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Bitcoin vs Gold, The Hedge Game

Volatility and Portfolio Analysis of Bitcoin and Gold during Market Distress

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Abstract

Over the recent years the interest in cryptocurrencies and Bitcoin has increased significantly. The price of Bitcoin has gone up substantially since the end of 2020 and more individual and institutional investors are incorporating Bitcoin to their portfolio. This thesis explores the features of Bitcoin as another financial asset, specifically compared to Gold as Bitcoin has been referred to as the new "Digital Gold". The main appeal of Gold is its properties as a hedge and safe haven. The recent market distress caused by COVID-19 provides the perfect conditions to examine if Gold and Bitcoin can provide investors with these features in a time of market turmoil. This is done by using ARCH models to examine the volatility behavior of Bitcoin and other assets. Thereafter a BEKK-GARCH model will be used to examine the relationship of Bitcoin and Gold to other assets. Finally, to examine the hedging properties, a portfolio analysis will be done using the minimum variance portfolio. The results indicate that Gold does act as a hedge and safe haven as has been claimed in earlier literature. Interestingly for Bitcoin, the results show that it increases the return and reduces the volatility on average when included in a portfolio with the OMXS 30 and NIKKEI 225. Bitcoin also provides an increased return in a minimum variance portfolio during market turmoil, consequently meaning that it provides a sort of "mini-hedge".

Keywords: Bitcoin, Gold, Hedge, Volatility, Portfolio Analysis

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1. Introduction

In this section, the background of the paper is presented, as well as the purpose, the research question and the structure. This section should give the reader an overview of how the thesis is structured, why the subject is important and the questions that will be answered in the paper.

1.1 Background

Bitcoin and the concept of cryptocurrencies was presented in a whitepaper by Nakamoto (2008). Since inception several thousands of unique cryptocurrencies have been created. The one holding the largest market cap and is most widely used is Bitcoin.

The paper by Nakamoto (2008) was introduced after the financial crisis, when the trust in the financial system had taken a hit (Weber, 2014). There have been several attempts at digital currencies before Bitcoin, but none of these were successful in solving the "Double spending problem" without a central authority (Dwyer, 2015). This problem simply means that if money is digital and easily replicable, what is to stop someone from copying it, making more and then spending it two or more times. This problem is the digital version of counterfeiting physical currency. The most common solution to this problem is to have a trusted third party monitor it, so that no one is abusing the system, which is the solution we use today with financial institutions. But giving someone control over the money supply gives them enormous power. Having a centralized system can lead to problems such as: corruption (what is to stop the bank from creating more money), mismanagement and misalignment of interest (someone takes a risk and the government needs to save a bank in opposition of what the people want and maybe thereby creating inflation), and that the control of your money is given to someone else (the government can choose to freeze your money and can even ban cash) (Weber, 2014). The trust required for this system is what was damaged after the financial crisis in 2008, therefore further analysis of Bitcoin is of great interest.

Cryptocurrencies and Bitcoin are based on the concept of a peer-to-peer network with a distributed ledger technology, making online transactions viable without financial institutions as third parties (Nakamoto, 2008). As there are no third parties the system is decentralized and to establish trust it uses cryptographic proof, which is where the name comes from, as well as digital signatures and

proof-of-work (Dwyer, 2015). The network is created and controlled by computer algorithms that record the transactions. Cryptocurrencies use blockchain technology which means that its users, the miners, all keep the record of all the transactions. The miners also create blocks which are then added to a ledger, eventually forming a blockchain. The transactions are made public information to all with the help of a transparent ledger, but the identities are kept anonymous to protect its users. To create new coins the miners, solve complex computer problems that become exponentially harder over time. Meaning that less and less Bitcoin can be mined over time, with a cap of 21 million (Weber, 2014). This is in contrast to our conventional system that is centralized and unlimited, meaning that the government can issue how much currency it wants, leading to the decline of value for the currency implying an inflationary system. The restricted supply of Bitcoin is then the opposite with a deflationary system, meaning that if demand increases over time and the supply is staying relatively constant, it will lead to an increased value of Bitcoin.

The interest in cryptocurrencies and Bitcoin comes from its independence from a third party and its popularity as an investment likely stems from the tremendous price increase that Bitcoin has seen (Klein, Thu & Walther, 2018). Since its inception in 2008, Bitcoin has had an incredible run, but this is not without its due share of volatility. From 2010-2017 the price of one Bitcoin went from $0.01 to $20000 followed by a drop down to $3800 in 2018 and then with its recent surge in 2021 up to $60000.

Bitcoin and cryptocurrencies, which started as an experiment of a decentralized currency that is not linked to any monetary policy, are growing. It is becoming an asset for investors and for institutions to include in their portfolios, as well as a new innovative way of payment.

1.2 Purpose, research question and structure

Recently, Bitcoin has gained increased interest from individual investors as well as institutional investors. Due to similarities with Gold such as decentralization, low correlation towards stocks and it being an inflation hedge, it has been named the new “Digital Gold”, it is therefore of interest to examine if this notion is correct. There have been previous studies that examine the volatility of Bitcoin, its role in a portfolio, its ability to affect the risk-return of a portfolio and its ability to hedge and act as a safe haven, such as Klein, Thu and Walther (2018). Which finds that Bitcoin is not the new digital Gold.

The purpose of this study is to have an econometric perspective of Bitcoin and Gold and to build on previous work done in this field. Since Bitcoin and cryptocurrencies are such a young subject, each year brings a lot of new data points to examine which increases the total sample amount by a great deal as compared to Gold which has existed for a long time. Thus, the increased sample improves the accuracy of the analysis done. Furthermore, with the recent COVID-19 pandemic, the financial markets have been highly volatile. This volatility brings with it risk and opportunity for investors. The market turmoil supplies the first global bear market condition for Bitcoin since the beginning of active trading in 2013. This is then a perfect time period to examine the properties of Bitcoin during a real widespread crisis in the market and investigate its use as a tool in a portfolio for hedging or use as a safe haven, in the same way that Gold has been used. This opportunity has not presented itself prior to this and is thus an important time period which is not covered by previous studies like Klein, Thu and Walther (2018).

The contribution and reason for writing this thesis is then to build on previous research on the economic aspects of Bitcoin as an investment asset. This since a significant amount of new data points has appeared which improves the analysis. Furthermore, such a widespread market crash as the COVID-19 brought with it has not been seen previously in the history of Bitcoin which gives an excellent period to examine Bitcoins behavior as a hedge during market distress. In addition, this study will also expand on previous research by comparing several stock market indices to examine the difference between countries.

The research questions are:

- *How does the volatility of Bitcoin compare to Gold and stock indices?*

- *What are the hedge and safe haven capabilities of Bitcoin as compared to Gold?*

This will be done by examining the volatility of Bitcoin and other assets with univariate ARCH models as well as a multivariate Bivariate Diagonal BEKK-GARCH model. Finally, the study will examine the hedging properties of Bitcoin and Gold in a portfolio of different indices by using a minimum variance portfolio.

The structure of this paper is divided into five sections. Following the introduction, section 2 describes the previous literature regarding Bitcoin, its volatility and performance in a portfolio.

Section 3 explains the methodology, the variables and the data. Section 4 presents the results from the study and an analysis of them. Finally, section 5 provides our conclusion.

2. Literature review

The aim of this section is to inform the reader of previous works that have been done on the subject. As to give the reader insight into how Bitcoin has been previously researched and reviewed.

2.1 History of money

In the beginning money was seen in the form of coins made of precious metals. The value of the currency then was inherent from the value of the metal. The next iteration of money saw it turned into bank notes, but the value of the note was still linked to precious metals that were contained in a bank vault. With the abandoning of the Gold standard currencies are no longer linked to any commodity. It is now more a gauge of the health and prosperity of a country according to Bariviera, Basgall, Hasperué and Naiouf (2017). With currencies presently most used in a digital form, the differences with crypto currencies might not seem that large. However, by definition, the requirements for an asset being a currency are: it is a store of value, a medium of exchange and unit of account. In their study Bariviera et al. (2017) find that Bitcoin does not display these properties and should therefore not be regarded as a currency, rather that it should be thought of as a speculative asset. Baek and Elbeck (2015) agrees that Bitcoin is a speculative asset and Yermack (2015) claims that Bitcoin is not a “real currency”. Cheah and Fry (2015) claims that the main appeal of Bitcoin is not its possible use as a money, rather that it is an object of speculation.

2.2 Valuing Cryptocurrencies and the volatility of Bitcoin and bubbles

Bitcoin is an asset which is hard to value as it has no clear intrinsic value and there is no consensus whether it is a commodity or a currency (Cheung, Roca & Su, 2015). This feature then relates to the creation of price bubbles which has often been investigated in the literature (Blau, 2018; Camerer, 1989; Shiller, 1990;). The occurrence of bubbles has been a phenomenon that has long existed in our history and in a lot of different markets. Regarding bubbles, they are very difficult to assess (Chaim & Laurini 2019), there is also no agreed upon definition of a bubble (Cheung, Roca & Su, 2015). Grinberg (2011) says that Bitcoin will always be portrayed as a bubble because of its nature, as a speculative commodity.

As with most financial assets the volatility of Bitcoin is an area that has been subject to a large degree of investigation. A high degree of volatility in an asset can be evidence of a bubble (Cheung, Roca & Su, 2015). Regarding its volatility, Bitcoin exhibits almost double the volatility of 51 other currencies (Blau, 2018) and it appears to be 26 times more volatile than the S&P 500 index (Baek & Elbeck 2015). Dwyer (2015) observed that Bitcoins volatility is much higher than that of Gold and some traditional currencies. He also finds that the volatility and excess return gives proof that it is a speculative asset and also that Bitcoin has a higher idiosyncratic risk than that of Gold. Blau (2018) investigates if the high level of volatility of Bitcoin is connected to speculative trading. Blau (2018) does however not find any relationship between Bitcoins volatility and speculative trading. This result contradicts those of Cheung, Roca and Su (2015) and Cheah and Fry (2015) who observe that Bitcoin seems to form bubbles during the same time period. More recently performed studies done by Chaim and Laurini (2019) examine the volatility of Bitcoin and find evidence of a bubble during the time period of these previous studies, but do not find any signs from that point in their sample up to their publication. Kristoufek (2019) also finds proof of bubbles in Bitcoin during these periods but none at the end of his sample either.

Kristoufek (2015) finds that Bitcoin is similar to speculative assets but also traditional financial assets. He observes that Bitcoin is often subject to bubbles and that Bitcoin and its price dynamics is not similar to those of Gold. Corbet, Lucey and Yarovaya (2018) also observes that the price of Bitcoin displays bubble behavior at times, but they do not find any persistent bubbles.

Bouri, Shahzad and Roubaud (2019) notices that cryptocurrencies display evidence of co-explosivity, which usually depends on the explosivity in other cryptocurrencies, Bitcoin in particular. Camerer (1989) claims that bubbles, with their large price swings, will result in excess kurtosis and negative skewness, giving a leptokurtic distribution. Chaim and Laurini (2019) finds that Bitcoin returns are leptokurtic, meaning that the returns of Bitcoin are not normally distributed, which gives evidence that there might exist some bubbles.

2.3 Diversification, hedge and safe haven capabilities of Gold and Bitcoin

Gold

One of the first studies examining Gold as a hedge is Capie, Mills and Wood (2005). The authors found that Gold works as a hedge against the dollar, but that there is some time varying to this result. They do however not differentiate between average and extreme shocks during this study. Capie, Mills & Wood (2005) claims that the reason that Gold has served as a hedge is because it is a homogenous asset, which can be traded easily on an open market. Neither can it be produced as easily as the fiat currencies used by governments.

Baur and Lucey (2010) define a hedge as an asset that is uncorrelated or negatively correlated on average with another asset. It is important to note that this property does not hold during severe market conditions, where the relationship might be positive. A safe haven is an asset that is uncorrelated or negatively correlated during extreme conditions with another asset. Where again it is important to note that this property does not hold on the average and the relationship might be another.

A number of studies use these definitions and examine the hedging and safe haven capabilities of Gold and other assets during market turmoil and events that could be classified as black swan events. These studies found that Gold could be used as a hedge against stocks on average and also as a safe haven during extreme market conditions, the capabilities of Gold is the strongest amongst assets classified as safe havens during black swan events (Baur & Lucey, 2010; Baur & McDermott, 2010; Baur & McDermott, 2016). Important to note that these results work for US and European investors but not for emerging markets.

Other studies find that metals could be used as a hedge in a portfolio, but these results mainly apply to Gold (Hood & Malik, 2013; Klein, 2017), important to note is that these results do not hold during extremely low or high periods of volatility and that these findings seem to dissolve over time. Studies have also found that investors could improve the performance and the risk management of their portfolios by adding precious metals (Conover, Jensen, Johnson & Mercer, 2009; Hammoudeh, Mali & McAleer, 2011).

Bitcoin

The correlation of an asset is important as it conveys the possibilities of the asset having diversification benefits, hedging capabilities and the ability to act as a safe haven. There have been several studies examining the relationship between cryptocurrencies and other financial assets. These studies find that the correlation in between cryptocurrencies are positive, but between cryptocurrencies and other financial assets classes it is low and that this offers diversification benefits and could improve the risk return profile in a portfolio (Baur, Dimpfl & Kuck, 2018; Bouri, Molnár, Azzi, Roubaud & Hagfors, 2017a; Brière, Oosterlinck & Szafarz, 2015; Corbet, Meegan, Larkin, Lucey & Yarovaya, 2018; Dyhrberg, 2016a; Eisl, Gasser & Weinmayer, 2015; Härdle, Harvey & Reule 2019). Gajardo, Kristjanpoller and Minutolo (2018) investigates the cross-correlation between Bitcoin and major currencies such as the Euro, GBP and YEN, when compared to other major financial assets such as Gold, crude oil and the DJIA index. They find that Bitcoin clearly has a different relationship with the financial assets as compared to the major currencies and that this should be considered when investing due to the possibility of creating diversification benefits in a portfolio. Regarding the low correlation between cryptocurrencies and other asset classes and its following diversification benefits the literature seems fairly unanimous.

However, when it comes to the hedging capabilities there is a large disagreement. Dyhrberg (2016a) found that Bitcoin is similar to both Gold and the dollar and that it can be classified as something in between a currency and a commodity. A number of studies have found that Bitcoin can be used as a hedge against stocks and the dollar on average and during times of uncertainty (Bouri, Gupta, Tiwari & Roubaud, 2017b; Dyhrberg, 2016a; Dyhrberg, 2016b). Accordingly, it is argued that Bitcoin provides investors with another tool in risk management, gauging market sentiment and being perfect for risk averse investors expecting bad news. Bouri, Azzi and Dyhrberg (2017) found that Bitcoin has safe haven properties, although after the crash in 2013 these properties disappeared. Guesmi, Saadi, Abid and Ftiti (2019) found that short selling Bitcoin offers both diversification and hedging benefits if included into a portfolio which consists of stocks, Gold and oil and helps reduce the portfolio risk.

On the other side, a number of studies found that Bitcoin exhibit hedging or safe haven capabilities for only a few markets or not at all (Baur, Dimpfl & Kuck, 2018; Bouri et al, 2017a, Klein, Thu & Walther, 2018). Bouri et al (2017a) find that the hedging properties of Bitcoin is only present in

Japanese or Chinese stocks and that these results vary depending on time period chosen. Baur, Dimpfl and Kuck (2018) argues that Bitcoin is different from a traditional currency and that its volatility is much higher than that of Gold or the dollar. They also argue against Dyhrbergs (2016a) claim that Bitcoin is something in between Gold and the dollar, they state that even though both Bitcoin and Gold are uncorrelated to the stock market, this does not equate to them being equal. Klein, Thu and Walther (2018) argues that Bitcoin and Gold could barely be more different and that Bitcoin is not the new "Digital Gold".

The recent pandemic and the market turmoil it brought produces the first bear market condition for Bitcoin since active trading began. Conlon and McGee (2020) examines the behavior of Bitcoin during the following market downturn from COVID-19. Their analysis indicates that holding Bitcoin in a portfolio increases the downside risk relative to holding only the S&P 500. Conlon and McGee (2020) show that during the recent market turmoil, Bitcoin did not act as a safe haven, but rather it actually increased the risk of the portfolio during a Black swan event that investors were seeking shelter from. Bitcoin seems to move together with the S&P 500 during this crisis, acting in opposite of a safe haven.

In closing, the literature seems to agree regarding Golds function as a hedge. For Bitcoin, the articles find that it is uncorrelated with all other asset classes and therefore bring diversification benefits in a portfolio. Finally, the hedging capabilities of Bitcoin is a subject in which there is no consent. It is then this disagreement that this thesis will address.

3. Methodology

In this section we describe our data and the theoretical framework and methodology we use for our thesis. The purpose is to inform the reader about how the data was collected, the use of data and why the variables were chosen. Furthermore, introduce the different models and concepts used in this paper.

3.1 Data collection and management of variables

The data for this study consists of eleven different time series. The variables are: Bitcoin (USD per BTC), Gold and Silver (USD per oz), crude oil WTI, S&P 500, OMXS30, NIKKEI 225, FTSE 100, MSCI World, MSCI Emerging Markets and MSCI Europe. The data for the variables is collected from DataStream, except for Bitcoin which is collected from CoinDesk.com. The sample consists of 1947 observations, during the time period between 2013-10-31 and 2021-03-31. Although Bitcoin was created in 2008, the interest in the asset did not become significant until 2013 when the volume started to increase. This low volume might affect the analysis. For this reason, the analysis and observations are based from the end of 2013. All the observations are in dollars and based on the historical daily closing prices of the variables, which is useful to note because of the fact that Bitcoin is traded continuously. Moreover, the observations only include the weekdays.

The returns r of the variables was calculated on the daily closing prices for all of the time series using the log-returns.

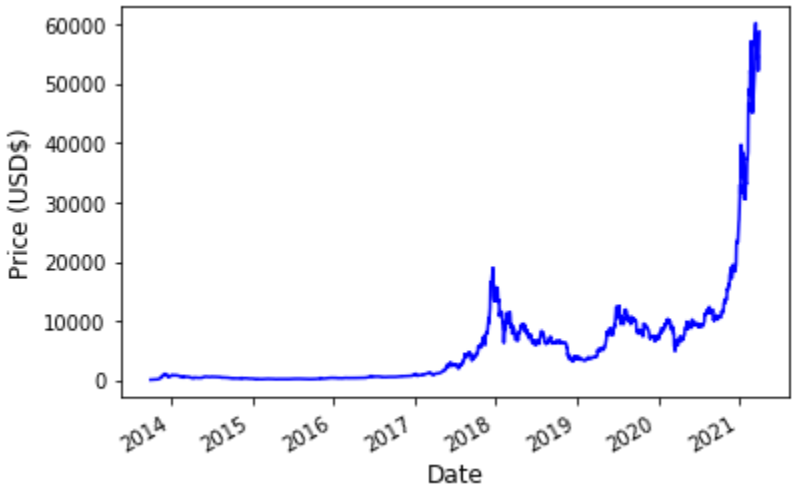
$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) * 100 \qquad (2)$$

3.1.1 Bitcoin

Since the emergence of cryptocurrencies in 2008, the interest for the asset class has grown over time. The original and go-to leader for the space is Bitcoin, with a market capitalization of $850 Billion, it is the world's largest digital currency accounting for almost 50% of the market capitalization of all cryptocurrencies. The interest for cryptocurrencies has skyrocketed, likely due to the recent rapid price increase as seen in Figure 1. Leading to more individuals and firms, incorporating Bitcoin into their portfolio. This paper is then interested in Bitcoin, as it is the leader and largest cryptocurrency,

for its use as an additional financial asset with it's supposed diversification and hedging properties, as it has been called the "New Digital Gold".

Figure 1: Price of Bitcoin, from October 2013 to March 2021



3.1.2 Gold

A commodity is a basic good, which is commonly used as the input for goods and services. For example: Gold, grain, beef or oil. These are then traded with future contracts, where the quantity and date are predetermined, on established markets. These contracts are used by traders looking to either hedge or speculate. As to hedge against price changes or simply making a profit. Commodities have been shown to have low or negative correlation with stocks, bonds and other types of financial assets (Baur & Lucey, 2010; Conover, Jensen, Johnson & Mercer, 2009; Hammoudeh, Mali & McAleer, 2011). Commodities are also considered an inflationary hedge, since they have a positive correlation on changes in inflation. As a result, commodities are frequently used by portfolio managers to manage risk and diversify investments. The most common commodity to use to diversify a portfolio is Gold. The reason is that Gold is one of the easiest commodities to store and it is not as commonly used as other commodities as a raw material in industries leading to less exposure in business cycles.

This paper is mainly interested in Gold, instead of all the other commodities, as it is the most commonly used commodity in a portfolio to hedge or as a safe haven against the market (Baur & Lucey 2010; Baur & McDermott, 2010). As can be seen in Figure 2, Gold gained a lot in price during the uncertainty that the COVID-19 meant. Gold and Bitcoin also share a lot of the same

characteristics in that neither are centralized so the prices do not depend on political decision. They are both scarce and costly to extract as well as easy to store (Dyhrberg 2016b).

Figure 2: Price of Gold from October 2013 to March 2021.

3.1.3 Stock Indices

The data sample consists of daily stock returns from the indices: S&P 500, OMXS 30, NIKKEI 225, FTSE 100, MSCI World, MSCI Emerging markets and MSCI Europe. The stock market indices are selected as they are representative of different investment opportunities around the world. These indices were chosen to gauge the different countries' businesses and their economies. Furthermore, they were also selected to expand the scope of earlier studies, which often focus on the S&P 500. As to see if there is a difference between the different markets and countries and their ability to use either Gold or Bitcoin as a part of their portfolios.

S&P 500: Is a stock market index that is weighted on the market capitalization of the 500 largest companies listed on the New York Stock Exchange.
NIKKEI 225: Is a stock market index that is price weighted on the 225 largest publicly traded companies on the Tokyo Stock Exchange.
OMXS30: Is a stock market index that is weighted on the market capitalization of the 30 most traded stocks on the Stockholm Stock Exchange.
FTSE 100: Is a stock market index that is weighted on the market capitalization for the 100 largest companies that are listed on the London Stock Exchange.

MSCI World: Is a stock market index that is weighted on market capitalization of 1583 companies in 23 different developed countries around the world.
MSCI Emerging Markets: Is a stock market index that is weighted on market capitalization of 1391 companies in 27 countries in emerging markets.
MSCI Europe: Is a stock market index that is weighted on market capitalization of 434 companies in 15 developed countries in Europe.

3.2 Econometric models

To examine our research question, we will apply econometric models with the same methodology and structure at large as Klein, Thu and Walther (2018), although the details will be different, such as having a different data set and including more variables. Initially we will apply an Augmented Dickey Fuller test to examine if our series are stationary. Then different ARCH-type models will be applied to the data to examine the volatility. Thereafter a Bivariate Diagonal BEKK-GARCH model will be used to study the relationship between Bitcoin and Gold and the other assets. Finally, the Minimum variance model will be applied to investigate the feature of Bitcoin as a hedge in a portfolio as compared to Gold.

3.2.1 Stationarity

A common assumption for econometric models and time series is that the data is stationary. The properties of a stationary time series are that the process does not change over time. This means that a stationary process has a constant mean, constant variance and a constant auto-covariance. It is a desirable trait and often a requirement for a time series to be stationary to apply different econometric models. Furthermore, if a time series is stationary the process of forecasting becomes much simpler as the change of the process is known over time (Enders, 2015).

Regressing two independent nonstationary time series onto each other might cause a spurious regression. Spurious regression means that the coefficients will be none zero, even though the processes are independent of each other (Enders, 2015). Leading to false results and a high R^2. This spurious regression bias depends on the fact that there is a similar local trend. Both of the variables are drifting up, which is often the case with finance and macroeconomic variables such as GDP and prices. To adjust for this potential problem the first difference is taken as to create stationary variables.

To examine if the time series are stationary, this study will apply an Augmented Dickey Fuller test to evaluate whether or not the series are stationary. Where under the null hypothesis the time series is non-stationary. If the null hypothesis is rejected then the time series is considered stationary.

3.2.2 Volatility ARCH-type models

The returns of an asset are generally thought of as being unpredictable, giving little use in estimation and forecasting. However, this unpredictability refers to the returns and mean of an asset and not the variance. A lot of financial and macroeconomic time series commonly exhibit heteroscedasticity, this in the form of volatility clustering, which is a stylized fact of finance. Subsequently meaning that the variance of the time series is time varying, with periods of high and low volatility clustered together. So that the variance behaves like an AR model, where it depends upon its own past shocks.

The autoregressive conditional heteroskedasticity (ARCH) model introduced by Engle (1982) is then based on this observation and is used to study the conditional variance of a time series that is conditioned on its own past. The ARCH model uses an AR(p) process in order to forecast the error term based on its past values. The problem is that in practice the ARCH often requires a large number of lags to capture the time variation in the volatility. The Generalized ARCH (GARCH) model introduced by Bollerslev (1986) expands the ARCH model by including an MA(q) component, allowing for a more flexible lag structure where the conditional variance is able to depend on its own lags, thereby reducing the number of lags required.

Moreover, Ding, Granger and Engle (1993) presented the Asymmetric Power ARCH (APARCH) model, to consider the asymmetry which is in the conditional variance. Asymmetry in the variance is in turn the observed phenomenon in which positive and negative shocks of equal size can lead to changes in the variance which are not of equal size. Finally, Baillie, Bollerslev and Mikkelsen (1996) created the Fractionally Integrated Generalized ARCH (FIGARCH), which aims to represent the long memory feature. This feature simply put, means that a time series depends to an extent on its own past values. The conditional variance will be depicted using the GARCH, APARCH and FIGARCH. These models have been chosen to capture the asymmetry and long memory in the volatility, as to show the properties and structure of the volatility behavior.

For the models, the asset return r_t is modeled by an AR (1) model as seen in equation (2).
The residual, ε_t in equation (2) is in itself modeled as in equation (3), h_t represents the conditional variance and η_t is Student's t-distributed as is seen in equation (4). The conditional variance is depicted for the different models in table 1.

$$r_t = \theta_0 + \theta_1 r_{t-1} + \varepsilon_t \quad (2)$$

$$\varepsilon_t = \sqrt{h_t \eta_t} \quad (3)$$

$$\eta_t \sim St - t_v(0,1)\ i.i.d.\ for\ all\ t = 1,\ldots,n \quad (4)$$

Table 1: ARCH models used to model the conditional variance.

Model	Definition
GARCH (1,1)	$h_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$
APARCH (1,1)	$h_t^\delta = \omega + \alpha_1(\varepsilon_{t-1} - \gamma_1 \varepsilon_{t-1})^\delta + \beta_1 h_{t-1}^\delta$
FIGACRH (1,1)	$h_t^2 = \omega + [1 - \beta_1 L - (1 - \phi_1 L)(1 - L)^d]\varepsilon_t^2 + \beta_1 h_{t-1}^2$

The most important parameters in the table are the asymmetry or leverage parameter $\gamma \in (-1,1)$, which measures the residuals asymmetric behavior and its impact on the conditional volatility. The Power parameter $\delta \in (0.25,5.0)$ determines how to model standard deviation (δ= 1), the variance (δ= 2) or any other real valued degree, and indicates the behavior of long memory in the volatility. The fractional integration parameter d ∈ (0,1) gives another possibility on how to measure long memory. To make sure of stationarity and non-negative variance $\alpha, \beta, \omega \geq 0$ have to hold. FIGARCH requires $0 \leq \beta \leq \varphi + d$ and $0 \leq d \leq 1-2\varphi$.

3.2.3 BEKK-GARCH

In addition to the univariate modeling, there will also be a comparison of Bitcoin and Gold from a multivariate perspective examining their relationship to other assets. As the financial markets often are connected, price movements in one market could lead to an effect in another. Therefore, it is important to investigate this dependence. This is done by modelling the dynamic correlation between either Bitcoin or Gold and one of the stock indices. In order to obtain these values, the Bivariate Diagonal Baba-Engle-Kraft-Kroner (BEKK-GARCH) model (Engle & Kroner, 1995) will be used.

The conditional variance covariance matrix H_t in the BEKK model is defined as:

$$H_t = C^T C + A^T \varepsilon_{t-1} \varepsilon_{t-1}^T A + G^T H_{T-1} G \tag{5}$$

In the model, as mentioned above H_t is the conditional covariance matrix of the price returns. A, C and G are K x K matrices. In the Diagonal BEKK model both of the parameters A and G are diagonal matrices, so all their off diagonal elements are set to zero. So, our Bivariate Diagonal BEKK model looks like:

$$\begin{aligned} H_t &= \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \\ &= \begin{bmatrix} c_{11} & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2{}_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon^2{}_{2,t-1} \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \\ &\quad + \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \end{bmatrix} \begin{bmatrix} \sigma_{11,t-1} & \sigma_{12,t-1} \\ \sigma_{21,t-1} & \sigma_{22,t-1} \end{bmatrix} \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \end{bmatrix} \end{aligned} \tag{6}$$

3.2.4 Portfolio-based comparison, Minimum variance

For the final part of the methodology the hedging properties of Bitcoin and Gold are compared. This is done by constructing a number of minimum variance portfolios including either Bitcoin and Gold as well as a stock market index and then comparing these.

We start by calculating the time varying weights w_t for a minimum variance portfolio that is composed of two components, being either Bitcoin or Gold and a stock market index. These weights are then optimized for each time period in our sample period as:

$$\underset{w_t}{Min} = w_t^T H_t w_t \tag{7}$$

Where $H_t$is the covariance matrix obtained from the Bivariate Diagonal BEKK-GARCH model. This is then with the restriction that the weight must sum up to one. Thereafter the historical Value-at-Risk (VaR) is calculated for the different indices over the sample. This is done with the empirical quantile at 1%, 5% and 10%. For our indices, given that they have T returns, the VaR is then the T x 0.01-th return. The times of distress for the index is when the returns are more negative than the VaR_q. Finally, the portfolios consisting of an index and either Bitcoin or Gold are evaluated. Examining the portfolio returns, that can be regarded as an Expected Shortfall. This allows us to investigate the properties of our assets as a hedge or safe haven.

3.3 Descriptive statistics

In table 1 the descriptive statistics for all of the variables included in the analysis can be seen. For the time period of the dataset all of the variables included have yielded a positive mean return. Bitcoin has had the highest mean return by far whereas FTSE100 has had the lowest. Looking at the standard deviation Bitcoin once again yields the highest value while Gold is at the other end of the spectra yielding the lowest standard deviation. It can be seen that Bitcoin has more than double the standard deviation of WTI as was found in Klein, Thu and Walther (2018), and the standard deviation for the rest of the variables is considerably lower. Thereafter the minimum and maximum values can be seen, where Bitcoin once again has the most extreme values.

Finally, the higher order moments as well as the tests are presented. It can be seen that all of the variables have negative skewness as well as excess kurtosis, this in combination with the Jarque-

Bera test, it can be concluded that none of the series are normally distributed. This is in line with the previous findings discussed in section 2, concluding that Bitcoins distribution is Leptokurtic (Chaim & Laurini, 2019). The Ljung-Box shows that for all of the variables there is autocorrelation present except for NIKKEI 225 which is not significant. The ARCH test concludes that the volatility of the series is heteroscedastic and that the ARCH models can be applied. Finally, by the Augmented Dickey-Fuller statistic it is clear that all of the log return variables are stationary on a 99 % level of significance.

Table 2: Descriptive statistics for the returns of the variables

	Mean	Std	Min	Max	Skewness	Kurtosis	JB	ARCH (25)	L-B (25)	ADF
Bitcoin	0,316	4,897	-31,594	30,637	-0,271	5,973	2901,02***	204,46***	39,52**	-43,94***
Gold	0,013	0,886	-5,264	5,133	-0,086	3,758	1140,40***	169,36***	52,55***	-44,55***
Silver	0,008	1,579	-13,527	8,48	-0,622	6,472	3503,15***	167,34***	41,29**	-43,00***
WTI	0,043	2,016	-19,319	10,509	-0,812	9,354	7272,49***	365,17***	54,86***	-42,62***
S&P500	0,043	1,093	-12,765	8,968	-1,049	22,517	41267,86***	726,45***	182,82***	-13,74***
FTSE 100	0,006	1,202	-14,214	11,167	-1,208	21,058	36254,41***	380,53***	85,68***	-45,57***
NI225	0,030	1,161	-8,241	7,337	-0,262	5,132	2146,16***	261,08***	29,69	-50,44***
OMXS30	0,011	1,29	-14,649	8,356	-1,079	12,744	13479,59***	245,87***	63,94***	-42,78***
MSCI W	0,030	0,922	-10,441	8,406	-1,493	25,069	51436,09***	730,55***	228,47***	-13,39***
MSCI EM	0,014	0,975	-6,943	5,581	-0,716	5,798	2877,48***	607,58***	49,99***	-38,79***
MSCI EU	0,007	1,084	-14,061	8,523	-1,553	21,272	37293,87***	309,48***	72,68***	-43,00***

*Note: In the table the Std is the standard deviation, Min and Max are the minimum and maximum values of the variables. The Jarque-Bera test examines if the returns are normally distributed, if the value is different from 0 then the variables are not normally distributed. The Ljung-Box test checks the 25th lag in the time series for autocorrelation. ARCH (25) is the ARCH LM test at the 25th lag. Finally, the ADF tests the stationarity of the time series at the 25th lag. Statistical significance indicated by asterisk on the 1 % ***, 5 % **, and 10 % * level.*

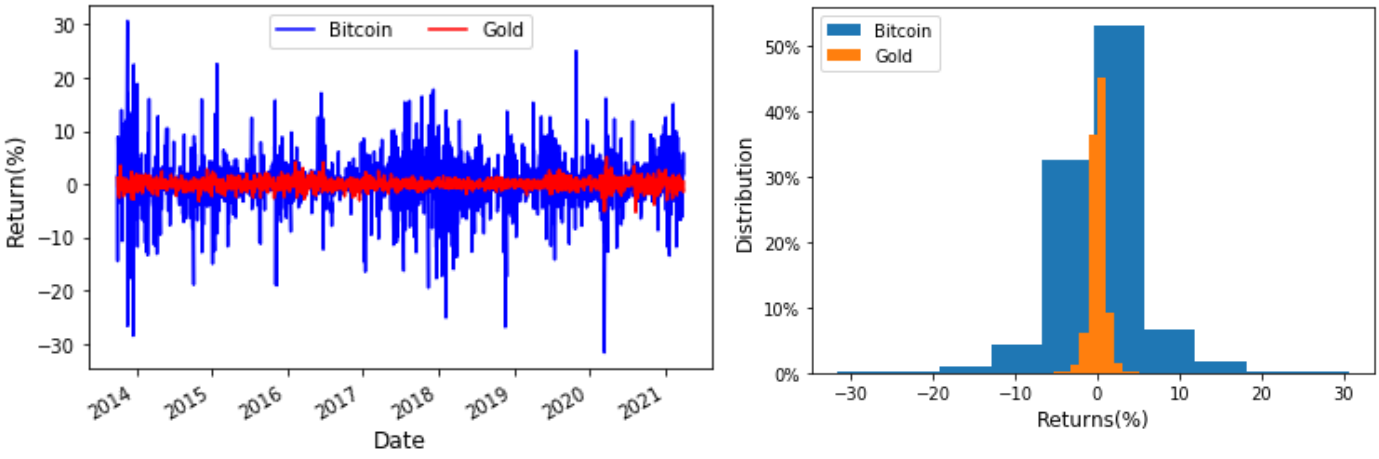
3.4 Preliminary analysis

The returns of Gold and Bitcoin are illustrated in Figure 4. It can be seen that the returns of the two assets are mean reverting, meaning that the two-time series are stationary, which is also concluded by the ADF test. By examining the volatility of the returns the stylized fact of volatility clustering is clear, both for Gold and Bitcoin, although the volatility of Bitcoin is much higher than that of Gold. Examining the data, it can be seen that Bitcoin is prone to large fluctuations and drawdowns, the largest and most recent one was yielded from the COVID-19 market crash.

It can be seen from the histogram that neither of the assets are normally distributed. The higher kurtosis of Bitcoin is clear, in that its tails are much fatter and denser than those of Gold. Furthermore, examining Bitcoin it can be seen that it follows a leptokurtic distribution. Meaning that the probability

of extreme events is likelier than those of a normal distribution. For Gold its return distribution is much more peaked, this is also a feature of the higher kurtosis of that of a normal distribution.

Figure 3: Daily returns of Bitcoin and Gold over time (left) and histogram of the returns of Bitcoin and Gold (Right).



Looking at the correlation matrix of the variables, it is interesting to see that Bitcoin has a consistently low correlation with all of the other variables. The only variable for which Bitcoin has a negative correlation is S&P 500 where the correlation is -0.0109, this is consistent with the result of Klein, Thu and Walther (2018). This is the only negative correlation found throughout all of the variables included in the analysis. Bitcoin has the highest correlation with the NIKKEI 225 index for which the value is 0.0726. Looking at Gold we can see that the commodity has a relatively higher correlation to every other variable compared to Bitcoin. Gold has its highest correlation with Silver, a correlation of 0.683 and its lowest correlation is with Bitcoin where the correlation is 0.0256.

Table 3: Correlation matrix between assets.

	Bitcoin	**Gold**	**Silver**	**WTI**	**S&P500**	**FTSE100**	**NI225**	**OMXS30**	**MSCI W**	**MSCI EM**	**MSCI EU**
Bitcoin	1										
Gold	0,026	1									
Silver	0,057	0,683	1								
WTI	0,040	0,052	0,166	1							
S&P500	0,011	0,061	0,123	0,338	1						
FTSE100	0,037	0,086	0,229	0,490	0,566	1					
NI225	0,073	0,091	0,159	0,118	0,091	0,091	1				
OMXS30	0,037	0,079	0,219	0,423	0,540	0,816	0,212	1			
MSCI W	0,008	0,100	0,198	0,040	0,953	0,750	0,260	0,708	1		
MSCI EM	0,053	0,116	0,247	0,420	0,488	0,645	0,426	0,606	0,639	1	
MSCI EU	0,041	0,087	0,233	0,384	0,590	0,945	0,246	0,897	0,782	0,660	1

4. Empirical Result and Analysis

In this section the results are provided along with an analysis of them. The structure follows that of the methodology. First the volatility structure of the different assets is presented and compared using the different univariate ARCH models. Thereafter the multivariate version, the BEKK-GARCH model, is examined. Finally, the portfolio analysis is presented considering the hedging properties of the two assets.

4.1 ARCH-type models

The univariate analysis as presented in the methodology consists of GARCH (1,1), APARCH (1,1), FIGARCH (1,*d*,1)

Table 4 Estimation results from the GARCH (1,1) Model

	θ_0	θ_1	ω	α_1	β_1	Df	LL	BIC	JB
Bitcoin	0,201***	-0.017	0,019**	0,226***	0,869***	2,586***	-5381,87	5,554	5021,56***
Gold	0,007	0.009	0,008**	0,039****	0,952***	4,629***	-2336,14	2,424	363,36***
Silver	0,028	-0.018	0,019**	0,003***	0,968***	3,049***	-3352,7	3,469	989,16***
WTI	0,022	0,0004	0,023**	0,067***	0,932***	5,173***	-3788,69	3,917	316,13***
S&P 500	0,091***	-0,072***	0,027***	0,216***	0,781***	4,424***	-2195,28	2,279	1064,75***
FTSE 100	0,031*	-0,004	0,039***	0,139***	0,835***	4,991***	-2607,14	2,703	451,72***
NI225	0,069***	-0,150***	0,052***	0,110***	0,857***	4,460***	-2763,6	2,863	676,11***
OMXS30	0,042*	-0,053**	0,031***	0,007***	0,905***	6,244***	-2954,59	3,060	497,64***
MSCI W	0,065***	0,085***	0,018***	0,184***	0,802***	5,132***	-1931,75	1,990	702,46***
MSCI EM	0,042**	0,184***	0,022***	0,088***	0,885***	10,710***	-2439,4	2,530	89,20***
MSCI EU	0,047***	-0,011	0,031***	0,123***	0,846***	5,071***	-2488,05	2,580	584,85***

*Note: Statistical significance indicated by asterisk on the 1 % ***, 5 % **, and 10 % * level. n=1947 LL is the log likelihood and Df is the degrees of freedom.*

In Table 4 the estimation results for the GARCH (1,1) model are displayed. For the constant, omega, all of the assets have a rather low value. The highest being MSCI Europe with a value of 0,0318 and the lowest being Gold with a value of 0,0082. Looking at the ARCH parameter, α, and the GARCH parameter, B, we can observe that for all of the included assets the sum of these parameters is close to 1. This indicates that shocks to the conditional variance will be highly persistent. The ARCH parameter, α, captures to which extent the value at time t is affected by its previous value at time t-1 while the GARCH parameter, β, captures to which extent the value today

is affected by the volatility in the previous period. The precious metals Gold and Silver as well as oil all observe low values of the α parameter with a corresponding higher value of the β parameter, this indicates that the value of these assets to a greater extent affected by their previous volatility. The different market indices have a higher α parameter accompanied by a lower β parameter indicating that they are to a greater extent affected by their past value. Looking at Bitcoin its structure is similar to that of the different market indices, particularly S&P 500, rather than Gold or Silver.

Table 5: Estimation results of the APARCH (1,1) Model

	θ_0	θ_1	ω	α_1	γ_1	β_1	δ	Df	LL	BIC	JB
Bitcoin	0,209***	-0,048***	0,064***	0,172***	-0.110	0,874***	0,633***	2,583***	-5365.987	5.546	1157,26***
Gold	0,009	0,009	0,007**	0,023	-0.059	0,948***	3,021***	4,640***	-2334.248	2.430	371,85***
Silver	0,031	-0,020	0,062***	0,027**	-0,203***	0,943***	3,092***	3,113***	-3347.005	3.471	1078,61***
WTI	-0,004	-0,001	0,016***	0,049***	0,688***	0,954***	1,109***	5,842***	-3772.221	3.908	254,38***
S&P500	0,056***	-0,057***	0,034***	0,145***	0,974***	0,864***	0,856***	4,929***	-2146.982	2.238	4857,66***
FTSE100	0,006	0,005	0,029***	0,097***	0,756***	0,896***	1,062***	5,634***	-2580	2.683	994,16***
NI225	0,045**	-0,134***	0,051***	0,103***	0,750***	0,879***	0,938***	4,922***	-2738.875	2.846	460,40***
OMXS30	0,017	-0,046**	0,022***	0,046***	0,999***	0,933***	1,388***	6,726***	-2929.56	3.043	455,30***
MSCIW	0,039***	0,093***	0,025***	0,118***	0,956***	0,877***	0,978***	5,837***	-1871.606	1.955	3618,08***
MSCI EM	0,018	0,202***	0,025***	0,053**	0,698**	0,898***	1,838***	13,26***	-2419.139	2.517	54,95***
MSCI EU	0,019	0,003	0,030***	0,079***	0,999***	0,898***	1,195***	5,580***	-2452.429	2.552	863,61***

*Note: parameter results of the APARCH (1,1) model. Statistical significance indicated by asterisk on the 1 % ***, 5 % **, and 10 % * level.*

Next, we look at the results of the APARCH model which results are presented in table 4. Here the γ parameter is of special importance since it displays the asymmetric effect in the model. A positive (negative) γ indicates that negative (positive) shocks have a higher impact on the volatility compared to positive (negative) shocks. Here we find an interesting result. For Gold, Silver and Bitcoin, the γ parameter although not significant in the case of Bitcoin or Gold, is negative. A negative γ is a result of an inverse leverage effect, suggesting that negative shocks have a larger impact on the volatility than positive shocks, and is an established property of Gold and Silver as shown by (Klein, Thu, & Walther, 2018). For the other asset's γ is shown to be positive between 0,688 and 0,999 and significant suggesting the presence of a leverage effect in their volatility i.e. subject to asymmetric volatility where a negative shock have a higher impact on the volatility compared to a positive

shock. When it comes to power parameter δ. WTI S&P 500, FTSE100, NI225, MSCI World and MSCI EU are all around one which suggests modelling the standard deviation appears better than the variance. Bitcoin has the lowest value of 0,633 while the other assets have return values between 1,388 and 3,092.

Table 6: Estimation results from the FIGARCH(1,d,1)

	θ_0	θ_1	ω	Φ	β_1	*d*	Df	LL	BIC	JB
BTC	0,216***	-0,012	0,539**	0,158**	0,712***	0,725***	3,267***	-5387,20	5,563	3720,33***
Gold	0,008	0,012	0,073**	0,143*	0,450***	0,293***	4,723***	-2333,80	2,426	300,85***
Silver	0,027	-0,023	0,171**	0,484***	0,728***	0,350***	3,195***	-3352,09	3,472	952,35***
WTI	0,018	0,002	0,077**	0,251***	0,739***	0,538***	5,315***	-3785,76	3,918	408,14***
S&P 500	0,091***	-0,073***	0,046***	-0,056	0,387***	0,611***	4,533***	-2187,57	2,276	1088,7***
FTSE 100	0,032*	-0,002	0,057**	0,144*	0,556***	0,537***	4,744***	-2604,04	2,704	463,06***
NI225	0,072***	-0,150***	0,188**	-0,124	0,081	0,350***	4,463***	-2759,91	2,864	546,19***
OMXS 30	0,043*	-0,054**	0,075**	0,146*	0,555***	0,469***	5,917***	-2954,46	3,064	416,28***
MSCI W	0,066***	0,087***	0,033***	-0,036	0,430***	0,620***	5,174***	-1907,58	1,988	643,80***
MSCI EM	0,043**	0,182***	0,082**	-0,056	0,221	0,346***	10,786***	-2434,55	2,529	103,017***
MSCI EU	0,049***	-0,009	0,057***	0,090	0,492***	0,516***	4,911***	-2483,39	2,579	600,56***

*Table: parameter results of the FIGARCH(1,d,1) model. Statistical significance indicated by asterisk on the 1 % ***, 5 % **, and 10 % * level.*

Finally, we look at the results of the FIGARCH model. The parameter results are presented in Table 5. In the FIGARCH the α parameter is replaced by ϕ but will still be interpreted as the ARCH effect. The most central feature of the FIGARCH model is its ability to captures long memory effects which are facilitated by the *d* parameter, where a *d* close to 1 indicates a high occurrence of long memory. The results show that Bitcoin has the largest *d* parameter out of all of the assets suggesting that it shows the greatest occurrence of long-term memory. Interestingly Gold is the asset which displays the lowest value of the *d* parameter suggesting that it shows the least occurrence of a long-term memory. The other assets display presence of long memory somewhere in between Bitcoin and Gold, where S&P500 and MSCI World are the closest to Bitcoin.

In order to determine which of the ARCH-type models are most beneficial in analyzing the volatility structure of the different assets we compare the Bayesian information criterion (BIC) of the asset in the different models. The BIC suggests that Gold and Silver are best analyzed with the GARCH model. S&P 500 is best analyzed with the FIGARCH model and Bitcoin, WTI and the remaining market indices are best analyzed by the APARCH model.

4.2 Bivariate Diagonal BEKK

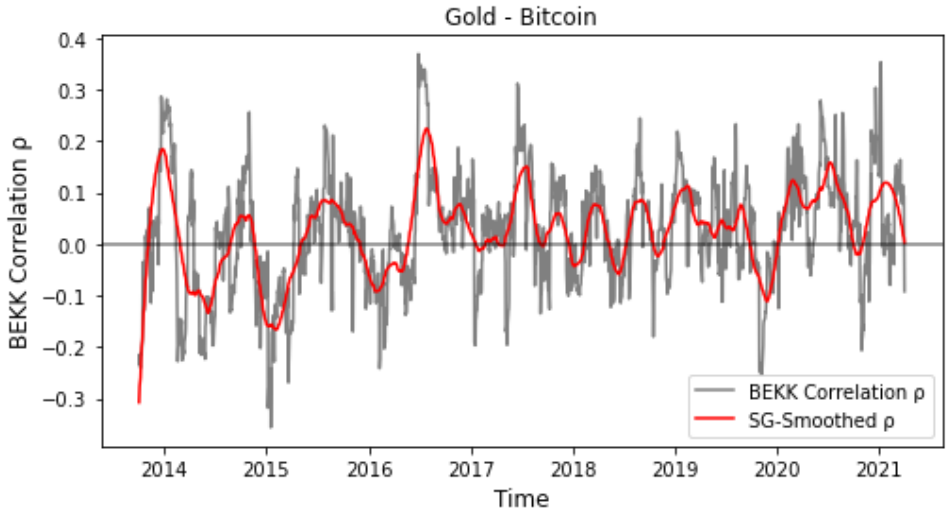
In this section we move away from the univariate analysis and focus on the multivariate results. Bitcoin and Gold are analyzed with each of the seven market indices by using the Bivariate Diagonal BEKK-GARCH model returning dynamic daily correlation for the purpose of investigating their relationship with the other assets. In the graphs below the obtained daily conditional correlations are presented and discussed. To better visualize the correlation between Gold and Bitcoin and the indices the data is fitted with a Savitzky-Golay filter for a clearer visualization of the results.

Table 7: Summary statistics of the BEKK-GARCH Correlations

	Mean	**Std.**	**Max**	**Min**
Gold-S&P500	-0,0655	0,1625	0,6611	-0,5353
Gold-FTSE100	0,0592	0,1737	0,6900	-0,5924
Gold-NI225	0,0939	0,1215	0,4307	-0,6250
Gold-OMX30	0,0278	0,1785	0,6701	-0,4877
Gold-MSCI W	0,0147	0,1837	0,7145	-0,5992
Gold-MSCI EM	0,0914	0,1803	0,6760	-0,5572
Gold-MSCI EU	0,0640	0,1871	0,6942	-0,6267
Gold-Bitcoin	0,0249	0,1122	0,3696	-0,3565
Bitcoin-S&P500	0,0246	0,1981	0,7614	-0,6545
Bitcoin-FTSE100	0,0346	0,1603	0,7858	-0,5692
Bitcoin-NI225	0,0486	0,1863	0,8240	-0,5486
Bitcoin-OMX30	0,0255	0,1315	0,6525	-0,4013
Bitcoin-MSCI W	0,0347	0,1897	0,6833	-0,6226
Bitcoin-MSCI EM	0,0249	0,1417	0,6011	-0,5459
Bitcoin-MSCI EU	0,0358	0,1608	0,7879	-0,5918

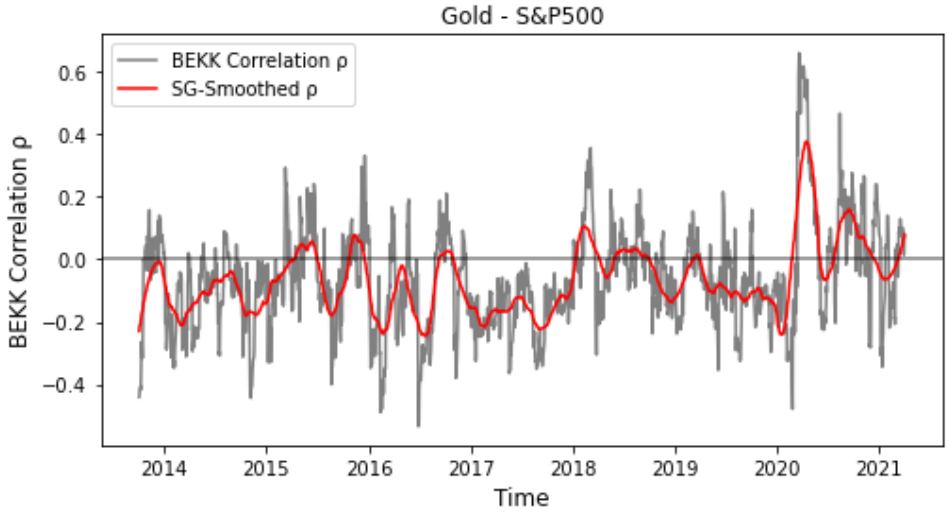
While Table 7 suggested that Bitcoin has a rather low correlation with the other assets over the whole sample period, analyzing the daily conditional correlation gives us more insight as to how the correlation has changed over time. The returns of Gold and Bitcoin showed a correlation of 0,026 over the whole sample period. Looking at the figure below we see that the daily conditional correlation moves frequently and takes different values between 0,369 and 0,356. The correlation between Bitcoin and Gold is interestingly the correlation with the smallest maximum value and the largest minimum value.

Figure 4: Daily conditional correlation between Gold and Bitcoin, calculated with the BEKK-GARCH



Looking at the correlation between Gold and S&P 500, which over the whole sample period was 0,06 we can see that it seems to move rather stable between 0,2 and -0,2 with a few outliers. In the beginning of 2020 it shot up and reached its peak of 0,661 this is particularly interesting to this investigation. The drastic increase in the correlation between Gold and S&P 500 is a result of the COVID-19 Pandemic and the following financial turmoil between 20th of February and 7th of April.

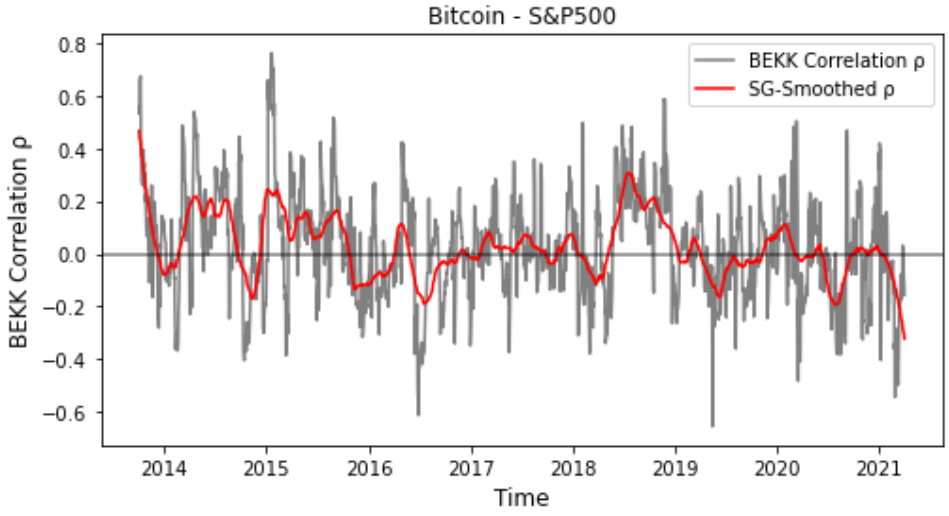
Figure 5: Daily conditional correlation between Gold and S&P 500, calculated with the BEKK-GARCH



Note: Unfiltered correlations are plotted in grey while the red line is Savitzky-Golay smoothing filter applied to the data.

Comparing the correlation between Gold and S&P 500 to the correlation between Bitcoin and S&P 500 we see that the latter has a higher volatility. Bitcoin reached a maximum positive correlation with S&P500 at 0,761, however this peak was observed already in 2015. During the financial turmoil following COVID-19 it seems as if Bitcoin has had a lower correlation to S&P 500 than that of Gold.

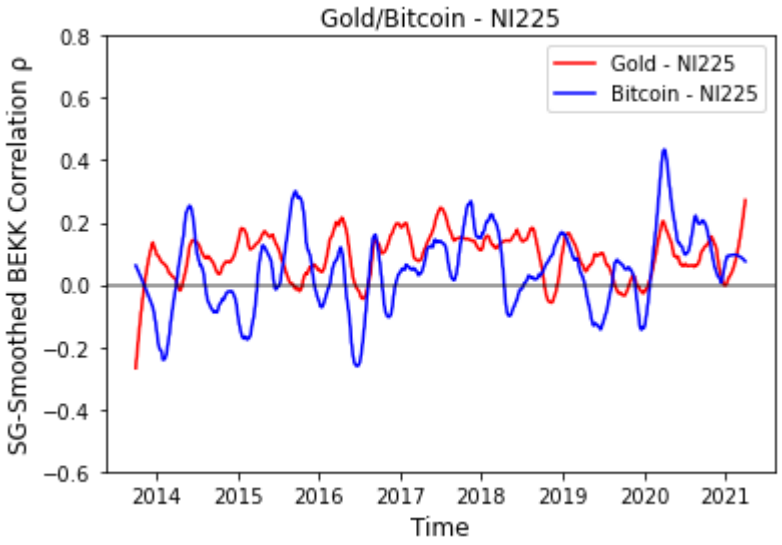
Figure 6: Daily conditional correlation between Bitcoin and S&P 500, calculated with the BEKK-GARCH



Note: Unfiltered correlations are plotted in grey while the red line is Savitzky-Golay smoothing filter applied to the data.

When comparing Bitcoin and Gold to other indices the results are largely similar, figures that visualize these can all be found in the appendix. The increase in correlation between Gold and market index during the Covid-19 financial turmoil can be found in FTSE 100, OMXS30, MSCI World index, MSCI EM, and MSCI EU. Similarly, Bitcoin seems to have a lower correlation to all of these indices during the same time period or at least it does not display such a drastic increase. Corresponding figures to the other market indices can be found in the appendix. This effect is however not as strong in the other markets as in the comparison with S&P 500. There is an exception to this seeming trend and that is in the daily correlations of Bitcoin and Gold to the NIKKEI 225 index. In this case the figures seem almost reversed from the S&P 500 case. The correlation between Gold and NIKKEI 225 moves frequently between 0,4 and -0,2 with a few notable low points further back in time. Bitcoin on the other hand often moves between 0,2 and -0,2 with recurring movements between 0,4 and -0,4 and few outliers upward of 0,7. The most interesting period to look at is once again the financial turmoil following the COVID-19 pandemic where the relationship between Gold, Bitcoin and NIKKEI 225 seems almost interchanged from the other market indices. Here the correlation to Gold does not seem to be that influenced by the market turmoil while Bitcoin sees a drastic increase reaching a peak correlation of 0,82.

Figure 7: S-G Smoothed daily conditional correlation between Bitcoin/Gold and NIKKEI 225.



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Applying the definition of a hedge and safe haven as described by Baur and Lucy (2010), the averages from Table 7 suggests that the only obvious hedge is that of Gold against S&P 500 which is the only correlation that is negative on average. For the rest of the correlations they are all positive albeit small is some cases where Bitcoin can be said to be a better hedge than Gold against some of the market indices. When it comes to safe haven capabilities during the Covid-19 pandemic neither asset can be clearly defined as a safe haven as both of them has a positive correlation with each market index. However, for all of the indices except for NIKKEI 225 Bitcoin shows a smaller increase in the correlation than Gold for this period.

4.3 Portfolio based analysis of the hedging property.

In this section the hedging properties of Bitcoin and Gold are examined. The portfolios that are examined consists of two asset and they are: Bitcoin-S&P 500, Gold-S&P 500, Bitcoin-FTSE 100, Gold-FTSE 100, Bitcoin-NIKKEI 225, Gold-NIKKEI 225, Bitcoin-OMXS 30, Gold-OMXS 30, Bitcoin-MSCI World, Gold-MSCI World, Bitcoin-MSCI Emerging Markets, Gold-MSCI Emerging Markets, Bitcoin-MSCI Europe and Gold-MSCI Europe. The structure of the results is as outlined in section 3.2.4.

First the time varying portfolio weights are represented in Table 8 for the 14 portfolios. There is a large variation between the different indices as to the proportion which is given to Gold. For the OMXS 30 which is the index with the largest amount has 65,65% of Gold on average in the portfolio, while the index with the lowest amount of Gold on average is the MSCI World with 39,99%. For all of the different stock indices the weight given to Gold is almost an order of magnitude bigger than that of Bitcoin. Bitcoin is generally given a small proportion of the portfolio, as is reported by Guasti et al. (2019) and Klein, Thu and Walther (2018). Where the MSCI World

index has the lowest amount with 4,33% of Bitcoin in the portfolio on average, while the OMXS30 has the highest amount of Bitcoin with 10,82%. Interestingly the OMXS30 index has the highest concentration of both Bitcoin and Gold while the MSCI World index has the lowest concentration of both Bitcoin and Gold. The size of Bitcoin in the portfolios and its standard deviations are about the same size, it can be noted that the standard deviation for Bitcoin is higher than the weights for all the indices except for OMXS30 and MSCI Emerging markets. This is likely due to the volatility of Bitcoin.

Table 8: Portfolio weights of the minimum variance portfolio containing either Gold or Bitcoin and a market index

Panel A: Descriptive statistics of portfolio weights

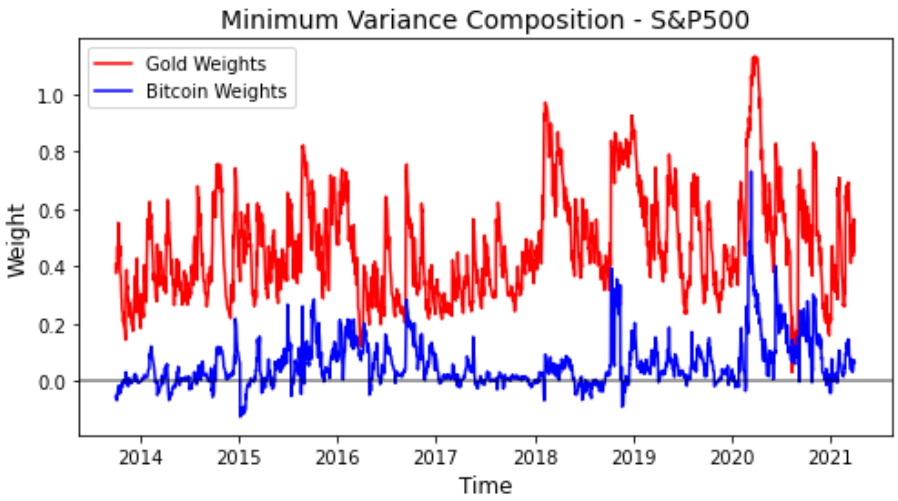
	S&P 500		**FTSE 100**		**NIKKEI 225**		**OMX 30**	
	Bitcoin	**Gold**	**Bitcoin**	**Gold**	**Bitcoin**	**Gold**	**Bitcoin**	**Gold**
Mean	0,0586	0,4687	0,0815	0,5661	0,0794	0,6158	0,1082	0,6565
Std. d	0,0822	0,1834	0,0915	0,1475	0,0905	0,1231	0,0981	0,1155
Min	-0,1243	0,0312	-0,1007	0,1961	-0,2365	0,2733	0,072	0,3703
Max	0,7317	1,1316	0,55	0,1735	0,5328	0,9653	0,4494	1,1736

Panel B: Descriptive statistics of portfolio weights

	MSCI World		**MSCI Emerging Markets**		**MSCI Europe**	
	Bitcoin	**Gold**	**Bitcoin**	**Gold**	**Bitcoin**	**Gold**
Mean	0,0433	0,3999	0,0676	0,543	0,0702	0,5359
Std. d	0,0639	0,1852	0,0666	0,1472	0,0794	0,1489
Min	-0,1223	0,0368	-0,0779	0,1211	-0,1018	0,2071
Max	0,6605	1,1746	0,3199	1,1711	0,5584	1,1878

In addition, the weights of Bitcoin and Gold together with the S&P 500 are illustrated in Figure 8, as to give a sense of the weights over time. The other figures of the portfolio weights generally follow the same pattern and can be found in the appendix. Figure 2 shows that the variation in the weights can vary from having more than 100% of the portfolio in Gold and short selling the S&P 500, to going the opposite way and having low amounts of Gold or even short selling Bitcoin during times.

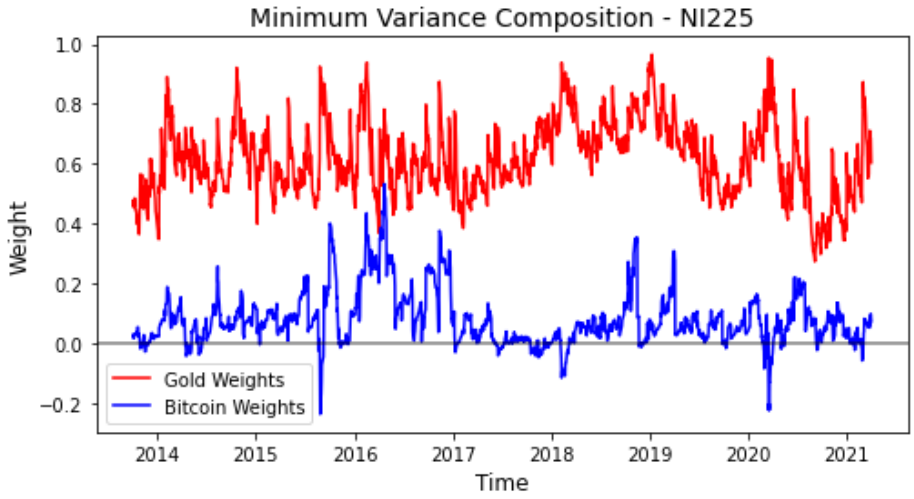
Figure 8: Minimum variance portfolio weights of Bitcoin/Gold and S&P 500



Note: The time varying weights of the minimum variance portfolio of Bitcoin and Gold with the S&P 500 index, based on the results from the BEKK-GARCH model.

An interesting result is that all the portfolios see a spike in the proportion of Gold during the COVID-19 market crash, where it is recommended to have more than 100% of the portfolio in Gold and short selling the indices. Except for the NIKKEI 225 where the amount of Gold does not go over 100%. Another interesting result is that the amount of Bitcoin in the portfolios also spikes up with Gold during the COVID-19 market crash except for the MSCI Emerging Markets which mostly stays flat, and the NIKKEI 225 which actually recommends short selling Bitcoin as can be seen in Figure 9.

Figure 9: Minimum variance portfolio weights of Bitcoin/Gold and NI225



Note: The time varying weights of the minimum variance portfolio of Bitcoin and Gold with the NIKKEI 225 index, based on the results from the BEKK-GARCH model.

Table 9: Value at risk of the different market indices at the 0,01; 0,05 and 0,10 level

	S&P 500	**NI225**	**FTSE100**	**OMXS30**	**MSCI W**	**MSCI EM**	**MSCI EU**
VaR0,01	-3,29002	-3,5146	-3,60973	-3,62875	-2,66667	-2,69798	-3,28504
VaR0,05	-1,56832	-1,85081	-1,70294	-1,99509	-1,2653	-1,54175	-1,5903
VaR0,10	-0,92737	-1,19941	-1,15049	-1,39797	-0,82796	-1,11357	-1,06094

The historical Value at Risk can be seen in Table 9. The highest values are given by the MSCI World index, where MSCI Emerging markets are close at the 1% and 5% level but are higher at the 10% level. The lowest VaR values are given by the OMXS 30 at the 1%, 5% and the 10%.

Table 10: Average portfolio and conditional portfolio returns.

	Return	Std.	rt|Var0,01	rt|Var0,05	rt|Var0,10
S&P500	0,0437	1,0930	-5,0154	-2,7465	-1,9842
Bitcoin	0,0431	1,1010	-3,5478	-2,2550	-1,6952
Gold	0,0290	0,6110	-1,3577	-0,7693	-0,6463
NIKKEI 225	0,0300	1,1610	-4,5304	-2,8379	-2,1462
Bitcoin	0,0390	1,1500	-4,0957	-2,6276	-2,0071
Gold	0,0180	0,7310	-1,3763	-0,8963	-0,7663
FTSE 100	0,0060	1,1610	-5,4798	-2,9690	-2,1788
Bitcoin	0,0080	1,2190	-4,9838	-2,6423	-1,9223
Gold	0,0070	0,6980	-1,5957	-0,9013	-0,7858
OMXS 30	0,0110	1,2900	-5,3173	-3,0748	-2,3656
Bitcoin	0,0370	1,2840	-4,7117	-2,7221	-2,0969
Gold	0,0100	0,7360	-1,2995	-0,8566	-0,7568
MSCI World	0,0300	0,9220	-5,0154	-2,7465	-1,9842
Bitcoin	0,0330	0,9710	-3,5478	-2,2550	-1,6952
Gold	0,0280	0,5870	-1,3577	-0,7693	-0,6463
MSCI EM	0,0140	0,9750	-4,5304	-2,8379	-2,1462
Bitcoin	0,0300	0,9750	-4,0957	-2,6276	-2,0071
Gold	0,0190	0,6700	-1,3763	-0,8963	-0,7663
MSCI EU	0,0070	1,0840	-5,4798	-2,9690	-2,1788
Bitcoin	0,0200	1,1310	-4,9838	-2,6423	-1,9223
Gold	0,0100	0,6780	-1,5957	-0,9013	-0,7858

Note: The returns belonging to respective market index is calculated as the returns of a portfolio holding 100 % of that index, while the respective asset below the index is calculated as the minimum variance portfolio of the index and that asset.

Finally, the Value at Risk from Table 9 are used to determine the periods when there is distress in the markets. Then examine how a portfolio which includes Bitcoin or Gold reacts during this time period of market turmoil. Table 10 shows the average return, the volatility and the average return given that the portfolio is in a time of distress at the 1%, 5% and 10% level.

We can see that Gold reduces the volatility for all portfolios when included with the indices, and reduces the return of the portfolio with the S&P 500, NIKKEI 225, OMXS30 and MSCI World, while it on the contrary increases the return in a portfolio with the FTSE 100, MSCI Emerging

Markets and MSCI Europe. These results are mostly in line with Klein, Thu and Walther (2018) who finds that gold reduces the volatility and return of all portfolios. For Bitcoin the results are more mixed, for the S&P 500 and the MSCI Europe the returns are decreased while the volatility is increased. For portfolios with FTSE 100 and MSCI World the returns as well as the volatility are increased. Interestingly for the MSCI Emerging Markets the volatility is constant while the return is more than doubled by including Bitcoin in a minimum variance portfolio. While for the NIKKEI 225 the return is increased with a decrease in volatility, and for the OMXS 30 the volatility is decreased while the return is more than tripled. These results about Bitcoin largely contrast those of Klein, Thu and Walther (2018) who find that Bitcoin increases the return at the cost of increasing the volatility.

Lastly the time periods when the market is in distress, which is when the returns are lower than the VaR presented in Panel B. We can see that both Bitcoin and Gold increase the average return of a minimum variance portfolio during market turmoil. These results are then in contradiction to those of Klein, Thu and Walther (2018) and Conlon and McGee (2020) who both find that Bitcoin contributes to the downside risk. We can see that Gold does provide a hedge by the much-increased returns when it is included in a portfolio. This then agrees with the earlier findings described in section 2. Although it does not provide full protection from a market downturn there are still clear hedging properties of Gold. The most interesting finding is that Bitcoin, although small, provides a reduction in risk for investors during times of distress in the market. It is most pronounced for the S&P 500 and the MSCI World index at the 1% level. Bitcoin, interestingly enough, does seem to provide a “mini-hedge” for all the indices.

5. Conclusion

The objective of this thesis was to answer questions regarding the volatility of Bitcoin and its hedging properties as compared to those of Gold. This was done by first analyzing the univariate volatility of the different variables using three different ARCH models. Similar to previous studies it was found that Bitcoin shows similar asymmetric behavior as Gold and Silver, although these results were not statistically significant. This finding of asymmetric behavior also suggest that both Bitcoin and Gold hold a desired property during negative shocks impacting the market. Examining the assets for presence of long memory they could not have been more different. Bitcoin displayed the highest presence of long memory being closest linked with the S&P 500 while Gold displayed the lowest. These results yielded the conclusion that Bitcoin and Gold best be modeled with different models. Where Gold favors the GARCH model and Bitcoin in turn favors the APARCH model.

The results from the multivariate analysis, the BEKK-GARCH model, found that the only distinct hedge was that of Gold against the S&P 500. Except for that, neither asset had any clear properties of being a hedge, where the results varied between indices for the assets. During the most recent market downturn following the COIVD-19 pandemic it was found that neither asset is a safe haven, however Bitcoin had a lower correlation to all of the indices except for NIKKEI 225.

Finally, the minimum variance portfolio analysis showed that the weights given to Gold in a portfolio was substantially higher than that of Bitcoin for all portfolios. Where the weight of Gold during certain periods exceeded 100 % of the portfolio and it was recommended to short sell the index. The index with highest recommended proportion of Gold and Bitcoin was the OMXS 30 and the lowest was the MSCI World. Interestingly all the portfolios had a spike in the amount of Gold and Bitcoin in the portfolio during the market distress following the COVID-19 pandemic, except for the NIKKEI 225 which inversely recommended short selling Bitcoin during this period.

For the return and volatility of the portfolios it was found that Gold, as in previous research, reduces the volatility of a portfolio on average. However, this study also found that Gold increased the return of some portfolios as well. This likely being due to the market turmoil observed under 2020. Previous studies like Klein, Thu and Walther (2018) had found that when included in a portfolio Bitcoin increases the returns and the volatility on average. This study found mixed results, where some portfolios with indices were in line with previous research. While for others it was the opposite

and it was found that Bitcoin reduced the volatility of a portfolio while substantially increasing the returns. These findings were especially for the NIKKEI 225 and the OMXS 30.

Furthermore, regarding the hedging properties, it was found that Gold substantially increased the returns of a portfolio during market distress, or inversely reduced the losses. Although not providing full protection from downturns, Gold exhibits hedging properties which is in line with previous studies. One of the most interesting findings in this study is that in contrast to previous studies who found that Bitcoin contributes to the downside risk and does not provide any hedge, this study does. Bitcoin is found to improve the portfolio during the worst 1, 5 and 10% of times. Where for when the market is in extreme distress, the worst 1% of times, it is found that Bitcoin provides a significant increase in gains for the S&P 500 and the MSCI World. Bitcoin, although not to the same extent as Gold, interestingly enough provides a sort of "mini-hedge" for all our indices.

Rounding up, some notions are put forward. Regarding the weights of the portfolios, one reason that OMXS 30 likely holds such high amounts of Gold and Bitcoin is because the index is so small, only consisting of 30 companies. Holding so few companies likely leads to higher volatility from swings in different sectors. Comparing this to the MSCI World index which has the lowest amount of the two assets, holds 1583 companies, which most likely reduces the volatility and swings from specific businesses and countries. In regards of our results, previous studies analyzing the properties of Bitcoin often include the very early years of Bitcoin. During these years the trading and usage of Bitcoin was very limited, this might lead to misleading results. This is one of the reasons that this study has chosen to start from October 2013, where the trading volume started to pick up. This study also had the advantage, compared to a lot of the previous studies, to be examining Bitcoin with more data points during a more active period in Bitcoins lifespan as well as examining Bitcoin during a time of global market distress as induced by the COVID-19 pandemic.

In light of what this study has concluded, this paper is still limited by the data available, Bitcoin and other cryptocurrencies are still a rather new addition to the financial markets. The impact of the COVID-19 pandemic has been the first major market downturn in the history of Bitcoin. Therefore, as time progresses and more data is made available it will be interesting to further study its properties. Our conclusion of Bitcoin being able to be used to boost returns and reduce volatility on average as well as during market distress, most likely will need revisiting.

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7. Appendix

Unsmoothed daily conditional correlation figures of Gold, Bitcoin and the indices FTSE100, OMXS30, MSCI World, MSCI Emerging markets and MSCI Europe are all available upon request.

Figure 10: Daily conditional correlation between Gold and NI225, calculated with the BEKK-GARCH.

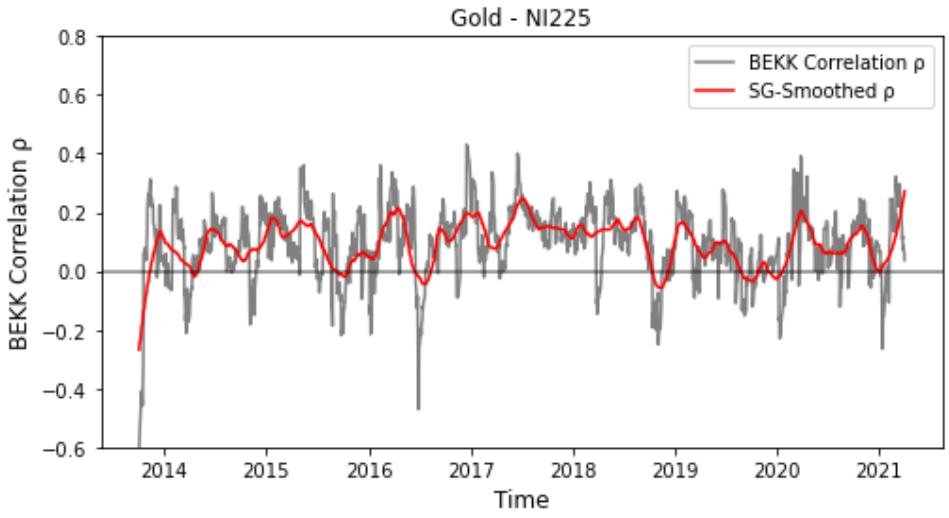

Figure 11: Daily conditional correlation between Bitcoin and NI225, calculated with the BEKK-GARCH.

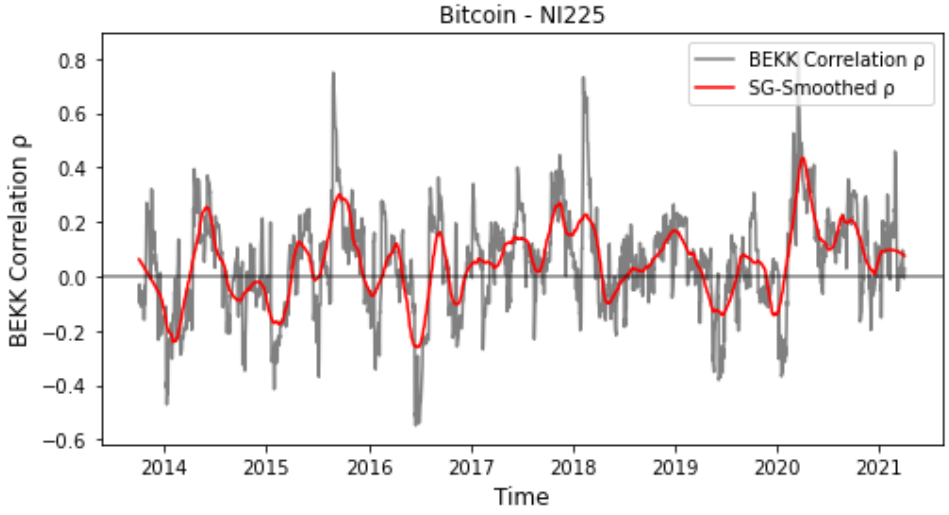

Figure 12: S-G Smoothed daily conditional correlation between Bitcoin/Gold and FTSE 100.

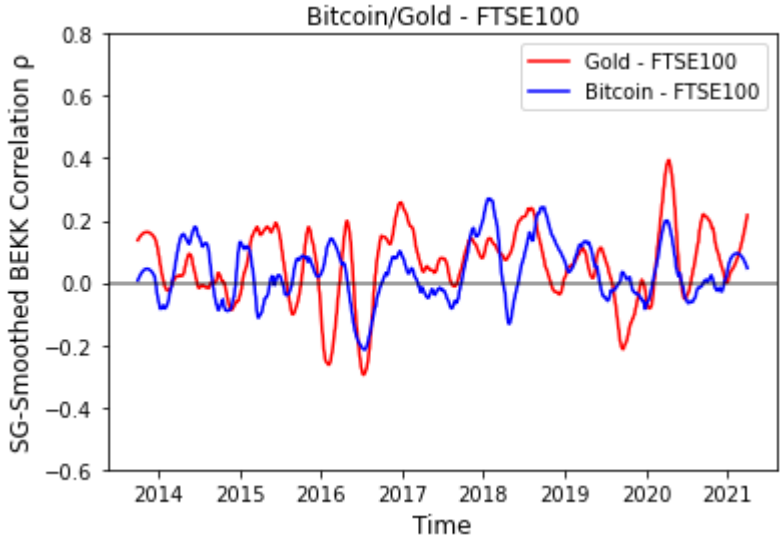

Figure 13: S-G Smoothed daily conditional correlation between Bitcoin/Gold and OMXS 30.

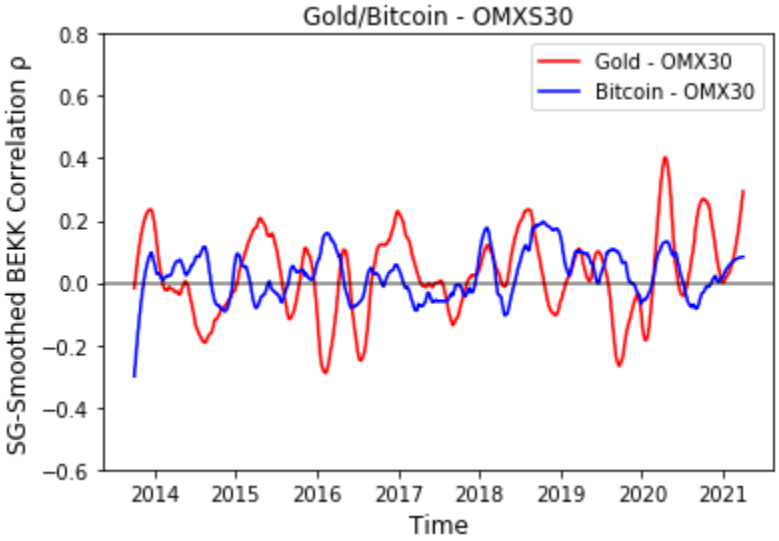

Figure 14: S-G Smoothed daily conditional correlation between Bitcoin/Gold and MSCI World index.

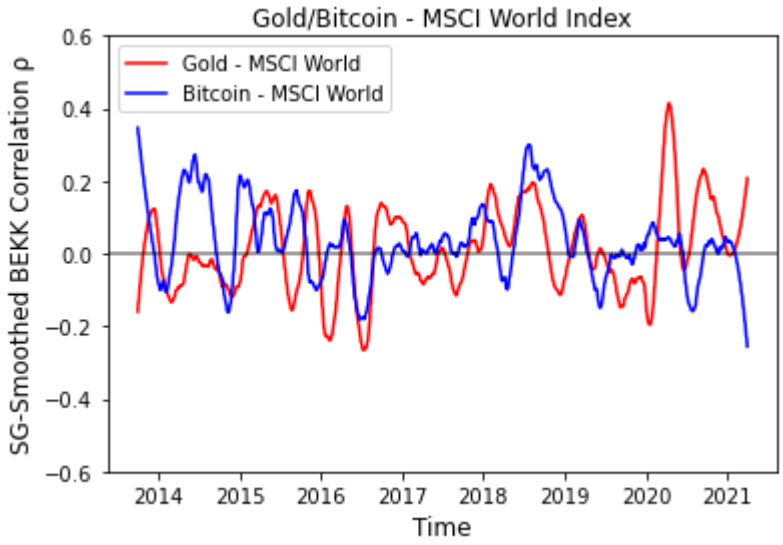

Figure 15: S-G Smoothed daily conditional correlation between Bitcoin/Gold and Emerging markets index.

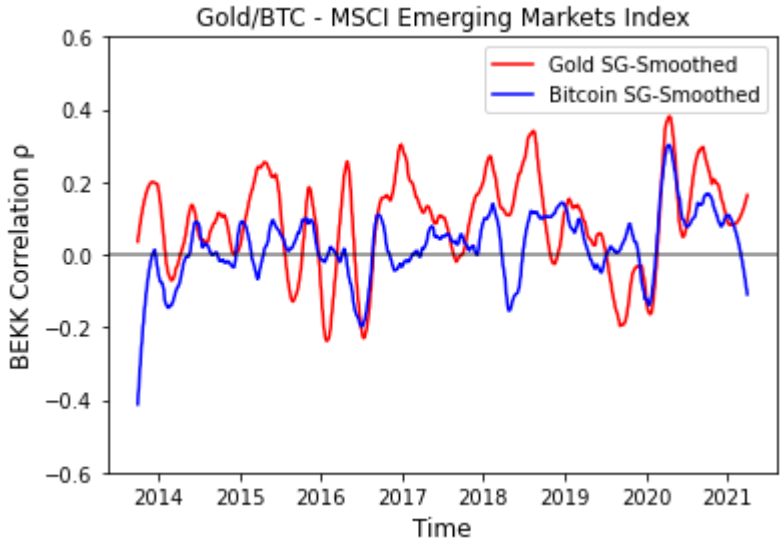

Figure 16: S-G Smoothed daily conditional correlation between Bitcoin/Gold and MSCI European index.

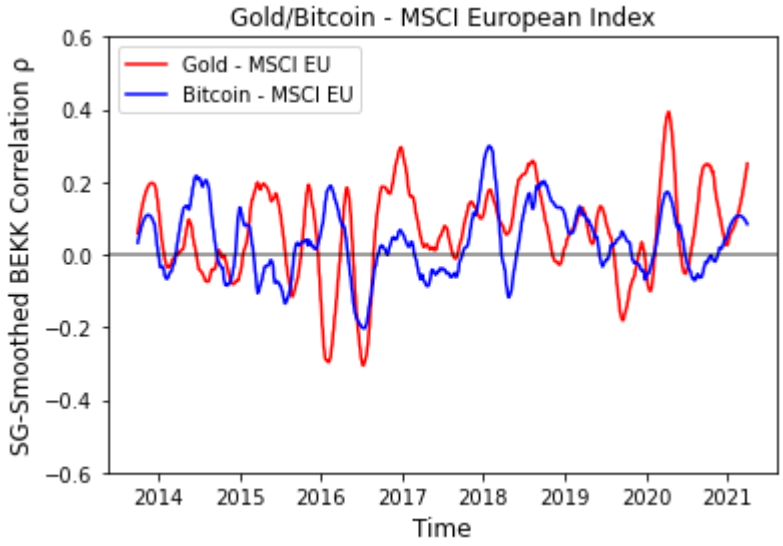

Figure 17: Minimum variance portfolio weights of Bitcoin/Gold and FTSE 100.

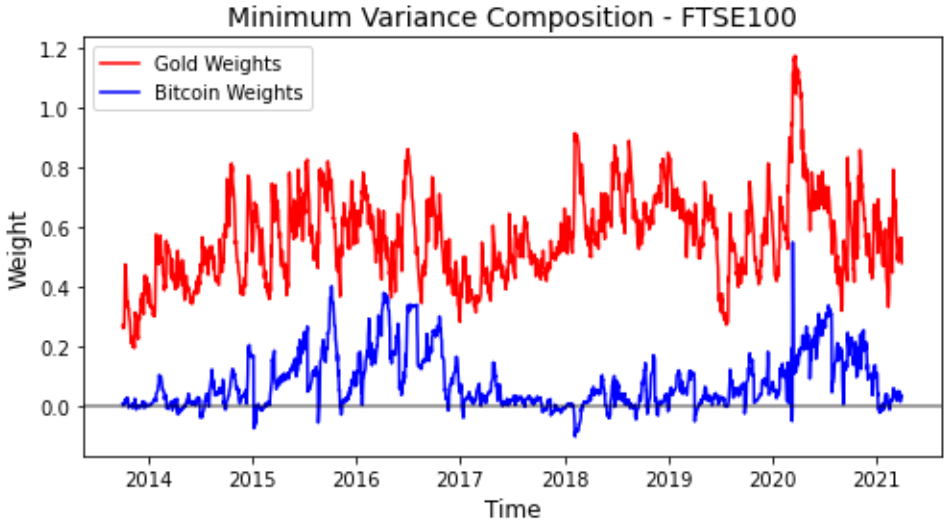

Figure 18: Minimum variance portfolio weights of Bitcoin/Gold and OMXS 30.

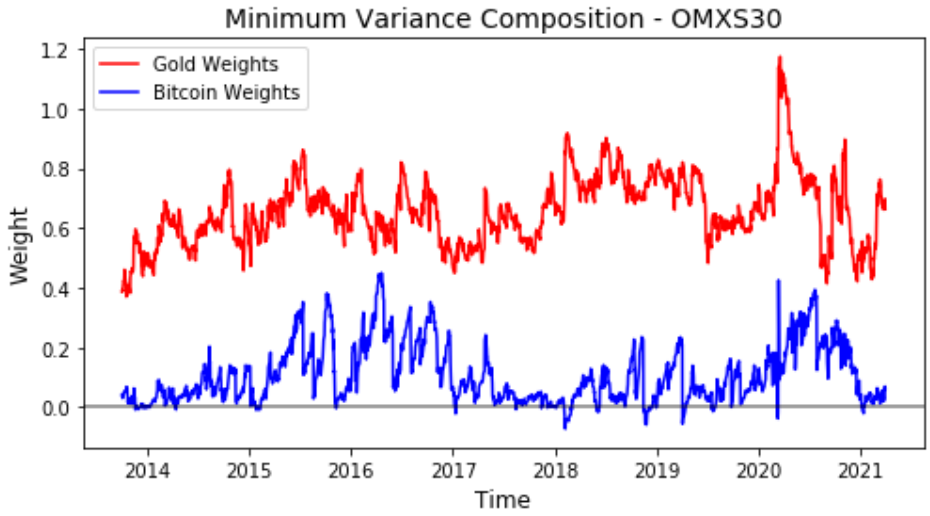

Figure 19: Minimum variance portfolio weights of Bitcoin/Gold and MSCI World index.

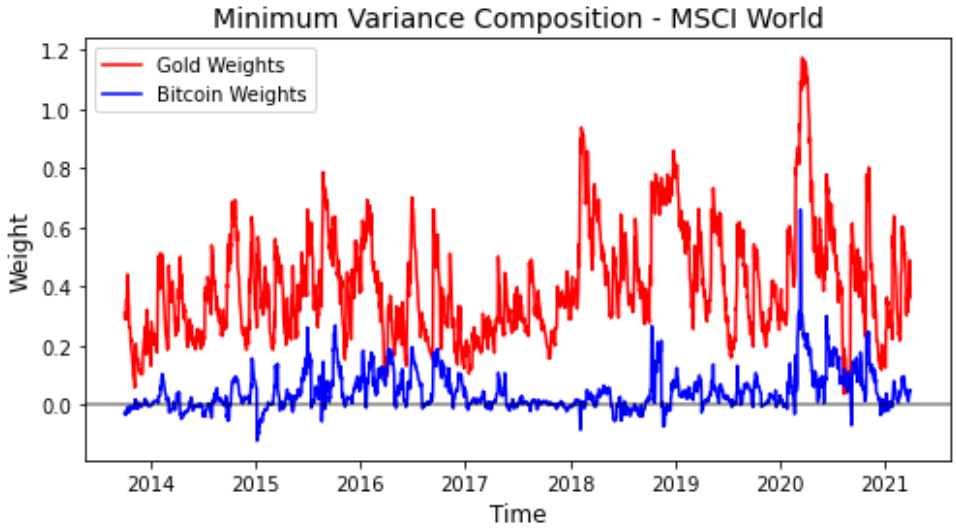

Figure 20: Minimum variance portfolio weights of Bitcoin/Gold and MSCI Emerging markets index.

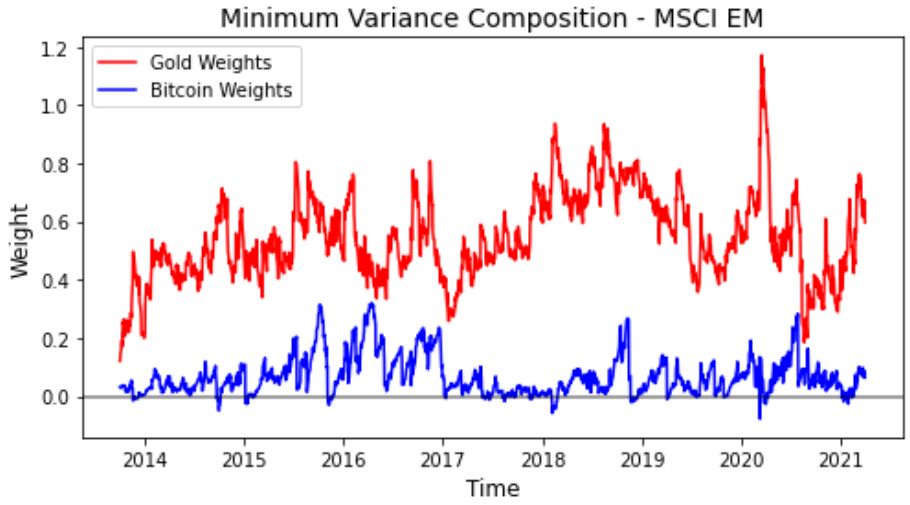

Figure 21: Minimum variance portfolio weights of Bitcoin/Gold and MSCI Europe index.