SCHOOL OF ECONOMICS AND MANAGEMENT

LUND UNIVERSITY

# Introspective equilibrium for coordination games with changing group size

A theoretical approach to growth groups behavior in minimum-effort coordination games

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January 2021
Master Essay 1 (NEKN01)
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Abstract

In the laboratory, minimum-effort coordination games routinely reach low levels for larger group sizes. Weber (2006 American Economic Review) showed that by simply starting with a small group and adding players that are exposed to the group's history over time, one can "grow" larger groups with high effort levels. Adapting the concept of introspective equilibrium, I create a model to replicate the findings of minimum-effort games with growing player counts. While remaining simple and flexible for further extensions it manages to explain almost all findings of previous growth experiments.



*Keywords: coordination game, minimum effort, weakest-link, introspective equilibrium*

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# 1 Introduction

Coordination problems are common feature of human interactions, from where to meet a friend to working with your coworkers to large scale challenges like merging two organizations. While they do not show incentive problems, miscoordination or inefficient equilibrium can and do occur. Inefficient equilibria can be costly, especially for repeated interactions, where for every round you lose out compared to more efficient outcomes. Understanding why some groups coordinate on efficient outcomes while others do not is needed in order to avoid less efficient ones.

One particularly striking setting in which coordination problems have been studied is the so-called minimum-effort coordination game. In this game players choose between different levels of effort and receive a payoff based on their choice and the group minimum. The higher the minimum the higher the payoff, while matching the minimum gives the highest individual payoff.

Previous experiments on minimum-effort coordination games (Van Huyck et al. (1990), Knez and Camerer (1994) among others) have repeatedly shown that large groups do not coordinate successfully during lab experiments. Yet, as pointed out by Weber (2016) we can observe large groups in the real world that do just that. He suggests growth as one reason for the contrast between large groups in the laboratory and the real world. Whereas groups in laboratories start at a large size, groups in the real world often start small and increase in size over time.

The idea in Weber's experiment is straightforward; fewer players in the beginning make it easier to coordinate on a (more) efficient equilibrium. When expanding the group, entrants' (the players joining throughout the game) uncertainty is alleviated through the exposure to the group's previous results. His experiment demonstrates that through such a treatment (more) efficiently coordinated groups can be created in lab experiments.

Using a simplified version of the game in Weber's experiment and adapting Kets and Sandroni's (2019) concept of introspective equilibrium to a finitely repeated game where players enter over time, I build a model that reproduces the main findings of Weber's experiment with growth groups.

Kets and Sandroni use (cultural) salience of certain behavior and introspection (taking the perspective of others) to explain why groups (culturally diverse or not) coordinate on an equilibrium (hence introspective eq.) for a wide variety of strategic interactions.

As Weber's and other experiments on minimum-effort coordination games do not test for the effect of cultural diversity, I am abstracting from this factor. This allows for the model to remain simple while also being more fitting to the experimental data.

My model manages to reproduce almost all of the key findings of Weber's experiment while remaining simple. Most importantly, it explains the improved performance of growth groups where entrants are informed, their relatively lower performance compared to the starting pair and the unimproved performance of growth groups where entrants are uninformed at the same time. It is also flexible and allows for the implementation of extensions such as player types for example to add more complexity if needed.

The paper is organized as followed. Chapter 2 shortly presents Weber's (2006) growth experiment with minimum-effort coordination games and their key findings. In Chapter 3 I introduce Kets' and Sandroni's concept of introspective equilibrium and adapt it for extensive form games with entrants. Chapter 4 presents the propositions drawn from my new definition as well as their proofs and Chapter 5 demonstrates how my model can explain the key findings from Chapter 2. Chapter 6 concludes.

# 2 Weber's experiment and findings

## 2.1 The experiment:

Weber (2006) experimentally investigates a minimum-effort coordination game. In this game there are n players who simultaneously and independently choose between seven actions: 1,2,3,4,5,6,7, which are interpreted as effort levels. Payoffs are given by Table 1, Weber (2006)

TABLE 1—PAYOFFS (IN DOLLARS) FOR MINIMUM-EFFORT GAME

| | | Minimum choice of all players | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| Player's choice | 7 | 0.90 | 0.70 | 0.50 | 0.30 | 0.10 | −0.10 | −0.30 |
| | 6 | | 0.80 | 0.60 | 0.40 | 0.20 | 0.00 | −0.20 |
| | 5 | | | 0.70 | 0.50 | 0.30 | 0.10 | −0.10 |
| | 4 | | | | 0.60 | 0.40 | 0.20 | 0.00 |
| | 3 | | | | | 0.50 | 0.30 | 0.10 |
| | 2 | | | | | | 0.40 | 0.20 |
| | 1 | | | | | | | 0.30 |

(Note that there is no incentive problem). In the standard version the number of players is fixed equal to 12 and the game is repeated 12 times. In another version (growth), the game starts with 2 players that play the game for 22 rounds with new players joining over time, until the group consists of 12 players as well.

Describing the growth version more precisely, Weber varies two dimensions. First, he establishes growth groups. As the name suggests, these groups start small at a size of 2 and grow in size over time. The starting pair plays 5-6 rounds before the new agents, called entrants, join. These entrants join at given times until the group as a whole reaches 12 players (max. size) and only one entrant enters the group simultaneously, with the exception of the last two[1]. Also, all agents stay until the last round has been played. The growth path, deciding at which points entrants join the game is predetermined and common knowledge as well as the total number of rounds. The second dimension that Weber varies is the exposure to the group's history which includes all the previous minima reached by the specific group. If treated, entrants observe said history before joining and actively playing themselves and incumbents are informed about this.

Groups that vary in both dimensions are called **history groups** and are the main focus of this paper. Groups that only vary in the second dimension are called **no-history groups**. Both in history groups and no history groups, it is common knowledge if entrants are exposed to history or not**.** Lastly, groups that are not altered in any dimension, meaning they start with 12 players, are **control groups.** Besides checking for the effect of variating the two dimensions, control groups are there to check if the sample behaves differently in relation to previous weakest-link coordination experiments (they do not).

Besides the control groups in Weber's experiment there is data from other weakest link experiments on fixed groups of varying size, which are useful for comparison with growth groups at different stages in time. From here on, I will refer to all such fixed groups as control groups. There, all groups that start with 8 players or more end up at the least efficient equilibrium after 5 rounds or earlier. (see Table 2, Weber (2006), Appendix)

[1] Weber experiments with more than one growth path. However, they only differ marginally so I do not treat them differently

## 2.2 The experimental results:

Weber's experiment exhibits some key findings that I aim to explain with my model. These are:

1. History groups show significantly higher minima on average than control groups.
2. History groups tend to perform lower than the starting pair (not 100% efficient).
3. No-history groups do not achieve higher minima than control groups.

Although not as prominent as the findings above, two other points are worth noting.

4. There seems to be more instability concerning the (growth) groups equilibrium in later stages of the game.
5. Correlation between an entrant joining and the minimum decreasing in both forms of growth groups.

The first three findings are the clearest ones. Weber (2006) and prior experiments [Van Huyck et al. (1990), Camerer and Knez (2000), Knez and Camerer (1994); Gerard P. Cachon and Camerer (1996), Chaudhuri et al. (2001)] demonstrate that with a larger fixed group size, minima decrease quite drastically. Control groups from the size of 8 end up at a minimum of 1 after 5 rounds in every case. For Weber's history groups, 5 out of 9 cases have a minimum higher than effort level 1 four rounds after reaching their max. size, two of them being at the highest level possible (see Table 4, Weber (2006), Appendix).[2] Here you can also observe finding 2. That is even though entrants observe all previous minima of the group and it improves the average minima compared to control groups, most groups do not manage to maintain the minimum reached by two player groups (86% level 7, see Table 2, Weber (2006), Appendix). Further it is clearly visible, that no-history groups do not sustain higher levels of coordination. By the time these groups reached a size of 8 all of them reached the lowest level, coinciding with the findings from previous experiments on control groups seen in Table 2 of Weber (2006).

Finding 4. comes from the Table 3, Weber (2006) (see Appendix) describing the course of group minima in Weber's treatment, where there seems to be more instability in the middle to later stages of a game. In Weber's chapter "Growing Efficient Coordination, he argues that the variance decreases from the previous round if all entrants are informed of the history. This

[2] Important to keep in mind is, growth groups play significantly more rounds than control groups, so there are more opportunities for the minimum to drop

would imply that as long as all players are informed, the variance decreases round by round, making a decrease less and less likely the longer the game goes on.
The last finding is the correlation between an entrant joining and the minimum decreasing mentioned by Weber in one of his footnotes. 71% of all decreases in minima occurred in rounds an entrant joined.

# 3 Theory

## 3.1 The game

To analyze this coordination game of seven effort levels, I am simplifying it to two choices: High, representing effort level 7 and Low, representing effort level 1.
One reason to simplify it this way is that both effort level 7 and effort level 1 stand out, as 7 is the payoff dominant action and level 1 is the risk dominant action. Further assume that players are risk neutral and they maximize their expected payoffs

Simplifying Weber's weakest link coordination game gives a one-shot game of the form:

| | | **minimum** | |
|---|---|---|---|
| | | H | L |
| **P1** | H | 0.9 | $-0.3$ |
| | L | 0.3 | 0.3 |

Introducing more rounds to it leads to an n-player, m-rounds version of the one-shot game depicted above. I will simplify the strategy space of the repeated game by restricting attention to two strategies, "start high"(H*) and "start low"(L*) where in every round but the first, both strategies play the lowest number of the previous round.

| | | minimum | |
|---|---|---|---|
| | | H* | L* |
| **P1** | H* | $0.9 * m$ | $-0.3 + 0.3 * (m-1)$ |
| | L* | $0.3 * m$ | $0.3 * m$ |

I will analyze the interactions with the aid of Kets and Sandroni's (2019) notion of an introspective equilibrium. In the set-up of Kets and Sandroni (2019) all players of a game receive an impulse prior to the start. These impulses are drawn independently from a given distribution, are privately observed and do not affect the payoff function. They represent the "instinctive reaction" to the game and are an element of the set of actions players can take in a game.

I assume that impulses can take one of two values, H with probability p or L with probability 1 – p. This way players take the sucker's payoff into account, inspired by Blonski et al. (2011). In my simplified version impulses of players are also independent of each other, whereas in Kets and Sandroni (2019) cultural factors such as group membership matter. In my model I abstract impulses from such factors.

In the one-shot game, impulses H and L correspond to the one-shot actions H and L respectively, for the repeated game they correspond to the strategies H* and L*

## 3.2 Noise

Another addition to my model is the existence of noise during the game. Noise (denoted with ε) is the exogenous random error probability every player has, and in accordance with Crawford (1991), it causes agents to deviate from their action with said probability. As there are only two actions in my model, the action from an error is clear. Also, errors are unintentional (they don't choose to make a mistake) and can occur both to players intending to choose H or L. The noise can be understood as a residual inability of players to best respond, summing up factors such as players inattention and forgetfulness. From a player's perspective it means the remaining lack of trust about other players (to best respond).

## 3.3 Solution concepts.

### 3.3.1 Introspective Equilibrium

Kets and Sandroni's (2019) theory states that in an environment of strategic uncertainty players resolve said uncertainty by taking another person's perspective. This is based the theory of mind, a concept from psychology (see Apperly (2012)). Taking another's person perspective requires players to reflect on their impulse (introspection). Kets and Sandroni define a concept of introspective equilibrium for strategic form games. Before choosing their actions each player privately observes a signal (called impulse) drawn from a commonly known distribution. A player's set of signals is equal to her set of actions, and the drawn signal is interpreted as an impulse to take the action in question. However, players do not act on the impulse. Instead, they go through an introspective process by which they

(i) calculate the best response assuming that all other players follow their impulses,

(ii) calculate the best response assuming that all others behave as described by (i)

(iii) calculate the best response assuming that all others behave as described by (ii)

… and so on forever.

This repeats itself until players choose to no longer alter their action. The limit of this process is the introspective equilibrium. For this Kets and Sandroni (2019) assume that the distribution of impulses is common knowledge.

Or to put it differently: An IE means that all players of a game have an instinctive reaction to the game before the first round starts, they reflect on their instinct, realize that others also have an instinctive reaction and choose a strategy based on this process. All players choosing their strategy based on an impulse at the start of a game works well for groups where every player participates from the start. In an environment where informed entrants join over time it is however insufficient. There entrants are confronted with the actual response of others instead of their beliefs about those. To extend this concept to extensive form games with entrants, I have to adapt it.

### 3.3.2 Defining "introspective equilibrium with informed entrants"

To extend the introspective equilibrium to an extensive form game I introduce the concept of introspective equilibrium with informed entrants.

**Definition:** An Introspective Equilibrium with informed entrants (IEie) is a strategy profile such that
(1) Incumbents strategy profile constitutes an IE in the game that would result from removing all periods except the ones in which only the incumbents participate.

(2) All players best respond to each other (in the game as a whole)

(3) The strategy profile does not require incumbents to change their behavior when new players enter, given that such a strategy profile respects (1) and (2)

This definition does not apply to no-history groups where uninformed entrants would have trouble best responding to history as they do not observe previous minima. As a consequence, all active participants (incumbents and entrants) draw a signal (impulse) whenever an entrant joins, and the introspective process repeats itself, given the current player count $n_t$. This means that a game with **un**informed entrants can be analyzed as a series of games with fixed player sets where the ordinary notion of IE can be applied.

In general, the intuition is that without history players build an introspective eq., whereas with history players choose to best respond to it.

# 4 Results

As mentioned in 3.1 the available strategies are "start high"(H*) and "start low"(L*). Thus, in the very first round with two incumbents, the possible strategy profiles are: (a) (H*; H*), (b) (H*; L*), (c) (L*; H*), (d) (L*; L*). Directly following the strategy implied by the impulse defines the level-0 strategy.

Let $p$ be the probability of impulse H and let all players choose their impulse as their action (level-0 strategy). Since the game at hand is a weakest link game, probabilities of the minima (if all players are level 0) are:
$P(min = H) = P(X = n) = p^n \qquad P(min = L) = P(X < n) = 1 - P(H) = 1 - p^n$

## 4.1 Results without noise

The first result establishes a condition under which different strategy profiles constitute an IEie of the repeated game with informed entrants.

*Proposition 1:*

- (a) (H*; H*) constitutes an IEie if $\boldsymbol{p}^* \geq \sqrt[n-1]{\frac{\mathbf{1}}{\boldsymbol{m}+\mathbf{1}}}$

- (d) (L*; L*) constitutes an IEie if $\boldsymbol{p}^* \leq \sqrt[n-1]{\frac{\mathbf{1}}{\boldsymbol{m}+\mathbf{1}}}$

The IEie is unique unless $P(H) = \boldsymbol{p}^*$

*Proof of proposition 1:*

*Given the probabilities mentioned in the beginning of this chapter, the expected utilities for a n-player one-shot game are:*

$E[U(H)] = 0{,}9p^{n-1} - 0{,}3(1 - p^{n-1}) = 1{,}2p^{n-1} - 0{,}3$

$E[U(L)] = 0{,}3$

*If incumbents consider multiple rounds* (1) *together with* (2) *"players best respond in the game as a whole" it changes to:*

$E_{n,m}[U(H^*)] = 0{,}9mp^{n-1} + (0{,}3m - 0{,}6)(1 - p^{n-1})$

$= 0{,}6mp^{n-1} + 0{,}6p^{n-1} + 0{,}3m - 0{,}6$

$E_{n,m}[U(L^*)] = 0{,}3m$

*It holds that* $E_{n,m}[U(H^*)] \geq E_{n,m}[U(L)]$ *if and only if:*

$0{,}6mp^{n-1} + 0{,}6p^{n-1} + 0{,}3m - 0{,}6 \geq 0{,}3m$

$\Leftrightarrow 0{,}6mp^{n-1} + 0{,}6p^{n-1} \geq 0{,}6$

$\Leftrightarrow p^{n-1}(m + 1) \geq 1$

$\Leftrightarrow p^{n-1} \geq \frac{1}{m+1}$

$\Leftrightarrow p \geq \boldsymbol{p}^* := \sqrt[n-1]{\frac{\mathbf{1}}{\boldsymbol{m}+\mathbf{1}}}$

*Hence* H* *is the best response to the level-0 strategy if the inequality "$\geq$" is fulfilled and* L* *if "$\leq$" is fulfilled. If both inequalities hold at the same time, that is,* $p = \boldsymbol{p}^*$ *both* H* *and* L* *are best responses and the equilibrium is not unique.*

QED

Note that to $\boldsymbol{p}^*$also applies:

For $n \to \infty,\ p^* \to 1$

For $m \to \infty,\ p^* \to 0$

If n and m grow at the same rate, for $n \wedge m \to \infty,\ p \to 1$

This demonstrates that the more player there are, the higher $p$ needs to be to establish the efficient equilibrium, while more rounds facilitate it.

Other implications from proposition 1 are that strategy profiles (b) and (c) "only exist as a result" at level-0, meaning before any incumbents introspective process has started. When they consider the other incumbent having an impulse (level-1), they realize that, for a given probability p, switching strategies might be optimal to match the other's anticipated action (based on the calculation in the proof of proposition 2.) While strategy profiles (a) and (d) are equilibria of the game, they are only an IEie if the respective inequalities hold. Otherwise, incumbents best respond by switching from (a) to (d) or vice versa. Switching from (a) to (d) corresponds to the inefficient lock-in in Kets and Sandroni (2019), where players choose what action they believe to be salient rather than payoff-efficient, even though they themselves had the impulse to do so.

Furthermore, the introspective equilibrium with informed entrants is reached at level-1 at latest. This is due to the combination of introspection and coordination game without multiple types of players. As explained before level-1 is the best response to level-0. With common beliefs about $p$ incumbents reach the same "conclusion" at level-1. And since it is the best response to match the action of others, the level-2 strategy of oneself is equal the level-1 strategy of the other incumbents which is the same as one's own level-1 strategy.

## 4.2 Results with noise

The second result establishes the condition for which values of $\varepsilon$, IEie (H*; H*) remains stable

*Proposition 2 (introspective equilibrium with informed entrants under noise):*

When considering errors, nobody deviates intentionally from the introspective equilibrium (H*; H*) (from best responding to it) if the error probability $\boldsymbol{\varepsilon}$ fulfills: $\mathbf{1} - \sqrt[n-1]{\frac{\mathbf{1}}{\boldsymbol{m}+\mathbf{1}}} > \boldsymbol{\varepsilon}$

*Proof of proposition 2:*

*The error lies in the execution of an action and not the introspective process so the process itself is not directly affected. The error probability needs to be sufficiently low so that players do not deviate from the IE in expectation of an error occurring. For this,* $E[U(H^*)] > E[U(L^*)]$ *needs to hold after considering for errors. The probability of no error occurring is* $1 - \varepsilon$*, so I calculate* $E[U(H^*)] > E[U(L^*)]$ *with the probability of no error occurring instead of* $P(min = H)$ *giving me* $1 - \varepsilon > \sqrt[n-1]{\frac{1}{m+1}} \Leftrightarrow \mathbf{1} - \sqrt[n-1]{\frac{\mathbf{1}}{\boldsymbol{m}+\mathbf{1}}} > \boldsymbol{\varepsilon}$.

QED

Proposition 2 states the error probability needed so players do not deviate from the IEie (H*; H*) in expectation of an error happening. If $\varepsilon$ would be too high, then it is players best response to switch from action H to action L in anticipation of somebody else making an error. It can be understood as an upper limit for the error probability, so action H remains a best response to the IEie (H*; H*)

The third result constructs the probability of H being the minimum effort in round $t$, given IEie = (H*; H*)

*Proposition 3:*

If (H*; H*) is the IEie of the first round, the probability of the efficient IEie to remain at period T is:

$$\prod_{t=2}^{T} (1 - \varepsilon)^{n_t}$$

From that it follows that the probability of having reached low at period T when starting high is:

$$1 - \prod_{t=2}^{T} (1 - \varepsilon)^{n_t}$$

<u>*Proof of proposition 3:*</u>

*The probability of making no error occurring in a round is* $1 - \varepsilon$ *and since it is a weakest link game not a single player can make a mistake so* → $(1 - \varepsilon)^{n_t}$. *This has to repeat over consecutive rounds for the equilibrium to remain at high after being established in the first round, so with a changing number of participants the probability of no error occurring in any round so far is*

$$\prod_{t=2}^{T} (1 - \varepsilon)^{n_t}$$

*The probability of having reached low is the probability of at least one error occurring in the rounds so far, which simply is the complement of no error occurring, so*

$$1 - probability\ of\ no\ error = 1 - \prod_{t=2}^{T} (1 - \varepsilon)^{n_t}$$

QED

Given that the incumbents have coordinated on strategy profile (H*; H*), proposition 3 describes the probability of a group to have a minimum of H (or L) at round $t$, dependent on *T* (number of rounds played so far), $\varepsilon$ (error probability) and $n_t$ (number of players in round t).

# 5 Explaining the experimental findings with the theory

Next, with the definition of IEie and the propositions above I am going to explain Weber's key findings listed in 2.2. First in a scenario without noise and secondly with noise.

## 5.1 Without noise:

Without noise is a benchmark scenario of sort and it is apparent this is too optimistic.

Proposition 1 ($\boldsymbol{p}^* > \sqrt[\boldsymbol{n-1}]{\frac{\boldsymbol{1}}{\boldsymbol{m+1}}}$) together with players best responding (definition (2)) explains finding 1. Control groups start with $n = 12$ and as a consequence the critical value p* needed for the introspective process to result in a High equilibrium is larger. Thus, it is less likely compared to growth groups that start with $n = 2$. This holds even after considering that players in control groups might take more future rounds into account. Unless the number of rounds for control groups massively outnumbers the incumbents' in history groups, the negative effect of more players dominates the positive effect of more rounds.

No-history groups being analyzed as a series of games with ordinary IE explains finding 3. since there, no-history groups are modelled to play similar to control groups of the same size. What it cannot explain is finding 2., that is that history groups tend to perform lower than the starting pair. Without an error probability, my model predicts that the minimum reached by the IEie is maintained without fail. This contradicts finding 2.

By the same argument it cannot explain finding 4., the seemingly higher instability in later stages of the game as well as finding 5. (correlation between entrants joining and minimum decreasing). 5. is not fulfilled only for history groups. No-history groups' decrease can be explained by the repeated introspective process when uninformed entrants join.

## 5.2 With noise:

Adding noise to the model, proposition 1 now together with proposition 3 still explains finding 1. and 3. The lower $n$ facilitates the payoff efficient equilibrium during the introspective process while a sufficiently low error probably allows for a share of groups to maintain it. As an example, with an error probability of 0.01, this model predicts 26.27% of history groups to maintain H as the minimum with proposition 6 being fulfilled at the same time (2 out of 9 ≈ 0,22 in Weber's experiment). The highest tolerable error probability in this game is $1 - \sqrt[12-1]{\frac{1}{1+1}} \approx 0.061$ (max. number of players ($n = 12$) and min. number of rounds ($m = 1$))

Regarding finding 3., with noise scenario predicts that no-history groups perform at most as good as control groups. Their introspective process equals that of controls groups of the same size but since they play more rounds, they are predicted to make more mistakes.

Proposition 3 explains finding 2. and 4. because with every period as well as with every additional entrant the probability of an error occurring rises. With every additional round played $1 - \prod_{t=2}^{T}(1-\varepsilon)^{n_t}$ (probability of reaching "Low") increases (finding 2.).

And with every added entrant the probability of the minimum dropping rises as $(1-\varepsilon)^n$ decreases with an increase in "n" and hence $1-(1-\varepsilon)^n$ (probability of at least one error) increases (finding 2. and 4.).

Finding 5. regarding history groups is still not explained. To keep in mind is that an error occurring and a player joining can coincide especially if an entrant joins every round for 8 or more consecutive rounds (As it happens in Weber (2016) experiment). However, the correlation is still a concern.

# 6 Conclusion

I created a model to explain the different behavior of groups in weakest link coordination game based on their growth path (starting large vs. starting small and growing large). It shows that smaller groups are more likely to coordinate on high effort levels while at the same time allowing them to maintain it if informed entrants join the group, resulting in high effort levels despite being a large group later on. The model indicates the importance of the starting pair to establish a high level of effort as a minimum which informed entrants orientate on. Projecting this on the real world would implicate that a group's performance is significantly shaped by their initial members. This appears intuitive as it is another form of "success breeds success".

However, it is a simplified model, so limitations are unavoidable. By simplifying the game from seven choices to two, it loses out on range. Mentioned previously, this theory can model how many groups will maintain a coordination level of 7, represented by High in my model. On the other hand, as one error means the equilibrium will go to Low, representing effort level 1, effort levels from 2 to 5 cannot be modelled and are aggregated together with level 1 instead. Another limitation is introduced through the introspective process. It is assumed that this happens instantaneously or to put it differently, it finishes with the first round. As sometimes seen in Weber's experiment and more thoroughly explained in Crawford (1991), a fixed two player pair needs a bit longer to coordinate on one equilibrium. Typically, they tend to increase

their coordination level over time, if it is not already at the maximum. This is however not impactful on my model as groups performance in all experiments usually is given by their fifth period minimum which my model compares to.
Lastly, with the error probability comes a limitation as well as implications. It models that players deviate but does not explain why. While it is realistic that mistakes happen, be it through inattention or forgetfulness, it is limited in explaining why it happens. Also, with an error probability groups are predicted to end up at the lowest effort level. This implies that a successful team of incumbents and informed entrants is not enough to assure sustained levels of high performance indefinitely. Without a tool to sort out errors, effort levels of group members would eventually dwindle leading to the lowest effort level. One possible way of groups to solve this is the creation of social norms, which appears to happen in one of Weber's (2006) sessions.
Furthermore, the prospect of inevitable errors is less dire in the real world since there not every interaction fails due to one single error. In a less error sensitive scenario, a single error is simply not enough to "torpedo" the efforts of every other group member which makes it easier to correct one's own action in future rounds.

The paragraph above already hinted at some avenues one could research and people are already doing. Better understanding the cause of errors allows to take precautions compared to an exogenous error probability that you do not know the source of. How to enable groups to "repair" errors through social norms using evolutionary game theory is another. When considering ways of extending my model, implementing player types and/or heterogenous beliefs about impulses is one option. The closest example for this can also be found in Kets and Sandroni (2019) where players belong to cultural distinct groups causing heterogeneity between types and cultural strength allowing for heterogeneity inside a type. Another could be by accounting for inattention or forgetfulness, which in my model would be informed entrants ignoring a group's history (through higher error probabilities for new entrants for example).

# Appendix

Table 2 from Weber (2006)

TABLE 2—DISTRIBUTIONS OF FIFTH-PERIOD GROUP MINIMA IN VARIOUS 7-ACTION MINIMUM-EFFORT STUDIES
(*1 = inefficient; 7 = efficient*)

| Minimum choice in fifth period | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 6 | 5 | 4 | 3 | 2 | 1 | Group size | Number of groups | Source |
| 86% | 3% | 3% | 3% | 0% | 0% | 5% | 2 | 37 | VHBB, CK |
| 18% | 4% | 0% | 11% | 15% | 15% | 37% | 3 | 27 | KC, CK |
| 0% | 0% | 0% | 0% | 10% | 10% | 80% | 6 | 10 | KC |
| 0% | 0% | 0% | 0% | 0% | 0% | 100% | 8 | 5 | CSS |
| 0% | 0% | 0% | 0% | 0% | 0% | 100% | 9 | 2 | CC |
| 0% | 0% | 0% | 0% | 0% | 0% | 100% | 14–16 | 7 | VHBB |

*Sources:* Van Huyck et al., 1990 (VHBB); Camerer and Knez, 2000 (CK); Knez and Camerer, 1994 (KC); Gerard P. Cachon and Camerer, 1996 (CC); Chaudhuri et al., 2001 (CSS).

Table 4 from Weber (2006)

TABLE 4—DISTRIBUTIONS OF SUBJECT CHOICES IN FOURTH PERIOD AS 12-PERSON GROUPS

| | | Control | Growth and history | Growth and no history |
|---|---|---|---|---|
| Choice | 7 | 3 (5%) | 32 (30%) | 0 (0%) |
| | 6 | 0 (0%) | 3 (3%) | 0 (0%) |
| | 5 | 6 (10%) | 13 (12%) | 0 (0%) |
| | 4 | 22 (37%) | 8 (7%) | 0 (0%) |
| | 3 | 7 (12%) | 10 (9%) | 0 (0%) |
| | 2 | 9 (15%) | 9 (8%) | 1 (3%) |
| | 1 | 13 (22%) | 33 (31%) | 35 (97%) |
| Total | | 60 | 108 | 36 |
| Minima | | 1, 1, 1, 1, 4 | 1, 1, 1, 1, 3, 4, 5, 7, 7 | 1, 1, 1 |

Table 3 from Weber (2006)

TABLE 3—AVERAGE MINIMA (MEDIANS) BY SESSION FOR RANGES OF GROUP SIZE

| | Growth path 1: Group size (number of periods at that size) | | | | | First $n = 12$ minimum |
|---|---|---|---|---|---|---|
| | 2 (6) | 3 (2) | 4–6 (4) | 7–11 (5) | 12 (5) | |
| Session 1 (h) | 7.0 (7) | 6.0 (6) | 4.5 (4.5) | 2.0 (2) | 1.0 (1) | 1 |
| Session 2 (h) | 6.3 (6.5) | 5.5 (5.5) | 5.3 (5) | 5.0 (5) | 4.2 (5) | 5 |
| | Growth path 2: Group size (number of periods at that size) | | | | | |
| | 2 (5) | 3 (4) | 4–6 (4) | 7–11 (5) | 12 (4) | |
| Session 3 (h) | 7.0 (7) | 5.0 (5) | 5.0 (5) | 3.4 (5) | 1.0 (1) | 1 |
| Session 4 (h) | 7.0 (7) | 7.0 (7) | 7.0 (7) | 7.0 (7) | 5.5 (5.5) | 7 |
| | Growth path 3: Group size (number of periods at that size) | | | | | |
| | 2 (5) | 3 (4) | 4–6 (4) | 7–10 (4) | 12 (5) | |
| Session 5 (h) | 6.6 (7) | 7.0 (7) | 7.0 (7) | 3.3 (3) | 2.6 (3) | 3 |
| Session 6 (h) | 7.0 (7) | 7.0 (7) | 7.0 (7) | 3.5 (3.5) | 1.0 (1) | 1 |
| Session 7 (h) | 6.0 (6) | 6.0 (6) | 4.8 (6) | 4.0 (4) | 2.0 (1) | 4 |
| Session 8 (h) | 7.0 (7) | 7.0 (7) | 7.0 (7) | 7.0 (7) | 5.8 (7) | 7 |
| Session 9 (h) | 7.0 (7) | 7.0 (7) | 7.0 (7) | 7.0 (7) | 7.0 (7) | 7 |
| Session 10 (nh) | 5.8 (6) | 7.0 (7) | 3.8 (4) | 1.8 (1) | 1.0 (1) | 1 |
| Session 11 (nh) | 7.0 (7) | 7.0 (7) | 2.8 (2.5) | 1.0 (1) | 1.0 (1) | 1 |
| Session 12 (nh) | 5.6 (6) | 1.0 (1) | 1.0 (1) | 1.0 (1) | 1.0 (1) | 1 |

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