

# The bounds of Korenblum's constant

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**Korenblum's constant is an unknown mathematical constant. It has been proven that it is somewhere between 0.28 and 0.68. But what is the sharp value? We have found an optimization algorithm that can find good results for the upper bound. It could potentially be used with new ideas to push the bound down.**

Imagine two functions on a disc with radius 1. Then imagine that one of them,  $f$ , is smaller than the other,  $g$  outside of an inner circle. According to Korenblum's conjecture then the average of  $f$  on the whole disc is smaller than the average of  $g$ . This sounds weird, since  $f$  could be arbitrarily larger than  $g$  inside the circle. But if we demand that  $f$  and  $g$  are holomorphic then it is actually true. Functions are holomorphic if we can differentiate them in the complex plane. It is more special for a function to be holomorphic than to be differentiable in the real numbers. They behave nicely in many ways, and we know a lot about holomorphic functions.

The biggest possible radius of the inner circle is known as Korenblum's constant. Today we know that it has to be somewhere between 0.28 and 0.68. This is quite a large gap for a mathematical constant. Naturally it is interesting to improve the bounds and hopefully some day know the sharp value of the constant.

To improve the upper bound it is enough to find functions  $f$  and  $g$  that contradict Korenblum's conjecture for some radius. I have written an optimization algorithm to improve the upper bound. The algorithm tries to find counter examples for as small radii as possible, given specific families of functions  $f$  and  $g$ . It can find bounds as close as 0.002 to the best currently known bound. This algorithm could be combined with future ideas of functions to potentially improve the upper bound.

Korenblum stated his conjecture in 1991. He also provided an upper bound of  $\frac{1}{\sqrt{2}} \approx 0.71$ , but it was unknown whether there existed a lower bound. In 1999 Hayman proved the conjecture and provided a lower bound of Korenblum's constant at 0.04. A key in his proof is to use hyperbolic geometry. That means that instead of using a flat plane we have lines and shapes on a hyperboloid. A hyperboloid looks like a cone with a rounded top.

The upper bound has been improved in several small steps by Wang. The key here is to find specific functions that contradict the conjecture. In the most recent examples the functions have been polynomials, and the key has been to optimize for some of the coefficients of the polynomials.