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Bitcoin as an enhancer of performance in an All Weather inspired portfolio

"*An analysis of Bitcoin in a Sharpe-ratio optimized All Weather inspired portfolio*"

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Bachelor Thesis in Economics

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Abstract:

The thesis employs a number of tools originating from the field of Modern Portfolio Theory to assess the appropriate weight of Bitcoin in addition to several assets commonly found in the All-Weather Portfolio. Utilizing Sharpe Ratio optimization, the thesis compares two main portfolios to determine if the inclusion of Bitcoin can improve the performance and risk-return ratio of an optimized All Weather Portfolio. A short introduction to the asset class is provided in combination with a basic explanation of the methodology of portfolio optimization and the origin of the All Weather portfolio. The results of the optimization suggest that a portfolio weight of approximately 1% in an optimized All Weather inspired portfolio results in a higher Sharpe Ratio and a higher expected annual return in comparison with a traditional All Weather inspired portfolio. Besides the increased Sharpe Ratio, both the Treynor Measure and Jensen's Alpha were affected positively by the inclusion of Bitcoin. All the portfolios including Bitcoin exhibited better performance than their traditional predecessors. The thesis then moves on to conclude that due to the rejected normal distribution of Bitcoin and utilization of the Sharpe Ratio, the results should be interpreted with some scepticism.

Table of content

List of figures:

List of tables:

List of equations:

1 Introduction

January 2009 saw the birth of the first known cryptocurrency, the now infamous Bitcoin. A person or potentially persons under the name “Satoshi Nakamoto” was the first to bring forward the concept of a decentralized digital currency based on the ledger system known as blockchain. Relying on authentication system called “proof of work”, where the authentication of transactions is based on a peer-to-peer system (Nakamoto, 2008). Since then Bitcoin has proved a relatively volatile asset setting record highs constantly during the last months since the COVID-19 crash, as of today its highest recorded price exceeds $63,000 (CoinmarketCap, nd-a). Being presented to the world as an alternative to the traditional fiat currencies, Bitcoin has been branded a speculative asset more than a valid currency for transactions (Yermack, 2015).

Years prior to the original paper for the theory behind Bitcoin was published by Satoshi Nakamoto, Ray Dalio was working on constructing a portfolio which achieved the level of diversification to perform even in the direst of times. The strategy he ultimately settled on is today known as the All Weather Strategy, the strategy then manifested in a portfolio of asset classes called the All Weather Portfolio (Podolsky, et al., 2012).

The All Weather Portfolio is collection of asset classes chosen to achieve stable performance even during a turbulent market environment, hence the correlation between the assets of choice is of great importance. Using the findings from economists such as Harry Markowitz and William Sharpe, an optimization of a number of portfolios will be performed, and their returns analysed respectively. The question to answer is:

“Will the inclusion of Bitcoin in an All Weather inspired portfolio improve the performance?”.

This paper focuses on the implications of including the relatively new asset class, cryptocurrency, or more specifically the first known kind of them all, Bitcoin, into a theoretical portfolio consisting of assets inspired by the All Weather Strategy. In chapter 2 an introduction to the different theories, performance measurements and strategies are presented along with a brief description of the properties of Bitcoin.

The thesis presents two theoretical combinations of assets in portfolios and compare their different characteristics alongside their performance respectively. By using data concerning the assets included and optimizing the portfolio weight of each asset in a number of portfolios, the thesis analyse the outcome and ultimately pass on judgement of Bitcoins fit in a diversified investment portfolio based on the Sharpe Ratio, Treynor Measurement and Jensen's Alpha. The scope of the thesis is to conclude if Bitcoin enhances the different measurements of performance presented in chapter 2 and determine the appropriate portfolio weight in an All Weather inspired portfolio for an American investor.

2 Theoretical Background

2.1.1 Modern Portfolio Theory

Harry Markowitz published in 1952 an article in the Journal of Finance with focus on asset selection and risk in portfolios which laid the ground work for the current modern portfolio theory, an article which earned him an Nobel prize in 1990 for his contribution to the financial field of research alongside Sharpe (Nobelprize, n.d).

Markowitz introduced the concept of mean-variance to the field of portfolio theory and proved that by choosing assets through expected return and the correlation between the returns of other assets, one could achieve the same expected return of a portfolio with lower risk than one focused solely risky assets with a high expected return and high volatility. Markowitz theory builds on the notion that individuals are rational by nature and always seek maximum expected return and minimized risk of a portfolio. In the paper Markowitz uses the volatility of returns as a measurement of risk, the probability of an asset/portfolio moving away from its expected return. Markowitz's research on correlation between assets helped form the concept of diversification within portfolio theory (Markowitz, 1952).

Markowitz visualized this concept through the efficient frontier, plotting possible combinations of assets associated with a certain level of expected return and risk. As a result, all efficient portfolios lie on the efficient portfolio line. Markowitz findings made it possible for investors to choose an efficient portfolio given the level of risk one could individually tolerate (ibid). Markowitz's findings and researched was built on several estimates given by the assets included, ultimately resulting in estimates of the portfolio. These are given by:

Expected return of the portfolio:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i) \qquad \textit{Equation 1}$$

Where *(w)* denotes the weight of each asset in the portfolio and $E(r_i)$ refers to the expected return of asset (*i*), following the formula:

$$E(r_i) = \bar{r} = \frac{1}{n}\sum_{t=0}^{n} r_t \qquad \textit{Equation 2}$$

The variance of the portfolio is given by:

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) \qquad \textit{Equation 3}$$

The covariance between two assets *i* and *j* daily returns is calculated as:

$$Cov(r_i, r_j) = 1/(n-1) \sum_{t=0}^{n} (r_{i,t} - \bar{r}_i)(r_{j,t} - \bar{r}_j) \qquad \textit{Equation 4}$$

William F. Sharpe further developed Markowitz theories and introduced the *Sharpe Ratio.* The Sharpe Ratio puts the expected return of a portfolio in relation to the potential risk-free return of a portfolio and the volatility of the portfolio in question. Sharpe used mathematics to formulate a problem of optimization from Markowitz findings, in turn creating an optimal Sharpe portfolio (Sharpe, 1994).

The Sharpe Ratio for an investment portfolio is calculated as:

$$Sharpe = (R_P - R_f)/\sigma_P \qquad \textit{Equation 5}$$

Where R_P is the return of the portfolio

R_f is the risk-free rate

σ_P is the standard deviation of the portfolio.

By using Markowitz and Sharpe's findings one can construct a portfolio which optimizes the Sharpe Ratio, hence providing the highest expected return associated with minimum risk. This can be done through numerical optimization, in this thesis Excel Solver will be used to perform the optimizations of the portfolios. The Excel solver also provides the service of locating an optimal portfolio when short selling is included in the portfolio. Research shows that by excluding short sales restrictions, an investor may double his/hers Sharpe Ratio of a portfolio when including up to approximately 100 assets (Moshe, L & R, Yaacov, 2001). Since the goal of this thesis is to examine Bitcoins properties as an enhancer of performance, we will thus include short selling in the portfolios to achieve the highest possible Sharpe Ratio given the data available.

2.1.2 Measuring performance of portfolios

To add further substance to the analysis the results will also include two other measures of portfolio performance besides the Sharpe Ratio. Both these measurements focus on systemic risk unlike the Sharpe Ratio which focus on the standard deviation of the portfolio. Systemic risk is measured as the beta of the portfolio which provides investors with more information to base their decisions regarding portfolio choices on.

2.1.3 Treynor Measure

The Sharpe Ratio focuses on the risk associated with the volatility of the portfolio, on the contrary, Treynor instead focuses on the systemic risk associated with the portfolio. Systemic

risk can be described as the likeliness that an event affects the economy or the industry. This is measured as the Beta (β) of the portfolio or an asset. The beta tells an investor the sensitivity of a certain stock or portfolio to the overall market, thus if the market moves a certain amount in either direction, the asset or portfolio moves the same amount times the beta. Beta can be calculated with respect to any overall market, in this thesis we will use the S&P 500 as a benchmark. The beta is calculated as:

$$\beta_s = Cov(r_s, r_m)/Var(r_m) \qquad \textit{Equation 6}$$

Where:

β_s denotes the beta of a stock or asset

r_s denotes the returns of the stock or asset

r_m denotes the returns of the market or benchmark the investor chooses

Treynor's measurement bears likeness to the concept of the Sharpe Ratio, the difference being that Treynor substitutes the variance of the portfolio with the beta of the portfolio. The beta of the portfolio (β_P) is calculated as the respective weight of each asset in the portfolio multiplied with the asset's beta towards the market (Treynor, 1965).

The Treynor measure is given by:

$$Treynor\ Measure = (R_P - R_f)/\beta_P \qquad \textit{Equation 7}$$

The Treynor Measure excludes the risk associated with variance of the portfolio and thus assumes that the portfolio which performance is being measured is sufficiently diversified. The Treynor Measure is best suited to be analysed alongside the Sharpe Ratio to gain an overlook of both measurements.

2.1.4 Jensen's Alpha

Jensens's Alpha is unique in that it measures the excess return as the return of a portfolio excluding the CAPM, where the CAPM is calculated as:

$$CAPM = r_f + \beta_P(E(r_m) \qquad \textit{Equation 8}$$

Jensen's Alpha suggests that the excess return is subject to not only the risk-free rate but also the beta of the portfolio multiplied with the expected return of the market. The alpha is then calculated as:

$$Jensen's\ Alpha = R_P - CAPM \qquad \textit{Equation 9}$$

Jensen's Alpha measures the return of a portfolio or fund that is in excess of the market, portfolios relying heavily on exposure towards the market will earn a lower percentage as a result. Jensen's Alpha is most suitable for actively managed funds or portfolios where it estimates the managers ability to obtain returns adjusted for market risk, in the case of an

optimization, the Alpha will serve as measurement of the applicability of the optimization method of the Sharpe Ratio (Jensen, 1967).

2.2 The All Weather Portfolio and Strategy

The theory of the All Weather Portfolio or the "All Seasons Portfolio" as it is occasionally called, originated from the founder of the world's largest hedge fund to date, Ray Dalio. Ray founded Bridgewater Associates in 1975, initially working with risk consulting but in 1987 taking the step into managing assets and eventually becoming the world's biggest hedge fund (Podolsky, et. al., 2012).

Ray Dalio and his co-workers at Bridgewater created the strategy as a response to the standard 90's institutional portfolio's high correlation to the domestic equities and lack of diversification. The strategy is based on the notion that all markets can be shrunken down to a few driving factors, in the All Weather Portfolio these factors are described as *Inflation* and *Growth*, both these factors can be either expected to rise or fall, thus a portfolio should be constructed to perform during these possible scenarios. Bridgewater constructed a matrix to visualize the expected performance of asset classes during the circumstances of the market as can be seen in Figure 1 along with Bridgewaters perception of the asset's properties.

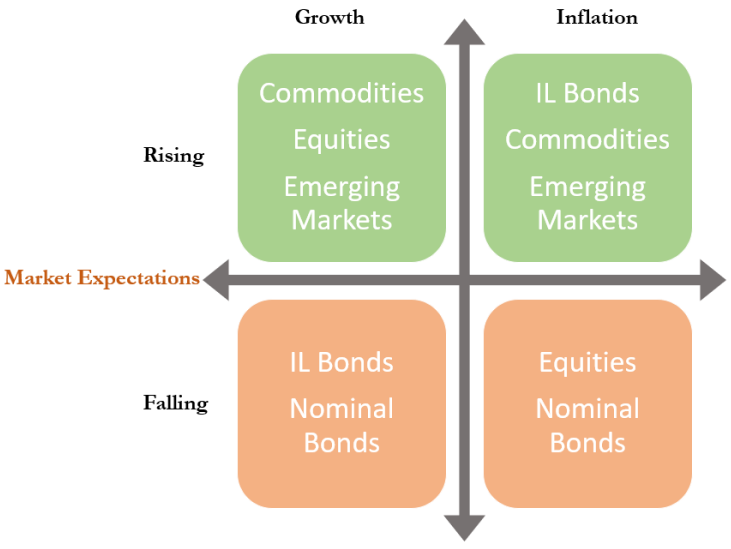

Figure 1: The All Weather Matrix. Source: Podolsky, et. al., 2012.

**Where IL Bonds refers to "Inflation Linked Bonds" and Nominal Bonds refers to bonds without a pay-off structure involving coupons.*

The All Weather Strategy focuses on the uncertain nature of investment and chooses to allocate between assets with the realisation that the future is unknown to investors and thus a risk averse investor should hedge for each possible scenario while simultaneously taking advantage of the capitalistic structure of central banks printing currencies, making intrinsic assets appreciate over time. The original strategy rebalances the portfolio depending on the market sentiment and the position in the existing economic cycle, it is not constructed to solely maximize the risk adjusted returns in the way that the Sharpe Ratio suggests.

This thesis instead draws inspiration from the assets chosen in the All Weather Portfolio and optimizes their respective weights through the Sharpe Ratio, hence the title of this thesis.

2.3 Bitcoin (BTC): A new phenomenon

Bitcoin first emerged from a whitepaper by a programmer or possibly programmers published under the name "Satoshi Nakamoto" in 2008, the paper described in detail the outlining of what came to be the block-chain technology (Nakamoto, 2008). Since then several other cryptocurrencies has accompanied Bitcoin in trying to claim market shares in various markets using either tokenization or simply trying to become the next digital currency of the future. Tokenization refers to the act of using block-chain technology to split up ownership of art, companies or voting rights, this is however, a property of other cryptocurrencies than the main asset of this thesis.

Bitcoin was the first known cryptocurrency and is currently the one with the largest market capitalization compared to its competitors. The top five biggest cryptocurrencies can be seen in Table 1.

Name	Market Cap (Billions) $	Circulating Supply	Type
Bitcoin (BTC)	749	18,713,700 BTC	Currency
Ethereum (ETH)	325	115,954,362 ETH	Platform
Binance Coin (BNB)	60	153,432,897 BNB	Currency
Tether (USDT)	58	58,019,510,515 USDT	Stablecoin
Cardano (ADA)	58	31,948,309,441 ADA	Platform

Table 1: Top cryptocurrencies by market capitalization. Source: Coinmarketcap.com (2021b)

The first vision for Bitcoin was that of a decentralized peer-to-peer payment system cutting out the middle hand which in today's society is provided by banks, meaning that Bitcoin is

not backed by any form of central bank and also has a limited supply. A standard fiat currency is subject to counterfeiting which it controls by usage of water stamps, silver striping and specific materials. For a completely digital currency, none of these alternatives are an option. Blockchain solves this issue by blocks of transactions which is authenticated by nodes or so-called miners. In a traditional monetary system, this authentication service is performed by banks, as a transaction is performed in the blockchain, however, the transactions are collected in blocks which is sent out to the nodes for authentication. This act of authentication is known as mining. Mining refers to the activity of solving complex mathematical problems to legitimize transactions of a Bitcoin block, each block contains 10 minutes worth of transactions. Solving a block will earn the miners different quantities of Bitcoin. As per 2019 each blocked mined is worth 12.5 BTC, approximately every 5^{th} year this reward is halved, making it more difficult for miners to profitable keep authenticate transactions. (Kaur et al., 2021) Mining serves an important role in the blockchain as it solves both the issue of counterfeiting and ownership through the peer-to-peer authentication. Mining is also the action which supplies the market with more Bitcoin, however there is a predetermined existing amount of Bitcoin which can't be exceeded in theory, the only individuals possessing the opportunity to alter the existing cap on Bitcoin would be the person responsible for the original source code, i.e. Satoshi Nakamoto (Nakamoto, 2008).

Research has shown that the original vision of Bitcoin as an alternative to fiat currency is hard to achieve since it largely fails to fulfil the criteria of a functioning currency. A currency must fulfil the criteria of functioning as a medium of exchange, a unit of account and a store of value (Yermack, 2015). Bitcoins high level of volatility makes the latter two of the criteria difficult to achieve. A currency which fluctuates in value makes transactions of predetermined values difficult. Storage and access to individual's Bitcoins is also a growing issue. While all Bitcoins are stored in the blockchain, companies provide the service of storing the access to the blockchain on their platforms. As the access information is stored online, it makes for a high value target for hackers. Bitcoin has been associated with illegal activity due to its decentralization, Bitcoin serves as hackers' primary currency of choice when demanding ransom for hostage situations regarding computerized systems or pipelines (Segendorf, 2014). An alternative is the so-called "cold storage", which stores the Bitcoin offline on a ledger. Cold storage is the safest alternative, but it makes transactions with Bitcoin difficult as the ledger needs access to a computer to function adding to the problem of Bitcoin as a store of value (Kaur et al., 2021).

In present time a number of companies has adopted Bitcoin as a mean of purchase, these include major actors such as Microsoft, Tesla and Starbucks (Tayeb, 2021). Thus, as a medium of exchange, Bitcoin seems promising. Outside of the data sample relevant to this optimization, Tesla withdrew their option to pay using Bitcoin citing concerns of environmental impact from mining. This concern has been voiced earlier among academics, going as far as to speculate in Bitcoin mining being one of few factors responsible for potentially driving the global warming temperature over the 2 Celsius thresholds (Badea & Mungiu-Pupazan, 2021). In addition to this the lack of fulfilment in the other criteria makes Bitcoin more of a speculative asset than a full-fledged currency (Wu & Pandley, 2017). Recent findings suggest that Bitcoin's price is significantly correlated to the investor attention, more so than fundamental properties (Zhu, 2021). Simultaneously, the very driver of the returns in Bitcoin is also found to be the driver behind the realized volatility in the asset in turn complicating a potential stabilization of prices in Bitcoin as it gains social acceptance/attention (ibid).

2.4 Previous research on the topic

Since Bitcoin or cryptocurrencies are a young asset class, academic research on the topic in a portfolio theory perspective is scarce. Research range from detailed analysis of the blockchain technology to comparison between gold and Bitcoin. Since the first major increase in investor attention at approximately 2017, more research has seen the light of day. However, several of the existing papers also evaluate Bitcoin's properties as a potential currency and leave more to be desired in a portfolio perspective.

Kaur et al. (2021) analyses Bitcoin's properties as an investment management tool and concludes that from a portfolio perspective, even though Bitcoin boasts impressive returns, it must make way for Japanese bonds in a portfolio from a risk-reward aspect, the paper is written from an Indian investors perspective and does not compare Bitcoin with bonds of other nationalities. According to Kaur et al. (2021) the volatility of Bitcoin has also seen a decrease during recent months at the date of the paper's origin, February 2020, making Bitcoin a more tempting alternative for investors. Since this paper was originally written a substantial depreciation and recovery has occurred in Bitcoin's price during the time of the COVID-crash of March 2020. Research on diversification focusing on Bitcoin has concluded that the optimal weight in an optimized global portfolio varies from a 0 to 5 % weight of a portfolio depending on assets included (Klabbers, 2017). Ahnhem & Lindberg (2017)

examines Bitcoin's role in a standard Swedish portfolio, their findings are in line with Klabbers (2017), a portfolio with an optimized Sharpe Ratio contains approximately 1,2 % Bitcoin. Klabbers also found that the risk inherited from Bitcoin is related to the very usage or media coverage of Bitcoin, further enforcing the findings of Zhu (2021) that Bitcoin's volatility is connected to investors' attention and not to the currency's fundamentals. Baek and Elbeck (2015) present evidence to further support this theory, showing that by comparing the volatile returns of Bitcoin to those of the S&P500 one may conclude that the driving forces in the price of Bitcoin is not based on fundamental properties but rather on internal attributes. Wu & Pandley (2014) concludes that Bitcoin is an interesting asset which could increase gains in a well-diversified portfolio and like Klabbers (2017) recommends a minor weight in the portfolio as long as Bitcoin is associated with the level of risk and uncertainty seen during the writing of the paper.

In conclusion, most of the papers have found that Bitcoin possesses positive traits to enhance portfolio performance when included in a portfolio consisting of assets associated with diversification. Inclusion in a portfolio is recommended to be no higher than 5 %. This thesis will serve as a further development of modern portfolio theory with Bitcoin as a main component while at the same time considering the development of the Bitcoin price data during the COVID-19 crash of 2020.

2.5 Purpose of thesis

Bitcoin is a unique and young asset, as earlier concluded, Bitcoin was originally visualized as a decentralized alternative to traditional fiat currency but has ultimately become a speculative asset for investors (Wu & Pandley, 2017). The appropriate weight of Bitcoin in a portfolio has been examined in academic literature before, many of which is discussed in 2.4. By the time this thesis has reached a conclusion, new data and properties of Bitcoin or other cryptocurrencies will have emerged, warranting additional research in the field.

Since COVID-19 began, a surge in speculative assets has occurred, this thesis aims to determine the appropriate portfolio weight of Bitcoin in an All-Weather inspired portfolio. Proving through financial theory and application that today's level of speculative mania in cryptocurrencies might not be financially sound for a rational investor, or even based on proper fundamentals.

3 Methodology

3.1 Portfolios

Three different portfolios will be analysed in this thesis, one containing a diversified combination of assets from the All Weather Strategy, including Bitcoin. This portfolio will be denoted AW (BTC) and will be optimized to maximize the Sharpe Ratio of the portfolio. Another portfolio will be created containing all assets as AW (BTC) with the exclusion of Bitcoin. This portfolio will serve as the main comparison component to AW (BTC) to analyse the effect of Bitcoin's inclusion. The portfolio optimized using only standard assets from the All Weather Strategy will be denoted AW. In addition to the two portfolios above, both will also be optimized without short sales restrictions, denoted (SS) after the portfolio name.

Two minimum variance portfolios will also be estimated to add substance to the increase in volatility an investor may expect when optimizing for the Sharpe Ratio. The minimum variance portfolio provides the theoretical minimum risk an investor may experience with the assets included and facilitates comparison between expected return and Sharpe Ratio between the both alternatives.

Volatility, i.e. standard deviation will be used as a proxy for risk in this thesis. Using the standard deviation as a measurement of risk has been debated in literature earlier and a discussion regarding this issue will be supplied in the discussion chapter of this thesis.

All assets in this optimization is originally denoted in USD to represent assets easily accessible by an American investor.

3.1.1 Returns

The daily returns of asset *i* are given by:

$$r_{i,t} = ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \qquad \textit{Equation 10}$$

3.1.2 Standard deviation of returns

The standard deviation of the daily returns is estimated by:

$$\sigma_{i,r} = \sqrt{\sum \frac{\left(r_{i,t} - \bar{r}_{i,t}\right)^2}{n-1}} \qquad \textit{Equation 11}$$

The standard deviation of the daily returns is then annualized by assuming 252 trading days per year and using this number to estimate the annual standard deviation of the asset. This is estimated by:

$$\sigma_{Annual} = \sigma_r * \sqrt{252} \qquad \textit{Equation 12}$$

3.1.3 Optimization Problem

Using the formula for the Sharpe Ratio and the equations presented in the theoretical background as well as above, the portfolio is then optimized using the Excel solver to choose the optimal weights to maximize the Sharpe Ratio. The Excel solver is tool included in Excel which performs optimizations given certain restrictions such that:

$$\max_{w} \; Sharpe_{P} = (R_P - R_f)/\sigma_P$$

A minimum variance portfolio is also estimated using the Excel Solver:

$$\min_{w} \; \sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)$$

All portfolios are optimized under the restriction that the sum of all weights for assets included must be equal to 100% of the portfolio such that:

$$\sum_{i=1}^{n} w_i = 1$$

4 Data

4.1 Data Collection

All data regarding assets in this thesis has been collected from Yahoo Finance. The period in question is 17th of September 2014 to 2nd April 2021, the period was chosen mainly since Yahoo Finance does not provide data from earlier but also since appropriately, 2014 marks the beginning of the era of BTC's emergence as an opportunity to investors (Kaur et al., 2021).

4.2 Selection of assets

The selection of assets to optimize is performed to follow a basic All Weather Portfolio allocation along with the inclusion of Bitcoin. The main inspirational components are shown in Figure 1. The assets included are presented in Table 2 along with the exposure their constructed to mirror.

Assets:	Classification	(Replicated) Content
SPX	ETF	S&P 500
TLT	ETF	US Treasuries with a length of 3-7 years
IEI	ETF	US Treasuries with a length of 20+ years
GLD	ETF	Gold
BTC/USD	Cryptocurrency	Bitcoin denoted in USD
MME	ETF	Emerging markets
GSG	ETF	Commodities (Low exposure towards gold)
VNQ	ETF	Real estate

Table 2: Summary of assets included in the portfolio.

In all cases of assets except for Bitcoin a proxy will be used for exposure towards the asset in the form of exchange traded funds. Exchange traded funds (ETFs) offer accessibility and liquidity to the average investor, they're also used as proxies to retain all assets in the same currency notation, while simultaneously offering a more diversified exposure towards assets of interest.

4.3 Description of asset included

Bitcoin (BTC):

BTC is easily accessible to smaller private investors through actors as CoinBase or Bittrex and naturally dividable into so called "*satoshi*"-units to allow for purchases of smaller

quantities than a whole BTC. The price development is denoted in USD. In this optimization, BTC will serve as the potential inclusion to the original All-Weather Portfolio.

SPDR Gold Trust (GLD):

The exchange traded fund GLD is constructed to mirror the development in the spot price of gold, it does so by purchasing and storing physical gold bars and bullions. GLD was chosen as exposure towards gold in the portfolios since it's the largest and most liquid alternative to holding physical gold for a private investor. GLD also liberates the investor from other costs associated with holding physical gold such as storage and handling fees (SPDR®, 2020a). In this optimization the GLD will serve as exposure towards Commodities.

Real Estate Van Guard (VNQ):

Van Guard Real Estate ETF is a broad exchange traded fund with exposure mainly towards the US real-estate market. VNQ is well diversified between different branches in the real-estate sector predominately through real-estate investment trusts (The Vanguard Group, 2018). In this optimization the VNQ will serve as exposure towards Equities.

SPDR Standard & Poor's 500 ETF Trust (SPY):

SPY is dedicated to replicate the US index S&P 500. The S&P 500 is an index consisting of the 500 largest actors by market capitalization in the US market (SPDR®, 2021b). In this optimization the SPY will serve as exposure towards Equities.

MSCI Emerging Markets Index (MME):

MME is a diversified ETF investing within several geographical areas including China, South Korea, Russia and Thailand among others. MME is also allocating between a wide array of sectors in each country (MSCI, 2020d). In this optimization the MME will serve as exposure towards Emerging Markets.

iShares S&P GSCI Commodity Indexed Trust (GSG):

GSG uses future contracts to replicate the returns of S&P GSCI, the future contracts are weighted according to the production world-wide. The index houses a lower level of exposure towards precious metals (5%) to not interfere with the portfolios existing exposure to GLD (iShares, 2020a). In this optimization the GSG will serve as exposure towards Commodities.

Bonds

iShares 3-7 Year Treasury Bond ETF (IEI):

IEI holds a portfolio of US treasuries with maturities of 3 – 7 years. The fund uses ICE US Treasury 3-7 Year Bond Index as benchmark (iShares, 2020b). In this optimization the IEI will serve as exposure towards Nominal Bonds.

iShares 20+ Year Treasury Bond ETF (TLT):

TLT seeks to mirror the performance of ICE US Treasury 20+ Year Index. The index is a market-weighted collection of US treasury notes with a remaining maturity of 20+ years. TLT is also subject to annual coupons from the US Treasury Bills (iShares, 2020c). In this optimization the TLT will serve as exposure towards inflation linked bonds.

4.4.3 Risk Free rate:

The US 3-month Treasury Bond has been utilized as the risk-free rate in the optimization of the portfolios. The rate has been annualized to fit an optimization of expected annual returns and volatility.

4.4 Data Issues

All assets in the data sets are assumed to have 252 trading days a year. This is true on average for the standard assets traded on regular markets, this is not true for cryptocurrencies as they are traded at all hours of the day during all days of the year. This is thus a simplification of the reality and is performed solely to be able to fairly estimate the correlation between the assets in each portfolio. The consequence of these actions is that Bitcoin appears slightly less volatile than it is and thus the results from this thesis should be treated with caution and not be interpreted as an absolute truth.

Since traditional assets are subject to holidays the data was cleaned by removing these days to keep the number of observations the same. This method was used on all assets and thus affected the standard deviation equally for all assets.

4.5 Descriptive statistics of the data

Ln Daily Returns of assets (Annualized)

	Mean Return (Annual)	**Standard Deviation (Annual)**	**Skewness**	**Excess Kurtosis**	**Shapiro-Wilk Test**	**Normality (Significance level of 5%)**	**β**
Gold	5%	2,3%	-0.12545	3.6493	0.960619	Rejected*	0.0665
Bitcoin	111%	89%	-0.68462	9.8145	0.895939	Rejected*	0.3885
S&P 500	11%	5,5%	-1.0220	20.480	0.823415	Rejected*	1.0
RealEstate	8%	10%	-2.2339	34.862	0.807957	Rejected*	0.6410
Commoditites	-11%	13%	-0.90009	8.1170	0.939353	Rejected*	0.2844
Bonds 3 – 7 Y	3%	0,8%	0.094694	5.8534	0.956459	Rejected*	-1.3147
Bonds 20+ Y	6%	3,6%	0.0044565	9.0135	0.930831	Rejected*	-0.5128
EmergingMarkets	4%	29%	-1.0855	10.574	0.919455	Rejected*	0.5263

Table 3: Descriptive statistics of the assets included in the portfolio.
Summary Statistics, using the observations 1 - 1646
(missing values were skipped)

From Table 3 can we conclude that none of the selected assets exhibit normal distributional properties when exposed to the Shapiro-Wilk test, the test rejects the null hypothesis that the returns are normally distributed at the significance level of 5 %. All assets returns in the portfolios presents a kurtosis exceeding 3.0, which is the standard for the normal distribution, as a consequence of this, the assets returns has heavier tails than a normally distributed asset resulting in larger deviations in the price occurring more frequently than the normal distribution assumption suggests (Ivanovski et al, 2015). Table 3 suggests that Bitcoin boasts the highest volatility of all assets included along with the highest expected annual return. In contrast to Bitcoin is the 3 – 7-year maturity Bonds asset, exhibiting a volatility of 0.8 %.

4.6 Correlation among assets

The correlation between the daily returns of each asset is visualized in Figure 2. Most noteworthy are the negative correlation between bonds and the SPY ETF. The inverse correlation between bonds and several of the other assets suggests that bonds will fill an important role as risk stabilizer in the portfolios. If one re-examines equation 3 in chapter 2.1, it can be concluded that the addition of an asset with zero or inverse correlation to an asset already included in the portfolio will decrease or at the minimum does not affect the portfolio variance.

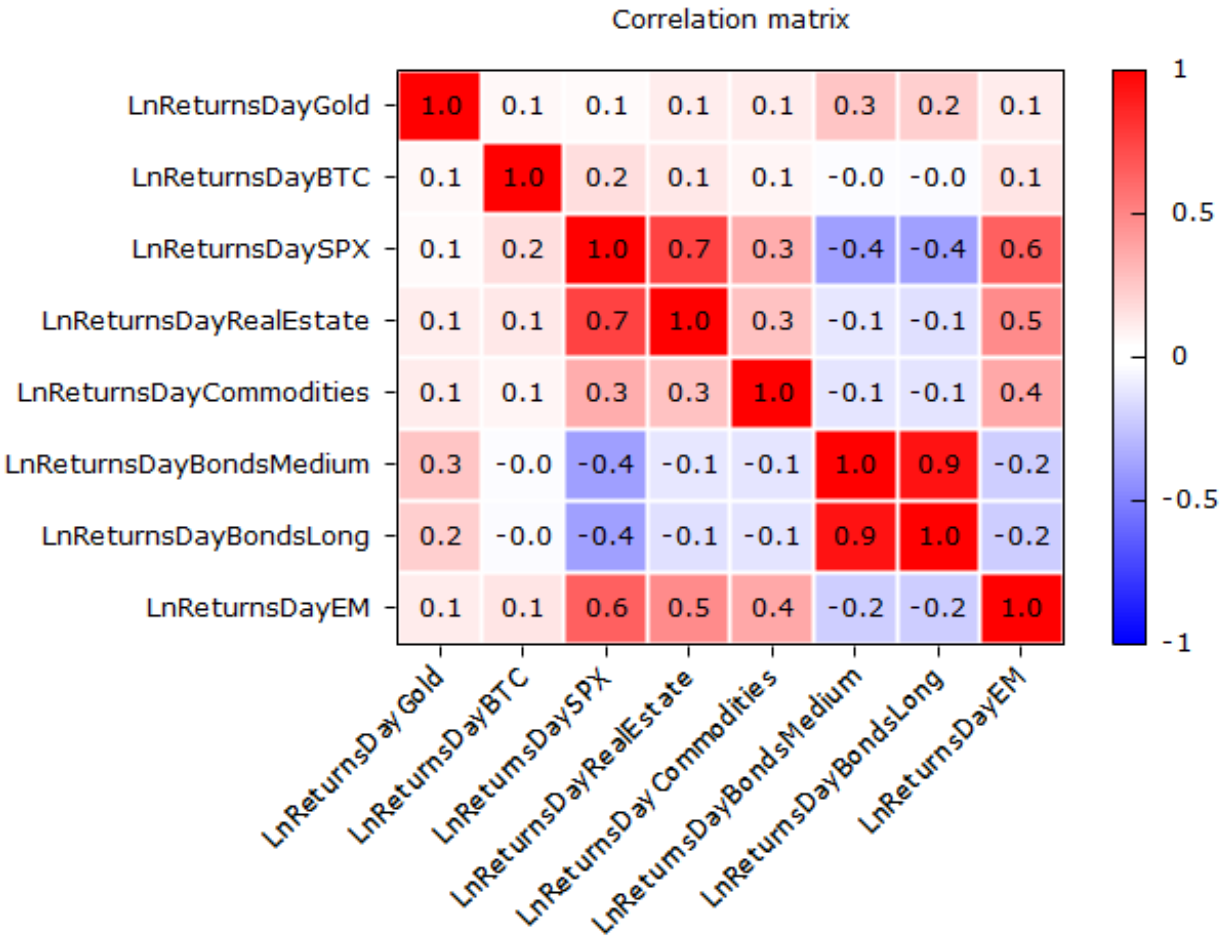

Figure 2: Correlation between daily returns of assets included

5 Results

In this section the different portfolios will be presented along with their expected values and ratios. The comparison will be made using the two main portfolios, All-Weather with and without Bitcoin included. A presentation of the two portfolios will also be made without short sales restraints. Alongside all portfolios two minimum variance portfolio will also be presented to determine if Bitcoin would be an inclusion when optimizing the minimum portfolio variance. The All-Weather can be found under (AW), minimum variance (MV) when allowing for short sales the portfolio will be denoted with (SS). All portfolios except for the minimum variance portfolios (AW-Min.Var) will be maximizing the Sharpe Ratio (Max SR).

5.1 The All Weather Portfolio Optimized:

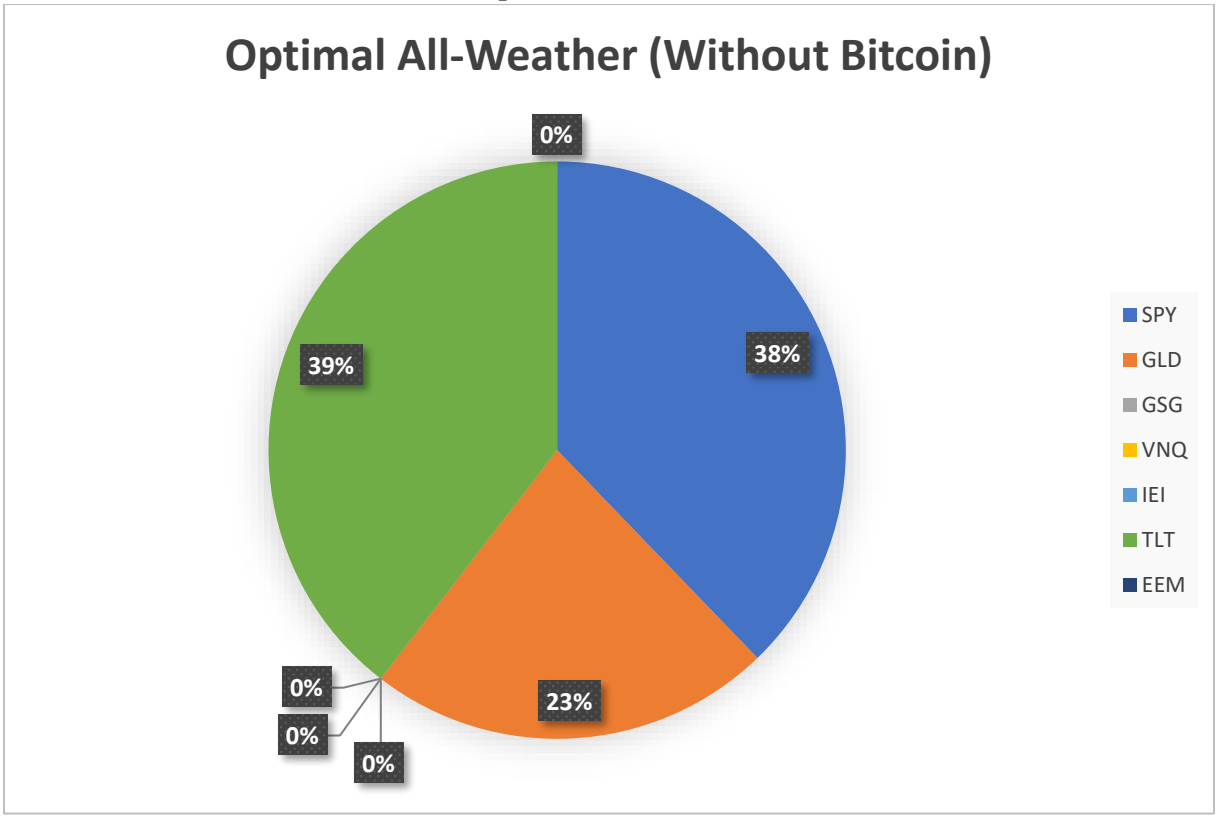

Figure 3: Weights of an optimal All Weather portfolio visualized

Asset	AW	AW-SS	AW-Min.Var
SPY	38%	86%	7%
GLD	23%	33%	-
GSG	-	-12%	-
VNQ	-	-19%	-
IEI	-	-35%	93%
TLT	39%	51%	-
EEM	-	-4%	-
BTC (X)	X	X	X
Expected Return	7.6%	12.7%	3.9%
Standard Deviation	2.2%	3%	0.7%
Variance	0.04%	0.08%	0.004%
Beta(P)	0.1903	0.9027	-1.16
Sharpe(P)	2.13	3,22	1,36
Jensen's Alpha	0.0245	-0.0058	0.13989
Treynor Measure	0.2423	0.1070	-0.0077

Table 4: Weights and performance of optimized traditional All-Weather portfolio (Estimates are annual)

The AW-Min.Var portfolio does unsurprisingly hold a large amount of the asset IEI, which we can conclude from Table 3, is also the asset from the selection with the lowest standard deviation. The small inclusion of SPY in the AW – Min.Var portfolio is due to the negative correlation between the two assets observed in Figure 2. The Treynor Measure is estimated to be negative due to the portfolio weight in IEI. IEI exhibits negative covariance with the SPY

and thus the beta for IEI is negative, which may also be concluded from Table 3. The AW-Min.Var portfolio does provide a solid expected return of 3.9 % along with the lowest systemic and volatility associated risk of the All-Weather inspired portfolios.

The optimized AW portfolio follows a similar construction with a weight distributed between the SPY, GLD and TLT. Like the IEI, the TLT is also inversely correlated with the SPY but offers a higher expected return than the 3-7-year bonds. Table 3 suggest that GLD exhibits the lowest annualized standard deviation but does so with a higher expected return than IEI. The combination between TLT and SPY grant the AW with a low portfolio beta estimate despite the portfolio weight in SPY, making it subject to low level of systemic risk.

AW-SS is subject to a higher level of systemic risk while at the same time offering a higher level of expected return than the AW. The results are in line with Moshe & Yaacov (2001)'s findings regarding short sales in an optimized Sharpe Ratio portfolio. The removal of the short sales constraint allowed the Solver to allocate a higher amount of portfolio weight in the SPY, which resulted in the highest beta estimate among the three portfolios along with the highest estimated Sharpe Ratio.

Considering all estimates of performance between the two main portfolios, the AW boasts both the lowest beta and the highest Jensen's Alpha and Treynor Measure. It does so while exhibiting a lower level of volatility than the AW – SS.

5.2 Optimized All Weather Portfolio with Bitcoin included:

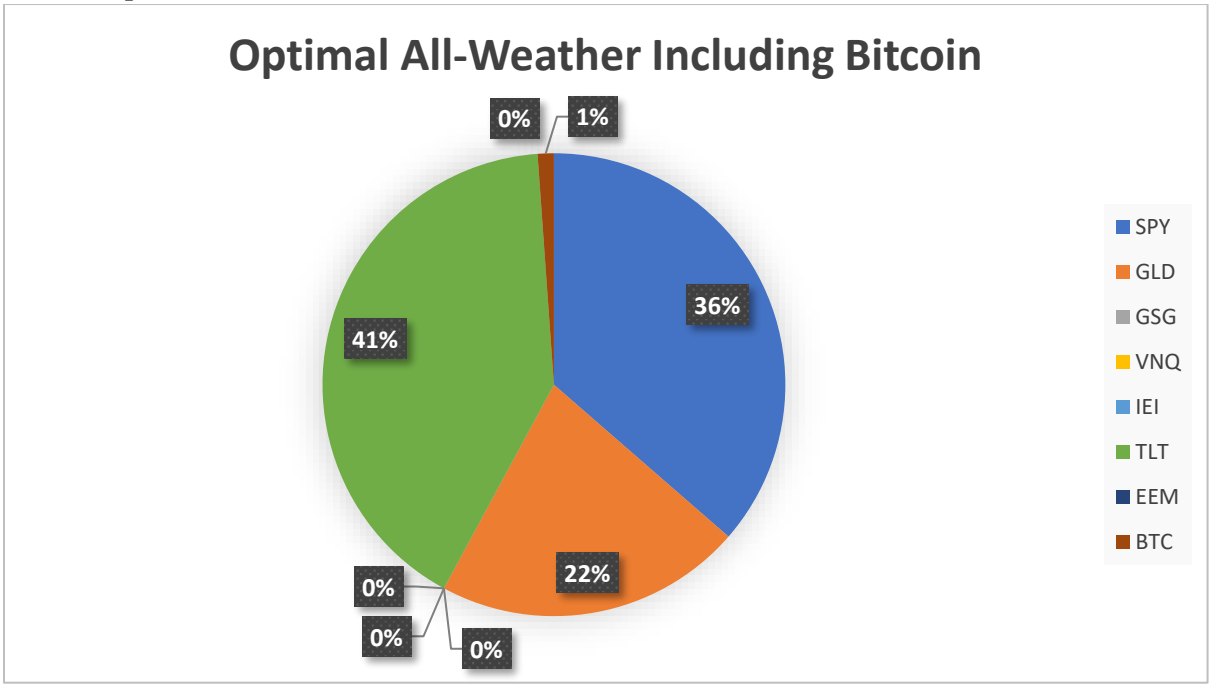

Figure 4: Weights of an optimal All Weather portfolio including Bitcoin.

Asset	AW(BTC)	AW(BTC)-SS	AW(BTC)-Min.Var
SPY	36%	82%	7%
GLD	22%	30%	-
GSG	-	-12%	-
VNQ	-	-19%	-
IEI	-	-27%	93%
TLT	41%	48%	-
EEM	-	-5%	-
BTC	1%	1%	-
Expected Return (P)	8.7%	13.3%	3.9%
Standard Deviation (P)	2.4%	3.1%	0.7%
Variance	0.04%	0.09%	0,004%
Beta (P)	0.1686	0.7747	-1.16
Sharpe (P)	2.32	3.36	1.36
Jensen's Alpha (P)	0.0380	0.0102	0,1399
Treynor Measure (P)	0.3386	0.1326	-0.0077

Table 5: Weights and performance of the optimized All Weather portfolio containing Bitcoin (Estimates are annual)

When the solver is allowed to allocate into Bitcoin, it does so with 1% of the total weight of the portfolio. AW (BTC) provides a higher return than its predecessor with only a 0.2% higher volatility. This allocation into Bitcoin enhances all of the available performance estimates displayed in Table 5 when compared to the AW portfolio. At the same time the beta, or sensitivity to systemic risk also decreases because of the smaller allocation in SPY.

The removal of the short selling constraint has a similar effect as in the first optimization. The allocation into Bitcoin forces the Solver to substitute weight in SPY for weight in BTC, resulting in a lower beta than the AW-SS previously presented. The short selling has the same effect as in earlier instances as both the expected return and Sharpe Ratio increase. Once again, we observe a decrease in the Treynor Measure and Jensen's Alpha as a result of the increased weight in SPY, causing the AW(BTC) – SS to be subject to higher level of systemic risk than the AW (BTC) portfolio.

Bitcoin is due to its high level of volatility and positive correlation to SPY, unsurprisingly not included in the minimum variance portfolio when optimizing for lowest possible risk.

5.3 Comparison of portfolios

In summary, all the performance measures are positively affected by the inclusion of Bitcoin into the potential assets. When comparing AW and AW (BTC), the inclusion results in all measures to increase along with the expected return and portfolio volatility, while at the same time decrease the level of systemic risk through the portfolio beta. The same can be concluded about the AW (BTC) – SS portfolio when compared to its predecessor AW-SS.

The minimum variance portfolio AW-Min.Var remains the same allocation-wise throughout all optimizations, it does prove to be a potential choice for individuals of extreme risk-averse nature.

6 Discussion

6.1 Findings

Before examining the different properties of the portfolios, we once again state the main question we want to answer in this thesis:

"Will the inclusion of Bitcoin in an All Weather inspired portfolio improve the performance?"

The results show that a small weight in an optimized All Weather inspired portfolio will increase the expected return and Sharpe Ratio of the portfolio in question. The inclusion of Bitcoin in an All Weather inspired portfolio will increase the standard deviation while simultaneously increase the expected annual return and lead to a higher Sharpe Ratio. Out of a pure Max Sharpe Ratio perspective of the portfolio, this suggests that the inclusion of Bitcoin in said portfolio will serve as a diversifier with a positive impact on the performance ratios of the portfolio. When comparing the two portfolios with and without short sale restriction the same conclusion can be reached. When Bitcoin is included in the short sales portfolio the expected annual return increased along with the Sharpe Ratio and volatility of the portfolio.

The portfolio including Bitcoin with short sales constraint exhibits both the highest Treynor Measure and the highest Jensen's Alpha, this is a consequence of the larger weight in SPY in the portfolio without short sales constraint which suffers from a higher systemic risk than the previously mentioned portfolio.

In all portfolios maximizing Sharpe Ratio a weight of 1% of the total portfolio in Bitcoin have shown favourable for the Sharpe Ratio and expected annual return. This weight is also favourably for the Treynor Measure and Jensen's Alpha with or without short sales constraint. All portfolios exhibiting superior performance measures included a 1% weight of allocation into Bitcoin. The highest Sharpe Ratio was observed in the AW (BTC) – SS portfolio, however this portfolio was also subject to the highest systemic risk and highest portfolio volatility. The portfolio AW (BTC) exhibits the highest Treynor Measure and Jensens's Alpha while at the same time being subject to a lower portfolio volatility than AW(BTC) – SS. As noted in the theoretical background, the Sharpe Ratio does not account for the systemic risk in the same way as the Treynor Measure and we thus end up with two different results regarding the superior portfolio depending on if the investor prioritises systemic risk or portfolio volatility. Regarding the main research question and with the tools and measurements utilized

in this thesis we may concluded that Bitcoin does serve as an enhancer of performance with respect to Sharpe Ratio, Treynor Measure and Jensen's Alpha.

6.2 Relevance of results

The conclusion reached builds on the findings of Wu & Pandley (2014) along with Klabbers (2017). Their findings suggested that a minor weight of Bitcoin in a portfolio enhanced performance of a portfolio if the portfolio exhibits a sufficient level of diversification between assets. The findings of Klabbers (2017) and Wu and Pandley (2014) are still far from irrelevant due to the several years since their publication, however the life span of Bitcoin has more than doubled since Wu & Pandley's conclusions were reached. The relevance of this thesis's results lies in the rediscovery of the diversificational properties of Bitcoin along with the reinsurance that previous research regarding Bitcoin and Sharpe Ratio still holds to this day. The findings also suggest that a conservative weight of the total portfolio should be allocated towards Bitcoin and serves as a reminder for common investors to assess the appropriate level of risk when constructing a portfolio.

It also points to the importance of short selling in modern portfolio theory when optimizing the Sharpe Ratio and adds further strength to earlier research regarding short sales and the Sharpe Ratio (Moshe & Yaacov, 2001).

6.3 Shortcomings in the methodology

6.3.1 Sample size and estimations

The Sharpe Ratio assumes correlation to be a fair predictor of the future, this not always the case. Correlation, mean, standard deviation and covariance is estimated based on the size of the sample. As in the case of cryptocurrencies overall, this data is limited to the lifetime of the asset and even less when downloading historical data from Yahoo Finance. With a data set limited to cryptocurrencies the probability of these estimates to fairly predict the future and become realized estimates decreases as compared with a more complete dataset. Bitcoin is a young asset, as stocks have existed for many years, cryptocurrency is yet to reach the 15-year mark (Nakamoto, 2008). While other asset classes can be viewed from a historical standpoint, Bitcoin has less data to offer to establish valid estimates of correlation, variance and standard deviation.

In excess to the problems presented, the estimates for standard deviation in this thesis assumes fewer tradeable days than Bitcoin is subject to, resulting in a lower standard deviation. Because of this, the minor weight conclusion reached in this thesis along with other research on the topic may be an over-estimation and the true appropriate weight in a portfolio may be

zero. In addition to the lack of normality in the daily returns of assets the results and conclusions of this thesis should not be considered rigours.

6.3.2 Problem with evaluating an asset's properties using only one form of optimization.

When establishing if an asset is a legitimate inclusion to a portfolio there are several measurements to consider beyond the Sharpe Ratio. More sophisticated methods of optimization may estimate the portfolios performance more accurately than the Sharpe Ratio which assumes standard deviation to be a fair assessment of risk. Standard deviation measures both the positive and negative deviation from the mean, the positive deviation is regarded as beneficial for investors since this would result in higher return except for short positions. Apart from the problem of what is beneficial for an investor, standard deviation also demands normal distributed returns of assets, we may conclude from Table 3 that none of the assets used in this thesis exhibits normality. This may lead to results that are misleading or inaccurate. Standard deviation is still a useful measurement for assessing risk, however it is crucial to understand its shortcomings when applied to investment management (Wander & D'Vari, 2003).

6.4 Specific risk associated with Bitcoin

Bitcoin is subject to other forms of risk than traditional assets regular financial risk. An investor may gain exposure towards Bitcoin using a number of methods, several of them associated with specific forms of risk. In modern markets, futures with Bitcoin as an underlying asset or ETFs will replicate the returns associated with ownership of Bitcoin and may even prove financially beneficial for the investor due to more clear legal guidelines regarding speculative activities. As presented by Kaur et al. (2021), investors may also gain exposure through cold-storage or platform ownership. The risk associated with cold storage is that of physical destruction of the wallet (ledger) or that of the investor misplacing or forgetting the password to the ledger. New York Times estimates that approximately 20% of all Bitcoin are lost in stranded wallets and may never be accessed due to lost passwords (Albrecht, 2021).

Investors turning to ownership of Bitcoin through platforms instead face the risk of hackers targeting larger providers of the service. The market for cryptocurrency providers is still unregulated and new actors emerge on a constant basis resulting in a questionable quality of IT security and regulation of investors account insurance.

Geographic risk may also be observed in Bitcoin, several nations have banned usage of cryptocurrencies, citing fear of a mounting losses for speculators and illegal activities taking place within the blockchain. China and India both issued statements regarding cryptocurrency activity to discourage the usage of them, proving problematic for Bitcoin since a majority of all miners or nodes reside in China (Cang, 2021). Another issue regarding the illegal activity is that of taxes. Since cryptocurrencies are decentralized, it is hard for the government to keep track of ownership and potential taxes on returns from investments amongst their citizens. The IRS reclaimed 25 million dollars in unpaid taxes in 2016, using solely the information from Coinbase Global (Saunders, 2021).

7 Conclusion:

The research finds that the inclusion of Bitcoin can enhance the performance of an All Weather inspired portfolio. The inclusion increases the volatility along with the expected return of the portfolio, the increase in the expected return out-weights the increase in portfolio volatility resulting in a higher Sharpe Ratio than was previously estimated in the original All-Weather inspired portfolio. The results show that an allocation equal to approximately 1% of the portfolio weight is beneficial to the All Weather inspired portfolio. The inclusion of Bitcoin results in a higher Jensen's Alpha in both the portfolio subject to short sales restriction and the portfolio where short sales is allowed. The same is true for Treynor Measure with both portfolios including Bitcoin boasting higher Treynor Measure than the portfolios where Bitcoin is excluded.

The thesis concludes that there is significant risk associated with the inclusion of Bitcoin into a portfolio, to enjoy the benefits of including such an asset the portfolio must achieve a high level of diversification. It also presents solutions to avoid certain types of risk associated with direct ownership of Bitcoin. Cryptocurrencies may have different properties as shown in Table 1, the thesis suggests further research since other cryptocurrencies may exhibit more beneficial characteristics than Bitcoin.

Though the thesis suggests a minor weight of maximum 1 % of the total portfolio weight in Bitcoin, it also urges to prudence when interpreting the results citing the lack of normality distribution among returns of assets.

7.1 Further Research

Suggested future research on the subject should include alternative measures of portfolio performance and optimization. The usage of Conditional VaR-framework has been used to establish a conclusion regarding Bitcoin before citing concerns regarding the rejection of normal distributions of Bitcoin returns (Eisl et al., 2015).

In addition to extended use of portfolio theory, additional research should examine alternative cryptocurrencies such as for example any of the known "stable coins", coins whose values are pegged against other cryptocurrencies, regular fiat currencies or commodities.

Bitcoin is associated with a lot of risk as presented in the discussion. As nature has taught us, the first species is not always the one to emerge as a dominating one. Development is regularly occurring and hopefully an alternative to Bitcoin will emerge which fixes the risks associated with a decentralized digital currency.

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