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Signal background interference effects in lepton trident events with a new low mass mediator

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Abstract

In this project we present a phenomenological study of signal to background interference effects in leptonic tridents produced in parton showers. The interference effects are introduced by linking matrix element corrections of a kinetically mixed dark photon model to the parton shower Dire. Using different dark photon masses and coupling strengths positron+proton collisions are simulated and different observables including the invariant mass of the lepton pair from the decay of the new vector boson mediator are studied. We find good agreement between the photon and massless dark photon effects.

Popular science introduction

What is the origin of the Universe? What is it made from? How can we study it? These questions are not only a common concern among the scientific community but also something that most people ask themselves at least once in their life. Nowadays it is known that the universe is made by three elemental components: normal matter, dark matter, and dark energy. Normal matter forms all the matter that we can see and touch and sense and the fundamental particles that form it are included in the so-called Standard Model. It accounts for the 4.9% of the total composition of the Universe and it is the only part that we know well. The 95.1% left is purely unknown and undiscovered. In this scenario, there are many theories as well as experiments that try to prove the existence of dark matter as a particle that interacts weakly or does not interact at all, with normal matter.

Earlier studies have bet on the weakly interacting massive particles (WIMPs) theories. These theories postulate the existence of dark matter as very massive particles that interact with normal matter through the weak force (one of the forces described in the Standard Model). However, no experimental evidence has proven the existence of WIMPs and alternative theories to continue investigating the fundamental composition of the Universe are gaining popularity.

One of the new theories is the "hidden sector dark matter". This sector is a subcategory of a broader spectrum of dark matter theories called light dark matter. They propose that dark matter is lighter than what we previously thought. Hidden sector theories postulate the existence of not only dark matter but also "dark photons". This concept may sound strange, but all physicists know that if it is possible to build a theory that does not violate some important checks in our models, it may be true. Dark photons are a mediator since they can interact with both normal matter and dark matter.

This project aims to explore in detail which observable consequences the production of dark photons could have in an experiment using computer simulations. You may ask, how is it possible to study the observables of a particle that has not even been proved to exist? This is the everyday job of many theoretical physicists. Once you have a valid equation, you can predict different consequences with it. Using computer simulations, complex physics theories with very complicated or no solution can give approximate results that can be compared with experimental results. In this project, we go from theory to simulations and provide the basis for future experimental analysis. Searching for variants of dark matter is challenging and exciting and it contributes to deep in the understanding of our Universe as well as the physics laws that govern it.

Contents

1 Introduction: A new light Beyond the Standard Model

The search for dark matter particles is one of the most pressing challenges in the history of particle physics. Discovering the nature of dark matter could answer a wide range of questions that arise from important physical observations of the Universe. From rotational curves of galaxies, via gravitational lensing in galaxy clusters, to the Cosmic Microwave Background spectrum, dark matter is always the missing piece of the puzzle.

Over the past several decades, the experimental dark matter program has been focused on weakly-interacting massive particles (WIMPs) partially because of Supersymmetry and the WIMP paradigm [1]. The mass range of WIMPs is typically in the range 1 GeV- 100 TeV and the interactions are done through Standard Model (SM) gauge bosons. However, so far, no observational evidence has been obtained. Nowadays, the field of DM has shifted attention towards the so-called Light Dark Matter (LDM) in the few GeV and sub-GeV mass range. Reasons for this is that LDM is not excluded and that the effects of it are attenuated.

Hidden-Sector Dark Matter theories [2] can explain the existence of LDM. In them, LDM is part of a dark sector that is neutral under the Standard Model forces but that can interact through new forces. One important model of hidden sector theories is the kinetic mixing model [3], in which a new U(1) massive gauge boson (popularly called "dark photon") mixes with the U(1) gauge boson of the SM. The mixing of the two bosons induces a coupling between the dark photon and SM particles. Extensions of this theory allow the coupling of dark photons with dark matter particles. As a starting point, this thesis will be focused on the study of the interaction between the dark photon and the SM particles.

One interesting effect applicable in the kinetic mixing model is the "dark bremsstrahlung" process, in which electrons in the multi GeV range lose energy by radiation of invisible dark photons. Dark photons can then decay into a pair of dark matter or SM particles (non-visible or visible decay). If the SM particles from the decay are leptons, the initial electron and the pair of leptons form a *lepton trident*. Dark bremsstrahlung and lepton trident production from dark photons are implemented in Dire [4], which is a parton shower that can act as an extension of the Event Generator PYTHIA [5].

Using parton showers and event generators one can study how the addition of lepton tridents produced by the emission of dark photons affects the initial formation of tridents from photon emission. However, the frequency of the first type of tridents is heavily suppressed due to the small coupling of the dark mediator particle to the SM matter. One possible way to enhance these Beyond the Standard Model (BSM) effects is to use parton showers corrected with matrix element corrections. These corrections change the parton shower evolution and take into account the matrix element interference effects between tridents produced from dark photons (signal) and tridents produced from photons (background). The interference contributions have less suppression since, in them, the dark coupling is mixed with the SM coupling of photons with leptons. This increase in BSM effects also comes with an increase in the complexity of its detection. The way interference terms contribute to the formation of leptonic tridents in a parton shower is not trivial and to fish out the BSM contributions is a challenge.

The goal of this project is to generate lepton tridents using parton showers and matrix element corrections and define observables capable of extracting the signal-background interference effects. For producing the matrix elements with interference terms, the matrix element generator `Madgraph5_aMC@NLO` [6] has been used together with the set of Feynman Rules from the kinetic mixing and SM model obtained with FeynRules [7]. Then, the matrix elements have been linked to PYTHIA+Dire and multiple simulations of positron-proton collisions have been performed. Inspired by the lepton-proton collider HERA [8] the collisions were done with the positron energy set to 27.5 GeV and the proton energy to 1 GeV. Most of the time, the dark photon emitted had a mass of $m_{A'}$=500 MeV and a mixing parameter of $\epsilon_D = \sqrt{0.3}$. The resulting events were analyzed in Rivet [9] by defining personalized observables.

The structure of this thesis project is divided as follows. First, the relevant physics of the new low mass mediator studied in this project is introduced in section 2. This includes an introduction of the kinetic mixing model, the phenomenology of the dark mediator when collinear dark bremsstrahlung is produced and a description of the signal of interest which includes interference terms. Then, the basic formalism of the parton showers used for the generation of collinear dark photons and lepton trident production is presented in section 3 as well as some insights about the main characteristics of Dire. From the following section, our developments are presented. In section 4, the derivation of matrix element corrections of the leptonic tridents from BSM and SM origins and how to use them to correct parton showers are shown. In the next section, the implementation of the project using different software packages is explained as well as the definition of interesting observables that can enhance the interference effects of interest. Finally, the presentation of the results obtained and its discussion is done in section 6. This includes the validation of the parton shower with dark photon emission, the differences in the observables when a parton shower is used with and without matrix element corrections and the effect of including matrix elements with different interference effects. With the set of parameters chosen, small differences due to signal-background interference effects arise. Nevertheless, an extensive discussion of the changes in the changes of the observables related to leptonic trident production has been done. Lastly, section 7 includes a discussion of future directions of extension of the project, where the conclusions are also summarized.

2 Hidden sectors

As introduced in the previous section, lack of experimental observation of WIMPs is shifting the interest towards Light Dark Matter (LDM), where the mysterious particle has a lower mass range between few MeV and a few GeV. A set of gauge theories that include LDM are called Hidden Sector Dark Matter or Dark Sector theories.

These theories are originated from the so-called hidden-valley models [10],[11], in which the Standard Model (SM) gauge group is extended by an additional group. This leads to new types of interactions under which dark matter particles are charged while remaining neutral to the interactions from the Standard Model. Hidden sector theories can explain the current dark matter abundance, the neutrino masses and the baryon asymmetry [3].

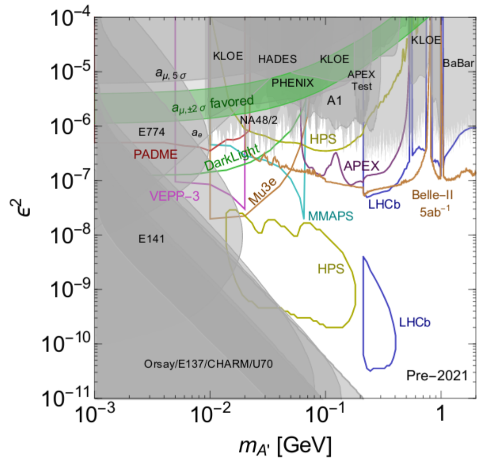
The new interactions are connected to SM particles via *portals*, which are operators that connect dark sector mediators and SM particles. A popular type of portal that respects the gauge and Lorentz symmetries of the SM Lagrangian is the vector portal [2]. This portal arises by the extension of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ by an Abelian $U(1)_D$ group and mixes the B boson with the new $U(1)_D$ vector boson. It is expressed as:

$$\mathcal{L} \subset -\frac{\epsilon_D}{2\cos\theta_W} B_{\mu\nu} A'^{\mu\nu} \tag{2.1}$$

Where $B_{\mu\nu}$ is the SM hypercharge field strength, $A'_{\mu\nu} \equiv \partial_\mu A'_\nu - \partial_\nu A'_\mu$ is the dark $U(1)_D$ vector boson field strength, ϵ_D is the mixing parameter which gives the strength of the mixing between the two bosons and θ_W the Weinberg angle.

Dark sector vector bosons have an unconstrained parameter space which current experiments have not been able to explore yet. Figure (1) shows the free parameter space for the mixing parameter ϵ_D and the mass of the dark boson $m_{A'}$. The gray areas correspond to the excluded values by experimental data and the color lines mark the regions that future experiments will be able to reach in the next two years. The available parameter space changes significantly depending on if the dark boson decays to dark matter particles (invisible decay modes).

The vector portal gives rise to QED-like processes which can be studied in particle collider experiments. The experiment that serves as the basis for this project is the Light Dark Matter eXperiment (LDMX) [13]. LDMX is a fixed target experiment with an electron beam at 4-16 GeV and a fixed target of Tungsten material. The interaction with the material produces dark matter particles through dark bremsstrahlung and the tracking detectors can reconstruct the kinematics of each electron. The goal of LDMX is to search for dark matter particles in the KeV to low GeV scale through missing momenta and displaced electromagnetic showers .



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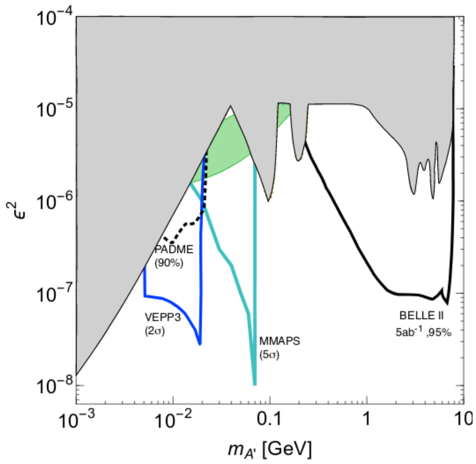

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Figure 1: Parameter space for dark boson. Gray areas show excluded parameter space for the mixing parameter ϵ and the dark boson mass $m_{A'}$. Colored lines show the expected reach of the current experiments in the next 2 years. **Left**: Sensitivity for experiments seeking visible decay modes $A' \to l + l-$. The green band gives the range where the dark boson could explain the discrepancy between theoretical and experimental value of the muon $g-2$. **Right**: Sensitivity for experiments seeking missing mass after A' emission and allowing invisible decay modes [3].

2.1 Kinetic Mixing Model

The vector portal interaction previously introduced is included in the so-called kinetic mixing model [15]-[21]. In this model, an Abelian gauge dark symmetry $U(1)_D$ is added to the SM group. The new dark sector couples to the SM through a kinetic mixing with the hypercharge gauge boson B_μ. The Langrangian of the kinetic mixing model in the flavour eigenbasis can be expressed as:

$$L = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{A}'_{\mu\nu}\hat{A}'^{\mu\nu} + \frac{1}{2}\frac{\epsilon_D}{\cos\theta_W}\hat{A}'_{\mu\nu}\hat{B}^{\mu\nu} + \frac{1}{2}m^2_{A'}\hat{A}'^{\mu}\hat{A}'_{\mu} \tag{2.2}$$

where the first two terms are the gauge kinetic terms, the third term represents the kinetic mixing term between the dark and SM U(1) bosons and the last term corresponds to the mass of the dark boson. In general, it is assumed that the mass of the new mediator $m_{A'}$ is induced by a Higgs potential with a hidden sector dark Higgs [16]. One can diagonalize the gauge and mixing terms by transforming the gauge fields and defining $B = \hat{B} - \frac{\epsilon_D}{\cos\theta_W}\hat{A}'$. After electroweak symmetry breaking the interaction Lagrangian of the bosons with the fermions has an additional term proportional to:

$$\mathcal{L}_{f\bar{f}A'} = -\epsilon_D e A'_\mu J^\mu_{EM} \tag{2.3}$$

Where J_{EM} is the electromagnetic current and A'_μ is the dark vector boson in the mass eigenstate. This term only differs from the interaction term of the photon A with the fermionic matter by the mixing parameter ϵ_D. Because of that, the dark boson of the kinetic mixing model is often referred as dark photon.

If we take into consideration the diagonalization of the mass matrix of the vector bosons (including the new dark boson) in the Higgs mechanism, the dark photon mixes with the Z boson. As a result, the interaction Lagrangian of the dark photon with the fermions becomes similar to the Z boson one but, for low energies and ϵ_D <<1, recovers the photon-like shape [15], [17].

2.2 Radiation of collinear dark photons

From the interaction Lagrangian between the fermions and the dark photons, one can determine the coupling strength. This is proportional to $-\epsilon_D e Q_f$, where e is the dimensionless electric charge of the electron and Q_f is the charge of the fermion interacting. The similarities with the interaction of photons with fermions allow us to describe the radiation of dark photons like in Quantum Electro Dynamics (QED). Here, the derivation of the cross-section of the emission of a dark photon with negligible mass from an electron is sketched but we refer to [22] for a complete overview.

For a hard process $ei \to j$, with an incoming electron e scattering with a particle i and producing a final state particle j, the matrix element is:

$$\mathcal{M}^{ei\to j}(p, p_j) = \bar{u}\bar{A}(p, p_j) \tag{2.4}$$

where $\bar{u}$ is the spinor of the electron and $A(p, p_j)$ is the remaining amplitude.

The emission of a dark photon A' from an incoming electron at high energy $E \gg m_e$ and $E \gg m_{A'}$(Figure 2) gives a matrix element:

$$\mathcal{M}^{ei\to A'j}(p, k, p_j) = \bar{A}^{ei\to j}(p-k, p_j)\frac{i(\not{p} - \not{k})}{(p-k)^2}[i\epsilon_D e \not{\epsilon}^*(k)]u(p) \tag{2.5}$$

where the mass of the electrons has been neglected. The momenta of the electron before and after the emission is expressed as p and p' respectively. The momenta of the dark photon and the final state particle j are given by k and p_j. The polarization vector of the vector boson is $\epsilon^*(k)$.

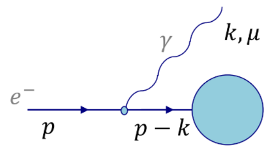


Figure 2: Collinear emission of a photon from an initial electron.

This expression can be simplified by expressing the different four-momenta in terms of the energy of the incoming electron E, the transverse momentum $k_\perp$ of the dark photon and an energy sharing variable z. The last variable defines the energy carried by the intermediate fermion $E' = zE$ and the dark photon energy $E_D \simeq (1-z)E$. The transverse momentum $k_\perp$ measures how collinear is the emitted dark photon with respect to the direction of the initial electron and determines how off-shell the intermediate fermion is $(p-k)^2 = p'^2 = k_\perp^2/(1-z)$.

When the dark photon emitted is collinear to the direction of the initial electron and $|k_\perp| \ll (1-z)E$, the momenta of the intermediate electron p' can be replaced by:

$$\not{p}' = \sum_\kappa u(p', \kappa)\bar{u}(p', \kappa) \tag{2.6}$$

This allows us to factorize the matrix element of the dark photon emission into the matrix element of the hard process with reduced momentum zp and an additional terms dependent on the z, $k_\perp$ the spinors and the polarization vectors:

$$\mathcal{M}^{ei\to A'j}(p,k,p_j) = \sum_{\kappa'} \mathcal{M}_{\kappa'}^{ei\to j}(zp,p_j)\frac{1-z}{k_\perp}\epsilon_D e\bar{u}(p',\kappa')\not{\epsilon}^*(k)u(p',\kappa') \tag{2.7}$$

Using the appropriate representations for the spinors and the polarization vectors the expression can be simplified even further. As usual, the cross-section is obtained by squaring the matrix element and integrating over the phase space of the emitted dark photon:

$$d\sigma^{ei\to A'j} = \epsilon_D^2\frac{\alpha}{2\pi}\int_{k_\perp{}^2_{min}}^{k_\perp{}^2_{max}}\frac{dk_\perp^2}{k_\perp^2}\int_0^1 dz\left[\frac{1+z^2}{1-z}d\sigma_\kappa^{ei\to j}(zp,p_j) + \mathcal{O}(|k_\perp|)\right] \tag{2.8}$$

where $\alpha = e^2/4\pi$ is the fine structure constant.

As a result of the collinear approximation, the cross-section has been factorized into the integral of the transverse momentum, the hard-scattering cross-section with momentum zp and a function $\frac{1+z^2}{1-z}$ independent of the hard cross-section. This function is known as the splitting function P_{ee}, and it represents the probability of emitting a photon from an electron. The factorization of the cross-section in the collinear regime is a key result that will be used later on.

Apart from the cross-section factorization equation, (2.8) has additional important properties. First, the integration of the transverse momentum gives rise to a logarithmic term with ${k_\perp}^2_{min}$ in the denominator. The lower limit of the transverse momentum is given by the mass of the intermediate electron. For zero or very small electron masses, the cross-section has a collinear singularity or enhancement. Second, the denominator of the splitting function $P_{ee} = \frac{1+z^2}{1-z}$ has a singularity when $z = 1$. Thus, emission of soft dark photons with low energy enhances the cross-section. Thus, in the collinear approximation, the radiation of dark photons can also lead to collinear and soft-radiation singularities or enhancements. These divergences can be canceled by including virtual perturbative corrections.

2.3 Lepton tridents and interference effects

After bremsstrahlung, off-shell photons or dark photons can decay into a pair of leptons and together with the initial scattered electron produce lepton tridents. If the mass of the dark photon is higher than twice the mass of the electron, on-shell dark photons can also produce tridents. Lepton tridents originated from dark photon emission are the signal of interest. For simplicity, this project is centered on detecting the BSM effects in the generation of tridents with two positrons and one electron or with one positron and a muon pair.

The matrix element of a lepton trident originated from a dark photon has two couplings to fermions. This makes its matrix element proportional to ϵ_D^2. Nevertheless, one also needs to take into account the tridents originated from photon emission in the total matrix element of the lepton trident. Symbolically, the transition probability of the process would be:

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM} + \mathcal{M}_D|^2 = |\mathcal{M}_{SM}|^2 + |\mathcal{M}_D|^2 + \mathcal{M}_{SM}^*\mathcal{M}_D + \mathcal{M}_D^*\mathcal{M}_{SM} \tag{2.9}$$

where $\mathcal{M}_{SM}$ is the matrix element of the lepton trident originated from the photon and $\mathcal{M}_D$ is the one originated from the dark photon. The associated Feynman diagram is illustrated in Figure (3).

Assuming a small mass of the dark photons, the second term in the transition probability, which is the transition probability of lepton tridents originated from dark photons, only differs from the SM term in the coupling of fermions to dark photons. As a consequence, this "dark" pure term is approximately $\epsilon_D^4|\mathcal{M}_{SM}|^2$.

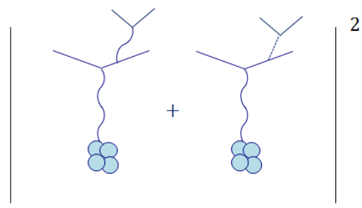


Figure 3: Interference terms of interest recovered when using matrix element corrections. The matrix elements of tridents produced from dark photons mixed with matrix elements of tridents produced from normal photons are less suppressed than the weight of the trident process when only dark photons are taken into account.

The last two terms of the expression are the mixed terms. They are the signal-background interference terms and are proportional to ϵ_D^2. If the mixing parameter ϵ_D is smaller than one, the interference terms will have a larger contribution to the total transition probability of the trident than the pure term associated with dark photon emission.

The example illustrated in Figure (3) only takes into account one type of Feynman diagram that can produce a lepton trident in the final state. Including other types of Feynman diagrams would increase the number of mixed terms to take into account. However, it is important to realize that only the terms that mix a lepton trident originated from a dark photon and a lepton trident originated from a photon are the signal-background interference terms of interest with the desired ϵ_D^2 dependence.

3 Event generators and Bremsstrahlung

Up until now, we have considered the general theoretical framework of the kinetic mixing theory for Light Dark Matter and the consequences of it in QED-like processes. However, to test the accuracy and reality of this theory it is necessary to use advanced programs to simulate collisions between particles at high energies and the consequent products of it. These programs are called Event Generators and the leading role of the Theoretical Particle Physics division at Lund University in the development of PYTHIA is worldwide recognized.

Parton evolution and emission of collinear particles are integrated into Event Generators. The output of these programs is a distribution of events according to cross-section calculations. The events contain a list of particles with its 4-momentum and quantum numbers. This project is focused on the initial parts of the particle collision simulations meaning the simulation of the hard scattering and subsequent parton showers. However, Event Generators can also simulate secondary hard scatterings, parton to hadron transitions, hadron decay and beam remnants.

The calculations of the cross-section in the parton shower can be improved by using Matrix Element Corrections (MECs). The next sections will introduce the necessary parton shower formalism as well as explain the functioning of MECs.

3.1 Parton shower basics

Parton showers are the part of Event Generators responsible for simulating succession of emissions from the incoming and outgoing particles or partons. These higher-order real emission corrections approximate the hard scattering process. Generally, flavor and four-momentum are locally conserved in parton showers. Unitarity is also respected, meaning that the sum of the probability of having an emission and not have it is equal to one [23].

The factorization of the cross-section in the collinear approximation shown in sub-section 2.2 can be generalized for any number of emissions and be used to calculate the cross-section of processes with N-emitted particles in the final state [24]. This universal factorization can be implemented in parton showers.

However, the cross-section of 2.8 is inclusive. One can calculate exclusive cross-sections with the additional requirement of ordered emissions. This means that, first, if a first photon is emitted at the scale t, no other first photon can be emitted at $t' > t$, and second, following emissions will occur at a scale $t'' < t$. The first ordering condition is enclosed in what is known as Sudakov factors, which represent the probability of not having an emission between the starting factorization scale t_{max} and the scale t at which the first emission occurs [23]. The Sudakov factor is defined as:

$$\Delta(t_{max}^2, t^2) = exp\left(-\int_{t^2}^{t_{max}^2} \frac{dt'^2}{t'^2}\int_0^1 dz \frac{\alpha_s}{2\pi}\frac{f_e\left(\frac{x}{z}, t\right)}{f_e(x,t)} P_{ee}(z)\right) \tag{3.10}$$

where the general scale variable t has substituted the previously used transverse momentum $k_\perp$, the function $f_e(x,t)$ represents the probability for resolving an electron with momentum fraction x in a flux of initial electrons and at the factorization scale t, and the splitting function is now a more general Altarelli-Parisi splitting function [24]. Apart from ensuring exclusive final states, these factors are also the equivalent approximation of virtual corrections in the parton shower context. Thanks to them, logarithmic virtual enhancements are resummed.

The second ordering condition implies that the emissions occur in a decreasing factorization scale $t_0 = t_{max} > t_1 > t_2 > t_n$ and it is implemented via a theta function $\Theta(t_i - t_j)$ where $i < j$. Multiplying the cross-section by the Sudakov factor and taking into account the second ordering condition one gets the exclusive cross-section of emitting a photon, before or after a hard scattering $ei \to j$, between the scales t_{max} and t_{min} as:

$$d\sigma^{ei\to j\gamma} = \int_{t_{min}^2}^{t_{max}^2} \frac{dt^2}{t^2} \int_0^1 dz \frac{\alpha_s}{2\pi} \frac{f_e\left(\frac{x}{z}, t\right)}{f_e(x,t)} P_{ee}(z)\Theta(t_{max} - t)\Delta(t_{max}^2, t^2)|\mathcal{M}(ei \to j)|^2 d\Phi_j \tag{3.11}$$

where the cross-section of the hard scattering process has been expressed in terms of its matrix element squared $|\mathcal{M}|^2$ and its phase-space $d\Phi_j(p_e p_i; p_j)$.The latter one ensures 4-momentum conservation and its integration is Lorentz-invariant.

The cross-section of the whole process (emission cross-section) has one part connected to the emission probability, formed by the matrix element squared of the hard scattering and the splitting function of photon emission, and a part connected to the no-emission probability, formed by the Sudakov factor. The combination of both parts makes the emission cross-section represent the probability of having a photon emission with no other photon emissions between the hard scale and the scale of the emission.

Even if only one type of splitting function and matrix element have been taken into account in this simplified example, the calculation of the cross-section of any process needs to consider the different ways or "paths" one could have arrived to the final state of interest. For example, one should take into consideration if the first photon emission occurs before or after the hard scattering process.

In a general case, the shower transitions from a $n-$particle state with momentum configuration Φ_n, to a $n + 1$ state with a $n + 1$-momentum configuration Φ_{n+1}, at a transition scale $t = t(\Phi_{n+1}/\Phi_n)$. Each possible transition or path, has an associated branching weight proportional to:

$$P_{ij}(\Phi_{n+1}/\Phi_n)\Theta(t(\Phi_n) - t(\Phi_{n+1}/\Phi_n))|\mathcal{M}(\Phi_n)|^2 d\Phi_{n+1} \tag{3.12}$$

where $P_{ij}(\Phi_{n+1}/\Phi_n)$ is an universal splitting function which tells the probability of a parton of tipe j emitting a collinear parton of type i, the Θ function restricts the showers to decreasing factorization scales t, $|\mathcal{M}(\Phi_n)|^2$ is the matrix element of the n-particle state and $d\Phi_{n+1}$ represents the phase-space of the process after the transition.

Effectively, when calculating the cross-section of a process one needs to consider all the configurations that can lead to the final state and sum over all the branching weights. In the example of a hard scattering with a subsequent photon emission it could be that the photon was emitted before the hard scattering or after it and one should sum both branching weights when calculating the cross-section of the process.

The main difference of Dire with respect of other parton showers is enclosed in the dipole-like structure, in the splitting functions [4] and the corrections of cross-sections with Matrix Elements [25]. First, the emissions occur according to a dipole radiation pattern where the dipoles are formed by pair of partons connected by their color or by electric charges. Second, the splitting functions used to cover not only the collinear region but also the soft singularities outside it. This is done by partial fractioning of the splitting functions using the Catani-Seymour approach [26].

4 Matrix element corrections

The accuracy of the parton shower depends on how well the sum of splitting probabilities and matrix elements approximates the full real emission probability. In phase-space regions that are accessible by an ordered sequence of splittings, one can use the so-called Matrix Element Corrections (MECs) [25]. The calculation and implementation of MECs is one of the main features of Dire. In a process with $n+1$ particles originated from a n-particle state the MEC factor is defined as:

$$\mathcal{R}(\Phi_{n+1}) = \frac{\mathcal{M}_{n+1}}{\sum_{\Phi'_n} P^{n'}_{n+1} \Theta(t_{n'} - t^{n'}_{n+1}) \mathcal{M}_{n'}} \tag{4.13}$$

where the shortcuts used are $\mathcal{M}_i = |\mathcal{M}(\Phi_i)|^2$, $P^n_{n+1} = P_{ij}(\Phi_{n+1}/\Phi_n)$, $t_n = t(\Phi_n)$ and $t^n_{n+1} = t(\Phi_{n+1}/\Phi_n))$.

The numerator of the matrix element correction factor $\mathcal{R}(\Phi_{n+1})$, contains the full matrix element at leading order of the $n+1$-configuration $\mathcal{M}_{n+1}2$ while the denominator contains a sum over all the branching weights (equation (3.12)) which are the paths the shower can populate that $n+1$-configuration. Furthermore, the matrix elements of the branching weights inside the MEC are also recursively corrected.

The calculation of the denominator is one of the key difficulties of the method. Parton showers are Markov processes, in which the next state only depends on the weight of the previous state. Thus, to get the denominator an explicit reconstruction of the different terms and their weights is needed.

Correcting the parton shower evolution using MECs can improve the calculation of the cross section. Theoretically, MECs can replace the parts of the cross-section which approximate the emission probability of the different states by the corresponding full fixed-order matrix element of them, which includes interference effects and contributions non-reachable by a parton shower.

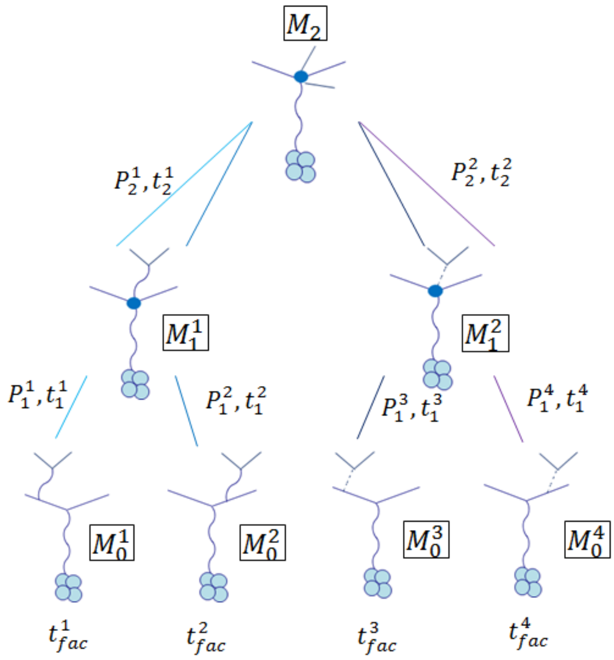


Figure 4: Schematic representation of trident process originated from a parton shower at different resolution scales. Each level (horizontal layer) has its own correction factor. The abbreviations used are: $\mathcal{M}_i = |\mathcal{M}(\Phi_i)|^2$, $P_i^{i-1} = P(\Phi_i/\Phi_{i-1})$ and $t_i^{i-1} = t(\Phi_i/\Phi_{i-1})$. The top level shows the trident process with indistinguishable origin. The medium layer shows how the trident is produced by dark photon or photon splitting. The bottom layer shows how the photon or dark photon radiation is radiated before or after the scattering with a nucleus.

MECs are implemented in Dire by a modification of the Sudakov-Veto algorithm commonly used in PYTHIA [12]. This algorithm is used to reach the full weight of a process without the explicit calculation of it by re-scaling an overestimated and simplified version. During the Sudakov-Veto algorithm MECs are implemented as a redefinition of the splitting functions. Changing the splitting functions using MECs does not only change the emission probability but also the no-emission probability, securing the unitarity of the method.

To understand more intuitively how these corrections work one can come back to the trident process introduced in subsection 2.2 and take a look to Figure (4). Starting from the scattering of a positron with a nucleus, one ends up with two positrons and one electron in the final state. In a simplified picture and for energies below 90GeV these three leptons could have been produced by the scattering of the positron and subsequent emission of a dark photon or a photon, which then decays into a pair of e+ e- particles. At the same time, the boson emission could have been produced before or after the hard scattering of the positron and the nucleus.

The final state is produced by a first emission of a photon or a dark photon and by a second emission where the photon or the dark photon split into a lepton pair. Calculating the cross-section of the process would require to know if the lepton pair is originated from a photon or a dark photon and to know if the photon or the dark photon have been obtained before or after the hard scattering process, which is known as initial or final state radiation (ISR or FSR).

The order of the different paths is defined with the variable t, which in Dire is the soft transverse momentum. All the integration variables introduced in section 3.1 are taken into account but are not displayed to simplify the explanation.

To correct the shower with MECs one needs to start by correcting the first emission. The MEC factors take into account if the emissions are ISR or FSR. The MEC factor associated to photon emission is defined as:

$$\mathcal{R}_1^1 = \frac{\mathcal{M}_1^1}{\mathcal{M}_0^1 P_1^1 \Theta(t_{fac}^1 - t_1^1) + \mathcal{M}_0^2 P_1^2 \Theta(t_{fac}^2 - t_1^2)} \tag{4.14}$$

and the one associated to dark photon emission is:

$$\mathcal{R}_1^2 = \frac{\mathcal{M}_1^2}{\mathcal{M}_0^3 P_1^3 \Theta(t_{fac}^3 - t_1^3) + \mathcal{M}_0^4 P_1^4 \Theta(t_{fac}^4 - t_1^4)} \tag{4.15}$$

In this case, P_1^1 is the splitting function associated to the emission of one ISR photon and P_1^2 is the splitting function associated to a FSR photon emission. $\mathcal{M}_0^1$ is the matrix element squared of the hard scattering process which originated the ISR emission. The term $\mathcal{M}_1^1$ represents the transition probability of the hard scattering with an additional photon emission. This last term is not calculated with the parton shower but with external matrix element generators like Madgraph and contains not only the paths to the final state included in the parton shower but additional contributions and interference terms between process involving a photon in the final state and the hard scattering between the positron and the nucleus. The meaning of the rest of the terms can be generalized.

To simplify the MECs, one can assume that $t_2^j < t_1^i < t_{fac}^i$ which means that the sequence of branching scales decreases from bottom to top and all the paths contribute to the final state. The MEC factor of the second emission becomes more complex. The denominator is formed by all the paths that can arrive to the final state, starting from the hard scattering. It is defined as:

$$\mathcal{R}_2 = \frac{\mathcal{M}_2}{P_2^1 \mathcal{R}_1^1 (P_1^1 \mathcal{M}_0^1 + P_1^2 \mathcal{M}_0^2) + P_2^2 \mathcal{R}_1^2 (P_1^3 \mathcal{M}_0^3 + P_1^4 \mathcal{M}_0^4)} = \frac{\mathcal{M}_2}{P_2^1 \mathcal{M}_1^1 + P_2^2 \mathcal{M}_1^2} \tag{4.16}$$

It is important to realize that the branching weights in the denominator are ME corrected with the MECs from the first emission process. This corrections simplifies the expression

of the denominator and recovers the full matrix elements squared of the first order emission in the denominator. The use of corrections from first-order emissions in the MECs of the second-order emissions illustrates the recursivity of MECs.
The cross-section of the process, corrected with the matrix element will be:

$$d\sigma_2 = \int_{t_{min}^2}^{t_1^2} \frac{dt_2^2}{t_2^2} \int_0^1 dz \frac{\alpha}{2\pi} \mathcal{R}_2(P_2^1 \mathcal{M}_1^1 + P_2^2 \mathcal{M}_1^2) \Delta(t_1^2, t_2^2) d\Phi_1 \tag{4.17}$$

Applying the MEC factor to the sum of branching weights will cancel the denominator of the correction factor and recover the full matrix element of the leptonic trident process:

$$\mathcal{R}_2(P_2^1 \mathcal{M}_1^1 + P_2^2 \mathcal{M}_1^2) = \mathcal{M}_2 \tag{4.18}$$

The corrected weight now has the full matrix element with the interference terms of interest included as it was illustrated in section 2.3. Furthermore, the matrix elements are also exactly calculated instead of approximated using collinear factorization and splitting functions.

It is important to realize that the Sudakov factor also changes when the MECs are introduced. In this example the corrected Sudakov factor will have a sum over the splitting functions that arrive to the final state through the splitting of a photon into a lepton pair or the splitting of a dark photon. The expression is given by:

$$\Delta(t_1^2, t_2^2) = \exp\left(- \int_{t_2^2}^{t_1^2} \frac{dt^2}{t^2} \int_0^1 dz \frac{\alpha}{2\pi} R_2(P_2^1 + P_2^2) \right) \tag{4.19}$$

Thus, both emission and no-emission probabilities are affected by the introduction of the MECs.

5 MECs in Pythia+Dire

In these sections, we describe how the matrix element corrections have been implemented in PYTHIA+Dire. A small introduction for each of the different software used is also presented. Lastly, the construction of observables that are used to extract the interference effects of MECs is presented.

5.1 FEYNRULES implementation

The first step before assembling the matrix elements is to introduce the theoretical model, meaning the Lagrangian and its parameters like masses or couplings.

For this, one can use FEYNRULES, which is a MATHEMATICA[33] package that allows for the calculation of Feynman Rules of any quantum field theory model directly from its Lagrangian [7] - [31]. In principle, it can work with an extensive range of models that have fields with spin up to 2 as long as the Lorenz and gauge invariances of the Lagrangian are respected. To implement Beyond the Standard Model theories into matrix element generators one can define the Lagrangian and its parameters in FeynRules and output the information of the model in the form of a Python library named Universal FeynRules Output (UFO) [32]. FEYNRULES can also load various restrictions. In this case, we have imposed a diagonal CKM matrix and the approximation of light particles to be massless.

To generate the Feynman rules of the kinetic mixing model we have defined the Lagrangian of the kinetic mixing as well as the different parameters of it. This includes the type of interactions, the gauge fields, the different particles, mixing parameter and so on. As an example, we define the dark photon as:

```
V[14] == {
    ClassName        -> DA,
    SelfConjugate    -> True,
    Mass             -> 0.1,
    Width            -> 0.1,
    ParticleName     -> "da_p",
    PropagatorLabel -> "da_p",
    PDG              -> 9000001
}
```

Which describes a vector field `V[14]` expressed as `DA` (our dark photon A'), equal to its own antiparticle, with a mass of $M_{DA} = 0.1$ GeV a width $W_{DA} = 0.1$ GeV and a `PDG` index of `9000001`.

Even if dark photons can interact with any type of fermions, the target of study are tridents with leptons in the final state. Thus, we define the Lagrangian, in MATHEMATICA notation as:

```
  LDA = -1/4 FS[DA,mu,nu] FS[DA,mu,nu] + 1/2 M_DA^2 DA[mu]DA[mu]

      - emix ee DA[mu](-lbar.Ga[mu].l)

      LTOTAL = LDA + LSM
```

where `FS[DA,mu,nu]`$= A'_{\mu,\nu}$ denotes a field strength tensor where A' is the gauge dark photon boson, `mu`, `nu` are the indices carried by the tensor, `l` is the lepton symbol, and

`Ga[mu]`$=\gamma^{\mu}$ represents the gamma matrices. Note that interaction of dark photons and quarks has not been included.

Using this model together with the SM Lagrangian LSM from the model database of FeynRules, we obtain a UFO output that contains a set of python scripts with all the Feynman Rules. Thus, all the vertices, parameters, couplings, propagators and so on can be found here. The UFO interface gives the connection to Madgraph; the next program used to generate Matrix Elements. Madgraph will allow us to change the values of the parameters defined in the model.

5.2 Madgraph Implementation

MADGRAPH5_AMC@NLO is a program written in Python used to generate matrix elements. It is capable of calculating cross-sections and matrix elements as well as generating events at leading order in perturbation theory [6]. It can also do these calculations with BSM models by using the associated set of Feynman Rules contained in the UFO output of FeynRules.

For generating events with dark photons a private version of MADGRAPH5_AMC@NLO supplied by S.Prestel has been used. This version generates outputs compatibles with the parton shower Dire. To compute the matrix elements of the trident events the next steps have been followed:

```
>> ./bin/mg5_aMC --mode PY8Kernels
    >> import model photonlike_UFO
    >> generate p e+ > j e+
    >> add process p e+ > j e+ a
    >> add process p e+ > j e+ da_p
    >> add process p e+ > j e+ e+ e- NP<=2
    output MEs_photonlike --PY8MEs
```

What Madgraph does is to import the simplified version of the Kinetic Mixing Model, here called photonlike model, to be able to generate processes with dark photons. Then it generates the hard scattering process, with a proton and positron beams generating a positron and a jet. On top of it, the emission of a photon, the emission of a dark photon and the 2 $\rightarrow$4 scattering process necessary to generate a proton-positron scattering which generates an e+, e-, e+ trident and a jet are also added. The number of dark photon vertices is set with `NP<=2`. In practice, all the matrix elements at tree level that can lead to the final process are generated.

If one wants to study a different type of trident new outputs need to be generated. For instance, we compared the MECs effects generated when the SM and the photonlike model were involved with the effects generated when ONLY the SM-matrix element corrections are present. For obtaining the SM MECs we generated the same trident process in Madgraph excluding the addition of the process involving a dark photon. Furthermore, tridents, where the photon or the dark photon decays into a pair of muons, are also explored and this required the generation of the correspondent matrix elements too.

The output file `MEs_photonlike` has the files necessary files to link the matrix elements with Pythia+Dire. From all of them, we use the param.card file to change the value of the dark photon parameters. If one wants to study tridents with different intermediate states it is necessary to link the corresponding output with the matrix elements against Dire and repeat the installation of Dire.

5.3 Dire implementation

The parton shower Dire is an open source algorithm which can be found in `http://dire.gitlab.io/`. During the development of this thesis project several parts of this algorithm were modified to work with the matrix elements containing dark photons calculated with Madgraph. Thus, we mainly focused on the code connected to the showers with QED splittings, the Dark QED splittings as well as the ones connected to parton shower evolution. To link the calculated matrix elements to Pythia+Dire it is necessary to specify the path to Pythia, the path to the matrix elements and the name of the model during the configuration of Dire as:

```
./configure --with-pythia8=/path_to_ythia/pythia8240
            --with-mg5mes=/path_to_ME/MEs_photonlike
            --mg5mes-model=photonlike_UFO
            --with-openmp-lib=/usr
```

During this process, some sources of error were identified. For instance, the existing splitting functions of the dark photon shower (U1new) were equated to the QED ones (except for the dark coupling), the PDG for the dark photon was set to the PDG defined in the UFO output and the function for having decay of resonances with unique probability was de-activated.

To run Pythia+Dire, one needs to configure the parameters of the simulation like the energy of the beams, the minimum invariant mass, the cut-off in the transverse momenta, etc. Also, the run file allows us to run Dire with different showers switched on. For instance, if one wants to, it is possible to only run Dire with photon emission originated from initial-state radiation. The dark photon emission in the parton shower evolution is done through the activation of the so-called **U1New** shower in Dire.

Apart from that, the beam remnants, hadronization and resonance decays can be switched on and off as well as the matrix element corrections. To use this last feature it is necessary to also specify the path to the folder that contains the matrix element corrections that Dire has been configured with and that will be used to perform the simulations. For the interested reader, a run file with all the parameters used when generating events with tridents coming from dark photons and matrix element corrections activated is included in the appendix.

The simulation of events was accompanied by a subsequent Rivet [9] analysis. In this project we defined our own analyses to obtain interesting observables capable of showing the interference terms of the matrix element corrections.

5.4 Trident observables

When defining analyses with Rivet, we selected the ones containing leptonic tridents in the final state. The production of tridents from the emission of a dark photon from a positron and subsequent splitting into a pair of muons should differ from the trident originated from a photon emission and splitting because of the effect of the mass of the dark photon and the coupling of the dark photon to the leptons. Thus, the first interesting observable was the rate of tridents in the final state.

To build the rest of the observables we used the 4-momenta of the particles. For a trident made of a positron and a muon pair they are expressed as:

$$\begin{cases} p^{\mu}_{\mu+} = (E_{\mu+}, p_x^{\mu+}, p_y^{\mu+}, p_z^{\mu+}) \\ p^{\mu}_{\mu-} = (E_{\mu-}, p_x^{\mu-}, p_y^{\mu-}, p_z^{\mu-}) \\ p^{\mu}_{e+} = (E_{e+}, p_x^{e+}, p_y^{e+}, p_z^{e+}) \end{cases} \tag{5.20}$$

The effect of the dark photon mass is expected to change the total energy of the trident which is simply defined as:

$$E_t = E_{\mu+} + E_{\mu-} + E_{e+} \tag{5.21}$$

Another interesting observable that can be affected by the presence of an intermediate dark photon is the transverse momenta of the trident since it's invariant under Lorentz boost along the beam direction in the z-axis:

$$p_{Tt} = \sqrt{p_x^{\mu+2} + p_y^{\mu+2}} + \sqrt{p_x^{\mu-2} + p_y^{\mu-2}} + \sqrt{p_x^{e+2} + p_y^{e+2}} \tag{5.22}$$

Furthermore, the effect of the invariant mass of the muon pair is also investigated. When calculating the cross-section of a decay as a function of the invariant mass of the decay products one expects a peak around the invariant mass of the particle that decayed.

For a two-particle decay 0→1+2, the invariant mass is defined as:

$$M^2 = p_0^\mu p_{0\mu} = (p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) \tag{5.23}$$

Lastly, the angular distance between the lepton pairs was constructed. This quantity is also invariant under boost along the beam direction and is defined as:

$$\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} \tag{5.24}$$

where η is the pseudorapidity and typically describes the angle of a particle relative to the beam axis and ϕ is the azimuthal angle between the particle and the transverse plane. In terms of 4-momenta components they can be expressed as:

$$\eta = \frac{1}{2}\ln\left[\frac{|\mathbf{p}| + p_z}{|\mathbf{p}| - p_z}\right] \qquad \phi = \tan^{-1}\frac{p_y}{p_x} \tag{5.25}$$

On top of these five observables, combinations of them, as well as the individual particle 4-momentum, were also explored but we chose these as the most relevant ones. Furthermore, for the case of tridents made by an electron and a positron pair the way of defining some of the observables became less trivial as we will see in the discussion of the results.

6 Results and discussion

6.1 Validation tests

Before testing interference effects, detailed validation tests of the dark photon shower of Dire are necessary. For this purpose, we first study the production of dark photons in positron-proton collisions. Then, we tune the study to the cases where the shower produces dark photons which then decay into a lepton pair. We use different masses of the dark photon in the shower as well as different mixing parameters and compare against the background from only QED emissions.

6.1.1 Validation of e+ p → e+ j A'

First, for validating the U1New shower we simulate positron-proton collisions in PYTHIA+Dire at particle energies of 27.5GeV (HERA energies) and 1GeV respectively. After completing the evolution we analyze the events and looked at the rate of dark photons and positrons in the final state as well as their energy and transverse momentum distribution. We test different masses of the dark photon as well as different values of the mixing parameter. All the observables are compared against a photonic background coming from simulations using the QED shower of Dire.

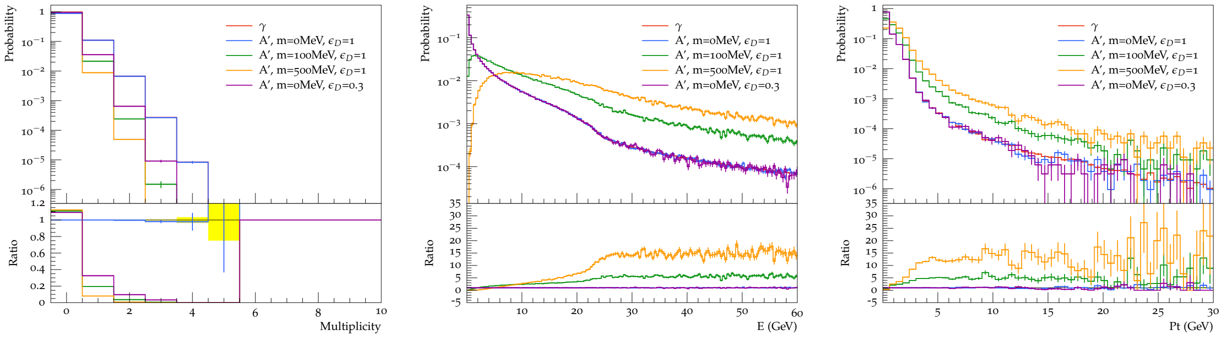
Before starting the validation tests one needs to ensure that the QED and the U1New showers have the same phase-space available. Since the dark photon coupling to quarks is not taken into account, we adjusted the Dire shower to disallow quarks from being radiators or recoilers. Moreover, the coupling of the dark photon is always constant. As a consequence, the running coupling of QED was fixed to the fine structure constant $\alpha_0 = e^2/4\pi$. To study photon vs. dark photon production we need to forbid the splitting of those bosons into other particles. With this aim, the resonance decays were switched off and the splitting kernels of photons into leptons were set to zero.

To get the probability of each observable we define histograms and normalize them to make the sum of bins equal to one. The probability is a relative probability and is defined as the number of times an event occurs (e.g. dark photon with a 500MeV mass with 3 GeV energy) divided by the total number of events (events that have a dark photon with 500MeV mass).

Figure (5) illustrates the multiplicity, energy and transverse momentum of the photon or dark photons in the final state generated with Pythia+Dire after simulating 10 million positron-proton collisions. For clarity of the discussion, we select one type of dark photon at a time and comment on the respective observables.

For the case of massless dark photons with ϵ_D^2=1 (blue line), we can see how their multiplicity, energy and transverse momentum measured are identical to the photon ones (red line). In the three observables, the ratio plot shows how the differences between the dark photon and the photon are negligible. The multiplicity plot shows how, in most cases, we do not get any photon or dark photon in the final state. If we look at events with one photon or dark photon in the final state, we find one in ten events that have an emission of these particles. Thus, the probability of having a final state with a large number of photons or dark photons is low. The similarity in the multiplicity between the photons and dark photons is explained by the choice of dark photon parameters. First, the unit mixing parameter ϵ_D ensures dark photon couplings to leptons identical to the SM photon ones. Then, the zero mass of the dark photons makes their splitting functions in the U1New shower identical to the splitting functions of photons in the QED shower. Therefore, if dark photons and photons have identical emission and no-emission probabilities the QED and U1New shower evolution will produce these particles at an equal rate.

When it comes to the energy and transverse momenta of the dark photons, the similarities with the photon case are also not surprising. The evolution of two showers that emit photons or dark photons with identical splitting functions and couplings will also result in the same distribution of 4-momentum of the two bosons independently of the shower used. If we look at the distributions themselves, we see a decrease in probability with energy. This means that soft photons and massless dark photons are more likely to be emitted. The emission of high energy photons and dark photons is less likely but still occurs, even



(a) Multiplicity (b) Energy (c) Transverse momentum

Figure 5: Comparison between e+ p → e+ j γ (red) and e+ p → e+ j A' with different masses and mixing parameter. A mixing parameter of one has been used with $m_{A'}$=0MeV (blue), 100MeV (green), 500MeV (yellow) and ϵ_D^2=0.3 and $m_{A'}$=0MeV in the purple line. A particle energy of 1GeV has been used for the proton, while the positron has been set to 27.5GeV, and 10^7 events have been used. Subfigure(a) shows the multiplicity of photons and dark photons obtained when using a photon shower or a dark photon shower separately. Subfigure(b) shows the energy of the emitted photons or dark photons and subfigure(c) shows the transverse momentum of them.

if the initial energy of the collision is much lower.

This can be explained by the boost of the 4-momenta of the final state particles into the lab frame and by the configuration of Dire when running the simulations. Emissions of particles from the positron before the collision with the proton are allowed. The initial positron has no restrictions in the initial energy as long as it has 27.5GeV in the moment of the collision with the proton. Both showers could start with a positron at very large energy and emit a photon or a dark photon with large energy and momenta as long as the positron that scatters with the proton had the desired energy. For the transverse momenta, the same behavior occurs. We can see how the decay in transverse momenta occurs more rapidly than in the energy by comparing the probability at 30 GeV.

Continuing the discussion of Figure (5), we move to the results obtained when the dark photon shower evolution is done with a dark photon mixing parameter of ϵ_D=1 and masses of 100 MeV (green line) and 500 MeV (yellow line). We find significant differences when comparing with the results obtained with the QED shower.
First, like in the massless case, the probability of having dark photons in the final state decreases with the multiplicity. However, now we find that regardless of the multiplicity, the probability of having massive dark photons in the final state is lower than the probability of having a photon. This can only be a consequence of a decrease in the splitting functions associated with dark photon emission. Physically this can be understood as the

emission of a particle with a large mass being more costly and less likely to happen.

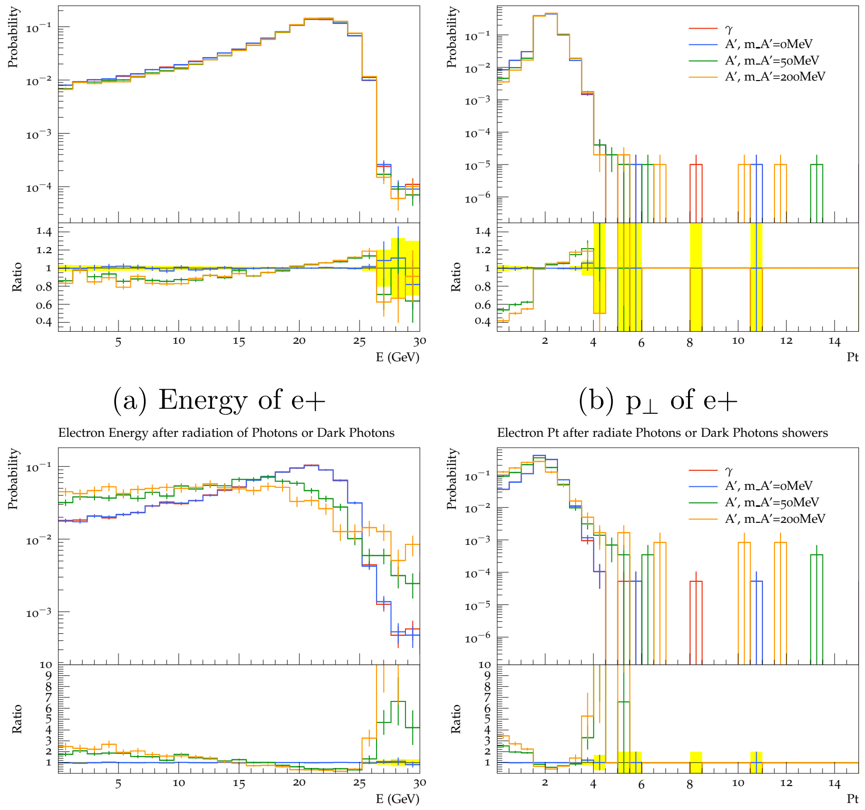
When comparing the dark photon energies and transverse momenta (sub-figures (5b) and (5c)) in the final state, the condition of the sum of the bins of each histogram being equal to one makes necessary to stress the meaning of probability in this case. Here the histograms are not obtained using the whole 10^7 events but they are filled with the condition of having a dark photon or a photon in the final state. For the observables of interest, the probability is the singular probability of emitting a massive dark photon of certain energy (or transverse momenta), given that the emitted dark photon has that specific mass. The normalization and the condition to fill the histograms implies that only the distribution between different histograms can be compared, not the rate.

For both energy and transverse momenta, the mean of the distribution of dark photons is larger for more massive dark photons(green and yellow lines). The emission of massive dark photons is more costly and occurs at a lower rate than photon emission, but when it occurs, those dark photons will, on average, have higher energies than the photons because of their mass. The lowest values of energies and transverse momenta in massive dark photons are determined by their rest mass.

Lastly, the way we define the probability can also explain the behavior obtained when the dark photons emitted have a mixing parameter of $\epsilon_D = \sqrt{0.3}$ and a zero mass (purple line). The difference in the mixing parameter only affects the coupling and this decreases the overall rate of the emission. However, other than the rate, the emitted dark photons still "behave" as photons and because of that the energies and transverse momenta distributions are identical.

The next step in the validation of the U1New shower is to look at the positrons in the final state. The same observables were repeated but now only 100.000 events were used to obtain the histograms. However, now we studied the observables with and without the condition of having an emitted photon or dark photon in the final state. Even if, in practice, one does not know when a dark photon has been emitted it is still interesting to observe how the distributions change when this extra condition is added.

Figure (6) shows the energy and transverse momenta of the scattered positron after simulating a positron-proton collision 100.000 times and allowing only photon emission or dark photon emission with different masses and mixing parameter set to one. The multiplicity of the positron is always 1 since the resonance decays, hadronization and beam remnants are switched off. We will first discuss the results when taking all the emitted positrons into account and then the positrons in the final state after a photon or a dark photon emission.



(a) Energy of e+ (b) $p_\perp$ of e+

(c) Energy of e+ after emission (d) $p_\perp$ of e+ after emission

Figure 6: Singular Energy and transverse momenta of the positron after an e+ + p collision with beam energies at 27.5GeV and 1GeV respectively and 100.000 events. The parton showers with only photon emission or with only dark photon emission have been used separately. The mixing parameter is ϵ_D^2=1. Subfigures (a) and (b) show the energy and $p_\perp$ for all positrons using showers with photon emission (red curve), massless dark photon emission (blue), dark photon emission with $m_{A'}$=50MeV and dark photon emission with $m_{A'}$=200MeV. Subfigures (c) and (d) show same curves but now the positron selected has photons or dark photons in the final state.

Figures (6a) and (6b) show identical energy and transverse momenta distributions when contrasting the positrons originated from the QED shower or the U1New shower with massless dark photons with unit mixing parameter. This, as explained, is a consequence of the choice of parameters of the dark photon, which makes the evolution of the two types of showers identical. In the case of the energy, we can see how the probability increases steadily with the energy of the positron until reaching a peak around 20GeV and then decreases promptly, with positrons above 30GeV one time each 10^4 events. This means that most of the positrons end up with energy near 20GeV after the collision with the protons while some events transfer most of the initial positron energy to the proton products.

Similar behavior occurs for the transverse momenta in Figure (6b), which peaks around 2GeV. The fact that it peaks at much lower energy indicates that most of the momenta remain in the initial z-direction. If we look at the energy and transverse momenta distribution of the positrons when allowing for the emission of massive dark photons (green and yellow lines) we can see some differences when comparing them to the positrons from the QED shower. When it comes to the energy, we see how the distribution of the positrons produced from massive dark photons has a larger mean than the energy distribution of the positrons originated from the QED shower. Now, in the normalization, the number of events that we normalize in each histogram is equal to the total number of events. Since the emission of massive dark photons is more costly, it will occur fewer times and there will be fewer positrons with low energy. Emitting a photon takes away part of the energy and because of that, there is less probability of having a positron above 20GeV compared to the cases when there is almost never dark photon emission.

If we assume that is experimentally possible to know when a photon or a dark photon has been emitted we could test some physical assumptions that LDMX aims to test [13]. Now, in the normalization of histograms, the total number of events will be the events that fulfill this extra condition.

Figures (6c) and (6d) show the energy, and transverse momenta of the positrons that come from events with an emitted a photon or a dark photon. We can see how, for the energy, the mean of the distribution of positrons that have emitted massive dark photons (green and yellow lines) seems slightly lower than the mean of the positrons that have emitted photons (red line) or massless dark photons (blue line). This effect in the energy due to massive dark photon emissions is expected since the massive dark photons carry away most of the initial energy of the positron. When it comes to the transverse momenta, higher transverse momentum in the positron indicates that the direction of it is very different from the original direction along the beam axis in the z-direction, which has null transverse momenta. One would expect that the emission of a massive dark photon changes the direction of motion of the positron more abruptly if the dark photon is very massive. This could be translated into high transverse momenta of positrons that have emitted a massive dark photon being more likely to occur than if the positrons had emitted massless particles. The distribution of transverse momenta of positrons that have emitted massless particles (red and blue lines in Figure (6d)) almost never reached values beyond 4 GeV while the positrons that have emitted massive dark photons do. This would confirm the expectations that we had about the transverse momenta behavior. However, the number of events used for those results is just 100.000. Therefore, the events that contain massive dark photons in the final state is so low that the error bars of the associated curves in the ratio plot are very large.

One has to realize that all these simulations were done with a mixing parameter of ϵ_D^2=1.

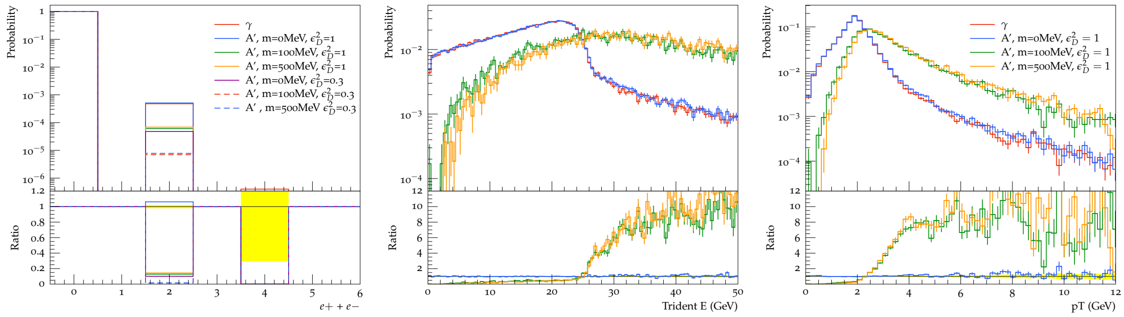
We have already seen a significant decrease in the rate of dark photons in Figure (5a) when massive dark photons have been studied. Lowering down the mixing parameter when using the U1New shower with massive dark photon emissions will decrease the rate of processes connected to dark photons even further.

6.1.2 Validation of trident events with e+ e- and $\mu+$ $\mu-$ pairs

After testing the dark photon emission of the U1New shower of Dire it is necessary to also validate the behavior of the shower when the splitting of dark photons into lepton pairs is allowed. With this aim, we again simulated positron-proton collisions using different configurations of the shower. Then, we selected the events that contained e+ e- e+ or e+ $\mu+$ $\mu-$ tridents in the final state.

To account for the emission of leptonic tridents, the multiplicity of new leptons in the final state has been used. Thus, events that contain one leptonic trident will have two extra leptons in the final state while two leptonic tridents will have four (recall that the initial lepton multiplicity is equal to one which corresponds to the scattered positron). Figure (7) illustrates the multiplicity of new leptons and the energy and transverse momenta of leptonic tridents obtained using the QED shower and the U1New shower separately. For the U1New shower different masses and mixing parameters of the dark photon have been tested. The multiplicity plot has been obtained after performing 10^7 events, while for the energy and transverse momenta plots it was necessary to increase the statistics to $15{\cdot}10^7$ events. Again, all the histograms have been normalized to make the sum of bins equal to one. When it comes to the energy and transverse momenta, each line in the histogram is normalized to the number of tridents obtained with their respective shower configuration. Thus, the probability shows how likely it is to have a trident at a certain energy or transverse momentum given that the dark photon or photon that originates it has a particular mass.

In the three observables, we can see identical distributions when comparing the tridents obtained from the splitting of massless dark photons with unit mixing parameter (blue line) with the tridents originated from the splitting of photons (red line). For the multiplicity case, the lepton multiplicity seems slightly higher for the tridents originated from the emission of massless dark photons. However, an increase in statistics is needed to check if this effect is statistically significant.



(a) Multiplicity (b) Energy (c) Transverse momenta

Figure 7: Comparison between trident events originated from a parton shower after an e+ p collision with only photons or only dark photons. Masses of 0MeV, 100MeV and 500MeV as well as ϵ_D^2=1 and 0.3 have been tried. The beam energies are set to 27.5GeV and 1GeV respectively. Subfigure(a) shows the multiplicity of new leptons that appear after 10^7 events obtained when using a photon shower (red) or a dark photon shower separately. Subfigure(b) shows the energy of the tridents and subfigure(c) shows the transverse momentum of them. In these cases we use $150{\cdot}10^7$ events in the simulation and dark photons with ϵ_D^2=1 and masses of 0MeV(blue), 100MeV(green) and 500MeV(yellow).

Next, we look at how the mass and the mixing parameter affect the multiplicity rate in Figure (7a). First, we can see how the rate of tridents originated from a dark photon with ϵ_D^2=0.3 and $m_{A'}$=0MeV (purple line) decreases by ∼90% compared to the tridents originated from massless dark photons with unit mixing parameter (blue line) or from SM photons (red line). This is because the trident process originated from a dark photon has two vertices that connect the dark photon with a fermion. The coupling, proportional to the rate, differs by ϵ_D^4=0.09 which would give roughly 10% of the tridents with a photon-like origin. This difference can be also observed between the rate of the tridents originated from massive dark photons with unit mixing parameter and the respective ones with ϵ_D^2=0.3 (green and yellow curves vs. discontinuous red and blue curves). One can see how, for both mixing parameters, the rate is reduced significantly compared with the QED background formed by tridents originated from SM photons (red line). However the difference between the rate of tridents originated from a 100MeV dark photon or a 500MeV one is of the order of ∼10%. As an important conclusion, we highlight that the combination of a small mixing parameter and a massive dark photon reduces the rate to one trident event in 10^5 collisions.

Finally, we look at how the mass of the dark photons affects the energy and transverse momenta of the trident in Figure (7b) and (7c). We can see how the mean of the distribution of the energy of the trident becomes larger when a massive dark photon originates it (green and yellow lines). The larger mean of energies of the tridents is explained by the effect of the mass of the dark photons in combination with the settings of the Dire shower that are used in this thesis. As explained before, the initial positron can have an arbitrarily large energy as long as it is lowered down to 27.5GeV by the emission of energetic photons or dark photons. Furthermore, the energies of the particles are measured in the lab frame. The tridents originated from massive dark photon emission have a minimum energy above the mass of the dark photons and a larger energy than the tridents originated from photon emissions. A mass of few hundred MeV can also increase the mean of the distribution of transverse momenta of the tridents originated from massive dark photons compared to the transverse momenta distribution of tridents originated from SM photons.

6.2 Differences Between Parton Shower and Matrix Element corrections

Once the agreement between the U1New and the QED parton showers has been proven, we can recover the original QED shower by allowing quarks to be radiators and recoilers of photons. To produce physical results we also need to simulate collisions with simultaneous U1new and QED showers, meaning that the parton shower can radiate photons or dark photons that then produce the trident signal. Special care needs to be taken when allowing dark photons to decay into leptons. Dire, by default, has a function enabled that makes all the massive dark photons and other resonances decay with unit probability to ensure their detection in an experimental setup. Besides, we have tested the effects of introducing matrix element corrections. To differentiate the BSM effects of the simple kinetic mixing model (KMM), one needs to first calculate separately the MECs from only the Standard Model and include them in the QED parton shower Dire.

Apart from the previous observables, the invariant mass (Eq. 5.23) and the angular separation ΔR (Eq. 5.24) of the pair of leptons from single trident events have been studied. The definition of these two observables changed depending on the type of leptons considered to calculate the observables.

For a muon pair, the three particles forming the trident were different and the invariant mass and angular separation of the pair of muons were calculated. In the case of the trident formed by a positron and an e+ e- pair, it was necessary to distinguish between the two positrons and establish which positron emitted the photon (radiator positron) and which positron was originated from the splitting of it (emitted positron). As an initial hypothesis, the positron with the largest energy or the one with the largest transverse momenta was chosen to be the radiator positron. As we saw, most of the dark photons emitted are collinear with the beam direction and have low energy and transverse momenta. Therefore, the radiator positron is expected to retain most of its energy and momenta. Following this logic, the invariant mass in this type of leptons was calculated using three different methods. The first two used the positron with the lowest energy or transverse momenta. The third method consisted in calculating two invariant masses of the e+e- pair using the two positrons of the trident separately and picking the minimum invariant mass. This method gave the clearest invariant mass peaks. Finally, the last method used, which is also the one presented in this thesis, consisted of calculating the invariant mass of the two leptons with the lowest energies in the final state, independently on the type of lepton. This last method had the largest statistics. For defining ΔR in e+ e- e+ tridents, we calculated the angular separation between the two combinations of e+ e- and chose the positron that has the smallest angular separation.

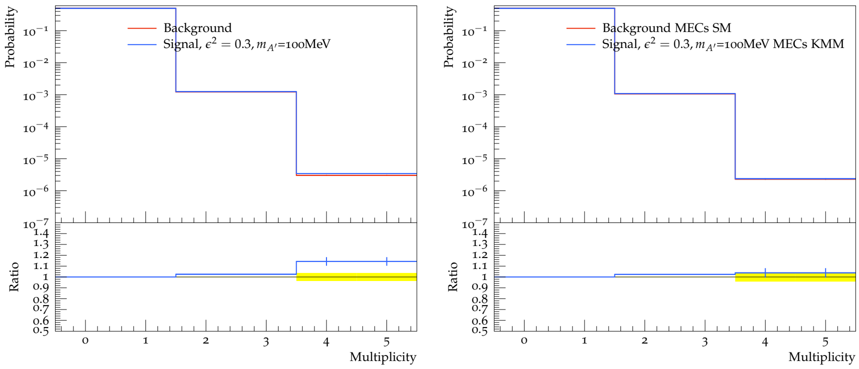
The observables were obtained after simulating $1{,}2{\cdot}10^7$ collisions using different shower configurations and most of the time, selecting the events which contained a single lepton trident in the final state. The methodology for defining the observables was the same as before except by the fact that the multiplicity was built using all the leptons in the final state. All the histograms were normalized to make the sum of bins equal to one. To study how MECs affected the shower evolution the results with MECs are presented separately than the results without MECs.

Figure (8) illustrates the multiplicity of leptons, and the energy and transverse momenta of leptonic tridents originated using the shower with only photon emission (QED shower) or from the shower with photon and dark photon emissions (U1New+QED shower) with a dark photon mass of 100MeV and a mixing parameter squared of ϵ^2=0.3. The effects of the correction of the QED shower using SM MECs and the correction of the QED+U1New shower using the KMM MECs is displayed in the observables shown on the right side of the page.

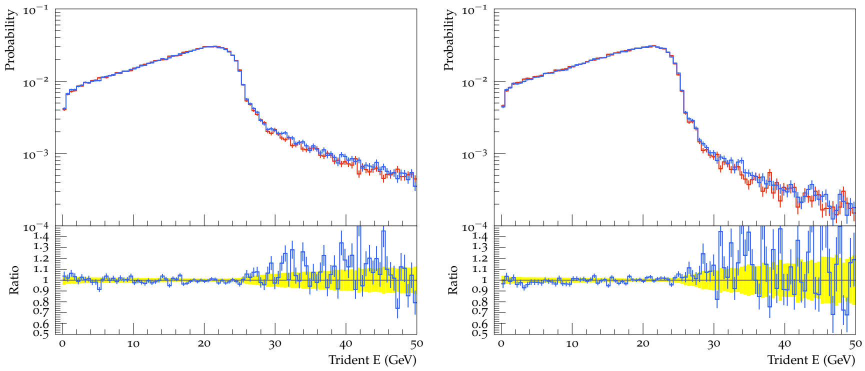
Starting from the multiplicity plots in Figures (8a) and (8b), we can see how, when comparing the multiplicity rate of the leptons originated from the QED shower (red curve) vs. the ones originated from the QED+U1New shower (blue curve), the differences are negligible for all the final states that have less or equal to three positrons in the final state. This can be seen in the ratio plot and it occurs independently on if the MECs are correcting the shower (Figure (8b)) or not (Figure (8b)). Intuitively, one would expect an increase in the rate when the two showers are switched on. The cross-section of the lepton trident process without MECs is proportional to the sum of the product between matrix elements of the previous states and the splitting probability that connects those with the final trident state. This can be seen in equation (4.17) if we eliminate the MEC factor $\mathcal{R}_2$. The U1New+QED shower has twice the number of splitting function that can produce a leptonic trident than the QED shower, and this would increase the trident cross-section. However, the cross-section is also dependent on the Sudakov factor (Eq. 4.19), which, eliminating the MEC factor $\mathcal{R}_2$, has an exponential function whose argument is the sum over the number of splitting functions that can arrive at the final state. A larger sum in the exponential implies a smaller Sudakov factor compared to the QED shower case. This reduces the emission probability of the photons produced with the U1New+QED shower. If both effects balance each other, we would not see significant changes in the rate of tridents compared to the original scenario where only the QED shower is activated. To be able to see major differences one could change the parameters of the dark photon or introduce cuts, like in energy or transverse momentum, to the leptons in the final state. For example, one could introduce a cut to eliminate all the leptons with energy below the invariant mass of the dark photon of consideration.

One could determine the approximate change in the lepton trident rate due to only the U1New shower by looking to Figure (7a) in the previous section. The multiplicity plot showed that the combined effect of having a 500 MeV dark photon and a coupling with a mixing parameter squared of ϵ_D^2=0.3 reduced the multiplicity rate of tridents to one trident in 10^5 events. Assuming a linear relation when photon and dark photon emission occurs together, in 1,2$\cdot$ 10^7 events, only 120 of them would have tridents originated from the splitting of a dark photon of 500 MeV and this would not be enough to produce a visible difference in rate. Even if this result is valid for dark photons of 500 MeV, we expect rates of the same order of magnitude for the current 100 MeV dark photons.

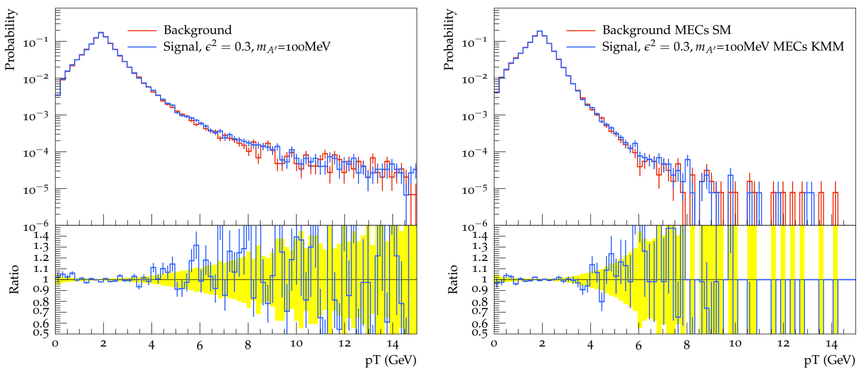
Continuing the discussion, we look at the effect of the MECs in the multiplicity of leptons in the final state (Figure (8b) vs. the multiplicity when the parton showers are not corrected (Figure (8a)). For states with less than six leptons, meaning up to two leptonic tridents in the final state, we can see that the rate of leptons in the final state decreases slightly when the MECs are switched on, independently on the type of MECs correcting the parton shower. To understand this effect we focus on the events with a single lepton trident in the final state. Assuming that, in the part of the cross-section associated to the emission probability, the cancellation of the MEC denominator is done as explained in section (4), the parton shower with MECs would approximate the emission probability of the lepton trident process to the full matrix element squared of the lepton trident, that was calculated using Madgraph. As we saw, the parton shower without MECs would approximate the emission probability to a sum of splitting functions and matrix elements squared of the previous states. This MEC factor would also change the Sudakov factor associated with the no-emission probability. Assuming that the dominant contribution to the change in rate is in the change in the emission probability a decrease in rate would imply that, in general, the full matrix element squared is smaller than the sum over splittings and approximated matrix elements. However, this seems counter-intuitive since the full matrix element includes Feynman diagrams that do not exist in the parton shower as we will see in the next section. Furthermore, interference terms between lepton tridents originated from dark photons and lepton tridents originated from photons, are also present in the full matrix element squared. Still, one would expect these terms to give small contributions. The largest differences are enclosed in the "pure terms" of the full matrix element squared which are associated with SM emissions. They were conceptually defined in equation (2.9). These terms calculate the emission probability of the process accurately independently on the energy and transverse momentum of the particles. The splitting functions, used when the MECs are switched off, give the right results only when the particles emitted are



(a) Multiplicity (b) Multiplicity with MECs

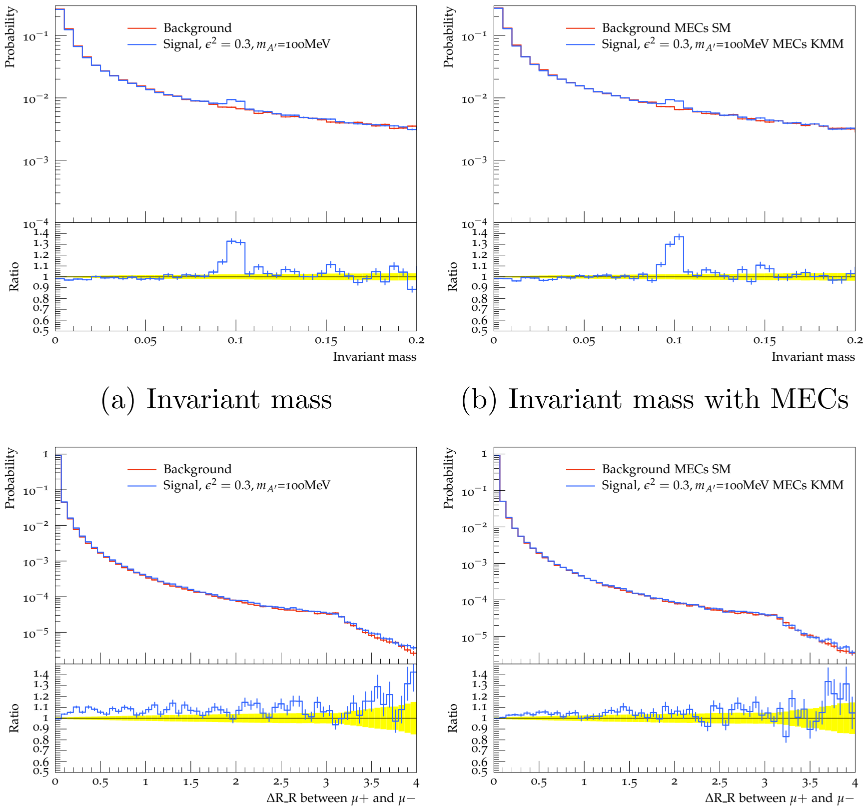


(c) Energy (d) Energy with MECs



(e) $p_\perp$ (f) $p_\perp$ with MECs

Figure 8: Singular probability of the multiplicity, energy and transverse momenta of tridents originated from a p + e+ collision with beams at 1GeV and 27.5GeV respectively and 1,2$\cdot 10^7$ events. In the left side, the observables are originated without MECs and from a parton shower with only photon emission from leptons and quarks allowed (red line) and from a parton shower with both photon and dark photon emissions allowed, with the dark photons emitted only from leptons (blue line). In the right side, the same observables are defined and corrected with SM MECs in the case of the photon emissions (red line) and with KMM MECs in the case of photon and dark photon emissions (blue line). The parameters of the dark photon are $m_{A'}$=0.1GeV and ϵ_D^2=0.3.



(a) Invariant mass (b) Invariant mass with MECs

(c) ΔR (d) ΔR with MECs

Figure 9: Singular probability of the invariant mass of the pair of leptons with minimum energy, and the angular separation ΔR between muon pairs originated from a p + e+ collision with energies of 1GeV and 27.5GeV respectively and 1,2·10^7 events. In the left side, the observables are originated without MECs and from a parton shower with only photon emission from leptons and quarks allowed (red line) and from a parton shower with both photon and dark photon emissions allowed, with the dark photons emitted only from leptons (blue line). In the right side, the same observables are defined and corrected with SM MECs in the case of the photon emissions (red line) and with KMM MECs in the case of photon and dark photon emissions (blue line). The parameters of the dark photon are $m_{A'}$=0.1GeV and ϵ_D^2=0.3.

collinear or soft.

The last thing to comment is how the activation of the two different MECs affects the final multiplicity of tridents, which is shown in Figure (8b). We can see that independently on the number of leptons in the final state, the change in rate when the QED+U1new shower is used with KMM MECs (blue line), does not change significantly compared to the rate of leptons obtained when the QED shower with SM MECs is used. At the most, an increase in the order of 1% is observed when the dark photon emission is included. Assuming that with higher statistics this difference becomes significant, one can think about reasons for these two rates to be different. We will dedicate the next section to the discussion of this type of effect.

When it comes to the energy and transverse momenta of the trident there is a visible difference in the shape of both distributions when the MECs are switched on (Figures (8d) and (8f)) and off (Figures (8c) and (8e)). Before discussing the change in the distributions we need to understand again how the histograms were constructed. They were normalized to make the sum of bins equal to one. For this, they were divided by the total number of events used to fill the histograms. When comparing the multiplicity of leptons in the final state we saw how, whether the MECs were switched on or off, the introduction of dark photon emission did not change the rate of leptons significantly. Independently of the type of emissions allowed, the number of leptonic tridents in the final state is almost the same. Thus, the energy and transverse momenta will be normalized by roughly the same quantity in the background from photon emissions and the signal from photon and dark photon emissions. This means that the rates can be compared between distributions.

When looking at the observables shown in the left side only in Figures (8c) and (8e), we can see how the distribution of energies and transverse momenta is very similar when comparing the tridents from the QED shower (red line) to the ones from the shower with photon and dark photon emission QED+U1New (blue line). As explained when studying the change in multiplicity, this can happen if the emission of dark photons increases the trident rate in a small quantity and this change is made less significant because of the effect of the smaller Sudakov factor. If the splittings coming from the dark photons increased the overall trident rate, one would expect the tridents to have higher energies and momenta due to the effect of the massive particle than the background. Even if the rate of energies above 30 GeV and transverse momenta above 6 GeV increases slightly when the QED+U1New is used, the uncertainties in both signal and background make this effect not statistically significant.

Coming back to the shape of both distributions when the MECs are switched on and off, one can see that the activation of MECs decreases the probability of having a leptonic trident with high energy or transverse momentum. In the region of large transverse momentum, the contribution from the Sudakov factor (equation (4.19)) to the overall rate is highly suppressed. This is because the integration limits of the argument in the exponential function of the Sudakov factor are very small since the trident is produced with large transverse momentum $p_{\perp 2}^2 = t_2^2$, and the emissions decrease in transverse momentum $p_{\perp 2} < p_{\perp 1}$. Small integration limits would give a number close to zero in the argument of the exponential factor that forms the Sudakov factor. This would make the Sudakov factor equal to one and the cross-section in this transverse momentum region would be determined by the part associated with the emission probability. Thus, if the shower corrected with

MECs decreases the probability of having a leptonic trident with high transverse momenta, it means that the emission probability corrected with MECs is smaller than the emission probability when the MECs are switched off. Using the notation defined in section 4 this means that:

$$|\mathcal{M}|_2^2 < P_2^1|\mathcal{M}|_1^1 + P_2^2|\mathcal{M}|_1^2 \tag{6.26}$$

where $|\mathcal{M}|_2^2$ is the full transition probability of the lepton trident, $P_2^1|\mathcal{M}|_1^1$ is the emission probability connected to the emission of a photon and its splitting into a pair of leptons and $P_2^2|\mathcal{M}|_1^2$ is the one connected to dark photon emissions.

Reasons for explaining this inequality were explorer when discussing the change in energy and transverse momentum. It is important to notice that in the example explained in section 4, emission of photons from quarks was not explicitly taken into account. However in this project, the QED shower has the radiation of photons from quarks switched on, and the SM MECs also include this type of radiation. On the other hand, dark photons are only emitted from leptons. The transition probabilities associated with radiation of photons from quarks, from leptons, or radiation of dark photons from leptons is approximated through splitting functions and approximated matrix elements when the parton shower is used, and is calculated exactly using Madgraph when MECs are used. Splitting functions give good results when the particles emitted are soft and collinear to the original beam direction. However, some splitting functions connected to one of the three types of radiations could give better approximations to the full transition probability than other splitting functions.

Finally, the last interesting observables to discuss are the invariant mass of the lepton pair and its angular separation ΔR shown in Figure(9). The sub-figures (a) and (b) show the invariant mass using the two leptons in the final state with the lowest energies. This means that both muons and positron-electron pairs are taken into account. Furthermore, events with two tridents are also considered in this observable. When calculating the cross-section of the lepton trident as a function of the invariant mass of the decay products, a peak around the invariant mass of the particle that decayed is expected. In this case, independently on if the MECs are activated or not, a peak around 100 MeV can be seen in both Figures (9a) and (9b). If the dark photons emitted did not have mass, the maximum increase in the rate of the invariant mass due to single lepton tridents originated from the U1New+QED shower would be roughly 10% of the original QED rate. This is because tridents coming from dark photon emission and subsequent splitting have two couplings to leptons and their emission probability is proportional to $\sim \epsilon_D^4 \approx 0.1$ times the emission probability of tridents from QED. Since tridents from two types of leptons are taken into account, as well as events with two tridents, for calculating the invariant mass of the lepton pair, the increase in the rate of the invariant mass near the rest mass of the dark photon is larger than 10 %. The difference in the shapes of the invariant mass peaks can be explained by the interference terms, as we will see in the next section.

When it comes to ΔR, it is helpful to think in the dark photon decaying into a pair of muons in the center of mass frame and translate that picture into the lab frame. Because of the conservation of 4-momenta and the mass of the dark photon, the muons originated from the decay of a massive dark photon at rest will have a higher magnitude of the 4-momenta in the rest frame than the muons originated from a photon decay. Also, the muons decay back-to-back forming an angle of 180°. Boosting the products of the decay into the lab-frame, which for this explanation we assume it moves in the z-direction from the rest frame, would decrease the angle between the muons. However, the mass of the dark photon makes the boost of the associated z-component of the muon momentum smaller than if the muons are originated from a massless particle. Thus, the muon pairs from dark photons are less collinear to the z-direction than the ones from photons. This would make the angular separation ΔR between muons from dark photons larger than the muons originated from photon decay. The x and y-components of the momentum are not affected by this boost. The effect of using MECs in the shower evolution is shown in the angular separation between muons when MECs are switched off and on (Figures (9c) and (9d) respectively). One can see how the angular separation between the muon pair originated from the QED+U1New shower (blue line) is larger than the angular separation when only the QED shower is used (red line). This difference is caused by the contributions from the dark photon emission and is larger when ΔR is large and MECs are switched off. In the parton shower without the MECs scenario, the changes in Δ_R are justified by an increase in the emission probability proportional to the splitting function of the dark photon splitting into a pair of muons and the transition probability of emitting a dark photon. In the scenario where the parton shower is corrected using MECs, the increase in the emission probability is proportional to the full transition probability of emitting a lepton trident that originated from a dark photon and the interference terms between the lepton trident from dark and SM origins. Therefore, the difference between the use of MECs and the parton shower is due to the approximated emission probability connected to dark photon emission being larger than the full emission probability together with the interference terms. It would also be interesting to see the effect of the massive dark photon in the angular separation between the lepton pair and the radiator positron or see the effect of the mass of the dark photon in the rapidity of the lepton pair.

6.3 Interference effects in parton showers with MECs

This final section is dedicated to the discussion of the results that can be connected to interference effects corresponding to the Feynman diagrams associated with trident production from dark photons and the equivalent ones with photonic origin.

To understand what kind of interference effects are the focus of interest one can look at the possible Feynman diagrams that have tridents when calculating the matrix elements with Madgraph. For example, when generating the KMM+SM matrix elements of the **e+ e- e+** trident process we get 96×4 different diagrams with the condition of having at the most 2 vertices where the dark photon couples to. However, there are many equivalent diagrams that only differ in the type of quark involved in the positron-proton collision. Also, the energy of our process allows us to exclude all the diagrams that involve a Z boson. Ultimately, only six processes from QED are unique and can occur in the system of study as Figure(10) displays. For the processes involving dark photons, we get four unique diagrams if the dark photons are not allowed to interact with quarks.

When the MECs calculate the rate of the trident process they take into account all these diagrams and square them to get the full transition probability of the process. This would include pure terms from squaring identical Feynman diagrams and mixed terms from the product of different ones. In this illustrative example the transition probability expression would have:

1. Six pure terms and 30 mixed ones with QED=8 and NP=0
2. Four pure terms and 132 mixed ones with QED=4 and NP=4
3. 2x4x6 mixed terms with QED=6 and NP=2

Where NP=2 and QED=6 means that the term has two vertices with dark QED coupling (New Physics) and six from QED.

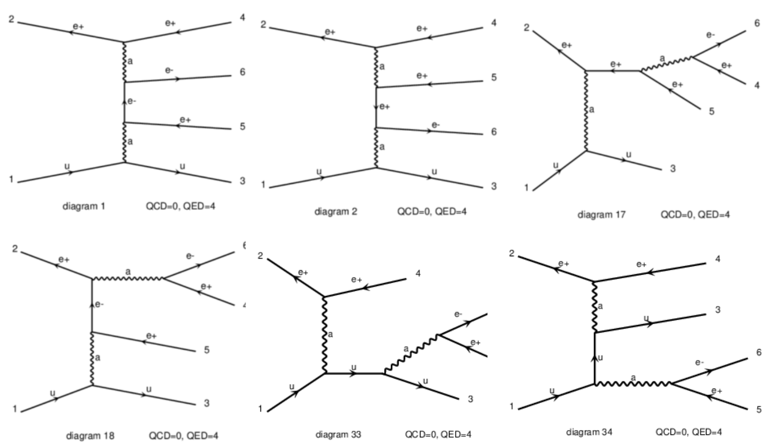


Figure 10: Feynman diagrams resulting from an e+ u interaction that generate e+ e- e+ tridents in different ways.QED=4 and NP=0.

The first type of contributions are purely connected to the Standard Model and the associated matrix elements are calculated separately and incorporated in the SM MECs that we use in the photon shower. The third type of contributions to the transition probability are the interference terms of interest. To study the effects of them we compared the Dire parton shower with only QED allowed and SM MECs switched on against the shower with photon and dark photon emission allowed and KMM MECs switched on. Even if the second type of terms also contribute, they have four vertices where the dark photon interacts with fermions. The contribution of these terms to the total transition probability is proportional to ϵ_D^4. For small mixing parameters, it would be almost negligible. The interference terms of interest have two vertices where the dark photons couple to fermions and are proportional to ϵ_D^2. They are expected to contribute significantly to the total transition probability. Besides the interference terms, MECs include the contributions from Feynman diagrams that can't be generated through a parton shower like the first and second diagrams of Figure (10).

In the previous section, we saw in Figure (8b) that the multiplicity rate of tridents is increased by $\sim$1% when comparing the tridents induced by the photon shower with SM MECs switched on (red line) to the ones from the photon and dark photon shower when the KMM MECs are switched on (blue line). Also, in Figure (9d) we saw how the ΔR distribution mean and rate increased slightly in the presence of the QED+U1New shower corrected with KMM MECs. Assuming that with higher statistics these effects become statistically significant, this would mean that the presence of interference terms in the transition probability is increasing the rate of tridents originated from dark photons. One should also note that the changes achieved by the parton shower without MECs are greater in the case of ΔR and for events with more than three leptons in the final state.

This result can be understood by considering the meaning of the MECs. MECs are introduced as a correction factor to the splitting functions used when calculating the cross-section of a trident process. Thus, they affect the emission and the no-emission probability of the photons and the dark photons and, as a consequence, the rate of induced tridents. The difference of using KMM MECs instead of SM MECs is that the transition probability has contributions from photon emissions, from dark photon emissions and interference terms which contain a mix of them, while the SM MECs have contributions from only photon emissions. However all the terms of the emission probability are accompanied by a Sudakov factor which has the same additional contributions from dark photon emission and interference terms. The exponential decay behavior of the Sudakov factor decreases the magnitude of the BSM effects introduced in the emission probability. Even if one gets an increase in the cross-section calculation when contributions from dark photon emission and interference terms are included, this increase is smaller than the increase one would get if the Sudakov factor, which has also changed, was not present.

To understand how the interference terms can be positive we need to compare the Feynman diagram of a lepton trident that originated from a photon and the one originated from a massive dark photon. In Figure (10) we can look at the Feynman diagram situated in the up-right corner as an example of the lepton trident shape. The difference between the matrix element of a lepton trident originated from a photon and one originated from a dark photon is not only in the couplings but also in the propagator of the mediator particle. In low energy processes, the propagator of a massive gauge boson is [29] :

$$\langle A'^{\mu}(p)A'^{\nu}(-p)\rangle \propto \frac{-ig^{\mu\nu}}{p^2 - m_A^2} \tag{6.27}$$

while the massless photon one is:

$$\langle A^{\mu}(p)A^{\nu}(-p)\rangle \propto \frac{-ig^{\mu\nu}}{p^2} \tag{6.28}$$

If the four-momentum of the outgoing lepton pair is smaller than the rest mass of the dark photon, the propagator would give a negative contribution when calculating the matrix element of the lepton trident originated from a dark photon. Multiplying this matrix element by the corresponding matrix element of the lepton trident originated from a photon would give an interference term with a negative sign. On the other hand, if the four-momentum of the lepton pair is larger than the rest mass of the dark photon, the respective interference term would have a positive contribution. In some regions of four-momentum the interference terms give a positive contribution while in other regions they give a negative one. When calculating the rate of lepton tridents using the parton shower with MECs switched on the positive contributions and the negative contributions of the interference terms are summed. An increase in rate compared to the background corrected with SM MECs would mean that the contribution from the positive interference terms is larger than the negative contributions. The pure terms associated with the emission probability of lepton tridents originated from only dark photons is suppressed by ϵ_D^4 but it is always positive and contributes to the increase in the rate.

In general we see that, for other observables, the QED+U1new shower with the activation of the KMM MECs generates similar distributions than the ones obtained with the QED shower and SM MECs. This can be seen in the energy and transverse momenta distribution (Figures (8d) and (8f)). This could mean that the observables are not sensitive to interference effects or that the interference terms are too small to be disentangled from the background.

The last observable to comment is the invariant mass of the lepton pair formed by the two leptons with the lowest energies. One can see a peak near the mass of the dark photon when the QED+U1New with KMM MECs switched on is used. This is because when the four-momentum of the lepton pair is very close to the rest mass, the propagator of the dark photon that produced it has a tiny number in the denominator. The matrix element of the lepton trident with this dark photon propagator will be very large. The contribution of this matrix element to the MEC numerator is enclosed in the terms associated with lepton tridents originated from pure dark photon emission and splitting, and in the interference terms between lepton tridents from photon and dark photon origin. When the four-momentum of the pair of leptons originated from a dark photon splitting is equal or slightly larger than the rest mass of the dark photon, we get an increase in the probability of producing a lepton trident at those four-momentum configurations. However, in regions of four-momentum slightly smaller than the rest mass of the dark photon, we expect the interference terms to have a negative sign and reduce the probability of having a lepton trident compared with the background case. The shape of the peak in the invariant distribution mass should be similar to a wiggle, with the lepton trident rate changing from smaller to bigger than the QED rate as the invariant mass of the lepton pair increases. To capture this behavior, the invariant mass definition needs to be improved, since the two leptons with the lowest energies do not necessarily need to come from the same mediator. Reducing the bin size could also help to observe this behavior.

We find that the BSM effects included in the KMM MECs do not automatically increase the cross-section and thus the rate of the lepton tridents, but that the increase is affected by the effect of the Sudakov factor change, the positive and the negative interference terms and the coupling of the dark photons to the fermions, which gives interference terms proportional to ϵ^2 and transition probabilities from only dark photon emission and splitting proportional to ϵ^4. In other observables, we can conclude that the presence of interference terms, with the dark photons parameters chosen, do not give significant differences compared with the results from the QED shower with SM MECs. Increase the tests using different dark photon configurations and improving our analyses could make interference effects visible in these observables.

7 Outlook and conclusions

Light Dark Matter theories are an exciting growing field with an extensive room for theoretical, experimental and phenomenological pioneering studies. A generic possibility is that DM particles couples to SM through a dark photon kinetically mixed with a SM photon. For low energies and masses, this dark photon has a coupling to the fermionic matter that differs from the photon coupling by only a proportionality factor called mixing parameter.

In this project we have performed a phenomenological study of the interference effects between leptonic tridents from SM photon origins, and dark photon originated ones using parton showers and matrix element corrections. For this, we have first included the simple and popular model of a kinetically mixing photon in the Standard Model Lagrangian and generated the set of Feynman rules needed to calculate the transition probability of the trident process with the interference terms. Then, using the parton shower Dire together with the Event Generator PYTHIA we have tested the dark photon shower implemented in Dire based on the sum of splitting functions and matrix elements to approximate the full rate of the process. Finally, using the matrix elements calculated with Madgraph, we have searched for the effects of the new matrix elements components in the parton shower evolution. For a mixing parameters of ϵ_D^2=0.3 and massive dark photons of 100MeV we have found lower multiplicity rate, trident energies and transverse momenta when including matrix elements corrections in the shower. This implies a lower emission probability that could be explained by lower values of the transition probabilities calculated with Madgraph than the ones calculated using the sum of splitting kernels and approximated matrix elements. Looking at regions with high transverse momentum in the lepton tridents helped us to confirm this hypothesis since the influence of the Sudakov-factor there was negligible.

When comparing the SM model against the kinetic mixing and SM model together we have found very small differences independently on if the matrix element corrections are included or not. In both cases, this could be a consequence of the effect of the dark photons being suppressed by the small coupling to fermions and the Sudakov factor change. The change in the no-emission probability also affects to the original rate of QED-induced tridents and this could balance the modifications induced by the rate of tridents originated from dark photons. For the invariant mass distribution, we have found a peak near 100 MeV when dark photons were taken into account.

In the case of shower evolution corrected with MECs, we have found that an overall positive contributions of the interference terms could increase the multiplicity rate of leptons in the final state as well as the angular separation between lepton pairs when comparing with the background obtained from the QED shower corrected with SM MECs.

Narrowing further the problem could make the interference effects become visible. Trying different values of the mixing parameter and increasing the number of simulations could increase the final number of dark photons that split into leptonic pairs. To improve our analysis, one could try to only take into account leptons originated from dark photons or decrease the type of emissions of consideration. For example, for studying these observables using leptons originated from dark photons, and from FSR radiation one could exclude leptons with small rapidity and with small angular separation between them and any quark. In FSR radiation, one expects the lepton pairs originated from dark photons being less collinear to the beam direction than if they were originated from dark photons emitted from a positron before its interaction with the proton. If the angular separation between quarks and leptons is very small, one can assume that they have been originated from a photon emission and subsequent splitting, since dark photon coupling to quarks was not allowed.

After this interference effects are detected, extensions of this project would take into account the treatment of beam remmants, hadronization and other advanced features in Event Generators through the analysis of the events using Rivet.

The MECs used by Dire contain matrix elements calculated with Madgraph and this allows to include interference effects in the shower evolution not only from the KMM model, but also from more complicated models which include Z-like interactions of the dark photon, or other type of portal interactions. This project is just the beginning of a promising algorithm that links theoretical models with particle collision simulations.

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A Appendix A

A.1 Run file of Dire

```
# Number of events to generate.
Main:numberOfEvents       = 100000

#Number of seed
Random:setSeed = on
Random:seed = 13712

# No distribution for incoming leptons. Added 15-10
PDF:lepton                = off

# Set reference value of alphaS(Mz).
```

```
SpaceShower:alphaSvalue   = 0.1201
TimeShower:alphaSvalue    = 0.1201

# Set hard process.
WeakBosonExchange:ff2ff(t:gmZ) = on
Beams:idA                 = -11
Beams:idB                 = 2212
Beams:frameType           = 2
Beams:eA                  = 27.5
Beams:eB                  = 1
PhaseSpace:Q2min          = 4
PhaseSpace:mHatMin        = 1

Dire:MG5card = /home/william/m19_patricia/physics/MG5_aMC_v2_6_0-hack/
PY8K_MEs_photonlike_e+_p_to_e+_j_e+_e-/Processes_photonlike_UFO/
param_card_photonlike_UFO_500MeV_eps01.dat
SLHA:file = /home/william/m19_patricia/physics/MG5_aMC_v2_6_0-hack/
PY8K_MEs_photonlike_e+_p_to_e+_j_e+_e-/Processes_photonlike_UFO/
param_card_photonlike_UFO_500MeV_eps01.dat

# Matrix element correction settings
Dire:doMECs = on
Merging:Process = e+p>e+j

# Do not use "power shower".
SpaceShower:pTmaxMatch    = 1
TimeShower:pTmaxMatch     = 1

# Set QED cut-offs and switch on QED shower.
TimeShower:pTminChgL = 0.001 #without this QED gives me nan
SpaceShower:pTminChgL = 0.001
TimeShower:U1newShowerByL = on
SpaceShower:U1newShowerByL = on
TimeShower:U1newShowerByQ = off
SpaceShower:U1newShowerByQ = off

TimeShower:QEDshowerByQ = on
TimeShower:QEDshowerByL = on
SpaceShower:QEDshowerByQ = on
SpaceShower:QEDshowerByL = on

11:m0 = 0.
13:m0 = 0.
```

```
# Use masses of PDF sets also for shower evolution.
# Note: Only correctly handled when using external PDF sets from LHAPDF
ShowerPDF:usePDFalphas    = off
ShowerPDF:useSummedPDF    = off
ShowerPDF:usePDFmasses    = off
DireSpace:forceMassiveMap = on

ProcessLevel:resonanceDecays = on
#! Switch off event generation steps
PartonLevel:MPI              = off
HadronLevel:all              = off #Turns off hadronization
BeamRemnants:primordialKT    = off
PartonLevel:Remnants         = off
Check:event                  = off
```