Development of a Quasi-Transient Lap Time Simulation Tool for evaluation of Torque Vectoring in Motorsport Applications

Alexander Söderholm Mullaart

Thesis for the degree of Bachelor of Science in Engineering

Division of Combustion Engines
Department of Energy Sciences
Faculty of Engineering | Lund University

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This degree project for the degree of Bachelor of Science in Engineering has been conducted at the Division of Combustion Engines, Department of Energy Sciences, Faculty of Engineering, Lund University.

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Examiner at Lund University was Dr. Sebastian Verhelst.

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"*Driving a car as fast as possible is all about maintaining the highest possible acceleration level in the appropriate direction.*"

Peter G. Wright
Former Technical Director at Team Lotus

Abstract

Quasi-static lap time simulation tools are widely used in Motorsport for parameter sensitivity analysis. Such tools, however, do not account for the yaw acceleration limitations of the vehicle and are not able to provide insights into the vehicle's driveability. In this thesis, the vehicle is characterised by its most fundamental components and modelled as a limit acceleration surface with three degrees of freedom. A quasi-transient lap time simulation tool that uses the identified vehicle model is developed. The concepts and modelling techniques used to characterise the vehicle and simulate vehicle lap time described in depth. Moreover, the capabilities of an active differential, which enables torque vectoring by distributing tractive force unequally between the drive wheels, are compared to those of an open differential. It is shown that the developed vehicle model and lap time simulation tool provides insights into yaw dynamics and relative parameter effect on lap time.

Acknowledgements

Firstly, I would like to thank my supervisor Martin Tunér for his commitment to me and as faculty advisor to the Lund Formula Student team. The team has provided an incredible educational platform for future engineers such as myself and has enabled me to write this thesis. Therefore, I would like to thank everyone in the team that I have worked with throughout the years.

Secondly, I would like to express gratitude to the organizers and volunteers that make Formula Student possible. I am very grateful for those involved in Formula Student who have openly shared their knowledge to the benefit of everyone. Moreover, I would like to acknowledge the Formula SAE Tyre Testing Consortium and Calspan Tyre Research Facility for providing me with the necessary tyre data used in this thesis.

Finally, I would like to thank my family, and especially my fiancée, Sahar Torabi, for always supporting me and encouraging me to follow my dreams.

Abbreviations and acronyms

CoP - Centre of pressure

$C_d A$ - Coefficient of drag times vehicle frontal area

$C_l A$ - Coefficient of lift times vehicle frontal area

CG_h - Centre of gravity height

FTLLTD - Front total lateral load transfer distribution

LAS - Limit acceleration surface

MF - Magic Formula

FSAE TTC - The Formula SAE Tyre Test Consortium

LFS-19 - The 11^{th} car of Lund Formula Student

MMD - Milliken Moment Diagram

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1. Introduction

This thesis investigates if it is possible to develop a simple and efficient lap time simulation tool that takes yaw dynamics into account. Such a tool would provide greater insights into the relative importance of the parameters that affect lap time, and perhaps most importantly, the driveability of the vehicle. Such knowledge would be instrumental in making the right decisions during the development phase of a vehicle, i.e. to focus on the most important areas.

In order to accurately model the vehicle, the tyres need to be modelled. Therefore, the thesis begins with tyre modelling and thereafter addresses vehicle modelling and lap time simulation. Finally, a design study is performed and torque vectoring by means of an active differential is compared to an open differential. Moreover, the vehicles modelled in the thesis uses parameters and tyres typical for Formula Student. The track that is later used in lap time simulation is sized appropriately for such a vehicle. Therefore, section 1.2 provides a brief overview of Formula Student. Moreover, unless otherwise specified, all presented data has been generated in MATLAB by the author.

1.1 Objective and limitations

The objective of this thesis is, as described in the introduction, to develop a simple lap time simulation program that takes yaw dynamics into account. This is to be done by the author from scratch in MATLAB. The method of implementing the yaw degree of freedom, which is described in depth in section 3.3, was conceived when the author studied Milliken Moment Diagrams, or MMDs. Such diagrams plot lateral acceleration against yaw acceleration for different driving situations, please refer to reference [1] for more information on the subject. If the longitudinal degree of freedom is introduced to such a diagram, the vehicle performance envelope can be nearly fully described. The extension of an MMD to three degrees of freedom and its use in lap time simulation is apparently not novel, however the author has not found any sources that implements it in the exact same way as is described in this thesis.

In order to meet the overall objective of the thesis, the program should feature the following:

- Non-linear tyres.
- Aerodynamic downforce and drag.
- Torque vectoring.
- The possibility of investigating the lap time sensitivity of the following parameters:
  - Mass
  - Yaw inertia
  - CG_h
  - Front weight proportion
  - Power (including the full power curve and gearing)
  - FTLLTD
  - Front downforce proportion
  - $C_l A$
  - $C_d A$

In order to keep complexity low, the following limitations were imposed:

- Rigid chassis without compliance effects.
- No rolling or pitching movement of the body.
- No suspension movement apart from steering angle.
- Constant camber and toe angles.
- Constant effective tyre radius.
- No road gradient, i.e. planar motion.
- No aerodynamic yaw sensitivity.
- No aerodynamic yaw moment.
- Rear-wheel drive.

1.2 Formula Student

Formula Student is an international student engineering competition held in different locations around the world. The task is to design, manufacture and race a small open-wheel race car. The teams compete in both static and dynamic events. In the static events, the car is not driven, instead the students are tested on their knowledge and understanding of different areas. In the dynamic events the car is driven in time-trial events. The overall winner of the competition is the team with the highest score out of a maximum of 1000. The events are as follows:

Static Events

- Engineering design: The students are questioned by design judges regarding choices made in the design of their car. Emphasis is put on engineering comprehension and the quality of reasoning.

- Cost and manufacturing: Tests the students' knowledge and understanding of manufacturing processes and related costs.

- Business plan presentation: The team presents a business case which involves selling their race car and are judged on its feasibility and potential success.

Dynamic Events

- Acceleration: An event testing the straight-line capabilities of the car. Standing start with a 75m long straight.

- Skid pad: Tests the steady-state cornering capabilities of the car on an 18.25m diameter circle.

- Autocross: Driving one lap from a standing start on a short transient track with an average speed of approximately 55 kph.

- Endurance: Driving 22km on the same track as autocross with a driver change half-way through.

- Efficiency: Scores the fuel efficiency to lap time ratio during endurance.

1.2.1 LFS-19

The LFS-19 is the 11th car developed and manufactured by Lund University's Formula Student team. The team competed with the car during the 2019 season and finished in 2nd and 8th place at the Formula Student Netherlands and Formula Student Germany competitions respectively. The baseline vehicle modelled in this thesis uses the parameters of LFS-19. The car features a steel-space frame, 600cc motorcycle engine, rear-wheel drive, disc-brakes, double-wishbone suspension, 10" wheels and an aerodynamic package.

Figure 1: LFS-19 competing at Formula Student Germany.

2. Tyre modelling

2.1 Formula SAE Tyre Test Consortium

The Formula SAE Tyre Test Consortium is a volunteer-managed organization funded by Formula Student teams with the objective of obtaining empirical tyre force and moment data. The organization is supported by the Calspan Tyre Research Facility which tests tyres on a continuous flat belt machine. The machine can control normal load, slip angle, camber angle, slip ratio, inflation pressure and speed. Moreover, the thermal state of the tyre is measured continuously during a test. All empirical tyre data used in this thesis has been provided by the FSAE TTC and follows the axis system shown in fig. 2.

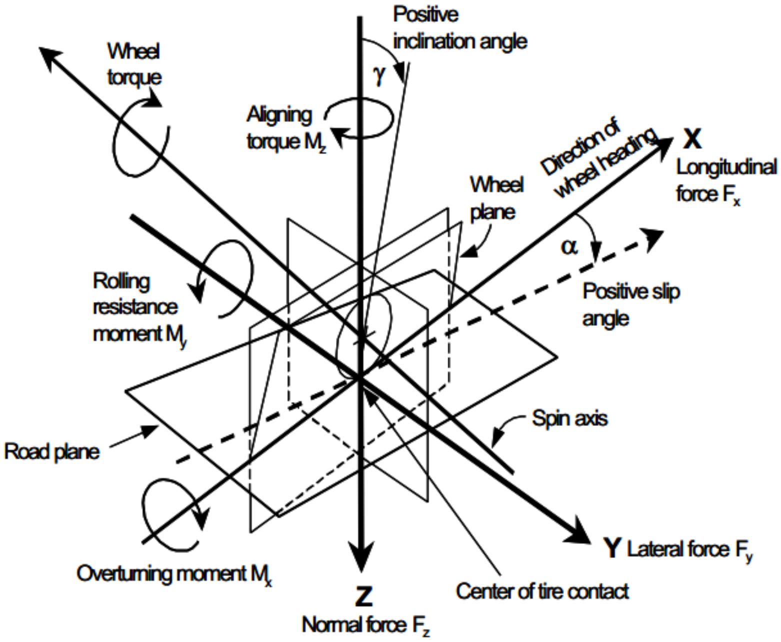

Figure 2: The SAE tyre axis system as defined in SAEJ2047.

2.2 Pacejka Magic Formula

The "Magic Formula" developed by Hans B. Pacejka is a semi-empirical model used to describe the force and moment outputs of pneumatic tyres given a certain set of operating conditions. In the model, the tyre is characterised by a set of coefficients that are fit to empirical data. Several versions, of differing complexity, of the MF models exist. The general form of the model is:

Eq. 1

$$R(\alpha) = D \sin\{C \arctan[B(1-E)\alpha + E \arctan(B\alpha)]\}$$

where *B*, *C*, *D* and *E* are coefficients fit to test data. *R* is a force or moment generated by the tyre and α is either slip ratio or slip angle. The coefficients govern the shape of the curve where *B* is the stiffness factor,

C the shape factor, D the peak value and E the curvature factor. These can be constant or vary with input parameters depending on which tyre characteristics that are included in the model. Please review reference [2] for an in-depth explanation of the model.

The MF model used in this thesis is a modification of the general formulation with the addition a load sensitivity term. It was chosen to include load sensitivity, which means that the coefficient of friction of the tyre decreases with increased normal load, as this has a big effect on the handling behaviour of the vehicle. Other effects, for example due to changes in tyre pressure or camber angle, have been neglected in order to keep complexity down but should also be modelled if an even more realistic model is required. The used formula is described as:

Eq. 2

$$R(\alpha, F_z) = (D_1 F_z + \frac{D_2}{1000} F_z) \times \sin(C \arctan(B\alpha))$$

where F_z is the normal force acting on the tyre. The coefficients B, C, D_1 and D_2 are fit to test data and $E=0$.

2.3 Fitting procedure

In order to model the vehicle appropriately for the purposes of this thesis, the following tyre forces and moments need to be modelled: lateral force F_y, longitudinal force F_x and self-aligning moment M_z. As parameters such as tyre pressure and camber angle are not included as inputs to the MF model of choice, the optimal combination of these parameters was identified by analysing raw data. Thereafter, the coefficients were fit using a non-linear least squares solver in MATLAB.

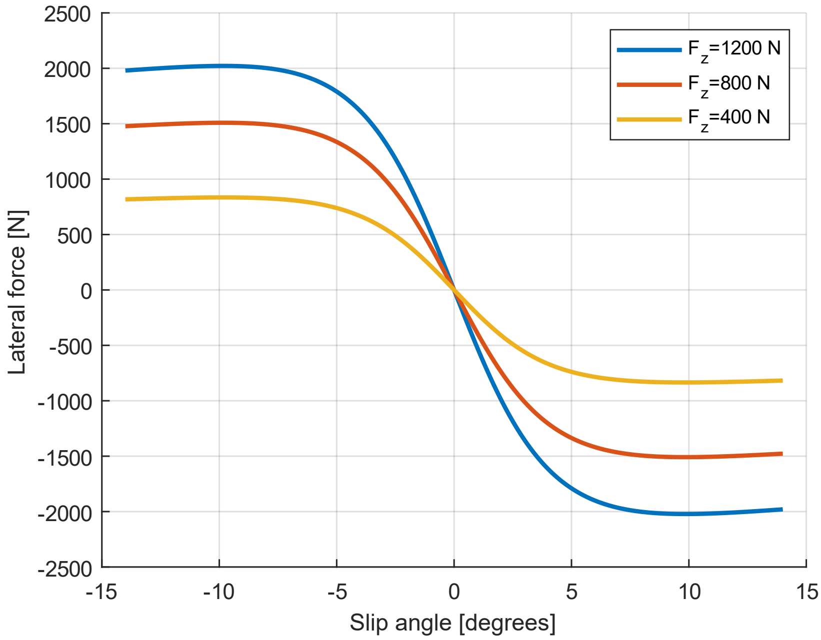

Figure 3: Lateral force vs. slip angle for different normal loads.

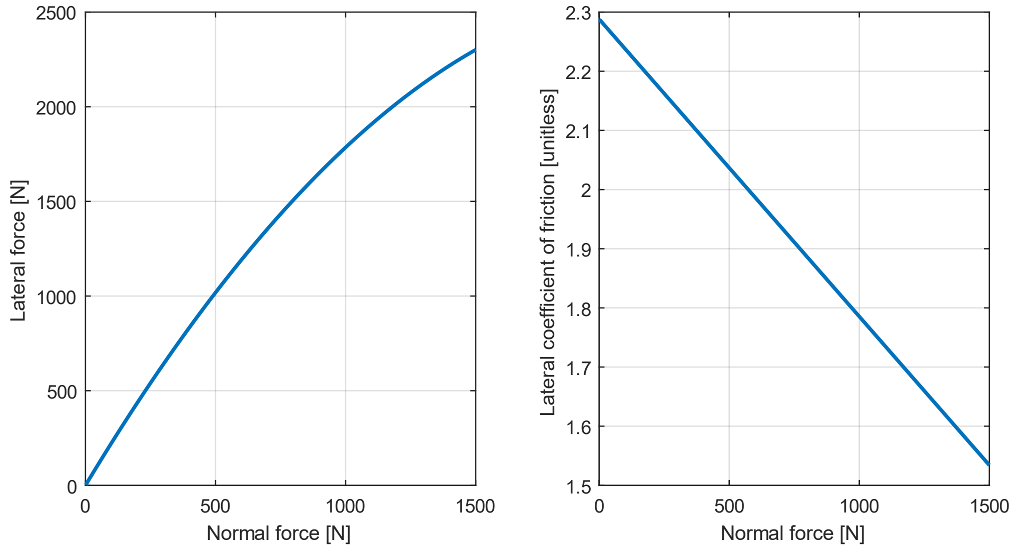

Figure 4: Peak lateral force vs. normal force is shown on the left and peak lateral coefficient of friction vs. normal force is shown on the right.

2.4 Combined lateral and longitudinal force

When lateral and longitudinal force is demanded simultaneously from the tyre, for example in corner exit, the maximum possible lateral and longitudinal force will be less than in a pure cornering or driving situation. For example, if the tyre is generating maximum lateral force and longitudinal slip is introduced, the generated lateral force will decrease and vice versa. This behaviour can be reasonably accurately modelled by a "friction ellipse" in which the semi-axes represent the maximum possible lateral and longitudinal forces possible at that normal load. All points within the ellipse represent possible resultant tyre forces when the tyre is rolling. If the tyre starts sliding, the coefficient of friction decreases, which means that the ellipse cannot be exceeded.

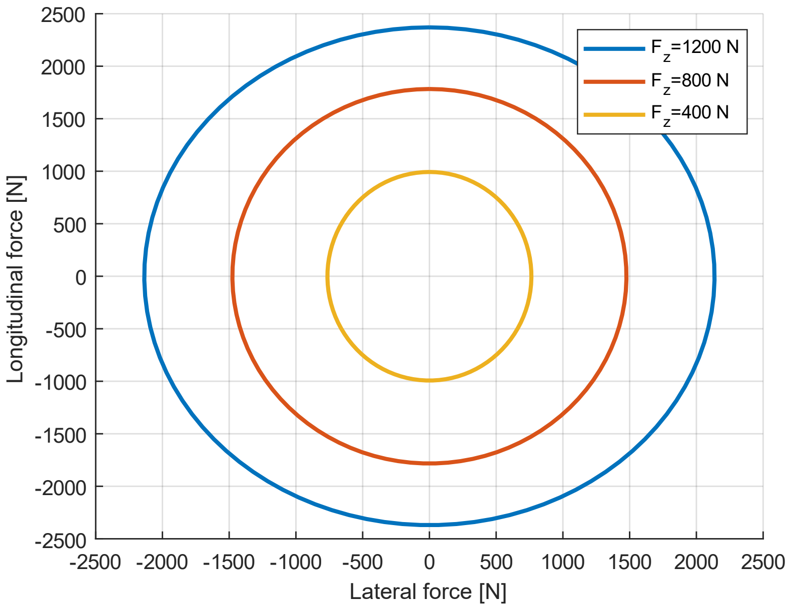

Figure 5: Friction ellipses derived from tyre data.

3. Vehicle modelling

3.1 Introduction

The vehicle performance envelope describes the region in terms of accelerations in which the vehicle can operate given a certain driving situation. These accelerations are caused by the forces and moments acting on the vehicle and are what causes vehicle movement. Therefore, in order to study the performance envelope, a model of the vehicle that deals with these forces and moments needs to be used.

In order to include the yaw dynamics of the vehicle, the yaw degree of freedom needs to be introduced. When a vehicle is taking a corner, the yaw velocity will vary from entry to exit. For this to be possible, yaw acceleration must be present. The yaw acceleration is a function of the unbalanced moments around the vehicle's centre of gravity and its yaw moment of inertia. In the following sections, the model used to include yaw dynamics in the vehicle's performance envelope is described.

3.2 Background

A commonly used method of describing the performance envelope is to generate a GG-V plot. This entails plotting the maximum simultaneous lateral and longitudinal acceleration over different speeds which results in a three-dimensional surface. The GG-V plot is a is an extension of the GG-diagram. Figure 6 shows logged lateral and longitudinal acceleration from LFS-19 being driven around a small track and a GG-diagram derived from tyre data and a simple vehicle model. The GG-diagram shows the theoretical acceleration limit of the car if it was only grip limited. The lowest possible lap time around a track will be achieved when the vehicle operates on this line. However, as this method only includes two degrees of freedom, it is only valid in steady-state driving situations where there, for example, is no yaw acceleration.

It should be noted that for the logged data in fig. 6, the positive longitudinal accelerations are of a lower magnitude than the negative ones as the vehicle is only rear wheel driven and limited by the power output of the engine. In braking, all four tyres generate longitudinal forces. In addition, the tyres will be closer to the grip limit as the moments generated by the brakes can be big enough to lock all wheels. Moreover, the shape traced by the logged data is highly dependent on the layout of the track and the skill of the driver.

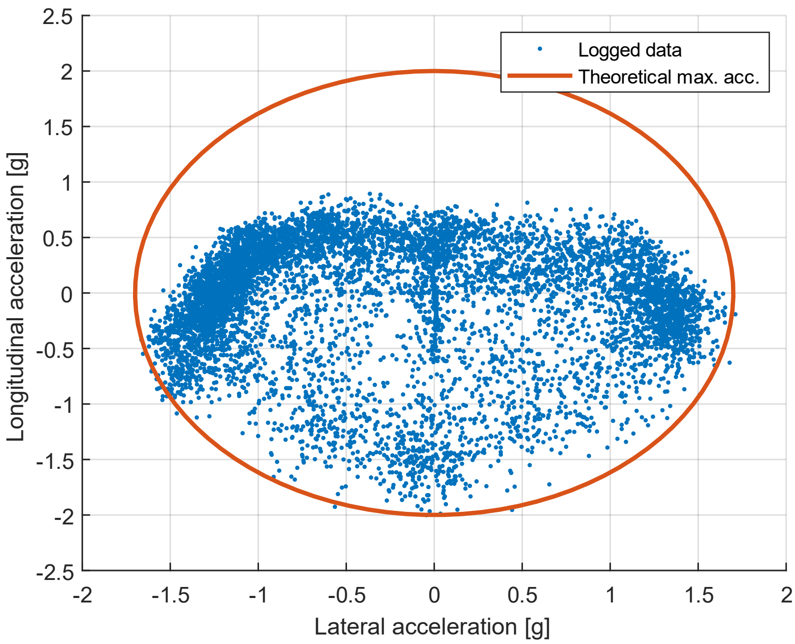

Figure 6: Logged data from an accelerometer and a GG-diagram derived from tyre data and a simple vehicle model.

3.3 Vehicle model

The model has the following features and limitations:

- Lateral, longitudinal and yaw degrees of freedom.
- Rigid chassis without compliance effects.
- Lateral and longitudinal load transfer.
- Equal front slip angles and parallel steering.
- Equal rear slip angles.
- No suspension movement.
- No rolling or pitching movement of the body.
- Aerodynamic downforce and drag.
- No aerodynamic yaw moment.
- Rear-wheel drive with possibility of unequal left to right tractive forces.
- Equal right to left braking forces on each axle.

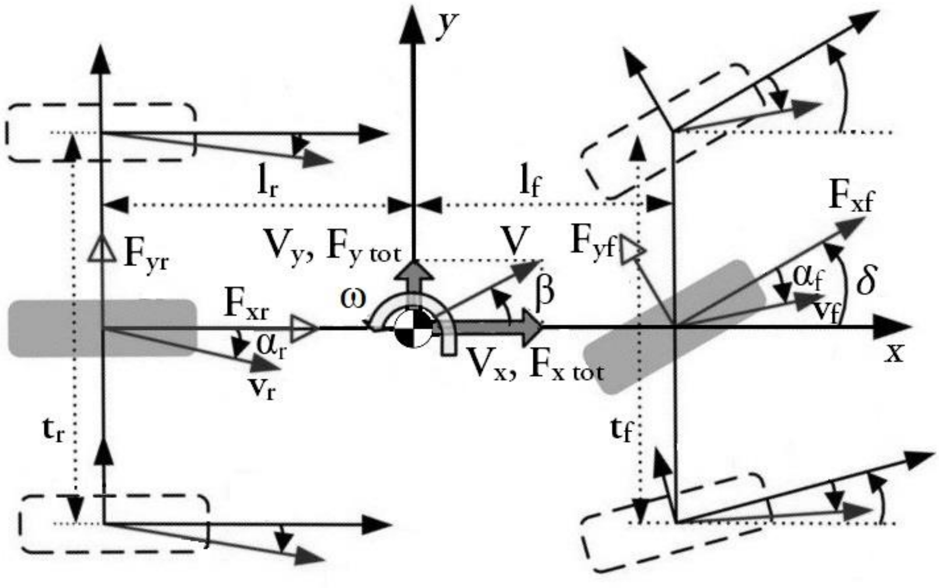

Figure 7: Schematic diagram of a bicycle model.

A schematic diagram of the vehicle model, commonly called a bicycle model [1], is shown above. The coordinate system is body fixed with its origin at the centre of gravity. The equations of motions governing the dynamics in the degrees of freedom are given by:

Eq. 3

$$ma_x = (F_{xfl} + F_{xfr})\cos(\delta) - (F_{yfl} + F_{yfr})\sin(\delta) + (F_{xrl} + F_{xrr}) - F_{drag}$$

Eq. 4

$$ma_y = (F_{xfl} + F_{xfr})\sin(\delta) + (F_{yfl} + F_{yfr})\cos(\delta) + (F_{yrl} + F_{yrr})$$

Eq. 5

$$\begin{gathered} I_z\dot{\omega} = l_f(F_{xfl} + F_{xfr})\sin(\delta) + l_f(F_{yfl} + F_{yfr})\cos(\delta) - l_r(F_{yrl} + F_{yrr}) \\ - 0.5t_r(F_{xrl} - F_{xrr}) + 0.5t_f(F_{yfr} - F_{yfl})\sin(\delta) - M_{zsa} \end{gathered}$$

where the subscripts *fl*, *fr*, *rl* and *rr* represent the front left, front right, rear left and rear right tyres respectively. The term M_{zsa} represents the sum of the self-aligning moment of all tyres, I_z is the yaw moment of inertia, V is velocity, ω is the yaw velocity and F_{drag} is aerodynamic drag. Due to steering angle, the front lateral tyre forces have components in the x direction and the front longitudinal tyre forces have components in the y direction. If steering angles are assumed to be small, these terms can be neglected. In Formula Student, the steering angles are often large and have a considerable effect on handling behaviour. However, neglecting these terms allow for easier computation in the following sections. Moreover, achieving high precision in absolute terms is outside the scope of this thesis. The equations can then be reduced to:

Eq. 6

$$ma_x = F_{xfl} + F_{xfr} + F_{xrl} + F_{xrr} - F_{drag}$$

Eq. 7

$$ma_y = F_{yfl} + F_{yfr} + F_{yrl} + F_{yrr}$$

Eq. 8

$$I_z\dot{\omega} = l_f(F_{yfl} + F_{yfr}) - l_r(F_{yrl} + F_{yrr}) - 0.5t_r(F_{xrl} - F_{xrr}) - M_{zsa}$$

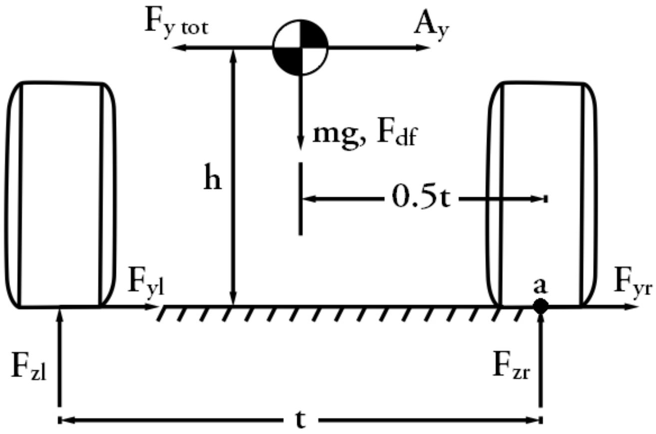

Figure 8: Rear view of the vehicle model. The magnitude of the lateral load transfer in steady state can be calculated from taking the equilibrium of moments about point *a*.

When the vehicle is undergoing lateral or longitudinal acceleration, the normal load on each tyre will change from the static value due to inertial forces if the centre of gravity is located above the ground plane. The distribution of the lateral load transfer between the front and rear axle can be affected by suspension parameters. The proportion of the lateral load transfer taken by the front axle is usually denoted front total lateral load transfer distribution, or FTLLTD. Moreover, the normal loads will also vary with speed due to aerodynamic forces. The distribution of these forces among the wheels is determined by the location of the centre of pressure, or CoP. If the aerodynamic downforce is distributed equally among the wheels, and the FTLLTD does not affect the rigid body equilibrium of moments, the wheel loads during steady-state driving can be described as:

Eq. 9

$$F_{zf} = mg[\frac{l_r}{2l} - \frac{ha_x}{2gl} \pm \frac{hl_r a_y}{glt_f}] + \frac{F_{df}}{4}$$

Eq. 10

$$F_{zr} = mg[\frac{l_f}{2l} - \frac{ha_x}{2gl} \pm \frac{hl_f a_y}{glt_r}] + \frac{F_{df}}{4}$$

where F_{zf} and F_{zr} represent the normal force acting on a front and rear tyre respectively and F_{df} is the aerodynamic downforce.

The tyre slip angle is derived geometrically from wheel velocity vectors. If the velocity at the contact point between tyre and ground is known, the slip angles can be calculated as follows [2]:

Eq. 11

$$\alpha_f = -\delta + \arctan(\frac{v_y + l_f \omega}{v_x \pm 0.5 t_f \omega})$$

Eq. 12

$$\alpha_r = \arctan(\frac{v_y - l_r \omega}{v_x \pm 0.5 t_r \omega})$$

Eq. 13

$$\beta = \arctan(\frac{v_y}{v_x})$$

where α_f and α_r represent a front and rear tyre slip angle respectively and β is the sideslip angle of the vehicle. For purposes that will be apparent later, it is desirable to reduce the above equations. If the track width is considered small relative to the corner radius, and the corner radius is large, the geometric and yaw velocity terms can be neglected. The equations reduce to:

Eq. 14

$$\alpha_f = -\delta$$

Eq. 15

$$\alpha_r = \beta$$

3.3 Limit acceleration surface

The vehicle performance envelope in the lateral, longitudinal and yaw degrees of freedom, at a certain speed, can be described by a three-dimensional region. At each point of this region, the vehicle is in steady state, i.e. undergoing constant acceleration. The outermost points of the region can be used to form a surface. This surface represents the acceleration limitations of the vehicle. An example of such a limit acceleration surface, or LAS, is shown in the figures below.

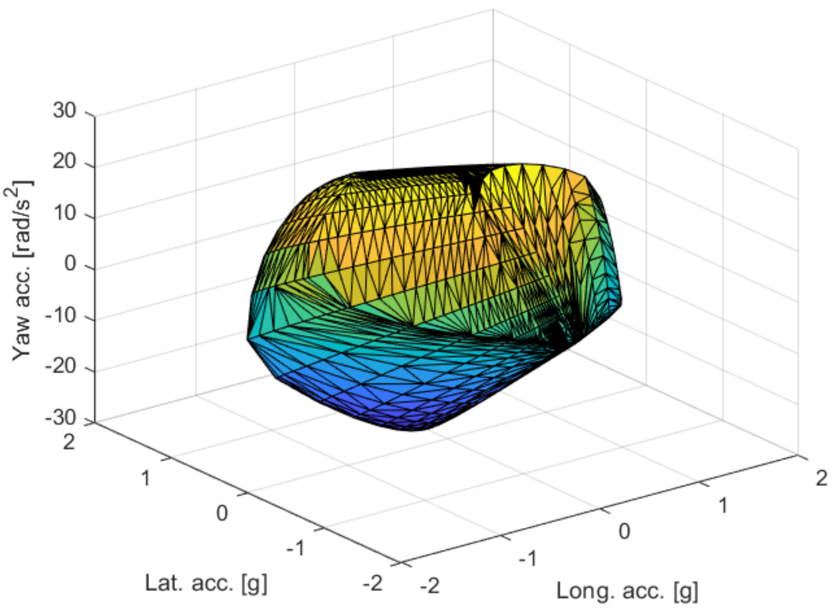

Figure 9: A limit acceleration surface, or LAS.

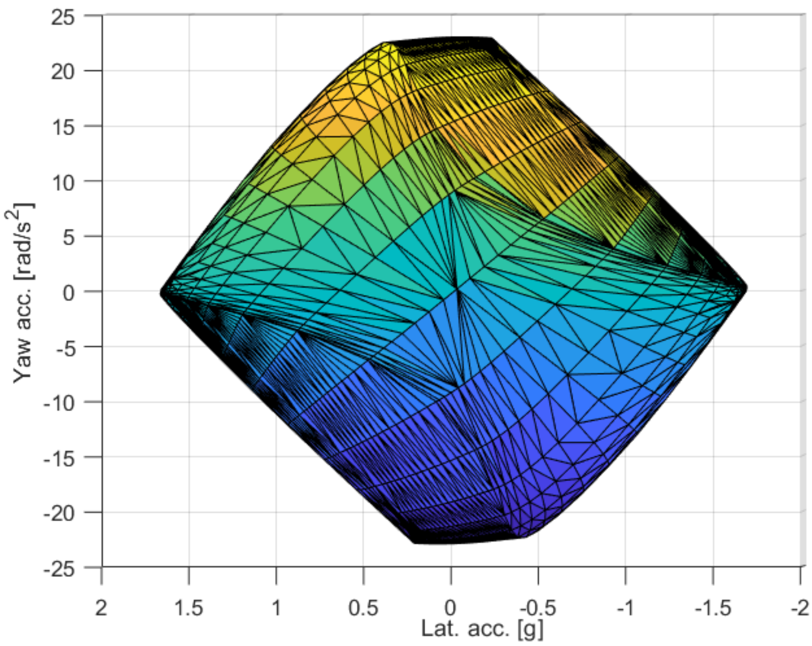

Figure 10: Lateral acceleration vs. yaw acceleration (side-view of the LAS).

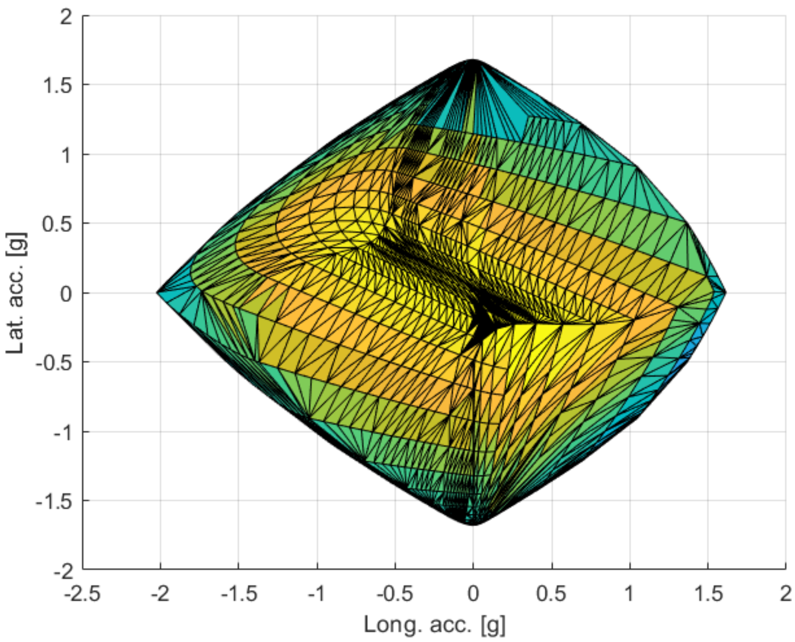

Figure 11: Longitudinal acceleration vs. lateral acceleration (side-view of the LAS).

In order to generate the points that make up the surface, it is necessary to study all possible force and moment combinations that may act on the vehicle model. By sweeping the steering angle, δ, and vehicle sideslip angle, β, (both of which are independent variables in the simplified model) all possible slip angle, and thus tyre output, combinations are generated. In doing so, the sweep range must be chosen to generate slip angles at which the tyres are saturated, i.e. the output reaches and goes beyond maximum values. Some of the slip angle combinations will not be realistic but are fine for the purpose of this thesis.

The process of calculating the resulting steady-state acceleration is iterative as the tyre outputs vary with normal load due to load-sensitivity. Therefore, the resulting vehicle acceleration cannot be known just from the tyre slip angles. The general algorithm for generating all possible steady-state acceleration combinations is described below:

1. Define a β and δ sweep range.
2. Define a vehicle speed and calculate aerodynamic forces.
3. For each β, sweep δ.
   a. For each β and δ combination, sweep slip ratio.
      i. Start with an initial normal load and vehicle acceleration guess.
      ii. Calculate tyre force and moment outputs.
      iii. Calculate vehicle acceleration.
      iv. Calculate wheel loads due to load transfer.
      v. Compare the results to those of the previous step. If the difference is low enough, the solution has converged. If not, update the normal load and acceleration estimate and return to step ii.
      vi. The solution has converged.
4. All possible steady-state accelerations in the sweep range have been computed.

Step 3.a involves sweeping slip ratio in a way that simulates the vehicle accelerating (with an open or active differential) and decelerating. The details of these steps will be explained in the sections below.

If the sweep range step size is chosen to be infinitely small, the outermost generated points would form a surface that is smooth but has sharp corners. However, in order to reduce computation time, the sweep step-size can be chosen to be arbitrarily large depending on the desired accuracy. In order to form the LAS, the convex hull of the region is calculated. When the convex hull is known it is used to form a triangular mesh. This mesh represents the LAS. If more points are required, the mesh elements can be filled with points. If the sweep range step size is about 1-2 degrees or smaller, the generated mesh closely approximates the smooth surface that describes the vehicle performance envelope. Moreover, since the front tyre forces are not broken into components to due steering angle, the region is shaped in such a way that the convex hull is a good geometrical fit. If these force components were to be accounted for, a properly adjusted concave hull would have been a better fit as the region takes on a more irregular shape.

3.3.1 Longitudinal grip usage

The limitations of the vehicle's drivetrain and brake system needs to be accounted for in order to produce a realistic LAS and will be dealt in the following sections.

Acceleration

For each β and δ sweep step, the tyre slip angles are known. The maximum possible lateral tyre force that can be generated by each tyre is calculated iteratively as explained in the previous section. When the lateral tyre force is known, the maximum possible simultaneous longitudinal tyre force can be calculated using the friction ellipse approach. Moreover, there will be an infinite amount of possible longitudinal forces smaller in magnitude than the maximum. In order to generate all points that will be used to make up the LAS, the usage of the tyres' longitudinal grip potential is swept from 0 to 100%. The sweep range step size depends on the accuracy desired.

Open differential

An open differential always distributes torque equally among the drive wheels. This means that the tractive forces have no effect on yaw acceleration as they are equal in size. However, this also means that the total possible tractive force is limited by the tyre with the least grip. For example, if a tyre loses traction and starts to spin it will only generate a small amount of tractive force. In the case of our model, the inner wheel will limit the tractive force as it has the least normal force acting on it (assuming equal grip levels across the road). Therefore, the usage of the inner tyre grip potential will be swept as explained in the previous section and the outer tyre is assumed to generate the same amount of tractive force.

Active differential

For the purposes of this thesis, an active differential is defined as a device which enables torque vectoring by unequally distributing engine torque among the drive wheels. It is assumed that such a device is capable of arbitrary torque distribution. An unequal tractive force distribution means that there will be an effect on yaw acceleration. For example, this means that the differential can transfer all the engine torque to just one drive wheel. In order to include this feature in the LAS, all possible tractive force combinations at a given lateral acceleration must be generated.

Deceleration
When the vehicle is in a corner and braking simultaneously, all four tyres will be subjected to different normal loads. This means that the maximum possible braking force will differ among the tyres. Therefore, the braking torque needs to be unequally distributed among the wheels in order to achieve maximum braking performance. This is called brake biasing and can be achieved in numerous ways. The LFS-19 is only capable of front-to-rear brake biasing of a fixed ratio, as is often the case for race cars. This means that when the brake pedal is applied, the braking torque on a front and rear wheel always differ with a certain ratio (neglecting other effects). Therefore, the tyre braking forces are equal left-to-right and do not affect yaw acceleration. However, this also means that the inner tyres will lock up and start to slide before the outer ones. If this is to be avoided, the inner tyres will limit the possible deceleration. Therefore, at a given lateral acceleration and known brake bias ratio, the inner tyre with the lowest braking force potential is found and used to calculate the total possible braking force without wheel slippage. The grip usage is also swept from 0 to 100%.

3.3.2 Aerodynamics

The LAS represents the vehicle performance envelope at a certain speed. However, due to the aerodynamic forces acting on the car, the shape of the LAS will vary with speed. In order to include the effect of aerodynamic drag and downforce in the lap time simulation program, while minimizing computational time, the entire LAS is re-scaled using polynomial functions. This is achieved by sweeping the vehicle speed from zero to top speed and generating a LAS for each step. At each step, the maximum lateral, longitudinal and yaw accelerations of the LAS are saved. These data points are then used to fit polynomials that outputs the vehicle's maximum acceleration capabilities as a function of speed. As an approximation, the functions are then used to re-scale an entire LAS where the vehicle speed is zero.

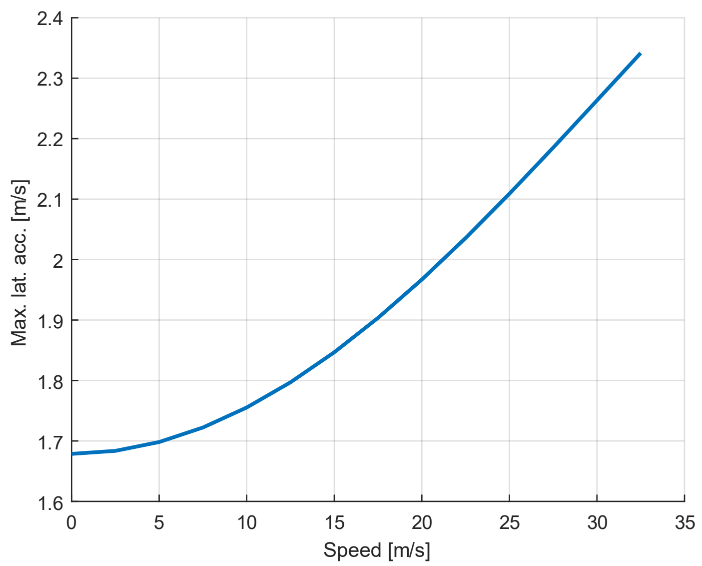

Figure 12: Maximum lateral acceleration vs. speed.

3.4 Engine power limit

The acceleration capability of the vehicle model is limited by the torque output of the engine, gearing, aerodynamic drag and grip of the tyres. In the case of an engine with a gearbox, the tractive force as a function of vehicle speed for different gears can be illustrated in a tractive effort plot. For maximum possible acceleration, the area beneath these curves is to be maximized. If the driver changes gear at the points where the different gear curves intersect one another, a maximum amount of power will have been used to accelerate the vehicle.

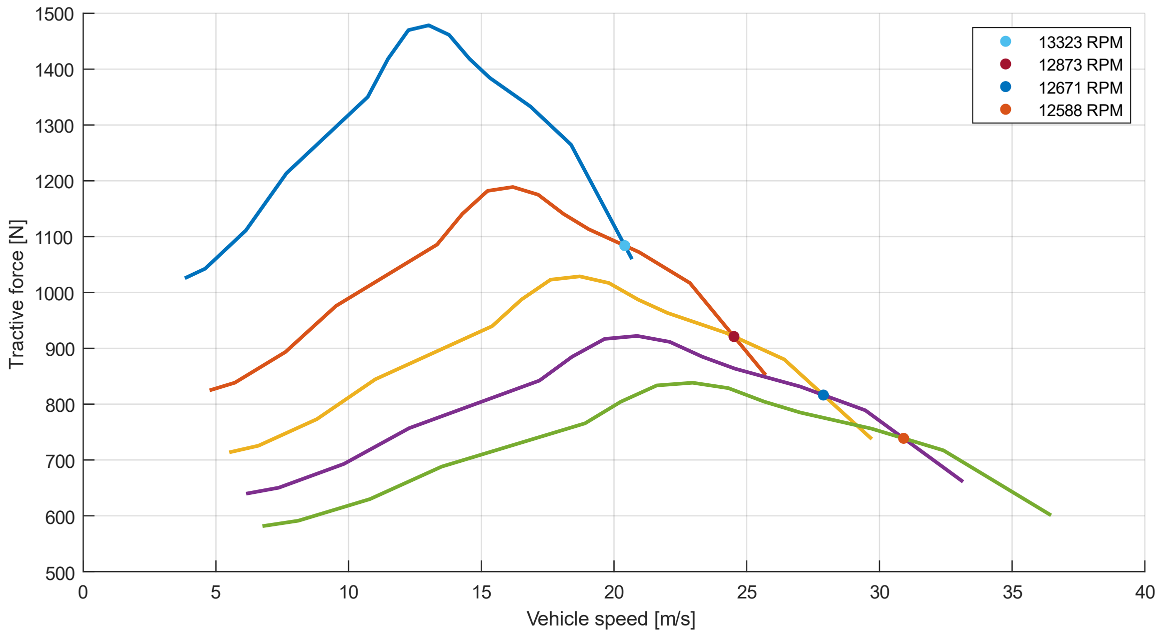

Figure 13: A tractive effort plot showing optimal shift points.

Moreover, in order to describe the acceleration performance of the vehicle model from standstill, the initial stages of a launch sequence need to be accounted for. The general algorithm for calculating the maximum acceleration of the vehicle as a function of speed is explained below:

1. Choose an engine RPM at which the clutch should be released.
2. Release clutch at chosen RPM.
    a. Calculate tractive force.
        i. If the tractive force is greater than the maximum tyre grip allows, assume that the tyre is operating at the grip limit. Hold engine RPM. The rear wheels are now spinning.
3. For a small time-step, do the following.
    a. Calculate tractive force, acceleration, load transfer and aerodynamic forces.
    b. Update vehicle speed. If the rear wheels are still spinning at that vehicle speed, hold RPM. Otherwise, update RPM. Break loop if top speed is reached, otherwise, return to previous step.

For the sake of simplicity, it was assumed that the tyres are operating at their grip limit if the vehicle is traction limited, i.e. if the engine torque is greater than the possible motive torque. The LFS-19 is only traction limited during the initial stages of a launch sequence, and as such, this is a reasonable approximation. It should be noted that the RPM at which the clutch should be released needs to be chosen with care, as to not produce unnecessarily unrealistic results. Below is a plot of the acceleration limit of the vehicle model when generated by the algorithm above.

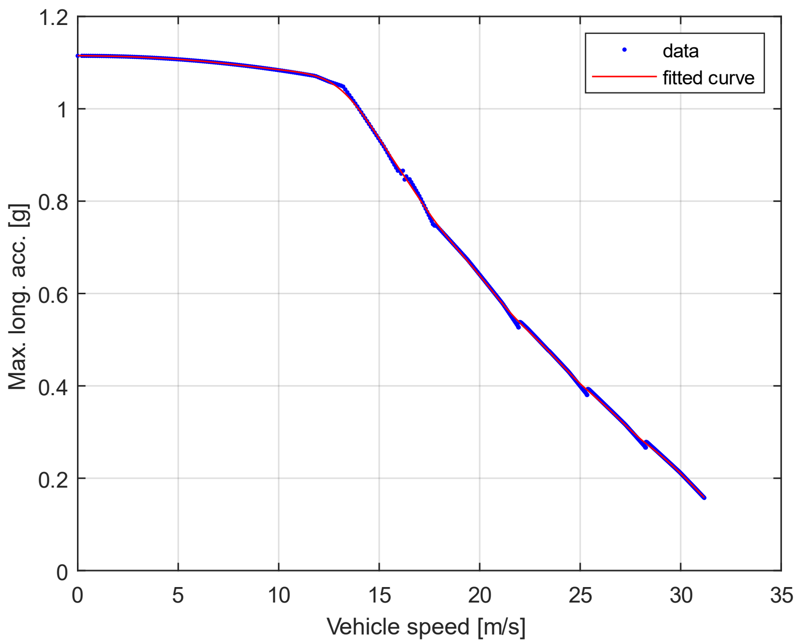

Figure 14: Maximum longitudinal acceleration vs. speed.

4. Quasi-transient lap time simulation

4.1 Introduction

The quasi-transient lap time simulation program calculates the theoretical minimum time it would take for a vehicle to complete a lap around a track given a pre-determined path. This is done in the lateral, longitudinal and yaw degree of freedom by using the limit acceleration surface described in the previous section. The path is split up into small segments of a certain length and curvature. At each segment, it is assumed that the vehicle is undergoing constant acceleration. If the segments are small enough, the results of the simulation closely approximate transient situations. As this approach requires the vehicle path to be calculated beforehand, the program is intended to be used for sensitivity analysis purposes rather than finding the optimal racing line, braking points and so on. In order to carry out the simulation, some assumptions regarding how the vehicle should manoeuvre along the path needs to be made:

- The vehicle brakes into a corner, up until corner apex.
- The vehicle reaches maximum possible lateral acceleration at zero yaw acceleration at the apex.
- The vehicle is accelerating out of the corner, after the apex.

The method used to determine where the vehicle should start to brake and accelerate, and at which rates, is explained in detail in section 4.3.

4.2 Track map

In order to simulate lap times, the path of the vehicle must first be determined. The path will be two-dimensional and is calculated from a set of coordinates in the plane. These coordinates can be generated in numerous ways, for example, they could be derived from logged GPS-data or from a picture of the track layout. As an approximation, the path is split up into small segments with start and end points. For each segment, the curve radius is assumed to be constant and the segment length is known. The curvature, which is defined as the inverse of the curve radius at that point, can be calculated analytically using:

Eq. 16

$$k = \frac{y'x'' - y'x''}{(x'^2 + y'^2)^{\frac{3}{2}}}$$

The equation above requires a continuous and smooth set of coordinates, and therefore, it is often necessary to filter the raw track coordinates to produce good results. Moreover, the curvature profile needs to be continuous and smooth for physical reasons. If the curvature profile is discontinuous or kinked, infinite or non-smooth yaw accelerations would be necessary to travel along the path [4].

When the curvature has been calculated, it is necessary to find the apexes of the path. An apex is the minimum radius of a corner of varying radius. It is calculated by differentiating the path curvature and finding the extreme points. Below are two graphs of a generic Formula Student autocross track.

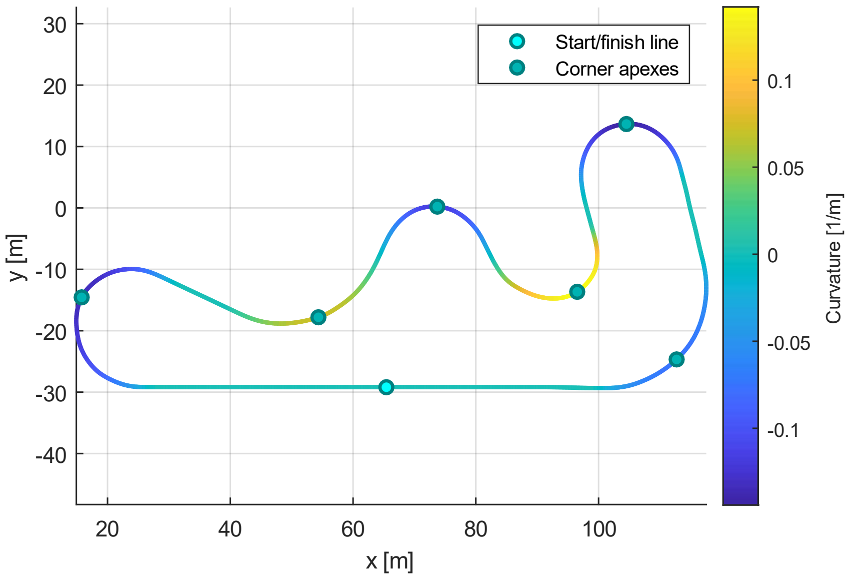

Figure 15: Apex location and curvature of a generic Formula Student autocross track.

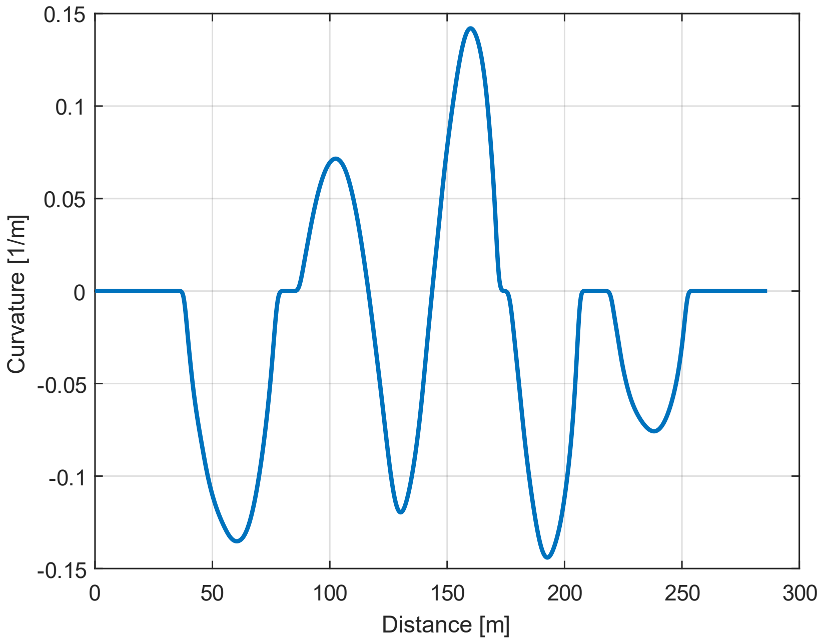

Figure 16: The curvature profile of the generic track.

4.3 General algorithm

Using the assumptions imposed in the introduction, lap time can be simulated. Below, the general algorithm for determining the vehicle's speed profile along the pre-determined path while doing a standing start is explained.

1. At the start of the lap, do a launch.
   a. Traverse through segments *forwards* while accelerating as much as possible.
   b. Break loop if the maximum possible lateral acceleration has been exceeded.
2. Calculate corner entries.
   a. For each apex, calculate maximum lateral acceleration at zero yaw acceleration.
      i. Traverse through segments *rearwards* while decelerating as much as possible (we are looking rearwards in time in order to find the optimal braking point).
      ii. Break loop if the maximum possible lateral acceleration has been exceeded.
3. Calculate corner exits.
   a. For each apex, calculate maximum lateral acceleration at zero yaw acceleration.
      i. Traverse through segments *forwards* while accelerating as much as possible.
      ii. Break loop if the maximum possible lateral acceleration has been exceeded or the end of the lap has been reached.

Each step of the algorithm produces a vehicle speed profile along the path. In order to find the only possible profile combination, the intersections of the different profiles are found and the profile with the lowest speed at that path segment is used to produce the final result.

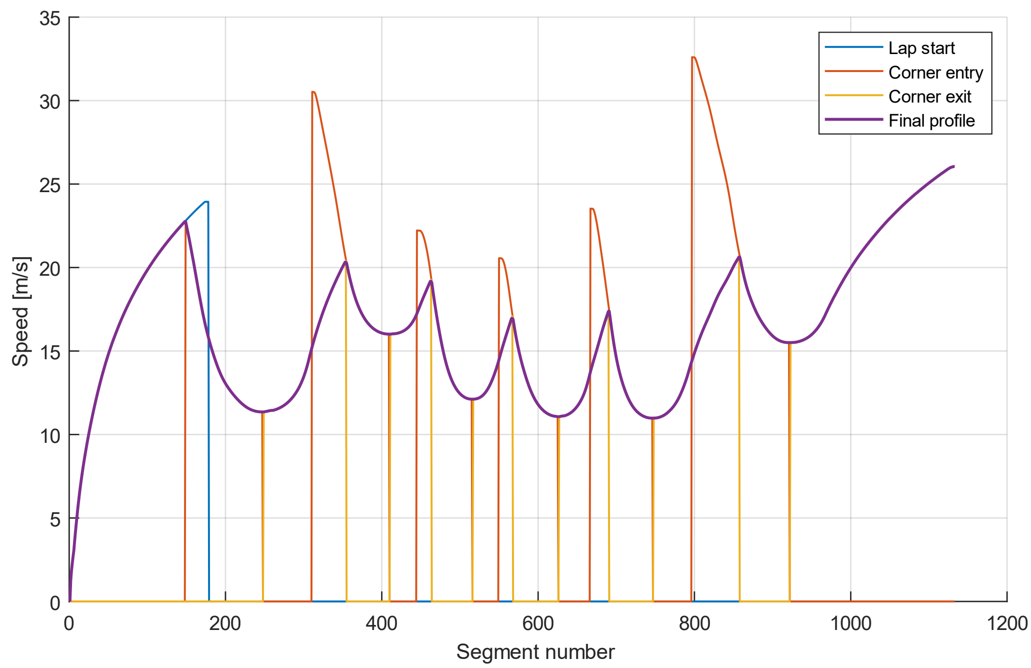

Figure 17: The velocity at the different stages of the lap time simulation algorithm.

4.4 Time convergence

When the program traverses through each segment of the path, it finds the minimum possible time, d_t, it takes for the vehicle to travel a small distance, d_s, and rotate a small angle, d_θ. The time it takes for the vehicle to travel linearly and rotate along the path segment must be equal. At the start of each segment, the vehicle's speed, yaw velocity and lateral acceleration is known. The LAS is sliced at the initial lateral acceleration and all possible longitudinal and yaw acceleration combinations are retrieved. In order to find the optimal combination, the program goes through all points along the slice profile and calculates the time it would take to travel d_s and rotate d_θ separately. It then finds the acceleration combinations that produce equal times and chooses the optimal one. Furthermore, the points on the slice that exceeds the maximum possible longitudinal acceleration of the vehicle at the initial speed are not used.

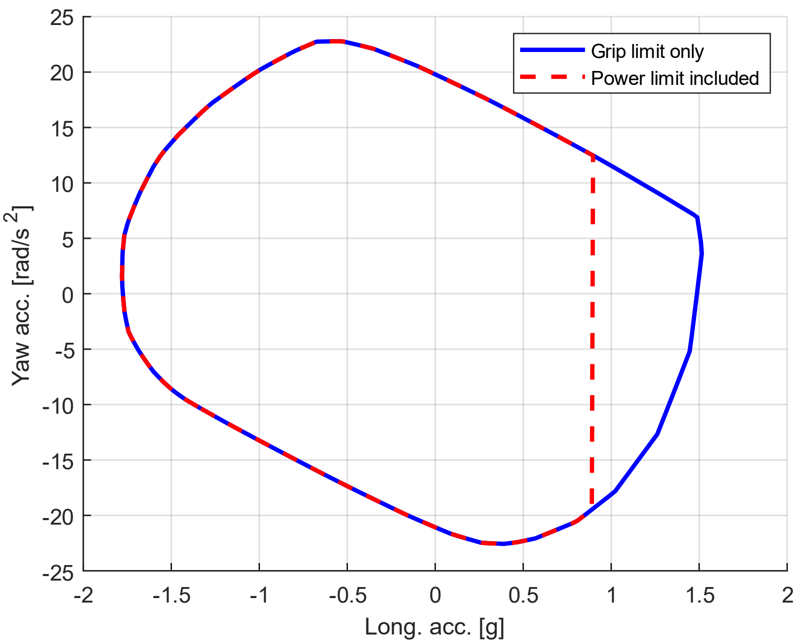

Figure 18: LAS slice at 0.3 *g* lateral acceleration and 15 *m/s* speed.

4.5 Validity and accuracy

As the vehicle model lacks the complexity to be fully realistic, the absolute values of the results will not be correct. However, for the purposes of identifying trends it is complex enough. Below is a list of notable phenomena that effects vehicle dynamics and have not been modelled.

- Wheel angle changes due to suspension movement.
- Wheel angle changes due to compliance effects.
- Wheel angle changes due to varying tyre effective radius.
- CG movement which affects load transfer.
- Tyre temperature and pressure effects.
- Aerodynamic sensitivity to chassis movement.
- Transient effects due to damping.
- Time delay in braking moment build-up.
- Time delay in delivered engine torque.
- Engine braking effects.
- Driveline and wheel inertia.

It should be noted that several of the points above, such as suspension movement, could be implemented relatively easily in future work.

5. Results and discussion

5.1 General results

The purpose of this subsection is mainly to inform the reader of the general capability of the lap time simulation program, no deeper analysis of the results is intended. Figure 19 and 20 shows results at the generic track. As can be seen, the vehicle does a standing start and drives clockwise along the track. As the track is very short, a lap only takes 19.3s to complete. The results can be used for more than just performance analysis purposes. For example, the energy absorbed by the brakes or the fuel consumption over a lap could be derived. Another example would be the possibility of estimating the cooling demand of the engine. In other words, the program could be used as an aid in the development of numerous vehicle subsystems.

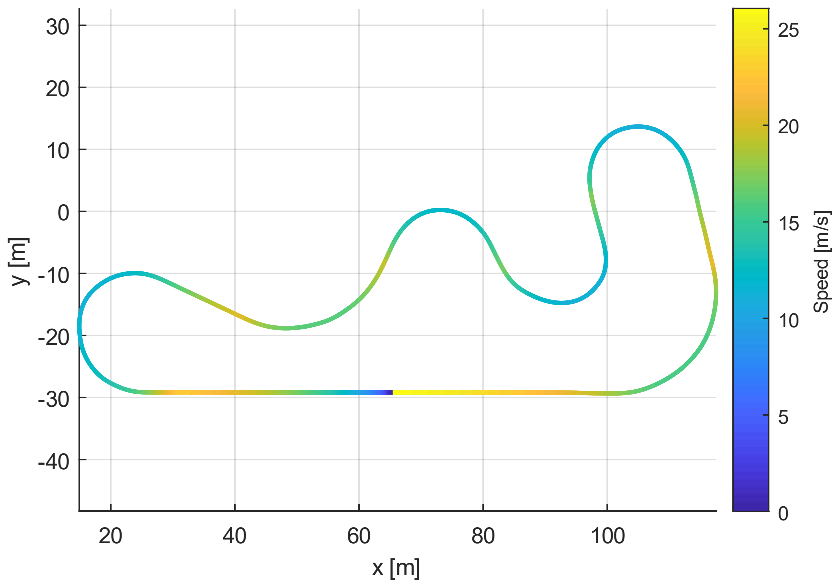

Figure 19: Speed over a lap of the generic track.

5.2 Computation time

The lap time simulation only takes a few seconds to run, however, the generation of an accurate LAS is more computationally expensive and takes about 20 s to compute. The computations have been performed on a laptop equipped with an Intel Core i5-4210 CPU and 8 GB of RAM running a 64-bit version of Windows 7. The generation of the LAS would be less computationally expensive if only the points used to create the surface are generated. Currently, most of the points generated by the algorithm presented in section 3.3 lie within the LAS and are not used. However, most computation time is spent on generating unique points that lie on the mesh elements of the LAS. This increase in resolution is necessary in order to produce accurate slices of the LAS at different lateral accelerations. It should be possible to reduce the computation time by improving the method by which these points are generated.

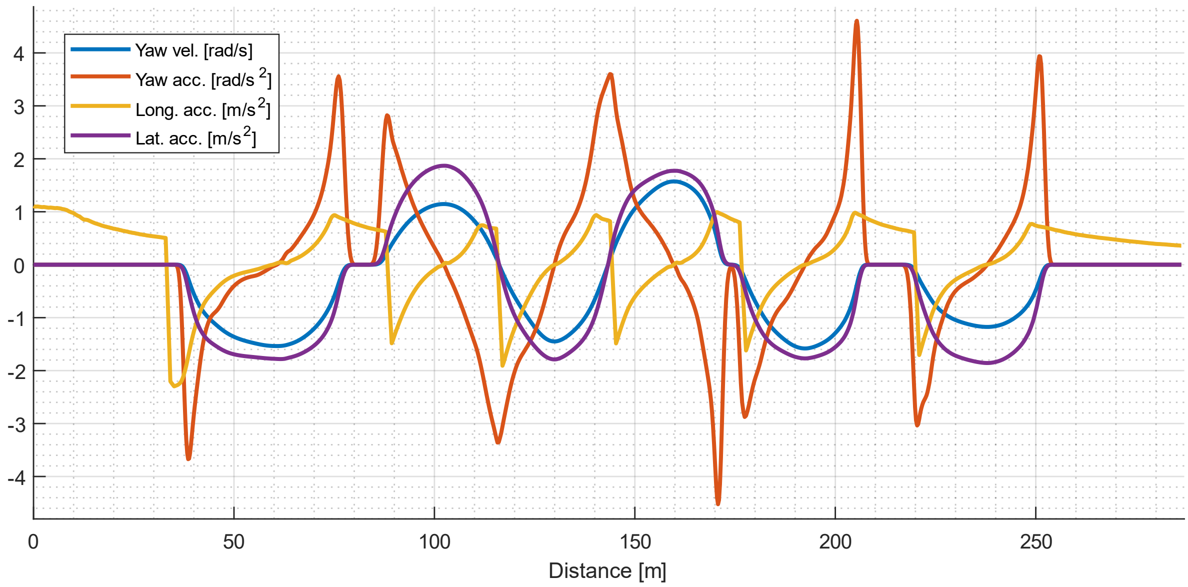

Figure 20: Yaw velocity, yaw acceleration, longitudinal acceleration and lateral acceleration over a lap of the generic track.

5.3 Design study

Below is a table of the results of a design study where key vehicle parameters where swept +- 5% from their nominal value while holding all other parameters constant. The results represent the percentage change in lap time at the generic track presented earlier compared to a baseline vehicle using the nominal values.

Parameter	Nominal value	-5%	+5%
Mass	290 *kg*	-1.04%	1.03%
CG_h	300 *mm*	-0.69%	0.75%
$C_d A$	1.5 m^2	-0.03%	0.04%
Yaw inertia	180 *kg* m^2	-0.01%	0.01%
Front downforce proportion	47.5 %	-0.01%	0.01%
$C_l A$	3 m^2	0.14%	-0.13%
Front weight proportion	47.5 %	0.23%	0.08%
Power	47.5 *kW*	0.49%	-0.44%
FTLLTD	47.5 %	0.49%	-0.13%

Table 1: Design study results.

Interestingly, yaw inertia has a very small effect on lap time. It should, however, be noted that the effect of yaw inertia on lap time will be highly dependent on the curvature profile of the path. At a more transient track, yaw inertia would likely be of more importance, but still less important than most of the other parameters.

5.4 Differential comparison

Holding all parameters but the differential constant, the results show that an active differential produced a 0.93% reduction in lap time compared to an open differential for the baseline vehicle (using the nominal values in table 1). The graph below shows the difference in lateral and yaw acceleration in the second last corner at the generic track. As expected, the torque vectoring capabilities of the active differential enable a better use of the tyre grip potential in corner exit as it is not limited by the grip of the inner rear tyre. This is evident from the increased longitudinal acceleration after corner apex which means that it is possible to increase the torque demand at a faster rate without causing the inner wheel to slip. Moreover, it can produce larger yaw acceleration.

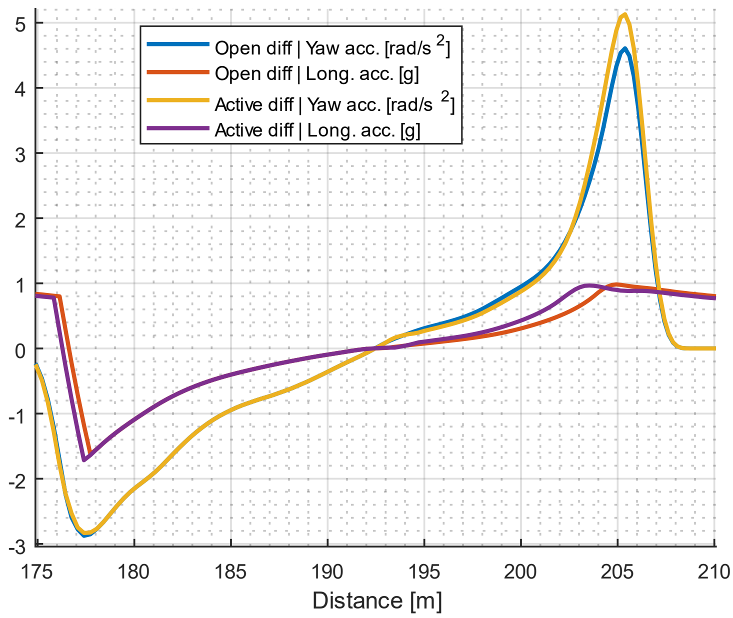

Figure 21: Yaw and longitudinal acceleration for an open and active differential.

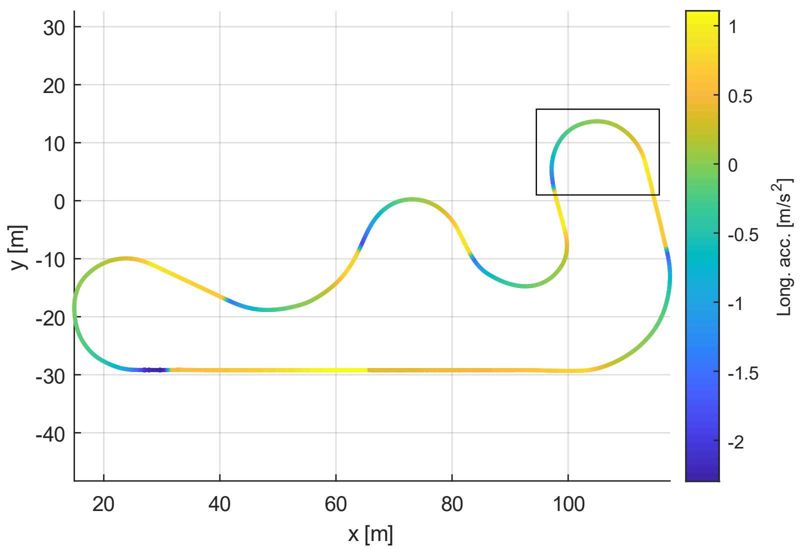

Figure 22: Long. acc. over a lap of the generic track with an open differential. The rectangle shows the corner in fig. 22.

6. Conclusion

It was shown that the limit acceleration surface successfully implements the yaw degree of freedom which enables the study of yaw acceleration and torque vectoring effect on lap time. Furthermore, it was shown that yaw inertia has a very low effect on lap time compared to, for example, vehicle mass and engine power. This implies that, in the case of an open differential, quasi-static and quasi-transient lap time simulation programs will produce almost equal lap times.

As expected, torque vectoring allows for better usage of tyre grip potential and therefore lower lap times. However, the model has not provided deeper insight into the driveability of the vehicle apart from producing absolute values of yaw acceleration. In further work, the LAS and the quasi-transient lap time simulation program could be expanded to include the metrics of stability and control (the derivative of yaw acceleration with regard to sideslip and steering angle). Currently, these metrics cannot be derived as the sideslip and steering angle over a lap are not known.

It was also shown that the lap time simulation program solves quickly but that the generation of the LAS is more computationally expensive. The generation of the LAS could likely be sped up considerably by means of more effective programming.

To conclude, the objective of the thesis has been met but there is room for improvement in future work.

7. References

[1] W.F. Milliken and D.L. Milliken, *Race Car Vehicle Dynamics*, UK: Society of Automotive Engineers, 1995.

[2] H.B. Pacejka, *Tire and Vehicle Dynamics*, 3rd ed. Oxford, UK: Society of Automotive Engineers, 2012.

[3] E.M. Kasprzak and D. Gentz, "The Formula SAE Tire Test Consortium-Tire Testing and Data Handling," Society of Automotive Engineers, SAE Paper No. 2006-01-3606, 2006.

[4] C. Patton, "Development of Vehicle Dynamics Tools for Motorsports", Oregon State University, PHD Dissertation 2013.