%, we expect the time scale to decrease by roughly 70% just like it did for the spin evolution. This would give us a time scale of around 9 Myr instead of 30 Myr, which seems to agree reasonably well with our plot.

Semi-major axis evolution for the nominal, increased mass, and increased radius.

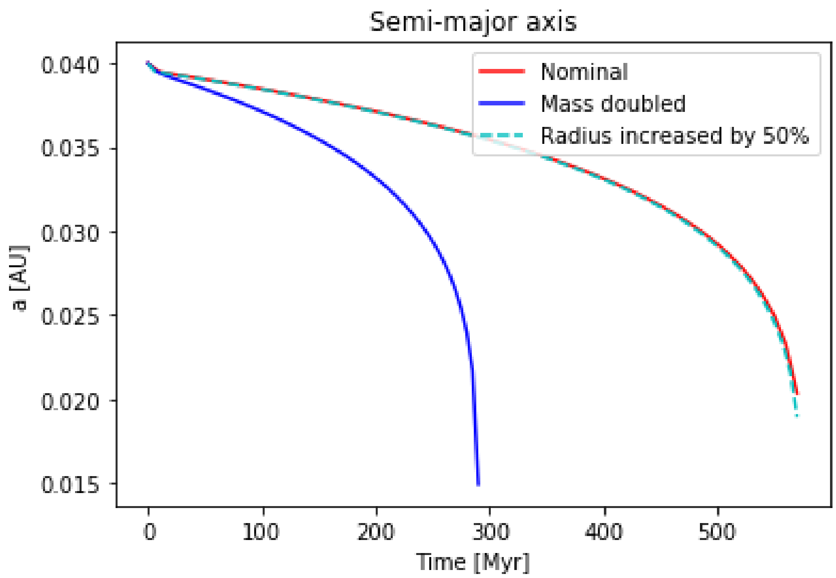

Figure 3.7: The semi-major axis evolution for different mass and radius.

The semi-major axis evolution is plotted in figure 3.7. The nominal model is shown in red, the simulation for an increased mass is showed in blue, while an increased radius is shown in dotted cyan. The semi-major axis have the same factor (M_J/M_p) that the time scale for the spin evolution and eccentricity evolution depend on. Just as for the other two effects the time scale will be half of the original value if the mass is doubled since the time scale depends directly on the inverse of the mass of the planet. We therefore see the time scale in the above figure go from about 600 Myr to 300 Myr. A higher mass will make the force from the star stronger, so the pull will be greater and therefore the process will be much quicker.

The semi-major axis does not depend on the radius of the planet at all. In equation (1.11) one can see that the radius of the planet isn't included, but the radius of the star compared to the solar radius is included. The one way the semi-major axis could perhaps be affected by the planets radius is via the eccentricity damping which happens in the beginning of the plot. This will however not affect the time scale for the semi-major axis very much since the time scale of the eccentricity damping is so short compared to the semi-major axis time scale that it won't affect much in the long run.

3.3 Variation of semi-major axis

In each plot in this section there are two simulations: the nominal, and one where the semi-major axis to the star have been decreased from 0.04 AU to 0.03 AU.

Spin evolution for a close in planet.

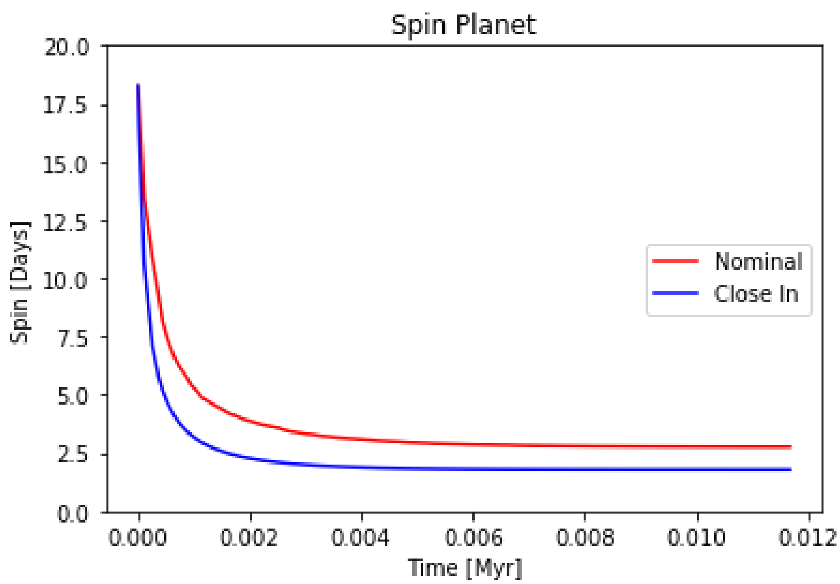

Figure 3.8: The spin evolution for a close in planet.

In the above plot there's the nominal model in red and the simulation for a close in planet with a semi-major axis of 0.03 AU. Looking at equation (1.9) we can see that the time scale depends on the semi-major axis in two different places in the equation. We have one term which depends on a to the power of $9/2$, and we have another one which depends on inverse of a to the power of $3/2$. Overall these two terms together gives us a decrease of the time scale which in figure 3.8 shows as being around 0.0025 Myr. This is a 83% decrease compared to the nominal value of 0.015 Myr.

The final spin period is also changed from roughly 3 days to 1.9 days. This is because of how the mean motion n is defined. In equation (1.4) we see that the value n depends on inverse of the semi-major axis to the power of $3/2$. This can then be put into equation (1.3) which describes the period. By decreasing the semi-major axis a we're actually decreasing the period too, and by how this is defined the mean motion has to decrease. Instead of the spin converging to 3 days it will converge to 1.9 days, which is exactly what we can see in figure 3.8.

Eccentricity evolution for a close in planet.

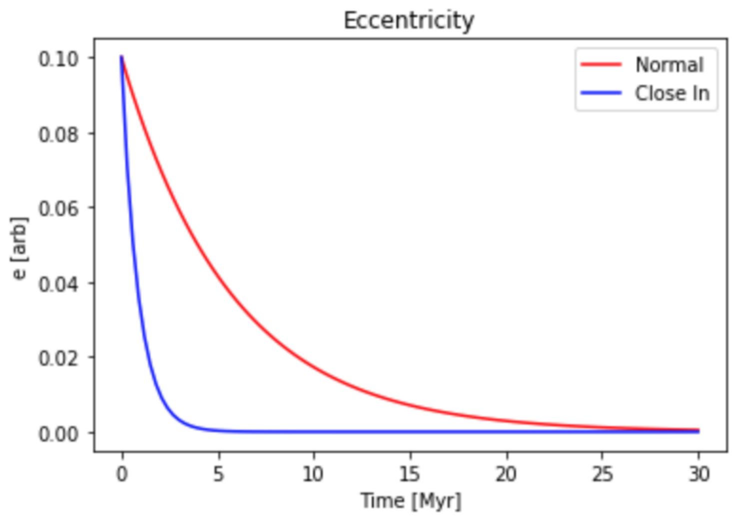

Figure 3.9: The eccentricity evolution for a close in planet.

In figure 3.9 we see the nominal model represented in red and the simulation with a semi-major axis of 0.03 AU represented in blue. The time scale for a close in planet with a semi-major axis of 0.03 AU is significantly shorter than for 0.04 AU with the circularization done after just 4 Myr. This is the case because of the term containing a in equation (1.10) which is to the power of 13/2. By changing the semi-major axis with 0.01 AU the time scale decreases from 30 Myr to about 4 Myr which can be seen in figure 3.9. This is a roughly 85% decrease of the time scale compared to the nominal model.

Semi-major axis evolution for a close in planet.

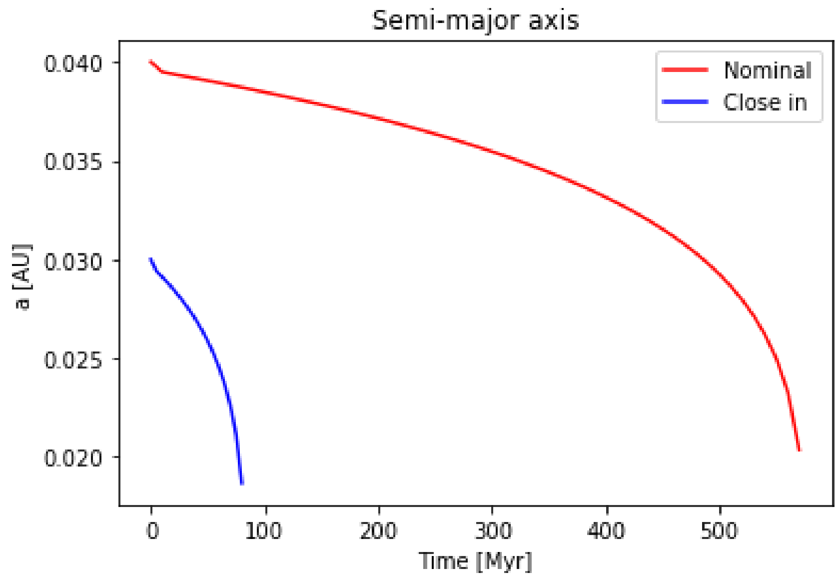

Figure 3.10: The semi-major axis evolution for a close in planet.

Figure 3.10 shows the semi-major axis evolution for an initial semi-major axis value of 0.03 AU in blue and the nominal model in red. Most notably for a close in planet is that the initial value will of course be lower since the planet is closer to the star. Instead of starting at 0.04 AU we now start at 0.03 AU. This means that the distance to the lower values of the semi-major axis will be shorter, giving the damping process a sort of head start compared to the nominal model. In equation (1.11) we can see the term discussed for the eccentricity too, namely (a/0.04AU) to the power of 13/2. This will greatly decrease the time scale of the semi-major axis damping.

In equation (1.11) we have that the timescale is defined as the semi-major axis a over the rate of change for the semi-major axis $\dot{a}$, and this rate of change is defined in equation (1.8). The change in the semi-major axis from 0.04 AU to 0.03 AU will affect the rate of change and make it shorter. Also, since the eccentricity now ends already at 4 Myr, this parameter will reach lower values quicker. This in turn will increase the time scale since the rate of change is in the denominator in (1.11). The semi-major axis itself however decreases which we can see from equation (1.11) decreases the time scale, so what we have is that the semi-major axis and the rate of change for the semi-major axis both affect the time scale but in opposite ways. The decrease of a is bigger than the decrease of $\dot{a}$ so the time scale as a whole will still decrease. Instead of reaching values below 0.02 AU after approximately 600 Myr it now reaches these values after under 100 Myr. If one studies the curve for the close in simulation and compare to others we can see that the curve is steeper. The change in a is therefore more rapid for lower initial semi-major axis values than for higher initial values.

3.4 Variation of tidal Q-value

In this section we have increased our planetary Q-value from 10^5 to 10^6. This is not the same as the $Q'_p = 3Q_p/2k_p$ which is the parameter used in the equations, but the change will still be proportional to the Q-value.

Spin evolution for a planet with larger Q-value.

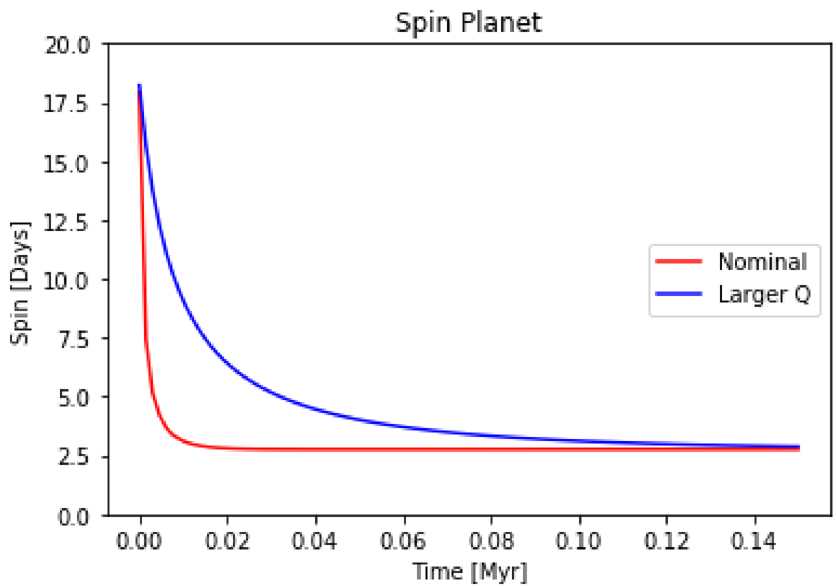

Figure 3.11: The spin evolution for a different Q-value.

The nominal model is represented in red in figure 3.11 while the simulation for the planet with a larger Q-value is represented by the blue line. We can see that the blue line takes significantly longer time to reach 3 days than the red line. The time scale for the tidal locking becoming roughly 10 times slower with an increased Q-value. The blue line in figure 3.11 reaches 3 days after roughly 0.15 Myr compared to the nominal model in red which reaches 3 days after 0.015 Myr. This is also what we can expect from equation (1.9) where the Q'_p is put to be 10^6 instead of 10^5 like in the nominal model. If we increase Q'_p with a factor of 10 then the timescale will also increase by that factor.

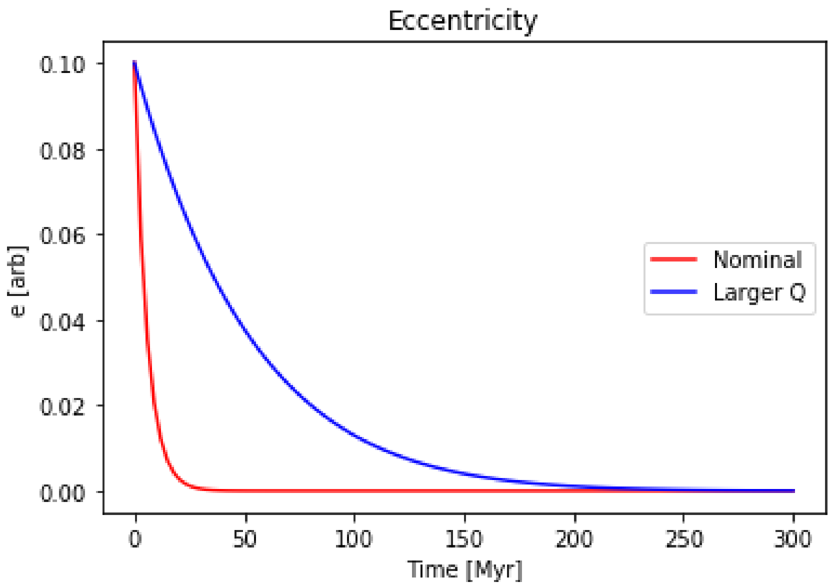

Figure 3.12: The eccentricity evolution for a different Q-value.

For the eccentricity we expect the change in timescale to be directly proportional to the change in Q. This we can see from equation (1.10) where we see that the change in Q is direct to the time scale change. Therefore we expected the time scale to increase by a factor 10 when we increased the Q-value by a factor 10, which is also what we can see in figure 3.12. Instead of reaching 0 eccentricity at 30 Myr we now reach it at 300 Myr which is 10 times longer.

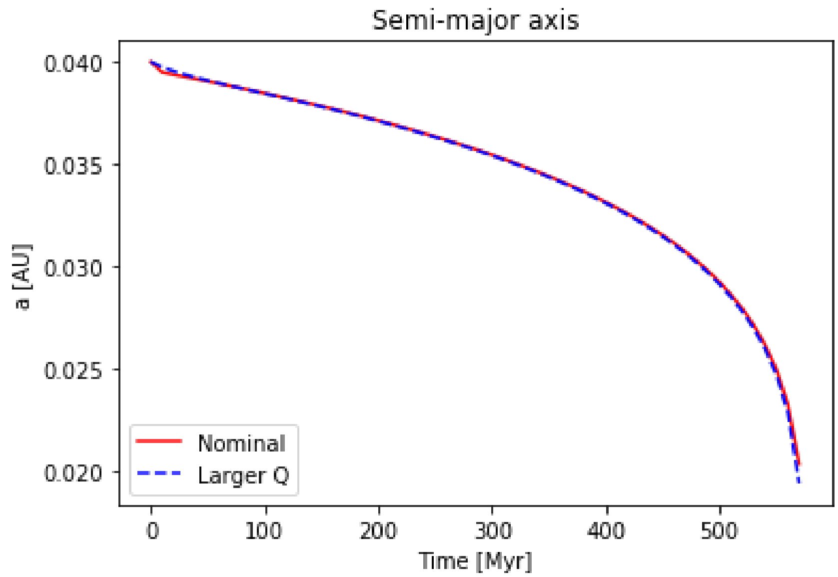

Figure 3.13: The semi-major axis evolution for a different Q-value.

In figure 3.13 we don't see any difference for the time scale for a higher Q-value compared to the nominal model, and that is because the semi-major axis depend on the Q'-value of the star rather than the planet as seen in equation (1.11). We do however see a difference in the beginning of the plot where the cirularization dominates and affects the slope. Since the eccentricity damping time scale will be 10 times longer for the new Q-value of 10^6 compared to the nominal model the contributions from the first term in equation (1.8) will last 10 times longer. In the long run however the difference in the beginning won't matter much since they are too small to affect the total time scale significantly.

Chapter 4

Conclusion and discussion

4.1 Conclusion of the nominal case

The results show quite clearly that there are different orders of magnitude for the time scale of the three different phenomena. The spin synchronization has the shortest time scale and is usually finished after just a few thousand years, and for the nominal model we get that the spin synchronization is complete at 15 000 years. The eccentricity damping time scale is a factor 10^3 longer than the spin synchronization and thus usually have values of tens of millions of years. In our nominal model we got that the eccentricity damping took 30 million years to complete. Lastly we then have the semi-major axis damping which have the longest time scale of all of the phenomena. The time scale is usually several hundred million years and is thus a factor 10 longer than the eccentricity damping and a factor 10^4 longer than the spin synchronization. In our nominal model we got that a decrease from 0.04 AU to 0.02 AU took roughly 580 million years. After reaching a value of 0.02 AU however our simulation was unable to compute any more values. The reason for this isn't completely clear, but I suspect it has to do with how close the planet comes the host star. The star's radius is approximately 0.004 AU, so the star's radius stands for 20% of the distance to the planet. It is unclear though if this is the reason why and other configurations of the variables in table 3.1 could be tested to see if that is the case. We were in any case unable to reach lower values for the semi-major axis and do not know what happens at even lower values.

4.2 Conclusion of the increased mass and radius cases

Thereafter we looked at how an increase in mass and radius respectively of the planet changed the time scale. For an increase in mass the time scale increased for the spin synchronization and the eccentricity damping, but decreased the time scale for the semi-major axis damping. For an increase of the radius on the other hand decreased the time scale for the spin and eccentricity but left the semi-major axis damping time scale unchanged.

4.3 Conclusion of the decreased semi-major axis case

The third thing we checked was how a decrease of the semi-major axis affected the time scale. For all three cases a decrease in the semi-major axis also meant a decrease for the time scale. For the spin synchronization the semi-major axis depended on the power of 9/2, while for the eccentricity the change for the semi-major axis depends on the power of 13/2. For the semi-major axis evolution it of course decreases if we actively push it down from 0.04 Au to 0.03 AU, but the time scale also changes by a factor depending on the value of the semi-major axis to the power of 13/2.

What's interesting about a change for the semi-major axis is that it changed the final value for the spin. Instead of going towards a value of 3 days it changed to 1.9 days. This is the only case amongst all simulations where a change of the variables actually changed the final value of the parameter. In all cases for the eccentricity we reached a value of 0, and for the semi-major axis we always reached below 0.020 AU before the simulations were unable to continue.

4.4 Conclusion of the increased Q-value case

The last thing we checked was how a change for the Q-value would change the time scales. For both the spin synchronization and the eccentricity damping the time scales are increased proportionally to the increase of the Q-value. The time scales both increase by 10 since the Q-value was increased by a factor 10. The semi-major axis time scale however is not notably changed by the change in Q-value except at the beginning of its evolution where it still depends on the eccentricity.

4.5 Discussion

In this report we have mainly been using a model of a planet similar to so-called "hot Jupiters" which are Jupiter-sized planets orbiting closely around stars. Therefore the timescales which have been discussed in this report won't be accurate for an Earth-like planet. We can however estimate the time scales using our results. A summary of the results obtained can be seen in table 4.1. If we instead had an Earth-like planet then the mass, radius and Q-value would all be smaller and the total time scale would be smaller than it has been for the cases in this report. It will of course depend on the magnitudes of these variables since a smaller radius will increase the time scale while a smaller mass and Q-value will decrease it and it's a matter of what affects the time scale the most.

Planetary parameter	τ_{Ω_p}	τ_e	τ_a
M↑	↑	↑	↓
R↑	↓	↓	-
a↓	↓	↓	↓
Q↑	↑	↑	-

Table 4.1: Summary of results.

We only checked what would happen for an increase of the value for the variables and not how a decrease would affect the time scale except for in the case of the semi-major axis. The reason is because all of the time scale equations are linear and that the change would be in the same magnitude but with opposite effect if we checked for a decrease. Future studies in this area could show how a decrease of these values would affect the time scale. In the case for the semi-major axis it could be interesting to see how far away a planet can be to experience these phenomena during the host stars lifetime. Perhaps one could find a threshold for the phenomena, and if the threshold would be the same for them all or if we could experience only one of these. It could also be interesting to see how different stellar parameters, such as the spin, mass, and radius, could change the time scale.

What would also be interesting to check is how an inclusion of a third and fourth body would affect the system, something that we simply did not have time to do in the time span given. As mentioned in section 2.1 the orbit averaging depends on perturbations from outer bodies, and the code is actually written to include more than just 2 bodies. It would be interesting to see just how much other bodies would affect the time scale and how the distance between the bodies would change the interaction. The inner bodies would also affect the outer ones, and perhaps that would decrease the time scale for the outer planets.

As mentioned in the introduction these results can give us a way to roughly determine the ago of close in exoplanets. If we see a planet that has achieved tidal locking but not quite yet achieved a circular orbit we know the time span in which it could have been formed. This of course depends on all the planetary parameters which can be hard to obtain if we're looking at an exoplanet far away. We also get a time line which we can expect exoplanets to follow. As we have seen the time scales haven't been random but follow the equations and the variables very well. We will therefore know how the planets are expected to move in the future which will not only affect itself but other planets in the system too.

For just the planet itself a change of spin, eccentricity and semi-major axis will greatly affect the climate on the planet. When a planet becomes tidally locked only one side of the planet will receive the flux of energy and heat of the host star, and unless there is any weather or winds on the planet this heat and energy is likely to remain on that side. A planet could therefore be several hundreds of degrees Kelvin on one side and near absolute zero on the other side. By having a circular orbit the planet would not experience a change in velocity and acceleration over one orbit. A change of the semi-major axis will greatly affect the climate on the planet. Since the semi-major axis decreases the planet will move closer to the host star and increase in temperature in all cases.

Chapter 5

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Appendix A

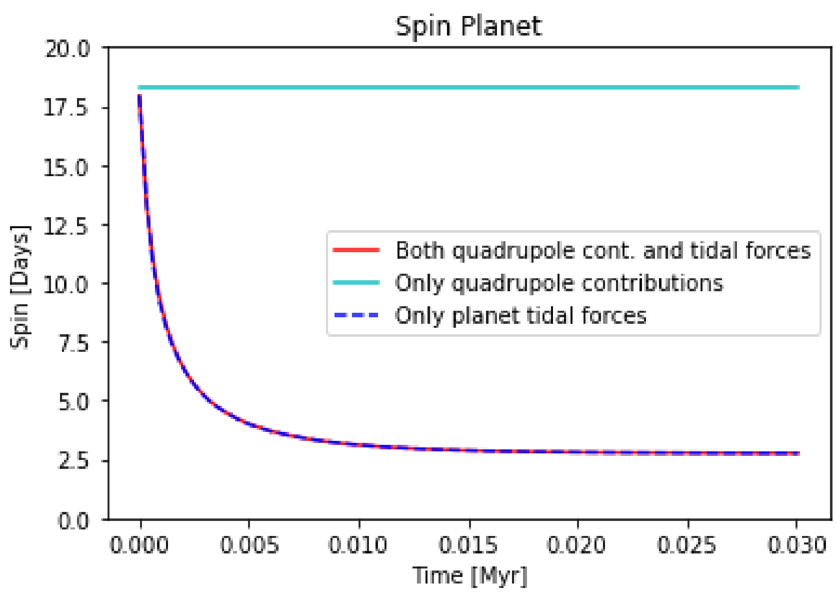

Figure A.1: The spin evolution.

There are several contributions to the tidal forces which govern the time scales for the spin synchronization, the circularization of the orbit and the semi major axis damping. In figure A.1 one can see simulations for different configurations of enabled forces. From what can be seen in the plot for the spin is that the main contributions to the effect come from the tidal friction contributions. There is no effect at all when the tidal friction contributions have been removed which shows us that the tidal friction forces play a more vital part than the quadrupole contributions.

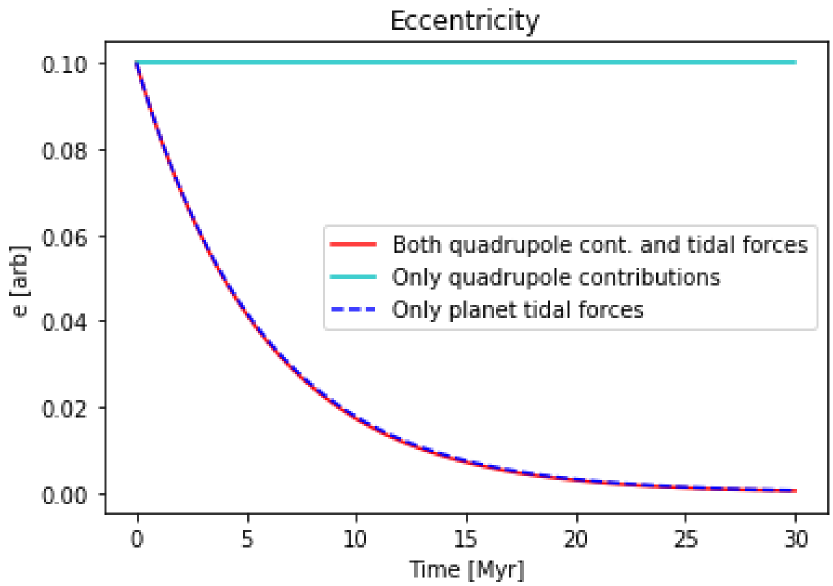

Figure A.2: The eccentricity evolution.

We see the same phenomena in figure A.2 for the circularization of the orbit as for the spin synchronization in figure A.1. When we only have the quadrupole contributions we do not see any effect, but as soon as we have tidal forces then the eccentricity will start to dampen. This then depends same type of forces as used for the spin synchronization, in other words the tidal friction forces. That explains why there is no effect when only the quadrupole contributions are enabled and no tidal friction forces. As can be seen the graphs looks the same for when both the forces are enabled and when only the planetary tidal friction forces are included, which shows that the planetary forces are the ones that govern the time scale.

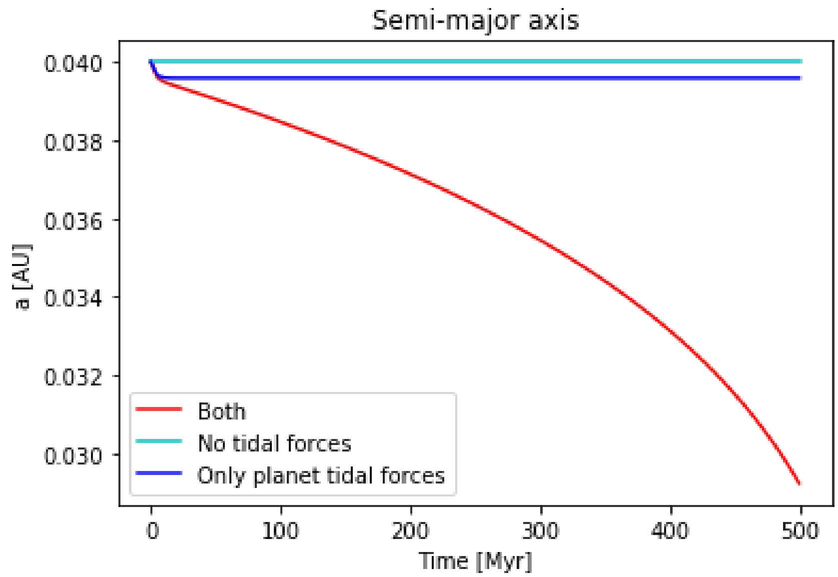

Figure A.3: The semi-major axis evolution.

In figure A.3 the tidal friction forces from the star is much more important than those from the planet. Here we have, just like for the spin and eccentricity, that the semi major axis does not change when there are no tidal forces in play. When only the planetary tidal forces are enabled the semi-major axis decays at first during the first 30 Myr but then remains constant. This is because the planetary tidal forces affect the eccentricity which makes the semi-major axis decay, but once the eccentricity evolution is finished the semi-major axis remains unchanged.