LUND UNIVERSITY
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Cellular automata analysis of life as a complex physical system

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Abstract

This thesis regards the study of cellular automata, with an outlook on biological systems. Cellular automata are non-linear discrete mathematical models that are based on simple rules defining the evolution of a cell, depending on its neighborhood. Cellular automata show surprisingly complex behavior, hence these models are used to simulate complex systems where spatial extension and non-linear relations are important, such as in a system of living organisms. In this thesis, the scale invariance of cellular automata is studied in relation to the physical concept of self-organized criticality. The obtained power laws are then used to calculate the entropy of such systems and thereby demonstrate their tendency to increase the entropy, with the intention to increase the number of possible future available states. This finding is in agreement with a new definition of entropic relations called causal entropy.

List of Abbreviations

CA cellular automata.

GoL Game of Life.

SOC self-organized criticality.

Contents

1 Introduction

The concept of cellular automata (singular, automaton) dates back to the 1950s, but it did not gain its widespread popularity until 20 years later. The sudden increase in interest, undeniably arose in association with the discovery of periodic behavior of certain configurations, in a specific cellular automaton [1].

A possible connection between cellular automata and life[1] is supported by the fact that life is a biological system in which spatial extensions are commonly involved to some extent, e.g. through population distribution. Furthermore, in biological systems, spatial extensions would experience nonlinear local interactions between its different components. In turn, these are likely to stimulate natural pattern formations (i.e. dynamic and static spatial patterns undergoing self-organization). This kind of self-organization plays an essential role in both the functionality and structure of biological systems [2], and it is also well described by cellular automata.

1.1 Cellular Automata and The Game of Life

The concept of cellular automata (CA) can be described as a lattice system, in which the state of each lattice point is determined by local rules. Particularly, CA is a discrete computational model that has been demonstrated to be feasible for modeling a variety of complex systems, for instance, in physics and biology (such as the dynamical evolution of a society of living organisms) [3, 4]. The most popular is a two-dimensional CA known as The Game of Life (GoL), introduced by mathemati-

[1]The term "life" here, in a biological sense, refers to the condition that distinguishes animals, plants, and their composite, from inorganic matter. The term, in general, refers to an entity having the ability to undergo e.g. reproduction and growth. In this thesis, "life" may further be seen as self-organizing autonomous entities, that is, including artificial and biological life.

cian John Conway [5]. To further understand the concept of CA, it is convenient to explain and consider a specific CA; due to the popularity of GoL, this is described below.

Consider a square lattice system of indefinite size, where each lattice site can attain one of two possible states, either 'dead' or 'alive' (as in the absence or presence of an alive individual); for clarification see Figure 1 below. The definite state of each lattice site is determined by a number of predefined sets of local rules. In GoL, the local rules are defined in terms of the state of the nearest neighbors[2] for each lattice site. The rules are defined as follows: (i) A live site will 'die' if it has less than two live neighbors (as if by 'solitude'). (ii) A live site with two or three live neighbors will live on (i.e. 'survive' to the next generation). (iii) A live site with more than three live neighbors will die (overpopulation). (iv) A dead site with exactly three live neighbors will become 'born'.

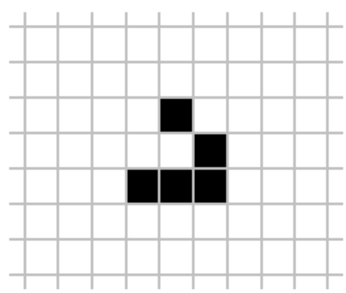


Figure 1: Illustration of the square lattice system in GoL with a "glider"-pattern. Each black lattice site represents a live site, while white represents dead sites. Note: The specific pattern shown here is called a "glider" due to its distinct propagation properties over time.

[2]Considering a square lattice, a lattice site's "nearest neighbors" are the eight surrounding squares which it is in direct contact with, i.e. not what is normally referred to as lattice distance.

By starting with an initial pattern of live and dead sites, and by considering each of the four rules above, timesteps can be taken in order to perform updates of the system, see illustrations of the process in Figure 2 below.

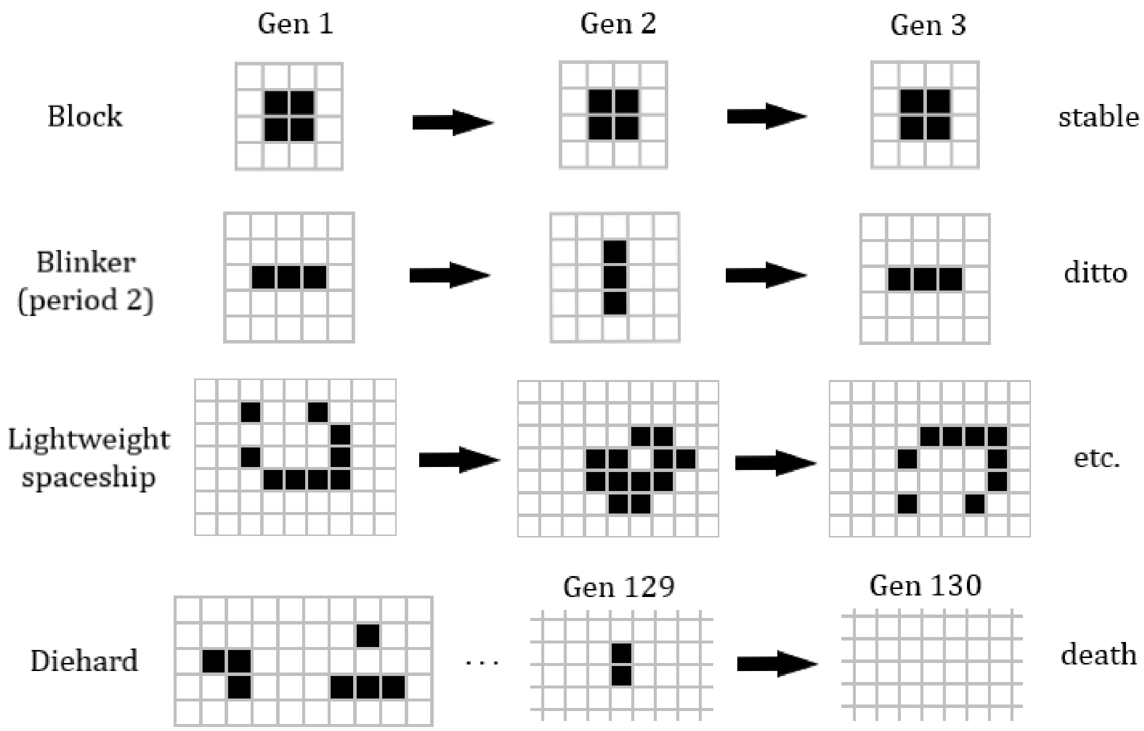


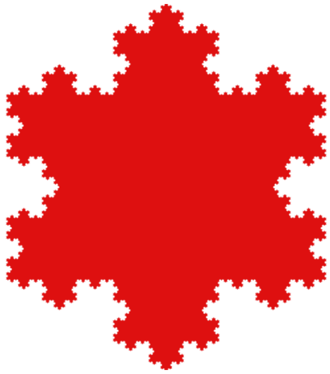
Figure 2: Illustration of four differently named initial patterns and their generational evolution. Each pattern here belongs in a certain category; from top: Still life, Oscillator, Spaceship, and Methuselah (patterns that become stable after many timesteps) [6]. Each arrow represents one timestep and the three dots indicate that several timesteps have been taken. Note: The absence of borderlines in Gen 129 and 130, of the Diehard-pattern, indicates that the configuration is located in a different area of the grid, relative to the initial pattern.

For a better understanding of the concept of CA and the evolution process of specific configurations, the reader is further encouraged to visit an online-based CA simulator[3] to try own patterns and study their evolution. At this point, it is also worth mentioning that GoL does not represent any particular system of a real model, rather it is supposed to be seen as a concept demonstrating the possibility of emerging large-scale structures in complicated extended dynamical systems [4].

1.1.1 Fractals in Nature and CA

The essence of fractals is that the observed system sustains the same characteristics on all scales, thus a major magnification of such a system should be comparable (or identical) to the initial (non-magnified) system. This property is known as self-similarity and is the characterization that differs fractals from Euclidean shapes [7, 8]. Examples of fractal patterns can be seen below in Figures 3 and 4, the figures depicts artificially constructed and naturally occurring fractals; by studying these images closer, it can be seen that all of the images illustrate the self-similarity characterization. The existence of naturally occurring fractal geometry, similar to those simulated by computers, is important evidence for the significant role which fractals play in nature [7].

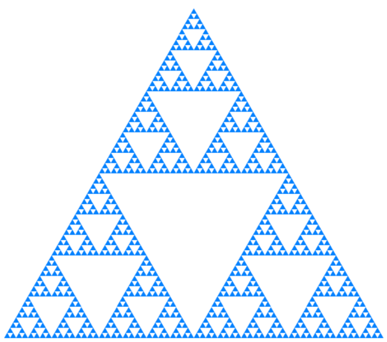
[3]Considering GoL, see for example `https://playgameoflife.com/` [Retrieved 18th of February 2020]



(a)

(b)

Figure 3: Images of fractal patterns[4], (a) known as the von Koch Snowflake and (b) Romanesco broccoli.



(a)

(b)

Figure 4: Images of fractal patterns, (a) known as Sierpinski Gasket[4], and (b) a shell of a snail[5].

It has previously been described that a CA is constructed upon a simple set of rules and, despite this, CA can generate varied and complex behavior [9]. For

[4]Images from https://mathigon.org/world/Fractals Accessed: 2020-02-18

[5]Image from http://www.gmilburn.ca/2009/07/16/i-see-sierpinski-shapes-by-the-sea-shore/ Accessed: 2020-02-18

example, it has long been known that fractals can generate in CA [10] and it has also been found in many different forms, in a variety of CA models [11, 12].

Consider a simple one-dimensional CA, which is using a sequence of sites with two possible states (similar to GoL's 'dead' and 'alive'). The state of each site, after every timestep, is deterministic and defined by a set of rules, which considers the state of the nearest neighboring sites. By considering the specific set of rules known as *Rule 90*[6], and giving the system a random initial configuration, the CA will generate self-similar patterns as illustrated in Figure 5 below [13, 14]. The pattern is familiar from Figure 4, i.e. this set of rules will generate the Sierpinski Gasket fractal.

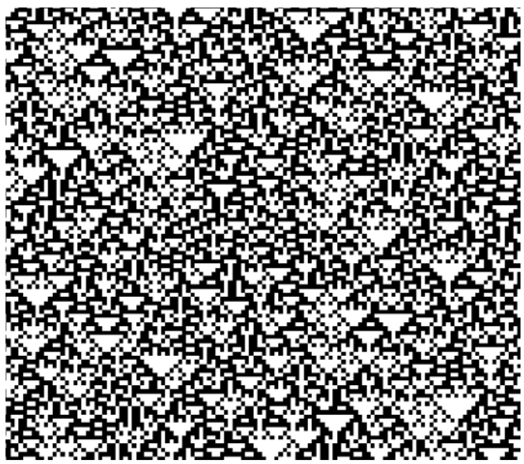


Figure 5: Illustration[7] of the outcome of Rule 90. Note: The used CA is one-dimensional but the illustration of the result is two-dimensional, this is the case since the generations are 'stacked' on each other (similarly, GoL could be displayed in three-dimensions, considering it as a simulation in time).

[6]See explicit explanation at `https://mathworld.wolfram.com/Rule90.html` Accessed: 2020-06-05

[7]Illustration from `https://www.johndcook.com/blog/2017/09/23/sierpinski-triangle-strikes-again/` Accessed: 2020-02-18

1.2 Self-organized Criticality

Self-organizing systems are commonly associated with life-like behavior and are convenient to consider in discussions regarding life [15]. In this thesis, self-organizing systems are regarded in the context of self-organized criticality (SOC), a mechanism that has been proposed to exist in nature and relates to systems' natural evolution into a self-organized critical structure, of a system state that could be perturbed by a single perturbation. It has further been advocated as the underlying mechanism of naturally occurring fractals, and the contingency in nature has been considered to be an indication of its existence [4, 16].

In the context of GoL, it has been numerically shown that local configurations can self-organize into a critical state. Considering SOC, a general mechanism for the emergence of scale-free structures is provided and, consequently, possibly allows for pioneering within the understanding of biological systems. This realization strengthens the importance of SOC in biological systems since there are no locally preserved quantities in these systems [4]. Furthermore, in a large class of nonequilibrium systems, SOC could explain the dynamic origin of nontrivial scaling. The concept of nontrivial scaling relates to self-similarity, which is possible to demonstrate in systems with well-defined individual events [17]. A remark regarding the general concept of 'self-organized' is put in the following way by Bak et. al (1989) [4, p. 780]:

> The idea of 'self-organized' is that it is in the nature of nonlinear processes to organize mathematical systems into structures that have order on all length scales. If this tendency is generally present in such mathematical systems, then we would also expect the natural world to contain structures on all scales.

The argument emphasizes that it is likely for SOC (or self-organization) to apply to natural systems, e.g. biological. The reasoning is further supported by the fact that studies on advanced mathematical models of biological systems, in particular co-evolution[8], have suggested that biology operates at a self-organized critical state [4].

Besides the many indications pointing towards an existing connection between SOC and life, one main counter-argument is that there has been extensive debate on whether a conservation law for SOC is a necessity. As previously mentioned, there are no locally preserved quantities in a biological system, thus the debate is directly connected to the discussion of the possibly existing connection between SOC and biological systems. Furthermore, all SOC models demand an explicit introduction of the system size in their definition; the system size has also been shown to have a vital effect on the behavior of the observed scaling. This size restriction will cut off large-scale features, and thus hinders the generality of SOC [4, 17].

1.3 The Role of Entropy

The word 'entropy' is referring to the concept of 'disorder', that is, entropy is a measurement of the lack of knowledge or information, about a certain process or system. Furthermore, entropy is one of the most important quantities in physics, and it has been proven to be useful when describing the long term behavior of chaotic and random processes. [18, 19, 20].

In thermodynamics, a process is said to be spontaneous if it, in an arbitrary time, will occur without external influence from the outside of the considered system [21]. Coupled to this, is the second law of thermodynamics, which allows the outcome of a process or system to be predicted. Specifically, the law states that:

[8]The term 'co-evolution' refers to the process used to describe how two or more species reciprocally affect each other's evolution, due to interactions between the species.

> *During any spontaneous process, the total entropy of a system and its surroundings must increase in time.* [22]

To concertize the concept of entropy, it can be described as a measure of the number of ways that some energy can be distributed in a system of molecules. In order to do this, it is necessary to consider the microstates, Ω, available to these molecules. In general, microstates are the instantaneous microscopic configuration of a thermodynamical system, for example, the position of atoms in a gas, the energy of each molecule in a system, or the individual elements in a sequence of numbers. On the other hand, macrostates represent the global thermodynamical properties of the system, therefore, e.g. the sum of all elements in the sequence of numbers would be a macroscopic view of the sequence. [23, 24]

Since a microstate gives information about the individual conditions of the system components, e.g. molecules, it is expected that the entropy somehow will be proportional to the number of available microstates of the system. In fact, the entropy is commonly defined as,

$$H = k_B \ln(\Omega), \tag{1}$$

where k_B is the Boltzmann constant [25]. However, note that the entropy is commonly denoted by S, but in order to avoid confusion with the (later defined) cluster size variable, S, the entropy is instead referred to as H in this thesis.

Furthermore, calculations of microstates in real systems easily become computationally heavy or very challenging to perform. Therefore, generally speaking, the entropy will often be calculated in terms of measurable macroscopic quantities. [25, 26]

The role of entropy in this thesis is further discussed in Method-section 2.3, The Causal Entropic Force.

1.3.1 Causal Entropic Forces

To explain causal entropic forces, it is here divided into the concepts of *entropic forces* and *causal entropy*. The concept of *entropic forces* has emerged from thermodynamic systems' tendency to maximize their entropy production. The array of macroscopic variables describing the system tends to evolve from a state to one with higher statistical probability. The entropic force will thus increase simultaneously with an increasing vigorous thermal motion [27, 28]. The systems' evolution, have been observed to appear as being modulated by some kind of force (the entropic force). A convenient analog is the popular paradigm of polymers contracting in a warm bath [29]. A more explicit definition of the entropic force can be seen in Section 2.3, along with its mathematical definition in Eq. (8), also in Section 2.3. *Causal entropy* can be defined "on the set of possible future evolutions of a system within a finite time" [30, p. 2]. Therefore, considering this definition, causal entropy is a quantity connected to a systems' diversity of future available options. In order to maximize the diversity of options, to make the system attain non-restricted states, one talks about causal entropy maximization [30].

1.3.2 Entropy Maximization and Intelligence

Several scholars have for more than 100 years been observing the connection between intelligence and entropy [31, 32]. In a recent study by Wissner-Gross and Freer in 2013 [31], a first step was taken in quantitatively formulating such a connection using a causal generalization of entropic forces. These forces were shown by Wissner-Gross and Freer to "spontaneously induce remarkably sophisticated behaviors", in simple physical systems. Regarding these behaviors, the authors make comparisons to human characteristics, such as social cooperation between individuals and tool use.

Furthermore, they argue that their results indicate a "potentially general thermodynamic model of adaptive behavior as a nonequilibrium process in open systems".

The importance of causal entropic forces, in the context of biology, is emphasized by the essence that the understanding of concepts within nonequilibrium physical systems, provides insight in numerous physical principles in a variety of biological systems (e.g. evolution, ecosystems and cellular dynamics in general) [33]. Furthermore, in the article from 2019 where Fang, et al. [33] investigates nonequilibrium physics in biological systems, the authors stress that apart from their findings, they expect several fundamental insights related to the phenomenon of life to emerge in the future. This reasoning is directly related to causal entropy since the preferred feature of maximum instantaneous entropy production for nonequilibrium physical systems [34] follows its evolution towards a higher-entropy macroscopic state [35]. However, despite the speculative connection between entropy maximization and intelligence, there has not yet been a formal formulation of any physical connection, thus further studies are needed to establish the nature of this relationship [36].

2 Method

2.1 Self-organized Criticality in CA

In the study of cellular automata behavior, most of the results are presented using the distribution of a measured quantity or feature. This distribution is defined by

$$D(x) = \frac{\text{occurrence of property } x}{\text{total number of events}} \tag{2}$$

Eq. (2) is used for $x = S$, A, t and r, where S is the cluster size, A is the total momentary activity (number of state changes in the considered generation), t is the time (i.e. the number of generations) it takes for a cluster to become static after a perturbation was made, and r is the distance from the initial perturbation to a specific site, z. Moreover, r is defined by

$$r = \sqrt{(x_p - x_z)^2 + (y_p - y_z)^2}, \tag{3}$$

with index p referring to the perturbed site, and index z referring to the considered active site.

Perturbations are here defined as the addition of an 'alive' cell in the system, i.e. one of the dead cells changes its state. If a perturbation is random, it implies that it is added at a random location in the system. This random placement of the perturbation may sometimes cause the system to be unaffected by the perturbation, in this case, the random placement of the perturbation is remade until it interacts with some other cells, i.e. causes the system to leave its static state.

In the measurements of activity in a single site, a, one site is regarded as active ($a = 1$) if it changed its state from e.g. dead to alive, the activity over time is thus measured by counting how many changes the site experienced within a given time (i.e. over a number of generations). The total momentary activity A, of the system in each generation, represents the sum of the instant changes due to the last time step, thus it is simply a sum of zeroes and ones, i.e.

$$A = \sum_{n=1}^{N} a_n \quad , \tag{4}$$

where subscript n represents each of the N different sites in the system.

Mathematically, the overall activity in one site, a_g, is defined as

$$a_g = \begin{cases} a_{g-1} + 1 & \text{if the state changed} \\ a_{g-1} & \text{if the state was unchanged} \end{cases} \tag{5}$$

and the total activity of the system, A_g, is given by

$$A_g = \sum_{n=1}^{N} (a_g)_n \quad , \tag{6}$$

where index g represents the generation and $n \in [1, N]$ represents each individual site of the N different sites in the system.

2.2 Power Laws

A power law is described by the following equation

$$y = kx^{\alpha}, \tag{7}$$

where y and x are the variables and k and α are constants.

Features that follow power laws can be seen in a variety of systems, both in nature-driven systems such as physics and geography, but also in man-made systems like economics. Systems with power law behavior are also commonly considered as scale-free systems when $\alpha < -1$ in Eq. (7). For example, many social systems are weakly scale-free, while many biological systems appear strongly scale-free. A famous example of the essence of power laws and scale-freeness is the Pareto law (also known as the 80/20-principle), this principle simply states the fact that e.g. roughly 20% of the citizens in a city possess 80% of the total wealth. The scale-freeness of this relation is apparent since the same relation holds for every scale of the system; instead of a city, it would be possible to consider a country, and yet find the same relation. [37, 38]

The occurrence of power laws in biological systems appear to be very common, it has for example been found that power laws could identify real-life tree crown construction processes in dynamic models that make use of structural features of the trees' crowns. Furthermore, scale-freeness has been found in complex biological systems such as protein-, metabolic-, and gene-interaction (considering the distribution of the number of connections of the network nodes) [39, 40, 41]. Moreover, in a study of power law distributions by Khanin R. and Wit E. [40], all of the 10 studied datasets (corresponding to different biological interactions) were found to be coupled to a power law distribution, to some extent. However, it has been emphasized by Brodio A. D. and Clauset A. [37], that most scale-free structures do vary depending on the chosen network domain.

2.3 The Causal Entropic Force

In a nonequilibrium physical system, the tendency to have a maximized instantaneous entropy production can be coupled to the systems' evolution toward higher-entropy macroscopic states. This process is characterized by the formalism of entropic forces $\mathbf{F}$, (associated with a macro-state partition, $\{\mathbf{X}\}$) of a canonical ensemble, which is described by

$$\mathbf{F}(\mathbf{X_0}) = T\boldsymbol{\nabla}_{\mathbf{X}} H(\mathbf{X})|_{\mathbf{X_0}}, \tag{8}$$

where T is the reservoir temperature, $H(\mathbf{X})$ is the entropy associated with macro-state $\mathbf{X}$, and $\mathbf{X_0}$ is the present macro-state. Specifically, this relates to partitioning the system according to micro or macro statistical ensembles [31]. Explicitly, the microstates here represent stable clusters, implying that the ensemble of the microstates is the set of all possible stable clusters. The macrostates are represented by the collective CA state-variables, i.e. $D(S)$, $D(A)$, $D(t)$, $D(r)$ and the entropy.

A natural generalization of Eq. (8), with the aim of uniformly maximizing the entropy production (with respect to the time between a future and present time), could mean that the generalized entropic forces over the configuration space, instead would go over paths *through* the configuration space. Microstates would be possible to develop from instantaneous configurations to fixed-duration paths through configuration space. This as continuously partitioning these microstates into macrostates, considering the initial coordinate of each path. For any open thermodynamic system, phase-space paths taken by the system $\mathbf{x}(t)$, over some time interval, can be treated as microstates. By partitioning these into macrostates $\{\mathbf{X}\}$, using that $\mathbf{x}(t) \sim \mathbf{x}'(t) \iff \mathbf{x}(0) = \mathbf{x}'(0)$, every macrostate $\mathbf{X}$ is identified by a unique system state $\mathbf{x}(0)$. Causal path entropy H_c, of a macrostate $\mathbf{X}$ with system state $\mathbf{x}(0)$, can then be defined as a path-integral

$$H_c(\mathbf{X}, t) = -k_B \int_{x(t)} \Pr(\mathbf{x}(t)|\mathbf{x}(0)) \ln \Pr(\mathbf{x}(t)|\mathbf{x}(0)) \mathcal{D}\mathbf{x}(t), \tag{9}$$

where Pr is the conditional probability that a state evolves from an initial state to another, which can be considered as the number of states in the system. [31]

Generally, the entropy is given by

$$H = \sum_{i}^{M} p_i \log p_i, \tag{10}$$

where p_i is the probability for each of the M possible outcomes. However, note that the minus sign normally seen in front of the summation is omitted, this is solely for illustrative purpose, with the intention to emphasize the later observed behaviour of the entropy, see Figure 10. Furthermore, since the metric p does not exist in the obvious sense in this case, the number of clusters, and their sizes can be used as a proxy. Thus, the instant entropy, H, of the CA system can be calculated by

$$H \approx \sum_{i}^{N} {S_i}^{\alpha} \log({S_i}^{\alpha}), \tag{11}$$

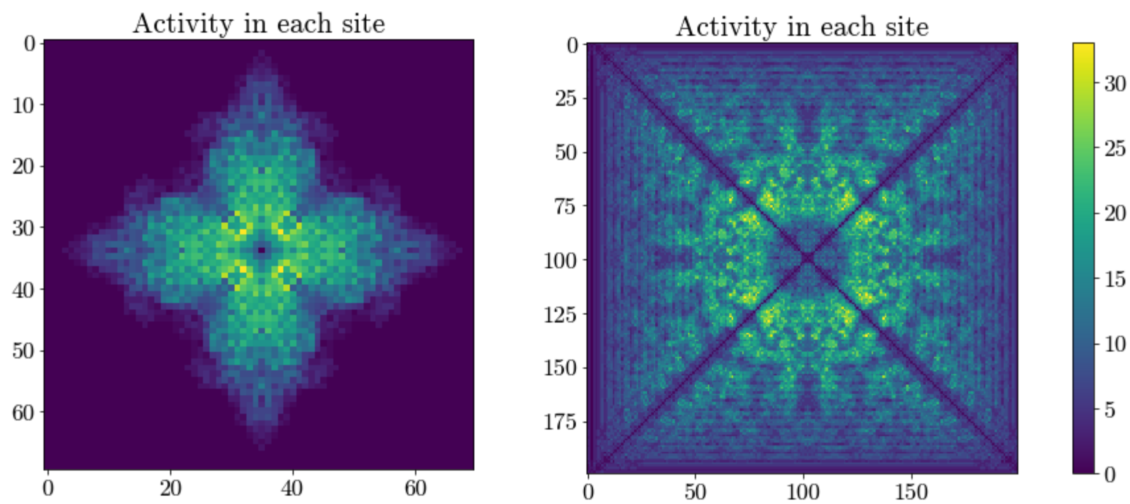
where S_i is the size of cluster i, N is the total number of clusters in the system at a specific time, and α is the exponent from Eq. (7). The addition of the α-exponent is motivated by results from simulations of the cluster distribution, $D(S)$, (see below in Section 3.1). In other words, the number of available states in the CA system is here interpreted by using the number of clusters and their respective size.

The results from the $D(S)$ measurements indicate that, for larger clusters, the size distribution is less likely to be described by a given power law. Therefore, for the specific set of rules, the same power law as found for $D(S)$ (see for example Figure 9 (a) in the Results section) is needed to calculate the entropy properly. However, the scaling factor in the power law for $D(S)$ can be omitted since the general trend of the entropy plot would be independent of this factor.

When investigating the behavior of the entropic force in the CA, Eq. (11) is used to measure the entropy, which according to Eq. (8) is directly related to the entropic force. In order to establish how the entropy behaves, the entropy (H_0) of the first stable state of a randomly initiated system, is plotted against the difference $H_1 - H_0$, where H_1 is the entropy of the stabilized system after the addition of a randomly placed perturbation. The main objective of this measurement is to investigate a possible trend of increase in such a plot. In case of an observed entropy increase after a perturbation, it has been shown that an entropic force is present in the CA; otherwise, it just yields knowledge in how CA behaves from an entropic aspect.

3 Results

In this thesis, we want to study how to quantify the emergence of complexity in CA for given rules. An important parameter in this study is the *activity* (A), which is a measurement of how much change there is between two generations (i.e. the change caused by the deaths and births of cells due to one update of the system). The activity thus measures how active the system is. Considering the first update (Gen 1 to Gen 2) of the top three configurations in Figure 2: the Block has $A = 0$, the Blinker has $A = 2$ and the Lightweight spaceship has $A = 9$. In the study of a system's evolution, the activity over time is an important property to consider, it tells how the system evolves and what parts of the system that are more active than others. The activity over time is here studied in two different ways. Below in Figure 6, the activity is illustrated using a method analogous to heat distribution. The maps are color-coded, where the more vibrant colors correspond to high activity, as seen on the color scale to the right of the activity maps.



(a) (b)

Figure 6: Color-coded mapping of the activity in each site in a 70×70 grid (a) and a 200×200 grid (b), both with periodic boundaries. The two different systems ran for (a) 120 and (b) 100 generations. The more vibrant colors correspond to a higher activity, for example, the sites with the most vibrant color in (b) have undergone a total of about 30 changes during its 100 generations.

For Figure 6 (a), birth occurred when a cell had 3 or 6 live neighbors, and a cell experienced death if it had 0, 1, 4, 7 or 8 live neighbors. The initial configuration of live cells was a 3×3 square, with one dead cell in the middle; as seen in the figure, the configuration results in a symmetric activity map. The rules used to obtain Figure 6 (b) are essentially based on the rules of GoL, however, it is here favorable to have 8 live neighbors (i.e. cells fulfilling this condition will remain static). These rules, in combination with a grid of only live cells, except one dead in the middle of the grid, results in the activity evolution seen in Figure 6 (b).

One other way to study the activity of a system is to measure the activity in each generation, this is illustrated below in Figure 7.

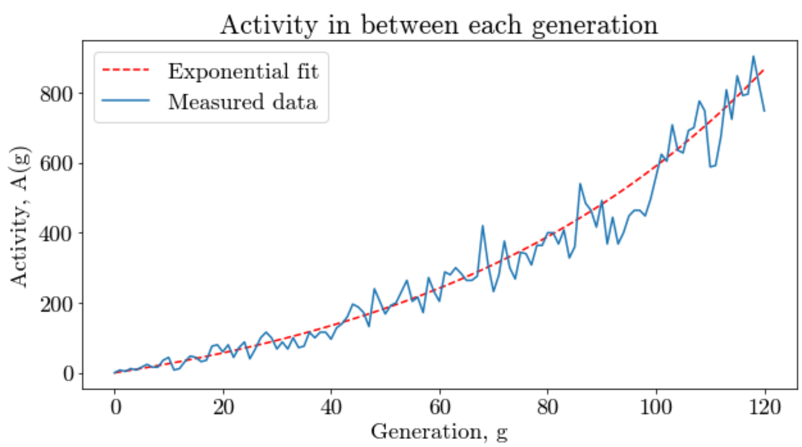


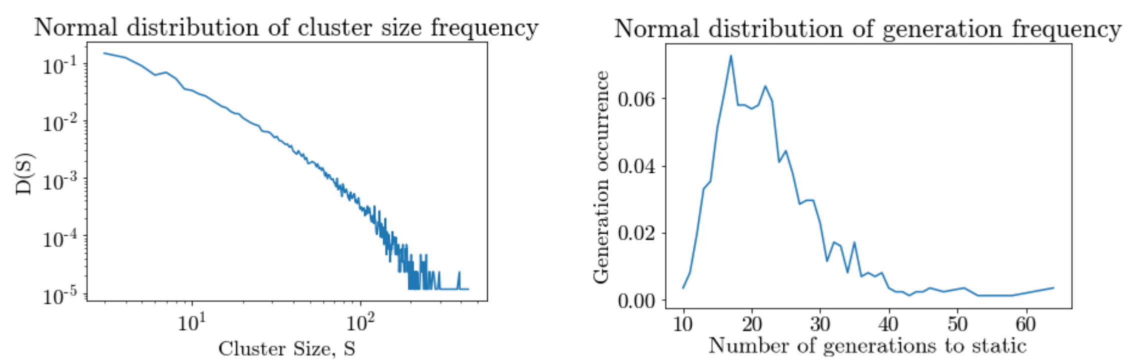
Figure 7: A plot of the activity between each generation. The plot corresponds to the same system and data as Figure 6 (a) and thus have the same rules and properties. The activity is observed to undergo an exponential increase, which is quantified by the function $A(g) = (1.52 \cdot 1.016^g - 1.52)10^2$

By illustrating the activity in time, as in Figure 7, it can be seen if the system becomes more or less active in time, the latter would cause the system to eventually reach a static state (i.e. an activity of zero). The corresponding $A(g)$-plot for Figure 6 (b), was also observed to be exponentially increasing, this time by the function $A(g) = (6.95 \cdot 1.01^g - 7.51)10^3$. Other CA like GoL and the one to be described in Section 3.1, undergo exponentially decreasing activity, see Appendix A.1.

3.1 Self-organized Criticality

From the discussion about fractals in Section 1.1.1, the discussion about SOC in Section 1.2, and the deep dive into power laws in Section 2.2, it can be concluded that, if power law behavior is seen in specific features of a studied CA, the system is scale-free and will self-organize into a critical state. The features of interest, are the distribution of cluster size $D(S)$, the time until the perturbed system comes to rest $D(t)$, the activity caused by random perturbations $D(A)$, and the distance from the initially perturbed site to all affected sites $D(r)$.

The following figures correspond to data gathered from a CA system based on the following set of rules: A cell will die if it has 0, 1, 7 or 8 live neighbors, a cell will be born if it has exactly 4 live neighbors, and for each other case a cell will remain unchanged. Moreover, periodic boundaries were used along with an initial density of 20% of randomly distributed live cells, and a general feature for this specific set of rules was that a variety of stable clusters were possible, making it favorable for the measurement of stable cluster distribution. The data was obtained by studying the clusters created by 1,000 random initial configurations, all with a density of 20% live cells. In Figure 8 below, the first studied features of the above stated set of rules are displayed.

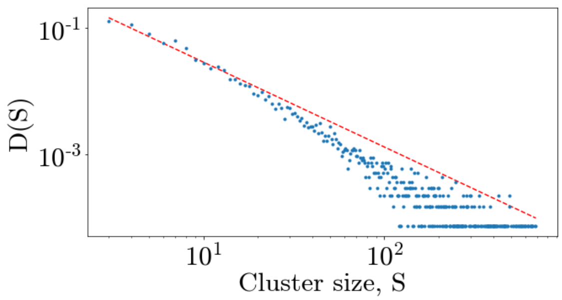


(a) (b)

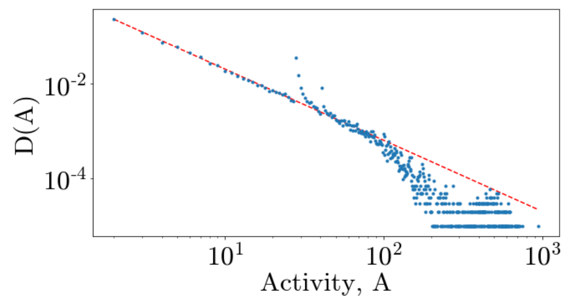
Figure 8: (a) Cluster size distribution in a log-log scale and (b) distribution of generations needed to reach a static state after system-initialization, both in a 100×100 grid.

The plot in Figure 8 (b), depicts the number of generations that were needed for the system to become static after initialization, i.e. this plot gives a good idea of how the specific CA evolves in time. It can be seen that most systems, initiated with the density of 20% live cells, will come to rest after roughly 20 generations. This characteristic of the system implies that the studied CA is suitable for further data collection; the rather low lifetime means that the data-gathering will be fast. If the maximum of the plot was located at a larger generation, the used program (see Reference [42]) would have to run for a long period of time to obtain a lot of data.

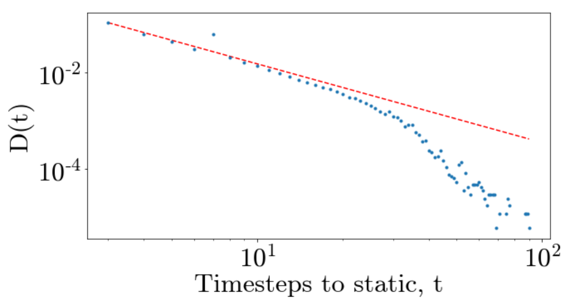
Below in Figure 9, all the previously mentioned distributions are displayed in log-log plots. To each of the obtained plots, a power law has been fitted according to Eq. (7), to approximately describe the behavior of the observed data. The simulated CA is based on the same rules that yielded the results in Figure 8, however, here a density of 30% live cells was used in the random initial configurations. Moreover, the distributions are calculated using Eq. (2), and the data in Figure 9 (b) - (d) are from randomly perturbed static systems. More results from the same CA, but with a density of 20% live cells in the random initial configurations, can be seen in Appendix A.2.



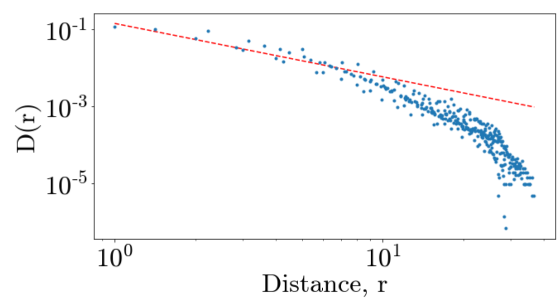
(a) Data corresponding to 12,483 studied clusters, 371 of unique size. The cluster size distribution is described by $D(S) = 0.696S^{-1.35}$.



(b) Data corresponding to 103,170 measured activities, whereof 488 were unique. The activity distribution is described by $D(A) = 0.668A^{-1.51}$.



(c) The data corresponds a total of 167,076 measured times, t, whereof 76 were unique. The behavior is approximated by $D(t) = 0.665t^{-1.63}$.



(d) Data corresponding to 1,461,293 measured distances, whereof 335 were unique. The trend is approximated to $D(r) = 0.139r^{-1.38}$.

Figure 9: The data (blue dots) in (a) corresponds to 1,020 random initial configurations in a system of size 100×100, while (b), (c) and (d) corresponds to data from 380 randomly initiated systems of size 30×30. The red-dashed lines are approximate power laws that describes the behavior of respective system.

There is an obvious trend of power law behavior in all of the above distribution plots. This indicates that the CA is scale-free, and as discussed previously, this power law behavior is a promising sign of similarities to biological systems. Other sets of rules, than the one described here, were found to yield similar results as above (i.e. could be represented by power laws). The data from some other sets of rules can be found in Appendix A. Furthermore, according to the previous discussion in Section 1.2, these findings demonstrate the existence of SOC in CA, for these specific sets of rules. The results also seem to agree well with previous studies on the specific cellular automaton "Game of Life", which also has been proven to possess SOC [4, 43, 44, 45].

3.2 Entropic Force

In section 3.1 it was discovered that the CA-rule described there, created a system of clusters that could be described by the function $D(S) = 0.696S^{-1.35}$, see Figure 9 (a) or in Appendix A.2, Figure A.5 . Considering the same system, the exponent in the power law for $D(S)$, $\alpha = -1.35$, can be inserted in Eq. (11) and the entropy change after a perturbation can be measured. Displayed in Figure 10 below, is the result for this specific set of rules applied in a grid of size 100×100.

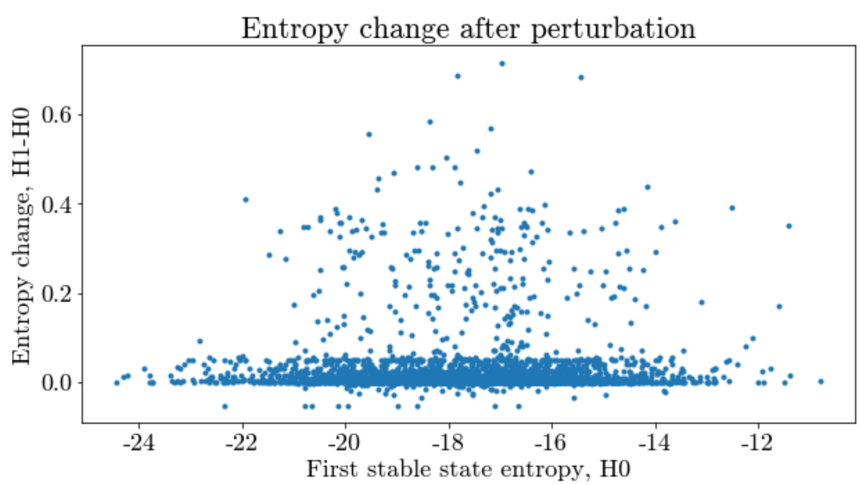


Figure 10: Entropy change caused by random perturbations of 2,560 systems, initialized with a random distribution of 20% of live cells.

4 Conclusions

It has been proven that SOC exists in all of the studied CA, this follows by the evident power law behavior seen in both Figure 9, Appendix A.1, A.2 and A.3. However, there is a recurring deviation from the power law behavior, seen in all of the approximated power laws. The origin of these large-value deviations is most certainly due to the finite sizes of the systems, in which the simulations were done. Therefore, with this realization, the finite system-size deviations can be disregarding when concluding that power laws are able to describe certain CA-behavior. Along with previous discussions about SOC and its relationship to biological systems, these are promising findings in before considering the last studied feature of CA in this thesis, namely entropy maximization.

The data in Figure 10 clearly showed an obvious entropy change, specifically, there is an increase of entropy corresponding to an increase in the possible number of states (i.e. cluster-count and cluster sizes). According to the reasoning in Section 2.3, this property of the entropy can be coupled with the entropic force via Eq. (8). By considering the discussion regarding entropy maximization and intelligence in Section 1.3.2, this finding suggests that CA-systems possibly share a deep connection with intelligent life-like systems. A similar result was also found for another set of rules, see Figure A.12 in Appendix A.3. This finding of intelligence traits in CA (via entropy maximization) appears to be previously unobserved and is thus considered a new finding. Therefore, this is the major contribution of this thesis towards a possible future formulation of a relation between CA and life-like systems.

Outlook

Future studies within this subject are a matter of course since the findings in this thesis indicate that there is a very clear sign of causal entropy in various cellular automata. The idea that life and various biological systems can be described by simple sets of rules, just like CA, yields many interesting opportunities for future studies. For example, the possibility of investigating nearest-neighbor rules in systems of, say, unicellular organisms, or even the neurons and their networks in the human brain. Moreover, in a recent study of a CA with continuous spacetime-state, the author states that CA ”could give answers to life, the universe, and everything.” [46, p. 2]. This, perhaps extravagant, statement is given even more background, now considering that causal entropy has been found in CA.

Acknowledgement

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Appendices

A Extra Data

The figure below is a result from the same system as for Figure 9 (b), however, the data here corresponds to a smaller simulation.

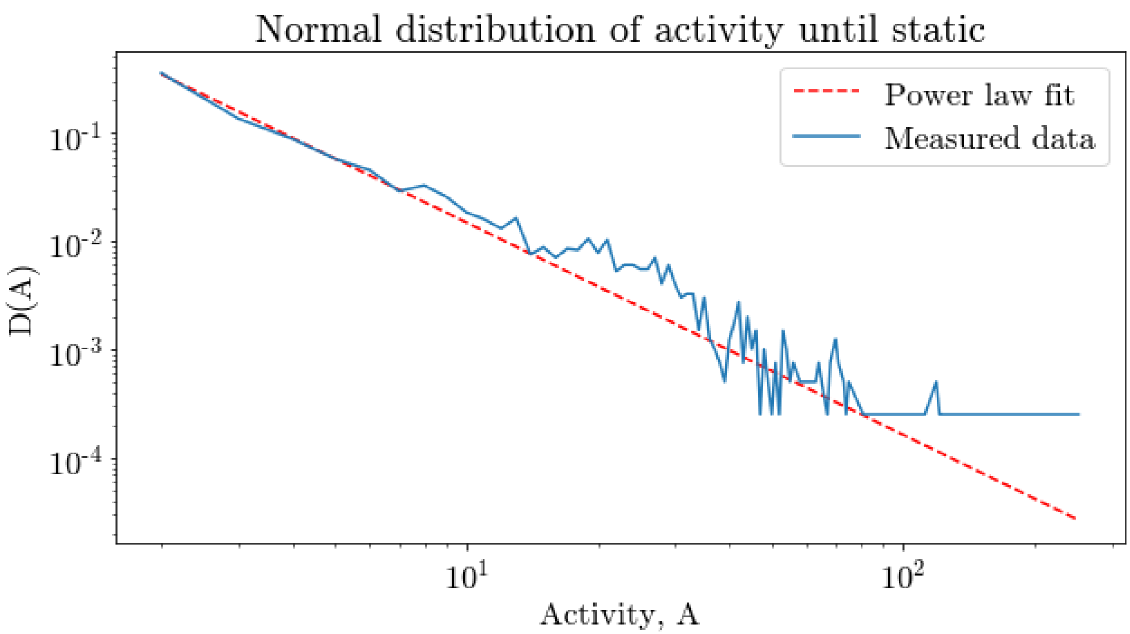

Figure A.1: The plot shows data from 30 different initial conditions, which resulted in a total of 3,969 measured activities, whereof 88 were unique. The distribution is described by $D(A) = 1.35A^{-1.96}$.

A.1 Decreasing Activity CA

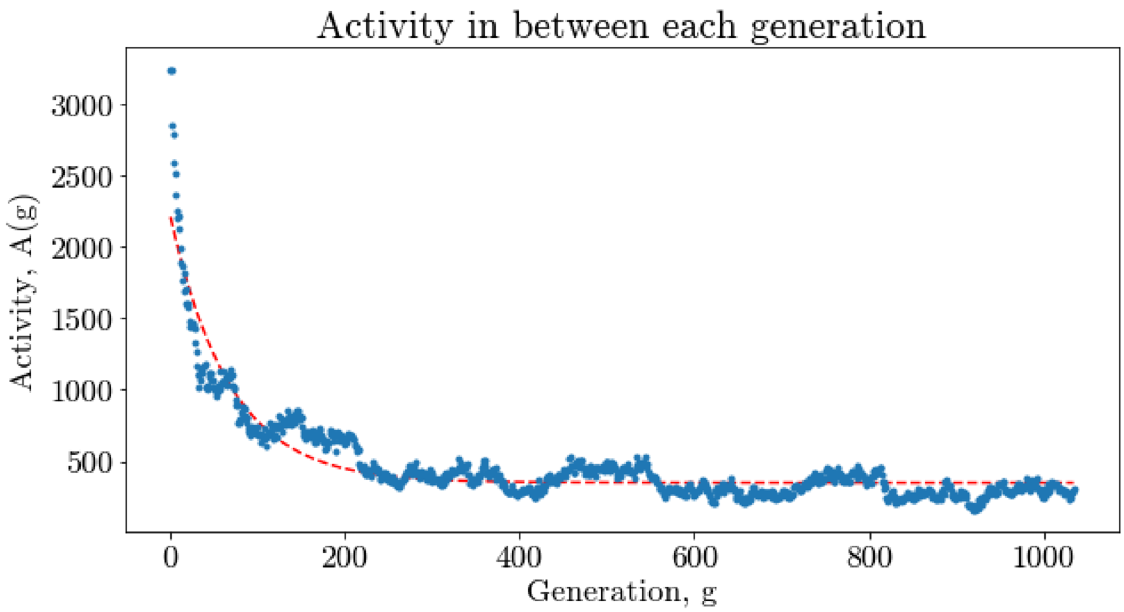

Figure A.2: A plot of the activity between each generation. The data is gathered from a GoL system with a size of 100×100. The activity is exponentially decreasing according to the function $A(g) = (18.8 \cdot 0.985^g + 3.51)10^2$

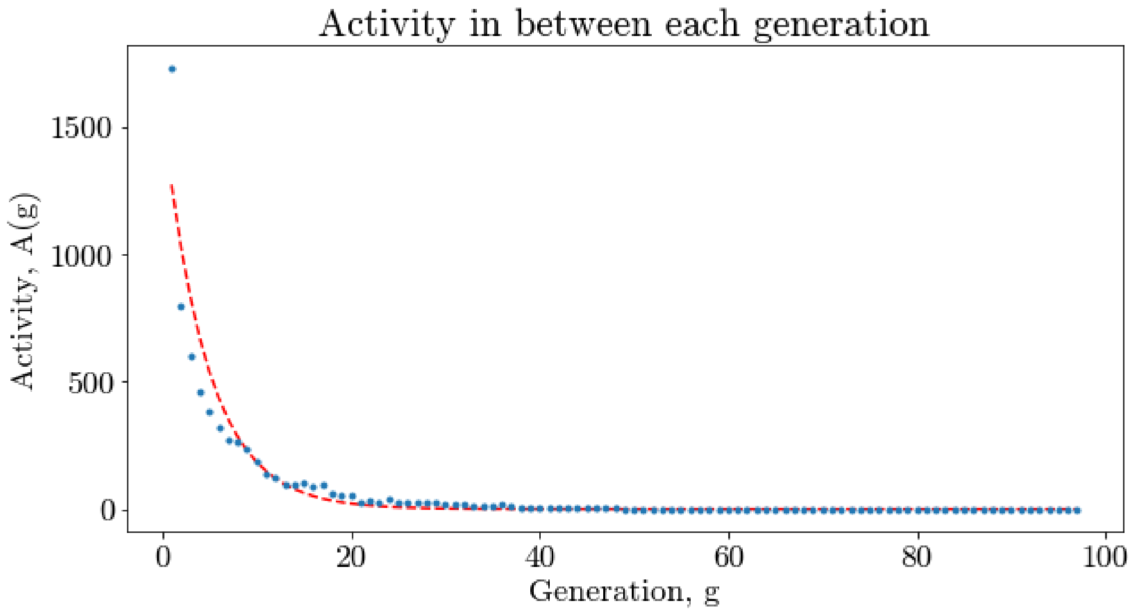

Figure A.3: A plot of the activity between each generation. The data is gathered from a 100×100-system, driven by the rules described in Section 3.1. The activity is exponentially decreasing according to the function $A(g) = 1.58 \cdot 0.807^g \cdot 10^3$

A.2 CA Density 20%

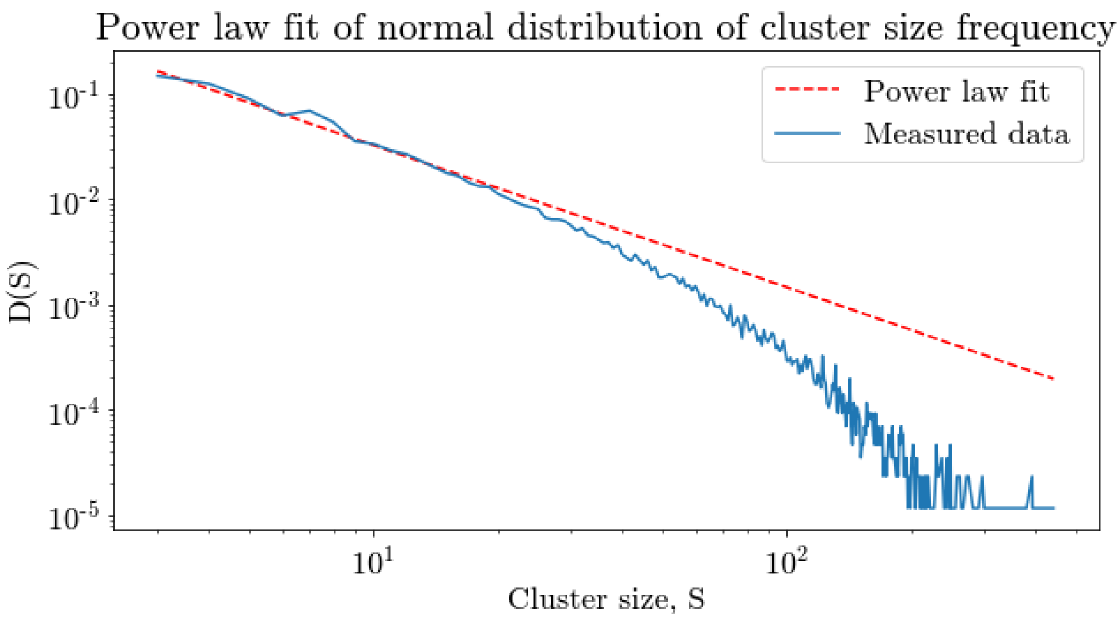

Figure A.4: Data from Figure 8 (a) with a power law fit. The red dashed line corresponds to the power law $D(S) = 0.722S^{-1.35}$

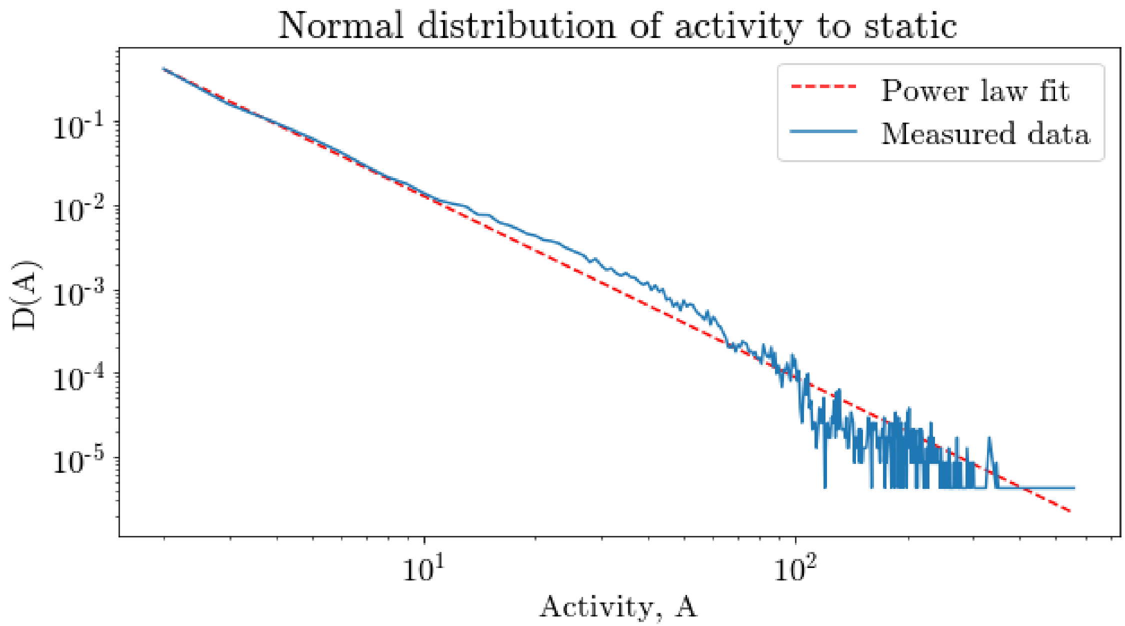

Figure A.5: Data from 1,200 different initial conditions in a 30×30-system, which resulted in 236,922 measured activities, whereof 295 were unique. The activity distribution is described by the power law $D(A) = 1.86t^{-2.16}$.

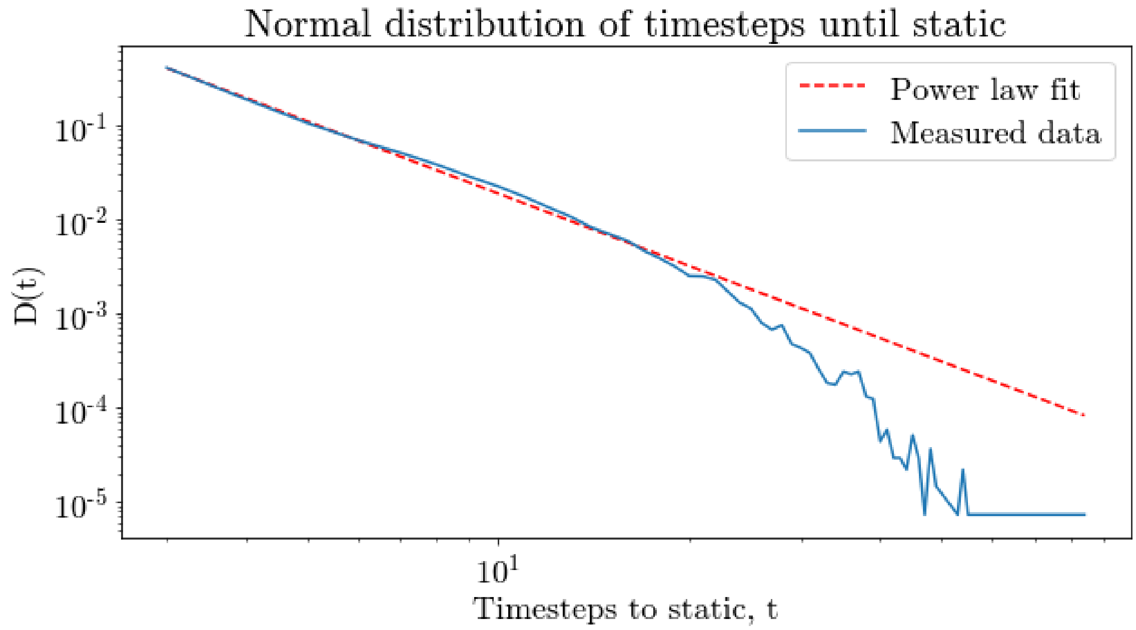

Figure A.6: The data corresponds to 1,200 different initial conditions in a system of size 30×30, and a total of 137,252 measured times, t. The approximated power law function is given by $D(t) = 6.67t^{-2.55}$

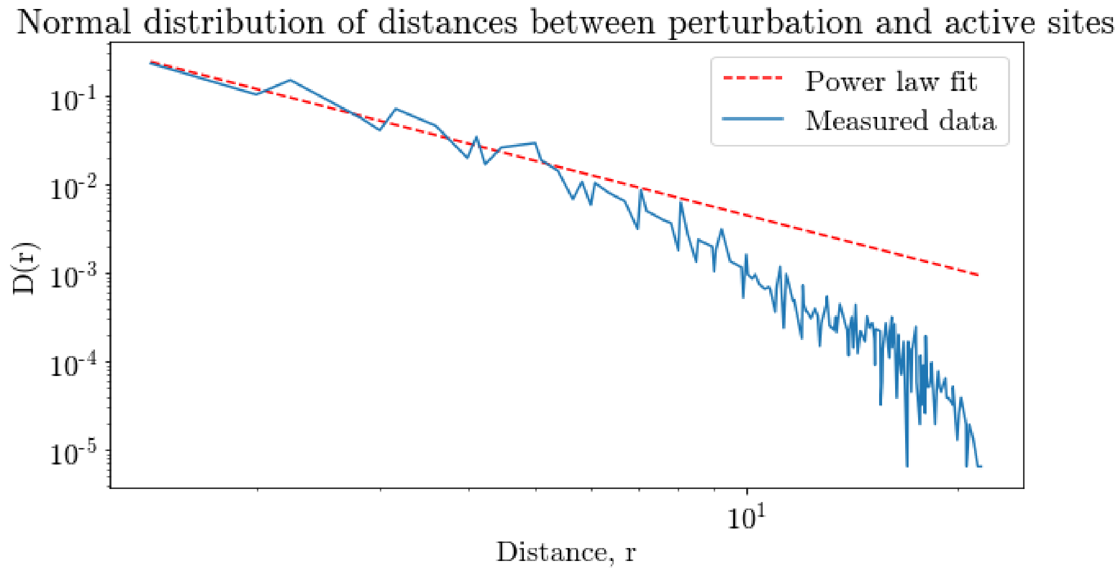

Figure A.7: Data corresponding to 15 different initial conditions in a system of grid-size 100×100, the number of distances measured was 155,160, whereof 144 were unique. The trend is approximated by the power law $D(r) = 0.494r^{-2.043}$.

A.3 Power-Laws and Entropy

The following five figures are based on data gathered using the rules: A cell will die if it has 0, 1 or 7 live neighbors, a cell will be born if it has 4 or 8 live neighbors, and for each other case a cell will remain unchanged. The measured quantities were $D(S), D(A), D(t)$, $D(r)$, and the entropy change $(H_1 - H_0)$, all data were collected from a system of size 70×70 with an initial density of 30% live cells.

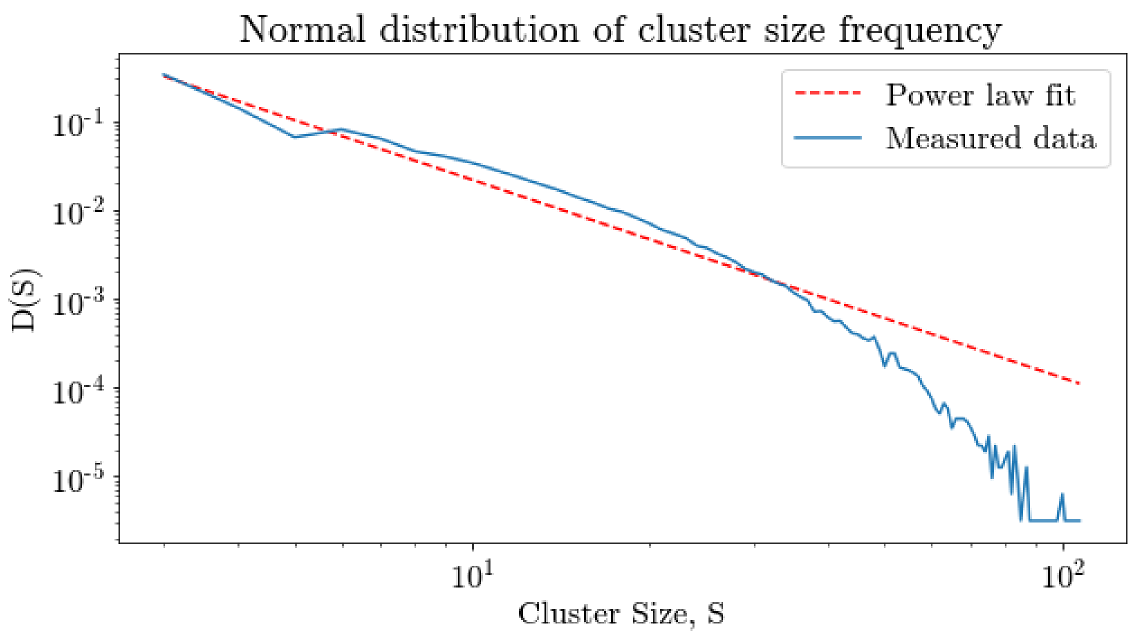

Figure A.8: Behavior described by $D(S) = 3.65S^{-2.23}$. Data from 6,100 different initial conditions, which resulted in 315,619 measured clusters whereof 93 unique.

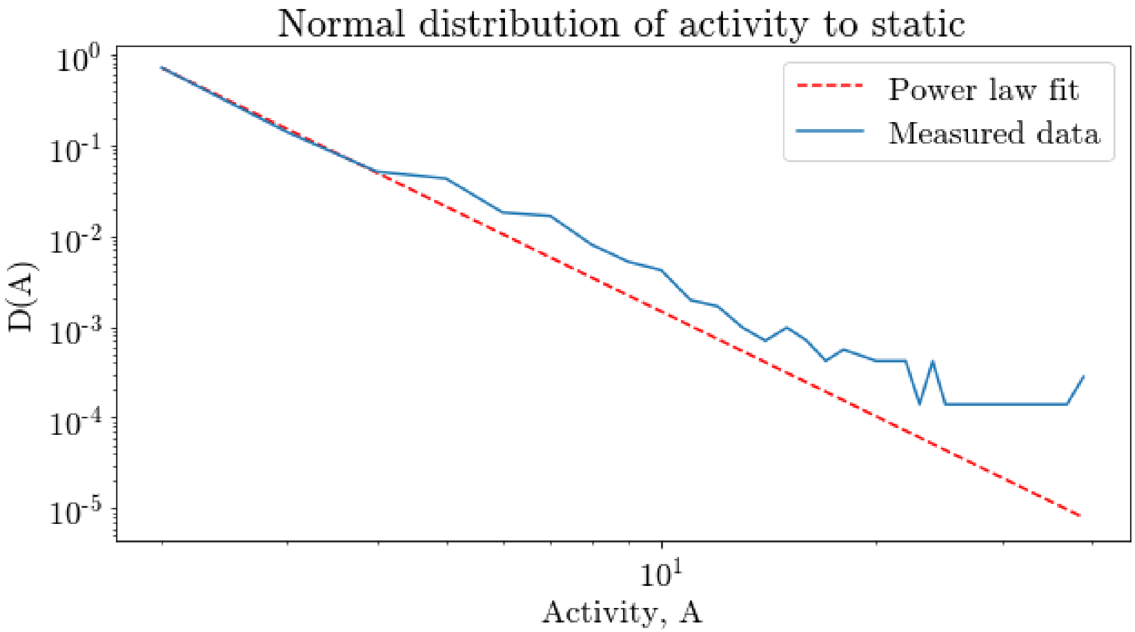

Figure A.9: Measured data is roughly following the power law $D(A) = 10.05A^{-3.83}$. The data corresponds 50 randomly initialized systems, which resulted in 7,145 measured activities whereof 29 unique.

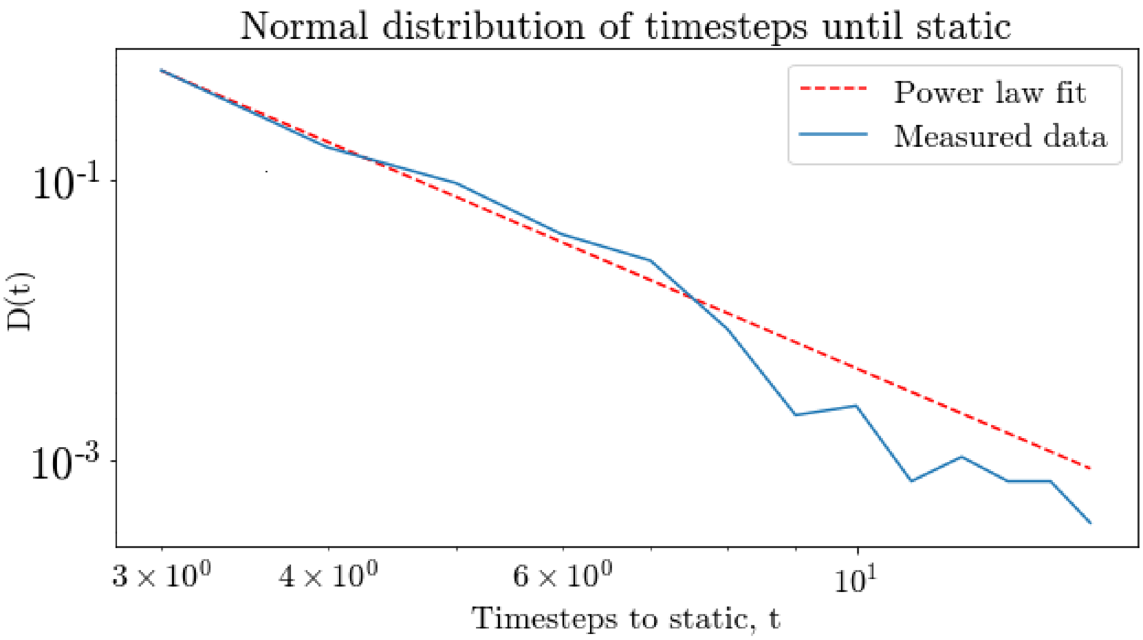

Figure A.10: Data described by $D(t) = 57.8t^{-4.11}$. From 50 randomly initiated systems, 169,609 times were measured, whereof 14 unique

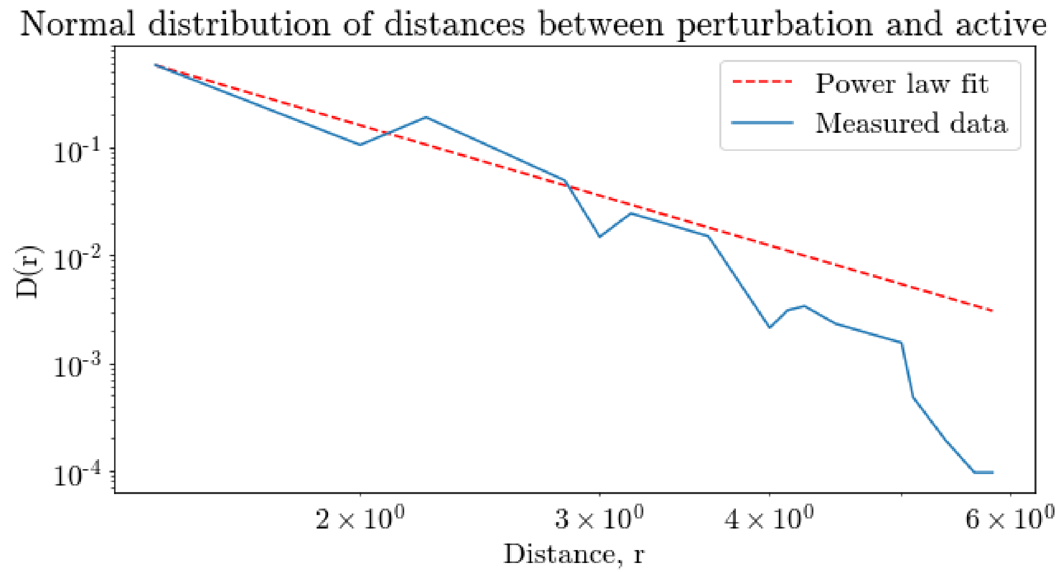

Figure A.11: The data roughly follows $D(r) = 2.11r^{-3.71}$, and corresponds to 17,527 measured distances, whereof 17 unique.

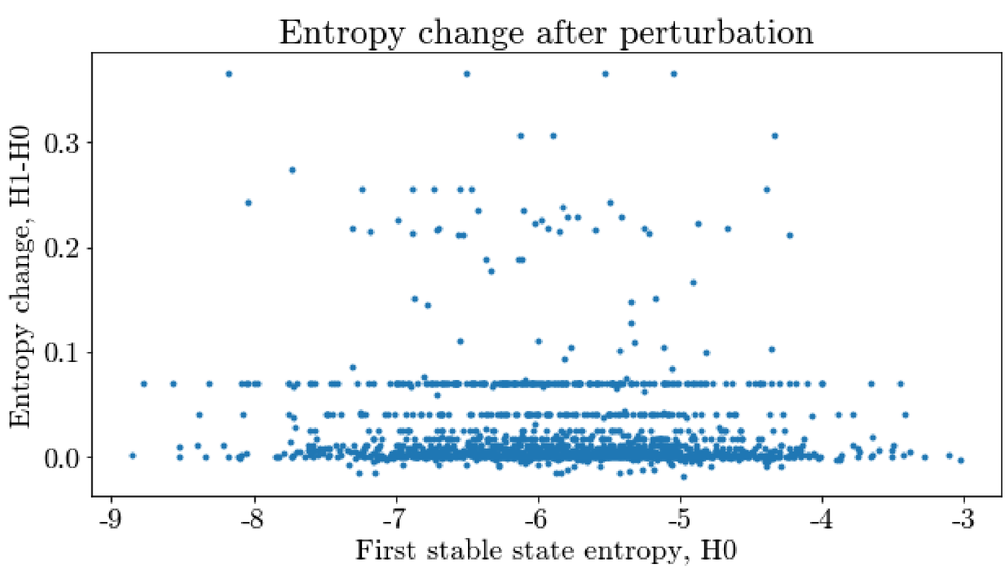

Figure A.12: Entropy change from 1,578 randomly perturbed systems. A clear increase in entropy is observed.

B List of Investigated Rules

On the next page, some of the more thoroughly investigated CA-rules are shown. The conditions were written in a .xlsx-file, and its content was directly read by the CA-program, a link to the program can be found in Reference [42].

The first column (A) represents the dimension of the system, specifically, the system is a grid of squares, thus a *Grid dim* of 40 means that the system contains a total of $40 \times 40 = 1,600$ cells. The parameter in column B corresponds to the visualized size of each square cell, a higher value gives larger cells. Column C takes a *True* or *False* argument on whether or not closed-/absorbing boundaries should be used; if set to *False*, periodic boundaries will be used. Columns D and E take the lists of the number of neighbors needed for a cell to experience any of the two changes. Column F takes the density for the initial random distribution of live cells, the density should be given in decimal-form. Columns G and H are for when it would be interesting to observe the activity in a specific region of the system; if *Loc. act* is set to *True*, the coordinates of the corners should be inserted according to x_1, y_1, x_2, y_2, where the top left corner of the system has coordinates $(x_1, y_1) = (1, 1)$ and bottom right corner has coordinate $(x_1, y_1) =$ (dim, dim). Column I and H are for the user's notes on the specific system, column I is mainly for noting initial configurations (other than the random ones) that yielded interesting patterns, or peculiar evolution of the system. In H, notes from observations or system specific features were written down. Furthermore, the main sets of rules investigated in this thesis are the ones in displayed row 13 and 18, and in row 1 and 2 are the rules for Game of life.

	A	B	C	D	E	F	G	H	I	J
1	**Grid dim.**	**Cell size**	**Abs. bound.**	**Birth**	**Death**	**Density**	**Loc. act.**	**Loc. act**	**Initial config. of alive cells**	**Comment**
2	40	15	False	[3]	[0,1,4,5,6,7,8]	0	False			Game of Life
3	40	15	True	[3]	[0,1,4,5,6,7,8]	0	True	1,1,20,20		Game of Life
4	40	15	False	[3,6]	[0,1,4,7,8]	0	False		Arbitrary	Exponentially growing activity
5	100	5	False	[3,6]	[0,1,4,7,8]	0	False		3-high and 4-wide H	Intresting evolution, very nice exp. activity-growth
6	40	15	False	[3,4]	[0,1,2,5,6,7]	0	False		Horizontal line of alive cells	Intresting fractal-like behaviour
7	30	20	True	[1,4]	[0,2,5,6,7]	0	True	10,10,20,20	2 by 2 square	Every gen is symmetric, almost periodic in time
8	50	15	False	[2,3]	[0,4,5,6,7,8]	0	False		vertical line of height 5	Symmetries and intresting activity-distribution
9	40	15	False	[3]	[0,1,4,5,6,7]	1	False		1 dead in the middle	Several symmetries. Beautiful activity-distrubution.
10	40	15	False	[3,6]	[0,1,4,7,8]	0	False		3 by 3 empty square in middle, lines 8-13	Amazing pattern evolution, see activity distr. plot
11	40	15	False	[3,6]	[0,1,2,4,7,8]	0.5	False		"glider" from [[1,1,1,0],[0,0,1,1],[1,1,1,0]]	Exponentially decreasing activity
12	30	20	False	[3]	[0,1,7,8]	0.3	False			
13	30	20	False	[4]	[0,1,7,8]	0.2	False			Variety of stable clusters
14	40	20	False	[5]	[0,1,6,7,8]	0.2	False			Variety of stable small clusters
15	40	15	False	[6]	[0,1,2]	0.4	False			lots of stable clusters, but a rare apperenece of size 5
16	70	5	False	[6,8]	[0,1,2]	0.4	False			lots of stable clusters, but a rare apperenece of size 5
17	70	5	False	[6,8]	[0,1,3]	0.4	False			odd behaviour for small clusters
18	70	5	False	[5,8]	[0,1,7]	0.2	False			nice power law behaviour