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Comparing the Liquidity-Adjusted Expected Shortfall Models Over High and Low Liquid Stocks Portfolios: Empirical Results on Thailand Stock Market

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Abstract

The stylized fact that stock markets are not perfectly liquid propels banks to incorporate liquidity risk in the risk metrics so that market risk can be managed properly. Disregarding liquidity risk can lead to an underestimation of overall risk and substantial losses. This is particularly true in emerging markets where illiquidity problem is more severe than in developed market economies. Liquidity risk can be measured by exogenous and endogenous variables, such as liquidity horizon, bid-ask spread and liquidity discount. This thesis attempts to evaluate the performance of liquidity-adjusted models incorporating exogenous and endogenous variables to estimate the expected shortfall (ES) over high and low liquid stocks portfolios in Thailand exchanges over the last 10 years. The evidences from model estimations, statistical inference and backtesting show that the regulatory liquidity-adjusted ES model, which uses liquidity horizons as liquidity adjustment, is sufficient for the banks holding low liquid stocks (small market capitalization stocks) over the evaluation period. However, all the liquidity-adjusted ES models in this study underestimate the losses on high liquid stocks (large and mid market capitalization stocks). One potential reason is that high liquid securities in Thailand have greater risk in terms of their quicker reaction to market changes and more transactions than low liquid securities, which can be seen in more volatile movements.

Keywords: expected shortfall, liquidity adjustment, bid-ask spread, liquidity discount, liquidation time

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Table of Contents

1. Introduction

Banks are exposed to risks when they trade market instruments in portfolios, and they can incur losses due to the uncertainty of the market. In addition, they are required to measure and quantify the level of the risk exposure and determine whether they have sufficient capital reserves in place to compensate the losses. Generally, value-at-risk (VaR) is used as a standardized risk measure to calculate the minimum capital requirements. However, prior studies, such as Lawrence and Robinson (1995), Jarrow and Subramaniam (1997) and Bangia et al. (1999), suggest that a drawback of the traditional VaR model is that it is not able to capture liquidity risk. This risk metric is based on the assumption that all assets are equally liquid and that traders can liquidate their assets without causing a significant price change, which is seldom verified in practice. Since the liquidity risk is neglected in this risk metric, which leads to an underestimation of the real risk and cannot comprehensively reflect the real risk, the recent subprime crisis has led researchers and analysts to cast liquidity risk in the role of the culprit (Dullien et al., 2010). As a result, in 2012, the Basel Committee on Banking Supervision (BCBS) decided to retire VaR and proposed to use liquidity-adjusted expected shortfall (ES) model (BIS, 2013). This model incorporates five categories of liquidity horizons and takes the effects of illiquidity into consideration. Therefore, it is used as a regulatory treatment to revise and improve the framework.

According to Bangia et al. (1999), disregarding liquidity risk would underestimate the overall risk by 25 to 30 percent. An efficient risk measure should, therefore, involve liquidity risk, which motivates the recent academic papers to incorporate liquidity risk into the VaR-model to fill the gap between conventional VaR concept and practical considerations. Some liquidity-adjusted VaR models, introduced by Jarrow and Subramaniam (1997), bid-ask spread model, proposed by Bangia et al. (1999), and market price response model, suggested by Berkowitz(2000b) and Cosandey(2001), are becoming popular. These models use liquidity risk proxies, such as liquidity discount, execution time lag, bid-ask spread, market price responding to the trading, to calculate the liquidity risk. Meanwhile, these models take into account endogenous or (and) exogenous liquidity risk and are, therefore, expected to improve the existing measurement of total market risk, before the official launch of the regulatory liquidity-adjusted ES model. According to Roy (2005), incorporating liquidity risk in the risk measure is particularly important for emerging stock markets, such as stock exchange of Thailand (SET), where the illiquidity problem is more severe than in developed markets and the process of financial sector reform is currently taking place. One of the evidences is the low turnover ratio of stock shares. *Figure 1* shows the gap between Thailand exchanges and the world market between 2007 and 2018.

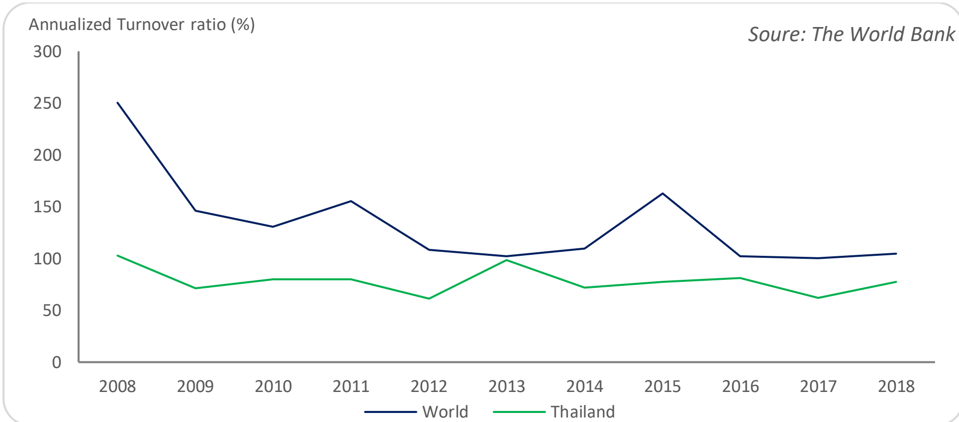

Figure 1: Stock share turnover (world vs. Thailand)

According to the World Bank, turnover ratio is defined as the value of domestic shares traded divided by their market capitalization. The value is annualized by multiplying the monthly average by 12. The higher the ratio, the more liquid the stock shares are. As shown in *Figure 1*, Thailand has a lower share turnover throughout the period, which reflects a less liquid stock market. In addition, it poses a challenge for banks that trade illiquid assets, because the illiquidity problem in this market can lead to the underestimation of overall risk. Banks will experience more-than-expected violations of capital requirements and will occur substantial losses if they hold portfolios in this market. Therefore, it is necessary for more and more banks to prepare for more sophisticated risk measurement models than conventional VaR model in the context of the Basel II.5 recommendations.

This thesis attempts to give an updated perspective on the implementation of regulatory liquidity-adjusted ES model, bid-ask spread model and liquidity discount model on all the stocks listed in Thailand exchanges over a 10-year period (March 31 2010 to March 31 2020). It examines and compares their performance on high and low liquid portfolios in Thailand exchanges by backtesting. It focuses on whether the regulatory approach outperforms other two liquidity-adjusted ES models on high and low liquid Thai stocks over the evaluation period and whether it is sufficient for banks to implement only the regulatory approach for Thailand exchanges.

The contribution of this study is that it shows how liquidity is incorporated into the ES measures. It also compares the performance of regulatory liquidity-adjusted ES model, bid-ask spread model and liquidity discount model and examines their accuracy, which provides practical information to banks holding portfolios of Thai stocks.

Due to the complexity of this study, there are several boundaries that are inevitable. In this study, the parameters, such as the scaling factor used in the bid-ask spread model and the parameters used in the price-drop function, are theory based. They follow the original assumptions in recent studies and are applied to the models in the same way. The hypothetical portfolios (high, mid, and low liquid portfolios) are constructed based on the assumptions that (1) a bank allocates equal weight of money in each security, (2) a bank re-invests in the portfolio every day, (3) it is possible to invest in any security in the market at any time, and (4) banks trade stocks at the end of each day (no intraday trading), to simplify the estimation. Meanwhile, the large, mid and small-caps are separated based on the criteria defined by Financial Times Stock Exchange (FTSE). However, the definition may differ across markets.

The thesis is divided into 7 sections. Sections 2 discusses theoretical background and literature review. The focus is set on the development of existing methodologies for incorporating liquidity risk into VaR. Section 3 describes the models and backtests that are implemented in this study. Section 4 discusses the data and descriptive statistics and Section 5 provides the empirical results. Section 6 concludes the thesis. In the Appendix, numerical examples of liquidity-adjusted ES methods are given, and the issue of estimating 10-day ES using overlapping data is discussed.

2. Literature Review

2.1 Liquidity

The notion of liquidity was first developed by John Maynard Keynes in 1936. According to Keynes, liquidity refers to the ease with which an asset can be converted into cash and people prefer to have liquidity to assure basic transactions, especially when they face social unexpected problems and when they might bear unusual costs . In early papers on liquidity, Bagehot (1971) and Black (1971) describe liquidity as the trade-off between price and immediacy. If an investor needs great volume of money immediately, it will cause a remarkable change in the stock price. In other words, the higher the immediacy, the higher the price.

Black (1971) uses the concepts of depth (the number of stocks that can be traded at a given price), breadth (the ability to trade across assets without price effects) and resilience (the time needed to recover from random shocks) to define different dimensions of liquidity. He also suggests market participants, policymakers, and regulators to consider these market characteristics to encourage price stability and ensure sufficient liquidity.

Today, as the market is getting more complicated and developed, more factors should be taken into account when describing liquidity. Dowd (2002) defines liquidity as a function of the market and it depends on many factors, such as the number of traders in the market, the time it takes for the trade to be carried out, the size of trades, the cost of transacting and economic climate. A market is more liquid and complete if there are a large number of traders in the market and the frequency of trades is high compared to a small market. However, even if a market is large and liquid, Getter et al. (2007) suggest that, during a financial crisis, the liquidity can fall remarkably.

2.2 Market Liquidity Risk

Grossman and Miller (1988) describe liquidity risk[1] that investors in the market trade their assets at prices different from those currently quoted. Lawrence and Robinson (1995) are among the first to establish that traditional VaR models often ignore liquidity risk. They also argue that liquidity should be captured in the VaR framework to give more accurate estimate of market risk. One year later, they implement a stochastic horizon model and apply a unique horizon to all the positions. They conclude that this method is inappropriate and that the shorter the holding period, the more one underestimates the VaR.

Later, Fernandez (1999) argues that rapid structural changes of financial markets result from liquidity risk and these changes increase concentration along with rising homogeneity of traders' behavior. Fernandez also argues that liquidity cannot be easily captured in financial markets and suggests to measure the liquidity by incorporating quantitative and qualitative factors.

To measure the liquidity risk, Bangia et al. (1999) split the liquidity risk into two components. The first part is exogenous component which is the result of the market movement and it cannot be affected by any player in the market. When a trader executes his stock position in the market, it has no significant effect

[1] Liquidity risk can be divided into two groups: market (asset) liquidity risk and funding (cash flow) liquidity risk. The focus in this study is on the market liquidity risk.

on the price movement because this transaction is too small relative to the market, so the market price movement cannot be controlled by any participant. Transaction costs, such as bid-ask price quote, are usually used as a good proxy for this component. The second part is endogenous liquidity risk which means that each individual action has an impact on asset price and that it varies across time periods and markets. If a trader's bid (ask) exposure is large and he wants to open the position immediately, then it is difficult for him to find the counterparty and the market price may increase (decrease) to reflect the high demand (excess supply). Since the endogenous illiquidity influences specific positions and investors' reactions may be different, some researchers suggest that it can be captured by unwinding costs. Le Saout (2002) suggests that financial institutions should neither ignore exogenous liquidity risk nor endogenous liquidity risk to comprehensively reflect overall risk.

2.3 Liquidity-adjusted Models

Bangia et al. (1999) measure the exogenous liquidity risk by using half bid-ask spread of the asset price and identify time-varying proportional bid-ask spread as the cost of liquidity. To implement this in the VaR model, the liquidity cost is added, resulting in a higher VaR. It is noted that if the correlation between the mid-price and the spread is low, it becomes more essential for analysts to incorporate liquidity component due to the fluctuating market conditions. Le Saout (2002) extends Bangia et al. (1999) model by applying the weighted average spread (WAS) which is functional of the stock trading volume. Ernst et al. (2012) argue that the liquidity cost is not normally distributed and tend to underestimate the worst-case losses. They apply the Cornish–Fisher approximation to incorporate the non-normality in the liquidity risk.

Jarrow and Subramanian (1997) estimate liquidity-adjusted VaR by solving an optimal liquidation problem – *the traders must liquidate their position within a fixed horizon to maximize the expected utility*. This approach uses (1) liquidity discount, (2) the volatility of liquidity discount, and (3) the time horizon until liquidation to define the optimal liquidation strategy. The first and second components are set as endogenous factors and represent the price discount due to the price impact after selling shares. The third component is set as exogenous factor and shows that traders cannot execute the position whenever they want. The authors assume that the stock prices follow Geometric Brownian Motion (GBM) process and all traders are risk neutral. The problem is then when traders want to unwind the position S shares by a fixed time T. They will be offered a lower buying price and delayed exposure, which will modify the parameters in GBM, adding the discount factor $c(S)$, and the execution time lag $\Delta(S)$.

Almgren and Chriss (1999) identify the strategy to execute portfolios in an illiquidity situation. Since the traders are faced with a situation of higher cost to sell the position quickly but lower execution time to reduce their exposure or vice versa, they suggest that the optimal trading strategies is to trade-off between the transaction cost and exposure cost. They also examine endogenous liquidity risk using their strategy, and their analysis provides new insights into optimal portfolio trading incorporating liquidity risk.

Hisata and Yamai (2000) generalize the model of Almgren and Chriss (1999) and formulate an optimal strategy by minimizing liquidation cost impact while using a non-linear relation between the traded volume and market impact variable. The market impact is caused by traders' behavior and the size of their positions relative to the size of the market. However, this approach assumes a simple valuation of the impact cost and it is, therefore, deficient.

Damodaran (2005) measures the effect of illiquidity on firm valuation. His study mainly focuses on private stocks. He runs the regression on bid-ask spread which is used as the proxy of illiquidity against the size of firm and trading volume. He also values the illiquidity as an option. The liquidity discount on the underlying asset whose owner is restricted from trading for a certain period is modeled as a put option. He concludes that the liquidity discount varies across firms and that the median of liquidity discount is at about 35 percent.

Lau and Kwok (2006) use price drop function parameter to define the liquidity discount introduced in Jarrow and Subramanian (1997) model. They consider a trader holding a portfolio of riskless and risky assets with the objective to maximize the expected value of riskless assets at the end of a fixed time horizon T. They simulate price movements with trinomial tree and assume that selling S units of risky assets at inter-period will affect the price drop in the deterministic function $\alpha(S)$. The price drop is a non-negative and non-increasing function, valued between zero and one. They conclude that the difference between the market value of the asset and its value when liquidated is characterized by the liquidation discount. They also suggest that the proportion of the loss in the liquidation process depends on the price impact function. Botha (2008) extends the model from stock level to portfolio level and incorporates EWMA technique in Jarrow and Subramania (1997) liquidity-adjusted VaR model.

In May 2012, BCBS published the Fundamental Review of the Trading Book (FRTB). The aim was to improve resilience of the bank sector by strengthening the capital standards for market risks, and the document has reflected Basel Committee's increased focus on reforming bank regulatory standards in response to the financial crisis.

Due to the aforementioned 2008 financial crisis, the Basel Committee has realized the importance of assessing the market liquidity risk. Before the introduction of the Basel II.5, the market risk framework was based on an assumption that all assets are equally liquid. However, this assumption turned out to be false in the financial crisis when banks were forced to hold risk positions for a much longer period of time, which led to substantial losses. Therefore, in the FRTB, in addition to a shift from VaR to ES, the Committee required to compute the ES on a daily basis and use a holding period of at least 10 days. The Committee also proposed an approach which incorporates the market liquidity risk comprehensively in regulatory market risk metric under the assumption that banks are able to shed the risk in a defined period of time. This approach is based on the concept of "Liquidity Horizon", defined as the number of trading days required to sell a position under stressed market conditions without moving market prices. Banks' exposures are assigned into five liquidity horizon categories: 10, 20, 40, 60 and 120 days (LH_1, LH_2, LH_3, LH_4, LH_5, respectively).

In addition, the Committee has discussed the feasibility of incorporating endogenous liquidity risk in the framework. It means that when a bank is assigning the liquidity horizons, it would be required to take into account trading behavior and the size of its exposures relative to the market. If the market environment is not favorable and the bank holds a large position, the liquidity horizons have to be extended to reflect higher liquidity risk.

3. Methodology

3.1 A Summary and Comparison of Different Approaches

As outlined in the literature review, there are three main groups of different approaches. It might be a good idea to give a summary of the features of these groups before moving on. The features that are compared are time to liquidation, spread cost and liquidity discount. *Table 1* shows the summary of the models.

Model	Time to Liquidation *(Exogenous)*	Spread Cost *(Exogenous)*	Liquidity Discount *(Endogenous)*
Regulatory liquidity-adjusted	Yes	No	No
Bid-ask spread	No	Yes	No
Liquidity discount	Yes	No	Yes

Table 1: Summary of the features of the liquidity-adjusted models

As shown in *Table 1*, different models have different properties. For example, although the regulatory liquidity-adjusted ES model does not integrate spread cost and liquidity discount, it incorporates varying liquidity horizons. Meanwhile, it requires banks to calculate capital using a constant level of risk. It is a new departure from current framework and provides further guidance to banks on incorporating liquidity in measuring overall market risk.

Most existing bid-ask spread models are developed on the basis of Bangia et al. (1999) model that incorporates the cost of liquidity. One of the advantages of these models is that they require relatively low data. The spread cost is available for most stocks in the markets. In addition, they are easy to implement, because the liquidity adjustment can be simply added in the risk metrics. Although they do not integrate varying liquidity horizons and liquidity discount, they can perform well, particularly when data is scarce.

Liquidity discount models incorporate both exogenous and endogenous liquidity risk. However, one drawback is that these models are sophisticated and require much more information than the regulatory liquidity-adjusted ES model and bid-ask spread models, so it can be difficult to implement.

As mentioned above, the objective of this study is to identify whether the regulatory liquidity-adjusted ES model outperforms academic liquidity-adjusted ES models, hence, one model from each group discussed above will be selected in order to simplify the estimation, implementation and comparison. With regard to bid-ask spread models, since Bangia et al. (1999) model forms the foundation of existing exogenous liquidity models, and considering the advantage of this model that it is easy to implement, Bangia et al. (1999) model will be applied and extended in this study. Jarrow and Subramanian (1997) model (JS model) will be selected from liquidity discount model group because this model incorporates both exogenous and endogenous liquidity risk, despite the difficulty of implementation. In addition, this model will be extended using a price drop parameter introduced by Lau and Kwok (2006). To be in line with Basel II.5 requirements, all the models that are selected will be adjusted from VaR to ES.

In short, the regulatory liquidity-adjusted ES model, the bid-ask spread model based on Bangia et al. (1999) model and the liquidity discount model based on JS (1997) model will be implemented in this study. The models will be incorporated with simple moving average (SMA), exponentially weighted moving average

(EWMA, simplified GARCH), normal distribution and student t-distribution assumptions in assets' return. In addition, a 97.5 percentile and one-tailed confidence level is used in calculating ES.

3.2 Regulatory Liquidity-Adjusted ES (Standard) Model

We assume the losses follow a continuous distribution and we focus on parametric approaches to estimate ES. Equation 3.1 shows the ES estimation following a normal distribution and equation 3.2 shows the standard model following the FRTB method.

$$ES_\alpha = \mu + \sigma \frac{f(z_\alpha)}{1-\alpha} \tag{3.1}$$

$$ES_{BIS} = \sqrt{ES_1^2 + \sum_{j=2}^{5} ES_j^2 \frac{LH_j - LH_{j-1}}{10}} \tag{3.2}$$

where μ and σ: daily mean and simple moving average (SMA) volatility of the loss;
α: the confidence interval level;
z_α: the α quantile for standardized normal distribution;
$f(z_\alpha)$: the pdf of standardized normal distribution;
ES_1: ES estimate which includes all the assets in the portfolio;
ES_j: ES estimate which removes assets in categories 1 to $j-1$;
LH_j: liquidity horizon for category j.

In equation 3.2, as mentioned earlier, there are five liquidity horizon categories for the risk factors across all the assets. According to the new framework, ES is scaled based on a 10-day base horizon as a viable method to arrive at the liquidity-adjusted ES. Then varying liquidity horizons are incorporated in the ES to capitalize the risk that banks are unable to exit positions in a short period of time. For example, if an investor holds small-cap stocks only, the liquidity-adjusted ES will be scaled up by $\sqrt{2}$ as shown below.

$$ES_{BIS} = \sqrt{ES_1^2 + ES_2^2 \frac{20-10}{10}} = \sqrt{ES_1^2 + ES_1^2 \frac{20-10}{10}} = \sqrt{2} ES_1$$

where ES_1 is computed by including all the assets in the portfolio and ES_2 is computed by taking away high liquid assets (assets with a 10-day horizon). In this case, since we only have small caps, $ES_1 = ES_2$.

Considering the fact that generally, financial returns are not normally distributed, the parametric model in equation 3.1 is, therefore, extended to equation 3.3 which incorporates volatility clustering and fat-tail on losses distribution.

$$ES_\alpha = \mu + \sqrt{\frac{v-2}{v}} \sigma_{EWMA} \frac{f^*(t_{\alpha,v})}{1-\alpha} \left(\frac{v + t_{\alpha,v}^2}{v-1} \right) \tag{3.3}$$

where σ_{EWMA}: exponentially weighted moving average (EWMA) asset loss volatility;
v: the degree of freedom;
$t_{\alpha,v}$: the α quantile for standardized student t-distribution;
$f^*(t_{\alpha,v})$: the pdf of standardized student t-distribution.

EWMA volatility is estimated by the following equation where lambda is the smoothing parameter.

$$\sigma_{EWMA}^2 = \frac{1-\lambda}{1-\lambda^T}\sum_{t=1}^{T}\lambda^{t-1}\,\sigma_t^2 \tag{3.4}$$

As shown in equation 3.4, EWMA gives greater weight on recent variance and it can be interpreted as a simplified version of a GARCH (1,1) model. According to *RiskMetrics^{TM}*, the lambda is set to 0.94. The length of estimating the volatility in this study is 245 days, which is the average annual working days for Thailand financial market.

3.3 Bid-Ask Spread (BAS) Model

Bid-ask spread model is a practical approach which incorporates liquidity risk. The bid price is quoted for the trader who wants to sell and the ask price is quoted for the buyer. The existence of two on-going market prices is due to the fact that market orders (purchases and sales) are not immediately executed even in the liquid market. These two market prices are considered as the additional risk that traders have to face when they execute their positions. The quicker the sell or the purchase, the lower or the higher price the trader has to submit. Hence, in this model, we use the half of the difference between the ask price and the bid price (spread) to be the liquidity cost. This cost is the liquidity premium paid beyond the market price and the spread is considered as the exogenous liquidity risk because the spread itself is random.

The cost of liquidity proposed by Bangia et al. (1999) is used as the exogenous liquidity component in the model (model in equation 3.1). The model assumes that the relative spread is normally distributed and has excess kurtosis, so the liquidity cost (LC) is described as following:

$$LC = \frac{P}{2}\left(\mu_{Spread} + k\sigma_{Spread}\right) \tag{3.5}$$

where P: the current asset value;
$Spread = (Ask\ Price - Bid\ Price)/Mid\ Price$: the relative spread;
μ_{Spread}: the sample mean of relative spread;
σ_{Spread}: the sample standard deviation of relative spread;
k: the scaling factor to accommodate a heavy-tail on the spread distribution.

According to equation 3.5, the spread is assumed to be normally distributed with some level of excess kurtosis, so the ES would be then adjusted as the following equation:

$$ES_{BAS} = \underbrace{\left\{\mu + \sigma\frac{f(z_\alpha)}{1-\alpha}\right\}}_{\text{ES model - eq 3.1 (only market risk)}} + \underbrace{\frac{P}{2}\left(\mu_{Spread} + k\sigma_{Spread}\right)}_{\text{Liquidity cost (exogenous liquidity adjustment)}} \tag{3.6}$$

In this equation, the liquidity cost is added to the standard ES as the liquidity adjustment. It means that if the spread rises, the total cost of closing contract will increase far off the normal market risk and ES will then rise. Also, if the volatility of relative spread becomes higher, the ES will also be larger. An example of how liquidity cost and bid-ask spread adjust the ES estimation is shown in *Appendix A*.

3.4 Liquidity discount (LD) Model

This extended model takes into account not only exogenous but also endogenous liquidity risk by implementing Jarrow and Subramanian (1997) model and using Lau and Kwok (2006) function and parameter. The mean, volatility, time scaling-up and dropping value will be adjusted to follow the characterization in the JS model. The ES model plus liquidity adjustment is expressed in equation 3.7, and a numerical example of ES estimation based on this equation is given in *Appendix B*.

$$ES_{LD} = \mu[\mu_{\Delta S}] + \mu_{ln\ c(S)} + \left\{\sigma\sqrt{[\mu_{\Delta S}]} + \mu\sigma_{\Delta S} + \sigma_{ln\ c(S)}\right\}\frac{f(z_\alpha)}{1-\alpha} \tag{3.7}$$

where $\mu_{\Delta S} = \frac{S}{\mu_{vol}}$: the expected execution time lag to liquidate the position of underlying asset;

S: the share amount of the underlying asset that trader wants to liquidate;

μ_{vol}: average trading volume for the period, for example 3 months;

$c(S) = \alpha(S)e^{\mu\Delta S}$: the liquidity discount due to the S shares sold;

$\alpha(S) = \frac{0.5}{1-0.5e^{-aS}}$: the price drop function;

a : the instantaneous price drop parameter, 0.000007 in this study, which is the optimal value solved based on a certain average selling shares applying Lau and Kwok (2006) study.

It is obvious that the mean and the variance in this equation are bigger than in equation 3.1. The component ΔS represents the liquidity time horizon as traders cannot execute transactions immediately. It takes more time for large positions to be sold, and consequently, risk arises. $c(S)$ stands for the quantity impact on asset price after an order is placed as the endogenous liquidity, $0 \le c(S) \le 1$. This adjustment factor can be considered as how the stock price changes after the liquidation. In other words, it is regarded as a price discrepancy between the price of security at the time when the order is placed and the price when the order is executed. For example, when a market participant places a selling order which has large positions to liquidate, he will naturally lower the offer price, and the price is, to some extent, discounted. Given the current time is 0, and the price at time t is $p(t)$, the stock price with liquidity adjustment at time $t + \Delta S$ is $p(t + \Delta S) = \alpha(S)p(t)e^{\mu\Delta S}$. The price drop function, $\alpha(S)$, is then added to decrease the stock value, and the price process is assumed to grow at the level $e^{\mu\Delta S}$ during the liquidation time lag. Again, a larger selling of S shares generates a greater price drop impact.

Lau and Kwok (2006) suggest an optimal liquidation strategy and construct a specific price drop function with certain parameters. They define the level of parameter for their portfolio and explain that an appropriate value depends on the number of units sold and the target level of cash in portfolio. Therefore, in this study, the same function introduced in the literature review will be used and the price drop parameter level will be solved based on the average selling shares units for all the portfolios.

It should be noted that if the assets are liquid, traders can unwind any transactions immediately and there will be no price impact from the transactions. It is because the uncertainty in asset price is due to market movements, $\mu_{\Delta S} = 1, \sigma_{\Delta S} = 0$ and $c(S) = 1$ (which implies $\mu_{ln\ c(S)} = 0$ and $\sigma_{ln\ c(S)} = 0$). Then the ES estimation will be the same as the conventional model.

The general assumption on average portfolio returns to zero is allowed because it is difficult to find a good estimation of true mean (JP Morgan, 1996) and incorporate both the EWMA volatilities and student t-distribution into the ES estimation. Therefore, the full-scale final model is:

$$ES_{LD,EWMA,t-dist} = \sqrt{\frac{v-2}{v}}\left\{\sigma_{EWMA}\sqrt{[\mu_{\Delta S}]} + \mu\sigma_{\Delta S} + \sigma_{ln\ c(S)}\right\}\frac{f^{*}(t_{\alpha,v})}{1-\alpha}\left(\frac{v+t_{\alpha,v}^{2}}{v-1}\right) \tag{3.8}$$

The distributions of loss, execution time lag, logarithm of liquidity discount in equation 3.8 are assumed to have a fat-tail at the degree of excess kurtosis of underlying traded assets' losses. The degree of freedom will be defined based on the tail behavior of the underlying assets for the respective evaluation period.

Table 2 summarizes the liquidity-adjusted ES models that will be implemented in this thesis.

ES Model	Normal dist. & SMA volatility	Normal dist. & EWMA volatility	Student t-dist. & SMA volatility	Student t-dist. & EWMA volatility
Conventional*	$\sigma\frac{f(z_\alpha)}{1-\alpha}$	$\sigma_{EWMA}\frac{f(z_\alpha)}{1-\alpha}$	$\sqrt{\frac{v-2}{v}}\sigma\frac{f^*(t_{\alpha,v})}{1-\alpha}\left(\frac{v+t_{\alpha,v}^2}{v-1}\right)$	$\sqrt{\frac{v-2}{v}}\sigma_{EWMA}\frac{f^*(t_{\alpha,v})}{1-\alpha}\left(\frac{v+t_{\alpha,v}^2}{v-1}\right)$
BAS	$\sigma\frac{f(z_\alpha)}{1-\alpha}+\frac{P}{2}k\sigma_{Spread}$	$\sigma_{EWMA}\frac{f(z_\alpha)}{1-\alpha} + \frac{P}{2}k\sigma_{Spread}$	$\sqrt{\frac{v-2}{v}}\sigma\frac{f^*(t_{\alpha,v})}{1-\alpha}\left(\frac{v+t_{\alpha,v}^2}{v-1}\right) + \frac{P}{2}k\sigma_{Spread}$	$\sqrt{\frac{v-2}{v}}\sigma_{EWMA}\frac{f^*(t_{\alpha,v})}{1-\alpha}\left(\frac{v+t_{\alpha,v}^2}{v-1}\right) + \frac{P}{2}k\sigma_{Spread}$
LD	$\left\{\sigma\sqrt{[\mu_{\Delta S}]}+\mu\sigma_{\Delta S} + \sigma_{ln\,c(S)}\right\}\frac{f(z_\alpha)}{1-\alpha}$	$\left\{\sigma_{EWMA}\sqrt{[\mu_{\Delta S}]}+\mu\sigma_{\Delta S} + \sigma_{ln\,c(S)}\right\}\frac{f(z_\alpha)}{1-\alpha}$	$\sqrt{\frac{v-2}{v}}\left\{\sigma\sqrt{[\mu_{\Delta S}]}+\mu\sigma_{\Delta S} + \sigma_{ln\,c(S)}\right\}\frac{f^*(t_{\alpha,v})}{1-\alpha}\left(\frac{v+t_{\alpha,v}^2}{v-1}\right)$	$\sqrt{\frac{v-2}{v}}\left\{\sigma_{EWMA}\sqrt{[\mu_{\Delta S}]}+\mu\sigma_{\Delta S} + \sigma_{ln\,c(S)}\right\}\frac{f^*(t_{\alpha,v})}{1-\alpha}\left(\frac{v+t_{\alpha,v}^2}{v-1}\right)$

Table 2: Conventional model and liquidity-adjusted ES models incorporate different moving average and distribution assumptions

* The regulatory liquidity-adjusted approach can be applied to all the ES models by scaling with a base liquidity horizon of 10 days and computed by:
$\sqrt{ES_1^2+\sum_{j=2}^{5}ES_j^2\frac{LH_j-LH_{j-1}}{10}}$.

3.5 Backtesting ES

ES models are useful only if they predict future risk accurately. In this study, Acerbi and Szekely (2015) backtest and the Traffic Light backtest are performed on ES models to examine the performance of the aforementioned models.

According to Acerbi and Szekely (2015), three model-independent, non-parametric backtests can be used for expected shortfall, and the tests do not assume any asymptotic convergence. The first test assumes that VaR has already been tested before testing ES so that the magnitude of the realized exceptions against the model predictions can be tested separately. The second test follows from the unconditional expectation, and by implementing this test, ES can be tested directly. According to the authors, the second test exhibits a remarkable stability of the critical levels across different tail shapes. The third test backtests the tails of the model and estimates ES from realized ranks. Since the first and the third tests require Monte Carlo simulation of the distributions of the test statistics, which are considered less applicable compared to the second test, the second test is therefore used to backtest the ES in this thesis.

According to the authors, the second test only requires the estimated one day ahead ES for day t and the magnitude of the loss if a VaR violation happens day t. In addition, the authors assume that independence of arrival of tail events is tested separately, and the losses are independently but not identically distributed and follow a continuous distribution.

The hypotheses can be written as:
H_0: $ES^P_{\alpha,t} = ES^F_{\alpha,t}$, saying that ES is correctly estimated by the model for each day t
H_1: $ES^P_{\alpha,t} < ES^F_{\alpha,t}$, saying that ES is underestimated at least one day.

The test statistic Z is defined as:

$$Z = -\frac{1}{T(1-\alpha)}\sum_{t=1}^{t=T}\frac{L_t I_t}{ES_{\alpha,t}(L_t)} + 1$$

where L_t denotes the stochastic loss for day t and I_t is an indicator function which takes the value 1 if a VaR violation happens day t and 0 otherwise:

$$I_t = \begin{cases} 1 \; if \; L_t > VaR_{\alpha,t}(L_t) \\ 0 \; if \; L_t \leq VaR_{\alpha,t}(L_t) \end{cases}$$

If ES method is correct, we expect $\hat{Z} = 0$, because:

$$E[Z] = -\frac{1}{T}\sum_{t=1}^{t=T}\frac{\frac{E\left[L_t : I_{\{L_t > VaR_{\alpha,t}\}}\right]}{1-\alpha}}{\widehat{ES}_{\alpha,t}} + 1$$

$$= -\frac{1}{T}\sum_{t=1}^{t=T}\frac{ES_{\alpha,t}}{\widehat{ES}_{\alpha,t}} + 1$$

Since under the null hypothesis, the ES estimate is correct every day t, which means that $\widehat{ES}_{\alpha,t} = ES_{\alpha,t}$, hence,

$$E[Z] = -\frac{1}{T}\cdot T + 1 = 0$$

If ES method underestimates, we expect $\hat{Z} < 0$ and if ES method overestimates, we expect $\hat{Z} > 0$.

The Traffic Light backtest was originally proposed by the Basel Committee in 1996 (BCBS, 1996). The model can be classified into 3 different backtesting zones: green, yellow and red, and be assigned to one of the zones based on the critical values. As is known, the critical value at the usual 5 percent level of the test statistic is $Z_{crit} = -0.70$. Following a Basel-type traffic light test for underestimation and the suggestion in Acerbi and Szekely (2015), for the ES estimates:

$$Green\ light: Z \geq -0.70$$

$$Yellow\ light:\ -1.80 < Z < -0.70$$

$$Red\ light: Z \leq -1.80$$

The backtests will be performed on the aforementioned models and the empirical results will be discussed in Section 5.

4. Data and Descriptive Statistics

As mentioned above, it is noted that the stocks in Thailand have lower turnover ratio than the world market and that the illiquidity of this emerging market can lead to the underestimation of overall risk. In this study, we will analyze hypothetical portfolios of listed stocks in the Stock Exchange of Thailand (SET) – Thailand national stock market. The close price, bid and ask price, volume and outstanding shares of the stocks are extracted from Bloomberg database.

To estimate the ES and perform annual backtesting, we need at least 245 data length (annual basis = 245 working days), and the data from March 31 2010 to March 31 2020 is collected. This sample period is chosen because both tranquil and volatile periods are incorporated. In addition, this period should be large enough for the backtests. As of Mar 31 2020, the SET has 601 listed companies. *Figure 2* illustrates the movement of SET index and its percentage change over the last 10 years. It shows a stock's return time series and it is noted that the volatility clustering exists.

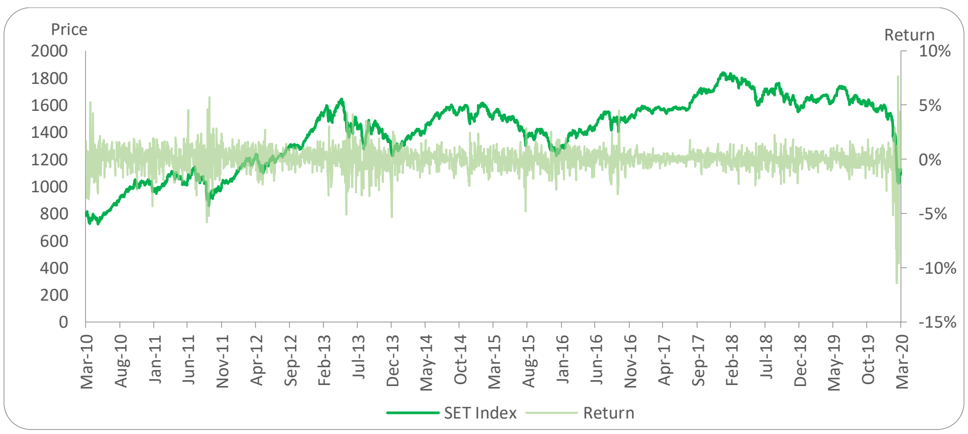

Figure 2: SET index and return (daily)

As we can see in *Figure 2*, the market developed with mild volatility, and there were some relatively stable periods. After the subprime crisis in 2008, the Thai market grew up and the index increased slowly. In the first half of 2013, it reached a new high at 1643.43, and many IPOs were announced. In addition, the market had a significantly high trading volume. However, the market was crashed by the political crisis in May 2014 and the terrorist attack in August 2015, so the price was marked by negative reaction, dropping to 1301.06 on August 24, 2015. When the Brexit was announced in 2016, Thai exchange followed the global trend and the index decreased by 42 point on June 24, 2016. Then, the price rebounded, rose and reached the peak at 1838.96 on January 24, 2018. Both internal factors – the export business and tourism – and external factors – good sign of crude price and the US economy – stimulated the investments in Thai equities. At the beginning of 2019, after the Thai general election, the index moved to around 1600-1700, before declining to less than 1000 due to the spread of Covid-19.

Table 3 shows the descriptive statistics of 9 sample stocks: name, number of available data, market capitalization, the highest-lowest-recent price, annualized average return and volatility, kurtosis, and the highest-lowest-average trading volume during the period. The first 3 highest market caps stocks for each group (large, mid, small caps) are selected as representatives. This gives the empirical evidence that the stock prices are not normally distributed and have excess kurtosis.

Securities**	#N	Market cap. (%)	----------------Price----------------			-------------Return-------------			-------------------Volume-------------------		
			Highest	Lowest	Latest	Mean (%)	Standard deviation (%)	Kurt	Highest	Lowest	Avg. last 3 months
(1) Large Caps.											
PTT	2442	7.3831	58.80	19.80	30.75	1.65	29.14	29.46	643.8360	6.3090	103.0424
AOT	2442	6.0944	81.00	3.13	50.75	26.67	30.59	7.06	465.6460	2.8540	46.9989
ADVANC	2442	5.0232	310.00	73.25	201	8.73	28.18	12.40	94.7518	0.7716	10.4920
(2) Mid Caps.											
TRUE	2442	0.8808	14.54	1.50	3.14	5.68	45.68	8.47	2,178.3454	11.3926	122.6126
BGRIM	662	0.8711	65.50	17.40	39.75	32.56	40.00	5.48	254.7613	1.2312	23.8026
BH	2442	0.6984	253.00	28.25	114	13.52	29.96	4.29	24.2033	0.0011	2.7024
(3) Small Caps.											
BCT	2442	0.0990	67.25	19.70	39.25	4.45	28.70	14.93	5.9501	0.0000	0.1124
PLANB	1250	0.0966	9.55	2.54	2.96	-8.62	39.21	5.29	507.1356	0.2364	38.5509
MAJOR	2442	0.1015	36.00	8.25	13.5	4.13	31.30	4.79	86.2077	0.1143	4.1070

Table 3: Descriptive statistics of 9 sample securities from three different groups of market capitalization[2]

** Abbreviation of the securities:

PTT: PTT Public Company Limited, state-owned oil and gas company;
AOT: Airports of Thailand Public Company Limited, state-owned airport operator;
ADVANC: Advanced Info Service Public Company Limited, mobile operator;
TRUE: TRUE Corporation Public Company Limited, telecommunications;
BGRIM: B. Grimm Power Public Company Limited, electric utilities company;
BH: Bumrungrad Hospital Public Company Limited, healthcare services provider;
BCT: Birla Carbon (Thailand) Public Company Limited, carbon manufacturer;
PLANB: Plan B Media Public Company Limited, advertising company;
MAJOR: Major Cineplex Group Public Company Limited, cinema operator and entertainment services provider.

To investigate the liquidity problem in the market, (1) the correlation between the bid-ask spread and mid-price, (2) the correlation between bid-ask spread and volume, (3) the share turnover ratio, and (4) the average free float are calculated and presented in *Table 4*.

Group (Sorting by market cap.)	---Correlation | Bid-ask spread---		Annualized Share turnover	Free float
	vs. Mid price	vs. Volume		
Large Caps (Top 30)	0.5880	-0.0273	61.57%	48.08%
Mid Caps (~90%)	0.4729	-0.0278	71.55%	44.44%
Small Caps (~98%)	0.4023	0.0063	56.57%	40.83%
All	0.4538	-0.0106	62.60%	43.43%

Table 4: Statistics on different market caps groups

For the correlation between the bid-ask spread and mid-price, it is noted that many stocks have high positive value, which means that their movements might be treated in a way which includes both market risk and liquidity risk, particularly for large-cap securities as they have a high correlation. However, some

[2] Using the same criteria defined by the Financial Times Stock Exchange (FTSE) SET index

primary stocks (large market caps), such as Bangkok Bank Public Company Limited (BBL) and Intuch Holdings Public Company Limited (INTUCH), have low correlation, so incorporating liquidity component is more significant for the banks that hold this kind of assets. Since their spreads' changes (liquidity risk) might not be in line with the price changes (market risk), the liquidity component should therefore be included in the model.

In addition, most of the correlations between the bid-ask spread and trading volume are negative but low. It is consistent with the assumption that a large bid-ask spread implies a low trading volume and less liquid stock. When a stock has a low trading volume, it is usually regarded as illiquid because it cannot be converted to cash easily. As a result, the trader requires more compensation for the transaction, resulting in a large bid-ask spread. Thus, bid-ask spread is considered to be a good proxy for the estimation of the liquidity risk.

Another factor associated with illiquidity is the share turnover. It is noted that mid-cap stocks, on average, have the highest ratio (71.55 percent) which implies that they are likely to be high liquid stocks. Large-cap stocks, on the other hand, have a lower-than-average share turnover, which means that large-cap stocks are not traded as frequently as expected in the Thai market. For example, Central Pattana Public Company Limited (CPN) has an annualized average ratio at about 35 percent comparing to the market average at about 62.60 percent. Our findings are consistent with Morck et al. (2000). They suggest that large-cap stocks in emerging stock markets have a lower-than-expected share turnover, compared to major stock markets in the world. This is due to poor minority investor protection and imperfect regulation in the emerging markets. Compared to developed markets, disclosure rules, accounting standards and judicial systems are relatively poor in these markets.

Due to the fact that sometimes large stocks might have a large number of inactive shares and that they are, therefore, illiquid, the free float of listed stocks is also collected to investigate the illiquidity. Free float refers to the shares of a firm available to public investors, excluding insiders (Rezaei & Tahernia, 2012). In other words, it is used to describe the common shares outstanding to the public for trading in the secondary market. Generally, the percentage of free float is expected to have a positive relationship with the liquidity, and stocks with higher levels of free float are usually associated with higher levels of liquidity and lower liquidity risk (El-Nader, 2018). In our dataset, it is noted that the mean of the free float of large caps is the highest, at 48.08 percent and that the mean of the free float of mid and small caps are 41.13 percent and 36.46 percent, respectively. So, different levels of free float are associated with different levels of liquidity as well as market cap.

Before composing the portfolios, the stocks are classified into 3 groups based on the size of market capitalization (large cap, mid cap and small cap). Usually, large-cap stocks are expected to be more liquid and small-cap stocks are expected to be more illiquid. The criteria defined by Financial Times Stock Exchange (FTSE) Group are shown as following:

Group 1: large-cap stocks which are the first 30 stocks sorting by the market cap on the main board;

Group 2: Mid-cap stocks which are the first 90 percent of stocks sorting by the market cap on the main board, excluding the stocks in Group 1;

Group 3: Small-cap stocks which are the first 98 percent of stocks sorting by the market cap on the main board, excluding the stocks in Group 1 and Group 2.

Meanwhile, the banks invest in three portfolios based on the assumptions that (1) banks will invest in the hypothetical portfolio with initial wealth of 10 million Thailand Baht (THB) and allocate their budget to the position on a daily basis, (2) they allocate and invest money in the stocks on equal weight, (3) they re-invest in the portfolios on a daily basis, and (4) they trade stocks at end of each day (no intraday trading).

Table 5 presents the descriptive statistics on 10-day profit and loss (P&L) of the trading portfolios based on overlapping data over the last 10 years.

Portfolio	10-day P&L			
	Average (THB)	Standard deviation (THB)	Kurtosis	Skewness
(1) Large Caps.	7,368	365,759	14.23	-1.86
(2) Mid Caps.	8,542	371,403	12.81	-1.71
(3) Small Caps.	-8,133	348,731	11.19	-1.64

Table 5: Descriptive statistics on the 10-day portfolios' profit & loss over the 10-year period

As shown in *Table 5,* Thai mid-cap stocks portfolio has the highest average P&L (8,542) and volatility (371k), which contradicts the traditional assumption that small caps should have a higher return and volatility. All 10-day portfolio P&L are not normally distributed with high kurtosis (fat-tail) and left (negative) skewness.

5. Empirical Results

According to Basel regulations, ES should be computed on a daily basis and an instantaneous price shock equivalent to a 10-day movement in prices should be used. However, using 10-day ES based on overlapping data leads to serial correlation and dependency problems[3], and it is noted that none of the models can pass the backtests. Therefore, this study estimates one-day ES and scales up the ES using the square root rule (multiply by $\sqrt{10}$) instead of measuring the risk on a 10-day overlapping return in order to avoid serial correlation and dependency problems.

To follow the regulatory liquidity adjustment, in this study, large-caps portfolio is grouped in the LH_1 according to the regulatory table. Small-caps portfolio is mapped to the LH_2 group due to the less-liquid characteristic. Since there is no specific rule of mapping the mid-cap equity, this study maps mid-cap stocks to the LH_1 group because mid-cap stocks have very high share-turnover (at 71.55 percent) and higher free float (at 44.44 percent) than the average, as shown in the *Table 4*, which implies high liquidity.

5.1 Backtesting ES

Table 6 summarizes the results of the backtests on three liquidity-adjusted ES models which incorporate different distribution and volatility assumptions over the full evaluation period. The test statistics Z are calculated for each model and the models are assigned to three traffic lights based on respective Z values.

Distribution	Normal						Student t					
Weight	SMA Volatility			EWMA Volatility			SMA Volatility			EWMA Volatility		
Model	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD
(1) Large Caps.												
Z test stat	-1.76	-1.53	-1.39	-1.76	-1.47	-1.24	-1.55	-1.37	-1.19	-1.57	-1.23	-1.04
Traffic light	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow
(2) Mid Caps.												
Z test stat	-3.03	-2.31	-2.28	-3.34	-2.42	-2.43	-2.61	-2.03	-1.99	-2.87	-2.11	-2.08
Traffic light	Red	Red	Red	Red	Red	Red	Red	Red	Red	Red	Red	Red
(3) Small Caps.												
Z test stat	-0.68	-2.52	-3.19	-0.59	-2.65	-3.34	-0.50	-2.25	-2.80	-0.46	-2.42	-2.99
Traffic light	Green	Red	Red	Green	Red	Red	Green	Red	Red	Green	Red	Red

Table 6: Backtests on scaled ES for the entire period

It is noted that none of the models testing on the large-caps and mid-caps portfolios can be grouped in the green zone. This means that all the models underestimate the actual losses. To investigate this problem, backtests on individual years are performed in order to find out whether there exist some problems in specific years. *Table 7 and 8* shows the results of annual backtests on large-caps and mid-caps portfolios.

[3] In this study, the ES using the 10-day overlapping returns is also estimated. Serial correlation is noticed and the empirical results are shown in the Appendix C.

Distribution	Normal						Student t					
Weight	SMA Volatility			EWMA Volatility			SMA Volatility			EWMA Volatility		
Model	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD
(1) Large Caps.												
31-Mar-11	0.69	1.00	0.85	-0.80	0.37	0.06	0.72	1.00	0.87	**-0.64**	0.69	0.40
31-Mar-12	-1.75	-1.55	-1.26	-1.19	-1.14	-0.76	-1.61	-1.29	-1.14	-1.08	-0.91	**-0.67**
31-Mar-13	0.55	0.56	0.72	**-0.52**	-0.46	0.05	0.58	0.59	0.87	-0.42	-0.11	0.12
31-Mar-14	-4.18	-3.87	-3.54	-3.13	-3.01	-2.85	-3.95	-3.67	-3.21	-2.95	-2.84	-2.68
31-Mar-15	-0.03	0.28	0.32	-1.92	-1.39	-0.93	0.03	0.34	0.50	-1.57	-1.06	**-0.63**
31-Mar-16	-1.42	-0.91	-0.87	-2.23	-1.65	-1.58	-1.03	-0.80	-0.76	-2.05	-1.37	-1.31
31-Mar-17	0.53	0.67	0.83	**-0.39**	-0.35	0.03	0.68	0.68	0.83	-0.34	-0.30	0.06
31-Mar-18	**-0.07**	-0.03	0.00	0.47	0.48	0.50	-0.03	0.01	0.04	0.49	0.50	0.52
31-Mar-19	-1.45	-1.39	-1.30	-1.39	-1.19	-1.11	-1.39	-1.33	-1.11	-1.20	-1.14	-1.07
31-Mar-20	-10.33	-9.99	-9.56	-6.35	-6.25	-5.74	-9.40	-9.10	-8.68	-5.80	-5.71	-5.09

Table 7: Annual back-testing ES on large-caps portfolio

Distribution	Normal						Student t					
Weight	SMA Volatility			EMMA Volatility			SMA Volatility			EWMA Volatility		
Model	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD
(2) Mid Caps.												
31-Mar-11	-0.45	0.19	0.22	-1.49	-0.82	-0.70	-0.34	0.25	0.28	-1.04	**-0.69**	-0.57
31-Mar-12	-2.81	-2.49	-2.41	-2.14	-1.79	-1.74	-2.49	-2.22	-2.13	-1.75	-1.57	-1.51
31-Mar-13	-0.05	0.32	0.63	-1.85	-0.94	-0.30	-0.01	0.34	0.64	-1.39	**-0.68**	-0.22
31-Mar-14	-5.47	-4.24	-4.16	-4.47	-3.98	-3.89	-4.82	-3.94	-3.85	-4.12	-3.56	-3.33
31-Mar-15	**-0.56**	0.12	0.10	-3.06	-1.12	-1.60	-0.32	0.30	0.16	-2.40	-0.87	-1.17
31-Mar-16	-1.62	-1.07	-1.09	-3.61	-2.11	-2.58	-1.32	-0.95	-0.96	-3.18	-1.92	-2.21
31-Mar-17	-1.12	-0.80	-0.83	-2.10	-1.48	-1.53	-1.05	-0.74	-0.77	-1.99	-1.40	-1.45
31-Mar-18	-1.08	0.00	-0.30	-0.86	-0.42	-0.43	**-0.55**	0.19	0.18	-0.50	-0.21	-0.35
31-Mar-19	-5.55	-4.47	-4.61	-4.91	-3.88	-4.01	-4.87	-3.87	-4.12	-4.51	-3.44	-3.55
31-Mar-20	-11.46	-10.58	-10.25	-8.80	-7.56	-7.42	-10.20	-9.54	-9.21	-7.64	-6.65	-6.37

Table 8: Annual back-testing ES on mid-caps portfolio

As shown in *Table 7 and 8*, the ES models pass the test in 2011, 2013, 2015, 2017 and 2018, but they fail in the year 2012, 2014, 2016, 2019 and 2020 when the market is affected by negative events and consequently, the losses exceed the expected values. Looking back to 2012, Thailand experienced a difficult time. The global economy continued to fluctuate due to the financial crisis, followed by the worst flooding which affected Thai economy and social development. The industrial and commercial sectors were disrupted by the natural disaster, causing unprecedented damage to the business and making stock market volatile. Between March 2013 and March 2014, the Thai economy slowed down due to weakness in domestic demand and structural limitations, such as labor and production constraints. Some public investment projects were delayed and real GDP experienced a more-than-expected decline (about 2 percent). Even if housing price grew and the government launched some projects to encourage the investment, the stock market rally fizzled. The 2015 terrorist attack had a negative impact on Thailand exchanges. Many foreign investors unloaded their shares (about a net of 156 billion THB worth of Thai equities). In addition, some domestic institutional investors moved out of Thailand and looked for foreign shares in order to diversify away from the sluggish Thai economy. All these factors increased the volatility of the stock market in 2016. Between March 2018 and March 2019, although the general election ushered a boost of confidence among investors, due to the impact of Federal Reserve's interest rate normalization, the Sino-US trade dispute, protracted Brexit negotiations and the global financial volatility, the stock market fell quickly. As of March 2020, because of the Covid-19 outbreak and the sluggish world economy, Thai exchanges remains volatile.

One potential reason why the models fail for particular years is that large-cap and mid-cap stocks are more sensitive and react more quickly to market changes than small-cap stocks. One of the evidences discussed in *Table 5* shows the high return volatility of large-caps and mid-caps portfolios (365k and 371k respectively), compared to 348k for small-caps portfolio over the evaluation period. It is also supported by the descriptive statistics that large-cap stocks have a higher daily return volatility (35.61 percent), compared to small-cap stocks (34.59 percent), although it is theoretically questionable that small-cap stocks should have a higher volatility.

Another potential reason is that high liquid stocks have more transactions than small-cap stocks in Thailand because of their attraction to both local and foreign investors, leading to higher volatility. According to Jones et al. (1994) and Huang and Masulis (2002), the volatility of stocks is correlated with the number of transactions. If a stock has a higher frequency of trades, the volatility tends to be higher. Usually large-cap firms have more resources, making them less vulnerable to negative events. This makes large-cap stocks to be a safer investment for investors, particularly when the economy is sluggish. Also, today, more and more Thai mid-cap firms are showing steady year-on-year growth with high profits. This boosts steady returns to investors, and in turn, a great number of investors prefer to invest in Thai mid-cap stocks, believing that these firms have potential for growth and will perform well in the future. As a result, mid-cap firms grow in size and the stocks tend to have more transactions as well. Although small-cap stocks are considered to be theoretically more volatile and riskier investments because small-cap stocks tend to offer greater returns that are always associated with higher volatility, the results in *Table 5* are consistent with prior studies.

It is also noted that in tranquil years when the models implemented on large-caps portfolio pass the test, the standard model outperforms as it returns the closest test statistic to the expected value. As of March 2011, the standard model with student t-distribution assumption and EWMA volatility performs the best (z-stats -0.64). As of March 2013 and March 2017, the standard model with normal distribution assumption and EWMA volatility outperforms other models (z-stats -0.52 and -0.39 respectively). As of March 2018, the standard model with normal distribution assumption and SMA volatility works well (z-stats -0.07). However, as of March 2012 and 2015, the liquidity discount model (student t distribution and EWMA volatility) outperforms the regulatory approach with the test stats of -0.67 and -0.63 respectively.

In comparison, as shown in *Table 8*, regulatory adjustment and BAS models on mid-caps portfolio are able to estimate the risk properly. For example, BAS model with EWMA volatility and student t-distribution performs well for 2011 and 2013. The standard model outperforms other models in 2015 and 2018. Therefore, there is no single model that performs best throughout the period.

Since the standard model outperforms other models during most of the period, the question arises as to with which volatility model and distribution assumption the model works better for high liquid portfolios. *Figure 3 and 4* shows the comparison of standard ES calibrated with SMA and EWMA volatility on large-caps portfolio and mid-caps portfolio and their profits and losses.

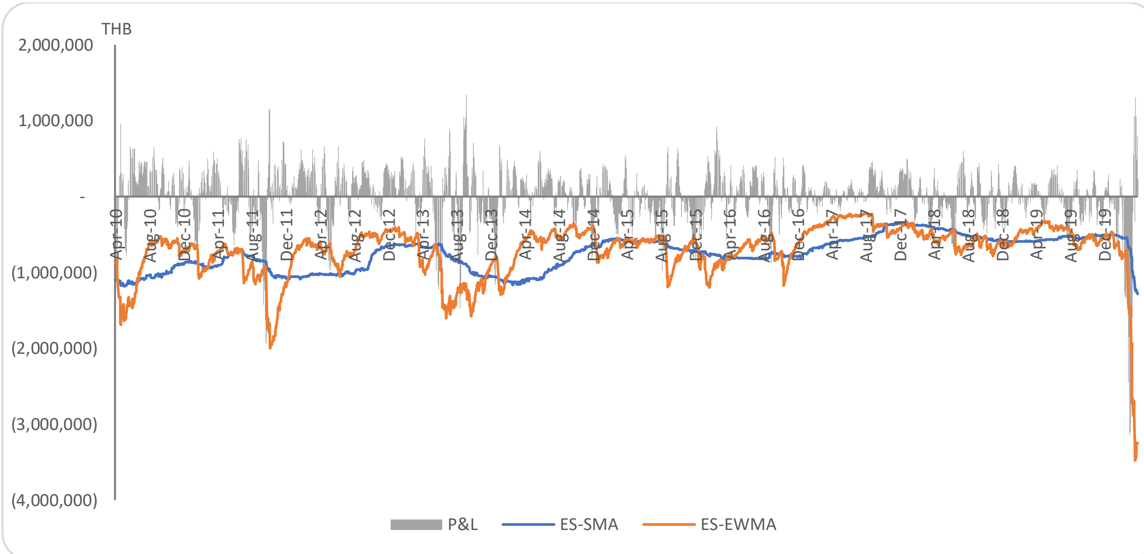

Figure 3: Comparison of the standard ES estimation versus P&L for large-caps portfolio

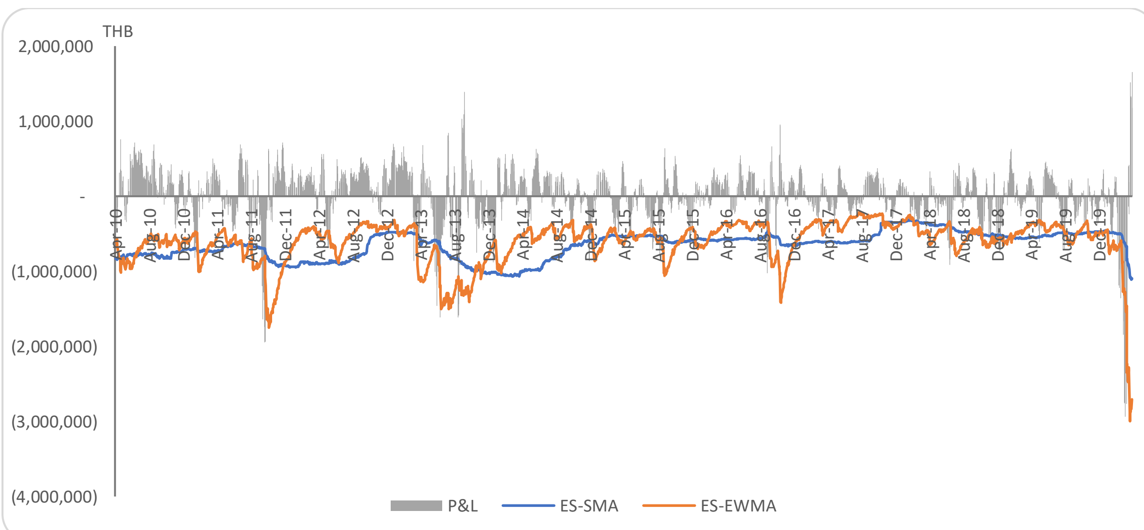

Figure 4: Comparison of the standard ES estimation versus P&L for mid-caps portfolio

As shown in the *Figure 3 and 4*, generally, mid-caps portfolio has a similar pattern of volatility on profit and loss as large-caps portfolio. It is also noted that both SMA and EWMA help highlight trends, estimate risk and forecast the future observations. SMA gives equal weight to the past "n" days and smoothens out volatility. EWMA gives the latest observation the largest weight, and weights associated with previous observations decay exponentially over time. Compared to SMA, EWMA is more sensitive to recent events and captures the volatility timely when the market is volatile. For example, between August 2011 and December 2011 and between April 2013 and December 2013, EWMA captures volatility clustering and works better than SMA. However, when the market is tranquil, for example, between April 2014 and December 2014 and between August 2018 and December 2018, SMA works better. Therefore, under different market conditions, SMA and EWMA give different views of risk. EWMA gives early warning signals of market changes while SMA can be used to assess the daily return during look back period because it has a good memory of historical observations.

With regard to the distribution assumptions, it is obvious that the performance of the models depends on the distribution assumptions. Taking proper distribution assumption into consideration can boost the efficiency of the models. As shown in *Table 5*, the kurtosis of large-caps portfolio return is larger (14.23) than small-caps portfolio (11.19). In addition, high liquid portfolios have a greater fat-tail and higher volatility than low liquid portfolio. Hence, generally, the student t-distribution tend to be more favorable because it can capture the excess kurtosis of the loss distribution.

Moving to low liquid portfolio (small-caps), it is noted that it is sufficient for the banks to implement the conventional model and scale with the liquidity horizon following the regulatory approach. As shown in *Table 6*, the standard model (with normal distribution & SMA volatility, normal distribution & EWMA volatility, student t-distribution & SMA volatility, and student t-distribution & EWMA volatility) returns the nearest actual z-stats (i.e. -0.68, -0.59, -0.5, and -0.46 for small-caps portfolio) to the expected value (-0.7). Particularly, the model with normal distribution and SMA volatility performs the best (-0.68 for small-caps) among all the approaches. The bid-ask spread and liquidity discount ES models underestimate the actual losses of the low liquid portfolios over the entire period, even though they work better than the standard model during specific period of time.

Breaking down the backtests from the entire period to a yearly basis, the annual backtesting on low liquid portfolio gives an interesting finding that different models outperform in difference market situations. *Table 9* shows the results of annual backtests on small-caps portfolios.

Distribution	Normal						Student t					
Weight	SMA Volatility			EWMA Volatility			SMA Volatility			EWMA Volatility		
Model	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD
(3) Small Caps.												
31-Mar-11	1.00	1.00	1.00	**0.85**	1.00	0.83	1.00	1.00	1.00	0.85	1.00	0.84
31-Mar-12	-1.37	-2.89	-3.50	-0.88	-2.36	-2.85	-1.26	-2.74	-3.28	**-0.66**	-2.23	-2.66
31-Mar-13	0.82	0.61	0.56	0.23	**-0.08**	-0.87	0.83	0.63	0.59	0.27	-0.05	-0.79
31-Mar-14	-1.43	-3.04	-4.04	**-0.24**	-2.84	-3.96	-1.03	-2.85	-3.48	-0.17	-2.65	-3.66
31-Mar-15	0.86	-0.02	**-0.67**	0.70	-0.22	-0.96	0.86	0.01	-0.60	0.85	-0.17	-0.87
31-Mar-16	0.57	-1.91	-0.94	-0.73	-3.59	-1.87	0.73	-1.61	-0.81	**-0.64**	-3.22	-1.58
31-Mar-17	**-0.47**	-1.29	-1.58	-0.99	-2.50	-2.95	-0.41	-1.20	-1.46	-0.89	-2.37	-2.76
31-Mar-18	0.85	-0.64	-1.24	1.00	-1.32	-2.24	0.86	**-0.44**	-0.87	1.00	-1.08	-1.95
31-Mar-19	-1.15	-4.98	-7.06	-1.14	-4.91	-6.13	-1.04	-4.71	-6.38	-1.03	-4.52	-5.63
31-Mar-20	-6.43	-11.91	-14.30	-4.67	-9.57	-12.24	-5.52	-10.45	-12.60	-4.11	-8.75	-10.69

Table 9: Annual backtesting ES on small-caps portfolio

As shown in *Table 9*, generally, the models pass the test in most years except 2019 and 2020 when the world economy was volatile and is now facing an economy turndown due to Covid-19. In 2011, the standard model with EWMA and normal distribution works best and returns the nearest z-stat (0.85) to the expected value (-0.7). In 2012 and 2016, the standard model with EWMA and student t-distribution outperforms other models (-0.66 and -0.64). The BAS model works well in 2013 and 2018 (-0.08 and -0.44). The standard model with SMA and normal distribution returns the nearest z-stat (-0.24) to the expected value in 2014, and in the following year the liquidity discount model with SMA volatility and normal distribution outperforms other models (-0.67). Overall, the models estimate the risk properly for most of the period, and different models perform well in different market situations.

Figure 5 shows the comparison of regulatory liquidity-adjusted ES with SMA and EWMA volatility and normal distribution assumption on small-caps portfolios and their profit and loss.

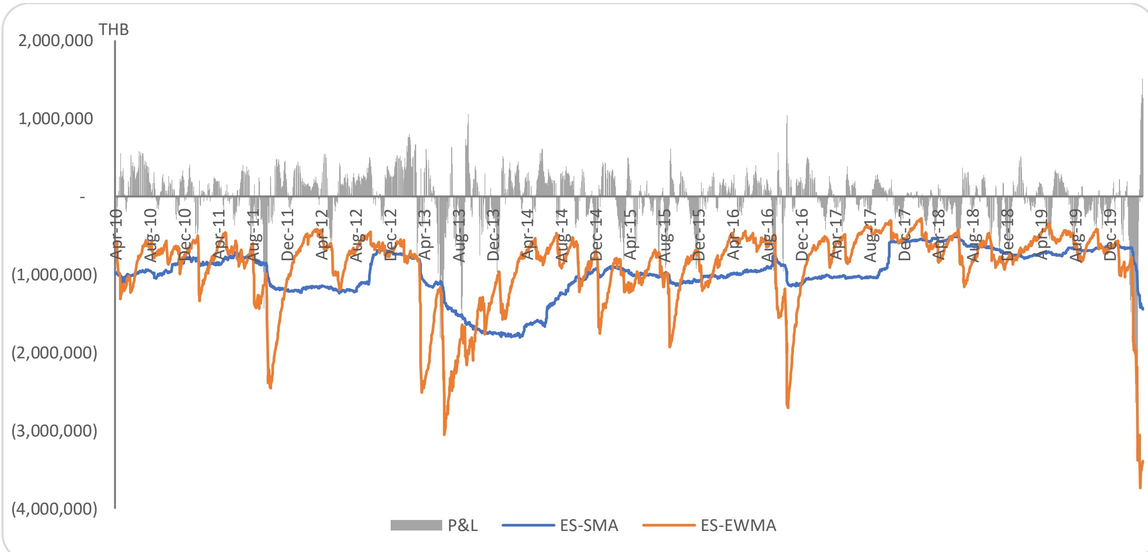

Figure 5: Comparison of the standard ES estimation versus P&L for small-caps portfolio

It is noted that EWMA volatility is sensitive to recent events and captures the volatility clustering. For example, between April 2013 and December 2013 and between April 2015 and March 2016, EWMA works better than SMA. However, as discussed above, which volatility model works better depends on market situations.

Meanwhile, *Table 5* shows that low liquid portfolio is not normally distributed and that explains why the models with a student t-distribution generally perform better than normal distribution for specific years. However, sometimes this assumption is quite vulnerable for Thailand exchanges because whether the model estimates the risk properly also depends on market situations and other external factors.

5.2 The Limitations of Liquidity Discount Model

Before moving to the conclusion, it is notably noted that although the regulatory liquidity adjustment on the parametric conventional ES model outperforms other models during most of the period, it underestimates the losses for high liquid portfolios for particular years. Therefore, liquidity discount model becomes an alternative to improve the estimation because it incorporates not only the liquidity horizon but also the liquidity discount factors in the model. In addition, the LD model usually returns higher z-stats (for high liquid portfolios) which implies higher unexpected losses and more bank capital reserves in place to compensate the losses. However, the LD model has some limitations. It depends on the size of trading positions required to liquidate, while the conventional parametric model does not. To further investigate this limitation, this study changes the assumption of the portfolio wealth from the initial 10 million THB to 100 and 500 million THB.

Table 10 shows the numbers of violations and results of backtesting ES on LD model over different sizes of trading portfolio.

Distribution	Normal						Student t					
Weight	SMA Volatility			EWMA Volatility			SMA Volatility			EWMA Volatility		
Portfolio size (million THB)	10	100	500	10	100	500	10	100	500	10	100	500
(1) Large Caps.												
#Violations	101	86	88	114	95	103	97	83	84	109	89	96
Z test stat	-1.39	-1.00	-1.07	-1.24	-0.82	-0.97	-1.19	-0.83	-0.88	-1.04	-0.64	-0.76
(2) Mid Caps.												
#Violations	135	103	44	166	99	33	130	98	39	157	92	30
Z test stat	-2.28	-1.33	0.09	-2.43	-0.94	0.40	-1.99	-1.10	0.22	-2.08	-0.72	0.48
(3) Small Caps.												
#Violations	169	60	0	198	52	0	160	58	0	191	47	0
Z test stat	-3.19	-0.22	1.00	-3.34	0.13	1.00	-2.80	-0.11	1.00	-2.99	0.25	1.00

Table 10: Tests on the liquidity discount ES model over different portfolio sizes

According to the results shown in *Table 10*, the performance of the LD model depends on different levels of liquidation portfolio. Overall, the violation numbers decrease and the test stats increase as the size of portfolio that is required to liquidate increases, except for 500 million THB on large caps.

6. Conclusions

Stock markets are not perfectly liquid and investors cannot unwind positions at going bid and ask prices. The way to estimate the market risk should, therefore, be revised to accommodate illiquidity effects. Liquidity risk can be measured by exogenous factors, such as liquidation time and spread cost, and endogenous factor, such as liquidity discount. The models which incorporate exogenous or (and) endogenous liquidity risk variables are able to measure the liquidity risk and adjust the estimates.

This thesis implements the regulatory liquidity-adjusted ES model, which banks are required to use to estimate the capital charges when they trade assets, and two academic liquidity-adjusted ES models, bid-ask spread (BAS) model and liquidity discount (LD) model, to estimate liquidity-adjusted ES over high and low liquid stocks portfolios in Thailand exchanges over a 10-year period. The evidence from model estimations, statistical inference and backtesting suggests that the standard ES model passes the backtests on low liquid stocks and that this model is sufficient for the banks to estimate the unexpected losses on low liquid stocks. However, for high liquid equities, none of the models can pass the tests, implying that the models cannot capture the losses properly. The potential reason is that large-cap stocks in Thailand are more sensitive to the unexpected events. The movements of large-cap stocks tend to be affected at a higher degree than small-cap equities. Therefore, the banks holding Thai stocks should take into consideration this characteristic of high liquid stocks apart from the liquidity risk when they estimate the ES. Meanwhile, based on this study, additional models can be implemented to improve the estimation.

In addition, this study suggests that even though the LD model is an alternative to improve the estimation for high liquid stocks, users should consider the constraint that the performance of this model depends on the size of the positions to liquidate. Also, the way to define the price-drop function should be taken into account when implementing this model. Since the implementation of the LD model requires more information and efforts, a trade-off between the performance of the model and cost-effectiveness should be taken into consideration. It remains interesting and challenging for further studies to identify optimized assumptions for the implementation of this model on portfolios with different properties and markets. Another finding is that the standard ES calibration with the overlapping sample leads to the serial correlation and dependency problems. Hence, to avoid these problems and perform the backtests, this thesis suggests that the estimation can be computed based on non-overlapping data.

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Appendix

A: Numerical Example of Bid-Ask Spread (BAS) Model

Suppose we invest in a large market capitalization stock. The average daily stock return equals to -0.12 percent and we re-balance our portfolio to invest in this asset with value of 1 THB everyday ($\mu = -0.0012$). The daily stock volatility is 2.28 percent ($\sigma = 0.0228$), the mean of relative spread is 0.36 percent ($\mu_{Spread} = 0.0936$), the volatility of relative spread is 0.05 percent ($\sigma_{Spread} = 0.0005$), and the scaling factor suggested by Bangia et al. is 3 ($k = 3$).

Using equation 3.6, the one-day expected shortfall at 99 percent confidence interval when holding this stock is:

$$ES_{BAS,0.99} = \left\{-0.0012 + 0.0228\frac{f(z_{0.99})}{1-0.99}\right\} + \frac{1}{2}\big(0.0936 + 3(0.0005)\big)$$

$$ES_{BAS,0.99} = \left\{-0.0012 + 0.0228\frac{0.0267}{1-0.99}\right\} + 0.0025 \ = 0.0569 + 0.0025 = 0.0622$$

The normal ES for this stock is 0.0569. Allowing for the exogenous liquidity risk will enlarge the ES by 0.0025. So, the uncertainty in the spread will affect the total market risk, increasing by 0.0025/0.0622 = 4.05 percent.

B: Numerical Example of Liquidity Discount (LD) Model

Suppose we have the same parameters as in the *Appendix A* but new initial wealth of 10 million THB to invest in the portfolio every day. Given the current stock price of 50.75, we hold 197,044 shares today. The average of last 3 months trading volume is 102,656,318 and the price drop parameter for this stock is 0.000007.

So, today's execution time lag to unwind position equals to $\Delta S = \frac{197044}{102656318} = 0.0019$ day, and the liquidity discount is $c(S) = \frac{0.5}{1-0.5e^{-0.000007(197044)}}e^{-0.12\%(0.0019)} = 0.57$. These new variables will be calculated in every day during the estimation period (usually one-year length of data). We then get the parameters: mean, volatility of logarithm of liquidity discount $\left(\mu_{ln\,c(S)} = -0.0003\,, \sigma_{ln\,c(S)} = 0.0048\right)$, and for time lag ($\mu_{\Delta S} = 0.0028, \sigma_{\Delta S} = 0.0003$).

To estimate one-day ES_{LD} for this stock, we plug in these parameters: mean and volatility of execution time lag and (logarithm of) liquidity discount, into equation 3.7.

$$ES_{LD} = -0.0012[0.0028] - 0.0003 + \left\{0.0228\sqrt{[0.0028]} - 0.0012(0.0003) + 0.0048\right\}\frac{f(z_{0.99})}{1-0.99} = 0.0722$$

By using the formula 3.7, the ES is lifted from 0.0596 to 0.0722 (21.12 percent). *Table A1* shows different ES estimates resulting from three parametric models.

Model	Mean	Volatility	LC	Mean (LD adj.)	Volatility (LD adj.)	ES	Ratio to conventional model
Conventional	-0.0012	0.0228				0.0596	
Bid-ask spread	-0.0012	0.0228	0.0025			0.0622	1.0422
Liquidity discount				-0.0014	0.0276	0.0722	1.2112

Table A1: Comparison of ES estimation

C: Backtesting 10-day ES (Using Overlapping Data)

According to the regulatory requirements, we should estimate the 10-day ES over the overlapping data sample. However, as shown in *Table A2,* none of the models that are under different volatility and distribution assumptions passes the test. The main problem is that the exceptions are not independent, which results in serial correlation.

Distribution	Normal						Student t					
Weight	SMA Volatility			EWMA Volatility			SMA Volatility			EWMA Volatility		
Model	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD	Std	BAS	LD
(1) Large Caps.												
Z test stat	-2.09	-1.98	-1.70	-3.57	-3.40	-2.67	-2.08	-1.96	-1.68	-3.55	-3.37	-2.65
Traffic light	Red	Red	Yellow	Red	Red	Red	Red	Red	Yellow	Red	Red	Red
(2) Mid Caps.												
Z test stat	-2.47	-2.26	-1.94	-4.47	-4.02	-3.08	-2.43	-2.24	-1.92	-4.44	-3.99	-3.03
Traffic light	Red	Red	Red	Red	Red	Red	Red	Red	Red	Red	Red	Red
(3) Small Caps.												
Z test stat	-0.94	-2.46	-2.17	-2.29	-4.55	-3.46	-0.92	-2.40	-2.13	-2.23	-4.50	-3.40
Traffic light	Yellow	Red	Red	Red	Red	Red	Yellow	Red	Red	Red	Red	Red

Table A2: Backtesting on 10-day ES

Figure A1, A2 and A3 show that the model cannot capture the losses timely and that there is usually a 10-day lag when a number of violations occur during this period. Since the models use overlapping observations, the serial correlation is induced and then the Acerbi and Szekely (2015) ES statistical tests are biased. Therefore, to perform the model validation, in this study, we estimate the one-day ES and then scale the ES by using square root rule to avoid this issue.

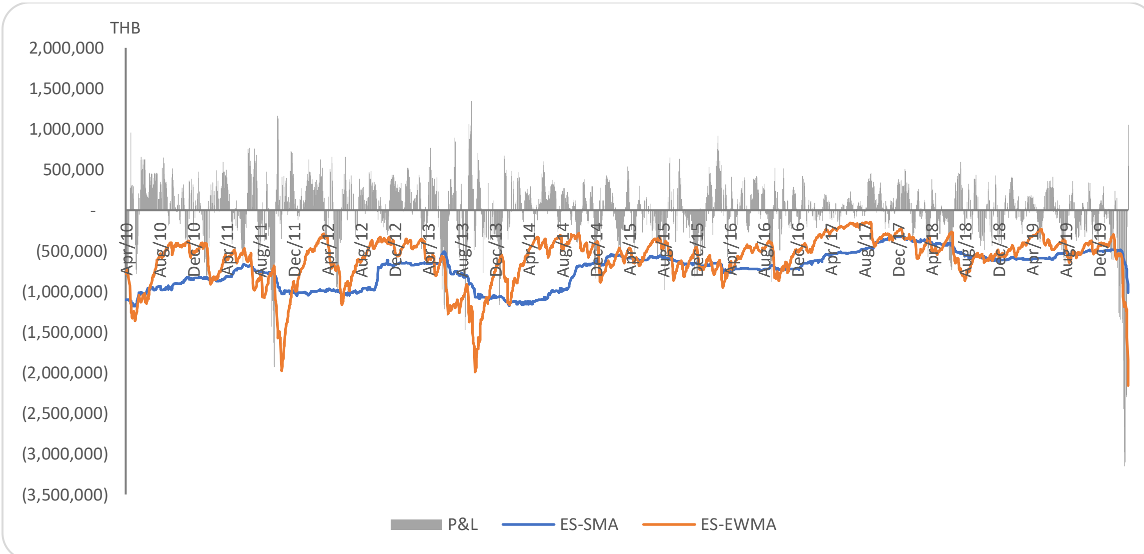

Figure A1: Comparison of standard 10-day ES estimation versus P&L on large caps portfolio

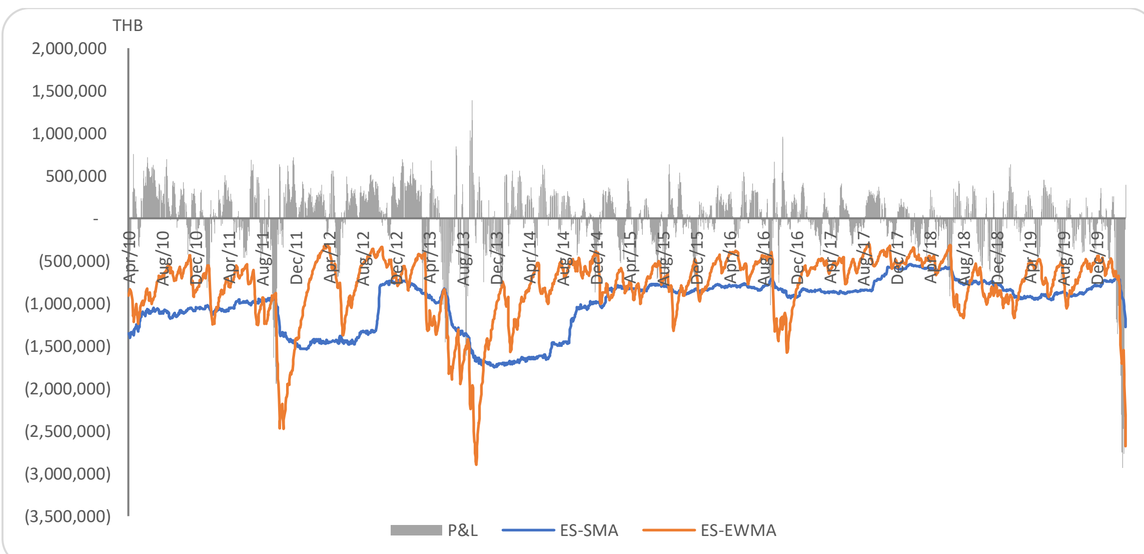

Figure A2: Comparison of standard 10-day ES estimation versus P&L on mid-caps portfolio

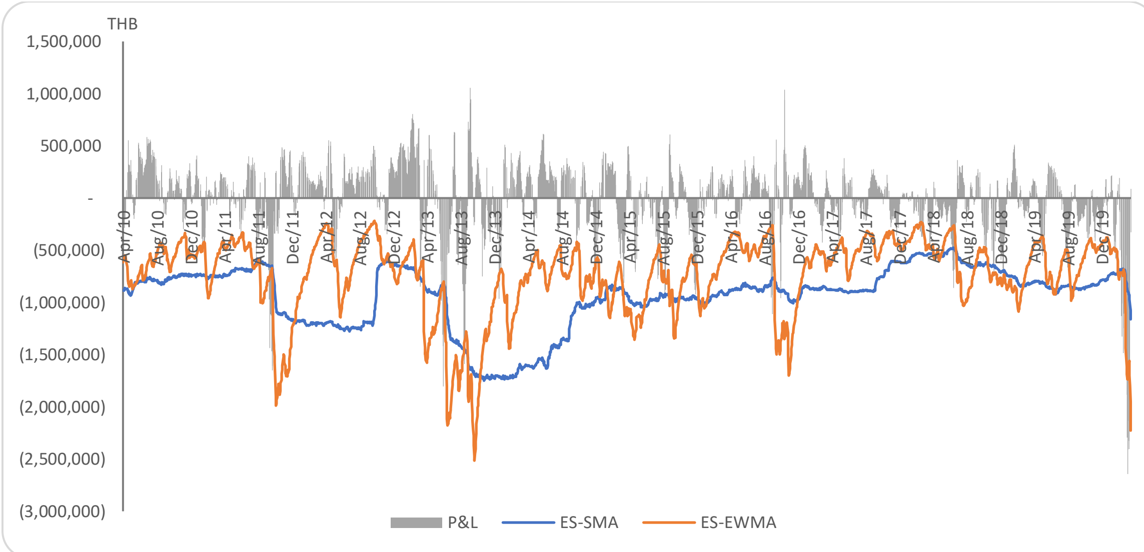

Figure A3: Comparison of standard 10-day ES estimation versus P&L on small-caps portfolio