Illiquidity and Its Threats

A Study of the U.S.Corporate Bond Market

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Abstract

In recent times of market turmoil, liquidity risk has become a big talking point. As certain Swedish fixed income funds, which were advertised as safe investment options, closed for a few trading days in March of 2020 due to the extremely high stress on the market, questions about how illiquidity affects risk and return were asked. There has been plenty of previous research on the subject and it all shows that a liquidity risk premium is present. However, most of this research has been done on stocks which is a relatively liquid type of asset and indicates that liquidity risk might be much more problematic for less liquid assets, such as the ones traded on the OTC-market. OTC data from the U.S. market is publicly available on the Trade Reporting and Compliance Engine and a data set containing around three million transactions over two years is used in this study. To find a suitable liquidity measure for the sample data at hand, a multiple linear regression is made for three different and widely used liquidity measures as dependent variables and known liquidity factors as explanatory variables. The measure is then incorporated into return models as well as analysed in times of stressed market. When incorporating the best liquidity measure into return models, it improves the results significantly which suggests that the bonds are traded at a premia. Finally, there is a clear increase in illiquidity as the volatility on the market increases which gives further understanding as to why the Swedish funds had to close.

Keywords: Liquidity, price dispersion, Hui-Heubel liquidity ratio, ILLIQ, TRACE, U.S. corporate bonds, Over-the-Counter market, regression analysis, return modelling, stressed market conditions.

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Chapter 1

Introduction

In March of 2020, by the beginning of the Covid-19 pandemic, the financial market's condition deteriorated rapidly and the stock market experienced severe daily drops that was compared to the Black Monday in 1987. Some Swedish fixed income funds were forced to make the controversial decision to close for a few trading days as they considered it impossible to accurately estimate the market value of its underlying assets and their risk exposure. Investors started questioning whether fund managers are aware of the actual risk of investing in illiquid assets and if the risk is reflected in the expected return. Once again, market liquidity raised concerns and became highly interesting for financial research. As for the Covid-19 pandemic, the Global Financial Crisis of 2007-2009 was associated with severe liquidity shocks in global stock and bond markets, forcing the Basel Committee on Banking Supervision to develop a framework containing a liquidity coverage ratio and liquidity risk monitoring tool. Basel III suggests that banks should hold sufficient high-quality liquid assets to cover its cash flows and it clearly indicates the importance of market liquidity to accurately estimate the risk in order to maintain stability on the financial market.

Keynes [1914] wrote one of the first books establishing the definition of liquidity of a financial asset. Keynes stated that a security is more liquid than another asset if "[...] it is more certainly realisable at short notice without loss.". Liquid assets are easily liquidated even at larger positions without affecting its fair market value, while Amihud et al. [2013] argue that illiquid assets are associated with indirect transaction costs, such as search and delay costs, that significantly increases the risk and should therefore be reflected in the expected return. Jankowitsch et al. [2011] propose two types of variables effecting liquidity, trading activity and bond characteristics. Trading activities include trade volume, quantity and bid-ask spread, while the bond characteristics include credit rating, maturity and age. Berkowitz [2000] and Bao et al. [2011] examine liquidity and argue that the trading volume is of great importance as it describes the interest of an asset and the possible transaction volume that can be liquidated in a single trade. Datar et al. [1998] use the turnover ratio which is closely related to the trading volume and Amihud and Mendelson [1986] suggest that the bid-ask spread is a good proxy for liquidity. Chakravarty and Sarkar [1999] confirm the existence of relationship between most of the mentioned factors, suggesting that they are suitable variables for describing the liquidity.

Securities traded on large stock exchanges as New York Stock Exchange or NASDAQ are considered liquid and from a liquidity perspective it is therefore more interesting to examine assets traded on thinner markets, such as the OTC market. In addition, as Lee [2011] argues that the U.S. market is an important driving force of global liquidity risk, data from the U.S. OTC market is a suitable choice for liquidity research. Trade Reporting and Compliance Engine has made this possible as it provides a large set of real-time transaction data for the fixed income market. The data set used in this paper contains 2,929,690 transactions for 167 corporate bonds from July of 2017 to July of 2019. To estimate the liquidity of each asset on a daily basis, three widely accepted liquidity measure are first evaluated by multiple linear regression models and then applied to the corporate bonds. As the measures are proved working on the examined market based on the dependence of factors suggested in previous research, the market liquidity is incorporated into return models to see whether liquidity risk is priced and if the liquidity-based models improves predictions of asset returns. Our main interest is to develop a wider understanding of market liquidity and to be able to accurately estimate the fair value of assets. Finally, the market liquidity is compared at normal and stressed market conditions by using VIX values as an estimate for increased market volatility to determine how the liquidity changes as the uncertainties on the financial market increases. The results will hopefully be able to explain the difficulties of pricing illiquid asset during true global crises and make investors realise their actual risk exposure.

We conclude that the liquidity measure proposed by Jankowitsch et al. [2011] is the most suitable choice of measure when estimating the liquidity of corporate bonds traded on the OTC market. When applying the estimator on asset return models we can prove that liquidity is of fundamental importance as it is significant for a high percentage of the bonds when incorporated into the CAPM as well as the Fama and French factor models. More importantly, as it is significant in almost as many cases as the market premium, it is clearly a central factor for illiquid assets. Furthermore, the liquidity of corporate bonds changes significantly as the market volatility increases, even for low levels of market stress and during short periods. The results undoubtedly prove that market liquidity should be incorporated when estimating the total risk exposure of portfolios containing less liquid assets to maintain stability of the global financial market and reduce the risk of future financial crises.

The structure of the paper is as follows. Chapter 2 provides a comprehensive background to market liquidity, commonly suggested liquidity factors and liquidity adjusted risk measures. Chapter 3 describes the data examined in the paper, different methods that are used to replace and estimate missing data and the overall methodology applied in the analysis. Chapter 4 provides the results as well as a discussion of the main results found in the study. Chapter 5 concludes and provides suggestions for further research.

Chapter 2

Theory

2.1 Market Liquidity

Market liquidity is commonly discussed in finance and is widely acknowledged as an important feature of a market, especially since many of the latest financial crises have been liquidity related. Yet, it is rarely incorporated into any risk measurements, valuation models or trading strategies. Market liquidity in financial markets refers to how easy an asset can be liquidated without having an impact on its fair price and is modelled by being dependent on the demand and supply of immediacy [Grossman and Miller, 1988]. It is something that affects all participants on the market and extensive work has been made to examine its true impact on both return and risk. One of the challenges with trying to quantitatively explain the liquidity is that it is multi-dimensional. Trading cost, trading quantity and resiliency among others all have an impact on liquidity. It has proven difficult to include all these dimensions when trying to measure the liquidity so the majority of existing studies focus on only one dimension. For example Datar et al. [1998] used trading quantity and Amihud and Mendelson [1986] used trading cost. Furthermore the existing studies concentrate on a specific financial asset which is equity. Most equities are very liquid and even though their liquidity still have an impact on return and risk it doesn't affect them as much as for truly illiquid assets such as the assets traded on the Over-The-Counter (OTC) market. There are also assets on growth markets and MTF-markets that are relatively illiquid. It is when holding truly illiquid assets that liquidity risk becomes a real threat.

There arises a number of direct and indirect costs when trading with less liquid assets that need to be taken into account when evaluating the risk of a portfolio or an individual asset. In particular, as the need of cash or a margin call takes place, a forced liquidation of a position causes transaction losses which, if poorly or not accounted for, may result in enormous financial losses. In addition, as the cost of trading an asset increases due to illiquidity, the value of the financial asset decreases and one would require a higher expected return to compensate for the increased costs. The direct trading cost, e.g. brokerage fees and transaction taxes, do not necessarily change substantially when trading less liquid assets. The indirect and less intuitive transaction costs on the other hand, do change the costs and value of the security. The indirect costs include search and delay costs that occurs because of the lack of buyers or because of the size of the position. In other word, if a holder of a position wishes to liquidate the position but cannot find a buyer the search for a or multiple seller(s) is associated with an indirect transaction cost that occurs due to the additional risk of holding the position and/or the risk of not being able to invest the money into another security [Amihud et al., 2013].

Chordia et al. [2001] argues that liquidity and trading activity are of fundamental importance and a greater understanding should increase investors confidence in financial markets and consequently enhance the efficiency of corporate resource allocation. Also, a wider understanding improves investors trading strategies and the precision in estimating the exposed total market risk.

2.2 Over-the-Counter Market

An OTC-market is defined as a market where two parties trade assets without a centralized platform. The dealers on the market set a price for a security for which they will sell or buy the asset. The assets which are most commonly traded on the OTC market are bonds, derivatives and currencies. One of the big risks with OTC-trading is the liquidity risk as there might not always be a buyer for the security a dealer is holding [Institute, 2021]. This risk is often over-looked when determining an asset's risk. There is also a probability of seeing transaction prices that are different from the market valuation of the asset. This is due to the prices being negotiated between the dealers on the OTC market. The differences between transaction price and market valuation as well as price differences in different transactions for the same asset are very interesting from a liquidity perspective. Another interesting perspective from a liquidity stand point is comparing the movements of the liquidity premium on equities and the transaction prices on the OTC market.

2.3 Liquidity Risk Premium

The liquidity risk premium is defined as the extra return an agent demands to take on the liquidity risk. As mentioned above there are several studies that have examined liquidity based on different liquidity dimensions. According to Amihud and Mendelson [1986] an asset's return is increasing in its bid-ask spread. The bid-ask spread is defined as the spread between the price the buyer is willing to pay and the price the seller is willing to sell for. By intuition this leads to an increase in the spread with illiquidity. Datar et al. [1998] wanted to test Amihud and Mendelson theory with a different approach. They use the turnover rate as a proxy for liquidity to prove the presence of a liquidity premium. They define the turnover rate as number of shares traded as a fraction of the number of shares outstanding. Their results supported Amihud and Mendelson's conclusion that a liquidity premium does in fact exist. Several other studies have approached this issue in different ways with different proxies for liquidity and they have all supported Amihud and Mendelson's results. This proves that liquidity is a complex factor with many dimensions and that it has a definitive impact on an asset's return and risk. Jong and Driessen [2006] explores the importance of liquidity risk when estimating the expected return for corporate bonds. They find evidence that the liquidity risk, examined by performing a cross-sectional regression of the expected return on market and liquidity, has a significant impact on the corporate bond return. Their results suggests that a liquidity risk premia should be incorporated when modelling the expected return for corporate bonds, especially since the regression analysis shows that the liquidity risk premia contributes similar size as the market risk premia to the expected return. As one would expect, they also find that the premia is higher for high yield bonds ($\sim$1.5%) compared to investment grade bonds ($\sim$0.6%) in terms of annual expected returns.

2.4 Stressed Market Conditions

Financial markets are normally relatively stable and therefore predictable. However, once in a while something unpredictable happens. It can be a newly elected president in a country with big influence, a couple of solo investors creating a hype over a specific stock on a forum like Reddit and Twitter or as in 2020 when the Covid-19 virus spread over the world. In the spring of 2020 financial markets all over the world took a big hit, the uncertainties of the virus led to a crash on the stock markets. Investors wanted to sell their assets but there was a lack of buyers and the uncertainty led to some of the worst numbers in modern history. Selling was intensified during the beginning of March and in mid-March one could observe several severe daily drops in the global stock market. On 12th of March, the Stockholm Stock Exchange dropped 11.1% and was compared to the Black Monday in 1987 which is globally known as one of the largest stock market crashes in history [Rognerud, 2020]. Overall, the OMXS30 index dropped by 32% from an all time high during one month. Similar to OMXS30, the S&P 500 had lost 34% of its market value due to the pandemic. In addition to the intensified selling pressure, minor markets, such as the OTC market and growth markets, suffered liquidity problems and some fixed income funds in Sweden even closed. The reason was that the fund managers were unable to make an accurate valuation of the underlying assets, mainly corporate bonds, due to the uncertainty in the market [Finansinspektionen, 2020]. Large stock exchanges are considered liquid even during uncertain times, however, in the growth market or the corporate bond market, a crash might mean that one is unable to find a buyer and you are forced to hold an asset with a decreasing value.

In 2007-2009, during the Global Financial Crisis, several global stocks and bond markets suffered severe liquidity shocks similar to the situation during early Covid-19. The liquidity shocks made the trading cost nearly multiply by three during the fourth quarter of 2008, at the time of the collapse of the Lehman Brothers. Additionally, there were single trading days with even higher transaction costs and thereby a substantial increased risk. The transaction costs for small cap securities were higher than the costs for large cap securities on average during the crisis and one would therefore expect OTC traded assets to have even higher costs [Amihud et al., 2013]. As in the case with the Swedish fixed income funds, the valuation of the underlying securities becomes very complex and hard as the market stress increases and the risk of holding a position on a far less liquid market increases remarkably. Consequently, large investors will expect a higher rate of return when investing in less liquid corporate bonds to compensate for the risk. Friewald et al. [2013] examines the liquidity in the U.S. corporate bond market and further confirms that in crises the economic impact of liquidity is significantly larger, in particular for high yield corporate bonds. They also find that ∼14% of the explained corporate yield spread changes can be explained by liquidity effects. Lang and Maffett [2011] suggests that a firms transparency lower correlations between its liquidity and both market liquidity and market returns. Nevertheless, even though the effects are less extensive for transparent firms, they argue that extreme illiquidity events and liquidity variance increase substantially during crises.

2.5 Liquidity Factors

Jankowitsch et al. [2011] argues that there are two types of variables that have an effect on liquidity. There are trading activity variables as well as bond characteristics variables. The trading activity variables include factors such as trade volume, trade quantity and the bid-ask spread. The bond characteristics variables include credit rating, age and maturity. Similarly, Aitken and Comerton-Forde [2003] suggest that liquidity measures normally falls into two categories: trade-based measures or order-based measures. They argue that the trade-based measures are the most tempting and widely accepted measures due to their simplicity and ease to use. The trade-based measures includes trading volume, trading frequency, trading value, etc., and are often referred to in literature. However, as these factors are *ex post* rather than *ex ante*, i.e. they are post trade factors describing transactions in the past rather than what would happened if one would sell the same position in the future, they argue that trade-based factors are not necessarily a good indication of future trades. Order-based measures on the other hand, are normally a more appropriate choice as they capture the ability as well as the cost associated with immediate trading. The bid-ask spread and order depth are two examples of order-based factors.

2.5.1 Trade Volume

Trade volume directly indicates the market interest of the asset. It therefore has a big impact on liquidity. Naturally, an asset with lower trade volume is more exposed to liquidity risk than an asset with high trade volume. Trading volume can be divided into two groups dependent on which factor(s) one would like to examine: the total daily trading volume and the transaction volume. The total daily trading volume is interesting when investigating the interest of a particular asset over time, while the transaction volume is a suitable choice when examine intra-day movements. From a historical perspective, they are both of substantial interest. However, it is hard to use trade volume as an indication of future liquidity and therefore it is a suitable choice for research rather than incorporating into future investment strategies. One of the most commonly applied and widely accepted liquidity measures, ILLIQ, published in 2002 by Yakov Amihud, uses the daily ratio of absolute return to dollar volume. The measure is closely related to the Amivest Liquidity Ratio which together with ILLIQ suggests that the liquidity risk depends on the trading volume [Amihud, 2002]. Berkowitz [2000] modelled liquidity price impact on past trades. His model is a time-series of trades in a linear regression where the coefficient signifies the return as a function of changes in volume. Bao et al. [2011] study illiquidity on the OTC market by using transaction data provided by TRACE and argues that the most interesting variable is the average trade size. They conclude that smaller trade volumes implies higher illiquidity and that the number of trades, contrary to the intuition, does not necessarily imply high liquidity. That is, on a less liquid market, an investor holding a large position might be forced to find several buyers. Accordingly this will increase the number of transactions as well as decrease the transaction volume compared to a more liquid market with investors willing to buy larger positions.

2.5.2 Trade Quantity

Trade quantity is related to trade volume. However, instead of looking at the total volume of assets traded, the number of trades is taken into account. This is interesting since on the OTC-market some trades involve a significant volume of assets. It also eliminates the problem of differences in amount outstanding of different assets. Fleming [2001] finds only a weak correlation between trading frequency and a number of liquidity measures tested in his paper. Fleming therefore argues that the trading frequency is a poor proxy for market liquidity. As previously mentioned, large investors might be forced to sell their assets in different transactions which will increase trading frequency without necessarily increasing the actual liquidity. Contrary, a smaller trading volume at higher frequency could rather be indicating a more illiquid asset.

2.5.3 Turnover Ratio

Turnover ratio is defined as the dollar volume traded divided by the amount outstanding. Datar et al. [1998] were among the first to use turnover ratio as a liquidity proxy. They performed the study as a successor to Amihud and Mendelson [1986] initial study on liquidity using the bid-ask spread as liquidity factor. They argue that there are two advantages of using the turnover ratio as proxy. The first one being that theoretically it seems more than reasonable as liquidity is correlated to trading frequency. The second advantage being that the data required to calculate the ratio is very accessible. Furthermore, it can be noted that liquidity is an increasing function of the turnover ratio.

2.5.4 Bid-ask Spread

The bid-ask spread is a widely used proxy for market liquidity of an asset and essentially describes the difference between the minimum price a holder is willing to sell an asset for (ask) and the maximum price a buyer is willing to buy the asset for (bid). On a liquid market, such as the New York Stock Exchange (NYSE), the spread is relatively small on average, i.e. a position is normally traded close to the market valuation of the asset independent of the transaction size. For a less liquid asset, e.g. those examined in this paper, one would expect a larger spread due to the lack of buyers and increased risk. Amihud and Mendelson [1986] were first in examining the effect liquidity had on asset returns and quantifying the liquidity premium. They did this by examining how the bid-ask spread correlates with expected returns. They found that an asset's return is an increasing function of the spread. Aitken and Comerton-Forde [2003] studies the liquidity of the economic crisis on the Jakarta Stock Exchange (JSX) during 1997 and 1998 and finds that the spread is a good indication for small investors. However, for larger investors, it must be combined with the order book data as the spread can only be applied for a specific trading volume range. Chakravarty and Sarkar [1999] investigates the liquidity of the U.S. fixed income markets and finds that the (realized) bid-ask spread is decreasing in trading volume and increasing in remaining time-to-maturity. For the corporate bond market in particular, the realized spread is also increasing in credit risk and age of the bond. The most interesting result from the paper suggests that the liquidity is an important determinant of the realized bid-ask spread for all of the examined fixed income markets, i.e. the corporate, municipal and government bond markets.

2.5.5 Credit Rating

Credit rating can be directly translated to the quality of a bond. It is therefore also a measure of the bond's risk. A high rated bond is a safer option but generally provides a lower yield than a lower rated bond. Bonds are divided into two groups, investment grade and high yield. Bonds with a rating of BBB- or better on Standard and Poor's scale are rated as investment grade. The bonds with lower rating are put in the high yield category and are also referred to as "junk bonds". There is a strong reason to believe that a bond's liquidity correlates with it's rating. The intuition is that a better credit rating means better liquidity and lower liquidity risk. Ericsson and Renault [2006] suggest that the liquidity spread is likely positively correlated with credit risk (likelihood of default) on the corporate bond market.

From a practical market perspective one can observe a correlation between credit rating and the issuers total amount of bonds outstanding. It is easier for a highly rated corporate to issue more and larger bonds. This gives entities with higher credit rating a larger presence in the bond market which enables have a more diversified investor basis. Which in turn makes it easier to match the interests of buyers and sellers and increases liquidity. Bonds from the same entities issued in a different market, where they are previously unknown, will usually have less liquidity (as less investors trade their bonds) despite having the same credit rating. For example, bonds issued by the Kingdom of Sweden will have lower liquidity than bonds issued by the United States Treasury in the USD-market, despite the fact that the Kingdom of Sweden has a higher credit rating [Nordenskjöld, 2021].

2.5.6 Maturity and Age

In Jankowitsch et al. [2011] study it was proven that for corporate bonds on the OTC-market a longer maturity relates to lower liquidity. From their study they can conclude that bonds with a maturity of 0-10 years have a minimal effect on liquidity while maturities over 10 years have a significant effect. Similarly they find that older bonds have lower liquidity compared to newly issued bonds. They argue that this is expected using the effect of "on-the-run"- and "off-the-run" bonds. The intuition is that newly issued bonds are "on-the-run" and are traded more frequently. Mizrach [2015] analysed transactions in TRACE from 2003 to 2015 and it showed a significant drop in par volume of trading volume 90 days after an active issue entered the secondary market. This confirm the suggested "on-the-run" phenomenon. The reduction in trading volume becomes extremely large for the last four years studied by Mizrach, more precisely, during these years the decrease after 90 days is approximately 35%. Bao et al. [2011] further confirms the findings proposed by Jankowitsch, Nashikkar and Subrahmanyam, i.e. older bonds tends to have higher illiquidity on average. Additionally, they confirm that the liquidity decreases as the maturity increases.

2.6 Liquidity Adjusted Risk Measure

In order to keep stability on the financial market, it is most central to estimate an accurate risk of financial assets. Particularly, this is important for investment companies which may be exposed for enormous risk and suffer severe losses in case of extreme events, such as the financial crisis in 2008 and pandemics. Value-at-risk (VaR) is a widely accepted quantitative tool used by institutional as well as individual investors to evaluate portfolio risk and risk of single financial assets. It can be measured by different methods, for instance by using a historical method or by Monte Carlo simulation, and the measurement is designed to assess how much one can loose at most given a pre-chosen level of confidence and time horizon. VaR is, from a statistical perspective, the distribution of the stochastic loss variable L. Mathematically the definition of VaR can be written by one of the following equations depending on if the loss distribution is discrete or continuous:

$$VaR^h_\alpha(L) = \min\{l : Pr(L > l) \leq 1 - \alpha\} \tag{2.1}$$

$$Pr(L > VaR^h_\alpha(L)) = 1 - \alpha \tag{2.2}$$

A breakthrough in the adoption of VaR as a risk measurement tool came when J.P. Morgan made its RiskMetric system publicly available through Internet in 1995 with the aim of improving transparency of market risk and how to manage the risk. However, VaR has been criticized by investors due to its sensitivity to incorrect assumptions and lack of information about potential losses in case of extreme (tail) events. In addition, the measurement has been criticized for only taking the market risk into account. Therefore, researchers have continued to further develop and modified the measurement in endless numbers of ways. Since the first paper on market liquidity by Amihud and Mendelson [1986], several papers have suggested different parameters representing liquidity on the financial market and proposed liquidity adjusted VaR measures. Amihud and Mendelson argued that assets should be compensated for being less liquid when determining asset returns and tested the effect of the bid-ask spread on asset pricing. They proved by empirical testing that the expected return is an increasing and concave function of the spread, i.e. a high-spread asset yields a higher net return. Bid-ask spread is a commonly used parameter when determining liquidity risk in recent liquidity risk models together with volume/transaction data and limit order book data.

2.6.1 Bid-ask Spread

An asset's fair value is normally defined as the middle of the bid-ask spread, referred to as the mid-price. The spread is a positive value that describes the difference between the lowest price the holder of the asset is willing to sell for and the highest price a buyer is willing to pay on the market. There are several factors that influence the size of the spread, such as the size of the trade, liquidity of the asset and the need of immediate trading, e.g. in illiquid markets the seller might have to accept a lower price if he/she need to execute the deal immediately. This spread, i.e. the cost of immediate trading, symbolizes the cost for investors to constantly be guaranteed a counterpart for trades and is therefore a useful measure for estimating liquidity of an asset. Bangia et al. [2001] developed a liquidity risk methodology based on the bid-ask spread which can be integrated into the standard VaR model. According to the paper, ignoring liquidity can underestimate market risk by as much as 25-30% in emerging markets. Further, the authors argue that models tends to focus on the mid-price and therefore misses the realized traded price which, in many markets, rather is the mid-price minus the bid-ask spread. By incorporating the uncertainty in market value into the model, i.e. combining the market risk and liquidity risk, they present the following liquidity adjusted VaR model:

$$L_{adj} - VaR = 1 - \exp(z\sigma_r^2) + (\mu_S + \hat{z}_S\sigma_S) \tag{2.3}$$

where σ_r^2 is the variance of the continuous mid-price return, μ_S the mean of the spread and σ_S the standard deviation of the bid-ask spread. The terms z and $\hat{z}_S$ is the percentile of the normal distribution for the given confidence and the empirical percentile of the spread distribution respectively. The strength of the model is that it is easy implemented and only requires spread data which is available in most markets. However, when investigating the liquidity on minor markets or the OTC market that are considered far less liquid than the stock exchange market, this is not necessarily the case and one is forced to use proxies from transaction data. Hence, the results are dependent on the choice of proxy which makes the model less accurate for these markets. In addition, the model assumes that the position size is without significant influence and the assumption tends to underestimate the actual liquidity risk for large order sizes. Ernst et al. [2009a] improved the model proposed by Bangia et al. [2001] by applying a Cornish-Fisher expansion to determine percentiles instead of using historical empirical distributions. As financial assets rarely follow a normal distribution, especially during periods of market stress, the Cornish-Fisher approximation adjusts percentiles from the normal distribution to account skewness and kurtosis, i.e. the Cornish-Fisher expansion transform the standard Gaussian random variable into a non-Gaussian random variable. The total risk, when accounting for non-normality, should according the paper be calculated as:

$$L_{adj} - VaR = 1 - \exp(\mu_r + \hat{z}_\alpha(r) \cdot \sigma_r) \cdot \left(1 - \frac{1}{2}\left(\mu_S + \hat{z}_\alpha(S) \cdot \sigma_S\right)\right) \tag{2.4}$$

where $\hat{z}_\alpha(r)$ and $\hat{z}_\alpha(S)$ is the Cornish-Fisher return distribution percentile and the spread distribution percentile respectively. Even though the model provides more precise estimations compared to the one proposed by Bangia et al., the model still assumes questionable features, e.g. that the size of the traded position does not significantly effect the price or spread.

2.6.2 Transaction Volume

The trading volume, or size of position, is another common liquidity measure and can be interpreted as the total volume traded of an individual asset or the size of one particular position. Liquid assets traded on the larger exchange markets are often traded many times every hour and considered liquid. Even if they are classified as mid-cap or even small-cap, they have a large total volume traded every day and if you are interested in selling an asset you will find a buyer relatively fast. Hence, even if you require an immediate trade of a large position, you will not be forced to sell far from the current fair valuation of the asset, i.e. the bid-ask spread is small independently of the size of the position. Contrary to this, on thin and less liquid markets, a large position will likely sell at a price closer to bid price compared to mid-price due to the delay costs of searching a counter-party and bearing the risk of holding the position while searching. Cosandey [2001] suggests a liquidity adjusted VaR model based upon trading volume data. The problem with the standard VaR model, Cosandey argues, is that one is using the effective historical price changes for when the portfolio was not sold. In order to accurately adjust for the liquidity effects, one should incorporate the daily price changes that would have occurred, given the market depth of the day, if one had sold the portfolio. By doing that, you will have the normal expected price movements as well as the liquidity-driven price movement. Cosandey assesses the price impact theoretically by investigating the supply and demand curves and graphically illustrates how the price changes given the size of the position you intend to sell. In practice however, one needs to approximate daily conditions and thereby the price impact, which is a weakness of the proposed model. Berkowitz [2000] argued, similarly to Cosandey, that the transaction volume has an impact on asset pricing and suggests a method to incorporate the liquidity risk into the traditional VaR model. By linear regression of historical transactions, Berkowitz measures the liquidity price impact while controlling for the influence from other risk factors. The transaction price, p_{t+1}, is measured according to the following equation:

$$p_{t+1} = p_{mid,t} + \alpha + \theta q_t^* + x_{t+1} + \varepsilon_t \tag{2.5}$$

where q_t^* is the amount of asset sold and x_{t+1} is the market-wide changes in asset prices, i.e. rational changes due to information on the market. As transaction volume represents the liquidity risk of the proposed method, the coefficient θ can be seen as the absolute liquidity cost per share traded. A problem is that the regression might result in a negative θ which is counter-intuitive as one would expect the price to decrease in order size. Further, the model has been criticized for assuming linear and non time-varying price impact. Still, the model has the advantage of taking the order size into account and since intraday data has become more and more available, the model becomes less approximate. One important feature mentioned in the article is that incorporating liquidity risk into the traditional VaR model does not necessarily change the total risk exposure in all cases, the total risk can still be more or less equal to the market risk. However, in some days, or for some assets during a period, the total risk can drastically differ from the market risk calculated by using the traditional VaR model. In the financial market an investor or fund decides what risk they are willing to take for a given expected rate of return and therefore, one needs to know the actual risk exposure to make an accurate decision, i.e. one is interested in the total risk in relation to the return rather than just knowing the risk exposure. This becomes even more important for financial institutions, banks and pension funds since an underestimation of risk exposure may have catastrophic outcomes in case of extreme events. In these cases, there are very large transactions and the money is normally very important for every individual investor, especially pension funds that investors trust with their savings.

2.6.3 Limit Order Book Data

One alternative to measure the liquidity of a market is to use the limit order book which is a record of outstanding limit orders, i.e. an order to buy or sell at a pre-set price or better. The record is maintained by a specialist at the exchange and the specialist is responsible for keeping track of market movements and the priority of the limit order book. By exploring the limit order book data, it is possible to determine the interest of a financial asset as well as specific variables of interest, such as the bid-ask spread and order volume. Further, from a liquidity perspective, it is possible to estimate how easy it would be to trade a particular position and the potential price impact of the trade. Assuming a limit order book with few limit buy/sell orders or where all orders are valid for low-volume trades only, the asset would be considered illiquid and one would expect that trades may incur considerable price impacts. Giot and Grammig [2005] suggests a liquidity adjusted VaR model based on volume-dependent transaction prices. The authors uses data from an automated auction system to construct a real-time order book and compute time series of potential price impacts given a portfolio position. Two different measures, based on frictionless and actual VaR, are defined to quantify liquidity risk. Frictionless returns are, according to the definition in the paper, the log ratio of consecutive mid-quotes and the actual returns are the log ratio of mid-quotes and consecutive bid price (given a selling volume). The first equation measures the difference and the second the ratio of VaR:

$$\Lambda_t = VaR_{mm,t} - VaR_{mb,t} \tag{2.6}$$

$$\lambda_t = \frac{\Lambda_t}{VaR_{mm,t}} \tag{2.7}$$

The method relies on the availability of intraday data, which, in many markets, can be hard to get and is normally very time consuming and tedious to analyse. However, when available, and by adding liquidity theory to the frictionless (standard) VaR methodology, the calculations becomes manageable.

2.6.4 Empirical Comparison

A common problem when applying the suggested models is the availability of real-world data, in particular intraday data. Assets traded on the large stock exchanges are considered liquid and the market liquidity risk is therefore relatively small in the context of total risk. In contrast, for assets traded on thinner markets and the OTC market the liquidity risk is of greater interest since these assets are considered far less liquid compared to those traded on the stock exchange. The lack of data in these markets makes the modelling difficult and the choice of methodology is dependent on the available data. Additionally, the availability of data may differ even in cases where one is examining the same market or asset. An investor can therefore be forced to use several models to estimate the risk or exclude observations that do not contain the required data and thereby risk to get a misleading estimation. Ernst et al. [2009b] conducts a comparative test of many of the aforementioned liquidity models and provides recommendations for which model that is most suitable in practice based on an empirical evaluation. They used daily data of stocks from major German indices from 2002 to 2007 and find that models on limit order data generally outperform models based on bid-ask spread and volume data. The models are ranked by the overall acceptance rate according to the Kupiec-statistic and the four top models, with an acceptance rate between 71% and 74% on average, are all based on limit order book data. Although the test suggest that models based on limit order book data shows superior performance compared to other measures, the authors find that the availability of data is the main driver of the preciseness of risk forecasts. In addition, the test compute acceptance rate based on models rather than the underlying parameters used in the models and it is therefore possible to create a new model that outperforms the tested model independent of the choice of liquidity parameter.

2.7 Fama and French Factor Models

In 1993 Eugene F. Fama and Kenneth R. French defined five risk factors that define average returns on financial assets. Within these five factors there are three stock-market factors and two bond-market factors. The stock-market factors are firm size, book-to-market equity and an overall market factor. The bond-market factors are related to maturity and default risk. Fama and French defined a three-factor model to calculate returns. The model is an expansion of the Capital Asset Pricing Model, more known as CAPM. The reason to further develop CAPM was that Fama and French found that the slope (β) in the regression of a stock's return on a market return only has weak information of the actual average return. Fama and French's model includes size and value risk factors as well as the market risk factor in CAPM. By doing so the Fama and French model adjusts for the over-performance of small-cap stocks compared to large-cap stocks. The three factor model looks as follows

$$R_{i,t} - R_{f,t} = \alpha_{i,t} + \beta_1 \left(R_{M,t} - R_{f,t}\right) + \beta_2 SMB_t + \beta_3 HML_t + \varepsilon_{it} \tag{2.8}$$

where $R_{i,t}$ is the total return of asset i at time t, $R_{f,t}$ is the risk-free rate, $R_{M,t}$ is the return of the market portfolio, SMB is the small minus big factor, HML is the high minus low factor also known as the value premium and the betas are the factor coefficients. The factors are constructed using value-weighted portfolios. There are six different portfolios divided by size and book-to-market. The portfolios are first divided in two categories (small and big) based on market equity, within these two categories the portfolios are divided in to three groups (growth, neutral and value) based on the ratio between book equity and market equity. The small minus big factor is the average return of the three small portfolios minus the average returns of the big portfolios. The high minus low factor is the average return of the two value portfolios minus the average return of the two growth portfolios [Fama and French, 1993]. The three factor model has been evaluated and developed in endless number of papers, e.g. Rahim and Nor [2006] tries to improve the Fama & French model by creating two different liquidity-based three factor models. In their first model they replace the HML factor by a liquidity factor and in the second they replace the SMB factor. Although they fail to statistically proof any differences, they argue that the liquidity three factor model, when replacing the SMB factor, indicates slightly improved predictions of stock return. The original three factor model has also been expanded to the Fama & French five factor model as follows

$$\begin{aligned} R_{i,t} - R_{f,t} = \alpha_{i,t} + \beta_1 \left(R_{M,t} - R_{f,t}\right) + \beta_2 SMB_t + \beta_3 HML_t \\ + \beta_4 RMW_t + \beta_5 CMA_t + \varepsilon_{i,t} \end{aligned} \tag{2.9}$$

The five factors are built on 18 different portfolios based on size and book-to-market, size and operating profitability and size and finally, size and investment. The three first factors stay the same as for the previous model. The two factors added are the Robust Minus Weak factor (RMW) as well as the Conservative Minus Aggressive factor (CMA). The RMW-factor is constructed by taking the average return of the two robust operating portfolios minus the average return of the two weak operating portfolios. Similarly the CMA-factor is built by taking the average return of the two conservative portfolios minus the two aggressive portfolios.

Chapter 3

Method and data

3.1 Data

As trades on the OTC market are made without a centralised platform, the transaction data is often unknown. However, since October 2004, all trades of U.S. corporate bonds have to be reported to the Trade Reporting and Compliance Engine (TRACE) operated by the Financial Industry Regulation Authority (FINRA). This allows all transaction data, including quantity, price and date, to be public and analysed. TRACE provides two data sets, Standard and Enhanced, where Enhanced is the more developed set which gives additional transaction-level information, such as buy/sell indicator and counter party information. The detailed transparency now allows for in-depth analysis and research on the U.S. OTC market.

3.1.1 TRACE Enhanced

To ensure we have sufficient data to perform our analysis we have chosen the top 50 ranked bonds in the category of investment grade (IG) as well as the top 50 ranked bonds in the category of high yield (HY) by number of S1 trades, where S1 indicates a second market trade at the market price. IG is a rating that implies minimal risk for investors while HY bonds pay higher interest rates but are more likely to default. These are most likely the most liquid bonds on the U.S. market which is the most liquid market in the world. Proof of liquidity risk with these bonds would suggest that there is a substantial liquidity risk overall. The lists of the top bonds come from the FINRA annual fact book of corporate bonds during the years 2017-2019. The available TRACE Enhanced data stretches up to 18 months before the data is retrieved. To get a substantial data set we chose data from two years. The time period, based on trade execution date, stretches from 1/7/2017 to 1/7/2019. In this set there are 2,929,690 transactions from 167 corporate bonds. Theoretically, as we chose the top 50 IG and top 50 HY corporate bonds during three years, we expect to get 300 corporate bonds. However, since most of the studied bonds did not mature during our period of interest, many corporate bonds did occur on the annual reports two or even three times. Hence, the number of corporate bonds was reduced from the theoretical value of 300 to 167. Table 3.1 illustrates how the bonds spread over the possible credit ratings as well as over investment grade and high yield ratings. Bonds rated "NR" do not have any rating because they have not been rated or because they matured within the sample period.

	S&P rating	Number of bonds
Investment grade	AAA	1
	AA	4
	A	11
	BBB	63
High yield	BB	50
	B	22
	CCC	6
	CC	3
	D	-
	NR	5

Table 3.1: Distribution of the data set by S&P credit rating.

In order to accurately analyse the TRACE data, it is central to clean the disseminated data according to the steps suggested by Dick-Nielsen [2014]. The TRACE data contains corrections, cancellations and agency transactions where the principal transaction has the same price as the agency transaction, i.e. to avoid double counting when analysing the data we only want to keep one side of the transaction. Dick-Nielsen argues that failing to correct the errors in the data will bias a popular liquidity measure, such as Amihud price impact, towards a more liquid market. In the Standard TRACE data set, Dick-Nielsen [2009] claims that as much as 18% of the transactions in some cases can be deleted due to changes and cancellations in the market or to avoid double counting in the data set. The cleaning suggested by Dick-Nielsen follows three main steps. First, we delete all reports that have been cancelled or corrected within the same day of reporting. Second, as it is possible to report a cancellation or correction later than the actual reporting day, we need to remove the reversals. Third, to avoid double counting of agency trades, we remove one part of the trade. Table 3.2 below illustrates the filtering and cleaning process of the data set and give detail of the number of observations remaining after each step as well as the percentage, with the initial sample set after the filtering process.

Filtering and cleaning steps	Observations	Remaining (%)
TRACE Enhanced as of February 2021	261,304,867	
Period sample (2017-07-01 - 2019-07-01)	45,533,984	
Initial sample (167 corporate bonds)	4,474,747	100
Cancellations and corrections	4,322,666	96.60
Reversals	4,321,255	96.57
Double counting of agency trades	2,929,690	65.47
Clean TRACE Enhanced data set	2,929,690	65.47

Table 3.2: Filtering and cleaning TRACE Enhanced data set.

To complete the data set at hand, further data was gathered from Thomson Reuters Datastream and Cbonds. The data from Datastream includes bid and ask prices, mid prices, original offering and amount outstanding. Cbonds provided original offering and amount outstanding data for those bonds that did not have any data from either TRACE or Datastream. Finally, the corporate bonds' credit ratings were gathered from the FINRA annual fact book of corporate bonds. Since Datastream provides daily data rather than intraday data as in the case with TRACE, we created a separate data set of daily data with 77,206 observations. When counting the actual days of trading, 78 of the bonds have been traded at least 500 of the possible trading days, 140 of the bonds have been traded at least 400 of the days and only two of the corporate bonds have been traded fewer than 100 days. The most and least traded bonds have been traded 508 days and 66 days respectively. Although the number of trading days gives an indication of the liquidity of an asset, it does not say anything about the number of intraday trades. For the full (filtered and cleaned) TRACE Enhanced data set of intraday trades, the corresponding numbers are 97,145 and 1,414 transactions, i.e. the most traded bond has been traded 97,145 times during the two year period and the least traded has been traded 1,414 times. It should be noted that the most/least traded bonds are not the same on daily basis as the ones on intraday basis. An interesting observation is the major jump between the most traded and the second most traded corporate bond, since the second most traded has only been traded 36,034 times.

3.1.2 Benchmarks

To identify market movements a number of indices are used from different segments. In addition to analysing market movements, the correlation between the different indices, liquidity measures and transaction parameters were calculated. The first index is Russell 3000, a very broad index measuring the performance of the largest 3000 U.S. companies. The advantage of Russell 3000 is that it includes many equities, more precisely, it represents about 98% of the investable U.S. equity market [FTSE Russell, 2021]. As it is one of the most comprehensive indices from the U.S. market, it is an obvious and suitable choice of benchmark for the analysis. The second index is the well-known S&P 500 which is one of the most used indices in financial and economical research papers. The index, created in 1957, is a market capitalisation weighted index including 500 large companies in leading industries listed on the stock exchanges in the United States. It covers approximately 80% of the market capitalisation and information technology is by far the largest sector in the index (∼27%) [S&P Global, 2021b]. The equities included in the S&P 500 are some of the most liquid financial assets in the world and therefore, to compare with less liquid indices, we included two indices from small-cap and bond market. First, as a small-cap benchmark, we used the Dow Jones U.S. small-cap total stock market index. The index is a market capitalisation weighted index measuring the performance of small-cap U.S. equity securities. The largest sector is health care, closely followed by financial, industrial, information technology and consumer discretionary. Compared to the S&P 500, with a median total market capitalisation of USD 27,141M, the small-cap index has a much lower median (USD 1,804M) [S&P Global, 2021a]. Second, we used S&P 500 bond index which is design to be a corporate bond counterpart to the S&P 500. The index brings transparency to the bond market by providing data throughout trading day instead of closing-level data like most bond indices. It is built upon debts issued by corporates from the S&P 500 and include investment grade as well as high yield bonds [S&P Global, 2021c]. The index is an appropriate choice of benchmark as several of the 167 corporate bonds are issued by companies included in the S&P 500.

It is difficult to define stressed market conditions as one has to choose an appropriate measure and level for when the market enters a period with stress, i.e. increased volatility and uncertainty in the financial market. However, the Chicago Board Options Exchange's (CBOE) Volatility Index, more known as VIX or *fear index*, is a widely accepted and popular when measuring the markets' expected volatility over the coming 30 days. The index is based upon S&P 500 index options which can be traded to hedge against movements in the S&P 500, i.e. if you are holding a position in S&P 500 and expect a decrease you can buy an option and hedge against the potential loss. Accordingly, to simplify the idea behind VIX, if investors expect increased volatility the interest of options increases as well as the VIX. Therefore, a high VIX value indicates an increased (expected) volatility on the stock market which is equal to stressed market conditions. An index value below 20 is considered a stable and stress-free period, while values above 30 corresponds to high volatility and increased risk (stress) [Kuepper, 2021]. VIX has been used in several papers examining liquidity, e.g. Chung and Chuwonganant [2014] studies the relationship between market movements and stock liquidity and Bams and Honarvar [2021] argues that investors' willingness to pay a liquidity premium increases as the market volatility (VIX value) increases. In this paper, VIX is mainly used to determine liquidity changes during different market conditions.

3.1.3 Fama and French Factors

Kenneth French provides a data library on his website with several data sets available for research, e.g. different portfolios and Fama and French factors. We use the daily data set of historical Fama and French factors collected from the website. Due to the relatively short sample period, the daily set is preferred as a longer horizon would heavily decrease the number of data points. The data set is originally based on the Center for Research in Security Prices (CRSP) database and the risk-free rate compounds to 1-month treasury bill rate from Ibbotson and Associates Inc.

3.1.4 Estimation of Bid-ask Spread

The data collected from Datastream was only available for a small number of bonds and since bid-ask spread and mid-quotes are central parameters in the analysis, the values were estimated by using the TRACE data instead. The TRACE Enhanced data has the advantage of having a buy/sell indicator that can be useful to estimate the bid-ask spread since it is essentially defined as the difference between the maximum price someone is willing to pay for the asset and the minimum a holder is willing to sell for. Therefore, as a first attempt, we took the difference between the minimum transaction price given a sell indicator and the maximum transaction price of a buyer for each trading day and bond. The estimation resulted in a high percentage of negative values, most certainly because of large intraday movements. Although, a negative value is theoretically possible, the economical intuition of the spread suggest a positive value and we would not expect a high percentage of negative spreads for several bonds and over a time period of two years. Corwin and Schultz [2012] derived a bid-ask spread estimate from daily max and min prices. They argue that their model has advantages compared to similar daily models and outperforms several of those. From a practical perspective, there are some problematic implicit assumptions underlying the estimator. First and foremost, it assumes continuous trading during market hours. In addition, it expects the value to be unchanged while the market is closed. In the stock market the model would probably be a reasonable estimator as the assumptions are more or less fulfilled. For illiquid assets in the OTC market however, an estimator cannot be based on these assumptions. As previously mentioned, the least traded assets have been traded 66 days and 1,414 times respectively during the sample period which is far from the requirements. Despite the fact that the models assumptions could not be fulfilled on the corporate bond market, the estimator was tested on some bonds. The result of the test was, as expected, unsatisfying and we considered the model inapplicable on our data.

Instead, inspired by the estimator proposed by Thompson and Waller [1988], we calculated the spread by using transaction data and the buy/sell indicator as in the first attempt. In contrast to the first estimate, we instead used the difference between the average transaction price for sellers and the average transaction price for buyers on a daily basis. The differences were mainly positive, more precisely, 98.8% of the calculated differences were positive. To smooth out the variance, reduce influence from extreme values and isolate movements of the individual asset we had to correct for market movements. A suitable choice would be to use credit default swap indices from the U.S. market, such as iTraxx or CDX provided by IHS Markit, or a future contract of one of the indices. Another option would be a future of one of the benchmarks previously mentioned. However, the method requires high frequency tick data to correct for market movements on transaction level. As the tick data is not publicly available, we tried to improve the estimator by deleting the percentiles representing the 10% highest and 10% lowest transaction prices on daily basis for seller and buyers respectively. This improvement helps reduce the influence from extreme values, i.e. the distribution of intraday trades smooths out. Although it was assumed to improve the model, it heavily reduced the number of observation due to the relatively few transactions and strongly increased the number of missing spread values. The main and finite reason for the decision to choose another strategy was the increased number of consecutive missing values. The final and applied estimator was calculated by replacing the average daily transaction values for buyers and seller with the difference between the median transaction prices. The advantage of the applied strategy is that we, similar to the exclusion of high/low percentiles, avoid strong influence from extreme values. The median methodology resulted in 98.1% positive spreads and in those cases we observed negative values, the value was excluded from the data set.

$$spread_{i,t} = \texttt{median}(p_{i,t}^{Seller}) - \texttt{median}(p_{i,t}^{Buyer}) \tag{3.1}$$

One problem with the suggested estimation model is that the cleaned data set does not guarantee both sell and buy indicators for each trading day and bond. That is, for illiquid assets, we might end up with only one of the two possible indicators. These days, together with the excluded negative median difference values, had to be replaced with substituted values. An alternative strategy would be to exclude the observation from the analysis with the risk of losing valuable data points. We applied the first approach due to the short sample period as well as the liquidity feature, i.e. some corporate bonds did only trade on a few occasions and we do not want to risk losing important observations. In total, the methodology of using buy/sell indicator resulted in 6.8% missing values with an average gap size of 1.35, i.e. most of the missing values were foregoing and followed by a known value. The maximum series of consecutive missing values was 11 days and occurred twice in the data set. A computed histogram illustrating the distribution of calculated spreads showed a high percentage of values in the range from 0 to 1 ($\sim$ 92%) and very few values exceeding 1.5 ($\sim$ 2.4%). Therefore, we set 1.5 as the maximum spread and let values exceeding 1.5, all negative values and missing values being replaced with substituted values. This gives an average spread value of 0.37 across all bonds and with an average transaction price of $ 97.7, the spread is approximately 37.4 basis point. Mizrach [2015] finds that the bid-ask spread is between 30-40 basis point on average for the relevant corporate bond market during 2012-2015 which confirm our calculated values and methodology.

Imputation is a statistical process of replacing missing data and there are many different variants of implementation, e.g. mean substitution or replacing missing values with zero. A method to improve the performance of imputation is to use a

Kalman Filter commonly used in time series analysis which was primarily developed by Rudolf E. Kálmán in 1960. The original paper offers a recursive solution to the discrete-data linear filtering problem which has been subject to further research. Due to its robustness of using series of measurements together with statistical noise makes it a suitable choice for imputation. The filter mainly uses two steps. Initially, it predicts the current state variable and then, once the outcomes of the next measurements is observable, it updates estimations by average weighting of the predicted and observable measurements [Bishop, 2006]. `R` offers a built-in function `na_kalman()` from the `imputeTS` package, a package for imputation of missing values in univariate time series. The filter uses an underlying state space model to predict future values in the time series and we chose the autoregressive integrated moving average (ARIMA) model for implementation of the filter. Implementing the ARIMA model requires estimation of three main components: p for the auto-regression (AR) component; d for the integration (I) component; and q for the moving average (MA) component [Elshenawy et al., 2018]. The `auto.arima` model in `R` returns the best fitted ARIMA model for the data set and we used the default method. Thus, the first value is found by using conditional sum-of-squares and then maximum likelihood. The general equation for the ARIMA model is:

$$X_t = c + \sum_{i=1}^{p} \alpha_i x_{d_{n-i}} + \sum_{i=0}^{q} \beta_i \varepsilon_{n-i} \tag{3.2}$$

where x_d is X differenced by d times and ε is the estimation error. The auto-estimated components p, d and q represents the number of lag observations, the degree of differentiating and order of moving average respectively [Elshenawy et al., 2018]. Figure 3.1 is a visualization of the Kalman filters replacement of missing values on a randomly chosen corporate bond from the data set.

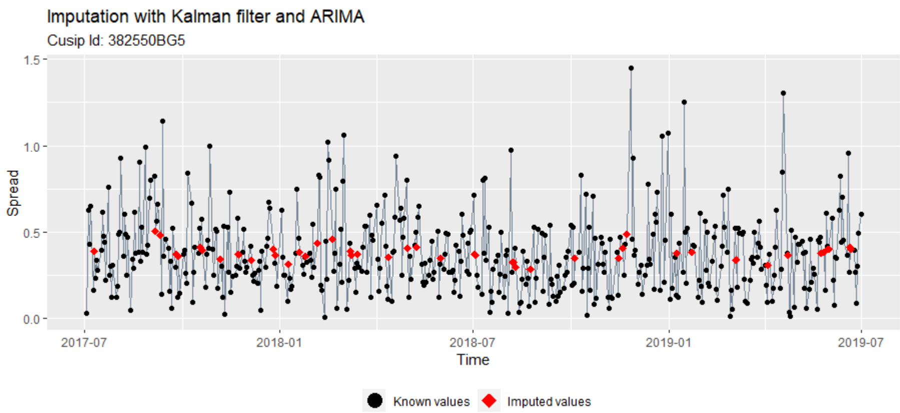

Figure 3.1: Imputation visualization of one randomly chosen corporate bond over the sample period when using Kalman filter and ARIMA as state space model.

As can be seen in the figure, the replaced values (red diamond) are very close to the known values (black points) and follows seasonal differences well. In addition, it tends to follow a mean estimation of its closest values and thereby smooths the data set. However, contrary to the known values, it never takes any extreme values. This trend repeats itself when examining the replaced values for other bonds. The reason for the filtering to follow a mean pattern rather than taking extreme values is probably because it weights the predicted values based on seasonal movements together with the observed values. Also, extreme values from the pre-known set rarely occurs more than once at a time and therefore it is expected that the replaced values rarely occurs.

3.2 Liquidity Measures

To assess the liquidity risk on the chosen bonds a liquidity measure has to be defined. There are several more or less developed measures suggested and used in research papers. However, as they heavily depend on the available data and the relevant market, it is hard to rely on a single measure. Consequently, to improve the analysis, we examine and compare three different measures that uses transaction data and can be measured on a daily basis.

3.2.1 Volatility of Price Dispersion

Jankowitsch et al. [2011] who also examined liquidity of U.S. corporate bonds on the OTC-market defined a new liquidity measure. First and foremost, the model focuses on the price dispersion effects on the OTC-market and should reflect the transaction cost associated with market liquidity, such as inventory and search costs. The proposed model measures the illiquidity of an asset and can be interpreted as the volatility of price dispersion. That is, a higher value suggests a less liquidity asset compared to a lower value. Acccordingly, an illiquid asset is characterised by a high price dispersion, i.e. the difference between transaction price and market valuation is large on average, whereas the liquid asset is associated with a negligible difference. The measure is modelled as follows

$$d_{i,t} = \sqrt{\frac{1}{\sum_{k-1}^{K_{i,t}} v_{i,k,t}} \cdot \sum_{k=1}^{K_{i,t}} \left(p_{i,k,t} - m_{i,t}\right)^2 \cdot v_{i,k,t}} \tag{3.3}$$

where $v_{i,k,t}$ is the transaction volume for bond i on day t and k is the transaction. $p_{i,k,t}$ is the reported price and $m_{i,t}$ is the market valuation of the bond at the time. We chose to interpret the daily mid price of the bond as the market valuation. The measure is used to measure correlation to factors that have been proven to impact liquidity such as transaction volume, bid-ask spread and credit rating by performing a cross-sectional regression.

Since the definition of liquidity states how easy a position can be liquidated without changing the fair price, i.e. the price of immediate trading, one expect the measure ($d_{i,t}$) to be low for liquid assets. From this perspective, the model seem to fulfill the literature suggested characteristics. Another advantage of the model developed by Jankowitsch, Nashikkar and Subrahmanyam is that it is independent of whether the trading price is above or below the market valuation as well as the incorporation of trade volume. Lastly it was developed by examining the price dispersion on the same market as analysed in this paper which makes it a good complement to the other liquidity measures mainly developed to determine stock liquidity.

3.2.2 Hui-Heubel Liquidity Ratio

Heubel and Hui [1984] developed their liquidity ratio to estimate stock liquidity and has been widely applied in liquidity research. This liquidity measure is built so that the impact of volume traded and its impact on the assets' price is captured. A high Hui-Heubel liquidity ratio suggests a lower liquidity for an asset during a particular time period. The ratio is formed as follows

$$HHI = \frac{\frac{(P_{MAX}-P_{MIN})}{P_{MIN}}}{\text{Turnover ratio}} \tag{3.4}$$

where P_{MAX} and P_{MIN} are the highest and lowest reported transaction price for an asset reported over the last five days. The turnover ratio is defined as the traded volume in dollars over the last five days divided by the amount outstanding. According to the equation, the ratio relates the turnover ratio (mainly the transaction volume) to its impact on prices. In other words, a larger volume relative to price changes suggests a more resilient and liquid market. The ratio decreases as the trading volume increases and/or as the difference between maximum and minimum transaction prices decreases. As stated in theory, several papers argues that the liquidity increases in trading volume which confirms the roll of the denominator in the equation. However, if we assume a non-zero correlation between the market and an asset, the maximum and minimum prices can theoretically fluctuate heavily in case the market is volatile. Especially, this is the case during times of stress. Consequently, the price difference will increase over a five day period and suggest a period of low liquidity which is not necessarily consistent with the true liquidity of the asset [Su, 2013]. Although the measure has some potential shortcomings, the advantage of its usefulness and the fact that the market is not highly volatile during the sample period makes it a suitable choice for analysing the liquidity.

3.2.3 ILLIQ

The $ILLIQ$-measure was constructed by Amihud in 2002 and is the most commonly used measure for stock liquidity. As the other two measures, $ILLIQ$ is a measure of illiquidity. The intuition of the measure is rather simple, an asset is considered illiquid if the return (price changes) is high in response to small trading volume. It is therefore closely related to HHI that measures a similar relationship, i.e. the one between price changes and trading volume. The measure is calculates as follows

$$ILLIQ = \frac{1}{N}\sum_{t=1}^{T}\frac{|r_t|}{V_t} \tag{3.5}$$

where T is the amount of days, V_t is the volume traded in dollars for day t and r_t is the return of the asset on day t. The measure is often used since it requires a minimum amount of data which is often very accessible. Goyenko and Ukhov [2009] adds the sample time period as an additional important determinant when choosing liquidity measure. The measure can be used on a daily, weekly or monthly basis which simplifies analysis over a longer time period. Acharya and Pedersen [2005] state two main problems while using $ILLIQ$: stationarity and costs. The measure does not measure the dollar invested, rather it measures the total dollar value which means that the measure for instance misses inflation and is therefore not stationary. For a smaller sample period, as examined in this paper, this does not become a problem. $ILLIQ$ measures the costs of selling a position while one normally is interested in the cost of a trade.

3.2.4 Replacement of Missing Mid Quotations

Similar to the situation with bid and ask quotations, Datastream did only provide a small set of mid quotations. As the percentage of missing values was too high, imputation could not be applied to replace the missing values. Especially since the mid quotations were mostly missing for bonds throughout the whole sample period and would in those cases be replaced by values from other bonds. Therefore, we calculated the daily volume weighted average transaction price for each bond and found that the difference between mid quotations provided by Datastream and the calculated values was very small on average. Thus, we used the mid quotation in those cases these were available and replaced it with the volume weighted average transaction price if they were not. The reason for choosing the volume weighted average transaction price instead of average transaction price was because most literature, as mentioned in the theory, argue that the trading volume has a large impact on transaction prices for less liquid assets, i.e. a larger position will likely be sold at a lower price compared to a smaller volume and to be sure that the theory is reflected in our data, the volume weighted average is a suitable choice.

3.2.5 Expected Impact From Explanatory Liquidity Variables

As all of the examined estimators measure the illiquidity, we expect the same sign for the explanatory variables as we run regression models with the different liquidity measures as continuous dependent response variable. Table 3.3 gives an overview of the different factors and their expected sign when running multiple linear regression models.

Liquidity factor	Description	Sign
Transaction volume	Daily trading volume in U.S. Dollar	-
Turnover ratio	Daily turnover ratio	-
Trades	Number of daily transactions	+ / -
Bid-ask spread	Estimated bid-ask spread in U.S. Dollar	+
Maturity	Time between maturity date and transaction date in years	-
Rating category	1 if it is a high yield bond, otherwise (investment grade) 0	+
Rating AAA	1 if the S&P rating of the bond is AAA, otherwise 0	
Rating AA	1 if the S&P rating of the bond is AA, otherwise 0	+*
Rating A	1 if the S&P rating of the bond is A, otherwise 0	+*
Rating BBB	1 if the S&P rating of the bond is BBB, otherwise 0	+*
Rating BB	1 if the S&P rating of the bond is BB, otherwise 0	+*
Rating B	1 if the S&P rating of the bond is B, otherwise 0	+*
Rating CCC	1 if the S&P rating of the bond is CCC, otherwise 0	+*
Rating CC	1 if the S&P rating of the bond is CC, otherwise 0	+*
Rating D	1 if the S&P rating of the bond is D, otherwise 0	+*
Rating NR	1 if the S&P rating of the bond is NR, otherwise 0	+*

* Given that AAA rating is incorporated into the intercept.

Table 3.3: Overview of the liquidity factors studied in this paper and their impact on the liquidity measures. To clarify, as all estimators measures illiquidity rather than the liquidity, the sign indicates the expected impact on illiquidity.

3.3 Bond-level Multiple Linear Regression

To see if known liquidity factors have a significant impact on our liquidity measure we perform a bond-level multiple linear regression. By doing this we can analyse the relationship between the liquidity measure and the known liquidity factors as well as determine whether or not the liquidity measures do work for U.S. corporate bonds traded on the OTC market. Finally, the methodology simplifies the identification of problematic corporate bonds because of firm specific risks and/or characteristics.

3.3.1 Methodology and Assumptions

The initial step of the regression analysis was to examine the liquidity measures and determine if they are appropriate measures. We expect a liquidity measure to be dependent on liquidity factors suggested in the literature, such as the bid-ask spread and transaction volume, which are further described in the theory. Considering the frequent use of $ILLIQ$ in similar papers and that the volatility of price dispersion, d, has been developed on the same market and data (different sample period), the two measures are expected to be suitable choices to determine liquidity. In addition, the Hui-Heubel liquidity ratio is expected to perform well on the data. Nevertheless, to prove the measures' relevancy, they need to be investigated on the actual data set. The daily data set was used in the regression analysis on bond level, i.e. the relevant models were created and evaluated for each bond. Consequently, each model resulted in 167 different estimation for each β parameter, p-value, confidence interval etc. The models were all created with liquidity measures as continuous dependent response variable assumed random and liquidity factors as independent non random explanatory variables according to the following equation:

$$Y_i =_0 +\beta_1 x_{i,1} + \cdots + \beta_p x_{i,p} + \varepsilon_i \tag{3.6}$$

Models indicating statistical significance at conventional levels (5%) for a t-test of the slope (β parameter) and expected direction (sign) of the relationship between the explanatory variable and response variable were considered an indication of suitable liquidity measure. The results from the bond-level regression were neither considered proof nor did they definitely establish how good a liquidity measure is, they rather provide an indication of if the measure is suitable choice for further in-depth analysis. Each one of the three measurements were investigated by the same number of models, i.e. all models were created and investigated with the different liquidity measures as response variable. By analysing each model on bond level we will also, in addition to determine whether the liquidity measures are suitable choices or not, be able to identify corporate bonds that are inappropriate for further analysis. It would be very time consuming and hard to do identify these bonds in the cross-sectional multiple linear regression analysis because of the large number of observations in our data set. Further reasoning for the exclusion of bonds and the methodology is described in Subsection 3.3.3.

To evaluate the result and relevancy of each estimator we calculated the percentage of significant parameters across all of the 167 bonds, i.e. the calculated result represent how many of the bonds that have significant parameters for each model and factor. Thus, even though we might have single data points or bonds acting unexpectedly due to firm specific factors, the result will still give an accurate proof of the performance of a model. Given that a model consisted of more than one explanatory variable, as in most cases, the percentage of significant parameters were calculated for each β individually. In addition to the significance, for each β parameter we calculated the percentage of bonds having the expected direction in relation to the response variable.

3.3.2 Residual Analysis

To further analyse the models, a residual analysis was performed. Considering that there were 167 different results for each model and performing further analysis for each bond would be too time consuming, three bonds for each of the categories IG and HY were chosen at random. It is assumed that considering that different bonds' data were relatively similar, the residuals should not significantly differ across the bonds. Therefore, if the residuals of the randomly chosen bonds were assumed normally distributed, it could be concluded that this was the case for most bonds.

3.3.3 Excluding Corporate Bonds

We apply a two-step approach to identify and potentially exclude corporate bonds from the full set cross-sectional multiple linear regression analysis. The methodology simplifies the identification of corporate bonds with strange data points based on the expected liquidity behavior proposed in financial literature. We have three liquidity measures and since a regression analysis requires a large number of models to study, it would be very time consuming to analyse each bond individually by plotting it for each model and liquidity measure. Also, it would be hard to find a suitable decision rule for excluding bonds or single observations that would work on the entire data set and throughout the analysis. The two-step approach is structured as follows:

1. Summarise corporate bonds that exclusively have insignificant explanatory variable coefficients at 5% significance level in at least 50% of the created models, given that the model has three independent explanatory variables or more.

2. Identify corporate bonds from the first step that, in addition to the individual factor-level insignificance, fail to reject the null hypothesis of a global F-test.

Bonds collected in the first step will be analysed in order to find common characteristics and features that potentially could explain which kind of corporate bonds that do not seem to fit the regression. To avoid excluding too many bonds summarised in the first step, we add a global F-test and choose to only exclude bonds identified by both steps. Accordingly, as we add the second step, we also avoid to exclude corporate bonds that have variables with significance close to our pre-chosen level of significance, i.e. the first step could theoretically exclude bonds with several explanatory variables very close to the significance level and by incorporating a global F-test we decrease that risk.

3.4 Full Set Cross-sectional Multiple Linear Regression

After excluding problematic bonds a regression of the remaining bonds is performed. This part of the analysis is extensive as simple linear regressions were performed first with the liquidity measures as dependent variables and known liquidity factors as explanatory variables separately. Secondly, multiple linear regression was performed with two or more liquidity factors as explanatory variables. Contrary to the bond-level regression analysis, this part allows modelling with constant parameters, such as the credit rating, which is a major difference and interesting since one would theoretically expect high yield corporate bonds to be less liquid compared to investment grade bonds. The regression-betas were then examined to see if they had expected sign and were significant. Furthermore, the models are analysed in regards on how well they fit the data and how they perform against each other. As this part of the analysis is only to confirm that the liquidity measures are in fact good measures of liquidity, only the best performing models will be analysed further. To determine the best performing model, the different models were compared using four commonly used estimators for model selection. R^2 and R^2_{adj} were used to determine the fraction of the variability of the response variable, the liquidity measure, that can be explained by the regression model. Thus, a higher value is preferred to a lower. Since the R^2

increases as one adds covariates to a model, the R^2_{adj} was applied to compare models with different number of explanatory variables as this measure, in contrast to R^2, can decrease when variables are added to the model. Furthermore, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were used as complement to the aforementioned measures as criterion for model selection. The estimators are very similar with the main difference that the latter punishes larger models harder. They use the likelihood function to assess the goodness of fit which in this case describes the probability of observing the true (observed) response variables as a function of the β's estimated by the model. The log-likelihood is then compared to the number of parameters used in the model. In other words, the estimators are a trade-off between small residual error and large number of parameters, i.e. a trade-off between the goodness of fit and simplicity of a model. Non of the estimators calculate the absolute quality, rather they calculate the relative quality of a model that can only be used to compare models. Hence, they are not able to suggest whether the models are good. For statistical model validation, an in-depth residual analysis were performed and it is further described in Subsection 3.4.1.

3.4.1 Residual Analysis

In order to find outliers, observations with large influence, and to statistically validate models, a regression analysis was performed. One basic assumption in multiple linear regression is normally distributed measurement errors, ε_i, with mean zero and variance σ^2, i.e. $\varepsilon_i \sim N(o, \sigma^2)$, and that they are pairwise independent. To test for normality, a histogram as well as a Q-Q plot of the residuals are plotted. Given a non-normal error distribution, both prediction and confidence intervals will be effected. The second measurement error assumption, homogenous variance, which otherwise may lead to a high influence of uncertain observations, is examined by plotting the residuals against the predicted response variable $\hat{Y}$. In addition, the residuals are plotted against each explanatory variable individually as this allows us to identify high residuals for different factor characteristics, e.g. one could see that high residuals often occurs for observations with low volume. Moreover, as we assume pairwise independent errors, we check for correlation with a plot of the autocorrelation versus the time lag. The assumed covariance matrix looks as follows

$$\mathtt{var}(\varepsilon) = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix} = \sigma^2 \mathbf{I}$$

To identify potentially influential observations the leverage (v_{ii}), which measures the impact of Y_i on its own estimated value $\hat{Y}_i$, is plotted for each model. For each plot there are two horizontal benchmarks, at $v_{ii} = 1/n$ and $v_{ii} = 2(p+1)/n$, where leverage above the latter is normally considered high. However, it is not a finite decision rule as each regression model heavily depends on the data set as well as the modelling and a high leverage observation is therefore not necessarily influential. For each model, a maximum level is set dependent on the plot and these observations are then highlighted in the rest of the analysis to see whether they are influential or not and if they should be excluded from the data set to improve the model. The residuals are, as the measurement error, assumed normally distributed with mean zero. The difference is that they have different variances and are dependent on the structure of the explanatory variables, $\mathbf{X}$, which make them hard to compare. By subtracting the mean and dividing by the estimated standard deviation of the residuals we get the studentized residuals with variance approximately equal to one. The estimated standard deviation comes from a regression where its own observation has been excluded, in contrast to the methodology when calculating the standardized residuals, to reduce influence from large residuals. This helps to make both the residual and standard deviation independent and the residuals are now comparable. Absolute values above two are considered high and two benchmarks are therefore set at ± 2 and ± 4. The high leverage observations collected in the previous step are highlighted to see if the same observations have large residuals that together indicate problematic observations. The final step of the regression analysis is to plot Cook's distance to conclude whether the potentially influential points in fact are influential or not. Given an individual observation with high leverage and a high Cook's distance, it will be excluded from the model to reduce its large influence on the estimates of β and σ^2, and consequently on predictions and statistical conclusions.

3.5 Modeling Returns

An interesting part of liquidity analysis is to examine if there is a presence of a liquidity risk premium. To examine this and, if present, to quantify the premium the correlation between excess returns and the liquidity measure is calculated. It is also possible to incorporate the liquidity measures into known return estimation models as the CAPM or the Fama and French factor models. In our analysis our liquidity measures were added as a factor to both these models and bond-level multiple linear regressions with excess returns as response variable were made to check for significance with the factors.

3.5.1 CAPM

To test the CAPM model together with the liquidity measures two regression models were made, one with and one without the liquidity measure as a factor, see Equation (3.7) and Equation (3.8). As mentioned above, significant factors on bond-level was checked for and a high percentage of significance across all bonds for both excess market return and the liquidity premia indicated that liquidity should, in some way, be incorporate into the return model to improve estimations of expected return. However, as this results in an indication rather than proof, we computed AIC, BIC, R^2 and R^2_{adj} on bond level. To propose proof of improvement, the four measures were calculated for both models on bond level and the results were then summarised to see the percentage of corporate bonds performing better as a liquidity premium was added to the original CAPM. Given a high percentage of improved models, we should be able to conclude that market liquidity should be incorporated into return estimating models. CAPM is rewritten in time-series regression form as follows

$$R_{i,t} - R_{f,t} = \beta_0 + \beta_1(R_{m,t} - R_{f,t}) + \varepsilon_{i,t} \tag{3.7}$$
$$R_{i,t} - R_{f,t} = \beta_0 + \beta_1(R_{m,t} - R_{f,t}) + \beta_2 LIQ_{i,t} + \varepsilon_{i,t} \tag{3.8}$$

where LIQ is the liquidity measure, i is a corporate bond and t is a trading day.

3.5.2 Fama and French

Previous studies suggests that the SMB-factor in the Fama & French factor models can be explained by the asset's liquidity. To test this and whether the Fama & French three factor model is applicable on OTC traded corporate bonds or not, the three factor model is applied on our data at bond-level, see Equation (2.8). The estimated beta for the SMB-factor is then compared to our liquidity measure for correlation. Furthermore, the correlation between the SMB-factor and our liquidity measures is tested to see if they follow the same pattern. The final step of analysing the three factor model is to replace the SMB-factor by our liquidity measures when modelling the data, Equation (3.9), to see if it could better explain the excess returns and the results were evaluated in the same fashion as for the CAPM.

$$R_{i,t} - R_{f,t} = \beta_0 + \beta_1(R_{m,t} - R_{f,t}) + \beta_2 HML_t + \beta_3 LIQ_{i,t} + \varepsilon_{i,t} \tag{3.9}$$

where LIQ is the liquidity measure, i is a corporate bond and t is a trading day. As the three-factor model is most commonly used to model returns for stocks and its precision for other types of assets is relatively unknown, we also try to model returns with the five-factor model in the same way as with the three factor model.

3.6 Stressed Market Conditions

Another interesting point of view is analysing if stressed market conditions has a noticeable impact on the liquidity measures. This is done by analysing how the liquidity measures differ over time and if it is affected by market turmoil. To do this a stressed market period needs to be defined in the sample data set. To define such a period the VIX index is used. The first part of the analysis consists of observing if there is a significant difference in the size of the liquidity measure. Secondly an analysis of the impact of the liquidity factors during this period compared to normal market conditions gives an answer to how stressed market conditions affects liquidity.

To perform an analysis of how stressed market conditions affect liquidity, two periods were chosen. One period where the market conditions are considered "normal", i.e volatility below 20 and one period where the conditions are considered "stressed", i.e volatility above 20. In our data sample there were only a few periods that could be considered as "stressed". The longest of those periods lasted a month between the 30th of November 2018 and the 31st of December 2018. The choice of a "normal" period was made as the closest period to the "stressed period" with no values over 20. Therefore the "normal" period starts on the 5th of September and ends on the 2nd of October. The reasoning behind not choosing a full month for the ”normal” period is that the two periods need to have the same amount of trading days which in this case is 18.

Chapter 4

Results

4.1 Liquidity Factors

Correlations between the different liquidity factors incorporated in the best performing multiple linear regression models are presented in the correlation matrix below. In addition, three plots of the relationship between the factors are presented in Figure 4.1.

	Daily volume	Daily trades	Turnover ratio	Bid ask spread
Daily volume	1.000			
Daily trades	0.212	1.000		
Turnover ratio	0.131	0.095	1.000	
Bid ask spread	-0.055	-0.012	-0.007	1.000

Table 4.1: Correlation matrix for the liquidity factors.

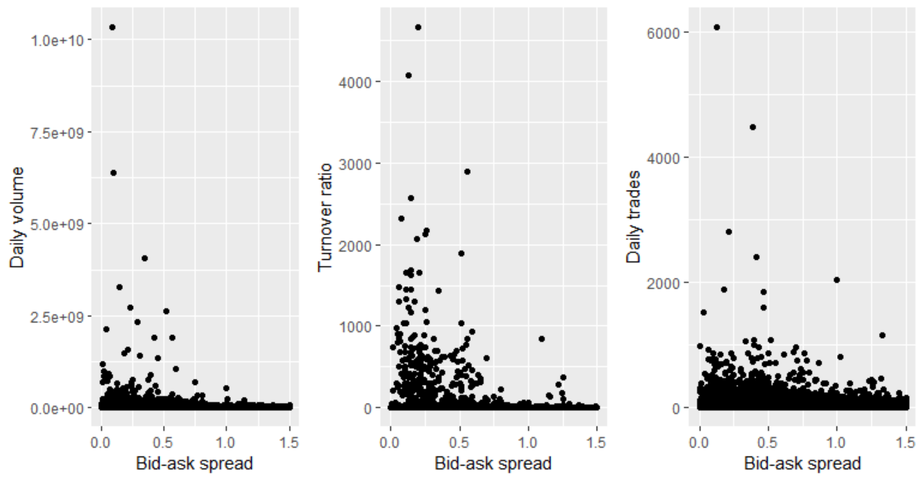

Figure 4.1: Liquidity factors plotted against bid-ask spread to visualise a potential relationship between them.

Looking at the correlations between the liquidity factors in Table 4.1 they are pretty weak. The strongest correlation is found between daily volume and daily trades which is rather expected. There is also a bit of correlation between daily volume and and turnover ratio which is also expected as the turnover ratio is a function of the daily volume. Even if the correlations are low between the bid-ask spread and the other liquidity factors, Figure 4.1 shows tendencies of lower bid-ask spread connected to higher daily volume as well as turnover ratio. Theoretically this makes sense since a high spread is a strong indicator of illiquidity and illiquidity is a decreasing function of volume and turnover ratio. This further confirms that the bid-ask spread methodology seems to fit the data set well. In the figure below a histogram of each factor is plotted to see how the data is distributed.

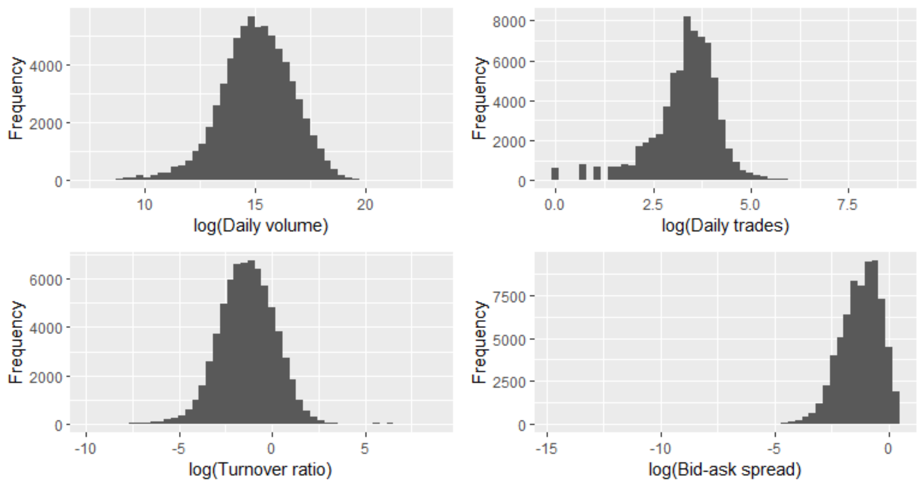

Figure 4.2: Histogram of the liquidity factors.

There are several observations with high values of daily trading volume, number of trades and the turnover ratio which forces a logarithmic scale to visualise the distribution. The most interesting histogram is the one for the bid-ask spread since it could not be observed, rather it had to be estimated which is further described in Subsection 3.1.4. Clearly, the majority of the observations have a spread below \$0.5 and only a very low percentage have a spread above \$1. This finally confirms the estimation methodology suggested in this paper.

4.2 Liquidity Measures

4.2.1 Basic Analysis

As a first analysis of the liquidity measures a couple of simple tests are performed to make sure the measures behave as expected. Firstly the mean, median, maximum and minimum values of all measures over the whole sample period are calculated for all of the bonds but also for the HY bonds and the IG bonds separately.

Data	Measure	Mean	Min	Median	Max
Full set	$ILLIQ$	$4.50 \cdot 10^{-10}$	$9.49 \cdot 10^{-18}$	$4.59 \cdot 10^{-12}$	$4.85 \cdot 10^{-6}$
	HHI	2.97	$4.05 \cdot 10^{-4}$	1.81	357.17
	d	0.36	$4.71 \cdot 10^{-7}$	0.28	10.66
HY	$ILLIQ$	$8.24 \cdot 10^{-10}$	$1.48 \cdot 10^{-16}$	$7.75 \cdot 10^{-12}$	$4.85 \cdot 10^{-6}$
	HHI	2.57	$4.05 \cdot 10^{-4}$	1.57	357.17
	d	0.45	$4.71 \cdot 10^{-7}$	0.37	10.66
IG	$ILLIQ$	$3.28 \cdot 10^{-11}$	$9.49 \cdot 10^{-18}$	$2.64 \cdot 10^{-12}$	$3.63 \cdot 10^{-8}$
	HHI	3.40	$2.57 \cdot 10^{-3}$	2.22	286.30
	d	0.27	$1.92 \cdot 10^{-4}$	0.20	2.97

Table 4.2: Table of liquidity measures and their mean, median, minimum and maximum over the sample period.

By this first analysis we can conclude that for $ILLIQ$ and d the average is significantly higher for HY-bonds than for IG-bonds. This is expected since both are illiquidity measures and are increasing in illiquidity and HY-bonds should be less liquid than IG-bonds. For HHI we conclude that the opposite occurs which is problematic since it is, as the two other measures, decreasing in liquidity. To further analyse the measures over the sample period we plot the daily means of the measures. In the plots the red dots are HY-bonds and the black dots are IG-bonds.

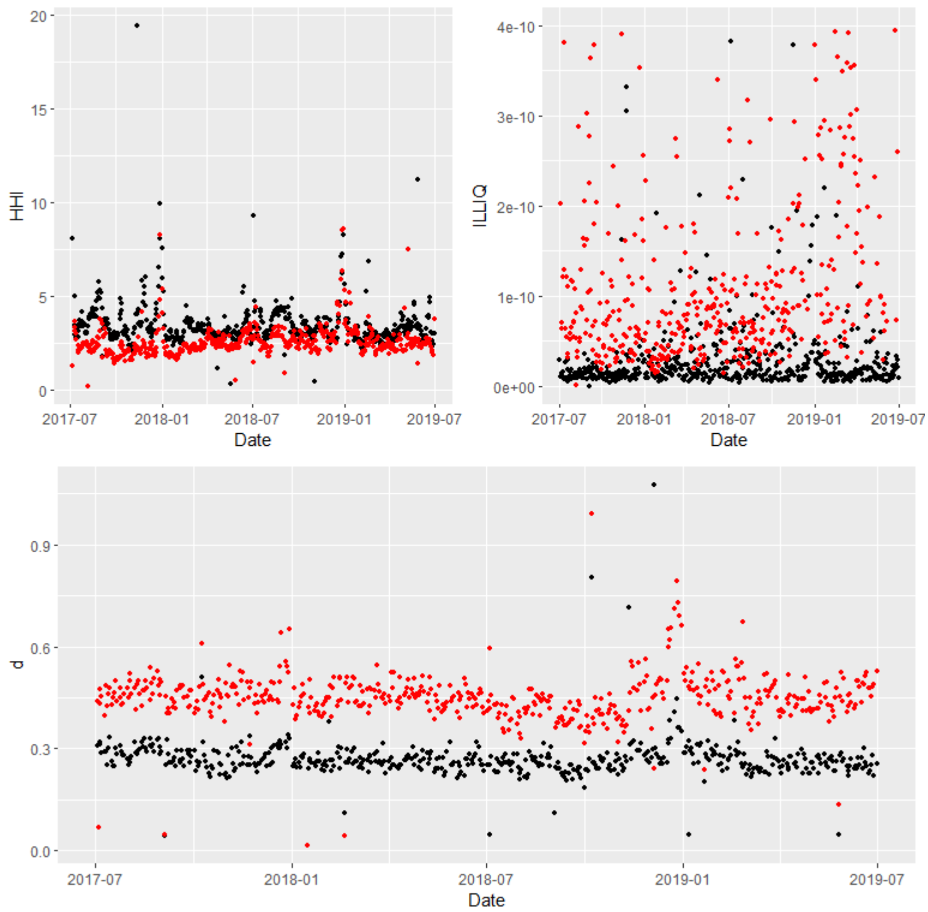

Figure 4.3: Daily averages of the liquidity measures over the time period by rating category. Investment grade bonds (black) and high yield bonds (red).

The plots further confirm that $ILLIQ$ and d behave expectedly when comparing IG-bonds to HY-bonds. However, the plot for $ILLIQ$ looks like it could be rather problematic. The daily means for HY-bonds are, relatively, more scattered compared to the other two measures and the daily means for IG-bonds are very close to zero. It also gives further indication that HHI is not behaving as expected on the sample data.

4.2.2 Bond-level Regression

For all three liquidity measures several models, including liquidity factors as explanatory variables, were analysed to see which model fit the data the best. To proceed with a liquidity measure to a more in-depth analysis the regression betas need to be significant for the majority of the bonds. For d and HHI such models were easily constructed with only a few bonds showing insignificance. However, for $ILLIQ$ the betas were insignificant for almost all bonds and models, even when transforming the response variable as well as the explanatory variables. Therefore, the rest of the analysis will only be applied to the other two measures. Even though HHI does not behave as expected, a more in-depth analysis is interesting as a clear difference can be seen between HY-bonds and IG-bonds and further analysis might give some answers. To further control the models applied on the measures, a few bonds were picked at random to perform a basic residual analysis. Looking at the residual plots we can conclude that they are marginally right skewed but the residuals can be assumed normally distributed.

4.2.3 Analysis of Excluded Bonds

The corporate bonds identified by the first step of the two-step excluding approach described in Subsection 3.3.3 are presented below for d and HHI respectively.

d-measure

To get further understanding of why certain bonds did not show significance in our regression we chose to analyse these bonds further, especially if we could find that the liquidity factors somehow differed from the other bonds. By looking at the bonds excluded by the first step of our approach, see Table 4.3, we could quickly conclude that the bonds whose maturity date was within our data sample period caused a problem. This is something we can relate to maturity and age being liquidity factors but most likely it's the fact that there are less trading days during our sample period that causes the issue. Furthermore, we could identify two bonds that were traded very few days compared to the median value, however the daily trading volume was a lot higher than the median value. Considering, trading volume and amount of trades were both explanatory variables in our regression, the unusual values are expected to lead to insignificance. After the second step of our approach we could conclude that only two bonds should be excluded. One of which is the ALTRIA GROUP INC bond which was one of the bonds with unusual trading data.

Cusip id	Insign (%)	Issuer	Coupon	Maturity	Rating	IG/HY	Trades	Average daily trades	Trading days	Median daily volume	Median daily spread
02209SBD4	100.0	ALTRIA GROUP INC	4.8	2029-02-14	BBB	IG	6,246	64.4	97	37,748,000	0.221
06051GFD6	100.0	BANK AMER CORP	2.65	2019-04-01	A	IG	15,824	36.5	434	6,492,500	0.082
126650CV0	66.7	CVS HEALTH CORP	3.7	2023-03-09	BBB	IG	15,954	48.2	331	22,169,000	0.084
126650CX6	86.7	CVS HEALTH CORP	4.3	2028-03-25	BBB	IG	22,021	66.5	331	62,029,000	0.119
172967HC8	100.0	CITIGROUP INC	2.5	2018-09-26	BBB	IG	11,083	35.9	309	3,411,000	0.129
172967KE0	100.0	CITIGROUP INC	2.05	2018-12-07	NR	HY	12,922	36.1	358	4,184,500	0.112
38145XAA1	93.3	GOLDMAN SACHS GROUP INC	2.625	2019-01-31	BBB	IG	13,935	35.5	393	3,257,000	0.107
46625HHS2	66.7	JPMORGAN CHASE CO	4.4	2020-07-22	A	IG	21,757	43.6	499	3,739,000	0.079
71654QCK6	53.3	PETROLEOS MEXICANOS	5.35	2028-02-12	BBB	IG	4,176	30.9	135	30,164,000	0.293
Median (all bonds)							17,130	36.6	499	3,464,000	0.300

Table 4.3: Corporate bonds and their relevant factors identified when applying the first step on the bond-level multiple linear regression for the price dispersion (d) liquidity measure.

HHI

A first glance at Table 4.4 which shows the bonds excluded by the first step of our approach for HHI shows that most of the bonds that were excluded for d also show insignificance for HHI. This further confirms that these bonds are unusual. Furthermore, we can conclude that there are significantly more bonds to be excluded for this measure. For plenty of these bonds there seems to be no apparent reason for exclusion. However, when performing the second step of our approach we can conclude that no bonds should be excluded.

Cusip id	Insign (%)	Issuer	Coupon	Maturity	Rating	IG/HY	Trades	Average daily trades	Trading days	Median daily volume	Median daily spread
02209SBD4	100.0	ALTRIA GROUP INC	4.8	2029-02-14	BBB	IG	6,246	64.4	97	37,748,000	0.221
03674XAC0	100.0	ANTERO RES CORP	5.125	2022-12-01	B	HY	6,402	13.0	494	3,170,000	0.291
053773AV9	100.0	AVIS BUDGET CAR RENTAL LLC/FINANCE INC	5.5	2023-04-01	B	HY	9,463	18.9	500	2,528,000	0.428
060505FL3	60.0	BANK AMER CORP	5.875	3999-12-31	BBB	IG	19,620	59.3	331	10,878,000	0.560
06051GFD6	60.0	BANK AMER CORP	2.65	2019-04-01	A	IG	15,824	36.5	434	6,492,500	0.082
10922NAC7	73.3	BRIGHTHOUSE FINL INC	3.7	2027-06-22	BBB	IG	9,473	33.1	286	4,198,500	0.434
126650CV0	100.0	CVS HEALTH CORP	3.7	2023-03-09	BBB	IG	15,954	48.2	331	22,169,000	0.084
126650CX6	100.0	CVS HEALTH CORP	4.3	2028-03-25	BBB	IG	22,021	66.5	331	62,029,000	0.119
163851AD0	100,0	CHEMOURS CO	7	2025-05-15	B	HY	4,008	8.6	467	2,056,000	0.284
20341WAD7	66.7	COMMUNICATIONS SALES & LEASING INC	8.25	2023-10-15	C	HY	10,470	21.5	487	6,896,000	0.333
34984VAB6	73.3	FORUM ENERGY TECHNOLOGIES INC	6.25	2021-10-01	B	HY	3,913	9.9	397	320,000	0.496
369622SM8	60.0	GENERAL ELEC CAP CORP	5.3	2021-02-11	BBB	IG	17,678	35.3	501	1,513,000	0.180
37045XCA2	60.0	GENERAL MTRS FINL CO INC	5.75	3999-12-31	BB	HY	36,034	79.5	453	5,511,000	0.635
37045XCM6	100.0	GENERAL MTRS FINL CO INC	6.5	3999-12-31	BB	HY	11,184	57.1	196	4,355,500	0.816
38148BAD0	100.0	GOLDMAN SACHS GROUP INC	5	3999-12-31	BB	HY	23,882	55.7	429	10,072,000	0.492
402635AE6	86.7	GULFPORT ENERGY CORP	6.625	2023-05-01	B	HY	1,499	3.9	385	800,000	0.375
48128BAD3	86.7	JPMORGAN CHASE & CO	4.625	3999-12-31	BBB	IG	22,145	51.3	432	7,496,500	0.602
561234AE5	80.0	MALLINCKRODT INTL FIN SA	4.75	2023-04-15	CC	HY	17,705	35.5	499	2,296,000	0.686
71647NAF6	86.7	PETROBRAS GLOBAL FIN B V	4.375	2023-05-20	BB	HY	21,002	41.3	508	9,195,500	0.323
71647NAM1	100.0	PETROBRAS GLOBAL FIN B V	6.25	2024-03-17	BB	HY	15,222	30.3	503	8,071,000	0.318
71647NAQ2	100.0	PETROBRAS GLOBAL FIN B V	8.75	2026-05-23	BB	HY	13,922	27.8	501	19,172,000	0.228
71647NAS8	100.0	PETROBRAS GLOBAL FIN B V	7.375	2027-01-17	BB	HY	19,478	38.8	502	32,351,000	0.217
71647NAY5	100.0	PETROBRAS GLOBAL FIN B V	5.999	2028-01-27	BB	HY	15,682	79.2	198	38,307,000	0.343
71647NAZ2	100.0	PETROBRAS GLOBAL FIN B V	5.75	2029-02-01	BB	HY	13,981	38.9	359	30,440,000	0.227
71654QCG5	86.7	PETROLEOS MEXICANOS	6.5	2027-03-13	BBB	IG	14,206	43.8	324	35,064,500	0.255
71654QCP5	93.3	PETROLEOS MEXICANOS	6.5	2029-01-23	BBB	IG	5,456	39.3	139	43,101,000	0.301
78412FAP9	100.0	SESI L L C	7.125	2021-12-15	CC	HY	12,974	26.1	498	1,993,000	0.474
88167AAD3	66.7	TEVA PHARMACEUTICAL FIN NETH III B V	2.8	2023-07-21	BB	HY	15,043	30.1	499	17,991,000	0.271
912909AN8	60.0	UNITED STS STL CORP NEW	6.25	2026-03-15	B	HY	13,974	43.4	322	4,757,500	0.364
92857WBQ2	100.0	VODAFONE GROUP PLC NEW	7	2079-04-04	BB	HY	6,166	93.4	66	16,825,500	0.439
958102AM7	100.0	WESTERN DIGITAL CORP	4.75	2026-02-15	BB	HY	17,130	48.1	356	15,567,000	0.256
966387BG6	100.0	WHITING PETE CORP	6.625	2026-01-15	B	HY	5,976	24.7	242	10,868,000	0.313
984121CD3	100.0	XEROX CORP	4.5	2021-05-15	BB	HY	13,246	26.5	499	1,257,000	0.320
N6945AAK3	100.0	PETROBRAS GLOBAL FIN BV	5.999	2028-01-27	BB	HY	23,125	68.0	340	34018000	0.290
Median (all bonds)							17,130	36.6	499	3,464,000	0.300

Table 4.4: Corporate bonds and their relevant factors identified when applying the first step on the bond-level multiple linear regression for the Hui-Heubel liquidity ratio.

4.3 Multiple Linear Regression

This part of the analysis consists of controlling the liquidity measures by concluding significant dependence of known liquidity factors and analysing the regression betas.

4.3.1 Regression Modelling

HHI

For HHI a multiple linear regression was performed with the credit rating, bid-ask spread, logarithmic transformation of the daily volume, logarithmic transformation of the daily amount of trades and the square root transformation of the turnover ratio as explanatory variables. The credit ratings are incorporated in the regression as dummy variables taking the value of 0 or 1. Furthermore it can be noted that the credit rating AAA is incorporated into the intercept.

Factor	Estimate	p-value
Intercept	17.707	$< 2 \cdot 10^{-16}$
AA	1.45694	$2.78 \cdot 10^{-11}$
A	-1.608	$4.06 \cdot 10^{-15}$
BBB	-0.899	$5.02 \cdot 10^{-6}$
BB	-2.175	$< 2 \cdot 10^{-16}$
B	-2.748	$< 2 \cdot 10^{-16}$
CCC	-2.299	$< 2 \cdot 10^{-16}$
CC	-4.255	$< 2 \cdot 10^{-16}$
C	-3.196	$< 2 \cdot 10^{-16}$
NR	1.809	$4.18 \cdot 10^{-16}$
Bid ask spread	1.025	$< 2 \cdot 10^{-16}$
log(daily volume)	-0.866	$< 2 \cdot 10^{-16}$
log(daily trades)	-0.147	$6.12 \cdot 10^{-9}$
sqrt(turnover ratio)	-0.042	$9.970 \cdot 10^{-4}$

Table 4.5: Table of regression betas and their corresponding p-values with HHI as response variable.

In Table 4.5 above we can see that all factors are significant at 1% significance level. Looking more closely at the β-estimates, starting with the credit rating variables, further conclusions can be drawn. As HHI is increasing in illiquidity it is expected that the estimates should have a bigger positive impact for the poorly rated bonds compared to the good rated bonds. Although the absolute values of the estimates are not linearly increasing over the ratings, there is a clear pattern that the poorly rated bonds have a bigger negative impact on the measure. This is problematic as it goes against theory but it offers an explanation to as why HY-bonds had a lower average rating than IG-bonds. Less surprisingly, the estimate for the bid-ask spread is positive. Intuitively, a greater bid-ask spread should lead to more illiquidity. Similarly, the estimate for the log-transformation of the daily volume is expected. According to the theory, liquidity is an increasing function of traded volume. A first glance at the log-transformation of the daily trades may seem unexpected. Intuitively, more daily trades should indicate higher liquidity. However, as discussed in Subsection 2.5.2 in the theory, more trades might indicate smaller transactions and less liquidity. Finally, the estimate for the square root transformation of the turnover ratio is also backed up by theory. According to theory, liquidity is an increasing function of the turnover ratio.

d-measure

For the volatility of price dispersion a multiple linear regression was performed with the credit rating, bid-ask spread, logarithmic transformation of the daily volume and the logarithmic transformation of the daily amount of trades as explanatory variables. As for HHI it can be noted that the credit ratings are incorporated as dummy variables and AAA is incorporated into the intercept.

Factor	Estimate	p-value
Intercept	0.464	$< 2 \cdot 10^{-16}$
AA	0.057	$1.75 \cdot 10^{-14}$
A	-0.034	$8.48 \cdot 10^{-7}$
BBB	0.075	$< 2 \cdot 10^{-16}$
BB	0.132	$< 2 \cdot 10^{-16}$
B	0.143	$< 2 \cdot 10^{-16}$
CCC	0.158	$< 2 \cdot 10^{-16}$
CC	0.091	$< 2 \cdot 10^{-16}$
C	0.120	$< 2 \cdot 10^{-16}$
NR	0.210	$< 2 \cdot 10^{-16}$
Bid ask spread	0.291	$< 2 \cdot 10^{-16}$
log(daily volume)	-0.047	$< 2 \cdot 10^{-16}$
log(daily trades)	0.094	$< 2 \cdot 10^{-16}$

Table 4.6: Table of regression betas and their corresponding p-values with d as response variable.

In Table 4.6 above it is easy to see that the regression betas correspond better to the theory than for HHI and it can be concluded that all factors are significant at a 1% significance level. Similarly to HHI, the absolute value of credit rating betas are expected to be increasing in illiquidity. As this measure is increasing in illiquidity the betas are also expected to be positive. Once again, the regression coefficients do not increase linearly but the overall pattern corresponds well to the theory with a significant difference between the betas for the IG-bonds and HY-bonds. Secondly, we can see that the measure is an increasing function of the bid-ask spread. As the bid-ask spread is one of the strongest liquidity factors as discussed in Section 2.5 of the theory, this estimate is important and is expected to be positive. Moving on to the estimate of the log-transformation of the daily volume, the sign of the estimate once again is in line with the theory in the sense that liquidity is increasing in volume. The final estimate, the one for the log-transformation of the daily trades, is harder to interpret but as discussed for HHI above as well as in the Subsection 2.5.2 in the theory, earlier studies suggests liquidity is a decreasing function of trades as more trades may be the cause of each trade being of smaller quantity. It can be concluded, that for the sample data in this study, and for this liquidity measure the liquidity is a decreasing function of amount of trades.

4.3.2 Residual Analysis

HHI

In Figure 4.4 a histogram and a Q-Q plot of the residuals, i.e. the differences between observed and predicted values, are plotted for the model presented in Table 4.5 above.

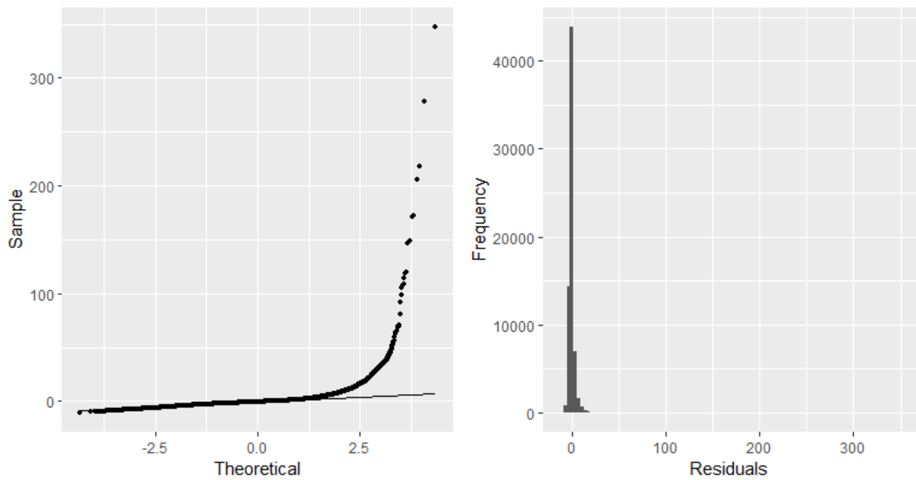

Figure 4.4: Q-Q plot (left) and histogram (right) of the residuals of the multiple linear regression model presented in Table 4.5.

The x-axis in the Q-Q plot (left plot) represents the theoretical quantiles of a standard normal distribution, while the y-axis represents the observations in the data set. Given that the observations are assumed to be normally distributed, we would expect that all data points would lay close to the perfectly percentile matched line in the Q-Q plot. As can be seen in the figure, the observations follows the line quite well on the left hand side whereas on the right hand side of the graph, the points lay above the line and are strictly exponentially increasing. This suggests that the errors are right skewed (fat tailed) and follows a Student's t-distribution rather than a normal distribution. This is further confirmed by the histogram (right plot) that has a fat right tail proposing extreme values affecting the regression model. The non-normality affects the prediction interval as well as the confidence interval. However, the estimated beta parameters can still be acceptable for large data sets. In addition, a data set of 2,929,690 observations is expected to have outliers that will effect the model. To test the robustness of the model two complementary models are implemented and evaluated. First, the top and bottom 2.5% are cut off the data set dependent on the HHI value. In other words, only a subset containing the interval between the 2.5th and the 97.5th percentile is examined for the same multiple linear model. Secondly, instead of applying the ordinary least square principle and assuming normally distributed errors, the regression model is assumed to have t-distributed error terms. Neither the first robustness model nor the second changes the betas noticeably which indicates that the model is working although the basic assumptions are not fulfilled. Moreover, when excluding the extreme values (first robustness model) the residuals are more normally distributed as expected since the fat tails are cut off. The result proofs that even though the error terms do not follow a normal distribution, the predicted betas can still be acceptable for large data sets. For the second robustness model the results are even more intuitive as a Student's t-distribution converges toward a normal distribution as the degrees of freedom increases. The skewed distribution of the residuals is expected to improve if the response variable, i.e. the liquidity measure, would be transformed using the logarithm. However, the readability of the β estimates would severely weakened and as the model will not be further used in the analysis, rather are used to determine whether the liquidity measure is appropriate, the model will remain untransformed.

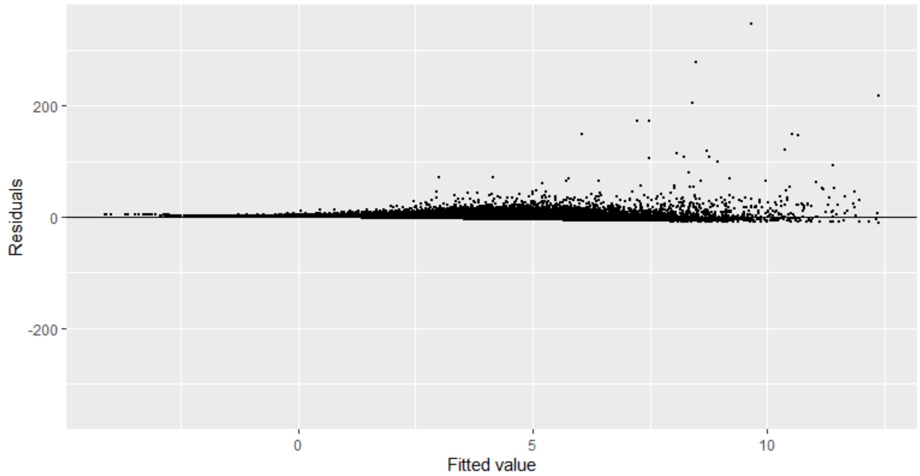

Figure 4.5: Ordinary residuals plotted against the fitted $\hat{Y}$ for the multiple linear regression model presented in Table 4.5.

In Figure 4.5 the residuals are plotted against the fitted values to determine whether the error terms have homogenous or heterogenous variance and to detect outliers. The figure shows that the residuals are small on average for lower fitted values, while there are several high residuals and a wider spread for higher values and the result suggest that the model fail to explain extreme values. The mean value of the residuals is very close to zero as assumed in the model although there are many large positive residuals according to the residual plot. Appendix B.1 provides additional valuable plots to further analyse the model and detect outliers. In Figure B.1 the residuals are plotted against all the explanatory variables separately to identify observation characteristics causing the high residuals. Because the trading volume, number of daily trades and turnover ratio have some very high values a logarithmic scale is used for these variables. The different plots proves that high residuals occurs at trading days with low trading volume, few trades and of course a low turnover ratio as it is closely related to the trading volume. Additionally, it often occurs when the bid-ask spread is in the range from 0 to 0.6 and for non-rated corporate bonds or bonds rated AA. Together, especially when considering the trading volume and non-rated bonds, the figure suggests that it is hard to predict the liquidity for corporate bonds with a very low liquidity level. It should also be noted that there are very few bonds rated AA and NR in the data set, see Table 3.1, making it harder for the model to predict the liquidity for those. Hence, higher residuals are expected for those bonds.

The correlogram, Figure B.2 in Appendix B.1, indicates that the residuals are positively correlated in the regression model which occurs when there is autocorrelation or dependency between the errors. Hence, they are not independent as assumed in the model. The autocorrelation is high (~ 0.7) for lag 1 and then stays stabile at approximately 0.2 for lag 4 and higher. When plotting the autocorrelation for higher lags it decreases and converges towards zero. Just below lag 300 it is within the confidence interval illustrated by the blue line in Figure B.2. Autocorrelation is a common problem for time series data as it is difficult for the regression model to effectively capture time trends. To confirm or potentially improve the model, statistical tests and alternative techniques for linear regression modelling were used as robustness checks. Initially, as a first order autocorrelation of the errors is highly suspected, a Durbin-Watson test was performed using the `lmtest` package in `R`. The test rejects the null hypothesis at a very high level of significance, i.e. the test suggests that there is a correlation between the error at one period and the error in a previous period. In addition, a Ljung-Box test confirms that there are non-zero autocorrelation for at least one of the lags (tested for lag 5, 10 and 20) and therefore we conclude that the error terms are not independently distributed, rather they exhibit serial correlation. Two different approaches that are better to handle residual correlation are used to potentially improve the model. First, the generalized least square (GLS) technique is used to estimate the beta parameters of the model. Second, the model is transformed into an autoregressive model by applying the Cochrane-Orcutt procedure in package `orcutt` in `R`. The GLS model is more or less identical to the model in Table 4.5 while the parameters of Cochrane-Orcutt transformed model differs from the original model. Despite the fact that the model has changed, the transformed model still rejects the null hypothesis of a Durbin-Watson test and the technique therefore failed to remove serial correlation between error. In total the robustness models and statistical tests suggest that the model is acceptable although it has correlation between the error terms which contradicts the model assumption.

The leverage is plotted in Figure B.3 and most values are significantly larger than the benchmarks set at $v_{ii} = 1/n$ and $v_{ii} = 2(p+1)/n$. However, as mentioned in the method, it does not necessarily imply influential points. A high leverage occurs as one or more predictors are extreme in comparison to the average values, i.e. far from the center of the x space, or if the combination of the explanatory variables is unusual. Observations with $v_{ii} \geq 0.005$ are collected and highlighted in the following analysis to determine whether these observations have a large influence and thereby should be excluded from the data set. Figure B.4 illustrates the studentized residuals against the fitted values with the high leverage observations highlighted with green triangles. The observations identified in the leverage plot (Figure B.3) do not have high residuals according to the studentized residuals plot and therefore is there no indication that they are in fact problematic. This is further confirmed by the Cook's distance plot, see Figure B.5, as all the high leverage observations are low and below the benchmark set at $D_i = 4/n$. Accordingly, non of the observations indicates a high influence that negatively affects the regression model. There are still outliers and observations with high influence, however, as they cannot be proved causing trouble, they are not excluded from the data set. To summarise, the residual analysis of the model presented in Table 4.5 suggests that the basic assumptions of the multiple linear regression, such as independent and normally distributed measurement error with homogenuos variance, are not fulfilled causing problem when estimating the parameters using the ordinary least square principle. Furthermore, there are outliers with high residuals as well as influential points. Although they exists, they cannot be proved problematic and are therefore not excluded from the sample. One expect these results when examining a very large data set on a market far less liquid and transparent compared to large stock exchanges and as neither the model nor the liquidity measure will be applied in the return modelling or stressed market analysis, they will not be further developed.

d-measure

A Q-Q plot and a histogram of the residuals from the model presented in Table 4.6 are plotted in the figure below.

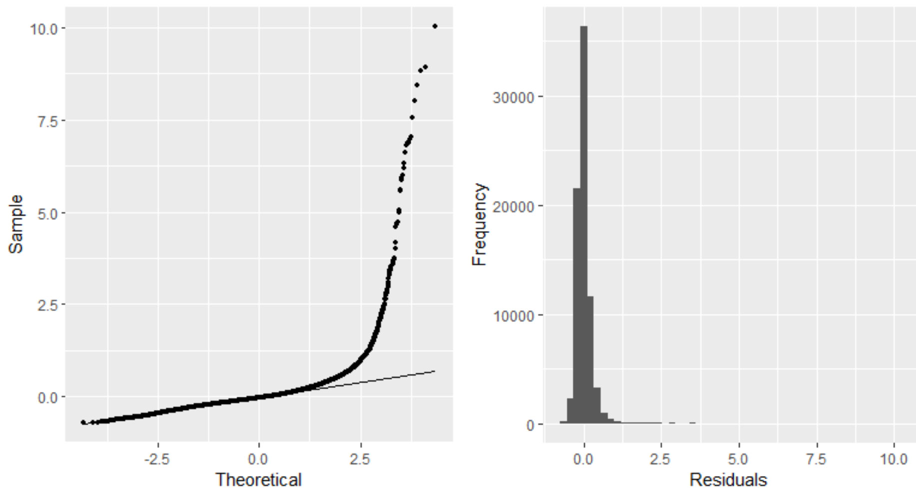

Figure 4.6: Q-Q plot (left) and histogram (right) of the residuals of the multiple linear regression model presented in Table 4.6.

Similar to the regression model for the Hui-Heubel liquidity ratio above, the residuals are right skewed suggesting fatter tail and a non-normal distribution according to the plots in Figure 4.6. Therefore, as for HHI, two additional models are created to test the robustness of the results proposed in this section. The first model assumes t-distributed residuals instead of normally distributed to deal with the fat tails. The estimated parameter are close and the signs identical to the original model assuming normally distributed error terms. The second model cuts of the tails by removing the 2.5% highest and lowest observations based on their liquidity measure (response variable). The parameters in this model are similar to the original and together with the first robustness model it proves that the model is robust from this perspective. As for the HHI model above, a logarithm transformation of the response variable is expected to improve the skewness of the residuals. Nevertheless, the model remains untransformed since it will not be further applied in the analysis and does not affect the results or conclusions of the paper.

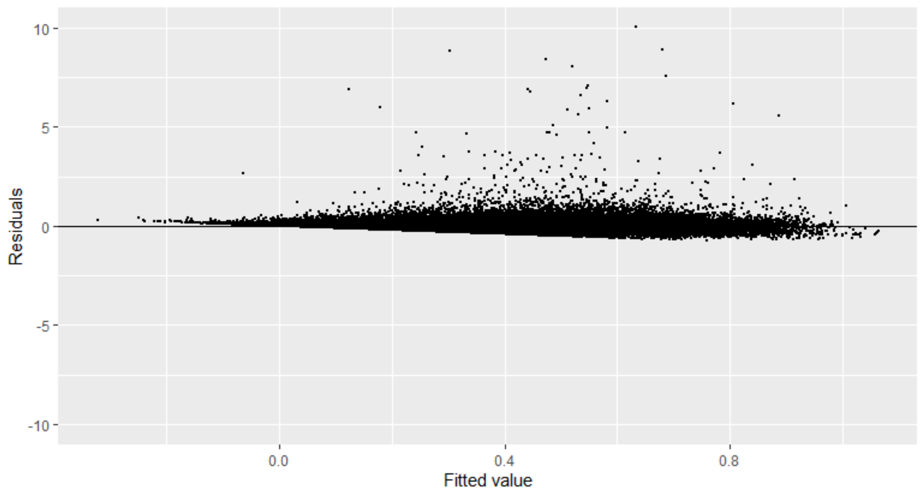

Figure 4.7: Residuals plotted against the fitted $\hat{Y}$ for the multiple linear regression model presented in Table 4.6.

In Figure 4.7 we can see that there is a good spread of the residuals and the large residuals are mainly positive, i.e. in those cases the residuals are large it is because the observed value is higher than the predicted. The mean value for the residuals are very close to zero as assumed in the model which is a good first indication. Similar to the residuals analysis for the HHI model, Appendix B provides additional plots to further analyse the model. In Figure B.6 are the residuals plotted against each explanatory variable separately and one can see that the residuals have a better spread over the factors compared to the residuals for HHI. Furthermore, as for HHI, the residuals are lower for higher bid-ask spreads contrary to the intuition. However, as a very high percentage of the observations have a bid-ask spread below one it is highly expected that the residuals occurs within this range. Contrary, one would perhaps expect some outliers in the upper area as fewer observations within a range makes it generally more difficult for the regression model to predict the outcome. The spread of residuals are good for trading volume and daily trades as mentioned, while the spread over credit ratings are harder to explain. Most of the large residuals are observations of high yield corporate bonds, especially bonds rated BB, B and NR. Since the non-rated bonds have high residuals for this model as well as for the model for HHI it is suspected that these bonds are harder to predict in general. In addition, as some of the non-rated bonds mature during the sample period, they have less data point which makes the prediction more difficult.

The autocorrelations have been calculated for lag 1 to 30 and they are illustrated in the correlogram in Figure B.7 in Appendix B.2. They are slowly decreasing in lag but the autocorrelations are very high even at high lags suggesting that the correlation between errors are not zero as assumed in the regression model. The autocorrelation is further confirmed by a Durbin-Watson test which fails to reject the null hypotheses. In other words, there is a true autocorrelation greater than zero between the residual at one period and the residual in the previous period. To test whether the high correlations are problematic and could be improved two robustness models are created as for HHI, i.e. a GLS regression and a regression following the Cochrane-Orcutt procedure. The GLS regression model is almost identical to the original model in Table 4.6 using original least square technique. The Cochrane-Orcutt methodology did in fact remove the first order autocorrelation at a high level of significance according to a Durbin-Watson test. Most of the estimated parameters in the transformed model are very close to the values in the original model and the signs are all the same. Consequently, the rest of the analysis will keep focusing on the original model.

Lastly, the leverage, studentized residuals and the Cook's distance are plotted to identify outliers and potentially influential observations. There are a lot of data point above the benchmarks in the leverage plot in Figure B.8 indicating that there are influential observations affecting the model. Data points with leverage above 0.0015 are highlighted as green triangles in the plot of studentized residuals, Figure B.9, and in the Cook's distance plot, Figure B.10. According to the studentized residuals, one of the observations with high leverage is an outlier far from the central which further indicates that it is influential. The same observation is confirmed influential in the Cook's distance plot and therefore removed from the data set to improve the model. The effects on the model are very small since it is only one observation of the total 69,132 daily data points based on almost three million transactions. The other data point with high leverage are however not influential as they neither have high (studentized) residual values nor do they have a high Cook's distance.

4.4 Return Models

The regression analysis suggests throughout the following subsections that a liquidity premium should be incorporated into return models, given that the security is a U.S. corporate bond traded in the OTC market and that the liquidity measure is a suitable proxy for liquidity. An in-depth analysis on bond level is provided in the subsections below.

4.4.1 Correlation Between Liquidity Measures and Excess Return

By performing a single linear regression model on bond level, we can conclude that there are clear indications that liquidity should be incorporated when estimating the excess return of corporate bonds on the U.S. OTC market. For each liquidity measure, d and HHI, the excess return was modelled as response variable and the liquidity measure as a single independent explanatory variable. The price dispersion estimator, d, was significant for approximately 65% of the corporate bonds, while the Hui-Heubel liquidity ratio was only significant for slightly below 4% of the cases. Hence, the results indicates that the liquidity should most likely be incorporated into an asset return model, given an appropriate choice of liquidity measure. Clearly, the Hui-Heubel liquidity ratio is a poor choice as its percentage of significance is close to the error expected from white noise at conventional levels of significance. In addition to the percentage of significant models, the estimated slope (β) of the liquidity measure was positive in 85% of the possible cases for d when the estimated parameter was significant. The slope for d is expected to be positive since an increase in illiquidity should, according to the liquidity theory, increase the expected return. That is because illiquidity increases the risk and should thereby increase the expected return of a financial asset. This further confirms the result that a liquidity factor should be incorporated when estimating asset returns. To finally test the relationship between the liquidity measures and the excess return, the correlations between the two different liquidity measures and the excess return were calculated on the full set, a subset of investment grade bonds and a subset of high yield bonds respectively. The correlation between d and excess return is -0.0073 for the full data set, 0.0745 for investment grade bonds and -0.0107 for high yield bonds. The corresponding values for the correlation between HHI and excess return are -0.0117, -0.0016 and -0.0174. It is difficult to make any conclusions from the correlations except that they are close to zero independent of rating category. For d we have a small positive value for investment grade bonds, while it is negative for high yield bonds. For both measures, high yield bonds have a lower correlation with the liquidity measure compared to investment grade bonds. In total the results suggest that d is the best liquidity measure to use when estimating returns on the relevant market and it is therefore the only measure used in the rest of the return estimating analysis.

4.4.2 CAPM with a Liquidity Premium

In this step of the return model analysis, we create two linear regression models that are compared to examine if the performance of the CAPM improves when a liquidity premium is added as an explanatory independent variable. The excess market return is collected from Kenneth Frenchs' website, see Subsection 3.1.3, and the variable is significant for 61% of the corporate bonds when modelling the excess return as a function of excess market return, i.e. the CAPM. This simple linear regression model was then compared to a second model with excess market return and d as two independent explanatory variables, still with the excess asset return as response variable. As expected, adding one explanatory variable, especially one that has been proved important when calculating asset excess returns, did increase the R^2 value. This result was true for all corporate bonds in the data set. As for the R^2, the R^2_{adj} was higher in most cases ($\sim 79\%$). Similar to the result in the previous section, the liquidity measure was significant for 61% of the bonds. The two models were also evaluated by comparing their AIC and BIC values on bond level. AIC indicated that the second model, with a liquidity premium, was better for 74% of the bonds and the corresponding value for BIC was 56%.

The results suggests that the CAPM alone is a poor choice for estimating the excess return of securities examined in this paper. By incorporating a liquidity premium, calculated as in Equation (3.3), the modelling improves in most cases. When summarising the results in this and the previous section it is however important to notice that the liquidity premium is dependent on whether the liquidity measure is a good choice for the relevant market or not. We cannot, based on the results provided in this paper, say if a liquidity premium should be incorporated when estimating asset returns on a different market or for other securities. In addition, we cannot say that the other measures, HHI and $ILLIQ$, do not work.

4.4.3 Fama and French

The Fama and French three factor model was examined in the same fashion as the CAPM but with one important difference, instead of adding an additional variable to the model, the liquidity premium replaced one of the factors suggested by Fama and French in their original paper from 1993. The SMB factor suggested in the original three factor model was significant in very few cases, more precisely for 7 of the 167 corporate bonds. Similar to the reasoning of the relationship between excess return and the Hui-Heubel liquidity ratio, the low percentage of significant models is expected from white noise and the result therefore suggests that the factor does not affect the excess return. The HML factor was significant for 37% of the bonds. Although it is undoubtedly a higher percentage of significance, indicating that it is a better fitted factor, it is still a relatively low percentage. Given that a perfectly working Fama and French three factor model is defined as a model of exclusively significant variables, the model does only work for three corporate bonds. When examining the Fama and French five factor model instead, both the RMW and CMA factors have a higher percentage of significant betas compared to the SMB factor. More precisely, they are significant for 10% and 22% of the corporate bonds respectively while the other factors, included in the three factor model, do not change substantially. To summerise, both the Fama and French three factor model and the five factor model seems to be inappropriate choices to model excess return on the relevant market.

In the modified three factor model the SMB factor was replaced by d and once again modelled on bond level. If we assume the same definition as for the original model when determining a perfectly working model, this model works for 23 corporate bonds compared to the three bonds for the original model. This is a major improvement even though it is still far from a clear indication of a good model. The liquidity premium itself is significant in 72% of the models and that is also an improvement compared to the very few significant SMB factors. When comparing the two models it is obvious that the modified model, incorporating a liquidity premium, is a better choice. Both R^2 and R^2_{adj} is higher in 86% of the possible cases. Similar, both AIC and BIC suggest that the modified model is better fitted for 86% of the bonds. One important difference between the original factors and the liquidity measure is that the latter is not only time dependent, it is dependent on which corporate bond that is examined. Hence, we expect a higher percentage of significance given that the return is dependent on the liquidity as suggested in the theory.

There does not seem to be any relationship between the average daily value of d and the SMB factor when plotting them against each other which is further confirmed when evaluating the result of a simple linear regression model with d as response variable and the SMB factor as explanatory variable on bond level. The result is expect since we have previously concluded that the liquidity heavily depends on different bond characteristics and individual risk associated with the bond issuer. Moreover, finding a relationship between an individual measure and a factor, especially a factor that does not measure the liquidity, would be a surprising result suggesting that the SMB factor is truly liquidity. Similarly, there is no relationship between the estimated betas for the SMB factor from the original Fama and French three factor regression model and the daily average of d. Even if a relationship was found, the result would still be questionable as the factor was only significant for a very few bonds and the estimated betas are therefore not proved statistically non-zero.

Model	R^2	R^2_{adj}	AIC	BIC
Liquidity adjusted CAPM (Equation 3.8)	100%	79%	74%	46%
Liquidity adjusted Fama and French model (Equation 3.9)	86%	86%	86%	86%

Table 4.7: The percentage of corporate bonds improved by a liquidity adjusted model compared to the original return models by four different measures for model selection.

4.5 Stressed Market

During the sample period from July 2017 to July 2019, there are shorter periods with increased market volatility based on the VIX data. This is also confirmed when looking at the log-returns of S&P 500, Russell 3000 and Dow Jones U.S. small cap index as one can see increased volatility when the VIX value increases. In contrast, the log-return of the S&P 500 bond index is rather stable over the sample period. Over the total sample period, however, the volatility is relatively low on average according to VIX and all the indices. The average closing value for VIX was 14.9 with a maximum of 37.3 and minimum of 9.1. The median closing value was 13.6. As values below 20 is considered benchmark for a stable and stress-free market, the values clearly suggests a time period of low stress. When incorporating intra-day movements, the highest value during the sample period was 50.3. The closing values did exceed 20 for 66 of the total 502 trading days, 25 for 17 trading days and 30 for 5 trading days. The corresponding intra-day numbers are 99, 32 and 11 trading days.

The first step of the analysis is to see if there is significant difference in the liquidity measure d for the two time periods. By taking the mean of the liquidity measure over the two time periods it can be concluded that the mean for the "normal" period is 0.31 and 0.44 for the "stressed" period. This equals an increase of 40% of the measure. It can also be concluded that the mean for the "stressed" period is 22% larger than the mean over the full set time period. Looking closer at the difference it can be concluded that the measure increased by 42% for HY-bonds and 38% for IG-bonds. This is a substantial increase for both types of bond and by looking at Figure 4.3 the increase is visible. To analyse the periods further and to get an understanding of why the liquidity measure increases so significantly a comparison of the liquidity factors, included in the multiple linear regression, over the two periods is made.

	Normal market			Stressed market		
Factor	Full set	HY	IG	Full set	HY	IG
Daily volume	$9.11 \cdot 10^6$	$7.58 \cdot 10^6$	$10.83 \cdot 10^6$	$8.58 \cdot 10^6$ (-5.8%)	$6.50 \cdot 10^6$ (-14.2%)	$10.86 \cdot 10^6$ (+0.3%)
Daily trades	36.25	31.23	41.92	40.29 (+11.1%)	28.70 (-8.1%)	52.93 (+26.3%)
Bid-ask spread	0.49	0.60	0.35	0.59(+20.4%)	0.72(+20%)	0.45(+28.6%)

Table 4.8: Table of liquidity factors and their average daily values for a normal period and a stressed period.

In Table 4.8 there are a few interesting conclusions to make. First of all it is clear that the bid-ask spread is highly correlated to the liquidity measure. As for the liquidity measure there is a substantial increase in the spread for the stressed period. Furthermore, it seems that HY-bonds and IG-bonds have different attributes when it comes to daily trading volume as well as daily trades. For HY-bonds there is a decrease of about 14% in daily volume during the stressed period which is in line with theory as a decrease in volume leads to less liquidity. There is also a small decrease in amount of daily trades which intuitively makes sense. Looking at the numbers for IG-bonds it is clear that the daily volume is not affected much by stressed market conditions. However, there is a substantial increase in amount of daily trades. This is a clear indication of that illiquidity might lead to more trades but trades of smaller quantity as discussed in Subsection 2.5.2 in the theory. Finally, it is worth mentioning that during the stressed period the average transaction price went down by 4.1%. Looking more closely at the price difference it becomes clear that the HY-bonds (5.8%) are affected more than IG-bonds (2.4%). However, the conclusion that a stressed market leads to higher illiquidity and subsequently a lower price can be made.

In total, the results shows that stress market conditions, defined as a high VIX values, has a major impact on market liquidity and thereby total risk of financial assets. During the sample period examined in this paper we did not experience any drastic market volatility increase compared to the volatility of true market crises, such as the Global financial crisis in 2007-2008 and the Covid-19 pandemic. The maximum value was 50.3 when incorporating intra-day data which is significantly lower than the values experienced during the aforementioned crises. The corresponding values for the financial crisis and Covid-19 was 89.5 and 85.5 respectively, with the proviso that Covid-19 is not over. Figure 4.8 below illustrates the VIX closing prices from July 2007 to December 2020 with the two crises as well as the sample period marked as coloured areas. The areas represents the approximate dates of the individual crises with the red area representing the approximate dates for the financial crisis, green the sample period and blue the ongoing pandemic.

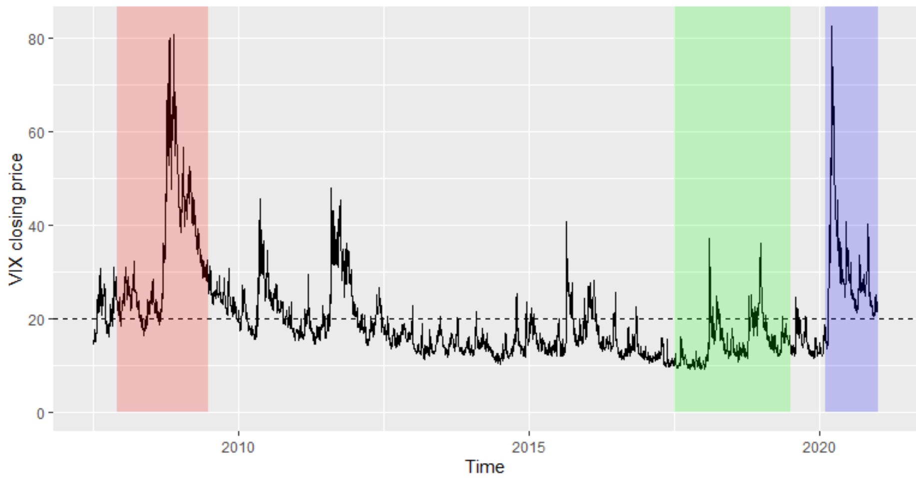

Figure 4.8: VIX closing prices from July 2007 to December 2020 with coloured areas representing approximate time period for interesting market crises. The first area (red) represents the Financial crisis, the second (green) is the sample period used in this paper and lastly (blue) the Covid-19 pandemic.

As can be seen in the figure, the market volatility over the sample period is low on average compared to the volatility experienced in true global crises. Especially when looking at the peak periods where the values are very high compared to the horizontal dashed stress-free benchmark. From September to December of 2008, by the time of the bankruptcy of Lehman Brothers and the climax of the financial crisis, the average closing price was 51.6 and in the beginning of Covid-19, from March to May 2020, the average closing price was 43.8. This further confirms that the temporary stress periods experience in the sample period is very small in comparison to global crises. Although the expected market volatility was stable, the market liquidity changed significantly as well as the liquidity factors. Hence, the results proofs that the securities examined in this paper are very sensitive to increased (expected) market volatility even at small differences and explains why Swedish income funds had to closed in early Covid-19. There are a major difference in stress condition between the sample period and Covid-19 and yet we see an increase in illiquidity by 40%. Consequently, we can assume that the liquidity drops significantly during crises causing problem to accurately estimate market values and the total risk exposure.

Chapter 5

Conclusion

In this paper we examine the liquidity of 167 corporate bonds traded on the U.S. OTC market by analysing 2,929,690 cleaned transactions provided by the Trade Reporting and Compliance Engine from 2017 to 2019. Liquidity factors supported by literature and different liquidity measures have been analysed over time and against each other using multiple linear regression methodology. Three widely accepted measures have been evaluated: (i) $ILLIQ$ developed by Amihud [2002]; (ii) HHI proposed by Heubel and Hui [1984]; and (iii) d, a volume weighted measure of the volatility of price dispersion, developed by Jankowitsch et al. [2011]. Several multiple linear regression models have been created and compared on bond level and on the entire data set separately on a daily basis. The best performing models were evaluated by an in-depth residual analysis and tested by comparing with different robustness models. Given that the measures were applicable on the sample data, they were then used to model asset return and finally analysed during stressed market conditions. To determine whether liquidity is priced for, i.e. if a liquidity premium should be incorporated when estimating asset return, a liquidity premium is added to the CAPM in the first part and replaces the SMB factor in the Fama and French factor model in the second. Lastly, the liquidity effects are examined during times of increased market volatility, referred to as stressed market conditions, by examining the changes of liquidity as the VIX value changes and as the volatility of commonly used indices increases.

After the analysis is made there are certain conclusions that can be made. Firstly, it can be concluded that the widely used liquidity measure $ILLIQ$ cannot be applied to corporate bonds on the OTC-market. The measure is most commonly used on stocks and that might explain the weak results obtained on the sample data. The HHI measure fits the data a lot better than $ILLIQ$ but the overall results are confusing since they say high yield bonds are more liquid than investment grade bonds. When performing a multiple linear regression with the measure as response variable and liquidity factors as explanatory variables all regression coefficients have expected sign except for the credit rating which explains the counter-intuitive results. Finally the d-measure that was created to fit OTC-data gives the most expected results. When performing a multiple regression, all regression coefficients are significant and their signs are supported by theory. This concludes the first part of the analysis, which consisted of finding an appropriate liquidity measure to perform further analysis on.

The second part of the analysis consisted of relating the liquidity measure to returns and by doing so proving that corporate bonds on the OTC-market are traded at a liquidity risk premium. This was done in a few different ways. Firstly the liquidity measure was incorporated in a simple linear regression with excess returns as response variable and the liquidity measure as explanatory variable. This was done for both HHI and d at bond level. Over the 167 bonds, 4% and 65% were significantly correlated with excess return for the two measures respectively. The low correlation between HHI and excess return, a percentage that is most likely explained by white noise, suggested the measure was not fit for further analysis. The rest of the analysis was only made for d and it consisted of adding the measure to the CAPM as well as replacing the SMB-factor in the Fama and French models (3-factor and 5-factor). Adding the measure to CAPM gave improved results compared to CAPM without the measure for the majority of the bonds which suggests a liquidity premium should be incorporated in return modeling. When applying the Fama and French models to the data sample the results were relatively weak. There was low significance between the factors and the excess return. Replacing the SMB-factor with the liquidity measure gave a small improvement but it can be concluded that the models do not fit the sample data very well.

Finally the effect of stressed market conditions on the liquidity measure was analysed. Two shorter sample periods within our sample periods were chosen. One period with normal market conditions and one with stressed conditions. The results were very clear, there was a significant increase in illiquidity during the stressed period. This was the case for both high yield bonds (42% increase) and investment grade bonds (38% increase). The increase was so significant that it was clearly visible when plotting the measure over the full sample period. When examining liquidity factors during the periods to see what caused the increase, the most noticeable change was an increase in the bid-ask spread (20.4% for all bonds). The conclusion that can be drawn is that stressed market conditions have a big impact on liquidity even though the stress is not overly exaggerated.

Following the analysis on stressed market conditions we want to suggest that further analysis should be done on this phenomena during periods of true economic crisis. Such as the crisis in 2007-2008 and during the outbreak of the Covid-19 pandemic. Furthermore, a more in-depth analysis of the HHI-measure can be done to get further understanding of why the credit rating affects the measure in the opposite way of intuition. Finally, out of a financial point of view, it would be interesting to see if fund performance could be increased.

Bibliography

Viral V. Acharya and Lasse Heje Pedersen. Asset pricing with liquidity risk. *Journal of Financial Economics*, 77(2):375–410, 2005. ISSN 0304-405X. doi: https://doi.org/10.1016/j.jfineco.2004.06.007.

Michael Aitken and Carole Comerton-Forde. How should liquidity be measured? *Pacific-Basin Finance Journal*, 11(1):45–59, 2003. ISSN 0927-538X. doi: https://doi.org/10.1016/S0927-538X(02)00093-8.

Yakov Amihud. Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*, 5(1):31–56, 2002. ISSN 1386-4181. doi: https://doi.org/10.1016/S1386-4181(01)00024-6.

Yakov Amihud and Haim Mendelson. Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2):223 – 249, 1986. ISSN 0304-405X. doi: https://doi.org/10.1016/0304-405X(86)90065-6.

Yakov Amihud, Haim Mendelson, and Lasse Heje Pedersen. *Market Liquidity: Asset Pricing, Risk, and Crises*. Cambridge University Press, 2013. URL `https://books.google.se/books?id=ZHi4wwEACAAJ`.

Dennis Bams and Iman Honarvar. VIX and liquidity premium. *International Review of Financial Analysis*, 74, 2021. ISSN 1057-5219. doi: https://doi.org/10.1016/j.irfa.2020.101655.

Anil Bangia, Francis X. Diebold, Til Schuermann, and John D. Stroughair. Modeling liquidity risk, with implications for traditional market risk measurement and management. In *Risk Management: The State of the Art*, pages 3–13. Springer US, 2001. doi: https://doi.org/10.10072F978-1-4615-0791-8_1.

Jack Bao, Jun Pan, and Jiang Wang. The Illiquidity of Corporate Bonds. *The Journal of Finance*, 66(3):911–946, 2011. doi: https://doi.org/10.1111/j.1540-6261.2011.01655.x.

Jeremy Berkowitz. Breaking the silence. *Risk*, 13(10):105–108, 2000. URL `https://www.risk.net/sites/risk/files/import_unmanaged/risk.net/data/Pay_per_view/risk/technical/2000/risk_1000_liquidity.pdf`.

Christopher M. Bishop. *Pattern Recognition and Machine Learning*. Springer, January 2006. ISBN 0-387-31073-8.

Sugato Chakravarty and Asani Sarkar. Liquidity in U.S. Fixed Income Markets: A Comparison of the Bid-Ask Spread in Corporate, Government and Municipal Bond Markets. 1999. doi: https://dx.doi.org/10.2139/ssrn.163139.

Tarun Chordia, Richard Roll, and Avanidhar Subrahmanyam. Market Liquidity and Trading Activity. *The Journal of Finance*, 56(2):501–530, 2001. doi: https://doi.org/10.1111/0022-1082.00335.

Kee H. Chung and Chairat Chuwonganant. Uncertainty, market structure, and liquidity. *Journal of Financial Economics*, 113(3):476–499, 2014. ISSN 0304-405X. doi: https://doi.org/10.1016/j.jfineco.2014.05.008.

Shane A. Corwin and Paul Schultz. A Simple Way to Estimate Bid-Ask Spreads from Daily High and Low Prices. *The Journal of Finance*, 67(2):719–759, 2012. URL `https://www.jstor.org/stable/41419709?seq=1`.

David Cosandey. Adjusting value-at-risk for market liquidity. *Risk*, (14):115–118, 2001. URL `https://www.riseofthewest.net/papers/200110_RiskMag_AdjustingValueAtRiskForMarketLiquidity.pdf`.

Vinay T. Datar, Narayan Y. Naik, and Robert Radcliffe. Liquidity and stock returns: An alternative test. *Journal of Financial Markets*, 1(2):203–219, 1998. ISSN 1386-4181. doi: https://doi.org/10.1016/S1386-4181(97)00004-9.

Jens Dick-Nielsen. Liquidity Biases in TRACE. *Journal of Fixed Income*, 19(2): 43–55, 2009. ISSN 1059-8596. doi: https://dx.doi.org/10.2139/ssrn.1424870.

Jens Dick-Nielsen. How to Clean Enhanced TRACE Data. 2014. doi: https://dx.doi.org/10.2139/ssrn.2337908.

Mohamed Elshenawy, Mohamed El-darieby, and Baher Abdulhai. Automatic Imputation of Missing Highway Traffic Volume Data. In *2018 IEEE International Conference on Pervasive Computing and Communications Workshops (PerCom Workshops)*, pages 373–378, 2018. doi: 10.1109/PERCOMW.2018.8480120.

Jan Ericsson and Olivier Renault. Liquidity and Credit Risk. *The Journal of Finance*, 61:2219 – 2250, 2006. doi: https://doi.org/10.1111/j.1540-6261.2006.01056.x.

Cornelia Maria Ernst, Sebastian Stange, and Christoph Kaserer. Accounting for Non-Normality in Liquidity Risk. *The Journal of Risk*, 14(3):3–21, 2009a. doi: https://dx.doi.org/10.2139/ssrn.1316769.

Cornelia Maria Ernst, Sebastian Stange, and Christoph Kaserer. Measuring Market Liquidity Risk - Which Model Works Best? 2009b. doi: http://dx.doi.org/10.2139/ssrn.1328480.

Eugene F. Fama and Kenneth R. French. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3–56, 1993. ISSN 0304-405X. doi: https://doi.org/10.1016/0304-405X(93)90023-5.

Finansinspektionen. Dialogmöte med fondbolag, 2020. URL `https://fi.se/sv/publicerat/nyheter/2020/dialogmote-med-fondbolag/`. Accessed 10 February 2021.

Michael J. Fleming. Measuring Treasury Market Liquidity. 2001. doi: https://doi.org/10.2139/ssrn.276289.

Nils Friewald, Rainer Jankowitsch, and Marti G. Subrahmanyam. *Illiquidity or credit deterioration: A study of liquidity in the US corporate bond market during financial crises*, pages 159–200. 2013. doi: https://doi.org/10.1142/9789814417501_0006.

FTSE Russell. *Russell 3000 Index*. FTSE Russell, 2021. Accessed 2021-03-09.

Pierre Giot and Joachim Grammig. How large is liquidity risk in an automated auction market? *Empirical Economics*, 30(4):867–887, 2005. doi: https://doi.org/10.1007/978-3-7908-1992-2_6.

Ruslan Y. Goyenko and Andrey D. Ukhov. Stock and Bond Market Liquidity: A Long-Run Empirical Analysis. *The Journal of Financial and Quantitative Analysis*, 44(1):189–212, 2009. ISSN 00221090, 17566916. URL `http://www.jstor.org/stable/40505920`.

Sanford J. Grossman and Merton H. Miller. Liquidity and Market Structure. *The Journal of Finance*, 43(3):617–633, 1988. doi: https://doi.org/10.1111/j.1540-6261.1988.tb04594.x.

Barbara Heubel and Bladwin Hui. *Comparative Liquidity Advantages Among Major U.S. Stock Markets*. Number 84081 in Financial Information Group Study Series. Data Resources Inc., 1984.

Corporate Finance Institute. What is over-the-counter?, 2021. URL `https://corporatefinanceinstitute.com/resources/knowledge/trading-investing/over-the-counter-otc/`. Accessed 4 February 2021.

Rainer Jankowitsch, Amrut Nashikkar, and Marti G. Subrahmanyam. Price dispersion in OTC markets: A new measure of liquidity. *Journal of Banking Finance*, 35(2):343–357, 2011. ISSN 0378-4266. doi: https://doi.org/10.1016/j.jbankfin.2010.08.016.

Frank De Jong and Joost Driessen. Liquidity Risk Premia in Corporate Bond Markets. 2006. doi: https://doi.org/10.2139/ssrn.686681.

John Maynard Keynes. *A Treatise on Money*, volume 2. Macmillan and CO., Limited, ST. Martin's Street, London, 1914.

Justin Kuepper. Cboe Volatility Index (VIX). 2021. URL `https://www.investopedia.com/terms/v/vix.asp`.

Mark Lang and Mark Maffett. Transparency and liquidity uncertainty in crisis periods. *Journal of Accounting and Economics*, 52(2):101–125, 2011. ISSN 0165-4101. doi: https://doi.org/10.1016/j.jacceco.2011.07.001.

Kuan-Hui Lee. The world price of liquidity risk. *Journal of Financial Economics*, 99 (1):136–161, 2011. ISSN 0304-405X. doi: https://doi.org/10.1016/j.jfineco.2010.08.003.

Bruce Mizrach. Analysis of Corporate Bond Liquidity. Technical report, 2015. URL `https://www.finra.org/sites/default/files/OCE_researchnote_liquidity_2015_12.pdf`.

Erik Nordenskjöld. Interview on the 27th of May, 2021. Citroneer AB.

Ruzita Adbul Rahim and Abu Hassan Shaari Mohd Nor. A comparison between fama and french model and liquidity-based three-factor models in predicting the portfolio returns. *Asian Academy of Management Journal of Accounting and Finance*, 2(2):43–60, 2006.

Knut Kainz Rognerud. Mörkaste dagen på stockholmsbörsen i modern tid, 2020. URL `https://www.svt.se/nyheter/ekonomi/fem-ars-borsuppgang-utraderad-1`. Accessed 10 February 2021.

S&P Global. *Dow Jones U.S. small-cap total stock market index Fact Sheet*. S&P Global, 2021a. Accessed 2021-03-10.

S&P Global. *S&P 500 Fact Sheet*. S&P Global, 2021b. Accessed 2021-03-10.

S&P Global. *S&P 500 bond index Fact Sheet*. S&P Global, 2021c. Accessed 2021-03-10.

Youjin Su. *Liquidity and asset pricing*. PhD thesis, University of Aberdeen, 2013. URL `https://ethos.bl.uk/OrderDetails.do?uin=uk.bl.ethos.606440`.

Sarahelen Thompson and Mark Leonard Waller. Determinants Of Liquidity Costs In Commodity Furures Markets. 1988. URL `https://ideas.repec.org/p/fth/colufu/172.html`.

A

Appendix 1 - Bond-level Residual Analysis

A.1 Hui-Heubel Liquidity Ratio

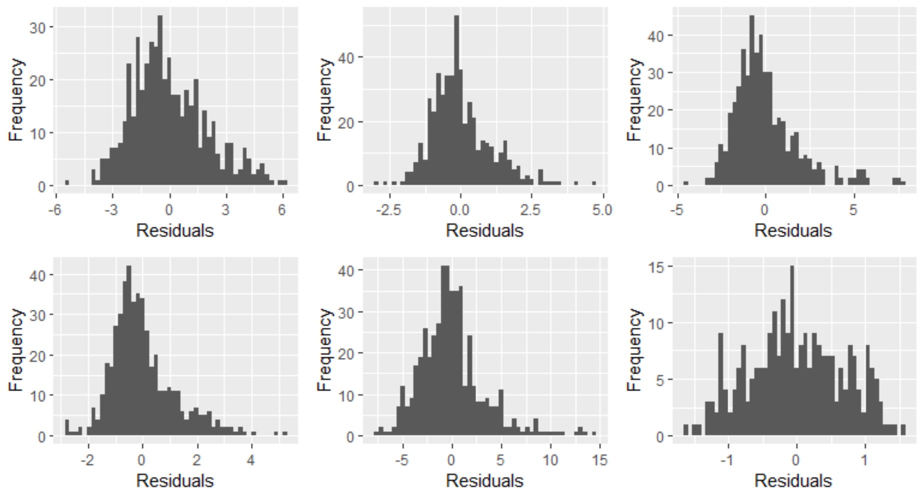

Figure A.1: Histogram of ordinary residuals for three randomly chosen HY-bonds (bottom) and three IG-bonds (top).

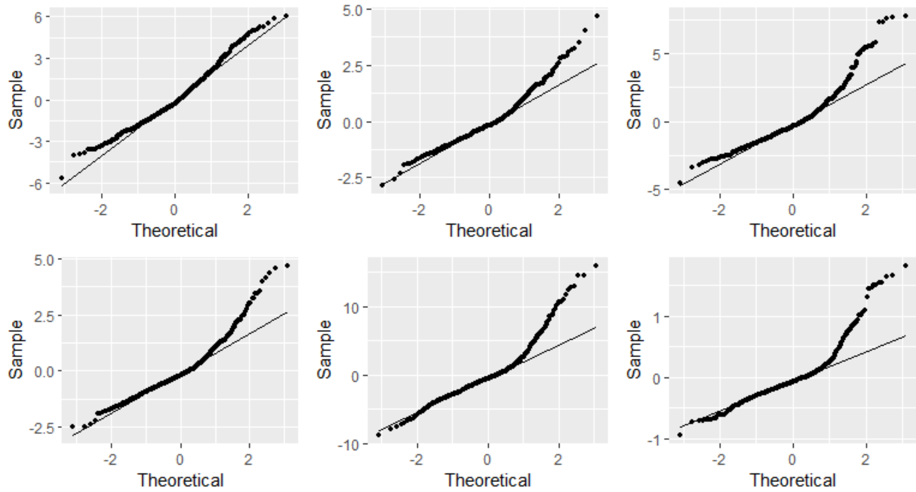

Figure A.2: Q-Q plots of residuals for three randomly chosen HY-bonds (bottom) and three IG-bonds (top).

A.2 d-measure

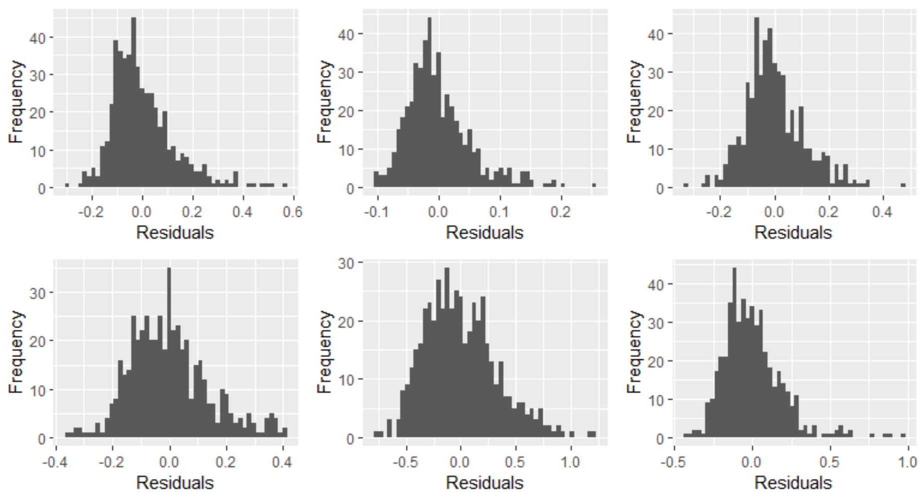

Figure A.3: Histogram of ordinary residuals for three randomly chosen HY-bonds (bottom) and three IG-bonds (top).

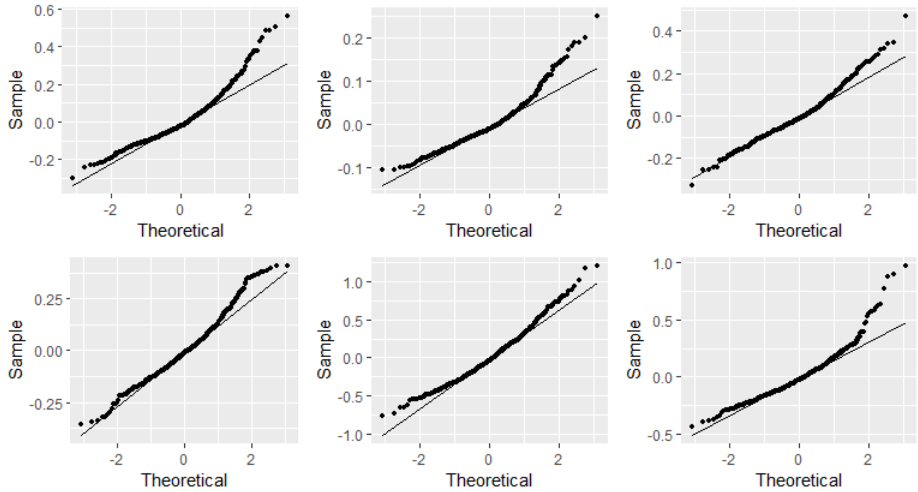

Figure A.4: Q-Q plots of residuals for three randomly chosen HY-bonds (bottom) and three IG-bonds (top).

B

Appendix 2 - Full Set Residual Analysis

B.1 Hui-Heubel Liquidity Ratio

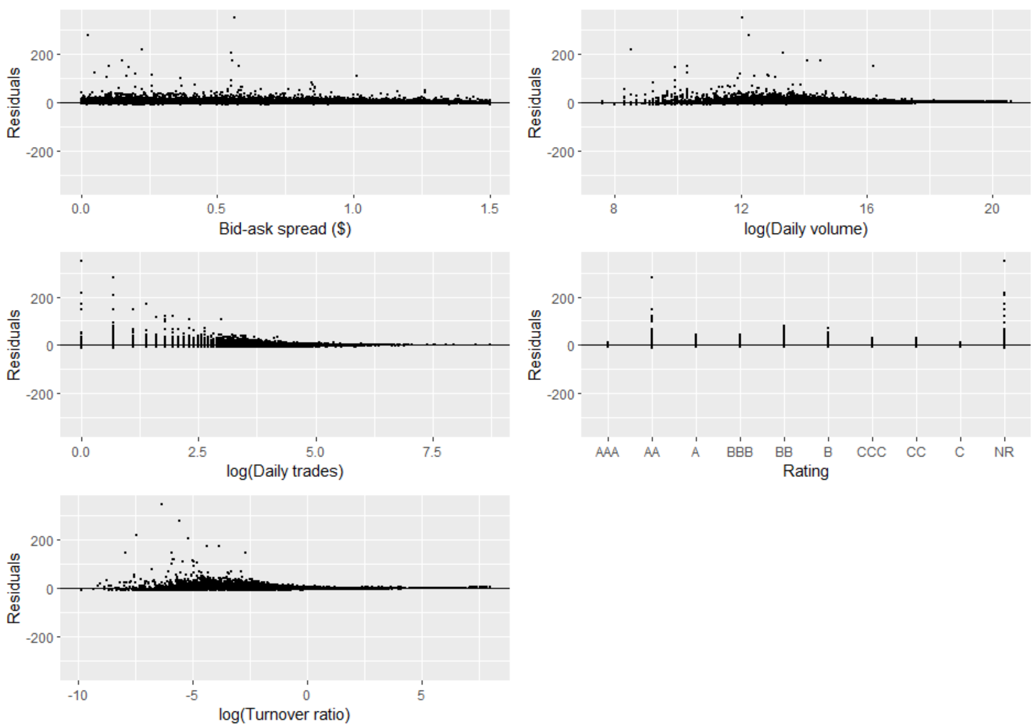

Figure B.1: Ordinary residuals plotted against the different variable for the multiple linear regression model presented in Table 4.5.

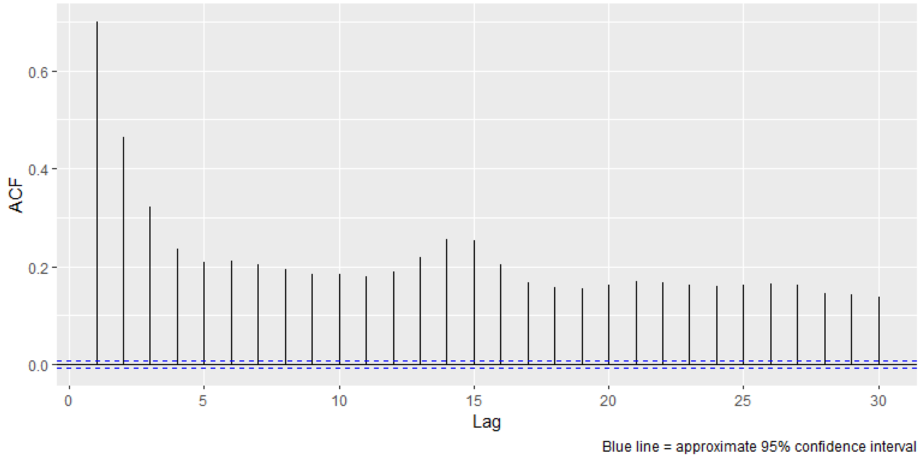

Figure B.2: Correlogram of the residuals of the multiple linear regression model presented in Table 4.5. The blue line represents a 95% confidence interval.

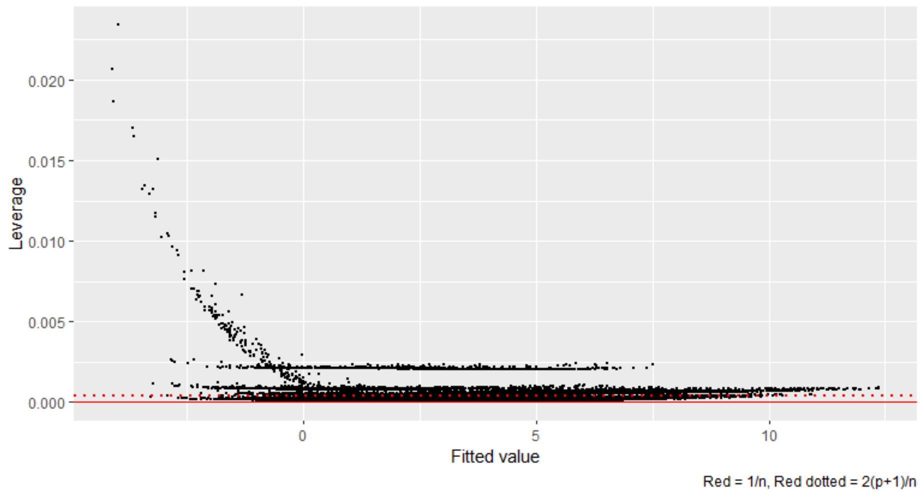

Figure B.3: Leverage plotted against the fitted $\hat{Y}$ for the multiple linear regression model presented in Table 4.5 with horizontal lines at $v_{ii} = 1/n$ (red line) and $v_{ii} = 2(p+1)/n$ (red dotted line).

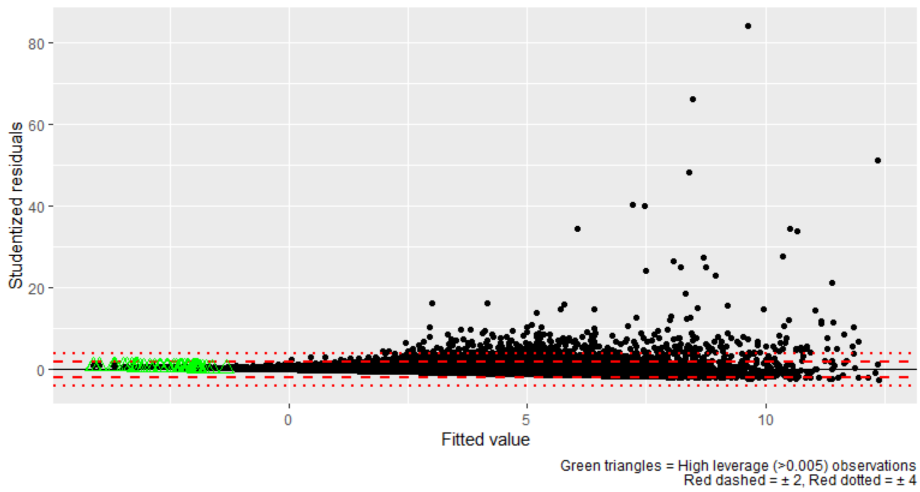

Figure B.4: Studentized residuals plotted against the fitted $\hat{Y}$ for the multiple linear regression model presented in Table 4.5 with horizontal lines at ± 2 (red dashed line) and ± 4 (red dotted line). Observations with $v_{ii} > 0.005$ are identified by the green triangles.

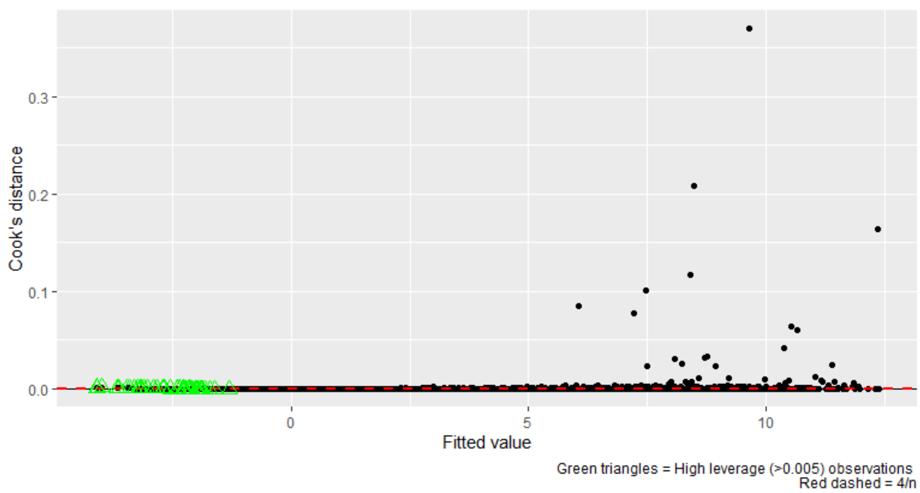

Figure B.5: Cook's distance for the multiple linear regression model presented in Table 4.5 with a red dashed horizontal line at $4/n$. Observations with $v_{ii} > 0.005$ are identified by the green triangles.

B.2 d-measure

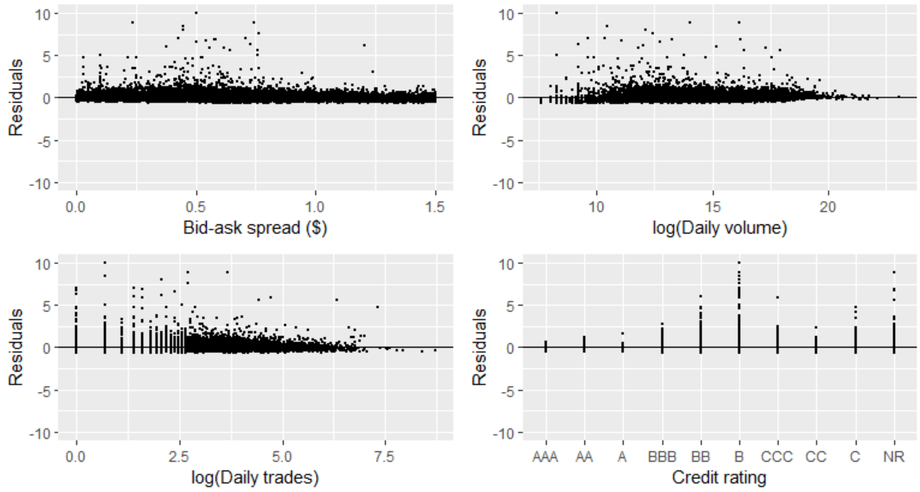

Figure B.6: Ordinary residuals plotted against the different variable for the multiple linear regression model presented in Table 4.6.

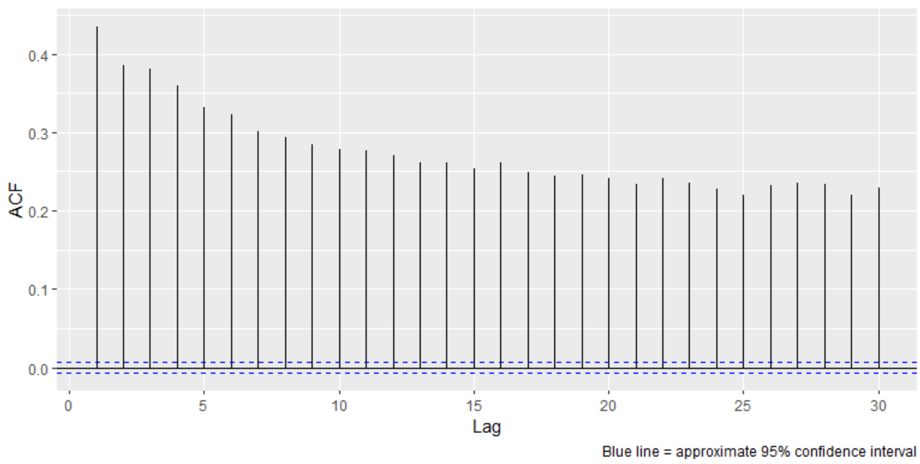

Figure B.7: Correlogram of the residuals of the multiple linear regression model presented in Table 4.6. The blue line represents a 95% confidence interval.

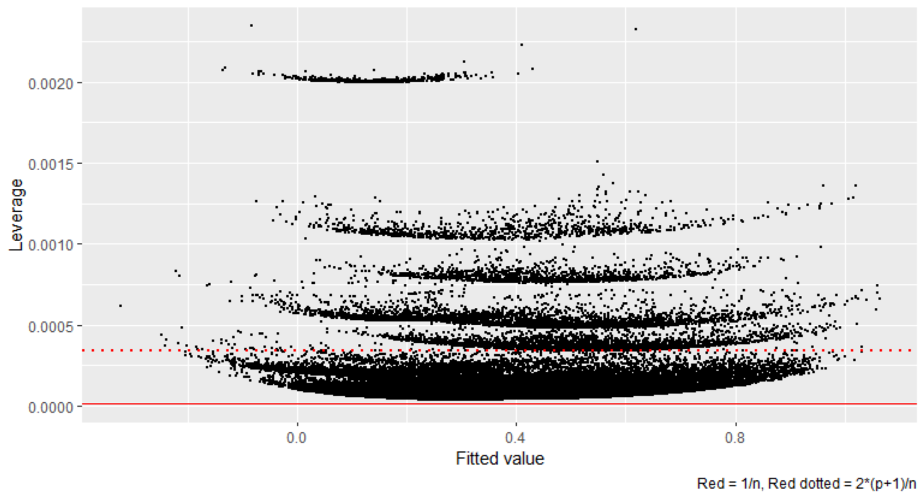

Figure B.8: Leverage plotted against the fitted $\hat{Y}$ for the multiple linear regression model presented in Table 4.6 with horizontal lines at $v_{ii} = 1/n$ (red line) and $v_{ii} = 2(p+1)/n$ (red dotted line).

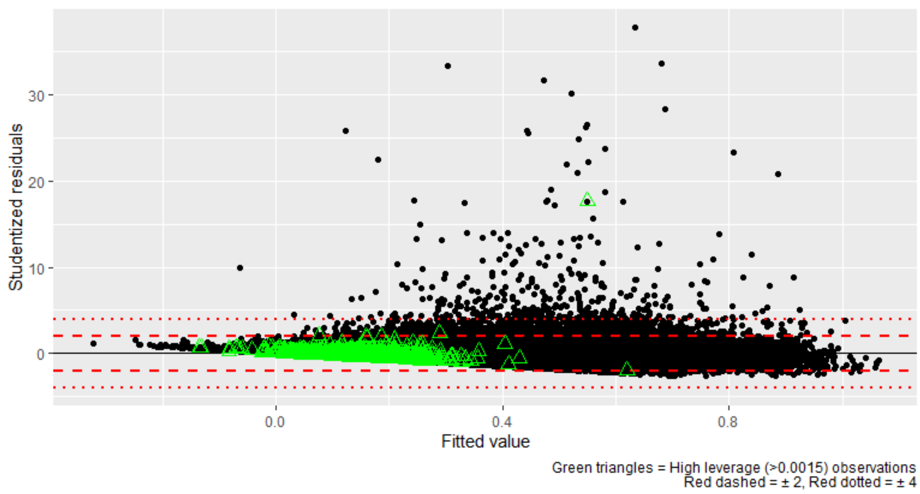

Figure B.9: Studentized residuals plotted against the fitted $\hat{Y}$ for the multiple linear regression model presented in Table 4.6 with horizontal lines at ± 2 (red dashed line) and ± 4 (red dotted line). Observations with $v_{ii} > 0.0015$ are identified by the green triangles.

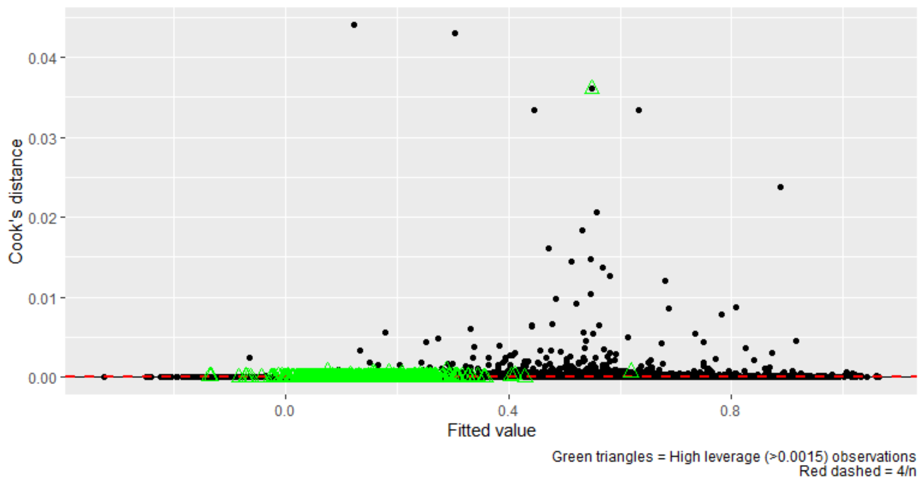

Figure B.10: Cook's distance for the multiple linear regression model presented in Table 4.6 with a red dashed horizontal line at $4/n$. Observations with $v_{ii} > 0.0015$ are identified by the green triangles.