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Optimal Control Actions in an Electrical Grid with Variable Renewable Energy Sources

Evaluation of the ANM4L Control Algorithms

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Preface

I would like to express my gratitude and appreciation to my supervisors Professor Olof Samuelsson and Martin Lundberg for all the time and energy they have put into supervising me. Their understanding and positive manner have helped me and made me comfortable. These times of pandemic have made this thesis work remote, but it has worked out very well.

This work is dedicated to my late father who left us three years ago. Having studied the same program as me, albeit at Uppsala University, it is sad that you never got to experience me as a graduated engineer in the same field. I am sure that wherever you are now you follow me.

Erik Boman, May 18th 2021

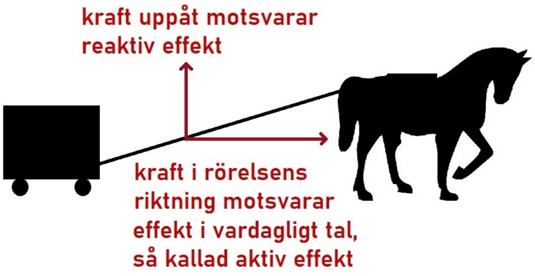
Abstract

The increasing amount of renewable energy in the electrical grid comes with several challenges, of which overvoltage and congestion (overcurrents) are the subjects of this work. One flexible way of handling these problems is active management by some control system. There is a trade-off between simplicity when implementing the control system and the performance. In the project Active Network Management for All (ANM4L), an algorithm using PI controllers for management of voltage has been developed. It can control active power and reactive power. To make it easy to implement, PI controllers at different buses cannot communicate. This work is a comparison between control actions acquired from optimization in the program GAMS and algorithm control actions to see how far from optimal the latter ones are. In a five-bus low voltage test network with a 30 kW photovoltaic generator at each bus, the least curtailment of active power compared to the optimal actions was obtained when the voltage management algorithm controlled curtailment of active power and decided the reactive power based on a constant power factor of 0.8. This simulation represented a case of today. When the generation was increased to 50 kW per bus, representing a distant future scenario, the least curtailment of active power compared to the optimal actions was obtained when the active and reactive power were independently controlled. Control of active and reactive power independently could, in this respect, be regarded as the best choice in the long term. However, the consumption of reactive power is not entirely independent as the inverter of a photovoltaic generator poses a limit on the power flow from and to that generator. Thus, if the consumption of reactive power cannot be increased further because of this limit, active power has to be curtailed. If a fair distribution of the control actions is prioritized, using a constant power factor of 0.8 is best in both the case of today and the future scenario. There is also an ANM4L algorithm for congestion management. Congestion management demands some communication between buses but the aim has, analogous to the approach in the voltage management, been to make it easy to implement. In a medium voltage test network, the CIGRE medium voltage distribution network benchmark, the congestion management algorithm came very close to the optimal actions when being set on prioritizing equal sharing between the buses.

Populärvetenskaplig sammanfattning

Den allt större mängden förnybar energi i elnätet för med sig ett antal utmaningar. Sol- och vindkraft är nyckfulla till sin natur. Ibland kan generering av effekt vara större än efterfrågan och detta kan orsaka för stora spänningar och strömmar. Projektet ANM4L har utvecklat algoritmer som ska användas i elnätet för att förhindra överspänning och överströmmar eftersom dessa skadar nätutrustningen.

Ett centralt begrepp här är reaktiv effekt. Det som i vardagligt tal kallas för effekt heter egentligen aktiv effekt och är i själva verket bara en del av den totala effekten i ett trefas-system. En bra liknelse ses i figur I. Den vardagliga (aktiva) effekten motsvarar den delen av kraften som faktiskt drar vagnen framåt. Det finns också en del av kraften som är riktad uppåt, men den är meningslös i sammanhanget eftersom den inte hjälper till att dra vagnen framåt. På samma sätt är det med reaktiv effekt - den kan inte konsumeras för att driva en spis eller en vattenkokare men den finns där ändå.



Figur I: Hur man kan se på reaktiv effekt

Reaktiv effekt kan dock konsumeras, om än inte för något så handfast som att driva en spis, och det som händer då är att spänningen sänks i elnätet på det aktuella stället. Algoritmen för att styra spänning reglerar spänningen genom att i första hand konsumera reaktiv effekt. Om det inte finns möjlighet att konsumera mer sådan börjar den minska den aktiva effekten. Men aktiv effekt var ju den användbara delen av effekten och minskas den får vi väl mindre effekt ut i elnätet? Ja, så är det men alternativet att låta spänningen vara för hög skadar utrustningen i nätet. Den versionen av algoritmen som förhindrar överströmmar kan inte konsumera reaktiv effekt utan reglerar endast genom att minska den aktiva effekten.

Algoritmerna har utvecklats med förhållningssättet att de ska vara lätta att tillämpa för nätoperatörerna och att det ska vara möjligt att använda dem länge, trots ett föränderligt elnät där andelen förnybart förväntas öka mer och mer. Om man tillämpar spänningsalgoritmen i ett nät där flera hushåll har solcellsanläggningar ska de olika anläggningarna inte behöva ha ett kommunikationssystem. Det vore förvisso bättre om anläggningarna hade kunnat koordinera med varandra men det hade gjort systemet svårare att tillämpa. Frågan är om algoritmen är för enkel och därmed för långt ifrån att vara optimal? Detta var frågan Martin Lundberg och Professor Olof Samuelsson i projektet ANM4L ställde till undertecknad, som utvärderade detta genom att i datorprogrammet GAMS räkna ut hur en optimal reglering skulle reglera spänning och ström i två modeller av elnät. Detta resultat jämfördes sedan med hur ANM4L-algoritmerna skulle reglera i samma elnätsmodeller för att se hur optimal algoritmerna är.

Den första elnätsmodellen bestod av fem solcellsanläggningar anslutna i rad till en transformatorstation som kopplar dem till "resten av elnätet". Resultatet av utvärderingen visade att algoritmen presterar tillfredsställande. Det prioriterade målet är att man ska klara av att få spänningar och strömmar inom de tillåtna gränserna genom att bara konsumera reaktiv effekt. Detta är för att man inte vill behöva minska den användbara aktiva effekten, men tyvärr behöver man ibland trots allt göra just detta. Om man tittar på den totala minskningen av aktiv effekt, alltså om man lägger ihop minskningarna på respektive solcellsanläggning, visar det sig att den, då algoritmen reglerat, inte är så långt ifrån densamma i det optimala fallet. Detsamma gällde i den andra elnätsmodellen, som innehöll vindkraft istället för solceller.

Om man däremot tittar på fördelningen av konsumtion av reaktiv effekt mellan de olika solcellsanläggningarna är det stor skillnad mellan algoritmens reglering och det optimala ditot. Den förra koncentrerar konsumtionen till de anläggningar som ligger längst bort från transformatorstationen medan den senare har en inte helt rättvis fördelning men mer åt det hållet. Vid konsumtion av reaktiv effekt slits en solcellsanläggnings utrustning (mer specifikt inverteraren) och är du solcellsägaren längst bort tycker du naturligtvis detta är orättvist. Det går dock i teorin att kompensera för detta på marknaden; idén är då att de som råkar bo längst in nära transformatorstationen och har solceller ger ekonomisk ersättning till den som råkar bo längst ut. Om ett sådant ersättningssystem kan utvecklas skulle den orättvisa fördelningen inte göra så mycket, men detta område behöver undersökas mer för att se om det är en lämplig väg att gå.

Den version av algoritmen som reglerar ström testades i den andra elnätsmodellen som innehöll vindkraft på två ställen. Som tidigare nämnts kan denna version av algoritmen inte konsumera reaktiv effekt utan reglerar endast genom att minska den aktiva effekten. Till skillnad från versionen som styr spänning måste algoritmen som styr ström ha viss form av kommunikation mellan de olika delarna av nätet. Detta gör det möjligt att ha en inställning som, om den är aktiverad, fördelar regleringsbördan rättvist mellan de båda vindkraftsgeneratorerna. Det visade sig att när rättvis fördelning användes blev den totala minskade aktiva effekten lika liten som i den optimala regleringen – och dessutom var bördan väldigt rättvist fördelad. Även om man inte ska dra för stora växlar på ett test i bara en elnätsmodell är det ett lovande resultat!

Contents

1 Introduction

The issue of climate change is one of the greatest long term challenges of today. One central part of it is that humanity needs to switch from fossil energy sources to sustainable ones. This is not easy in any sector of the society, but it is harder in some parts than others. Furthermore, the challenges that this transition creates are very different in different sectors. The sector which is relevant regarding this thesis, the power sector, faces many challenges when more and more renewable energy is connected to the electrical grid. It is important to note that renewable energy sources like solar panels or wind turbines are often connected at the low or medium voltage levels and this is in contrast with the traditional way of having generation at the high voltage level and load at the low voltage level [1].

Connecting renewable energy at the low or medium voltage levels can lead to overvoltages and overcurrents. The intermittency of photovoltaics and wind power means that generation sometimes can be higher than demand. This increases the voltage and can cause congestion. Congestion means that there will be a too large power flow compared to what the equipment is designed for, or equivalently it can be seen as an overcurrent. The voltage and current problems limit how much renewable energy can be connected. Since these problems have not been encountered in the low and medium voltage grids before, convenient solutions are yet to be developed. One approach to control the voltage and current, to be able to connect more renewables, is to adjust the reactive power (Q) and the active power (P). The project Active Network Management for All (ANM4L) is, among other subjects, treating this. ANM4L is a European project which works towards active management of electrical grids to make integration of large amounts of renewable energy sources possible [2]. Project partners are RISE Research Institutes of Sweden (coordinator), municipality of Borgholm, Lumenaza GmbH, Lund University, RWTH Aachen University, E.ON Energidistribution AB, E.ON Észak-dunántúli Áramhálózati Zrt., and E.ON Solutions GmbH. The active network management solutions treated in this thesis will be demonstrated in the Swedish island Öland and in Hungary.

Since overvoltage has not been a problem in the low and medium networks before, there has been no strategy to handle it apart from the automatic disconnecting by generator inverters when the voltage reaches a certain set point [3]. This means unfortunately that the power output will become zero although there could have been some generation without reaching overvoltage. If a grid has seen a considerable risk of overvoltage, grid reinforcement has traditionally been used to reduce that risk to an acceptable level but this approach is expensive [1].

The ANM4L voltage control algorithm provides an active management solution, as it consumes Q and, if needed, curtails some or all P to adjust the voltage in the grid [4]. This results in more P available which is economically favorable. The algorithm uses PI controllers which adjust Q and P with the voltage as input. When the maximum consumption of Q at a bus is reached, that controller will start curtailing P. Simulations have shown that the algorithm can reduce the curtailment of P during peak production and that further installation of renewable energy in the grid is possible [4].

Another ANM4L algorithm is targeted at handling congestion or, equivalently, overcurrent [5]. This algorithm cannot consume Q; it has curtailment of P as its only control action. It has been tested in a medium voltage network model and brought the current at an overloaded line to below the maximal limit.

The question remains how optimal the algorithms' control actions are. In the ANM4L project, grid owner aspects have been prioritized: fixed parameters and low communication requirements between different generation sources. The actions are therefore not optimal considering generating unit owner aspects such as curtailing as little P as possible. Furthermore, the voltage algorithm does not consider the issue of fairness – some generation sources may end up contributing much more to the control than others. Different approaches can be taken when seeking optimal control. Indeed, the field of mathematical optimization is vast and only a brief part will be treated in this work. The choice of objective function well affects the result and thus there can be several definitions of ”optimal”.

1.1 Problem Formulation

The aim of this work is, with the description above as background, to evaluate the control algorithms developed in the ANM4L project against optimal control actions. This will be done in two test networks: one low voltage network and one medium voltage one. The performance of different versions of the algorithms will also be compared, for voltage and congestion management respectively. In the optimization, it is examined which control actions are optimal for P and Q and at which buses in the systems the actions should be taken. Generally, one ideally wants to prevent overvoltage or overcurrents while minimizing active power curtailment and this should be done with minimal control effort. From this, two questions are asked:

- How optimal are the ANM4L control algorithms?
- For voltage and congestion management respectively, which version of the ANM4L control algorithm comes closest to the optimal actions?

It is of course important to reflect upon if the optimum found by an optimization method is global or not. This will be discussed later in this thesis.

1.1.1 Delimitations

The ANM4L algorithms will only be evaluated in two test network models. Three levels of generating power will be examined in the low voltage network and one level of generation in the medium voltage network. Furthermore, although it would have been interesting, the results of the optimization will not be used to improve the ANM4L algorithms; only evaluation of the algorithms will be performed. It should be stressed that the optimization problems in this thesis are non-convex which means that the optima found are locally optimal but cannot be guaranteed to be globally optimal. This is a flaw that this thesis has to live with. Yet, locally optimal control actions can serve as a reasonable benchmark to compare the ANM4L control actions with. Lastly, it should be said that most focus will be set on the voltage management and less focus on the congestion management.

1.2 Related Work

The amount of photovoltaics (PV) in the grid has increased considerably past years. [3] stress the challenges of ever-increasing distributed PV in the Australian electrical grid. There are times during the day when distributed PV in aggregate is the largest type of generation in the Australian national electricity market. As overvoltage at the distribution level is a new phenomenon, there is no other handling of it than the automatic disconnection of solar panel inverters when the voltage reaches a certain level. The authors of [3] propose several more sophisticated control systems for voltage. Among those is control of P and Q. In contrast to the PI controller of the ANM4L algorithm, their suggestion is a droop-based controller. In other words this can be seen as a proportional controller with deadband.

The radial structure of the grid means that households in remote areas will see their power curtailed more often than households at the inner part of a feeder. This raises the question of fairness. [6] treat the problem of fairness when controlling voltage in an electrical grid. To evaluate the conflict between fairness and having better overall performance, a quantification of the cost of fairness is proposed.

There are several previous studies treating optimization of voltage control systems in electrical grids. [1] examines two droop-based control algorithms of an electric grid. Both adjust Q to maintain acceptable voltages as power generation by PV increases. The input to decide control actions is P for the first algorithm and voltage for the second algorithm. Q will be adjusted according to the droop-based relationship with the input. This will affect the voltage as well as the active power losses. One focus of [1] is to optimize parameters in the droop-based relationships. The optimized methods were tested in a power system model and both of them showed a reduction of reactive power consumption and reduction of active power losses. While, in the case of [1], optimal control *parameters* were sought, the focus in this work will be optimal control *actions*.

Similarly to the unfair distribution of curtailment of P, the consumption of Q when using a control system can be unevenly distributed. The question could be asked if the aim should be to have equal contribution to consumption of Q. [1] treats this issue. The result of having unequal distribution of consumption of Q, which means letting some parts of the system have higher consumption than others, can cause excessive loading in those parts. This is proposed to be solved by introducing constraints on the consumption of Q. If, on the other hand, equal distribution of consumption of Q is chosen this results in higher total consumption of Q and greater line losses. It also leads to broader activation ranges which means that there could be consumption of Q although there is no overvoltage. Furthermore, the gains of the control system are lower. With the now mentioned aspects as background, [1] proposes that it is perhaps better to share the costs of the consumption of Q in another way than having equal distribution.

The ANM4L voltage control system is a decentralized system. That is, PI controllers at different buses do neither communicate with each other, nor with a central controller. This makes it easier to implement, but it also makes the control non-optimal. [7] treat, similarly to this thesis, a decentralized system for controlling voltage by Q. They compare its actions to optimal actions and conclude that in some cases the decentralized system can fail to bring the voltage inside the allowed interval although there exists a solution considering available P and Q that can be controlled. They propose what could possibly be called a middle way between centralized and decentralized control which is a system letting neighboring buses communicate. This system is also tested in simulations and it finds a solution in the case in which the decentralized system fell short. While this solution is not something which will be implemented by ANM4L at the moment, it is interesting to see other perspectives.

Congestion management has gained more and more interest in research when the amount of renewable energy in the electrical grid has increased. There are different ways of handling congestion. Economic policies for creating flexible demand can be used [8]. Another sort of load management is letting an intelligent control system decide, in real time, where to curtail load [9]. Using energy storage as a buffer is another possible solution [10]. These strategies will be treated some more in Section 2.3.

2 Theory

In this chapter, an introduction to the problems of overvoltage and overcurrents will initially be given. Then focus will be shifted to possible ways of solving them and after that the ANM4L algorithms will be presented. Last, the subject of optimization will be briefly introduced.

2.1 Power Systems with Variable Renewable Energy

Variable renewable energy (VRE) is renewable energy which cannot be dispatched on demand. Typically, it is used when speaking about solar and wind power. The increasing amount of VRE in electrical grids leads to several challenges. One of those is that the voltage can rise when a VRE generator is installed in a distribution line [11]. Observe the single line diagram in Figure 1.

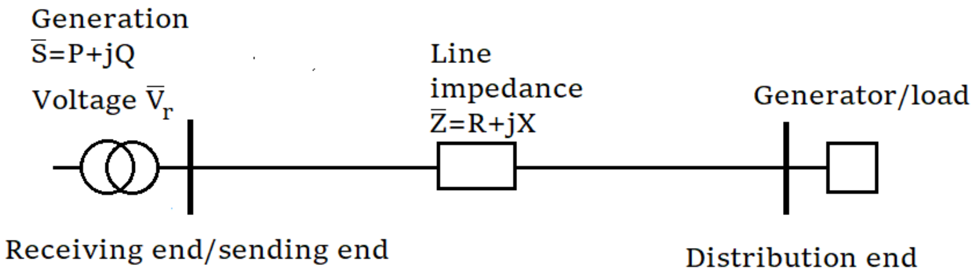


Figure 1: Illustrated here is a single line diagram for a line with a generator/load to the right. To the left is a transformer connecting it to the external grid. The right end is thus the distribution end and the left end is the receiving/sending end.

A bar over a variable means that it is a vector. The generation at the receiving bus is $\overline{S} = P + jQ$ and the voltage at the receiving bus is $\overline{V}_r$. The line impedance is $\overline{Z} = R + jX$. The voltage change $\overline{\Delta V}$ between a point at the receiving end and a point at the distribution end at a line can be written as in Equation (1) [12]. $\overline{I}$ is the current at the line. An asterisk means complex conjugate.

$$\overline{\Delta V} = \overline{IZ} = \frac{\overline{S}^*}{\overline{V}_r}\overline{Z} = \frac{1}{\overline{V}_r}((RP + XQ) + j(XP - RQ)) \tag{1}$$

Equation (1) gives the following implications. With large generation of P (and $Q = 0$) at the end of a line, the voltage will increase at that position. With too little generation of P compared to a load at the end of the line, the voltage will decrease instead. This leads to a behaviour of the voltage at a line as in Figure 2:

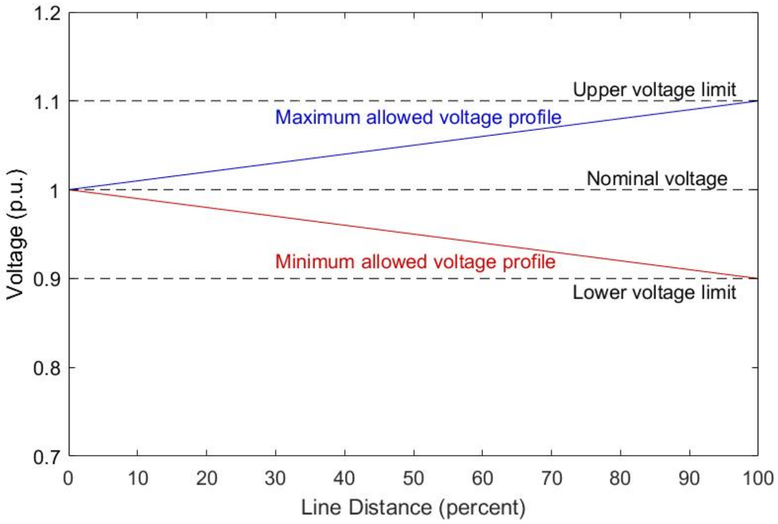


Figure 2: Illustrated here are allowed voltage profiles in a distribution line with only one generator/load positioned as in Figure 1. The distribution end is to the right and the receiving/sending end to the left.

If consumption of Q is introduced at the end of the line, that is negative Q, the real part of Equation (1) will decrease. The imaginary part, on the other hand, will increase but as $|\overline{V}_r|$ will increase, $\overline{\Delta V}$ over the line will decrease.

If a line would be half as long, it would have half as large impedance [5]. Analogously, if a generator or load is connected at an intermediate point of a feeder the impedance between that generator/load and the beginning of the feeder will be proportional to the distance to the beginning of the feeder. This means that a generator connected to the end of the feeder affects the voltage more than a generator connected to the middle part. Similarly, curtailment of P and consumption of Q to reduce the overvoltage are most effective at the end of the feeder.

Another possible problem when introducing VRE in a distribution network is that the currents at the lines could become too large because of a too large power transfer. That will, in turn, cause too much heating of the equipment [5]. This reduces the life time of the equipment and long periods of overcurrents should therefore be avoided.

2.1.1 Grid Reinforcement

If an electrical grid has experienced a risk of overvoltage and/or congestion and that risk has been considered too large, the traditional solution has been grid reinforcement [1]. Grid reinforcement means installation of new lines or upgrading/replacing the existing lines to increase the transfer capacity [12]. This is, however, expensive. The problems coming with the great increase in distributed generation would preferably be handled in other ways. Active network management can be used to avoid, or postpone, grid reinforcement.

2.1.2 Grid Codes

The European standard EN50160 states that the voltage in a low or medium voltage grid is allowed to deviate ±10 % from nominal voltage. The measurements of voltage in the grid are used to create root mean square (rms) avarage values over 10 minute periods. It is allowed that these rms values deviates more than ±10 % in up to 5 % of the periods during a week, according to the same standard. In this work, these deviations are not considered.

2.2 Voltage Management

The tap changers in transformers can be used to raise or lower the voltage profile [12]. A tap changer is a system which allows the ratio between the primary and secondary windings in a transformer to be varied. This makes it possible to decide the ratio between the voltages at the primary and secondary sides. It can change the voltage about ±10 % in steps of about 1 %, the exact numbers being decided by the physical components. It does not change the slope of the voltage profile, but displaces the curve upwards or downwards. If two feeders are connected to one transformer with one of them having problems with overvoltage and the other one having problems with undervoltage, it may not be possible to set the tap changer in the transformer so that both feeders stay inside the allowed voltage interval. This is because one of the feeders demands raising of the voltage profile and the other one demands lowering.

It is also possible to use control of load in the grid to control the voltage [12]. An increase of load would decrease the voltage and vice versa. This demands a control system to decide if a load is going to be increased or decreased. To optimize the system, some communication between the different loads can be used but this makes it complex and harder to implement.

Furthermore, battery energy storage systems can be used to control voltage [13]. In simulations made by [13], such a system performed well as it kept the voltage within the allowed interval although the test scenario had a level of PV installation which was much higher than it is today.

2.2.1 The ANM4L Algorithm for Voltage Management

The ANM4L-algorithm provides another alternative for voltage management in the grid by using consumption of Q in first hand. The consumption is done by the inverters of the PV-generators. If the inverter of a PV generator does not allow to increase the consumption of Q further, P is curtailed. At each controlling bus, there is a PI controller. The PI-controllers take the voltage as input and the outputs are control actions for P and Q. The algorithm is developed with the approach that it should not be too complicated to implement. Therefore, the different PI controllers do not communicate neither with each other nor with a central controller.

A version of the algorithm which uses constant power factor will also be examined. This will be treated more in Section 4, but the basic idea is that a constant power factor could possibly make the algorithm control actions come closer to the optimal actions. The reason it could work is because a constant power factor prevents the algorithm from curtailing P at the outer buses at the feeder although there is still plenty of potential to consume Q at the inner buses. In theory a power factor of P/Q=-X/R, with X being the reactance of the lines and R the resistance, would eliminate the voltage rise entirely [5]. However, the rated transfer capacity S_{max} of the inverters of the PV stations poses a limit on how much Q that can be consumed because $P^2+Q^2 \leq S_{max}^2$ must hold.

The allocation of control actions between the different generators in the grid is an aspect which should be considered before implementing the ANM4L control system in a real network. If equal sharing of Q is used, this may lead to higher total consumption of Q and greater line losses [1]. If unequal sharing of Q is used, it could cause heavy load at the components which end up with the greatest burden in the control and thus cause an unfair situation among the generator owners in the grid. There are thus drawbacks with both equal and unequal sharing of Q.

The issue of fairness can be tackled in many ways. One way is to share the control effort unequally and then use the market to compensate the ones which contribute more to the control. A second way to treat the fairness issue is to distribute the consumption of Q equally and live with the higher total consumption of Q and greater losses.

2.3 Congestion Management

Congestion means transfer of too much power at a line compared to what the line withstands. The problem of congestion can equivalently be seen as an overcurrent problem. Similar to the case of voltage management, there are different ways of preventing congestion. Some examples will be given here to serve as a background.

One way is to focus on the load. Economic policies which create incentives to increase or reduce demand (depending on the type of problem) when there is a risk of congestion is a possible strategy [8]. Customers in a flexibility program can be asked to change their demand if there is such a risk. Power producers in such a program could be asked to change their generation. Examples of projects of this type are the CoordiNet project in the EU [14] and the sthlmflex project in Stockholm by Svenska kraftnät, Ellevio and Vattenfall [15].

A second way of preventing congestion by load management is the use of real-time load control. An intelligent system is then responsible for deciding which loads are convenient to curtail, if curtailment is needed [9].

Another strategy is to use energy storage. The idea is then to store electricity from times of excess generation to use in times of too low generation [10]. The system would also increase flexibility in the network.

2.3.1 The ANM4L Algorithm for Congestion Management

Unlike the ANM4L voltage control algorithm, the corresponding one for preventing overcurrents cannot be completely local [5]. To understand why, consider the following example: a feeder having almost 100 % of its generation at the most remote bus (that is the bus furthest out at the feeder) but with an overcurrent at the beginning of the feeder cannot prevent the congestion, unless there is some communication between the beginning and end parts of the feeder. The aim has been to make the amount of communication in the algorithm as low as possible to, analogously to the approach in the voltage management, make an implementation of the system as easy as possible. The congestion management algorithm examined in this work uses, similarly to the voltage one, PI controllers. This one takes current as input and curtails P as control action.

The sharing of curtailment between buses can be decided in several ways. Two versions of the algorithm are treated in this work, representing two ways of distributing the control actions:

- The first one uses prioritization by power transfer distribution factors (PTDF). $PTDF_{ij}$ is a factor which tells how a change in P at bus i affects the flow at line j [5]:

  $$PTDF_{ij} = \frac{\Delta S_{line,j}}{\Delta S_{bus,i}}$$

  It can be used to make sure that the control is effective by curtailing P at the buses which affect the flow the most.

- The second one uses equal sharing, which will distribute the actions completely equal.

2.4 A VRE Network Emerges

The ANM4L algorithms have been developed with the continuous growth of the network in mind and should therefore be scaleable [5]. In other words, all the VRE will not appear at once in a network but it is a process going on for many years. If the ultimate goal is that the algorithms should perform well in the "final" network, one has to live with that they perform less well in the beginning.

2.5 Optimization

Optimization is, in simple terms, to minimize or maximize an objective function subject to a set of constraint functions which must be fulfilled [16]. A general formulation of such a problem can be written as in Equation (2):

$$\begin{aligned} \text{minimize (or maximize)} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0,\ i = 1, ..., p \\ & h_j(x) = 0,\ j = 1, ..., q \end{aligned} \tag{2}$$

The function $f(x)$ is the objective function. The functions $g_i(x)$ are inequality constraints which are traditionally written on the form: $g_i(x) \leq 0$ and the functions $h_j(x)$ are equality constraints which are traditionally written one the form $h_j(x) = 0$. Various methods exist for solving this problem and different solvers use different approaches. The solvers used in this work are briefly treated in Section 4.5.

2.5.1 The Issue of Finding a Global Optimum

It is relevant to know if the optimum found is global or not. In other words the question is if it is only locally optimal or if it is the optimal solution on the whole domain of the objective function. To treat this area, one has to be familiar with the concept of convexity. A function y is said to be convex if it fulfills (3) [16]:

$$\begin{cases} \boldsymbol{a}, \boldsymbol{b} \in S,\ S \subseteq \boldsymbol{R}^n \\ 0 < \lambda < 1 \end{cases} \Rightarrow y(\lambda \boldsymbol{a} + (1-\lambda)\boldsymbol{b}) \leq \lambda y(\boldsymbol{a}) + (1-\lambda) f(\boldsymbol{b}) \tag{3}$$

To understand this definition, a geometrical representation of a one-dimensional function can be used as an example. (3) then means that a straight line which connects the two points $(a, y(a))$ and $(b, y(b))$ must lie above the function curve, or be equal to it [16]. If an optimization problem is convex, a minimum found is guaranteed to be a global minimum. A problem is convex if and only if its objective function and all its constraint functions are convex.

In this work, the non-convex functions sine and cosine are used in the optimization problem. This means that it cannot be guaranteed that the optimum found will be global. In past years, some papers have focused on how non-convex power flow problems can be relaxed to convex ones and thus guarantee that the optimization finds a global optimum. [17] examines the results of solving convex relaxations of power flow problems which includes relaxing the constraints. No details about the method will be treated here, but the idea is that after obtaining a solution to the relaxed problem, the solution is evaluated against the constraints which were relaxed. Then it is possible to see if the solution to the relaxed problem solves the unrelaxed problem and if so, a global optimum has been found to the unrelaxed problem as well.

The size of this thesis work has not made such an interesting approach possible to consider. This leads to that an optimum found is a local optimum but not necessarily a global optimum. This problem is not possible to bypass with the approach used in this work.

3 Test Networks

Two test networks were used, one low voltage (LV) and one medium voltage (MV) network. In the LV network, only voltage management was examined. In the MV network, both voltage and congestion management were examined. The reason that congestion management was examined only in the MV network is that there was no model of the congestion management algorithm in that network. Combined management of voltage and congestion was not examined for the same reason; there was no model in which the algorithm manages both these problems.

The LV network has shorter lines than the MV network. With that as background, the LV network lines are modeled having impedances with a resistance and a reactance part whereas the MV network lines are modeled having impedances with a resistance part, a reactance part and a shunt capacitance.

3.1 The LV Test Network

The LV test network is used for control development in the ANM4L project. It consists of five buses in the LV part of the network. *Bus MV* was introduced in the MV part, which corresponds to the external grid. This test network is used to examine control in a distribution network with a large amount of photovoltaics. All buses in the LV part have a PV-generator and all generators have identical characteristics. Each PV-generator represents one household during peak production at noon, that is a worst-case scenario. The household loads are assumed to be small and thus neglected. The network can be seen in Figure 3.

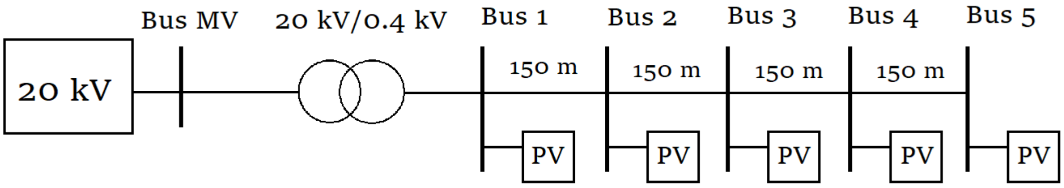


Figure 3: The LV Test Network

The parameters in the network are:

- The nominal voltage is 0.4 kV line to line in the LV part and 20 kV in the MV part.
- The lines in the LV part of the grid have R=0.346 Ω/km and X=0.0754

Ω/km and the distance between two neighboring buses in the LV part is 150 m.

- The line impedance between bus 1 and bus MV has R=2.828 Ω and X=2.8274 Ω.
- The transformer impedance on both the 0.4 kV-side and the 20 kV-side has R=0.0095 Ω and X=0.0175 Ω.
- The transformer apparent power (S) transfer rating, that is the maximum S the transformer is designed to transfer, is 150 kVA.
- When the PI controllers can control P and Q independently, the PV inverter rating S_{max} was set to 5 kVA above the rating of one generator. That is, S_{max} changed with the generator rating. When the PI controllers could control only P and Q was decided by a constant power factor, then $S_{max} = P/\cos(\varphi)$ in which $\cos(\varphi)$ is the power factor. This S_{max} was chosen because the output from a generator could never be the full rated P if S_{max} is 5 kVA above the rated P, unless the rated output is smaller than the assumed ratings in this work or if the power factor is higher than 0.85 or so. The lower the power factor is, the larger S_{max} needs to be. The reason for that is that with a lower power factor, the consumption of Q will be larger and thus the apparent power flow through the inverter will increase.
- When S_{max} was changed in the algorithm control case, it was also changed in the optimization program to give both control systems the same ability to transfer power from and to the PV stations.
- The voltage deviation limit was set to 0.1 p.u. from nominal voltage.

The operating points used in the simulations are:

- Three cases with three different PV-generator output ratings were examined. The first case was 30 kW per generator, the second was 40 kW per generator and the third was 50 kW per generator. These levels were chosen because they are feasible given the network parameters; the low-generation case of 30 kW demands some but not too large control effort and the high-generation case of 50 kW demands a fairly large control effort. Whether a certain output level demands a large control effort or not could be understood by studying the ANM4L control actions for that output level.
- The voltage at bus MV, that is the external grid, was set to 1.02 p.u..

3.2 The MV Test Network

The MV test network is a part of the MV benchmark network defined by the International Council on Large Electric Systems (CIGRE). The MV benchmark network consists of two feeders of which only the right one, having three buses, is used in this work. It could represent a European rural feeder. Although only a part of the CIGRE network is used, the bus numbers 12, 13 and 14 are nevertheless kept to be consistent with the original network. The network consists of three buses in the MV part of the network. *Bus HV* (high voltage) was introduced in the HV part, and corresponds to the external grid. This test network is used to examine voltage management and congestion management in an MV network with wind power. The network can be seen in Figure 4.

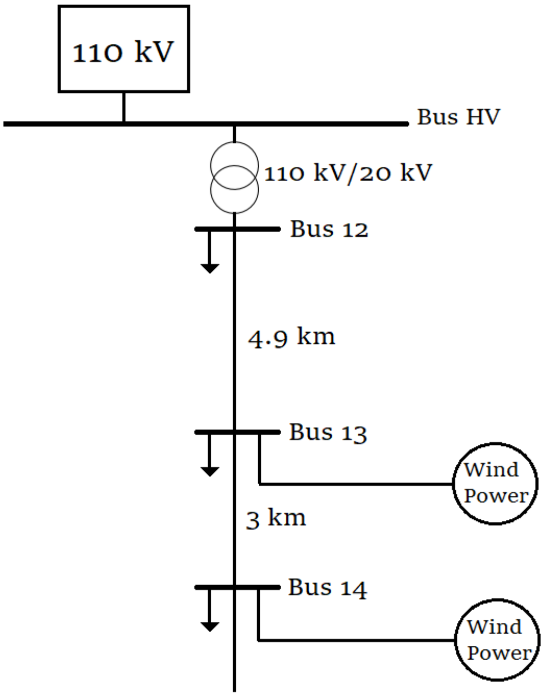


Figure 4: The MV test network

There are two types of load relevant here: residential load and commercial load. The former corresponds to aggregated households and the latter corresponds to business operations. There are daily load profiles for the loads in the network. The simulation is assumed to take place around 1 pm and the residential loads are, with the profiles as background, assumed to be 65.3 % of their maximal values. The corresponding number for the commercial loads is 85 %. The loads presented below are adjusted for this.

The parameters in the network are:

- There is a residential load of 9.79 MW + 1.99 Mvar and a commercial load of 4.26 MW + 1.40 Mvar at bus 12.
- There is a commercial load of 0.029 MW + 0.018 MW at bus 13.
- There is a residential load of 0.14 MW + 0.034 Mvar and a commercial load of 0.28 MW + 0.17 Mvar at bus 14.
- The nominal voltage is 20 kV line to line in the MV part and 110 kV in the HV part.
- The lines 12–13 and 13–14 have R=0.510 Ω/km, X=0.366 Ω/km and C=10.09679 nF/km. Line 12–13 is 4.89 km and line 13–14 is 2.99 km.
- The line impedance between bus 12 and bus HV has R=0.0064 p.u. and X=0.4800 p.u., using an S_{base} of 100 MVA. This impedance includes transformer impedance and line impedance.
- The transformer S transfer rating, that is the maximum S the transformer is designed to transfer, is 25 MVA.
- The voltage deviation limit was set to 0.05 p.u. from nominal voltage.
- It was assumed that there is a congestion risk at the line between bus 12 and 13. The stated line current limit of that line is assumed to be 195 A.

The operating points used in the simulations are:

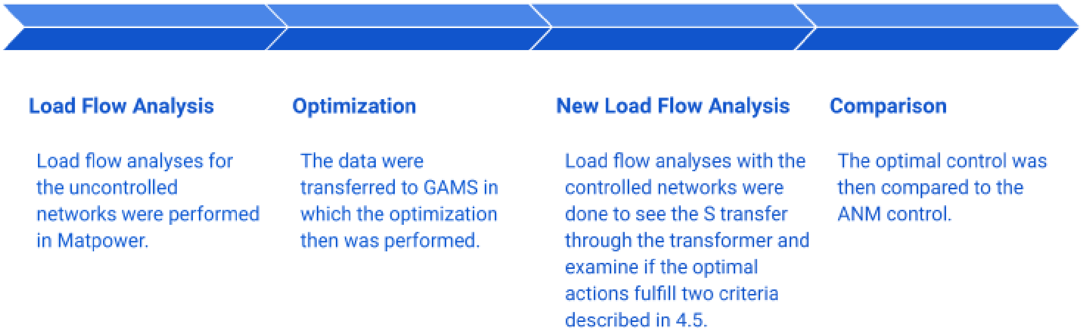
- Only one wind power generation case was examined: generation of 3 MW at bus 13 and generation of 6 MW at bus 14. This case was chosen since it leads to both overvoltage and congestion.
- The voltage at bus HV, that is the external grid, was set to 1.02 p.u. but the tap changer in the transformer was set so that the voltage at the MV side was 1.05 p.u..

4 Method

The optimization task was done in two steps. First, a load flow analysis for the uncontrolled LV network and MV network respectively was performed in Matpower which is a power system simulation package in Matlab. Then the data were transfered to GAMS in which the optimization was done. GAMS is a modeling program focused on solving optimization problems. Load flow analyses were then performed once again in Matpower with the controlled networks as input, to acquire the transfer of S through the transformers and examine if the optimal control actions fulfill two criteria which will be presented in Section 4.5. Four aspects were then focused on when comparing the active network management (ANM) control and the optimal control:

- The distribution of the optimal control actions was compared to the distribution of the ANM control actions.
- The total curtailment of P with optimal control was compared to the corresponding value for ANM control. This aspect was regarded as more important than the total consumption of Q. In other words, it was regarded more important to retain a large transfer of P to the grid than keeping the consumption of Q low.
- The total consumption of Q with optimal control was compared to the corresponding value for ANM control.
- The transfer of S through the transformers with optimal control was compared to the corresponding value for ANM control to see if any of those values exceeded the transformer rating.

The workflow can be seen in Figure 5.



Load Flow Analysis

Load flow analyses for the uncontrolled networks were performed in Matpower.

Optimization

The data were transferred to GAMS in which the optimization then was performed.

New Load Flow Analysis

Load flow analyses with the controlled networks were done to see the S transfer through the transformer and examine if the optimal actions fulfill two criteria described in 4.5.

Comparison

The optimal control was then compared to the ANM control.

Figure 5: Workflow

Two sorts of comparisons regarding the voltage management were made:

- one comparison in which both the PI controllers and the optimization program could control P and Q independently
- one comparison in which the PI controllers could control P, using a constant power factor to decide Q, whereas the optimization program could still control P and Q independently

The reason why a case was examined in which the PI controllers use a constant power factor is, as was explained in Section 2.2.1, because this could possibly make the algorithm control come closer to the optimal control actions. That is, if an adequate power factor is used. A too small power factor would consume an unnecessarily large amount of Q. A too large power factor would lead to the consumption of Q being far from solving the overvoltage problem and thus an unnecessarily large amount of P will be curtailed.

Two different versions of the voltage algorithm with constant power factor were tested in the LV network: a power factor of 0.8 and one of 0.9. Also in the MV network, two different versions of the voltage algorithm with constant power factor were tested: a power factor of 0.9 and one of 0.95. The power factors respectively were chosen because they were believed to be adequate given the reasoning above. Regarding the congestion management, only P could be controlled in both the algorithm and the optimization.

4.1 The LV Network

4.1.1 Load Flow Analysis

To acquire bus voltage absolutes, V, and bus voltage phase angles, θ, a load flow analysis was performed in Matpower. At some buses only θ was unknown and at some, both V and θ were unknown. Matpower uses Newton's method and an initial guess (V_{init}, θ_{init}) to iterate until V and θ at the concerned buses are found that leads to the known P and Q of the buses. At buses with only θ unkown (called PV-buses), P is known and at buses with both V and θ unknown (called PQ-buses), P and Q are known.

Bus MV was labeled reference bus, thus having known voltage absolute and phase, and buses 1-5 were labeled PQ-buses. A seventh bus, a dummy bus, labeled as PV-bus had to be introduced in the load flow analysis, because Matpower demands that there must be one reference bus, at least one PV-bus and at least one PQ-bus. The dummy bus was connected on the left side of bus MV. It had neither generation nor load, thus not affecting the result.

4.1.2 Acquiring the ANM4L Algorithm Control Actions

A Matlab Simulink model of the LV network with the voltage control algorithm was recieved from the ANM4L project. The control actions from the ANM algorithm in the LV network were acquired by running the Simulink model.

4.1.3 The LV Network Optimization: Objective Function

The objective function consists of two terms. The first one sums the squares of the curtailed P at the buses respectively. It should be minimized. The bus number is indicated by k and the total number of buses is N.

$$f_1 = \sum_{k=1}^{N} \Delta P_k^2 \tag{4}$$

The second term sums the squares of the consumed Q at the buses respectively. It should also be minimized. Once again, the bus number is indicated by k and the total number of buses is N.

$$f_2 = \sum_{k=1}^{N} \Delta Q_k^2 \tag{5}$$

Squares of ΔP_k, ΔQ_k are used to make the optimization program control as little as possible regardless of the sign of the actions. If sums without squaring would be used as the objective function, the optimization program would prefer to make the actions positive at the inner buses at the feeder and thus even out the negative variables at the remote buses. Since the goal is to minimize the control effort, whether the actions are negative or not, the variables are squared. An alternative to this would have been to use absolute values but this made the problem discontinuous, which in turn made it harder for the solvers to find an optimum.

To sum up, the objective function has the form:

$$f_{obj} = w_1 \cdot f_1 + w_2 \cdot f_2 \tag{6}$$

Weight factors w_k can be multiplied with the terms of the objective function respectively. w_1 was set to 1 and w_2 was set to 0.001. This was to retain a large transfer of P to the grid by avoiding curtailment of P. If both weight factors would have been set to one, curtailment of P would be used primarily because the network has a small X/R-ratio. A small X/R-ratio makes control using P more effective than control using Q.

4.1.4 The LV Network Optimization: Constraint Functions

In the constraint functions below, P_k is the active power generation at bus k. V_k is the absolute value of the voltage at bus k and θ_k its phase angle. Y_{ki} is the absolute value of the line admittance between bus k and i and α_{ki} its phase angle, with the admittance being equal to $1/(R + jX)$. The values of P_k, V_k and θ_k that are acquired from the load flow analysis are modified during the optimization iteration as:

- $\widetilde{P}_k = P_k + \Delta P_k$
- $\widetilde{V}_k = V_k + \Delta V_k$
- $\widetilde{\theta}_k = \theta_k + \Delta\theta_k$.

Below, the constraint functions 1 and 2 are equality constraints and the constraint functions 3 and 4 are inequality constraints, as explained in Section 2.5.

Constraint 1

The relation between the active powers, voltages and admittances must hold:

$$\widetilde{P}_k = \widetilde{V}_k^2 Y_{kk} \cos(-\alpha_{kk}) - \sum_{i \neq k}^{N} \widetilde{V}_k \widetilde{V}_i Y_{ki} \cos(\widetilde{\theta}_k - \widetilde{\theta}_i - \alpha_{ki}) \tag{7}$$

This is the standard load flow equation for P.

Constraint 2

The relation between the reactive powers, voltages and admittances must hold. Note that there is only ΔQ_k on the left hand side because there is neither any consumption nor generation of reactive power in the uncontrolled network:

$$\Delta Q_k = \widetilde{V}_k^2 Y_{kk} \sin(-\alpha_{kk}) - \sum_{i \neq k}^{N} \widetilde{V}_k \widetilde{V}_i Y_{ki} \sin(\widetilde{\theta}_k - \widetilde{\theta}_i - \alpha_{ki}) \tag{8}$$

This is the standard load flow equation for Q.

Constraint 3

The voltage must be above the lower allowed limit (in p.u.):

$$\widetilde{V}_k \geq 0.9 \tag{9}$$

The voltage must be below the upper allowed limit (in p.u.):

$$\widetilde{V}_k \leq 1.1 \tag{10}$$

Constraint 4

The relation between P, Q and S must hold. S_{max} is the inverter transfer rating of the PV units respectively.

$$\widetilde{P}_k^2 + \Delta Q_k^2 \leq S_{max}^2 \tag{11}$$

Optimization Variables

In GAMS the optimization variables were: ΔP_k, ΔQ_k, ΔV_k and $\Delta\theta_k$. That is, those are the variables which GAMS can vary. Since ΔV_k and $\Delta\theta_k$ are dependent on ΔP_k, ΔQ_k, the latter variables were the optimization variables in the mathematical sense.

4.2 The MV Network

4.2.1 Load Flow Analysis

In the same way as for the LV network, a load flow analysis was performed in Matpower to acquire bus voltages, V, and bus voltage phase angles, θ. Bus HV was reference bus and bus 12, 13 and 14 were PQ-buses. As in the LV network, a dummy bus labeled as PV-bus was connected to the external grid with neither generation nor load. The reason was the same here; Matpower demands one reference bus, at least one PQ-bus and at least one PV-bus. Currents in the network were calculated in Matlab and included in the data passed to GAMS.

4.2.2 Acquiring the ANM4L Algorithm Control Actions

The control actions from the ANM algorithm in the MV network were recieved from the ANM4L project.

4.2.3 The MV Network Optimization: Objective Function

The objective function consists of the same squared terms as in the LV network:

$$f_1 = \sum_{k=1}^{N} \Delta P_k^2 \tag{12}$$

$$f_2 = \sum_{k=1}^{N} \Delta Q_k^2 \tag{13}$$

The objective function thus has the form:

$$f_{obj} = w_1 \cdot f_1 + w_2 \cdot f_2 \tag{14}$$

The weight factors were set to the same values as in the LV network because otherwise unnecessarily much P would be curtailed despite the higher X/R-ratio.

4.2.4 The MV Network Optimization: Constraint Functions

Let the buses be labeled $k = 1, 2...K$ from the beginning to the end of the feeder. Let the lines be labeled $m = 1, 2...M$ from the beginning to the end of the feeder. The difference between the generation and load at a bus is calculated so that the net generation/load at that bus is acquired. The current through the net generation/load is labeled $I_{gen/load}$. The current named $I_{gen/load,k}$ at bus k is illustrated in Figure 6.

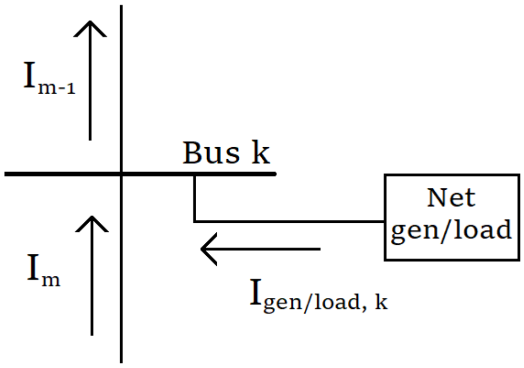


Figure 6: Current through generator/load at bus k

In the constraint functions below, P_k and Q_k are the active power generation/load and reactive power generation/load at bus k respectively. V_k is the absolute value of the voltage at bus k and θ_k its phase angle. I_m is the absolute value of the current per phase at line m and ϕ_m its phase angle. Note that a symmetrical three-phase system is assumed. $I_{gen/load,k}$ is the absolute value of the current per phase through the generator/load at bus k and $\phi_{gen,k}$ its phase angle. Y_m is the absolute value of the line admittance at line m and α_m its phase angle. The values of P_k, Q_k, V_k and θ_k that are acquired from the load flow analysis are modified during the optimization iteration as:

- $\widetilde{P_k} = P_k + \Delta P_k$
- $\widetilde{Q_k} = Q_k + \Delta Q_k$
- $\widetilde{V_k} = V_k + \Delta V_k$
- $\widetilde{\theta_k} = \theta_k + \Delta\theta_k$
- $\widetilde{I_m} = I_m + \Delta I_m$
- $\widetilde{\phi_m} = \phi_m + \Delta\phi_m$
- $\widetilde{I}_{gen/load,k} = I_{gen/load,k} + \Delta I_{gen/load,k}$
- $\widetilde{\phi}_{gen/load,k} = \phi_{gen/load,k} + \Delta\phi_{gen/load,k}$.

The constraint functions 1, 2, 3 and 4 are equality constraints and the constraint functions 5, 6 and 7 are inequality constraints, as explained in Section 2.5.

Constraint 1

The current $I_{gen/load,k}$ should, with Figure 6 as background, be defined as:

$$\widetilde{I}_{gen/load,k} e^{j\widetilde{\phi}_{gen/load,k}} = \widetilde{I}_{m-1} e^{j\widetilde{\phi}_{m-1}} - \widetilde{I}_m e^{j\widetilde{\phi}_m} \tag{15}$$

Constraint 2

V, I, P and Q are dependent. The absolute values have this relation:

$$3 \cdot \widetilde{V}_k^2 \cdot \widetilde{I}_{gen/load,k}^2 = \widetilde{P}_k^2 + \widetilde{Q}_k^2 \tag{16}$$

The currents are defined as current per phase, which leads to the factor of 3. Note that $\widetilde{P}_k^2 + \widetilde{Q}_k^2 = \widetilde{S}_k^2$.

Constraint 3

That V, I, P and Q are dependent also decides the relation between the phase angles. The phase angle of the apparent power must be equal to the difference in phase between the voltage and the current:

$$\arctan(\widetilde{Q}_k/\widetilde{P}_k) = \widetilde{\theta}_k - \widetilde{\phi}_{gen/load,k} \tag{17}$$

Constraint 4

I, V and Y have the following relation:

$$\widetilde{I}_m e^{\widetilde{\phi}_m} = (\widetilde{V}_k e^{j\widetilde{\theta}_k} - \widetilde{V}_{k+1} e^{j\widetilde{\theta}_{k+1}}) \cdot Y_m e^{j\alpha_m} \tag{18}$$

In other words, the current at line m must be equal to the voltage change at that line times the admittance of that line.

Constraint 5

The relation between P, Q and S must hold. S_{max} is the inverter transfer rating of the wind power plants respectively.

$$\widetilde{P}_k^2 + \widetilde{Q}_k^2 \leq S_{max}^2 \tag{19}$$

Constraint 6

When examining voltage management, this constraint is included while constraint 7 is not included. The voltage must be above the lower allowed limit (in p.u.):

$$\widetilde{V}_k \geq 0.95 \tag{20}$$

The voltage must be below the upper allowed limit (in p.u.):

$$\widetilde{V}_k \leq 1.05 \tag{21}$$

Constraint 7

When examining congestion management, this constraint is included while constraint 6 is not included. The current at the line(s) which are at risk of congestion must be constrained:

$$\widetilde{I}_m \leq I_{limit} \tag{22}$$

As stated in Section 3.2, the current at the line between bus 12 and 13 in Figure 4 is of concern. That line is assumed to have a limit $I_{limit} = 195$ A. The reason why the voltage constraints were not included when the current was limited is that there were yet no ANM4L simulation results in which the algorithm manages both voltage and congestion. Thus, there would have been no algorithm actions for combined voltage and congestion management to compare with.

Optimization Variables

The optimization variables for voltage management in the MV network are ΔP_k, ΔQ_k, ΔV_k, $\Delta\theta_k$, ΔI_m, $\Delta\phi_m$, $\Delta I_{gen,k}$ and $\Delta\phi_{gen,k}$. Since ΔV_k, $\Delta\theta_k$, ΔI_m, $\Delta\phi_m$, $\Delta I_{gen,k}$ and $\Delta\phi_{gen,k}$ are dependent on ΔP_k, ΔQ_k, the latter variables were the optimization variables in the mathematical sense.

The optimization variables for congestion management are the same in all but one case: ΔQ_k is not controllable.

4.3 Minimization of Line Losses

In addition to minimization of curtailment of P and consumption of Q, it was suggested that minimization of line losses of P could be tried as objective function or added as a term to the existing objective function. To only minimize the line losses of P is not suitable because this is a goal which is seldom desirable. The minimum of losses of P, which is 0, is when all generation of P is reduced to 0. Yet, in that case one does of course not generate any P. It is probably more often desirable to acquire as much utilizable P as possible and then the expression $\mathrm{P}_{rated} + \Delta\mathrm{P} - \mathrm{P}_{losses}$ can be maximized, the expression being interpreted as generated P minus curtailed P and line losses. That is, the P one can actually use. In regard to the voltage management, this optimization problem was examined but as the optimal control actions involve increasing P at the inner buses at the feeder and reducing it at the outer buses, it is very far from the ANM algorithm control actions. Therefore, this problem was not treated further.

4.4 Comparing Objective Function Values

It was tested to insert the algorithm control actions ΔP and ΔQ in the objective function to compare the resulting objective function value to what value the objective function took when the optimal actions were used, for a given simulation case. This approach is similar to the comparison of total curtailed P, described in Section 4. This is because the weight factors in the objective function were chosen so that the total curtailment of P was much more significant than the total consumption of Q, described in Section 4.1.3. Hence, one could suspect that comparing the objective function values and the total curtailment of P give the same ranking of the versions of the algorithm (independent control of P and Q, constant power factor). When testing comparison of the values of the objective function, this was indeed the case. Since no new significant information was acquired, the comparison of objective function values will not be treated further.

4.5 Motivation of Accuracy

To verify the result, all sets of control actions given by GAMS were tested in regard to two criteria. The first criterion was that the voltage actually should be reduced to below the limit after control action, when voltage management was examined, or that current should be reduced to below the limit, when congestion management was examined. This was tested by running a power flow analysis in Matpower for a case in which the control actions were used. Criterion two was that an optimal solution should not cause redundant reduction of voltage or current. In the case of voltage management, the bus furthest out at the feeder has the highest voltage so that voltage should be exactly at the limit after the control. In the case of congestion management, the current at the line with a too high current should be reduced so that it is exactly at the limit. If these values are not at the limit, excessive control has been used. Since the objective function implied minimizing of the control effort, this means that the solution found is not optimal.

In optimization, different initial values of the variables can be used to see if that affects the results. This approach was not used, but it should be stressed that in regard to this it is important to note that delta-variables together with initial operating points were used in this work. If no operating points would have been used and the initial values of the variables would have been zero, the optimization program would have had to find a suitable solution starting with no information about what reasonable values of the variables would be. In this work, the operating points described in Section 3.1 and Section 3.2 provided reasonable points to start the search of an optimum from, because the operating points corresponded to the state in the network which was to be controlled.

Three different solvers were used too see if a change in solver affected the results. The solvers used are presented briefly in the following paragraphs.

4.5.1 KNITRO

KNITRO is developed to find local solutions to continuous optimization problems [18] (and, which is not relevant here, discrete problems). There are several algorithms it can choose from when solving. In this work, the choice of algorithm was set to auto, which let KNITRO decide which algorithm is most suitable to use. It chose a barrier method with a simple backtracking method for the linesearch.

4.5.2 MINOS

MINOS is developed to find locally optimal solutions to nonlinear optimization problems [19]. The nonlinear functions must be smooth. If a problem contains a nonlinear objective function and nonlinear constraints as in this work, the problem is divided by MINOS into subproblems having linearized constraints and a Lagrangian objective function. These subproblems are solved iteratively.

4.5.3 CONOPT

CONOPT is a solver which can find locally optimal solutions to nonlinear problems. It uses a traditional method of finding a feasible path by performing line searches in adequate directions [20]. CONOPT and MINOS are recommended to use in parallel because they are complementing each other. [21] states that in a test with 196 large and difficult models, both CONOPT and MINOS failed on 14 of them but only four of these were in common. With this as background, both CONOPT and MINOS are used in this work.

5 Results and Discussion

All sets of control actions GAMS suggested fulfilled the two criteria described in Section 4.5. This ensures that the solution actually solves the problem with overvoltage or overcurrent and that no redundant actions are used. The accuracy of the result is further enhanced by the fact that the three solvers gave the same control actions for a given choice of parameters throughout the whole results.

5.1 Voltage Management in the LV Network

Note that the optimization program can control P and Q independently throughout the results. However: the optimal control actions change when the algorithm version changes (P and Q independent, constant power factor of 0.8 and constant power factor of 0.9) despite the fact that the optimization uses the same way of control. This is because S_{max} of the PV inverters varies, as was described in Section 3.1. When the algorithm could control both P and Q, S_{max} was set to the generator rating P_{rated} plus 5 kW. When the algorithm could control only P, S_{max} was set to $P_{rated}/\cos(\varphi)$. The value of S_{max} affects how much the consumption of Q can increase before P has to be curtailed. It is of course important that the optimization program has the same S_{max} as the algorithm because otherwise, the one with a greater S_{max} will be able to increase the consumption of Q more before starting to curtail P which results in an unfair comparison.

These differences in S_{max} between the sections stated above means that it is not possible to compare curtailment or consumption in absolute terms between the different diagrams (P and Q independent, constant power factor of 0.8 and constant power factor of 0.9). Yet, what is possible to compare between them is how far the algorithm actions are from the optimal actions in the diagrams respectively.

The results regarding the LV network are presented divided into the parts "Generation of 30 kW per PV-generator" (Section 5.1.1), "Generation of 40 kW per PV-generator" (Section 5.1.2) and "Generation of 50 kW per PV-generator" (Section 5.1.3). The reason is that this makes it straightforward to see how the performance of the algorithm changes when the generation per PV-generator increases. The case of 30 kW per generator can be used to represent the situation today whereas the case of 50 kW per generator can represent a future scenario.

5.1.1 Generation of 30 kW per PV-generator

Note that, throughout the LV network results, ΔP is a change in P but ΔQ is the total reactive power since Q is zero in the LV network before control.

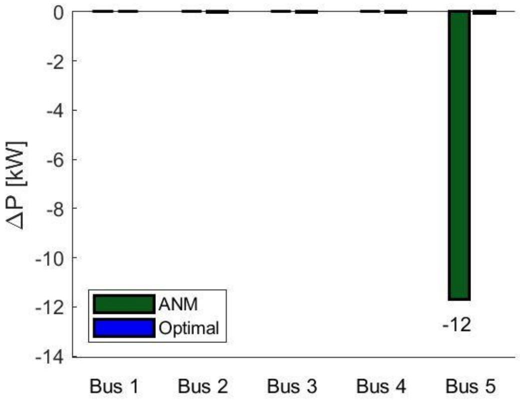


Figure 7: Curtailment of P with generation 30 kW per bus, the ANM algorithm controls P and Q independently, LV network

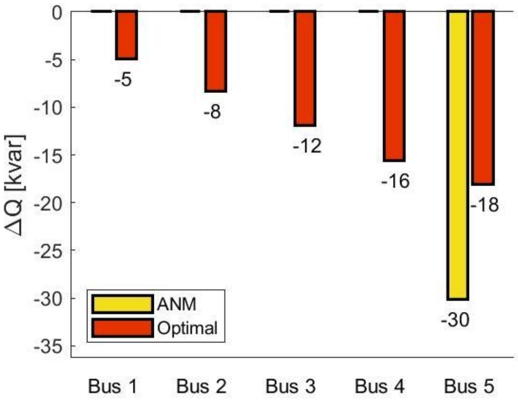


Figure 8: Consumption of Q with generation 30 kW per bus, the ANM algorithm controls P and Q independently, LV network

The curtailment of P when the ANM algorithm controls P and Q independently can be seen in Figure 7. The consumption of Q when the ANM algorithm controls P and Q independently can be seen in Figure 8. It is clear that the ANM control does not at all have an equal distribution of the actions. The ANM control gives the most remote bus the whole burden of managing the voltage with both curtailment of P and consumption of Q as actions. The optimal control actions are more equally distributed, but the optimal control could not be called equal given the differences in consumption of Q between the buses. Both the ANM actions and the optimal actions are concentrated to the most remote buses. This is an expected result; the optimization program decides upon actions at the remote buses because it is the most effective way. The reason for that is that a change in power at the remote buses affects the voltage more than a change at the buses at the beginning of the feeder. The ANM algorithm controls at the remote buses because it is only the PI-controllers at those buses that notice the overvoltage.

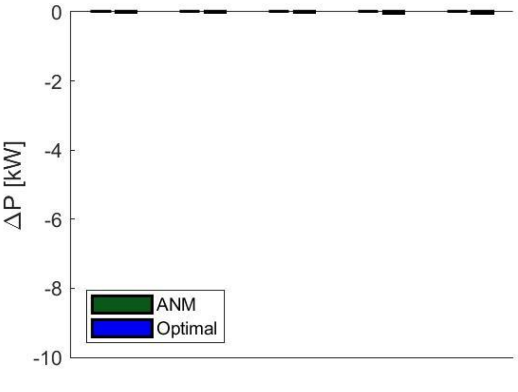


Figure 9: Curtailment of P with generation 30 kW per bus, the ANM algorithm uses a constant power factor of 0.8, LV network

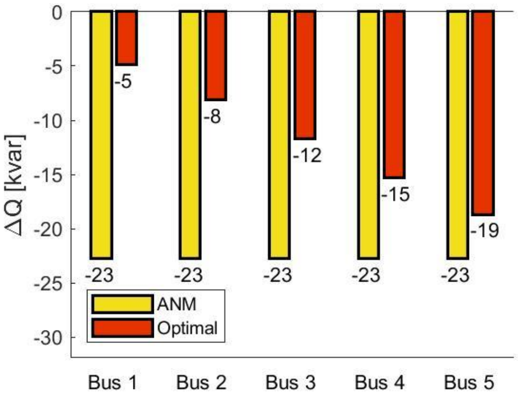


Figure 10: Consumption of Q with generation 30 kW per bus, the ANM algorithm uses a constant power factor of 0.8, LV network

The curtailment of P when the ANM algorithm uses a constant power factor of 0.8 can be seen in Figure 9. The consumption of Q when the ANM algorithm uses a constant power factor of 0.8 can be seen in Figure 10. The ANM algorithm using a constant power factor of 0.8 is an improvement compared to when P and Q were controlled independently, as it curtails no P and comes closer to the optimal actions. Furthermore, it can be stated that the consumption of Q are completely equally distributed.

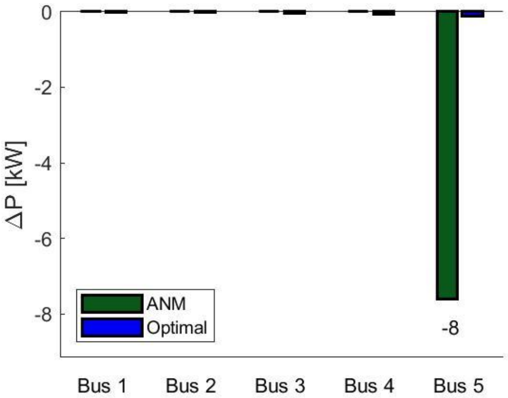


Figure 11: Curtailment of P with generation 30 kW per bus, the ANM algorithm uses a constant power factor of 0.9, LV network

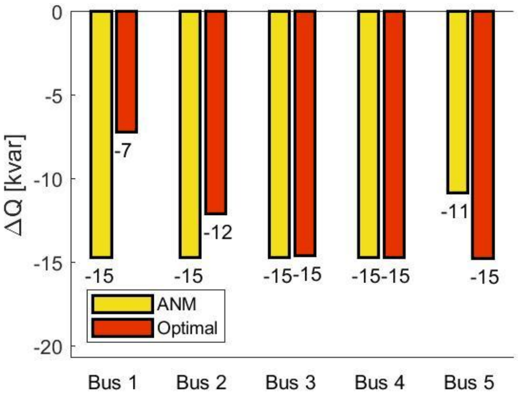


Figure 12: Consumption of Q with generation 30 kW per bus, the ANM algorithm uses a constant power factor of 0.9, LV network

The curtailment of P when the ANM algorithm uses a constant power factor of 0.9 can be seen in Figure 11. The consumption of Q when the ANM algorithm uses a constant power factor of 0.9 can be seen in Figure 12. It can be seen that the curtailment of P at the most remote bus is further from the corresponding optimal value than when using 0.8 as power factor. Yet, using a constant power factor of 0.9 still comes closer to the optimal control than when the algorithm controls P and Q independently. The consumption of Q for the ANM algorithm is rather equally distributed.

In Table 1 below, the total curtailment of P, total consumption of Q and transferred S through the transformer are presented in the different cases respectively.

	indep.	pf=0.8	pf=0.9
$\|\Delta P_{tot}\|$, optimal (kW)	0	0	0
$\|\Delta P_{tot}\|$, ANM (kW)	12	0	8
$\|\Delta Q_{tot}\|$, optimal (kvar)	59	59	64
$\|\Delta Q_{tot}\|$, ANM (kvar)	30	115	71
S through transformer, optimal (kVA)	152	152	157
S through transformer, ANM (kVA)	133	184	151

Table 1: Comparison of total curtailment of P and total consumption of Q when the generation per PV-generator is 30 kW. The three columns correspond to three cases: the ANM algorithm controlling P and Q independently, the ANM algorithm using a constant power factor of 0.8 and the ANM algorithm using a constant power factor of 0.9. Note that the absolute numbers are not possible to compare between the columns, as was described in the beginning of Section 5.1, but it is possible to compare how far the ANM control is from the optimal control. Also information about the transferred S through the transformer can be seen in the table. Transformer rating: 150 kVA.

All three versions of the algorithm perform fairly well in terms of total P curtailed compared to the corresponding optimal value. The version which controls P and Q independently curtails the most P, 12 kW, which is still not very much compared to that P would be curtailed to zero at bus five if there were no ANM because then the PV unit at that bus would be disconnected. While the control becomes more equally distributed with a constant power factor, it increases the total consumption of Q.

When the ANM algorithm controls P and Q independently, the S transfer through the transformer is below the transformer S rating. When a power factor of 0.9

is used, the S transfer is just at the limit. In the case of using a power factor of 0.8, the rating is exceeded by 34 kVA which corresponds to 23 % of the S rating. All in all, if the transformer flow is to be less than the rating it is possible to resolve that by choosing an adequate version of the algorithm. Exceeding of the transformer S rating is not a very severe problem when the generation is 30 kW per bus compared to the scenarios with higher level of generation, as will be seen in the following sections.

5.1.2 Generation of 40 kW per PV-generator

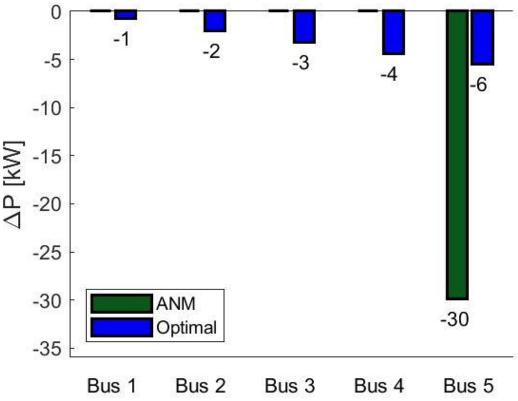


Figure 13: Curtailment of P with generation 40 kW per bus, the ANM algorithm controls P and Q independently, LV network

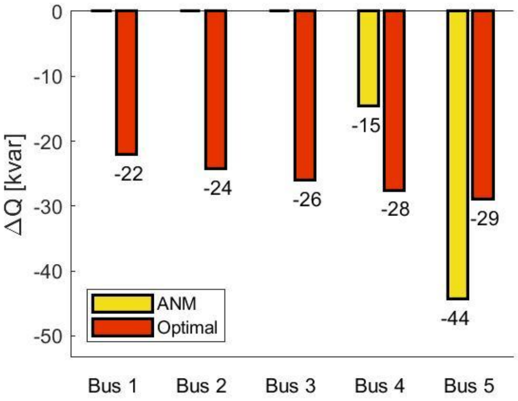


Figure 14: Consumption of Q with generation 40 kW per bus, the ANM algorithm controls P and Q independently, LV network

The curtailment of P when the ANM algorithm controls P and Q independently can be seen in Figure 13. The consumption of Q when the ANM algorithm controls P and Q independently can be seen in Figure 14. The optimal control actions are much more equally distributed than the ANM control. The ANM control actions are concentrated to the remote buses.

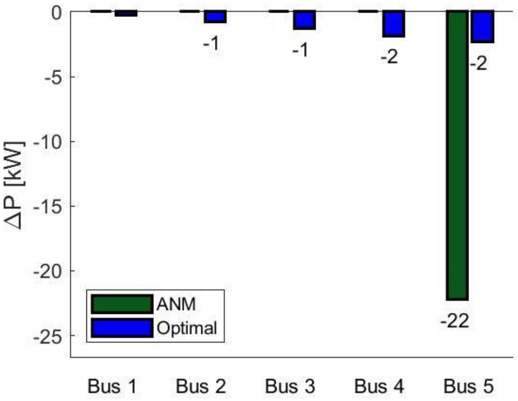


Figure 15: Curtailment of P with generation 40 kW per bus, the ANM algorithm uses a constant power factor of 0.8, LV network

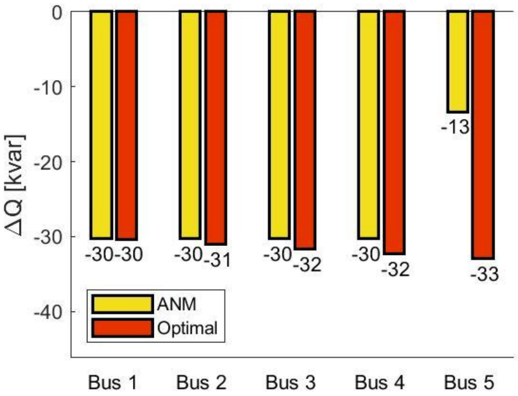


Figure 16: Consumption of Q with generation 40 kW per bus, the ANM algorithm uses a constant power factor of 0.8, LV network

The curtailment of P when the ANM algorithm uses a constant power factor of 0.8 can be seen in Figure 15. The consumption of Q when the ANM algorithm uses a constant power factor of 0.8 can be seen in Figure 16. It can be stated that the optimal control actions are rather equally distributed. The ANM control curtailment of P at bus five comes closer to the corresponding optimal value than when P and Q were controlled independently. The ANM distribution of the consumption of Q is equal on bus 1-4 but bus five consumes about half as much Q as the other buses. On the other hand, bus five has its P curtailed to half of the rated output. All in all, the ANM control here should be regarded as much more equally distributed than when P and Q were controlled independently.

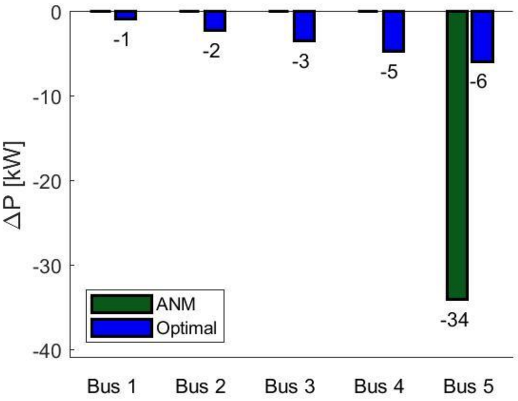


Figure 17: Curtailment of P with generation 40 kW per bus, the ANM algorithm uses a constant power factor of 0.9, LV network

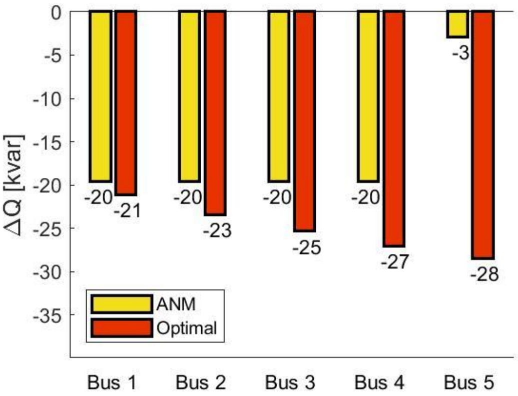


Figure 18: Consumption of Q with generation 40 kW per bus, the ANM algorithm uses a constant power factor of 0.9, LV network

The curtailment of P when the ANM algorithm uses a constant power factor of 0.9 can be seen in Figure 17. The consumption of Q when the ANM algorithm uses a constant power factor of 0.9 can be seen in Figure 18. Using a power factor of 0.9 leads to that the ANM control actions are further away from the optimal ones than when a power factor of 0.8 was used. Bus five sees its P being curtailed almost entirely.

In Table 2 below, the total curtailment of P, total consumption of Q and transferred S through the transformer are presented in the different cases respectively.

	indep.	pf=0.8	pf=0.9
$\lvert\Delta P_{tot}\rvert$, optimal (kW)	16	6	17
$\lvert\Delta P_{tot}\rvert$, ANM (kW)	30	22	34
$\lvert\Delta Q_{tot}\rvert$, optimal (kvar)	129	158	124
$\lvert\Delta Q_{tot}\rvert$, ANM (kvar)	59	133	83
S through transformer, optimal (kVA)	220	240	214
S through transformer, ANM (kVA)	171	213	184

Table 2: Comparison of total curtailment of P and total consumption of Q when the generation per PV-generator is 40 kW. The three columns correspond to three cases: the ANM algorithm controlling P and Q independently, the ANM algorithm using a constant power factor of 0.8 and the ANM algorithm using a constant power factor of 0.9. Note that the absolute numbers are not possible to compare between the columns, as was described in the beginning of Section 5.1, but it is possible to compare how far the ANM control is from the optimal control. Also information about the transferred S through the transformer can be seen in the table. Transformer rating: 150 kVA.

The differences in total curtailment of P between the optimal control and the ANM control are about the same, around 15 kW, regardless of which version of the ANM algorithm that is used. Given that, it can be said that all three versions perform well compared to a case in which there were no active management; in such a case the generator at bus five would be disconnected.

It should be noted that the transformer S rating is exceeded in all cases. The ANM algorithm exceeds the transformer rating with the least margin, 21 kVA, when it uses independent control of P and Q. 21 kVA corresponds to 14 % of the S rating.

5.1.3 Generation of 50 kW per PV-generator

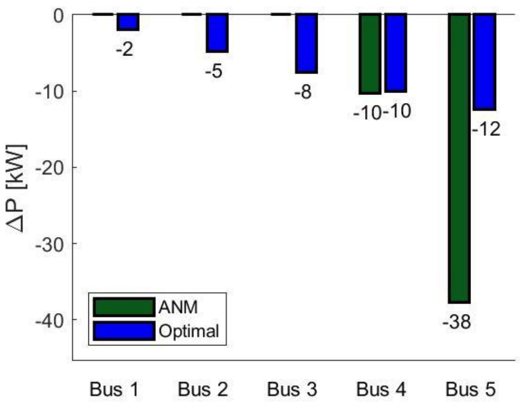


Figure 19: Curtailment of P with generation 50 kW per bus, the ANM algorithm controls P and Q independently, LV network.

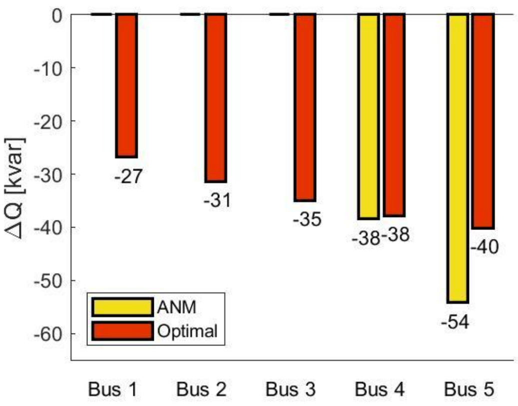


Figure 20: Consumption of Q with generation 50 kW per bus, the ANM algorithm controls P and Q independently, LV network.

The curtailment of P when the ANM algorithm controls P and Q independently can be seen in Figure 19. The consumption of Q when the ANM algorithm controls P and Q independently can be seen in Figure 20. The distribution of the optimal actions is not equal but at least much closer to equal than the ANM actions. Again, the ANM actions are heavily concentrated to the remote buses.

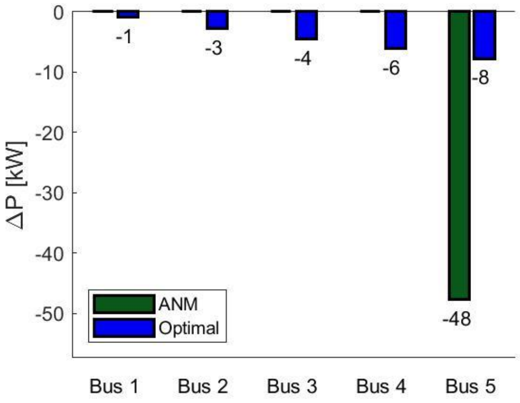


Figure 21: Curtailment of P with generation 50 kW per bus, the ANM algorithm uses a constant power factor of 0.8, LV network

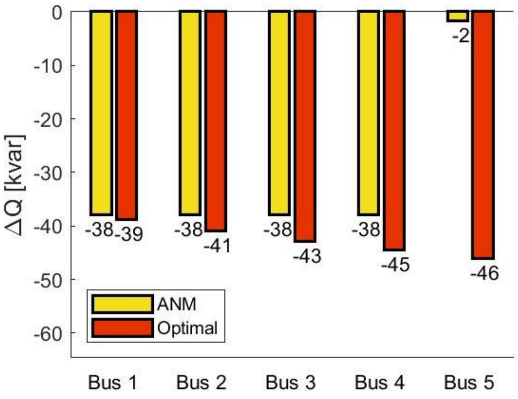


Figure 22: Consumption of Q with generation 50 kW per bus, the ANM algorithm uses a constant power factor of 0.8, LV network

The curtailment of P when the ANM algorithm uses a constant power factor of

0.8 can be seen in Figure 21. The consumption of Q when the ANM algorithm uses a constant power factor of 0.8 can be seen in Figure 22. The optimal actions are not equally distributed, but no generator has its P curtailed with more than 24 % in the optimal control . The ANM actions are equally distributed on bus one to four but bus five sees its generation of P being curtailed down to almost zero. The consumption of Q is almost zero at bus five. This is because the curtailment of P drags the consumption of Q down, as a result of the constant power factor.

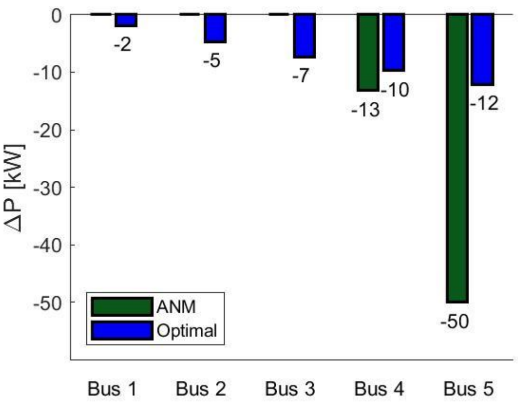


Figure 23: Curtailment of P with generation 50 kW per bus, the ANM algorithm uses a constant power factor of 0.9, LV network

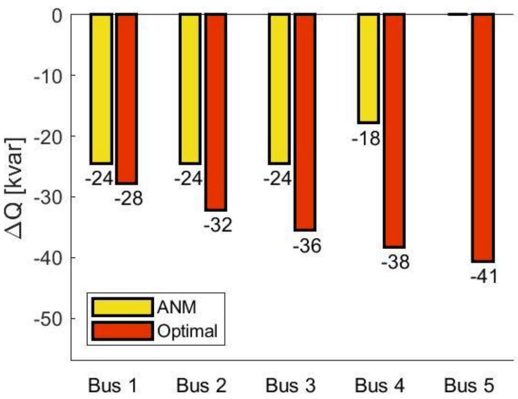


Figure 24: Consumption of Q with generation 50 kW per bus, the ANM algorithm uses a constant power factor of 0.9, LV network

The curtailment of P when the ANM algorithm uses a constant power factor of 0.9 can be seen in Figure 23. The consumption of Q when the ANM algorithm uses a constant power factor of 0.9 can be seen in Figure 24. Using a power factor of 0.9 does not work as well as using a power factor of 0.8 in this case. Not only is the P at bus five curtailed to zero, the P at bus four is also curtailed down to 37 kW.

In Table 3 below, the total curtailment of P, total consumption of Q and transferred S through the transformer are presented in the different cases respectively.

	indep.	pf=0.8	pf=0.9
$\lvert\Delta P_{tot}\rvert$, optimal (kW)	37	22	36
$\lvert\Delta P_{tot}\rvert$, ANM (kW)	48	48	63
$\lvert\Delta Q_{tot}\rvert$, optimal (kvar)	171	214	175
$\lvert\Delta Q_{tot}\rvert$, ANM (kvar)	92	154	90
S through transformer, optimal (kVA)	262	305	269
S through transformer, ANM (kVA)	206	187	206

Table 3: Comparison of total curtailment of P and total consumption of Q when the generation per PV-generator is 50 kW. The three columns correspond to three cases: the ANM algorithm controlling P and Q independently, the ANM algorithm using a constant power factor of 0.8 and the ANM algorithm using a constant power factor of 0.9. Note that the absolute numbers are not possible to compare between the columns, as was described in the beginning of Section 5.1, but it is possible to compare how far the ANM control is from the optimal control. Also information about the transferred S through the transformer can be seen in the table. Transformer rating: 150 kVA.

The differences in total curtailment of P between the optimal control and the ANM control are larger when a constant power factor is used. The difference is about the same regardless of which of the two power factors that is used. Thus, in the sense of total curtailment of P the ANM algorithm controlling P and Q independently is performing best.

Yet, all the cases exceed the transformer S rating with fairly large margins. The ANM algorithm using a power factor of 0.8 exceeds it by the least margin, 37 kVA, which corresponds to 25 % of the S rating. The other versions of the algorithm exceed the S rating by over 35 %.

5.2 Voltage Management in the MV Network

In none of the voltage control simulations in the MV network was the transferred S through the transformer close to the transformer rating. Therefore, the numbers in regard to this are not presented.

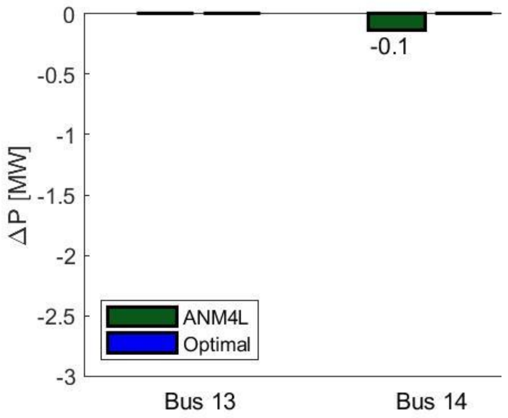


Figure 25: Curtailment of P, the ANM algorithm controls P and Q independently, MV network

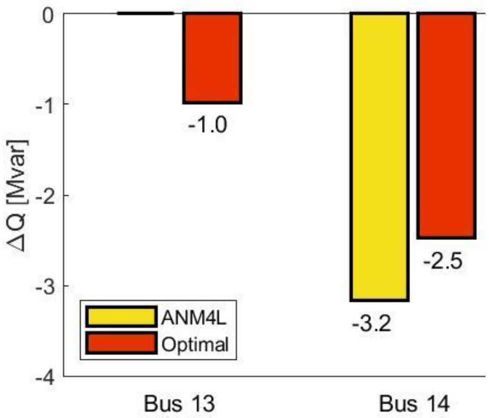


Figure 26: Consumption of Q, the ANM algorithm controls P and Q independently, MV network

The control actions for curtailment of P when the ANM algorithm controls P and Q independently can be seen in Figure 25. The control actions for consumption of Q when the ANM algorithm controls P and Q independently can be seen in Figure 26. The optimal actions use, in total, 0.3 Mvar more than the ANM actions but the former curtail no P whereas the latter curtail 0.1 MW. This curtailment is rather small as it is around 1 % of the total rated generation, 9 MW, of the two wind power generators. Since the curtailment of P is low, the ANM algorithm should be assessed as performing well in this context. Yet, one drawback in the perspective of fairness is that the control actions are concentrated entirely to the most remote generator.

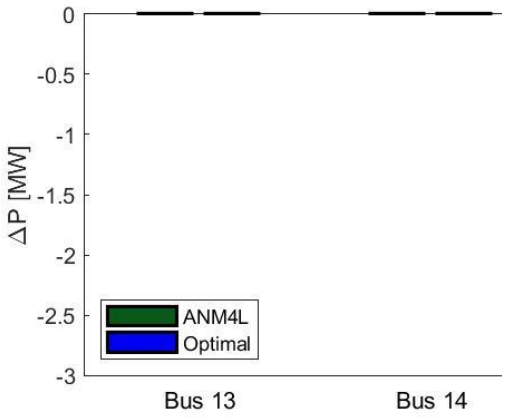


Figure 27: Curtailment of P, the ANM algorithm uses a constant power factor of 0.9, MV network

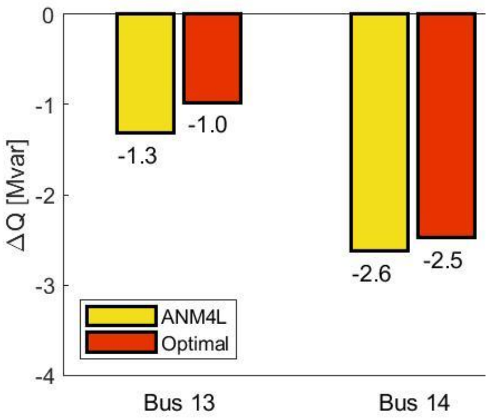


Figure 28: Consumption of Q, the ANM algorithm uses a constant power factor of 0.9, MV network

The control actions for curtailment of P when the ANM algorithm uses a constant power factor of 0.9 can be seen in Figure 27. The control actions for consumption of Q when the ANM algorithm uses a constant power factor of 0.9 can be seen in Figure 28. Using a constant power factor of 0.9 improves the performance compared to control of P and Q independently as no P is curtailed in the actions of the ANM algorithm and the ANM actions are closer to the optimal actions. In fact, the sharing of consumption of Q is fairly similar to the optimal actions. Given the above, it is seemingly favorable to use a power factor of 0.9 in this network at this operating point.

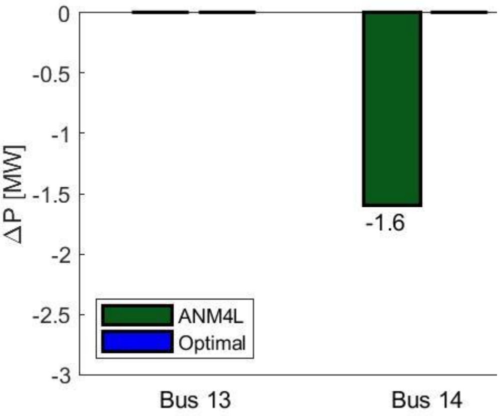


Figure 29: Curtailment of P, the ANM algorithm uses a constant power factor of 0.95, MV network

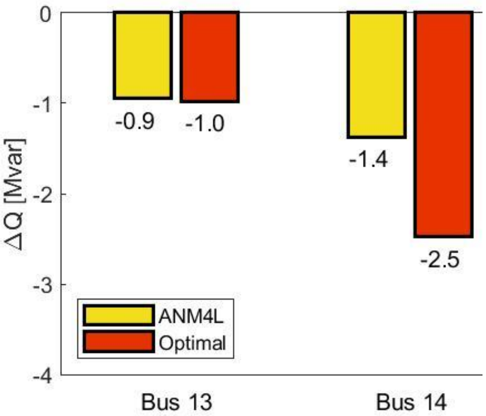


Figure 30: Consumption of Q, the ANM algorithm uses a constant power factor of 0.95, MV network

The control actions for curtailment of P when the ANM algorithm uses a constant power factor of 0.95 can be seen in Figure 29. The control actions for consumption of Q when the ANM algorithm uses a constant power factor of 0.95 can be seen in Figure 30. The PI controller at bus 14 curtails 1.6 MW. This is clearly worse both when comparing to independent control of P and Q and using a power factor of 0.9.

5.3 Congestion Management in the MV Network

5.3.1 PTDF Prioritization

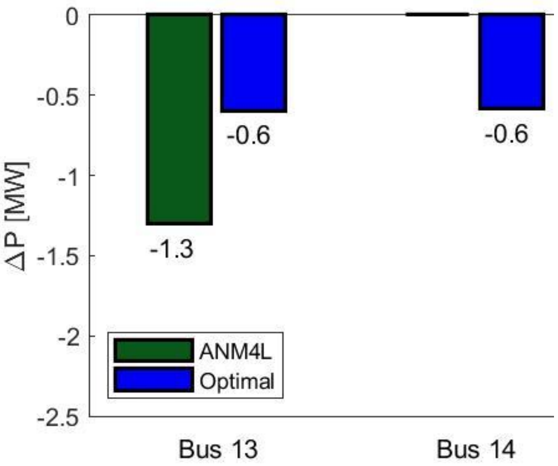


Figure 31: Curtailment of P, the ANM4L algorithm controls P with PTDF prioritization, MV network

The control actions for curtailment of P can be seen in Figure 31. The optimal control curtails 0.1 MW less than the ANM control. This is a rather small amount; compared to the total rated output of the generators it corresponds to just over 1 %. However, the ANM curtailment is concentrated entirely to the less remote generator whereas the optimal control has an almost equal sharing of curtailment. The ANM algorithm should be assessed as performing fairly well regarding the total curtailment of P, which is close to optimal.

5.3.2 Equal Sharing

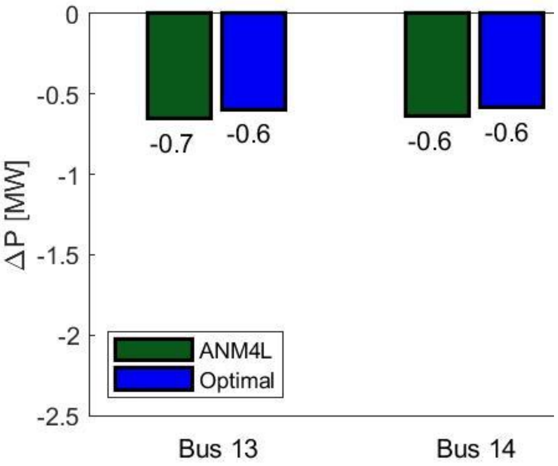


Figure 32: Curtailment of P, the ANM4L algorithm controls P with equal sharing setting, MV network

The control actions for curtailment of P can be seen in Figure 32. The total curtailed P is about the same as when PTDF prioritization was used but the actions are much more equally shared. Moreover, the actions are very close to the optimal ones. The equal sharing setting thus works very well in this case.

6 Conclusion

The conclusion will be divided into one section about voltage management and one section about congestion management.

6.1 Voltage Management

That the transformation of the electrical grid will be a rather slow process taking many years, instead of an overnight process, is important when evaluating the results. Let the case of 30 kW generated per bus in the LV network play the role as the case of today and let increasing generation per bus represent future scenarios. In the case of today, the voltage without any control will raise above the voltage limit. With no ANM there would be disconnecting of the most remote bus which leads to no generation there. The ANM algorithm using independent control of P and Q curtails 12 kW at the most remote bus but saves it from being disconnected. The algorithm using a constant power factor of 0.8 curtails no P and distributes the consumption of Q equal. The algorithm using a constant power factor of 0.9 has an almost equal distribution of consumption of Q and curtails 8 kW which is better than the one using independent control of P and Q. All three versions of the algorithm can thus be said to perform well in the 30 kW-case of today. If one version has to be chosen in this case of today, the one using a constant power factor of 0.8 should be regarded as best. This is because it does not curtail any P and is by doing so closest to the corresponding optimal actions. Furthermore, it has a totally equal distribution of consumption of Q. However, there is one caveat – the transformer S rating is exceeded by 34 kVA, with the rating being 150 kVA, when using that version of the ANM algorithm.

In the case of 40 kW generated per bus, which could represent a scenario not very far away in the future, all three versions of the algorithm perform well in regard to total P curtailment. The one using a constant power factor of 0.8 has the most equal distribution of control actions. However, the transformer S rating is exceeded by the least margin, 21 kVA, if P and Q are controlled independently.

In the future scenario of 50 kW generated per bus, the ANM algorithm works acceptably. It could possibly postpone grid reinforcement even at this high level of generation. It must yet be stressed that it is uncertain how rapid the transition to a high level of PV generation will be, or how high it will end up being. The best version of the algorithm in this scenario is probably the one which controls P and Q independently. That approach gives the least difference between the optimal actions and ANM actions in regard to total curtailed P. Yet, one disadvantage is that the actions will not at all be equally distributed if P and Q are independent. It is possible to compensate for this at the market; the PV unit owners contributing less to the control would then economically compensate the owners contributing more. The costs of having a compensating system must be examined further to see if it is favorable to have such a system. If fairness in regard to sharing the physical actions would be prioritized in the future, using a constant power factor of 0.8 is best because although it does not distribute the actions equally it does it more equal than the other algorithm versions. In the case of 50 kW per bus the buses at the beginning of the feeder take on the burden to consume Q whereas the buses at the end of the feeder curtail P. The transformer S rating is exceeded heavily by all three versions of the algorithm, so some upgrading of the transformer is needed in a future scenario like this one.

In the MV network, only one generation level was examined. The results imply that using a constant power factor of 0.9 makes the ANM control actions come closest to the corresponding optimal actions.

6.2 Congestion Management

The use of equal sharing seems to work very well in the network used in this work. Not only are the actions very equally shared but they are close to the optimal actions. Although this is just the result from one simulation, it is promising. After all, the physics of congestion is easier than the physics of voltage control. If network losses can be disregarded, curtailment of P can be done at any bus on a feeder with the same reduction in current at the beginning of the feeder as a result. As long as the power flow is decreased, it does not matter where power is curtailed. However, depending on the network properties the network losses cannot always be disregarded and if not, PTDF can be used to reduce the total curtailment of P. The MV test network has a rather uniform distribution of line losses and thus the version of the algorithm using PTDF curtails about as much total P as the one using equal sharing. If only minimizing of the total curtailment of P is of interest, then the PTDF version of the algorithm works about as well as the one with equal sharing, but it gives bus 13 the whole burden of managing the congestion.

7 Future Studies

Relaxation of the optimization problem, which was mentioned in Section 2.5.1, was not used in this work. It is a clever way to be able to guarantee that the optimum found is global. To examine this area further would be interesting. The significance of guaranteeing a global optimum can, however, be questioned. Sometimes a local optimum can be a reasonable enough benchmark to compare with.

A very interesting question is how a market-based system for compensating unfairness regarding control actions would be designed. It is important to make the costs of implementing such a system as low as possible. If the costs would be too large, it might be better to use physical equal sharing of the control actions. While a compensating market-system may not be needed today, it will possibly (probably) be needed in the future so there is no reason not to start thinking about it. Morten Hemmingsson at the IEA institution at LTH suggested that maybe the whole issue of controlling voltage with reactive power can be solved at the market.

It should be said that the most important future study in regard to this work is to use it in the continuous development of active network management, in particular in the ANM4L project, to make further utilizing of renewable energy in the electrical grid possible.

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Appendix

Matpower Cases

LV Network

```
function mpc = anm_network_PV
%ANM_NETWORK    Power flow data for the anm pv test network.
%   Please see CASEFORMAT for details on the case file format.

%% MATPOWER Case Format : Version 2
mpc.version = '2';

%%-----  Power Flow Data  -----%%
%% system MVA base
mpc.baseMVA = 1;

%% bus data
%   bus_i   type    Pd  Qd  Gs  Bs  area    Vm  Va  baseKV  ...
    zone    Vmax    Vmin
mpc.bus = [
    1   1   0   0   0   0   1   1.02    0   0.4 1   10  0;
    2   1   0   0   0   0   1   1.02    0   0.4 1   10  0;
    3   1   0   0   0   0   1   1.02    0   0.4 1   10  0;
    4   1   0   0   0   0   1   1.02    0   0.4 1   10  0;
    5   1   0   0   0   0   1   1.02    0   0.4 1   10  0;
    6   3   0   0   0   0   1   1.02    0   20  1   10  0;
    7   2   0   0   0   0   1   1.02    0   20  1   10  0;
];

%% generator data
%   bus Pg  Qg  Qmax    Qmin    Vg  mBase   status  Pmax    ...
    Pmin    Pc1 Pc2 Qc1min  Qc1max  Qc2min  Qc2max  ramp_agc    ...
    ramp_10 ramp_30 ramp_q  apf
mpc.gen = [
    1   0.03    0   0       0       1.02    1   1   0.035   0 ...
        0   0   0   0   0   0   0   0   0   0   0;
    2   0.03    0   0       0       1.02    1   1   0.035   0 ...
        0   0   0   0   0   0   0   0   0   0   0;
    3   0.03    0   0       0       1.02    1   1   0.035   0 ...
        0   0   0   0   0   0   0   0   0   0   0;
    4   0.03    0   0       0       1.02    1   1   0.035   0 ...
        0   0   0   0   0   0   0   0   0   0   0;
    5   0.03    0   0       0       1.02    1   1   0.035   0 ...
        0   0   0   0   0   0   0   0   0   0   0;
    6   0       0   1000    -1000   1.02    1   1   1000    ...
       -1000   0   0   0   0   0   0   0   0   0   0   0;
```

```
    7   0       0   0       0       1.02    1   1   0       0 ...
             0   0   0   0   0   0   0   0   0   0   0;
    ];

%% branch data
%   fbus    tbus    r   x   b   rateA   rateB   rateC   ...
    ratio   angle   status  angmin  angmax
mpc.branch = [
    1   2   0.346*0.15/0.16                         ...
        0.0754*0.15/0.16 ...
                                                              ...
        0   1   1   1   0   0   1   -360    360;
    2   3   0.346*0.15/0.16                         ...
        0.0754*0.15/0.16 ...
                                                              ...
        0   1   1   1   0   0   1   -360    360;
    3   4   0.346*0.15/0.16                         ...
        0.0754*0.15/0.16 ...
                                                              ...
        0   1   1   1   0   0   1   -360    360;
    4   5   0.346*0.15/0.16                         ...
        0.0754*0.15/0.16 ...
                                                              ...
        0   1   1   1   0   0   1   -360    360;
    1   6   2.828/400 + 0.0095/0.16 + 0.0095/400    ...
        0.0090*2*pi*50/400 + 5.5863e-05*2*pi*50/0.16 + ...
        5.5863e-05*2*pi*50/400   0   1   1   1   0   0   1   ...
        -360    360;
    6   7   1   1   0   1   1   1   0   0   1   -360   360;
];

%%-----  OPF Data  -----%%
%% generator cost data
%   1   startup shutdown    n   x1  y1  ... xn  yn
%   2   startup shutdown    n   c(n-1)  ... c0
mpc.gencost = [
    2   1   1   3   0.11    5   150;
    2   1   1   3   0.11    5   150;
    2   1   1   3   0.11    5   150;
    2   1   1   3   0.11    5   150;
    2   1   1   3   0.11    5   150;
    2   1500    500 3   0.5 10  300;
    2   1500    500 3   0.5 10  300;
];
```

MV Network

```
function mpc = anm_network_WP_small
%ANM_NETWORK    Power flow data for the anm wind power test ...
    network.
%   Please see CASEFORMAT for details on the case file format.

%% MATPOWER Case Format : Version 2
mpc.version = '2';

%%-----  Power Flow Data  -----%%
%% system MVA base
mpc.baseMVA = 100;

%% bus data
%   bus_i   type    Pd  Qd  Gs  Bs  area    Vm  Va  baseKV  ...
    zone    Vmax    Vmin
mpc.bus = [
    1   2   0                                   0 ...
                                            0   0   1   1.03    0   ...
        110 1   10  0;
    2   3   0                                   0 ...
                                            0   0   1   1.03    0   ...
        110 1   10  0;
    3   1   0.653*14.99400+0.85*5.01600     ...
        0.653*3.044662+0.85*1.648679     0   0   1   1       0 ...
          20  1   10  0;
    4   1   0.85*0.03400                        0.85*0.021071 ...
                            0   0   1   1       0   20  1   10  0;
    5   1   0.653*0.20855+0.85*0.33150      ...
        0.653*0.052268+0.85*0.205445     0   0   1   1      ...
        0   20  1   10  0;
];

%% generator data
%   bus Pg  Qg  Qmax    Qmin    Vg  mBase   status  Pmax    ...
    Pmin    Pc1 Pc2 Qc1min  Qc1max  Qc2min  Qc2max  ramp_agc    ...
    ramp_10 ramp_30 ramp_q  apf
mpc.gen = [
    2   0     0   0   0   1.05    100 1   5000    -5000   0   ...
        0   0   0   0   0   0   0   0   0   0;
    4   3     0   0   0   1.03    100 1   0         0       0   ...
        0   0   0   0   0   0   0   0   0   0;
    5   6     0   0   0   1.03    100 1   0         0       0   ...
        0   0   0   0   0   0   0   0   0   0;
    ];

%% branch data
```

```
%   fbus    tbus    r   x   b   rateA   rateB   rateC   ...
    ratio   angle   status  angmin  angmax
mpc.branch = [
    1   2   1               1               0 ...
                                    250     250    250  0    0    ...
        1   -360    360;
    2   3   0.0064          0.4800          0 ...
                                    250     250    250  0    0    ...
        1   -360    360;
    3   4   4.89*0.510/4    4.89*0.366/4    ...
        4.89*2*pi*50*10.09679*1e-9*4   250     250    250  0    0    ...
        1   -360    360;
    4   5   2.99*0.510/4    2.99*0.366/4    ...
        2.99*2*pi*50*10.09679*1e-9*4   250     250    250  0    0    ...
        1   -360    360;
    ];

%%-----  OPF Data  -----%%
%% generator cost data
%   1   startup shutdown    n   x1  y1  ... xn  yn
%   2   startup shutdown    n   c(n-1)  ... c0
mpc.gencost = [
    2   1   1   3   0.11    5   150;
    2   1   1   3   0.11    5   150;
    2   1   1   3   0.11    5   150;
];
```

Matlab Scripts for Passing Data to GAMS

LV Network

```
%choose if constant power factor should be used
constant_pf=false;
%if true, what should it be?
pf=0.9;

mpopt = ...
    mpoption('pf.enforce_q_lims',0,'pf.nr.max_it',100,'out.all',0);
mpc = runpf('anm_network_PV',mpopt); %run power flow analysis
Y_abs=abs(full(makeYbus(mpc))); %make admittance absolutes ...
    matrix for network to use in GAMS
Y_ang=angle(full(makeYbus(mpc))); %make admittance angles ...
    matrix for network to use in GAMS
for n=1:3
    baseMVA=mpc.baseMVA; %MVA base
    bus=mpc.bus; %bus information
    gen=mpc.gen; %generator information
```

```
    branch=mpc.branch; %branch information
    nbr_of_buses=length(mpc.bus(:,1)); %number of buses
    if constant_pf %if constant pf, S_max is set to generator ...
       P_rating/pf
        S_max=(mpc.gen(1,2))/pf;
    else %if varying pf, S_max is set to generator P_rating + ...
       5 kW
        S_max=mpc.gen(1,2)+0.005;
    end
    iwgdx(append('case_', sprintf('%g',10*(n+2)),'kW'), ...
       'baseMVA', 'bus', 'gen', 'branch', 'S_max', 'Y_abs', ...
       'Y_ang') %save data to GAMS
    mpc.gen(1:5,2)=mpc.gen(1:5,2)+0.01; %increase P_rating at ...
       PV-buses with 10 kW to next iteration
    mpc = runpf(mpc,mpopt); %run power flow analysis
end
```

MV Network

```
%choose if constant power factor should be used
constant_pf=false;
%if true, what should it be?
pf=0.9;

mpopt = ...
   mpoption('pf.enforce_q_lims',0,'pf.nr.max_it',100,'out.all',1);
mpc = runpf('anm_network_WP_small',mpopt); %run power flow ...
   analysis
if constant_pf %if constant pf, S_max is set to generator ...
   P_rating/pf
    S_max=(mpc.gen(1,9)-0.005)/pf;
else %if varying pf, S_max is set to something very large
    S_max=10;
end
for n=1:1
    baseMVA=mpc.baseMVA; %MVA base
    bus=mpc.bus; %bus information
    gen=mpc.gen; %generator information
    nbr_of_buses=length(mpc.bus(:,1)); %number of buses
    vabs=[mpc.bus(:,8); 0]; %voltage absolutes at buses with ...
       dummy added at the end as padding
    vang=[mpc.bus(:,9); 0]; %voltage angles at buses with ...
       dummy added at the end as padding
    branch=[inf inf 0; mpc.branch(:,3:5); inf inf 0]; %branch ...
       information with dummy ended at beginning and end as ...
       padding
```

```
    Y_abs=abs(1./(branch(:,1)+1i*branch(:,2))+1i*branch(:,3)); ...
       %line admittances absolutes
    Y_ang=angle(1./(branch(:,1)+1i*branch(:,2))+1i*branch(:,3)); ...
       %line admittances angles
    I_abs=[0]; %line current absolutes with dummy at the ...
       beginning as padding
    I_ang=[0]; %line current angles with dummy at the ...
       beginning as padding
    for k=2:nbr_of_buses+1
        I=(vabs(k)*exp(1i*deg2rad(vang(k))) - ...
           vabs(k-1)*exp(1i*deg2rad(vang(k-1)))) * ...
           Y_abs(k)*exp(1i*Y_ang(k))/sqrt(3); %line currents
        I_abs(end+1)=abs(I); %line current absolutes
        I_ang(end+1)=angle(I); %line current angles
    end
    I_gen_abs=[];
    I_gen_ang=[];
    for l=1:nbr_of_buses
        I_gen=(I_abs(l)*exp(1i*I_ang(l)) - ...
           I_abs(l+1)*exp(1i*I_ang(l+1))) / sqrt(3); ...
           %generator currents
        I_gen_abs(end+1)=abs(I_gen); %generator currents absolutes
        I_gen_ang(end+1)=angle(I_gen); %generator currents angles
    end
    S=-(bus(:,3)+1i*bus(:,4))/baseMVA; %load at buses
    S(gen(:,1))=S(gen(:,1))+(gen(:,2)+1i*gen(:,3))/baseMVA; ...
       %add generation at buses to get net gen/load
    P_bus=real(S); %P at buses
    Q_bus=imag(S); %Q at buses
    iwgdx('case_WP', 'baseMVA', 'bus', 'gen', 'branch', ...
       'S_max', 'Y_abs', 'Y_ang', 'nbr_of_buses', 'I_abs', ...
       'I_ang', 'I_gen_abs', 'I_gen_ang', 'P_bus', 'Q_bus') ...
       %save data to GAMS
    mpc.gen(2:3,2)=mpc.gen(2:3,2)+0; %increase XX to next ...
       iteration
    mpc = runpf(mpc,mpopt); %run power flow analysis
end
```

GAMS Optimization

Note that the constraint functions are numbered differently here than in the method section. This is because one constraint function had to be divided into two functions in some cases.

LV Network

```
$title Optimization of control actions for control of voltage.
*----------------------------------------------------------------------
* Filename: optimization_v_nonlin.gms
* Description: Optimization of control actions for control of ...
    voltage.
*
* Usage: gams optimization_v_nonlin
*----------------------------------------------------------------------

*===== SECTION: LOAD THE LOAD FLOW ANALYSIS DATA
Set s /1*20/;
Alias (bus_nbr1,s);
Alias (bus_nbr2,s);
Alias (bus_cols,s);
Alias (gen_nbr,s);
Alias (gen_cols,s);
Alias (branch_nbr,s);
Alias (branch_cols,s);

Scalars      baseMVA "SBase"
             V_limit "maximum allowed value for the voltage ...
                 deviation"
             S_max "maximum apparent power";

*maximum allowed value for the voltage deviation
V_limit = 0.1;

Parameters   bus(bus_nbr1,bus_cols) "data of buses",
             gen(gen_nbr,gen_cols) "data of generators",
             branch(branch_nbr,branch_cols) "data about branches",
             V_abs(bus_nbr1) "voltage absolutes at buses",
             V_ang(bus_nbr1) "voltage angles at buses",
             P_bus(bus_nbr1) "net active power generation/load ...
                 at buses",
             Q_bus(bus_nbr1) "net reactive power ...
                 generation/load at buses",
             P_max(gen_nbr) "rating of the generator",
             Y_abs(bus_nbr1,bus_nbr2) "admittance matrix with ...
                 absolutes for the network",
```

```
               Y_ang(bus_nbr1,bus_nbr2) "admittance matrix with ...
                  angles for the network";

$GDXIN case_50kW.gdx
$LOAD bus, branch, gen, baseMVA, S_max, Y_abs, Y_ang
$GDXIN

*voltage absolutes at buses
V_abs(bus_nbr1) = bus(bus_nbr1,"8");
*voltage angles at buses
V_ang(bus_nbr1) = bus(bus_nbr1,"9")*pi/180;
*active power at buses
P_bus(bus_nbr1) = gen(bus_nbr1,"2")/baseMVA - ...
   bus(bus_nbr1,"3")/baseMVA;
*reactive power at buses
Q_bus(bus_nbr1) = gen(bus_nbr1,"3")/baseMVA - ...
   bus(bus_nbr1,"4")/baseMVA;
*rating of maximal active power of the generators
P_max(gen_nbr) = gen(gen_nbr,"9");

*===== SECTION: INITIALIZE DATA NEEDED FOR THE OPTIMIZATION
Free variable
z "demanded by GAMS for the objective function",
delta_P(bus_nbr1) "curtailment of active power at buses",
delta_Q(bus_nbr1)  "reactive power consumption at buses",
delta_V_abs(bus_nbr1)  "voltage absolute change at buses",
delta_V_ang(bus_nbr1)  "voltage phase angle change at buses";

Equations
obj "sums of delta_Q and delta_P"
constraint_1 "The relation between P, V and Y must hold"
constraint_2 "The relation between Q, V and Y must hold"
constraint_3 "delta_V must be above the lower limit"
constraint_4 "delta_V must be below the upper limit"
constraint_5 "P^2 + Q^2 < S_max^2 at PQ-buses"
constraint_6 "P^2 + Q^2 < S_max^2 ar reference bus"
;

obj.. sum(bus_nbr1$(bus(bus_nbr1,"2") = ...
   1),power(delta_P(bus_nbr1),2)) + ...
   0.001*sum(bus_nbr1$(bus(bus_nbr1,"2") = ...
   1),power(delta_Q(bus_nbr1),2)) =E= z;

constraint_1(bus_nbr1)..  ...
   sum(bus_nbr2,Y_abs(bus_nbr1,bus_nbr2)*(delta_V_abs(bus_nbr2) ...
   + V_abs(bus_nbr2))*(delta_V_abs(bus_nbr1) + ...
   V_abs(bus_nbr1))*cos((delta_V_ang(bus_nbr1) + ...
   V_ang(bus_nbr1)) - (delta_V_ang(bus_nbr2) + ...
   V_ang(bus_nbr2)) - Y_ang(bus_nbr1,bus_nbr2))) =E= ...
```

```
    delta_P(bus_nbr1) + P_bus(bus_nbr1);
constraint_2(bus_nbr1)..  ...
    sum(bus_nbr2,Y_abs(bus_nbr1,bus_nbr2)*(delta_V_abs(bus_nbr2) ...
    + V_abs(bus_nbr2))*(delta_V_abs(bus_nbr1) + ...
    V_abs(bus_nbr1))*sin((delta_V_ang(bus_nbr1) + ...
    V_ang(bus_nbr1)) - (delta_V_ang(bus_nbr2) + ...
    V_ang(bus_nbr2)) - Y_ang(bus_nbr1,bus_nbr2))) =E= ...
    delta_Q(bus_nbr1) + Q_bus(bus_nbr1);
constraint_3(bus_nbr1)$(bus(bus_nbr1,"2") = 1).. ...
    delta_V_abs(bus_nbr1) + V_abs(bus_nbr1) =G= 1 - V_limit;
constraint_4(bus_nbr1)$(bus(bus_nbr1,"2") = 1).. ...
    delta_V_abs(bus_nbr1) + V_abs(bus_nbr1) =L= 1 + V_limit;
constraint_5(bus_nbr1)$(bus(bus_nbr1,"2") = 1).. ...
    power(delta_P(bus_nbr1) + P_bus(bus_nbr1),2) + ...
    power(delta_Q(bus_nbr1)+Q_bus(bus_nbr1),2) =L= power(S_max,2);
constraint_6(bus_nbr1)$(bus(bus_nbr1,"2") = 3).. ...
    power(delta_P(bus_nbr1) + P_bus(bus_nbr1),2) + ...
    power(delta_Q(bus_nbr1)+Q_bus(bus_nbr1),2) =L= ...
    power(S_max*10,2);

*lower limit is set to -P_bus at the desired buses, as you ...
    cannot curtail more P than there is generation
delta_P.lo(bus_nbr1)$(bus(bus_nbr1,"2") = 1 and ...
    P_bus(bus_nbr1)>0) = -P_bus(bus_nbr1);

*reference bus can consume as much P as is demanded
delta_P.up(bus_nbr1)$(bus(bus_nbr1,"2") = 3) = 1;

*bound delta_Q
delta_Q.lo(bus_nbr1) = -1;
delta_Q.up(bus_nbr1) = 1;

*it is not possible to change the voltage at MV-part buses
delta_V_abs.fx(bus_nbr1)$(not bus(bus_nbr1,"2") = 1) = 0;
delta_V_ang.fx(bus_nbr1)$(not bus(bus_nbr1,"2") = 1) = 0;

*===== SECTION: SOLVING THE PROBLEM
Model anm /
obj
constraint_1
constraint_2
constraint_3
constraint_4
constraint_5
constraint_6
/;

*choose one of the three solvers below
Option NLP = knitro;
```

```
*Option NLP = minos;
*Option NLP = conopt;

Solve anm using NLP minimizing z;
```

MV Network

```
$title Optimization of control actions for control of current.
*-----------------------------------------------------------------
* Filename: optimization_i_nonlin.gms
* Description: Optimization of control actions for control of ...
    current.
*
* Usage: gams optimization_i_nonlin
*-----------------------------------------------------------------

*===== SECTION: LOAD THE LOAD FLOW ANALYSIS DATA
Set s /1*20/;
Alias (bus_nbr1,s);
Alias (bus_nbr2,s);
Alias (bus_cols,s);
Alias (gen_nbr,s);
Alias (gen_cols,s);
Alias (branch_nbr,s);
Alias (branch_cols,s);

Scalars      baseMVA "SBase"
             I_limit "maximum allowed value for current"
             V_limit "maximum allowed value for the voltage ...
                 deviation"
             S_max "maximum apparent power"
             k "factor to multiply with P to acquire Q"
             nbr_of_buses "number of buses";

*maximum allowed value for current
I_limit = 0.0390;
*maximum allowed value for the voltage deviation
V_limit = 0.05;

Parameters   bus(bus_nbr1,bus_cols) "data of buses",
             gen(gen_nbr,gen_cols) "data of generators",
             branch(branch_nbr,branch_cols) "data about branches",
             V_abs(bus_nbr1) "voltage absolutes at buses",
             V_ang(bus_nbr1) "voltage angles at buses",
             I_abs(bus_nbr1) "current absolutes in which ...
                 element n is the current at the inner branch of ...
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               bus n at the radial",
            I_ang(bus_nbr1) "current angles in which element n ...
               is the current at the inner branch of bus n at ...
               the radial",
            I_gen_abs(bus_nbr1) "generator/load current absolute",
            I_gen_ang(bus_nbr1) "generator/load current phase ...
               angle",
            P_bus(bus_nbr1) "net active power generation/load ...
               at buses",
            Q_bus(bus_nbr1) "net reactive power ...
               generation/load at buses",
            Y_abs(bus_nbr1) "admittance absolutes in which ...
               element n is the current at the inner branch of ...
               bus n at the radial",
            Y_ang(bus_nbr1) "admittance angles in which ...
               element n is the current at the inner branch of ...
               bus n at the radial",
            P_max(gen_nbr) "rating of the generator";

$GDXIN case_WP.gdx
$LOAD baseMVA, bus, branch, gen, I_abs, I_ang, I_gen_abs, ...
   I_gen_ang, S_max, P_bus, Q_bus, Y_abs, Y_ang, nbr_of_buses
$GDXIN

*voltage absolutes at buses
V_abs(bus_nbr1) = bus(bus_nbr1,"8");
*voltage angles at buses
V_ang(bus_nbr1) = bus(bus_nbr1,"9")*pi/180;
*rating of maximal active power of the generators
P_max(gen_nbr) = gen(gen_nbr,"9");

*===== SECTION: INITIALIZE DATA NEEDED FOR THE OPTIMIZATION
Free variable
z "demanded by GAMS for the objective function",
delta_P(bus_nbr1) "curtailment of active power at buses",
delta_Q(gen_nbr)  "reactive power consumption at buses",
delta_I_gen_abs(bus_nbr1)  "generator/load current absolute ...
   change",
delta_I_gen_ang(bus_nbr1)  "generator/load current phase angle ...
   change",
delta_I_abs(bus_nbr1)  "current absolute change",
delta_I_ang(bus_nbr1)  "current phase angle change",
delta_V_abs(bus_nbr1)  "voltage absolute change",
delta_V_ang(bus_nbr1)  "voltage phase angle change";

Equations
obj "sums of delta_Q and delta_P"
constraint_1 "Real part of relation between branch currents ...
   and generator/load currents"
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constraint_2 "Imaginary part of relation between branch ...
    currents and generator/load currents"
constraint_3 "The relation between absolutes of P, Q, V and I ...
    must hold"
constraint_4 "The relation between phase angles of P, Q, V and ...
    I must hold"
constraint_5 "The relation between the real parts of I=V*Y ...
    must hold"
constraint_6 "The relation between the imaginary parts of ...
    I=V*Y must hold"
constraint_7 "P^2 + Q^2 < S_max^2 for PQ-buses"
constraint_8 "P^2 + Q^2 < S_max^2 for reference bus"
constraint_9 "Use for voltage control: V must be above the ...
    lower limit"
constraint_10 "Use for voltage control: V must be below the ...
    upper limit"
constraint_11 "Use for congestion control: current at the ...
    inner side of bus 4 must be below the stated limit";

obj.. sum(bus_nbr1$(bus(bus_nbr1,"2") = 1 or bus(bus_nbr1,"2") ...
    = 3),power(delta_P(bus_nbr1),2)) + ...
    0.001*sum(bus_nbr1$(bus(bus_nbr1,"2") = 1 or ...
    bus(bus_nbr1,"2") = 3),power(delta_Q(bus_nbr1),2)) =E= z;

constraint_1(bus_nbr1).. (I_gen_abs(bus_nbr1) + ...
    delta_I_gen_abs(bus_nbr1))*cos(I_gen_ang(bus_nbr1) + ...
    delta_I_gen_ang(bus_nbr1)) =E= (I_abs(bus_nbr1) + ...
    delta_I_abs(bus_nbr1))*cos(I_ang(bus_nbr1) + ...
    delta_I_ang(bus_nbr1)) - (I_abs(bus_nbr1+1) + ...
    delta_I_abs(bus_nbr1+1))*cos(I_ang(bus_nbr1+1) + ...
    delta_I_ang(bus_nbr1+1));
constraint_2(bus_nbr1).. (I_gen_abs(bus_nbr1) + ...
    delta_I_gen_abs(bus_nbr1))*sin(I_gen_ang(bus_nbr1) + ...
    delta_I_gen_ang(bus_nbr1)) =E= (I_abs(bus_nbr1) + ...
    delta_I_abs(bus_nbr1))*sin(I_ang(bus_nbr1) + ...
    delta_I_ang(bus_nbr1)) - (I_abs(bus_nbr1+1) + ...
    delta_I_abs(bus_nbr1+1))*sin(I_ang(bus_nbr1+1) + ...
    delta_I_ang(bus_nbr1+1));
constraint_3(bus_nbr1)$(bus(bus_nbr1,"2") = 1 or ...
    bus(bus_nbr1,"2") = 3).. power(I_gen_abs(bus_nbr1) + ...
    delta_I_gen_abs(bus_nbr1),2)*power(V_abs(bus_nbr1) + ...
    delta_V_abs(bus_nbr1),2)*3 =E= power(P_bus(bus_nbr1) + ...
    delta_P(bus_nbr1),2) + power(Q_bus(bus_nbr1) + ...
    delta_Q(bus_nbr1),2);
constraint_4(bus_nbr1)$(bus(bus_nbr1,"2") = 1 or ...
    bus(bus_nbr1,"2") = 3).. arctan2(Q_bus(bus_nbr1) + ...
    delta_Q(bus_nbr1),P_bus(bus_nbr1) + delta_P(bus_nbr1)) =E= ...
    (V_ang(bus_nbr1) + delta_V_ang(bus_nbr1)) - ...
    I_gen_ang(bus_nbr1) - delta_I_gen_ang(bus_nbr1);
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constraint_5(bus_nbr1).. (I_abs(bus_nbr1) + ...
    delta_I_abs(bus_nbr1))*cos(I_ang(bus_nbr1) + ...
    delta_I_ang(bus_nbr1))*1.7321 =E= ((V_abs(bus_nbr1) + ...
    delta_V_abs(bus_nbr1))*cos(V_ang(bus_nbr1) + ...
    delta_V_ang(bus_nbr1) + Y_ang(bus_nbr1)) - ...
    (V_abs(bus_nbr1-1) + ...
    delta_V_abs(bus_nbr1-1))*cos(V_ang(bus_nbr1-1) + ...
    delta_V_ang(bus_nbr1-1) + Y_ang(bus_nbr1)))*Y_abs(bus_nbr1);
constraint_6(bus_nbr1).. (I_abs(bus_nbr1) + ...
    delta_I_abs(bus_nbr1))*sin(I_ang(bus_nbr1) + ...
    delta_I_ang(bus_nbr1))*1.7321 =E= ((V_abs(bus_nbr1) + ...
    delta_V_abs(bus_nbr1))*sin(V_ang(bus_nbr1) + ...
    delta_V_ang(bus_nbr1) + Y_ang(bus_nbr1)) - ...
    (V_abs(bus_nbr1-1) + ...
    delta_V_abs(bus_nbr1-1))*sin(V_ang(bus_nbr1-1) + ...
    delta_V_ang(bus_nbr1-1) + Y_ang(bus_nbr1)))*Y_abs(bus_nbr1);
constraint_7(bus_nbr1)$(bus(bus_nbr1,"2") = 1).. ...
    power(delta_P(bus_nbr1)+P_bus(bus_nbr1),2) + ...
    power(delta_Q(bus_nbr1)+Q_bus(bus_nbr1),2) =L= power(S_max,2);
constraint_8(bus_nbr1)$(bus(bus_nbr1,"2") = 3).. ...
    power(delta_P(bus_nbr1)+P_bus(bus_nbr1),2) + ...
    power(delta_Q(bus_nbr1)+Q_bus(bus_nbr1),2) =L= ...
    power(S_max*10,2);
constraint_9(bus_nbr1)$(bus(bus_nbr1,"2") = 1).. ...
    delta_V_abs(bus_nbr1) + V_abs(bus_nbr1) =G= 1 - V_limit;
constraint_10(bus_nbr1)$(bus(bus_nbr1,"2") = 1).. ...
    delta_V_abs(bus_nbr1) + V_abs(bus_nbr1) =L= 1 + V_limit;
constraint_11..  I_abs("4")+delta_I_abs("4") =L= I_limit;

*bound delta_P
delta_P.lo(bus_nbr1)$(bus(bus_nbr1,"2") = 1) = -1;
delta_P.up(bus_nbr1)$(bus(bus_nbr1,"2") = 1) = 1;
*P cannot be changed at bus 3
delta_P.fx("3") = 0;

*bound delta_Q
delta_Q.lo(bus_nbr1)$(bus(bus_nbr1,"2") = 1) = -1.5;
delta_Q.up(bus_nbr1)$(bus(bus_nbr1,"2") = 1) = 1.5;
*Q cannot be changed at bus 3
delta_Q.fx("3") = 0;

*when congestion management is examined, uncomment these
delta_Q.fx("4") = 0;
delta_Q.fx("5") = 0;

*inner branch of the bus furthest in at the radial does not exist
delta_I_abs.fx("1")=0;
delta_I_ang.fx("1")=0;
*outer branch of the bus furthest out at the radial does not exist
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delta_I_abs.fx("6")=0;
delta_I_ang.fx("6")=0;
*this line is just a dummy line between dummy bus and ...
    reference bus
delta_I_abs.fx("2")=0;
delta_I_ang.fx("2")=0;

*it is not possible to change the voltage at HV buses
delta_V_abs.fx(bus_nbr1)$(bus(bus_nbr1,"2")  = 3)  = 0;
delta_V_ang.fx(bus_nbr1)$(bus(bus_nbr1,"2")  = 3)  = 0;
delta_V_abs.fx(bus_nbr1)$(bus(bus_nbr1,"2")  = 2)  = 0;
delta_V_ang.fx(bus_nbr1)$(bus(bus_nbr1,"2")  = 2)  = 0;

*===== SECTION: SOLVING THE PROBLEM
Model anm /
obj
constraint_1
constraint_2
constraint_3
constraint_4
constraint_5
constraint_6
constraint_7
constraint_8
*constraint_9
*constraint_10
constraint_11
/;

*choose one of the three solvers below
Option NLP = knitro;
*Option NLP = minos;
*Option NLP = conopt;

Solve anm using NLP minimizing z;
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