LUND UNIVERSITY
School of Economics and Management

The Effect of Coarse Reasoning on a Search and Matching Market With Transferable Utility

A Thesis in Microeconomic Theory

Ludvig Jonathan Andersson, 941014-8374
Lund University, Master in Science of Economics, NEKN01

May 2020

Supervision by Associate Professor Erik Mohlin

Contents

Abstract

This thesis investigates the effect of relaxing the assumption of full rationality on a frictional search and matching market with transferable utilities. It has previously been shown that such markets *without* transferable utilities may exhibit agents searching for a matching partner for longer than optimal due to them overestimating their prospects on the market. Reduced search frictions further enhance this negative effect. I find in this thesis that this is not the case for markets with transferable utility. Instead, agents become impatient and willing to accept larger set of matches, which is a result of them *underestimating* their prospects on the market. The equilibrium outcome with bounded rationality is less efficient than that with full rationality. With transferable utility and boundedly rational agents, reduced search frictions may reduce the suboptimality implied by the boundedly rational agents' equilibrium behaviour.

Keywords: Search Theory, Assortative Matching, Bounded Rationality, Transferable Utility, Matching Theory

Chapter 1

Introduction

This thesis investigates a search and matching market with transferable utility and a continuum of heterogenous and boundedly rational agents, to evaluate the effects of bounded rationality on the equilibrium behaviour and matching patterns. The agents engage in searching, which at a certain rate results in a proposed matching. Agents decide whether to accept or reject the proposed match. While unmatched, an agent does not produce any output. Only while in a pair can the agents produce a flow of divisible output. The divisibility of produced output implies transferability of utility within pairs. The division is assumed to conform with the Nash bargaining solution. The market is characterized by search frictions which, as opposed to frictionless searching, implies that searching is not straight forward, e.g. agents are not guaranteed to be proposed a match after having searched, and search is time consuming. When an agent is proposed a match, she is faced with the trade off of either continuing her search for a better option, *or* exit the search market to produce immediately with her proposed match. The general framework is based on that of Shimer & Smith (2000). To the difference of their approach, I relax the assumption of full rationality by introducing boundedly rational agents with *analogy based expectations* as introduced by Jehiel (2005). Previous work, e.g. Bloch & Ryder (2000), has found that, in a rational expectations model, reduced search frictions would make a large share of the participating agents strictly better off. In contrast, Antler & Bachi (2019) found that, with analogy based expectations in a search and matching framework with non-transferable utilities, search frictions are not necessarily bad for the participating agents. On the contrary, reduced search frictions may lead to agents of intermediate value searching forever after. That is, they never accept a proposed match, due to a general overoptimism and selection neglect. Antler & Bachi (2019) suggest thinking about reduced search frictions as advances in technology. For example, the introduction of dating apps provide a much greater supply of potential mates, and make it a lot easier (less time consuming) to keep searching for better options. The concept can of course be thought of in the context of a labor market as well. However, labor markets are arguably characterized by transferable rather than non-transferable utility. This is the setting of interest for the current thesis.

Analogy Based Expectations

As opposed to fully rational individuals, boundedly rational individuals behave in a likely more realistic manner. That is, agents have cognitive limitations and limited information and are, simply put, not perfect. There are several ways to model bounded rationality. The choice of framework for this thesis is one developed by Jehiel (2005) called the *analogy based expectations equilibrium*. In an analogy based equilibrium, as opposed to a Baysian-Nash equilibrium, agents understand only the average behavior of their opponents over bundles of states/types, called analogy classes. Only with the finest of partitioning of analogy classes, i.e. only one state per analogy class, do we expect the analogy based equilibrium to coincide with the Baysian-Nash equilibrium. After each round of play, every agent observes the others' actions and the analogy class where the underlying state belongs, but not the exact state. Hence, any one agent expects that the opponents condition their strategies coarsely on analogy classes rather than on types. In equilibrium, the agents are required to hold correct beliefs about the other agents' average behaviour in every analogy class. Analogy based expectations are appealing since they capture the fact that agents are, in reality, likely not able to register the details of every instant in which a match was rejected or accepted. Instead, agents form beliefs on the basis of feedback from aggregate behaviour. I will follow Antler & Bachi (2019) in that I will allow agents to weigh different contingencies differently, depending on the frequencies with which they are reached, as well as an exogenously determined probability with which the outcomes in these contingencies are observed. The exogenous variable determining this probability allows me to systematically *select* the information on which an agent's belief is based on. Agents estimate the acceptance likelihood of a potential match by averaging over *all* agents (i.e. both those they reject and those they accept), hence there is some neglection of correlation between agent-type and behaviour as a result of the coarsity of analogy-class partitioning.

Assortative Matching

Assortative matching is related to this thesis in the sense that I will, as my benchmark case, use a frictional search and matching market with transferable utility on which the core allocation is *positive assortative matching*. The core allocation should not be confused with the equilibrium. The equilibrium is determined by a sequentially rational system of beliefs and a *profile of the agents' strategies* in which any *one* agent cannot improve her expected outcome by deviating from the profile, given that *everyone else* play their strategy as determined by the profile and given the system of beliefs. The core allocation is, however, a feasible allocation in which no coalition of agents can improve upon it. Hence, in this thesis, core allocation refers to the optimal matching patterns and the equilibrium refers to the agents' choice of strategy given their beliefs. The benchmark case/market will correspond to the market analyzed by Shimer & Smith (2000) where conditions for positive assortative matching hold. Becker (1973) showed that positive assortative matching (PAM) will be the core allocation on a *friction free* market if the production function is supermodular. That is, one's marginal utility of a higher valued agent increases in one's own value. The PAM on the frictionless market implies that every agent matches with another agent of identical value. When introducing frictions, Shimer & Smith (2000) shows that there are further requirements for PAM to hold. Additionally, they show that instead of such identical-value matching as in the frictionless case, any one agent will be willing to match with another agent within a certain interval. To be able to analyze what bounded rationality implies for the benchmark case, I will present some results by Shimer & Smith (2000) in Section 3.1.3, determining the necessities and sufficiencies for the benchmark case characteristics, e.g. PAM.

Antler & Bachi (2019)

What they found may be somewhat of a surprise. While one might expect that it would be to the benefit of society to reduce search frictions so that agents face the maximum number of options available to them, the authors found that once relaxing full rationality and allowing selection neglect, intermediate valued agents search for a longer period of time than optimal. Some may even remain in the search process forever. The reason for agents remaining in the search longer than optimal is twofold. On the one hand, agents overestimate their prospects in the market, i.e. they overestimate the expected value of their potential future partner, and thus the expected value of remaining unmatched at any point in time. On the other hand, they underestimate the time it would take for them to get matched, which is a result of them failing to understand the mutual match acceptance rate.

Purpose and Structure

The purpose of this thesis is to investigate the effect of bounded rationality on equilibrium behaviour in a market with transferable utilities and how it compares to the effects on a market with non-transferable utility. Depending on how bounded rationality affects agents' behaviour, search frictions may be more or less desirable. Search frictions can, in turn, be affected by policy decisions, and so the conclusions of this thesis can inform such work. In chapter two I introduce the model and notation. First, I present the base framework of the search and matching market which is inspired by that of Shimer & Smith (2000). Second, I proceed to formally incorporate bounded rationality into this framework. This will form the model of this thesis. Chapter three will solve the model. I provide the equations determining agents' expected values as matched and unmatched, respectively, which are central for finding agents' mutually optimal strategies. The solution to how agents are assumed to transfer utility is then derived. I then present some results by Shimer & Smith (2000) which are important for the characterization of the core allocation, and serve as benchmark results. Next, I provide the solution for agents' optimal choice of strategy and show that this is unique. Finally, I show how bounded rationality affects the equilibrium behaviour of the agents and which parts of the benchmark results that bounded rationality has no effect on. The last chapter, chapter four, covers a conclusion and a comparison of these results to those found by Antler & Bachi (2019) on a market with non-transferable utility.

Chapter 2

The Model

In section 2.1 I present the framework of the market and the notation to be used throughout the thesis. Section 2.2 formally introduces the concept of analogy based expectations, how the framework should be understood on this market and how these analogy based beliefs will enter the model of this thesis. Section 2.3 presents the assumptions used for the benchmark case, which are necessary to obtain positive assortative matching as the core allocation[1].

2.1 Framework

There is a set of agents with values $x \in [0, 1]$ according to an atomless continuous distribution F. The corresponding density function is denoted f, for which $0 < \underline{f} < f(x) < \overline{f} < \infty, \forall x$, thus it is positive and boundedly finite. The value of an agent determines the type of that same agent, i.e. an agent of value x will be of type x, hence, type and value may be used interchangeably. For any type x there is a continuum of agents. If an agent is unmatched, his/her output is normalized to 0. For the sake of intuition, consider an unmatched agent being a person searching for a job. Output being zero would in this case mean that this agent is does not produce any value unless he/she becomes matched, i.e. finds a job. Once two agents (types), say x and y, are matched together, they will produce output $p(x, y)$. The output

[1]Whether or not these assumptions remain possible for the case of this thesis, with bounded rationality, will be investigated in chapter 3.

produced once matched depends on their types, $p : [0,1]^2 \to \mathbb{R}$.

2.1.1 Incorporating Search and Frictions

In continuous time and with infinite horizon, agents take part in time consuming search and randomly set up meetings with other agents.

Actions

At any instant, an agent will be either *matched* or *unmatched*. If an agent is *matched* then that agent will not be part of the matching or search conducted. Hence, only *unmatched* agents will engage in searching and possibly matching. Once a match is proposed to an agent, the agent immediately observes the proposed match's type. The agent then decides whether to *accept* or *reject* the match. Thus any one agent has a binary set of actions, being {*accept*, *reject*}. Only if both agents in a proposed match accepts, will the proposed match become the actual match, i.e. either one in the pair may veto the proposed match. If a match is mutually accepted, the pair exit the market together.

Match Dissolution

Any actual match, at any point in time, is exogenously dissolved with constant Poisson probability rate $\lambda > 0$. Hence, any match will last a period of time d_t with probability $e^{-\lambda d_t}$. At the time in which the match is destroyed, both agents re-enters the set of unmatched agents with identical individual characteristics as before.

Matching Flow Rate

Let $u \leq f$ be the density function associated with unmatched agents, i.e. $\int_X u(x)dx$ is the mass of unmatched agents of types $x \in X \subseteq [0,1]$. Let $\mu > 0$ be the flow rate with which an agent is chosen to be randomly matched to another agent. Since already matched agents do not engage in the search, an agent x will only match with another agent y at the rate $\mu \int_Y u(y)dy$. The rate can be intuitively understood as *the probability with which you are chosen to be matched to any other agent, and the probability with which whom you are assigned to match with, actually is available.*

Strategies

A strategy for any one agent of type $x \in [0,1]$ is defined as a set $\mathcal{A}_x$ of agents with whom x is willing to match, the *acceptance set*. Let $\mathcal{R}_x$ be the set of agents with whom x would reject a match. Let $\mathcal{B}_x$ be the set of agents who would be willing to match with x, hence the *opportunity set* of x[2]. From this, x's matching set can be defined as $\mathcal{M}_x = \mathcal{A}_x \cap \{y \mid x \in \mathcal{A}_y\} = \mathcal{A}_x \cap \mathcal{B}_x$. This implies that a match (x, y) is *mutually agreeable* if $y \in \mathcal{M}_x$. Hence, $y \in \mathcal{M}_x$ if and only if $x \in \mathcal{M}_y$, due to symmetry in matching sets. Agents maximize their expected payoff with a discount rate corresponding to an interest rate $r > 0$. The produced output $p(x, y)$ is shared between the agents matched within the pair, such that $\psi(x \mid y) + \psi(y \mid x) = p(x, y)$. Following Shimer & Smith (2000), how the produced output is split within the pair will be determined by the Nash bargaining solution.

Steady State

As introduced above, matches will dissolve with probability $\lambda > 0$. Following Shimer & Smith (2000), the creation and dissolution of matches for every type of agent need to be perfectly balanced to maintain a steady state population of available agents. Since the density of all agents $x \in [0,1]$ is $f(x)$ and that of unmatched is $u(x)$, we can simply describe the rate of dissolution of matches as $\lambda(f(x) - u(x))$. The rate with which matches are created by agents of type x is $\mu u(x) \int_{\mathcal{M}_x} u(y)dy$. Hence, steady state requires[3]

$$\lambda(f(x) - u(x)) = \mu u(x) \int_{\mathcal{M}_x} u(y)dy \tag{2.1}$$

[2] In general there is no need to keep track of an opportunity set ($\mathcal{B}_x$) and an acceptance set ($\mathcal{A}_x$) when only dealing with nonnegative match surplus (Chade et al. 2017). However, for a more intuitive comparison to cases without the possibility of equalizing matches with transfers, I will sometimes refer to *opportunity set* and *acceptance set*.

[3] Shimer & Smith (2000) notes that there could be alternative approaches to achieve steady state, e.g. allow an inflow of unmatched entrants and leaving matched pairs to be permanent.

2.2 Analogy Based Expectations in the Model

When an agent faces incomplete information under rational expectations a common approach to determine an agent's optimal strategy is to assume that an agent is at least able to understand the behaviour of other agents given each possible *state*. The uncertainty lies in the fact that a certain state is played with a probability, and an agent may not be able to see what state is being played at the time of choice. In a "few-possible-states" situation, rational expectations may not be too unreasonable an assumption. However, when we have a continuum of possible states, as illustrated by the continuum of agents' types in this model, rational expectations are not credible. Jehiel (2005) introduced the analogy based expectations to cope with this. With analogy based expectations, agents bundle several states at which other acting agents must move into so-called *analogy classes*. An agent then only tries to learn the average behaviour within each analogy class. The partitioning into analogy classes will determine the coarsity of reasoning, or sophistication of an agent if you will.

In this thesis the system of beliefs derived from analogy based expectations will, for an agent x, be denoted β_x which tells us the probability with which agent x *believes* every other agent to accept her. Just like under rational expectations, we will require the system of beliefs to be sequentially rational given a strategy profile σ. Formally, β_x is consistent with strategy profile σ if,

$$\beta_x = \frac{\int_{\mathcal{M}_x(\sigma)} u(y)dy + \xi \int_{\mathcal{R}_x(\sigma)\cap\mathcal{B}_x(\sigma)} u(y)dy}{\int_{\mathcal{A}_x(\sigma)} u(y)dy + \xi \int_{\mathcal{R}_x(\sigma)} u(y)dy} \tag{2.2}$$

I will follow Antler & Bachi (2019) in that I will allow agents to weigh different contingencies differently, which is appealing intuitively. When considering a search and matching market it is likely the case that if you reject a proposed match, you are not as likely to observe what your proposed match actually chose. However, if you accept a match, you will certainly know the answer of your partner, since either you exit the market together, or you continue your search. The variable ξ will reflect the probability with which an agent observes the choice of another agent, whom she rejected. If $\xi = 0$, an agent never observes the choice of an agent whom she rejected, thus, her belief is based solely on situations of *mutual* acceptance.

2.3 Assumptions

With Shimer & Smith (2000) and positive assortative matching as my benchmark case, I need to make certain assumptions concerning the production function. These assumptions, following below, are identical to those of Shimer & Smith (2000).

Assumption 1 *$p(x, y)$ is nonnegative, symmetric*[4]*, continuous, and twice differentiable, with uniformly bounded first partial derivatives*[5] *on* $[0, 1] \times [0, 1]$ *(Shimer & Smith 2000).*

Assumption 2 *The production function p is strictly supermodular. The own marginal product of any $x > 0$ is strictly increasing in her partner's type. That is, if $x' > x$ and $y' > y$, then $p(x', y') + p(x, y) > p(x', y) + p(x, y')$ (Shimer & Smith 2000).*

Assumption 3 *The first partial derivative of the log of the production function is supermodular: For all $x_1 \leq x_2$ and $y_1 \leq y_2$, $p'_x(x_1, y_1)p'_x(x_2, y_2) \geq p'_x(x_1, y_2)p'_x(x_2, y_1)$ (Shimer & Smith 2000).*

Assumption 4 *The cross-partial derivative of the log of the production function is supermodular: For all $x_1 \leq x_2$ and $y_1 \leq y_2$, $p'_{xy}(x_1, y_1)p'_{xy}(x_2, y_2) \geq p'_{xy}(x_1, y_2)p'_{xy}(x_2, y_1)$ (Shimer & Smith 2000).*

These assumptions are made as a benchmark. In chapter 3 I investigate whether the introduction of bounded rationality affects the possibility of making one or several of above mentioned assumptions.

---

[4] $p(x, y) \equiv p(y, x)$
[5] $\exists M \in \mathbb{R}$, such that $\mid p'_i(x, y) \mid \leq M, \forall i = x, y, \forall (x, y) \in [0, 1] \times [0, 1]$.

Chapter 3

Solving the Model

In section 3.1 I provide the equations for an agent's perceived expected present value as unmatched and matched, respectively. The reason for the values being as perceived is that a boundedly rational agent acts, by construction, only on the basis of coarsly processed information. Section 3.2 presents the solution concept for the transfer bargain, i.e. what determines how the production output is split between the two agents matched. In section 3.3 I briefly introduce some of the results found by Shimer & Smith (2000). These results are cornerstones of the benchmark case, determining the matching patterns and core allocation. It's necessary to understand the matching patterns of the benchmark equilibrium in order to analyze how, and to what extent bounded rationality affects these. In section 3.4 I present necessary and sufficient conditions for an agent to match in equilibrium which then can be expressed as a mutual optimality condition. This will enter an agent's perceived expected value as unmatched. Following this, I show that the solution, expressed in terms of reservation strategies, is unique. Finally, I end this section and chapter by showing how the exogenously determined variables affect the results, and what parts of the benchmark case that are changed by the introduction of bounded rationality, and which parts that remain intact.

3.1 Expected Values

3.1.1 Expected Unmatched Value

Let $\Omega(x)$ denote the *perceived* expected present value of an unmatched agent x. Let $\Omega(x \mid y)$ be the expected present value of x as matched with y, hence $\mathcal{S}(x \mid y) = \Omega(x \mid y) - \Omega(x)$ is agent x's personal surplus when matched with y. Given that an agent is unmatched, she expects to meet and match at the rate of $\mu\beta_x \int_{\mathcal{A}_x} u(y)dy$. Note that due to coarse reasoning according to analogy based expectations, I do not integrate over $\mathcal{M}_x$, as done by Shimer & Smith (2000) with rational expectations. Hence, we can express the perceived expected present value, over a time interval d_t, of an unmatched agent x as,

$$\Omega(x) = \frac{1}{1+rd_t}\Bigg[\Big(\mu\beta_x d_t \int_{\mathcal{A}_x} u(y)dy\Big)\mathbb{E}[max\{\Omega(x \mid y), \Omega(x)\} \mid y \in \mathcal{A}_x]$$
$$+ \Big(1 - \mu\beta_x d_t \int_{\mathcal{A}_x} u(y)dy\Big)\Omega(x)\Bigg]$$
$$= \frac{1}{1+rd_t}\Bigg[\Omega(x) + \Big(\mu\beta_x d_t \int_{\mathcal{A}_x} u(y)dy\Big)\Big(\mathbb{E}[max\{\Omega(x \mid y), \Omega(x)\} \mid y \in \mathcal{A}_x] - \Omega(x)\Big)\Bigg]$$
$$\Rightarrow \Omega(x)\frac{(1 - \frac{1}{1+rd_t})}{d_t} = \frac{1}{1+rd_t}\Bigg[\mu\beta_x \int_{\mathcal{A}_x} max\{\mathcal{S}(x \mid y), 0\}u(y)dy\Bigg]$$

Let $g(d_t) = 1 - (\frac{1}{1+rd_t})$ and $h(d_t) = d_t$. Since $\lim_{d_t \to 0} g(d_t) = \lim_{d_t \to 0} h(d_t) = 0$, from L'Hôpital's rule we get,

$$\lim_{d_t \to 0} \frac{g(d_t)}{h(d_t)} = \lim_{d_t \to 0} \frac{1 - (\frac{1}{1+rd_t})}{d_t} = \lim_{d_t \to 0} \frac{g'(d_t)}{h'(d_t)} = \lim_{d_t \to 0} \frac{\frac{r}{(rd_t+1)^2}}{1} = r$$

Hence, in the limit $d_t \to 0$,

$$r\Omega(x) = \mu\beta_x \int_{\mathcal{A}_x} max\{\mathcal{S}(x \mid y), 0\}u(y)dy \tag{3.1}$$

3.1.2 Expected Matched Value

Likewise, we can express the expected present value, over a time interval d_t, of an agent x matched with an agent y as,

$$\Omega(x \mid y) = d_t\psi(x \mid y) + \frac{1}{1+rd_t}\Big[(1-\lambda d_t)\Omega(x \mid y) + \lambda d_t \Omega(x)\Big]$$
$$\Rightarrow \Omega(x \mid y) = d_t\psi(x \mid y) + \frac{1}{1+rd_t}\Big[\Omega(x \mid y) + \lambda d_t(\Omega(x) - \Omega(x \mid y))\Big]$$
$$\Rightarrow \Omega(x \mid y)\frac{(1-\frac{1}{1+rd_t})}{d_t} = \psi(x \mid y) + \frac{1}{1+rd_t}\lambda(\Omega(x) - \Omega(x \mid y))$$

Let $g(d_t) = 1 - (\frac{1}{1+rd_t})$ and $h(d_t) = d_t$. Since $\lim_{d_t \to 0} g(d_t) = \lim_{d_t \to 0} h(d_t) = 0$, from L'Hôpital's rule we get,

$$\lim_{d_t \to 0} \frac{g(d_t)}{h(d_t)} = \lim_{d_t \to 0} \frac{1-(\frac{1}{1+rd_t})}{d_t} = \lim_{d_t \to 0} \frac{g'(d_t)}{h'(d_t)} = \lim_{d_t \to 0} \frac{\frac{r}{(rd_t+1)^2}}{1} = r$$

Hence, in the limit $d_t \to 0$,

$$r\Omega(x \mid y) = \psi(x \mid y) - \lambda \mathcal{S}(x \mid y)$$

As opposed to an unmatched agent in Shimer & Smith (2000), an unmatched agent's perception of her expected present value in this model is not the same as her actual. While an agent knows for sure her own strategy ($\mathcal{A}_x$), she can only coarsly predict other agents' behaviour. Namely, she believes with probability β_x that *any* agent of *any* type will accept her. Hence, the complete probability with which she *expects* to meet an agent accepting her, conditional on her accepting as well is given by $\mu\beta_x \int_{\mathcal{A}_x} u(y)dy$, while the *actual* probability is given by $\mu \int_{\mathcal{A}_x \cap \mathcal{B}_x} u(y)dy = \mu \int_{\mathcal{M}_x} u(y)dy$, which corresponds to that of Shimer & Smith (2000). Clearly, an agent being boundedly rational will then base her expected value as unmatched on the former rather than the latter. Additionally, note that β_x does not directly enter the expected value of a matched agent. Agents are not allowed to actively quit any relationship accepted, hence an already matched agent's belief of whether another agent would accept them or not is irrelevant, since them returning to the market as unmatched is determined exogenously by λ. The bounded rationality implies that an agent may have a perception of the market which does not correspond to the actual market. Hence, it enters, and distorts, the individual agent's expectations by β_x, which may have implications on individual and aggregate behaviour on the market.

3.2 Match Surplus and Nash Bargaining

Having transferable utilities in the model, we need to specify the solution concept we consider when it comes to the division of the surplus resulting from a match. Following Shimer & Smith (2000), I apply a Nash bargaining solution to the problem. I assume that agents seek to maximize the geometric mean of their surpluses by determining the split of the produced output, $p(x, y) = \psi(x \mid y) + \psi(y \mid x)$. First, note that,

$$r\Omega(y \mid x) = \psi(y \mid x) - \lambda\mathcal{S}(y \mid x) \equiv [p(x, y) - \psi(x \mid y)] - \lambda\mathcal{S}(y \mid x)$$

By maximization of the geometric mean, the objective function can be formalized as

$$\max_{\psi} \mathcal{S}(x \mid y)^{\gamma}\mathcal{S}(y \mid x)^{1-\gamma} \equiv \max_{\psi} [\Omega(x \mid y) - \Omega(x)]^{\gamma}[\Omega(y \mid x) - \Omega(y)]^{1-\gamma}$$

First order conditions yields,

$$\gamma[\Omega(y \mid x) - \Omega(y)]\Omega'(x \mid y) + (1 - \gamma)[\Omega(x \mid y) - \Omega(x)]\Omega'(y \mid x) = 0$$

Since $f(x, y) - \psi(x \mid y) = \psi(y \mid x)$ we get,

$$\Omega'(x \mid y) = \frac{1}{r + \lambda}, \Omega'(y \mid x) = -\frac{1}{r + \lambda} \Rightarrow \Omega'(x \mid y) = -\Omega'(y \mid x)$$

And thus,

$$\gamma\mathcal{S}(y \mid x) = (1 - \gamma)\mathcal{S}(x \mid y)$$

Bargaining power is determined by γ. If $\gamma = 0.5$, the two agents have identical bargaining power and the solution becomes $\mathcal{S}(y \mid x) = \mathcal{S}(x \mid y)$ which corresponds to the bargaining result used by Shimer & Smith (2000). For the results of this thesis I restrict my attention to situations in which the two agents have identical bargaining power.

For any given match (x, y) the transfer is determined by the Nash bargaining solution. Once x has been proposed a match with y, and vice versa, the transfer between the two is with certainty determined by above solution concept, leading to $\mathcal{S}(y \mid x) = \mathcal{S}(x \mid y)$.

Because of this, as well as agents immediately observing their potential partner's type once proposed to each other, the bounded rationality enters only during the search process, and not during the bargain. For the sake of intuition, this could be thought of as the search process being one game. If proposed a match which is mutually accepted, the two agents, say x and y, enter a second game being the Nash bargaining game. In this game there is no uncertainty. The transfer is such that $\mathcal{S}(y \mid x) = \mathcal{S}(x \mid y)$.

3.3 Benchmark of Shimer & Smith (2000)

Earlier work has found that when markets are free of friction, we need only make sure that supermodularity of the production function holds to achieve positive assortative matching (Becker 1973). However, more is required to maintain positive assortative matching when having a frictional market. How much more depends on how we choose to implement the frictions (Chade et al. 2017). *If* bounded rationality affects the market, we would like to analyze *how* it is affected. As I, as my benchmark, use a frictional search and matching market, with transferable utility and PAM as its core, it is important to present the appropriate necessities and sufficiencies.

3.3.1 Benchmark Results - (Shimer & Smith 2000)

Result 1 *Posit Assumption 1, then all matching sets $\mathcal{M}_x$ are nonempty and closed, and the matching correspondence $\mathcal{M} : [0,1] \rightrightarrows [0,1]$ is upper hemicontinuous (Shimer & Smith 2000).*

Result 2 *Assume symmetric, convex and nonempty matching sets $\mathcal{M}_x$ for all x, and an upper hemicontinuous matching correspondence $\mathcal{M} : [0,1] \rightrightarrows [0,1]$. There is positive assortative matching if and only if $0 \in \mathcal{M}_0$ and $1 \in \mathcal{M}_1$ (Shimer & Smith 2000).*

That $0 \in \mathcal{M}_0$ and $1 \in \mathcal{M}_1$ follows from the matching sets being symmetric by construction. However, Shimer & Smith (2000) additionally proves sufficency of these conditions.

Result 3 *Posit Assumption 1 and fix x. Provided Assumption 2, Assumption 3 and Assumption 4, the match surplus function $\mathcal{S}(x, y)$ is quasi-concave in y and all matching sets are convex (Shimer & Smith 2000).*

Provided Assumption 1 through 4, we get convexity by Result 3. We get closed and nonempty matching sets, as well as upper hemicontinuity in matching correspondence by Result 1. By Result 2 we get PAM only if $0 \in \mathcal{M}_0$ and $1 \in \mathcal{M}_1$. Since matching sets are convex, closed and nonempty we can perfectly define an agent's matching set by its *upper* and *lower boundary*. Hence we can let $a_x \equiv \min\{y \mid y \in \mathcal{M}_x\}$ denote the lower boundary, and $b_x \equiv \max\{y \mid y \in \mathcal{M}_x\}$ the upper boundary. Finally, by supermodularity and matching set boundaries being nondecreasing in agent's value, Shimer & Smith (2000) shows that $0 \in \mathcal{M}_0$, and similarily that $1 \in \mathcal{M}_1$. In summary, these are the conditions that guarantee a positive assortative matching as the core allocation.

3.4 Who To Accept, and When

3.4.1 Mutual Optimality

In this section I derive an agent's value equation based on a mutual optimality condition by proving that a nonnegative *personal* match surplus is equivalent to the *total* match surplus being nonnegative, *and* an agent's acceptance set, $\mathcal{A}_x$, is equivalent to her matching set, $\mathcal{M}_x$. Additionally, I prove that, although agents accept matching partners within an interval, the optimal solution in terms of lowest acceptable transfer is in fact unique.

From $r\Omega(x) = \mu\beta_x \int_{\mathcal{A}_x} max\{\mathcal{S}(x \mid y), 0\}u(y)dy$ it's clear that an agent accepts a proposed match only if $\mathcal{S}(x \mid y) = \Omega(x \mid y) - \Omega(x) \geq 0$. We can solve for $\Omega(x \mid y)$ in terms of $\Omega(x)$.

$$r\Omega(x \mid y) = \psi(x \mid y) - \lambda\mathcal{S}(x \mid y) = \psi(x \mid y) + \lambda\Omega(x) - \lambda\Omega(x \mid y) \tag{3.2}$$

And thus, $\Omega(x \mid y)$ in terms of $\Omega(x)$ is given by,

$$\Omega(x \mid y) = \frac{\psi(x \mid y) + \lambda\Omega(x)}{r + \lambda} \tag{3.3}$$

By equation (3.3) we can express $\mathcal{S}(x \mid y)$ as,

$$\mathcal{S}(x \mid y) = \frac{\psi(x \mid y) + \lambda\Omega(x)}{r + \lambda} - \Omega(x) = \frac{\psi(x \mid y) - r\Omega(x)}{r + \lambda} \tag{3.4}$$

Then we get,

$$r\Omega(x) = \mu\beta_x \int_{\mathcal{A}_x} max\{\frac{\psi(x \mid y) - r\Omega(x)}{r + \lambda}, 0\}u(y)dy$$

Hence, an agent accepts a match if and only if $\frac{\psi(x|y) - r\Omega(x)}{r+\lambda} \geq 0$. By the Nash bargaining solution we get that $\mathcal{S}(x \mid y) = \mathcal{S}(y \mid x)$ which implies that,

$$\frac{\psi(x \mid y) - r\Omega(x)}{r + \lambda} = \frac{\psi(y \mid x) - r\Omega(y)}{r + \lambda}$$

Being constrained by $p(x, y) = \psi(x \mid y) + \psi(y \mid x)$ it's clear from above equality condition that given an agent x, her part of the match surplus is not increasing in her partner's type y for all $y \in [0, 1]$, which hints about preferences being single-peaked as shown for the benchmark case. Denote the sum of the two personal match surpluses by $\mathcal{T}(x, y)$, i.e. the total match surplus. Then, $\mathcal{T}(x, y) = \mathcal{S}(x \mid y) + \mathcal{S}(y \mid x)$, which is equivalent to,

$$\mathcal{T}(x, y) = \frac{\psi(x \mid y) - r\Omega(x)}{r + \lambda} + \frac{\psi(y \mid x) - r\Omega(y)}{r + \lambda}$$

Using the constraint $p(x, y)$, this can be rewritten as,

$$\mathcal{T}(x, y) = \frac{p(x, y) - r\Omega(x) - r\Omega(y)}{r + \lambda} \tag{3.5}$$

Proposition 1 *A match with any two agents x and y is mutually accepted if and only if the sum of their two personal surpluses is greater than, or equal to zero. That is,*

$$\mathcal{T}(x, y) = \frac{p(x, y) - r\Omega(x) - r\Omega(y)}{r + \lambda} \geq 0$$

Additionally, any match accepted by one agent within a pair, has to be mutually accepted as well, i.e. $\mathcal{M}_x \equiv \mathcal{A}_x$ *for all* x.

Proof 1 *An agent* x *accepts a match with any other agent* y*, i.e.* $y \in \mathcal{A}_x$*, if and only if her personal match surplus is nonnegative,* $\mathcal{S}(x \mid y) \geq 0$*. By the Nash bargaining solution the total match surplus is split fifty-fifty between the two matched agents, and so* $\mathcal{T}(x, y) \geq 0 \Leftrightarrow \mathcal{S}(x \mid y) + \mathcal{S}(y \mid x) \geq 0 \Leftrightarrow \mathcal{S}(x \mid y) = \mathcal{S}(y \mid x) \geq 0$*. This implies that* $y \in \mathcal{A}_x$ *if and only if* $x \in \mathcal{A}_y$*, and specifically an agent's acceptance set,* $\mathcal{A}_x$*, will be equivalent to that of her matching set,* $\mathcal{M}_x$ *and accepting a match requires the total match surplus to be nonnegative.* ■

By proposition 1, I can now express a mutual optimality condition,

$$\mathcal{T}(x, y) = \frac{p(x, y) - r\Omega(x) - r\Omega(y)}{r + \lambda} \geq 0 \Leftrightarrow y \in \mathcal{M}_x, x \in \mathcal{M}_y \tag{3.6}$$

And an agent's value equation can be written as,

$$r\Omega(x) = \Phi\beta_x \int_{\mathcal{A}_x} \big[p(x, y) - r\Omega(x) - r\Omega(y)\big] u(y) dy, \Phi \equiv \frac{\mu}{2(r + \lambda)} \tag{3.7}$$

Given that $\mathcal{S}(x \mid y) \geq 0 \Rightarrow y \in \mathcal{A}_x$ we do not need to specify that an agent chooses to accept or reject based on $\max\{p(x, y) - r\Omega(x) - r\Omega(y), 0\}$. Given that we are integrating over $\mathcal{A}_x$, the agent will choose to accept. Following Shimer & Smith (2000) I will assume that an agent, if indifferent between accepting and rejecting, will choose to accept.

Uniqueness of Solution

Proposition 2 *Any agent has a reservation transfer which is the lowest acceptable transfer from a match. An agent* x *accepts other agents within a type interval, hence, more than one type of agent may provide a transfer greater than, or equal to* x*'s reservation transfer. However, the reservation transfer is in itself unique.*

Proof 2 *From section 3.4.1 and equation (3.4) we found that an agent* x *accepts a match only if,*

$$\mathcal{S}(x \mid y) = \frac{\psi(x \mid y) - r\Omega(x)}{r+\lambda} \geq 0 \Longrightarrow \psi(x \mid y) - r\Omega(x) \geq 0$$

We can therefore interpret an agents perceived expected present value as her reservation transfer. Denote agent x's reservation transfer by ψ_{RES}*. From this we can express a reservation transfer equation satisfying* $\psi_{RES} = r\Omega(x)$*,*

$$\psi_{RES} = \frac{\mu\beta_x}{r+\lambda}\int_{a_x}^{b_x} \big[\psi(x \mid y) - \psi_{RES}\big]u(y)dy \tag{3.8}$$

The lefthand side of the equation above represents the marginal cost of remaining in the search process, and the righthand side is the marginal gain of remaining in the search process. I will show that the solution to this, i.e. the reservation transfer solving equation (3.8), is unique.

Clearly, the lefthand side is increasing in reservation transfer. Rewrite the righthand side as a function of the reservation transfer, i.e. $\mathcal{H}(\psi_{RES})$*. Differentiating this function, and by Leibniz's rule we get,*

$$\begin{aligned}\frac{d\mathcal{H}(\psi_{RES})}{d\psi_{RES}} = \frac{\mu\beta_x}{r+\lambda}\Bigg[&\Big(\psi(x \mid b_x) - \psi_{RES}\Big)\frac{du(b_x)}{d\psi_{RES}} - \Big(\psi(x \mid a_x) - \psi_{RES}\Big)\frac{du(a_x)}{d\psi_{RES}} \\ &- \int_{a_x}^{b_x} u(y)dy\Bigg]\end{aligned}$$

Note that b_x (a_x) *denotes the upper (lower) boundary of an agent x's matching set, and thus her acceptance set as well. From previous section (see equation (3.6)) we found that an agent's personal match surplus is nonnegative only if her matching partner is in her matching set. At the boundaries the match surplus goes to zero. Because of this, above equation can be simplified,*

$$\frac{d\mathcal{H}(\psi_{RES})}{d\psi_{RES}} = -\frac{\mu\beta_x}{r+\lambda}\int_{a_x}^{b_x} u(y)dy \tag{3.9}$$

Equation (3.9) shows that $\mathcal{H}(\psi_{RES})$ *is decreasing in reservation transfer, and therefore, the solution is in fact unique.* ■

3.4.2 System of Beliefs

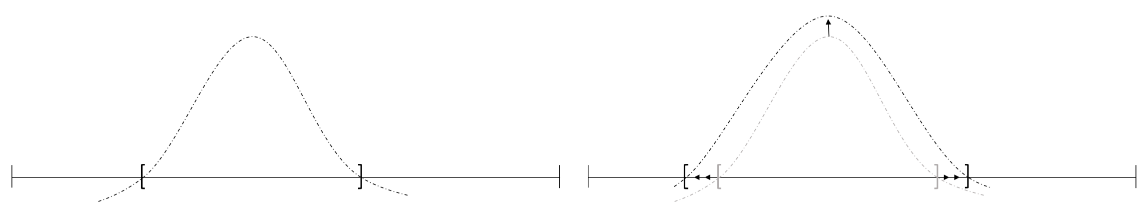
Consider the belief for any one agent as determined by equation (2.2). The equation tells the probability with which an agent believes every other agent to accept her. The agent averages the behaviour of other agents over the whole spectrum of types, thus it is the coarsest of possible reasonings in the framework of analogy based expectations with only one analogy-class. Consider the term $\int_{\mathcal{R}_x(\sigma)\cap\mathcal{B}_x(\sigma)} u(y)dy$. The intersection $\mathcal{R}_x \cap \mathcal{B}_x$ consists of agents whom x rejected, but they, on the contrary, accepted x. This would imply that $\mathcal{S}(x \mid y) \neq \mathcal{S}(y \mid x)$ which cannot be the case, as a result of Nash bargaining. Hence, $\mathcal{R}_x \cap \mathcal{B}_x = \emptyset$ and, in turn, $\int_{\mathcal{R}_x(\sigma)\cap\mathcal{B}_x(\sigma)} u(y)dy = 0$. The weighting must correspond to the actual frequencies with which the situation is visited, however, by construction the situation of non-mutual acceptance is impossible, thus the correct weighting is zero. However, due to the coarsity of reasoning agent x still averages over the the whole spectrum of types. Hence, the probability with which an agent x believes every other agent to accept her is,

$$\beta_x = \frac{\int_{\mathcal{M}_x(\sigma)} u(y)dy}{\int_{\mathcal{A}_x(\sigma)} u(y)dy + \xi \int_{\mathcal{R}_x(\sigma)} u(y)dy} \tag{3.10}$$

Conditional on x accepting, we can write her belief of being accepted as $\beta_x \int_{\mathcal{A}_x} u(y)dy$, and the actual probability with which another agent accepts her, conditional on her accepting, is $\int_{\mathcal{M}_x} u(y)dy$. Since we know that, by proposition 1, $\mathcal{M}_x \equiv \mathcal{A}_x$, we can easily compare,

$$\left[\frac{\int_{\mathcal{M}_x \equiv \mathcal{A}_x} u(y)dy}{\int_{\mathcal{A}_x} u(y)dy + \xi \int_{\mathcal{R}_x} u(y)dy}\right] \int_{\mathcal{A}_x} u(y)dy < \int_{\mathcal{A}_x} u(y)dy, \forall \xi \in (0, 1] \tag{3.11}$$

That is, an agent *underestimates* the probability with which she is accepted whenever she, with some frequency (i.e. $\xi \neq 0$), observes the action of those whom she rejected. Denote an agents *true* expected present value as unmatched by $r\Omega^\star(x)$, and as before, her *perceived* expected present value as unmatched by $r\Omega(x)$. As the agent underestimates the probability with which she is accepted, she will in turn underestimate her own expected present value, i.e. $r\Omega(x) < r\Omega^\star(x)$. If we consider the value equation (3.7), then clearly $[\mathcal{S}(x \mid y) \mid r\Omega(x)] > [\mathcal{S}(x \mid y) \mid r\Omega^\star(x)]$ which implies a shift upwards in the match surplus function. When agents underestimate their prospects on the market, they are willing to accept other agents within a larger interval. From Figure 3.1, with rational expectations, agents are only willing to accept other agents within the interval depicted in (a). When agents are boundedly rational, they mistakenly underestimate their prospects on the market and become willing to accept agents within a larger interval.



(a) $\mathcal{S}(x \mid y)$ with rational expectations (b) $\mathcal{S}(x \mid y)$ with bounded rationality

Figure 3.1: $\mathcal{S}(x \mid y)$ shifts up for all agents when the bounded rationality reduces every agent's value.

By relaxing the assumption of full rationality, we are moving further away from an optimal allocation being agents matching with other agents, as similar as possible. The optimal allocation is determined by the production function which remains the same as the benchmark case, however, the boundedly rational agent fails to perfectly understand the market which leads to suboptimal choice of strategy given this production function. Hence, *compared to the case of rational expectations, everyone are worse off in equilibrium and we have lower efficiency in the presence of bounded rationality.*

3.4.3 Exogenous Variables

The variable Φ (see equation (3.7)) is a collection of exogenously determined variables which sets the level of search frictions on the market. Reduced search frictions (increased Φ) makes it less costly for agents to search, and will, at least partly, compensate for their behaviour due to their bounded rationality. The variable ξ determines the probability with which an agent observes the choice of action of another agent, whom she rejected. From equation (3.11), an agent will underestimate her value as unmatched only if $\xi \neq 0$. Hence, if agents, for sure, never can observe the choice of their rejections, they will behave *as if* they have rational expectations. These variables are interesting as they are outcomes of policy decisions and how markets are designed, thus are possible to affect.

3.4.4 Matching Patterns

In section 3.3 I presented some important results from Shimer & Smith (2000) concerning the matching patterns. Bounded rationality reduces every agents unmatched value, which in turn shifts the match surplus function upwards. However, the property of single-peaked preferences remains and so, in turn, convexity of matching sets. Bounded rationality never entered the production function, and thus, Assumption 1, 2, 3 and 4 remains intact. The supermodularity conditions together with the fact that there still exists an upper- and a lower boundary of each agent's matching set, although being the boundaries of a now larger set, implies that the market remains to be characterized by positive assortative matching. In the non-frictional setting, PAM implied a preference for *identical type* matching. Introducing frictions reduced every agents unmatched value, and PAM became such that agents preferred matching to *similar* individuals. Bounded rationality further reduces an agents unmatched value, and agents become even less picky, accepting an even wider span of agents. Hence, bounded rationality does not affect the possibility of using assumptions 1 through 4 (see section 2.3), and thus results 1 through 3 (see section 3.3) will still hold. At this point we can compare the concept of core allocation and equilibrium. The core allocation remains to be positive assortative matching as the above mentioned assumptions, and thus also the benchmark results still holds. However, due to bounded rationality, agents equilibrium choice of strategy is changed. Although the new strategy profile is individually optimal given the system of beliefs, which in turn is sequentially rational given that strategy profile, the bounded rationality moves us away from the optimal allocation as seen from the whole economy as agents accept even less similar matching partners. Thus, bounded rationality affects equilibrium behaviour such that we move further away from the optimal allocation, which in itself is a point unaffected by the introduction of bounded rationality. Hence, we get worse outcomes in comparison to the case of rational expectations.

Chapter 4

Conclusion and Comparison

To my knowledge, only one study has included bounded rationality in a search and matching model, more specifically the study by Antler & Bachi (2019). These authors analyzed a search and matching model with search frictions and nontransferable utility and found that, once relaxing the assumption of full rationality, reduced search frictions led to some agents searching for a matching partner for longer than optimal. This was due to coarse reasoning and selection neglect which resulted in overoptimism concerning one's prospects on the market, and underestimation of the time to achieve a better match. The purpose of this thesis is to, in a similar manner, introduce bounded rationality, this time to a market with transferable utility. The baseline model is that of Shimer & Smith (2000) which has, as a benchmark, clear and proven matching patterns and properties, hence I can get straight to the point of including bounded rationality. In section 4.1 I cover the conclusions and interpretations of the results. Section 4.2 covers a comparison of the results in this thesis with those found in Antler & Bachi (2019) with nontransferable utilities. Finally, I end the thesis with some possible extensions of this study for future research.

4.1 Conclusions and Interpretations

Agents being boundedly rational is illustrated by β_x, which in turn was determined by the assumption that agents' reasoning was as coarse as it could be, i.e. only having one analogy class. Transfers work as match equalizers. That is, an agent matching with a relatively higher valued partner, expects to abstain from a relatively larger transfer such that the two agents can enjoy equal parts of the total surplus. Equal parts of the surplus was a result of agents having identical bargaining power. Because of the match equalizing property of transfers, agents has both upper and lower boundaries to what agents they will accept a match with. More specifically, agents has single-peaked preferences. Below or above these boundaries agents would never accept a match since it would imply a negative personal match surplus, and they would then be strictly better off by continuing their search for a better option. The boundedly rational agent knows her own acceptance set, however, she coarsly predicts other agents' acceptance sets, i.e. their behaviour. She will average the probability of any other agent accepting her over the whole spectrum of agents. As the total match surplus is split equally between the two matched agents, only mutual acceptance is possible, hence, an agent's acceptance set is equivalent to her matching set. The bounded rationality implies that the agent does not understand that whomever she accepts, actually, in turn accepts her. Thus, by failing to note this, and treating all agents and possible situations in one analogy class, she will underestimate her own expected value as unmatched. This implies that the agent sets her reservation transfer below that of the benchmark case and she will accept other agents within a larger interval. Reduced search frictions (e.g. increasing μ) will act against this underestimation implied by the bounded rationality, and may thus positively contribute towards agents behaving *as if* they were more rational. Bounded rationality does not affect the actual production function, nor the convexity of matching sets. This implies that positive assortative matching remains to be the core allocation, although, agents now accept even *less similar* matching partners.

4.2 Comparing to Antler & Bachi (2019)

In the market depicted by Antler & Bachi (2019) bounded rationality and selection neglect led to agents believing that all other agents were achieveable when, in fact, some were out of their league. Utility was increasing in types across the whole spectrum, which implied that agents overestimated their own expected value as unmatched, and in turn their prospects on the market. Hence, some agents found it worthwhile remaining in the search process for longer than optimal since their expected value of being unmatched, partly being based on their belief of achieving a top-valued agent, exceeded that of being matched (to someone who was *actually* achievable). Agents' *perception* of the probability with which they were accepted by all other agents was thus greater than the *actual* probability. However, the top-valued agents, still boundedly rational and neglecting selection, did actually behave as if they were fully rational. The reason for this is that the top-valued agents were actually correct in their belief. They thought that everyone was achievable, and everyone was as well, i.e. everyone would accept the top-valued agents. Additionally, agents had only a lower boundary of their acceptance set, since the higher the value of their match partner, the better. These differences make clear that, in this thesis, agents have single-peaked preferences as opposed to Antler & Bachi (2019) where utility is strictly increasing in matching partner's type. As opposed to this thesis, reduced search frictions in Antler & Bachi (2019) will not work against agents' flawed perceptions, but rather enhance what the bounded rationality implies. In such a case, it may actually be desirable to keep agents down to earth by implementing search frictions. When agents are able to transfer utility within the matched pair, the opposite is true. When agents underestimate their own unmatched value, they would thus tend to remain in the search process *shorter* than optimally so. No agent in this model can escape their shortcomings in reasoning, as seen in top-valued agents in Antler & Bachi (2019). Finally, note that selection neglect in Antler & Bachi (2019) was illustrated as agents believing that anyone whom they would accept, would in turn accept them. Higher valued agents were more selective, and so this perception was in fact not true. In this thesis, this perception is, on the contrary, true which implies that that kind of selection neglect does not have any effect. In conclusion, the results of this thesis are opposite those of Antler & Bachi (2019).

4.3 Future Research and Extensions

In this thesis I considered the bargaining being solved by the Nash bargaining solution. A possible extension would be to consider another solution concept to the bargaining problem, as there are studies showing that the Nash solution is not really supported by experimental results (Schellenberg 1990), although being a good general description of how agents tend to approach cooperative agreements. Additionally, the bargain could, instead of being a certain game, be characterized by uncertainty in which agents form beliefs. This would allow for extending the concept of bounded rationality into the bargaining problem. This could in turn alter an agent's expectations of her final payoff, and potentially change her behaviour in the previous search equilibrium.

In this thesis, search frictions were modelled as time-discounting and probabilities of agents matching after having searched. Depending which specific market we would like to model, other implementations may be more or less appropriate. Different assumptions on search frictions lead to different requirements for maintaining certain matching patterns (see e.g. Atakan (2006) for fixed search costs and requirements for PAM). Hence, the equilibrium behaviour may also be affected by how we introduce search frictions.

Bibliography

Antler, Y. & Bachi, B. (2019), 'Searching forever after'.

Atakan, A. E. (2006), 'Assortative matching with explicit search costs', *Econometrica* **74**(3), 667–680.

Becker, G. S. (1973), 'A theory of marriage: Part i', *Journal of Political economy* **81**(4), 813–846.

Bloch, F. & Ryder, H. (2000), 'Two-sided search, marriages, and matchmakers', *International Economic Review* **41**(1), 93–116.

Chade, H., Eeckhout, J. & Smith, L. (2017), 'Sorting through search and matching models in economics', *Journal of Economic Literature* **55**(2), 493–544.

Jehiel, P. (2005), 'Analogy-based expectation equilibrium', *Journal of Economic theory* **123**(2), 81–104.

Schellenberg, J. A. (1990), ''solving'the bargaining problem', *Mid-American Review of Sociology* pp. 77–88.

Shimer, R. & Smith, L. (2000), 'Assortative matching and search', *Econometrica* **68**(2), 343–369.