A WHOLE

MAKING FRACTIONS TANGIBLE

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LUND UNIVERSITY

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MAKING FRACTIONS TANGIBLE

Degree Project for Master of Fine Arts in Industrial Design,
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ABSTRACT

How can the tools used in elementary school facilitate mathematical learning and understanding? This project has been a study in how maths is taught and how to create a broader understanding of the subject.

This report covers the process of, and is part of, my Master thesis at the Industrial Design School in Lund, Sweden. It includes the research and the development of "A Whole", a mathematical tool to be used by elementary pupils when learning fractions.

Mathematics can be defined as an abstract and general science for problem-solving and method development. Mathematics is abstract as it's disengaged from the original problem. This is a precondition for it to be used generally, i.e applicable in several different situations. Since mathematics in itself is abstract but in great relation with the physical world it can be hard for pupils to see how they correlate.

This project originates from my time working as a substitute teacher where the greatest reward was when a pupil managed to process the information and make it his/her own. The purpose of "A Whole" is to facilitate this process and bridge an abstract subject to the physical world.

To develop a tool that benefits and is appreciated by both pupils and teachers a close collaboration with specialists in the chosen field was established as well as with teachers and pupils.

The final result, "A Whole", is a set of pieces that focuses on creating an overall understanding of fractions. It clarifies how fractions relate to mathematics and to the concrete, non-abstract world.

"A Whole" can be used individually and in pairs to develop an understanding of what fractions are, both as parts of a whole and as parts of quantities. With the different parts the pupils can explore how fractions relate to decimal numbers and percentage as well as quantities. Through the game, several pupils can collaborate or compete to fulfil challenges.

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INTRODUCTION

RESEARCH

SYNTHESIS

IDEATION

RESULT

INTRODUCTION

WHY THIS PROJECT?

MATHEMATICS AND PEDAGOGY

Mathematics is one of three core subjects in the Swedish educational system. As a discipline it is highly regarded and due to international comparisons it has been elevated as a subject which schools should focus more on. This has made it a political topic and many revisions of the curriculum have been made.

Within the discussion about what the maths curriculum should focus on there's a innate pedagogical question. Pedagogy is an important and natural part of everything that relates to teaching. It's the idea of how something should be taught and therefore also a part of any educational discussion.

MOTIVATION

On different occasions and during periods, I have worked as a substitute teacher in a couple of schools. I have taught most subjects and worked from 1st grade to 9th grade. The semester before starting my Masters I worked for some time in a 1st and 2nd grade class and later on got in charge of a 6th grade class. I have found this work to be very joyful and rewarding but also very hard. One of the most developing parts is to see great teachers in action giving the pupils that "aha" experience.

During my time as a substitute teacher I have found that various teachers' pedagogical knowledge isn't properly applied during the lessons since many teachers are alone in the class. It's hard to make time to assist every pupil that struggles and to make the process fit that specific pupil.

During lessons there are various tools that the pupil can use to make it easier to understand the subject. However, I find that there is a gap or disconnection between teachers' teaching methods and the tools used by the pupils. The most commonly used tools in the 1st and 2nd grade during maths are stones or matches for counting. There is a disconnection between seeing that if you remove two out of five stones there are three left and understanding the expression 5-2=3, connecting mathematics to reality.

GOALS

The goal for this thesis is to work with the target group in close collaboration with specialists and teachers in the chosen field of mathematics who will use the tool in the classroom. The aim is to develop a method or tool that benefits and is appreciated by both teachers and pupils. The tool should be a natural part of the lesson without stigmatizing.

DEMARCATIONS

The field of education is vast with many topics of study, each with its own problems. It ranges from the educational system with its overall organization to the different approaches which are required in the classroom.

To narrow down the focus of the thesis the field of mathematics was chosen. Mathematics is an abstract subject that can be hard to grasp but it is still a concrete and objective field of studies where there is a right or wrong answer.

Exploring mathematics and its curriculum further demarcations were made and the field of fractions was chosen as it often can be difficult to understand for beginners.

Focus is put on the lower to middle classes in elementary school. This since it is here the interest and fundamental understanding of the subject are developed.

METHODS

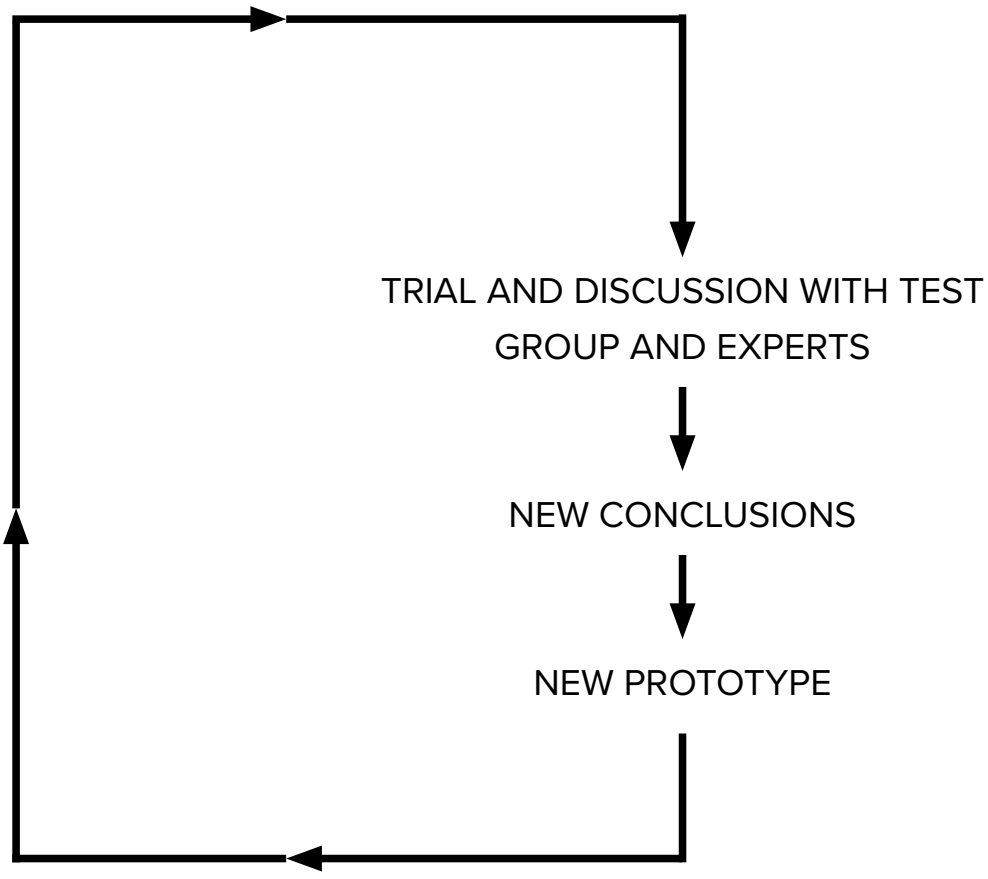
Secondary research was fundamental to create an understanding of the subject. What is mathematics and how is it taught? What is pedagogy and how do we learn and teach mathematics? This was the basis to make conclusions of what teaching and learning entails.

Interviews were conducted with several experts in the field. Their area of expertise varied and they were involved at different levels of the educational system. They were contacted several times during the process for evaluation and feedback.

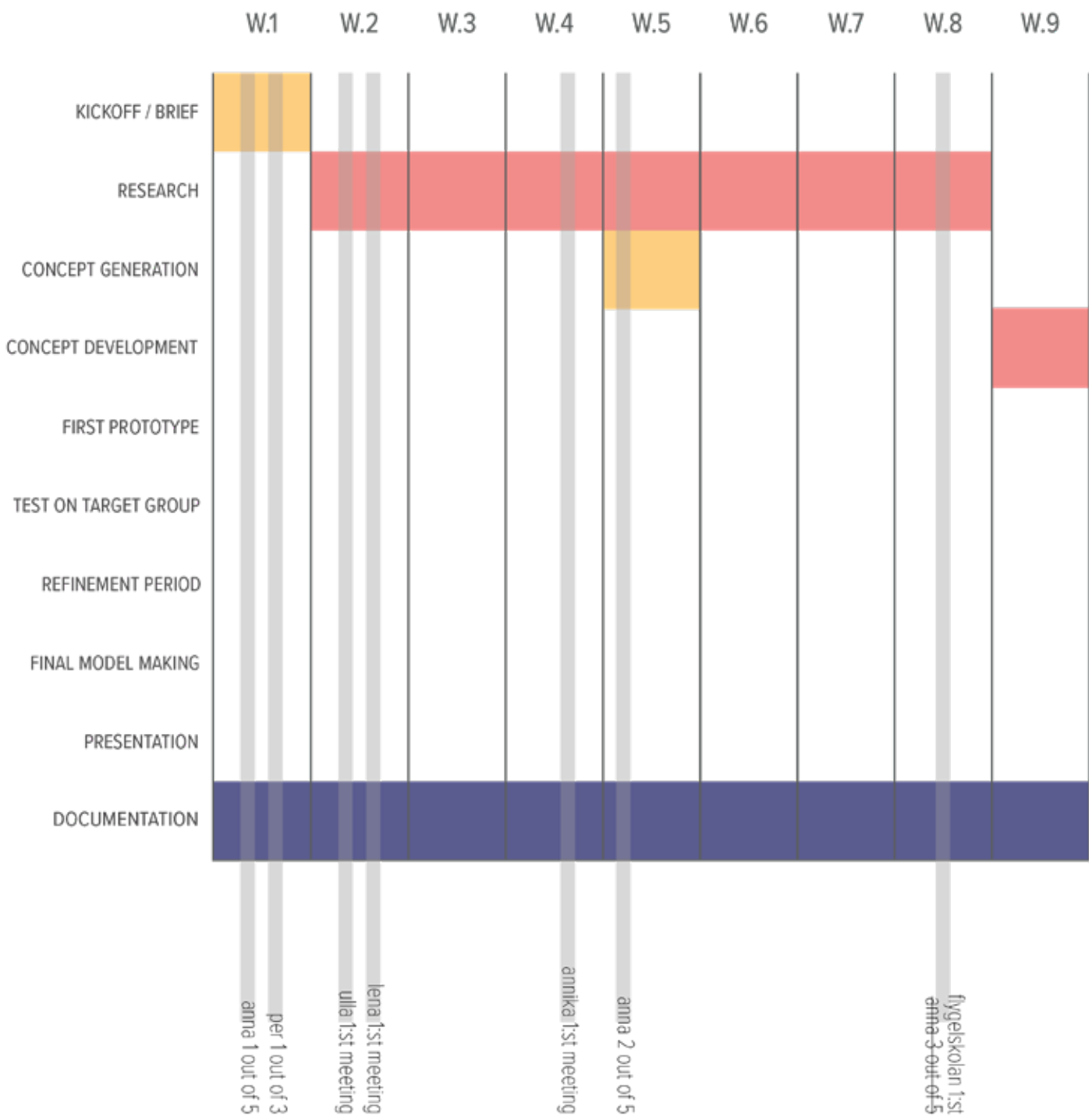
Observational studies at Flygelskolan. Early in the process I established contact with an elementary school, ages six to eleven. Here I conducted observational studies and discussed the topic with teachers as well as with pupils.

A test group was established in the targeted age group, this to make sure that the tool would serve its purpose.

During the development phase the process was looped, from building prototypes of the product to explore it with the pupils as well as with the experts. From the feedback new conclusions were drawn and a new prototype was made.


TRIAL AND DISCUSSION WITH TEST GROUP AND EXPERTS
NEW CONCLUSIONS
NEW PROTOTYPE

TIMETABLE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KICKOFF / BRIEF
RESEARCH
CONCEPT GENERATION
CONCEPT DEVELOPMENT
FIRST PROTOTYPE
TEST ON TARGET GROUP
REFINEMENT PERIOD
FINAL MODEL MAKING
PRESENTATION
DOCUMENTATION
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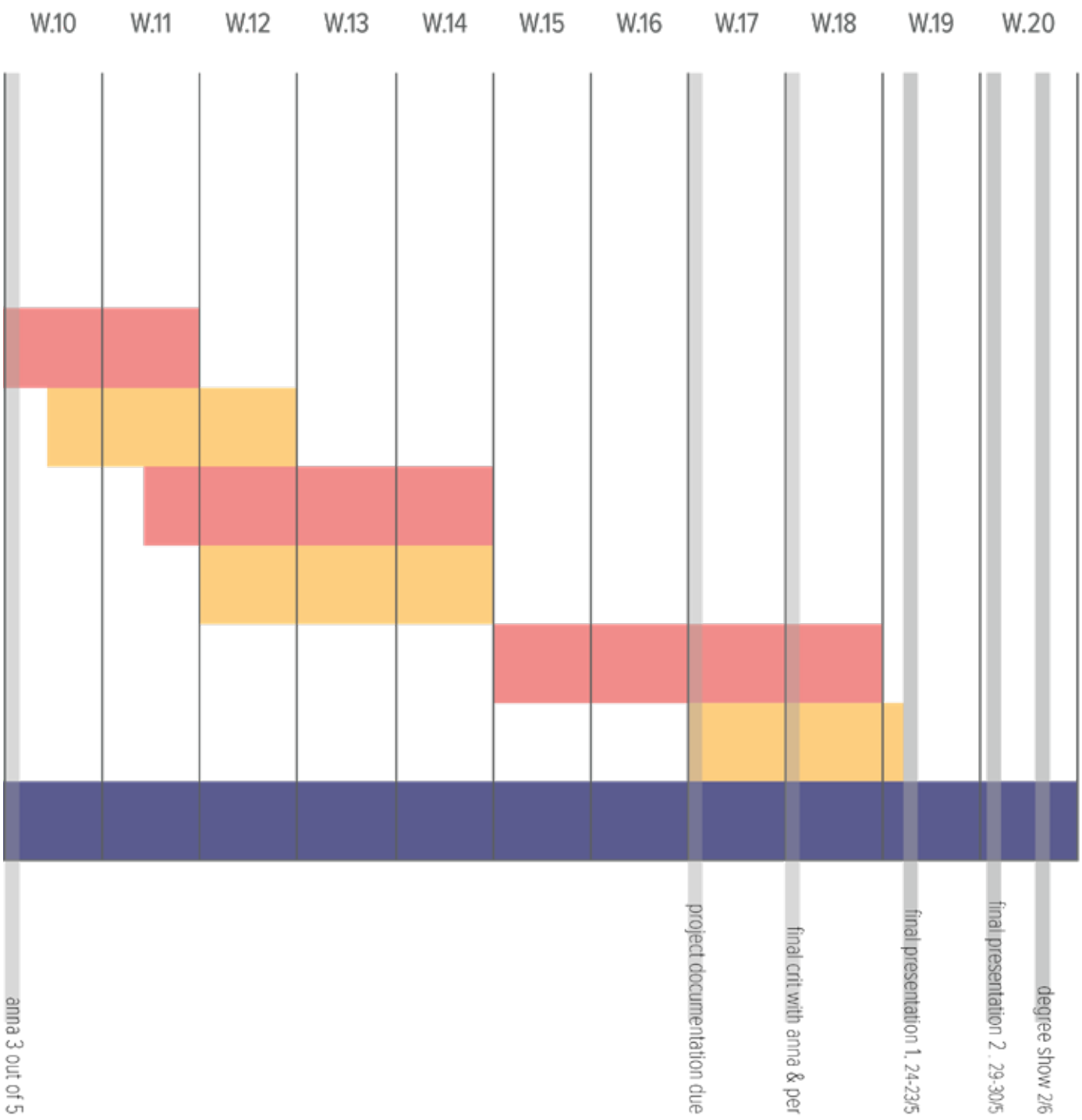
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final presentation 2 . 29-30/5
degree show 2/6

BRIEF 1.0

Develop a tool that makes the complexity of mathematics easier to understand. The tool should be used to bridge where and when mathematics gets more abstract.

RESEARCH

HOW WE TEACH MATHEMATICS

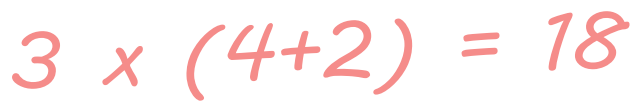
WHAT IS MATHEMATICS?

To get a clearer understanding of the field of mathematics it's essential to understand the building blocks and the essence that makes it a scientific field.

Mathematics can be defined as abstract and general science for problem solving and method development. Mathematics is abstract as it's disengaged from the original problem. This is a precondition for it to be used generally, i.e applicable in several different situations. The field of mathematics is centred around studies and the construction of patterns[1]. With these studies conjectures can be made and when proven and formulated into general methods they can be used to solve problems and create demarcations for these problems.[2]

In general, the field of mathematics can be divided in four categories, quantity, structure, space and change. These four fields are commonly known as arithmetic, algebra, geometry and mathematical analysis. Within these fields there are also subdivisions.[3]

Arithmetic (from the Greek "arithmos", number) is the oldest part of mathematics. It's the study of numbers and includes the "rules of arithmetic" i.e. the four traditional operations, addition, subtraction, division and multiplication.[4] Included in the field of arithmetic is number theory which is a division in modern mathematics.

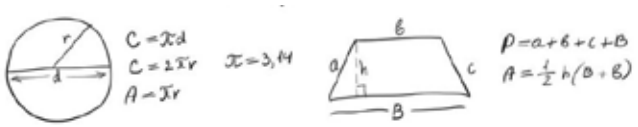


1 Nationalencyklopedin, Matematik
2 Wikipedia, Mathematics
3 Ibid
4 Wikipedia, Arithmetic

$$5x^2 - 6x = 13$$

Algebra (from the Arabic "al-jabr", reunion of broken parts) is the study of mathematical symbols and how to manipulate these symbols. In algebra letters are used to stand for numbers that are either unknown or can take different values. For example, X+5=7, X is unknown and trough the law of inversion it can be proven that X=2.

Geometry (from the Greek "geo-", earth and "-metron", measurement) studies questions regarding size, shape, relative positions of figures and properties of space. Several different early cultures independently developed geometry for a practical way to handle lengths, areas and volumes. Some of the fundamental concepts for geometry are the concepts of points, lines, planes, surfaces, angles and curves.[5]



Mathematical analysis is focused on limits and related theories, such as differentiation, integration, measure, infinite series and analytic functions. This is often studied in relation to real and complex function. Mathematical analysis derived from calculus [6] (from the Latin "calculus", small pebble used for counting)[7] which is the study of how things change. It makes it possible to formulate a model in which there are change and a way to understand the prediction of said model.[8]

5 Wikipedia, Geometry
6 Wikipedia, Mathematical_analysis
7 Wikipedia, Calculus
8 Kleitman D

HISTORICAL MATHEMATICAL TOOLS

According to the book "Mathematical Thought from Ancient to Modern Times", mathematics as an organized science did not exist until the classical Greek period from 600 to 300 B.C.[1] However, early humans performed simple calculations using tools as their fingers, notches on sticks and pebbles[2] and civilizations prior to the classical Greek period formed the rudiments of mathematics.

Throughout history humans have developed methods and tools to make calculations easier to handle and to understand.

PROTRACTOR - ANCIENT

The protractor is an ancient device used to measure and construct angles. It's a semicircular disk marked with degrees from 0º to 180º.[3]

COUNTING BOARD - ANCIENT

A counting board is a table or tablet with marked parallel lines. Ancient cultures such as the Greeks, the Babylonians and the Romans used these for counting. The lines represent numbers and pebbles or other counters were placed on the lines representing multiples of that number. It's the same place value system used today where the value of the counter is dependent on the line on which it is placed.[4]

ABACUS - 1200 B.C

The abacus was invented in China around 1200 B.C. and was one of the first tools for counting. It was used in many ancient civilizations as for example Persia and Egypt.[5]

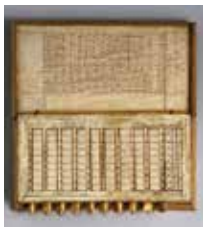
1 ThoughtCo
2 Encyclopedia.com
3 ThoughtCo
4 Encyclopedia.com
5 ThoughtCo

ACCOUNTING - 1458

When medieval Europe transitioned to monetary economy merchants depended on bookkeeping to oversee multiple transactions.

In 1458 Benedetto Cotrugli invented the double-entry accounting system, which revolutionized accounting.[6]

NAPIER'S BONES - 1617



Napier's bones is a manually operated calculation device invented by the Scottish mathematician John Napier (1550 -1617) to simplify calculation of products and quotients of numbers.

With the embedded multiplication tables, multiplications can be reduced to addition and division to subtraction. More complex use of the device can extract square roots.[7]

SLIDE RULERS - 1620

In the year 1620 the rectangular slide ruler was invented by mathematician William Oughtred (1574 - 1660).[8] It's a mechanical analogue computer and generally used for multiplication and division but can also handle exponents, roots, logarithms, and trigonometry.[9]

GRAPHS - 1786

William Playfair (1759 - 1823) is generally known as the inventor of most graphical forms to display data, including line plots, bar chart and pie chart.[10]

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6 ThoughtCo
7 Wikipedia,Napier's bones
8 ThoughtCo
9 Wikipedia, Slide rule
10 ThoughtCo

PEDAGOGY AND DIDACTICS

To better understand how an education is constructed and what the fundamental ideas of learning are we have to understand the meaning of pedagogy and didactics.

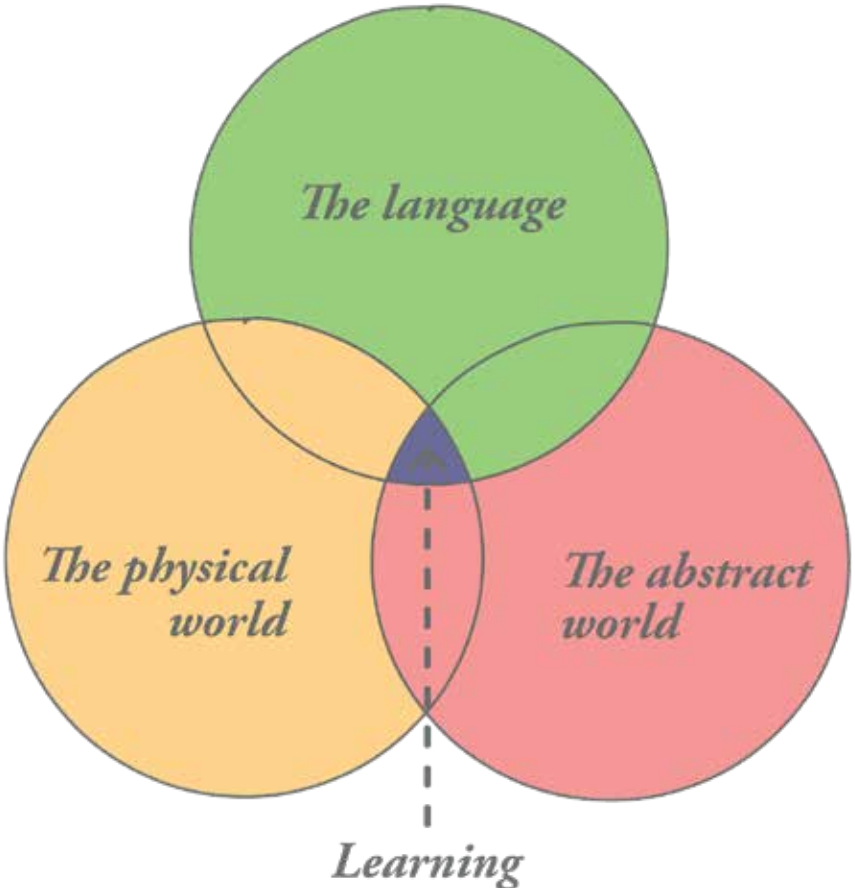
In the world of learning you often come upon the terms pedagogy and didactics. These two terms are greatly intertwined and easily can be mixed up. However, they have different significations of how and from what level we are to approach and evaluate learning and the educational system. Easily explained pedagogy is the theory of education and learning while didactics is how we approach it[1]. Another discrepancy is that pedagogy is based on behavioural science while didactics is based on educational science.[2]

Pedagogic comes from the Greek paidagōgikē[3], which means leading child.[4] This was the term for the older slaves in ancient Athens who were in charge of raising and educating the children of rich families.[5] Pedagogy can be seen as the science of teaching. It's a discipline that deals with both theory and practice of education, including intellectual, moral and personal growth. Pedagogy includes the overall aspects of an educational system as well as the direct communication between two persons where one is learning from the other or there's a mutual exchange.[6]

Didactics can be seen as a sub-genre of pedagogy.[7] Didactics is focused on the analysis and understanding of different factors that influence teaching and learning.[8] Questions that usually are involved when studying didactics are:

- Who will learn?
- What should the individual learn?
- With whom will the individual learn?
- How will the individual learn?
- Through what means will the individual learn?
- For what should the individual learn?[9]

1 Nationalencyklopedin, Pedagogik
2 Wikipedia, Pedagogik
3 Nationalencyklopedin, pedagogik
4 Wikipedia, Pedagogik
5 Wikipedia, Pedagogik
6 Nationalencyklopedin, Pedagogik
7 Högskolan i Gävle. *Didaktik - kunskapsområde och ämne.*
8 Wikipedia, Didaktik
9 Nationalencyklopedin, Didaktik


The language
The physical world
The abstract world
Learning

MATHS IN SWEDISH SCHOOL

To understand what the pupils are supposed to learn it's important to understand the Swedish educational system and how school approaches teaching of mathematics.

In the "Curriculum for the compulsory school, preschool class and school-age educare" you can read:

"Teachers should
- take into account each individual's needs, circumstances, experiences and thinking,
- reinforce the pupils' desire to learn as well as the pupil's confidence in their own ability,
- provide scope for pupils to exercise their ability to create and use different means of expression"[1]

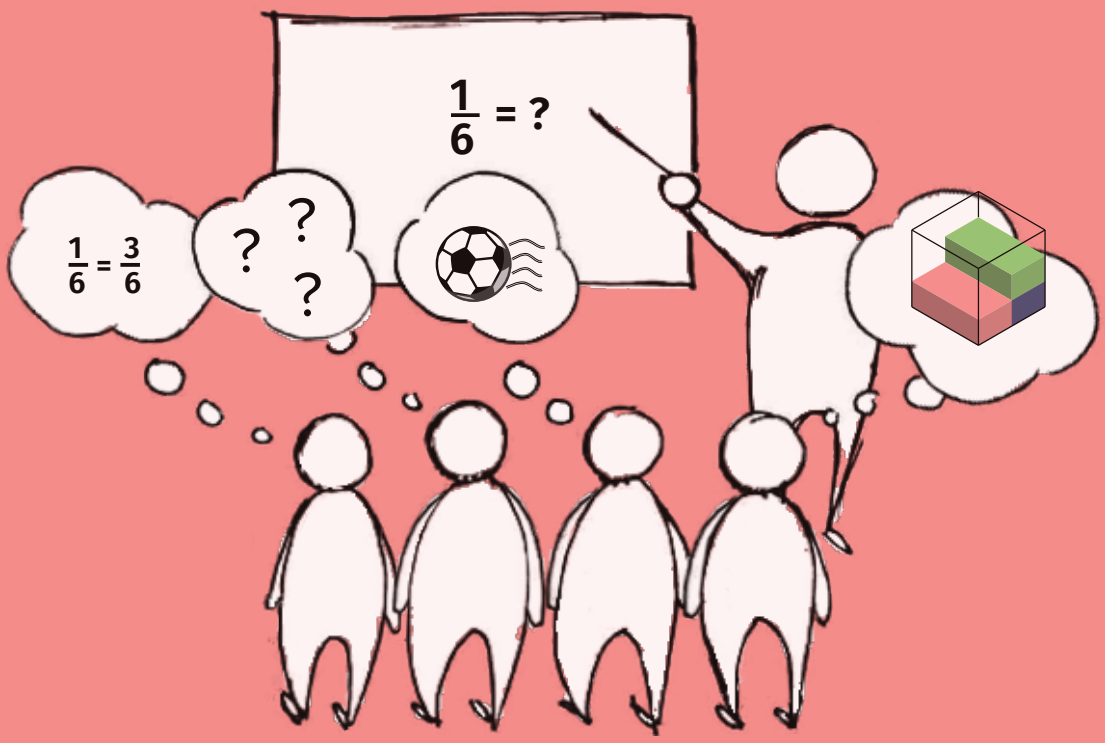
The individual pupils and how they learn differently has been taken into account. Focus is put on the importance of allowing different expressions to build learning and building the pupils confidence in the subject as well as other areas.

Some of the aims in teaching mathematics are expressed as follows,

"Teaching should help pupils to develop their interest in mathematics and confidence in their own ability to use it in different contexts. It should also provide pupils with the opportunity to experience aesthetic values in mathematical patterns, forms and relationships."[2]

The importance of building mathematical confidence is pointed out as well as understanding maths in relation to other subjects and to the real world.

1 Skolverket, page 12-13
2 Ibid, page 55


1/6 = ?
1/6 = 3/6
? ? ?

NUMBER SENSE

Number sense is recognized as one of the most fundamental goals in teaching mathematics. Number sense is a person's general understanding of numbers and operations.[1] This is the foundation for creating effective strategies when using numbers and operations. A good number sense shows an expectation of what will happen when performing an operation.[2]

A good number sense supports the pupils' mathematical competence and helps them to use their knowledge and understanding when solving problems that they meet in their surrounding.[3] Traditional assessments in school give little insight in a pupil's number sense. Most tests focus on calculation and problem solving but put little emphasis on other important aspects of number sense as mental calculation, estimation and number handling. A number of international studies have shown that pupils in spite of good results on pen and paper tests show weaknesses in their number sense. When pupils perform written calculations they do the algorithms as a mechanical procedure without a meaningful handling of numbers and operations.[4]

The importance of strengthening pupils' ability to reason, represent, communicate and discover correlations can't be understated.[5] They are all important parts in good mathematics teaching and develop a strong number sense. In order to practise these skills focus should be on letting the pupils develop their strategies by exploring, discussing and motivating their thinking and solutions.[6] In the classroom focus should be put on open discussions around problem solving. The discussion is more accessible when the pupils are offered different mathematical tools and solve the problem together.[7]

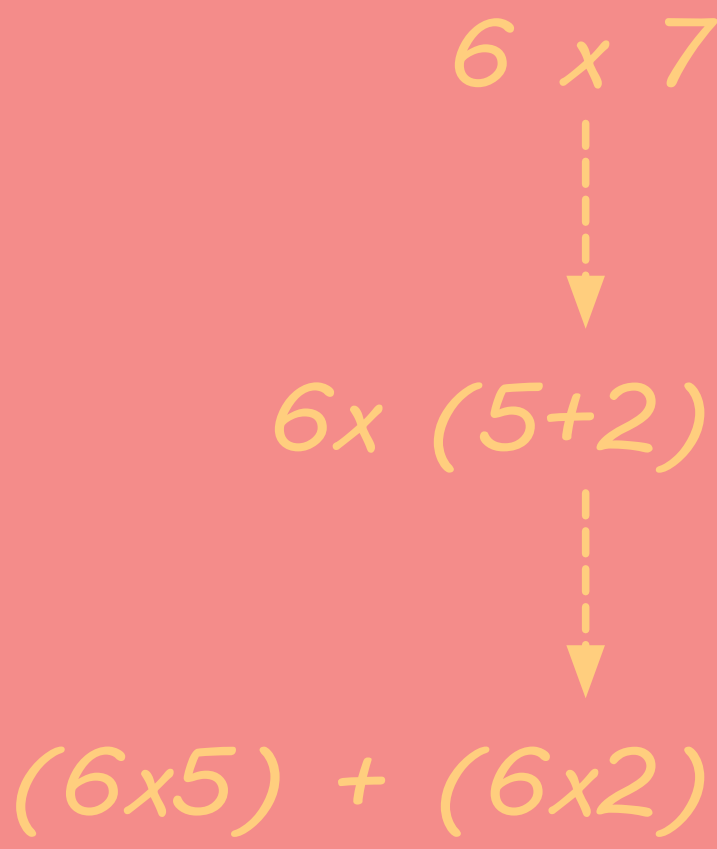
1 Reys B, et al. (1995), page 23
2 Ibid, page 23
3 Ibid, page 23
4 Ibid, page 24
5 Heirdsfield A (2014), page 3
6 Ibid, page 3
7 Eriksson H (2006)

DECOMPOSING AND COMPOSING

Decomposing and composing numbers are important parts of number sense and in understanding mathematical operations. Decomposing and composing quantities or numbers are related concepts. When composing you put together numbers or quantities while decomposing you break a quantity or number into parts. This makes them more manageable. It's important to understand that unlike partitioning, which cuts the number into equal parts, decomposing allows you to divide the number into different values. For example: 10 partitioned with 2 will give you 5 and 5 while if you decompose 10 it can give you 5, 4 and 1.[1]

It's important to develop this thinking early on and it's commonly practised during one's schooling. It's something that most adults do without thinking much about it. Decomposing and composing numbers make mathematics easier and friendlier and can be used in addition, subtraction, division, multiplication and even when working with fractions.[2]

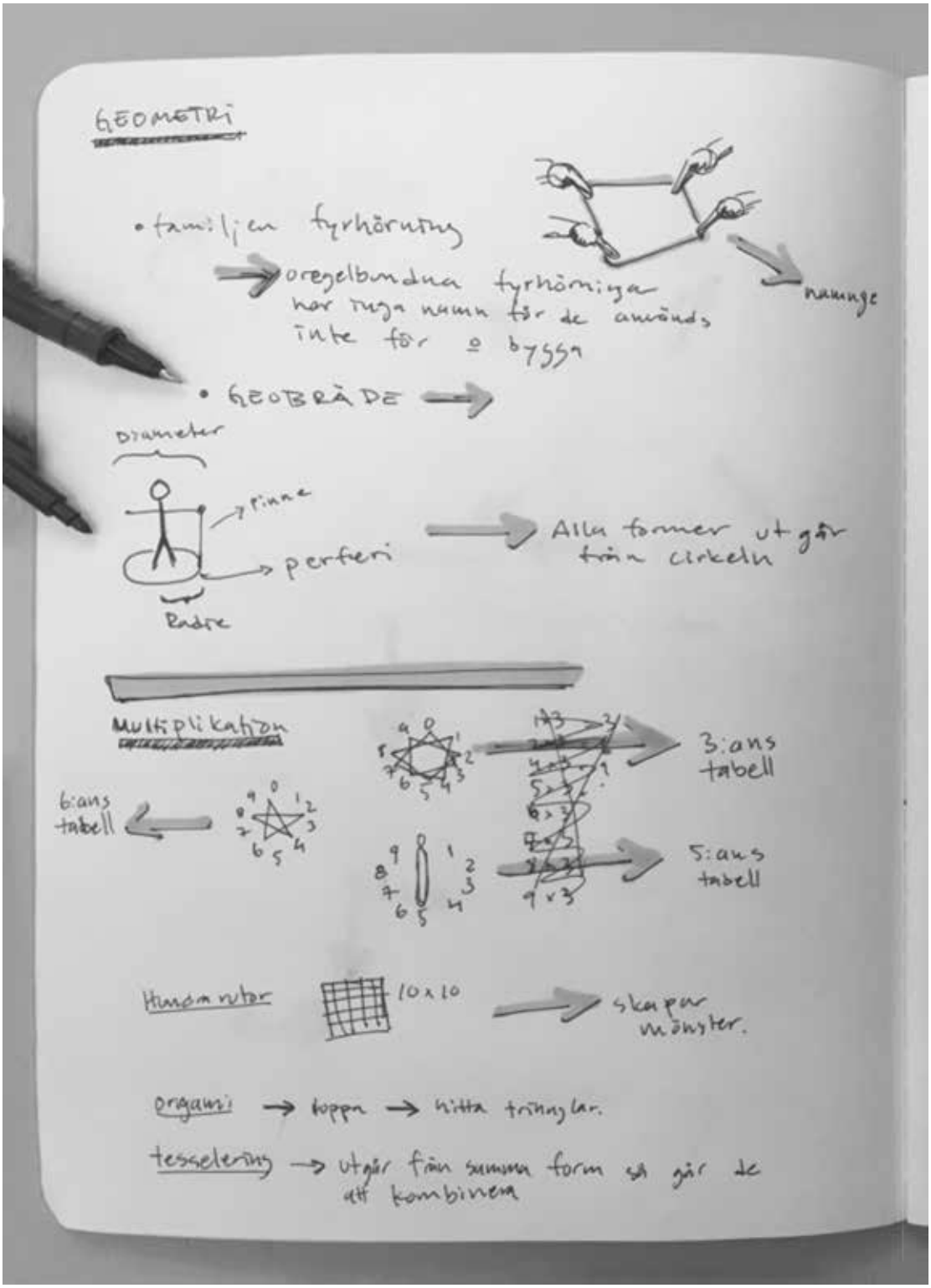
1 Decomposing Numbers.
2 Build Math Minds.


6 x 7
6x (5+2)
(6x5) + (6x2)

FIELD STUDY AT MALMÖ UNIVERSITY

Lena Andersson invited me to attend one of her lectures to future recreational instructors. This lecture focused on teaching the students different ways to stimulate mathematical thinking through play and games.

It was discussed how the instructors can help the pupils to understand mathematics in a more physical way. They used games and some exercises that create mathematical patterns and learned how to talk about what happens through out the games. This deepens the pupils' understanding and makes mathematics easier to relate to.


GEOMETRI
• familjen fyrhörning
oregelbundna fyrhörningar
har inga namn för de används
inte för ≥ 5-sida
namnge
• GEOBRÄDE
Diameter
Radie
perferi
Alla former utgår
från cirkeln
Multiplikation
3:ans
tabell
6:ans
tabell
5:ans
tabell
Hundrarutor
10x10
skapar
mönster.
origami → toppa → hitta trianglar.
tesselering → utgår från samma form så går de
att kombinera

INTERVIEWS

The field of mathematics is vast and to better understand how teachers approach mathematical education interviews were arranged with three teachers who are working at different levels in the educational system.

The interviews focused on educational approach in the classroom, tactics that are used and their views on teaching. We also discussed tools and exercises, what worked for them and what didn't.

Through these interviews key problems and areas were found. This led to a better understanding of how the educational system is built. By focusing on three interviewees who are working in the same field but at different levels the range of the problems varied. Because of their different experience their approach varied, however they still had a similar understanding of which the difficult areas are.

INTERVIEW

ULLA ÖBERG

Ulla Öberg has about 20 years of experience as an elementary school teacher and about 20 years of experience from educating teachers. Ulla is now retired but is still working with in-job training of maths teachers.

She's a maths enthusiast and as she is highly regarded in her field she continues to appear at seminars to hold presentations on her view on pedagogy and mathematics

KEY INSIGHTS

- The importance of the discussion, everything that is worth saying in the class room should come from the pupils.
- Catch the pupils interest through a story and relate it to the real world. This makes them relate to what the mathematics they are working with truly are about.
- Use open ended questions instead of direct questions. In this way you find out how far the pupils' understanding reaches.
- Use the mathematical language and its variations, for example, subtract is to remove.
- It is shown that to facilitate further understanding of mathematics it's important that the teaching is focused on the process instead of the result. In this way pupils develop confidence in their mathematical ability.

INTERVIEW

Lena Andersson

Lena Andersson works at the Teacher Education Program at Malmö University where she teaches mathematics and its pedagogy to future educators at different levels of the educational system.

KEY INSIGHTS

- The rules of arithmetic and numbers are isolated to the subject, which is a problem.
- What do I do when the page is done? We are too bound to the use of mathematics books.
- This leads to a situation where the literature dictates how we teach and it doesn't cover the whole picture.
- In Sweden we aren't good at developing our number sense.
- We need to work with mathematics more as a whole.
- In classes F-3 there's a lot of material but there isn't so much for higher grades.
- How can we work with investigating learning?
- Many teachers develop their own tools since they don't find anything that work for them on the market.

INTERVIEW

Annika Ericsson

Annika Ericsson works at Flygelskolan in first grade and as a remedial teacher in grades F-5.

KEY INSIGHTS

- To break away from the stigma of working with tools. Tools should be standard.
- It's important to work with tools and books simultaneously. In this way you build a physical understanding of mathematics.

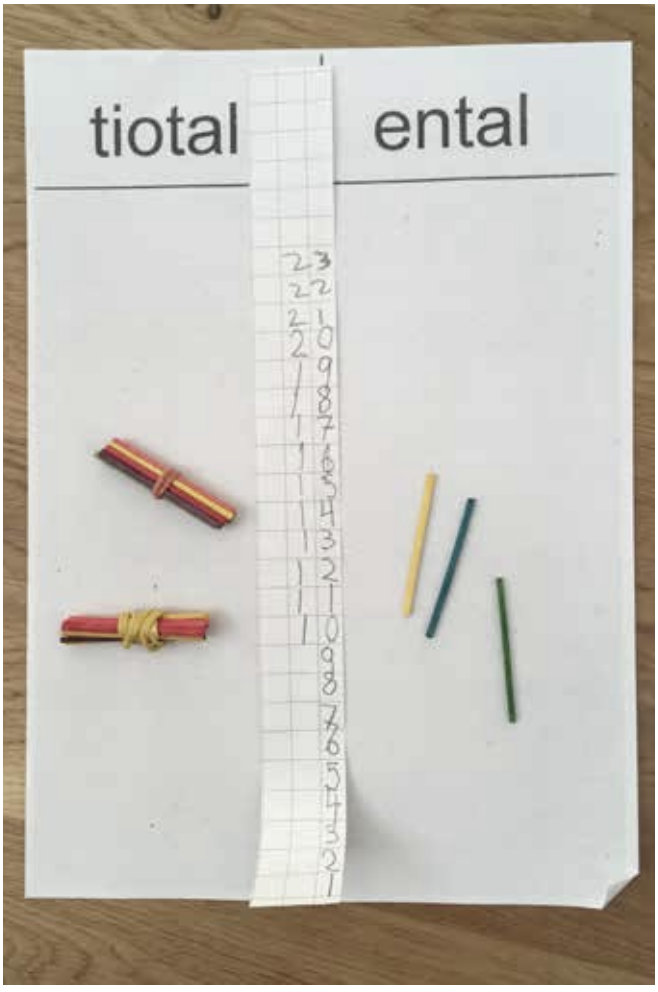
OBSERVATIONS AT FLYGELSKOLAN

Through Annika Ericsson I got in contact with the educational system and spent time in different classes, first to fifth grade. Here I observed the structure and tools used during mathematics lessons and discussed with pupils and teachers.

In lower grades a variety of teaching methods were used to clarify mathematical principals. They had typical blackboard teaching, then worked individually and in groups. Physical tools were used to deepen the understanding and digital applications to enforce knowledge.

In higher grades it got more common with blackboard teaching alternating with working in the textbooks.


FLYGELSKOLAN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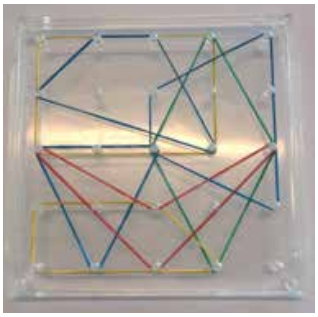
TOOLS TODAY AND DIY

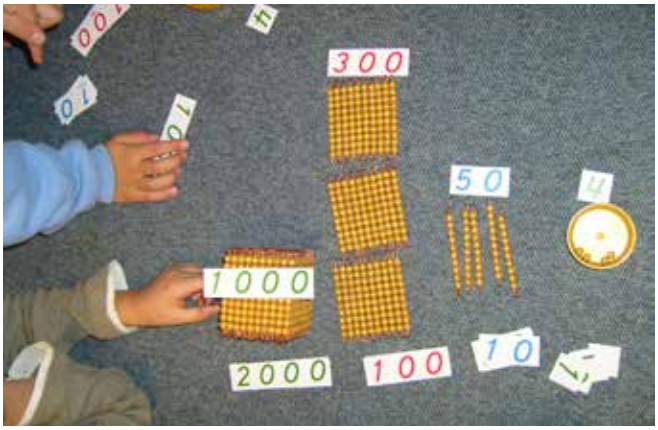
Discussing tools used today with various teachers I found that there are many tools to choose from. The Montessori school has had a big influence in this field and they are working hands on from an early age. Still there are many teachers that make and develop their own tools since they can't find anything that explains the problem in a satisfactory way.

The newest addition to the market are applications for tablets that are focusing on education. However, discussing this with teachers who are working with digital tools, they find that *digital games and exercises are great for repetitive learning but it's hard to find apps which are explanatory and deepen the understanding.*

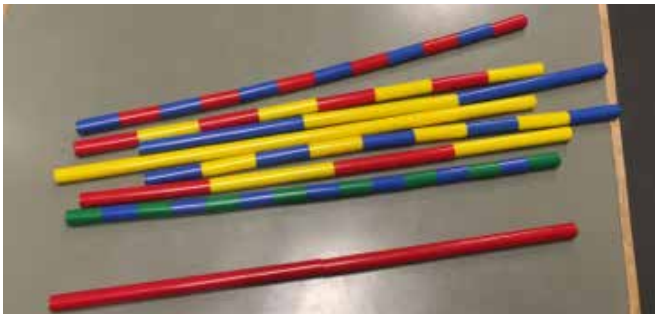
TESSELLATION / GEOMETRY



POSITIONS



DIY FRACTIONS



RESEARCH CONCLUSIONS

On what level in the structural hierarchy decisions are made will have different impact and different prospect of reaching the aimed target. The pupils' development should be the aim for schools and teaching.

The field of teaching mathematics is both vast and complex but there are some key elements for building a solid foundation. This can be achieved by developing a good number sense early on and focusing on the process. This is done by understanding how numbers and operations correlate and interact with each other. To build a solid fundamental understanding there are a lot of different areas that have to be taken into consideration and taught, such as place value, comparing as well as decomposing and composing numbers. Many of these parts are more easily understood with physical and, or illustrative exercises, connecting it to the real world.

During the interviews as well as when discussing with teachers at Malmö University and with teachers at Flygelskolan I found that one of the harder parts for pupils in earlier grades is fractions and how they correlate with the positional system and other kinds of number forms as percentage and decimal numbers.

THE EDUCATIONAL SYSTEM

- The pupil should always be the focus.

HOW TO BUILD A SOLID MATHS FOUNDATION

- develop a good number sense
- focus on the process
- understand how numbers and operations correlate and interact with each other.

CONNECT TO THE REAL WORLD

- Physical and, or illustrative exercises make it easier to relate maths to the real world.

FRACTIONS

- For pupils in the earlier grades, fractions is one of the more difficult parts in maths.
- It's hard to understand how they correlate with the positional system and other number forms.

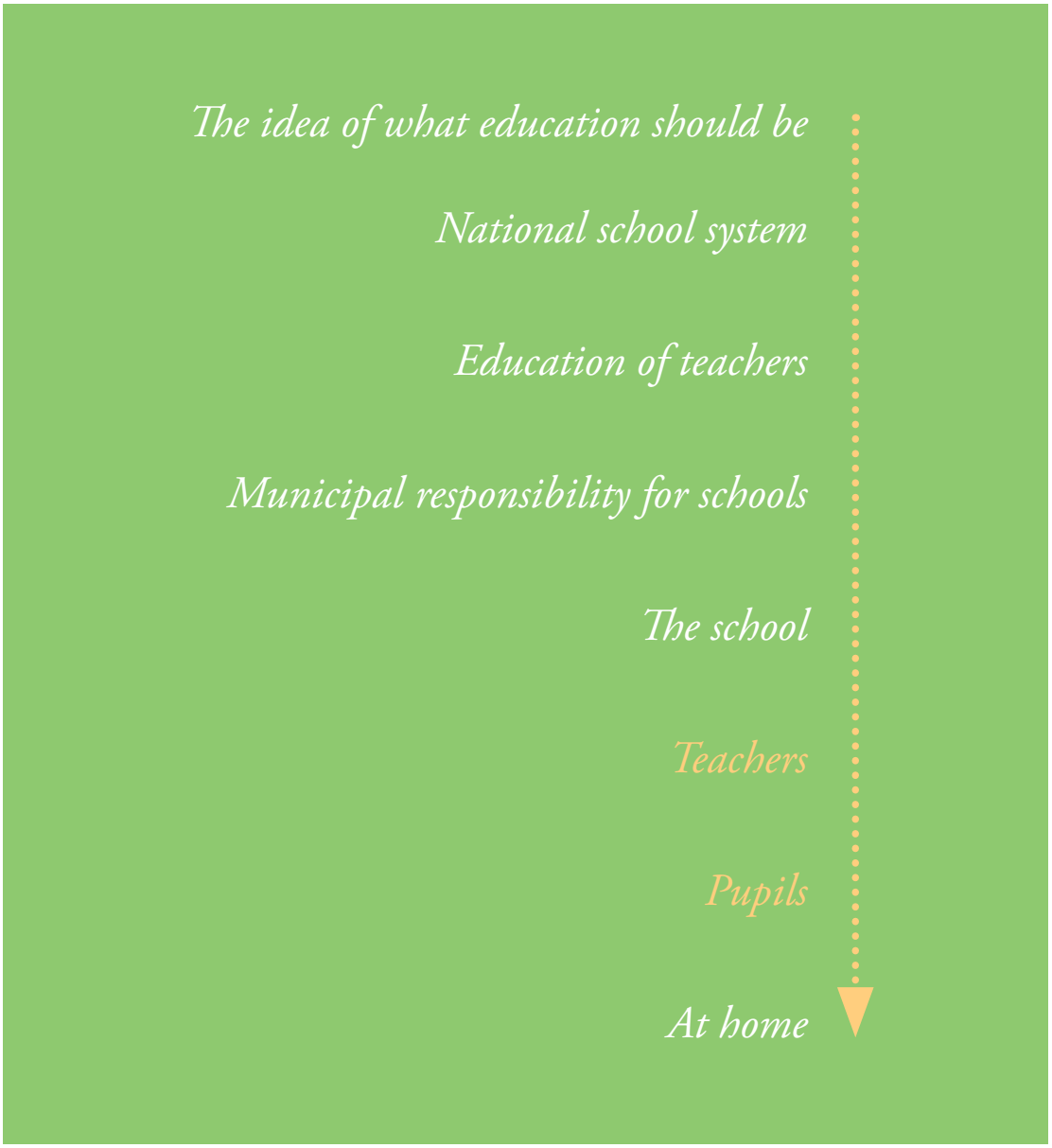
SYNTHESIS

CHOOSING DIRECTION

TARGET GROUP

The educational system is a big and complex organisation which is built on an idea of what education should be. The system spans from the idea of what the education should include, which is decided on parliamentary and governmental level and formulated in the school law, to what the pupils are taught at home. In between are authorities as national education administration, municipalities, principals and teachers who are responsible for the execution.

To narrow the range of the project decisions on what the target group should be had to be made. Earlier I had decided not to focus on system design but to explore the pupils interaction and understanding of the subject. Therefore the target group was decided to be pupils and teachers.


The idea of what education should be
National school system
Education of teachers
Municipal responsibility for schools
The school
Teachers
Pupils
At home

KEY FINDINGS AND OPPORTUNITIES

The possibilities and areas of interest are formulated from the conclusions and the questions of who, why and what. The questions were answered to define the area wherein the project would be focused.

OUTSIDE OF MATHS

- A way to work with other subjects inside the maths curriculum. This could show how maths is important in other fields than just maths which could help to keep up interest and relevancy.

OPERATIONS

- Show what happens when we do an operation. To clarify that an operation has impact on both sides of the equal sign.

BUILD AND PLAY

- A creative way to learn about the subject at hand. Preferably through play or by building. A building set or game that stimulates learning.

OVERVIEW

- To create understanding of how maths integrates with the world and other subjects. This should create an understanding of why maths is important and keep up the interest.

OPEN SOURCE

- An open source platform where the teachers can share and collect DIY tools and solutions.

FRACTIONS

- Introduction of fractions, decimals and percentage. This is one of the harder areas to understand and connect with other mathematical expressions of value.

STORYBOARDING

- By structuring the problem/equation/subject and build ing a story about it one could facilitate understanding.

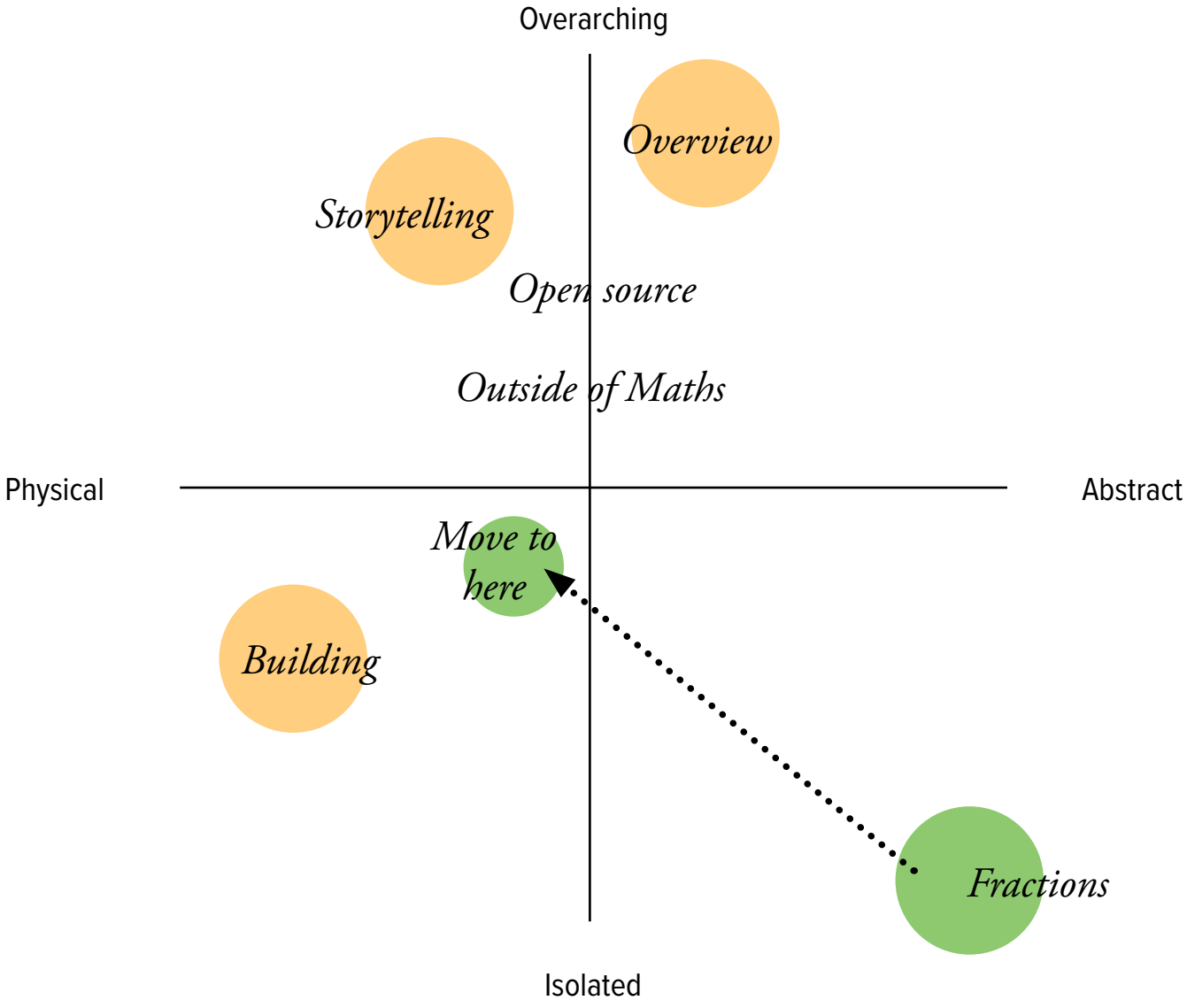
CHOOSING AREA

To choose the direction of the project the areas and opportunities were compared to each other and to different educational aims.

- **Physical:** To what extent is the area based in the physical world?
- **Abstract:** How abstract and ambiguous is the area?
- **Overarching:** To what extent is the area focussed on overall understanding and how does it relate to other fields?
- **Isolated:** To what extent is the area isolated?

It became clear that the aim was to present an option on how to make fractions more based in the physical world and how to give an overarching understanding of the area.

Three different approaches were chosen to be explored in combination with fractions.


Overarching
Overview
Storytelling
Open source
Outside of Maths
Physical
Abstract
Move to here
Building
Fractions
Isolated

DEFINING AREA AND TARGET GROUP

FRACTIONS

Through research the topic was narrowed down to fractions. This area is hard to grasp for many pupils in the beginning and it has room for a lot of improvement and exploration.

Three different pedagogical methods were chosen to be explored in combination with fractions: building, storytelling and overview.

PUPILS AND TEACHERS

Pupils and teachers were chosen as target groups since I believed that it was in these groups a product could have the greatest impact without working on a structural level.

BRIEF 2.0

To make the introduction of fractions smoother and to facilitate the understanding.

This should be done by using a physical object which makes fractions tactile.

IDEATION

CONCEPT DEVELOPMENT AND TESTING

IDEATION - STORYTELLING

Storytelling is a good way to learn and discuss mathematical thinking, how to approach a problem and the different ways to solve it. By letting the pupils create their own story through pre-set conditions they get more involved in the process.

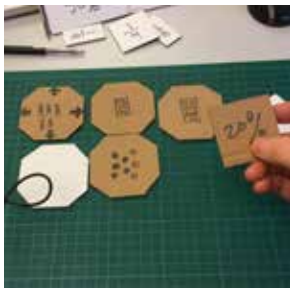
I explored how this could be done as a game and found inspiration from boardgames that focused on storytelling. Diving deeper into the concept it got clear that it lacked the explanatory idea of how to learn fractions. Storytelling in mathematics is good when you can be there and guide the discussion. Otherwise it lacks the overarching explanation of what fractions are.

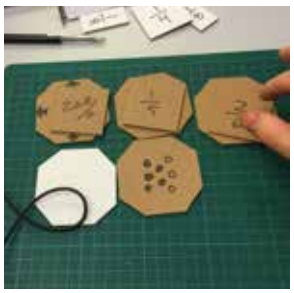
Inspiration and examples

- What can a board game revolving around storytelling look like?
- Draw cards to tell a story about a character.
- Draw a card and tell a story in an allotted time.

Fixed board game

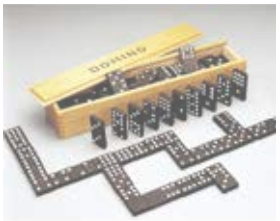
- A fixed board where you move a game piece or collect points by solving and/or telling stories revolving around mathematical questions.

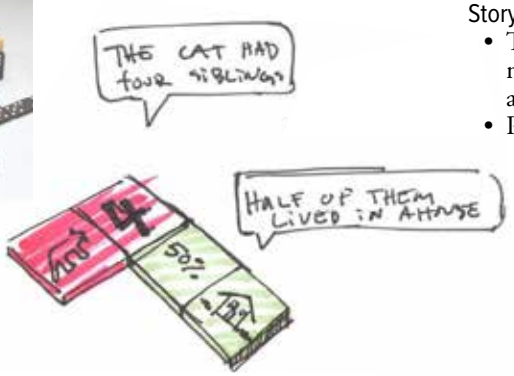




Build the game

- Draw story cards to build the game board.
- Draw maths/mission cards and tell a story that connects it to the story card.



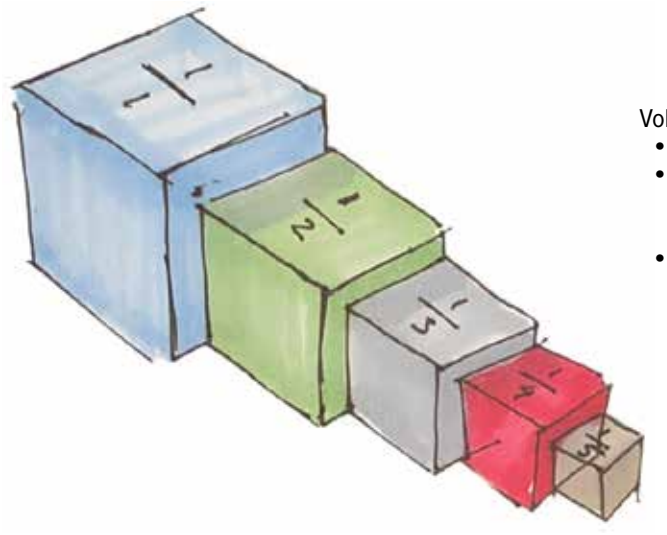


Storytelling domino

- The pieces have one side with a numeral value or maths questions and one storytelling side.
- Played like domino.

IDEATION - BUILDING WITH VOLUMES

If physical exercises can make mathematics easier to relate to, can three dimensional models be an option? I explored how fractions can be represented in volumes with the cube as base. It's an interesting area but it quickly gets really intricate and you lose the overarching representation of what fractions are.

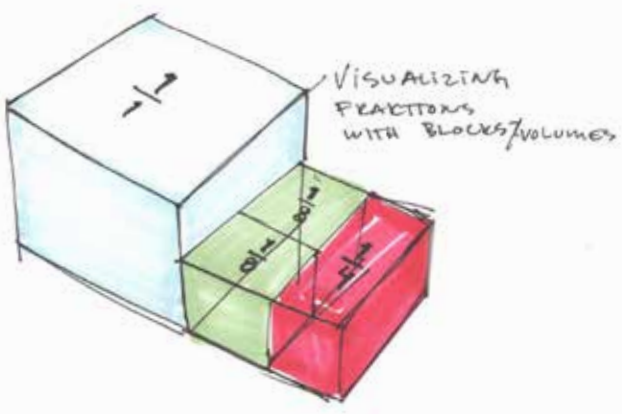


Volumes 1

- One big cube represents 1/1.
- The other cubes are the fractions of that cube. A 1/4 cube is 25% of the volume of the first cube.
- What can it be used for?

Volumes 2

- One big cube represents 1/1.
- The other volumes are then made to be fractions of the 1/1 cube.
- The volumes will need to be in various forms, cubes as well as rectangular cuboids.



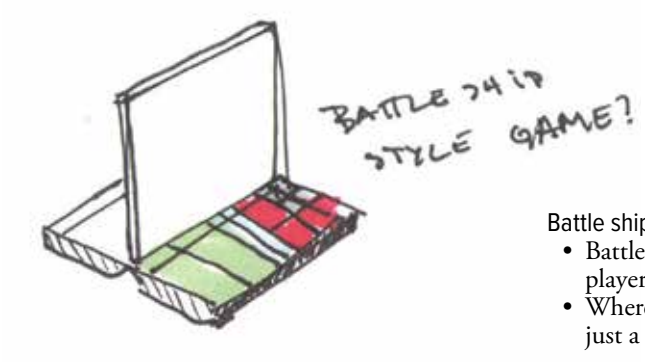
Volumes 3

- A cube that is divided into, for example, ten small cubes.
- One small cube represents 1/10 of the complete big cube and therefore five of the small cubes represent 1/2 of the big cube.
- There will have to be different combinations to create a 1/1 cube that represent fractions which aren't compatible with each other, as 1/3 and 1/7.

IDEATION - GAMES

After exploring storytelling and building with volumes I realized that many of the ideas and concepts were based on games. Gamification is an interesting approach that needed further exploration.

It was important to find a balance between game and learning and therefore some fun ideas weren't taken further since the educational aspects got lost in the game.

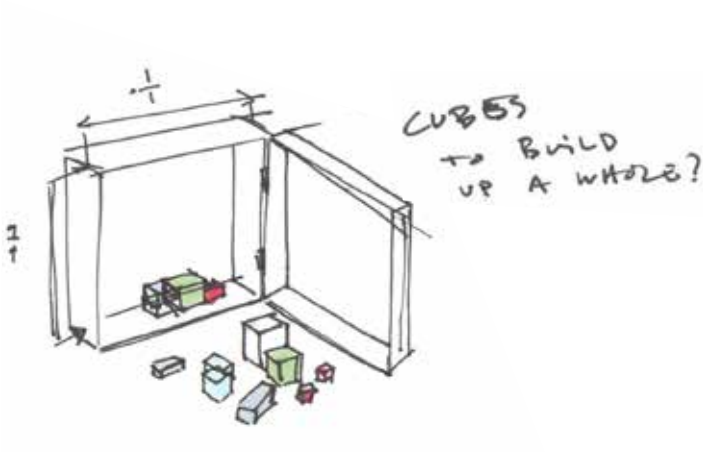


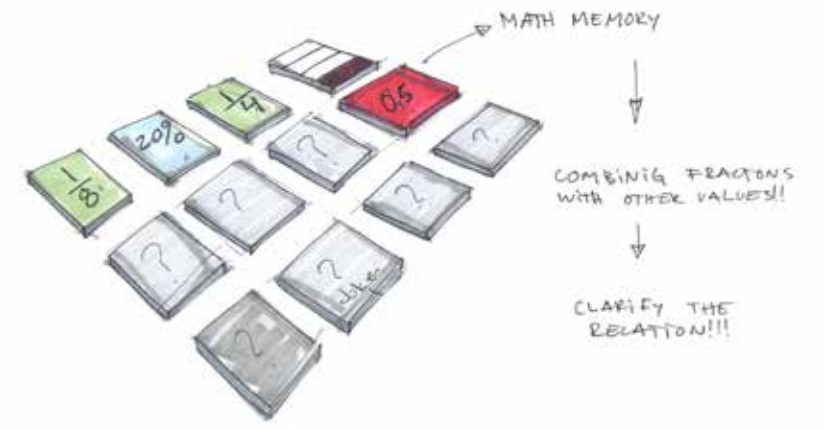
Battle ship fractions

- Battle ship inspired. Guess the other players board of fractions.
- Where is the educational value? Is it just a game?

Build a whole with cubes.

- The box represents a whole, 1/1.
- Stack smaller cubes in different sizes representing different fractions.
- To complex?

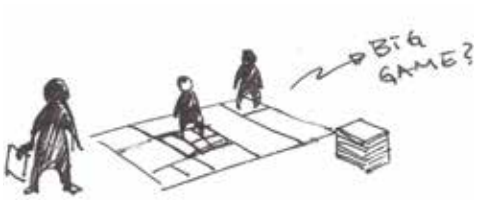




Maths memory.

- A lot of cards with different number values and number forms.
- They are turned up side down and played like memory.
- Match a number form with a certain value with another number form with the same value

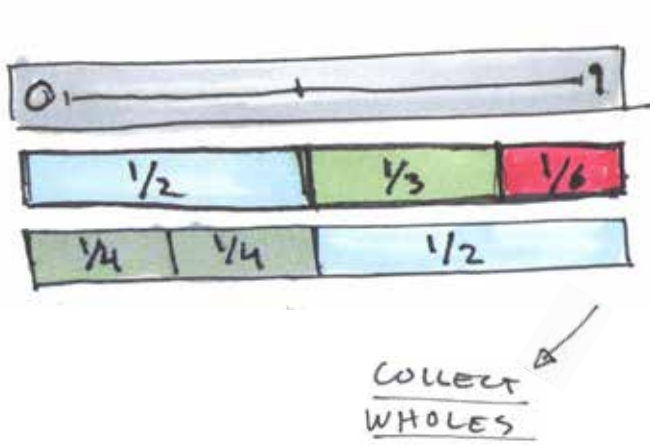
Would it be more engaging if the game had really big pieces?

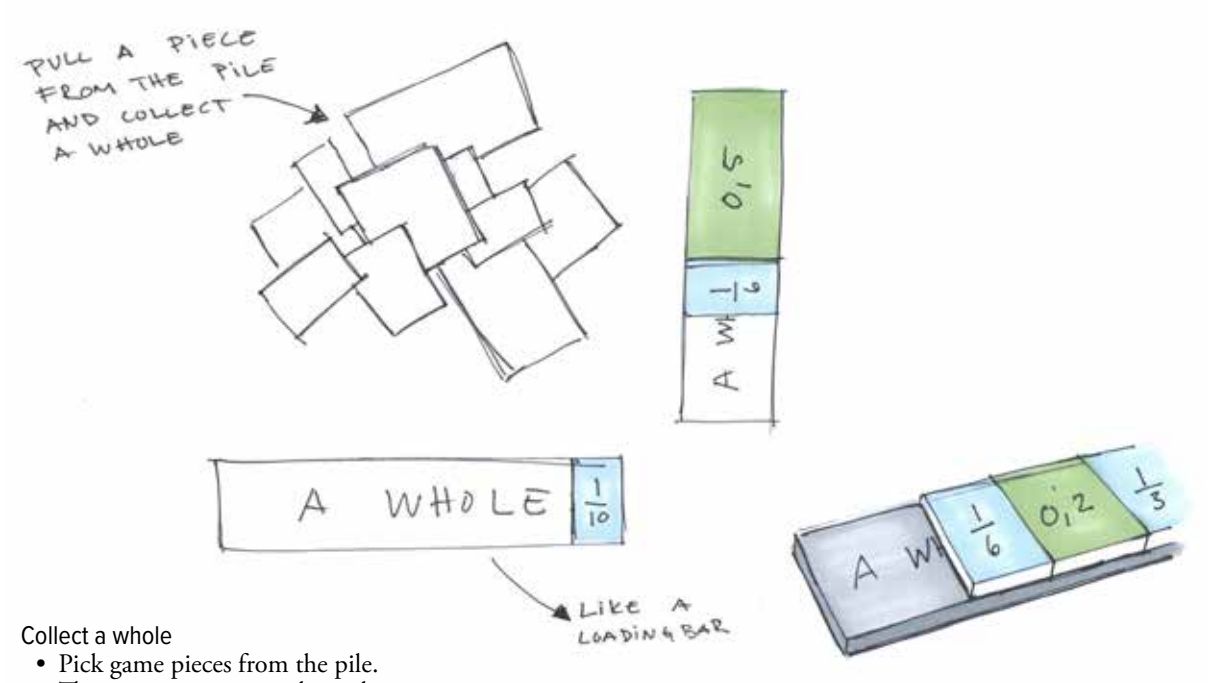


IDEATION - COLLECT AND BUILD

The concept that was chosen to be taken further to iteration and explored with the pupils was "Collect a Whole". It was a mix between building with volumes and maths memory.

The concept centred on game pieces with different sizes depending on the value of the fraction represented. The aim was to build a whole and in that manner collect points. It could also be used to clarify fractions in relation to other number forms as percentage and decimal numbers.

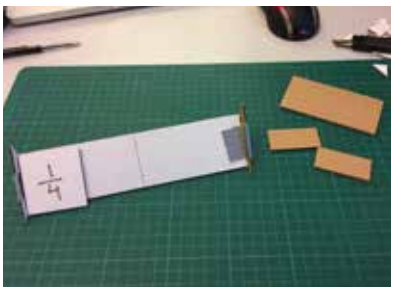


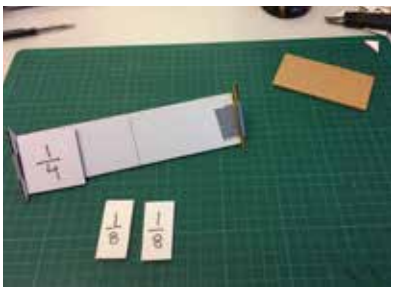


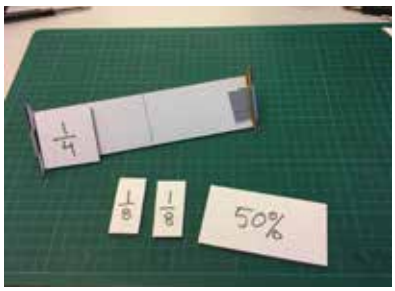
Collect a whole

- Pick game pieces from the pile.
- The pieces vary in size depending on their numeral value.
- The value can be shown in fractions, percentage or decimal form.
- The goal is to collect points by building wholes.

What it could look like







"A WHOLE" VERSION #1

The first model of "A Whole" was fairly simple and made to test the concept with pupils to see if the idea would work. The pieces were cut out of cardboard and all values and number forms had the same colour and were marked in the same way.

Each player gets a plate which represents a whole. The plate is a form of answer key and is supposed to make it easier for the players to measure how close they are to collect a whole.

One after another the players pick cards from the pile. The cards are cut into fractions of the plate, one half to one tenth. In the pile there are also cards with the same value as the fractions but in percentage and in decimal form.

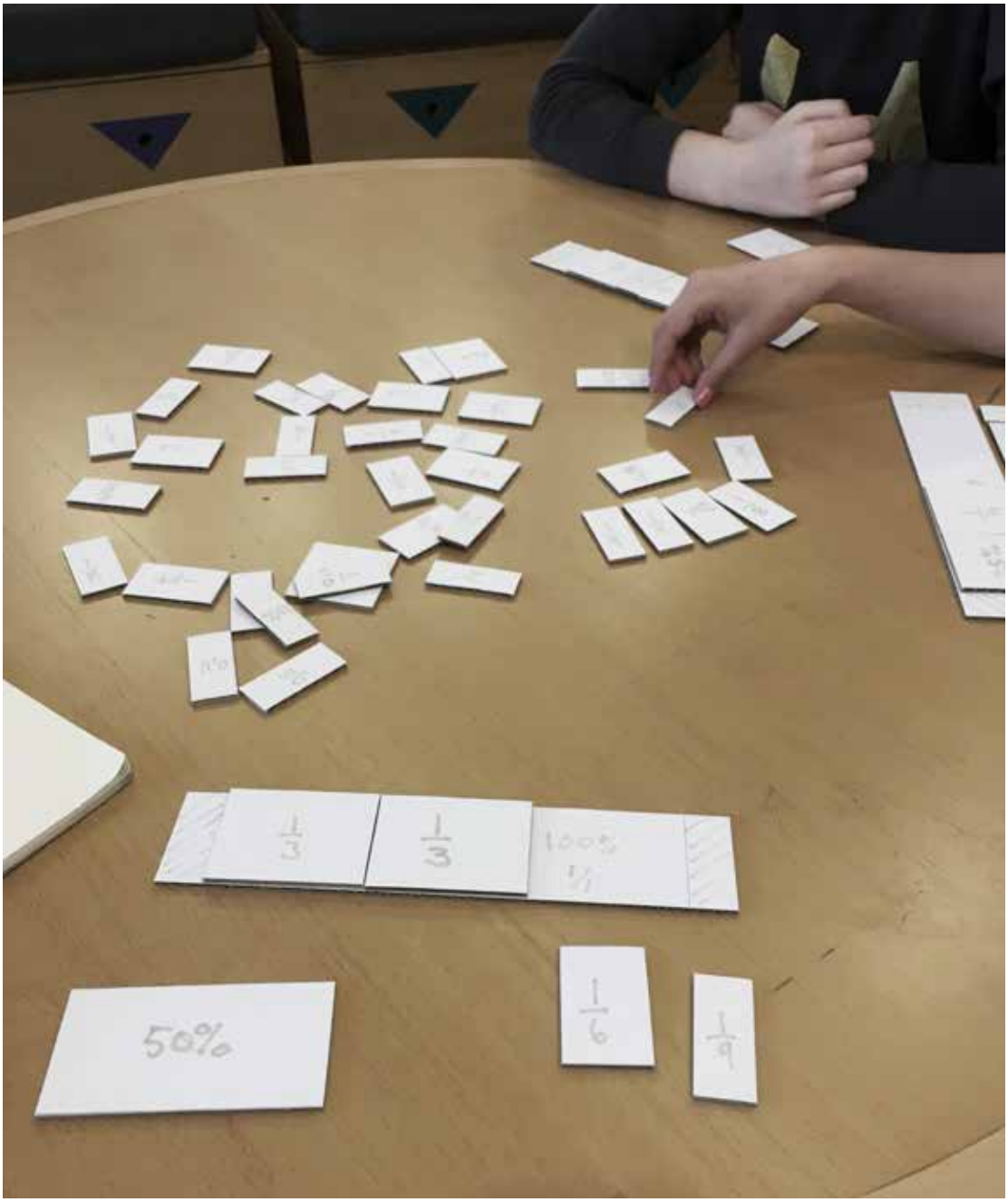
The aim is to collect wholes with the cards they pick and for each whole they get one point.


50%
14%
45%
20%
0,33
16,6%

TRIAL SESSION WITH PUPILS #1

At an early stage of the concept development "A Whole" version #1 was tried in a fifth grade class. Together with pupils from this class it went through several iterations. It concerned different aspects as the understanding of the game, how it looked and what worked and didn't work.

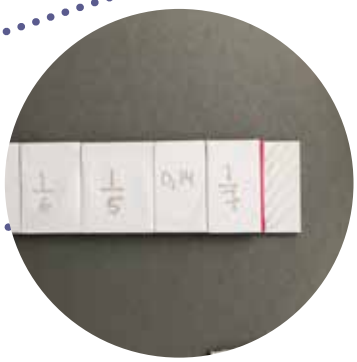
We tried two different variations, with values facing up and with values facing down. The task was to collect a whole and gain points for each whole collected.


1/3
1/3
100%
1/1
50%
1/6
1/9

FEEDBACK

- Messy with everything (fractions, percentage, decimals) at the same time.
  - › Allow for different levels of difficulty?
  - › Work with one number form at a time?
- All the small values are left.
  - › Higher points for more pieces used, more complex solution?
  - › Only allow to use one “1/2” for each whole?
- Hard to differentiate between pieces.
  - › Colour after value or number form?
- Hard to collect a whole when some values only match themselves, as “1/7”.
  - › Incorporate/work with estimation?
- It gets repetitive quickly and the pupils lose interest/ engagement in the game.
  - › Incorporate task cards?
- You can see the value from the size of the piece and target the desired one.
  - › Same size on all pieces?
- Incorporate number axis to show more of the relation?
  - › Could this improve the task cards?


20%
0,25
25%
10%




20%

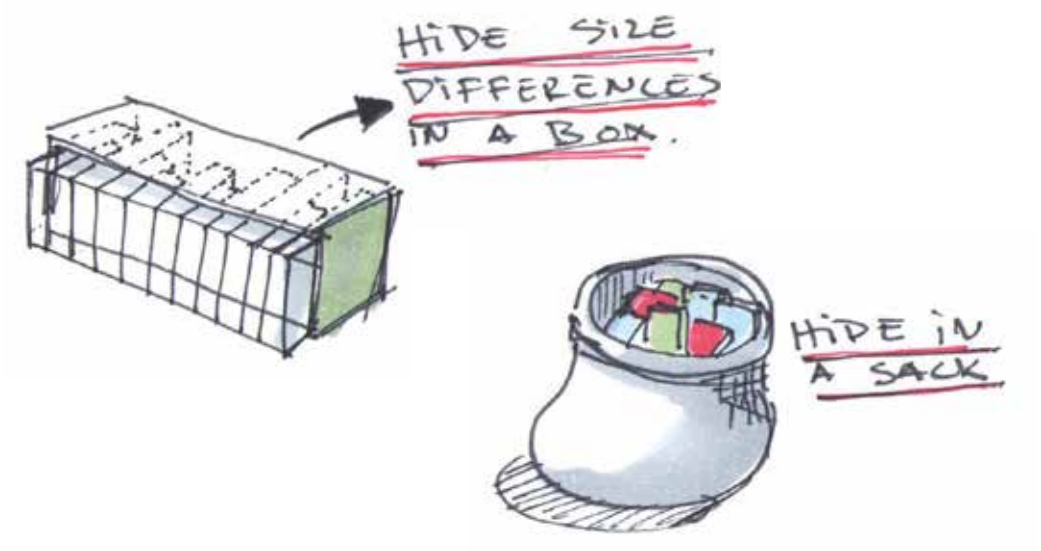

100%

CONCEPT DEVELOPMENT

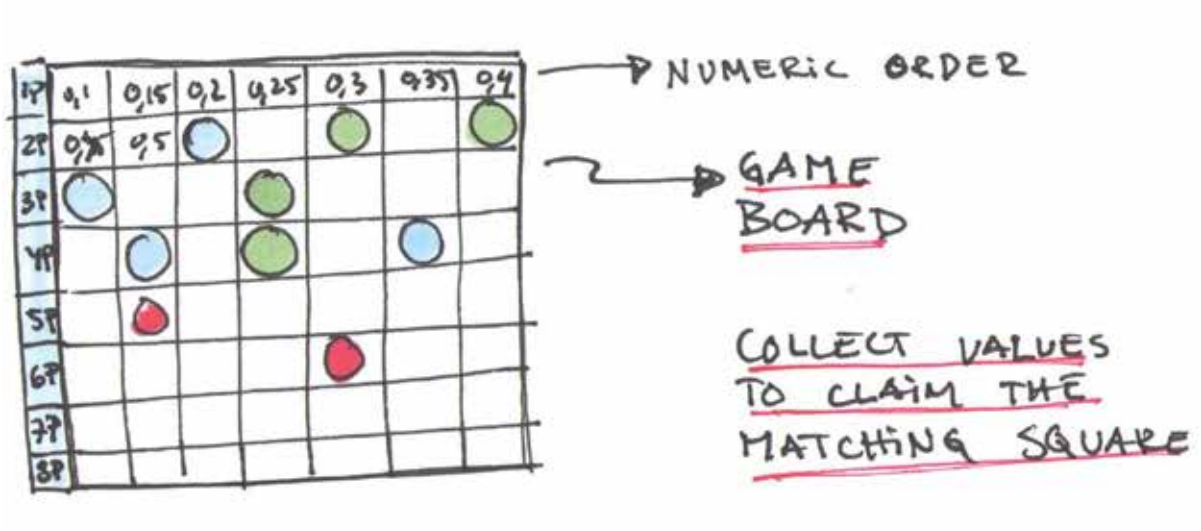
After the first feedback the aim was to explore improvements of the game experience.

- Develop the pieces, make them more distinct.
- Hide the size so the players can't aim for a specific value.
- Develop the game aspect, a game board or task cards?
- Explore how the gathering of pieces and how to build a whole can be improved.

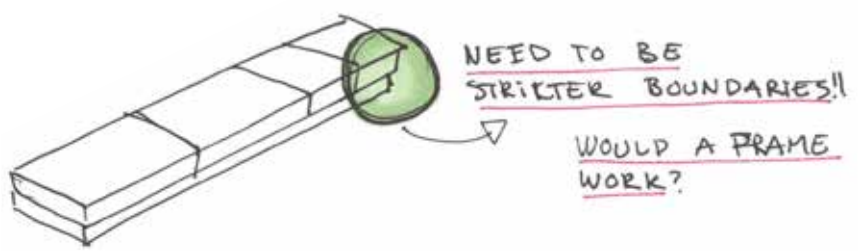
How can the pieces be hidden?

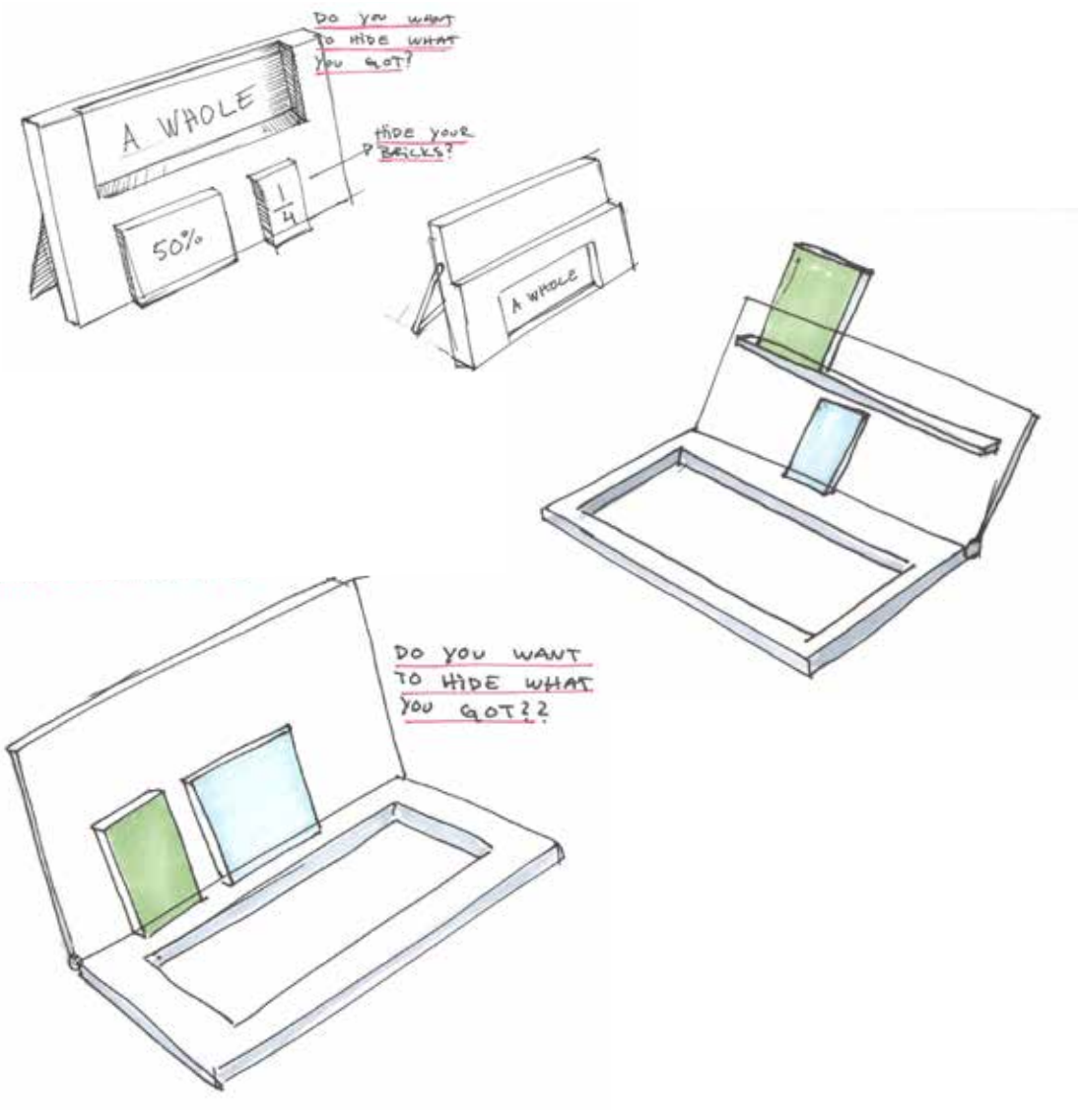


How can the game be improved?



The frame was lacking in precision.
How can it be improved?





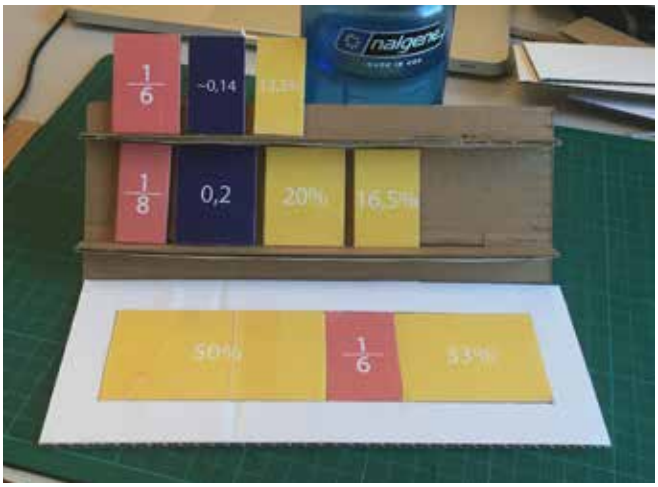
To make it easier to differentiate the game pieces three colours were picked to distinguish the number forms.

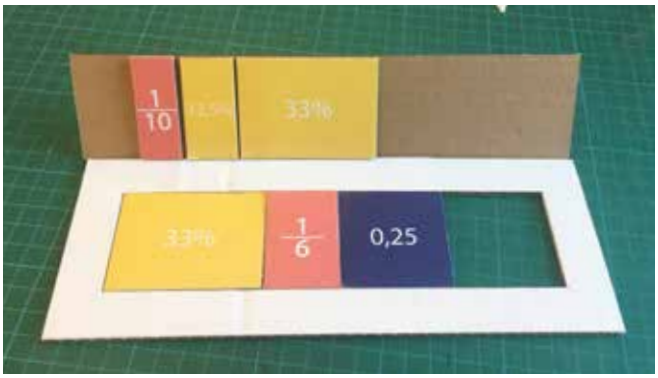
FRACTIONS

DECIMALS

PERCENTAGE

Test of colours for the number forms and mockups of more advanced frames.





"A WHOLE" VERSION #2

For the second version of "A Whole" the cards were coloured after their number form, this made it easier to differentiate between them.

The plate was replaced by a frame which made the boundaries of a whole clearer.

The game aspect was developed and a simple game board was made. The board consisted of fields with numbers in decimal form. The first row of fields started with the number 0,1 and further down the board the higher the number. Each line on the board was marked with a point, starting with the lowest one, at the top line since that line had the lowest values in its squares.

The goal for the players was to pick cards from a bag and build the values corresponding to the fields on the board and claim them. Once a square was claimed the player earned the point that was marked for that line. Higher numbers were worth more points.


0,1
0,15
0,6
0,65
1,15
1,2
1,25
1,65
1,7
1,75
1,3
1,8
1,85
1,35
1,9
1,4
1,95
2
0,75
0,8
0,85
0,9
0,95
1,5
1,55
1,6
0,35
0,4
0,45
0,5
0,55
1,1
E
P
P
E
0,25
0,5
0,2
25%
16,5%
0,165
0,165
25%
1/9
1/5
1/8
1/4
1/7
1/9

TRIAL SESSION WITH PUPILS #2

In the second session with the pupils at Flygelskolan we tested "A Whole" version #2. There was a lot of improvement compared to the first trial session. The coloured cards and the frame worked a lot better than before but the game aspect was still lacking a lot.

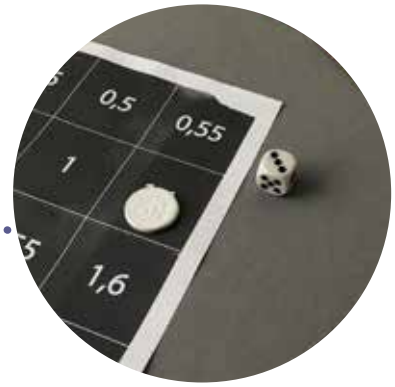

1,15
1,2
1,25
1,3
1,35
0,8
1,65
1,7
1,75
1,8
1,85
1,9
0,35
0,4
0,85
1,4
1,45
1,5
1,55
1,6
0,55
1,1
2
1,95
~0,33
1/4
-0,165
0,5
0,25
1/4

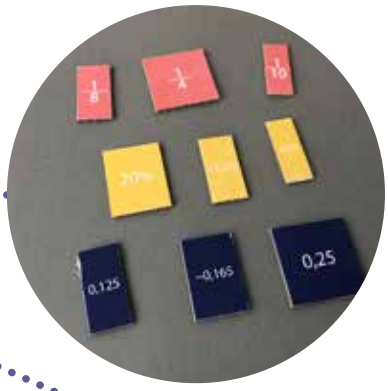
FEEDBACK

- Hard to understand when it's uneven, 1/3, 1/7, 0,33 etc.
- It's better now with more alternatives.
- It's better now that you can differentiate the pieces.
- Could still be good to add task cards.
- Alternatively, roll a dice, move game piece according to the dice and fulfil the task before moving on.
- For first time players, the aim of the game and the rules are unclear.
- They find it hard to combine the different number forms, fractions, percentage and decimals.
  - › Introduce different levels of how to play the game?
- When using a bag to hide the pieces you could still feel the size of the pieces which reveals the value.


1/6
1/3
1/9
~0,165
~0,33
~0,1


~0,33
0,25


0,5
0,55
1
1,6

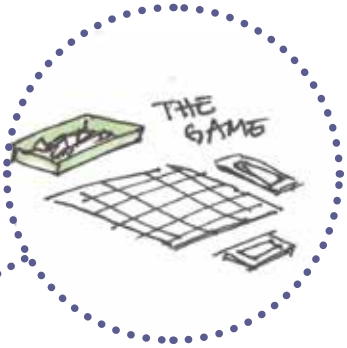

0,125
~0,165
0,25

TRIAL SESSION WITH EXPERTS

To understand how this tool could be used in a classroom and what works and doesn't work from a pedagogue's view I had new meetings with Lena Andersson and Ulla Öberg discussing "A Whole" version #2. They provided important feedback that wasn't focused on during the target group trials. It included for example how the product can be used. They suggested that it should be possible to use it individually, in pairs and in groups. The tool could be adaptable to provide different usage.

FEEDBACK FROM LENA

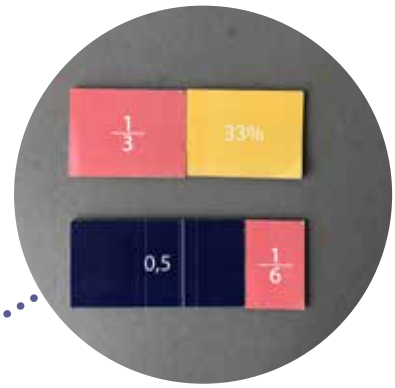
- Add paper strips to create own values or to fill in the lacking value to make a whole.
- Not just a game. The tools can be used in other ways in the classroom. How?
  - › Should the game be complemented with more tools for the teacher to use in the classroom?
- Packaging, how do you store the game in the classroom?
- Chart to show/compare the fractions, percentage and decimals.
  - › Expand the tools with blank pieces?
- Working with the game/tools individually, in pairs and the whole class. How does this change the game/tools?
  - › Show three exercises to display the function of the tools.
  - › Build many wholes with the material, individually or help each other.
  - › Remake from fractions to decimals and/or percentage.
  - › Work with finding the common denominator.
  - › Incorporate calculator to work with bigger numbers.
- What do pupils learn from building a whole? To reach the learning purposes there is a need for discussion and explanation of thought process.

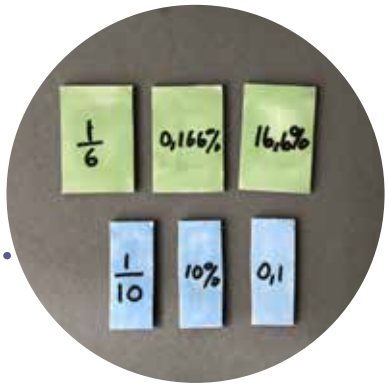

THE
GAME


A WHOLE
A WHOLE

FEEDBACK FROM ULLA

- Do you get more points for showing several solutions for one task?
- Is it cheating if the same value, regardless of number form, has the same colour?
- Fractions of quantity? How to add it to the game?
  - › Add “buttons” to work with in correlation to the pieces during classroom tasks. For example, the buttons could have 5 different colours, 2 forms, 2 or 4 holes.
- Develop a story around the game to catch interest.


1/3
33%
0,5
1/6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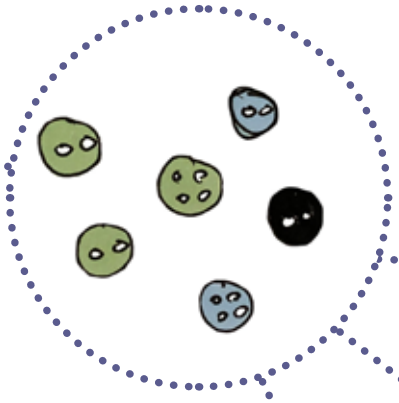
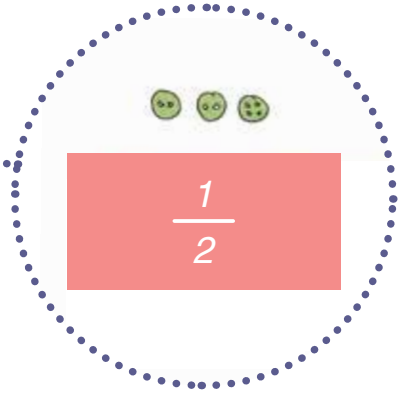
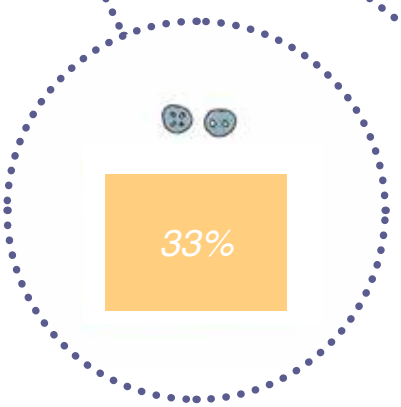
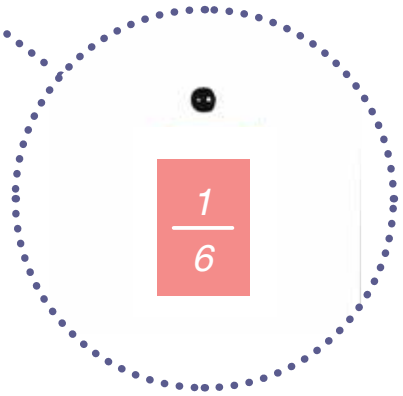
0,166%
16,6%
1/10
10%
0,1




1/2


33%

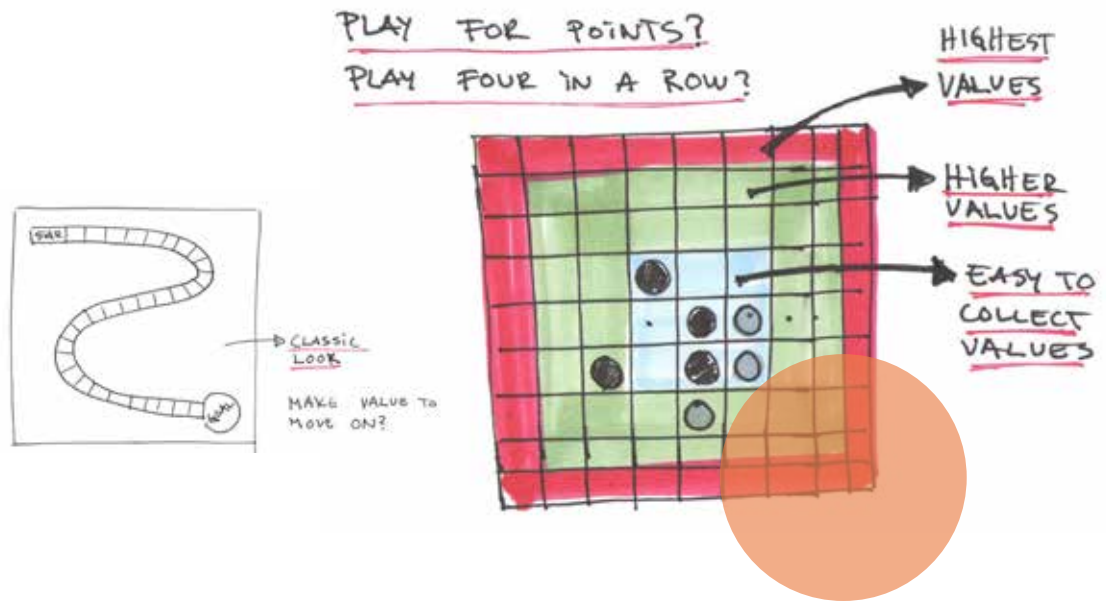

1/6

SIMPLIFYING AND ADDING VERSATILITY

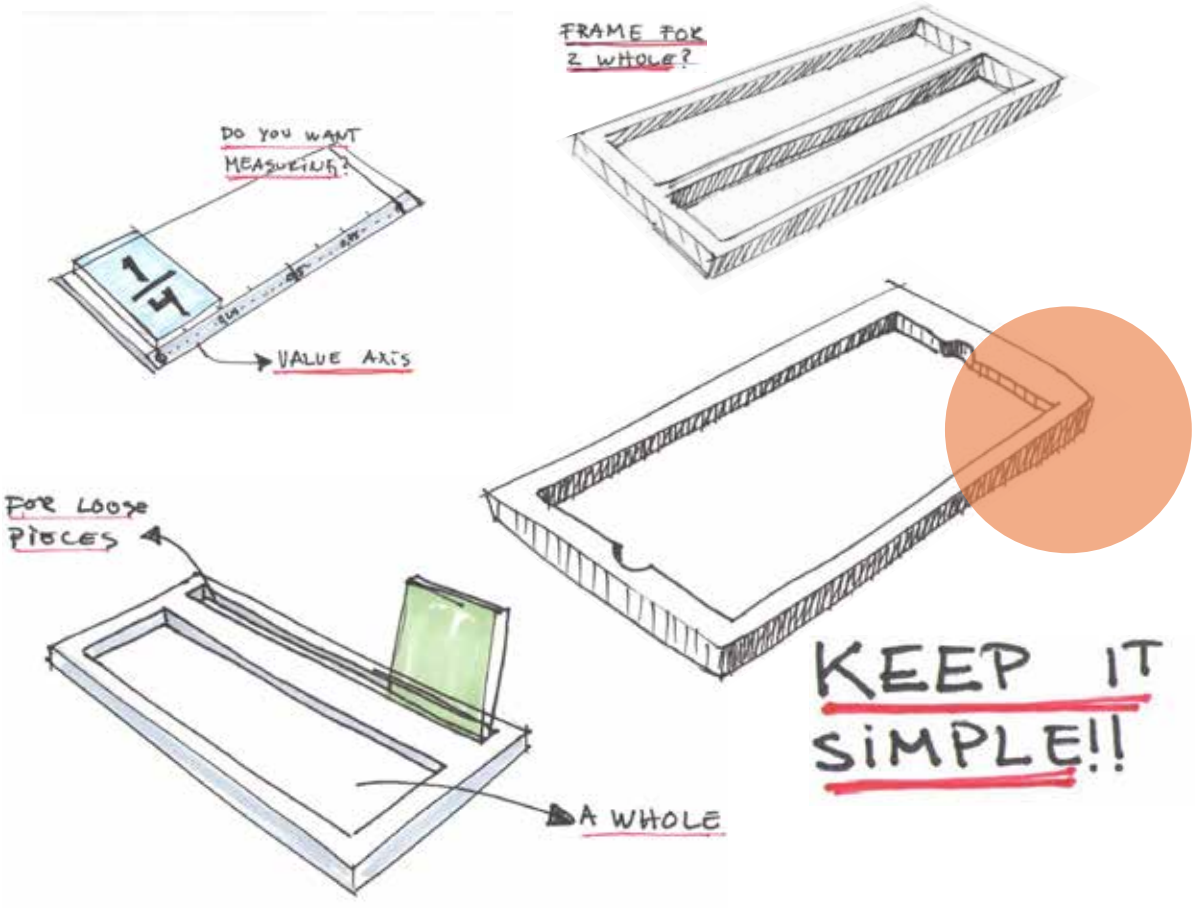
After the latest sessions with pupils and the experts, Lena and Ulla, focus took a new turn. Through the feedback I could conclude that the product should not just be a game but rather a tool that can be used in several different ways in the classroom, including a game. The focus of this development phase was to simplify the product and design for versatility.

The frame was kept to a bare minimum, only symbolizing a whole.

HOW SHOULD THE GAME BE FORMULATED AND PLAYED?



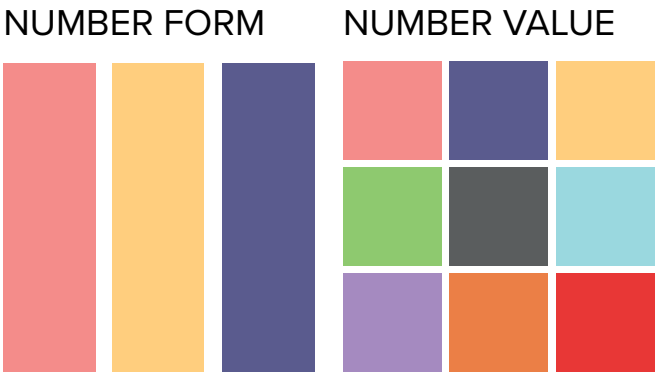
THE FRAME - SHOULD IT HAVE MORE OPTIONS?



I decided to add “buttons” to incorporate fractions of quantity. With buttons in different shapes, colours and with varied numbers of holes you can make a large variety of combinations and hence show different fractions.

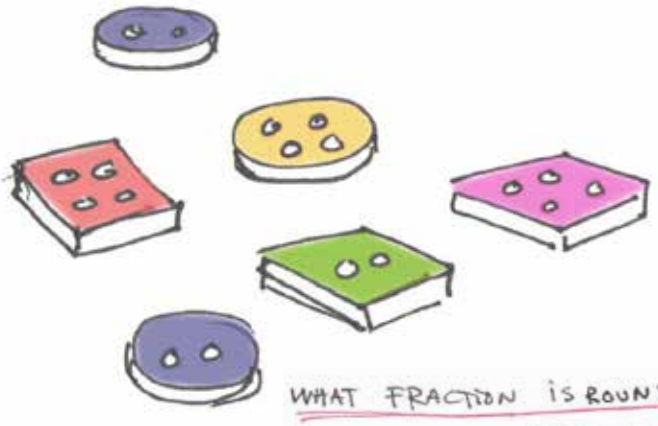
The box was designed to have the same function as a bag would have, hiding the pieces. Through the discussion and the design of the game it was clear that it was not important to hide the size. To connect the size of the piece with the value is an essential part of relating maths to the real world.

It was also decided that the pieces should be coloured according to their value and not to their number form. This makes it clear that 1/10 is the same as 10% and 0,10.

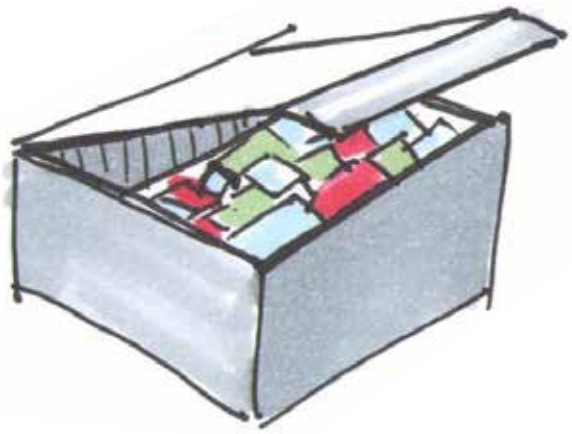


FRACTIONS OF QUANTITY

5 COLOURS
2 FORMS
2 VARIATIONS OF HOLES



WHAT FRACTION IS ROUND? → $\frac{1}{2}$
WHAT FRACTION HAVE 2 HOLES? → $\frac{1}{2}$
WHAT FRACTION IS BLUE → $\frac{1}{3}$
WHAT FRACTION IS GREEN? → $\frac{1}{6}$



NO NEED FOR BAG OR SPECIAL BOX?

HIDE PIECES IN THE BOX!!

MATERIAL

Choosing materials for "A Whole" I explored three different options which had an easy production line and with a low investment cost, acrylic sheets, painted wood and foamed PVC sheets.

It was important to consider how and if schools would be able to produce the material themselves and with what precision it's possible to do this in a casual workshop. How available is the material and is it possible to order a semi-finished product?

ACRYLIC SHEET

Acrylic sheets come in different thicknesses, a variation of colours and they can be both opaque and translucent. The material is easy to work with. It can be sawn, water cut, laser cut, it's easy to glue and good for milling, bending and hot forming.

A school would have good options to produce "A Whole" in acrylic sheet. Most schools have access to a workshop where they could work the material and it's easy to order the pieces laser cut from a manufacturing company.

Exploring the material there were some issues. The numbers would have to be painted or etched, however if etched they needed to be painted anyway. If using a translucent acrylic the numbers are seen through the material and the pieces would need to be glued to an opaque piece. It would be difficult to make it look good.

A important factor is the noise it makes when used, acrylic is hard and could make a lot of noise in a classroom.

PAINTED WOOD

Wood with painted numbers is a good option if everything would be produced at the school. Most schools have a workshop and are used to handling wood. However, wood is time consuming to work in and not as easy to order as a semi-finished product. It's also a less optimal material to set up as a low quantity production for sale.

PAINTED AND ETCHED WOOD

ACRYLIC SHEET

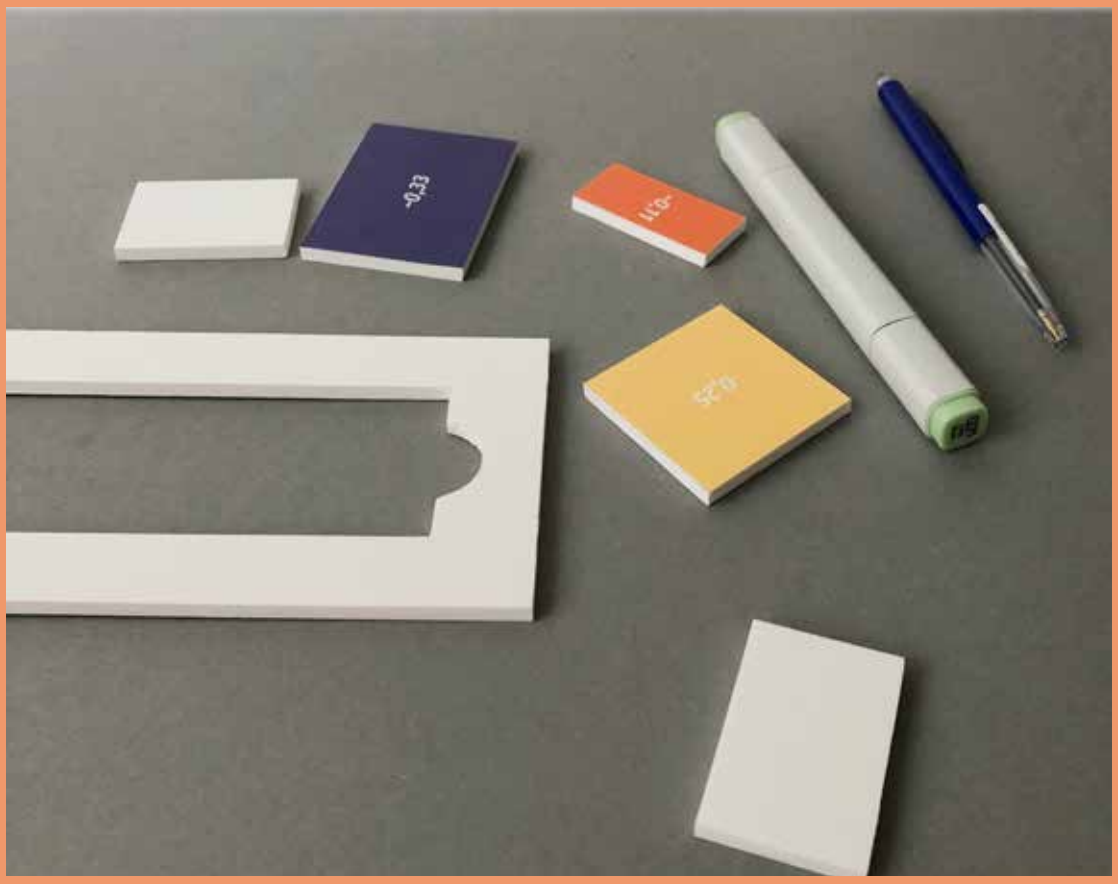
FOAMED PVC BOARD

Foamed PVC board is a lightweight, rather stiff material but with some flexibility. It comes in different colours and thicknesses. It has some water resistance and because of its qualities it's often used for signs. It can be printed on as well as painted and worked in many ways. It can be water cut, sawn, worked with sharp blade and allows for hot forming, however it can't be laser cut.

Due to how easy it is to work within in a workshop as well as by hand, it's a good option for schools to produce by themselves. Since it's a common material for signs there are several companies working with this material and it's easy to order, either as a complete product or semi-finished.

Because of its characteristics, foamed PVC board make less noise than the acrylic or wooden pieces.

I believe foamed PVC board is the optimal choice to produce a whole.



FOAMED PVC BOARD

OPEN SOURCE

To make "A Whole" an optimal solution for teachers I wanted to be able to take full advantage of the teachers' creativity. Many teachers come up with new ways of using their old material or simply develop their own tools. To be able to share the future possibilities of "A Whole" I want to make it all available on an open source platform.

With the open source format the teachers can either order the product on the website or download drawings and create their own product. Here they would also find extra material as different board games, the buttons and suggestions on activities.

The main point of the platform would be to collect and share ideas with the users and allow for a creative discussion amongst teachers.

BUY AND ORDER

DOWNLOAD MATERIAL

GAMES AND ACTIVITIES

FORUM

RESULT

"A WHOLE" - A MATHEMATICAL TOOL TO MAKE THE LEARNING OF FRACTIONS MORE TANGIBLE

"A WHOLE"

The final result is “A Whole”, a product that facilitates the teaching and learning of fractions. It focuses on creating an overall understanding. By visualizing what a fraction is in relation to size, how it connects to other mathematical forms and the non abstract world a deeper understanding can be reached.

The product allows for blackboard teaching, group work, teacher to pupil discussion and individual studies.

The idea of how to learn with the product is to collect a whole. “A Whole” can be used individually, in pairs or in groups to develop an understanding of what fractions are, both as parts of a whole and as parts of quantities. With the different pieces the pupils can explore how fractions relate to decimal numbers and percentage. Through the game, several pupils can support each other or compete to fulfil challenging assignments.

THE PRODUCT INCLUDES:

- A frame that symbolizes a whole
- Pieces which vary in size and colour depending on their value. These pieces come in three number forms, fractions, percentage and decimal numbers and can be mixed or worked with separately depending on the pupil’s level of knowledge.
- A box, which is constructed to store all the parts as well as hiding the game pieces while playing.

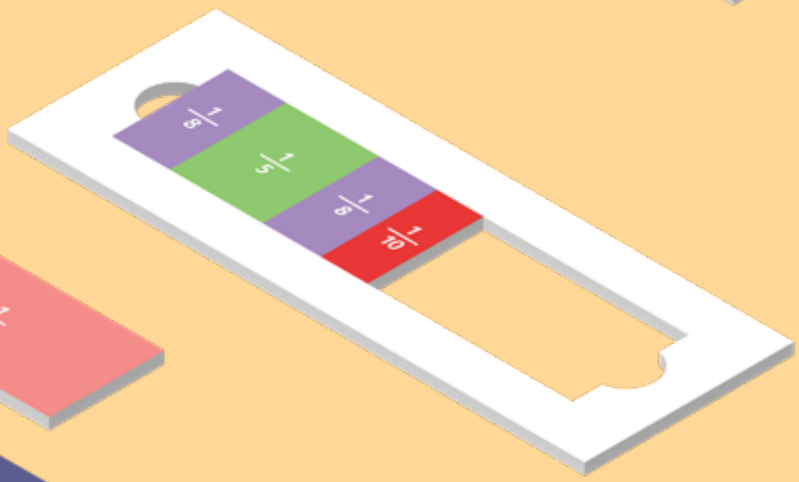
COMPLEMENTARY ITEMS, NOT IN THE FINAL MODEL

- Buttons that make it easier to work with fractions of quantities.
- A game board, with which the pupils can play “Four in a row” or play games where they collect points by fulfilling tasks.


EN
HEL
-0,33
-0,71 - 11%

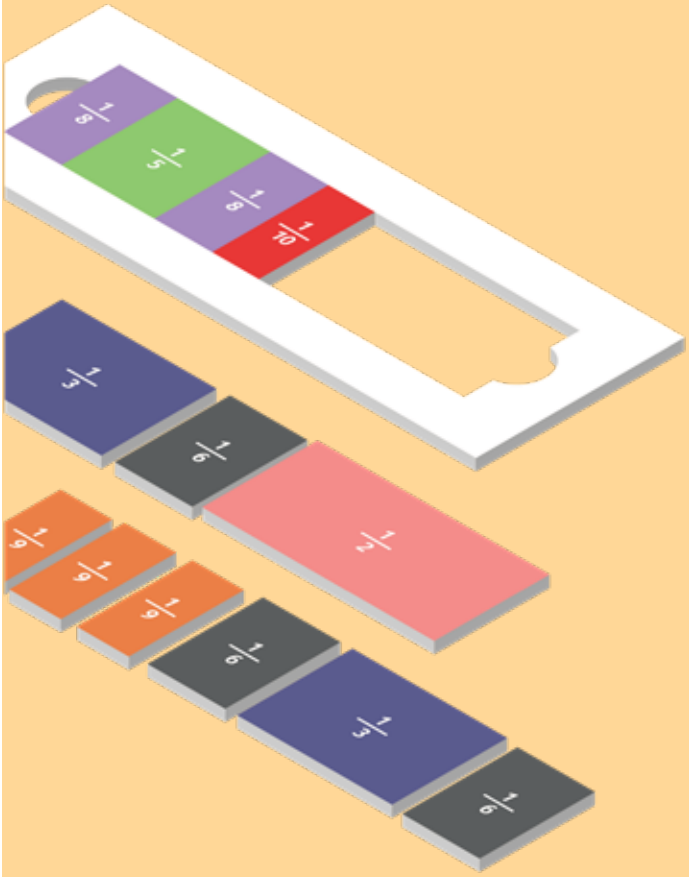
WORKING INDIVIDUALLY

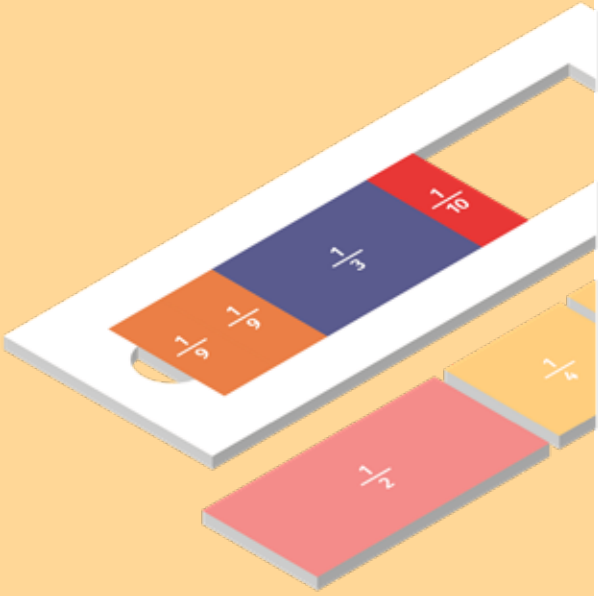
Create as many wholes as you can with the pieces you have got. The degree of difficulty can be varied depending on the pupil´s level of knowledge. During introduction to fractions the pupil can work with the fraction pieces exclusively and as he/she progresses both percentage and decimal numbers can be added.



WORKING IN PAIRS

Working in pairs you can either start collecting as many wholes as you can individually and later compare the results with your partner or you can collaborate and work together from the beginning.



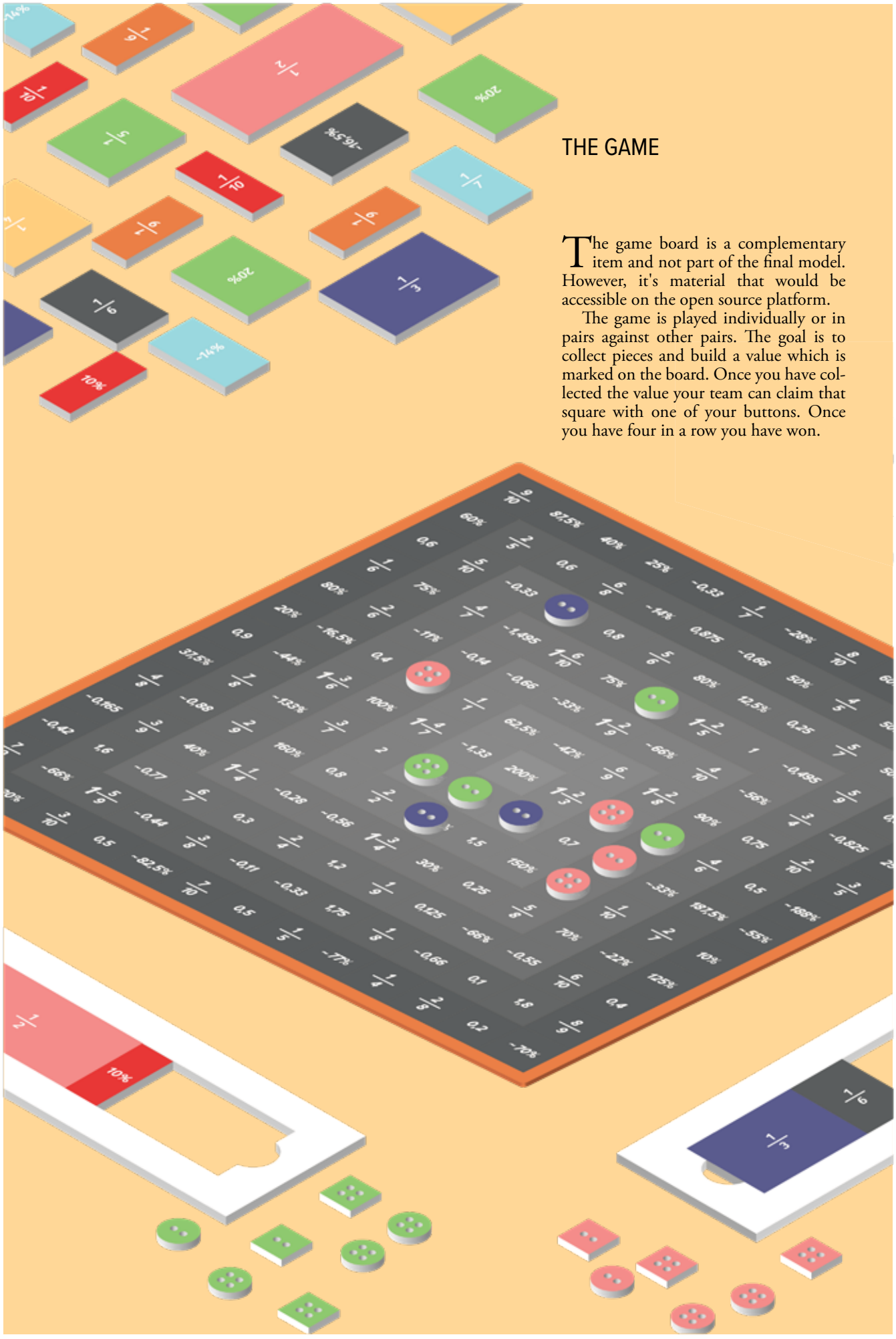


FRACTIONS OF QUANTITY

The buttons for these exercises are complementary items and not part of the final model. However, it's material that would be accessible on the open source platform.

Working with fractions of quantities you build the value out of some of the optional variations that you can create with the buttons. For example, build the fraction that represents the buttons with two holes and build the percentage that represents the blue buttons.

This activity can also be done with fractions only or with decimal numbers as well as with percentage.



THE GAME

The game board is a complementary item and not part of the final model. However, it's material that would be accessible on the open source platform.

The game is played individually or in pairs against other pairs. The goal is to collect pieces and build a value which is marked on the board. Once you have collected the value your team can claim that square with one of your buttons. Once you have four in a row you have won.

WORKING INDIVIDUALLY WITH FRACTIONS

WORKING IN PAIRS WITH FRACTIONS

WORKING WITH ALL THREE NUMBER FORMS

FUTURE DEVELOPMENT

There are a couple of development ideas for "A Whole". It would be nice to test the final model and the final board game with pupils as well as discuss it with teachers. From there the next step would be to establish it as a product amongst teachers. I think a good start to do that would be to reach out to NCM, "Nationellt Centrum för Matematikutbildning". They have a wide national reach towards mathematics teachers. NCM produces maths magazines, literature for education of teachers as well as organize seminars, courses and conferences. By getting featured by them I would get a wide reach.

The main development aspect would be to realize the open source platform. I believe the open source platform is the key to make "A Whole" a successful product. By making it a convenient solution it stands apart from the completion.

CONCLUSION AND REFLECTION

IN WHAT DID I SUCCEED?

In general I'm satisfied with the outcome of the project. I believe that the final result fulfils the demands and expectations that were set at the beginning. During the process and evaluating the prototypes with experts and pupils I realized that it was a product that had potential to be a useful tool in the classroom.

At the start my goal was to work closely with the targeted age group as well as with professionals who would have use of the product. I'm glad that I made this happen during the project and I believe I couldn't have reached the conclusions and the result without these collaborations.

WHAT COULD HAVE BEEN DONE DIFFERENTLY?

A problem during this project was me not managing my time and priorities correctly and staying in the research phase for too long. This created a situation where I was short of time for the later phases.

I would have liked to have a final discussion with the experts about the product as well as seeing how the pupils would interact with it.

Although I'm content with both the material and the form choices that were made I believe it would have been worth spending some more time exploring various options.

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