A Financial Market Segmented New-Keynesian Macro Model

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This essay complements the monetary literature by estimation and simulation of a New-Keynesian macro model featuring financial market frictions and long bond portfolio policy. The model is an extended version of the canonical three-equation New-Keynesian model with segmented financial markets distinguishing the short-term money market from the long-term bond market. Our data is U.S. quarterly data spanning between 1996:Q4-2019:Q3. The analysis yields the following findings. The structural estimation delivers a high parameter steering the Fed's easing on the bond market. Credit shocks boost output, prices and short-term interest rates in simulations of the DSGE model, consistent with empirical evidence from our VAR. The impact of conventional monetary policy shocks is generally stronger than QE shocks.

Keywords: Monetary Policy, Financial Market Segmentation, Credit Conditions, QE, IS Curve.

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1. Introduction

The wake of the financial crisis 2007-2009 has led to a resurgent interest in understanding the link between the real economy and the financial sector. The strong tightening of U.S. credit conditions 2007-2008 caused considerable effects on the economy on which the Federal Reserve responded by easing policy (Bernanke, 2009). Large-scale asset purchase programmes aiming at stabilizing and stimulating the economy have become part of the policy instruments of many Central Banks in contemporary times. Under the effective zero bound 2008-2015 the Fed's balance sheet increased from some $850 millions to $4500 millions. This essay seeks to understand the simultaneous behavior of financial frictions, unconventional quantitative easing (QE) policy, and the business cycle. What are the dynamics of prices, output, interest rates, credit frictions and balance sheet policies? How does a sudden credit boom affect the economy? How does the transmission of QE policy work in a model with segmented financial markets?

While the topic is not new *per se*, we study such questions using a class of structural forward-looking macro models that did not until recently stressed an integration of the financial markets with the business cycle. The theory-based dynamic stochastic general equilibrium (DSGE) models widely used in research and at policy institutions have been critized for not putting enough emphasize on financial market disruptions prior to the great recession (Christiano, Eichenbaum and Trabandt 2018; Lindé, 2018; Stiglitz, 2018). The criticism has concerned both an allegedly inadequate forecasting ability to the crisis (Christiano et al., 2018) and a belief of both model-builders and policymakers that frictions on the markets was not crucial for simulation purposes (Lindé, 2018). Post-crisis development of medium-scale DSGE models accounts for financial frictions and QE policy (Gertler and Karadi, 2011; Carlstrom, Fuerst and Paustian, 2017; Del Negro, Eggertsson, Ferrero and Kiyotaki, 2017). The simple three-equation New-Keynesian model has been extended with credit frictions (Cúrdia and Woodford, 2016) and asset purchases policy (Cúrdia and Woodford, 2011).

The present essay studies business cycle phenomena by estimating and simulating a New-Keynesian DSGE model featuring financial market segmentation, credit frictions and QE policy. Our study complements the New-Keynesian macro literature threefold, with contributions related to estimation, policy analysis and model evaluation. First, whereas some of the previous literature calibrates the parameters governing credit frictions and QE policy in estimation and simulation purposes (Carlstrom et al., 2017; Del Negro et al., 2017), we estimate all parameters of a complete model.[2] To carry out full structural estimation is important for policy analysis. Second, the model is small and has a structure which enables to study how QE and credit shocks favor a boost of the real economy but how it could have competing effects on the nominal side.[3] Its parsimony also enables to trace many of the mechanisms to the benchmark three-equation New-Keynesian model, the workhorse in monetary economics textbooks (Woodford, 2003; Galí, 2015; Walsh, 2018). Third, DSGE models are often built and developed on evidence from Vector Autoregression (VAR) models (see Ravenna, 2007; Christiano et al., 2018). We estimate a VAR to compare with the dynamics of the DSGE.

The DSGE model we use is developed by Sims and Wu (2019) and has six equations consisting of a Phillips curve, an IS curve, a Taylor-type rule for conventional monetary policy, an AR(1) for credit conditions in the market, an AR(1) for unconventional QE policy, and an AR(1) for the natural interest rate. We estimate the equations jointly on U.S. quarterly data 1996:Q4-2019:Q3 with Full Information Maximum Likelihood (FIML). We discuss the structural parameters before solving the model numerically. As noted, we also estimate a VAR model. The empirical performance of DSGE models and VARs are frequently compared on various metrics (e.g. Edge and Gürkaynak, 2010). The VAR is considered as a complement to the DSGE; we study the interrelationship of the variables with another type of model with less restrictions, and make a contrast with the structural model's assumptions. We compare the impulse-response functions of the two models and study if the structural model's equilibrium analytics is broadly consistent with empirical shocks. This is important for DSGE models to be useful for policy analysis.[4]

The remainder of this essay is organized as follows. Section two introduces the New-Keynesian model and the VAR model. Section three discusses the data. In section four we specify our estimation approach. Section five presents the empirical results. In section six we analyze shocks based on impulse-response functions. Section seven concludes.

[2] Cooley (1997) describes calibration as *"a strategy for finding numerical values for the parameters of artificial economic worlds"*. One strand of the DSGE literature uses calibration to match model-dynamics with actual data.

[3] To our knowledge there is no precise categorization between small- or medium-scale models. A typical small-scale model is the New-Keynesian hybrid model with 7 parameters. A well-known medium-scale model is the Smets and Wouters (2007) model with 19 parameters.

[4] See for example Ravenna (2007), Chari, Kehoe and McGrattan (2009), Christiano et al., (2018), and Lindé (2018).

2. The Models

2.1 The Four Equation New-Keynesian DSGE Model

The model is the log-linearized four equation New-Keynesian model developed by Sims and Wu (2019). We have augmented the Phillips curve and the IS curve with endogenous lags of inflation and output making the equations become hybrid variants. In addition, we set expected future inflation in the Taylor rule rather than contemporaneous inflation and abstract from the elasticity of real marginal costs to the output gap.[5] The model consists of a Phillips curve (1) characterizing aggregate supply, an IS curve (2) characterizing aggregate demand, a Taylor-type policy rule (3) for the short-term money market interest rate describing conventional monetary policy, and AR(1) processes for the evolvement of credit conditions in the financial markets (4), unconventional QE policy (5), and the natural interest rate (6). Sims and Wu (2019) name it a four-equation model as they do not consider behavioral assumptions for (4) and (6), but take them to be exogenous. We follow them and say the four equation New-Keynesian model.

$$\pi_t = \delta E_t\pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda\hat{y}_t - \frac{\theta\lambda\sigma}{(1-\theta)}\left[\beta^{FI}cr_t + \beta^{CB}qe_t\right] + \varepsilon_t^{PC}, \quad \varepsilon_t^{PC} \sim N(0, \sigma_{PC}^2) \tag{1}$$

$$\hat{y}_t = \mu E_t\hat{y}_{t+1} + (1-\mu)\hat{y}_{t-1} - \frac{(1-\theta)}{\sigma}[i_t - E_t\pi_{t+1} - r_t^n] - \theta\left[\beta^{FI}(E_t cr_{t+1} - cr_t) + \beta^{CB}(E_t qe_{t+1} - qe_t)\right] + \varepsilon_t^{IS}, \tag{2}$$

$$\varepsilon_t^{IS} \sim N(0, \sigma_{IS}^2)$$

$$i_t = (1-\rho^{TR})[\phi_\pi E_t\pi_{t+1} + \phi_{\hat{y}}\hat{y}_t] + \rho^{TR} i_{t-1} + \varepsilon_t^{MP}, \quad \varepsilon_t^{MP} \sim N(0, \sigma_{MP}^2) \tag{3}$$

$$cr_t = \rho^{CR} cr_{t-1} + \varepsilon_t^{CR}, \quad \varepsilon_t^{CR} \sim N(0, \sigma_{CR}^2) \tag{4}$$

$$qe_t = \rho^{QE} qe_{t-1} + \varepsilon_t^{QE}, \quad \varepsilon_t^{QE} \sim N(0, \sigma_{QE}^2) \tag{5}$$

$$r_t^n = \rho^{NR} r_{t-1}^n + \varepsilon_t^{NR}, \quad \varepsilon_t^{NR} \sim N(0, \sigma_{NR}^2) \tag{6}$$

Parameter restrictions

$$0 < \delta < 1 \quad \lambda > 0 \quad 0 \le \theta < 1 \quad \sigma > 0 \quad 0 \le \beta^{FI} \le 1 \quad 0 \le \beta^{CB} \le 1 \quad \beta^{FI} + \beta^{CB} = 1$$

$$0 < \mu < 1 \quad 0 < \rho^{TR} < 1 \quad \phi_\pi > 0 \quad \phi_{\hat{y}} > 0 \quad 0 < \rho^{CR} < 1 \quad 0 < \rho^{QE} < 1 \quad 0 < \rho^{NR} < 1$$

[5] We so suppose that the elasticity between the real marginal cost and the output gap is constant. See Woodford (2003) for a derivation.

The full model (7) is a system of 13 parameters $\{\delta\ \lambda\ \theta\ \sigma\ \beta^{FI}\ \beta^{CB}\ \mu\ \rho^{TR}\ \phi_\pi\ \phi_{\hat{y}}\ \rho^{CR}\ \rho^{QE}\ \rho^{NR}\}$ with 6 variables $[\pi_t\ \hat{y}_t\ i_t\ cr_t\ qe_t\ r_t^n]$. Where π_t is inflation, $\hat{y}_t$ is the output gap, i_t is the short-term nominal interest rate, cr_t is credit conditions in the market, qe_t is the Central Bank's asset holdings, and r_t^n is the natural interest rate. Small letters with index denote log levels (except interest rates) as deviations from a steady state level.

This is a DSGE model characterizing the behavior of firms, households, the Central Bank and financial intermediates. The E_t is the time *t* conditional expectational operator. The parameters δ and μ governs the degree of forward-looking behavior of inflation and output, respectively. The parameter λ measures the effect of the real driving variable of inflation, also the slope of the Phillips curve. The parameter σ is the inverse intertemporal elasticity of substitution (IES) and θ steers the impact of credit frictions and QE policy on output. The parameters β^{FI} and β^{CB} indicates long-term bond holdings of financial intermediates and the monetary authority in relation to total outstanding bonds, respectively. The parameter ρ^{TR} captures interest rate smoothing and ϕ_π and $\phi_{\hat{y}}$ describes the monetary authority's long-run reaction to expected future inflation and the contemporaneous output gap, respectively. The three parameters ρ^{CR}, ρ^{QE} and ρ^{NR} captures the persistence of credit conditions in the financial markets cr_t, asset holdings of the Central Bank qe_t, and the natural interest rate r_t^n. The equations are subject to shocks denoted by ε_t^{PC} (supply shock), ε_t^{IS} (demand shock), ε_t^{MP} (conventional policy shock), ε_t^{CR} (credit shock), ε_t^{QE} (QE shock), and ε_t^{NR} (natural rate shock). The shocks are assumed to be multivariate normal; it is allowed with contemporaneous cross-correlations of the shocks.

If $\theta = \rho^{TR} = 0$ there is no scope for credit frictions and QE policy to affect the economy and no endogenous smoothing of the short-term interest rate. If $\delta = \mu = 1$ inflation and output is purely forward-looking. These two restrictions would make the model collapse to the canonical three-equation New-Keynesian model, the workhorse in monetary economics textbooks (Woodford, 2003; Galí, 2015; Walsh, 2018). The parameters $\{\delta\ \lambda\ \sigma\ \mu\ \rho^{TR}\phi_\pi\ \phi_{\hat{y}}\}$ are therefore standard. The forward-looking behavior of inflation δ and output μ is determined by optimizing decisions of firms and households. We have extended the Phillips curve and the IS curve to include endogenous persistence of inflation and output. This means that we suppose that some share of the firms $0 < \delta < 1$ are backward-looking, reflecting rule-of-thumb behavior (Galí and Gertler, 1999) or price indexation (Woodford, 2003; Christiano, Eichenbaum and Evans, 2005). The $0 < \mu < 1$ means that we assume that households are partly backward-looking, reflecting habit formation of consumption (Fuhrer, 2000; Smets and Wouters, 2007; Ascari, Magnusson and Mavroeidis, 2019) or adjustment costs to investments (Christiano et al., 2005). The IES σ determines how households responds to changes in the ex-ante short real interest rate $r_t = [i_t - E_t\pi_{t+1} - r_t^n]$ in the consumption-saving decision. The $\sigma > 0$ holds as the substitution effect is assumed to dominate the income effect. The parameter $\lambda > 0$ transmits output to inflation and is what drive changes in inflation, which is the real variable that depends on how firms reset their prices over some time horizon (see Woodford, 2003; Galí, 2015). When the ex-ante real interest rate becomes higher households decides to save more which depresses current output. This effect is transmitted to the Phillips curve via the parameter λ, leading to a disinflation. If instead the following would hold $\theta = 0$ but $0 < \rho^{TR}, \delta, \mu < 1$, the model becomes the New-Keynesian hybrid model, the benchmark in much of the recent monetary and macro-finance literature.[6]

The natural interest rate is the ex-ante real interest rate when potential (natural) output equals actual output (see e.g. Holston, Laubach and Williams, 2017). Interest rate smoothing ρ^{TR} reflects the tendency of Central Banks to gradually adjust short-term interest rates (Clarida, Galí and Gertler, 1999, 2000). The $0 < \rho^{TR} < 1$ holds because the Central Bank is assumed to choose ρ^{TR} to avoid large swings in the short rate due to an aversion of volatility in the financial markets. The $(1 - \rho^{TR})$ therefore represents the weight assigned for interest rate changes to economic conditions (see Clarida et al., 1999, 2000). The rule is the Clarida et al., (2000) version with expected future inflation; the Central Bank is assumed to respond to the probable path of future inflation rather than contemporaneous inflation. It is recognized that policymakers consider expectations of inflation when conducting policy (Boivin and Giannoni, 2006; Bernanke, 2010, Walsh, 2017, chapter 1). The Taylor-principle is satisfied if the Central Bank raises the short rate more than one-for-one with (expected future) inflation $\phi_\pi > 1$. Fulfilment of the principle is important for determinacy in a general setting (see Bullard and Mitra, 2002).

There is scope for credit frictions and QE policy to affect the economy only if $0 < \theta < 1$. This is because of segmentation of the financial markets (see Gerali, Neri, Sessa and Signoretti, 2010; Cúrdia and Woodford, 2016; Carlstrom et al., 2017; Sims and Wu, 2019). The general idea is that there is some fraction of the households $0 \leq \theta < 1$ that cannot borrow and save according to the

[6] An incomplete list of papers that studies the New-Keynesian hybrid model include Gürkaynak, Sack and Swanson (2005), Lindé (2005), Söderström, Söderlind and Vredin (2005), Cho and Moreno (2006), Hördahl, Tristiani and Vredin (2006), Benati (2008), Bekaert, Cho and Moreno (2010), Bikbov and Chernov (2013), Baele, Bekaert, Cho, Inghelbrecht and Moreno (2015), Buncic and Lentner (2016).

real interest rate from a short bond because of other type of constraints. This is why we have the term $\frac{(1-\theta)}{\sigma}$ in front of the short real interest rate in the IS curve. The share of the population with these borrowing constraints θ instead responds in their consumption-saving decision to long-term conditions governed by long bond holdings of financial intermediates and the Central Bank. The last term in square brackets in the IS curve is essentially the spread between the interest from a bond with a high credit rating, such as a government bond, and a bond with lower rating, typically a lower rated corporate bond, known as a credit spread (see Sims and Wu, 2019).[7] The segmentation of the credit market is a consequence of making a distinction between the money market and the bond market. Conventional monetary policy involves adjusting the short rate for short-debt in the money market while unconventional policy entails adjusting the bond portfolio for long-debt in the bond market. This distinction is related to the classical segmentation theory in which preferences for instruments over different maturities are differentiated, making short and long bonds independent.

Provided that $0 < \theta < 1$ holds, the parameters β^{FI} and β^{CB} are weights steering the impact of credit frictions and QE policy on output. They measure long-term bond holdings of financial intermediates β^{FI} and the Central Bank β^{CB}. Total outstanding long bonds in the economy are owned by financial intermediates or the Central Bank, why $\beta^{FI} + \beta^{CB} = 1$ holds.[8] This means that the long bonds held by the financial intermediates and the Central bank, and ultimately total long bonds in the economy, are perfect substitutes. If $\beta^{FI} > \beta^{CB}$ then financial intermediates own more bonds and if $\beta^{FI} < \beta^{CB}$ the Central Bank owns more bonds.

Expansionary QE policy behavior would be to lower the credit spread by decreasing the long-term yield to support the long-term borrowing individuals, which happens when the Central Bank purchases more bonds. So, the mechanism $0 < \theta < 1$ works as $\sigma > 0$, but the θ is for households responding to conditions in the bond market and the σ is for households responding to money market conditions. Expectations of tighter credit conditions tomorrow relative to today makes it is easier to access credit today ($E_t cr_{t+1} < cr_t$), leading θ to consume and invest more in the current period, increasing current output. Expectations of higher long yields tomorrow relative to today by an anticipated sell-off in the Central Bank's bond portfolio from the current to the next period ($E_t qe_{t+1} < qe_t$) functions similarly. If ($E_t cr_{t+1} > cr_t$) and if the Central Bank intend to stabilize and

[7] With a credit spread it is common to use the difference in the interest paid from a corporate BAA bond and a 10-year treasury (see Lindé, Smets and Wouters, 2016, and references therein).
[8] This equality is not unique for this model, Carlstrom et al., (2017) considers a similar case.

prevent a decline in current output, they need to purchase more bonds today ($E_t qe_{t+1} < qe_t$) to off-set the worse credit conditions today. Whenever θ becomes higher for a constant σ, the transmission of unconventional QE policy becomes stronger relative to conventional policy. It also makes the economy more vulnerable to suddenly tighter credit conditions. The persistence of these effects is captured by ρ^{CR} and ρ^{QE}.

Credit and QE enters the Phillips curve with a negative sign because it has a competing impact on inflation (see Sims and Wu, 2019). The term in front of square brackets in (1) is a combination of other parameters; the θ is mainly motivated for the IS curve. The Phillips curve parameter λ is at work in favor of a boost of inflation when credit conditions in the market becomes looser (cr_t higher) or if the Central Bank purchases more bonds (qe_t higher) because λ transmits higher output to inflation when $0 < \theta < 1$ holds. If, on the other hand, the combined term in front of the square brackets in the Phillips curve is high (which includes λ) and dominates this effect, then higher cr_t and qe_t functions disinflationary.

Credit and QE enters the Phillips curve and the IS curve as "shocks" because they are modeled outside these behavioral equations. Along with (4)-(5) the natural rate (6) might be called shocks, not just their residuals. We could see θ as the response to credit and QE shocks where β^{FI} and β^{CB} are weights steering its relative effects, similar to the terminology used by Sims and Wu (2019). While (5) is a rule for QE, (4) is simply assumed to follow an AR(1) (see Sims and Wu, 2019). To let some variables be exogenous to some of the equations (1)-(3) is not new. In the monetary literature, Ireland (2004) let technology shocks in the IS curve follow an AR(1). In the macro-finance literature, Gürkaynak et al., (2005) let the inflation target be determined by some weight of past inflation in relation to the past inflation target; Hördahl et al., (2006) let the inflation target follow an AR(1); and Bekaert et al., (2010) let potential output follow an AR(1) and model the inflation target based on expectations of future inflation.

Fig. 1 shows the historical relationship between $\beta^{FI} + \beta^{CB} = 1$. Fig. 2 depicts the funds rate and asset holdings of the Fed. The β^{CB} has risen (or equivalently β^{FI} has fallen) relative to the sum of commercial bank credit and the Fed's asset holdings 1996:Q4-2019:Q4. The Fed has purchased long-term bonds in the market which means that there are fewer available to hold by the commercial banks. The pattern of β^{CB} follows the increase in the balance sheet during the effective lower bound of the funds rate 2008-2015.

We can write the model in matrix notation

$$\begin{bmatrix} 1 & -\lambda & 0 & \beta^{FI}\dfrac{\theta\lambda\sigma}{(1-\theta)} & \beta^{CB}\dfrac{\theta\lambda\sigma}{(1-\theta)} & 0 \\ 0 & 1 & \dfrac{(1-\theta)}{\sigma} & -\theta\beta^{FI} & -\theta\beta^{CB} & -\dfrac{(1-\theta)}{\sigma} \\ 0 & -(1-\rho^{TR})\phi_{\hat{y}} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ \hat{y}_t \\ i_t \\ cr_t \\ qe_t \\ r_t^n \end{bmatrix} = \begin{bmatrix} \delta & 0 & 0 & 0 & 0 & 0 \\ \dfrac{(1-\theta)}{\sigma} & \mu & 0 & -\theta\beta^{FI} & -\theta\beta^{CB} & 0 \\ (1-\rho^{TR})\phi_{\pi} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} E_t\pi_{t+1} \\ E_t\hat{y}_{t+1} \\ E_t i_{t+1} \\ E_t cr_{t+1} \\ E_t qe_{t+1} \\ E_t r_{t+1}^n \end{bmatrix} + \begin{bmatrix} (1-\delta) & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho^{TR} & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho^{CR} & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho^{QE} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho^{NR} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \hat{y}_{t-1} \\ i_{t-1} \\ cr_{t-1} \\ qe_{t-1} \\ r_{t-1}^n \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{PC} \\ \varepsilon_t^{IS} \\ \varepsilon_t^{MP} \\ \varepsilon_t^{CR} \\ \varepsilon_t^{QE} \\ \varepsilon_t^{NR} \end{bmatrix} \tag{8}$$

And in compact matrix notation

$$\Psi X_t = \Omega E_t X_{t+1} + \Lambda X_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Sigma) \tag{9}$$

Where $X_t = [\pi_t \ \hat{y}_t \ i_t \ cr_t \ qe_t \ r_t^n]^T$ is the (6 x 1) vectors of the variables; Ψ, Ω and Λ are the (6 x 6) matrices of the parameters; and $\varepsilon_t = [\varepsilon_t^{PC} \varepsilon_t^{IS} \ \varepsilon_t^{MP} \varepsilon_t^{CR} \ \varepsilon_t^{QE} \ \varepsilon_t^{NR}]^T$ is the (6 x 1) vector of the shocks where $\sum$ denotes the diagonal in the variance-covariance matrix.

The equilibrium solution for linear rational expectations models like (7) are derived from the compact matrix notation and takes the form of a VAR(1)

$$X_{t+1} = \Xi X_t + I\varepsilon_t \tag{10}$$

Where Ξ and I are (3 x 3) matrices with non-linear parameters (see Sims, 2001; Cho and Moreno, 2006, 2011, and references therein).

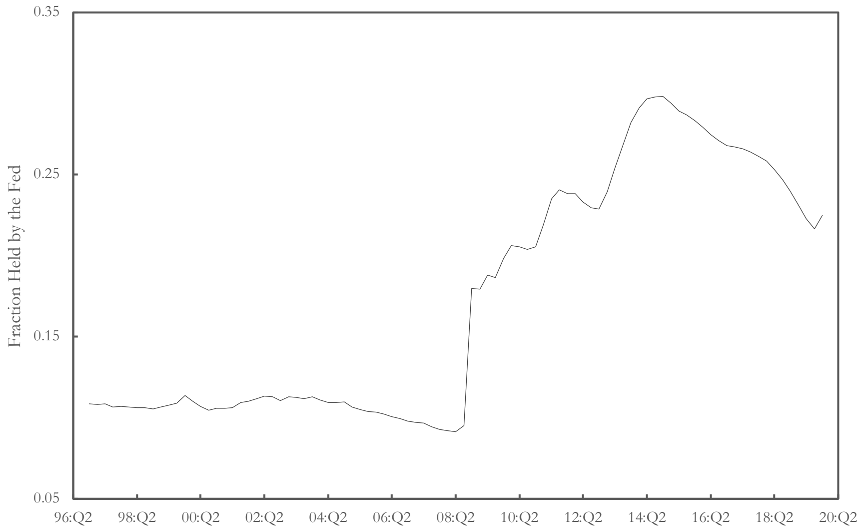

FIGURE 1. Long-term bond holdings of the Fed relative to total outstanding long bonds 1996:Q4-2019:Q4.
Source: Board of Governors of the Federal Reserve System and the author's calculations.

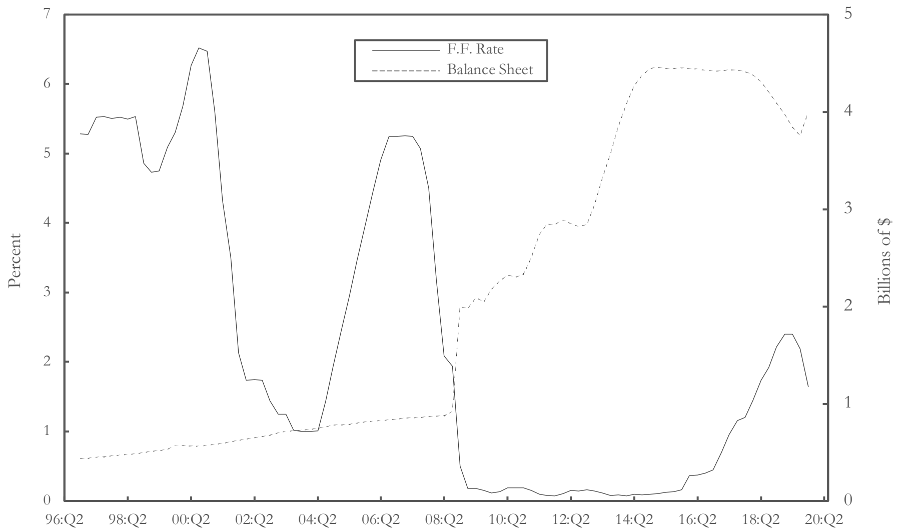

FIGURE 2. The Federal Funds Rate (left axis) and Asset holdings of the Fed (right axis) 1996:Q4-2019:Q4.
Source: Board of Governors of the Federal Reserve System and the author's calculations.

2.2 The VAR Model

We have seen that the New-Keynesian model has many restrictions and mentioned in the introduction that DSGE models are sometimes constructed on empirical findings from VAR models. We estimate a VAR to complement the DSGE in our study of the dynamics of the 6 variables using another type of model with less restrictions. We then compare impulse-response functions between the two models and analyse similarities and trace dissimilarities, mainly qualitatively.[9]

We estimate an unrestricted VAR as proposed by Sims (1980). In VARs one does not have to *a priori* distinguish if the variables are endogenous or exogenous Sims (1980), why Sims (1980) advocated the use of VARs rather than the structural models at that time with many restrictions. Another advantage with VARs is the rich parameterization which makes them popular for forecasting purposes (Karlsson, 2013). An advantage of structural models is the possibility to carry out systematic policy analysis and trace the behavior of economic agents. We choose to not impose restrictions of the VAR making it for example a structural VAR but rather let it be unrestricted to let the data speak (Sims, 1981). The drawback is that the two models becomes less comparable since we do not identity structural shocks.[10] We base our choice on that we do not know if the structural model characterizes the economy adequately (in the data) because it is small and has to our knowledge not been estimated previously. To impose restrictions in a VAR is most efficient if one is certain that a structural model accurately describes the economy (Bernanke, Boivin and Eliasz, 2005).

The unrestricted VAR(p) in matrix notation is

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \\ i_t \\ r_t^n \\ cr_t \\ qe_t \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{bmatrix} + \begin{bmatrix} \varphi_{1,1}^1 & \varphi_{1,2}^1 & \varphi_{1,3}^1 & \varphi_{1,4}^1 & \varphi_{1,5}^1 & \varphi_{1,6}^1 \\ \varphi_{2,1}^1 & \varphi_{2,2}^1 & \varphi_{2,3}^1 & \varphi_{2,4}^1 & \varphi_{2,5}^1 & \varphi_{2,6}^1 \\ \varphi_{3,1}^1 & \varphi_{3,2}^1 & \varphi_{3,3}^1 & \varphi_{3,4}^1 & \varphi_{3,5}^1 & \varphi_{3,6}^1 \\ \varphi_{4,1}^1 & \varphi_{4,2}^1 & \varphi_{4,3}^1 & \varphi_{4,4}^1 & \varphi_{4,5}^1 & \varphi_{4,6}^1 \\ \varphi_{5,1}^1 & \varphi_{5,2}^1 & \varphi_{5,3}^1 & \varphi_{5,4}^1 & \varphi_{5,5}^1 & \varphi_{5,6}^1 \\ \varphi_{6,1}^1 & \varphi_{6,2}^1 & \varphi_{6,3}^1 & \varphi_{6,4}^1 & \varphi_{6,5}^1 & \varphi_{6,6}^1 \end{bmatrix} \begin{bmatrix} \hat{y}_{t-1} \\ \pi_{t-1} \\ i_{t-1} \\ r_{t-1}^n \\ cr_{t-1} \\ qe_{t-1} \end{bmatrix} + \ldots + \begin{bmatrix} \varphi_{1,1}^p & \varphi_{1,2}^p & \varphi_{1,3}^p & \varphi_{1,4}^p & \varphi_{1,5}^p & \varphi_{1,6}^p \\ \varphi_{2,1}^p & \varphi_{2,2}^p & \varphi_{2,3}^p & \varphi_{2,4}^p & \varphi_{2,5}^p & \varphi_{2,6}^p \\ \varphi_{3,1}^p & \varphi_{3,2}^p & \varphi_{3,3}^p & \varphi_{3,4}^p & \varphi_{3,5}^p & \varphi_{3,6}^p \\ \varphi_{4,1}^p & \varphi_{4,2}^p & \varphi_{4,3}^p & \varphi_{4,4}^p & \varphi_{4,5}^p & \varphi_{4,6}^p \\ \varphi_{5,1}^p & \varphi_{5,2}^p & \varphi_{5,3}^p & \varphi_{5,4}^p & \varphi_{5,5}^p & \varphi_{5,6}^p \\ \varphi_{6,1}^p & \varphi_{6,2}^p & \varphi_{6,3}^p & \varphi_{6,4}^p & \varphi_{6,5}^p & \varphi_{6,6}^p \end{bmatrix} \begin{bmatrix} \hat{y}_{t-p} \\ \pi_{t-p} \\ i_{t-p} \\ r_{t-p}^n \\ cr_{t-p} \\ qe_{t-p} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{\hat{y}} \\ \epsilon_t^{\pi} \\ \epsilon_t^{i} \\ \epsilon_t^{nr} \\ \epsilon_t^{cr} \\ \epsilon_t^{qe} \end{bmatrix} \quad (11)$$

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[9] The intention is not to match DSGE-functions from numerical solutions with functions from a VAR. Fernández-Villaverde, Rubio-Ramírez, Sargent and Watson (2007) shows under what conditions shocks in a structural model is consistent with shocks in a VAR.

[10] Rudebusch (2002a) also compare impulse-response functions from a structural model with an unrestricted VAR.

And in compact matrix notation

$$Y_t = \Gamma_0 + \Phi_1 Y_{t-1} + \ldots + \Phi_P Y_{t-p} + \epsilon_t\,, \quad \epsilon_t \sim N(0,\Sigma) \tag{12}$$

Where $Y_t = [\hat{y}_t\ \pi_t\ i_t\ r_t^n\ cr_t\ qe_t]^T$ is the (6 x 1) vector of the contemporaneous values of the variables; $\Gamma_0 = [\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4\ \gamma_5\ \gamma_6]^T$ is the (6 x 1) vector of constants; each Φ_P for $1 \leq p$ is the (6 x 6) matrices of persistence parameters of the associated Y_{t-p} $1 \leq p$ lags of the variables; and $\epsilon_t = [\epsilon_t^{\hat{y}}\ \epsilon_t^{\pi}\ \epsilon_t^{i}\ \epsilon_t^{nr}\ \epsilon_t^{cr}\ \epsilon_t^{qe}]^T$ is the (6 x 1) vector of the error terms for each of the six equations where $\sum$ denotes the diagonal in the variance-covariance matrix. As with FIML, VAR allows contemporaneous cross-correlations of residuals why these are assumed to be multivariate normal. The variances of the error terms are denoted by $[\sigma_{\hat{y}}^2\ \sigma_{\pi}^2\ \sigma_{i}^2 \sigma_{nr}^2\ \sigma_{cr}^2\ \sigma_{qe}^2]$.

As in much of the monetary policy literature we set the real driving variable first, prices and interest rates (Christiano et al., 1999; Bernanke et al., 2005). We then follow Boivin, Giannoni and Stevanovic (2020) and set credit. QE is last. The ordering is relevant for the impulse-response functions as we are going to use the Cholesky decomposition, implying here that a shock to the output equation set first may have an impact on all the 6 variables simultaneously; a shock to the credit equation set fifth could jointly affect credit and QE; a shock to the QE equation set last only has an effect on QE contemporaneously. The ordering does not matter for the estimates because the VAR is of the reduced form so the equations could efficiently be estimated as a system with ordinary least squares (OLS).

The DSGE model is theory-based and forward-looking. Our VAR model is backward-looking and more flexible, and allows us to study and detect any relationship across the 6 variables. We will be able to study if credit and QE are appropriately modeled as AR(1), if there is a positive relationship of credit and QE on output, and if there is a one-way relationship of credit and QE shocks being predominantly inflationary or disinflationary. The order of lags p is to be determined by economic theory or some statistical criterion. Given that the natural interest rate is persistent it could bias the joint statistics for the optimal number of lags. We shall come later to this after having shown how the natural rate is estimated.

3. The Data

The data used in the empirical analysis is quarterly U.S. data spanning over the period 1996:Q4-2019:Q3.[11] The variables are measured as deviations from a non-stochastic trend, which is defined as de-meaned data.[12] To de-mean the series is the last step prior to estimation.

The data of inflation, output and the short-term nominal interest rate with computations are standard. The data for inflation and output are retrieved at quarterly frequency from the Bureau of Economic Analysis and the nominal interest rate at monthly frequency is from the Board of Governors of the Federal Reserve System, all three via Fred, Federal Reserve Bank of St. Louis. Inflation is defined as a quarterly annualized change of the implicit GDP deflator $\pi_t = 400(\ln P_t - \ln P_{t-1})$ where P_t is the GDP deflator. The GDP deflator is a price index for total output and not only consumption goods and services. The index is often used for the U.S. in related research (Lindé, 2005; Cho and Moreno, 2006). The data for output is real GDP at 2012 chained prices. The series is detrended with the HP-filter setting the multiplier to 1600. Denoting the log of real GDP by y_t and the potential by y_t^n the output gap is computed as $\hat{y}_t = 100(y_t - y_t^n)$. We choose to not use other measures of the output gap as various estimates for the U.S. yields similar results (Lindé, 2005; Cho and Moreno, 2006).[13] Another limitation of our study relates to the use of the HP-filter. It is proposed that one should remove the first twelve and the last twelve estimates when using the HP-filter on quarterly data (Sørensen, and Whitta-Jacobsen, 2010, chapter 13). Since we would like to include as many observations as possible, we remove only the first two and last two estimates in our main tests. A sub-sample test is later carried out when excluding the last 12 observations applies.

The short-term money market nominal interest rate is the policy rate of the Fed, the effective Funds rate. Quarterly observations are obtained by taking the average over the associated months. The natural interest rate is estimated with the HP-filter of the ex-ante short real interest rate $r_t = [i_t - E_t\pi_{t+1}]$. Several methods are employed to compute the natural interest rate and could lead to imprecise estimates (Laubach and Williams, 2003). The HP-filter is however widely used (Garnier and Wilhelmsen, 2005; Stracca, 2010; Krustev; 2018).

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[11] In the appendix we report FIML estimates of the New-Keynesian model on semi-annual data. With semi-annual we set the penalty parameter of the HP-filter equal to 400.

[12] Most of the variables used in the econometric tests are stationary according to the augumented Dickey-Fuller test. Test results are available in the appendix.

[13] See also Fuhrer and Rudebusch (2004) who considers five different definitions of potential output in a study of the IS curve.

At this point it is worth emphasizing that our definitions of some of the variables in the equations (1)-(3) may not be optimal. In separate estimation of the Phillips curve, the IS curve or the Taylor rule the literature sometimes uses different definitions of inflation, the ex-ante real interest rate or the "inflation gap". If we take inflation as an example, in the Phillips curve inflation should be defined as a quarterly change since it is a short-run supply curve. In the IS curve, one could use the definition of an annual change (i.e., $\pi_t^a = 100(\ln P_t - \ln P_{t-3})$ because the short-term interest rate is in units of percent per year. In the Taylor rule, the definition π_t^a is frequently used too because Central Banks inflation targets are defined over an annual change. For reference, Rudebusch (2002a) estimate the equations separately and uses π_t in the Phillips curve and expectations data of π_t^a in the IS curve. Other authors estimating the equations simultaneously uses the same definitions of the variables across all equations (Lindé, 2005; Cho and Moreno, 2006; Bunic and Lentner, 2016).

The data to represent credit conditions in the financial markets is the total credit (volume) of all commercial banks in the U.S. The sample for each observation is based on reported values by some 875 banks and institutions. Weekly data expressed in billions of U.S. dollars is retrieved seasonally adjusted from the Board of Governors of the Federal Reserve System via Fred. Quarterly observations are the average over the associated weeks. Credit and the balance sheet should both be in real values as with the ex-ante real interest rate for money market conditions. Denoting total bank credit by CR_t the real market value is calculated as $cr_t = CR_t/P_t$. Then we take logs $\ln(CR_t/P_t)$. The data is upward trending during the sample period and our analysis concerns business cycles; we compute the first-difference of the series $\Delta cr_t = 100(cr_t - cr_{t-1})$.[14] Other variables such as investments and wages are subject to the log first-difference in the literature (Lindé et al., 2016).

The data of the Central Bank's long-term bond holdings is the balance sheet of the Federal Reserve, i.e., the reserve balance credit. The weekly releases of the Fed's balance sheet announced by the Fed dates back to June 27, 1996.[15] Both the weekly average and the Wednesday level expressed in millions of dollars is retrieved each week. So, we have two measures of the balance sheet which will be tested separately.[16] Quarterly observations are the average over the associated weeks. The succeeding computations follows the same procedure as credit. Denoting the quarterly face value

[14] Even though credit volume and large-scale asset purchases are incorporated in the equations it is yet a stochastic business cycle model with focus on equilibrium conditions. Galí (2018) discusses the need for DSGE models to in the future work with accumulations.
[15] Available at the following link: https://www.federalreserve.gov/releases/h41/
[16] The data available as series at weekly frequency via Fred is usually the Wednesday levels.

of the balance sheet by QE_t the real market value is calculated as $qe_t = QE_t/P_t$. The series is then subject to logs $qe_t = \ln(QE_t/P_t)$. The final step is to take the first-difference $\Delta qe_t = 100(qe_t - qe_{t-1})$. The data for QE is chosen quite strictly based on the New-Keynesian equations. An increase in the balance sheet could lead to lower long yields, which is an effect of more bond purchases by the Central Bank. One may instead use a long-short spread, however some of the VAR literature also uses Central Banks total asset holdings for QE (e.g. Beck, Duca and Stracca, 2019).

One observation across all variables are removed due to a large outlier of Δqe_t in 2008:Q4.[17] This is due to a significant easing by the Fed during the financial crisis. This can be seen in fig. 1. The spike would have a considerable effect on all the parameters in the tests. The observations are excluded before de-meaning the series.

[17] The Δqe_t Wednesday level goes from 0.023 at 2008:Q3 to 0.754 at 2008:Q4, then down to -0.031 at 2009:Q1.

4. Methodology

4.1 Estimation Strategy and Simulations of the DSGE

The New-Keynesian model is a DSGE model which are often estimated as a system rather than equation-by-equation (Blanchard, 2016). The equations are estimated simultaneously with FIML allowing contemporaneous correlations between the shocks.[18] FIML is maximum likelihood (ML) techniques that generates model-consistent predictions of forward-looking variables but uses all information available from a whole system of equations to maximize the likelihood function. Instrumental variables techniques such as system GMM is also considered in the literature (Bekaert et al., 2010). Whereas Bayesian methods are used for both small-scale models (e.g. Benati, 2008) and medium-scale models (e.g. Smets and Wouters, 2007), there could be situations when FIML is appropriate (Mickelsson, 2015). In all specifications we set the diagonal covariance matrix, BHHH optimization and the Hessian information matrix.

The first thing we do is some algebraic manipulation of the IES. We rewrite σ because $1/\sigma = \sigma^{-1}$, but the -1 is dropped for simplicity as in the empirical literature (e.g. Fuhrer and Rudebusch, 2004; Stracca, 2010). This means that the term in front of the last square brackets in the IS curve (2) becomes $\frac{(1-\theta)}{\sigma} = (1-\theta)\sigma$. For the combined Phillips curve parameter we use the shorthand notation $\eta = \frac{\theta\lambda\sigma}{(1-\theta)}$

We have 6 specifications for our parameter estimates. We choose to not experiment with the equations other than discussed below. It would make it hard to discuss the magnitude of the parameter estimates; our analysis concerns a complete model with relatively high parameter interdependency. The Phillips curve, the IS curve or the Taylor rule have separately been subject to extensive robustness analysis (Fuhrer and Rudebusch, 2004; Goodhart and Hofmann, 2005; Galí, Gertler and Salido-Lopez, 2006; Mavroeidis, 2010; Stracca, 2010, among others).

Our first specification is to estimate the model unconstrained; the equations are written as a system exactly as in this essay. So, it is unconstrained given how it is specified from the very beginning. Next, given $\beta^{FI} + \beta^{CB} = 1$, we set $\beta^{CB} = (1 - \beta^{FI})$ and $\beta^{FI} = (1 - \beta^{CB})$, respectively. Even though this functions as a constraint for the parameter being the "residual", the same is not true for the parameter to be estimated. Put differently, if we set $\beta^{CB} = (1 - \beta^{FI})$, then β^{FI} is still allowed be become as high or low as possible, i.e. is unconstrained. Whenever Δcr_t is in front of Δqe_t in the Phillips curve and the IS curve, we set $\beta^{CB} = (1 - \beta^{FI})$ and vice versa. From an econometric point

[18] The empirical analysis is carried out in Eviews 11.

of view this is not conventional as we base on theory why $\beta^{FI} + \beta^{CB} = 1$ should hold even when taken to the data. This case has its similarities with letting lagged inflation and output be equal to $(1-\delta)$ and $(1-\mu)$ and the weighted response of the Central Bank be $(1-\rho^{TR})$. These specifications are however justified econometrically (Lindé, 2005; Mavroeidis 2005) though not without criticism (Rudebusch, 2002b; Rudd and Whelan, 2005; Mavroeidis, 2010). We leave open a discussion about how β^{FI} and β^{CB} could be most efficiently estimated.

In our final specification we set $\sigma = 1$ so there is no competing substitution and income effect. This is set in calibrations in both the canonical New-Keynesian model (Woodford, 2003; Galí, 2015; Walsh, 2018) and in larger models (Carlstrom et al., 2017; Del Negro et al., 2017). Here we try it partly to relax the parameter dependency as the consequence is that σ disappears. Since ML estimation results in a low IES, Baele et al., (2015) constrain it to $\sigma = 0.1$. We set $\sigma = 1$ for the unconstrained case, for $\beta^{CB} = (1-\beta^{FI})$ and $\beta^{FI} = (1-\beta^{CB})$, which adds up to our 6 specifications.

The FIML estimates will be subject to diagnostic tests. These include testing the strong assumptions of normality and non-serial correlations of the residuals, testing some parameters jointly with the Wald test and making a structural break test. Given that credit, the balance sheet and the natural rate does not include forward-looking terms, one could estimate these equations with OLS which is more efficient than ML (Verbeek, 2012, chapter 6). We assume that FIML and OLS estimates of the AR(1) would not be too different. As with the lags of inflation, output and the short rate these AR(1) can be contrasted with the estimates from a VAR(1). For the simulations we use the Dynare to numerically solve the model.[19] We choose to depict the impulse-response functions from the simulations and the VAR over a 20-period horizon because variance decompositions sometimes become constant after this time (see e.g. Bekaert et al., 2010).

4.2 Specification of the VAR

The ordering of the variables in the VAR was discussed in section 2.1 and we showed how the natural interest rate is estimated in section 3. The natural rate estimated as the HP-trend of the ex-ante short-term real interest rate is very persistent and is impactful on the statistics for the optimal number of lags. When the natural rate is included in the VAR the lowest criterion proposes that 4 is the optimal number of lags. When the natural rate is excluded the optimal number of lags is 1 according to the Schwarz information criterion (SC) and 2 according to the likelihood-ratio test

[19] Available at the following link: https://www.dynare.org. Dynare is used in textbooks (Galí, 2015; Walsh, 2018) and in papers (Smets and Wouters, 2007; Lindé et al., 2016; Del Negro et al., 2017).

(LR), Final prediction error (FPE), Akaike information criterion (AIC) and Hannah-Quinn information criterion (HQ).[20] To choose a VAR(4) could lead to overfitting the equations. Too many lags may disturb inferences. We report estimates and statistics from VARs of two orders: a VAR(1) and a VAR(2).[21] We choose these orders also because linear rational expectations models can be derived into a VAR(1) with non-linear parameter restrictions as indicated previously (see Sims, 2001; Cho and Moreno, 2006, 2011, and references therein). As with the FIML estimates, diagnostic tests for normality and autocorrelation will be listed. Finally, since our data is de-meaned we do not include any constants $\Gamma_0 = 0$.

[20] The test statistics of lag selection and exclusion are available in the appendix.
[21] Some of the monetary policy literature uses 4 lags with quarterly data (Christiano et al., 1999). We have a relatively short sample period.

5. Empirical Results

5.1 FIML Parameter Estimates and VAR(1) Estimates

Table 1 presents the parameter estimates of the New-Keynesian DSGE model yielding correctly signs in all our 6 specifications. It also lists the estimated volatilities of the shocks. The VAR(1) estimates are shown in table 2 which reports significant estimates of roughly 40 percent of the first-order lags.

TABLE 1. FIML ESTIMATES OF THE FOUR EQUATION NEW-KEYNESIAN DSGE MODEL ON U.S DATA 1996:Q4–2019:Q3. Δqe_t = THE QUARTERLY AVERAGE OF THE WEDNESDAY LEVEL

Specification Parameters	Unconstrained (1)	$\beta^{CB} = (1-\beta^{FI})$ (2)	$\beta^{FI} = (1-\beta^{CB})$ (3)	As (1) + $\sigma = 1$ (4)	As (2) + $\sigma = 1$ (5)	As (3) + $\sigma = 1$ (6)
δ	0.498***	0.498***	0.498***	0.497***	0.500***	0.500***
	(0.077)	(0.077)	(0.077)	(0.077)	(0.077)	(0.077)
λ	0.064	0.060	0.060	0.010	0.091	0.091
	(0.078)	(0.077)	(0.077)	(0.030)	(0.061)	(0.061)
θ	0.977***	0.109***	0.109***	0.986***	0.570***	0.570***
	(0.077)	(0.040)	(0.040)	(0.036)	(0.056)	(0.056)
σ	0.244	0.007	0.007	1.000	1.000	1.000
	(0.805)	(0.041)	(0.041)	-	-	-
β^{FI}	0.080**	0.606***	0.606	0.076**	0.805***	0.805
	(0.038)	(0.162)	-	(0.037)	(0.052)	-
β^{CB}	0.040**	0.394	0.394**	0.040**	0.195	0.195***
	(0.018)	-	(0.162)	(0.017)	-	(0.052)
μ	0.492***	0.490***	0.490***	0.488***	0.324***	0.324***
	(0.047)	(0.047)	(0.047)	(0.047)	(0.094)	(0.094)
ρ^{TR}	0.942***	0.942***	0.942***	0.942***	0.925***	0.925***
	(0.021)	(0.021)	(0.021)	(0.021)	(0.022)	(0.022)
ϕ_π	2.232**	2.230**	2.230**	2.210**	1.424*	1.424*
	(1.205)	(1.203)	(1.203)	(1.191)	(0.791)	(0.787)
$\phi_{\hat{y}}$	1.687**	1.688**	1.688**	1.703**	2.278***	2.278***
	(0.720)	(0.720)	(0.720)	(0.717)	(0.641)	(0.641)
ρ^{CR}	0.490***	0.490***	0.490***	0.490***	0.490***	0.490***
	(0.092)	(0.092)	(0.092)	(0.092)	(0.092)	(0.092)
ρ^{QE}	0.524***	0.524***	0.524***	0.524***	0.524***	0.524***
	(0.093)	(0.093)	(0.093)	(0.093)	(0.093)	(0.093)
ρ^{NR}	0.975***	0.975***	0.975***	0.975***	0.975***	0.975***
	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)
Volatilities						
σ_{PC}	0.801	0.800	0.800	0.798	0.787	0.787
σ_{IS}	0.378	0.376	0.376	0.376	0.766	0.766
σ_{MP}	0.385	0.385	0.385	0.385	0.391	0.391
σ_{CR}	0.902	0.902	0.902	0.902	0.902	0.902
σ_{QE}	2.208	2.208	2.208	2.208	2.208	2.208
σ_{NR}	0.106	0.106	0.106	0.106	0.106	0.106

NOTES: The table shows the FIML estimates of the New-Keynesian model on U.S. quarterly data 1996:Q4–2019:Q3 (2008:Q4 excluded) using the annualized log first-difference of the GDP deflator, HP-detrended real output 2012 chained prices, the quarterly average of the weekly effective Federal funds rate, the log first-difference of the total bank credit of all commercial banks in the U.S., the log first-difference of the total balance sheet of the Fed, and the HP-trend of the ex-ante real interest rate. Credit and the balance sheet are in real market values, GDP deflated. The columns shows how the model is specified prior to estimation. The optimization method is BHHH (max 5000 iterations; covergence is achieved), the covariance matrix is diagonal, and the Hessian Information matrix. Standard errors in parantheses. $^{*}p<0.1$, $^{**}p<0.05$, $^{***}p<0.01$. The equations in de-meaned form are

$$\pi_t = \delta E_t\pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda\hat{y}_t - \eta\left[\beta^{FI}\Delta cr_t + \beta^{CB}\Delta qe_t\right] + \varepsilon_t^{PC}, \quad \eta = \frac{\theta\lambda\sigma}{(1-\theta)}$$

$$\hat{y}_t = \mu E_t\hat{y}_{t+1} + (1-\mu)\hat{y}_{t-1} - (1-\theta)\,\sigma\left[i_t - E_t\pi_{t+1} - r_t^n\right] - \theta\left[\beta^{FI}(E_t\Delta cr_{t+1} - \Delta cr_t) + \beta^{CB}(E_t\Delta qe_{t+1} - \Delta qe_t)\right] + \varepsilon_t^{IS}$$

$$i_t = (1-\rho^{TR})\left[\phi_\pi E_t\pi_{t+1} + \phi_{\hat{y}}\hat{y}_t\right] + \rho^{TR} i_{t-1} + \varepsilon_t^{MP}$$

$$\Delta cr_t = \rho^{CR}\Delta cr_{t-1} + \varepsilon_t^{CR}$$

$$\Delta qe_t = \rho^{QE}\Delta qe_{t-1} + \varepsilon_t^{QE}$$

$$r_t^n = \rho^{NR} r_{t-1}^n + \varepsilon_t^{NR}$$

The parameters are correctly signed and most of them are significant.[22] The degree of forward-looking behavior of inflation δ and output μ is around 0.5, which is by and large in line with FIML or ML estimates of this class of models (Cho and Moreno, 2006; Bikbov and Chernov, 2013; Buncic and Lentner, 2016).[23] Since many use pre-crisis data, our results yields updated evidence that these parameters are structural. The slope of the Phillips curve λ has generally been found to be more variant, our estimates lies somewhere in between lower values around 0.001 (Cho and Moreno, 2006) and higher values like 0.100 (Baele et al., 2015). Its value in spec. (1)-(3) is in line with Bekaert et al., (2010). As in our case the literature has struggled to obtain significant estimates of λ (Cho and Moreno, 2006; Buncic and Lentner, 2016). Baele et al., (2015) finds a significant and higher value with actual expectations data.

The IES σ is correctly signed but not significant and low, implying a weak response of output to the money market short-term real interest rate, and ultimately a weak channel for conventional monetary policy. Estimates of σ are often rejected, which is problematic since this mechanism is fundamental in DSGE models (Christiano, Trabandt and Walentin, 2010). We recall that σ is the inverse and as our estimates suggests $\theta > 0$ with $(1 - \theta)\sigma$, the response of output to changes in the short interest rate becomes even lower. Recent estimates of the IES with newly developed econometric methods yields values close to zero for the U.S. (see Ascari et al., 2019).

The estimates of θ which measures the response of output to conditions on the bond market are significant, implying that the QE policy channel works in the model. The value in spec. (2)-(3) leads to a close value as in calibrations $\theta = 0.33$ by Sims and Wu (2019). Gerali et al., (2010) considers a similar parameter and calibrate it to 0.2. In the appendix we report estimates of $\theta = 0.2$ using semi-annual data. A higher θ *ceteris paribus* makes the transmission of QE policy stronger. A higher θ could also make the economy more vulnerable to credit frictions; if β^{FI} is high relative to β^{CB} then the Fed would need to purchase more bonds if the credit conditions in the market becomes tighter to prevent a slowdown in output. Not surprisingly the estimate varies across our specifications. The trio $\{\theta\ \beta^{FI}\ \beta^{CB}\}$ requires to set $\beta^{FI} + \beta^{CB} = 1$ to reach theoretically interpretable values. This also

[22] The estimates with Δqe_t = the quarterly average of the weekly average are not so different. They are found in the appendix. The empirical analysis is with Δqe_t = the quarterly average of the wednesday level.

[23] For previous estimates of the parameters $\{\delta\ \lambda\ \sigma\ \mu\ \rho^{TR} \phi_\pi\ \phi_{\tilde{y}}\}$, see also Rudebusch (2002a), Lindé (2005), Söderström et al., (2005), Hördahl et al., (2006), and Benati (2008).

yields the most realistic dynamics with respect to the data.[24] We find that these parameters are significant nevertheless also when unconstrained.

The weights of U.S. commercial banks β^{FI} and the Fed's β^{CB} long-term bond holdings are positive and significant but also varies. Together with $\theta > 0$, looser credit conditions in the market in the current quarter relative to expectations about the next quarter ($E_t\Delta cr_{t+1} < \Delta cr_t$) boosts current output. More credit is granted which stimulates consumption and investments, increasing output in the current quarter. An expansion in the Fed's balance sheet in the current quarter relative to expectations about the next quarter ($E_t\Delta qe_{t+1} < \Delta qe_t$) functions in a similar way. Our estimates yields $\beta^{FI} > \beta^{CB}$ meaning that U.S. commercial banks hold more bonds (assets) than the Fed, in line with the historical trend over the sample. Carlstrom et al., (2017) considers a similar parameter as β^{CB} and calibrate it to 0.4.

The parameters that makes up the Phillips curve parameter η have the correct signs, meaning that credit and QE shocks are disinflationary by the value of η. The parameter is however not stable since it depends on λ and σ, both of which are rejected. The estimates of λ and σ are by and large in line with the empirical literature, as discussed. If we instead contrast with calibrated values by Sims and Wu (2019), they set so that $\eta = 0.042$. In spec. (1) we have $\eta = 0.663$ and in (2)-(3) $\eta = 0.00005$. For the data, the value $\eta = 0.00005$ seems most plausible in comparison with the dynamics of $\{\theta\ \beta^{FI}\ \beta^{CB}\}$ steering log-differences of credit and QE on output. We shall return to a discussion of the competing effects of credit and QE shocks on inflation.

The conventional policy-response parameters ϕ_π and $\phi_{\hat{y}}$ are high and could be categorized as consistent with the activist-regime in Baele et al., (2015). We find that the Fed reacts strongly to the output gap, $\phi_{\hat{y}}$ is well above one. The degree of interest rate smoothing is high, in line with previous findings (e.g. Clarida et al., 1999, 2000). The persistence of credit conditions ρ^{CR} in the market and of QE policy ρ^{QE} is around 0.5 indicating a relatively persistent effect of these shocks, but not so compared to calibrated values of 0.8 (Sims and Wu, 2019). The natural rate is very persistent, the high ρ^{NR} makes natural rate shocks have a long-lived effect in its propagation through the economic system.

[24] The combined effects in the IS curve are however not so different across some of the specifications. In spec. (1) we have $0.977[0.080(E_t\Delta cr_{t+1} - \Delta cr_t) + 0.040(E_t\Delta qe_{t+1} - \Delta qe_t)] = [0.078(E_t\Delta cr_{t+1} - \Delta cr_t) + 0.039(E_t\Delta qe_{t+1} - \Delta qe_t)]$. In spec. (2)-(3) we have $0.109[0.606(E_t\Delta cr_{t+1} - \Delta cr_t) + 0.394(E_t\Delta qe_{t+1} - \Delta qe_t)] = [0.066(E_t\Delta cr_{t+1} - \Delta cr_t) + 0.043(E_t\Delta qe_{t+1} - \Delta qe_t)]$

Comparing our estimates and the fit of the equations across the 6 specifications, it is clear that to impose $\beta^{FI} + \beta^{CB} = 1$ leads to the most reasonable values of these parameters from a theoretical perspective. However, the IS curve has worse fit in spec. (5)-(6) when we also set $\sigma = 1$. This is evident by the higher standard deviation of the IS shocks σ_{IS}. The forward-looking parameter of output μ is also lower. It is generally not conventional to set $\sigma = 1$ in the empirical literature. Instead comparing spec. (1)-(3), we see that the shock volatilities does not change despite different estimates of $\{\theta \ \beta^{FI} \ \beta^{CB}\}$. The volatilities of the credit σ_{CR} and QE shocks σ_{QE} are high; the AR(1) are too simple for these shocks. The standard deviation of the supply shocks σ_{PC} are high, implying a relatively bad fit. The IS curve and the Taylor rule have better fit with lower volatilities of the demand σ_{IS} and policy shocks σ_{MP}.

We pick spec. (2)-(3) for our subsequent analysis of shocks. This could be considered as our structural estimation; the estimates are theoretically interpretable and the standard parameters are by and large in line with previous findings. What can we say about the model's analytics with these parameters? It is solvable; the Taylor principle $\phi_\pi > 1$ is a general requirement for determinacy under the Blanchard-Kahn conditions for otherwise plausible parameter values. Our high $\phi_\pi = 2.230$ and $\phi_{\hat{y}} = 1.688$ implies strong reactions to expected future inflation and the contemporaneous output gap; the Fed adjusts the funds rate relatively aggressively to changed economic conditions, but in a sluggish fashion $\rho^{TR} = 0.942$. QE policy and credit frictions have a relatively high impact on output $\theta = 0.109$ relative to conventional policy $(1 - \theta)\sigma = 0.006$. The expectations of credit conditions Δcr_{t+1} and bond purchases Δqe_{t+1} in the next period however prevents an amplification. The competing effect of credit and QE shocks on inflation will be predominantly inflationary rather than disinflationary. This is because the effect of the response θ to higher Δcr_t and Δqe_t with the transmission of higher output to inflation λ is stronger than the disinflationary force η. Our finding $\beta^{FI} > \beta^{CB}$ implies that credit shocks have more profound effects than QE shocks for a 1 percent standard deviation increase in either of these variables. We shall return to an analysis of shocks but now contrast the relationships found from the FIML estimates of the structural model with the estimates of an unrestricted VAR(1).

TABLE 2. VAR(1) ESTIMATES

Equations/lags	$\hat{y}_t$	π_t	i_t	r^n_t	Δcr_t	Δqe_t
$\hat{y}_{t-1}$	0.671***	-0.020	-0.017	0.005	-0.017	-0.231
	(0.091)	(0.132)	(0.067)	(0.016)	(0.149)	(0.357)
π_{t-1}	-0.031	0.355***	0.103**	0.018	0.114	-0.053
	(0.073)	(0.107)	(0.054)	(0.013)	(0.120)	(0.289)
i_{t-1}	0.185**	0.156	0.898***	-0.004	0.066	0.043
	(0.093)	(0.135)	(0.068)	(0.016)	(0.151)	(0.365)
r^n_{t-1}	-0.193**	-0.199	0.074	0.973***	0.012	0.057
	(0.104)	(0.151)	(0.076)	(0.019)	(0.170)	(0.409)
Δcr_{t-1}	0.102*	0.104	0.075*	0.025**	0.429***	-0.569**
	(0.064)	(0.093)	(0.047)	(0.011)	(0.104)	(0.251)
Δqe_{t-1}	-0.017	0.000	-0.025	-0.011**	-0.004	0.463***
	(0.024)	(0.035)	(0.017)	(0.004)	(0.039)	(0.093)
Volatilities	$\sigma_{\hat{y}}$	σ_π	σ_i	σ_{nr}	σ_{cr}	σ_{qe}
St.dev.	0.551	0.801	0.402	0.098	0.901	2.163

NOTES: The table shows the unrestricted VAR(1) estimates. Standard errors in parentheses. *p<0.1, **p<0.05, ***p<0.01. R^2_{adj} [$\hat{y}_t$ π_t i_t r^n_t Δcr_t Δqe_t] = (0.737 0.186 0.965 0.997 0.232 0.301). D-W [$\hat{y}_t$ π_t i_t r^n_t Δcr_t Δqe_t] = (1.565 2.009 0.897 0.182 2.026 2.112).

Roughly 40 percent of the first-order lags are significant at conventional levels. We see that the inflation lag is lower in the VAR(1) while lagged output is higher. The output equation has the best fit of the 6 equations with respect to the 6 variables and its contemporaneous value is positively affected by the first-order credit lag. The positive impact of credit on output from the VAR(1) is in line with the DSGE model. The estimate of QE on output and the short-term nominal interest rate is negative, meaning that the VAR(1) predicts that an increase in the balance sheet would decrease output and the short rate. It is however not significant. Any relationship between inflation and output is rejected, as is also the case in the New-Keynesian equations. Inflation and credit only fit AR(1) of its own lagged values. The AR(1) of credit and QE are slightly lower from the VAR compared to the FIML estimates. It becomes clearer that inflation does not seem to fit so well with credit and QE, and especially that the combined parameter η in the Phillips curve is uncertain.

QE is negatively affected by the first-order lag of credit, implying that more credit in the previous quarter lowers the balance sheet. The structural model does not allow such direct dependency. We find no significant relationship between the short rate and QE in the VAR(1), consistent with the segmentation assumption. The short rate is however affected by credit, violating the assumption. We find in the backward-looking reaction function that the response of the interest rate to lagged inflation is lower than to expected future inflation in the Taylor rule.[25] Interest rate persistence is high, consistent across the two models. Overall, we find that many of the lags could efficiently be dropped from the VAR. The VAR(1) is parsimonious with high shock-volatilities and relatively low explanatory power, and we shall try a VAR(2) in section 5.3.

[25] $(1-0.942)2.230E_t\pi_{t+1} = 0.129E_t\pi_{t+1} > 0.103\pi_{t-1}$

Fig. 3 depicts the actual and one-time ahead forecasted values of the New-Keynesian equations. DSGE models have been criticized for not replicating data very well (Edge and Gürkaynak, 2010; Baele et al., 2015; Lindé, 2018). The critique applies for the Phillips curve which has low explanatory power. The IS curve and Taylor rule tracks the output gap and the funds rate well and outperforms the VAR(1) equations. The AR(1) of credit and QE are parsimonious; indeed, they are interpreted as shocks. The Δqe_t is volatile which pose problems for our empirical analysis.

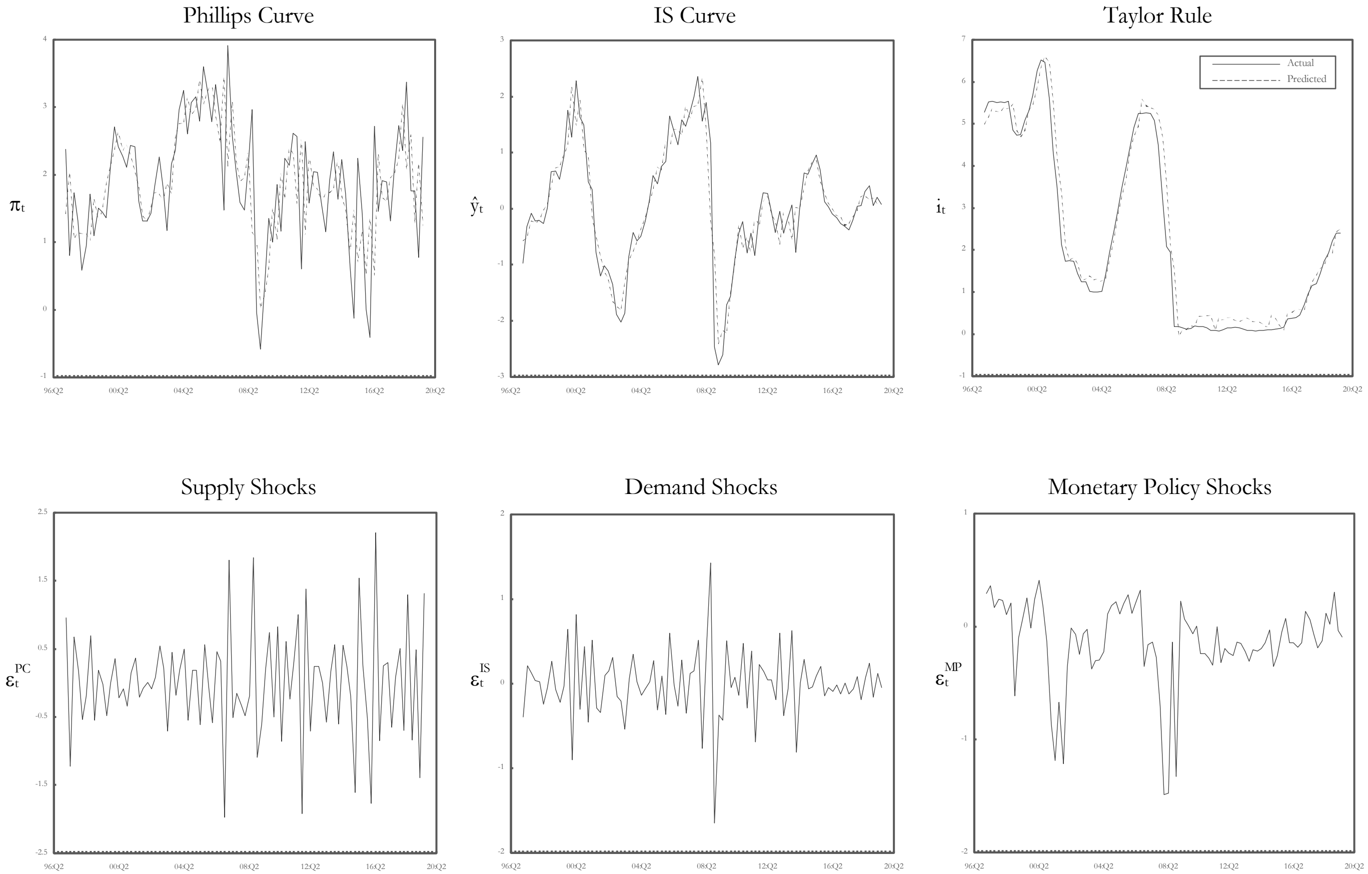

FIGURE 3. Actual and one-time ahead forecasted values of the four equation New-Keynesian DSGE Model.
R^2_{adj}: Phillips Curve 0.198; IS Curve 0.879; Monetary Policy Rule 0.969.
D-W: Phillips Curve 3.099; IS Curve 2.649; Monetary Policy Rule 0.922.
The plots are for non-de-meaned data.
Parameter set (2) from table 1.

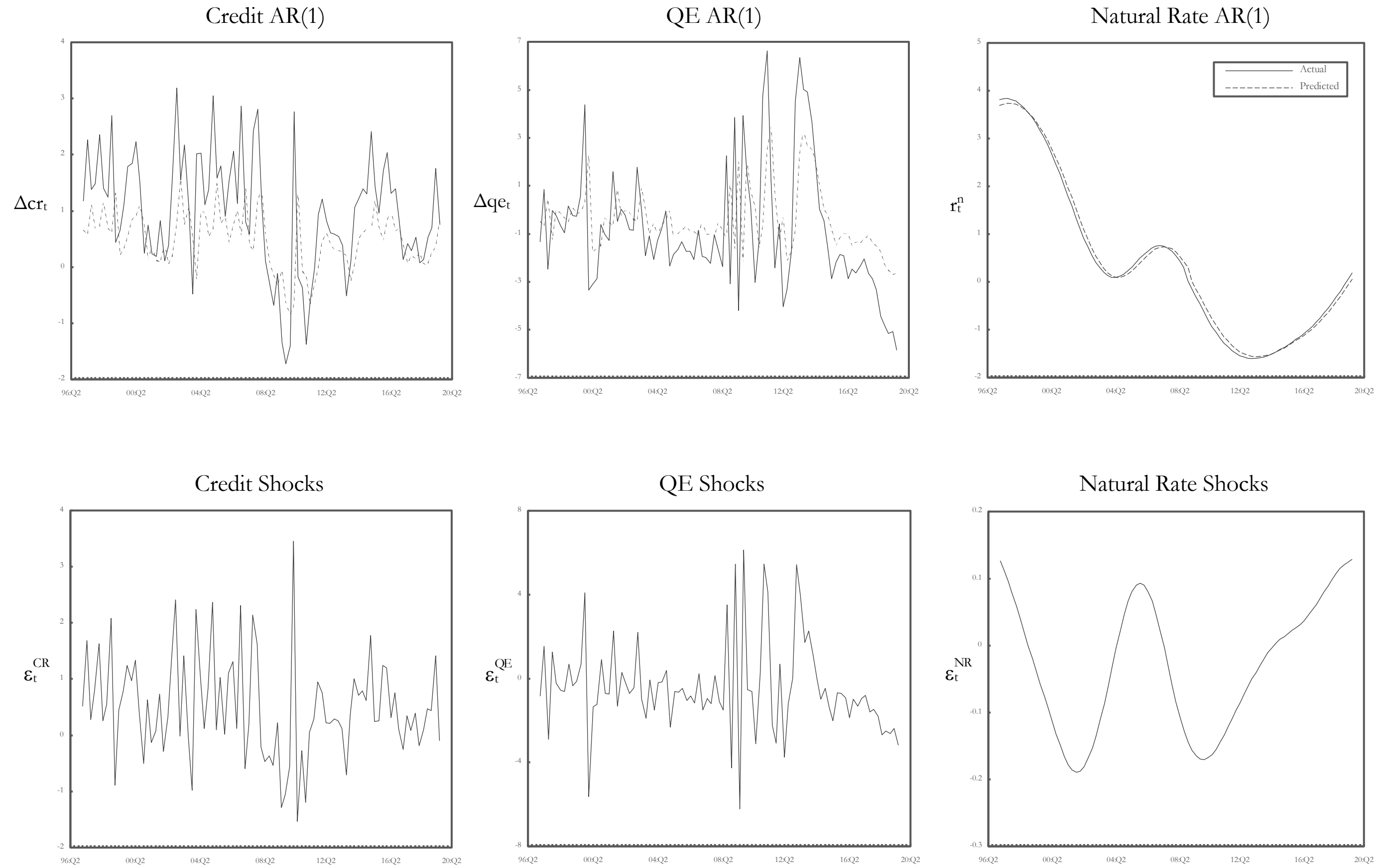

FIGURE 3. Actual and one-time ahead forecasted values with the associated shocks of the four equation New-Keynesian DSGE Model.
R^2_{adj}: Credit AR(1) 0.241; QE AR(1) 0.263; Natural Rate AR(1) 0.996.
D-W: Credit AR(1) 2.051; QE AR(1) 2.189; Natural Rate AR(1) 0.020.
The plots are for non-de-meaned data.
Parameter set (2) from table 1.

5.2 Diagnostics

We estimated the parameters of the New-Keynesian DSGE model and presented the VAR(1) estimates. Both FIML and VAR allows contemporaneous correlations and the methods assumes normality and non-serial correlations of the residuals. Table 3 and 4 lists contemporaneous cross-correlations and serial correlations of the residuals along with the Jarque-Bera test for normality. The Wald test is constructed to study whether some parameters contributes statistically to the model. The test results are shown in table 5. A sub-sample test of the New-Keynesian equations are reported in table 6.

TABLE 3. RESIDUAL DIAGNOSTICS OF THE NEW-KEYNESIAN DSGE MODEL

1. Correlations							
	$\varepsilon_t^{PC}\varepsilon_t^{IS}$	$\varepsilon_t^{PC}\varepsilon_t^{MP}$	$\varepsilon_t^{PC}\varepsilon_t^{CR}$	$\varepsilon_t^{PC}\varepsilon_t^{QE}$	$\varepsilon_t^{PC}\varepsilon_t^{NR}$	$\varepsilon_t^{IS}\varepsilon_t^{MP}$	$\varepsilon_t^{IS}\varepsilon_t^{CR}$
	0.164	0.127	-0.191	0.023	-0.006	0.069	-0.045
	$\varepsilon_t^{IS}\varepsilon_t^{QE}$	$\varepsilon_t^{IS}\varepsilon_t^{NR}$	$\varepsilon_t^{MP}\varepsilon_t^{CR}$	$\varepsilon_t^{MP}\varepsilon_t^{QE}$	$\varepsilon_t^{MP}\varepsilon_t^{NR}$	$\varepsilon_t^{CR}\varepsilon_t^{QE}$	$\varepsilon_t^{CR}\varepsilon_t^{NR}$
	0.138	0.012	0.155	0.028	0.442	-0.072	0.218
	$\varepsilon_t^{QE}\varepsilon_t^{NR}$						
	-0.229						
2. Autocorrelations							
Lag = i. P-value.	$\varepsilon_t^{PC}\varepsilon_{t-i}^{PC}$	$\varepsilon_t^{IS}\varepsilon_{t-i}^{IS}$	$\varepsilon_t^{MP}\varepsilon_{t-i}^{MP}$	$\varepsilon_t^{CR}\varepsilon_{t-i}^{CR}$	$\varepsilon_t^{QE}\varepsilon_{t-i}^{QE}$	$\varepsilon_t^{NR}\varepsilon_{t-i}^{NR}$	
1	0.000	0.002	0.000	0.788	0.321	0.000	
2	0.000	0.002	0.000	0.955	0.043	0.000	
3	0.000	0.005	0.000	0.875	0.038	0.000	
4	0.000	0.008	0.000	0.895	0.027	0.000	
5	0.000	0.017	0.000	0.952	0.051	0.000	
3. Normality Test							
Jarque-Bera	ε_t^{PC}	ε_t^{IS}	ε_t^{MP}	ε_t^{CR}	ε_t^{QE}	ε_t^{NR}	
P-value	0.350	0.000	0.000	0.071	0.000	0.041	
Value	2.099	18.956	92.18	5.290	18.91	6.405	

NOTES: The table shows residual diagnostics of the FIML estimates from table 1 set (2). The first row shows the contemporaneous correlations between the shocks. The second row lists the probability values from the Ljung-Q box test of autocorrelation. The third row reports statistics from the Jarque-Bera normality test.

The correlations between the estimated shocks are overall not high. One exception is the monetary policy shocks and the natural rate shocks which exhibit a relatively high co-movement. This is perhaps not surprising. According to the figures one would not lose much efficiency by instead carry out equation-by-equation estimation. The supply, demand and policy shocks suffers severely from autocorrelation. The literature has discussed the need for these shocks to be AR(1) for better diagnostics (Cho and Moreno, 2006). We see that credit can efficiently be modeled with an AR(1) since the shocks pass both the test for normality and non-serial correlations.

TABLE 4. RESIDUAL DIAGNOSTICS OF THE VAR(1)

1. Correlations						
$\epsilon_t^{\pi}\epsilon_t^{\hat{y}}$	$\epsilon_t^{\pi}\epsilon_t^{i}$	$\epsilon_t^{\pi}\epsilon_t^{cr}$	$\epsilon_t^{\pi}\epsilon_t^{qe}$	$\epsilon_t^{\pi}\epsilon_t^{nr}$	$\epsilon_t^{\hat{y}}\epsilon_t^{i}$	$\epsilon_t^{\hat{y}}\epsilon_t^{cr}$
0.252	0.292	-0.109	0.022	0.050	0.527	0.235
$\epsilon_t^{\hat{y}}\epsilon_t^{qe}$	$\epsilon_t^{\hat{y}}\epsilon_t^{nr}$	$\epsilon_t^{i}\epsilon_t^{cr}$	$\epsilon_t^{i}\epsilon_t^{qe}$	$\epsilon_t^{i}\epsilon_t^{nr}$	$\epsilon_t^{cr}\epsilon_t^{qe}$	$\epsilon_t^{cr}\epsilon_t^{nr}$
0.122	0.177	0.196	0.026	0.456	-0.081	0.202
$\epsilon_t^{qe}\epsilon_t^{nr}$						
-0.156						
2. Autocorrelations						
Lag = i. P-value.	$\epsilon_t^{\pi}\epsilon_{t-i}^{\pi}$	$\epsilon_t^{\hat{y}}\epsilon_{t-i}^{\hat{y}}$	$\epsilon_t^{i}\epsilon_{t-i}^{i}$	$\epsilon_t^{cr}\epsilon_{t-i}^{cr}$	$\epsilon_t^{qe}\epsilon_{t-i}^{qe}$	$\epsilon_t^{nr}\epsilon_{t-i}^{nr}$
1	0.924	0.042	0.000	0.917	0.564	0.000
2	0.950	0.055	0.000	0.988	0.040	0.000
3	0.880	0.115	0.000	0.897	0.074	0.000
4	0.500	0.202	0.000	0.948	0.064	0.000
5	0.553	0.178	0.000	0.980	0.114	0.000
3. Normality Test						
Jarque-Bera	ϵ_t^{π}	$\epsilon_t^{\hat{y}}$	ϵ_t^{i}	ϵ_t^{cr}	ϵ_t^{qe}	ϵ_t^{nr}
P-value	0.418	0.000	0.000	0.027	0.029	0.089
Value	1.747	428.0	106.9	7.182	7.063	4.838

NOTES: The table shows residual diagnostics from the VAR(1) estimates. The first row shows the contemporaneous correlations between the shocks. The second row lists the probability values from the Ljung-Q box test of autocorrelation. The third row reports statistics from the Jarque-Bera normality test.

The correlations between the residuals from the VAR(1) estimates are higher than from the FIML estimates. Since our VAR is reduced the equations can efficiently be estimated as a system with OLS. The higher correlations here imply that it makes sense for efficiency to allow correlations. The supply and demand shocks are non-serially correlated from the VAR(1). The policy shocks are non-normal and serially correlated. Table 5 reports statistics from the Wald test, examining whether some parameters contribute statistically to the model.

TABLE 5. THE WALD TEST

Tests	All Parameters = 0	$\theta=\beta^{FI}=\beta^{CB}=\rho^{CR}=\rho^{QE}=\rho^{NR}=0$	$\theta=\beta^{FI}=\beta^{CB}=0$	$\theta=0$
χ^2	26602	21773	14.449	7.291
p-value	0.000	0.000	0.001	0.007

NOTES: The table shows statistics from the Wald Test.

We reject the null for the tested parameters to be zero, suggesting that $\{\theta\ \beta^{FI}\ \beta^{CB}\}$ add explanatory power to the model. It is not easy to test these parameters one-by-one because of the interdependency. Also, we cannot base the question whether these parameters should belong to the model with the Wald test because it would strongly reject the important parameters λ and σ. We address another question, could $\{\theta\ \beta^{FI}\ \beta^{CB}\}$ be considered as structural in the sense of time-invariant?

We study if $\{\theta\ \beta^{FI}\ \beta^{CB}\}$ are stable over time in our sample. We base our following tests on QE. One could see from fig. 3 that there seems to be a trend break of the data somewhere between

2012:Q2-2016:Q2. Even though the full series is stationary according to the augmented Dickey-Fuller test (see the appendix), we use the breakpoint unit root test to ask if and if so where there are any breaks. The output is

H_0: Δqe_t has a unit root

t-stat. -6.565 $p < 0.01$ Break Date: 2014:Q1

The statistics suggests that there is no unit root, as expected, but if so it would be a break at 2014:Q1. This seems plausible by inspecting fig.2; Δqe_t trends downwards after this date. We therefore test the model between 1996:Q4–2013:Q4. We first set $\beta^{CB} = (1 - \beta^{FI})$ and then $\beta^{FI} = (1 - \beta^{CB})$ to test the significance of both these parameters.

TABLE 6. FIML ESTIMATES OF THE FOUR EQUATION NEW-KEYNESIAN DSGE MODEL ON U.S DATA 1996:Q4–2013:Q4

Parameters	δ	λ	θ	σ	β^{FI}	β^{CB}	μ	ρ^{TR}	ϕ_π	$\phi_{\hat{y}}$	ρ^{CR}	ρ^{QE}	ρ^{NR}
Value	0.496***	0.054	0.111***	0.019	0.578***	0.422**	0.484***	0.946***	2.971	1.603*	0.485***	0.361***	0.979***
S.E	(0.087)	(0.069)	(0.046)	(0.049)	(0.186)	(0.185)	(0.053)	(0.029)	(2.306)	(0.888)	(0.107)	(0.119)	(0.008)

NOTES: The table shows the FIML estimates of the model on U.S. quarterly data 1996:Q4 – 2013:Q4 (2008:Q4 excluded). The optimization method is BHHH (max 1000 iterations; covergence is achieved), the covariance matrix is diagonal, and the Hessian Information matrix. Standard errors in parantheses. *p<0.1, **p<0.05, ***p<0.01. The standard deviations of the shocks are $[\sigma_{PC}\ \sigma_{IS}\ \sigma_{MP}\ \sigma_{CR}\ \sigma_{QE}\ \sigma_{NR}]$ = (0.710 0.414 0.437 0.996 2.393 0.116). The jarque-bera probability values of normality are $[\varepsilon_t^{PC}\ \varepsilon_t^{IS}\ \varepsilon_t^{MP}\ \varepsilon_t^{CR}\ \varepsilon_t^{QE}\ \varepsilon_t^{NR}]$ = (0.065 0.052 0.000 0.241 0.041 0.091).

Our estimates points towards stability of $\{\theta\ \beta^{FI}\ \beta^{CB}\}$. This together with $\{\delta\ \lambda\ \mu\ \rho^{TR}\phi_{\hat{y}}\ \rho^{CR}\ \rho^{NR}\}$. The sub-sample test yields different values of $\{\sigma\ \phi_\pi\ \rho^{QE}\}$ compared to the full sample. The value of θ is slightly lower and the weight on β^{FI} is just lower compared to the full sample. The trio $\{\theta\ \beta^{FI}\ \beta^{CB}\}$ is again significant at conventional levels.

5.3 Additional lags and VAR(2) Estimates

Our estimates of the DSGE and the VAR(1) are not totally satisfying regarding distributional assumptions. We shall consider additional standard lags as a potential remedy. It is not easy to justify more lags in macro models (Clarida et al., 1999), why our extension is for testing empirical fit. Any standard approach of augmenting the New-Keynesian equations with lags does not help for studying the simultaneous behavior of the 6 variables because of the structure, but it might reach to better diagnostics. Most of the variables are not so persistent, partly due to the short sample period, so just one more lag is considered.[26] The real interest rate is usually not lagged and θ is also an exempt for more lags. We consider a weighted lag-structure of inflation and output ála Rudebusch (2002a). The lag of the interest rate is standard and we let credit, QE and the natural rate obey AR(2) processes. We have

$$\pi_t = \delta E_t\pi_{t+1} + (1-\delta)(\omega_1\pi_{t-1} + \omega_1\pi_{t-2}) + \lambda\hat{y}_t - \eta[\beta^{FI}\Delta cr_t + \beta^{CB}\Delta qe_t] + \varepsilon_t^{PC}, \quad \eta = \frac{\theta\lambda\sigma}{(1-\theta)} \tag{13}$$

$$\hat{y}_t = \mu E_t\hat{y}_{t+1} + (1-\mu)(\nu_1\hat{y}_{t-1} + \nu_2\hat{y}_{t-2}) - (1-\theta)\sigma[i_t - E_t\pi_{t+1} - r_t^n] - \theta[\beta^{FI}(E_t\Delta cr_{t+1} - \Delta cr_t) + \beta^{CB}(E_t\Delta qe_t - \Delta qe_t)] + \varepsilon_t^{IS} \tag{14}$$

$$i_t = (1 - \rho_1^{MP} - \rho_2^{MP})[\phi_\pi E_t\pi_{t+1} + \phi_{\hat{y}}\hat{y}_t] + \rho_1^{MP} i_{t-1} + \rho_2^{MP} i_{t-2} + \varepsilon_t^{MP} \tag{15}$$

$$\Delta cr_t = \rho_1^{CR}\Delta cr_{t-1} + \rho_2^{CR}\Delta cr_{t-2} + \varepsilon_t^{CR} \tag{16}$$

$$\Delta qe_t = \rho_1^{QE}\Delta qe_{t-1} + \rho_2^{QE}\Delta qe_{t-2} + \varepsilon_t^{QE} \tag{17}$$

$$r_t^n = \rho_1^{NR} r_{t-1}^n + \rho_2^{NR} r_{t-2}^n + \varepsilon_t^{NR} \tag{18}$$

We set set $\beta^{CB} = (1-\beta^{FI})$ and then $\beta^{FI} = (1-\beta^{CB})$. The FIML estimates on U.S. data 1996:Q4-2019:Q3 are

$$\pi_t = \underset{(***)}{0.307} E_t\pi_{t+1} + (1-0.307)(\underset{(***)}{0.404}\pi_{t-1} + 0.006\pi_{t-2}) + \underset{(**)}{0.176}\hat{y}_t - 0.0002[\underset{(***)}{0.610}\Delta cr_t + \underset{(**)}{0.390}\Delta qe_t] + \varepsilon_t^{PC}$$

$$\hat{y}_t = \underset{(***)}{0.494} E_t\hat{y}_{t+1} + (1-494)(\underset{(***)}{1.164}\hat{y}_{t-1} - 0.134\hat{y}_{t-2}) - (1-\underset{(***)}{0.112})0.009[i_t - E_t\pi_{t+1} - r_t^n] - 0.112[0.610(E_t\Delta cr_{t+1} - \Delta cr_t) + 0.390(E_t\Delta qe_t - \Delta qe_t)] + \varepsilon_t^{IS}$$

$$i_t = (1 - \underset{(***)}{1.499} + \underset{(***)}{0.547})[1.238 E_t\pi_{t+1} + 0.925\hat{y}_t] + 1.499\, i_{t-1} - 0.547 i_{t-2} + \varepsilon_t^{MP}$$

$$\Delta cr_t = \underset{(***)}{0.462}\Delta cr_{t-1} + 0.057\Delta cr_{t-2} + \varepsilon_t^{CR}$$

$$\Delta qe_t = \underset{(***)}{0.423}\Delta qe_{t-1} + \underset{(**)}{0.208}\Delta qe_{t-2} + \varepsilon_t^{QE}$$

$$r_t^n = \underset{(***)}{1.965} r_{t-1}^n - \underset{(***)}{0.971} r_{t-2}^n + \varepsilon_t^{NR}$$

Volatilities $[\sigma_{PC}\ \sigma_{IS}\ \sigma_{MP}\ \sigma_{CR}\ \sigma_{QE}\ \sigma_{NR}] = (0.756\ 0.378\ 0.323\ 0.910\ 2.187\ 0.011)$

[26] See the lag test of the VAR in the appendix.

R^2_{adj} [PC IS MP CR QE NR] = (0.288 0.878 0.977 0.234 0.285 0.999)

D-W [PC IS MP CR QE NR] = (2.508 2.845 2.180 1.989 1.908 0.037)

Jarque-Bera (p-value) [ε_t^{PC} ε_t^{IS} ε_t^{MP} ε_t^{CR} ε_t^{QE} ε_t^{NR}] = (0.431 0.000 0.000 0.041 0.000 0.450)

The degree of forward-looking behavior of inflation declines with an additional lag. Our estimate of 0.3 is in line with Rudebusch (2002a). We find that output is to a high extent forward-looking despite more lags, consistent with Lindé (2005) and Söderström et al., (2005). One major difference now is that the Phillips curve parameter λ is significant and also higher. A higher λ *ceteris paribus* makes the monetary policy transmission mechanism stronger for both conventional short rate policy and unconventional balance sheet policy. Following the substantially higher λ and the slightly higher σ, the combined Phillips curve parameter η goes up. A higher η all else equal means that the disinflationary forces relative to the inflationary forces of credit and QE shocks becomes stronger. The Phillips curve has better fit with the additional lag: the standard deviation of the shocks is lower, it has higher explanatory power and lower serial correlations. The fit of QE is not better despite a significant second-order lag. The other equations are not so different compared to the baseline case.

The VAR(2) estimates are

TABLE 7. VAR(2) ESTIMATES

Equations/lags	$\hat{y}_t$	π_t	i_t	r_t^n	Δcr_t	Δqe_t
$\hat{y}_{t-1}$	0.848***	0.384**	0.023	-0.002	-0.116	-0.360
	(0.116)	(0.187)	(0.071)	(0.001)	(0.206)	(0.517)
$\hat{y}_{t-2}$	-0.239**	-0.515**	-0.033	-0.003**	0.203	0.157
	(0.114)	(0.183)	(0.070)	(0.001)	(0.202)	(0.506)
π_{t-1}	-0.094	0.242**	-0.004	0.005***	-0.030	0.006
	(0.070)	(0.113)	(0.043)	(0.000)	(0.125)	(0.313)
π_{t-2}	0.016	0.135	0.095**	0.004***	0.245**	0.137
	(0.071)	(0.114)	(0.044)	(0.000)	(0.126)	(0.317)
i_{t-1}	0.416**	0.348	1.412***	-0.002	0.642**	-0.831
	(0.176)	(0.282)	(0.108)	(0.002)	(0.311)	(0.780)
i_{t-2}	-0.178	-0.128	-0.560***	-0.005**	-0.728**	0.870
	(0.169)	(0.271)	(0.104)	(0.002)	(0.299)	(0.750)
r_{t-1}^n	0.698	-0.795	1.045***	1.965***	0.914	-1.299
	(0.626)	(1.004)	(0.385)	(0.008)	(1.109)	(2.776)
r_{t-2}^n	-0.902	0.521	-0.874**	-0.963***	-0.689	1.277
	(0.609)	(0.977)	(0.375)	(0.007)	(1.079)	(2.701)
Δcr_{t-1}	0.007	0.018	-0.012	0.002***	0.343***	-0.582**
	(0.062)	(0.099)	(0.038)	(0.000)	(0.110)	(0.276)
Δcr_{t-2}	0.072	0.060	0.023	0.002***	0.027	0.329
	(0.063)	(0.102)	(0.039)	(0.000)	(0.113)	(0.282)
Δqe_{t-1}	-0.061**	-0.030	-0.031**	-0.000	-0.026	0.403***
	(0.025)	(0.040)	(0.015)	(0.000)	(0.044)	(0.111)
Δqe_{t-2}	0.107***	0.024	0.035**	-0.000	0.079**	0.170*
	(0.024)	(0.039)	(0.015)	(0.000)	(0.043)	(0.109)
Volatilities	$\sigma_{\hat{y}}$	σ_π	σ_i	σ_{nr}	σ_{cr}	σ_{qe}
St.dev	0.482	0.773	0.297	0.000	0.854	2.138

NOTES: The table shows the unrestricted VAR(2) estimates. Standard errors in parentheses. *p<0.1, **p<0.05, ***p<0.01. R^2_{adj} [$\hat{y}_t$ π_t i_t r_t^n Δcr_t Δqe_t] = (0.798 0.247 0.980 0.999 0.318 0.325). D-W [$\hat{y}_t$ π_t i_t r_t^n Δcr_t Δqe_t] = (2.066 1.968 2.326 0.590 1.902 1.905). Jarque-Bera (p-value) [$\epsilon_t^{\hat{y}}$ ϵ_t^{π} ϵ_t^{i} ϵ_t^{nr} ϵ_t^{cr} ϵ_t^{qe}] = (0.000 0.782 0.000 0.708 0.006 0.000).

Of the total 72 lags 40 percent are significant at conventional levels. The fit is better with the VAR(2) compared to the VAR(1) for all six equations as evident by lower volatilities of the error terms, higher explanatory power and non-serial correlations except for the very persistent natural rate residuals. The VAR(2) captures the additional persistence of inflation, output and the short rate, which is not possible for the structural model or the VAR(1). We find a strong relationship between inflation and output in the VAR(2), as is also the case in the extended New-Keynesian model. Current output is negatively by an increase in the balance sheet in the previous quarter, but the net effect is a boost for output due to a stronger positive effect in lag two. There is still no effect of credit and QE in the inflation equation. One could however note that there seems to be a competing effect of QE on inflation also in the VAR(2). In the inflation equation, the first lag is a reduction whereas the second lag is a boost. We find a direct link between the short rate and QE; the first lag lowers the short rate but the second is an increase. The New-Keynesian model does not incorporate any direct relationship between QE and the short rate, as discussed.

Overall, the dynamics of credit and QE with inflation, output and interest rates in the data are ambiguous. For one, credit and QE have better fit than inflation in relation to the other variables. One cannot rule out this is a potential data problem in terms of too short sample period and because of volatile series. Our sample covers the financial crisis which evidently increases the likelihood of studying data that disturb the estimates. Especially QE is a volatile series which is problematic for our analysis.

Our analysis yields the following conclusions regarding the empirical performance of the models. Inflation is better modelled with an additional lag; it is even between the augmented Phillips curve and the inflation equation from the VAR(2). Inflation does not fit with credit and QE, the parameter η makes the Phillips curve worse. In the DSGE case, the Phillips curve parameter η is a combined term, which includes the parameter θ that is mainly microfunded for the IS curve. Our original IS curve outperforms the output equation of the VAR(2) in terms of explanatory power and lower residual-volatility. The parameter restrictions however lead to serious serial correlations of the shocks which is not a problem of the unrestricted VAR(2). Output seems to fit generally well with credit and QE, the IS parameter θ may be justified empirically according to our analysis. The high persistence of the short-term interest rate in the data makes it better modelled with a second lag. The VAR(2) is better than the augmented Taylor rule with slightly higher explanatory power and lower shock-volatility. The downward skew of the short rate shocks from the original Taylor rule is not a concern for either the extended Taylor rule nor the VAR(2).

6. Shock Analysis

6.1 Calibration and (or) Estimation: A Short Discussion

We have estimated the parameters of the DSGE and the VAR models and are interested in studying monetary policy behavior to shocks of the economy. When solving DSGE models, the parameters are either estimated, calibrated or a combination of the two. Cooley (1997) describes calibration as *"a strategy for finding numerical values for the parameters of artificial economic worlds"*. While most of the parameters have been found to be significant, and the system is solvable using all our sets, the model has a rigorous theoretical foundation. If we set exactly some of our parameter values from table 1 or 2 prior to simulation it does not mean that this would correspond to how the model works in equilibrium because it is a dynamic system and the simulation is numerical.

Two parameters that are often set differently in calibrations compared to their estimated values are the slope of the Phillips curve λ and the IES σ. As mentioned, $\sigma = 1$ is sometimes set calibrations (Del Negro et al., 2017). In line with our empirical results, ML estimates of σ typically becomes very low and is often not significant.[27] A higher σ leads to stronger reactions of output to conventional policy shocks. In this model of Sims and Wu (2019), the response becomes lower as $(1 - \theta)\sigma$ with $\theta > 0$. This is worth to keep in mind as they set $\sigma = 1$ along with others' calibrating newly developed models incorporating financial frictions and QE (e.g. Del Negro et al., 2017). The slope of the Phillips curve λ is usually set higher in calibrations partly because the empirical literature supposes a constant elasticity of real marginal costs to the output gap. A higher λ implies a stronger transmission of output to inflation, leading to stronger effects of demand shocks on inflation.

Still, to use at least some estimated parameter values in simulations is not unusual (Smets and Wouters, 2007; Lindé et al., 2016; Carlstrom et al., 2017). Instead of changing one or more parameters we set our values from set (2)-(3) in table 1 $\{\delta\ \lambda\ \theta\ \sigma\ \beta^{FI}\ \beta^{CB}\ \mu\ \rho^{TR}\ \phi_{\pi}\ \phi_{\hat{y}}\ \rho^{CR}\ \rho^{QE}\ \rho^{NR}\}$ = (0.498 0.060 0.109 0.007 0.606 0.394 0.490 0.942 2.230 1.688 0.490 0.542 0.975). The standard deviations of the shocks are their estimated values $[\sigma_{PC}\ \sigma_{IS}\ \sigma_{MP}\ \sigma_{CR}\ \sigma_{QE}\ \sigma_{NR}]$ = (0.800 0.376 0.385 0.902 2.208 0.106). The Blanchard-Kahn conditions does not allow more lags and it is the structural parameters that are of interest, why the simulations are for our original model. Our complete dynare code is available in the appendix. The VAR is Cholesky decomposition and of order 2 since the empirical performance is better than with just one lag.

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[27] We find that FIML estimates of σ on U.S. data lies is in the interval $0.0009 < \sigma < 0.087$ by comparing findings from the following papers (Lindé, 2005; Cho and Moreno, 2006; Buncic and Lentner, 2016). This is the inverse, so it is $1111 < \sigma < 11.49$.

6.2 Impulse-response Functions

Figure 4 shows the impulse-response functions from simulations of the DSGE model with our empirical parameter values. Figure 5 depicts the impulse-response functions from the unrestricted VAR(2). The vertical axis is percentage points in both figures, the horizontal axis represents periods in fig. 4 and in fig 5. it is quarters.

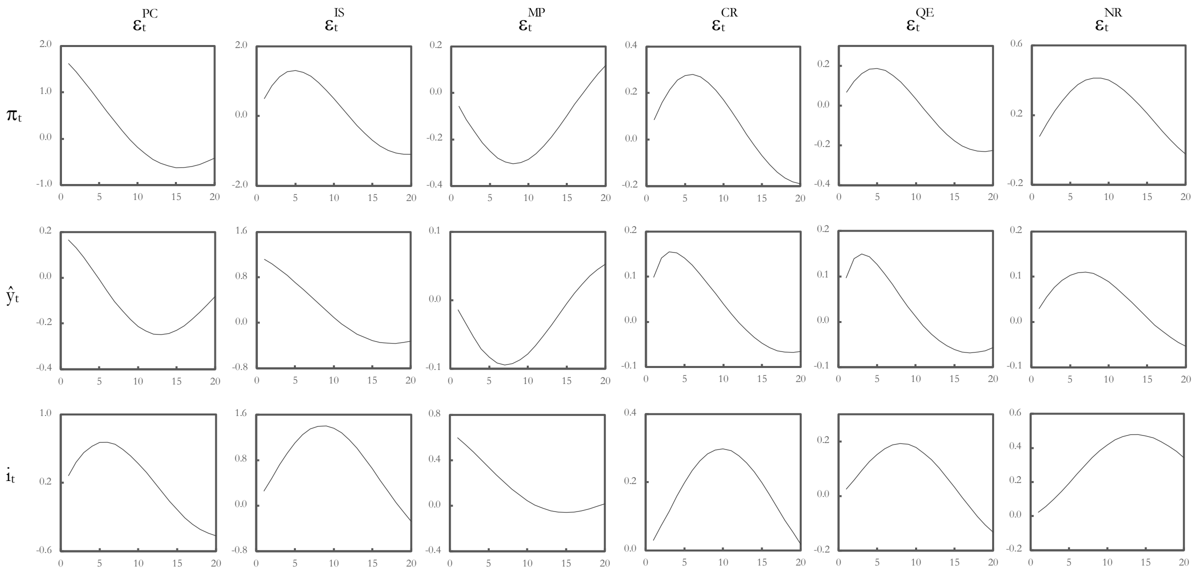

FIGURE 4. Impulse-Response Functions of the New-Keynesian DSGE Model. Simulated with $\{\delta\ \lambda\ \theta\ \sigma\ \beta^{FI}\ \beta^{CB}\ \mu\ \rho^{TR}\ \phi_\pi\ \phi_{\hat{y}}\ \rho^{CR}\ \rho^{QE}\ \rho^{NR}\}$ = (0.498 0.060 0.109 0.007 0.606 0.394 0.490 0.942 2.230 1.688 0.490 0.542 0.975).

$$\pi_t = \delta E_t\pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda\hat{y}_t - \eta\left[\beta^{FI}\Delta cr_t + \beta^{CB}\Delta qe_t\right] + \varepsilon_t^{PC}, \quad \eta = \frac{\theta\lambda\sigma}{(1-\theta)}$$

$$\hat{y}_t = \mu E_t\hat{y}_{t+1} + (1-\mu)\hat{y}_{t-1} - (1-\theta)\sigma\left[i_t - E_t\pi_{t+1} - r_t^n\right] - \theta\left[\beta^{FI}(E_t\Delta cr_{t+1} - \Delta cr_t) + \beta^{CB}(E_t\Delta qe_{t+1} - \Delta qe_t)\right] + \varepsilon_t^{IS}$$

$$i_t = (1-\rho^{TR})[\phi_\pi E_t\pi_{t+1} + \phi_{\hat{y}}\hat{y}_t] + \rho^{TR} i_{t-1} + \varepsilon_t^{MP}$$

$$\Delta cr_t = \rho^{CR}\Delta cr_{t-1} + \varepsilon_t^{CR}$$

$$\Delta qe_t = \rho^{QE}\Delta qe_{t-1} + \varepsilon_t^{QE}$$

$$r_t^n = \rho^{NR} r_{t-1}^n + \varepsilon_t^{NR}$$

The impact of supply, demand and conventional short-term interest rate shocks are by and large in line with economic intuition. A supply shock such as an energy price shock raises inflation by a significant amount. The Fed responds by hiking the funds rate $\phi_\pi > 1$ whereby the households responding to money market conditions decides to save more $\sigma > 0$ and output decreases. A demand shock such as higher government spending than expected or a shift in preferences stimulates aggregate demand and increase both output and prices $\lambda > 0$. The model's mechanism leads the Fed to hike the funds rate $\phi_{\hat{y}} > 0$ to cool down the economy. A natural rate shock functions as a positive demand shock. A conventional monetary policy shock could be an exchange rate appreciation, making the money market consumers save more which depresses current output. Inflation goes down via λ. The response of inflation and output to conventional short rate shocks takes the form a hump-shaped gradual declination. This is argued to be a criterion that make models useful for policy analysis (Lindé, 2018). The shocks have long-lived effects and cyclical pattern due to price indexation of firms $0 < \delta < 1$, habit persistence of consumption $0 < \mu < 1$, and smoothing of the funds rate $0 < \rho^{TR} < 1$.

A credit shock could be an increase in the supply of loans. More credit is granted, leading the households responding to bond market conditions $0 < \theta < 1$ save more which increases output as it stimulates consumption and investments. The effect is inflationary rather than disinflationary as the transmission of output to inflation dominates the disinflationary forces (partly a consequence of $\eta < \lambda$). The short rate increases which reflects the reaction of the Fed to inflation and output from its respective targets. The QE equation is an AR(1), a shock may be an announced large-scale asset purchase programme. An increase in the Fed's long bond portfolio boost output due to θ and leads to higher inflation via λ. The effect raises prices for the same reason as for the credit shocks. Sims and Wu (2019) also finds with calibration that inflation goes up to credit and QE shocks.

The short rate increases to QE shocks. If we see it as the Fed have full control over the short-term interest rate the segmentation assumption together with the model's structure could justify an increase in the funds rate; the short rate and the long bond portfolio are separate instruments targeted for different segments. If the Fed believes the households on the money market could be separated from the households on the bond market because of diverse circumstances, the hike in the short rate to QE shocks reflects the reaction to inflation and output from targets. The higher short rate would in turn make the households that responds to money market conditions postpone consumption.

A purchase of long bonds by the Fed put downward pressure on long yields, making the QE shocks to some extent the mirror of the short rate shocks in this model. Credit shocks have larger impact than QE shocks because $\beta^{FI} > \beta^{CB}$. For both credit and QE shocks the expectations of future values prevent an amplification of the responses and the effects are persistent due to $0 < \rho^{CR}, \rho^{QE} < 1$. We see that the impact of conventional policy shocks is generally stronger than unconventional policy shocks. One exception is the response of output which is slightly stronger on impact to QE shocks. The impact of QE shocks is by and large in line with a richer DSGE model by Carlstrom et al., (2017).

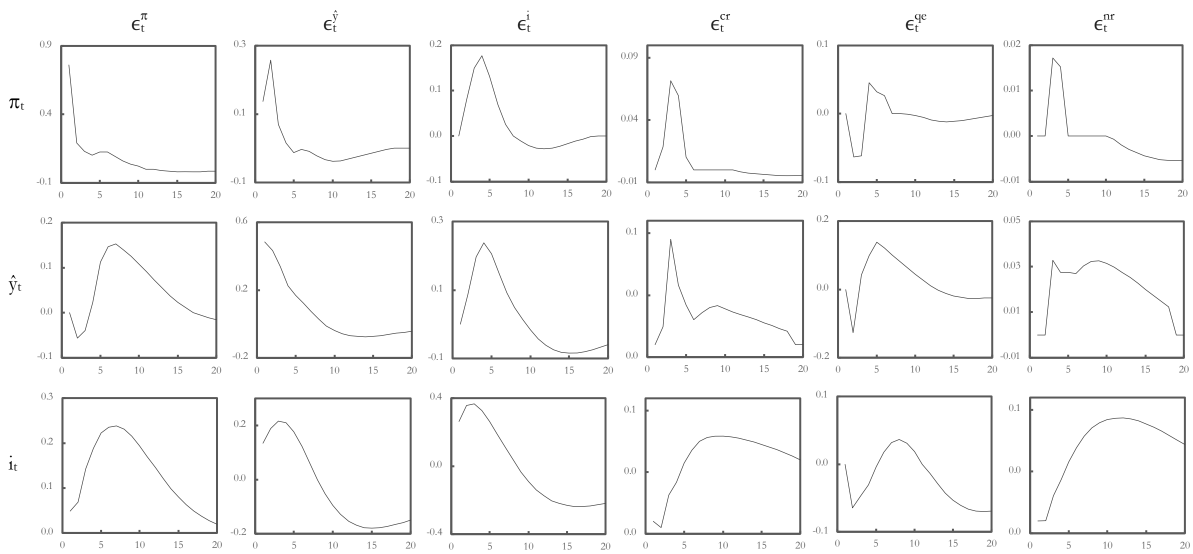

FIGURE 5. Impulse-Response Functions of the unrestricted VAR(2). Cholesky ordering ($\hat{y}_t$ π_t i_t r_t^n Δcr_t Δqe_t).

$$\hat{y}_t = 0.848\hat{y}_{t-1} - 0.239\hat{y}_{t-2} - 0.094\pi_{t-1} + 0.016\pi_{t-2} + 0.416i_{t-1} - 0.178i_{t-2} + 0.698r^n_{t-1} - 0.902r^n_{t-2} + 0.007\Delta cr_{t-1} + 0.072\Delta cr_{t-2} - 0.061\Delta qe_{t-1} + 0.107\Delta qe_{t-2} + \epsilon_t^{\hat{y}}$$

$$\pi_t = 0.384\hat{y}_{t-1} - 0.515\hat{y}_{t-2} + 0.242\pi_{t-1} + 0.135\pi_{t-2} + 0.348i_{t-1} - 0.128i_{t-2} - 0.795r^n_{t-1} + 0.521r^n_{t-2} + 0.018\Delta cr_{t-1} + 0.060\Delta cr_{t-2} - 0.030\Delta qe_{t-1} + 0.024\Delta qe_{t-2} + \epsilon_t^{\pi}$$

$$i_t = 0.023\hat{y}_{t-1} - 0.033\hat{y}_{t-2} - 0.004\pi_{t-1} + 0.095\pi_{t-2} + 1.412i_{t-1} - 0.560i_{t-2} + 1.045r^n_{t-1} - 0.874r^n_{t-2} - 0.012\Delta cr_{t-1} + 0.023\Delta cr_{t-2} - 0.031\Delta qe_{t-1} + 0.035\Delta qe_{t-2} + \epsilon_t^{i}$$

$$r_t^n = -0.002\hat{y}_{t-1} - 0.003\hat{y}_{t-2} + 0.005\pi_{t-1} + 0.004\pi_{t-2} - 0.002i_{t-1} - 0.005i_{t-2} + 1.965r^n_{t-1} - 0.963r^n_{t-2} + 0.002\Delta cr_{t-1} + 0.002\Delta cr_{t-2} - 0.000\Delta qe_{t-1} - 0.000\Delta qe_{t-2} + \epsilon_t^{nr}$$

$$\Delta cr_t = -0.116\hat{y}_{t-1} + 0.203\hat{y}_{t-2} - 0.030\pi_{t-1} + 0.245\pi_{t-2} + 0.642i_{t-1} - 0.728i_{t-2} + 0.914r^n_{t-1} - 0.689r^n_{t-2} + 0.343\Delta cr_{t-1} + 0.027\Delta cr_{t-2} - 0.026\Delta qe_{t-1} + 0.079\Delta qe_{t-2} + \epsilon_t^{cr}$$

$$\Delta qe_t = -0.360\hat{y}_{t-1} + 0.157\hat{y}_{t-2} + 0.006\pi_{t-1} + 0.137\pi_{t-2} - 0.831i_{t-1} + 0.870i_{t-2} - 1.299r^n_{t-1} + 1.277r^n_{t-2} - 0.582\Delta cr_{t-1} + 0.329\Delta cr_{t-2} + 0.403\Delta qe_{t-1} + 0.170\Delta qe_{t-2} + \epsilon_t^{qe}$$

We find some similarities of the impact of supply, demand and conventional policy shocks between the DSGE model and VAR model. Inflation, output and the short rate increases to shocks to its own variables with the magnitude of the responses as in the written order; demand shocks initially boosts prices followed by a fall below steady state; and supply and demand shocks raises the short rate. We find three differences. These are the impact of supply shocks on output and the reactions of inflation and output to short rate shocks. The three effects are related to the relationships governed by the two uncertain and rejected structural parameters λ and σ.[28] By economic intuition inflation and output should go down rather than up with higher interest rates. Contractional short rate shocks boosts prices in the VAR; we have the classical "price puzzle" problem (Hanson, 2004).[29] This is partly because the VAR(2) estimates yields wrong sign of the first-order lag of the short rate on inflation. How is this relationship different in the structural model? The estimate of the IES σ is positive and the DSGE has no direct link between the short rate and inflation other

[28] We found these two parameters to be correctly signed, but the uncertainty of the estimates is high. From table 1 spec. (2)-(3) we have the 95 percentage intervals λ = (-0.091 0.211) and σ = (-0.073 0.087). The literature has found negative estimates of both λ (Lindé, 2005) and σ (Stracca, 2010). The constraints $\lambda, \sigma > 0$ are requirements for numerical solutions of the New-Keynesian model under the Blanchard-Kahn conditions.
[29] There are several interpretations of the price puzzle problem. One strand of the literature sees it as a flaw of the model itself, others view it as empirically possible (see Dueker, 2006).

than that a hike first depresses output by $(1-\theta)\sigma$ and then transmit this to inflation via λ. A hike in the short rate also leads to higher output in the VAR, in part a consequence of the positive first-order lag of the short rate in the output equation. In the structural model there is a direct link by the term $(1-\theta)\sigma$ between interest rates and output in the IS curve. The response of output to short rate shocks in the VAR instead yields evidence of an income effect.

The impact of credit and natural rate shocks are by and large robust for the DSGE model against the empirical VAR. Credit and natural rate shocks boosts prices, output and the short rate in both models. Moreover, we find that the peak (through) of the response of inflation, output and the short rate to QE shocks are generally lower (higher) than to conventional policy shocks in the VAR. This is by and large also the case in the DSGE, as we mentioned. This implies that our estimated value of θ in the structural model which steers the relative magnitude of unconventional policy shocks relative to conventional policy shocks could be plausible. By plausible we mean that if θ would be set higher until some threshold all else equal, then the effect of QE shocks becomes stronger compared to short rate shocks.

There seems to be a competing effect of QE shocks on inflation in the VAR just as in the DSGE. Based on economic theory inflation and output should go up with higher QE since it puts downward pressure on long yields. In the DSGE, our value of η in relation to θ and λ pushes the effect to be predominantly inflationary in the Phillips curve. Beck et al., (2019) also finds that inflation initially decrease slightly and then goes above steady state in a VAR when using data of total assets of the Central Bank for QE. Assuming our impulse-responses from the VAR well represents the impact of QE shocks, it would probably be a daunting task to change the DSGEs parameters to match the initial small decrease and the following increase and at the same time motivate these parameter values as structural. The uncertainty is however high, which is because we do not find any significant relationship between inflation and QE from the VAR estimates.

The VAR functions have a more zig-zag pattern in general, reflecting that the equations are not specified and include more lags. Quantitative comparisons are not our main focus as put forward previously, as it could differ quite substantially between models (or in our case also between methods). For comparison of DSGE and VAR impulse-response functions in larger models, see for example Sveriges Riksbank (2009) and Christiano et al., (2018).

7. Conclusions

This essay studied financial frictions, QE policy, and business cycles. Whereas our topic was not new, we tested a newly developed New-Keynesian model where the QE policy channel comes from segmentation of the financial markets. Due to many theoretical restrictions and on the basis that DSGE models often are constructed on empirical findings from VARs, we estimated a simple VAR to test against the DSGE. Our three contributions to the monetary literature are related to estimation, policy analysis and model evaluation.

The structural estimation led to a high value of the share of long-term bonds held by the Fed relative to the U.S. commercial banks. The implication in the model is that the Fed has the possibility to off-set credit frictions with a smaller purchase from a period to another compared to if this share would be lower. We found in the VAR that output fit relatively well with credit and QE, providing support for its relevance in the IS curve. The low interest rate elasticity and real driving variable of inflation contributed to credit and QE shocks being predominantly inflationary. A key finding in the shock-analysis is the impact of conventional relative to unconventional policy shocks. The VAR were robust in that short-term interest rate shocks are more impactful than long-term bond purchases shocks; the DSGE shared much of the same features. The rejection and the uncertainty of the short rate elasticity and the real driving variable however caused some problems for the analysis, not only for the impact of credit and QE shocks. Uncertainty also comes from our lack of findings of any effect of credit and QE on inflation in the VAR.

There are several potential topics for further research. First, it would be desirable with more studies on the econometrics of the newly developed New-Keynesian models. Studies on both the robustness of estimates and cross-country evidence could be valuable. Second, consistent with our findings in the analysis of the DSGE, Carlstrom et al., (2017) and Sims and Wu (2020) documents that a QE shock leads to an initial increase of inflation followed by a fall below steady state. While an increase is in line with common thoughts, we found, as in Beck et al., (2019), that the effect in a VAR is an initial slight decrease followed by an increase above steady state. How to cope with the uncertainty of the impact of QE on inflation is important for policy analysis looking ahead.

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Appendix

1A. The Data

TABLE 1A. THE MACROECONOMIC DATA

Variables	Details	Source
Inflation	Gross Domestic Product: Implicit Price Deflator, Index, Quarterly	Bureau of Economic Analysis
Output	Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly	Bureau of Economic Analysis
Short Interest Rate	Effective Federal Funds Rate, Percent, Monthly	Board of Governors of the Fed System
Credit Conditions	Bank Credit, All Commercial Banks, Billions of U.S. Dollars, Weekly	Board of Governors of the Fed System
Central Bank Balance Sheet	Reserve Bank Credit, Weekly Releases, Weekly	Federal Reserve, Statistical Release

NOTES: The GDP deflator, real GDP, the effective Funds rate, and bank credit are retrieved via Fred, Federal Reserve Bank of St. Louis.

2A. FIML Estimates of the New-Keynesian Model

TABLE 2A. FIML ESTIMATES OF THE FOUR EQUATION NEW-KEYNESIAN DSGE MODEL ON U.S DATA 1996:Q4–2019:Q3. Δqe_t = THE QUARTERLY AVERAGE OF THE WEEKLY AVERAGE

Specification Parameters	Unconstrained (1)	$\beta^{CB} = (1 - \beta^{FI})$ (2)	$\beta^{FI} = (1 - \beta^{CB})$ (3)	As (1) + $\sigma = 1$ (4)	As (2) + $\sigma = 1$ (5)	As (3) + $\sigma = 1$ (6)
δ	0.498***	0.498***	0.498***	0.497***	0.498***	0.498***
	0.077	0.077	0.077	0.077	0.077	0.077
λ	0.086	0.060	0.060	0.038	0.087	0.087
	0.082	0.077	0.077	0.072	0.060	0.060
θ	0.982***	0.094**	0.094**	0.976***	0.577***	0.577***
	0.065	0.043	0.043	0.031	0.057	0.057
σ	0.290	0.007	0.007	1.000	1.000	1.000
	1.115	0.041	0.041	-	-	-
β^{FI}	0.078**	0.734***	0.734	0.072*	0.797***	0.797
	0.037	0.178	-	0.038	0.056	-
β^{CB}	0.016	0.266	0.266	0.017	0.203	0.203***
	0.018	-	0.179	0.019	-	0.056
μ	0.497***	0.495***	0.495***	0.490**	0.329***	0.329***
	0.048	0.048	0.048	0.048	0.095	0.095
ρ^{TR}	0.942***	0.942***	0.942***	0.942***	0.925***	0.925***
	0.021	0.021	0.021	0.021	0.022	0.022
ϕ_{π}	2.234**	2.213**	2.231**	2.187**	1.434*	1.434*
	1.206	1.205	1.205	1.176	0.795	0.795
$\phi_{\hat{y}}$	1.686**	1.687**	1.687**	1.720**	2.270***	2.270***
	0.720	0.720	0.720	0.714	0.642	0.642
ρ^{CR}	0.491***	0.490***	0.490***	0.490***	0.490***	0.490***
	0.092	0.092	0.092	0.092	0.092	0.092
ρ^{QE}	0.613***	0.613***	0.613***	0.613***	0.613***	0.613***
	0.086	0.087	0.087	0.087	0.087	0.087
ρ^{NR}	0.975***	0.975***	0.975***	0.975***	0.975***	0.975***
	0.007	0.007	0.007	0.007	0.007	0.007
Volatilities						
σ_{PC}	0.796	0.800	0.800	0.793	0.788	0.788
σ_{IS}	0.390	0.387	0.387	0.389	0.770	0.770
σ_{MP}	0.385	0.385	0.385	0.385	0.391	0.391
σ_{CR}	0.902	0.902	0.902	0.902	0.902	0.902
σ_{QE}	2.032	2.005	2.005	2.005	2.005	2.005
σ_{NR}	0.109	0.106	0.106	0.106	0.106	0.106

NOTES: The table shows the FIML estimates of the New-Keynesian model on U.S. quarterly data 1996:Q4–2019:Q3 (2008:Q4 excluded) using the annualized log first-difference of the GDP deflator, HP-detrended real output 2012 chained prices, the quarterly average of the weekly effective Federal funds rate, the log first-difference of the total bank credit of all commercial banks in the U.S., the log first-difference of the total balance sheet of the Fed, and the HP-trend of the ex-ante real interest rate. Credit and the balance sheet are in real market values GDP deflated. The columns shows how the model is specified prior to estimation. The optimization method is BHHH (max 5000 iterations; covergence is achieved), the covariance matrix is diagonal, and the Hessian Information matrix. Standard errors in parantheses. $^{*}p<0.1$, $^{**}p<0.05$, $^{***}p<0.01$.

3A. FIML Estimates of the New-Keynesian model on Semi-annual Data.

TABLE 3A. FIML ESTIMATES OF THE FOUR EQUATION NEW-KEYNESIAN DSGE MODEL ON U.S DATA 1997:H1–2019:H1. Δqe_t = THE QUARTERLY AVERAGE OF THE WEDNESDAY LEVEL

Parameters	δ	λ	θ	σ	β^{FI}	β^{CB}	μ	ρ^{TR}	ϕ_π	$\phi_{\hat{y}}$	ρ^{CR}	ρ^{QE}	ρ^{NR}
Value	0.527^{***}	0.065	0.200^{***}	-0.063	0.906^{***}	0.094	0.563^{***}	0.823^{***}	2.779^{**}	1.839^{***}	0.545^{***}	0.544^{***}	0.942^{***}
S.E	(0.095)	(0.054)	(0.077)	(0.091)	(0.107)	(0.107)	(0.070)	(0.046)	(1.106)	(0.445)	(0.131)	(0.140)	(0.015)

NOTES: The table shows the FIML estimates of the New-Keynesian model on U.S. semi-annual data 1997:H1–2019:H1. The optimization method is BHHH (max 1000 iterations; covergence is achieved), the covariance matrix is diagonal, and the Hessian Information matrix. Standard errors in parantheses. $^{*}p<0.1$, $^{**}p<0.05$, $^{***}p<0.01$. The standard deviations of the shocks are [ε_t^{PC} ε_t^{IS} ε_t^{MP} ε_t^{CR} ε_t^{QE} ε_t^{NR}]= (0.454 0.578 0.577 1.360 3.544 0.154). Explanatory power R^2_{adj} [PC IS MP CR QE NR] = (0.568 0.793 0.935 0.296 0.267 0.989). Jarque-bera probability values of normality [ε_t^{PC} ε_t^{IS} ε_t^{MP} ε_t^{CR} ε_t^{QE} ε_t^{NR}] = (0.443 0.000 0.129 0.001 0.000 0.000). Specification $\beta^{CB} = (1 - \beta^{FI})$.

4A. Stationarity Test

TABLE 4A. AUGMENTED DICKEY FULLER TEST

Variables	π_t	$\hat{y}_t$	i_t	Δcr_t	Δqe_t^{WL}	Δqe_t^{AVR}	r_t^n
Level	0.000	0.008	0.006	0.000	0.000	0.000	0.018
First Difference	0.000	0.000	0.000	0.000	0.000	0.000	0.294

NOTES: The table shows the probability values from the Augmented Dickey-Fuller of stationarity. The lag length is chosen by the Schwarz Criterion.

5A. VAR(p) Lag Selection

TABLE 5.1A. VAR(p) LAG SELECTION NATURAL RATE INCLUDED

Lag/Criterion	LogL	LR	FPE	AIC	SC	HQ
1	1910.869	NA	0.000	-45.178	-44.128	-44.756
2	2200.457	495.441	0.000	-51.288	-49.190	-50.445
3	2382.521	285.160	0.000	-54.808	-51.660	-53.543
4	2666.132	403.206	0.000	-60.774	-56.577^{*}	-59.088^{*}
5	2705.157	49.839	0.000	-60.847	-55.601	-58.740
6	2768.619	71.876^{*}	0.000^{*}	-61.509	-55.214	-58.980
7	2811.704	42.565	0.000	-61.680	-54.336	-58.729
8	2850.535	32.748	0.000	-61.747^{*}	-53.355	-58.376

NOTES: The table shows the lag selection statistics of a VAR(p) with the variables ($\hat{y}_t$ π_t i_t r_t^n Δcr_t Δqe_t).

TABLE 5.2A. VAR(p) LAG SELECTION NATURAL RATE EXCLUDED

Lag/Criterion	LogL	LR	FPE	AIC	SC	HQ
1	1434.126	NA	0.000	-33.955	-33.228^{*}	-33.662
2	1479.551	79.945^{*}	0.000^{*}	-34.447^{*}	-32.990	-33.862^{*}
3	1496.467	27.717	0.000	-34.252	-32.067	-33.374
4	1517.806	32.394	0.000	-34.164	-31.243	-32.991
5	1539.336	30.090	0.000	-34.080	-30.438	-32.617
6	1565.555	33.484	0.000	-34.110	-29.738	-32.354
7	1584.589	22.016	0.000	-33.966	-28.866	-31.917
8	1609.348	25.655	0.000	-33.961	-28.132	-31.619

NOTES: The table shows the lag selection statistics of a VAR(p) with the variables ($\hat{y}_t$ π_t i_t Δcr_t Δqe_t).

6A. VAR(4) Lag Exclusion

TABLE 6.1A. VAR(4) LAG EXCLUSION NATURAL RATE INCLUDED

Lag/Variable	$\hat{y}_t$	π_t	i_t	r_t^n	Δcr_t	Δqe_t	Joint
1	58.471	14.134	119.624	461164	17.491	31.516	-
	(0.000)	(0.028)	(0.000)	(0.000)	(0.008)	(0.000)	
2	17.147	8.931	18.830	115773	5.179	17.411	-
	(0.009)	(0.177)	(0.005)	(0.000)	(0.521)	(0.008)	
3	4.392	3.356	17.068	50872	4.806	11.312	-
	(0.624)	(0.763)	(0.009)	(0.000)	(0.569)	(0.079)	
4	7.757	0.935	10.784	28669	13.32	7.048	-
	(0.256)	(0.988)	(0.095)	(0.000)	(0.038)	(0.317)	

NOTES: The table shows χ^2. Probability values in parentheses.

TABLE 6.2A. VAR(4) LAG EXCLUSION NATURAL RATE EXCLUDED

Lag/Variable	$\hat{y}_t$	π_t	i_t	Δcr_t	Δqe_t	Joint
1	63.230	14.035	150.316	17.177	34.796	292.436
	(0.000)	(0.015)	(0.000)	(0.004)	(0.000)	(0.00)0
2	14.954	7.902	7.621	6.688	15.039	47.570
	(0.011)	(0.162)	(0.178)	(0.244)	(0.010)	(0.004)
3	2.451	5.457	4.939	2.137	8.960	26.375
	(0.784)	(0.363)	(0.423)	(0.829)	(0.111)	(0.388)
4	6.316	3.391	4.703	7.533	6.015	32.051
	(0.277)	(0.640)	(0.453)	(0.184)	(0.305)	(0.157)

NOTES: The table shows χ^2. Probability values in parentheses.

7A. Dynare Code

```
[Code by Erik Hjort, April 26, 2020]

Var pi, y, r, CR, QE, r_nr;

Varexo shock_supply, shock_demand, shock_policy, shock_credit, shock_QE, shock_natural;

Parameters delta,lambda,theta,sigma,b_FI,b_CB,mu,rho_tr,phi_pi,phi_y,rho_cr,rho_qe,rho_nr;

delta=0.498; lambda=0.060; theta=0.109; sigma=0.007; b_FI=0.606;
b_CB=0.394; mu=0.490; rho_tr=0.942; phi_pi=2.230; phi_y=1.688; rho_cr=0.490; rho_qe=0.542;
rho_nr=0.975; sigma_supply=0.800; sigma_demand=0.376; sigma_policy=0.385; sigma_credit=0.902;
sigma_QE=2.208; sigma_natural=0.106;

Model(linear);

pi=delta*pi(+1)+(1-delta)*pi(-1)+lambda*y-((theta*lambda/(1-theta)*sigma)*(b_FI*CR+b_CB*QE))+shock_supply;
y=mu*y(+1)+(1-mu)*y(-1)-(1-theta)*sigma*(r-pi(+1)-r_nr)-theta*(b_FI*(CR(+1)-CR)+b_CB*(QE(+1)-QE))+shock_demand;
r=(1-rho_tr)*(phi_pi*pi(+1)+phi_y*y)+rho_tr*r(-1)+shock_policy;
CR=rho_cr*CR(-1)+shock_credit;
QE=rho_qe*QE(-1)+shock_QE;
r_nr=rho_nr*r_nr(-1)+shock_natural;

End;

Initval; pi=0; y=0; r=0; CR=0; QE=0; r_nr=0; end;
Endval; pi=0; y=0; r=0; CR=0; QE=0; r_nr=0; end;

Shocks;
var shock_supply=sigma_supply;
var shock_demand=sigma_demand;
var shock_policy=sigma_policy;
var shock_credit=sigma_credit;
var shock_QE=sigma_QE;
var shock_natural=sigma_natural;
corr shock_supply, shock_demand = 0.163;
corr shock_supply, shock_policy = 0.127;
corr shock_supply, shock_credit = -0.191;
corr shock_supply, shock_QE = 0.020;
corr shock_supply, shock_natural = -0.006;
corr shock_demand, shock_policy = 0.069;
corr shock_demand, shock_credit = 0.138;
corr shock_demand, shock_QE = -0.104;
corr shock_demand, shock_natural = 0.012;
corr shock_policy, shock_credit = 0.155;
corr shock_policy, shock_QE = 0.028;
corr shock_policy, shock_natural = 0.442;
corr shock_credit, shock_QE = -0.072;
corr shock_credit, shock_natural = 0.204;
corr shock_QE, shock_natural = -0.229;

End;

Stoch_simul;
```