Detailed Balance in Quantum Dot Tunneling

Martin Plate

LUNDS UNIVERSITET

Supervisor: Ville Maisi
Co-supervisor: David Barker

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Department of Physics
Division of Solid State Physics

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Abstract

Previously recorded data of electron tunneling in and out of a quantum dot (QD) is used to extract knowledge of the state of the system. Using the detailed balance of the system, it is shown that information about the electron temperature in the lead T_e, as well as knowledge about the degeneracies of the states in a quantum dot can be obtained from measurements of the tunneling rates in and out of the QD. Combining the measurements of the tunneling rates with temperature measurements of the surrounding environment T_d, the lever arm of the system is determined. It is shown that knowing the lever arm T_e can be directly measured. A dependence of the lever arm on the filling of the quantum dot N is also observed.

Contents

Abbreviations

SET: Single electron transistor
WZ: Wurtzite
ZB: Zincblende
QD: Quantum dot
FD: Fermi-Dirac
SEM: Scanning electron microscope
DOS: Density of states

1 Introduction

Using the concept of microstates it can be shown that a thermodynamic system out of equilibrium will evolve so that the system takes on the macro state which has the most amount of corresponding microstates, giving the well-known second law of thermodynamics. This holds true as far as the system is sufficiently large, if this is the case small fluctuations are shown to be inconsequential.[1] For a long time this has proved sufficient, but as devices have started to take on sizes on the mesoscopic scale the limit for where classical thermodynamics is able to describe the system has been reached. To properly describe these systems, thermodynamic theory needs to be reformulated to take into account fluctuations. It has been shown both in theory and experiment that for systems sufficiently localised in space and time the second law of thermodynamics does no longer necessarily hold.[2–5]

In order use classical thermodynamics the system has to be larger than the thermodynamic limit.[1] The system considered in this thesis is on the other end of the scale only consisting of two possible microstates. A central idea in classical thermodynamics is that no knowledge about the specific microstate of the system is needed and that the system is instead described by its macrostate. The aim in this thesis is instead the opposite, to use precise knowledge of the microstate of part of the system as a function of time to derive knowledge about the macrostate of the system.

The system analysed in the thesis consists of a quantum dot (QD), coupled to a single lead that acts as a reservoir, supplying electrons that tunnel in to the QD, as well as empty states in the lead that electrons from the QD can tunnel to. Similarly to atoms QDs have quantised energy states that the electrons are allowed to occupy, leading some to call them artificial atoms.[6] For this thesis the discrete energy levels are used to sample electrons depending on their energy. It has been suggested that quantum dots could be used as qubits for quantum computing.[7] This would be achieved by using two electron levels in the QD, giving a two level system, where one of the levels correspond to 0 and the other level corresponds to a 1.[7]

1.1 Quantum dots

QDs are potential wells where electrons are confined in all directions.[8] In order for the electrons to be confined their wavelength has to be comparable to the width of the well. To construct a QD, an electron gas is confined within a semiconductor. Confinement in a direction is achieved either by varying the material leading to discontinuities in the band structure, or by patterning gates close to the electron gas.[9] The confinement of the electrons in all directions gives a discrete energy spectrum, and only electrons with a specific energy can enter the QD.[8] If the QD is approximated as a rectangular box with infinite potential on the edges the energy of electrons in the dot is given by [8]

$$E(k_z) = \frac{\hbar^2 k_z^2}{2m_e^*} + E_{xy} \tag{1}$$

Here m_e^* is the effective mass of electrons in the semiconductor and k_z the wave vector of the electron along the nanowire. E_{xy} is the energy contribution from the transverse confinement. The wave vector k_z is quantised meaning the allowed energy states in the dot will be as well. Due to the Pauli principle only one electron is allowed per state. As there is a spin degree of freedom two electrons will be allowed per energy state in the QD.

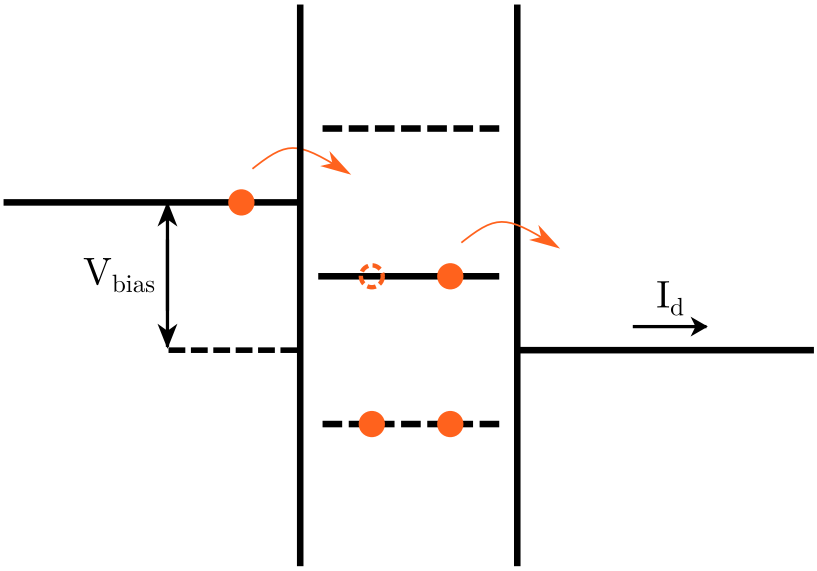

Figure 1: A schematic of a single electron transistor (SET). The middle section is a QD. If the energy level is in either of the positions marked by dotted lines no current will flow, while if it is in the middle current is allowed to flow.

If the QD is coupled to two leads a single electron transistor (SET) can be formed, allowing only one electron to pass at a time. This is done by applying a voltage over the QD creating a small bias window. The voltage over the QD creates a small bias window where electrons can tunnel between the leads allowing a current to flow. However, this only happens if at least one of the QD energy levels is within the bias window. An electron tunneling onto the QD will repel other electrons from tunneling in before that electron has tunnelled out, this is referred to as coulomb blockade.[10] A plunger gate located close to the QD can be used to shift the position of the energy levels by changing the potential in the QD. If the bias window is smaller than the spacing between energy levels and the plunger gate voltage is swept, this will give rise to oscillations in the current through the SET known as coulomb oscillations.[10] When the voltage applied to a plunger gate is changed by an amount ΔV the amount that the energy level of the corresponding QD changes E with respect to the applied voltage is given by the lever arm

$$\alpha = \frac{E}{\Delta V} \tag{2}$$

2 The device

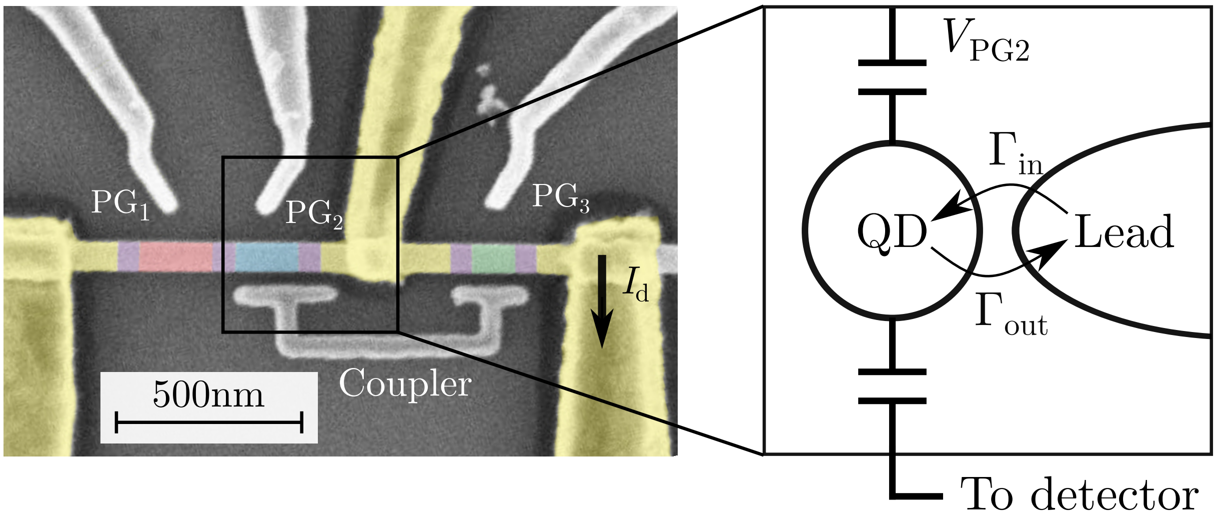

Figure 2: A scanning electron microscope (SEM) image of the device used. Parts marked PG denote plunger gates. The square to the right is a schematic over the system as used in this thesis. Adapted from a SEM image by David Barker.

The device consists of a nanowire with three QDs (red, blue and green in Figure 2) grown into it, each one with a corresponding plunger gate. To construct the device a nanowire with QDs grown into it is placed on a wafer and using epitaxial markers on the nanowire for positioning, the gates and leads are patterned and deposited.[9] The nanowire is an epitaxially grown InAs nanowire. Tunneling barriers are constructed in the nanowire by varying the crystal phase between the zincblende (ZB) polytype and the wurtzite (WZ) polytype. With the barriers being of WZ type and the dots as well as the leads being of ZB type.[9] The plunger gates allow for tuning the energy level within each QD. Dots 1 and 2 (red and blue) are capacitively coupled to the third QD (green), this QD is used to detect changes in occupation of the other two dots. The yellow parts are the leads and purple denotes tunnel barriers.

For the measurements used in this thesis only QD 2 is used (marked in blue). If voltages are applied to plunger gates 1-3 $V_{PG1,2,3}$ each dot can be tuned by displacing the energy levels of the QDs. By putting QD 1 in coulomb blockade, any measured tunneling will be between QD 2 and the middle lead. This means that the system analysed will be a QD coupled to a single lead, a schematic of this is shown in the zoom-in in Figure 2. Using a single energy level in QD 2 the energy distribution of the electrons in the middle lead can be probed. As the QDs have discrete energy levels, only electrons in the lead matching the energy level of QD will be able to tunnel into the QD. Conversely, any electrons tunneling from the QD to the lead will have the same energy as the energy level of the QD.

The middle lead acts as a reservoir supplying electrons that tunnel between the QD and the lead. If an electron tunnels into the QD the dot will be placed in coulomb blockade, stopping further tunneling into the dot until this electron tunnels out of the QD. So the dot will alternate between having N and $N+1$, where the first situation is referred to as the electron being out of the QD, while in the second case it is referred to the electron being in the QD.

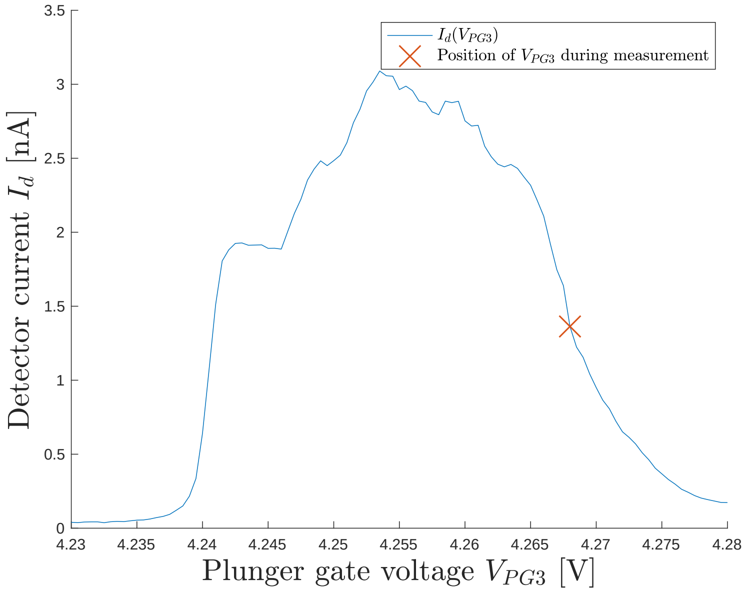

Figure 3: A coulomb oscillation of the SET. The red dot denotes the positioning of the energy level within the SET.

2.1 The detector

To measure the occupancy of QD 2 the third dot is used as a detector, with a capacitive coupler between dots 2 and 3. The third dot is tunnel coupled to two leads. If a small voltage is applied over QD 3 it will act as a single electron transistor, and if V_{PG3} is swept the current through dot I_d will display coulomb oscillations. It is this change in detector current I_d that is used to determine when there is an extra electron in QD 2. For I_d to be as sensitive as possible to changes in the potential, the differential current $\frac{\mathrm{d}I_d}{\mathrm{d}V_{PG3}}$ should be maximised. In order to maximise the differential current V_{PG3} is adjusted so that it corresponds to the edge of a coulomb oscillation (see Figure 3). In turn this makes I_d very sensitive to changes in the V_{PG3}, and thus to charges nearby the detector such as those in the coupler. This allows for detecting a single electron difference in occupancy. Due to the closer proximity between the dot and the coupler QD 2 will be more coupled, yielding a larger displacement of the SET energy level, which can be used to differentiate between a change in occupation of either dot.

3 Theory

Electrons tunnel between the QD and the reservoir at rates described by [2, 11].

$$\Gamma_{\text{in}} = d_{\text{in}}\Gamma_0 f(E, T_e) \tag{3}$$

$$\Gamma_{\text{out}} = d_{\text{out}}\Gamma_0(1 - f(E, T_e)) \tag{4}$$

Here Γ_0 accounts for the tunnel-coupling as well as the density of states (DOS). While the DOS should ideally be that of a 1D system it can be approximated as constant over a small range ΔE. The plunger gate voltage not only alters the position of the energy level within the QD but also has an effect on the tunneling barrier. However, this effect is negligible over a small range ΔE. $f(E, T_e)$ is the FD distribution. The factors $d_{\text{in(out)}}$ accounts for any degeneracies in the states the electron can tunnel to(from).

Assuming that the system satisfies $E_{ch} \gg k_B T_e$ where E_{ch} is the charging energy when an electron is added, then there will at most be one level that electrons can tunnel to and from.[12] An electron tunneling onto the dot will put the QD in coulomb blockade, stopping further tunneling into the dot. This means that once an electron has tunneled into the dot, an electron has to tunnel out of the QD before any tunneling in can happen. There are then two possible states for the system, the first with the electron in the QD and the second one with the electron out of the QD. For such a system the rate equation is then

$$\begin{bmatrix} \dot{p}_{\text{in}} \\ \dot{p}_{\text{out}} \end{bmatrix} = \begin{bmatrix} -\Gamma_{\text{out}} & \Gamma_{\text{in}} \\ \Gamma_{\text{out}} & -\Gamma_{\text{in}} \end{bmatrix} \begin{bmatrix} p_{\text{in}} \\ p_{\text{out}} \end{bmatrix} \tag{5}$$

Where p_{in} is the probability of finding the electron in the QD and p_{out} the probability of it being out of the QD. For a system at equilibrium $\dot{p}_{\text{in}} = \dot{p}_{\text{out}} = 0$, by inserting this into equation (5) the detailed balance condition is obtained

$$p_{\text{in}}\Gamma_{\text{out}} = p_{\text{out}}\Gamma_{\text{in}} \tag{6}$$

Using the detailed balance of the system along with equations (3) and (4) it is concluded that

$$\frac{p_{\text{in}}}{p_{\text{out}}} = \frac{\Gamma_{\text{in}}}{\Gamma_{\text{out}}} = \frac{d_{\text{in}}}{d_{\text{out}}} \frac{f(E, T_e)}{1 - f(E, T_e)} = \frac{d_{\text{in}}}{d_{\text{out}}} e^{-E/kT_e} \tag{7}$$

which is just the Boltzmann factor times a constant. It is this form of the detailed balance that allows for determining the lever arm of the system without knowing Γ_0.

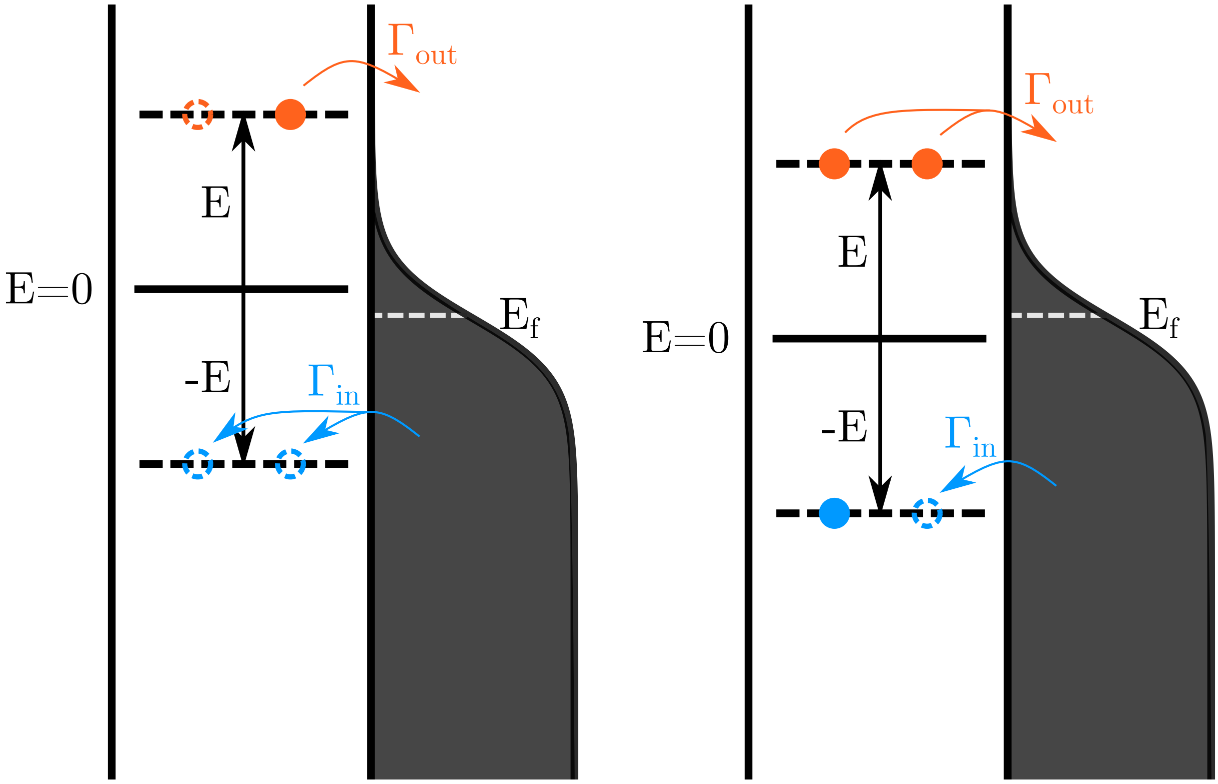

Figure 4: To the left the degeneracies for an even N is shown, to the right N is odd. Depending on if N is odd or even either Γ_{in} or Γ_{out} will be two fold degenerate. Choosing the $E = 0$ when $\Gamma_{\text{in}} = \Gamma_{\text{out}}$ leads to E_f being offset by $\pm \ln(2)$.

3.1 The effect of degeneracies

As mentioned earlier each one of the energy levels has a degeneracy of 2. Therefore, if a QD is filled with an even number of electrons N (see left part of Figure 4), there will be two unoccupied states on the lowest unfilled energy level that an electron can tunnel to, meaning that $d_{\text{in}} = 2$. If an electron tunnels onto the dot the coulomb blockade stops any further tunneling in, and as there is only one electron that can tunnel out from the highest occupied level $d_{\text{out}} = 1$. If instead N is odd (see left part of Figure 4), there will only be one unoccupied state on the lowest unfilled level, meaning that $d_{\text{in}} = 1$. Once an electron tunnels into the empty state there will then be two electrons on the highest occupied level that can tunnel out, meaning that $d_{\text{out}} = 2$.

By using equation (7) the energy E where the in and out rates are equal is found to be.

$$\frac{\Gamma_{\text{in}}}{\Gamma_{\text{out}}} = \frac{\Gamma_{\text{in}}}{\Gamma_{\text{in}}} = \frac{d_{\text{in}}}{d_{\text{out}}} e^{-E/kT_e} = 1 \tag{8}$$

this implies that if $\Gamma_{\text{in}} = \Gamma_{\text{out}}$ then

$$e^{-E/kT_e} = \frac{d_{\text{out}}}{d_{\text{in}}} = e^{\ln(d_{\text{out}}/d_{\text{in}})} \tag{9}$$

So if the energy is chosen so that $E = 0$ when $\Gamma_{\text{in}} = \Gamma_{\text{out}}$ the Fermi energy of the reservoir E_f will be placed at $E = kT_e \ln(d_{\text{out}}/d_{\text{in}})$. For a system with $d_{\text{in}} = 2$ and $d_{\text{out}} = 1$ the

Fermi energy will then occur at $E = -kT_e \ln(2)$, if the degeneracies are instead reversed the Fermi energy will instead occur at $E = kT_e \ln(2)$.

4 Measurements

In total four data sets are analysed, the data sets are recorded at different device temperatures T_d and filling of the QD N. This allows for analysing how the detailed balance varies with T_d and N by using equation (7). By varying V_{PG2} the energy level of the QD is tuned to a specific energy, as the level is discrete this allows for probing the rates at that specific energy. Each data set consists of time traces, with each time trace corresponding measuring either the in or out rate at a specific energy.

To determine the degeneracies the tunneling rates need to be measured far from the Fermi energy. This presents a problem as either Γ_{in} or Γ_{out} will tend to 0 as the energy level moves further from E_f, negatively affecting the statistics of the measurement. As a tunneling in event has to be followed by an out, and vice versa, the one with the lower rate will limit the accuracy of both rates. While the tunneling rate one way goes to 0 the other one approaches its maximum value, by using a feedback technique the high rate can be measured with good accuracy without being limited by the slower process.[12] For example, if we want to measure the tunneling out rate, the energy is first lowered to a position where the tunneling in rate is high, and it is kept there until an electron tunnels in, then the level is quickly raised to where the out rate is to be measured. This is repeated every time the electron tunnels out. By allowing it to tunnel in where Γ_{in} is high, good statistics for the measurement of Γ_{out} can be achieved.

To implement the feedback technique an Arduino microcontroller was used to monitor the detector current. If I_d was above a certain threshold the microcontroller would then send a voltage signal to the voltage source used to supply V_{PG2}. The detector current was also sent via a digital acquisition system onto a computer used to record the time traces. The measurements were performed in a dilution refrigerator, the temperature of which was monitored using a thermal probe.

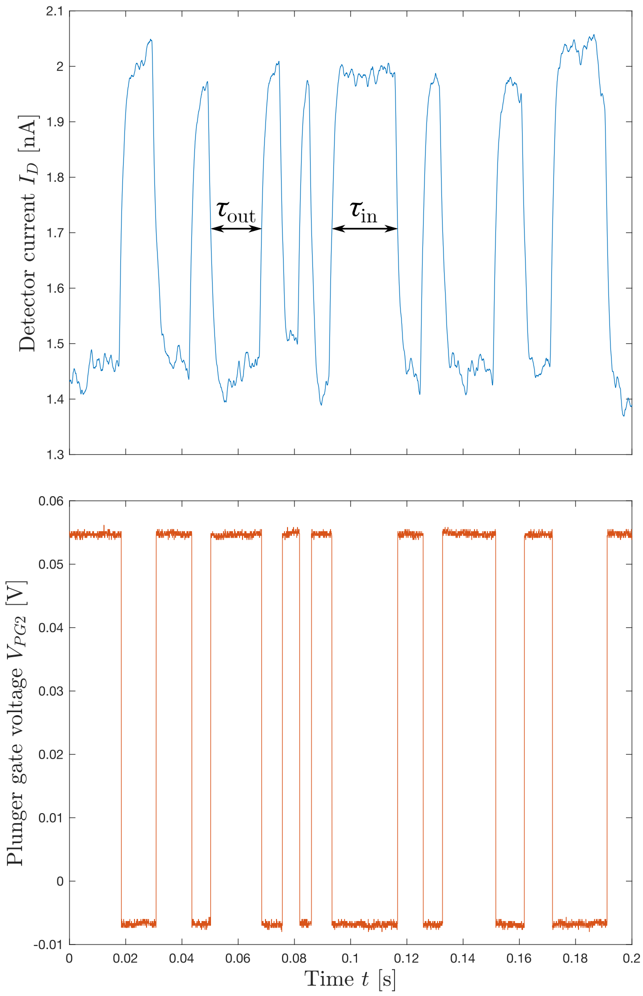

Figure 5: Above: Part of a time trace. The detector current varies depending on if there are N or $N+1$ electrons in the QD. $\tau_{\text{in(out)}}$ is the time the electron spends in(out of) the quantum dot. Below: The feedback applied during the same time as the time trace.

Figure 5 shows part of a time trace along with the corresponding applied feedback. The waiting times are extracted from the time traces. In the time trace of Figure 5 the waiting times have been marked as $\tau_{\text{in(out)}}$ for the time the electron spends in(out of) the dot. The events are independent [12, 13], thus the tunneling the rates can be extracted from the average of the waiting times $\langle \tau_{\text{in(out)}} \rangle$ [11]

$$\Gamma_{\text{in}} = \frac{1}{\langle \tau_{\text{out}} \rangle} \quad , \quad \Gamma_{\text{out}} = \frac{1}{\langle \tau_{\text{in}} \rangle} \tag{10}$$

To determine the degeneracies of the system equations (3) and (4) are fitted to the data, with

$$E = \alpha \Delta V_{\text{PG2}} \pm k_B T_e \ln(2) \tag{11}$$

where $\pm \ln(2)$ offsets the Fermi energy so that $\Delta V_{\text{PG2}} = 0$ when $\Gamma_{\text{in}} = \Gamma_{\text{out}}$.

4.1 Extracting the lever arm

According to equation (7) the ratios between Γ_{in} and Γ_{out} depend exponentially on E. Hence, the ratios are fitted to the function

$$C_1 e^{C_2 \Delta V_{\text{PG2}}} \tag{12}$$

Where C_1 and C_2 are fitting constants. Comparing (7) and (12) it is seen that

$$C_1 = \frac{d_{\text{in}}}{d_{\text{out}}} \tag{13}$$

$$C_2 \Delta V_{\text{PG2}} = -\frac{E}{k_B T_e} \tag{14}$$

From (14) it can be observed that C_2^{-1} depends linearly on T_e

$$C_2^{-1} = -\frac{1}{\alpha} k_B T_e = \lambda T_e \tag{15}$$

By performing a linear fit of C_2^{-1} against T_e the lever arm can then be extracted as

$$\alpha = -\frac{k_B}{\lambda} \tag{16}$$

5 Results

The four data sets differ in filling of the QD N, and cryostat temperature T_d. Measurements **c** and **d** (see Table 1) have the same N, the other two sets have different N with any other data set. The temperatures at which the data sets are recorded are listed in the Table 1.

Table 1: The temperatures the measurements were recorded at.

Measurement	T_d[mK]
a	10
b	100
c	100
d	200

For data sets **a** and **b** the voltage offset ΔV_{PG2} of the PG voltage goes in a range $\Delta V_{\text{PG2}} = \pm 0.35\,\text{mV}$, while for **c** and **d** the voltage steps are twice as big, and the range is instead $\Delta V_{\text{PG2}} = \pm 0.70\,\text{mV}$.

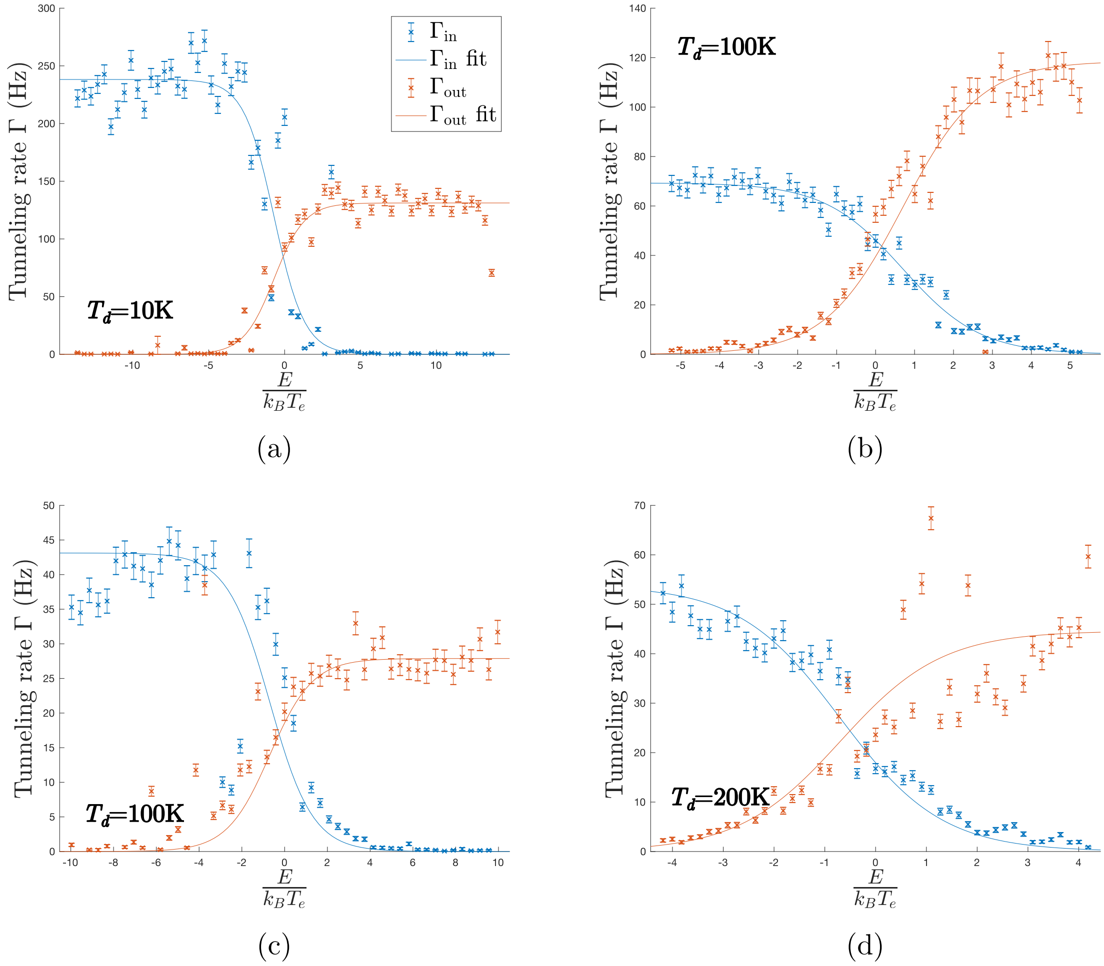

Figure 6: The experimentally determined tunneling rates $\Gamma_{\text{in(out)}}$ along with fits of equations (3) and (4) to the data. The x-axis is chosen so that $E = 0$ when $\Gamma_{\text{in}} = \Gamma_{\text{out}}$.

From the fits of equations (3) and (4) to the data seen in Figure 6 it is observed that for most of the data sets the distribution of the electrons in the lead follows a Fermi-Dirac (FD) distribution, in agreement with what is expected from an electron gas.[1] For measurement **d** the data and model Γ_{out} differ significantly, there are also many outliers in the data for Γ_{out}. Data set **d** is the one with the highest T_d of the measurements, the increased noise of the data could be due to a higher signal-to-noise ratio at the higher temperature. Another effect that could decrease the resolution of the data in **d** is that the temperature increase leads to a broadening of the FD distribution, possibly leading to excited states becoming available. Comparing $k_b T_e$ to the charging energy for similar QDs it is found that at $T_e = 200\,\text{mK}$, The width of the Fermi function $k_b T_e \approx 0.02\,\text{meV}$ while $E_{ch} \approx 2\,\text{meV}$ [9], so only one extra electron should be allowed on the QD at any time.

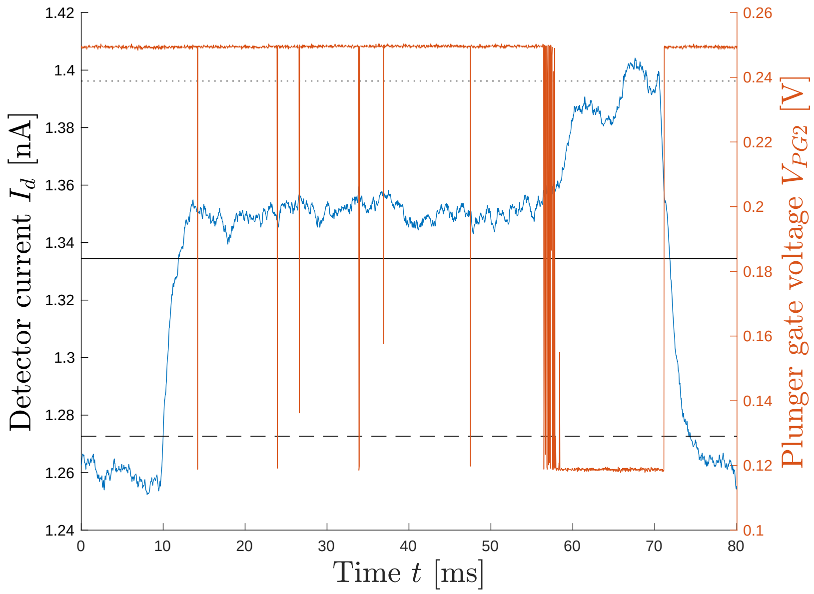

Figure 7: Part of a time trace from data set d. The detector current (blue) is overlayed with V_{PG2}. The the black black horizontal line in the middle is the threshold for when the electron is considered to be in the QD. The dotted lines is where the detector current should lay when the electron is either in or out of the dot.

When looking at the time traces for data set d, it was observed that I_d had a dependence on V_{PG2} (see Figure 7). This leads to steps forming in the time trace, and it is not until V_{PG2} has switched that the detector current reaches the correct value. In Figure 7 a step has formed right below the threshold for switching V_{PG2} (the threshold for switching is not displayed in the figure). While the electron tunnels in at $t = 10.76\,\text{s}$ it is not until $t = 10.81\,\text{s}$, meaning that the tunneling out rate is not measured at the expected energy. Thus, Γ_{out} will be too low in this case.

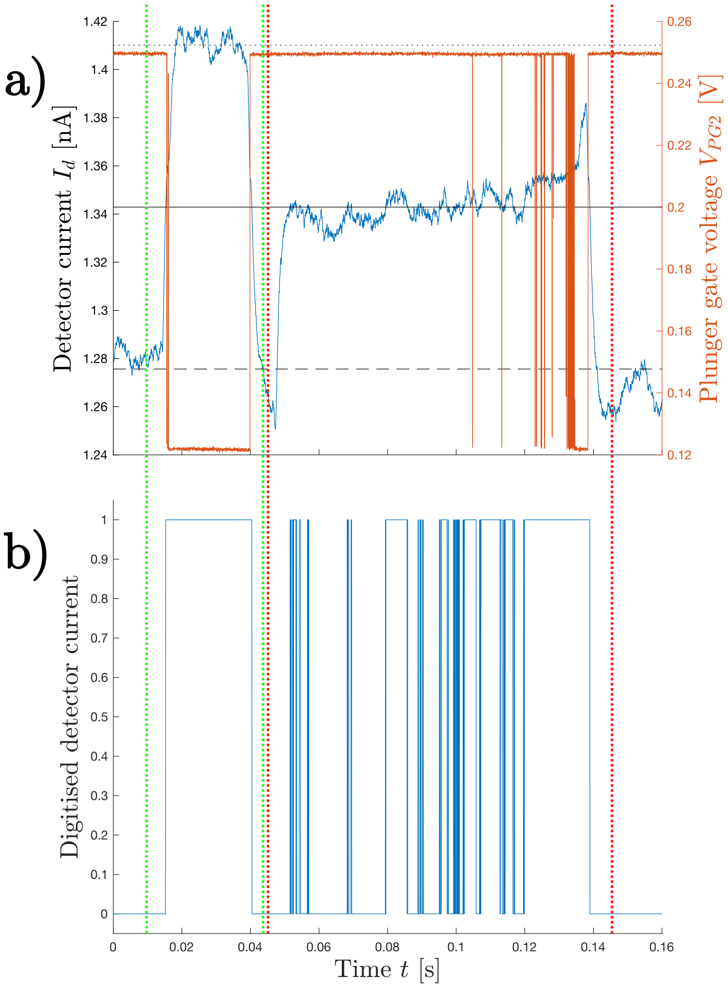

Figure 8: **a)** The detector current overlayed with the feedback voltage. The horizontal lines above and below is where the detector current should lay when the electron is in or out of the dot respectively, the middle line is the threshold for when the electron is considered in or out of the dot. **b)** The digitised time trace used to determine the waiting times.

To the left in Figure 8 the green vertical bars denote a good tunneling in event followed by a good tunneling out event. In the right part Figure 8 an electron tunnels in but the plunger voltage does not switch (denoted by the red vertical bars). This again leads to Γ_{out} not being measured at the correct E. However, in this case the detector current ends up in the middle of the threshold used for considering an electron in or out of the dot. So instead the noise of the measurement is measured, leading to the value determined for Γ_{out} being too high. Both these effects in combination gives the distribution of Γ_{out} observed in Figure 6d.

Looking at both figures 7 and 8 it can be noted that the signal is still significantly greater than the noise and so it should still be possible to get good data at these increased temperatures. The main problem seems to lay in that I_d has a dependence on V_{PG2}. The dependence probably occurs because of the close proximity between plunger gate 2 and

QD 3. By applying a compensating feedback to V_{PG3} simultaneously as V_{PG2} is switched this effect should disappear. While this does not seem to be a problem over the smaller range of ΔV_{PG2} used for data sets **a** and b, it could be useful if measurements over a larger range of ΔV_{PG2} is required. This could be useful for determining the degeneracies at higher T_e, where the FD distribution is broadened, requiring a larger range of ΔV_{PG2} to determine the rates where the FD distribution plateaus.

Below the determined ratios $d_{\mathrm{in}}/d_{\mathrm{out}}$ for the data sets are presented

Table 2: The ratio $d_{\mathrm{in}}/d_{\mathrm{out}}$ of the measurements.

Measurement	$d_{\mathrm{in}}/d_{\mathrm{out}}$
a	1.82
b	0.58
c	1.55
d	1.21

For a QD with an even number of electrons N when the QD is in the out configuration the degeneracies of the system is $d_{\mathrm{in}} = 2$, $d_{\mathrm{out}} = 1$, while for an odd number N it would be the opposite. Using this $d_{\mathrm{in}}/d_{\mathrm{out}}$ should be 2 in the first case while it should be 0.5 in the second. Using Table 2 it can then be deduced that N is even for measurements **a** and **c** while for measurement **b** it is odd. As discussed earlier data set **d** has a lot of problems with the measurement for Γ_{out}. This means the degeneracies can not be reliably determined for d. However, as **d** was recorded with the same amount of filling N it should have the same degeneracies.

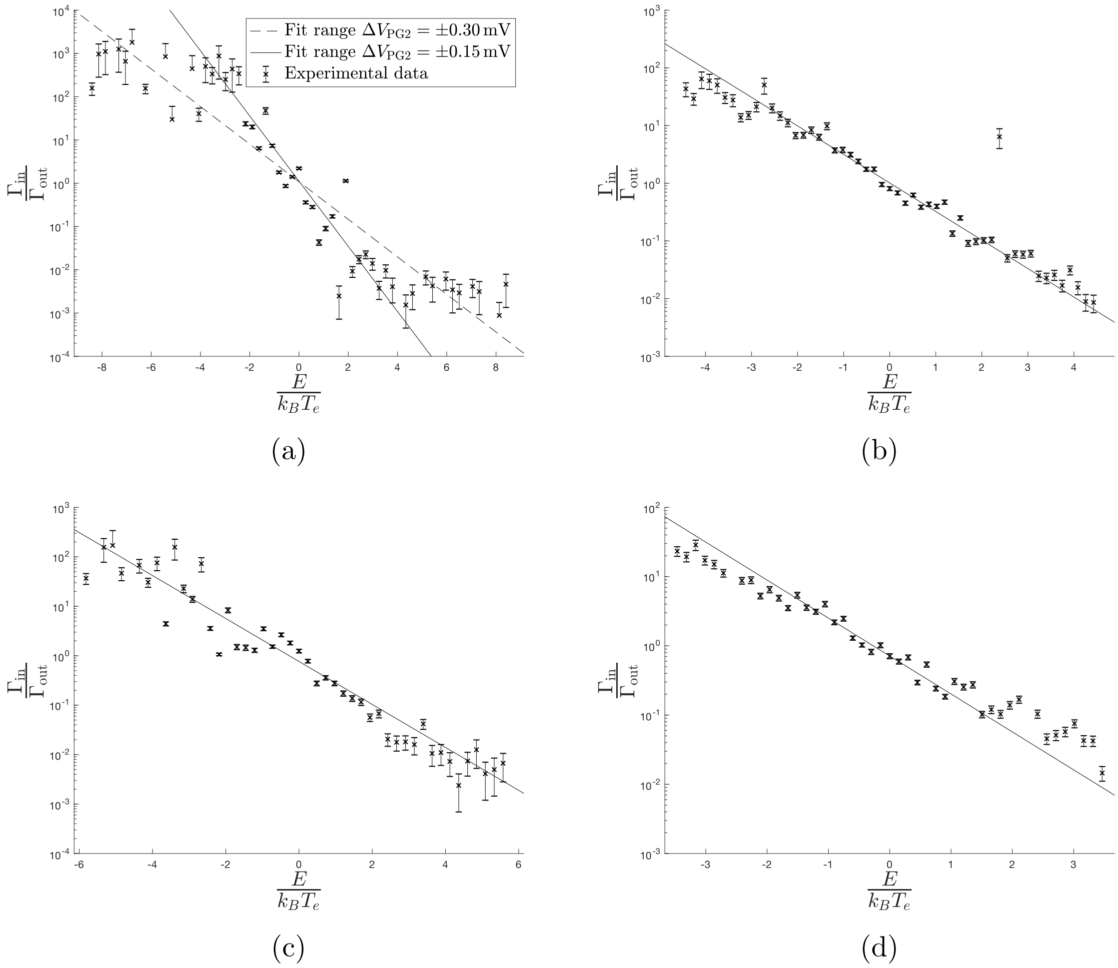

Figure 9: The ratio between the rates against the displacement of the QD energy level plotted in a semilogarithmic plot.

Plotting the ratio $\frac{\Gamma_{\mathrm{in}}}{\Gamma_{\mathrm{out}}}$ in a semilogarithmic plot (see Figure 9) **b** and **c** seems to have an exponential dependence on E, in agreement with equation (7). For measurement **a** the data appears to saturate as E gets further from 0. This happens because, as displacement goes further from $E = 0$, either Γ_{in} or Γ_{out} tends to zero, eventually getting so small that it becomes limited to by the time of a time trace. As E becomes big Γ_{in} becomes limited meaning that the ratio will become too large, while for small E the rate out Γ_{out} becomes limited so that the ratio becomes to small. This leads to the fitted slope not being steep enough (see Figure 9a). To make sure this does not affect the later results for the temperature the data sets are truncated so that only data corresponding to a plunger gate voltage of at most $\Delta V_{\mathrm{PG2}} = \pm 0.15\,\mathrm{mV}$ is used for the fit.

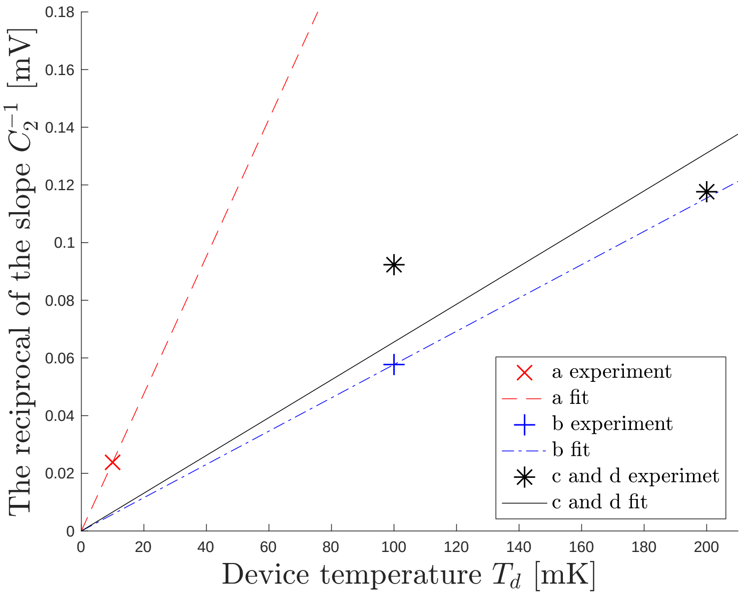

Figure 10: For each set of rates measured at a temperature cryostat temperature T_d lines are fitted to the logarithm of the ratio $\Gamma_{\text{in}}/\Gamma_{\text{out}}$. In the figure above the reciprocal of the slopes have been plotted as a function of the cryostat temperature T_d where that set of rates where recorded.

Looking at the data for **c** and **d** in Figure 10 it appears that C_2^{-1} has a dependence on the cryostat temperature T_d. For higher temperatures the electron and device temperature follow each other $T_e = T_d$.[14] This then implies that C_2^{-1} depends on T_e as in agreement with (15). Using equation (16) and setting $T_e = T_d$ the lever arm of the device was determined to be that listed in table 3

Table 3: The experimentally determined values for α.

Measurement	α
b	$-0.132(6)\,\text{eV/V}$
c and **d**	$-0.149\,\text{eV/V}$

A previous study using similar devices found a lever arm of $\alpha \approx -0.1\,\text{eV/V}$.[9] At $T_d = 100\,\text{mK}$ there are two measurements recorded corresponding to different filling of the QD N, with N referring to the amount of electrons in the QD when it is in the out state. Comparing the two different measurements at $T_d = 100\,\text{mK}$ it can be seen that C_2^{-1} differs by approximately $0.04\,\text{mV}$. This happens because the lever arm changes depending on the filling of the quantum dot N.[9]. To confirm that this is the case further studies could be conducted where the data sets are recorded both as a function of temperature and filling of the QD. The fit to C_2^{-1} for data set **a** gives a very steep slope, yielding $\alpha = -0.036\,\text{eV/V}$. This very low, however for lower temperatures it has been found that the electron temperature saturates, and $T_e = T_d$ can thus not be used. This happens because at low temperatures the electron-phonon coupling becomes very weak.[15–18].

6 Conclusions and outlook

A model for the tunneling rates as a function of E and T_e was successfully applied to find the degeneracies of a quantum dot system coupled to a reservoir, both for different amounts of filling of the QD N and for different temperatures of the surrounding environment. As expected from an electron gas the electrons are observed to follow a FD distribution. A decrease in accuracy of the rate measurements was observed when the range of ΔV_{PG2} was increased. Suggestions are made that by applying a compensating feedback to the plunger gate of the detector the accuracy may be maintained over a larger range of ΔV_{PG2}. By combining tunneling rate measurements with device temperature measurements a method for determining the lever arm of a device using the detailed balance was demonstrated, and it was found that the ratio of the tunneling rates can be described by the Boltzmann factor. The analysed data was shown to agree with this theory in a neighbourhood of $E = 0$, further away from $E = 0$ a saturation was observed due to limitations in the data. Finally the dependence of the detailed balance on both T_e and N was explored. Indications of a difference in lever arm depending on N is observed, this is in agreement with what has been found in previous studies. Future work could focus on determining the accuracy of the methods used in the thesis, as well as confirming some of the observations. To examine the accuracy of the extracted lever arm, more data points of C_2^{-1} as a function of temperature would be needed, this could then be compared with alternative methods of determining the lever arm. Altering the filling of the QD the dependence on α could be explored further. Once the lever arm has been determined the device would then be able to function as an electron thermometer, the accuracy of which could be investigated. In conclusion then the models used in the thesis was shown to be viable for determining both the degeneracies and the lever arm of a quantum dot.

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