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Market Efficiency for Bitcoin

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Abstract

The essence of market efficiency has been an interesting area for inspection by investors and scholars. In this study, we investigate the efficiency of a relatively new asset: Bitcoin. This paper examines the efficiency of Bitcoin by studying the impact of Bitcoin's so-called halving dates. To test for weak-form market efficiency, we check for the random walk, in addition to employing statistical tests of the martingale difference hypothesis in returns. Based on our results, we find evidence of the time-varying efficiency degree of the Bitcoin market. The return predictability is discovered to be driven by changes in market conditions, as implied by the adaptive market hypothesis. The results also show a decreasing trend in the inefficiency given the sequential halving dates. This means that Bitcoin is becoming more efficient over time, even though the evidence is relatively weak.

Keywords:

Bitcoin, Efficiency Market Hypothesis, Adaptive Market Hypothesis, Halving Events

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Contents

1. Introduction

Since the elapse of the 2008 financial crisis, international economies exposed substantial problems related to financial services operations. Financial innovation was highly demanded to sustain the continuity of global markets. On 3 January 2009, the first decentralized digital currency network came into existence, with Satoshi Nakamoto mining the genesis block of Bitcoin (Block 0). Bitcoin was found to address the need of an electronic payment system that utilizes cryptographic proof, for enabling any two willing parties to transact directly without the need for a trusted third intermediary (Nakamoto, 2009). Upon its launch, the economic values Bitcoin has grasped have compromised billions of dollars in terms of alternative financial services transactions. At an accelerated pace, Bitcoin forwarded a USD 1 trillion market cap in only twelve years. Most recently in 2021, the cryptocurrency market gained a modest market cap of USD 500 billion. Such growth contributed to extremely bullish sentiments, including CEOs of large firms' interest in cryptocurrency transactions, financial institutions easing access to cryptocurrency purchases for their customers, and the launch of approved regulatory ETFs on major exchanges such as the Toronto Stock Exchange TSX (Ali, 2021).

The distinction between Bitcoin and fiat currencies is that simply nobody controls Bitcoin. The concept of decentralization is what defines Bitcoin to be completely advanced and unique. In his white paper "Bitcoin", Nakamoto outlines that the goal behind mining Bitcoin turns to be a piece of a tremendous decentralized system. Unlike ordinary banking transactions that take few days, Bitcoin organizes instalments forms quickly for parties to receive cash within a couple of minutes. In addition, Bitcoin is an exceptionally inventive approach that guarantees for a sender the cash collection by the receiver party through transparent Blockchain technology (Encrybit, 2018). To ensure the monetary policy Bitcoin undertakes, approximately; every four years, the halving event occurs, where the number of generated Bitcoin rewards per block will be halved (divided by 2). This monetary system aims to provide gradual distribution of the 21 million Bitcoins over time. The halving event ensures that the Bitcoin cryptocurrency becomes scarcer with time. As Bitcoin's halving date approach, traders and miners will be more aware of Bitcoin's decreasing supply. Traders may enter the market to speculate on market prices, introducing higher volatility during this time frame. Given the appealing attributes and deflationary effect of Bitcoin, interest in investing or mining Bitcoin has been continuously growing.

Recent studies considered the efficiency of Bitcoin investigation in the sense of Fama (1970), to be essential for evaluating Bitcoin's price mechanism and momentum of growth. Several studies were prominent to study Bitcoin in terms of efficiency, yet this focus has a been a subject of dispute between proponents and opponents of the efficient market hypothesis. For instance, Nadarajah and Chu (2017), Khuntia and Pattanayak (2018), Kristoufek (2018), and Dimitrova et al. (2019) found that Bitcoin is almost efficient. In contrast, Yonghong et al. (2018), Cheah et al. (2018), Al-Yahyaee et al. (2018), and Vidal-Tomás et al. (2019) provide observed outcomes that do not back up the efficient market hypothesis for Bitcoin. Considering this controversy, an evolutionary alternative to efficient market hypothesis, the adaptive market hypothesis was proposed by Lo (2004), whereby he supported the view that the market develops over time, as does market efficiency. An important connotation of adaptive market hypothesis is that market efficiency can arise from time to time due to changing market conditions such as behavioral bias, structural change, and external events (Lo, 2004).

This study contributes to previous research that focused on studying the efficiency of Bitcoin from a whole market perspective through adding an assessment to establish the impact of halving events on Bitcoin efficiency. Since inception, Bitcoin has completed three halving dates, and our purpose is to understand how these events impact the pre and post halving efficiency behavior of Bitcoin. Such evaluation is particularly new and interesting for further examination, as the results from this study could help in devising opportunities for traders and investors. An inefficient form of Bitcoin signifies prospects to investors over informational efficiency and predictable patterns of Bitcoin price through helping them devise effective trading strategies. If investors understand the efficiency mechanism behind Bitcoin, they will be able to implement this knowledge and adjust portfolios or create new investment as well as hedging strategies.

Our research questions will try to answer: “Does Bitcoin returns follow a random walk or a martingale process?”, “Can Bitcoin be a weak form efficient asset?” and “Is there any impact of halving events on the efficiency level of Bitcoin?”. We expect that Bitcoin does not follow a a random process or a martingale and will not satisfy the weak form of market efficiency.

This study will investigate Bitcoin efficiency through implementing weak-form market efficiency tests on different dataset windows. The analysis will be conducted based on statistical tests to capture linear and non-linear dependance in the returns data frames. The data set windows will represent the logarithmic returns of Bitcoin in between three halving periods divided into six subsamples of pre and post halving dates.

Notably, the most recent period - subsample 6 - post the third halving event, has the lowest number of observations in our data set. Statistical inference on concluding about Bitcoin efficiency in this period may be a limitation to our overall study conclusion. Moreover, this study does not assess the Bitcoin efficiency relative to the whole cryptocurrency market; it only assesses efficiency based on Bitcoin daily returns. We may need to include and add a representative index for the cryptocurrency market and measure Bitcoin efficiency in relevance to it in future work.

The remainder of this paper is structured as follows: Section 2 reviews some previous literature on Bitcoin and its halving events. This section also includes the market efficiency theory and theoretical background in two aspects, the efficient market hypothesis, and the adaptive market hypothesis. Moreover, the random walk hypothesis and the martingale difference hypothesis are presented. The last part of the literature review gives the view of prior research of market efficiency using the time-varying autoregressive model. Section 3 covers the detailed framework of the market efficiency tests methodologies. The data sets of interest and the descriptive data are discussed in Section 4. The empirical results and the subsequent discussion are presented in Sections 5 and 6, respectively. Finally, the thesis ends with some concluding remarks in Section 7.

2. Literature and Theoretical Review

This section illustrates the literature and the main theories we refer to the Bitcoin market efficiency tests. The section starts with the presentation of Bitcoin and its scheduled halving event information. Then two views of the market efficiency hypothesis: efficient market hypothesis and adaptive hypothesis, are discussed. After that, stochastic processes employed to test the market efficiency of return series called random walk and martingale are explained. Lastly, time-varying autoregressive model literature used to test market efficiency in prior research is demonstrated.

2.1 Bitcoin and Halving Events

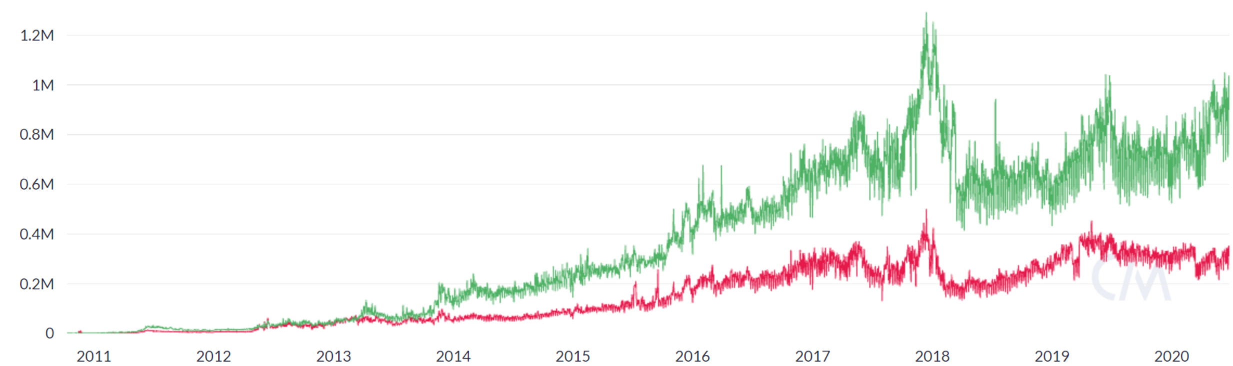
Bitcoin is a virtual medium of exchange that operates like a currency but with no underlying assets backing up. An anonymous inventor referred to be called Satoshi Nakamoto founded Bitcoin in 2009, intending to decentralize the world of finance. His goal is to free the financial infrastructure and be independent for both intermediaries and centralized institutions. Such decentralization enables peer-to-peer instant payment system on the internet, allowing users to engage in operations and transactions directly without requiring any third parties involved.

Each user has an own wallet with a specific address that contains digital keys (public keys and private keys). Antonopoulos (2017) explains that the public key is used to ensure that the user is the owner of the address and has the authority to receive the funds, while the private key is used to sign transactions for spending funds privately.

When a new transaction is requested, the transaction is disseminated to the Bitcoin network. The software starts a verification, and a valid transaction will be sent to the memory pool, waiting to be recorded in chronological order in the block. Nakamoto (2008) describes that the timestamp server is implemented to prove the existence of data at the time to get into the hash and form a chain of hash-based proof-of-work. Consequently, the block and chain are not able to change without redoing the proof-of-work. Each block has a limit size of 1 megabyte; thus, a new block is generated approximately every 10 minutes and is appended to the previous block. This application can be viewed as a shared public ledger, called "Blockchain".

However, how are new blocks generated? The blocks cannot be generated themselves but are mined by Bitcoin miners. Fund (2016) defines mining as the process based on distributed consensus system agreeing on how many units of the currency each member always holds to prevent double spending. Miners are rewarded the transaction fee for the record that they checked and confirmed. Also, they receive the incentive on the numbers of blocks they calculated successfully.

Yago (2021 cited in Divine & Reeth, 2021) said, "Satoshi Nakamoto had anticipated that as the transactions fees generated by the network increase over time, the need for the miner subsidy would decrease - and built the pre-determined, diminishing rate of newly minted BTC with this in mind".



Source: https://charts.coinmetrics.io/network-data/

Figure 1 Number of Bitcoin Active Addresses (Green) and Transactions (Red)

Trefis (2018) identifies two main factors behind the demand for Bitcoin: the number of active users and the number of transactions. As shown in Figure 1, there are upward patterns in both drivers. Because of the increasing demand for Bitcoin, the transaction fees also increased as expected. Thus, Bitcoin halving reduces half of the reward from mining Bitcoin every 210,000 blocks; this takes four years. The event impacts a decrease in the supply of new Bitcoins in the market, whilst the demand remains at the same level.

The literature on the effect of Bitcoin halving is limited in academic research. Meynkhard (2019) conducted a study to observe the effect of halving on Bitcoin's fair market value and found that reducing remuneration by half every four years leads to an increased market value of Bitcoin. Another research on the halving effect on the price regarding the halving in 2020 was conducted by Masters (2019). The results exhibit the possible decline in price in the short-term following the event and will increase afterwards.

2.2 Efficient Market Hypothesis

The efficient market hypothesis is a theory that explains the financial market movements presented by Eugene Fama's research. It states that a market is efficient at any given time since all available and relevant information to the asset's pricing is incorporated in its price.

According to Fama (1970), the paper categorized the efficient markets into three forms: weak, semi-strong, and strong form, by the level of available information reflected publicly.

- The weak form states that the asset price (or return) incorporates all know information on historical values. There is no relationship between past information and current market prices. Thus, it is impossible to attain superior profits by analyzing past returns consistently.
- The semi-strong form focuses on the speed of price adjustment to other publicly known information about the market or the particular asset, for instance, announcements of stock splits, annual reports, new security issues, etc. The prices will adjust immediately upon the release of public announcements, and it is impossible to attain superior profits by analyzing public information consistently.
- The strong form suggests that any investor has access to any information relevant to the formation of asset prices that have recently appeared. Therefore, no one or even company insiders can consistently attain superior profits.

In reality, it is difficult to achieve a strong version of market efficiency because of the legal barriers restricting the disclosure of private information to the public. However, weaker versions of the hypothesis are widely accepted. In addition, the weak-form market efficiency is the most tested comparing to other forms.

The weak-form efficient market hypothesis implies that the trend analysis or technical analysis is worthless. The return forecast is unpredictable or random; an investor cannot beat the market to retrieve the abnormal returns (Malkiel, 2003). Nevertheless, there are some counterarguments that patterns are presented, and many researchers have worked on testing the weak-form efficiency. Besides, return predictability studies are conducted on many financial markets, and so does the cryptocurrency market. The weak form efficiency of Bitcoin was initially observed by Urquhart (2016); the conclusion is that the Bitcoin returns between August 2010 and July 2016 are in an inefficient market under tests for randomness. Later, Nadarajah and Chu (2016) conducted follow-up research applying the odd integer power transformation to input data, but with the same methodology. The result is not consistent with the previous study: most tests exhibited support to the efficient market hypothesis by following the random walk process, except for the tests of independence. By applying the unit root tests to observe weak form efficiency in Bitcoin, the analysis showed that the Bitcoin model of GARCH (1,1) with structural break has predictable power. The market between April 2015 to April 2016 is inefficient (Alam, 2017).

As the popularity of the efficient market hypothesis study arises, it has been attacked by many papers since the 1980s. There are some pieces of evidence presenting market anomalies and irrational trait of investors against the hypothesis that the market is efficient at any given time. De

Bondt and Thaler (1985) discovered that stock prices overreact, evidencing substantial weak-form market inefficiencies. Jegadeesh and Titman (1993) found the momentum effect from the trading strategies leading to significant abnormal returns. Moreover, Barber and Odean (2001) emphasized the predictable investor behavior of overconfidence. Accordingly, these behavioral biases in human decision-making contradict the economic intuition and the efficient market hypothesis.

2.3 Adaptive Market Hypothesis

Unlike the old theory of market efficiency, behavioral economists have believed that the market is not statically efficient. Market participants are not perfectly rational; also, assets are not always traded at fair value. Lo (2004) proposed a new framework, namely the adaptive market hypothesis, to test market efficiency that varies over time. The study reconciled between the efficient market hypothesis and its behavioral critics and explained the time-varying market efficiency. Lo (2004) clarified that the economic systems involve human interaction, which is complex because human behavior is heuristic and adaptive. Although individuals can perform the predictability of returns occasionally due to market conditions, it cannot be claimed that the decision in an investment of market participants is entirely predictable, particularly in the changing environment.

The growing attention to the adaptive market hypothesis has motivated the emergence of several studies in a time-varying degree of market efficiency. In addition to the previous result that the Bitcoin market is inefficient from August 2010 to July 2016, Urquhart (2016) examined market efficiency in an adaptive manner by splitting this time into two periods to determine whether the efficiency level has changed differently over time. The result shows that the process moves towards an efficient market in the latter period. In the following year, Bariviera (2017) applied Hurst exponent to the study of Bitcoin market efficiency to detect the long-term memory in the Bitcoin return time series. The outcome of Bariviera's study is consistent with Urquhart's (2016). The first half of the studying period, from 2011 to 2014, shows persistent behaviors but the second period, from 2015 and 2017, shows the opposite, which means the data series is compatible with the efficient market hypothesis in the latter period of data only. Furthermore, Bariviera (2017) considered the price volatility in the study and found the long memory in the result. However, the long memory had decreased since 2014, implying that the informational efficiency level increased in the Bitcoin market after 2014. After that, a handful of studies for persistence in Bitcoin have been carried out to observe the market efficiency that varies from time to time due to changing market condition. According to Caporale, Gil-Alana and Plastun (2018), the Bitcoin returns series in the period between April 2013 and October 2017 did not follow the random walk and the degree of persistence decreased over time, which is consistent to Urquhart (2016) and Bariviera (2017). Bouri et al. (2019) investigated persistence in the level and volatility of Bitcoin prices. The data was broken into subsamples, and the evidence of long memory was found in almost all subsamples (Bouri et al. 2019).

2.4 Random Walk Hypothesis and Martingale Difference Sequence

Weak-form market efficiency can be evaluated by considering different types of dependence between an asset's prices or returns at two different times. The tests and models for examining weak-form market efficiency are based on two theories: random walk hypothesis and martingale difference sequence.

Firstly, the random walk hypothesis describes that the prices or returns move in an unpredictable way and cannot be distinguished from a random walk process (Fama, 1965). Samuelson (1965) and Fama (1970) both found the random character in share prices in their studies as the consequences of the concept of rational expectations, meaning that the current asset price contributes to the best-predicted price in the future. The hypothesis deduces that past asset movements cannot be used to forecast its movement in the future. The theory is closely related to the efficient market hypothesis as they agree that it is not possible to outperform the market without taking on additional risks (Samuelson, 1965; Fama, 1965; Malkiel, 1973). According to Campbell, Lo and Mackinlay (1997), the random walk process can be written as

$$p_t = \mu + p_{t-1} + \varepsilon_t \tag{1}$$

p_t is equal to the price at time t (today). μ represents an expected change in the price, or we can call it a drift term. ε_t can be interpreted as new information at time t of asset value, which is the random part.

There are three versions of the random walk hypothesis based on the dependence of the random shock (ε_t):

- Random Walk 1 defines that the random shock is independently and identically distributed (IID) with mean zero and variance σ^2 or $\varepsilon_t \sim IID(0, \sigma^2)$.
- Random Walk 2 defines that the random shock is independently but not identically distributed considering the long-time spans. This can be viewed as the relaxed assumption of random walk 1; also, the variance can be time varying.
- Random Walk 3 defines that the random shock is uncorrelated to past values, but the squared increments are correlated.

$$Cov[\varepsilon_t, \varepsilon_{t-k}] = 0 \quad \text{for all } k \neq 0 \tag{2}$$

$$Cov[\varepsilon_t^2, \varepsilon_{t-k}^2] \neq 0 \quad \text{for some } k \neq 0 \tag{3}$$

Random walk 3 is the weakest case of the random walk hypothesis.

Many empirical researches test the weak-form efficiency in a variety of financial markets, including Bitcoin and cryptocurrency markets, based on the random walk hypothesis. For example, Urquhart (2016), Nadarajah and Chu (2016) and Wei (2018) conducted Wald-Wolfwitz runs test, Ljung-Box test and Brock-Dechert-Scheinkman test to observe the random walk process for Bitcoin returns.

Secondly, the martingale process describes that the best forecast of tomorrow's price is today's price. Danthine (1977) offered to replace the random walk models with the martingale model in testing the weak-form market efficiency. Referring to the definition of a weak-form efficient market, Fama (1970) explained how the prices "fully reflect" available information with martingale price sequence. The price sequence (p_t) follows a martingale when the expected value of the next period's price sequence (p_{t+1}) is equal to the current price, given the information sequence of past prices $\Phi_t = \{p_t, p_{t-1,\dots}\}$.

$$E(p_{t+1}|\Phi_t) = p_t \quad \text{or equivalently,} \tag{4}$$

$$E(r_{t+1}|\Phi_t) = 0 \quad \text{where } r_{t+1} = p_{t+1} - p_t \tag{5}$$

This also implies a fair game meaning that the trading rules based on Φ_t cannot generate greater expected profits than a buy and hold strategy during the future period.

Equation (5) implies that non-overlapping price changes are uncorrelated at all leads and lags when p_t is a martingale.

$$cov(r_{t+1}, r_t) = 0 \tag{6}$$

For the sake of testing weak-form market efficiency in a dynamic situation, the martingale difference sequence tests are usually applied. The automatic variance ratio test is one of the martingale processes tests that has received attention in measuring weak-form efficiency for Bitcoin in many studies (Urquhart, 2016; Nadarajah and Chu, 2016; Wei, 2018). Besides that, automatic portmanteau test and generalized spectral test are adopted. These tests utilize spectral density function allowing the discovery of underlying periodicities in returns. Both are used for weak-form efficiency test in many financial markets. However, there are not many pieces of research focused solely on Bitcoin.

According to two theories supporting the weak-form market efficiency test, Campbell, Lo and Mackinlay (1997) summarized different kinds of dependence that can exist between asset's returns r_t and r_{t+k} at time t and t+k in the below table. Suppose $f(r_t)$ and $g(r_{t+k})$ are the random variables, where $f(\cdot)$ and $g(\cdot)$ are two arbitrary functions, and consider the covariance:

$$cov(\, f(r_t), g(r_{t+k})) = 0 \quad \text{for all t and } k \neq 0 \tag{7}$$

Table 1 Classification of Random Walk and Martingale Hypotheses

$cov(\, f(r_t), g(r_{t+k})$	$g(r_{t+k}), \forall g(\cdot)$ Linear	$g(r_{t+k}), \forall g(\cdot)$
$f(r_t), \forall f(\cdot)$ Linear	Random Walk 3 $Proj[r_{t+k} \vert r_t] = \mu$	-
$f(r_t), \forall f(\cdot)$	Martingale $E[r_{t+k} \vert r_t] = \mu$	Random Walks 1 and 2 $pdf(r_{t+k} \vert r_t) = pdf(r_{t+k})$

Note: Linear means f(·) or g(·) is restricted to arbitrary linear functions.
Proj[y|x] denotes the linear projection of y onto x.
pdf(·) denotes the probability density function of its argument.

2.5 Time-Varying Autoregressive Model

Statistical tests for the random walk hypothesis and martingale difference sequence are adopted to investigate the market efficiency. We can examine the adaptive market hypothesis by applying these tests under the sub-windows of data divided by time. Another approach is using the time-varying model. It is one of the dynamic econometric models that allow the parameters to vary over time. In this part, we present the literature on the time-varying model method to measure market efficiency.

The previous studies of the time-varying model relied on the Bayesian estimation technique (Primiceri, 2005) with the Kalman filtering and smoothing to find a likelihood value or unobservable state vector. Ito and Noda (2012) thereafter proposed a new method of the regression-based time-varying model without Bayesian estimation to research the market efficiency in the U.S. stock market from the S&P 500 stock price index. The study utilizes the time-varying autoregressive (TV-AR) model, together with the time-varying moving average (TV-MA) model. It is based on the impulse response theory for the propagation of shock in the model. They focused on the utility maximization problem and its corresponding Euler equation:

$$p_t = E_t[m_{t+1}(p_{t+1} + \kappa_{t+1})] \tag{8}$$

p_t is the stock price at t, m_{t+1} is a stochastic discount factor defined as $\delta\frac{u'(C_{t+1})}{u'(C_t)}$, and κ_{t+1} is the dividend. $E[.]$ represents the conditional expectation given the information available at t. Since m_{t+1} is close to 1 and $E_t[\kappa_{t+1}] = 0$, the price will follow a random walk process or a martingale (Ito, Noda & Wada, 2016). Then the equation can be simplified as follow:

$$E[x_t|I_{t-1}] = 0 \qquad (9)$$

where x_t is a log difference return, and I_{t-1} is the information set available at t-1.

Assuming the time series x_t is stationary, by Wold's decomposition,

$$x_t = \Phi(L)u_t \quad \text{where } \Phi(L) = \sum_{i=0}^{\infty}\Phi_i^2 < \infty \quad \text{with } \Phi_0 = 1 \qquad (10)$$

L is the lag operator, and $\{u_t\}$ is an IID process with a mean of zero and variance of σ^2. Ito, Noda, and Wada (2016) summarized that efficient market hypothesis is equivalent to $x_t = u_t$ and the long-run multiplier of a shock is the summation of the coefficients, i.e.

$$\Phi_\infty \equiv \Phi(1) = \Phi_0 + \Phi_1 + \Phi_2 + \cdots \qquad (11)$$

To derive the measure of market efficiency from the time-varying long-run multiplier, they employed the spectral density concept (Ito & Noda, 2012). Subsequently, they pointed out the superiority of their model, the non-Bayesian TV-AR model from the traditional Bayesian method:

1. The model is more straightforward as it does not need iteration in Kalman filtering and smoothing.
2. The model is more flexible than the model that contains random parameter variation because the state equation can handle these stochastic constraints.
3. Asymptotic assumptions of estimates are preserved due to the regression method.

At another time, Ito, Noda, and Wada (2017) researched the time-varying parameter model and referred to the equivalence of Kalman filtering and smoothing procedure and generalized least squares (GLS) regression approach. They proved that the GLS estimator of the TV-AR model yields the exact estimate as the Kalman-smooth estimates and its mean squared error.

The approach had been adopted by Noda (2020) to test the adaptive market hypothesis for Bitcoin's efficiency analysis. The author employed a GLS-based TV-AR model and proved that the degrees of Bitcoin efficiency changed at different times from April 2013 to September 2019. The level of efficiency becomes higher after 2013, which is consistent with the research from Urquhart (2016), who utilized the statistical tests for subsamples of data.

3. Methodology

This section outlines the methodology used in the study. We test the market efficiency by employing some statistical tests of the martingale difference hypothesis in returns. In addition, we check whether Bitcoin returns time series follow the random walk process or not. Finally, the GLS based TV-AR and its measure of market efficiency are adopted in the last subsection.

3.1 Prerequisite Tests

This subsection briefly describes diagnostic tests for univariate time series data, including normality and unit root tests.

3.1.1 Kolmogorov-Smirnov Goodness of Fit Test

Kolmogorov-Smirnov Goodness of Fit test (or one sample KS test) is a goodness of fit test determining how well the data fit the hypothesized distribution (uniform or normal) (Poshakwale, 1996). The test compares the cumulative distribution function (CDF) of a data series with a reference distribution from theoretical expectations. In this case, we used the normal distribution. Suppose the data contains N ordered data observations: $y_1, y_2, \ldots, y_N$, the empirical distribution function (ECDF) is

$$E_N = \frac{n(i)}{N} \tag{12}$$

where $n(i)$ is the number of observations less than the sample value y_i (NIST/SEMATECH, n.d.).

The hypothesis of the test is defined as:

H_0: The cumulative distribution function of samples equals the normal distribution function

H_1: The cumulative distribution function of samples does not equal the normal distribution function

The test statistic (D) is the least upper bound of all differences in the sample's cumulative distribution from the normal distribution function. It can be written as:

$$D = \sup|\Phi - E_N| \tag{13}$$

When D value is greater than or less than the critical value at a significance level of $\alpha/2$, the test rejects the null hypothesis at the α level of significance.

3.1.2 Jarque-Bera Test

Jarque-Bera (JB) test will be implemented to evaluate the normality of the time series returns. The test matches the skewness and kurtosis of data to see if it matches a normal distribution. A normal distribution has a skewness of zero and kurtosis of three. The JB test follows a chi-square (χ^2) distribution with two degrees of freedom, is calculated as:

$$\text{JB} = \frac{T}{6}\left(b_1^2 + \frac{1}{4}(b_2 - 3)^2\right) \sim (\chi^2) \tag{14}$$

where T is the sample size, b_1 the coefficient of skewness and $b_2 - 3$ the excess kurtosis.

The hypothesis of the test is defined as:

H_0: $Returns\ follow\ a\ normal\ distribution$

H_1: $Returns\ do\ not\ follow\ a\ normal\ distribution$

In general, a large JB value indicates that returns are not normally distributed.

3.1.3 Unit Root Test

A unit root is a stochastic trend in time series. The test is designed to reveal whether the return series Y_t is difference-stationary (null hypothesis) or trend-stationary (alternative hypothesis). The presence of a unit root exhibits a systematic pattern that is unpredictable such as the random walk model (Glen, n.d.). The random walk is a difference-stationary series because the first difference of y is stationary. For example, we consider the series as a simple autoregressive (AR) model:

$$y_t = \mu + y_{t-1} + \varepsilon_t \tag{15}$$

$$y_t - y_{t-1} = \mu + \varepsilon_t \tag{16}$$

$$(1 - L)y_t = \mu + \varepsilon_t \tag{17}$$

$$\Delta y_t = \mu + \varepsilon_t \tag{18}$$

where ε_t is a white noise disturbance.

The hypothesis of the test can also be written as:

H_0: $y_t = \mu + y_{t-1} + \varepsilon_t$

H_1: $y_t = \mu + \Phi y_{t-1} + \varepsilon_t$ where $\Phi \in (-1,1)$

The study adopted a well-known unit root test – the Augmented Dicker-Fuller (ADF) test. After all, it is criticised for low statistical power when a root Φ closes to one, and the sample size is small. The solution to the downside of the test is the complementary use of a stationarity test, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test.

3.1.3.1 Augmented Dicker-Fuller Test

Brooks (2019) illustrates the ADF model as:

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} a_i \Delta y_{t-i} + u_t \tag{19}$$

where u_t is a white noise disturbance and is assumed not to be autocorrelated.

The hypothesis of the test is defined as:

$H_0: \psi = 0$

$H_1: \psi < 0$

The test statistic (t_ψ) follows the student's t-distribution and can be calculated from:

$$t_\psi = \frac{\hat{\psi}}{SE(\hat{\psi})} \tag{20}$$

where $\hat{\psi}$ is the estimate of ψ, and $SE(\hat{\psi})$ is the standard error of the coefficient.

When t_ψ value is less than or equal to the critical value at a significance level of α, the test rejects the null hypothesis at the α level of significance.

3.1.3.2 Kwiatkowski-Phillips-Schmidt-Shin Test

KPSS test is a stationarity test applicable for both trend stationarity and level stationarity. It also can detect the existence of a random walk embedded in the series. If the random walk is presented, the series contains a unit root, which can be concluded that it is not stationary.

Because of the low power of ADF, when the process is stationary but with a root close to the non-stationary boundary, the failure to reject the null hypothesis might occur. Therefore, the confirmatory data analysis is applied using the KPSS test.

The hypotheses of KPSS are the reversing of ADF's hypotheses, which can be written in the form of I(d) as:

$H_0: y_t \sim I(0)$

$H_1: y_t \sim I(1)$

The rejection outcomes of KPSS should be in the opposite way to the ADF rejection results. One is to reject the null hypothesis, but another one is not. If both ADF and KPSS give the same result to reject the null hypothesis or to not reject the null hypothesis, they imply conflicting results (Brooks, 2019).

3.2 Random Walk Hypothesis Tests

In the following subsection, the relevant independence tests of random walk and statistical contributions to tests will be performed. This study tests random walk hypothesis focusing on the Random Walk 1 (Wald-Wolfowitz runs test and Brock-Dechert-Scheinkman test) and Random Walk 3 (Ljung-Box Test) aspects only.

3.2.1 Wald-Wolfowitz Runs Test

Wald-Wolfowitz runs test (runs test) presents the test for linear independence of the returns under the hypothesis of randomness (Bradley, 1968). The method uses median or mean as the reference point to split data, then assign "+" to the values larger than the reference and "-" to the smaller than or equal to the reference. We assume n_1 be the number of observations of "+" and n_2 be the number of observations of "-". The total number is $N = n_1 + n_2$.

A run represents a sequence of repeating signs, for instance, ++ or --, forming a binomial distribution. The number of runs (R) is the number of different subsets of the consecutively same sign in the sequence. The test measures how fast or slow the oscillation between positives and negatives (Tokić, Bolfek, & Peša, 2018). This can be viewed in terms of R: the faster oscillation, the higher value of R and vice versa. The hypothesis of the test is defined as:

H_0: Two samples are identically distributed

H_1: Two samples are NOT identically distributed

If R deviates from the expectation, there are some differences in either the shift or the distribution spread. Therefore, the null hypothesis is rejected. Moffitt (2017) explains that when the distribution characteristics in all permutations of two series observations are equal, the observed series is random.

The test statistic (Z) is approximated with a normal distribution when each sample size n_1 and n_2 is larger than 10. The expected number of runs is $\frac{2n_1n_2}{n_1+n_2} + 1$ and the variance is $\frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}$.

$$Z = \frac{R - \left(\frac{2n_1n_2}{n_1+n_2} + 1\right)}{\sqrt{\frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}} \tag{21}$$

When the Z value is either greater or less than the critical value at a significance level of $\alpha/2$, the test rejects the null hypothesis at the α level of significance.

3.2.2 Brock-Dechert-Scheinkman Test

Brock-Dechert-Scheinkman (BDS) test is a test of independence used for time series, which can be transformed to test the IID errors of the models (Brock et al. 1996), which means it tests whether the data series is IID or not. It has a feature to detect linear dependence, non-linear dependence, and chaos in the data series.

Assume the return series $x_1, x_2, \ldots, x_N$, we construct a set of pairs of data $\{(x_s, x_t), (x_{s+1}, x_{t+1}), \ldots, (x_{s+m-1}, x_{t+m-1})\}$ where s, t are an observation of the series and m is the continuing point used in the data called embedded dimension. The test suggests that:

$$P[|x_t - x_s| < \varepsilon]^2 = P[|x_t - x_s| < \varepsilon] \times P[|x_{t-1} - x_{s-1}| < \varepsilon] \tag{22}$$

for all s, t. The metric bound $\varepsilon > 0$ and the lagged pairs (x_t, x_{t-1}) must satisfy the above condition (Moffitt, 2017).

Moreover, Campbell, Lo and Mackinlay (1997) explain the test from another perspective: correlation integrals for dimension m ($c_{m,n}$). The property of the test is the ratio of correlation integrals which means it is the probability that two data observations conditional on the previous n data observations are close and within a certain length of ε apart.

When the data are IID, the conditional probability is simply the multiplication of probabilities.

The hypothesis of the test is defined as:

H_0: The data series are IID from a continuous distribution

H_1: The data series are from a continuous distribution, but not IID

The test statistic of independence ($b_{m,n}(\varepsilon)$) can be calculated based on Eviews (2020):

$$b_{m,n}(\varepsilon) = c_{m,n}(\varepsilon) - c_{1,n-m+1}(\varepsilon)^m \quad (23)$$

The $b_{m,n}(\varepsilon) \times \sqrt{n-m+1}$ is normally distributed with a mean of zero. Then, the BDS test statistic denoted by W or $W_{m,n}(\varepsilon)$ is given:

$$W_{m,n}(\varepsilon) = \frac{b_{m,n}(\varepsilon) \times \sqrt{n-m+1}}{\sigma_{m,n}(\varepsilon)} \quad (24)$$

When $W_{m,n}(\varepsilon)$ value is either greater than or equal to or less than or equal to the critical value at a significance level of $\alpha/2$, the test rejects the null hypothesis at the α level of significance.

3.2.3 Ljung-Box Test

It is crucial to assess autocorrelation in the time series data, which is a type of serial dependence. In this case, random errors are mostly positively correlated over time. Each random error is more likely to be like the previous random error than it would be if the random errors were independent of one another.

The Ljung-Box test will be used to detect the presence of autocorrelation being zero of return series, t. The test can detect autocorrelation up to any predesignated order k. In the case of IID returns, the Q statistic is considered asymptotically distributed as a χ^2 variable.

The autocorrelation coefficient at lag k is defined as the following:

$$\rho(k) = \frac{cov(r_t, r_{t+k})}{\sqrt{Var(rt)}\sqrt{Var(r_{t+k})}} \quad (25)$$

For perfectly uncorrelated returns, the autocorrelation function $\{\rho(k)\}$ should be equal to 0 for every k > 0.

The Ljung-Box test follows a chi-square (χ^2) distribution with the degrees of freedom of k. It tests for all autocorrelations up to lag m are different from zero and is computed as:

$$Q_m \equiv T(T+2)\sum_{k=1}^{m}\frac{\rho^2(k)}{T-k} \tag{26}$$

where T is the sample size.

The null hypothesis of the test is no serial autocorrelation presence. Rejecting the null hypothesis would mean rejecting the random walk assumption.

The hypothesis of the test is defined as:

H_0: $\rho_1 = \rho_2 = \cdots \rho_m = 0$ Returns do not exhibit serial autocorrelation.

H_1: $\rho_1 \neq \rho_2 \neq \cdots \rho_m \neq 0$ Returns exhibit serial correlation.

If the Q_m exceeds the critical value from the chi-squared statistical tables, reject the null hypothesis of no autocorrelation.

3.3 Martingale Difference Sequence Tests

In this subsection, we present the tests of predictability of Bitcoin returns based on past price changes through the martingale difference sequences.

3.3.1 Automatic Variance Ratio Test

Lo and Mackinlay (1988) variance ratio test can determine whether an asset follows a random walk by detecting whether an asset price exhibits autocorrelation. If the asset prices exhibit correlation, past prices can help predict future prices, which would violate the weak form of the efficient market hypothesis.

The central idea of the variance ratio test is based on the observation that when returns are uncorrelated over time, we must have:

$$Var\big(x_t(k)\big) = Var(x_t + \cdots + x_{t-k+1}) = kVar(x_t) \tag{27}$$

where x_t is the daily return of Bitcoin and k is the period difference number.

The test can be estimated as

$$V(k) = \frac{Var\big(x_t(k)\big)}{kVar(x_t)} = 1 + 2\sum_{j=1}^{k-1}\left(1 - \frac{j}{k}\right)\hat{\rho}_j \tag{28}$$

where $\hat{\rho}_j$ is the estimator for ρ_j. When $\rho_j = 0$ for all j, it means the series is a random walk and $V(k) = 1$.

The choice of the holding period k is completely arbitrarily and made without any statistical justification. To modify this weakness, Choi (1999) proposed a fully data-dependent method based on spectral density to estimate the optimal $\hat{k}$ called the Automatic Variance Ratio (AVR) test.

Under the assumption that returns are IID, the test would be:

$$AVR\big(\hat{k}\big) = \sqrt{\frac{T}{\hat{k}}}\,\frac{V(k) - 1}{\sqrt{2}} \xrightarrow{d} N(0,1) \tag{29}$$

Assuming that $T \to \infty, k \to \infty,\ and\ \frac{T}{k} \to \infty$, AVR converges in distribution to a normally distributed random variable under the null hypothesis of a martingale difference sequence.

A positive (negative) value of the AVR indicates overall positive (negative) autocorrelation in the asset return. However, its absolute value is often used as a more efficient price exhibits fewer autocorrelations in both directions.

The statistical significance of return predictability can be evaluated using the (1-α)% confidence interval based on the wild bootstrap for the AVR statistic. Also, the use of the wild bootstrapped AVR is robust to unconditional heteroskedasticity. If the AVR statistic lies outside its (1-α)% confidence interval, it is statistically different from zero at the α% level of significance. In this case, it indicates the presence of statistically significant return predictability.

3.3.2 Automatic Portmanteau Test

The automatic portmanteau test is one of the time series tests for autocorrelation developed by Escanciano and Lobato (2009b). The test is data-driven in selecting the order of the considered sample autocorrelations. It is robust to conditional heteroskedasticity, which is typically presented in the financial time series. The hypothesis of the test is defined as:

$H_0: \rho_j = 0$ for all $j \geq 1$

$H_1^K: \rho_1 = \cdots = \rho_{K-1} = 0\ ,\ \rho_K \neq 0$ for some $K \geq 1$

The initial research of autocorrelation contains the underlying assumptions regarding heteroskedasticity. Although the tests had been continuously developed to improve the finite sample performance (Ljung & Box, 1978; Li & McLeod, 1981), the independence assumption is needed. Also, another limitation is about the selection the autocorrelations number which is arbitrary.

Escanciano and Lobato (2009b) overcame the dependence conditions and proposed the automatic test statistic (AQ) to examine the optimal lag order as follows:

$$AQ = Q^*_{\tilde{p}} = T\sum_{i=1}^{\tilde{p}} \tilde{\rho}_i^2 \tag{30}$$

where $\tilde{p}$ is the optimal value of lag order which is data dependent.

The AQ statistic asymptotically follows a chi-square distribution with one degree of freedom. When AQ value is greater than the critical value at a significance level of α, the test rejects the null hypothesis at the α level of significance.

3.3.3 Generalized Spectral Test

Escanciano and Velasco (2006) proposed the dependence test to capture linear and non-linear dependency in returns series and allow conditional heteroskedasticity. The test was developed from a spectral test to investigate the martingale difference hypothesis from Durlauf (1991). The hypothesis of the test is defined as:

H_0: $m_j(y) = 0$ for all $j \geq 1$ where $m_j(y)$ are the pairwise regression functions

H_1: $P(m_j(Y_{t-j}) \neq 0) > 0$ for $j \geq 1$

To simulate the critical values for the test statistic (D_n^2), the Cramer–von Mises norm for minimum distance parameter calculation is adopted.

$$D_n^2 = \sum_{j=1}^{n-1}(n-j)\frac{1}{(j\pi)^2}\sum_{t=j+1}^{n}\sum_{s=j+1}^{n}\left(Y_t - \bar{Y}_{n-j}\right) \times \left(Y_s - \bar{Y}_{n-j}\right)\exp(-0.5\left(Y_{t-j} - Y_{s-j}\right)^2) \tag{31}$$

The null hypothesis of no return autocorrelation is rejected when D_n^2 is larger than the empirical (1-α)th sample quantile of test statistic or p value from the bootstrap is less than α at the α level of significance.

3.4 Generalized Least-Squares Time-Varying Autoregressive Model

The GLS regression extends the ordinary least square (OLS)'s estimation of the standard linear model by providing for possibly unequal error variances and correlations between different errors. This study will utilize a GLS TV-AR model of Ito, Noda, and Wada (2016, 2017) to analyze time-varying market efficiency in the Bitcoin return series. The model was developed based on the non-Bayesian TV-AR model that we explained in Section 2.5. The standard AR model that has been mainly used to analyze the time series of assets returns is as follow:

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \cdots + \alpha_q x_{t-q} + u_t \tag{32}$$

where $\{u_t\}$ satisfies $E[u_t] = 0$, $E[u_t^2] = 0$, and $E[u_t u_{t-m}] = 0$ for all m.

Normally, $\alpha_\ell's$ are assumed to be constant in time series analysis, but we assume that the coefficients of the TV-AR model change over time. Therefore, the GLS-based TV-AR model will be applied to analyze the Bitcoin time series returns. The GLS-based TV-AR is expressed as follows:

$$x_t = \alpha_{0,t} + \alpha_{1,t} x_{t-1} + \cdots + \alpha_{q,t} x_{t-q} + u_t \tag{33}$$

where $\{u_t\}$ satisfies $E[u_t] = 0$, $E[u_t^2] = 0$, and $E[u_t u_{t-m}] = 0$ for all m. As we estimate the GLS-based TV-AR model, we assume that the parameter dynamics are fixed whereby:

$$\alpha_{\ell,t} = \alpha_{\ell,t-1} + v_{\ell,t} \ , (\ell = 1, 2, \ldots, q) \tag{34}$$

where $\{v_{\ell,t}\}$ satisfies $E[v_{\ell,t}] = 0$, $E[v_{\ell,t}^2] = 0$, and $E[v_{\ell,t} v_{\ell,t-m}] = 0$ for all m and ℓ.

Ito, Noda, and Wada (2017) described the TV-AR model in the form of a basic state-space model:

$$y_T = Z_t \beta_t + \varepsilon_t \tag{35}$$

$$\beta_t = \beta_{t-1} + \eta_t \tag{36}$$

y_T is a $\mathrm{k} \times 1$ vector of observable variables, Z_t is a $\mathrm{k} \times \mathrm{m}$ matrix of observable variables, β_t is a $\mathrm{m} \times 1$ vector of time-varying coefficients, and ε_t and η_t are $\mathrm{k} \times 1$ and $\mathrm{m} \times 1$ vectors of normally distributed error terms with zero mean and covariance matrix H_t and Q_t respectively.

The model then is formulated into the matrix form of equations for t = 1, …, T as per Durbin and Koopman (2012):

$$Y_T = Z\beta + \varepsilon \qquad \text{where } \varepsilon \sim N(0, H) \tag{37}$$

$$\beta = C(b_0^* + \eta) \quad \text{where } \eta \sim N(0, Q) \text{ and } \beta_0 \sim N(b_0, P_0) \tag{38}$$

where the model set $Z_t = (y'_{t-1} \otimes I_k)$ and $\beta_t = vec(\alpha_t)$.

$$Y_T = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_t \end{bmatrix}, \; Z = \begin{bmatrix} Z_1 & & 0 \\ & Z_2 & \\ & & \ddots \\ 0 & & Z_t \end{bmatrix}, \; \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_t \end{bmatrix}, \; \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_t \end{bmatrix},$$

$$H = \begin{bmatrix} H_1 & & 0 \\ & \ddots & \\ 0 & & H_t \end{bmatrix}, \; C = \begin{bmatrix} I & 0 & \dots & 0 \\ I & I & & \vdots \\ \vdots & \vdots & \ddots & 0 \\ I & I & \dots & I \end{bmatrix}, \; b_0^* = \begin{bmatrix} b_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$P_0^* = \begin{bmatrix} P_0 & 0 & & 0 \\ 0 & 0 & & \\ \vdots & & \ddots & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \; \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_t \end{bmatrix}, \; Q = \begin{bmatrix} Q_1 & & & 0 \\ & Q_2 & & \\ & & \ddots & \\ 0 & & \cdots & Q_t \end{bmatrix}$$

The regression analysis is used based on below equation to generate the estimates under the assumption that the estimated TV-AR(q) model is a locally stationary by combining Equations (33) and (34) (Maddala & Kim, 1998):

$$\begin{bmatrix} Y_T \\ -b_0^* \end{bmatrix} = \begin{bmatrix} Z \\ -C^{-1} \end{bmatrix} \beta + \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix} \tag{39}$$

The resulting coefficients are then applied, together with the time-varying moving average model (TV-MA(∞)), to calculate the time-varying degree of market efficiency according to the Ito, Noda, and Wada (2014, 2016).

The underlying TV-MA(∞) model is given by

$$y_t = \mu_t + \Phi_{0,t} u_t + \Phi_{1,t} u_{t-1} + \cdots \text{ where } \Phi_{0,t} = I \text{ for all t} \tag{40}$$

Suppose a cumulative sum of the TV-MA coefficient matrices is denoted by $\Phi_t(1)$.

$$\Phi_t(1) = \sum_{j=1}^{\infty} \widehat{\Phi}_{j,t} \tag{41}$$

$$= (I - \sum_{j=1}^{q} \hat{\alpha}_{j,t})^{-1} \tag{42}$$

This matrix is called a "long-run multiplier" since it can measure the long-run effect of shocks on the returns y_t (Ito, Noda & Wada, 2014). When $\Phi_t(1) = I$, it suggests the efficiency of the market. Therefore, the degree of market efficiency (ζ_t) is measured by the distance between $\Phi_t(1)$ and I or in the sense that how near or far between the actual market and the efficient market based on the spectral norm.

$$\zeta_t = \sqrt{\max \lambda[(\Phi_t(1) - I)'(\Phi_t(1) - I)]} \tag{43}$$

$$= \left| \frac{\sum_{j=1}^{q} \hat{\alpha}_{j,t}}{1 - (\sum_{j=1}^{q} \hat{\alpha}_{j,t})} \right| \tag{44}$$

where $\lambda[(\Phi_t(1) - I)'(\Phi_t(1) - I)]$ is the eigenvalue of matrix $(\Phi_t(1) - I)' \times (\Phi_t(1) - I)$ for each t (Noda, 2016). From Equations (43) and (44), the deviation of ζ_t from zero indicates the market deviation from the efficient condition.

4. Data and Descriptive Statistics

The daily closing prices listed in USD have been extracted from Coinmetrics.io, one of the biggest transparent and accessible online cryptocurrencies data platforms. Coinmetrics tracks over 7000 coins with live price movements and reliable historical data collected from over 30 world's leading spot and derivatives crypto exchange platforms.

The daily returns of Bitcoin have been computed as logarithmic returns.

$$R_t = \ln\left(P_t / P_{t-1}\right) \tag{45}$$

P_t is the Bitcoin closing price on day t, and P_{t-1} is the closing price of Bitcoin on day t-1.

In this study, the data set covers the logarithmic returns of Bitcoin spans from 18 July 2010 to 18 April 2021, for a total of 3,927 observations.

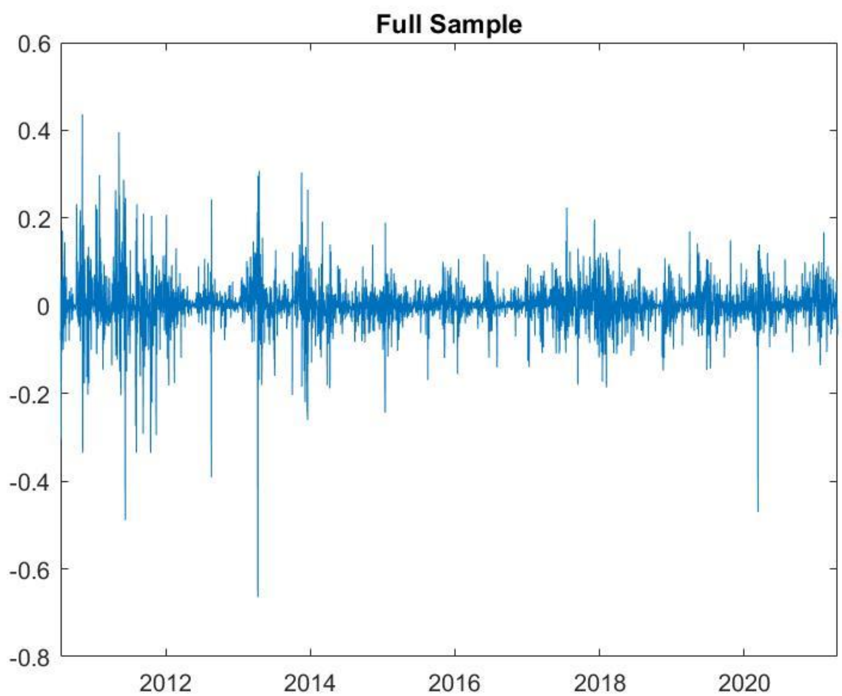


Figure 2 Daily Log Returns of Bitcoin

To observe the halving effect on the market efficiency, we first separated the data into four subsamples based on three halving dates. We split the period between the first halving (28 November 2012) and the second halving (9 July 2016) and the second halving (9 July 2016) and the third halving (18 May 2020) again, giving six subsamples in total.

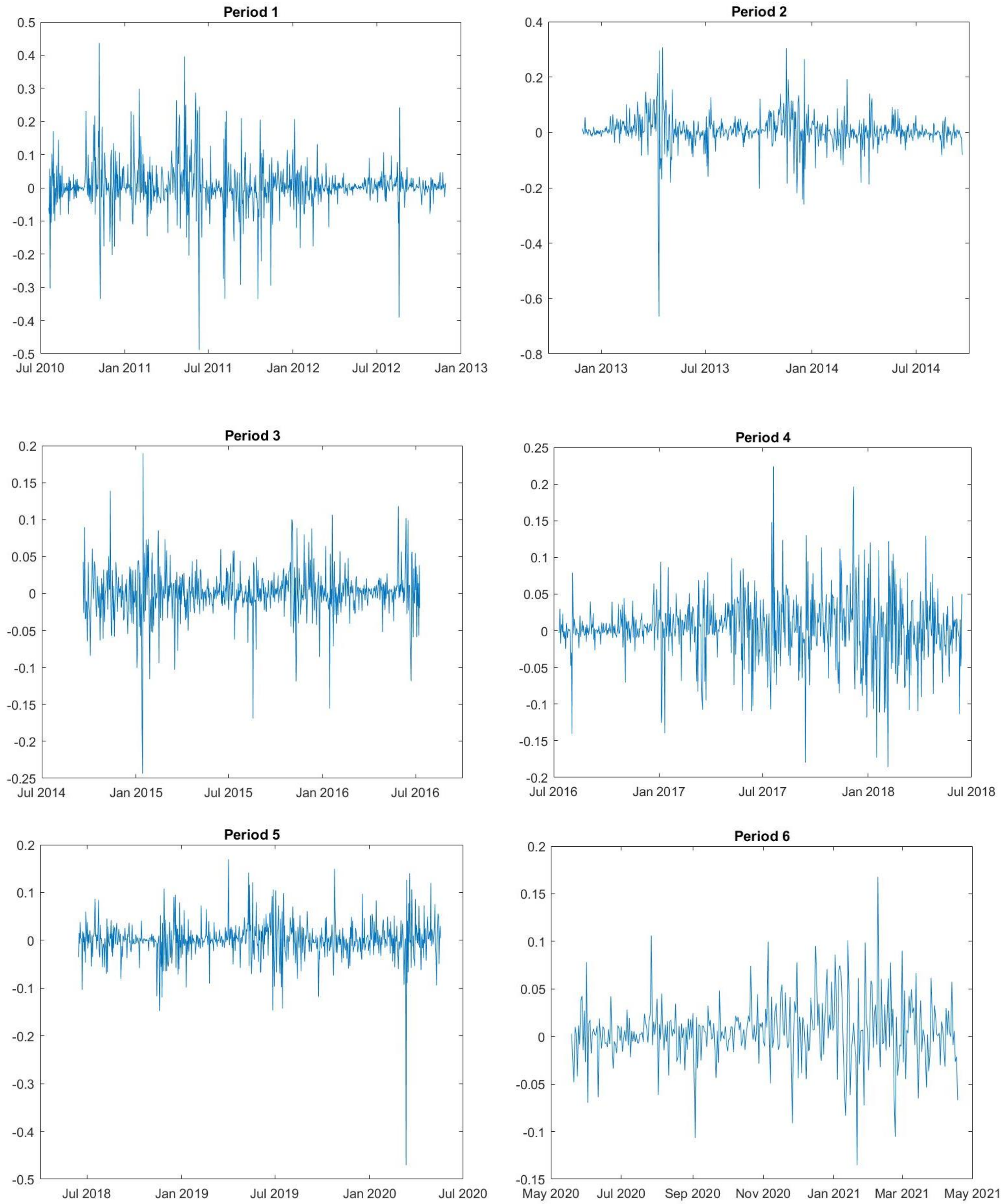


Figure 3 A Comparison of Daily Log Returns of Bitcoin in 6 Subsample Periods

Figure 3 illustrates the movement of Bitcoin returns. In the left column, line graphs display the returns before halving events. On the other hand, the returns after the halving events occurred are shown in the right column.

Table 2 Descriptive Statistics of Bitcoin Returns in Six Subsample Periods

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12*	19-Sep-14	9-Jul-16*	14-Jun-18	18-May-20*	18-Apr-21
Mean	0.5749%	0.5248%	0.0765%	0.3290%	0.0544%	0.5246%
Standard Error	0.2522%	0.2475%	0.1345%	0.1673%	0.1546%	0.1875%
Median	0.1074%	0.3846%	0.1191%	0.3502%	0.0581%	0.5057%
Standard Deviation	7.4118%	6.3572%	3.4537%	4.4413%	4.1018%	3.4311%
Sample Variance	0.5493%	0.4041%	0.1193%	0.1972%	0.1682%	0.1177%
Excess Kurtosis	8.9055	23.5495	8.5264	3.2593	26.1197	2.9535
Skewness	-0.1957	-1.7971	-0.7707	-0.1206	-2.0014	0.1769
Minimum	-48.8884%	-66.4948%	-24.3706%	-18.6095%	-47.0563%	-13.5241%
Maximum	43.6655%	30.7474%	18.9771%	22.4053%	16.9680%	16.7650%
Count	864	660	659	705	704	335

*Note: * indicates the exact halving dates occurred.*

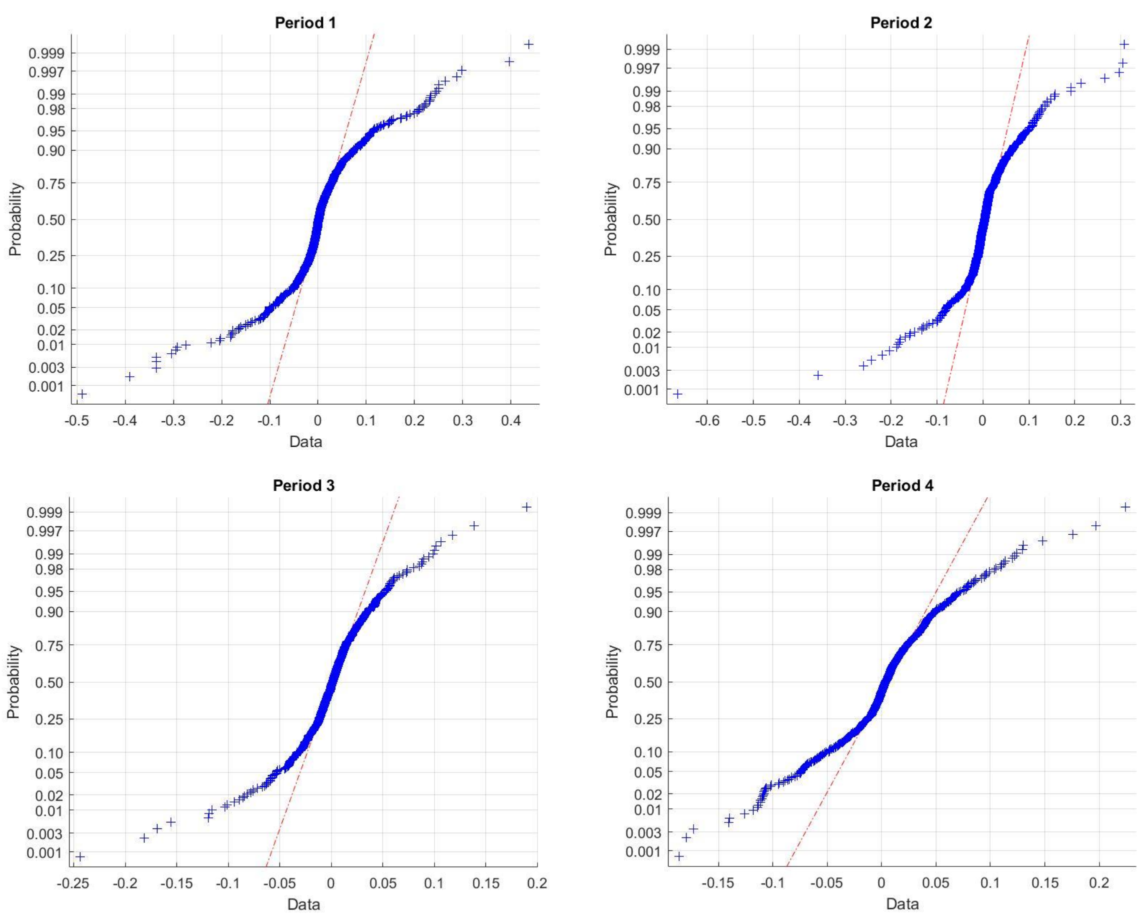
Table 2 presents a summary of descriptive statistics of the daily returns for Bitcoin. Average returns are found to be positive in all periods, ranging from 0.05% to 0.57%. The first period has registered the highest mean return (0.57%), followed by the second period (0.52%). The volatility measured by standard deviation varies between 3.43% and 7.41%. The data is especially volatile in the beginning parts, and then the volatility demonstrates a decreasing trend. Bitcoin returns are leptokurtic and negative-skewed in most subsamples, except for the most recent period that the returns have the least positive excess kurtosis and positive skewness. The positively skewed returns can be implied that the positive returns are more observed than negative returns.

5. Results

This section displays the findings from the various tests we have performed on the Bitcoin returns. The first subsection presents the normality and stationary test results. Subsequently, the random walk test outcomes are given, followed by the results from martingale difference sequence tests. Lastly, the empirical analysis of the GLS-based TV-AR model and its measure of market efficiency degree are presented.

5.1 Prerequisite Tests

From the previous section, we conducted the KS goodness of fit test and JB test to confirm the non-normality of data we observed from the excess kurtosis and skewness values in Table 2. As indicated in Figure 4, the normal probability plots for each data period are shown. If the sample data is normally distributed, the empirical cumulative distribution plot (blue mark) will appear linear.



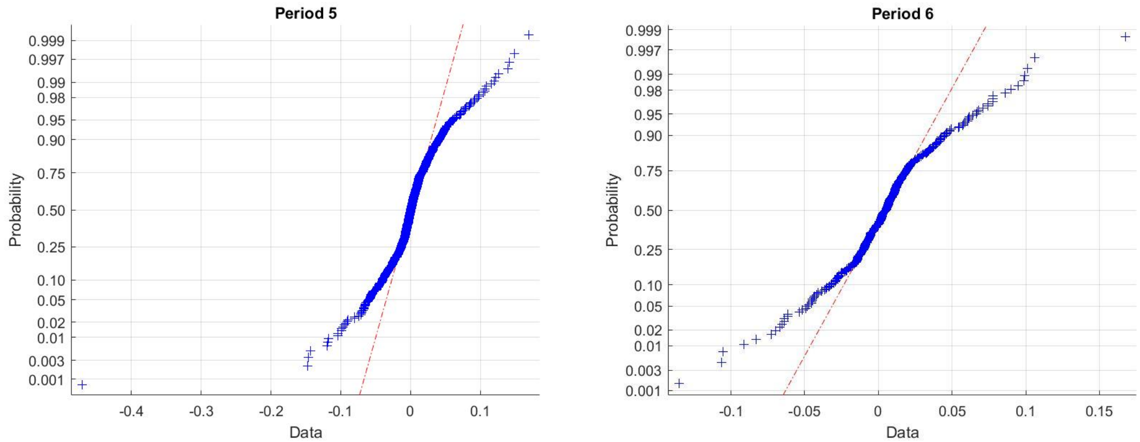


Figure 4 Normal Probability Plots of Bitcoin Returns for 6 Data Periods

Table 3 Kolmogorov-Smirnov and Jarque-Bera Test Results

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12	19-Sep-14	9-Jul-16	14-Jun-18	18-May-20	18-Apr-21
KS stat	0.4211	0.4296	0.4534	0.4446	0.4485	0.4562
KS p value	2.3168e-134	5.2539e-107	5.8703e-119	2.4343e-122	2.4736e-124	1.722e-61
JB stat	2823.2813	15363.1278	2026.7941	307.7390	20184.6940	118.4542
JB p value	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

Table 3 summarizes the test statistics and corresponding p values of 6 subsamples. Both tests reject the null hypothesis that the data comes from the normal distribution in every subsample.

Furthermore, we tested the stationarity of the data series by ADF test and KPSS test. The optimal number of lags based on the minimum value of Akaike Information Criteria (AIC) is 1. The results are shown in the table below.

Table 4 Augmented Dicker-Fuller and Kwiatkowski-Phillips-Schmidt-Shin Test Results

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12	19-Sep-14	9-Jul-16	14-Jun-18	18-May-20	18-Apr-21
ADF stat	-20.8979*	-18.5426*	-20.3978*	-18.1923*	-18.2860*	-12.5665*
ADF p value	0.0001*	0.0001*	0.0001*	0.0001*	0.0001*	0.0001*
KPSS stat	0.1226	0.0749	0.0236	0.1859*	0.0903	0.1106
KPSS p value	0.0933	0.1000	0.1000	0.0213*	0.1000	0.1000

*Note: * indicates the rejection of the null hypothesis of ADF or KPSS test at a 5% significance level*

The results of the ADF test of unit root for all periods are to reject the null hypothesis that the data series contains a unit root. This signifies the stationarity of Bitcoin returns in all periods. Consequently, we expect the results from KPSS test not to reject the null hypothesis.

Indeed, the results of the KPSS turned out to not reject the null hypothesis that the data does not contain a unit root, at the significance level of 5% with the exception for period 4 (after the second halving event). The KPSS p value for period 4 data is approximately 0.0213, which means there is around a 2.13% chance the series contains a unit root, and this is inconsistent with the results from the ADF test. However, period 4 data is stationary at a significance level of 1%.

From the above results, we can also imply that the Bitcoin returns market is inefficient based on the random walk hypothesis. All random walk processes are non-stationary; however, note that not all non-stationary series are random walks.

In the following subsections, the results from weak-form market efficiency tests based on the random walk hypothesis and martingale difference sequences are presented.

5.2 Random Walk Hypothesis Tests

5.2.1 Wald-Wolfwitz Runs Test

Because of the non-normal distribution of data, the non-parametric runs test is applied to the indices to observe the randomness to examine the efficiency of Bitcoin returns. Elango and Hussein (2008) mentioned that the use of mean as a reference in runs test is compatible and effective if the data distribution is symmetrical. However, it is weak when there is a presence of outliers. Thus, the median is more suitable in our case.

Table 5 Wald-Wolfowitz Runs Test Using the Sample Median Reference Results

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12	19-Sep-14	9-Jul-16	14-Jun-18	18-May-20	18-Apr-21
E[nruns]	424	330	331	354	353	169
nruns	424	312	362	336	378	193
n1	432	330	329	352	353	167
n0	432	330	329	352	351	168
Z statistic	-0.5787	-1.4413	2.4579*	-1.2446	1.8483	2.6266*
p value	0.5629	0.1495	0.0139*	0.2133	0.0645	0.0085*

*Note: * indicates the rejection of the null hypothesis of runs test at a 5% significance level*

The results are depicted in the above table. nruns represents the actual number of runs, n1 and n0 represent the number of values above and below the median, respectively. The estimated Z-values for periods 3 and 6 are significant at the 5% level. Also, the Z-statistic for period 6 rejects the null hypothesis at a 1% significance level. The negative Z value marks that the expected number of runs is greater than or equal to the actual observed number and implies that there is a positive serial correlation in the data series. Also, the positive Z value indicates that the expected number of runs is less than the actual observed number and implies a negative serial correlation in the data series. Squalli (2006) explained that a sample with a high number of runs suggests cyclical or seasonal fluctuations. Therefore, we can conclude that Bitcoin returns in periods 3 and 6 do not follow random walks, and then they are weak-form inefficient. However, other remaining periods results fail to reject the null hypothesis of the randomness. By comparing the corresponding p values between before and after halving event, there is no observable pattern of efficiency degree from the runs test.

5.2.2 Brock-Dechert-Scheinkman Test

BDS test is another non-parametric test to check the dependence in a non-linear fashion in data series. We performed the test of whether the Bitcoin returns in 6 periods are IID.

We employed the ε of 0.7 according to Belaire-Franch and Contreras (2002), as it was recommended to be the optimal value in the calculation for non-normal distribution series.

Table 6 Brock-Dechert-Scheinkman Test Results

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12	19-Sep-14	9-Jul-16	14-Jun-18	18-May-20	18-Apr-21
dimension (2)	11.1275	10.6253	5.1545	5.3541	4.3767	2.2191
	(0.0000)	(0.0000)	(2.543e-07)	(8.597e-08)	(1.205e-05)	(0.0265)
dimension (3)	13.5758	12.2759	5.5188	7.2745	4.8327	2.6972
	(0.0000)	(0.0000)	(3.414e-08)	(3.477e-13)	(1.347e-06)	(0.0070)
dimension (4)	14.9412	13.5833	6.3950	8.9737	5.1133	3.0933
	(0.0000)	(0.0000)	(1.605e-10)	(0.0000)	(3.166e-07)	(0.0020)
dimension (5)	16.3940	14.6573	7.5523	10.3948	5.6766	3.5196
	(0.0000)	(0.0000)	(4.286e-14)	(0.0000)	(1.374e-08)	(4.321e-04)
dimension (6)	17.6342	16.2607	8.4923	11.8857	5.9722	3.8523
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(2.341e-09)	(1.170e-04)

Note: The table shows the W test statistic with resulting p values displayed in parentheses

The results in the above table reported the rejection of the null hypothesis for all periods using dimensions up to 6 at a 5% significance level. There are signs of non-linear dependence found in the Bitcoin returns series in every period, indicating significant inefficiency in Bitcoin. Furthermore, it is worth mentioning that the test statistics are decreasing over time, and p values are rising. This can imply that Bitcoin becomes more efficient over time.

5.2.3 Ljung-Box Test

In detecting the presence of autocorrelation, the Ljung-Box test was performed with lags of k varying from 1 to 6. The number of lags selected is selected arbitrarily, as there is no optimal criterion for its detection.

Table 7 Ljung-Box Test Results

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12	19-Sep-14	9-Jul-16	14-Jun-18	18-May-20	18-Apr-21
Q_1 statistic	4.5263*	0.4115	0.0097	0.0060	7.8365*	0.2068
p value	0.0334*	0.5212	0.9215	0.9382	0.0051*	0.6493
Q_2 statistic	6.6480*	1.6241	9.8719*	0.2580	12.7013*	0.2188
p value	0.0360*	0.4440	0.0072*	0.8790	0.0017*	0.8964
Q_3 statistic	6.6568	1.9752	13.4103*	0.5335	13.6359*	0.6593
p value	0.0837	0.5776	0.0038*	0.9115	0.0034*	0.8827
Q_4 statistic	7.9046	3.3771	13.4315*	3.9304	17.9614*	0.7615
p value	0.0951	0.4968	0.0093*	0.4155	0.0013*	0.9435
Q_5 statistic	10.2554	28.1354*	16.4971*	6.0092	18.2476*	1.1871
p value	0.0683	3.425e-05*	0.0056*	0.3053	0.0027*	0.9461
Q_6 statistic	20.7140*	28.5122*	24.6618*	7.3969	19.4529*	1.8480
p value	0.0021*	7.523e-05*	0.0004*	0.2857	0.0035*	0.9331

*Note: * indicates the rejection of the null hypothesis of the Ljung-Box test at a 5% significance level*

Based on the results presented in Table 7, we can only see that in period 5, the null hypothesis has been rejected at the 95% confidence level, based on the low p values. This indicated that in period 5, Bitcoin returns exhibited serial correlation, thus rejecting the random walk assumption. Similarly, period 1 for lags 1 – 2, 6 and period 3, starting from lag 2, present rejection of the random walk assumption and the null hypothesis of serial correlation presence. Thus, period 1, 3 and 5 could be considered as weak-form inefficient.

Periods 4 and 6 fail to reject the null hypothesis, indicating that Bitcoin returns in those time frames were not serially correlated, which utilizes the random walk assumption.

Regarding period 2, at lag 5 and 6, Bitcoin returns reject the null hypothesis of no serial correlation and the random walk assumption.

5.3 Martingale Difference Sequence Test

5.3.1 Automatic Variance Ratio Test

AVR is used to assess the exhibition of autocorrelation in Bitcoin returns to determine whether past prices could predict future prices.

Under the assumption that returns are IID, AVR statistical values were computed simultaneously with a bootstrap of 500 iterations.

Table 8 Automatic Variance Ratio AVR Test Results

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12	19-Sep-14	9-Jul-16	14-Jun-18	18-May-20	18-Apr-21
AVR	1.3287	0.4893	-0.1680	-0.0214	-1.4188	-0.0029
p value	0.2360	0.7940	0.7960	0.9260	0.2040	0.9780

With optimal holding period value chosen by data-dependent procedure, periods 1 and 2 data exhibited positive AVR, which indicates an overall positive autocorrelation in the asset returns. However, from period 3 until period 6, AVRs exhibited negative values. All p values computed are greater than 5%, which signify non-rejection of the null hypothesis of the martingale difference hypothesis.

5.3.2 Automatic Portmanteau Test

By allowing the returns data to select the number of orders for autocorrelations automatically, the automatic portmanteau test maximizes the value of the robustified portmanteau statistic corrected by a penalty term increasing function of the included number of autocorrelations.

Using the vrtest package in RStudio, the function Auto.Q returns a robustified portmanteau test with automatic lag selection.

Table 9 Automatic Portmanteau Test

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12	19-Sep-14	9-Jul-16	14-Jun-18	18-May-20	18-Apr-21
AQ statistic	1.3625	0.0340	0.0017	0.0029	2.7738	0.1595
p value	0.2431	0.8537	0.9670	0.9574	0.0958	0.6896

The data-driven results provided in Table 9 show a conclusion of failing to reject the null hypothesis in all periods at the 95% confidence level. Thus, all periods exhibit independence in returns.

5.3.3 Generalized Spectral Test

The GS test for the martingale difference hypothesis is based on the generalized spectral distribution function. The test depends on the generating data process under dependence on the asymptotic null distribution. Hence, the bootstrap is implemented. The number of replications (B) is 300 as per the original experiment from Escanciano and Velasco (2006).

Table 10 Generalized Spectral Test Results

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12	19-Sep-14	9-Jul-16	14-Jun-18	18-May-20	18-Apr-21
GS p value	0.2667	0.4667	0.6000	0.3700	0.2367	0.2600

Table 10 presents the resulting p values for the GS test. The p values for all subsample periods are higher than 5%. Thus, there is no evidence against the martingale difference hypothesis.

5.4 Generalized Least-Squares Time-Varying Autoregressive Model

According to Equation (33), we calculated the optimal number of lags of the x variable by an information criterion based on the number of lags that minimizes the value of AIC. In our estimation, we employed the AR(6) model.

$$x_t = \alpha_{0,t} + \alpha_{1,t} x_{t-1} + \cdots + \alpha_{6,t} x_{t-6} + u_t \qquad (46)$$

Under the GLS TV-AR, the data is used for all six periods as one full sample. The GLS TV regression is applied to obtain the coefficients $\{\alpha_{0,t}, \alpha_{1,t}, \ldots, \alpha_{6,t}\}$ at each t = 1, …, 3,920.

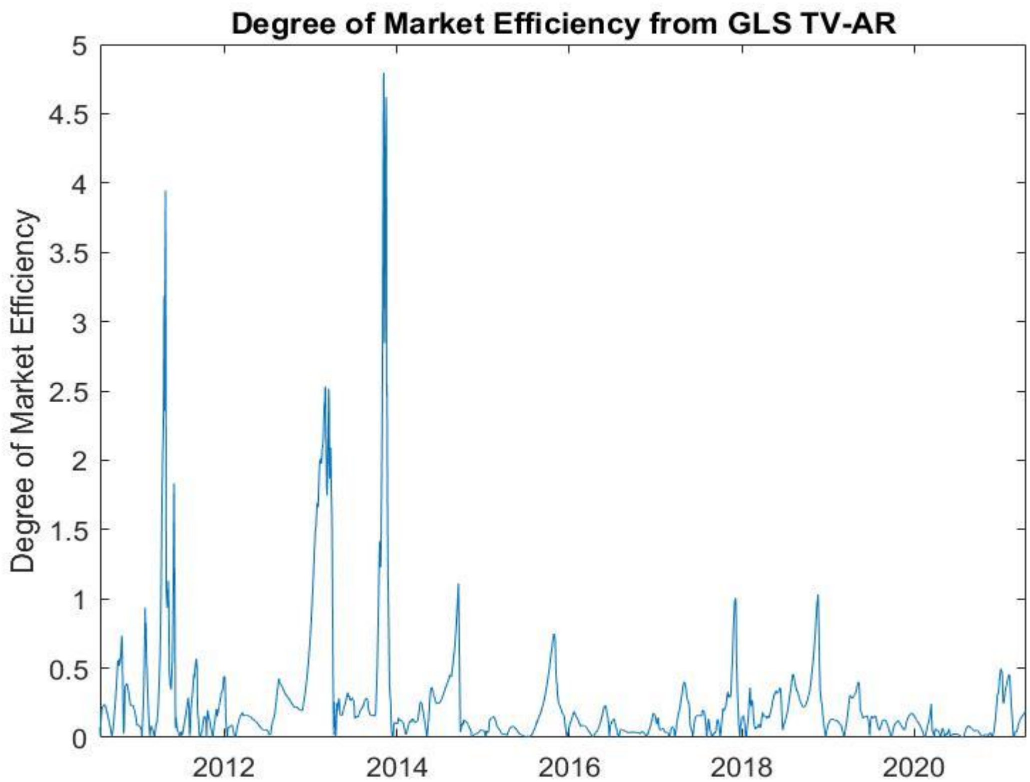


Figure 5 Time-Varying Degree of Bitcoin Market Efficiency

Figure 5 illustrates the degree of market efficiency (ζ_t) calculated from Bitcoin returns GLS TV-AR model. The more ζ_t deviates from zero; the less efficiency is at t. The line graph shows that ζ_t varies over time, and the inefficiency is detectable when a financial crisis or issue related to Bitcoin prices grows. The GLS TV-AR result is discussed in greater depth in Section 6.

Regarding the halving effect on the market efficiency degree, we summed up the average figures of ζ_t in three different durations before and after halving events, such as 30 days, 90 days, and 180 days.

Table 11 Degree of Market Efficiency (ζ_t) Observed Before and After Halving Events

Average ζ_t	Observable Duration		
	30 Days	90 Days	180 Days
Halving 1 (28-Nov-2012)			
Before	0.2058	0.2602	0.2035
After	0.5221	1.3355	1.0626
Halving 2 (09-Jul-2016)			
Before	0.1038	0.1085	0.1014
After	0.0411	0.0398	0.0560
Halving 3 (18-May-2020)			
Before	0.0295	0.0611	0.0844
After	0.0227	0.0305	0.0321

From Table 11, one can see that the average ζ_ts before the second halving are greater than the average figures after the event in all three observable durations. This is also true for the third halving. On the other hand, it is the other way round for the first halving. The after-halving 1 average degree is relatively high, especially for 90 days (1.3355) and 180 days (1.0626).

Moreover, it is noticeable from each column in Table 11 that the degree of efficiency increases over time as the measure ζ_t value decreases. The finding that the market efficiency of Bitcoin has evolved corroborates Noda's (2020), which used the same GLS TV-AR model. Furthermore, this finding resembles Urquhart's (2016) and Bariviera's (2017) that examine weak-form efficiency using time series subsample estimation.

6. Discussion

In this section, we discuss the results from the random walk hypothesis and martingale difference sequence statistical tests. Moreover, the GLS TV-AR model outcomes and the measure of the market efficiency are explained accordingly.

We categorized the statistical test results into two areas based on the hypothesis. The p values from six statistical tests are arranged into the below table:

Table 12 p values from Statistical Tests

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Starting Date	18-Jul-10	29-Nov-12	20-Sep-14	10-Jul-16	15-Jun-18	19-May-20
Ending Date	28-Nov-12	19-Sep-14	9-Jul-16	14-Jun-18	18-May-20	18-Apr-21
Random Walk Hypothesis						
Runs Test	0.5629	0.1495	0.0139*	0.2133	0.0645	0.0085*
Ljung-Box Test	0.0021*	7.523e-05*	0.0004*	0.2857	0.0035*	0.9331
BDS Test	0.0000*	0.0000*	0.0000*	0.0000*	2.341e-09*	1.170e-04*
Martingale Difference Hypothesis						
AVR Test	0.2360	0.7940	0.7960	0.9260	0.2040	0.9780
Automatic Portmanteau Test	0.2431	0.8537	0.9670	0.9574	0.0958	0.6896
GS Test	0.2667	0.4667	0.6000	0.3700	0.2367	0.2600

*Note: * indicates the rejection of null hypothesis at a 5% significance level*
Ljung-Box test p values are based on lag (m) = 6
BDS test p values are based on dimension (m) = 6

Under the random walk hypothesis test findings, we can draw three conclusions. First, periods 1-3 and 5 results show the rejection of Random Walks 1 and 3. Note that the runs test is based on linear dependence only; however, the BDS test additionally incorporates non-linear dependence. Second, Bitcoin returns in period 4 follow Random Walk 1 if we consider only linear dependency, whereas they do not if non-linear dependence is considered. Finally, in the last period, the Bitcoin returns satisfy the condition of Random Walk 3 only.

Furthermore, comparing period 1 with period 2, period 3 with period 4, and period 5 with period 6, no pattern is observed in the Bitcoin market efficiency as an effect of halving events from the resulting p values. However, we recognise that the Bitcoin market efficiency as per the random walk hypothesis follows the adaptive market hypothesis since the market switches between efficiency and inefficiency at different times.

For the martingale difference hypothesis, the AVR, the automatic portmanteau test and GS test show that the Bitcoin market is efficient according to the martingale difference hypothesis. It satisfies the condition that the return is a fair game. Bitcoin returns follow a martingale, and future return variations are entirely unpredictable given the current information set.

Overall, Bitcoin returns accept the martingale difference hypothesis; however, it does not follow Random Walks 1 and 3 in some periods. The random walk hypothesis is more restrictive than the martingale process, as outlined in Section 2.4. Thus, all information in past Bitcoin returns is useful for forecasting the next period's expected return however it is not able to forecast the probability distribution of the next period's return.

Concerning the GLS TV-AR model, we can see the movement of market efficiency degree evolving with time from Figure 5. The graph illustrates several peaks of the inefficiency of returns corresponding to the occurrences of Bitcoin-related events. The following events ascribe the period of inefficiency from the model.

During 2011, Bitcoin price significantly fluctuated. The price skyrocketed from USD 1 in April 2011 to USD 30 in June 2011, giving approximately 3000% gain in three months. Unfortunately, this was followed by a sharp drop in November 2011, reaching a semi-annual low at USD 2. In the year 2013, Bitcoin again underwent two price bubbles. In March, the Blockchain network encountered a technical glitch causing a temporary fall, and then the price rebounded to near previous highs. However, in the following month, a Bitcoin exchange called Mt. Gox grappled with a security breach and database leaked problems, leading to a market crash all over. The inefficiency measure ζ_t peaked at 4.8 in late 2013. The escalation corresponds to the shutdown of Silk Road, an illegal online market that exploited a loophole of Bitcoin technology to conduct transactions. In the following year, the theft of Bitcoins from Mt. Gox accounts continued, the company halted all withdrawal requests from clients and filed for bankruptcy in the end. Between 2017 and 2018, the price bubble occurred once more. The price was hovering around USD 1,000 at the beginning of 2017 but shot up to almost USD 20,000 in December 2017. Then, it was continuously tumbled until the end of the year 2018.

Correspondingly, we put up the measure of market efficiency from the GLS TV-AR model along the timeline to display the development of market efficiency degree over time in Table 13.

Table 13 Degree of Market Efficiency (ζ_t) from GLS Time-Varying Autoregressive Model

	Halving 1		Halving 2		Halving 3	
Duration	Before	After	Before	After	Before	After
30 Days	0.2058	0.5221	0.1038	0.0411	0.0295	0.0227
90 Days	0.2602	1.3355	0.1085	0.0398	0.0611	0.0305
180 Days	0.2035	1.0626	0.1014	0.0560	0.0844	0.0321

The measures reveal a decreasing trend in the inefficiency degree values, except for the term after the first halving. The inefficiency in the post-halving 1 timeframe could be ascribed to a range of Bitcoin's financial issues in the year 2013, as mentioned earlier. In general, the Bitcoin market becomes more efficient period over period. The model suggests that the Bitcoin market is less efficient before the halving and becomes more efficient after the event, yet this is not a strong evidence to conclude that the efficiency increase is due to the halving event.

In short, based on the GLS TV-AR model, we notice that the Bitcoin market appears to be efficient without the extreme crash of Bitcoin prices. Price bubble events, or crashes are exogenous news that have not been incorporated in the prices; hence, the market displays the inefficiency pattern. This finding is in line with the adaptive market hypothesis.

Because Bitcoin has no means for intrinsic valuation, its price is purely driven by supply and demand. The transaction management and money issuance operate through the mathematical process without a central authority. Therefore, only market participants are the ones who control the prices on a completely transparent trading system. Although halving is a reason causing a drop in the Bitcoin’s supply because the award given to miners is cut down, all investors are already aware about it. This means that halving information is already priced in the Bitcoin value, and the event does not constitute new information.

7. Conclusion

The efficiency in Bitcoin returns explains how Bitcoin prices reflect available information in the cryptocurrency market. Thus, one can carry out fundamental analysis to chart out a guaranteed return trading strategy. The main objective of this study was to investigate the efficiency of Bitcoin through implementing weak-form market efficiency tests on different dataset windows representing pre and post halving events. We assessed whether Bitcoin followed a random walk or a martingale process through these tests besides studying how Bitcoin's market efficiency varies over time. The characteristics of Bitcoin and the concept of halving dates are perceived to impact the performance of Bitcoin price, momentum, and trading volume. Thus, it turns to be appealing to investigate Bitcoin's efficiency through a halving scope.

In particular, we tested Bitcoin market efficiency by checking for the random walk process, employing statistical tests of the martingale difference hypothesis in returns, and employing a GLS-based TV-AR model. The statistical tests suggest that Bitcoin returns in all sub-periods follow the martingale difference hypothesis; however, the returns in periods 4 and 6 only that follow the Random Walk 3. We concluded that there was an unspecific pattern observed in the Bitcoin market efficiency as an impact from halving events. Nevertheless, we acknowledge that the Bitcoin market efficiency follows the adaptive market hypothesis since the market shifts between weak-form efficiency and inefficiency at different periods. Distinctly, the GLS TV-AR model supported the prior research findings in terms of the increasing degree of weak-form efficiency over time.

Over time, we expect Bitcoin to be more efficient as the market capitalization of Bitcoin is in continuous growth, which is driving more investors and traders to join the market. We propose further work to improve the dynamic market efficiency examination from splitting data into subsamples to utilizing the rolling window of data. Also, the further studies can empirically investigate the changing degree of market efficiency whilst comparing Bitcoin to alternative investments asset classes as a mean for hedging portfolios. In addition, we ought to expand this research and study Bitcoin efficiency in relevance to a constructed cryptocurrency index that represents the top-weighted market capitalization coins.

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