# Lund University

Master Thesis
Department of Industrial Management and Logistics

# Forming an Inventory Control Policy and Forecasting Model for an E-Commerce Company

A Study at LSBolagen


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# Abstract

**Title:** Developing an Inventory Control Policy and Forecasting Model for an E-Commerce Company

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**Background**: Increasingly competitive business environments forces companies to develop better control of the material flows in their supply chain. Inventories are often kept in large amounts in order to buffer against uncertainties, resulting in high average holding costs. An important part in controlling this flow of material and inventory is to decide upon when to order and how much. Two important components when evaluating these two, is the inventory policy used as well as the method of forecasting.

**Purpose:** Forming a basic inventory policy and forecasting model for the warehouse in Sweden.

**Research Questions:** (1) How can the inventory policy for the warehouse in Sweden be improved? (2) What is an adequate forecasting model for the purchasing department?

**Methodology:** In this thesis, frameworks presented by Yin (2009) and Hillier & Liebermann (2010) are used to construct the research approach. An initial analysis of the problem presented was conducted in order to properly identify the intended problem. Following this, an analytical analysis of gathered data was conducted to analyse the current performance of the company. Finally, models based on well-known theory were constructed to represent the new inventory policy and forecasting method.

**Conclusion:** A simple inventory policy could be constructed. Lack of historical data and previous analysis meant that an exact representation of the current situation could not be constructed. The demand could be approximated to follow a normal distribution. As the company currently orders in batches, a continuous review (R, Q) policy was chosen. It was found that the inventory levels could be decreased in almost all cases when setting 95 % fill rate constraint. Due to lack of historical data, an adequate forecasting model could not be decided upon. Instead Alternative Forecasting Application Models (AFAMs) representing the different models were developed for choosing a suitable forecasting model in the future.

**Keywords:** Inventory Policy, Forecasting Model, Stochastic Demand, Optimisation,

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# Table of abbreviations

A - Ordering cost (fixed)
AFAM – Alternative Forecasting Application Model
EOQ - Economic Order Quantity
ERP - Enterprise Resource Planning
h - Holding cost rate (SEK/year)
IL - Inventory Level
IP - Inventory Position
KS - Kolmogorov-Smirnov
L - Lead time
MAD - Mean Absolute Deviation
MOQ - Minimum Order Quantity
MTS - Make-To-Stock
OR - Operations Research
Q - Batch Quantity Size
R -Reorder Point
SCM - Supply Chain Management
SKU - Stock Keeping Unit
T – Review Period
WMS – Warehouse Management System
3PL – Third-party Logistics

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# 1. Introduction

*This section will describe the background of the master thesis as well as an introduction to the case company. The identified problem is discussed with the identified research questions and delimitation.*

## 1.1 Background

In today's business world, companies are facing very demanding customers that will take their business elsewhere if they are not satisfied. Requirements such as short lead times and the ability to be flexible with customisation of products puts a big pressure on competing organisations (Christopher, 2011). As a result, *Supply Chain Management* (SCM), i.e. the control of material flows from suppliers of raw material to customers, is a crucial problem for organisations and its strategic importance has been recognised by top management. Total investment in inventories is enormous, and as such, there is large improvement potential regarding the amount of tied-up capital in warehouses (Axäter, 2006).

However, sometimes large quantity of stocks is a necessity. The two main reasons for having large stocks are the economies of scale of ordering products in batches and the uncertainties within a supply chain. Demand can be difficult to forecast and could stem from different order patterns by customer resulting in high variability, proving that demand management is a key aspect to improve a company's SCM (Lambert, & Cooper, 2000). Supply is often dependent on actors upstream in the SC. These factors in combination with lead times in both production and distribution creates a need to apply safety stocks to operations. To mitigate the impacts of these factors, adopting appropriate inventory policies can reduce their inventory. (Axäter, 2006)

## 1.2 Company description

LSBolagen is an e-commerce company with web sites in the Nordics, UK, Germany and France. Their office is located in Ängelholm, Sweden, with approximately 30 employees as of 2018. They are specialised in durable goods and retail products such as wine cabinets, large range cookers and table tennis products. The main goal of LSBolagen is to deliver their range of products to all of Europe with short lead times and low cost.

LSBolagen is a growing company with a turnover of 100 MSEK in 2017. Their present business model involves purchasing from both Europe and China. One warehouse is located in Klippan, Sweden, and the other in the UK. The warehouse in the UK is rented through a 3PL, from where they have daily deliveries to the eight countries they are operating in.

## 1.3 Problem formulation

All products of LSBolagen are Made To Stock (MTS), resulting in high amount of tied up capital due to expensive SKUs. At present, purchasing of products at LSBolagen is done irregularly based on the company's own decision model. This decision model is based on a mean-average assessment, considering the sales data of previous months to produce a forecast. Procurement is then decided upon with a formula taking into consideration the following data on a three-monthly basis: stock-on hand, amount in transit to customer, historical sales and the expected quantity to be delivered from suppliers. The resulting value for each SKU is then individually assessed and compared to the general reorder points used. Long lead times and quantity limitations from suppliers further complicates the process as products often must be consolidated to fill up containers to a certain amount.

Recently, the company has found a need to assess and evaluate their inventory management process. There has been no previous analysis of whether the used reorder points are adequate for their processes or if they should be changed. As such, there is a need to develop an evaluation framework for the performance of SKUs in stock. Which SKUs have been over- and under-procured and in what periods, what implications does backorders have for sales and what could the optimal reordering points for the purchasing department be. Furthermore, as purchasing decision is done based on historical data, there is a need to develop a forecasting tool for decision making.

Additionally, as LSBolagen is an international company with a satellite warehouse in the UK, there is a need to further coordinate assortments between the main warehouse in Sweden and the satellite warehouse in the UK.

## 1.4 Purpose of the study and research questions

The purpose of the master thesis is first to analyse the current purchasing process and inventory policy used by the company, then to propose and evaluate an improved inventory and forecasting policy adequate for the company's operations. The thesis will answer the following research questions:

1. How can the inventory policy for the warehouse in Sweden be improved?
2. What is an adequate forecasting model for the purchasing department?

## 1.5 Delimitations

The study will be conducted for the case company's main warehouse in Klippan. As LSBolagen has around 400 SKUs, only key articles in the wine cabinet category presented by the case company will be analysed. Furthermore, as this is the first attempt to fit a theoretical inventory policy to their practice, the model will be kept simple. The focus will be to minimise the average stock levels following a service level constraint. We will not change the method of current operations, e.g. choice of suppliers. The final implementation of a new inventory policy will be done by the company.

# 2. Methodology

*This section will present, describe and motivate the chosen research strategy for this study. The purpose is to explain how the study has been performed and how valid and reliable the obtained results are.*

## 2.1 Research approach

As this study is based on practical inventory control problems identified by the company, a standardised method for operations research (OR) projects has been chosen for this thesis. The OR approach can be divided into the following 6 phases:

1. Define the problem and gather the relevant data
2. Formulate a mathematical model to represent the study
3. Develop a computer-based procedure for deriving solutions to the problem from the model
4. Test the model and refine it as needed
5. Preparing to apply the model
6. Implementation

(Hillier, & Lieberman, 2010)

### 2.1.1 Define the problem and gather relevant data

The nature of practical problems encountered by OR teams is often vague and not precisely defined. Therefore, the initial phase of any OR project is to develop a well-defined definition of the objective to be solved and what constraints can be identified. Before moving forward, it is important to maintain a holistic approach for the given objective as an OR study should aim to optimize the whole system and not just one function. As such, the definition of the objective should be conducted in collaboration with management. After the definition has been properly defined, necessary data has to be gathered from the company. As data from both inventory systems and ERPs can be quite extensive, necessary time has to be allocated to the data mining process including cleaning the data and identifying relevant data and interesting patterns. (Ibid.)

### 2.1.2 Formulate a mathematical model to represent the study

After the objective is properly defined, the next step is to re-formulate this problem to be convenient for analysis. The conventional OR-approach is to construct a mathematical model which represents the essence of the problem. A first step when constructing a model is to begin with a very basic model and then move towards more elaborate models - until a model which sufficiently covers the complexity of the problem is found. (Ibid.)

### 2.1.3 Develop a computer-based procedure

With a mathematical model created, the next step is to analyse the outcome of the model. If a model is created to find an optimal solution, it has to be recognized that it is only optimal for the model at hand and the assumptions that define it. As the model is an idealized rather than exact representation of the actual problem, there are no guarantees that the best possible solution to the real system has been found. However, a well formulated and valid model should be a good approximation of the real problem. Furthermore, research suggests that rather than focusing on optimization of the problem. Instead focus should be on satisficing, reaching results which enables multiple goals to be met. In order to mitigate the risk of the result being an optimal solution with low practical value, a thorough post-optimality analysis needs to be conducted. This can be seen as a what-if analysis to see what impacts other scenarios would have on the optimal solutions. By conducting this sensitivity-analysis, it is possible to identify which parameters are sensitive, i.e. they can’t be changed without changing the optimal solution. (Ibid.)

### 2.1.4 Test the model and refine it as needed

A first draft of a mathematical model often contains some flaws. It could be that it does not incorporate the right parameters or constraints, as it is hard to gain thorough understanding of all the aspects of the problem. However, as a mathematical model is an abstract construction of reality, it requires some extent of assumptions for the model to be tractable. Therefore, an important part of the analysis is to ensure that the model is a valid representation of the problem. A proper criterion when ensuring the validity of a model, is to evaluate the model outcome for alternative courses of action. The obtained results should follow what would happen in the real world. (Ibid.)

However, it is difficult to create a model that is valid for all cases. Because they are often problem specific. A systematic approach to validate a model is to use a retrospective test. This test involves reconstructing the past using historical data and then check how the model and the resulting solution would have performed in this scenario. A disadvantage of this test is that it uses the same data which were used to guide the construction of the model in the initial stage. However, if the past can be deemed as a satisfactory representation of the future, then this approach should work sufficiently. (Ibid.)

### 2.1.5 Preparing to apply the model

After the model has been developed and tested it is time to prepare for implementation. If the aim of the model is to be used repeatedly, the next step is to ensure that the user understands the model and how it should be used. The system describing the application should include, model, solution procedure and operating procedures for implementation. (Ibid.)

### 2.1.6 Implementation

The final step of the OR-study includes the final implementation of the results of the model. This should be carried out in a number of steps. First, the OR-team gives management an explanation of the new system and how it relates to operating procedures. Next, the two parties share responsibility when launching the new system. Throughout the new user period, it is important for the OR-team to obtain feedback on the performance of the system. This in order to be able to change it if the assumptions are found to no longer be valid. A final step is the documentation of the methodology used in the study in order for it to be reproducible in the future. (Ibid.)

To support this OR-study, a literature review and mapping of the current processes will be conducted in parallel to the study.

## 2.2 Research quality

According to Yin (2009), research design represents a set of logical statements. However, it is also possible to judge the quality of the design according to certain logical tests. Generally, four tests are used for establishing empirical research. These four tests are:

- Construct validity
- Internal validity
- External validity
- Reliability

### 2.2.1 Construct validity

The first step in testing the quality is to identify the correct measures for the study. In order to do this, there are three different tactics. The first tactic is using multiple sources of evidence for investigation, for encouraging convergent lines of inquiry. This tactic is relevant during the data collection phase. The second tactic is to establish a chain of evidence, which also is relevant during the data collection. Lastly, it is suggested that the study is reviewed by the key informants. (Yin, 2009)

### 2.2.2 Internal validity

For explanatory studies, researchers try to explain why and how events occur. The person researching on the subject wants to avoid making wrong conclusions regarding different factors of a phenomenon. In addition, internal validity is related to questioning the reached conclusion's reliability and also analysing different, potential conclusions. This is important to consider when there is lack of data for certain conditions that can't be detected by the investigator. There are some tactics for constructing internal validity. For instance, using logic models, patterns matching, address rival explanations and do explanation building. (Ibid.)

### 2.2.3 External validity

External validity considers whether the study's findings are applicable to other similar studies than the active one. This includes defining areas from the study that can be used in other cases. By conducting replications of the study, and expecting the same or similar results, the reliability of the study can be strengthened and claimed to support the implied theory. The recommended strategy for improving the external validity in single studies is applying well-known theory. (Ibid.)

### 2.2.4 Reliability

Testing for reliability focuses on ensuring that if the study would be conducted by another party, by following the same procedures, the outcome would be the same. Reliability aims to reduce the errors and biases in the study. The reliability of the study can be questioned, by the outside researcher, through investigation of the documentation of the procedure and the results. (Ibid.)

### 2.2.5 Research quality

In this master thesis, the following actions have been taken in order to ensure the quality of the study. The constructed validity is assumed by the case company who has identified specific problems in their operations. Internal validity is strengthened by gathering data from multiple sources within the case company, both quantitative data from the ERP and WMS, as well as qualitative data from observations. Furthermore, meetings have been conducted regularly with representatives of the different functions in order to verify whether the collected data is representative or not. Additionally, in order to strengthen the internal validity of the constructed model, retrospective tests have been used. This has been done in line with what is discussed in section 2.1.4, where the model has been tested with historical data to ensure that the results fall in line with what should happen in reality. The external validity is strengthened by applying well-known mathematical theory when constructing the models. Thus, the models are applicable to other studies with similar conditions. Finally, in order to construct reliability, documentation and meetings have been conducted throughout the work process, enabling the case company to understand and proceed with the study.

## 2.3 Data collection and data analysis

The following section will describe various sources of data that were available and the approach to analyse the information.

### 2.3.1 Types of data

Data that can be obtained in research studies may be classified as quantitative or qualitative. The quantitative data can be represented numerically and analysed with statistical models. This includes information that can be either classified or calculated that contains different characteristics. For instance, weight, colour, shares and amounts. Data that is qualitative involves information that for the most part consists of descriptions and words with nuances and details. Qualitative data needs different approaches for analysis when categorizing and sorting. To analyse complex issues, it is recommended to combine these two types of data. (Höst et al., 2006)

### 2.3.2 Sources of data

In a thesis, there are multiple ways of collecting data. According to Höst et al. (2006), the most common sources of data are; logbooks, surveys, interviews, observations, measures and data collected by others. Logbooks are often used for documenting data collected continuously throughout the process, such as data from informal meetings and interviews. Observation and interviews are often used to build and understanding of the processes and problems at the company. Observations can further be divided into either direct observation or participant observation, with the difference being that a participant observer is more involved in interaction with the studied item. Semi structured interviews can be conducted to gather relevant information. Benefits of interviews contra surveys is that the interviewee is more likely to provide extensive answers to questions compared to the obtained answers from a survey. Measures are useful to collect physical data such as the volume of an item. Finally, data collected by others in the form of archival records, statistics, registers and processed data. This data has to be critically analysed as it has been collected and used by another party.

## 2.4 Data Collection and analysis in this study

This section will describe the chosen sources of information used in the study as well as the approaches used for analysing it.

### 2.4.1 Data collection

As this study is a practical problem study, focus has been on conducting a quantitative analysis of the company's processes. Focus has been on collecting data already gathered by others, and to verify the reliability of this data though meetings with the employees where the analysis is continuously analysed and presented. Data regarding sales, procurement, lead times and item master data has been collected though the available ERP-system at the company. Inventory data has been collected from their separate WMS. As the company is relatively young and growing, the obtained data only stretched one year back. Data dating further back is deemed no longer representative of the current operations.

Documentation and observations are done to further understand and map the operations of the case company. The different steps of the inventory management process, such as the review of inventory, forecasting and replenishment, are observed and analysed through informal interviews and observation of employees at the purchasing department.

### 2.4.2 Approach of analysis

In order to construct an inventory policy for the case company, a substantial amount of quantitative data had to be collected. The collected qualitative data has been mainly used to verify and support the analysis conducted from the quantitative data. Sales, inventory and purchasing data has been collected and initially analysed by studying the mean and standard deviations to understand the current problem. Data had to be cleaned from conversion errors and unrealistic values both small and big. Furthermore, the analysis has been conducted by constructing a model in accordance to section 2.1. This model will provide an approximate inventory policy for the case company and is based on established literature and mathematical models which will be further discussed in section 3. The inventory policy and forecasting methods have been programmed in MATLAB and Excel respectively. In order to further analyse sales and lead times pattern, the distribution fitting program Stat::Fit has been used.

# 3. Theory

*This section will describe the important concepts of inventory control and the necessary components to develop a new inventory policy. In addition, the necessary theory and tools for constructing a forecasting model will be introduced.*

## 3.1 Ordering concepts

Axsäter (2006) argues that the purpose of inventory control systems is to, based on the stock situation and cost factors, determine when and how much to order. It is intuitive to consider only the physical stock on hand when discussing the stock situation. However, the ordering decision cannot be made just from the stock on hand. Instead the outstanding orders yet to arrive as well as possible backorders has to be included. In inventory control the stock situation is therefore usually characterized by the inventory position which can be described as:

$$Inventory\ position\ =\ stock\ on\ hand\ +\ outstanding\ orders\ -\ backorders \qquad (3.1)$$

In this project, the inventory position will be used to determine the new reorder points. However, the reorder points are developed by balancing the holding cost for inventory as well as the cost of shortage which both are dependent on the inventory level:

$$Inventory\ level\ =\ stock\ on\ hand\ -\ backorders \qquad (3.2)$$

When using an inventory control policy in practice, the system can be reviewed either continuously or periodically. With continuous review a new order is placed as soon as the inventory position is sufficiently low. The triggered order will then arrive to the warehouse after a certain lead time ($L$). In the case of periodic review, the inventory position is instead reviewed at certain points in time. Let $T$ denote the review period, i.e. time interval between reviews. As we do not know when during the review period an order will be placed, the time period in which the periodic review has to guard against variations in demand becomes $T + L$. Similarly, continuous review only has to guard for variations in demand during the lead time. However, a periodic review with a small review period is very similar to a continuous review.
(Axsäter, 2006)

## 3.2 An (R, Q) policy

With an (R, Q) policy, the inventory position (IP) is observed. When the inventory position declines to (or below) the reorder point (R), a batch quantity of size (Q) is ordered. If the Inventory position is exceedingly below the reorder point, it may be necessary to order a multiple of the batch quantity Q in order to get above R. To further illustrate this process, Figure 1 below illustrates the behaviour of an (R, Q) policy with periodic review and continuous demand. (Ibid.)

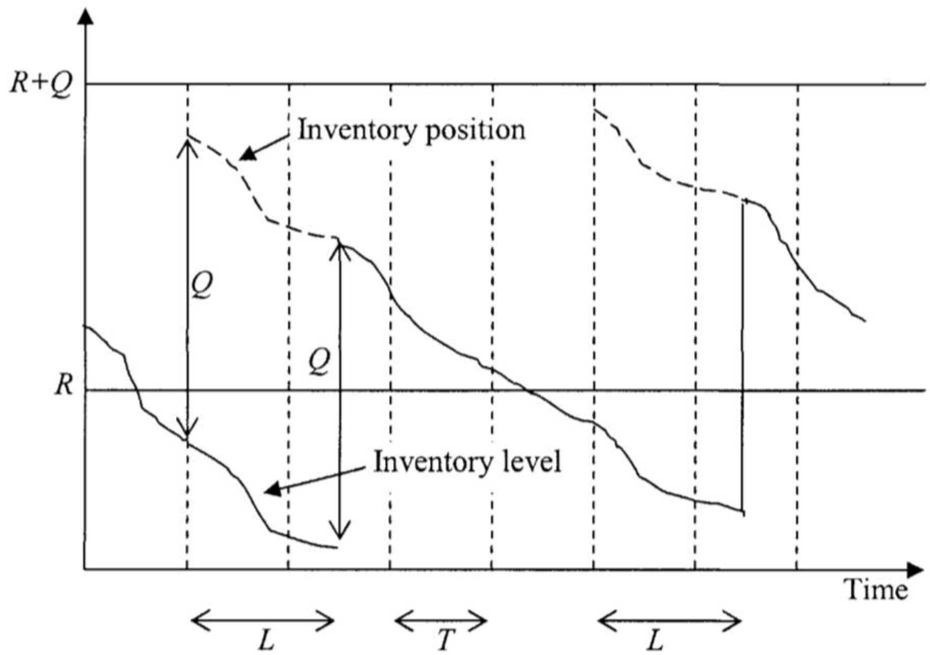



*Figure 1. Illustration of (R, Q) policy with period review and continuous demand. (Axsäter, 2006)*

A natural question to ask is whether there exists a policy better than the described (R, Q) policy. Axsäter, (2006) elaborates that in most situations for singe-echelon inventory systems, especially for situations with low to no ordering cost and fixed batch quantities, the (R, Q) policy is better to use. An alternative would be to replace the fixed quantity Q by always ordering up to a fixed inventory position. Such a policy is however not necessarily optimal when dealing with service constraints. Section 6.4 provides a further motivation as to why the (R, Q) policy has been deemed appropriate to model the current system by.

## 3.3 Stochastic demand distribution

In practice, the demand during a certain time is nearly always a nonnegative integer, i.e. it is a discrete stochastic variable. In general, for products with low demand, a discrete demand model is suitable to use. However, for products with a higher demand, a continuous demand approximation may be convenient. (Axsäter, 2006)

When modelling demand as continuous distribution, the most common distribution to use is the normal distribution. From the Central Limit Theorem (CLT), under general conditions, a sum of many independent, random variables will have a distribution that is approximately normally distributed (Blom, Enger, Englund, Grandell & Holst, 2005). If the demand in different time increments is mutually independent, and the lead time is the sum of time increments, an approximation can be made. According to CLT, the lead time demand can be approximated as normally distributed - given that the lead time is long enough. However, a problem with the normal distribution is that there is always a small probability for negative demand (Axsäter, 2006). Axsäter (2011) discusses that even lead time demand as low as 10 units, could be modelled successfully with a normal approximation as the distribution is very robust. Tyworth &

John (1997) further elaborates on this subject and argues that a model with normally distributed demand is very robust when calculating costs as well as service levels.

Given the mean and standard deviation of the demand during a specific time, a unique normal distribution can always fit into these kinds of parameters. A standardised normal distribution with a mean $\mu$ and standard deviation $\sigma$ has the following density:

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty \tag{3.3}$$

(Blom et. al., 2005)

As the demand in different time increments is mutually independent, the lead time demand can be seen as the sum of these time increments. Moreover, according to CLT, the lead time demand can be approximated as normally distribution given that the lead time is long enough.

## 3.4 Cost aspects to consider

This section will explain the different cost aspects to consider when constructing an inventory policy, such as: holding costs, ordering costs, backorder cost.

### 3.4.1 Holding cost

When holding stock there is an opportunity cost for tied up capital in inventory. In principle, holding cost should be similar to the return of an alternative investment. However, the return is not with all certainty expected to be equal to the holding cost. This is due to financial risks associated with alternative investments that need to be taken into account. The holding cost per unit and time unit is usually determined as a percentage ($\kappa$) of the unit value. This percentage is then multiplied with the variable replenishment cost per unit ($C$):

$$h = \kappa * C \tag{3.4}$$

(Axsäter, 2006; Hadley & Whitin, 1963):

This percentage can’t be allocated to all items, hence is product specific. For instance, computers should have high holding cost because of obsolescence. It is common that the percentage is considerably higher than the interest charged by banks (Axsäter, 2006). As Berling (2005) describes the holding cost (h), or carrying cost, it is the cost of carrying one unit of inventory for one unit of time. In accordance with the influential work of La Londe & Lambert (1975), the holding cost can be divided into four major cost components:

- Capital cost
- Storage space cost
- Inventory service cost

- Inventory risk cost

Occasionally, the last three mentioned components are viewed as one and then indicated as out-of-pocket holding cost. The inventory cost risk is associated with a decrease in the value of the inventory, because of changes in the goods' physical attributes. For instance, the stored goods could be affected by change in fashion, deterioration etc. However, this risk does not involve costs in association with financial risk i.e. the change in the value of the inventory in relation to the growth of the economy. (Berling, 2005)

### 3.4.2 Ordering cost

Ordering costs are often associated with fixed costs. Set up cost is synonymous with order cost, with the latter being more frequently used in manufacturing environments. This ordering cost must be weighed against the expected holding cost when determining the order/batch quantity. Ordering costs are typically used when the order is placed at an outside supplier. The costs that are included in ordering costs are clerical of:

- Preparing
- Monitoring
- Realising
- Receiving the order

Additionally, ordering costs frequently includes the costs of the physical handling of goods upon arrival and inspections. (Berling, 2005)

### 3.4.3 Backorder cost

In the event that demanded items cannot be delivered due to shortage, various costs may be incurred. A customer might be willing to wait for the product. Resulting in costs such as extra administration, discount for late delivery, transportation and handling to be incurred. However, if the customer is not willing to wait, the sale is considered lost and the contribution of the sale is subsequently also considered lost. As shortage and backorder costs are often hard to quantify in practice, it is common to replace the shortage costs with an adequate service level. (Axsäter, 2006)

### 3.4.4 Service level

In practice, the reorder point is often determined by either taking the backorder costs into consideration or by setting a service level constraint. In practice it is often regarded as easier to specify a service level. The three common service level definitions are:

$S_1 = probability\ of\ no\ stockout\ per\ order\ cycle$

$S_2 = fill\ rate\ - fraction\ of\ demand\ that\ can\ be\ satisfied\ immediately\ from\ stock\ on\ hand.$

$S_3 = "ready\ rate" - fraction\ of\ time\ with\ positive\ stock\ on\ hand$

$S_1$ can be interpreted as the probability that an order arrives in time, i.e., before the stock on hand is finished. It is a service level commonly used in practice. However, a major weakness is that $S_1$ does not take the batch quantity into consideration. The fill rate ($S_2$) and ready rate ($S_3$) are more complex to work with. Although, they will most often provide a more accurate description of the customer service level that the company measures. In this case the fill rate has been chosen to best describe the situation of the case company. (Ibid.)

## 3.5 Batch quantity

The classical Economic Order Quantity (EOQ) formula is one of the simplest and most well-known in the area of inventory control and lot sizing. The model is based on the following assumptions:

- Demand is constant and continuous
- Ordering and holding costs are constant over time
- The batch quantity does not need to be an integer
- The whole batch quantity is delivered at the same time
- No shortages are allowed

The relevant costs are the costs which varies with the batch quantity Q e.g. the holding cost *h* and the ordering cost *A*. With the demand *d* the resulting cost function can be obtained as:

$$C = \frac{Q}{2} * h + \frac{d}{Q} * A \qquad (3.5)$$

This resulting cost function is convex and by solving the cost minimisation problem with respect to Q we obtain the economic order quantity as:

$$Q^* = \sqrt{\frac{2 * A * d}{h}} \qquad (3.6)$$

By inserting (3.6) into (3.5) we get the optimal cost as:

$$C^* = \sqrt{2Adh} \qquad (3.7)$$

And can finally see how important the optimal order quantity is by combining (3.5), (3.6) and (3.7) into:

$$\frac{C}{C^*} = \frac{1}{2} * \left(\frac{Q}{Q^*} + \frac{Q^*}{Q}\right) \qquad (3.8)$$

From which it can be observed that the cost increase is a simple function of Q/Q*, which varies very little even with large deviations from the optimal order quantity.

## 3.6 Reorder point

In order to determine the reorder point, we must first choose a suitable demand model and review model. As mentioned in section 3.3, according to the CLT, the lead time demand data should follow a continuous normal distribution, given that the lead time is long enough. This hypothesis will be further tested in section 6.2. Regarding the choice of review model, is has been deemed appropriate to use continuous review. However, this is an approximation of the current process. As the review window of the case company is weekly, it can be approximated to follow a continuous review by adding half a review period to the total lead time. With a review period (T) of one week, in best case scenario, the order will be placed if the inventory position is observed to exactly hit the reorder point. In contrast, the worst case scenario is that the inventory position will not be observed to have surpassed the reorder point until T time units later. Thus, by assuming equal probability of observing an inventory position below R during the review period, we will on average place an order $T/2$ time units after the inventory position has surpassed the reorder point. The lead time can then be expressed as $\tilde{L} = L + \frac{T}{2}$. The reorder point will be determined by evaluating what reorder point is fulfilling a specific fill rate, denoted as $S_2$ in section 3.4.4

When dealing with normally distributed demand, we assume that the inventory position is uniformly distributed on the interval [R, R+Q]. Now, consider an arbitrary time t when a system is in steady state. Define $IP(t)$ as the inventory position an arbitrary time $t$, and $D(t, t + L)$ the lead time demand over the lead time $L$ between time $t$ and $t + L$, then the inventory level at time $t + L$ can be defined as:

$$IL(t + L) = IP(t) - D(t, t + L) \tag{3.9}$$

Next, we assume that the demand in mutually exclusive time periods is independent. For normally distributed demand and constant lead time, let $\mu' = \mu * L$ and $\sigma' = \sigma * \sqrt{L}$ , denote the mean and standard deviation of the lead time demand. For continuously, normally distributed demand, we can then obtain the inventory level distribution function as:

$$F(x) = P(IL \leq x) = \frac{1}{Q} * \int_{R}^{R+Q} \left[1 - \Phi\left(\frac{u - x - \mu'}{\sigma'}\right)\right] du \tag{3.10}$$

Given the inventory position $u$ at an arbitrary time $t$, the inventory level at time $t + L$ is less than or equal to $x$ if the lead time demand is $u - x$ (Axsäter, 2006). Next, the loss function, i.e., the amount of shortages that occur in one cycle, is defined as:

$$G(x) = \int_{x}^{inf} (v - x) * \varphi(v) dv = \varphi(x) - x * (1 - \Phi(x)) \tag{3.11}$$

By using that $G'(x) = \Phi(\mathrm{x}) - 1$ (3.10) can be reformulated as:

$$F(x) = \left(\frac{\sigma'}{Q}\right)\left[G\left(\frac{R - x - \mu'}{\sigma'}\right) - G\left(\frac{R + Q - x - \mu'}{\sigma'}\right)\right] \tag{3.12}$$

Furthermore, for continuous review, $S_2 = S_3$ and the service level can be expressed as the probability of positive stock on hand, which can be denoted as:

$$S_2 = S_3 = 1 - F(0) = 1 - \left(\frac{\sigma'}{Q}\right)\left[G\left(\frac{R - \mu'}{\sigma'}\right) - G\left(\frac{R + Q - \mu'}{\sigma'}\right)\right] \tag{3.13}$$

For a given service level, the reorder point can be calculated by using a bisection search. Starting with a lower bound for $\underline{R}$, e.g., $\underline{R} = -Q$ and an upper bound of $\overline{R}$. Next, $R = \frac{\underline{R}+\overline{R}}{2}$ is considered. If the resulting service level is too low $R$ can replace $\underline{R}$ and otherwise it can replace $\overline{R}$. This continues until the gap between $\overline{R}$ and $\underline{R}$ is sufficiently small.

## 3.7 Stochastic lead times

Liao & Shyu (1991) define lead times as the time from when an order has been placed, until it is has arrived in the warehouse and is ready to satisfy demand. According to Axsäter (2006), the most common type of stochastic lead time is sequential deliveries, i.e. orders cannot cross in time. While the stochastic lead times may be dependent on the previous demand due to congestions in the supply system - the demand perceived after the order has been placed will not affect the lead time. The distribution of the demand during a stochastic lead time can be replaced by a normal distribution with correct mean and variance. We denote the mean and standard deviation as (μ) and (σ) respectively. We obtain the mean of the stochastic lead time demand *D* during the as:

$$E(D) = \mu * E(L) \tag{3.14}$$

For a given $L$, $E(D(L))^2 = \sigma^2 + (\mu * L)^2$ , the variance of D can be determined as:

$$Var(D) = \sigma^2 E(L) + \mu^2 * Var(L) \tag{3.15}$$

By setting $\mu' = E(D)$ and $\sigma' = \sqrt{(Var(D))}$ we can, as an approximation, use expressions for constant lead times such as in section 3.6.

## 3.8 Distribution fitting and analysis of input data

The first step in order to construct and adequate inventory policy and forecasting model, is to determine the distribution of the demand. In order to evaluate what distribution to construct a model by, the ExtendSim-tool Stat::Fit has been used. It must be noted, however, that the test can in no way prove the hypothesis that a certain set of data points follow a particular distribution. Instead, the tested hypothesis is that the inspected data points are independent samples from a theoretical probability distribution. If the hypothesis is rejected, it can be concluded that the theoretical distribution is a not good representation of the data set. However, failure to reject the hypothesis does not imply that the hypothesis is true. There might be several candidate theoretical distributions that could be considered a good fit for the data set.

Two of the most well-known goodness of fit tests are the chi-square test and the Kolmogorov-Smirnov (KS) test, which are both used by the Stat::Fit tool. Specific definitions of both tests as well as examples and can be found in Laguna & Marklund (2013).

## 3.9 Forecasting

There are mainly two reasons why inventory control systems require products to be ordered before customers demand them. Firstly, there is almost always a lead time between the time of ordering and time of delivery. Secondly, depending on the ordering costs, it is usually necessary to order in batches rather than single units. Hence, it is vital to forecast the future demand. The demand forecast is an estimated average of the future demand for a specific period of time. However, it’s not sufficient to only estimate the average demand. There is a necessity to determine how uncertain the forecast is. If there is a great uncertainty in the forecast, there might be a need for a larger safety stock. Therefore, an analysis of the deviations in the forecast is also needed. (Axsäter, 2006)

### 3.9.1 Forecasting approaches

According to Axsäter (2006), there are generally two types of forecasting methods that are suitable for inventory control. Typically, these forecast approaches concerns a relatively short time horizon. It’s seldom necessary to look more than a year ahead. The two type of approaches are:

- Extrapolation of historical data
- Forecasts based on other factors

Forecast that is based on prior demand data, is extrapolated and based on statistical methods. Extrapolation is the most important and commonly used approach to acquire forecasts over shorter horizons. Forecasts based on other factors is used when the demand of an item is dependent on e.g. another item(s), sales campaigns and more. (Axsäter, 2006)

### 3.9.2 Demand models

As mentioned before, extrapolation of historical data is the most commonly used approach for demand-based forecasting method within inventory control. However, to find an appropriate technique, we need to model the stochastic demand. Some examples of these models are; constant, trend and trend-seasonal models (Axsäter, 2006)

#### 3.9.2.1 Constant model

The simplest demand model is constant. This means that demand in different periods are represented by independent random deviations from an average. This average is assumed to be relatively stable over time in comparison to the random deviations. To represent a constant model, the following notations need to be introduced:

$x_i = demand\ in\ period\ t,$
$a = average\ demand\ per\ period\ (assuming\ that\ it\ varies\ very\ slowly),$
$\varepsilon_i = independent\ random\ deviations\ with\ mean\ zero.$

With these notations, a constant demand model for period *t* can be represented as

$$x_t = a + \varepsilon_i \tag{3.16}$$

There are many products that can be well represented by a constant model, particularly for products that are in their mature stage of its product life cycle and used regularly. In cases where trends and seasonalities are not expected, it is reasonable to assume a constant model. (Ibid.)

#### 3.9.2.2 Trend model

In cases where the demand is assumed to decrease and increase systematically, it is possible to extend the constant model by considering a linear trend. Let

$a = average\ demand\ in\ period\ 0,$
$b = the\ systematic\ trend\ that\ increases\ or\ decreases\ per\ period\ (assuming\ it\ varies\ slowly)$

The trend model can be modelled as followed:

$$x_t = a + bt + \varepsilon_i \tag{3.17}$$

During stages such as initial growth stages and phase-out stages in a product life cycle, it is natural to assume that the demand follows a positive respectively negative trend model. (Ibid.)

#### 3.9.2.3 Trend-seasonal model

To describe a trend-seasonal model, the following notation needs to be introduced:

$F_t = seasonal\ index\ in\ period\ t\ (assuming\ that\ it\ varies\ slowly)$

The seasonal index indicates how the demand in period t is expected to change due to seasonal variations. For instance, $F_t = 1{,}2$ means that demand in period t is expected to become 20 percentage higher because of seasonal variations. The trend-seasonal demand model can be modelled as

$$x_t = (a + bt)F_t + \varepsilon_i \tag{3.18}$$

Here it is assumed that the variations of the seasons increase and decrease proportionally with the changes in the level of the demand series. This is in many cases a reasonable assumption. An alternative assumption to this is seasonal demand variations being additive. There are many products that have seasonal demand variations. For instance, demand for ice cream increases during the summer. However, a seasonal model is only essential if the demand follows the same pattern year after year. (Ibid.)

### 3.9.3 Choosing demand model

By studying the three demand models, the trend model is more general than the constant model. Consequently, the trend-seasonal model is even more general than the trend model. Even though a more general model seems to be more advantageous, it is not necessarily the case. Because a more general demand model covers a wider range of demand classes, more parameters need to be estimated. It might be quite difficult to determine accurate estimations of the parameters, especially if the independent deviations are relatively large. Thus, it might be more efficient to use a simpler demand model with fewer parameters. Therefore, a use of a simpler demand model, with fewer parameters, is more efficient. Moreover, it is worth noting that the independent deviations, $\varepsilon_t$, can't be forecasted. Hence, the best forecast for $\varepsilon_t$ is always zero. Consequently, if the independent deviations are relatively large, there is no possibility to avert large forecast uncertainties. (Ibid.)

A practical issue when using demand models, is that it is often very difficult to obtain demand data. The reason is that usually only sales are recorded. When using historical sales data instead of historical demand data for forecasting, some significant errors might occur in situations where larger portions of the total demand is lost because of shortages. (Ibid.)

The basic forecasting models in the following sections - moving average, simple exponential smoothing and exponential smoothing with trend - are commonly used and are in general suitable techniques for most items. (Ibid)

### 3.9.4 Moving average

The moving average technique is performed by taking the average over the *N* most recent demands. Here it is assumed that the underlying structure is described as a constant model. Due to the independent deviations $\varepsilon_t$ can't be predicted, the average demand $a$ needs to be estimated. In the case of $a$ being completely constant, the best estimate would be the average of all observations $x_t$. However, if $a$ vary slowly, then the most recent values of $x_t$ are the most relevant to consider. To present a moving average model, the following notations need to be introduced:

$\hat{a}_t = \ estimation\ of\ a\ after\ demand\ observation\ in\ period\ t$
$\hat{x}_{t,\tau} = forecast\ for\ period\ \tau > t\ after\ demand\ observation\ in\ period\ t$

Hence, the moving average can be described as:

$$\hat{x}_{t,\tau} = \hat{a}_t = \frac{(x_t + x_{t-1} + x_{1-2} + \cdots + x_{t-N+1})}{N} \tag{3.19}$$

Due to assuming a constant demand model, the forecast demand for any value $\tau > t$ is the same. The value of *N* depends on how slowly it is estimated that $a$ will vary, as well as the size of the stochastic deviations of $\varepsilon_t$. In the case of $a$ varying slowly and the stochastic deviations are relatively large, a higher value of *N* is more adequate. This is done in order to limit the influence of the stochastic deviations. In the other case, if the stochastic deviations are relatively small and $a$ varies rapidly, a smaller value of N is preferable. (Ibid.)

### 3.9.5 Exponential smoothing

Exponential smoothing, also known as simple exponential smoothing, is another forecasting method. Assuming a constant demand model, we strive to determine the average demand in the first period. To update the procedure of the forecast, in a specific period, a linear combination of the most recent demand and previous forecast is used. Resulting in the following model:

$$\hat{x}_{t,\tau} = \hat{a}_t = (1 - \alpha)\hat{a}_{t-1} + \alpha x_t \tag{3.20}$$

Where $\tau > t$ and $\alpha = smoothing\ constant\ (0 < \alpha < 1)$

When $\alpha = 0$, the forecast does not get updated. Consequently, when $\alpha = 1$, the most recent demand is chosen for the forecast. In monthly forecast updates, a typical value for the smoothing constant is between $\alpha = 0{,}1$ and $\alpha = 0{,}3$ (Axsäter, 2006). According to Silver, Pyke & Peterson (1998) a larger value of $\alpha = 0{,}3$ in simple exponential smoothing procedures should raise the question of validity of the assumed underlying model. In such cases, it should be considered to use an exponential smoothing with trend model instead.

### 3.9.6 Exponential smoothing with trend

An updated version of exponential smoothing is that the demand also follows a trend. In this case, two parameters need to be estimated - $\hat{a}_t$ and $\hat{b}_t$. As in the constant model, $\hat{a}_t$ denotes the average demand per period while $\hat{b}_t$ denotes the trend that is the systematic decrease or increase in demand per period. As mentioned before, the independent deviation, $\varepsilon_i$, cannot be predicted. There exist different methods for estimating $\hat{a}_t$ and $\hat{b}_t$. The considered approach was proposed by Holt (2004):

$$\hat{a}_t = (1-\alpha) * \left(\hat{a}_{t-1} + \hat{b}_{t-1}\right) + \alpha x_t \tag{3.21}$$

$$\hat{b}_t = (1-\beta) * \hat{b}_{t-1} + \beta * (\hat{a}_t - \hat{a}_{t-1}) \tag{3.22}$$

The smoothing constants $\alpha$ and $\beta$ are values between 0 and 1. Furthermore, the "average" $\hat{a}_t$coincide to period $t$ where the demand has been observed. For the future period, i.e. $t+k$, the forecast can be obtained by:

$$\hat{x}_{t,t+k} = \hat{a}_t + k * \hat{b}_t \tag{3.23}$$

Note that the trend could either be positive or negative. The purpose of exponential smoothing with trend is to follow systematic linear changes in demand more accurately. When the smoothing constants, $\alpha$ and β, are relatively large, the forecast reacts more quickly to changes. However, this also makes the forecast more sensitive to stochastic deviations. When the forecast is updated monthly, some conventional values for the smoothing constants are $\alpha = 0{,}2$ and $\beta = 0{,}05$. Additionally, when initiating the forecast, it is plausible to assume the trend $\hat{b}_0 = 0$. (Axsäter, 2006)

## 3.10 Human judgement

When deciding an adequate demand model, it is mostly based on historical data. However, in some cases, human judgement is more advisable to use for estimating the future demand. For instance, in situations when known factors will affect the future demand, although haven't affected the previous demand. Thus, forecasting systems should be designed so both automatic and manual forecast can easily be used. Cases where manual forecasting is suitable is when factors such as; sales promotions, price changes, new products, conflicts that influence the demand or new regulations influence the demand. (Ibid.)

## 3.11 Forecast errors

The following section describe forecast errors and how they should be updated. In addition, the section contains how manual forecasting should be considered.

### 3.11.1 MAD and standard deviation

In addition to estimating the mean of the future demand, it is also necessary to know how uncertain the forecast is - i.e. the size of the forecast errors. This is necessary for determining a suitable safety stock. Commonly, variations are described as the variations around the mean through the standard deviation. By letting $X$ be a stochastic variable with the mean, $m = E(X)$, the standard deviation σ is defined as:

$$\sigma = \sqrt{E(X-m)^2} \tag{3.24}$$

Another measure of variability is the *Mean Absolute Deviation* (MAD) which was originally recommended for its simple computational practices. MAD describes the expected value of the absolute deviation from the mean and defined as:

$$MAD = E\,|X-m\,| \tag{3.25}$$

(Ibid.)

According to Silver, Pyke & Peterson (1998), MAD can also be described as:

$$MAD = \sum_{t=1}^{n} \frac{|x_t - \hat{x}_{t-1,t}|}{n} \tag{3.26}$$

The original reason for its use was its computational simplicity. However, if the errors are assumed to be normally distributed, the standard deviation can be obtained. Standard deviation and MAD give similar picture of how the demand varies around the mean. In the case of assuming the forecast errors being normally distributed, which is very common, the relation between MAD and σ is described as:

$$\sigma = \sqrt{\frac{\pi}{2}} * MAD \;\approx\; 1{,}25 * MAD \tag{3.27}$$

The relation above is commonly used within forecasting. Even in cases when it is less reasonable that the errors are normally distributed (Axsäter, 2006). However, according to Silver, Pyke & Peterson (1998), using a normal distribution for forecast errors are recommended for three reasons. Firstly, empirically the normal distribution often gives a better fit to data than many other suggested distributions. Secondly, if the lead time is long and forecast errors in several periods are added, a normal distribution would be expected through the Central Limit Theorem. Lastly, the normal distribution leads to manageable and analytical results.

### 3.11.2 Updating MAD

Let $MAD_t$ denote the estimation of MAD after period $t$. At the end of period $t-1$, a forecast for period $t, \hat{x}_{t-1,t}$ can be obtained from the forecasting system. This could be regarded as a "mean" for the stochastic demand in period $t, x_t$. Although, this is not always accurate due to systematic errors in the forecast that frequently occurs. After period $t$, the value $x_t$ is obtained and consequently the absolute deviation from the "mean", $|x_t - \hat{x}_{t-1,t}|$. Generally, it is assumed that the absolute deviations can be observed as independent, stochastic deviations from the mean that varies somewhat slowly. Following a constant model in accordance to (3.16), $MAD_t$ can for example be updated by exponential smoothing. The forecast for the $MAD_t$ at the end of period $t$ is described as:

$$MAD_t = (1-\alpha) * MAD_{t-1} + \alpha * \left| x_t - \hat{x}_{t-1,t} \right| \tag{3.28}$$

where $0 < \alpha < 1$ is the smoothing constant. Note that this constant is not necessarily the same as in exponential smoothing but often it is used in a similar fashion. Due to the absolute deviation frequently varies a lot, the smoothing constant is chosen relatively small. (Ibid.)

Alternatively, to (3.28), is to update the $MAD_t$ as a moving average (3.19). When $MAD_t$ has been determined, the standard deviation can be calculated in similar way as in (3.27):

$$\sigma = \sqrt{\frac{\pi}{2}} * MAD_t \approx 1{,}25 * MAD_t \tag{3.29}$$

(Ibid.)

### 3.11.3 Manual forecast

The previously discussed forecasting models have been related to extrapolating historical data. However, in some cases, these forecasting techniques are less suitable to use. For instance, in cases where factors affecting the future demand, e.g. promotions, has not been observed in historical demand. In these situations, it is appropriate to let manual interactions with the forecast be conducted rather than automatic ones. Hence, forecasting systems should be designed so that switching between automatic and manual forecasts can be easily managed. (Ibid.)

Usually, the use of manual forecasts is conducted for a number of items during a relatively short time span. Some cases were manual forecasts can be considered are:

- Sales campaigns
- Conflicts that affect the demand
- Changes in price
- Newly introduced products without historical data
- New competitive products in the market
- New regulations

A frequent complication with manual forecasts is the systematic errors that occur due to optimistic or pessimistic beliefs by the people that forecasts. Resulting in over- respectively under-procurement of stock. (Ibid.)

### 3.11.4 The problem of overfitting

According to Kuhn & Johnson (2013), there exists many techniques that can follow the structure of a set of data very well. In a matter of fact, it follows the data so accurately that the applied model correctly predicts every sample. Furthermore, in addition to following the general patterns of the data, the model also has the capability to follow each characteristic of the sample's unique noises - i.e. some of the residual variations have been extracted as if they represent the underlying model structure. These types of models are said to be overfitted and will generally have poor accuracy when forecasting a new sample. According to Liu (2000), overfitting normally occurs when including too many regressors in a model or using more complicated non-linear models to estimate a linear or non-linear relationship.

## 3.12 Updating Order quantities and Reorder points

In earlier sections of the thesis, various techniques for forecasting and determining batch quantities and reorder points have been described. These techniques can be implemented into an inventory control system. Forecasts are usually updated in periods. Generally, it is practical to update the reorder points and batch quantities at the same time as the forecast is updates. Let $t_F$ denote the forecast period which can be set differently - days ($t_F = 30$), months ($t_F = 1$), years ($t_F = 1/12$) etc. Generally, it is favourable to use the same time unit in all inventory control calculations. Usually, forecasts are updated by either exponential smoothing (Section 3.9.5) or exponential smoothing with trend (Section 3.9.6).

In order to determine the reorder point, the distribution of the lead time demand has to be determined. By letting μ' and σ' be the mean and average of the lead time demand after the forecast update, the simple exponential smoothing and moving average is:

$$\mu' = \frac{\hat{a}_t}{t_F} * L \tag{3.30}$$

For exponential smoothing with trend

$$\mu' = g(L) \tag{3.31}$$

$$g(L) = \left(\hat{a} + \frac{\hat{b}_t}{2}\right) * \frac{L}{t_F} + \frac{\hat{b}_t * L^2}{2 * {t_F}^2} \tag{3.32}$$

The standard deviation is described as:

$$\sigma' = \sigma * L^c = \sqrt{\frac{\pi}{2}} * MAD_t * \left(\frac{L}{t_F}\right)^c \tag{3.33}$$

Where the parameter$c = 0{,}5$ if the forecast errors are assumed to be independent during different time periods. This is considered a standards assumption, in which the parameter $c$always is within the interval $[0{,}5\ ;\ 1]$.

The batch quantity is most commonly updated by demand per time unit (μ), holding cost rate per time unit (h) and ordering cost (A), similar as in section 3.5:

$$Q = \sqrt{\frac{2 * A * \mu}{h}} \tag{3.34}$$

(Axsäter, 2006).

Furthermore, it is not common to consider stochastic variations in lead times. A reason for this is the difficulty to determine the distribution of the lead time. However, if the lead time variations are known and deliveries are sequential, it is appropriate to utilize the approximation seen in section 3.7. With the newly updated mean and standard deviation for the lead time, the batch quantities and reorder points can be updated as described in section 3.5 and 3.6 respectively.

## 3.13 Performance evaluation and aggregation

The general purpose of an inventory control system is to reduce ordering and holding cost while still maintaining customer satisfaction. Therefore, it is essential to be able to evaluate the performance continuously. Evaluating performance is needed for the availability of adjusting the control when different changes in operations occurs and creating motivation for efficient application. The performance evaluation should consider several important aspects; e.g. costs regarding tied up capital in inventories and ordering costs or service levels. Normally, it’s not difficult to decide proper performance measures. The most difficult objective is to choose the level of aggregation for measurement. Due to often hundreds or thousands of items, it is not practical to follow individual items separately. Neither is it suitable to aggregate all items. It is advantageous to aggregate products that are similar and having similar inventory control. The evaluation of the performance should primarily be aggregated to relatively large groups and afterwards, if required, be divided into smaller groups. Additionally, it is important to be aware of potential measurement errors and take them into account when interpreting the results. (Ibid.)

# 4. Background to LSBolagen

*The following section gives an overview of the case company, particularly with regards to its products markets, customers and supply network.*

## 4.1 The company

LSBolagen is, as previously mentioned, distributing a wide range of products served in European countries. The company can be divided into four business units: Administration, Sales, Purchasing, Marketing and Economy. With the current business model, products are procured from suppliers in both Europe and Asia, with the largest suppliers operating in East Asia.

This study will focus on the key products of the two biggest suppliers located in China. Further explanation of the characteristics of the products analysed, as well as the different markets and supplier network, will be provided in the following sections.

## 4.2 Products

The products the company is selling are mainly high-end retail products of different types. The six products chosen for evaluation are from two highest grossing segments. Furthermore, these six products are among the highest turnovers within their specified category and have been chosen to be evaluated by the company. The products will henceforth be referred to as Product A-F, where products A-C are from one segment and products D-F from another. These products are mostly bought by customers by one piece at a time, but in some cases more than one per order.

## 4.3 Market and customers

The customers the company serves are mainly private consumers and companies interested in these retail products for long-term use. Additionally, some companies buy these products during events and are consequentially bought in larger quantities. The customers are, as mentioned, mainly resident in the Nordic countries, the UK, Germany and France. The different customer segments' geographical areas will not be analysed individually. The main focus is to develop an inventory policy for the warehouse in Sweden, which serves to all these customers.

## 4.4 Supply network

The company's suppliers for these six products are mainly located in China. The purchased products are then shipped and transported to either the company's warehouse in Sweden or to their satellite warehouse in the UK. These products are produced at the supplier when the case company makes an order. Hence, upon orders from the company, the production starts and are then transported to designated destination in either Sweden or the UK. The two suppliers analysed in this report are both situated in eastern Asia. Due to the production and long transportation time from China to Europe, the lead time is several months long.

# 5. Analysis of the current situation

*This section will describe the current situation of the case company and analyse their current inventory, sales and purchasing processes.*

## 5.1 Current inventory situation

An analysis of the current inventory was first conducted in order to gain suitable metrics to then compare with the results of the constructed model. The six products, Product A-F, are considered as key products by the case company, and are all within the top-ten highest grossing products of their segments. Furthermore, the company is experiencing concerns regarding over- and understocking of these products. The company has not performed any previous analysis of their stock situation. Nor have they set any holding cost to evaluate their tied-up capital or set a service level as a goal. As this is a first analysis of this kind for the company, and the products being high-end products, it was decided upon a fixed holding cost rate for all products. According to Berling (2005), a frequently occurring figure in literature is 25 %. This will be the considered holding cost rate henceforth. Furthermore, the desired fill rate target has been set to 95 % in accordance with the companies own future vision.

As of today, there is no measured service level to compare the modelled results with. Moreover, the received inventory data from the company still contains “goods in transit”, i.e. goods that are already sold but not yet shipped to customer. Consequently, the true inventory level (IL) cannot be obtained. However, an estimation of this can be done by reconstructing the inventory level for each week $t$ of the past year. This has been done in accordance to (3.1). By assuming that the inventory levels are registered at the beginning of the week, by subtracting the weekly demand $d_{t-1}$ from the past weeks ingoing inventory level $IL_{t-1}$ the inventory level for week $t$ $IL_t$ can be calculated as:

$$IL_t = IL_{t-1} - d_{t-1} \tag{5.1}$$

As $IL_0$ includes goods which have been sold but not transported to the customer, and there is no way of knowing the number of units, a flat-rate value has been removed to represent these units. This flat-rate is represented by the weekly average demand during the past year for each product. By assuming that the sales data is correct, the inventory levels have been reconstructed by; starting from the initial inventory level at the beginning of week 1 ($IL_0$), subtracting the past years observed sales ($d$) on a weekly basis. Inbound orders have been added according to registered date of arrival. The result is an inventory level free from transit data. Furthermore, by following this method, we get an estimation of how much of the total registered sales that would have had to be backordered during the previous year. The fill rate can then be calculated as:

$$S_2 = 1 - \frac{\#Backorders}{Total\ Sales} \tag{5.2}$$

Table 1 below shows the simulated stock situation over the past year (2018-2019) for the case company.

*Table 1. Simulated stock data of previous year for the case company*

| | Average Total Inventory | Holding cost (SEK) | Fill rate |
|---|---|---|---|
| **Product A** | 95 | 208 363 | 100 % |
| **Product B** | 73 | 176 302 | 91 % |
| **Product C** | 19 | 50 336 | 69 % |
| **Product D** | 26 | 58 835 | 88 % |
| **Product E** | 83 | 95 629 | 78 % |
| **Product F** | 16 | 18 993 | 73 % |

As can be seen in Table 1 above, the resulting service levels for the simulated year was in four cases, considerably lower than the company's own vision, suggesting there exists a problem with inventory control. However, it should be noted that the data used for simulating the past year is purely sales data and not the actual demand. Consequently, if the actual demand had been registered, the service levels would have been even lower than presented in Table 1.

## 5.2 Purchasing process

As of today, there is no clear inventory policy in place, the company has developed their own process for evaluating the need for restocking. Figure 2 below, illustrates the different steps in the process. This process takes place continuously during the purchasing month before ending with a final compilation of all orders placed during the month. First, the inventory position is calculated in line with the theory in section 3.1. Secondly, the forecasting is based on the moving average method. Due to very long lead times (up to 8 months), the company has to forecast for several months ahead. They analyse the sales data and take an average from sales data one lead time back in time. This average is then assumed to represent the monthly demand of the product. Finally, when conducting the forecasting, the personnel takes trends and seasonalities into consideration in different ways. In the event of campaigns, the observed sales during that month(s) is disregarded and not used in their moving average calculation. In case of seasonalities, they either add or subtract the number of items purchased by knowledge.

The monthly forecasted demand, in combination with the calculated inventory position, gives an estimate for the inventory level one lead time ahead in time. Thereafter, they investigate if they reach or surpass their own set reorder point of 5 units.

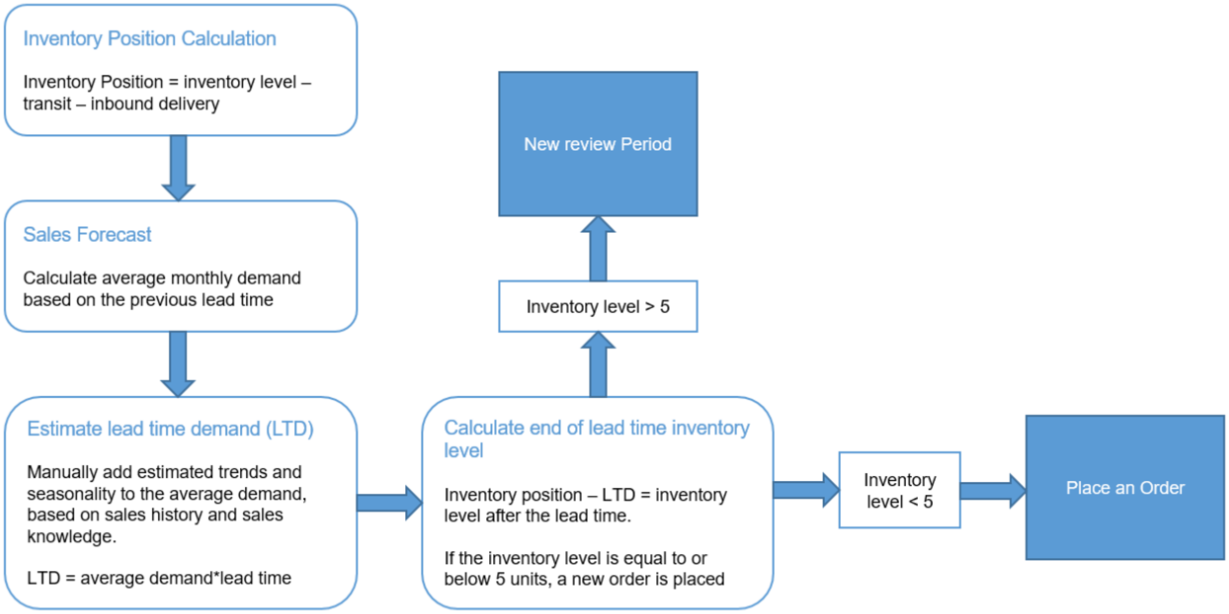



*Figure 2. Current purchasing process of the case company*

This process has been developed by the case company on their own and can be seen in Figure 2 above. Additionally, as the company is combining a number of products when declaring an order, it is up to the personnel to evaluate what quantities of each product that should be procured. These final restrictions of combinations of different products, as well as quantity restrictions from the suppliers, will not be addressed in this project. The focus is placed on optimizing the procedure for the key products separately. The shipment constraint will be addressed with the addition of a minimum order quantity for supplier 1 and supplier 2 of 30 and 20 units, respectively. Any additional factors such as price discounts will not be covered in this report.

### 5.2.1 Sales order pattern

The company has historically, without any extensive sales analysis, used a push-based policy in their sales model. Products have been procured without a fixed or analytically calculated order quantity, which will be discussed further in the next section, and promotions have been used to increase the turnover of goods in stock. The promotional periods can be seen in Table 2 below. It has been stated from the personnel that they are currently moving from this push-based sales policy, to a more pull-based sales policy. Thus, enabling the procurement to consider future promotions.

*Table 2. Promotional periods during the previous year*

| Product | mars-18 | apr-18 | maj-18 | juni-18 | juli-18 | aug-18 | sep-18 | okt-18 | nov-18 | dec-18 | jan-19 | feb-19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | | | | | X | X | X | | | | | X |
| | | | | | | | | | | | | |
| B | | | | | X | X | | | X | X | | |
| | | | | | | | | | | | | |
| C | | | X | X | X | X | | | | | | |
| | | | | | | | | | | | | |
| D | X | X | X | X | X | X | | | | | | X |
| | | | | | | | | | | | | |
| E | | | X | X | X | X | | | | | X | |
| | | | | | | | | | | | | |
| F | | | | | X | X | | | X | | X | |

## 5.2.2 Purchase pattern analysis

As previously discussed, the lack of structured forecasting and ordering processes has meant that no fixed batch quantities have been set or used in the organisation. Figure 3 below, illustrates the different order quantities used during the previous year. From the boxplot presented, it can be distinguished that several different order quantities are used, especially for Product A and Product E. These variations in order quantities can to some extent be accredited to the combination of products ordered and the purchasing discounts from the supplier. Furthermore, without a historical sales analysis, the impact of the promotional periods (e.g. Table 2) has not been thoroughly evaluated. This could lead to under- and overestimation of sales (see section 3.11.3).

Sales discounts can be added to an inventory policy when evaluating batch sizes according to an economic order quantity model. However, as the set holding costs and fixed ordering costs have not yet been evaluated or tested, the model will be constructed without additional factors such as sales discounts.


*Figure 3. Boxplot illustrating order quantities procured during the previous year. The cross represents the mean while the line represents the median.*

# 6. Inventory policy analysis

*This chapter describes the current state of the company's inventory situation, which will be the basis for constructing the new inventory control policy. Furthermore, the constructed mathematical model will be described and the results from this model will also be provided.*

## 6.1 Analysed system

The current purchasing process is highly complex, where product combinations and supplier restrictions are influencing decisions. However, as the company has expressed their concern for the key products A-F, the system in this report will focus on a simplified process for each product. The products are assumed to be ordered individually, in order to answer the research question of whether it is possible to construct a new inventory policy. The purpose is to optimize the purchasing process and reorder point for each product separately by only looking at the respective supplier and their connection to the warehouse in Sweden, as can be seen in Figure 4 below.

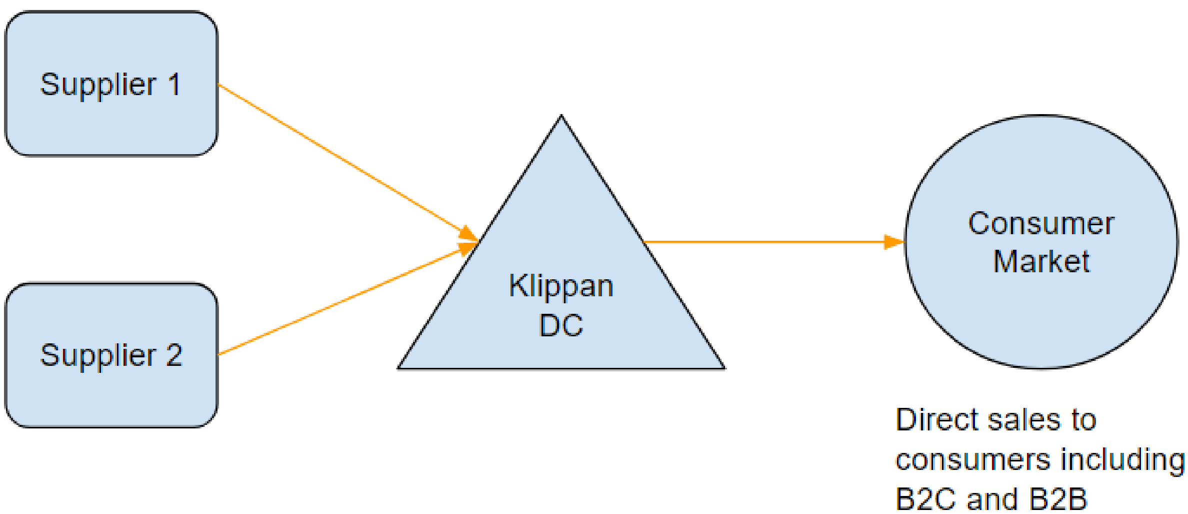



*Figure 4. Illustration of the analysed system*

Furthermore, as the Product A-F are from the company's two main suppliers. Product A-C and Product D-F are from supplier 1 and 2 respectively, two separate lead time analyses will be conducted, which will be further discussed in section 6.3.

## 6.2 Demand model analysis

In order to calculate new reorder points and construct a new inventory policy, an appropriate distribution describing the demand data has to be found. As the company does not have any data on the true demand of their products, sales data have been used instead to model the demand. In section 3.3 and 3.6, a hypothesis has been created that the lead time demand should follow a continuous normal distribution according to the Central Limit Theorem. This hypothesis has been further analysed using the distribution fitting tool Stat::Fit (bundled with the ExtendSim software) which showed that the hypothesis could not be rejected.

In section 3.3 it was discussed that the normal approximation was of interest when building a robust model with low complexity. Normal approximated demand is a viable option for both high demand products as well as low demand products. To verify this assumption, sales data for the products were gathered and put into Stat::Fit, which through a goodness of fit test, checks whether or not the normal approximation can be rejected as a distribution or not. As there is no true demand known to the company, the point of sales data was considered appropriate to represent the demand.

After cleaning the data of conversion errors from their ERP to Excel, the data was analysed in order to find a demand period with the length of a lead time, representative of a normal year. In order to do this we first had to take the promotional periods into consideration, which is presented in Table 2. When performing hypothesis tests with Stat::Fit, effects such as seasonalities should be excluded. This due to the resulting skewing effect on the data, resulting in an inadequate distribution fit. After choosing a time period with as little influence from promotions as possible, remaining extreme values were investigated and in some cases removed if found that the value does not represent the demand during a normal year.

The resulting data set was then inserted into Stat::Fit, which showed that the hypothesis, that the data sets are normally distributed, could not be rejected for either product. Thus, it will be assumed henceforth that the lead time demand follows a normal distribution. The resulting rankings of the considered distributions in the test can be seen in Appendix 2a.

## 6.3 Lead time analysis

No lead time analysis has been conducted previously at the case company. Therefore, historical data had to be gathered and checked whether it still represents the current situation or not. Without a structured process for lead time documentation, this data had to be manually interpreted from the purchase orders. As described in section 3.7, the lead time is defined as the time from order placement until it is available in the warehouse. Neither of these data points had been properly documented in the previous years. The date of order placement differed between the personnel as there were no previous need to document such data, resulting in possible deviation from the true order date. Likewise, the availability in the warehouse is not logged into the system. However, the time of arrival could be obtained for each delivery and was deemed representative for the availability of the products. As a result of lack of documentation, many orders were deemed invalid for analysis, and the obtained data points are to be considered an approximation.

### 6.3.1 Supplier lead time analysis

Regarding supplier 1, it was revealed that the historical data of 2016-2017, did not represent the current year or analysis (2018-2019) according to the purchasing department. Subsequently, with only a data set of 8 data points, as can be seen in Table 3, deemed to represent the current situation of a 255 days lead time, it was infeasible to use Stat::Fit to determine an appropriate distribution for the data set. Instead, we will use a deterministic approach to evaluate the impact of different lead times for this supplier. A three-step model will be used, were we will evaluate the worst case scenario of 10 months, the current scenario of 8 months as well as the suppliers own predicted future scenario of 5,5 months lead time.

*Table 3. Standard lead time for suppliers as well as collected data points for analysis*

| Category | Standard Lead Time (Days) | Number of Data Points |
|---|---|---|
| Supplier 1 | 255 | 8 |
| Supplier 2 | 150 | 25 |

Regarding supplier 2, as can be seen in Table 3, there were 25 data points available for lead time analysis, deemed to represent the current situation of a 150 days lead time. These data points have been cleaned and extreme values bigger than two standard deviations above the mean have been investigated and in one case removed. The resulting set of data point were large enough to analyse with the Stat::Fit tool. Figure 5 illustrates the compiled distribution fitting from Stat::Fit, presenting the five distribution which received highest rank according to Stat::Fit as well as the normal distribution. Each of these distributions are deemed to be a good fit to represent the lead time. The full results of the tests can be seen in Appendix 2b.

According to Axsäter (2006), it is often practical to use a simple approximation where the distribution of the demand, during a stochastic lead time, is replaced by a normal distribution. This is done by using the formulas presented in section 3.7, where the mean and standard deviations care simply calculated from the obtained sales data. This approximation will be used when calculating the reorder points.

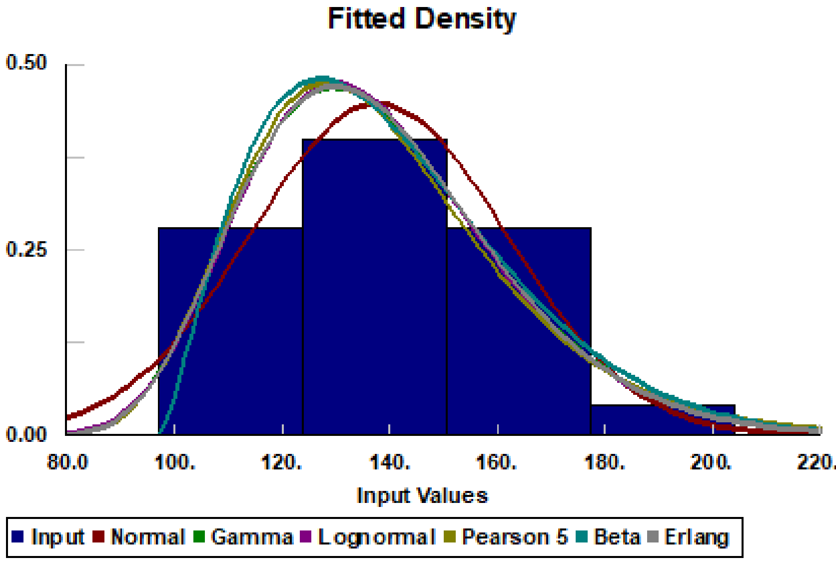



*Figure 5. Plot of the Stat::Fit analysis. The five highest ranked distributions are displayed as well as the normal distribution*

## 6.4 New inventory control policy

As the products are currently ordered in batches, with ordering cost relative to the total purchased value, an (R, Q) policy has been decided upon to best describe the system used at the company. However, as previously discussed, the different products are ordered independently of each other. Furthermore, only one location has been investigated namely the warehouse in Klippan, Sweden. As discussed by Axsäter (2006), having a fixed batch size as well as independently ordered products, implies that the (R, Q) policy is a preferable choice. Furthermore, the (R, Q) policy is suitable to use when using fill rate constraints.

As this is the first model constructed to represent the operations of the company, a simple model has been developed. Continuous review has been used in the model, as the short periods used by the purchasing department has been deemed short enough to represent a continuous review by following the reasoning in 3.6 of adding half a review period to the total lead time.

As described in section 6.2, the demand has been analysed to follow a continuous normal distribution. The robustness of this distribution, as well as the simplicity in implementing and using the normal distribution, makes it a suitable choice. Furthermore, the lead time demand will be assumed to be normally distributed this hypothesis could not be rejected when analysing the data (see Appendix 2b). The lead times has been modelled under the assumption that deliveries cannot cross in time, i.e., we have sequential deliveries.

Supplier constraints has set a lower limit to the possible batch quantity, a Minimum Order Quantity (MOQ), which will be used as the minimum allowed batch size. In combination with this, the economic order quantity (EOQ) formula, discussed in section 3.5, will be used to calculate the order quantities, rounded to nearest integer. With the batch quantities calculated, the reorder points are calculated to minimize holding costs by an iterative approach until a reorder point is found that satisfies the fill rate constraints.

## 6.5 Model formulation

The following section describes the process of compiling new reorder points and order quantities. The code has been programmed in MATLAB and can be found in Appendix 1. The different steps in the iterative model can be seen in Figure 6 below and describes a process which uses continuous review and normally distributed lead time demand.

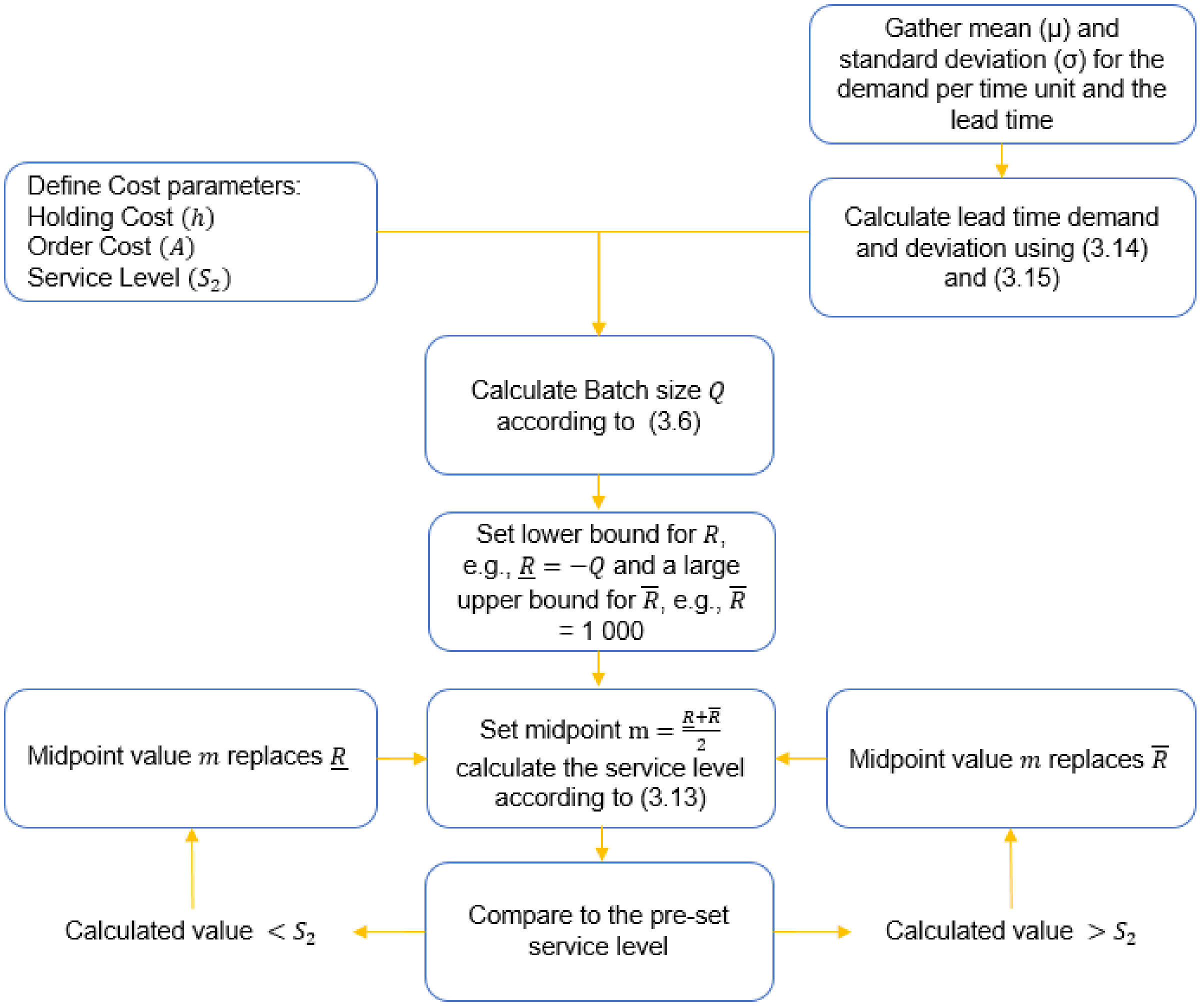



*Figure 6. Illustration of the iterative computation of reorder points satisficing the service level constraint*

## 6.6 Modell results

With the model, described above, the resulting optimal reorder points and order quantities has been calculated that fulfils a 95 % fill rate constraint. For the deterministic lead time analysis of supplier 1, Figure 7 below illustrates the effects the different lead times will have on the possible stock-on-hand. As can be expected, an increase in lead time requires the company to have more items in stock in order to mitigate the impacts of higher lead time deviations in the demand.

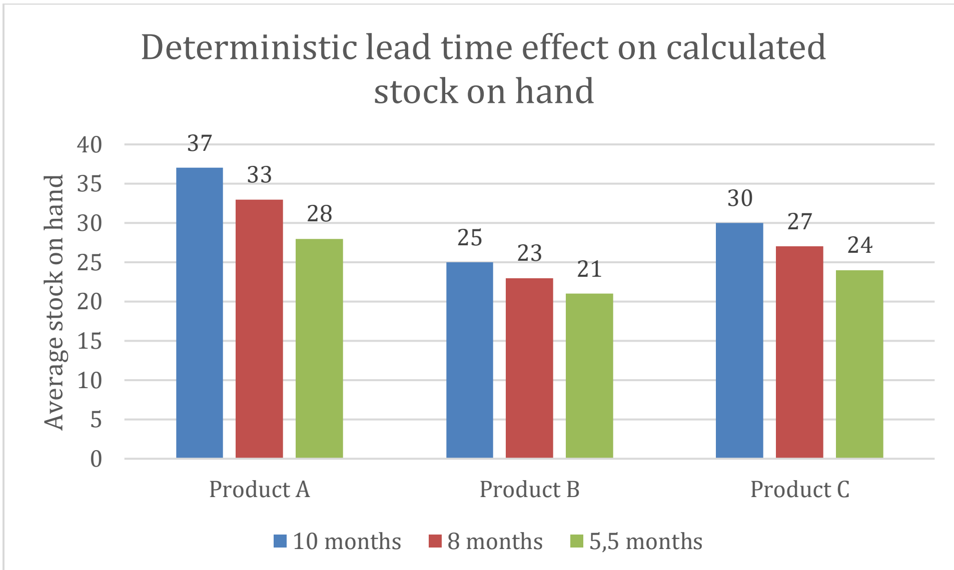



*Figure 7. Result of the deterministic lead time on the calculated stock on hand for supplier 1*

As previously discussed, we were able to conduct a lead time analysis for Products D, E, F of supplier 2. And even though this supplier has been perceived to have relatively stable lead times as compared to supplier 1, the observed lead time variations can still influence the stock levels. Thus, an analysis has been done for different lead time variations where the standard-deviation of the lead time has been reduced to a deterministic lead time in four stages. The result of this analysis can be seen in Figure 8 below. Notably, in this analysis is that even when assuming a constant lead time for Product F, the expected inventory level will still be higher than what has been the average in the current state. However, there exists significant improvements in reducing the variability of the supplier for this product.

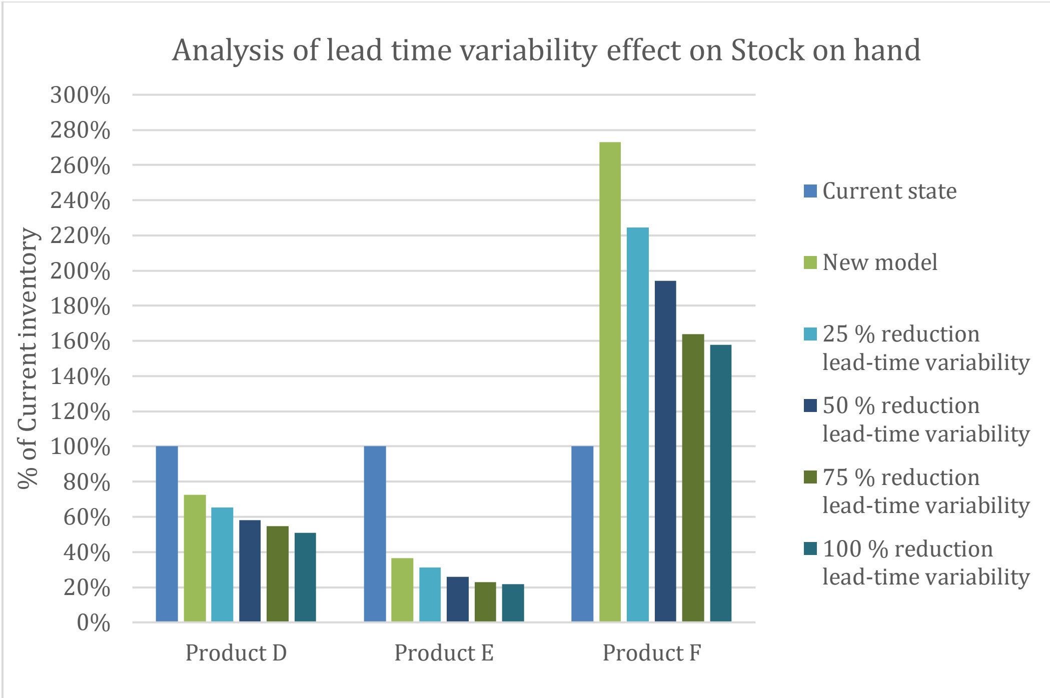



*Figure 8. Model results of lead time variability analysis for supplier 2*

The calculated inventory levels presented above should however be regarded as purely theoretical, and will be hard to reach in practice as they depend on a specific demand structure.

### 6.6.1 Comparison with current model

To further validate the calculated reorder points and batch quantities, the constructed inventory model will be tested for the past year's demand. The resulting average inventory and service level will then be compared with the performance of the current inventory policy, presented in section 5.1.

In order for the models to be comparable, the initial setup of both policies will be the same, i.e., the initial warehouse level is the same for both models. Additionally, as the lead times are five and eight months respectively, if we assume that the new inventory policy is put in place at the start of the year, we will have a lag time of one lead time before the effects of the new policy can be seen. Furthermore, mismatches between the observed inbound order sizes and the proposed batch sized of the new policy, will skew the results even further. In order to mitigate this, it will be assumed that the inventory policy has been set to practice beforehand. In order to replicate a (R, Q) policy, a multiple (n) of the batch quantity has been added to inbound in order to raise the inventory position above the reorder point at the start of the year. With the initial inventory level ($IL_0$) and the average weekly demand $(\mu)$ the time to arrival for the first batch can then be calculated as:

$$t_1 = \left(\frac{(IL_0 + n * Q) - R}{\mu}\right) \tag{6.1}$$

Subsequent batches are assumed to arrive $Q/\mu$ time units (corresponding to the average time between orders) one after another. Reorder points and batch quantities for all products all calculated using the demand discussed in section 6.2 and have been kept fixed throughout the year. Table 4 below presents the estimated difference a new policy would have had on the inventory levels during the previous year for supplier 1.

*Table 4. Results for supplier 1 of the simulated year for the newly constructed model and current model used by the company*

| | **Product A** | **Product B** | **Product C** |
|---|---|---|---|
| Average stock (units) | | | |
| New model | 95 | 44 | 31 |
| Company model | 95 | 73 | 19 |
| **Inventory changes** | 0 % | – 40 % | + 65 % |
| Service level (%) | | | |
| New model | 100 % | 100 % | 94 % |
| Company model | 100 % | 91 % | 69 % |
| **Service level changes** | 0 % | + 10 % | + 37 % |

As can be seen in the Table above, the new model is able to perform on par with or better than the current model in terms of service level for all three products. The inventory could be reduced for Product A & B, while the increased inventory for Product C is a result of increasing the service level significantly. Regarding Product A, the average inventory could not be reduced further during the year as there was significant stock going into the year, meaning it would take a longer time for the new model to have an impact on the inventory level. However, the average inventory for Product B & C can both be seen to converge towards the calculated theoretical value presented in section 6.6.

For supplier 2, Table 5 below presents the results from the simulated comparison. As with supplier 1 the model is able to perform better than the current model for both Product D and F with regards to service levels as well as showing inventory levels that are converging towards the theoretical inventory levels. However, the increased service level for Product F is achieved by a severe increase in total average inventory. Regarding product E, this is the product with the greatest variance in its demand of the analysed products. As the used lead time demand in the model represents the demand during a normal year, meaning big variations as a cause of promotions etc. have been removed, the model is not able to cope with the big variance of this product. In order to reduce the impact of higher demand in certain periods, as in this case, the reorder points should be recalculated by forecasting the demand and adjusting reorder points accordingly.

*Table 5. Results for supplier 2 of the simulated year for the newly constructed model and current model used by the company*

| | **Product D** | **Product E** | **Product F** |
|---|---|---|---|
| Average stock (units) | | | |
| New model | 25 | 70 | 43 |
| Company model | 26 | 83 | 16 |
| **Inventory changes** | – 7 % | – 18 % | + 145 % |
| Service level (%) | | | |
| New model | 98 % | 73 % | 96 % |
| Company model | 88 % | 78 % | 73 % |
| **Service level changes** | + 12 % | – 6 % | + 33 % |

From the simulation of the two suppliers we can see that the new model is able to perform as good as or better than the current model in all cases but one with regards to service levels. This suggests that an inventory model with set reorder points and order quantities could benefit the company. However, it might not always be possible to order in the batch quantities suggested in the new model. As the two suppliers are both based in China, events such as the Chinese New Year disrupts the normal order cycle as products has to be produced in advance to or after the event. This could explain why the inventory levels in the beginning of the year has either been very high, or very low. Furthermore, as the company has previously used promotions to push out excessive stock, there has not been a need to accurately forecast the demand and to use a set inventory policy. However, as the company has spoken about a desire to go from using promotions to push out excessive stock to a more pull based system, as discussed in section 5.2.1, the incentive to order large batches will be reduced. Furthermore, a new forecasting process should help the estimations of future demand.

### 6.6.2 Sensitivity of the constructed model

In the constructed model the batch quantity (Q) has been calculated using the classic economic order quantity model in combination with the supplier's minimum order quantity received from the company. However, this quantity might not be optimal from a practical point of view, as discounts and combinations add further complexity to the procurement process. However, as the cost function is convex and relatively flat, it is not sensitive to changes in batch sized as can be seen from the formulas presented in section 3.5. Axsäter (1996) further argues that in practice it is common to replace the stochastic demand with its mean and then calculate Q with a deterministic model. The reorder point can then be determined by a stochastic model. Under general assumptions, it is possible to show that by following this approximated procedure, the total general cost increase will always be lower than 11.8 percent of the optimal solution.

# 7. Demand forecasting

*The following segment describes the used approach for identifying, applying, testing and evaluating the different forecasting models suggested as potential tools for the case company.*

## 7.1 Identifying seasonalities and trends

Before testing the different forecast methods, the sales data for the different products will be investigated for obvious seasonalities and trends. Therefore, any systematic increases or decreases in product sales during specific periods has to be identified. Seasonalities will be investigated by identifying variations in demand during specific monthly periods in different years. As a result, the company can get a suitable benchmark of the variations of sales during different seasons for the products. Note that these systematic changes do not involve sales campaigns or any other factors that cannot be explained by historical data.

## 7.2 Choosing parameters and assumptions

The initial parameters will be set with guidance from Axsäter (2006) and Silver, Pyke & Peterson (1998). The considered forecasting models will need outside historical data for initializing $MAD_t$ i.e. an initial MAD-value ($MAD_0$) for each forecasting model will be calculated from historical data. However, the $MAD_t$ for the moving average will be updated by utilizing (3.19) and (3.28) - whereas (3.19) can alternatively be updated as a moving average. Hence, the forecast errors for the moving average model will be updated as:

$$\widehat{MAD}_t = \frac{(MAD_t + MAD_{t-1} + MAD_{t-2} + \cdots + MAD_{t-N+1})}{N} \tag{7.1}$$

For exponential smoothing with and without trend, The MAD smoothing constant in (3.28) will initially be set to $\alpha = 0{,}2$, as advocated by Axsäter (2006). The forecast errors are assumed to be normally distributed due to independent variables adding up towards a normal distribution according to the Central Limit Theorem (Blom et al., 2005). The normal distribution is robust and preferred due to the factors mentioned in section 3.1.2.

## 7.3 Process of choosing forecasting model

The used approach for choosing an adequate forecasting model is first to sample historical sales data over a specific period for each model; moving average, exponential smoothing and exponential smoothing with trend. Silver, Pyke & Peterson (1998) advocates to experiment with the parameters to establish suitable smoothing constants. Thus, the samples will need to be adjusted and reiterated.

The initial samples will consist of initial MAD-values ($MAD_0$) and parameters, as mentioned in section 7.2. The samples will be compared to historical data outside the sample-period and the initializing MAD-period. Subsequently, the parameters of the samples will be adjusted around the initial parameters to minimize the forecast errors in each model and then reiterated to advance a better forecast. Note the importance of avoiding overfitting when reiterating the parameters in the samples (see Figure 9). In accordance with Kuhn & Johnson (2013), the samples would not be optimized when replicating noises to accurately (see section 3.6.4). Lastly, the forecasting models will be compared in regards of different adequate aspects towards the company. The main factors that will be considered when choosing an appropriate forecasting model for the company is the following:

- Degree of forecast errors
- Difficulty of determining parameters
- Degree of how easily the forecasting model can be implemented into the company

By investigating these factors, the company could apply an adequate forecasting model into their system. The forecast approach can be summarized in the Figure 9 below:

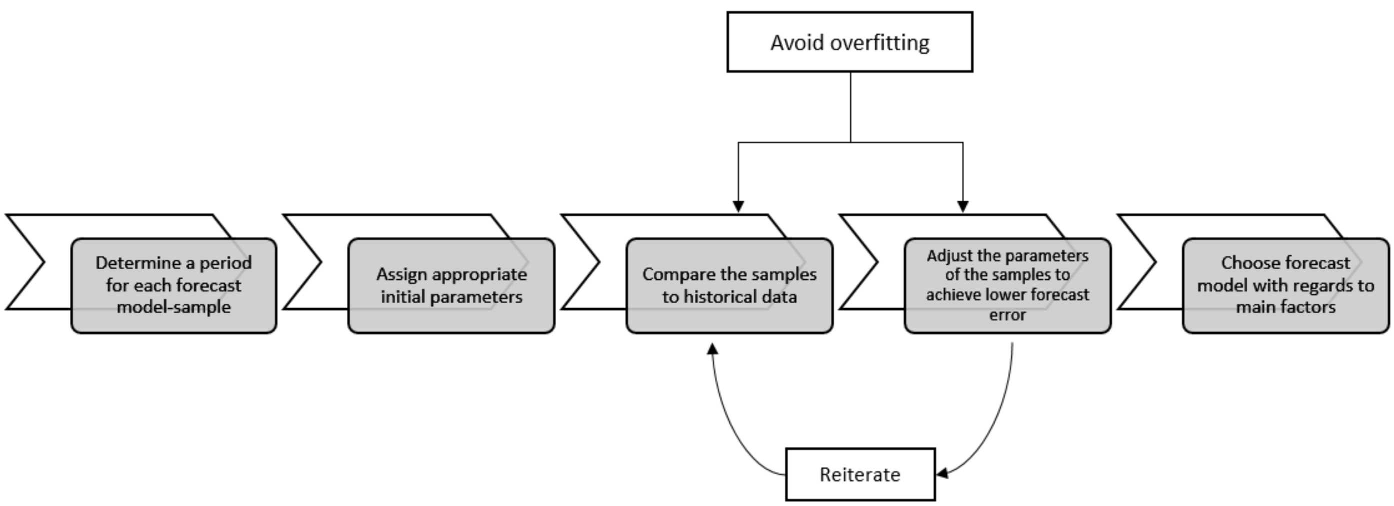



*Figure 9.Used approach for testing the forecasting models*

As mentioned in section 3.9.3, the forecasting models; moving average, exponential smoothing with and without trend are suitable techniques for most items. Therefore, these forecast methods have been used in the analysis.

## 7.4 Updating the reorder points and order quantities

After choosing an appropriate forecasting model with regards to the main factors, the company should use the obtained forecasted data to update their reorder points and order quantities. The updating process is dependent on a new average (μ’) and $MAD_t$ to obtain the new standard deviation (σ’), which has been mentioned in section 3.12. With the updated parameters, new order quantities and reorder points can be calculated - as described in section 3.12.

# 8. Forecast analysis

*The following part describes how a suitable forecasting model could be applied into the company. Due to lack of data, the following part will describe a fundamental ground with alternative forecasting application models for how the different forecasting models could be applied and what to consider when choosing between the different methods.*

## 8.1 Consequences of lack of data

Because of the lack of data, the execution of an appropriate forecasting model wasn't feasible. The products analysed only had sales data 11 months back, which limited the possible analysis. In accordance with Axsäter (2006), seasonalities must be confirmed by investigating trends during specific time intervals in different years - hence this analysis couldn't be conducted for these products. Additionally, the company has had frequent sales campaigns during the year (see Table 2) which added further complexity to distinguish trends. In accordance to Axsäter (2006), trend models are adequate in situations such as product life cycles and their phases. However, due to the campaigns and sporadic deviations for the products, the trends were difficult to identify.

The most significant consequence of the lack of data, was the complexity of choosing an adequate forecasting model. Consequently, a comparison between the chosen factors in section 7.3 could not be conducted. The insufficient data made it unfeasible to construct samples for each product, test and adjust the parameters to then choose between the forecasting methods. Another complication was that the lack of data did not give an indication whether the data was overfitted or not as a result of following noises too accurately. Therefore, the lack of data did not give sufficient material to conduct an adequate forecasting model for the company. Lastly, due to not being able to obtain an adequate forecasting model, the update of the product batch quantities and reorder points could not be conducted.

## 8.2 Practical applications of forecasting models

In the case of the analysed products having sufficient data in the future, the mentioned forecast approach can be applied to evaluate the different forecasting models. The following section will describe how the company could apply the analysis into their current system used for forecasting, Excel.

### 8.2.1 Moving average

As mentioned in section 5.2, the current period length, N, for the analysed products are their lead times. However, this does not have to be the optimal period length for the products. In accordance to Axsäter (2006), a way to evaluate a forecast is by looking at its errors. So, an appropriate way for the company to update their current moving average method is to include standard deviation or MAD into their evaluations. This gives an indication of how uncertain the forecast is and also an implication of determining a suitable safety stock (Axsäter, 2006). Thus, the company could use different period lengths and compare them between each other by observing the forecast errors.

### 8.2.2 Exponential smoothing with and without trend

By following the forecast approach in section 7.3, when sufficient data is available, the company can use exponential smoothing (with and without trend) into their current system. As a starting point, in accordance with Axsäter (2006), appropriate smoothing constants would be $\alpha = 0{,}2$. According to Silver, Pyke & Peterson (1998), if the value of $\alpha$ is larger than 0,3, it should raise the question of validity and consider if a trend model is more appropriate.

The company needs to identify the trends of the products. Note that a trend should try to be identified without interfering factors such as sales campaign. An important aspect to consider when using these forecast methods, is to avoid fitting the forecasts too well as a result of following unique noises that generally have poor accuracy when forecasting other samples (Kuhn & Johnson, 2013).

## 8.3 Exemplary applications

In order to give a fundamental application for a future forecasting model for the company, different Alternative Forecasting Application Models (AFAMs) were created. These AFAMs are compatible with their current system and can be observed in Appendix 3. The AFAMs are constructed for the forecasting models; moving average, exponential smoothing with and without trend. Moreover, the AFAMs can both be used for creating samples, adjusting the parameters, and forecasting future sales. In addition, the AFAMs also take forecast errors (MAD and σ) into consideration for evaluating the uncertainty of the models. The initial MAD-value ($MAD_0$) was randomly set due to its necessity for updating exponential smoothing with and without trend.

The forecasting models in Appendix 3 displays the interface of the AFAMs. The AFAMs are also easily handled for manual forecasting if necessary and also contains representative graphs to get an overview how well the forecasts follow the actual sales. An important note of these examples is that they currently consider monthly sales and therefore generates one monthly forecast - in these examples, Product A's monthly sales. When the company has more data, they can allocate monthly sales into quarters, half years etc. - which potentially is more adequate due to the longer lead times after purchasing.

### 8.3.1 Moving average example

The moving average is performed as described in section 3.4.4. When analysing how many periods to include in the moving average, different number of periods are compared with each other. The reason for this is to find a suitable number of periods when forecasting the demand. The comparisons were conducted by examining the differences in forecast errors - where the lowest errors were to prefer. The forecast errors are updated and compared by utilizing (7.1). The standard deviation is then calculated with (3.29)

A major disadvantage with updating the forecast errors as a moving average, is that the forecast errors becomes less valid the shorter the period of N is. As the MAD is updated in similar way, the mean of the forecast errors will have very few data points throughout its usage. For instance, a period of $N = 3$ gives three data points to determine the $MAD_t$ and standard deviation (see Appendix 3a). In addition, the moving average AFAM is the only forecasting model that has an internal ranking system, due to actively comparing different time periods.

### 8.3.2 Simple exponential smoothing example

The simple exponential smoothing is conducted as described in section 3.4.5. Due to the assumption that forecast errors are normally distributed, the standard deviation can be applied with (3.29) by updating the $MAD_t$ (3.28). In order to update the forecasts and connect it with their current inventory policy, the updated standard deviations (3.29) will be utilized to obtain new order quantities and reorder points as described in section 3.2.2 and 3.2.3 respectively (see section 3.7). The lead time here is either deterministic or stochastic and is used as described in section 3.2.4.

### 8.3.3 Exponential smoothing with trend

The exponential smoothing with trend is performed as described in section 3.4.6, by obtaining $\hat{a}$ and $\hat{b}$. In resemblance to section 8.3.2, the forecast errors are obtained with (3.28) and (3.29) in order to determine the new order quantity and reorder point - as described in section 3.7. Note that the smoothing factor for the trend has been set arbitrary to match a trend. Even though, a conventional value of the smoothing constant is $\beta = 0{,}05$, it does not imply it is suitable for the case company. As mentioned before, the trends of the products could not be identified and consequently the model could not be compared with the others.

## 8.4 Trade-offs between the forecasting models

When considering the different forecasting models, there are various trade-offs and differences when applying them and adjusting their parameters. For the moving average method, a larger value of N gives the consequence of putting more emphasis on old values (Axsäter, 2006). In comparison, exponential smoothing has essentially the same effect as moving average when $\alpha$ has a small value. When comparing simple exponential smoothing with moving average, there are some advantages that are minor but obvious. For instance, if the average is assumed to vary slowly, it is plausible to put more emphasis on the most recent demands by weighting most recent demands more as in simple exponential smoothing. Furthermore, a moving average over a full year may be advantageous for eliminating the influence of seasonal variations on the forecast when exponential smoothing only need to take previous forecast and recent demands into consideration (Axsäter, 2006). Additionally, in accordance with Axsäter (2006), simple exponential smoothing (or in some cases moving average) is in general an adequate forecasting technique for most items. However, items following trends and seasonalities need other methods.

Another disadvantage of using a more general forecasting model, such as trend and trend-seasonal models, is the wider class of demands that require more estimations (see section 3.4.3). Moreover, the complexity becomes even larger if the independent deviations are relatively large. In accordance with the main factors (see section 7.3), it might be difficult for the case company to adjust these parameters. Consequently, making a simpler forecasting model preferable.

When considering the chosen forecasting models for evaluation, it initially seems that some forecasting models are to prefer over others. The drawback of evaluating the moving average model is that the forecast errors that get less valid with lower periods (see section 7.3.1). Because a fewer number of periods results in less data points, the $\widehat{MAD}_t$ becomes more uncertain compared to a $\widehat{MAD}_t$ with a higher number of periods. This indicates difficulty to determine forecast errors for some cases. However, this forecast method is considered the easiest one to apply into their current system. The reason for this is that the company is already using a type of moving average in their current system. Simple exponential smoothing is considered marginally more difficult to implement into their current system. As for exponential smoothing, the forecasting model gives a more certain indication of the forecast errors than the moving average with a few number of periods. This does not however answer the question if the model is more adequate than the other due to the lack of data. Exponential smoothing with trend was not possible to use because of too little data. Hence, this is considered the most difficult forecasting model to apply. Nonetheless, it was not possible to determine if the exponential smoothing with trend model is the most suitable one for the company due to the lack of data.

A practical consideration when updating the batch quantities, is how frequent they should be updated. It might be difficult to change the batch quantities too often because of predetermined agreement with suppliers. However, the reorder points are more flexible to adjust as it can be updated when the observed demand differs greatly from the demand currently used in the model, without changing the batch quantities.

# 9. Conclusion

*This section consists of answering the research question connected to the purpose and a conclusion of the master thesis. In addition, the section mentions future investigations and applications for the company as well as other companies.*

## 9.1 Inventory policy

How can the inventory policy for the warehouse in Sweden be improved. The first question of this thesis was how the current inventory policy of the case company could be improved. This question could after analysis be further derived into two parts:

1. Could an inventory policy be constructed which performed better than the currently used inventory policy?
2. Is the constructed policy a good representation of the operations of the company?

As this is the first analysis of its kind for the case company, it was decided upon to construct a simple model. The first step when building such a model, is to describe the demand, which after analysis could be approximated to be represented by a normal-distribution, due to its properties of being a robust, capable of handling small and high demand, as well as being a simple distribution to use. Furthermore, as the company's current process involves ordering quantities in batches, an (R, Q) policy has been used as a suitable policy. Continuous review has been used to represent the operations of the case company, as the weekly review period can be approximated to follow a continuous review by adding half a review period to the total lead time. The lead time has been approximated to follow a normal distribution in order for the results, of both suppliers, to be based on same prerequisites. The conducted lead time analysis for supplier 2 can still be of value for the company as they can observe the supplier's performance.

With an (R, Q) policy and review method in place, the corresponding quantity (Q) and reorder point (R) are theoretically optimized for each product. This is done by an algorithm which iteratively calculates what R value corresponds to a specific service level, given that the order quantity is fixed. Through this it could be found that the total inventory levels could theoretically be for four items, in some cases significantly, whilst raising the service level. Finally, a retrospective test with historical data was made to test how the new models performs compared to the company's current model. Here it was found that the new model performed as good as or better than the current model in five out of six cases in terms of service level, and the stock level could be reduced in four cases.

As there had been no thorough study of inventory control at the company previously, many assumptions had to be made in order to construct the model. Cost parameters such as holding costs and fixed ordering costs were not known and had to be approximated using theory and company estimations. However, changes in order quantities as a result of different cost parameters do not incur a cost increase to any large extent. As previously discussed in the sensitivity analysis (section 6.6.2), if the reorder point is updated with the new batch quantity, the average cost increase will be lower than 11.8 % compared to an optimal solution. However, many assumptions have been made in order to construct the model, hence it should be considered to be an approximation of the current operations. Furthermore, the company does not normally order the products separately but instead in a mix to fill containers, as well as ordering based on discounts from the suppliers which were not considered. Finally, the analysis has been conducted for the six key items and not for all products in the company.

## 9.2 Forecasting

In order to answer the second research question, three applicable AFAMs were programmed in Excel for the case company (see Appendix 3). With these software application models, the company can themselves apply and evaluate the different forecasting models, as well as forecasting future demand. The different AFAMs will aid the company to adjust, evaluate and compare each forecast method by following the approach described in section 7.3. Additionally, the AFAMs also take forecast errors into consideration, by which the forecasting models' uncertainty can be evaluated and give an indication of an appropriate safety stock. Finally, the AFAMs' forecast periods are currently set as monthly (see Appendix 3), but the can be allocated into quarter years, half years and years etc.

Unfortunately, the data was not sufficient to evaluate and compare the performance of different forecasting models. Firstly, not being able to identify seasonalities due to not having data over particular time periods during different years. Secondly, the systematic trends could not be identified. Not only because of the lack of data, but frequent sales campaigns disguises any systematic trend for each product. Lastly, the selected forecasting models, that potentially could be applicable for the company, could not be sufficiently compared. The choosing of adequate parameters for the exponential smoothing methods, with and without trend, could not be conducted. Consequently, samples with adjusted parameters could not be tested for other demand periods and were therefore more prone of inaccurate parameters and overfitting. Due to lack of data, it has not been possible to determine which of the forecasting models that is more suitable for the company when updating reorder points and batch quantities.

## 9.3 Future recommendations and investigation

Regarding inventory policy, in order to use the new (R, Q) policy successfully at the company, batch calculations as a result of quantity discounts should be added. And finally, the products should be classified and grouped appropriately, as calculating the reorder points and order quantities for each product separately is not very practical. Tentatively, this grouping should be done according to sales volume, as this would enable matching products with their corresponding extra components.

For the company to conduct a forecast evaluation in the future, they need to gather and save more data. In addition, the company should investigate if there are any systematic changes in the products to develop an appropriate trend model. They also need to investigate if the products are exposed to any seasonality. If there are any seasonalities, the company should investigate in developing a trend-season model.

## 9.4 Contributions to theory

The project is a contribution to current inventory control literature as an empirical study regarding inventory policy and forecasting within e-commerce companies. The study has focused on several specific items but are also applicable to other products outside of this study. An extension of the study could eventually be to investigate its application to larger product groups and aggregations within the same industry. The models and methods are based on fundamental theory within inventory control and demand forecasting. Even though the models are developed for the case company, their applications can be implemented to other companies operating within and outside e-commerce environments.

# References

**Books:**


Axsäter, S. (2006). *Inventory control* (2nd ed.). New York: Springer.

Berling, P. (2005). *On determination of inventory cost parameters*. Department of Industrial Engineering Management and Logistics. Lund: Media-Tryck.

Blom, G., Enger, J., Englund, G., Grandell, J. & Holst, L. (2005). *Sannolikhetsteori och statistikteori med tillämpningar* (5th ed.). Studentlitteratur AB

Christopher, M. (2011). *Logistics and supply chain management* (4th ed.). Edinburgh Gate: Pearsons Education Limited.

Hadley, G., & Whitin, T. M. (1963). *Analysis of Inventory Systems.* Prentice-Hall.

Hillier, S.H. & Lieberman, G.J. (2010). *Introduction to Operations Research* (9th ed.). New York: McGraw-Hill Higher Education

Höst, M., Regnell, B. & Runesson, P. (2006). *Att genomföra examensarbete* (1st ed.). Lund: Studentlitteratur AB

Kuhn, M., & Johnson, K. (2013). *Applied predictive modeling*. New York: Springer.

Laguna M., & Marklund, J. (2013). *Business processes modeling,simulation and design* (3rd ed.). Boca Raton: Taylor & Francis Group.

Silver, E. A., Pyke, D. F., & Peterson, R. (1998). *Inventory management and production planning and scheduling* (3rd ed.). New York: Wiley.

Yin, R. (2009). *Case study research: design and methods* (4th ed.). Thousand Oaks: SAGE publications, Inc.


**Articles:**


Axsäter, S. (1996). Using the deterministic EOQ formula in stochastic inventory control. *Management Science*, *42*(6), 830-834.

Axsäter, S. (2013). When is it feasible to model low discrete demand by a normal distribution? *OR spectrum*, *35*(1), 153-162.

Holt, C. C. (2004). Forecasting Seasonals and Trends by Exponentially Weighted Moving *International Journal of Forecasting, 20(1),* 5-10

La Londe, B. J., & Lambert, D. M. (1975). Inventory carrying costs: Significance, components, means, functions. *International Journal of Physical Distribution*, *6*(1), 51-63.

Lambert, D. M., & Cooper, M. C. (2000). Issues in supply chain management. *Industrial marketing management*, *29*(1), 65-83.

Liao, C-J. & Shyu, C-H. (1991). An analytical Determination of Lead Time with Normal Demand. *International Journal of Operations & Production Management*, 11(9), 72-78

Liu, Y. (2000). Overfitting and forecasting: linear versus non-linear time series models.

Tyworth, J. E., & O'Neill, L. (1997). Robustness of the normal approximation of lead-time demand in a distribution setting. *Naval Research Logistics (NRL)*, *44*(2), 165-186

# Appendix

## Appendix1. MATLAB Code

```
% Below you find an example of a model in Matlab for computing
% service level

% Parameters which needs to be predefined, important that the time
% unit for lead time and demand is the same (weekly, monthly etc..)
% Cost Parameters

r =...          %Holding interest
c1 =...         %Item purchase value
A =...          %Ordering Cost
S2 =...         %Desired Fill rate service level
h = c1*r        %Holding cost per unit and time uni
T =…            % Length of the review period

%Demand Calculation
Lp = T/2        %Average review time to the added to the total lead time
Lt =...         %Mean of lead time
L = Lp + Lt
Lsigma =...     %Standard deviation of lead time (Lsigma = 0 with constant lead times)

my =...         %Mean of demand
sigma =...      %Standard deviation of demand

myprim = my*L;     %Lead time demand
sigmaprim = sqrt(sigma^2*L + my^2*temp^2) % Lead time standard deviation

% Order Quantity Calculation
MOQ =...        %Define minimum order quantity
Q = sqrt(2*A*myprim / h); % Economic order quantity

if Q < MOQ      %Bisection which checks if the minimum quantity is reached
        Q = MOQ;
end

   % Set the initial interval as well as midpoint to calculate SL2
   RL = -Q;
   RU = 1600;
   R = (RL+RU)/2;
```

```
    % Preparation for the lossfunction which calculates the loss function G(x
    GSS = (R-myprim)/sigmaprim;
    GSS2 = (R - myprim + Q)/sigmaprim;

    % Service level is calculated with the initial interval, compares to S2
    SL2 = 1 - (sigmaprim/Q)*(lost(GSS)-lost(GSS2));

    % Bisection method to calculate the lowest R-value which satisfies S2
    % S2 is interpreted as S2% probability of positive stock
    % Loops until the R-value is found

    while (SL2 ~= S2)
        R = (RL+RU)/2;
        GSS = (R - myprim) / sigmaprim;
        GSS2 = (R - myprim + Q) / sigmaprim;
        SL2 = 1 - ((sigmaprim / Q)*(lost(GSS) - lost(GSS2)));
        if SL2 > S2
            RU = R; % midpoint above S2, set RU to m to reduce the interval
            GSS = (R - myprim) / sigmaprim;
            GSS2 = (R - myprim + Q) / sigmaprim;
            SL2 = 1 - ((sigmaprim / Q) * (lost(GSS) - lost(GSS2)));
        end

        if SL2 < S2
            RL = R; % midpoint below S2, set RL to m to reduce the interval
            GSS = (R - myprim) / sigmaprim;
            GSS2 = (R - myprim + Q) / sigmaprim;
            SL2 = 1 - ((sigmaprim / Q)*(lost(GSS) - lost(GSS2)));
        end
    end

 Lager = (R + Q / 2 - myprim) % Calculates average inventory

%% The following bisection calculates the loss function G(x)

function [g] = lost(x) % x=GSS or GSS2

    g = normpdf(x) - x*(1 - normcdf(x));

end
```

# Appendix 2 – Ranking of Stat::Fit

## Appendix 2a. Ranking of distribution for demand in Stat::Fit

Stat::Fit ranking for product A

Document1: Automatic Fitting

Auto::Fit of Distributions

| distribution | rank | acceptance |
|---|---|---|
| Logistic(6.79, 1.7) | 87.4 | do not reject |
| LogLogistic(-42.5, 28.9, 49.2) | 79.9 | do not reject |
| Johnson SU(-3.09, 12.3, -3.86, 5.3) | 78.6 | do not reject |
| Cauchy(6.91, 1.65) | 76.1 | do not reject |
| Laplace(7., 2.31) | 74.7 | do not reject |
| Gamma(-8.94, 27.1, 0.583) | 73.1 | do not reject |
| Erlang(-8.94, 27., 0.585) | 72.6 | do not reject |
| Weibull(-6.98e-002, 2.43, 7.81) | 70.3 | do not reject |
| Lognormal(-14.4, 3.04, 0.143) | 69.3 | do not reject |
| Beta(-0.145, 18.5, 2.89, 4.8) | 65.3 | do not reject |
| Pearson 5(-16.9, 62.2, 1.46e+003) | 63.2 | do not reject |
| Normal(6.86, 3.03) | 59.4 | do not reject |
| Inverse Gaussian(-14.1, 993, 20.9) | 53.3 | do not reject |
| Chi Squared(1., 6.2) | 45.8 | do not reject |
| Triangular(0.146, 15., 6.01) | 42.8 | do not reject |
| Pearson 6(1., 5.72e+004, 3.89, 3.69e+004) | 37.7 | do not reject |
| Rayleigh(0.542, 4.95) | 35.5 | do not reject |
| Extreme Value IA(5.39, 2.72) | 32.9 | do not reject |
| Extreme Value IB(8.43, 3.2) | 4.6 | reject |
| Power Function(0.909, 14.1, 0.981) | 0.466 | reject |
| Uniform(1., 14.) | 0.183 | reject |
| Exponential(1., 5.86) | 7.59e-003 | reject |
| Pareto(1., 0.555) | 0. | reject |
| Johnson SB | no fit | reject |
| Inverse Weibull | no fit | reject |

Stat::Fit ranking for product B

Document1: Automatic Fitting

Auto::Fit of Distributions

| distribution | rank | acceptance |
|---|---|---|
| Extreme Value IA(2.17, 1.36) | 86.6 | do not reject |
| Logistic(2.8, 1.04) | 81.7 | do not reject |
| Inverse Weibull(-3.02, 4.17, 0.199) | 54.5 | do not reject |
| Pearson 5(-0.551, 4.1, 11.2) | 45.8 | do not reject |
| Lognormal(-1.2e+003, 7.09, 1.56e-003) | 28. | do not reject |
| Normal(3., 1.87) | 27.7 | do not reject |
| Rayleigh(-5.79e-002, 2.53) | 24.4 | do not reject |
| Inverse Gaussian(0.4, 3.12, 2.6) | 19.9 | do not reject |
| Cauchy(2.46, 1.06) | 17.3 | do not reject |
| Laplace(3., 1.44) | 9.65 | do not reject |
| Extreme Value IB(4.02, 2.23) | 2.75 | do not reject |
| Chi Squared(1., 3.13) | 1.16 | reject |
| Weibull(1., 1., 2.2) | 0.329 | reject |
| Beta(1., 1.89e+004, 2.67, 1.88e+004) | 0.141 | reject |
| Erlang(1., 1., 2.) | 0.109 | reject |
| Gamma(1., 1., 2.) | 0.109 | reject |
| Exponential(1., 2.) | 0.109 | reject |
| LogLogistic(1., 2.69, 2.19) | 6.9e-002 | reject |
| Pearson 6(1., 8.83e-002, 64.6, 3.05) | 2.1e-002 | reject |
| Power Function(0.8, 17., 0.397) | 1.34e-003 | reject |
| Uniform(1., 8.) | 3.8e-005 | reject |
| Triangular(1., 8.64, 1.) | 0. | reject |
| Pareto(1., 1.11) | 0. | reject |
| Johnson SB | no fit | reject |
| Johnson SU | no fit | reject |

Stat::Fit ranking for product C

Document1: Automatic Fitting

Auto::Fit of Distributions

| distribution | rank | acceptance |
|---|---|---|
| Johnson SU(-425, 645, -200, 319) | 99.9 | do not reject |
| Weibull(-1.61, 3.38, 8.12) | 99.4 | do not reject |
| Lognormal(-748, 6.63, 3.21e-003) | 99.1 | do not reject |
| Normal(5.67, 2.42) | 98.7 | do not reject |
| Logistic(5.66, 1.41) | 91.2 | do not reject |
| LogLogistic(-169, 123, 175) | 90.7 | do not reject |
| Triangular(-2.96e-002, 11., 6.) | 82.8 | do not reject |
| Beta(1., 10., 2.25, 2.06) | 74.4 | do not reject |
| Pearson 5(-17., 82., 1.84e+003) | 72.2 | do not reject |
| Cauchy(5.62, 1.52) | 55.1 | do not reject |
| Extreme Value IB(6.87, 2.23) | 37.7 | do not reject |
| Extreme Value IA(4.45, 2.33) | 28.6 | do not reject |
| Rayleigh(0.42, 4.09) | 20.6 | do not reject |
| Laplace(6., 1.97) | 11.2 | do not reject |
| Uniform(1., 10.) | 9.66 | do not reject |
| Power Function(1., 10.1, 1.36) | 6.23 | do not reject |
| Exponential(1., 4.67) | 2.37e-003 | reject |
| Pareto(1., 0.623) | 0. | reject |
| Erlang | no fit | reject |
| Inverse Gaussian | no fit | reject |
| Johnson SB | no fit | reject |
| Gamma | no fit | reject |
| Inverse Weibull | no fit | reject |
| Chi Squared | no fit | reject |
| Pearson 6 | no fit | reject |

Stat::Fit ranking for product D

Document1: Automatic Fitting

Auto::Fit of Distributions

| distribution | rank | acceptance |
|---|---|---|
| Extreme Value IA(1.78, 0.984) | 99.1 | do not reject |
| Inverse Weibull(-2.03, 4.35, 0.271) | 79.7 | do not reject |
| Logistic(2.18, 0.763) | 77.5 | do not reject |
| Cauchy(1.98, 0.61) | 77.4 | do not reject |
| Laplace(2., 1.) | 72.3 | do not reject |
| Pearson 5(0.297, 2.51, 3.43) | 29.2 | do not reject |
| Lognormal(-1.2e+003, 7.09, 1.28e-003) | 22.9 | do not reject |
| Normal(2.41, 1.54) | 22.7 | do not reject |
| Rayleigh(6.24e-002, 1.98) | 16.5 | do not reject |
| Weibull(1., 1., 1.66) | 3.75 | do not reject |
| Extreme Value IB(3.28, 2.) | 3.45 | do not reject |
| Inverse Gaussian(0.791, 0.824, 1.62) | 3. | reject |
| Erlang(1., 1., 1.41) | 1.36 | do not reject |
| Gamma(1., 1., 1.41) | 1.36 | do not reject |
| Exponential(1., 1.41) | 1.36 | do not reject |
| Chi Squared(1., 2.55) | 1.26 | reject |
| Power Function(0.75, 9., 0.487) | 0.868 | reject |
| LogLogistic(1., 2.99, 1.53) | 0.379 | reject |
| Beta(1., 420, 2.52, 517) | 0.298 | reject |
| Pearson 6(1., 2.98e-002, 179, 3.73) | 0.12 | reject |
| Pareto(1., 1.4) | 7.96e-002 | reject |
| Uniform(1., 7.) | 5.43e-005 | reject |
| Triangular(1., 7.47, 1.) | 0. | reject |
| Johnson SB | no fit | reject |
| Johnson SU | no fit | reject |

Stat::Fit ranking for product E

Document1: Automatic Fitting

Auto::Fit of Distributions

| distribution | rank | acceptance |
|---|---|---|
| Extreme Value IA(3.95, 2.12) | 98.4 | do not reject |
| Rayleigh(0.416, 3.76) | 85.9 | do not reject |
| Chi Squared(1., 4.61) | 73.4 | do not reject |
| Weibull(0.267, 2.12, 5.51) | 73.2 | do not reject |
| Uniform(1., 9.) | 68.5 | do not reject |
| Lognormal(-8.18, 2.57, 0.184) | 61.2 | do not reject |
| Pearson 5(-12.4, 52.3, 898) | 59.5 | do not reject |
| Triangular(0.247, 10.7, 4.) | 49. | do not reject |
| Gamma(-26.9, 168, 0.191) | 42.7 | do not reject |
| LogLogistic(-95.3, 67.6, 100) | 42. | do not reject |
| Erlang(-26.9, 173, 0.185) | 40.2 | do not reject |
| Logistic(5.06, 1.49) | 39.4 | do not reject |
| Normal(5.14, 2.44) | 33.6 | do not reject |
| Inverse Gaussian(-6.61, 261, 11.8) | 32.8 | do not reject |
| Cauchy(4.4, 1.67) | 30.3 | do not reject |
| Beta(1., 9., 1.58, 1.57) | 24.7 | do not reject |
| Power Function(1., 9.17, 1.24) | 19.4 | do not reject |
| Extreme Value IB(6.37, 2.25) | 15.9 | do not reject |
| Laplace(4., 2.1) | 2.41 | do not reject |
| Exponential(1., 4.14) | 2.31 | do not reject |
| Pareto(1., 0.667) | 2.43e-003 | reject |
| Pearson 6(1., 7.08e+003, 0.354, 605) | 1.58e-004 | reject |
| Johnson SB | no fit | reject |
| Inverse Weibull | no fit | reject |
| Johnson SU | no fit | reject |

Stat::Fit ranking for product F

Document1: Automatic Fitting

Auto::Fit of Distributions

| distribution | rank | acceptance |
|---|---|---|
| Inverse Gaussian(-120, 2.11e+005, 127) | 99.4 | do not reject |
| Logistic(6.57, 1.79) | 99.1 | do not reject |
| Normal(6.6, 3.1) | 98.9 | do not reject |
| Lognormal(-127, 4.89, 2.33e-002) | 98.3 | do not reject |
| Johnson SU(5.21, 75.1, -0.44, 24.1) | 98.2 | do not reject |
| LogLogistic(-420, 243, 426) | 97.8 | do not reject |
| Weibull(-1.83, 2.99, 9.44) | 93.9 | do not reject |
| Pearson 5(-22.7, 84.8, 2.46e+003) | 88.6 | do not reject |
| Laplace(6.5, 2.5) | 88.1 | do not reject |
| Triangular(-0.447, 13.9, 6.) | 87.5 | do not reject |
| Gamma(-148, 1.81e+003, 8.53e-002) | 85.1 | do not reject |
| Cauchy(6.32, 1.84) | 84.7 | do not reject |
| Erlang(-148, 1.81e+003, 8.53e-002) | 83.2 | do not reject |
| Chi Squared(1., 6.63) | 63.7 | do not reject |
| Beta(1., 12., 2.92, 2.72) | 56.7 | do not reject |
| Extreme Value IA(5.05, 2.93) | 54.8 | do not reject |
| Extreme Value IB(8.16, 2.95) | 49. | do not reject |
| Rayleigh(4.58e-002, 5.13) | 44.9 | do not reject |
| Pearson 6(1., 17.4, 7.05, 20.7) | 44.8 | do not reject |
| Uniform(1., 12.) | 17. | do not reject |
| Power Function(1., 12.1, 1.48) | 5.3 | do not reject |
| Exponential(1., 5.6) | 6.06e-002 | reject |
| Pareto(1., 0.582) | 1.4e-004 | reject |
| Johnson SB | no fit | reject |
| Inverse Weibull | no fit | reject |

# Appendix 2b. Ranking of distribution for lead times in Stat::Fit

Document1: Automatic Fitting

**Auto::Fit of Distributions**

| distribution | rank | acceptance |
|---|---|---|
| Lognormal(5.02, 2.66, 0.23) | 100 | do not reject |
| Erlang(9.53, 9., 1.13) | 99.9 | do not reject |
| Gamma(9.53, 8.91, 1.14) | 99.8 | do not reject |
| Pearson 5(4.09, 21.4, 319) | 99.2 | do not reject |
| LogLogistic(6.02, 7.03, 13.3) | 99.1 | do not reject |
| Beta(13.9, 39.5, 2.36, 7.58) | 98.5 | do not reject |
| Extreme Value IA(18.1, 2.88) | 97.4 | do not reject |
| Logistic(19.5, 1.93) | 96.6 | do not reject |
| Weibull(13.1, 2.02, 7.48) | 96.2 | do not reject |
| Rayleigh(13.1, 5.25) | 95.4 | do not reject |
| Pearson 6(13.9, 3.18e+003, 3.17, 1.68e+003) | 95.4 | do not reject |
| Normal(19.7, 3.41) | 92.1 | do not reject |
| Inverse Gaussian(5.05, 268, 14.7) | 88.9 | do not reject |
| Chi Squared(12.4, 7.37) | 88.1 | do not reject |
| Laplace(19.3, 2.67) | 75.7 | do not reject |
| Triangular(12.9, 30.1, 17.4) | 60.8 | do not reject |
| Cauchy(19., 1.96) | 57.1 | do not reject |
| Extreme Value IB(21.5, 3.86) | 22.2 | do not reject |
| Exponential(13.9, 5.85) | 0.191 | reject |
| Power Function(13.8, 34.4, 0.631) | 3.28e-002 | reject |
| Uniform(13.9, 29.1) | 3.21e-002 | reject |
| Pareto(13.9, 2.96) | 2.21e-002 | reject |
| Johnson SB(11.1, 15.2, -0.236, 1.15) | 0. | reject |
| Inverse Weibull | no fit | reject |
| Johnson SU | no fit | reject |

# Appendix 3. Interface of the forecasting models

## Appendix 3a. Interface of moving average

**Moving Average**

**Product A**

| Date | mars-18 | apr-18 | maj-18 | juni-18 | juli-18 | aug-18 | sep-18 | okt-18 | nov-18 | dec-18 | jan-19 | feb-19 | mars-19 | apr-19 | maj-19 | juni-19 | juli-19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sales | 22 | 34 | 35 | 15 | 27 | 25 | 27 | 42 | 32 | 21 | 31 | 33 | | | | | |
| | | | | | | | | | | | | | | | | | |
| Moving average N=6 | | | | | | | 26,3 | 27,2 | 28,5 | 28,0 | 29,0 | 29,7 | 31,0 | | | | |
| Abs. Error | | | | | | | 0,67 | 14,83 | 3,50 | 7,00 | 2,00 | 3,33 | | | | | |
| MADt | | | | | | | | | | | | 5,2 | | | | | |
| Moving average N=5 | | | | | | 26,6 | 27,2 | 25,8 | 27,2 | 30,6 | 29,4 | 30,6 | 31,8 | | | | |
| Abs. Error | | | | | | 1,6 | 0,2 | 16,2 | 4,8 | 9,6 | 1,6 | 2,4 | | | | | |
| MADt | | | | | | | | | | 6,5 | 6,5 | 6,9 | | | | | |
| Moving average N=4 | | | | | 26,5 | 27,75 | 25,5 | 23,5 | 30,25 | 31,5 | 30,5 | 31,5 | 29,25 | | | | |
| Abs. Error | | | | | 0,5 | 2,75 | 1,5 | 18,5 | 1,75 | 10,5 | 0,5 | 1,5 | | | | | |
| MADt | | | | | | | | 5,8 | 6,1 | 8,1 | 7,8 | 3,6 | | | | | |
| Moving average N=3 | | | | 30,3 | 28,0 | 25,7 | 22,3 | 26,3 | 31,3 | 33,7 | 31,7 | 28,0 | 28,3 | | | | |
| Abs. Error | | | | 15,3 | 1,0 | 0,7 | 4,7 | 15,7 | 0,7 | 12,7 | 0,7 | 5,0 | | | | | |
| MADt | | | | | | 5,7 | 2,1 | 7,0 | 7,0 | 9,7 | 4,7 | 6,1 | | | | | |

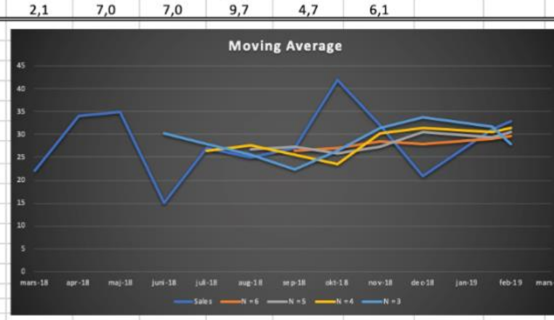
The following value is not representative for Product A. It is an arbitrary value to give a representation of MADt for N=6

The lead time for Product A is considered deterministic

| | |
|---|---|
| $t_F$ | 1 |
| L | 8 |
| E(L) | |

The Period with least forecast error

| Product A | | | | |
|---|---|---|---|---|
| **Period (N)** | **MADt** | **σ** | **μ** | **Rank** |
| 6 | 5,22 | 6,55 | 248 | **1** |
| 5 | 6,63 | 8,31 | 254,4 | **4** |
| 4 | 6,28 | 7,86 | 234 | **3** |
| 3 | 6,03 | 7,56 | 226,667 | **2** |



## Appendix 3b. Interface of simple exponential smoothing

**Simple Exponential Smoothing**

Smoothing constant from (3.28)

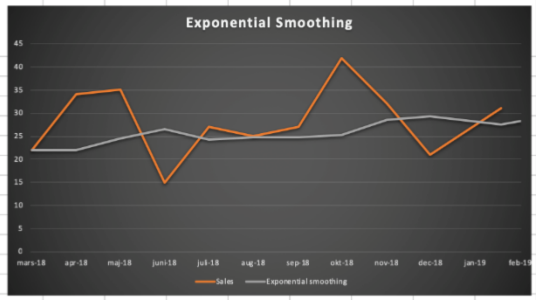
| | |
|---|---|
| k | 1 |
| α | 0,2 |
| $MAD_0$ | 6,5 |
| $α_{MAD}$ | 0,2 |
| $t_F$ | 1 |
| L | 8 |
| E(L) | |

Assume the first forecast is correct, as a starting point

$MAD_0$

The lead time for Product A is considered deterministic

| Product A | | | MADt | | |
|---|---|---|---|---|---|
| Period | Sales | Simple Exponential Smoothing ât | 6,5 | Stadard Deviation (σ') | μ' |
| mars-18 | 22 | 22,0 | 5,2 | 6,5 | 176,0 |
| apr-18 | 34 | 22 | 6,6 | 8,2 | 176,0 |
| maj-18 | 35 | 24 | 7,4 | 9,2 | 195,2 |
| juni-18 | 15 | 27 | 8,2 | 10,3 | 212,2 |
| juli-18 | 27 | 24 | 7,1 | 8,9 | 193,7 |
| aug-18 | 25 | 25 | 5,7 | 7,2 | 198,2 |
| sep-18 | 27 | 25 | 5,0 | 6,3 | 198,5 |
| okt-18 | 42 | 25 | 7,4 | 9,2 | 202,0 |
| nov-18 | 32 | 29 | 6,6 | 8,2 | 228,8 |
| dec-18 | 21 | 29 | 6,9 | 8,7 | 234,3 |
| jan-19 | 31 | 28 | 6,2 | 7,8 | 221,0 |
| feb-19 | | 28 | | | |
| mars-19 | | | | | |
| apr-19 | | | | | |



# Appendix 3c. Interface of exponential smoothing with trend

**Exponential Smoothing with Trend**

| | |
|---|---|
| k | 1 |
| α | 0,2 |
| β | 0,05 |
| MAD0 | 6,5 |
| αMAD | 0,2 |
| tF | 1 |
| L | 8 |
| E(L) | |

Assume β is 0,05

Smoothing constant from (3.28)

Assume the first forecast is accurate, as a starting point.

Initially it is assumed that there is no trend

Assume the first forecast is accurate, as a starting point.

$MAD_0$

**Product A**

| Period | Sales | ât | b^ | Exponential Smoothing with Trend | MADt 6,5 | Standard Deviation (σ') | µ' |
|---|---|---|---|---|---|---|---|
| mars-18 | 22 | 22,0 | 0 | 22,0 | 5,2 | 6,5 | 176,0 |
| apr-18 | 34 | 22 | 0,0 | 22,0 | 6,6 | 8,2 | 176,0 |
| maj-18 | 35 | 24 | 0,1 | 22,0 | 7,8 | 9,8 | 199,5 |
| juni-18 | 15 | 27 | 0,2 | 24,5 | 8,2 | 10,3 | 220,1 |
| juli-18 | 27 | 24 | 0,1 | 26,7 | 6,6 | 8,3 | 197,1 |
| aug-18 | 25 | 25 | 0,1 | 24,3 | 5,4 | 6,8 | 202,4 |
| sep-18 | 27 | 25 | 0,1 | 24,9 | 4,8 | 6,0 | 202,6 |
| okt-18 | 42 | 25 | 0,1 | 24,9 | 7,2 | 9,0 | 206,7 |
| nov-18 | 32 | 29 | 0,3 | 25,4 | 7,1 | 8,9 | 239,3 |
| dec-18 | 21 | 29 | 0,3 | 28,9 | 7,3 | 9,1 | 245,4 |
| jan-19 | 31 | 28 | 0,2 | 29,6 | 6,1 | 7,6 | 228,6 |
| feb-19 | | 28 | | 27,8 | | | |
| mars-19 | | | | | | | |