Master Thesis

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2021

Nominal GDP Targeting in Sweden

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A New Keynesian DSGE Model Approach

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Abstract

Since the end of the Great Moderation, there has been a great surge in studies proposing alternatives to the current dominant monetary policy regime of inflation targeting. One popular such alternative has become NGDP targeting. However, as studies tend to gravitate towards larger economies, an unintended side effect has become a significant degree of homogeneity in regards to the economic setting where the target is evaluated. With the purpose of addressing and highlighting this issue, this study investigates the target in the case of Sweden, a country where no empirical study on the relative merits of NGDP targeting has been done before.

The paper empirically explores the topic of NGDP targeting vis-á-vis other monetary policies within a New Keynesian DSGE model. The model is fitted to Sweden via a mix of calibration and Bayesian estimation of the parameters, employing data of the Swedish economy spanning the period 1993:2-2014:4. Evaluation is done in regards to welfare losses, where output gap targeting is found to produce the most desirable outcome followed by an estimated Taylor type rule. The finding of a Taylor type rule to outperform a NGDP target in such a setting goes against much of the current literature. The results thus perhaps suggest a significant degree of heterogeneity in the performance of the target across different economies. As such, the study motivates further investigation into the relative merits of NGDP targeting across a more diverse set of economies.

Keywords: NGDP, NGDP Targeting, DSGE, Monetary Policy, Sweden

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List of Abbreviations

DSGE	Dynamic Stochastic General Equilibrium
ECB	European Central Bank
FED	Federal Reserve
FOC	First Order Condition
GDP	Gross Domestic Product
IT	Inflation Targeting
MH	Metropolis-Hastings
NGDP	Nominal Gross Domestic Product
NGDPT	Nominal Gross Domestic Product Targeting
SCB	Statistiska Centralbyrån
US	United States
ZLB	Zero Lower Bound

1 Introduction

The history of monetary policy tends to follow the business cycle. A monetary regime rises in prominence as the economy is flourishing just to be brought down and replaced when the cycle collapses and the inadequacies of the current system is laid bare. Though the history of central banking thus is one of failure, the continued reforms show a historic willingness to do again and to do right. But what about the Great Recession? Does the apparent persistence of our current Inflation Targeting (IT) regimes perhaps display a new inertia in the realm of monetary policy. Is inflation targeting truly the best alternative, or is it perhaps living off past achievements, causing unwarranted and unwanted complacency?

The fact is that with the ushering in of the Great Moderation, commonly ascribed to the period 1984-2007 in the case of the United States (US) (though arriving arguably later in the case of Sweden), many considered the holy grail of monetary policy to have been found and the problem solved. Truth be told, had the Great Moderation continued, there would be little to no need for further research in the area. But, in December of 2007, the Great Recession came and it came to stay. For 19 months it ravaged the global economy and in its wake it left economies in a stagnant era of low interest rates and low growth. With it, however, came also a renewed and amplified interest in monetary policy from which new research came to flourish, spread, and proliferate. The debate was again on, flaws were exposed, remedies proposed, new ideas spawned, and as in the case of Nominal Gross Domestic Product (NGDP) targeting, old ones brought back off dusty shelves.

NGDP targeting in its modern theoretical form is often credited to Meade (1978) and Tobin (1980), whilst its resurgence tends to instead be credited to a blog post by Robert Sumner. An economist that like many others considered monetary policy solved and had moved on to other areas of research as the crisis hit. Sumner garnered the support of economic press and many colleagues alike with his post, and has since published extensively on the topic, summarizing his ideas in Sumner and Roberts (2018). NGDP targeting retains many of the desirable properties of our current inflation targeting, entails no real breaking from the Volcker era low inflation, and further offers up a brighter view on dealing with the Zero Lower Bound (ZLB).

In this thesis I explore NGDP targeting for the case of Sweden by employing the New Keynesian Dynamic Stochastic General Equilibirum (DSGE) Model of Garín, Lester and Sims (2016), there used to explore the alternative in the case of the US. The model features both wage and price rigidities, as well as several other aspects that have been shown to satisfactorily capture salient business cycle properties as well as the dynamic effects of monetary policy shocks. The model is fitted to Sweden using a mix of calibration and Bayesian estimation, where the data of the Swedish economy spans the period 1993:2 to 2014:4. No empirical DSGE study of NGDP targeting has been performed on Swedish data before to my knowledge, and as such, the fitting of the model to Sweden is motivated by the purpose of the study. Namely to contribute to the current literature by providing further insights into the performance of NGDP targeting vis-á-vis other policies under differing economic circumstances. The policies considered for comparison are output gap targeting, IT, and two Taylor type rules.

Output gap targeting is found to be the most desirable target across most settings in accordance with Garín et al. (2016), where the results even appear robust to the consideration of imperfect output gap estimations. However, contradictory to Garín et al. (2016), an estimated Taylor rule is found preferable to an NGDP target. This suggests perhaps significant heterogeneity in the performance of the target across differing economies, as studies tend to find NGDP the preferable alternative, but then to also be estimated on either US or Euro Area data (Garín et al., 2016; Beckworth and Hendrickson, 2020; Fackler and McMillin, 2020; Diallo, 2019). These findings are conditional on an estimated high degree of habit formation in Sweden. Due to differences in the models, no direct comparison can be done to the estimates of the parameter in the Swedish Riksbank´s DSGE models MAJA II, and RAMSES II. The parameter value is, however, within the range of prior findings in the literature (Havranek et al., 2017).

The remainder of the paper is structured as follows. In the next section the conceptual foundations of nominal GDP targeting is laid forth, followed by a review of the most relevant literature on the subject. The section wraps up with a look at the rule in the case of Sweden. In section 3 the model is presented, whilst section 4 looks at the calibration and estimation of the parameters. Section 5 looks briefly at solving the model given the objective of welfare evaluation. In section 6 the results are presented as well as contrasted to the prior literature, and lastly, section 7 concludes.

2 Theoretical/Literature Review

This section aims to present the literature and theoretical review, covering the basics of NGDP targeting, as well as try and place it in the current debate. This is followed by the related literature and previous empirical findings, before finally relating the target to the specific case of Sweden.

2.1 Nominal GDP Targeting

Just like inflation, the output gap, the exchange rate, or any number of economic indicators, NGDP can serve as a target and guide for monetary policy. As such, in its essence, NGDP targeting (NGDPT) is simply a monetary policy rule where the level or the growth of nominal spending is targeted. Given that for every unit of currency spent there is a unit earned, it is often also referred to as nominal income targeting. Whether indeed having the monetary policy target nominal spending is the best of possible targets, is essentially what the debate about NGDPT boils down to, and which will be covered in the adjacent section 2.2. In this section focus instead lies on basics of the target in and of itself.

A NGDP measure makes no adjustments for inflation as is done for real GDP, but is instead a compound measure of both inflation and real GDP growth. Targeting this compound rather than just the inflation rate, allows the respective components to vary. Assuming for example 3 percent real GDP growth to be the long run stable trend of an economy, a NGDP growth target of 5 percent would, similarly to the policy of many of today's central banks, imply 2 percent stable long run inflation. However, unlike with inflation targeting, these 2 percent are allowed to vary in the short run. Rising to 4 percent if growth is down to 1 percent (which should help heat up the economy), or similarly retracting to 0 if growth is 5 percent (which should help cool down the economy). Therefore, monetary policy will be focused on stabilizing the business cycle, rather than just prices. The target, however, does so without letting inflation completely loose, where emphasis can still be on a 2 percent long-run target, and where changes from it will be restricted to be kept inversely proportional to the changes of real GDP from its target. An attractive quality to those with memory of its rampant levels prior to the Great Moderation.

For increased clarity one can look at NGDP targeting through the lens of the simple *equation of exchange*:

$$NGDP = price\ of\ goods \times quantity\ of\ goods = money\ supply \times velocity \qquad (1)$$

For the equality to hold with a fixed NGDP target the quantity of transactions and the price level must have an inversely proportional relationship. That is, the money supply must move in the opposite direction to changes in the velocity to equate the path of NGDP with its target. As such, a NGDP target responds to demand shocks in the same fashion as IT. Where an increase or decrease in the velocity of money brought by the demand shock, as individuals increase or decrease the rate at which they spend, would entail a reduction or increase in the money supply that fully offset the change (Sumner and Roberts, 2018). It further also inherently cushions the economy to supply shocks, a quality not shared by IT, which is one of the main arguments in favour of NGDPT. Under an IT regime, increased prices on for example oil imports cause the central bank to tighten policy to keep inflation on target. Such a response has unfavourable effects on labour markets as it forces other non-oil related prices to drop to compensate for the increase. This in turn forces producers of these goods to lay off workers as they are unable to meet labour costs at the lower prices on their products. In fact, this is what happened in 2008, both at the Federal Reserve (FED) and the European Central Bank (ECB), but later also again in 2011 at the ECB (Hallet, Lechthaler, Reicher, Tesfaselassie, Blot, Creel and Ragot, 2015; Beckworth, 2015; Sumner and Roberts, 2018). Under a NGDP targeting regime, such a strict reeling in of the inflation spike caused by the increased import prices would not be necessary, as the burden can be shared between prices and output, which would be favourable for labour markets and the economy alike.

I will end the section with a slight transgression relevant both for inflation and NGDP targeting, which is whether to target the rate or the level of the variable. As it is a whole debate on its own, I will make only the small remark that the current literature tends to have started leaning over to the level targeting side since the financial crisis. The most striking indication of this might be the recent FED switch to inflation level targeting in 2021 (Martínez-García, Coulter and Grossman, 2021). The reason there is no real need to delve deeper into the subject is that in the model at hand, they are unconditionally equivalent. Targeting a zero rate or targeting the level t-1 is equivalent unless one condition on the target not being met the prior period, which is not done in this study. Instead an ability of the central bank to hit its target is assumed, similarly to Svensson (2003), and further a common assumption in theoretical studies (Hallet et al., 2015).

2.2 NGDP Targeting, an Interesting Alternative

2.2.1 The Current Critique of Inflation Targeting

There are essentially three aspects for which the current dominant monetary regime, IT, is being criticised in the current literature and ongoing debate according to Andersson and Claussen (2017). Though NGDP might not offer up a complete solution to either of these, I will try to briefly cover what role it might play to better show how it might fit into the debate.

Firstly, inflation targeting banks tend to be too focused on inflation, not putting sufficient weight on production and unemployment or in the terminology of Mervyn King, to be "inflation nutters". That is, however, a commonly held critique also amongst proponents of the framework. The aforementioned Mervyn King being one such clear example, who instead argues for a change of the relative weights on different economic indicators within a flexible IT framework. One such instance is his 2015 co-authored external and independent review of the Swedish Riksbank, where he argues that case for Sweden, which will be discussed more in section 2.4. If one is to accept, however, that the monetary policy ought to target the real economy more, NGDP would indeed be an alternative. As it can be thought of as a special case of flexible IT with a greater weight on output than traditionally put on it within the IT framework. This theoretical equivalence between the two targets can be seen as pro for NGDPT, in the sense of being rather easily implementable as it entails no radical departure from the current system as argued by Lechthaler, Reicher, Mewael, & Tesfaselassie (2015) in European Parliament notes on IT vis-á-vis NGDPT. But also a con, as argued by Blot, Creel, & Ragot (2015) in the same notes, as the relative closeness of the two regimes should offer only very marginal benefits which would not be likely to compensate for other complications entailed by a regime switch (Hallet et al., 2015).

Secondly, inflation targeting banks take too little notice of imbalances in the economy and financial risks. Andersson and Claussen (2017), however, argue that since it is possible under a flexible IT regime to target imbalances in the economy and financial risks, the problem again does not lie in the target in and of itself, but again rather in the relative weights. The critique thus perhaps only shows a relative difference in the importance put on these variables by the central bank and its critics. Andersson and Claussen (2017) further go on to argue that it is also difficult to tell if and when there are financial imbalances and risks, and that there is no clear view on an observable and quantifiable target for it in monetary policy strategy or mandate. An argument in favour of NGDP worth mentioning in regards to this, is its tendency to slightly reduce asset market instabilities by reducing asset bubbles. The reasoning being that these bubbles tend to form under periods of above trend NGDP growth. Though not exclusively Sumner (2012) argues, as shown by the housing bubble leading up to the Great Recession being accompanied by just slightly above average NGDP growth. Others, however, for instance Selgin, Beckworth and Bahadir (2015), find NGDP targeting likely to have reduced the severity of the housing bubble also, despite the only slightly above average NGDP growth, where they find FED policy over accommodative in the post-2001 dot.com crash period.

Thirdly, inflation targeting banks are unable to sufficiently stimulate their economies when close to the ZLB, as evident from their very contemporary struggles since the financial crisis. For this point of the debate they are less concerned with inflation vis-á-vis NGDP, or any other alternative economic indicator for that matter. Instead the focus lies on the earlier mentioned discussion on whether it is better to target the growth rate or the level of said indicator. By targeting the level, one introduces history-dependence to the target, and as it turns out, not letting bygones simply be bygones helps steer agents' expectations in the right direction. An aspect of NGDP worth mentioning here is the earlier discussed compound nature of it that allows the respective components to vary. When in a low growth period, inflation will make up for the slack in GDP, and the higher inflation in turn reduces the real interest rate, providing additional stimulus. As such it has a built-in mechanism from which one can make quite straight parallels to another popular suggestion to deal with the ZLB, namely raising the inflation target, as suggested by both Blanchard, Blanchard, Mauro and Dell'Ariccia (2010) and Krugman (2012).

2.2.2 The Argued Benefits of NGDP

In the literature there are two qualities of NGDP that come up frequently and which one might perhaps refer to as its primary strengths. The first relates to supply shocks as mentioned briefly already in section 2.1, and the second relates to the knowledge problem faced by central banks.

In the case of oil price shocks, as discussed in section 2.1, IT is in reality not completely hapless, with the proposed quick fix of looking at an inflation measure that excludes energy prices. A case in point could be the FED's ability to avoid tightening policy in response to oil and food price spikes in 2011, still within its IT framework. However, the European Central Banks's (ECB) inability to do the same, repeating its mistake from just a few years earlier in 2008, twice, perhaps puts the robustness of such a technique into question (Beckworth, 2015). The greater point to be made, however, is that supply shocks are not localised to the oil sector, but can instead hit the economy anywhere. The productivity spike brought by computers in the 1990s being one such example, and the housing boom in 2004-2006 being another (Sumner, 2012). And with no quick fix to the

IT framework for the general case of supply shocks, this is an aspect that puts NGDP in quite favourable light. By allowing the burden of unfavourable supply shocks to be equally borne by inflation and GDP, the economy is better cushioned against the shock and labour markets should operate smoother as discussed in section 2.1.

As for the knowledge problem, it was noted already by Hayek (1945) that due to the fact that the information necessary for optimal economic planning is distributed amongst many economic agents, the complete picture is not available to any one agent or authority, such as the central bank. This is the basis for the favour in much literature for rules over discretion, as the information for accurate active policy is simply not available. This knowledge problem has later been extended beyond the pure discretion case to also include constrained discretionary monetary policy, including the use of a Taylor Rule in a flexible IT regime, by Orphanides (2000, 2002*a*,*b*, 2004). He shows how the knowledge problem arises also for the variables in a Taylor rule, where the prime culprit is the output gap, the estimation of which is highly prone to errors. His 2004 study even suggests contemporary output gap estimation to perhaps be the reason for the rampant inflation of the 70s, and Beckworth and Hendrickson (2020) further suggests also the favourable economic outcomes during the Great Moderation. The reasoning being that the 70s were characterised by uncommonly large errors in the output gap estimates, whilst the Great Moderations was characterised by uncommonly accurate estimates. If the central bank were to instead follow a NGDP target, they would not require real time knowledge of the output gap, making it less prone to measurement errors and less reliant on an omnipotent policy maker (Beckworth and Hendrickson, 2020). In fact, Beckworth and Hendrickson (2020) find output gap estimation errors to be the cause of as much as 13 percent of the actual variation in the variable, thus creating additional and unnecessary volatility in the economy.

2.2.3 The Argued Problems of NGDP

One argument against NGDP is in favour of more discretion, in the sense that the target might be too restrictive by weighing inflation and the real economy equally in all situations. Another common concern is the potential weakening of the anchoring of inflation expectations, for which Volcker and others fought so hard. There is further a more practical issue of the target, related to the data quality of NGDP, namely that it is published with long lags, and often revised quite substantially (Andersson and Claussen, 2017). However, though inflation is accessible more frequently and perhaps accurately it does not come without its own set of problems. Key being the simple fact that there is no clear consensus on how to define it in most cases. Take a phone for instance, how much of its price increase in the last 10 years is inflation, and how much is due to qualitative improvements (Sumner and Roberts, 2018). There is also the fact that CPI has been shown to systematically overestimate inflation, and that the new measures employed since have not solved the problem, still perhaps overstating it by 1-0.5 percentage points (Sivák, 2013).

Another common argument against NGDP is the flip side of a common argument made for it by its proponents, namely the ease or difficulty of communicating the target to the public. Whilst critics argue that this less-known indicator of NGDP would be more difficult for agents to relate to than the well known target of inflation, proponents point at episodes such as during the Great Recession. When the public appeared outraged at the fact that the FED wanted to raise its cost of living in the middle of a recession where they were already struggling to make ends meet. Having the central bank communicate a goal of increasing economic activity by 5 percent rather than increasing prices by 2 percent would perhaps have been better for public relations. But as can be seen, the arguments going either way are in the end quite subjective to the observer's belief on the behaviour of the public.

2.3 Related Literature

NGDPT is by no means a new phenomenon, with its modern foundational roots often ascribed to Meade (1978) and Tobin (1980), and was around that time an integral part of the policy debate. Some, however, look even further back for its foundation, such as Christensen (2011) who credits Friedman (1971) for having laid the groundwork for something resembling nominal GDP targeting with his analysis of the connection between the money supply and level of nominal income. Or even Örn (1999), who finds NGDP targeting to be a modern take on Davidson´s norm, pointing to the practical equivalences between the two, and which he had laid fourth already in the early 1900s. However, to whomever you accredit NGDPT, as New Zealand took the lead in 1989 as the first country to adopt an official inflation target and the world's central banks followed suit, ushering in the Great Moderation, the target fell out of fashion (Graham and Smith, 2012).

As such, literature on the subject during the late 90s, and 2000s is sparse, with some notable exceptions in Hall and Mankiw (1993), Jensen (2002), and Kim and Henderson (2005). Hall and Mankiw (1993) investigated the target in a simple aggregate demand and supply model, simulations of which suggest that nominal income targeting would have produced lesser volatility in price levels and inflation than historical policy. They, however, find a hybrid target, having the monetary policy primarily target the level of real output and secondly the price level, to be even more desirable. Jensen (2002) employs a simple linear New Keynesian model with emphasis on forward-looking private agents to look at NGDP growth targeting. His focus lies on NGDPT producing more inertia in monetary policy than IT, a quality needed for optimal monetary policy in the New Neoclassical Synthesis framework. He concludes NGDPT to be favoured in his setup but draws no further conclusions than that it merits further research on the comparative performance of NGDPT vis-à-vis IT. Finally Kim and Henderson (2005) look at the target in a closed economy model with optimizing agents, monopolistic competition, and one period contracts, whilst keeping the model simple enough for exact solutions. They find NGDPT to dominate IT for plausible parameter values, and further also for the superiority of NGDPT to be positively related with the importance of productivity shock. Though the relationship had already been established by several theoretically, they are one of the first to provide added empirical proof (Selgin, 2018).

Though these studies are empirical they tend to favour simplicity, a desirable property to be sure, but more rigorous and contemporary methods have since then been employed in the literature since the newfound interest in the target following the Great Recession. One of the first such studies is by Garín et al. (2016), that investigates nominal GDP targeting in the workhorse framework of monetary policy, DSGE models. They employ two new Keynesian DSGE models, a basic calibrated one and a medium sized one with a mix of calibrated parameters and ones estimated on US data for the period 1984:1-2007:3. With welfare as the basis for comparison they contrast the target with IT, output gap targeting, and a Taylor rule. They find output gap targeting to produce the lowest welfare losses, but with nominal GDP targeting coming in at second place, ahead of both IT and an estimated Taylor rule.

Beckworth and Hendrickson (2020) similarly study the target in another New Keynesian DSGE model, notably adding on a focus on the imperfect estimates used by central banks. More precisely, they motivated by the systematic deviations of the FEDs real-time output gap estimates as documented by Orphanides (2000, 2002*a*,*b*, 2004), introduces forecast errors of the output gap in the model, thus having the monetary policy target an imperfect estimate of the gap rather than the actual gap. Their study shows nominal GDP targeting to outperform a Taylor Rule in both inflation and output volatility when accounting for this imperfect information.

Following the suggestions of McCallum (1988), that monetary policy rule alternatives ought to be considered within several frameworks, Fackler and McMillin (2020) explore nominal GDP targeting as well as price level targeting within a Vector Autoregression (VAR) framework, estimated on US data for the period 1979:4-2003:4. The evaluation is, similarly to Garín et al. (2016), done in respect to welfare losses. They find NGDPT to produce lower losses as compared to both price level targeting and a continuation of the implicit flexible inflation targeting that characterized the estimation period.

Diallo (2019) again investigates the topic in a New Keynesian DSGE model, this time, however, in the case of the Euro Area. They further also distinguish themselves in the choice of the comparative evaluation method, for which they use Bayesian model comparison. Their results suggest a strong preference for nominal GDP targeting in Euro area data as compared to a Taylor Rule.

Finally, Chen (2020) also investigate NGDPT, IT, and Taylor type rules in a New Keynesian DSGE framework calibrated to fit the US. Whilst they similarly to the aforementioned studies conclude NGDP to outperform IT and Taylor type rules in most settings, they do find a Taylor rule to weakly dominate an NGDP target from a welfare perspective, along which dimension this study aims to evaluate the different policies.

2.4 The Case of Sweden

As can be seen from the literary review, studies tend to focus on either the US or the Euro Area, where only the occasional study tend to investigate the target with estimation or calibration to a different economy. No such study has, however, been done in the case of Sweden to the best of my knowledge, distinguishing the study and making it highly relevant. Just as McCallum (1988) argues that new policies ought to be considered in a wide set of models, they ought also to be considered across a wide set of economies and their particular economical conditions.

Sweden jumped the bandwagon of inflation targeting in the early 1990s following a period of high and damaging inflation. The importance of bringing it down was thus the prime focus as the Swedish Riksbank, who had just gained its independence in the same time period, staked out its new policy. Over the years, as inflation was brought down and stabilized, confidence in the Riksbanks ability to attain its target grew. With this new found confidence it could, similarly to other inflation targeting central banks around the world, allow itself to also start focusing on the real economy. This new regime came to be referred to as flexible inflation targeting, which retains its dominance in Sweden, as in much of the developed world, to this day (Andersson and Claussen, 2017).

The latest review of the monetary policy of the Swedish Riksbank was performed by Goodfriend and King (2016), covering the period 2010-2015. In it NGDPT is brought up, however, only very briefly to mention that they find it an undesirable alternative in the context of extending the duties of the central bank beyond inflation control. They instead conclude that flexible inflation targeting offers up “perfectly acceptable” ways of making the tradeoff between volatility in output and in inflation, further also avoiding the issue of NGDP data revisions. A not too unpredictable conclusion given both their prior shown support for IT (Wallström and Isaksson, 2015). Looking more recently at the Swedish Riksbanks Governour, Stefan Ingves (2020) speech on “The monetary policy toolbox”, he even fails to mention the target amongst his list of internationally proposed tools for dealing with recessions. As such one might perhaps conclude that there appears to be at least no clear support for NGDPT at the Swedish Riksbank. However, looking again at the 2017 paper by Andersson and Claussen discussed in section 2.2.1, where both authors work at the monetary policy department of the Swedish Riksbank, part of the reason might perhaps be pinpointed. As what they conclude in their discussion on alternatives to inflation targeting, in regards to NGDPT, is that "*There are no real-world examples and the theoretical research gives no clear-cut answers as to what is preferable when it comes to choosing between flexible inflation targeting and nominal GDP growth targeting*" (Andersson and Claussen, 2017, p. 73). Whilst this paper can do little in regards to their first argument, their second argument can be seen to further motivate a study just as this, to better see how the target might perform in the case of Sweden.

As a final remark I would like to bring up the comment made by Claudia Sahm in the Macro Musings podcast episode “David Beckworth on Nominal GDP Targeting in the Wake of the COVID-19 Crisis”. From her work at the FED she could tell that the next thing the FED would have tried in 2011, had Europe continued to struggle, was indeed NGDPT (Beckworth, 2020). As such, the target ought to be implementable and not that far off from the academic consensus, as that tends to be what guides the FEDs policy (Sumner, 2012).

3 The model

This section aims to provide a detailed description of the model employed in the paper and its key characteristics.

The model employed is a fairly standard medium sized New Keynesian DSGE model with both wage and price stickiness, developed by Erceg, Henderson and Levin (2000), and the same one as employed by Garín et al. (2016) in their study on the desirability of NGDPT. Similarly to Garín et al. (2016) I will use the model to look at average welfare losses to compare NGDPT to several other common monetary policy rules. The main aspect in which this paper parts from their study is in the parametrization. Whilst they fit the model to US data, I will here aim to fit the model to the case of Sweden. I further also extend the model with imperfect output gap forecasting in later sections, in the spirit of Beckworth and Hendrickson (2020). This extension is motivated by the fact that the output gap is not readily observable, but must instead be forecasted. In the standard model, the output gap is, however, assumed observable to the central bank in their targeting of it.

The household side of the economy has a continuum of households on the unit interval supplying differentiated labour and are assumed monopolistically competitive in their supply of it. For convenience it also features a representative labour aggregator that combines the labour input of the households and sells this bundle of labour to the firms in the economy. Production takes place in two phases, first by a continuum of competitive producers on the unit interval producing differentiated intermediate goods, and secondly by a firm aggregating the differentiated outputs into final output, which it then sells to the households. There is also a simple government sector whose only source of financing is a lump sum tax, and who is further assumed to be running a balanced budget each period. Finally, monetary policy enters into the estimated model through a Taylor type instrument rule, which through different parameterization is isomorphic to the different targeting regimes under consideration. I will now outline the basic assumptions and problems of each sector and its agents, wrapping up with a discussion of the aggregation and equilibrium conditions of the model.

3.1 Households

3.1.1 Representative Labour Aggregator

We assume a continuum of households indexed on $h \in [0, 1]$, each being a monopoly supplier of their imperfectly substitutable labour represented by $N_t(h)$. The representative labour aggregator combines these differentiated labour inputs into an aggregate denoted

$N_{d,t}$, which we for convenience, similarly to Erceg et al. (2000), assume it equates to the proportions of each labour that firms would choose. As such, the representative labour aggregators demand for the labour of each household is equal to that of the firms. This aggregate is sold to the firms at real wage w_t, which the labour aggregator takes as given, and the real wage of household h is denoted $w_t(h)$. The bundling of labour inputs into the aggregate labour input, $N_{d,t}$, takes the Dixit and Stiglitz (1977) form:

$$N_{d,t} = (\int_0^1 N_t(h)^{\frac{\epsilon_\omega - 1}{\epsilon_\omega}} dh)^{\frac{\epsilon_\omega}{\epsilon_\omega - 1}}, \ \epsilon_\omega > 1 \tag{2}$$

where ϵ_ω is a parameter of substitutability between the different households' labour. The labour aggregator is also assumed a profit maximizing entity, and its optimization problem can be written as follows:

$$max_{N_t(h)} w_t N_{d,t} - \int_0^1 w_t(h) N_t(h) dh \tag{3}$$

By the First Order Condition (FOC) of the labour aggregators optimization problem we get a downward sloping demand curve for each variety of labour, given by (4). And by plugging the demand curve back into the labour aggregate $N_{d,t}$ we also get an expression for the aggregate real wage index (5).

$$N_t(h) = (\frac{w_t(h)}{w_t})^{-\epsilon_\omega} N_{d,t} \tag{4}$$

$$w_t^{1-\epsilon_w} = \int_0^1 w_t(h)^{1-\epsilon_\omega} dh \tag{5}$$

3.1.2 Households

As stated in the beginning, the model economy is one that features wage rigidities. These wage rigidities faced by households, if left unchecked, give rise to heterogenous decision making which comes with complications. However, following Erceg et al. (2000), I make two simplifying assumptions. Firstly, by introducing a means for households to insure against idiosyncratic wage risk, namely by assuming state-contingent securities. And secondly, I assume preferences to be separable into consumption and labour. As shown by Erceg et al. (2000), these two assumptions ensure that households are homogeneous in their non labour market choices, and the (h) notation of any non labour market variable can thus be ignored. The model also allows for capital accumulation and utilization, internal habit formation in consumption, as well as indexation of wages to lagged inflation, all in accordance with Garín et al. (2016). Now the optimization problem for the individual household can be written as follows.

$$max_{C_t,B_t,u_t,I_t,K_{t+1},w_t(h)N_t(h)} E_0 \sum_{t=0}^{\infty} \beta^t v_t [\ln(C_t - bC_{t-1}) - \psi \frac{N_t(h)^{1+\eta}}{1+\eta}] \tag{6}$$

Where E is the expectations operator, C_t and P_t is the consumption and price level in period t, $\beta \in (0,1)$ is the discount factor, η represents the inverse of the Frisch elasticity of labour supply, ψ is a scaling parameter on the disutility from labour, v_t is an exogenous preference shock common to all households (See section 3.4), and $b \in [0,1)$ is the parameter of habit formation in consumption. The household further faces four constraints when optimizing, firstly, the flow budget constraint in period t:

$$C_t + I_t + \frac{B_t}{P_t} + [\gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2]K_t \leq w_t(h)N_t(h) + R_t u_t K_t + \Pi_t + T_t + \\ + (1 + i_{t-1})\frac{B_{t-1}}{P_t} \tag{7}$$

Here I_t denotes investment, B_t is the stock of nominal bonds in the given period, which pays out the nominal interest rate i_t in periods $t+1$, and B_{t-1} is the stock of nominal bonds from the prior period, which pays out i_{t-1} in the given period t. K_t denotes physical capital, u_t is a parameter of capital utilization, and R_t is the real rental rate of capital services, where capital services is the product of utilization and physical capital, i.e. $u_t K_t$. Π_t is the real profit distributed from firms to households, and T_t is a lump sum tax. Finally, the resource cost of capital utilization is given by $\gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2$. The inclusion of a variable capital utilization cost is in the spirit of Christiano et al. (2005) who showed it essential to produce real inflation inertia and persistence in output.

The second constraint is the law of motion of the physical capital in the economy given by:

$$K_{t+1} = Z_t[1 - \frac{\tau}{2}(\frac{I_t}{I_{t-1}} - 1)^2]I_t + (1-\delta)K_t \tag{8}$$

Here $\tau \geq 0$ is the investment adjustment cost, which again draws upon Christiano et al. (2005), and the investment change is squared to represent the higher cost of sudden large changes as compared to incremental ones. The depreciation rate of physical capital is given by $\delta \in (0,1)$, and Z_t is an exogenous shock to the marginal efficiency of investment (see 3.4). Thirdly:

$$N_t(h) \geq (\frac{w_t(h)}{w_t})^{-\epsilon_\omega} N_{d,t} \tag{9}$$

Which simply restates equation (4) as the constraint that the supply of household labour equates at least to the demand for it at the given wage. And last:

$$w_t(h) = \begin{cases} w_t^{\#}(h), & \text{if } w_t(h) \text{ chosen optimally} \\ (1+\pi_t)^{-1}(1+\pi_{t-1})^{\zeta_w} w_{t-1}(h), & \text{otherwise} \end{cases}$$

Which represents the wage setting rule, which is assumed to follow a Calvo (1983) process, where the household can adjust its wage with probability $1-\theta_w$, each period, in which case it chooses $w_t^{\#}(h)$, or with probability θ_w be unable to adjust their wage. If unable to adjust their wage, it is equal to $(1+\pi_t)^{-1}(1+\pi_{t-1})^{\zeta_w} w_{t-1}(h)$, where $\zeta_w \in [0,1]$ is the parameter of indexation of wages to past inflation. An Euler equation for bonds that is the same across household emerges from optimization and it can be shown that all household, given the opportunity, would adjust their wage to the above common optimal wage $w_t^{\#}(h)$. That is, households are homogenous in their choice of the optimal wage (Garín et al., 2016).

3.2 Production

In the production side of the economy we have two types of producers, intermediate goods producers and final goods producers, thus production takes place in two stages. The intermediate goods market is assumed monopolistically competitive where the firms produce differentiated outputs and have some pricing power, but are subject to price rigidities and take wages as given. The final goods market is assumed competitive, where final goods producers bundle together the intermediate goods and sell them, taking prices of both intermediate and final goods as given.

3.2.1 Final Goods

The continuum of intermediate goods producers is represented on the unit interval indexed by $j \in [0,1]$, producing their differentiated outputs denoted $Y_t(j)$. These differentiated outputs are transformed into final output by the final good producers using the following process, where ϵ_p denotes the substitutability between the different outputs. Similarly to the labour aggregator the final good producers aggregation of the intermediate goods take the Dixit and Stiglitz (1977) form:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p-1}{\epsilon_p}} dj\right)^{\frac{\epsilon_p}{\epsilon_p-1}},\ \epsilon_p > 1 \tag{10}$$

Firms seek to maximize their profit by choosing an optimal quantity of each differentiated good $Y_t(j)$, taking prices, which we will denote $P_t(j)$, as given. As such the final goods firm's maximization problem can be represented as follow:

$$max_{Y_t(j)} P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj \tag{11}$$

From the FOC of the maximization problem we get the demand of each output, given by (12), and by plugging (12) back into the maximization problem we get the aggregate price index (13):

$$Y_t(j) = (\frac{P_t(j)}{P_t})^{-\epsilon_p} Y_t \tag{12}$$

$$P_t^{1-\epsilon_p} = \int_0^1 P_t(j)^{1-\epsilon_p} dj \tag{13}$$

3.2.2 Intermediate Goods

Each producer of intermediate goods faces the following production function:

$$Y_t(j) = max\{A_t \hat{K}_t(j)^\alpha N_t(j)^{1-\alpha} - F, 0\} \tag{14}$$

Here A_t is an exogenous productivity shock common to all firms in the economy (see section 3.4), K_t is capital services which are leased from households, $F \geq 0$ represents fixed costs, and $\alpha \in (0,1)$ is the capital share. It is further assumed that firms are not able to freely adjust their prices to the exogenous productivity shock, but that their price setting instead follows the same Calvo (1983) type contracts as the households wages. As such, similarly to the households, there is a probability $1-\theta_p$, each period that they can adjust their prices and a probability θ_p that they cannot, $\theta_p \in [0,1)$. Given the opportunity to adjust the firm will set the optimal price, denoted $P_t^{\#}(j)$, and if not, the price will equal the prior period's price, denoted $P_{t-1}(j)$ times $(1+\pi_{t-1})^{\zeta_p}$, where $\zeta_p \in [0,1]$ again denotes the degree of indexation of non-updated prices to lagged inflation. Similarly to households, it can be shown that firms are homogeneous in their price setting (Garín et al., 2016).

$$P_t(j) = \begin{cases} P_t^{\#}(j), & \text{if } P_t(j) \text{ chosen optimally} \\ (1+\pi_{t-1})^{\zeta_p} P_{t-1}(j), & \text{otherwise} \end{cases}$$

No matter what price state the firm ends up in, it will choose labour input to minimize costs whilst still producing enough to meet demand. Therefore, they are faced with the following minimization problem when optimizing:

$$min_{N_{d,t}(j),\hat{K}_t(j)} = w_t N_{d,t}(j) + R_t \hat{K}_t(j) \tag{15}$$

$$s.t. \quad A_t \hat{K}_t(j)^{\alpha} N_{d,t}(j)^{\alpha} - F = Y_t(j) \tag{16}$$

3.3 Government

Government spending in the model is given by:

$$\ln(G_t) = (1 - \rho_G)\ln(G^*) + \rho_G \ln(G_{t-1}) + \sigma_G \varepsilon_{G,t} \tag{17}$$

It is assumed an exogenous stochastic stationary process where the level of government spending in the steady state is given by G^*, whilst G_t and G_{t-1} simply denotes the government spending in period t and $t-1$ respectively. Furthermore, σ_G is a scaling parameter of $\varepsilon_{G,t}$ which in turn is the innovation which we draw from a standard normal, and $\rho_G \in (0, 1)$ is the persistence parameter. Further assumption of the government side of the economy is that the lump sum tax T_t is their only source of revenue, and that each period $G_t = T_t$, i.e. they are running a balanced budget each period.

3.4 Exogenous Variables

In addition to the government spending, there are three more exogenous processes in the model, the preference shock to households, v_t, the shock to the marginal efficiency of investment, Z_t, and the productivity shock to firms, A_t. All the shocks are assumed AR(1), following a stationary stochastic process with non-stochastic means and a steady state value normalized to unity in accordance with Garín et al. (2016).

$$\ln(v_t) = \rho_v \ln(v_{t-1}) + \sigma_v \varepsilon_{v,t} \tag{18}$$

$$\ln(Z_t) = \rho_v \ln(Z_{t-1}) + \sigma_Z \varepsilon_{Z,t} \tag{19}$$

$$\ln(A_t) = \rho_v \ln(A_{t-1}) + \sigma_A \varepsilon_{A,t} \tag{20}$$

All the processes are fairly similar to the government spending, where similarly to ρ_G, also ρ_v, ρ_Z, and ρ_A all lie between 0 and 1, assuring stationary. Again σ_v, σ_Z, and σ_A are all scaling parameters, or the standard deviation, of their respective innovations $\varepsilon_{v,t}$, $\varepsilon_{Z,t}$, and $\varepsilon_{A,t}$, which are all drawn from a standard normal.

3.5 Market Clearing and Aggregation

For market clearing we have that the labour aggregator's supply of labour equates to the aggregate demand for it by the intermediate goods producers, and that the supply of physical capital also equates its demand by the intermediate goods producers.

$$\int_0^1 N_{d,t}(j)dj = N_{d,t} \tag{21}$$

$$\int_0^1 \hat{K}_t(j)dj = u_t K_t \tag{22}$$

As such we can write real total firm profits as:

$$\Pi_t = \int_0^1 \Pi_t(j)dj = \int_0^1 [Y_t(j) - w_t N_{d,t}(j) - R\hat{K}_t(j)]dj = Y_t - w_t N_{d,t} - R_t u_t K_t \tag{23}$$

Integrating the households flow budget constraint (7) over h, and using the expression of total firm profits together with the simple government budget constraint $G_t = T_t$, and the fact that bond-holding is zero in equilibrium (as we assume government to not be issuing any debt), we can arrive at the aggregate resource constraint of the economy.

$$Y_t = C_t + I_t + G_t + [\gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2]K_t \tag{24}$$

The aggregate wage and price indexes are given by (25) and (26) respectively. Note that they can be written without household or firm subscripts.

$$w_t^{1-\epsilon_w} = (1-\theta_w)w_t^{\#,1-\epsilon_w} + \theta_w(1+\pi_t)^{\epsilon_w - 1}(1+\pi_{t-1})^{\zeta_w(1-\epsilon_w)}w_{t-1}^{1-\epsilon_w} \tag{25}$$

$$P_t^{1-\epsilon_w} = (1-\theta_p)P_t^{\#,1-\epsilon_p} + \theta_p(1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)}P_{t-1}^{1-\epsilon_p} \tag{26}$$

Using the market clearing conditions and integrating over the individual firm production function (14) we arrive at the aggregate production function (27), where v_t^p is a measure of price dispersion given by (28).

$$Y_t = \frac{A_t(u_t K_t)^{\alpha} N_{d,t}^{1-\alpha} - F}{v_t^p} \tag{27}$$

$$v_t^p = \int_0^1 (\frac{P_t(j)}{P_t})^{-\epsilon_p} dj \tag{28}$$

By setting $\theta_p = \theta_w = 0$ we construct a hypothetical flexible price level of output that we denote Y_t^f. This is the equilibrium level of output that would prevail in the absence of the stickiness we have introduced in prices and wages. From this we can define the output gap, X_t as the ratio of the realised output level and the flexible price level of output, i.e. $X_t = \frac{Y_t}{Y_t^f}$.

Aggregate welfare, along which criteria we aim to evaluate the different policies, is defined in the same manner as in Garín et al. (2016). Namely, as the sum of expected discounted value of flow utility (6) across all households.

$$W_t = \int_0^1 E_t \sum_{s=0}^{\infty} \beta^s v_{t+s}\{\ln(C_{t+s} - bC_{t+s-1}) - \psi \frac{N_{t,+s}(h)^{1+\eta}}{1+\eta}\}dh \tag{29}$$

By rewriting (29) recursively and solving we can express the aggregate welfare with aggregate variables as:

$$W_t = v_t[\ln(C_t - bC_{t-1}) - \psi v_t^w \frac{N_{d,t}^{1+\eta}}{1+\eta}] + \beta E_t\{W_{t+1}\} \tag{30}$$

Where v_t^w is the measure of wage dispersion given by (31), accounting for the discrepancy between aggregate labour demand and supply caused by these dispersions.

$$v_t^w = \int_0^1 (\frac{w_t(h)}{w_t})^{-\epsilon_w(1+\eta)} dh \tag{31}$$

3.6 Monetary Policy

Monetary policy enters the estimated model through the generalized Taylor (1993) type instrument rule seen in (32) Where i_t is the nominal interest rates, i.e. the policy instrument, $\rho_{int} \in [0, 1)$ a smoothing parameter, i^* the natural rate of interest, ϕ_π, ϕ_x, and ϕ_y are the coefficients on inflation, the output gap, and output growth respectively, which are all positive semi definite. Finally, $\varepsilon_{int,t}$ is a policy innovation, drawn from a standard normal, with standard deviation σ_t.

$$\ln(1+i_t) = (1+\rho_{int})\ln(1+i^*) + \rho_{int}\ln(1+i_{t-1}) + \phi_\pi \ln(1+\pi_t) + \phi_x \ln(X_t) + \\ + \phi_y \ln(\frac{Y_t}{Y_{t-1}}) + \sigma_{int}\varepsilon_{int,t} \tag{32}$$

The use of this type of Taylor rule has been seen to provide an accurate approximation of the conduct of monetary policy historically (or at least over the last few decades) (Garín et al., 2016). The instrument rule can be seen to represent all targeting rules considered via different parameter choices, as all rules considered can be thought of as special cases of the Taylor rule for increased clarity. First, the primary rule of concern in this paper, NGDP targeting. For this the target growth rate is set to zero for simplicity, rather than some positive trend number, which gives the following expression in growth rates:

$$\pi_t + \ln(Y_t) - \ln(Y_{t-1}) = 0 \tag{33}$$

Under this targeting rule, the monetary policy will set the nominal interest rate to uphold the equality, and is isomorphic to the Taylor rule in (32) when $\rho_{int} = \phi_x = 0$ and $\phi_\pi = \phi_y \to \infty$. That is with output gap and rate smoothing weights set to zero, and weights on inflation and output growth going to infinity. By rewriting inflation as $\pi_t = ln(P_t) - ln(P_{t-1})$, and noting that a perfect negative correlation between output and inflation is implicit of a zero nominal GDP growth target, we can also write a nominal GDP level targeting rule as:

$$E_t[\ln(P_{t+s}) + \ln(Y_{t+s})] = \ln(P_{t-1}) + \ln(Y_{t-1}) \tag{34}$$

This constant nominal GDP level target is equivalent to the earlier zero nominal GDP growth target unconditionally. But, conditionally on the $t-1$ realisation of nominal GDP being different than its target, the rules are not equivalent. However, given that above parameterisation of the instrument rule is employed, they are both unconditionally and conditionally equal, as the infinite weights on the inflation and GDP deviation will ensure that the target is always attained.

The second rule considered is a strict inflation targeting, where monetary policy is set only in concern to hitting the inflation target, which takes the following form:

$$\pi_t = 0 \tag{35}$$

Similarly to nominal GDP, the target is set to zero rather than some deterministic trend for simplicity. The strict inflation targeting rule is isomorphic to the Taylor rule in (32) when we set $\phi_\pi \to \infty$, and $\rho_{int} = \phi_x = \phi_y = 0$. That is we let the weight on the inflation deviation go to infinity, whilst setting the output gap, the GDP growth, and the interest rate smoothing parameters to zero.

The third rule is a strict output gap targeting, where the monetary policy is set solely to keep the output gap at zero, which takes the following form:

$$X_t = 1 \tag{36}$$

The rule is isomorphic to the Taylor rule in (32), with $\rho_{int} = \phi_\pi = \phi_y = 0$, and $\phi_x \to \infty$. As evident from the construct of the output gap, namely $X_t = \frac{Y_t}{Y_t^f}$, this is equivalent to keeping nominal output at its flexible price level Y_t^f.

Finally, we consider a standard Taylor rule with only an inflation and output gap parameter, which are set to 1.5 and 0.1 respectively, i.e. $\phi_\pi = 1.5$, $\phi_x = 0.1$. The original Taylor rule suggests a greater weight of 0.5 on the output gap, but 0.1 was favoured due to its use by Jonsson and Katinic (2017). They find such a parameterized rule to quite accurately describe the behaviour of the Swedish Riksbank, and they further motivate their choice with the fact that it is the estimate of the parameter in the Swedish Riksbanks DSGE model Ramses.

This concludes the description of the model economy. The full set of equation that make up the economy can be found in Appendix D.

4 Calibration & Estimation

This section aims to provide a descriptive presentation and motivation of the calibrated parameters, as well as present the method employed for the parameter estimations, and the estimation results.

Similarly to Garín et al. (2016), a mix of calibration and estimation of the model parameters will be employed. The mixing of these two methods is common practice in the literature, though it has received criticism, see for example Iskrev (2019). The basis of the criticism stems from the former method, calibration, as it assumes complete knowledge of the parameters, and whether this is a reasonable assumption or not can often be argued. Indeed, in most cases it is in fact an unreasonable and undesirable assumption, which has caused this method to fall out of fashion in favour of formal estimation techniques, which calibration is not. Instead calibration relies on choosing the parameters values to fit the steady state of the model to the data. The reason for still employing calibration tends to either be that the values of the parameters might be considered "known", having been to a significant degree already established in the literature. Or, the perhaps more undesirable but pragmatic rational that the estimation of the selected parameters is difficult with the given data. The reasoning for the use of calibration in this study is a mix of both, though one should be aware that even with well motivated and justified calibration of parameters, the method could still have a significant impact on the final estimated model results due to parameter interdependence (Iskrev, 2019). As for the parameters not calibrated, but instead estimated, Bayesian inference is employed. The basic procedure of the method is to treat the parameter vector as a random variable, and then using probability distributions try to find and describe the uncertainty of the parameter vector. The method is covered more in section 4.2 before it is employed.

4.1 Calibrated Parameters

The set of parameters to be calibrated rather than estimated is the same as in Garín et al. (2016), but in the effort of fitting the model to Sweden, the Swedish Riksbanks work on DSGE models is used extensively for these values. The parameter values and description can be seen in Table 1 below. Starting with the discount rate, it is set to $\beta = 0.999$, similarly to Corbo and Strid (2020) in the Swedish Riksbanks DSGE model MAJA II. The high value is motivated by the fact that a lower value would mean an implausible high model-implied interest rate. For example, the calibrated discount rate in Garín et al. (2016), $\beta = 0.995$, would imply a yearly steady state interest rate of about 2 percent rather than the 0.4 percent implied by our calibration. The disutility of labour scaling parameter is set such that labour hours in the steady state is between one half and one third, which is consistent with a parameter value of $\psi = 6$. The labour elasticity of substitution is set to $\epsilon_w = 10$, and the goods elasticity of substitution to $\epsilon_p = 10$. The steady state government spending G^* is set such that $\frac{G^*}{Y^*} = 0.32$, to match the data share. For the capital share I draw upon MAJA II, setting it to $\alpha = 0.25$, rather than the 0.35 seen in Ramses II. The reason being that it is set unconventionally high there to compensate for a positive external finance premium not present in this model (Corbo and Strid, 2020; Adolfson, Laséen, Christiano, Trabandt and Walentin, 2013). Fixed costs are calibrated such that profits are zero in the long run, and the utilisation linear cost parameter γ_1 is set to attain a steady state normalisation of utilisation of one, both in accordance with Garín et al. (2016). Finally, the depreciation rate is set to $\delta = 0.015$, in accordance with MAJA II and close to Ramses II´s 1.3 percent.

Table 1: Calibrated Parameters

Parameter	Description	Value
β	Discount Rate	0.999
ψ	Disutility from labour	6
ϵ_w	Labour Elasticity of Substitution	10
ϵ_p	Goods Elasticity of Substitution	10
G^*	Steady State Government Spending	$\frac{G^*}{Y^*} = 0.32$
α	Capital Share	0.25
F	Fixed Cost	$\Pi^* = 0$
γ_1	Utilisation Linear Cost	$u^* = 1$
δ	Depreciation Rate	0.015

4.2 Estimated Parameters

All remaining parameters that have not been calibrated are instead estimated with Bayesian methods. The section describes the data employed in these estimations as well as the basics of Bayesian inference. The estimates are presented at the end of the subsection.

4.2.1 Data

As the model is on a quarterly frequency, quarterly frequency is also employed for the observable variables in the model. These are consumption, output, inflation, investments, and the nominal interest rate. One for each exogenous shock to avoid stochastic singularity, as if there are more innovations than shocks some innovations would be exact linear combinations of others. The time period is 1993:2 to 2014:4, where the starting period is chosen to coincide with the introduction of IT in Sweden, and the end period as to avoid the introduction of a negative repo rate. The data for all but one variable is taken from Statistiska Centralbyrån (SCB), where the odd one out is nominal interest rates which instead are obtained from the Swedish Riksbank´s webiste. NGDP is defined as the sum of consumption, investment, and government spending. Furthermore, to deal with non-stationarity issues in the data, all series except inflation are log first differenced, where inflation is already defined as the log first difference of the implicit GDP deflator.

4.2.2 Bayesian Inference

For the basics of Bayesian inference I make heavy use of Griffoli (2010), which one is referred to for a more complete discussion. The basic premise of Bayesian methods is to through employing probability distribution, try to describe the uncertainty of the parameter vector. As such, it treats the parameter vector as a random variable, which can be put into contrast to classical estimation techniques where instead a fixed true variable value is assumed to exist and hence estimated. The method starts by specifying priors, which describes *a priori* knowledge or beliefs about the model parameters, that is, before accounting for the information in the data. By combining this prior density and the likelihood function, which is the distribution of the data given the model, with the equation that lends the method its name, Bayes Theorem, we obtain the posterior density, which is what we are trying to estimate:

$$p(\theta_M \mid Y_T, M) = \frac{p(Y_T \mid \theta_M, M)p(\theta_M \mid M)}{p(Y_T \mid M)} \tag{37}$$

Here $p(\cdot)$ is the chosen probability density function, θ_M is the parameters of the model, M is used to note the specific model at hand, and Y_T is the data up to and including period T. The denominator is the marginal density of the data, conditional on the model, and the numerator corresponds to the posterior kernel, as such:

$$p(\theta_M \mid Y_T, M) \propto p(Y_T \mid \theta_M, M)p(\theta_M \mid M) \equiv \mathcal{K}(\theta_M \mid Y_T, M) \tag{38}$$

The posterior kernel is the key building block of the method that allows one to rebuild all posterior moments of interest via Metropolis-Hastings (MH) simulations. However, as we do not know the likelihood function, we must first estimate it. This is done with the help of the Kalman filter, where through Kalman filter recursions we can derive the log-likelihood. The log posterior kernel in turn, is the sum of the log-likelihood and the log prior density:

$$\ln \mathcal{K}(\theta_M \mid Y_T^*) = \ln \mathcal{L}(\theta_M \mid Y_T^*, M) + \ln p(\theta_M \mid M) \tag{39}$$

Here Y_T^* is the set of endogenous observable variables and θ_M is a vector containing all the parameters we are trying to estimate. Since the prior distribution is known by assumption, and we obtain the log-likelihood by the performed Kalman filtering, we in effect also know the log posterior kernel. We can therefore proceed to estimate the posterior distribution, as it is given by the kernel function. An explicit form solution is, however, unfeasible due to the kernel being a non-linear and complicated function of the deep parameters. It is here the earlier mentioned MH algorithm comes in, or the random walk MH algorithm in this paper to be more precise. The method can be summarized in the following steps from An and Schorfheide (2007):

1. Find the posterior mode of the parameter, denoted $\tilde{\theta}$, by using numerical methods to optimize $\ln \mathcal{L}(\theta \mid Y_T^*) + \ln p(\theta)$.
2. Denote the inverse of the Hessian computed at this posterior mode $\tilde{\Sigma}$.
3. From the jumping distribution $\mathcal{N}(\tilde{\theta}, c_0^2\tilde{\Sigma})$, draw a starting value denoted $\theta^{(0)}$.
4. From the proposal distribution $\mathcal{N}(\theta^{(s-1)}, c^2\tilde{\Sigma})$, draw ϑ (for $s = 1, ..., n_{sim}$, and where n_{sim} is the chosen number of simulations). With probability $min\{1, r(\theta^{(s-1)}, \vartheta \mid Y)\}$ this jump is accepted, so that $\theta^{(s)} = \vartheta$, but otherwise rejected, and $\theta^{(s)} = \theta^{(s-1)}$ instead. Where:
$$r(\theta^{(s-1)}, \vartheta \mid Y) = \frac{\mathcal{L}(\vartheta \mid Y)p(\vartheta)}{\mathcal{L}(\theta^{(s-1)} \mid Y)p(\theta^{(s-1)})}$$

By then taking all the retained values from the algorithm and constructing a histogram, the histogram will after enough iterations become the smoothed posterior distribution which we seek. The above described methods are all employed with the aid of Dynare, a computational software which is run using Matlab (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto and Villemot, 2011).

4.2.3 The Estimated Parameters

The results of the Bayesian estimation can be found in Table 2 along with the priors, and the graphs of both can be found in Appendix B. The estimate of the internal habit formation parameter b and the investment adjustment cost τ suggest significant real inertia in the model. As for the Calvo parameters, they suggest less wage stickiness than assumed. The relatively stickier prices than wages are in line with other estimates on Swedish data such as in Ramses II (Adolfson et al., 2013). The monetary policy rule suggests the central bank to target inflation almost solely, which is consistent with the Swedish Riksbanks inflation targeting mandate, and to also display significant real inertia. The estimates even suggest a procyclical response to GDP growth, which is assumed unreasonable and thus set to 0 in the estimated model. It could be argued that a distribution bounded by zero might be more appropriate for the parameter given this reasoning, but as a Normal prior appears to be the praxis in the literature I opt to stay with this approach. The response to the output gap is very small, which might be explained by the earlier mentioned imperfect targeting of it by the central bank. As for the shocks, the estimates suggest both the preference shock v, and the marginal efficiency of investment shock Z to be much more volatile than assumed. Looking at the plots of the shocks in Appendix C, it further appears it might be caused by the financial crisis, as both shocks spiked around that time. I test for this by re-estimating the model over the shorter sample period 1993:2-2007:4 but obtain similarly high parameter values, and it does thus not appear to be the cause. Such higher estimates of the shocks can similarly be found in Diallo (2019) study of the Euro Area where a posterior mode of 0.5 is found for the investment shock with a prior mode of 0.01.

Table 2: Priors & Posteriors of Estimated Parameters

	Priors			**Posteriors**			
Parameter	Prior Dist	Mean	SD	Mode	Mean	SD	90% HPD Interval
b	Beta	0.7	0.1000	0.9093	0.8992	0.0346	[0.8452,0.9530]
τ	Normal	4	0.2000	4.0517	4.0593	0.1988	[3.7249,4.3801]
η	Normal	1.5	0.2000	1.5602	1.5993	0.1898	[1.3101,1.9049]
γ_2	Beta	0.1	0.1000	0.1936	0.2410	0.1207	[0.0709,0.3980]
θ_p	Beta	0.7	0.1000	0.7044	0.6962	0.0484	[0.6146,0.7759]
θ_w	Beta	0.7	0.1000	0.4621	0.4724	0.0812	[0.3314,0.6228]
ζ_p	Beta	0.5	0.1000	0.1602	0.1863	0.0512	[0.0811,0.2686]
ζ_w	Beta	0.5	0.1000	0.4632	0.4692	0.1060	[0.3041,0.6244]
ρ_{int}	Beta	0.7	0.1000	0.9221	0.9218	0.0195	[0.8925,0.9515]
ϕ_π	Normal	0.5	0.1000	0.3273	0.3440	0.0546	[0.2465,0.4365]
ϕ_y	Normal	0.2	0.1000	-0.0211	-0.0220	0.0102	[-0.0400,-0.0060]
ϕ_x	Normal	0.05	0.1000	0.0045	0.0040	0.0036	[-0.0018,0.0103]
ρ_A	Beta	0.7	0.1000	0.9013	0.8876	0.0345	[0.8355,0.9431]
ρ_v	Beta	0.7	0.1000	0.4598	0.4509	0.1080	[0.2683,0.6141]
ρ_Z	Beta	0.7	0.1000	0.5062	0.5025	0.0809	[0.3840,0.6321]
ρ_G	Beta	0.7	0.1000	0.8734	0.8670	0.0364	[0.8063,0.9258]
σ_A	Inv. Gamma	0.01	0.0100	0.0090	0.0098	0.0019	[0.0063,0.0130]
σ_v	Inv. Gamma	0.01	0.0100	0.1388	0.1421	0.0486	[0.0741,0.2200]
σ_Z	Inv. Gamma	0.01	0.0100	0.1587	0.1627	0.0195	[0.1335,0.1917]
σ_G	Inv. Gamma	0.01	0.0100	0.0118	0.0121	0.0010	[0.0105,0.0135]
σ_i	Inv. Gamma	0.002	0.0100	0.0012	0.0012	0.0001	[0.0011,0.0014]

5 Solving the Model

This section aims to provide a brief description of the method employed for solving the model.

The DSGE model at hand is non-linear and like many of today's DSGE models, too complex to be solved analytically for an exact solution. A common simple approach to the issues presented by nonlinearity has since the influential papers by Kydland and Prescott (1982), and King et al. (1988), been to log-linearise the model, to then enable standard solution techniques. The method, however, comes with complications, where a loss of accuracy is a commonly argued one, and the certainty equivalence property of the solution another. The reason for the latter is that linearization to the first order only retains the first moment of the shocks, which completely disappear when expectations are taken. This causes the unconditional expectations of the endogenous variables of the system to be equal to their non-stochastic steady state values Griffoli (2010). Though this might in some cases be an acceptable simplification, it does not carry through well to welfare comparison, such as the one aimed at in this paper, as shown by Kim and Kim (2003). In essence, the issue arises from the general fact that second-order approximation to the policy function is a prerequisite for correct second-order approximation of the equilibrium welfare function.

Given this, the solution method of Schmitt-Grohé and Uribe (2004) is employed, which is a commonly used pure perturbation approach that uses second-order approximation, as needed for the welfare comparison. The method gets quite computationally complex and notationally heavy and is as such not replicated here, where the curious reader instead is referenced to the original paper. The underlying concept of the method is the use of Taylor's theorem, taking second order approximation of the equations of the model, and then using the implicit function theorem to evaluate derivatives as necessary. The method is similarly to the Bayesian estimation employed with the aid of the computational software Dynare, which again is run using Matlab (Adjemian et al., 2011).

6 Empirical Findings and Analysis

This section aims to provide a descriptive presentation and analysis of the results and try to bridge the findings to the previously established research.

6.1 Model Fit

It is first important to establish how well the model relates to the real world, which it seeks to mimic. One popular way to do so is to look at the second moments of the estimated model, and to see if they are in line with the data. In the estimated model, output is less volatile than consumption which is less volatile than investment. Having consumption being more volatile than output suggests perhaps consumption smoothing to not be very prevalent, and it is further different from the original model and estimates of US data. However, it is fully consistent with data for Sweden as can be seen in Corbo and Strid (2020). One can further also look at the relative contribution of the different shocks to the business cycle dynamics, which are broadly consistent with Garín et al. (2016), despite the significantly larger preference and investment shocks estimates as compared to the priors. The investment shock being the most important, accounting for almost 60 percent of the variation, whilst the technology shock accounts for about 35 percent, and the preference and government spending only account for about two percent each. Finally, the monetary policy shock accounts for less than one percent of the unconditional variance in output. As such, though a model can never be a perfect fit, it appears to reproduce these key aspects satisfactorily.

Comparing the estimated parameters to those found in Garín et al. (2016) for the US, noticeably higher volatility estimates for both the investment and preference shocks are found. These findings can be related to how Swedish GDP and its components are found almost twice as volatile as their counterparts in US and Euro Area data by Corbo and Strid (2020). There are also two more key parameters that are estimated to be qualitatively different from theirs, namely a higher habit formation as well as lower wage stickiness. Additionally, there are also differences in the estimated Taylor rule, which, however, is to be expected, as each economy has its own independent central bank with different mandates. Apart from these instances, the estimations are broadly in line with theirs.

6.2 Welfare Performance

6.2.1 Comparison Method

To compare the different monetary policy rules performance in the estimated model I similarly to Garín et al. (2016) look at welfare, which is defined as the sum of expected discounted value of flow utility across all households as given by equation (30). By comparing the welfare from simply running the specified model to that obtained from running it with both price and wage stickiness set to zero, we through equation (40) get the compensating variation. This is the percent of consumption each period which would make a household indifferent between the two cases, as such a lower value is desirable, and zero indicates an optimal policy which obtains the flexible price outcome.

$$CV = 100 \times (\exp[(1 - \beta \times (W_{flex} - W_{sticky})] - 1) \tag{40}$$

When comparing the rules the interest rate innovation is set to zero in the estimated Taylor rule to better facilitate comparison, as none of the other rules are subject to interest rate shocks. Throughout the section the estimated Taylor rule is run as equation (32) with the estimated values found in Table 2, with the exception of ϕ_y, which as earlier discussed in section 4.2.3 is set to zero. The Calibrated Taylor rule is run as equation (32) with $\phi_x = 0.1$, $\phi_\pi = 1.5$, and the other parameters set to zero. Inflation is run as equation (35), NGDP is run as equation (33), output gap is run as equation (36).

6.2.2 Wage and Price Stickiness

The compensating variation for the different monetary policy rules from the estimated model can be found in the first column of Table 3, whilst subsequent columns look at the relative performance of the rules under different levels of price and wage stickiness. Output gap targeting achieves the best outcome in all but the case of completely flexible wages, where it shares the spot with IT, as in that case the divine coincidence holds and targeting either inflation or the output gap achieves the flexible price outcome. For all other levels of price and wage stickiness, IT is the worst performing rule. The second and third best rule tends to be the two Taylor rules, the estimated one outperforming the calibrated one for the estimated model, as well as when either prices or wages are fully flexible. Finally, NGDP tends to occupy the fourth spot, outperforming only IT with two exceptions. In the estimated model it is preferred to the calibrated Taylor rule, and in the case of completely flexible wages it outperforms them both, though it is there naturally dominated by the optimal policies of either inflation or output gap targeting.

Compared to the findings from the similar exercise by Garín et al. (2016), the Taylor rule when contrasted to NGDP appears to perform substantially better when the model is fitted to Sweden. The dominance of output gap targeting and the poor performance of inflation targeting is on the other hand in line with their findings. The favouring of a Taylor rule over NGDP further goes against Diallo (2019) comparison of the rules in Euro Area data, and Fackler and McMillin (2020) findings in their US estimated VAR framework. However, it is not unheard of, for example Chen (2020) finds a Taylor rule to outperform a NGDP growth target in welfare comparisons in their DSGE model, which similarily to Garín et al. (2016), and Fackler and McMillin (2020), also is fitted on US data.

Table 3: Welfare Performance: Price & Wage Stickiness

Policy Rule	Estimated Values	$\theta_p = 0.75$ $\theta_w = 0.75$	$\theta_p = 0.75$ $\theta_w = 0.5$	$\theta_p = 0.75$ $\theta_w = 0$	$\theta_p = 0.5$ $\theta_w = 0.75$	$\theta_p = 0$ $\theta_w = 0.75$
Estimated Taylor	1.6961	0.9626	1.6132	2.2374	0.8985	0.7931
Callibrated Taylor	2.0629	0.8459	1.8224	3.9639	0.8581	0.7990
Inflation	2.1944	4.9927	2.6258	0.0000	4.9927	4.9927
NGDP	2.0470	0.9874	1.9936	0.8961	0.9023	0.9289
Output Gap	0.1008	0.1220	0.1054	0.0000	0.0824	0.0000

Notes: *The table displays compensating variations for the different policy rules from running the model with the different values of selected parameters. All other parameters are kept at their baseline values found in Table 1 and 2.*

6.2.3 Policy Inertia

As noted in section 6.1, the estimated Taylor rule displays significant inertia with the smoothing parameter ρ_{int} set at over 0.92. As inertia is needed for optimal monetary policy in the New Neoclassical Synthesis framework, the importance of this aspect for the relative performance of the rule in the estimated model is tested (Jensen, 2002). This is done by plotting the compensating variation of the estimated Taylor rule for progressively smaller levels of policy inertia, as seen in Figure 1. As evident from the graph, the relative performance of the rule is indeed dependent on ρ_{int}, where the breaking point for making the estimated Taylor rule actually the worst rather than second best in the baseline model is somewhere around 0.8, which is still higher then its prior of 0.7. The estimated persistence parameter in Garín et al. (2016) is, however, 0.88, at which level the estimated Taylor rule is still preferred to all rules but the output gap targeting. As such it appears to not be the sole cause of the relatively better performance of the rule in Sweden.

6.2.4 Imperfect Output Gap Estimation

Given its prevalence in the literature on NGDP, the model is extended with imperfect output gap estimation. This exercise is performed by looking at historical revisions of the output gap for Sweden in the OECD dataset "Quarterly output gap revisions database", which was used by Tosetto (2008*b*,*a*) in her paper on the topic of output gap revisions. We start by taking the difference of the first estimate of the output gap to that of its revised values published at least 3 years later, which is available between 1991:1-2004:4

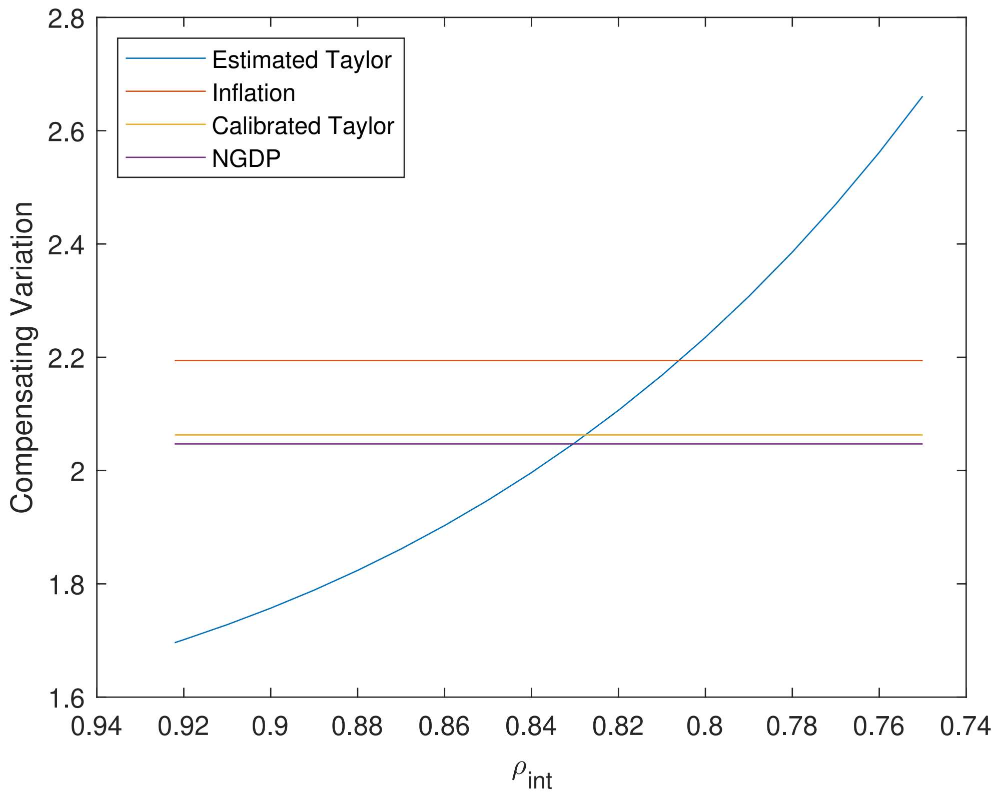

Figure 1: Different Levels of Policy Inertia

in the data set. We then fit an AR(1) process to this error series of the same form as the other exogenous processes in the model:

$$\ln GE_t = \rho_{gap} \ln(GE_t) + \sigma_{gap}\varepsilon_{gap,t} \tag{41}$$

By then having both the output gap targeting rule, as well as the two Taylor rules target the sum of the actual output gap and this newly constructed gap error GE, the behaviour of the rules should be more similar to that of any real world application. The output gap error has a persistence parameter estimate of about 0.75, the standard deviation of the shock process is estimated to be around 0.005.

The results of this exercise can be found below in Table 4, the addition of the estimation error appears to have a very marginal impact on the results, all rankings remaining the same as in the original model. At the given persistence parameter, the standard deviation of the shock would have to be roughly nine times larger than its estimate, 0.045, for nominal GDP targeting to outperform output gap targeting in the estimated model. What can be noted though, is that the data for the output gap coincides with the Great Moderation, a period as earlier mentioned characterized by unusually accurate estimations of the output gap. However, with the given data limitations the process can unfortunately not be fitted to any other time period in the case of Sweden. The results can be contrasted to those of Beckworth and Hendrickson (2020), where the output gap is found to have a much more significant impact on the results. As they have the error entered into the model in a much different process, no direct comparison of the relative magnitude of the error can unfortunately be performed. As such, it is difficult to say whether the differences are due to an estimated lesser error in this paper, or if it relates to the differences in having the model fitted to Sweden rather than the US.

Table 4: Imperfect Output Gap Estimation

Policy Rule	Baseline Model	Gap Error Added
Estimated Taylor	1.6961	1.6963
Calibrated Taylor	2.0629	2.0648
Inflation	2.1944	2.1944
NGDP	2.0470	2.0470
Output Gap	0.1008	0.1328

Notes: *The table displays compensating variations for the different policy rules from running the model with (right) and without (left) the output gap estimation error specified in equation* (41)

6.2.5 Comparison to Garín et al. (2016) - Estimation Outliers

Given that the model is the same as that employed by Garín et al. (2016) it is especially interesting to contrast their results to see what might be the cause of the differences in the welfare rankings. As such I look to explore if perhaps certain outliers in the estimation, as compared to their estimates, might be the cause. As discussed in section 6.1, four outliers in the estimations as compared to the fitting of the same model to the US economy in Garín et al. (2016) are found. These being the preference shock volatility, the investment shock volatility, the habit formation parameter, and the wage stickiness. As wage and price stickiness was already the topic of the prior sections, to which the results appear robust, it will not be covered any further in this section. Each of the other three outliers are tested for one by one, setting each to the estimated value in their study. The results can be found below in Table 5.

Table 5: Estimation Outliers

Policy Rule	Baseline Model	$b = 0.7508$	$\sigma_v = 0.0181$	$\sigma_Z = 0.0389$
Estimated Taylor	1.6961	2.3012	0.4022	1.4599
Calibrated Taylor	2.0629	2.5811	0.7877	1.5302
Inflation	2.1944	1.8068	2.1413	2.0223
NGDP	2.0470	1.0447	1.6868	0.6704
Output Gap	0.1008	0.0398	0.0395	0.0385

Notes: *The table displays compensating variations for the different policy rules from running the model with the different values of selected parameters. All other parameters are kept at their baseline values found in Table 1 and 2*

Whilst the results appear relatively robust to the change in the preference shock, setting either the habit formation parameter, or the technology shock to either of the estimates in Garín et al. (2016), causes NGDP targeting to get ranked second just as in their study. Changing the habit formation, however, has inflation targeting performing better than either Taylor rule, which again is different to their rankings. When the technology shock is reduced, however, the ranking of the rules indeed follows theirs.

The rankings thus seem to be dependent on the estimation of both the degree of habit formation, and the magnitude of the technology shock. Contrasting the results to those of the Swedish Riksbank's two DSGE models, MAJA II, and Ramses II, they find significantly lower habit formation parameters. There the parameter is estimated at around 0.75 in MAJA II, and even 0.53 in Ramses II. As noted by Adolfson et al. (2013), however, the low estimate in Ramses II might be explained by import and export dynamics in the model, where a lower habit formation parameter can be in effect without creating much larger fluctuations in consumption. As the presence of imports and exports also differentiates MAJA II from the model employed in this study, it is not unreasonable that the reasoning to a certain degree could motivate a higher degree of habit formation even compared to their higher estimate of 0.75. Therefore, it is difficult to make any direct comparisons, but it should be noted that the findings indeed are conditional on the attained high estimation of the parameter. From the meta-analysis of the parameter by Havranek, Rusnak and Sokolova (2017) it can, however, be noted that the estimate is within the 95 percent confidence interval for DSGE Macro models.

As for the investment shock, the better performance of the NGDP targeting is reasonable, as the technology shock, which in the model can be thought of as supply shocks, becomes the dominant shock. These results further corroborate that the performance of NGDP is positively related to the importance of technology shocks, empirically, similarly to Kim and Henderson (2005). With this new calibration, the technology shock account for around 80 percent of the variance in output and the investment shock for only around

8 percent. Such a variance decomposition goes against the literature, so whilst it does neatly display the strength of NGDP targeting in the presence of supply shock vis-á-vis the other rules, the original findings produce a much better model fit.

6.3 Robustness checks

To check the robustness of the results in regards to the estimation, setting all the estimated variables to their 10th percentiles and their 90th percentiles is both tested. The results can be seen in Table 6, the output gap dominance appears robust to the exercise, and the rankings as a whole are unchanged at the 90th percentile. At the 10 percentile there is more variation amongst the rules, where the performance of IT appears especially improved. It can, however, be concluded that it does not appear to improve the performance of NGDP targeting.

Table 6: Parameter Robustness: 10th and 90th Percentiles

Policy Rule	10th Percentile	Baseline Model	90th Percentile
Estimated Taylor	0.8438	1.6961	3.1647
Calibrated Taylor	0.7654	2.0629	4.5519
Inflation	0.5795	2.1944	8.6933
NGDP	1.0439	2.0470	4.3074
Output Gap	0.0335	0.1008	0.4154

Notes: *The table displays compensating variations for the different policy rules from running the model with all parameters set at either their 10th (left) or 90th (right) percentiles, as well as their baseline values (center)*

I further try conditioning on one parameter at a time, setting each to either its 10th or 90th percentile and checking for the compensating variation of each rule in each case. The results are to be found in Table 8 and 7 in Appendix A, where the changed parameter can be seen in the leftmost column. All other parameters are kept at their baseline values.

Looking first at the 90th percentiles in Table 7, the ranking of the output gap and the estimated Taylor rule as first and second best, appears robust. An exception can be found in the preference shock, where instead inflation targeting is found to be the second best rule. The change is consistent with the findings of Garín et al. (2016), in that the Taylor rule is found to handle preference shocks especially poorly. As the high ranking of IT is primarily due to the shock increasing the compensating variation of the Taylor rules quite significantly. It can, however, be concluded that it does not appear to improve the performance of NGDP targeting much.

Looking at the 10th percentiles in Table 8, the ranking of the output gap and the estimated Taylor rule as first and second best, again appears robust, except on three occasions. The first instance, the habit formation parameter, was already covered in the earlier comparison of the results to that of Garín et al. (2016), but it again highlights the dependence of the rankings on its estimated high value. The second instance, the wage stickiness, was covered in the section on price and wage stickiness, where the better performance of the inflation targeting is to be expected with lower wage stickiness, as IT then comes closer to implementing the flexible equilibrium via the divine coincidence. The final instance, the investment shock reduction improving the performance of NGDP, was also discussed in the comparison to the estimations in Garín et al. (2016) and again reasonably improves NGDP by increasing the relative importance of supply shocks.

7 Conclusion

This paper looks into NGDP targeting in the case of Sweden, by employing a New Keynesian DSGE model estimated on Swedish data for the period 1993:2-2014:4. By looking at the performance of the target in the case of Sweden it aims to provide further insights into the performance of the rule under differing circumstances, as no empirical DSGE study of NGDP targeting has been performed on Swedish data before to my knowledge. The rule is compared to IT, output gap targeting, an estimated Taylor rule, as well as a classical calibrated two parameter Taylor rule, where the evaluation method is welfare losses.

Output gap targeting is found to be the most desirable target across most settings in accordance with Garín et al. (2016). The target's preference even appears robust to the addition of imperfect output gap estimation, which goes against the findings of Beckworth and Hendrickson (2020). Though it should be noted that the error process for the estimation is taken from a particularly favourable period for output gap estimations, it cannot be excluded that the relative difference might also be due to differences in the calibration and estimation of the model to Sweden rather than the US. As for NGDP, it is found preferable to strict inflation targeting, but to produce greater welfare losses than the estimated Taylor rule in most settings. This includes conditioning on different levels of policy inertia, a quality needed for optimal monetary policy in the New Neoclassical Synthesis framework (Jensen, 2002).

Contrasting the results to those of Garín et al. (2016), NGDP targeting is found to be less desirable in the case of Sweden than that of the US. The result appears robust to differing levels of price and wage stickiness, where NGDP only outperforms the estimated Taylor rules in the case of completely flexible wages. In which setting inflation or output gap targeting is the optimal policy anyways. The results further appear robust to the estimation of the parameters as a whole, where setting all parameters to either their 90th or 10th percentile of their posteriors is tested. It is, however, found that the results are conditional on the unconventionally high estimate of habit formation, but that concrete comparison to other estimates of the parameter in Sweden is limited due to differences in models. The estimate is, however, within the range of prior estimations in the literature.

With this first study to investigate the performance of NGDP targeting in a DSGE model fitted to Sweden, what is found central to the purpose and that adds to the current empirical strand, is the relatively better performance of a Taylor type rule to that of NGDP. In many prior empirical studies NGDP is found preferable to Taylor type rules (Garín et al., 2016; Beckworth and Hendrickson, 2020; Fackler and McMillin, 2020; Diallo, 2019). But as these studies tend to focus on either the US or the Euro Area, these different rankings perhaps suggest significant heterogeneity in the performance of the target across differing economies. An aspect not too often discussed in the literature, but one that ought to be explored as the target is becoming more discussed in policy circles as evident from the FED´s consideration of it in 2011, and the proliferation of literature on the topic (Beckworth, 2020).

For future research I, in accordance with the purpose of the study, urge for a continued diversification of the economies explored when evaluating the target. As for the particular case of Sweden, three avenues are found to be of particular interest. Firstly, I would like to see the target tried in a two-region DSGE model such as MAJA II, to account for the significant impact foreign economies can have on such a small globalized economy as Sweden and incidentally the performance of the target. Secondly, the addition of a ZLB to the model would be of great interest, firstly as that is where NGDP targeting has been argued to be particularly helpful, but also as to be able to estimate the model on more recent data where the constraint is more likely to bind. Finally, further investigation into the output gap estimation errors in the case of Sweden could benefit much policy research, as even though output gap targeting is not too prominent in the current debate, the variable do enter into the Taylor rule which is still as relevant as ever. Moreover, all these three aspects would carry through well into the consideration of the target also in other economies.

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Appendix A

Here the tables from the parameter robustness tests are presented. The leftmost column shows the parameter changed, and all other parameters are kept at their baseline model values. The remaining columns display compensating variation from running the model with the rule specified at the top of each column. The Estimated Taylor rule is run as equation (32) with the estimated values found in Table 2 with the expection of ϕ_y *which is set to zero. The Calibrated Taylor rule is run as equation (32) with* $\phi_x = 0.1$ *and* $\phi_\pi = 1.5$. *Inflation is run as equation (35), NGDP is run as equation (33), and Output Gap is run as equation (36)*

Table 7: Parameter Robustness: 90th Percentile HPD

90th Percentile HPD	Estimated Taylor	Calibrated Taylor	Inflation	NGDP	Output Gap
$b = 0.9530$	1.5709	1.9677	2.2699	2.4563	0.1038
$\tau = 4.3801$	1.6707	2.0444	2.2326	1.9675	0.1007
$\eta = 1.9049$	1.6789	2.3279	2.3520	1.7369	0.0974
$\gamma_2 = 0.3980$	1.6994	2.0882	2.1650	2.0489	0.1003
$\theta_p = 0.7759$	1.8258	1.9990	2.1948	2.2919	0.1017
$\theta_w = 0.6228$	1.1759	1.2889	3.5328	1.3614	0.1111
$\zeta_p = 0.2686$	1.6579	1.9988	2.1948	2.0465	0.0920
$\zeta_w = 0.6244$	1.6630	1.9900	2.1948	2.0433	0.1226
$\rho_A = 0.9431$	1.7423	2.0753	2.1732	2.0742	0.1232
$\rho_v = 0.6141$	2.1844	2.5858	2.2485	2.4488	0.1061
$\rho_Z = 0.6321$	1.9106	2.7648	2.2718	2.7424	0.1093
$\rho_G = 0.9258$	1.6956	2.0626	2.1945	2.0437	0.1009
$\sigma_G = 0.01304$	1.8351	2.2811	4.3474	2.1704	0.1971
$\sigma_v = 0.2200$	3.7205	4.0565	2.2771	2.6048	0.1058
$\sigma_Z = 0.1917$	1.6965	2.3247	2.2793	2.7285	0.1046
$\sigma_G = 0.0135$	1.6981	2.0656	2.1979	2.0829	0.1009

Table 8: Parameter Robustness: 10th Percentile HPD

10th Percentile HPD	Estimated Taylor	Calibrated Taylor	Inflation	NGDP	Output Gap
$b = 0.8452$	1.9291	2.2702	2.0536	1.5358	0.0973
$\tau = 3.7249$	1.7276	2.0872	2.1561	2.0474	0.1011
$\eta = 1.3101$	1.7405	1.8707	2.0609	2.3596	0.0437
$\gamma_2 = 0.0709$	1.6888	1.9933	2.2900	2.0460	0.1033
$\theta_p = 0.6146$	1.5743	2.0900	2.1948	1.8182	0.0935
$\theta_w = 0.3314$	2.3678	2.9093	1.2883	2.6890	0.0892
$\zeta_p = 0.0811$	1.7253	2.1099	2.1948	2.0518	0.1071
$\zeta_w = 0.3041$	1.7368	2.1426	2.1948	2.0625	0.0860
$\rho_A = 0.8355$	1.6633	2.0383	2.2572	2.0175	0.0785
$\rho_v = 0.2683$	1.2981	1.6598	2.1652	1.8334	0.0988
$\rho_Z = 0.3840$	1.5883	1.7834	2.1524	1.6587	0.0972
$\rho_G = 0.8063$	1.6969	2.0624	2.1955	2.0546	0.1008
$\sigma_A = 0.0063$	1.6308	1.9596	1.1891	1.9892	0.0551
$\sigma_v = 0.0741$	0.7540	1.1343	2.1559	1.7852	0.0985
$\sigma_Z = 0.1335$	1.6228	1.8969	2.1411	1.6172	0.0984
$\sigma_G = 0.0105$	1.6954	2.0612	2.1928	2.0245	0.1008

Appendix B

Here the figures of the priors and posteriors of the Bayesian estimation discussed in Chapter 4: Calibration & Estimation is presented. The gray line is the prior density, the black line is the posterior density, and the dashed green line is the posterior mode. The x-axis displays part of the support of the prior distribution whilst the corresponding density is displayed on the y-axis.

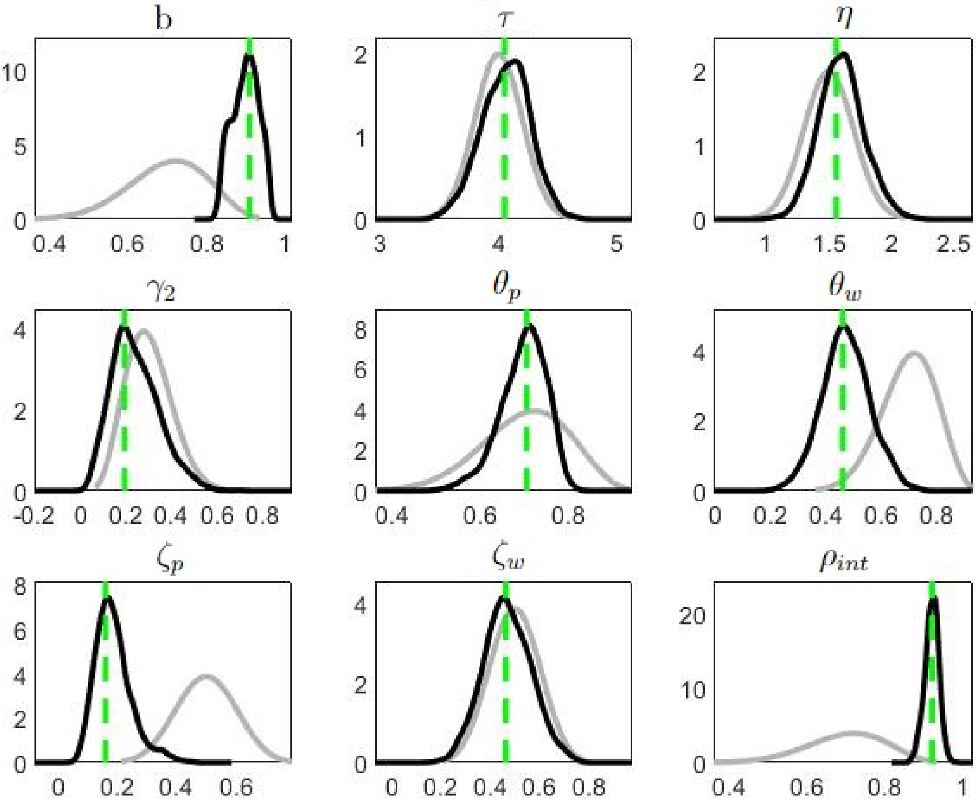

Figure 2: Priors & Posterior Densities 1

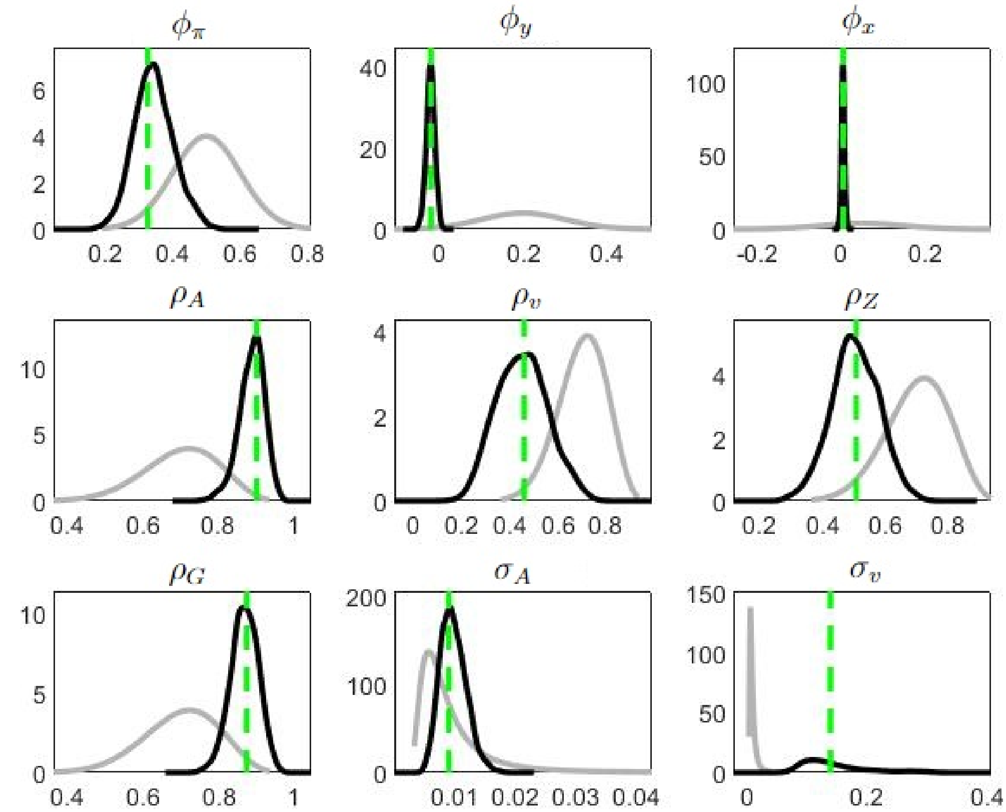

Figure 3: Priors & Posterior Densities 2

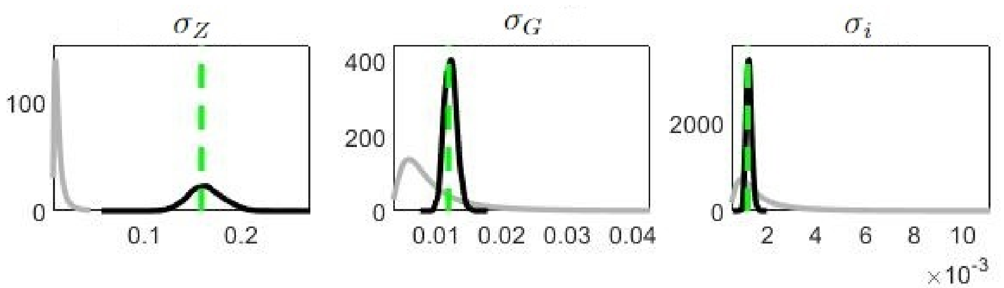

Figure 4: Priors & Posterior Densities 3

Appendix C

Here the figures of the data used in the estimation is presented, as well as the estimated smoothed structural shocks derived from the Kalman smoother at the posterior mean

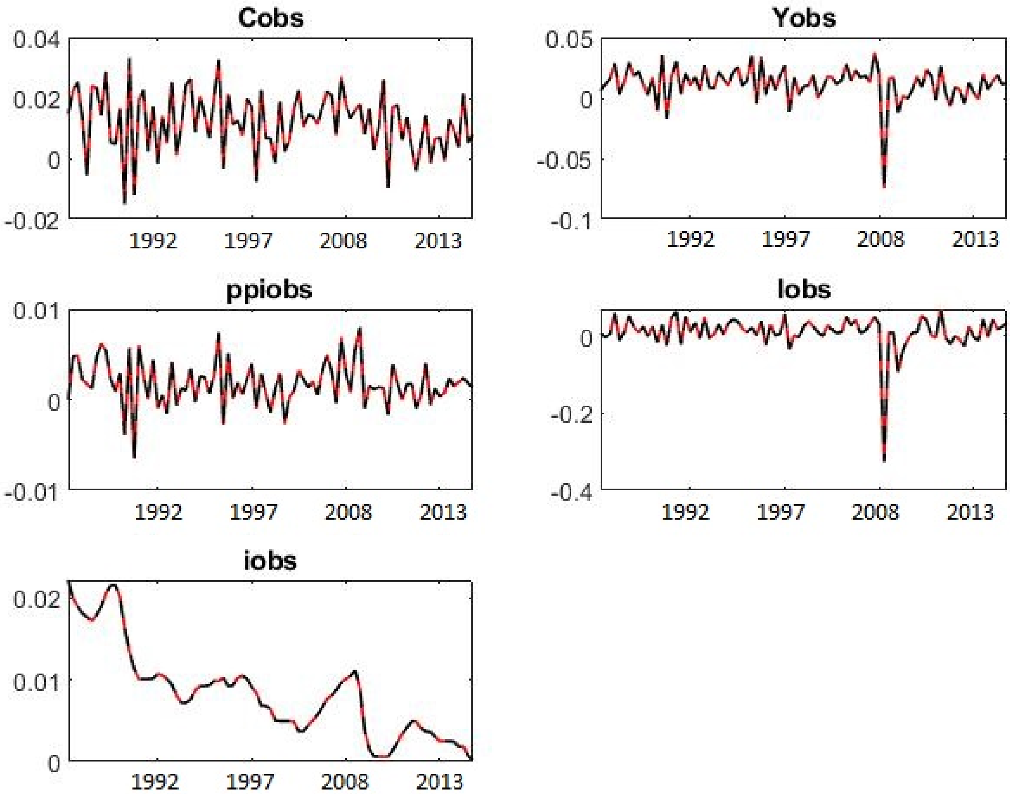

Figure 5: The Data

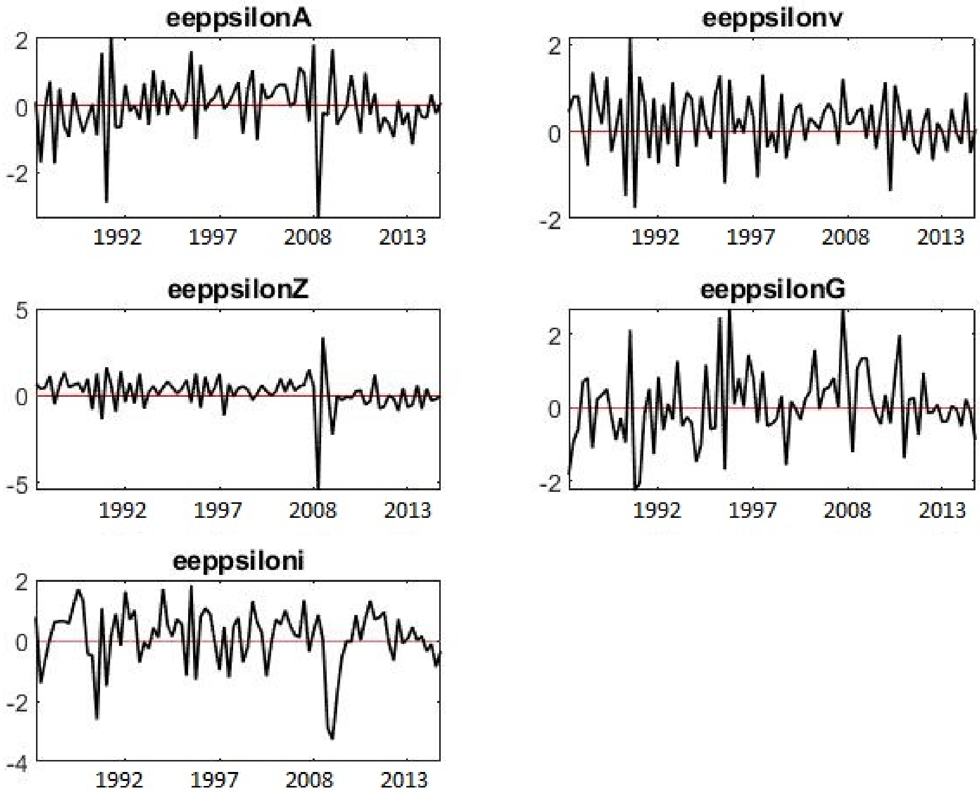

Figure 6: The Estimated Shocks

Appendix D

Here the equilibrium conditions of the model is presented. An equilibrium is a non-explosive sequence of: $\{\lambda_t,\ \upsilon_t,\ C_t,\ i_t,\ \pi_t,\ R_t,\ u_t,\ \mu_t,\ Z_t,\ I_t,\ w_t^{\#},\ H_{1,t},\ H_{2,t},\ w_t,\ N_{d,t},\ K_t,\ mc_t,\ \pi_t^{\#},\ X_{1,t},\ X_{2,t},\ G_t,\ A_t,\ \upsilon_t^p,\ W_t,\ \upsilon_t^w,\ Y_t\}$ *Such that Equation A1 through A26 all hold, given initial values of the state variables and current realization of the innovations in the shock process.*

$$\lambda_t = \frac{\upsilon_t}{C_t - bC_{t-1}} - \beta b E_t \frac{\upsilon_{t+1}}{C_{t+1} - bC_t} \tag{A1}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{1 + i_t}{1 + \pi_{t+1}} \tag{A2}$$

$$R_t = \gamma_1 + \gamma_2(u_t - 1) \tag{A3}$$

$$\mu_t = \beta E_t \lambda_{t+1}[R_{t+1}u_{t+1} - (\gamma_1(u_{t+1} - 1) + \frac{\gamma_2}{2}(u_{t+1} - 1)^2)] + \beta(1 - \delta)E_t\mu_{t+1} \tag{A4}$$

$$\lambda_t = \mu_t Z_t[1 - \frac{\tau}{2}(\frac{I_t}{I_{t-1}} - 1)^2 - \tau(\frac{I_t}{I_{t-1}} - 1)\frac{I_t}{I_{t-1}}] + \beta\tau E_t\mu_{t+1}Z_{t+1}(\frac{I_{t+1}}{I_t} - 1)(\frac{I_{t+1}}{I_t})^2 \tag{A5}$$

$$w_t^{\#} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{H_{1,t}}{H_{t,2}} \tag{A6}$$

$$H_{1,t} = \psi\upsilon_t(\frac{w_t^{\#}}{w_t})^{-\epsilon_w(1+\eta)}N_{d,t}^{(1+\eta)} + \theta_w\beta E_t(\frac{w_{t+1}^{\#}}{w_t})^{\epsilon_w(1+\eta)}(1 + \pi_t)^{\zeta_w\epsilon_w(1+\eta)}(1 + \pi_{t+1})^{\epsilon_w(1+\eta)}H_{1,t+1} \tag{A7}$$

$$H_{2,t} = \lambda_t(\frac{w_t^{\#}}{w_t})^{-\epsilon_w}N_{d,t} + \theta_w\beta E_t(\frac{w_{t+1}^{\#}}{w_t})^{\epsilon_w}(1 + \pi_t)^{\zeta_w(1+\epsilon_w)}(1 + \pi_{t+1})^{\epsilon_w - 1}H_{2,t+1} \tag{A8}$$

$$\frac{u_t K_t}{N_{d,t}} = \frac{\alpha}{1-\alpha}\frac{w_t}{R_t} \tag{A9}$$

$$mc_t = \frac{w_t^{1-\alpha} R_t^{\alpha}}{A_t}(1-\alpha)^{\alpha-1}\alpha^{-\alpha} \tag{A10}$$

$$\frac{1+\pi_t^{\#}}{1+\pi_t} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{X_{1,t}}{A_{2,t}} \tag{A11}$$

$$X_{1,t} = \lambda_t mc_t Y_t + \theta_p \beta E_t (1+\pi_t)^{-\zeta_p \epsilon_p}(1+\pi_{t+1})^{\epsilon_p} X_{1,t+1} \tag{A12}$$

$$X_{2,t} = \lambda_t Y_t + \theta_p \beta E_t (1+\pi_t)^{\zeta_p(1-\epsilon_p)}(1+\pi_{t+1})^{\epsilon_p - 1} X_{2,t+1} \tag{A13}$$

$$Y_t = C_t + I_t + G_t + [\gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2]K_t \tag{A14}$$

$$K_{t+1} = Z_t[1 - \frac{\tau}{2}(\frac{I_t}{I_{t-1}} - 1)^2]I_t + (1-\delta)K_t \tag{A15}$$

$$Y_t = \frac{A_t (u_t K_t)^{\alpha} N_{d,t}^{1-\alpha} - F}{v_t^p} \tag{A16}$$

$$(1+\pi_t)^{1-\epsilon_p} = (1-\theta_p)(1+\pi_t^{\#})^{1-\epsilon_p} + \theta_p(1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)} \tag{A17}$$

$$w_t^{1-\epsilon_w} = (1-\theta_w) w_t^{\#,1-\epsilon_w} + \theta_w (1+\pi_t)^{\epsilon_w - 1}(1+\pi_{t-1})^{\zeta_w(1-\epsilon_w)} w_{t-1}^{1-\epsilon_w} \tag{A18}$$

$$v_t^p = (1-\pi_t)^{\epsilon_p}[(1-\theta_p)(1+\pi_t^{\#})^{-\epsilon_p} + \theta_p(1+\pi_{t-1})^{-\zeta_p\epsilon_p}v_{t-1}^p] \tag{A19}$$

$$v_t^w = (1-\theta_w)(\frac{w_t^{\#}}{w_t})^{-\epsilon_w(1+\eta)} + \theta_w(\frac{w_t}{w_{t-1}}(1+\pi_t))^{\epsilon_w(1+\eta)}(1+\pi_{t-1})^{-\zeta_w\epsilon_w(1+\eta)}v_{t-1}^w \tag{A20}$$

$$\ln(v_t) = \rho_v \ln(v_{t-1}) + \sigma_v\varepsilon_{v,t} \tag{A21}$$

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \sigma_Z\varepsilon_{Z,t} \tag{A22}$$

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \sigma_A\varepsilon_{A,t} \tag{A23}$$

$$\ln(G_t) = \rho_G \ln(G_{t-1}) + \sigma_G\varepsilon_{G,t} \tag{A24}$$

$$W_t = v_t[\ln(C_t - bC_{t-1}) - \psi v_t^w \frac{N_{d,t}^{1+\eta}}{1+\eta}] + \beta E_t\{W_{t+1}\} \tag{A25}$$

$$\ln(1+i_t) = (1+\rho_{int})\ln(1+i^*) + \rho_{int}\ln(1+i_{t-1}) + \phi_\pi \ln(1+\pi_t) + \phi_x \ln(X_t) + \phi_y \ln(\frac{Y_t}{Y_{t-1}})$$
$$or\ \pi_t = 0,\ or\ \pi_t + \ln(Y_t) - \ln(Y_{t-1}) = 0,\ or\ X_t = 0 \tag{A26}$$

λ_t is the lagrange multiplier on the flow budget constraint of a household.
μ_t is the multiplier on the capital accumulation equation.
Equation A1 is the marginal utility of income.
Equation A2 is the standard Euler equation for bonds.
Equation A3 is the first order condition for capital utilisation.
Equation A4 is the Euler equation for physical capital.
Equation A5 is optimality condition for the choice of investment.
Equation A6, A7, and A8, describe optimal wage setting.
Equation A9 and A10 come out of cost-minimization for firms and establish that all firms face the same real marginal cost and hire capital services and labour in the same ratio, equal to the aggregate ratio.
Equation A11, A12, and A13 describe optimal price setting.
Equation A14 is the aggregate resource constraint.
Equation A15 is the law of motion for physical capital.
Equation A16 is the aggregate production function.
Equation A17 and A18 is the evolution of inflation and the real wage.
Equation A19 is the law of motion for price dispersion.
Equation A20 is the process for wage dispersion.
Equation A21 is exogenous process for the preference shock.
Equation A22 is the exogenous process for investment shock.
Equation A23 is the exogenous process for productivity.
Equation A24 is the exogenous process for government spending.
Equation A25 is the aggregate welfare.
Equation A26 is the different monetary policy rules.